A Parameterized Family of Equilibrium Profiles for Three-Player Kuhn Poker

Duane Szafron, **Richard Gibson**, and Nathan Sturtevant

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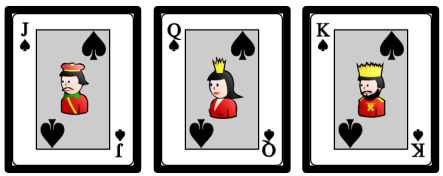
One Slide Summary

- Kuhn Poker is introduced and solved, leading to advances in:
  - Strategy representations [Koller and Pfeffer, 1997]
  - Opponent modelling [Hoehn *et al.*, AAAI 2005]
  - Equilibrium algorithms [Ganzfried and Sandholm, AAMAS 2010]

Image source: ebaumsworld.com

[Kuhn 1953]



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- 3-player Kuhn Poker introduced, but not solved

[Abou Risk and Szafron, AAMAS 2010]

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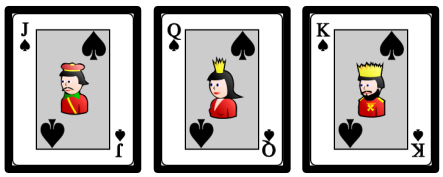

[Kuhn 1953]

Image source: toonpool.com

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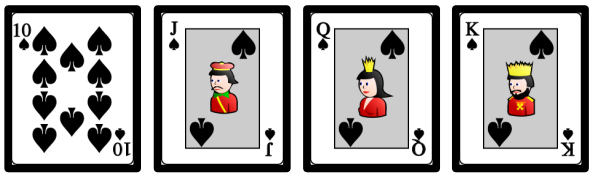

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- 3-player Kuhn Poker introduced, but not solved
- Here, we present a family of solutions for 3-player Kuhn Poker

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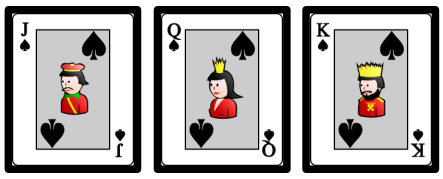

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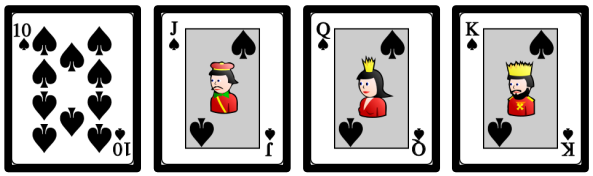

Outline of Talk

- Motivation and Rules of Kuhn Poker
- Nash equilibrium in Kuhn Poker
- Nash equilibrium in 3-player Kuhn Poker New!
- Conclusions and Future Work

Outline of Talk

- **Motivation and Rules of Kuhn Poker**
- Nash equilibrium in Kuhn Poker
- Nash equilibrium in 3-player Kuhn Poker New!
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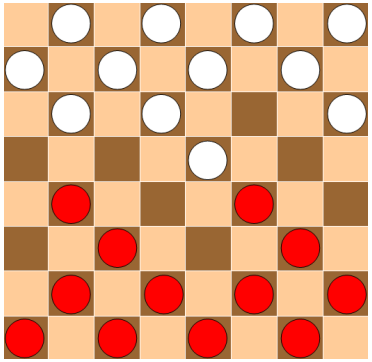
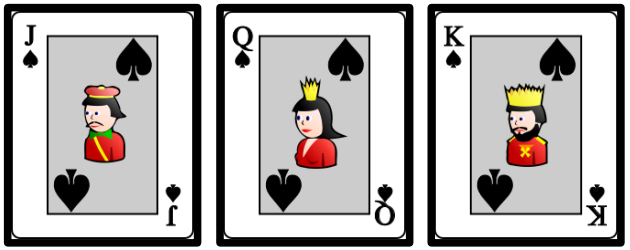
Why Poker Research?

