Locally Equivalent Weights for Bayesian MrP

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Are US non-voters becoming more Republican?

Blue Rose research says yes:

“Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate.”

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

“Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available.”

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- Different data sources
- ⋆⋆⋆ **Different statistical methods**
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Our contribution

We define "MrP local equivalent weights" (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key diagnostics that motivate calibration weighting.

$\Rightarrow$ **MrPlew provides direct comparisons between MrP and calibration weighting.**

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- Introduce the statisical problem
  - Contrast CW and MrP
  - Prior work: Equivalent weights for linear models
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  - Our key idea: Locally equivalent weights for non–linear models

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    - Describe covariate balance
    - Define MrPlew weights and connect them to covariate balance
    - Theoretical support
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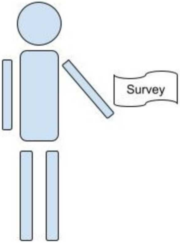
  - Example of real-world results
- Other uses of locally equivalent weights
  - Parital pooling
  - The meaning of negative weights
  - Frequentist variance estimation
- Future directions

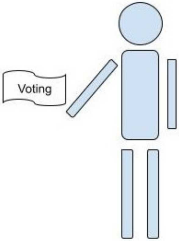
The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support Trump")

We want the average response in a target population, in which we observe only covariates.





Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

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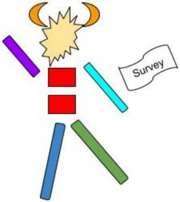

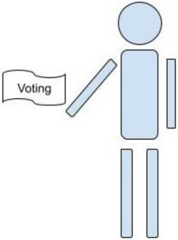

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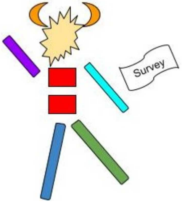

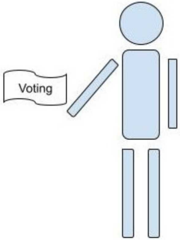

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The problem is that the populations may be very different.

Our survey results may be biased.

How can we use the covariates to say something about the target responses?

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j. Let $Y_{\mathcal{S}} = \{y_1, \ldots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

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Calibration weighting (CW)

▶ Choose "calibration weights" w_i using only the regressors $\mathbf{x}$ (e.g. raking weights)

Bayesian hierarchical modeling (MrP)

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- ▶ Weights give interpretable diagnostics:
  - Frequentist variability
  - Partial pooling
  - Regressor balance

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$\leftarrow$ We open this box, providing analogues of all these diagnostics

Prior work: Equivalent weights for linear models

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form $\hat{y}$:

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But what if you use a non–linear link function? Or a hierarchical model?

> *"It would also be desirable to use nonlinear methods ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods." — (Gelman 2007a)*

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Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}_{\mathrm{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *inherently nonlinear*.

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For $w_i^{\mathrm{MrP}} = \dfrac{N_T^c / N_T}{N_S^c / N_S}$ when $\mathbf{x}_i = c$.

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&\approx \int (\alpha^\intercal\mathbf{x})\, m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_S(\mathbf{x})d\mathbf{x} && \text{(By assumption)} \\
&\approx \alpha^\intercal \frac{1}{N_S}\sum_{i=1}^{N_S} \mathbf{x}_i m(\mathbf{x}_i^\intercal\hat{\theta}) && \text{(Law of large numbers)}
\end{aligned}
$$

Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \text{Logistic}(\mathbf{x}^\intercal\theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal\mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$
\begin{aligned}
\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) &= \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta}) \\
&\approx \int m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_T(\mathbf{x})d\mathbf{x} && \text{(Law of large numbers)} \\
&= \int \frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_S(\mathbf{x})d\mathbf{x} && \text{(Multiply by } \mathcal{P}_S(\mathbf{x})/\mathcal{P}_S(\mathbf{x})\text{)} \\
&\approx \int (\alpha^\intercal\mathbf{x})\, m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_S(\mathbf{x})d\mathbf{x} && \text{(By assumption)} \\
&\approx \alpha^\intercal \frac{1}{N_S}\sum_{i=1}^{N_S} \mathbf{x}_i m(\mathbf{x}_i^\intercal\hat{\theta}) && \text{(Law of large numbers)} \\
&= \alpha^\intercal \frac{1}{N_S}\sum_{i=1}^{N_S} \mathbf{x}_i y_i && \text{(Property of exponential family MLEs)}
\end{aligned}
$$

