Quaternary Golay Sequence Pairs

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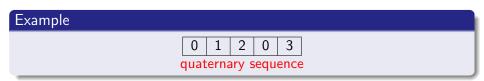
Background

- Background
- 2 Classifying Quaternary Golay Sequence Pairs

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- 3 Constructing a Binary Barker Sequence from a Quaternary Golay Sequence

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- 3 Constructing a Binary Barker Sequence from a Quaternary Golay Sequence
- 4 Summary and Open Problems

Background



Example

	0	1	2	0	3			
,	quatornary coquence							

quaternary sequence

$$A = (a_0, ..., a_{n-1})$$
 is a quaternary sequence if $a_i \in \mathbb{Z}_4$ for all $0 \le j < n$.



0 1 2 0 3 quaternary sequence

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Example

0 0 0 0 2 2 0 0 2 0

binary sequence

Example

2 quaternary sequence

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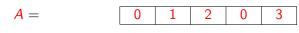
Example

2 0 2 0 0 binary sequence

 $A = (a_0, ..., a_{n-1})$ is a binary sequence if $a_i \in \{0, 2\}$ for all $0 \le j < n$.

$$A =$$

0	1	2	0	3
		•		



$$C_A(1) = i^{0-1} + i^{1-2} + i^{2-0} + i^{0-3} = -1 - i \text{ (where } i = \sqrt{-1})$$

$$C_A(2) = i^{0-2} + i^{1-0} + i^{2-3} = -1$$

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$$C_A(u)$$
 :=
$$\sum_{j=0}^{n-u-1} i^{\mathbf{a}_j - \mathbf{a}_{j+u}} \text{ for all } 0 \leq u < n$$

What is a Barker sequence?

A quaternary (binary) sequence A of length n is a quaternary (binary) Barker sequence if

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Applications include:

- radar
- pulse compression

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• Binary Barker sequences exist for lengths 2, 3, 4, 5, 7, 11, and 13.

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- Binary Barker sequences exist for lengths 2, 3, 4, 5, 7, 11, and 13.
- There are no binary Barker sequences of odd length > 13 (Turyn and Storer, 1961).
- Barker Sequence Conjecture: There are no binary Barker sequences of length > 13.
 - smallest open case is for length $> 10^{22}$ (Leung and Schmidt, 2005).

Let A and B be quaternary (binary) sequences of length n.

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A sequence pair (A, B) is a quaternary (binary) Golay sequence pair if

$$C_A(u) + C_B(u) = 0$$
 for all $1 \le u < n$

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Example

и	0	1	2	3	4
$C_A(u)$	5	-1-i	-1	0	i
$C_B(u)$	5	1+i	1	0	-i
$C_A(u) + C_B(u)$	10	0	0	0	0

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Applications include:

medical ultrasound, etc.



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 - i.e. each pair either derivable from some general construction method or identified as one of five "seed" pair

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 - Explains all known binary and quaternary Golay sequences of length 2^m .

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No nonexistence results for quaternary Golay sequences.

Classifying Quaternary Golay Sequence Pairs

Ordered quaternary Golay sequence pair counts

In 2002, Craigen, Holzmann, and Kharaghani exhaustively found all ordered quaternary Golay sequence pairs of small length *n*:

n	# pairs	n	# pairs	n	# pairs
1	16	8	6656	15	0
2	64	9	0	16	106496
3	128	10	12288	17	0
4	512	11	512	18	24576
5	512	12	36864	19	0
6	2048	13	512	20	215040*
7	0	14	0	21	0

^{*} Frank Fiedler, personal communication.

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 Using small "seed" pairs, how many of the pairs in the table above can be explained using the multi-dimensional construction process and other constructions?



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- Using small "seed" pairs, how many of the pairs in the table above can be explained using the multi-dimensional construction process and other constructions?
- How can we explain the existence of the seed pairs?

