Санкт-Петербургский
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Brute force solutions. Competitive Programming: Core Skills

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SPbSU

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Outline

- **Intuitive solutions**
- Search space
- Backtracking

Introduction

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- But sometimes it's not.
- In this lesson we are going to develop a method for designing solutions which are **always correct.**
- The catch is they are going to be slow.

Digits ordering

Largest number

Input: list of digits.
Output: the largest number that can be made of the digits.

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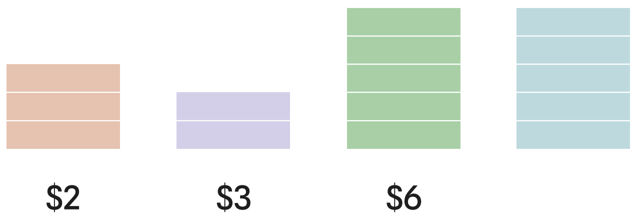
> The solution is to order the digits from the biggestone to the smallest one.

Robber's problem (aka knapsack problem)



Robber's problem

Input: a list of items with weights (kg) and costs ($) as well as the capacity of a bag (kg).

Output the maximum total cost of items that fit in the bag.

Robber’s problem: tempting approach

- It’s natural to process items in order of decreasing $ per kg.

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- Let's calculate utility $\frac{\text{cost}}{\text{weight}}$ for each item.
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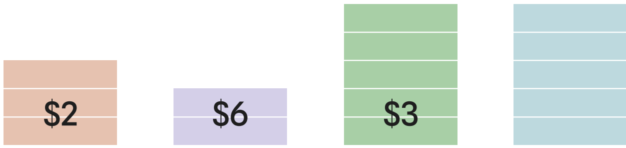
- Therefore we should try to put items with maximum utility first.
- Nice and easy. But, unfortunately, **wrong.**

Robber's problem: example



3 kg	2 kg	5 kg	The total cost is $0
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Robber’s problem: example

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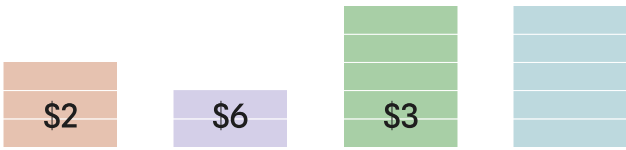

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But the third item doesn't fit to the knapsack.

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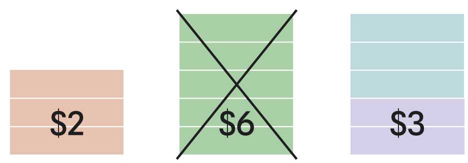

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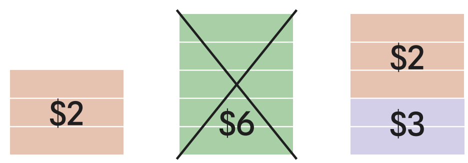

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- But we could do better with the third item only:

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- It’d be good to have a solution which is **always** conceptually correct.
- And that’s what we’ll do!

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- The approach yields slow solutions but it's **conceptually correct by definition.**
- Therefore it could be used to verify correctness of faster solutions for the same problem.

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1. Find an element of a set A satisfying some property (or a number of such elements).
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We will call the set A **search space.**

Superstring

Superstring

Input: m strings $s_1, \ldots, s_m$ consisting of letters “a” and “b” only and an integer n.

Output: a string s of length n containing each s_i (for all $1 \le i \le m$) as a substring.

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Sample

Input: $m = 2$; $n = 3$; $s_1 = \text{ab}$, $s_2 = \text{ba}$

Output: aba (**ab**a, a**ba**)
(another valid output is bab).

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Superstring: solution

- One way to solve a problem is to simply go through all possible candidate solutions. For the superstring problem, the search space consists of all strings of length n over the alphabet $\{a; b\}$.
  For each such string, we check whether it is indeed a superstring of $s_1, \ldots, s_m$

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- Let's consider another testcase: $n = 4$, s_1 = bab, s_2 = abb.
- There are only $2^4 = 16$ strings of four letters "a" and "b".

Superstring: search space

Candidate	bab	abb	Candidate	bab	abb
aaaa	×	×	baaa	×	×
aaab	×	×	baab	×	×
aaba	×	×	baba	**bab**a	×
aabb	×	a**abb**	**babb**	**bab**b	b**abb**
abaa	×	×	bbaa	×	×
abab	a**bab**	×	bbab	b**bab**	×
abba	×	×	abba	×	×
abbb	×	**abb**b	bbbb	×	×

Maximum subarray problem

Maximum subarray problem

Input: an array $a_1, \ldots, a_n$.

Output the largest possible sum $a_l + a_{l+1} + \ldots + a_{r-1} + a_r$ for $1 \leq l \leq r \leq n$.
Note that a_i could be negative.

Sample

Input: $a = (4, 1, -2, 3, -10, 5)$

Output: the best subarray is $(\underbrace{4, 1, -2, 3}, -10, 5$ and the sum is $4 + 1 + (-2) + 3 = 6$.

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- Enumerate all pairs (l, r) such that $l \leq r$, for each pair compute the sum $a_l + \ldots + a_r$, and take the maximum.

Robber's problem

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Input: n items with given weights $w_1, \ldots, w_n$ and costs $c_1, \ldots, c_n$.

Output the largest total cost of the set of items whose total weight does not exceed W.

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Sample

Input: $W = 5$; $n = 3$
$w_1 = 3 \ \ w_2 = 2 \ \ w_3 = 5$
$c_1 = 2 \ \ c_2 = 3 \ \ c_3 = 6$

Output: The best solution is to put the last item to the knapsack and get the total cost 6.

