PART A

(a) Output:

n=4 begins
_n=2 begins
__n=1 begins
__n=1 ends
__hi
__n=1 begins
__n=1 ends
_n=2 ends
_hi
_n=2 begins
__n=1 begins
__n=1 ends
__hi
__n=1 begins
__n=1 ends
_n=2 ends
n=4 ends

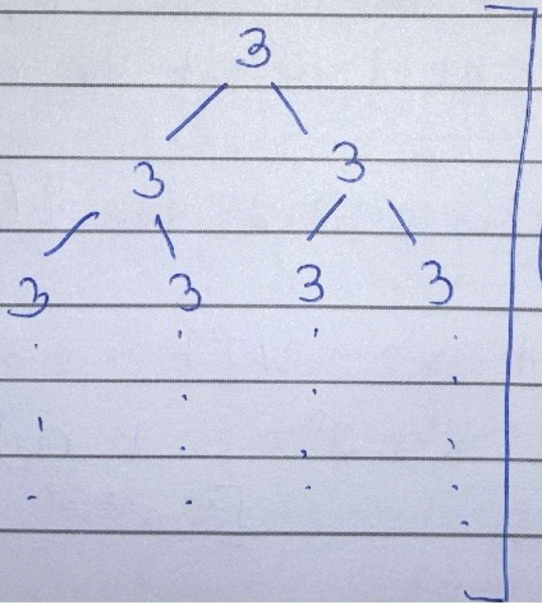
_ represents a space

(b)

$T(n) = 2$, for $n<=1$
$T(n) = 2T(n/2)+3$, for $n>1$

(c)

Recursion Tree:



($\log_2 n + 1$ levels)

Approximately 3×2^i lines are printed at each level i, starting at $i = 0$ at the root.

$\therefore$ Total lines printed for $f(n, 0)$ is:

$$3 \times 2^0 + 3 \times 2^1 + 3 \times 2^2 + \dots + 3 \times 2^{(\log_2 n)}$$

$$= 3(2^0 + 2^1 + 2^2 + \dots + n)$$

$[\because a^{\log_a n} = n]$

$$= k + 3n$$ (where k is a constant)

$\therefore$ We assume $T(n) = O(n)$

$\therefore$ We guess $T(n) \leq cn - k$, (for some constant $c > 0$)

[We subtract a lower-order term to prove by induction.]

$\therefore$ $T(n/2) \leq c(n/2) - k$.

$\therefore$ $T(n) \leq 2 \cdot c \cdot (n/2) - 2k + 3$

$\therefore$ $T(n) \leq cn - 2k + 3$

$\therefore$ $T(n) \leq cn - k$, $\boxed{\forall\ k \geq 3}$

$\therefore$ By induction we proved that

$T(n) = O(n)$