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%***** MATLAB TDR SYSTEM *****
%* File      : tdrExamples/DiffReac/DiffReac.tex
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%*
%* The TDR system in Matlab has been implemented by
%*   Mathias Franz (Oct 2004 - Feb 2005)
%*   Alf Gerisch   (Oct 2004 - )
%*****

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This example implements the scalar linear diffusion-reaction equation

$$\partial_t u = \nabla \cdot (D \nabla u) + g(u)$$

with constant positive diffusion coefficient  $D > 0$  and reaction function  $g(u)$ . We always use a value  $D = 1$ . The reaction function can be either  $g \equiv 0$ , or of logistic type  $g(u) = \alpha u^\beta (1 - u)$  with  $\alpha, \beta > 0$ , or of a generalised logistic type  $g(u) = -\alpha u(1 - u)(\beta - u)$ . In the generalised logistic case,  $\alpha > 0$  and  $-1 \leq \beta < 1$ ; the case with  $0 < \beta < 1$  is what the geneticists refer to as the heterozygote inferiority case; we refer to [T. Kawahara and M. Tanaka. *Interactions of traveling fronts: An exact solution of a nonlinear diffusion equation*. Physics Letters A, 97:311-314, 1983] and the references cited there for details.

The spatial domain is either the 2D domain  $(x, y) \in \Omega := (0, 1) \times (y_0, y_0 + 1)$  with parameter  $y_0 \geq 0$  or the 3D axi-symmetric domain with cross section  $\Omega$  and the  $x$ -axes as axis of symmetry, that is  $(x, r(y, z)) \in \Omega$ . The model is used to compare these two cases. The equation is considered for  $t \geq t_0 = 0$ .

The initial condition is always independent of  $x$  and in the 2D case can be either (`selectICandBC = 1`)

$$u(0, x, y) = (y - y_0)^2 \exp(-50(y - y_0 - 1)^2)$$

corresponding to a peak along  $y = 1 + y_0$  and rapidly decaying towards zero for  $y \rightarrow y_0$ , or (`selectICandBC = 2`)

$$u(0, x, y) = \exp(-50(y - y_0 - 0.3)^2)$$

corresponding to a peak along  $y = 0.3 + y_0$  and rapidly decaying towards zero for  $y \rightarrow y_0$  and  $y \rightarrow 1 + y_0$ . In the axi-symmetric case,  $y$  is simply replaced by  $r(y, z)$ .

In the case `selectICandBC = 1` we use constant Dirichlet boundary conditions (equal to one) along the boundary  $y = y_0 + 1$  in case of a 2D spatial domain (and along the boundary  $r(y, z) = y_0 + 1$  in case of the axi-symmetric spatial domain). On the remaining boundary parts we use zero-flux conditions. In the case `selectICandBC = 2` we use zeros flux condition on all the boundary.

The problem and its solution are independent of the  $x$ -coordinate. Therefor we use in that direction a coarse grid only.