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• Input: m + 1 Golay sequence pairs of length $n_0, n_1, ... n_m$, where m > 1, to create a multi-dimensional object.

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How do we use the multi-dimensional construction process?

- Input: m+1 Golay sequence pairs of length $n_0, n_1, ...n_m$, where $m \ge 1$, to create a multi-dimensional object.
- Process multi-dimensional object...
- Output: A collection of Golay sequence pairs, all of length $n_0 \cdot n_1 \cdot ... \cdot n_m \cdot 2^m$.

$$\left(\begin{array}{c|c} \hline 0 & , & \hline 0 &) \\ \hline \left(\begin{array}{c|c} \hline 0 & , & \hline 0 &) \\ \end{array} \right) & \mapsto & \begin{array}{c} 64 \text{ ordered pairs of} \\ \text{length } 1 \cdot 1 \cdot 2 = 2 \end{array}$$

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n	# pairs	# pairs left to explain	n	# pairs	# pairs left to explain	n	# pairs	# pairs left to explain
1	16	0	8	6656	512	15	0	0
2	64	0	9	0	0	16	106496	8192
3	128	128	10	12288	4096	17	0	0
4	512	0	11	512	512	18	24576	0
5	512	512	12	36864	0	19	0	0
6	2048	0	13	512	512	20	215040	67584
7	0	0	14	0	0	21	0	0

n	# pairs	# pairs left to explain	n	# pairs	# pairs left to explain	n	# pairs	# pairs left to explain
1	16	0	8	6656	512	15	0	0
2	64	0	9	0	0	16	106496	8192
3	128	128	10	12288	4096	17	0	0
4	512	0	11	512	512	18	24576	0
5	512	512	12	36864	0	19	0	0
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• "Shared autocorrelation property"

п	# pairs	# pairs left to explain	n	# pairs	# pairs left to explain	n	# pairs	# pairs left to explain
1	16	0	8	6656	512	15	0	0
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7	0	0	14	0	0	21	0	0

- "Shared autocorrelation property"
- Multi-dimensional construction process with special length 8 pairs and a trivial length 1 pair

		# pairs			# pairs			# pairs
n	# pairs	left to	n	# pairs	left to	n	# pairs	left to
		explain			explain			explain
1	16	0	8	6656	0	15	0	0
2	64	0	9	0	0	16	106496	0
3	128	128	10	12288	4096	17	0	0
4	512	0	11	512	512	18	24576	0
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n	# pairs	# pairs	n	# pairs	# pairs	n	# pairs	# pairs left to
-	1.0	explain		CCEC	explain	1.5		explain
1	16	0	8	6656	0	15	Ü	0
2	64	0	9	0	0	16	106496	0
3	128	128	10	12288	4096	17	0	0
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• Symmetry Lemma: (A, B) are a Golay pair $\Leftrightarrow (A + B, A - B)$ are a Golay pair (where A and B are in "multiplicative" notation).

		# pairs			# pairs			# pairs
n	# pairs	left to	n	# pairs	left to	n	# pairs	left to
		explain			explain			explain
1	16	0	8	6656	0	15	0	0
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- Symmetry Lemma: (A, B) are a Golay pair $\Leftrightarrow (A + B, A B)$ are a Golay pair (where A and B are in "multiplicative" notation).
- Explains all remaining length 10 pairs and 2048 of the remaining length 20 pairs from binary Golay seed pairs.

		# pairs			# pairs			# pairs
n	# pairs	left to	n	# pairs	left to	n	# pairs	left to
		explain			explain			explain
1	16	0	8	6656	0	15	0	0
2	64	0	9	0	0	16	106496	0
3	128	128	10	12288	0	17	0	0
4	512	0	11	512	512	18	24576	0
5	512	512	12	36864	0	19	0	0
6	2048	0	13	512	512	20	215040	65536
7	0	0	14	0	0	21	0	0

		# pairs			# pairs			# pairs
n	# pairs	left to	n	# pairs	left to	n	# pairs	left to
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 Multi-dimensional construction process with special length 10 pairs and a trivial length 1 pair

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n	# pairs	left to	n	# pairs	left to	n	# pairs	left to
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Let's look at lengths 5 and 13...

