Outline

- Logic, deduction, and induction.
- Hume, and a little Popper.

Logic

Logic studies the *validity* of an argument, not the *truth* of its conclusions.

An argument is valid if it is logically sound, and an argument can be valid without being true, and vice-versa.

In contrast, a proposition is a statement which is either true or false.

Example:
If James wants a job, then he will get a haircut tomorrow.
James will get a haircut tomorrow.
So: James wants a job.

If James wants a job, then he will get a haircut tomorrow.
James wants a job.
So: James will get a haircut tomorrow.

Logic

Logic studies the *validity* of an argument, not the *truth* of its conclusions.

An argument is valid if it is logically sound, and an argument can be valid without being true, and vice-versa.

In contrast, a proposition is a statement which is either true or false.

Example:
If James wants a job, then he will get a haircut tomorrow.
James will get a haircut tomorrow.
So: James wants a job.

If James wants a job, then he will get a haircut tomorrow.
James wants a job.
So: James will get a haircut tomorrow.

Questions:
Which argument is valid?
What are the propositions?
Which propositions are true?

Logic

Question: Which of these arguments are valid? (Hacking 2001, Ch.1 Question 7)

- I follow three major league teams. Most of their top hitters chew tobacco at the plate.
  $\Rightarrow$ Chewing tobacco improves batting average.
- The top six hitters in the National League chew tobacco at the plate.
  $\Rightarrow$ Chewing tobacco improves batting average.
- A study by the American Dental Association of 158 players on seven major league teams during the 1988 season showed that the mean batting average for chewers was 0.238 compared to 0.248 for non-users. Abstainers also had a higher fielding average.
  $\Rightarrow$ Chewing tobacco does not improve batting average.
- In 1921, every major league pitcher who chewed tobacco when up to bat had a higher batting average than any major league pitcher who did not.
  $\Rightarrow$ Chewing tobacco does not improve batting average.

Logic

Question: Which of these arguments are valid? (Hacking 2001, Ch.1 Question 7)

- I follow three major league teams. Most of their top hitters chew tobacco at the plate.
  $\Rightarrow$ Chewing tobacco improves batting average.
- The top six hitters in the National League chew tobacco at the plate.
  $\Rightarrow$ Chewing tobacco improves batting average.
- A study by the American Dental Association of 158 players on seven major league teams during the 1988 season showed that the mean batting average for chewers was 0.238 compared to 0.248 for non-users. Abstainers also had a higher fielding average.
  $\Rightarrow$ Chewing tobacco does not improve batting average.
- In 1921, every major league pitcher who chewed tobacco when up to bat had a higher batting average than any major league pitcher who did not.
  $\Rightarrow$ Chewing tobacco does not improve batting average.

None of them are valid. But some are better than others. In what sense? In any logical sense?

Logic

The Stoics identify the following syllogisms, purported patterns of valid inference:

- *Modus Ponens*: If A, then B. A. Therefore, B.
- *Modus Tollens*: If A, then B. Not B. Therefore, not A.
- The Hypothetical Syllogism: If A, then B. If B, then C. Therefore, if A, then C.
- The Conjunctive Syllogism: Not both A and B. A. Therefore, not B.
- The Dilemma: If A, then B. If C, then B. A or C. Therefore, B.
- The Disjunctive Syllogism: A or B. But not A. Therefore, B.

Logic

The Stoics identify the following syllogisms, purported patterns of valid inference:

- *Modus Ponens*: If A, then B. A. Therefore, B.
- *Modus Tollens*: If A, then B. Not B. Therefore, not A.
- The Hypothetical Syllogism: If A, then B. If B, then C. Therefore, if A, then C.
- The Conjunctive Syllogism: Not both A and B. A. Therefore, not B.
- The Dilemma: If A, then B. If C, then B. A or C. Therefore, B.
- The Disjunctive Syllogism: A or B. But not A. Therefore, B.

