

An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Make a Big Difference?



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Dropping data: Motivation

You're a data analyst, and you've

- Gathered some exchangeable data,
- Cleaned up / removed outliers,
- Checked for correct specification, and
- Drawn a conclusion from your statistical analysis
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Well done!

Would you be concerned if you could **reverse your conclusion** by removing a **small proportion** (say, 0.1%) of your data?

Dropping data: Mexico Microcredit

Consider Angelucci et al. [2015], a randomized controlled trial study of the efficacy of microcredit in Mexico based on 16,560 data points. The variable “Beta” estimates the effect of microcredit in US dollars.

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Question: Is the reported interval $-4.55 \pm (5.88)$ a reasonable description of the uncertainty in the estimated efficacy of microcredit?

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...but sometimes, surely yes.

For example, often in economics:

- Small fractions of data are missing not-at-random,
- Policy population is different from analyzed population,
- We report a convenient summary (e.g. mean) of a complex effect,
- Models are stylized proxies of reality.

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Question 1: How do we find influential datapoints?

The number of subsets $\binom{N}{\lfloor \alpha N \rfloor}$ can be very large even when α is very small.

In the MX microcredit study, $\binom{16560}{15} \approx 1.4 \cdot 10^{51}$ sets to check for $\alpha = 0.0009$.

We provide a fast, automatic approximation based on the **influence function**.

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Question 2: What makes an estimator non-robust?

Non-robustness to removal of $\lfloor \alpha N \rfloor$ points is:

- Not (necessarily) caused by misspecification.
- Not (necessarily) caused by outliers.
- Not captured by standard errors.
- Not mitigated by large N .
- Primarily determined by the **signal to noise** ratio
... in a sense which we will define.

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- We provide deterministic error bounds for small α .
- We show the accuracy in simple experiments.
- We show the accuracy in a number of real-world experiments.

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Conclusion: Related work and future directions

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How do we find influential datapoints?

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The influence function

- Weights as derivatives
- Influence function
- Simulation
- Experiments

The linear approximation.

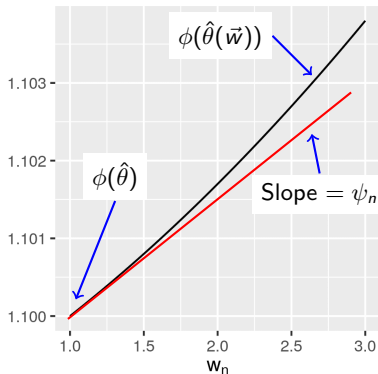
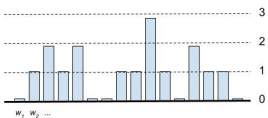
Original weights:



Leave-one-out weights:



Bootstrap weights:



$$\phi(\hat{\theta}(\vec{w})) = \phi(\hat{\theta}) + \sum_{n=1}^N \psi_n(\vec{w}_n - 1) + \text{Higher-order derivatives}$$

Key idea: Controlling higher-order derivatives can control the error.

The linear approximation.

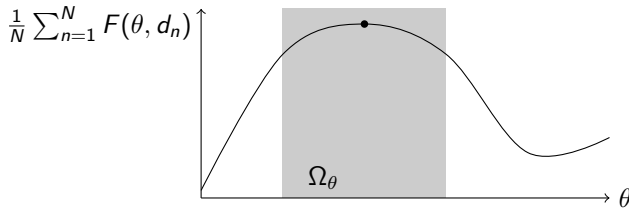
Let W_α be the set of weight vectors with no more than $\lfloor \alpha N \rfloor$ zeros.

Let $H(\theta, d_n) := \left. \frac{\partial G(\theta, d_n)}{\partial \theta^T} \right|_\theta$.

