A Parameterized Family of Equilibrium Profiles for Three-Player Kuhn Poker

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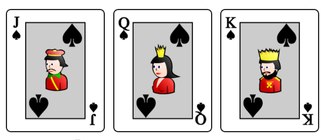
Poster available online at http://cs.ualberta.ca/~rggibson/

1. SUMMARY

2-player Kuhn Poker solutions led to advances in: [Kuhn 1953]

- **strategy representations** [Koller and Pfeffer, 1997]
- **opponent modelling** [Hoehn *et al.*, AAAI 2005]
- **equilibrium algorithms** [Ganzfried and Sandholm, AAMAS 2010]



We present the first set of analytical solutions to 3-player Kuhn Poker

2. MOTIVATION

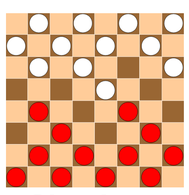
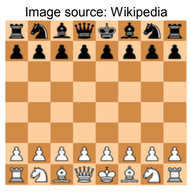


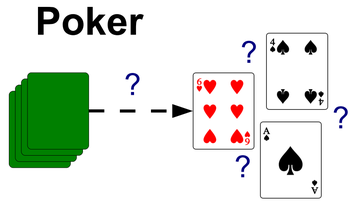
Image source: Wikipedia



Checkers **Chess**

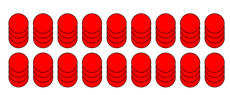
- Deterministic
- Binary outcomes
- Perfect Information
- **"Easy"**

Poker



- Stochastic elements

Pot 1



Pot 3

Pot 2

- Varying outcomes

Image source: ebaumsworld.com

- Imperfect information

- Poker research is applicable to other areas:

Airport Security
[Pita *et al.*, AI Mag. 2009]

Adaptive Treatment Strategies
[Chen and Bowling, NIPS 2012]

Sequential Auctions
[?]

RESEARCH SUPPORTED BY:

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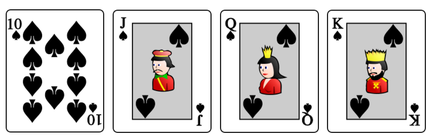
WestGrid
compute • calcul CANADA
NSERC CRSNG
UNIVERSITY OF ALBERTA
UNIVERSITY of DENVER

3. 3P KUHN POKER

[Abou Risk and Szafron, AAMAS 2010]

Deck:



Each player:
- dealt 1 private card
- antes one chip

Image source: toonpool.com

One betting round:
- no raises allowed
- bets worth 1 chip

1. Check.
2. Bet!
3. Call.
4. Fold

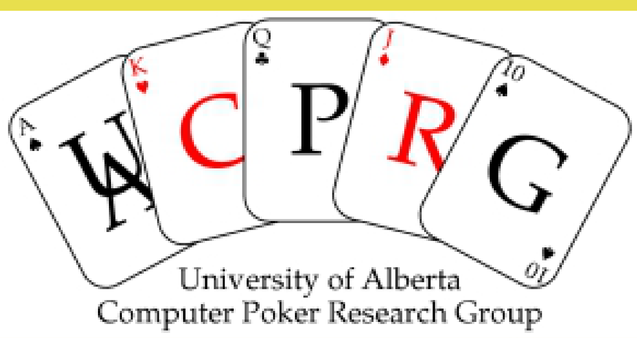
In a showdown:
- non-folds reveal
- highest card wins

Lose. -1

Lose. -2

Win! +3

- Players can bluff, slow play
  - strategic properties of Texas Hold'em
- Small enough to analyze by hand



4. NASH EQUILIBRIUM

Example: 3-player Matching Pennies

	P2: H	P2: T
P1: H	0, 0, 0	1,-2,1
P1: T	-2,1,1	1,1,-2

P3: H = Heads

	P2: H	P2: T
P1: H	1,1,-2	-2,1,1
P1: T	1,-2,1	0, 0, 0

P3: T = Tails

Nash equilibria:
- All players play **Heads** with probability 1
- All players play **Tails** with probability 1
- All players play **Heads** and **Tails** with probability 0.5

Definition of Nash equilibrium:
- "No player can change their strategy and do better"
  - assuming all other players' strategies are fixed
- Every game (matrix, sequential, ...) has at least one equilibrium [Nash 1950]

5. 3P KUHN EQUILIBRIUM HIGHLIGHTS

New!

Player 1:

Bet: 0

Call: 1

Call: $\frac{1}{2}$ if P3 folds
Call: 0 otherwise

Player 2 (after Player 1 Checks):

Bet: b_1 Bet: b_2

Bet: 0 Bet: $2b_1 + 2b_2$

$\beta = \max\{b_1, b_2\} \leq \frac{1}{4}$

Player 3 (after Players 1 and 2 Check):

Complicated

Bet: $p \in \left[0, \min\left\{\frac{1}{2}, \frac{2 - b_1}{3 + 2b_1 + 2b_2}\right\}\right]$

Utilities:

$-\kappa\left(\frac{1}{2} + \beta\right)$

$-\frac{\kappa}{2}$

Controls β!

$+\kappa(1 + \beta)$

$\kappa = \frac{1}{24}$

6. ROBUST EQUILIBRIA

New!

Which equilibrium should we play?
- Pick one with best worst-case performance, assuming opponents play some equilibrium strategies

Player 1:

Fixed strategy

Player 2:

Choose $b_1 = b_2 \leq \frac{1}{4}$

Player 3:

Choose $p = 0$

Utilities:

$\geq -\frac{3\kappa}{4}$

$-\frac{\kappa}{2}$

$\geq +\kappa$