Local Weighting–Based Diagnostics for Bayesian Poststratification

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Are US non-voters becoming more Republican?

Blue Rose research says yes:

“Politically disengaged voters have become much more Republican, And because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate.”

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

“Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available.”

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- The problem is very hard (it’s difficult to accurately poll non–voters)
- Different data sources
- **Very different statistical methods:** $\star$
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  - The CES uses calibration weighting (CW)

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Our contribution

We define "MrP local equivalent weights" (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key diagnostics that motivate calibration weighting.

$\Rightarrow$ **MrPlew provides direct comparisons between MrP and calibration weighting.**

Outline

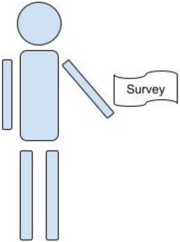
- Introduce the statisical problem and two methods (CW and MrP)
- Describe covariate balance, one of the classical CW diagnostics
- Define MrPlew weights and connect them to covariate balance
- Example of real-world results
- Future directions

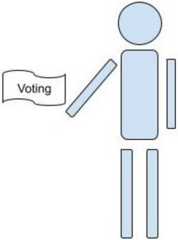
The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support Trump")

We want the average response in a target population, in which we observe only covariates.





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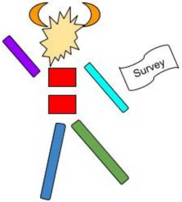

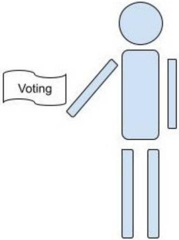

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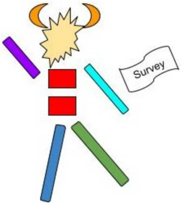

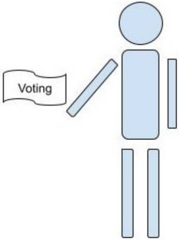

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Our survey results may be biased.

How can we use the covariates to say something about the target responses?

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

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Calibration weighting

▶ Choose "calibration weights" w_i
using only the regressors $\mathbf{x}$
(e.g. raking weights)

Bayesian hierarchical modeling (MrP)

▶ Choose $\mathbb{E}\left[y|\mathbf{x}, \theta\right] = m(\theta^\intercal \mathbf{x})$,
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- Weights give interpretable diagnostics:
  - Frequentist variability
  - Partial pooling
  - Regressor balance

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$\leftarrow$ (We open this box, providing analogues of all these diagnostics)

Prior work

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form $\hat{y}$:

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Most existing literature on comparing CW and MrP focus on such linear models. [1]

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But what if you use a non–linear link function? Or a hierarchical model?

> *"It would also be desirable to use nonlinear methods such as regression trees ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods." — (Gelman 2007a)*

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Our approach

For nonlinear models, we will *define* $w_i^{\mathrm{MRP}} = \frac{\partial \hat{\mu}_{\mathrm{MRP}}}{\partial y_i}$.

Our primary task is then to **rigorously justify** such weights' use in common diagnostics.

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The weights can look very different!

Does this mean anything? Are the differences important?

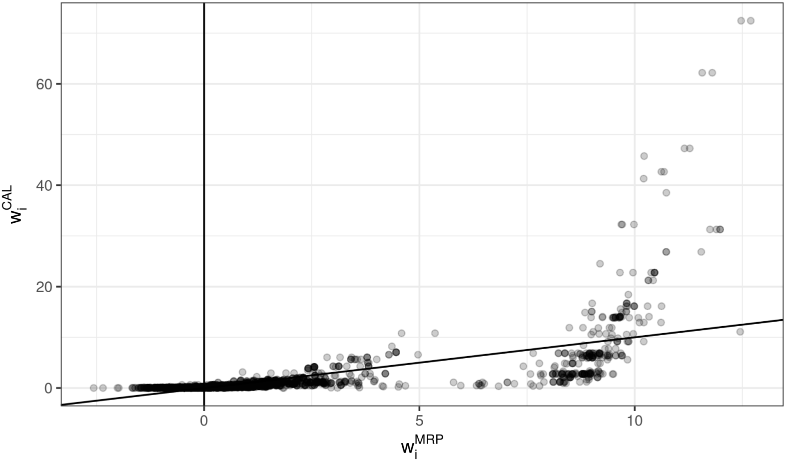

Figure 1: Comparison between raking and MrPlew weights

What are we weighting for?[2]