Assumption (Smooth Objective)

Fix the dataset. Assume there exists a compact $\Omega_\theta \subseteq \mathbb{R}^D$ with $\hat{\theta}(\vec{w}) \in \Omega_\theta$ for all $\vec{w} \in W_\alpha$. Assume that, for all $\theta \in \Omega_\theta$:

- $\frac{1}{N} \sum_{n=1}^N H(\theta, d_n)$ and $\frac{1}{N} \sum_{n=1}^N G(\theta, d_n)$ are bounded.
- $\frac{1}{N} \sum_{n=1}^N H(\theta, d_n)$ is uniformly non-singular and Lipschitz (in θ).
- $\phi(\theta)$ has a Lipschitz first derivative.



The linear approximation.

Theorem

Let Assumption 1 hold for a given dataset. Then there exists a sufficiently small α such that

$$\sup_{\vec{w} \in W_\alpha} \left| \phi^{\text{lin}}(\vec{w}) - \phi(\hat{\theta}(\vec{w})) \right| \leq C_1 \alpha \text{ and } \sup_{\vec{w} \in W_\alpha} \left| \phi(\hat{\theta}(\vec{w})) - \phi(\hat{\theta}) \right| \leq C_2 \sqrt{\alpha}.$$

where C_1 and C_2 are given by the quantities in the assumption.

Since $\alpha \ll \sqrt{\alpha}$ when α is small, Theorem 1 states that the linear approximation's error is of smaller order than the actual difference.

Corollary

Under standard conditions, Assumption 1 holds for fixed constants with probability approaching one. Then Theorem 1 applies with probability approaching one.

The linear approximation.

For $N = 5,000$ data points, compute the OLS estimator from:

Regressors
 $x_n \sim \mathcal{N}(0, \sigma_x^2)$

Residuals
 $\varepsilon_n \sim \mathcal{N}(0, \sigma_\varepsilon^2)$

Responses
 $y_n = \theta_0 x_n + \varepsilon_n$

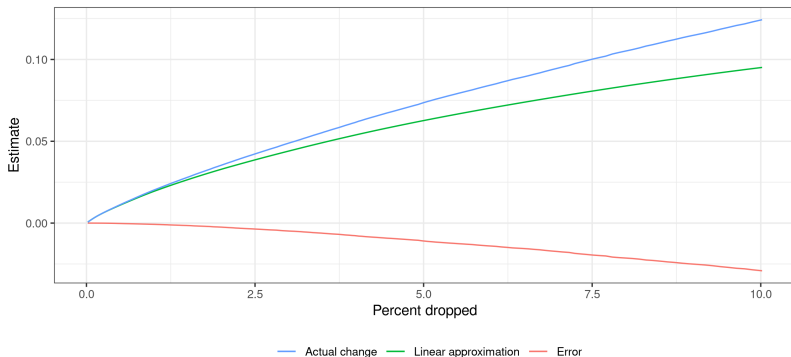


Figure: The actual change, linear approximation to the change, and approximation error. Here, $\sigma_x = 2$, $\sigma_\varepsilon = 1$, and $\theta_0 = 0.5$.

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Conclusion

- You may be concerned if you could reverse your conclusion by removing a $\lfloor \alpha N \rfloor$ datapoints, for some small α .
- Robustness to removing a $\lfloor \alpha N \rfloor$ datapoints is principally determined by the signal to noise ratio, does not disappear asymptotically, and is distinct from (and typically larger than) standard errors.
- Robustness to removing a $\lfloor \alpha N \rfloor$ datapoints is easy to check! We can quickly and automatically find an approximate influential set which is accurate for small α .

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“An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Change Conclusions?”

<https://arxiv.org/abs/2011.14999>

See the paper for applications to:

- Hierarchical meta-analysis of microcredit [Meager, 2020]
- Cash transfers randomized controlled trial [Angelucci and De Giorgi, 2009]
- Oregon Medicaid experiment [Finkelstein et al., 2012]
- Expository simulations

zaminfluence: R package with leave- α -out robustness for OLS and IV estimators

<https://github.com/rgiordan/zaminfluence>

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