Nearly equivalent weights for (some) logistic regression MrP

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Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal\mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta}) = \frac{1}{N_S}\sum_{i=1}^{N_S} \underbrace{w_i^{\text{MrP}}}_{\alpha^\intercal\mathbf{x}_i} y_i + \text{Small error}$$

But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

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But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

Key idea (informal)

If $\hat{\mu}_{\text{MrP}}$ is approximately linear, then $w_i^{\text{MrP}} \approx \frac{\partial\hat{\mu}_{\text{MrP}}}{\partial y_i}$.

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Nearly equivalent weights for (some) logistic regression MrP

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But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

Key idea (informal)

If $\hat{\mu}_{\text{MrP}}$ is approximately linear, then $w_i^{\text{MrP}} \approx \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$.

For logistic regression, could compute and analyze $\frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$ using the implicit function theorem.[2]

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \mathrm{Logistic}(\mathbf{x}^\intercal\theta)$.
- Set a hierarchical prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$, use MCMC to draw from $\mathcal{P}(\theta|\text{Survey data})$.
- MrP is $\hat{\mu}_{\mathrm{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T} \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_j^\intercal\theta)\right]$.

No reason to think $Y_{\mathcal{S}} \mapsto \hat{\mu}_{\mathrm{MrP}}(Y_{\mathcal{S}})$ is even approximately linear.

But we can still compute and analyze $w_i^{\mathrm{MrP}} := \frac{\partial \hat{\mu}_{\mathrm{MrP}}}{\partial y_i}$ using Bayesian sensitivity analysis![3]

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

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MrP locally equivalent weights (MrPlew)

For new data $\tilde{Y}_S$, form a series expansion

$$\hat{\mu}_{\text{MrP}}(\tilde{Y}_S) \approx \hat{\mu}_{\text{MrP}}(Y_S) + \sum_{i=1}^{N_S} w_i^{\text{MrP}} (\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MrP}} := \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}.$$

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

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Our task is to rigorously show that even such local weights can be used diagnostically.

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

The weights can look very different!

Does this mean anything? Are the differences important?

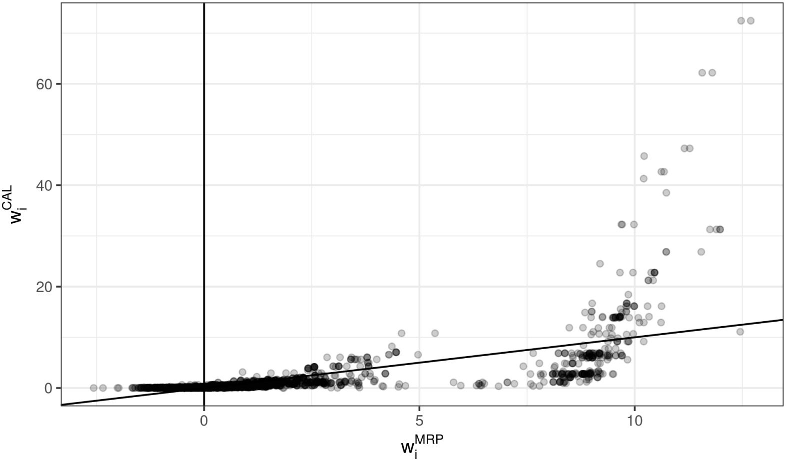

Figure 1: Comparison between raking and MrPlew weights for the Name Change dataset

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