Conceit: If we begin with true propositions, and combine them according to the above syllogisms, we necessarily reach a true conclusion.

Reasoning in this way is called **deductive reasoning**.

Example

Set theory can provide a means to visualize and analyze logical reasoning. In particular, deductive implication is equivalent to set inclusion.

Here is a running example for this presentation. Suppose a bag contains three coins: one regular coin (HT), one with both faces tails (TT), and one with both faces heads (HH). The coin is flipped, and either the first or second side comes up.

Exactly one possible outcome of coin $\times$ side occurs; call this the “truth.”

Example

Set theory can provide a means to visualize and analyze logical reasoning. In particular, deductive implication is equivalent to set inclusion.

Here is a running example for this presentation. Suppose a bag contains three coins: one regular coin (HT), one with both faces tails (TT), and one with both faces heads (HH). The coin is flipped, and either the first or second side comes up.

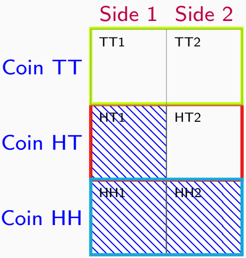
Exactly one possible outcome of coin $\times$ side occurs; call this the "truth."

	Side 1	Side 2
Coin TT	TT1	TT2
Coin HT	HT1	HT2
Coin HH	HH1	HH2 $\times$

Example:
Coin HH was picked and the second side came up (HH2).

In this context, classical propositional logic can be represented as set operations.

Example



We observe heads $= HT1 \vee HH1 \vee HH2$

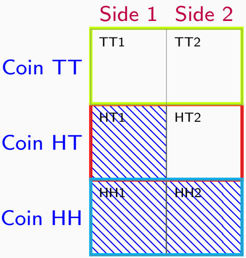
We chose the TT coin $= TT1 \vee TT2$

We chose the HT coin $= HT1 \vee HT2$

We chose the HH coin $= HH1 \vee HH2$

Recall that the truth $\times$ lies in exactly one cell.

Example



We observe heads $= HT1 \vee HH1 \vee HH2$

We chose the TT coin $= TT1 \vee TT2$

We chose the HT coin $= HT1 \vee HT2$

We chose the HH coin $= HH1 \vee HH2$

Recall that the truth $\times$ lies in exactly one cell.

Questions:

Suppose we observe heads (so we know $\times \in$).

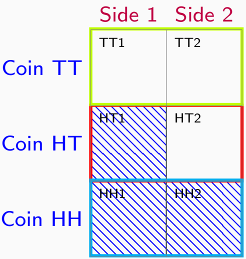
What can we deduce about whether we drew the TT coin?

What can we deduce about whether we drew the HT coin?

What can we deduce about whether we drew the HH coin?

Hint: Note that $\subseteq$ ^{c}.

Example



We observe heads $= HT1 \vee HH1 \vee HH2$

We chose the TT coin $= TT1 \vee TT2$

We chose the HT coin $= HT1 \vee HT2$

We chose the HH coin $= HH1 \vee HH2$

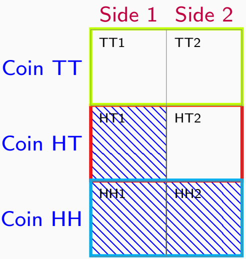
Recall that the truth $\times$ lies in exactly one cell.

In general: Set inclusion is the same as deductive implication:

A is true and $A \subseteq B$ implies that B is true. This is the set version of *modus ponens*.

(**Optional exercise:** Rewrite the Stoics' other logical syllogisms as set operations.)

Example



We observe heads $= HT1 \vee HH1 \vee HH2$

We chose the TT coin $= TT1 \vee TT2$

We chose the HT coin $= HT1 \vee HT2$

We chose the HH coin $= HH1 \vee HH2$

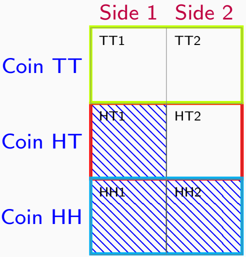
Recall that the truth $\times$ lies in exactly one cell.