Lengths 5 and 13

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$$B_5 = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$
 binary Barker sequence of length 5



$$B_{13} + G_{13,1} = egin{bmatrix} 0 & 0 & 0 & 1 & 2 & 2 & 2 & 3 & 0 & 0 & 1 & 1 \\ B_{13} + G_{13,2} & \hline{0} & 1 & 2 & 2 & 2 & 3 & 0 & 0 & 0 & 1 & 2 & 2 & 3 \\ \end{bmatrix}$$

$$B_{13} + G_{13,1} = egin{bmatrix} 0 & 0 & 0 & 1 & 2 & 2 & 2 & 3 & 0 & 0 & 1 & 1 \\ B_{13} + G_{13,2} = egin{bmatrix} 0 & 1 & 2 & 2 & 2 & 3 & 0 & 0 & 0 & 1 & 2 & 2 & 3 \\ \end{bmatrix}$$

$$X_m := ((0 1 2 3)^m 0 1)$$

$$Z_m := ((1 \ 2 \ 3 \ 0)^m \ 1 \ 2)$$

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Binary Barker to quaternary Golay

Theorem

Let $m \in \mathbf{N}$. Suppose $\operatorname{int}(A, B)$ is a binary Barker sequence of length 8m + 5 where $A = ((0\ 0\ 2)^m\ 0\ 0)$. Then the sequences

$$E := \inf(A + W_m, B + X_m),$$

$$F := \inf(A + Y_m, B + Z_m)$$

form a quaternary Golay pair of length 8m + 5.

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• Explains all length 5 and 13 quaternary Golay pairs!

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- Explains all length 5 and 13 quaternary Golay pairs!
- Unfortunately, this result does not give rise to any new quaternary Golay pairs.

Ordered quaternary Golay pairs left to explain

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• binary Barker to quaternary Golay theorem

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• ??

Constructing a Binary Barker Sequence from a Quaternary Golay Sequence

Goal

• We have seen that particular binary Barker of length $\equiv 5 \pmod{8}$ \Rightarrow quaternary Golay of length $\equiv 5 \pmod{8}$

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- \leftarrow quaternary Golay of length $\equiv 5 \pmod{8}$

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• We have seen that particular binary Barker of length $\equiv 5 \pmod{8}$ \Rightarrow quaternary Golay of length $\equiv 5 \pmod{8}$

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 particular binary Barker
 - particular binary Barker of length $\equiv 5 \pmod{8}$ \leftarrow quaternary Golay of length $\equiv 5 \pmod{8}$
- Since there are no binary Barker sequences of odd length greater than 13, this would prove that there are no more quaternary Golay sequences for these lengths.

Lemma

If A and B are sequences of length n and $A + B = (0 \ 1 \ 2 \ 3 \dots)$, then

$$C_B(u) = i^{-u} \cdot \overline{C_A(u)}$$

for all integers 0 < u < n.

Lemma

If A and B are sequences of length n and $A + B = (0 \ 1 \ 2 \ 3 \dots)$, then

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for all integers 0 < u < n.

Call a sequence A of length n good if

$$C_A(u) = -i^{-u}\overline{C_A(u)}$$
 for all integers $0 < u < n$.

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A partial Barker-to-Golay converse

Theorem

Let $A = (a_0, ..., a_{n-1})$ be a good sequence of length n = 8m + 5. Assume that

- (1) $a_{2u-1} + a_{2u+1} \equiv 1 \pmod{2}$, for all $1 \leq 2u 1 \leq \frac{n-7}{2}$, and
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Then there exists a binary Barker sequence of length n, and so $m \in \{0,1\}$.

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Proof: About 20 pages of lemmas.

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 - Alternative approach?



Thanks for listening!