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- For the given example, possible sets are the following: $\varnothing$, {1}, {2}, {3}, {1; 2}, {1; 3}, {2; 3}, {1; 2; 3}.
- Not all of these sets fit into the backpack, but it's easy to check: compute the total weight of the set and check whether it exceeds the capacity of the backpack.

Search space: summary

Problem	Problem Search space
Superstring	strings consisting of letters “a”and “b”

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Exploring the search space

- For the maximum subarray problem the search space gives us the solution instantly.
- We can try all possible pairs with two nested for cycles.
- For the substring problem we want to try all possible strings of n symbols.
- It’d be good to have n nested for cycles iterating through letters “a” and “b”.

Implementation

Your pseudo-Python code will look like this for the superstring problem:

```
for x[0] in ['a', 'b']:
    for x[1] in ['a', 'b']:
        # . . .
        for x[n−1] in ['a', 'b']:
            # check if x contains
            # all strings
            # s[1], . . .s[m]
```

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- Backtracking is roughly the way how to write n nested for cycles.

Recursion

Enumerating all strings x over *{a, b}* of length *n*:

```
for x[0] in ['a', 'b']:
    for x[1] in ['a', 'b']:
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The simplest possible way to simulate this “code” with an actual code is via recursion.

Recursion

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So we can implement the function recursively.

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Let's first do it for three nested for cycles.

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        print (n, x)
