An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Make a Big Difference?

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Dropping data: Motivation

You're a data analyst, and you've

- Gathered some exchangeable data,
- Cleaned up / removed outliers,
- Checked for correct specification, and
- Drawn a conclusion from your statistical analysis (e.g., based the sign / significance of some estimated parameter).

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Well done!
Would you be concerned if you could **reverse your conclusion** by removing a **small proportion** (say, 0.1%) of your data?

Dropping data: Mexico Microcredit

Consider **?**, a randomized controlled trial study of the efficacy of microcredit in Mexico based on 16,560 data points.
The variable "Beta" estimates the effect of microcredit in US dollars.

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Question: Is the reported interval $-4.55 \pm (5.88)$ a reasonable description of the uncertainty in the estimated efficacy of microcredit?

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...but sometimes, surely yes.
For example, often in economics:

- Small fractions of data are missing not-at-random,
- Policy population is different from analyzed population,
- We report a convenient summary (e.g. mean) of a complex effect,
- Models are stylized proxies of reality.

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Question 1: How do we find influential datapoints?

The number of subsets $\binom{N}{\lfloor \alpha N \rfloor}$ can be very large even when α is very small.

In the MX microcredit study, $\binom{16560}{15} \approx 1.4 \cdot 10^{51}$ sets to check for $\alpha = 0.0009$.

We provide a fast, automatic approximation based on the **influence function**.

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Question 2: What makes an estimator non-robust?
Non-robustness to removal of $\lfloor \alpha N \rfloor$ points is:

- Not (necessarily) caused by misspecification.
- Not (necessarily) caused by outliers.
- Not captured by standard errors.
- Not mitigated by large N.
- Primarily determined by the **signal to noise** ratio

  ... in a sense which we will define.

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- We provide deterministic error bounds for small α.
- We show the accuracy in simple experiments.
- We show the accuracy in a number of real-world experiments.

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