We want:

$$\text{Target average response} = \frac{1}{N_T}\sum_{j=1}^{N_T} y_j \approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i = \text{Weighted survey average response}$$

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Such weights satisfy "covariate balance" for **x**.

You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

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Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

Raking calibration weights

"Raking" selects weights that

- Are as "close as possible" to some reference weights
- Under the constraint that they balance some selected regressors.

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Generalized covariate balance checks

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Balance–informed sensitivity check (BISC) (informal)

Pick a small δ, and define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}\left[\mu(\tilde{y}) - \mu(y)|\mathbf{x}\right] = \frac{1}{N_T}\sum_{j=1}^{N_T}\left(\mathbb{E}\left[\tilde{y}|\mathbf{x}_p\right] - \mathbb{E}\left[y|\mathbf{x}_p\right]\right) = \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change for observed $\tilde{y}, y$:

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\text{Replace weighted averages with changes in an estimator}} \overset{\text{check}}{\approx} \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

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When $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{CAL}}(\cdot)$, BISC recovers the standard covariate balance check.

When $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{MRP}}(\cdot)$ and δ is small, BISC recovers our proposal.

Generalized covariate balance for MrP

Step one: Construct $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.

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Two possibilities:

- Allow $\tilde{y}$ to take values other than $\{0, 1\}$ and set $\tilde{y} = y + \delta f(\mathbf{x})$, or
- Use an estimate of $\mathbb{E}\left[y|\mathbf{x}\right]$ to draw new binary $\tilde{y}$.

Our approach:

- Use $\tilde{y} = y + \delta f(\mathbf{x})$ to identify problematic "imbalanced" $f(\mathbf{x})$
- Sanity check by generating binary $\tilde{y}$ using $f(\mathbf{x})$ (which is fast and easy)

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Problem: $\hat{\mu}_{\text{MRP}}(\cdot)$ is computed with MCMC.

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MrP Local Equivalent Weights (MrPlew)

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$$\hat{\mu}_{\text{MRP}}(\tilde{y}) - \hat{\mu}_{\text{MRP}}(y) \approx \sum_{i=1}^{N_S} w_i^{\text{MRP}}(\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MRP}} := \frac{d}{dy_i}\hat{\mu}_{\text{MRP}}(y).$$

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Use in BISC: For a wide set of judiciously chosen $f(\cdot)$, check

$$\underbrace{\delta \sum_{i=1}^{N_S} w_i^{\mathrm{MRP}} f(\mathbf{x}_i)}_{\approx \hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \hat{\mu}_{\mathrm{MRP}}(y)} \overset{\text{check}}{\approx} \delta \frac{1}{N_T} \sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

This a **sensitivity analysis** that formally coincides with a **balance check**.

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$$\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \hat{\mu}_{\mathrm{MRP}}(y) \approx \sum_{i=1}^{N_S} w_i^{\mathrm{MRP}}(\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\mathrm{MRP}} := \frac{d}{dy_i}\hat{\mu}_{\mathrm{MRP}}(y).$$

Computation: The weights are given by weighted averages of posterior covariances[3].

They can be easily computed with standard software[4] **without re–running MCMC**.

[3] **G.**, Broderick, and Jordan 2018.
[4] We use `brms` (Bürkner 2017).