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Outer call: threeFors (n, ['']*3).

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We will write the function nestedFors with additional parameter firstFor and it’ll behave like:

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This function simulates for cycles with numbers firstFor, firstFor + 1, ..., n − 1.

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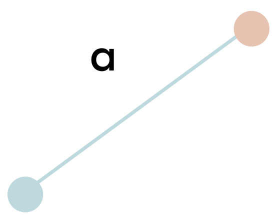

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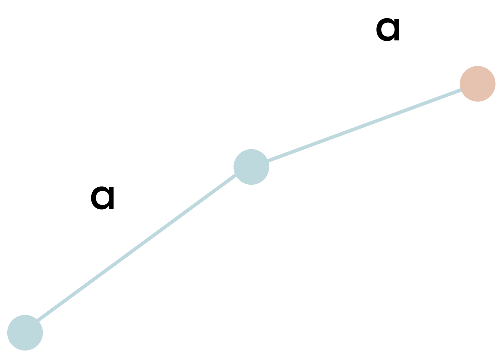

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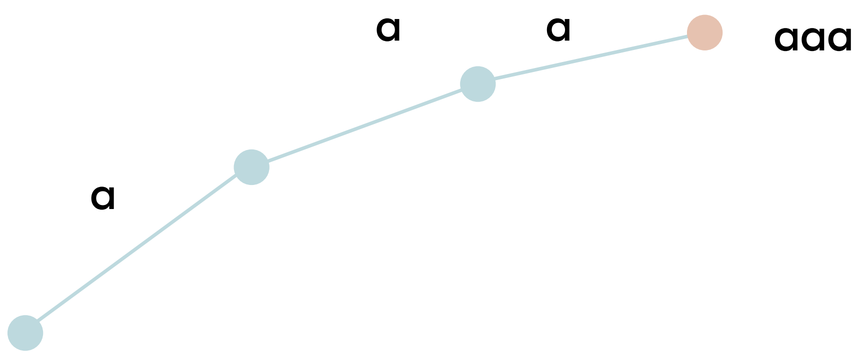

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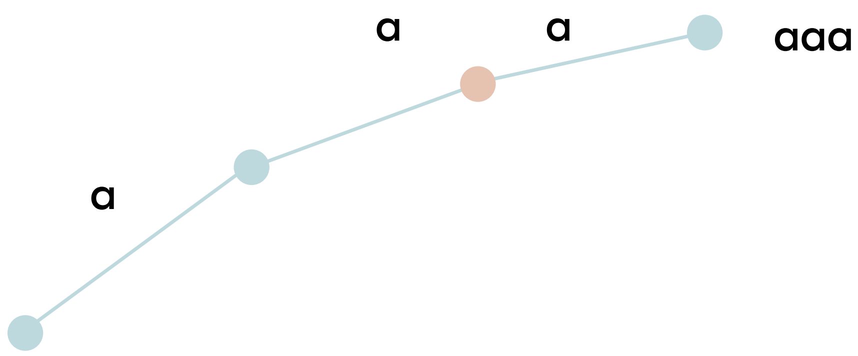

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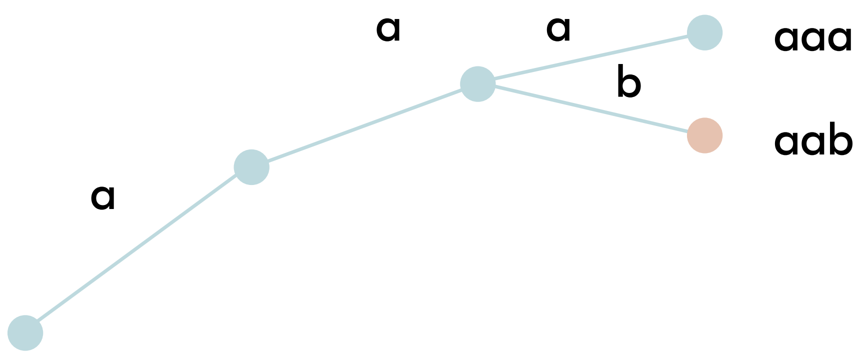

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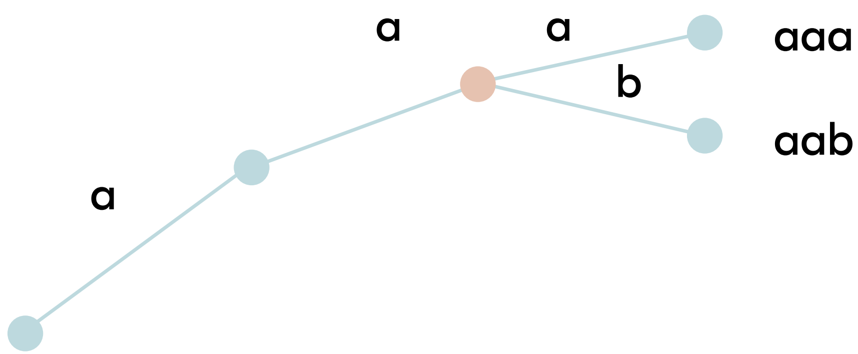