- Classic games, such as chess and checkers, are:
  - Deterministic
  - Binary outcomes (+ draw)
  - Perfect information



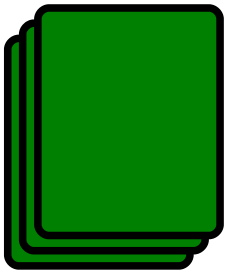
- On the other hand, poker contains
  - Stochastic events
  - Varying outcomes
  - Imperfect information

Kuhn Poker



[Kuhn 1953]

Kuhn Poker



[Kuhn 1953]

Kuhn Poker

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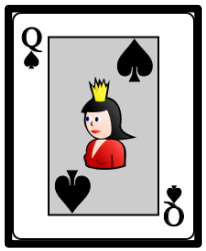
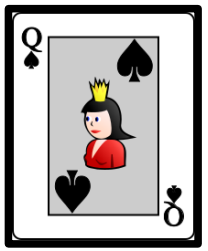


[Kuhn 1953]

Kuhn Poker

?

Check?
Bet?



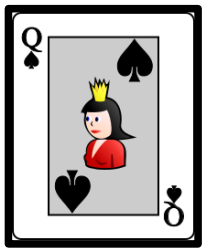
[Kuhn 1953]

Kuhn Poker

?

Bet!

Fold?
Call?

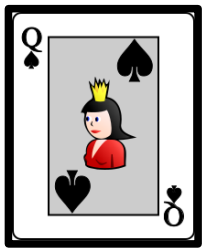


[Kuhn 1953]

Kuhn Poker

?

Call.



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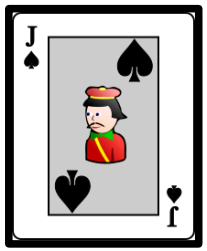

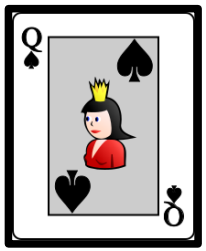

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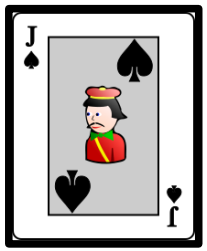
Kuhn Poker

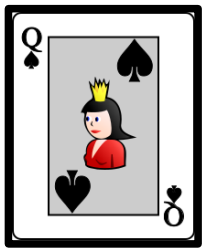


-2

Lose.

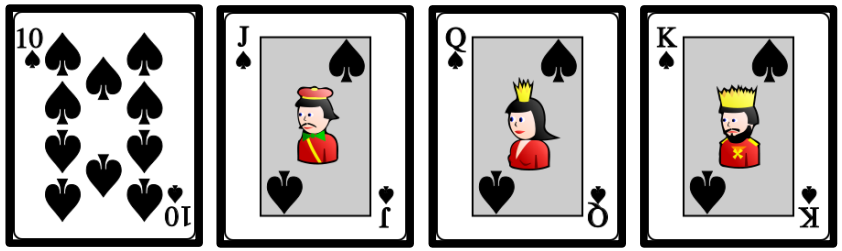
+2

Win!



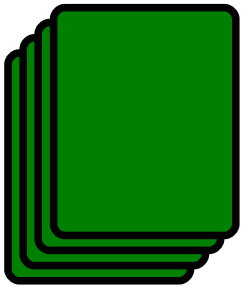
[Kuhn 1953]

3-Player Kuhn Poker



[Abou Risk and Szafron, AAMAS 2010]

3-Player Kuhn Poker



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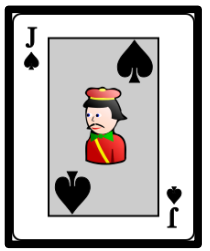
3-Player Kuhn Poker

?

?



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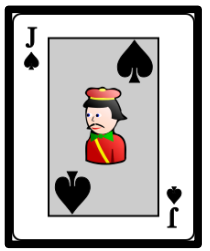
3-Player Kuhn Poker

?

?

Check.



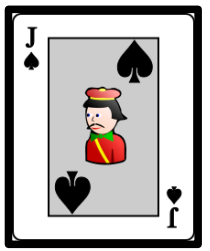
[Abou Risk and Szafron, AAMAS 2010]

3-Player Kuhn Poker

?

?

Bet!



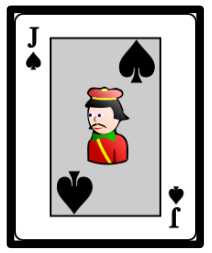
[Abou Risk and Szafron, AAMAS 2010]

3-Player Kuhn Poker

?

