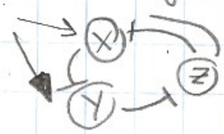
Problem 2

$\Delta Z = 1.0$

AC-DC syst



Oscillations and bi-stability!

Coherence resonance regulates oscillations

multifunctionality mapping is important

(w/ minimal param changes)

coh resonance

look at gene clock

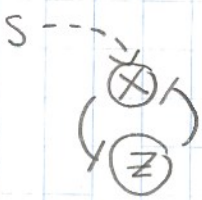
Bifurcation →

stem arm, shell oscillations

continuation

Part A

Figure 1B shows the following system



$$\frac{d\tilde{x}}{d\tilde{t}} = \underbrace{\text{generation of } X}_{\text{activate by } S,\ \text{inhibited by } Z} - \underbrace{\text{degredat of } X}_{\tilde{\delta}_x \tilde{X}}$$

← follow first order degra

(I will follow the same pattern as the paper where the cooperativity for activation with S is 1, and repression cooperativity for z is n_{zx})

$$\boxed{\frac{d\tilde{X}}{d\tilde{t}} = \frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1 + S + (\tilde{Z}/\tilde{Z}_x)^{n_{zx}}} - \tilde{\delta}_x \tilde{X}}$$

for z we have

$$\frac{d\tilde{Z}}{d\tilde{t}} = \text{generation of } z - \text{degredation of } z$$

$$\boxed{\frac{d\tilde{Z}}{d\tilde{t}} = \frac{\tilde{\alpha}_z}{1 + (\tilde{X}/\tilde{X}_z)^{n_{xz}}} - \tilde{\delta}_z \tilde{Z}}$$

Part B

This is the error!

we are now here to nondimensionalize the equations using the parameters given in equations (3) → (6).

first $t = \tilde{t}\,\delta x$ $\quad \tilde{t} = \frac{t}{\delta x}$ ⟶ t should be $\boxed{\tilde{t}\,\tilde{\delta}x = t}$

this is the error

$$\frac{d\tilde{X}}{d\left(\frac{t}{\delta x}\right)} = \frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1 + S + (\tilde{z}/\tilde{z}_x)^{n_{zx}}} - \tilde{\delta}_x \tilde{X} \qquad \frac{d\tilde{Z}}{d\left(\frac{t}{\delta x}\right)} = \frac{\tilde{\alpha}_z}{1 + (\tilde{X}/\tilde{X}_z)^{n_{xz}}} - \tilde{\delta}_z \tilde{Z}$$

$$\frac{d\tilde{X}}{dt} = \frac{1}{\tilde{\delta}_x}\left[\frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1 + S + (\tilde{z}/\tilde{z}_x)^{n_{zx}}}\right] - \tilde{X} \qquad \frac{d\tilde{Z}}{dt} = \frac{1}{\tilde{\delta}_x}\left[\frac{\tilde{\alpha}_z}{1 + (\tilde{X}/\tilde{X}_z)^{n_{xz}}}\right] - \delta_z \tilde{Z}$$

when $\delta_z = \frac{\tilde{\delta}_z}{\delta x}$

now substitute for $X = \frac{\tilde{x}\,\tilde{\delta}_x}{\tilde{\alpha}_z}$ and $\frac{\tilde{z}\,\tilde{\delta}_x}{\tilde{\alpha}_z} = Z$ into both expressions

$$\tilde{x} = \frac{X\tilde{\alpha}_z}{\tilde{\delta}_x} \qquad \tilde{z} = \frac{Z\tilde{\alpha}_z}{\tilde{\delta}_x}$$

$$\frac{\tilde{\alpha}_z}{\tilde{\delta}_x}\frac{\partial X}{\partial t} = \frac{1}{\tilde{\delta}_x}\left[\frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1 + S + (\tilde{z}/\tilde{z}_x)^{n_{zx}}}\right] - \frac{X\tilde{\alpha}_z}{\tilde{\delta}_x} \qquad \frac{\tilde{\alpha}_z}{\tilde{\delta}_x}\frac{\partial Z}{\partial t} = \frac{1}{\tilde{\delta}_x}\left[\frac{\tilde{\alpha}_z}{1 + (\tilde{x}/\tilde{x}_z)^{n_{xz}}}\right] - \frac{\delta_z\tilde{\alpha}_z Z}{\tilde{\delta}_x}$$

$$\frac{\partial X}{\partial t} = \frac{\frac{\tilde{\alpha}_x}{\tilde{\alpha}_z} + \frac{\tilde{\beta}_x}{\tilde{\alpha}_z}S}{1 + S + (\tilde{z}/\tilde{z}_x)^{n_{zx}}} - X \qquad \frac{\partial Z}{\partial t} = \frac{1}{1 + (\tilde{x}/\tilde{x}_z)^{n_{xz}}} - \delta_z Z$$

$$\alpha_x = \frac{\tilde{\alpha}_x}{\tilde{\alpha}_z} \qquad \frac{\tilde{\beta}_x}{\tilde{\alpha}_z} = \beta_x$$

we now need to subsitute for $\tilde{x}/\tilde{x}_z$ and $\tilde{z}/\tilde{z}_x$

$$\frac{\partial X}{\partial t} = \frac{\alpha_x + \beta_x S}{1 + S + (\tilde{z}/\tilde{z}_x)^{n_{zx}}} - X$$

$$z_x = \frac{\tilde{z}_x\tilde{\delta}_x}{\tilde{\alpha}_z} \Rightarrow \tilde{z}_x = \frac{z_x\tilde{\alpha}_z}{\tilde{\delta}_x}$$

$$\Downarrow$$

$$\frac{\tilde{z}}{\tilde{z}_x} = Z/z_x$$

$$x_z = \frac{\tilde{x}_z\tilde{\delta}_x}{\tilde{\alpha}_z} \qquad \tilde{x}_z = \frac{x_z\tilde{\alpha}_z}{\tilde{\delta}_x}$$

$$\frac{\tilde{x}}{\tilde{x}_z} = X/x_z$$

$$\boxed{\frac{\partial X}{\partial t} = \frac{\alpha_x + \beta_x S}{1 + S + (Z/z_x)^{n_{zx}}} - X} \quad (1)$$

$$\boxed{\frac{\partial Z}{\partial t} = \frac{1}{1 + (X/x_z)^{n_{xz}}} - \delta_z Z} \quad (2)$$

<u>Part C</u>

see plots in matlab.

@ s.s both eqn 1 and eqn 2 = 0

Yes the solid black lines are qualitatively reproducible.

$\boxed{\text{See the plot}}$

My plot has the same trends as those in the figure

See document for remaining parts, plots and analysis of the figures