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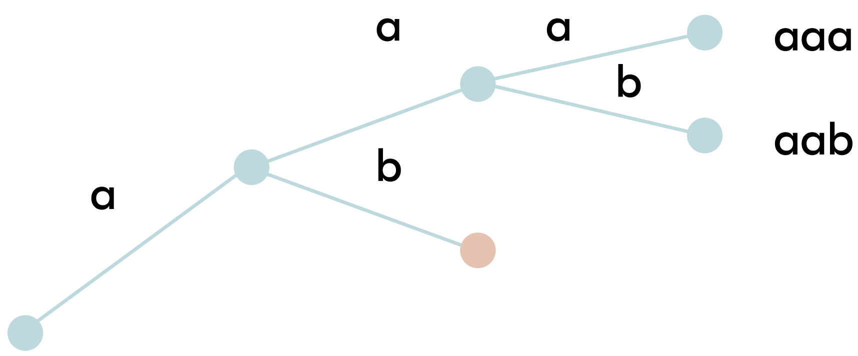

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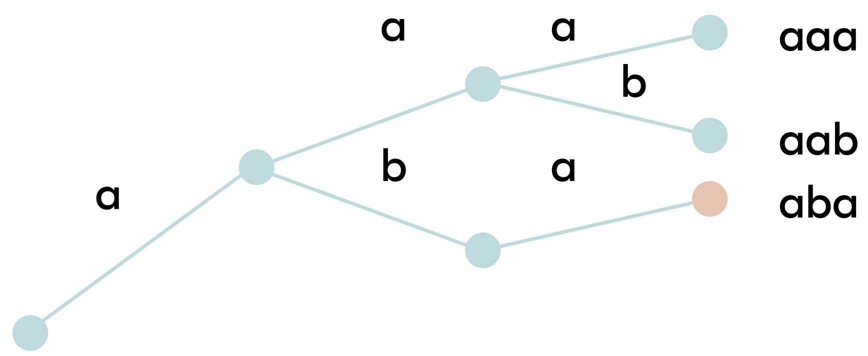

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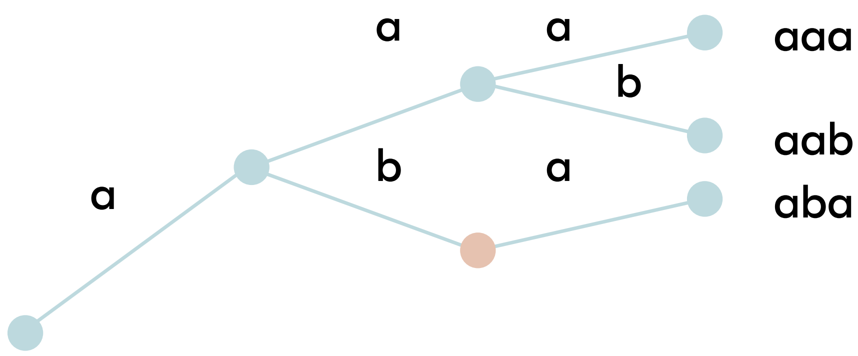

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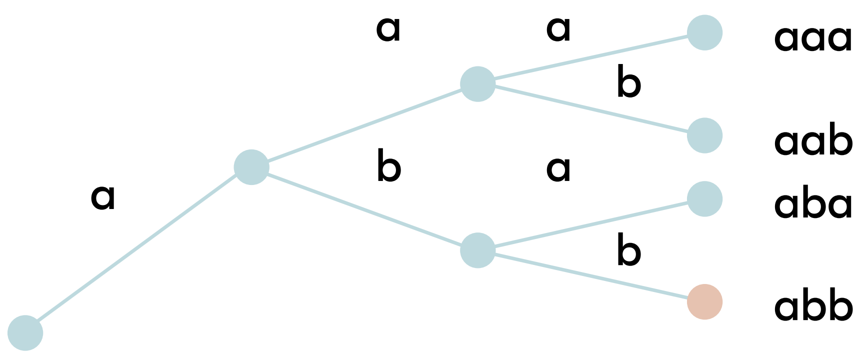

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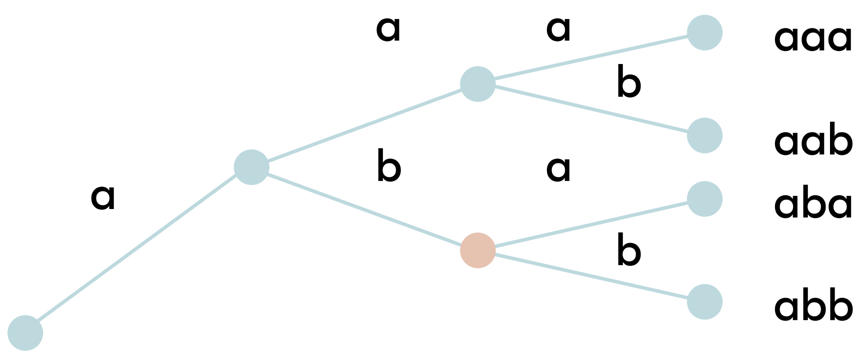

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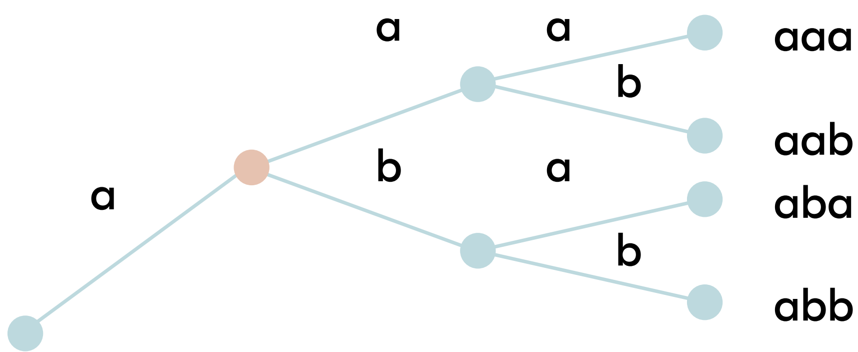

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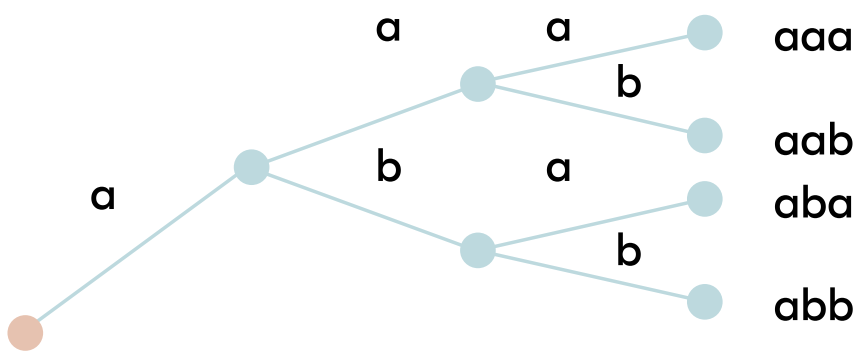

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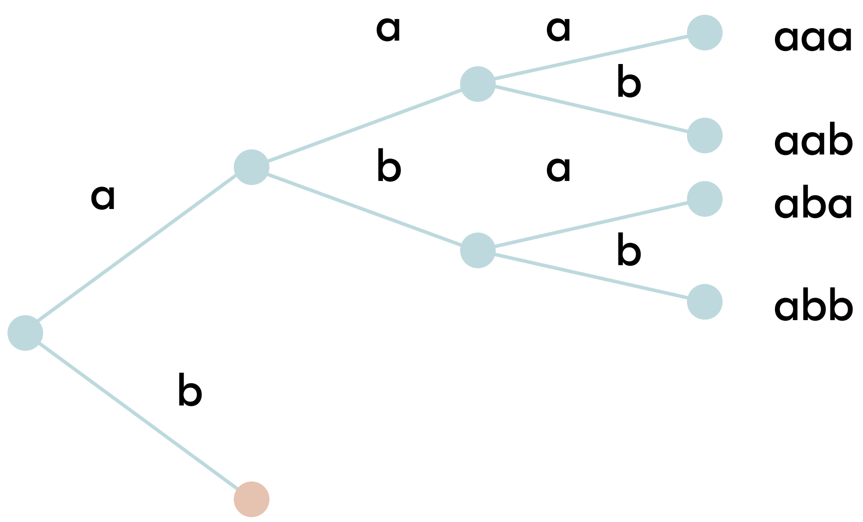

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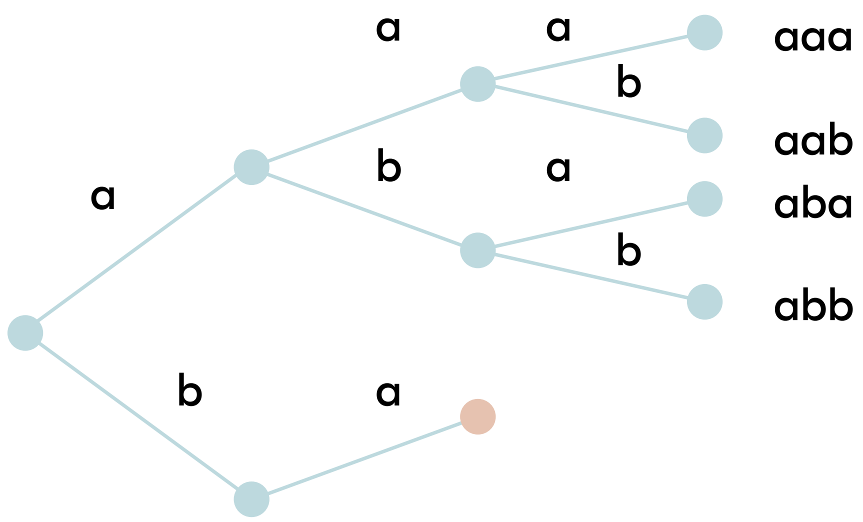