?

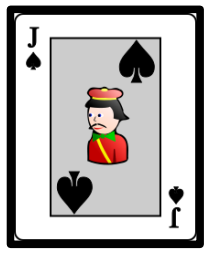
Call.



[Abou Risk and Szafron, AAMAS 2010]

3-Player Kuhn Poker

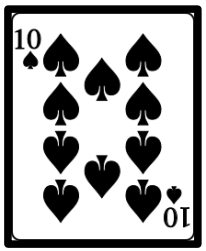
?

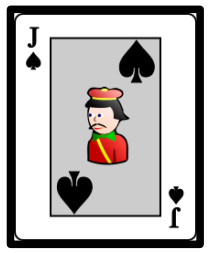


Fold.

[Abou Risk and Szafron, AAMAS 2010]

3-Player Kuhn Poker





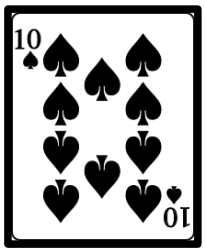
[Abou Risk and Szafron, AAMAS 2010]

3-Player Kuhn Poker

-1

Lose.

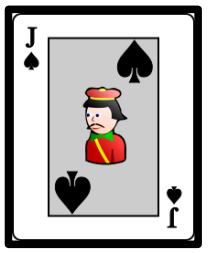
-2



Lose.

+3

Win!



[Abou Risk and Szafron, AAMAS 2010]

Kuhn Poker Games

- Toy poker games
- Players can bluff, slow play
  - Strategic elements of Texas Hold'em
- Small enough to analyze by hand

Outline of Talk

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- **Nash equilibrium in Kuhn Poker**
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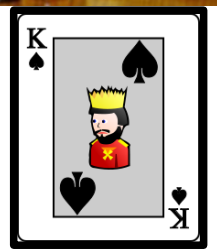
Definition of Nash Equilibrium

- Example: Rock-Paper-Scissors

		1/3 R	1/3 P	1/3 S
1/3	**R**	0	-1	+1
1/3	**P**	+1	0	-1
1/3	**S**	-1	+1	0

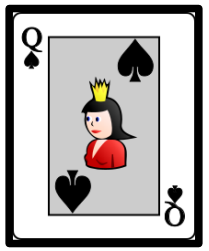
- "No one can change their strategy and do better"
  - assuming all other players' strategies are fixed
- Every game has at least one equilibrium [Nash 1950]

Equilibrium in Kuhn Poker



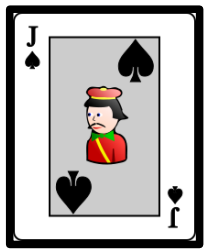
Bet: $p \in [0, 1]$

?



Bet: 0

Slowplay ($p < 1$)



Bet: $\frac{p}{3}$

Bluff ($p > 0$)

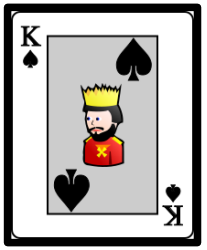
[Kuhn 1953]

Equilibrium in Kuhn Poker

?

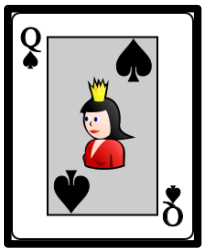
Bet!

Call: 1

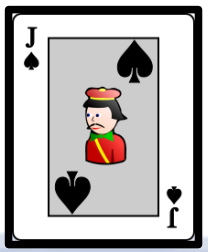


Mixed Strategy

Call: $\frac{1}{3}$



Call: 0



[Kuhn 1953]

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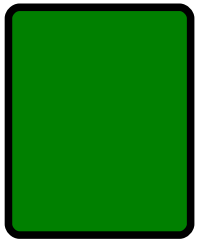
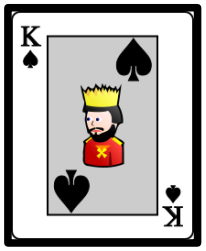
- Motivation and Rules of Kuhn Poker
- Nash equilibrium in Kuhn Poker
- **Nash equilibrium in 3-player Kuhn Poker** New!
- Conclusions and Future Work