In general: Set inclusion is the same as deductive implication:

A is true and $A \subseteq B$ implies that B is true. This is the set version of *modus ponens*.

(**Optional exercise:** Rewrite the Stoics' other logical syllogisms as set operations.)

If A is true, and A overlaps partially with B, **nothing** about B follows deductively, because **you might be wrong**, and **deduction cannot be wrong**.



We observe heads $= HT1 \vee HH1 \vee HH2$

We chose the TT coin $= TT1 \vee TT2$

We chose the HT coin $= HT1 \vee HT2$

We chose the HH coin $= HH1 \vee HH2$

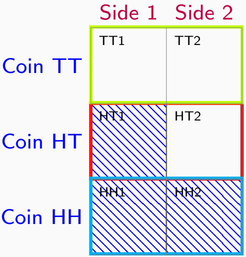
Recall that the truth $\times$ lies in exactly one cell.

And yet it seems silly to claim that observing a heads tells you nothing about whether you have chosen the HH coin.

After all, the "prior" $p(\mathrm{HH\ coin}) = 1/3$, but the "posterior" $p(\mathrm{HH\ coin}|\mathrm{heads}) = 2/3$.

[1] Or I think we will see this, I haven't read most of these papers yet either.

Example



We observe heads $= HT1 \vee HH1 \vee HH2$
We chose the TT coin $= TT1 \vee TT2$
We chose the HT coin $= HT1 \vee HT2$
We chose the HH coin $= HH1 \vee HH2$
Recall that the truth $\times$ lies in exactly one cell.

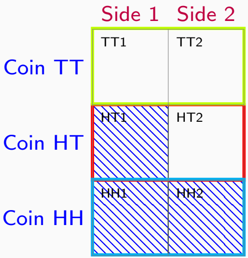
And yet it seems silly to claim that observing a heads tells you nothing about whether you have chosen the HH coin.

After all, the "prior" $p(\text{HH coin}) = 1/3$, but the "posterior" $p(\text{HH coin}|\text{heads}) = 2/3$.

Reasoning that goes beyond deduction is called "induction." (It is also sometimes called "ampliative" reasoning because it concludes more than is given by the premises.)

[1] Or I think we will see this, I haven't read most of these papers yet either.

Example



We observe heads $= HT1 \vee HH1 \vee HH2$

We chose the TT coin $= TT1 \vee TT2$

We chose the HT coin $= HT1 \vee HT2$

We chose the HH coin $= HH1 \vee HH2$

Recall that the truth $\times$ lies in exactly one cell.

And yet it seems silly to claim that observing a heads tells you nothing about whether you have chosen the HH coin.

After all, the "prior" $p(\text{HH coin}) = 1/3$, but the "posterior" $p(\text{HH coin}|\text{heads}) = 2/3$.

Reasoning that goes beyond deduction is called "induction." (It is also sometimes called "ampliative" reasoning because it concludes more than is given by the premises.)

Questions:

Is induction even possible? If so, how?

As we will see, early statisticians were extremely preoccupied with this question.[1]

[1] Or I think we will see this, I haven't read most of these papers yet either.

The European Enlightenment was an exciting time

Is induction even possible? If so, how?

We have asked this question in the setting when it has, arguably, the best chance of being possible (a well-defined probability setup).

Hume asked it in much, much greater generality.

The European Enlightenment was an exciting time

Is induction even possible? If so, how?

We have asked this question in the setting when it has, arguably, the best chance of being possible (a well-defined probability setup).

Hume asked it in much, much greater generality.