Generalized covariate balance for MrP

Step one: Construct $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.
Step two: Evaluate $\hat{\mu}_{\text{MRP}}(\tilde{y}) - \hat{\mu}(y)$.

Problem: $\hat{\mu}_{\text{MRP}}(\cdot)$ is computed with MCMC.

- Each MCMC run typically takes hours, and
- Output is noisy, and $\hat{\mu}_{\text{MRP}}(\tilde{y}) - \hat{\mu}_{\text{MRP}}(y)$ may be small.

MrP Local Equivalent Weights (MrPlew)

Form the first-order Taylor series approximation

$$\hat{\mu}_{\text{MRP}}(\tilde{y}) - \hat{\mu}_{\text{MRP}}(y) \approx \sum_{i=1}^{N_S} w_i^{\text{MRP}} (\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MRP}} := \frac{d}{dy_i} \hat{\mu}_{\text{MRP}}(y).$$

Theory: We state conditions under which, as $\delta \to 0$, and $N \to \infty$,

- The residual is of lower order than the MrPlew term,
- *Uniformly* over a very wide class of $f(\cdot)$.

Uniformity is the hard part, but this justifies using MrPlew to *identify* problematic $f(\cdot)$.

Builds on earlier work on uniform error bounds for Bernstein–von Mises theorem(–ish) results[3].

[3] **G.** and Broderick 2024; Kasprzak, **G.**, and Broderick 2025.

Real Data

Analysis of changing names after marriage (based on Alexander (2019)).

- **Target population:** ACS survey of US population 2017–2022[4]
- **Survey population:** Marital Name Change Survey[5]
- **Respose:** Did the female partner keep their name after marriage?
- For regressors, use bins of age, education, state, and decade married.

Survey observations:	$N_S = 4,364$
Target observations (rows):	$N_T = 4,085,282$

Uncorrected survey mean:	$\frac{1}{N_S}\sum_{i=1}^{N_S} y_i = 0.462$
Raking:	$\hat{\mu}_{\text{CAL}} = 0.263$
MrP:	$\hat{\mu}_{\text{MRP}} = 0.288 \quad (\text{Post. sd} = 0.0169)$

[4]Ruggles et al. 2024.

[5]Cohen 2019.