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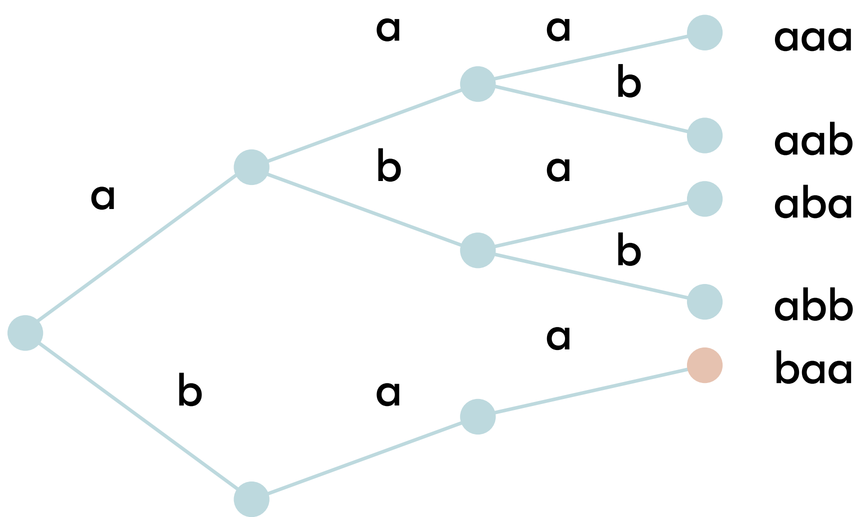

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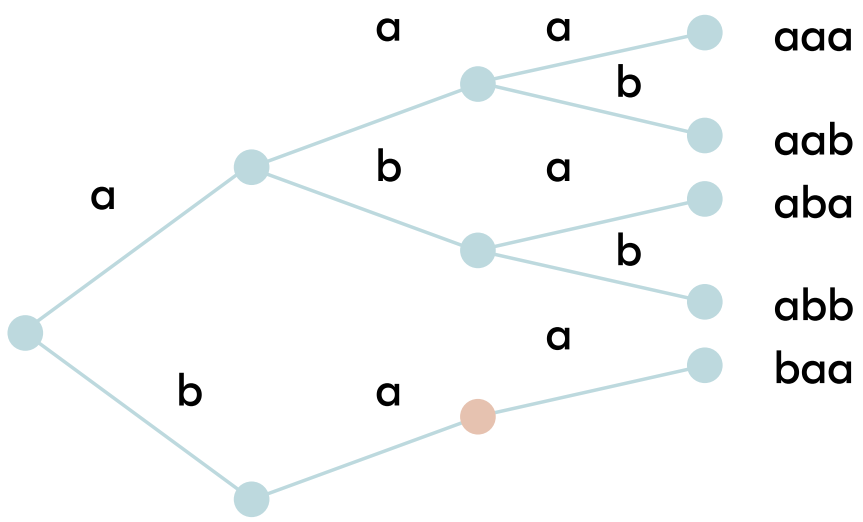

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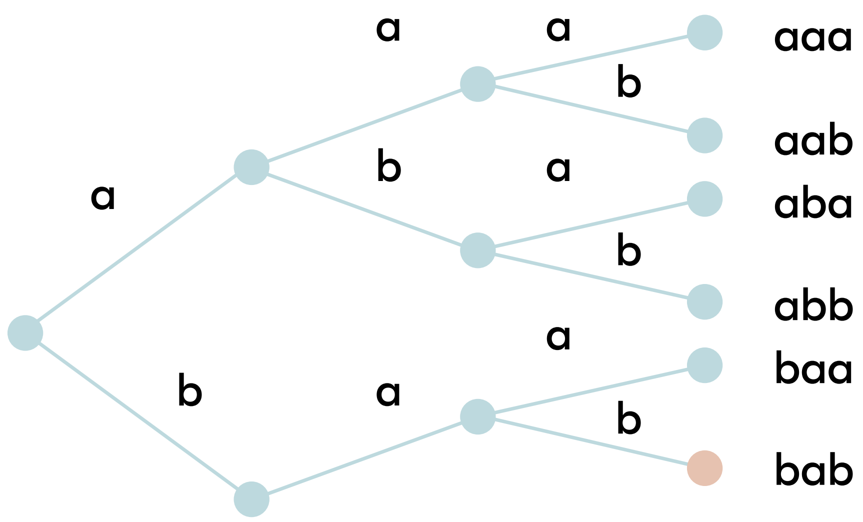
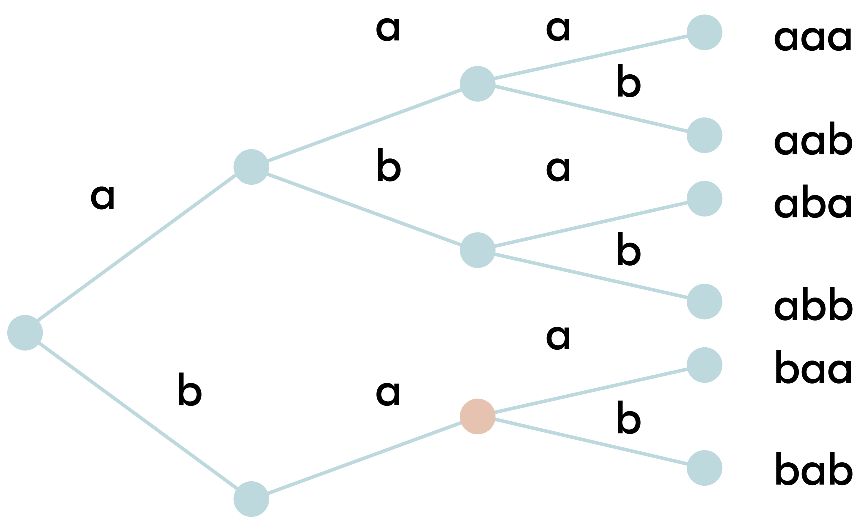
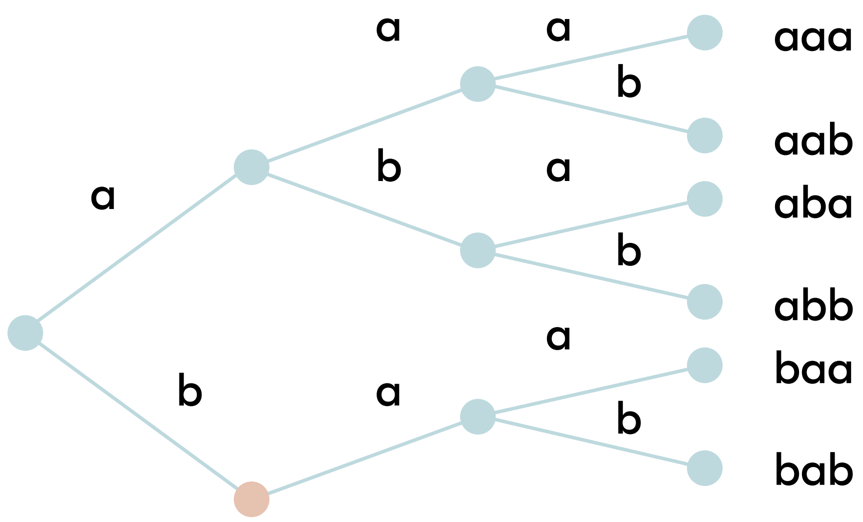
Recursion: visualization



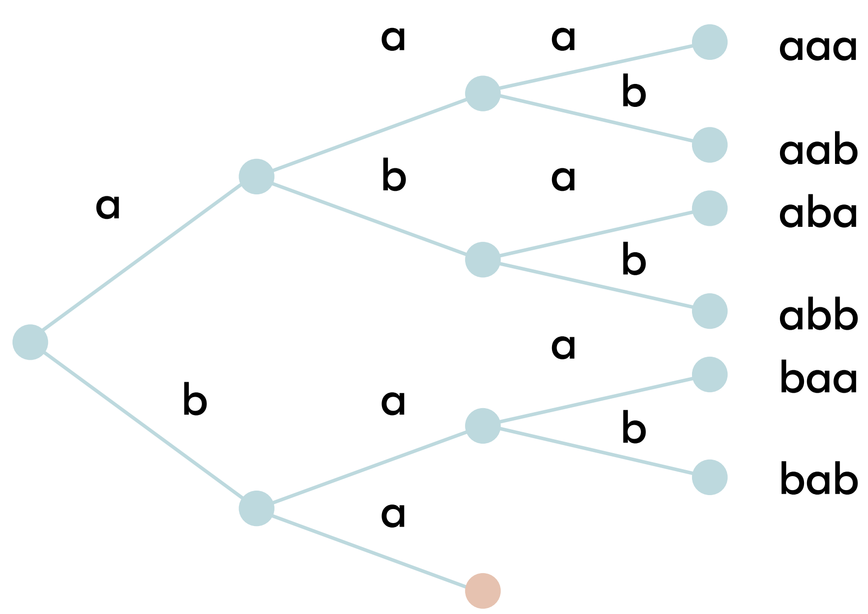
Recursion: visualization



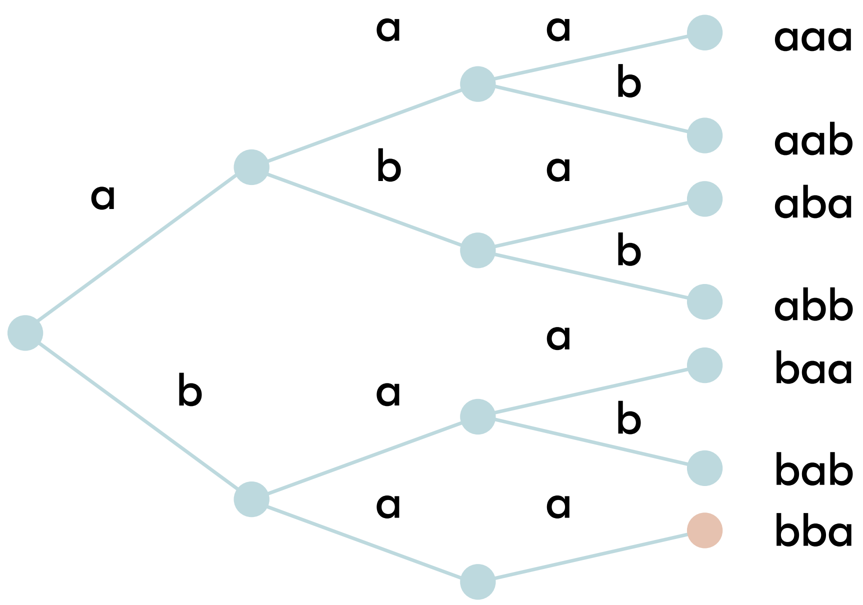
Recursion: visualization



Recursion: visualization



Recursion: visualization



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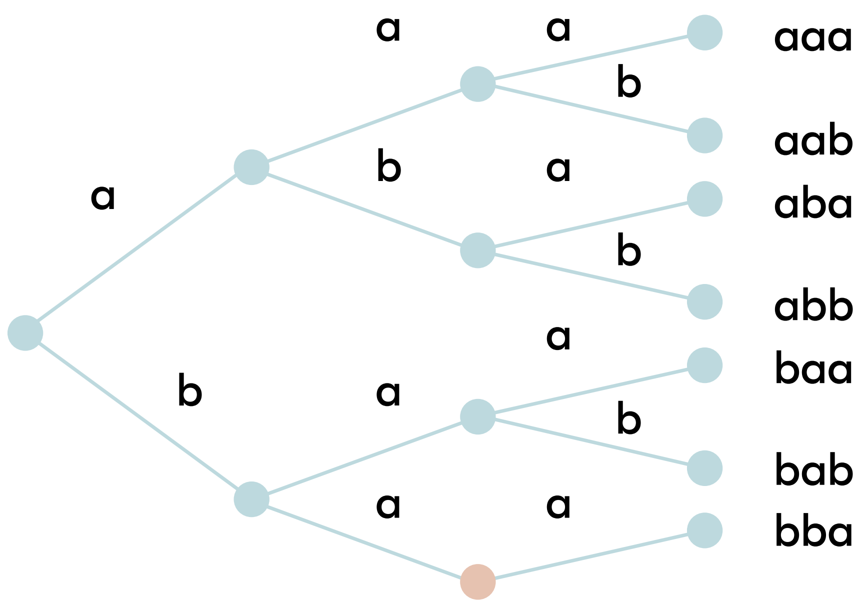

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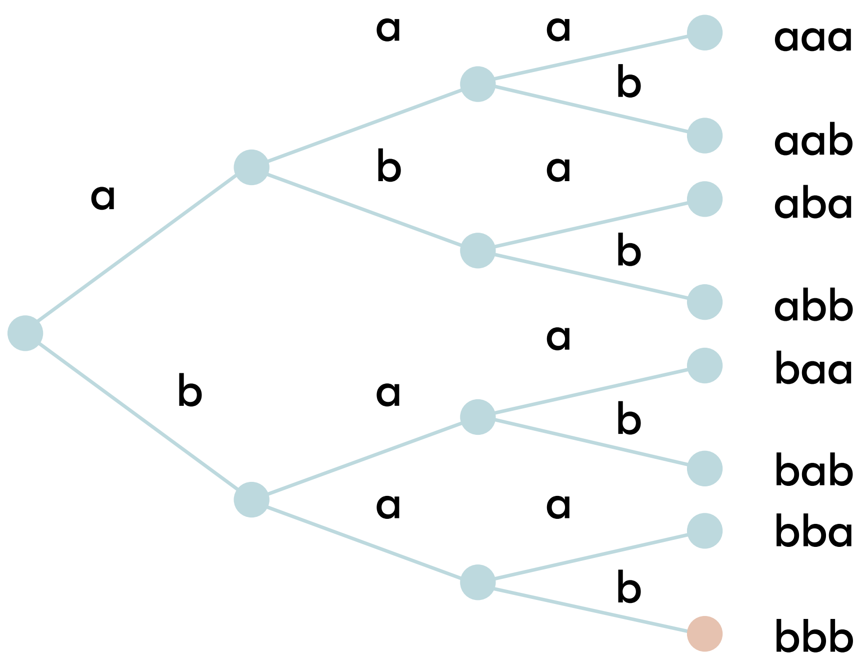

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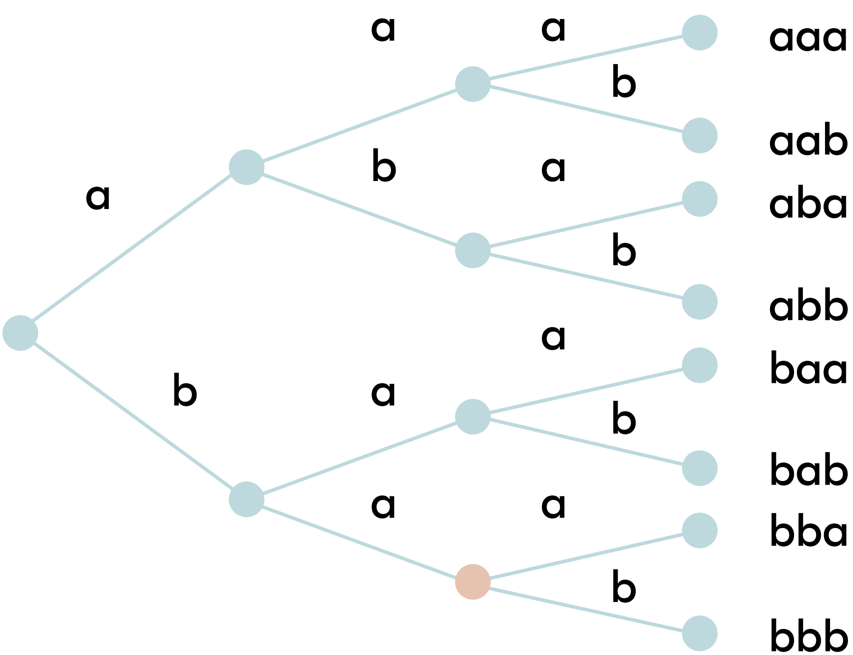