- 1657: Huygens' The Value of all Chances in Games of Fortune
- 1687: Newton's *Prinicipia Mathematica*
- 1690: Locke's Essay Concerning Human Understanding
- 1713 Bernoulli's The Art of Conjecturing
- 1718 De Moivre's The Doctrine of Chances
- 1739: **Hume's An Enquiry Concerning Human Understanding**
- 1751-1772: Diderot's Encyclopedia
- 1763: Bayes' An Essay towards solving a Problem in the Doctrine of Chances
- 1770: Euler's Foundations of integral calculus
- 1776: The American Revolution
- 1776: Adam Smith's The Wealth of Nations
- 1789: The French Revolution
- 1812: Laplace's Analytic Theory of Probabilities

The European Enlightenment was an exciting time

Is induction even possible? If so, how?

We have asked this question in the setting when it has, arguably, the best chance of being possible (a well-defined probability setup).

Hume asked it in much, much greater generality.

- 1657: Huygens' The Value of all Chances in Games of Fortune
- 1687: Newton's *Prinicipia Mathematica*
- 1690: Locke's Essay Concerning Human Understanding
- 1713 Bernoulli's The Art of Conjecturing
- 1718 De Moivre's The Doctrine of Chances
- 1739: **Hume's An Enquiry Concerning Human Understanding**
- 1751-1772: Diderot's Encyclopedia
- 1763: Bayes' An Essay towards solving a Problem in the Doctrine of Chances
- 1770: Euler's Foundations of integral calculus
- 1776: The American Revolution
- 1776: Adam Smith's The Wealth of Nations
- 1789: The French Revolution
- 1812: Laplace's Analytic Theory of Probabilities

Foundations, rigor, and revolution were all in the air.

Hume

Is induction even possible? If so, how?

1. All questions are “Relations of Ideas” or “Matters of Fact.”
2. “Relations of Ideas” are deductive, certain, mathematical.
3. “Matters of Fact” are sensory, experiental, contingent, local in time and space.
4. The content of ideas is entirely matters of fact.

Hume

Is induction even possible? If so, how?

1. All questions are “Relations of Ideas” or “Matters of Fact.”
2. “Relations of Ideas” are deductive, certain, mathematical.
3. “Matters of Fact” are sensory, experiental, contingent, local in time and space.
4. The content of ideas is entirely matters of fact.

Question: Do the components of Hume’s argument seem to have analogues in deductive logic?

Hume

Is induction even possible? If so, how?

1. All questions are "Relations of Ideas" or "Matters of Fact."
2. "Relations of Ideas" are deductive, certain, mathematical.
3. "Matters of Fact" are sensory, experiental, contingent, local in time and space.
4. The content of ideas is entirely matters of fact.

Question: Do the components of Hume's argument seem to have analogues in deductive logic?

Hume's question: We believe much more about the world than we experience directly. What is the foundation of this belief? Is it experience? Is it deduction? What combination of the two?

Cause and Effect

Hume specifies three ways ideas can be associated:

1. Resemblance
2. Co-occurence
3. Cause and effect

Hume argues that it is because of cause and effect that we extrapolate from local, transient experiences.

Cause and Effect

Hume specifies three ways ideas can be associated:

1. Resemblance
2. Co-occurence
3. Cause and effect

Hume argues that it is because of cause and effect that we extrapolate from local, transient experiences.

Question: What is the difference between Hume's co-occurence and cause and effect?

Cause and Effect

Hume specifies three ways ideas can be associated:

1. Resemblance
2. Co-occurence
3. Cause and effect

Hume argues that it is because of cause and effect that we extrapolate from local, transient experiences.

Question: What is the difference between Hume's co-occurence and cause and effect?

Question: What is the difference between Hume's cause and effect and Judea Pearl's?

Cause and Effect

Hume specifies three ways ideas can be associated:

1. Resemblance
2. Co-occurence
3. Cause and effect

Hume argues that it is because of cause and effect that we extrapolate from local, transient experiences.

Question: What is the difference between Hume's co-occurence and cause and effect?

