Generative Adversarial Networks

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Generator - Forward Pass

In our abstracted example with batch size of 64:

1. Generate a 64 by 100 (convention) array, with z $\sim (-1, 1)$
2. Pass each of the 1*100 vectors through the network (128-728) with Leaky ReLU
3. Output tanh activation of this vector of length 728 and reshape to 28*28

Where Leaky ReLU:

$$f(x) = 1_{x \leq 0}(\alpha x) + 1_{x > 0}(x) \tag{1}$$

For some small positive constant α

Discriminator - Forward Pass

1. Flatten 28*28 image to 1*728
2. Pass this through a network (728-128-1 for example)
3. Store both the raw output of the network as well as its sigmoid transformation (so you have two numbers for each image in the batch)

Crucially, we run this on the fake images from the generator as well as real images from our training set. This means that we end up with a set of outputs for both the fake and real images - we need to remember this for the backpropagation steps

Loss functions

General form

$$\min_G \max_D V(D, G) = \mathbf{E}_{x \sim p_{data}(x)}[log D(x)] + \mathbf{E}_{z \sim p_z}(z)[log(1 - D(G(z)))]$$

Discriminator

$$\frac{1}{m}\sum_{i=1}^{m}[log D(x^{(i)}) + log(1 - D(G(z^{(i)})))]$$

Generator

$$\frac{1}{m}\sum_{i=1}^{m}[-log(D(G(z^{(i)})))]$$

Backpropagation - Discriminator

For the first layer:

$$\nabla_\theta \frac{1}{m} \sum_{i=1}^{m} [log D(x^{(i)}) + log(1 - D(G(z^{(i)})))]$$

$$\frac{\partial f(x, z)}{\partial x} = -\frac{1}{1 + D(x) + \epsilon} \qquad \text{[Real inputs]}$$

$$\frac{\partial f(x, z)}{\partial z} = -\frac{1}{1 + D(G(z)) + \epsilon} \qquad \text{[Fake inputs]}$$

Backpropagation - Discriminator

- From here, we really just backpropagate like usual
- Only one change - we simulataneously calculate gradients for both the real and fake inputs
- Weight and bias updates using SGD then just consist of the sum of the gradients multiplied by the learning rate

Backpropagation - Generator

Again, step-by-step. For the first layer:

$$\nabla_\theta \frac{1}{m} \sum_{i=1}^{m} [-log(D(G(z^{(i)})))]$$

$$\frac{\partial f(x, z)}{\partial x} = -\frac{1}{D(G(z)) + \epsilon}$$

Backpropagation - Generator

A twist here is that we need some information from the discriminator to calculate the derivative of D(G(z))