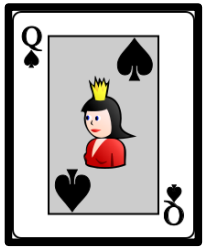
Equilibrium in 3-Player Kuhn Poker



Bet: 0

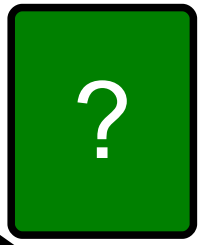


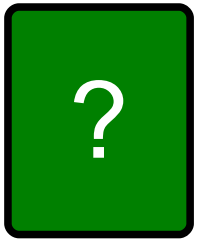
Call: 1



Call: $\frac{1}{2}$ if P3 folds

Call: 0 otherwise





Passive strategy

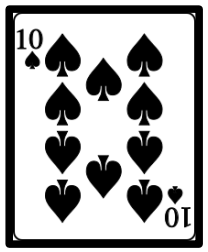
New!

Equilibrium in 3-Player Kuhn Poker

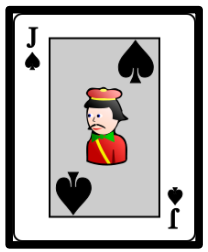
?

Check.

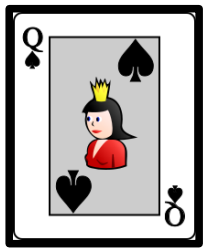
?



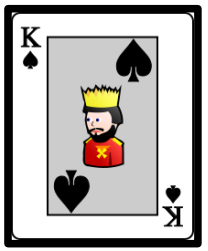
Bet: b_1



Bet: b_2



Bet: 0



Bet: $2b_1 + 2b_2$

$$\beta = \max\{b_1, b_2\} \leq \frac{1}{4}$$

New!

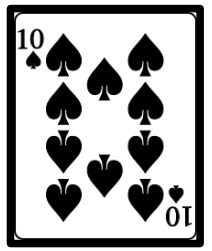
Equilibrium in 3-Player Kuhn Poker

?

Check.

Check.

?



Bet: $p \in \left[0, \min\left\{\frac{1}{2}, \frac{2-b_1}{3+2b_1+2b_2}\right\}\right]$

Complicated → ...

New!

Equilibrium in 3-Player Kuhn Poker

$$-\kappa\left(\frac{1}{2}+\beta\right)$$

$$-\frac{\kappa}{2}$$

$$+\kappa(1+\beta)$$

$$\kappa = \frac{1}{24}$$

New!

Equilibrium in 3-Player Kuhn Poker

$$-\kappa\left(\frac{1}{2}+\beta\right)$$

$$-\frac{\kappa}{2}$$

$$+\kappa(1+\beta)$$

Controls β!

$$\beta = \max\{\text{Bet:}\ \text{10♠},\ \text{Bet:}\ \text{J♠}\}$$

$$\kappa = \frac{1}{24}$$

New!

Outline of Talk

- Motivation and Rules of Kuhn Poker
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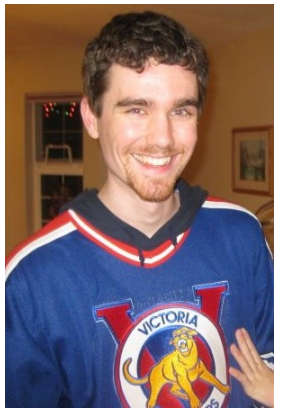
Conclusions

- First analytically-derived 3-player Kuhn Poker Nash equilibrium profiles
  - Largest game with more than 2 players to be solved analytically
- Profiles exhibit ability for second player to transfer utility between the two opponents
- Proved certain Nash equilibrium strategies are robust
- Future work:
  - Other 3-player Kuhn Poker equilibria?
  - Can we build insights into other multiagent domains?

Thanks for Listening!

- Computer Poker Research Group:
  - Website: http://cs.ualberta.ca/~poker
  - Twitter: @PolarisPoker
- Richard Gibson:
  - Email: rggibson@cs.ualberta.ca
  - Website: http://cs.ualberta.ca/~rggibson
  - Twitter: @RichardGGibson



Clip art images from this talk can be found at clker.com