Question: What is the difference between Hume's cause and effect and Judea Pearl's?

Question: What is the difference between Hume's cause and effect and the counterfactulas of the Neyman-Rubin framework?

Cause and Effect

Cause and effect differs from co-occurence by being **necessary**.

So how do we establish that something is necessary? Can we establish it by deduction? By experience?

Cause and Effect

Cause and effect differs from co-occurence by being **necessary**.

So how do we establish that something is necessary? Can we establish it by deduction? By experience?

Not by deduction:

> "I say then that, even after we have experience of the operations of cause and effect, our conclusions from that experience are not founded on reasoning, or on any process of the understanding." (28)

Cause and Effect

Cause and effect differs from co-occurence by being **necessary**.

So how do we establish that something is necessary? Can we establish it by deduction? By experience?

Not by deduction:

> "I say then that, even after we have experience of the operations of cause and effect, our conclusions from that experience are not founded on reasoning, or on any process of the understanding." (28)

And not by experience:

> "For all inferences from experience suppose, as their foundation, that the future will resemble the past, and that similar powers will be conjoined with similar sensible qualities. If there be any suspicion that the course of nature may change, and that the past may be no rule for the future, all experience becomes useless, and can give rise to no inference or conclusion." (32)

Cause and Effect

Cause and effect differs from co-occurence by being **necessary**.

So how do we establish that something is necessary? Can we establish it by deduction? By experience?

Not by deduction:

> "I say then that, even after we have experience of the operations of cause and effect, our conclusions from that experience are not founded on reasoning, or on any process of the understanding." (28)

And not by experience:

> "For all inferences from experience suppose, as their foundation, that the future will resemble the past, and that similar powers will be conjoined with similar sensible qualities. If there be any suspicion that the course of nature may change, and that the past may be no rule for the future, all experience becomes useless, and can give rise to no inference or conclusion." (32)

Hume's answer: "Custom."

Cause and Effect

Cause and effect differs from co-occurence by being **necessary**.

So how do we establish that something is necessary? Can we establish it by deduction? By experience?

Not by deduction:

> "I say then that, even after we have experience of the operations of cause and effect, our conclusions from that experience are not founded on reasoning, or on any process of the understanding." (28)

And not by experience:

> "For all inferences from experience suppose, as their foundation, that the future will resemble the past, and that similar powers will be conjoined with similar sensible qualities. If there be any suspicion that the course of nature may change, and that the past may be no rule for the future, all experience becomes useless, and can give rise to no inference or conclusion." (32)

Hume's answer: "Custom."

Question: What is Hume saying about inductive reasoning?

Question: In Hume's view, what classes of questions fall under the category of inductive reasoning?

Hume on science

Hume writes:

> "Elasticity, gravity, cohesion of parts, communication of motion by impulse; these are probably the ultimate causes and principles which we shall ever discover in nature." (26)

and

> "Our senses inform us of the colour, weight, and consistence of bread; but neither sense nor reason can ever inform us of those qualities which fit it for the nourishment and support of a human body." (29)

Hume on science

Hume writes:

> "Elasticity, gravity, cohesion of parts, communication of motion by impulse; these are probably the ultimate causes and principles which we shall ever discover in nature." (26)

and

> "Our senses inform us of the colour, weight, and consistence of bread; but neither sense nor reason can ever inform us of those qualities which fit it for the nourishment and support of a human body." (29)

Question: These observations have not held up well. What consequences does this have for Hume's view of science, if any?

Hume on everyday life

Question: In Hume's view, what consequences do his skepticism have for real life?

Hume on everyday life

Question: In Hume's view, what consequences do his skepticism have for real life?