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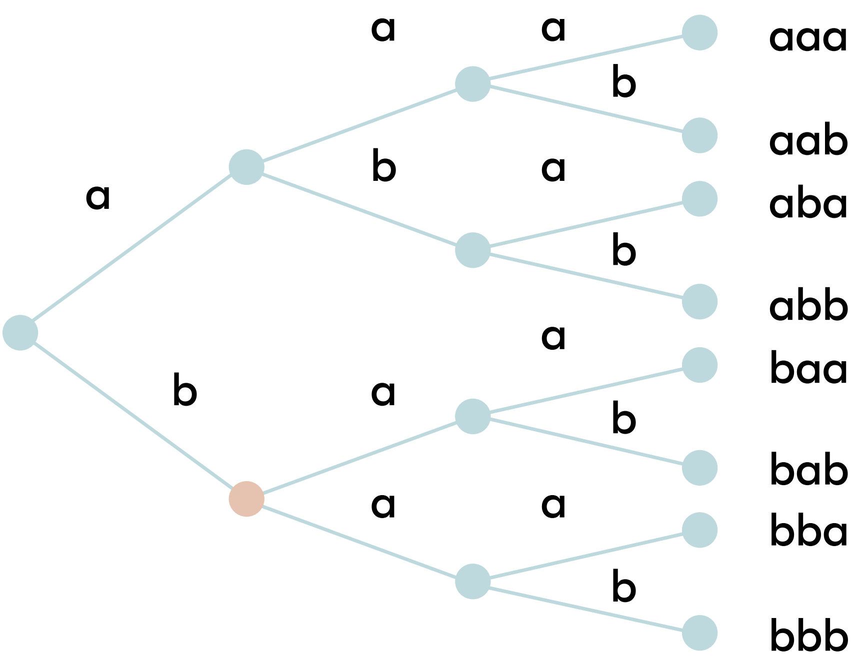

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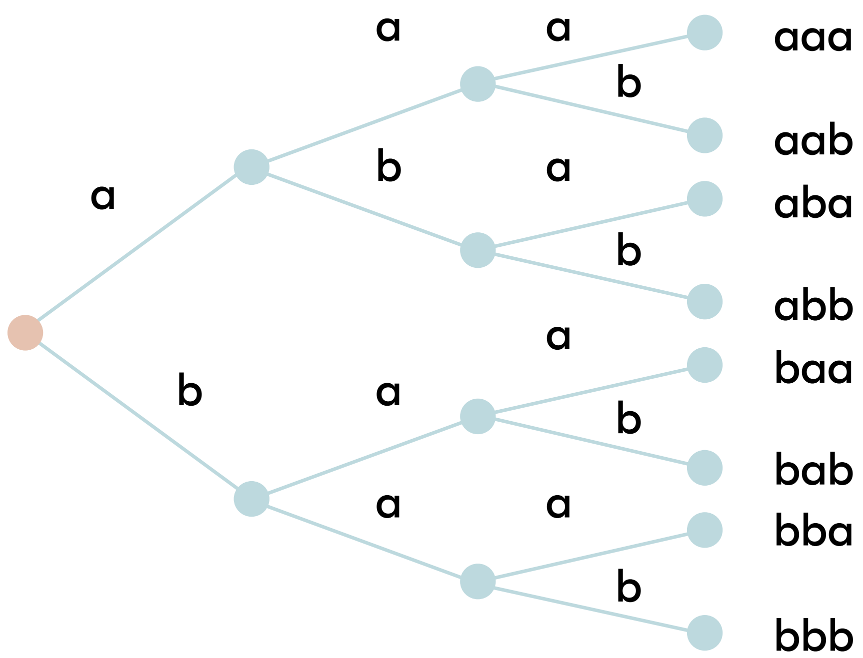

Robber’s problem

- Search space for the robber’s problem is **the set of all sets of items.**

СПбГУ

Robber's problem

- Search space for the robber's problem is **the set of all sets of items.**
- How to enumerate all sets of n items?

Robber's problem

- Search space for the robber's problem is **the set of all sets of items.**
- How to enumerate all sets of n items?
- Basically, it is the same as enumerating all strings over {0, 1} of length *n*!

Set to string

firstFor	Items 1 2 3
∅	0 0 0
{1}	1 0 0
{2}	0 1 0
{1, 2}	1 1 0
{3}	0 0 1
{1, 3}	1 0 1
{2, 3}	0 1 1
{1, 2, 3}	1 1 1

Robber’s problem: search space

Items 1 2 3	Set	Weight	Cost
0 0 0	∅	0	0
1 0 0	{1}	3	2
0 1 0	{2}	2	3
1 1 0	{1, 2}	2 + 3 = 5	2 + 3 = 5
0 0 1	{3}	5	6
1 0 1	{1, 3}	3 + 5 = 8	2 + 6 = 8
0 1 1	{2, 3}	2 + 5 = 7	3 + 6 = 8
1 1 1	{1, 2, 3}	2 + 3 + 5 = 10	3 + 2 + 6 = 11

Recall our example: $n = 3$; $W = 5$ and

$w_1 = 3 \quad w_2 = 2 \quad w_3 = 5$

$c_1 = 2 \quad c_2 = 3 \quad c_3 = 6$

Robber's problem: solution

Therefore we reduced robber's problem to the same n nested for cycles!

Search space

Designing a brute force solution:

Search space

Designing a brute force solution:

1. Identify the search space.

Search space

Designing a brute force solution:

1. Identify the search space.
2. Design a way of enumerating all its elements.

Search space

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Designing a brute force solution:

1. Identify the search space.
2. Design a way of enumerating all its elements.
3. Turn it into a solution.

> The resulting solution is usually slow,
> but it is clearlycorrect and can be used for debugging.