Figure

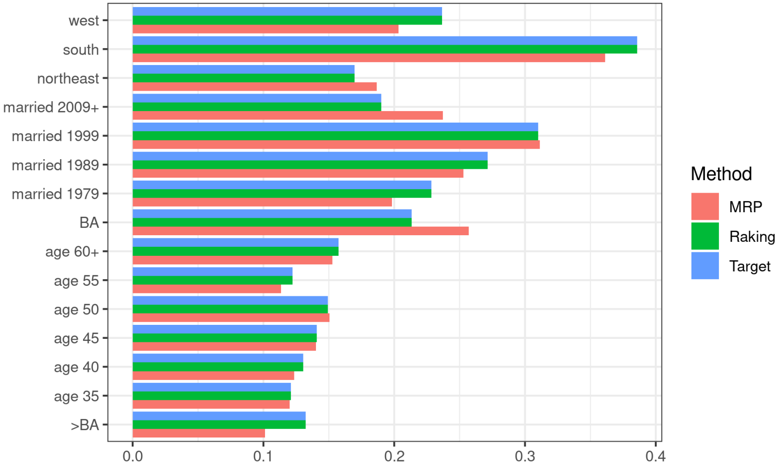

Figure 2: Imbalance plot for primary effects

Figure

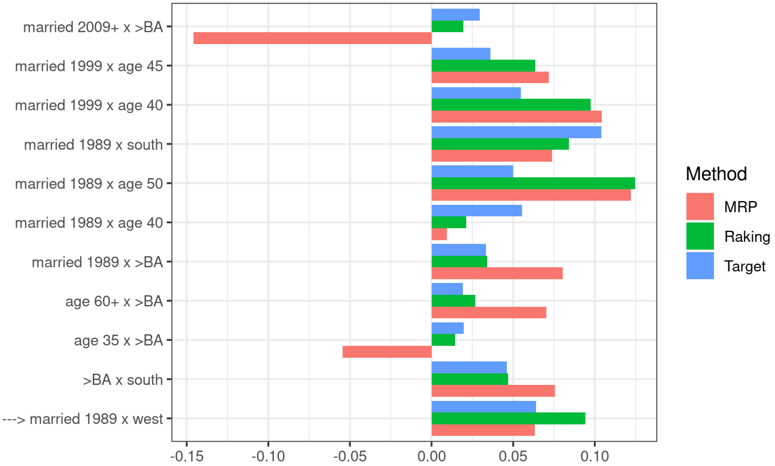

Figure 3: Imbalance plot for select interaction effects

Figure

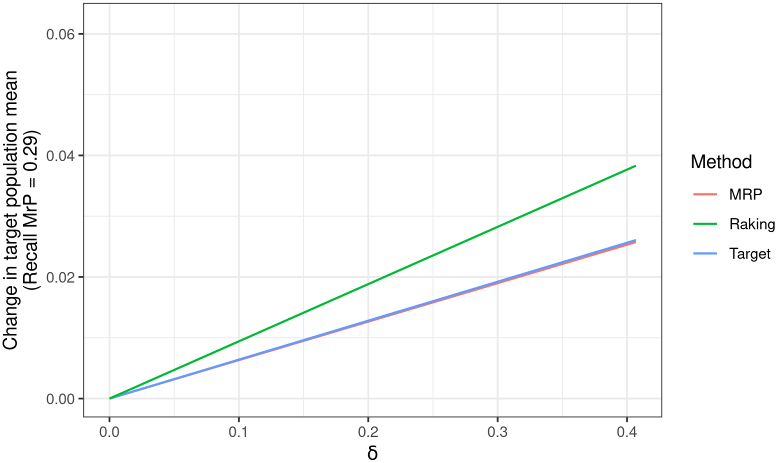

Figure 4: Continuous predictions Alexander

Figure

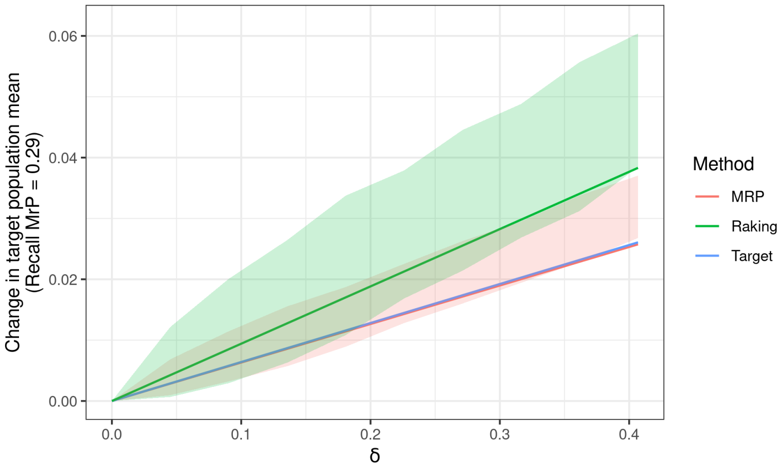

Figure 5: Continuous predictions Alexander

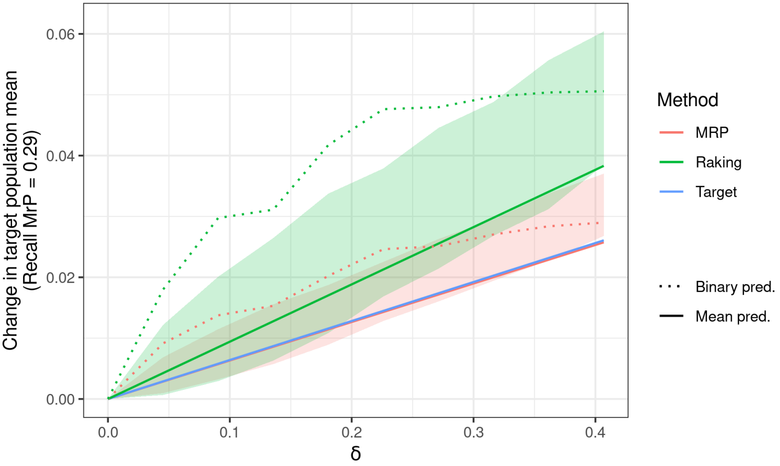

Figure 6: Continuous predictions Alexander

Figure

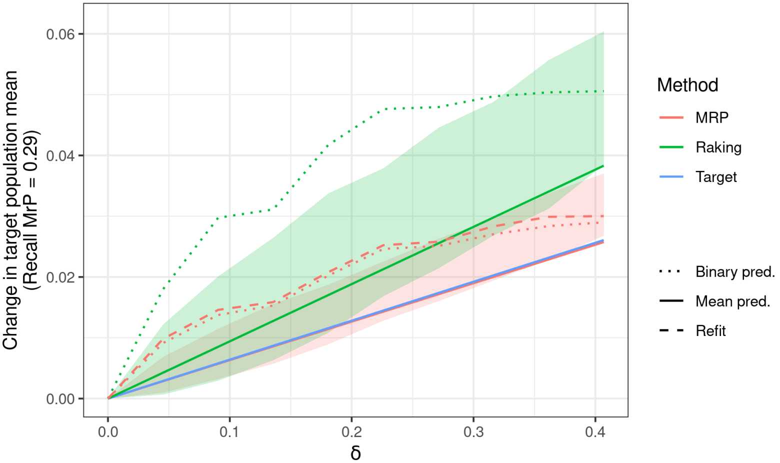

Figure 7: Continuous predictions Alexander

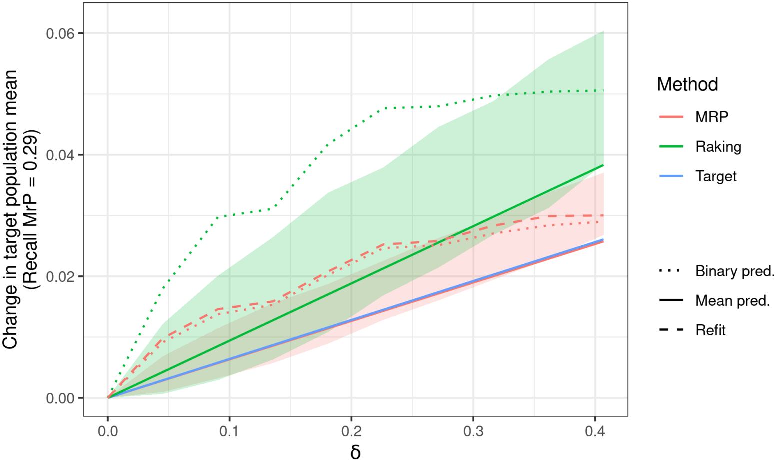

Figure 7: Continuous predictions Alexander

Running ten MCMC refits: 28 hours Computing approximate weights: 27 seconds

Related and future work

Today, I focused on covariate balance. In this work, we also provide rigorous justification for

- Frequentist covariance estimation
- Parital pooling
- Negative weights (extrapolation)

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Student contributions and future work:

- **Alice Cima** contributed significantly to this work
- **Vladimir Palmin** is working on extending MrPlew to `lme4`
- **Sequoia Andrade** is working on generalizing to other local sensitivity checks
- **Lucas Schwengber** is working on novel flow–based techniques for local sensitivity

Alice Cima

No picture!

Vladimir Palmin

Sequoia Andrade

Lucas Schwengber

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