Hume writes:

> "And though none but a fool or madman will ever pretend to dispute the authority of experience, or to reject that great guid of human life, it may surely be allowed a philosopher to have so much curiosity at least as to examine the principle of human nature, which gives this mighty authority to experience, and makes us draw advantage from the similarity which nature has placed among different objects." (31)

> "My practice, you say, refutes my doubts. But you mistake the purport of my question. As an agent, I ma quite satisfied in the point; but as a philosopher, who has some share of curiosity, I will not say scepticism, I want to learn the foundation of this inference." (32)

A way forward?

"There is certainly a probability, which arises from a superiority of chances on any side; and acoording as this superiority encreases, and surpasses the opposite chances, the probability receive a proportional increase, and begets still a higher degree of belief or assent to that side, in which we discover the superiority." (46)

A way forward?

> "There is certainly a probability, which arises from a superiority of chances on any side; and acoording as this superiority encreases, and surpasses the opposite chances, the probability receive a proportional increase, and begets still a higher degree of belief or assent to that side, in which we discover the superiority." (46)

Warning: Probable / probaility had a different meaning for Hume than for us!

probability (n.) mid-15c., probabilite, "likelihood of being realized, appearance of truth, quality of being probable," from Old French probabilite (14c.) and directly from Latin probabilitatem (nominative probabilitas) "credibility, probability," from probabilis (see probable).

Meaning "something likely to be true" is from 1570s; mathematical sense is from 1718, "frequency with which a proposition similar to the one in question is found true in the course of experience." (www.etymonline.com)

A way forward?

> "There is certainly a probability, which arises from a superiority of chances on any side; and acoording as this superiority encreases, and surpasses the opposite chances, the probability receive a proportional increase, and begets still a higher degree of belief or assent to that side, in which we discover the superiority." (46)

Warning: Probable / probaility had a different meaning for Hume than for us!

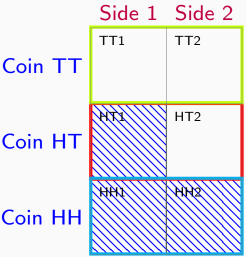
probability (n.) mid-15c., probabilite, "likelihood of being realized, appearance of truth, quality of being probable," from Old French probabilite (14c.) and directly from Latin probabilitatem (nominative probabilitas) "credibility, probability," from probabilis (see probable).

Meaning "something likely to be true" is from 1570s; mathematical sense is from 1718, "frequency with which a proposition similar to the one in question is found true in the course of experience." (www.etymonline.com)

Question: Is Hume putting this forward as a potential solution?

A different way forward?

Karl Popper answers Hume using a very different technique than probability.



We observe heads $= HT1 \vee HH1 \vee HH2$

We chose the TT coin $= TT1 \vee TT2$

We chose the HT coin $= HT1 \vee HT2$

We chose the HH coin $= HH1 \vee HH2$

Recall that the truth $\times$ lies in exactly one cell.

Question: Recall our coin example. Suppose we have two scientific theories, corresponding to "coin HH was chosen" and "coin TT was chosen." We observe heads. What does Popper say we can conclude? How does this relate to Hume's question of induction?

Conclusion

- Deductive logic is an ancient tradition designed to produce certainty.
  - Deduction generates guaranteed truths from true propositions
  - Deduction has a deep formal connection to set theory
- Modern epistemology (which owes a lot to Hume) requires more than deduction:
  - How to weigh inconsistent evidence?
  - How to treat random evidence?
  - How to extrapolate from old data to new settings?
  - How to evaluate and test scientific theories?
- All the above questions (and more) fall under the category of "induction."
  - Induction concludes more than is given in the premises.
  - Maybe induction is not even necessary (c.f. Popper)
- Can we form an "inductive logic?"
  - What would this even mean?
  - Can we exploit the connection between deduction and set theory?

References

Hacking, I. (2001). *An introduction to probability and inductive logic*. Cambridge university press.

— (2016). *Logic of statistical inference*. Cambridge University Press.

Shenefelt, M. and H. White (2013). *If A, then B: How the world discovered logic*. Columbia University Press.