Locally Equivalent Weights for Bayesian MrP

Ryan Giordano, Alice Cima, Erin Hartman, Jared Murray, Avi Feller University of British Columbia Statistics Seminar October 2025











Notice that there was no discussion of misspecification!

Calibration weights (typically) do not depend on Y_S .

Notice that there was no discussion of misspecification!

Calibration weights (typically) do not depend on Y_S .

But the high level idea can be extended much more widely:

- 1. Assume your initial model was accurate
- 2. Select some perturbation your model should be able to capture
- 3. Use local sensitivity to detect whether the change is what you expect
- 4. If the change is not what you expect, either (1) or (2) was wrong

Notice that there was no discussion of misspecification!

Calibration weights (typically) do not depend on Y_S .

But the high level idea can be extended much more widely:

- 1. Assume your initial model was accurate
- 2. Select some perturbation your model should be able to capture
- 3. Use local sensitivity to detect whether the change is what you expect
- 4. If the change is not what you expect, either (1) or (2) was wrong

Checks of this form give generalized versions of many standard linear model diagnostics.

Regression

Regression

General models

Regression

General models

Consistency / Unbiased

$$\begin{split} y &= \theta^\mathsf{T} \mathbf{x} + \varepsilon \\ \tilde{y} &= (\theta + \delta)^\mathsf{T} \mathbf{x} + \varepsilon \\ \hat{\theta}(\tilde{y}) &\stackrel{\mathsf{check}}{=} \hat{\theta}(y) + \delta \end{split}$$

Regression

General models

Consistency / Unbiased

$$\begin{aligned} y &= \theta^\mathsf{T} \mathbf{x} + \varepsilon \\ \tilde{y} &= (\theta + \delta)^\mathsf{T} \mathbf{x} + \varepsilon \\ \hat{\theta}(\tilde{y}) &\stackrel{\mathsf{check}}{=} \hat{\theta}(y) + \delta \end{aligned}$$

$$y = f(\mathbf{x}, \varepsilon, \theta)$$

 $\tilde{y} = f(\mathbf{x}, \varepsilon, \theta + \delta)$
check $\hat{\rho}(\mathbf{x}) + \delta$

$$\hat{\theta}(\tilde{y}) \stackrel{\mathrm{check}}{=} \hat{\theta}(y) + \delta$$

Regression

General models

Consistency / Unbiased

$$\begin{split} y &= \theta^\intercal \mathbf{x} + \varepsilon \\ \tilde{y} &= (\theta + \delta)^\intercal \mathbf{x} + \varepsilon \\ \hat{\theta}(\tilde{y}) &\stackrel{\text{check}}{=} \hat{\theta}(y) + \delta \end{split}$$

$$\begin{split} y &= f(\mathbf{x}, \varepsilon, \theta) \\ \tilde{y} &= f(\mathbf{x}, \varepsilon, \theta + \delta) \\ \hat{\theta}(\tilde{y}) &\stackrel{\text{check}}{=} \hat{\theta}(y) + \delta \end{split}$$

Exogonous residuals

$$y = \theta^{\mathsf{T}} \mathbf{x} + \varepsilon$$
$$\tilde{y} = y + \varepsilon z$$
$$\hat{\theta}(\tilde{y}) \stackrel{\mathsf{check}}{=} \hat{\theta}(y)$$

Regression

General models

Consistency / Unbiased

$$\begin{split} y &= \theta^\mathsf{T} \mathbf{x} + \varepsilon \\ \tilde{y} &= (\theta + \delta)^\mathsf{T} \mathbf{x} + \varepsilon \\ \hat{\theta}(\tilde{y}) &\stackrel{\mathsf{check}}{=} \hat{\theta}(y) + \delta \end{split}$$

$$\begin{split} y &= f(\mathbf{x}, \varepsilon, \theta) \\ \tilde{y} &= f(\mathbf{x}, \varepsilon, \theta + \delta) \\ \hat{\theta}(\tilde{y}) &\stackrel{\text{check}}{=} \hat{\theta}(y) + \delta \end{split}$$

Exogonous residuals

$$y = \theta^{\mathsf{T}} \mathbf{x} + \varepsilon$$
$$\tilde{y} = y + \varepsilon z$$
$$\hat{\theta}(\tilde{y}) \stackrel{\text{check}}{=} \hat{\theta}(y)$$

$$y \sim \mathcal{P}(y|\mathbf{x})$$
 and $\mathcal{P}(\mathbf{x}) = w$ $\tilde{w} = w + \delta z$ $\hat{\theta}(\tilde{w}) \stackrel{\text{check}}{=} \hat{\theta}(w)$

Regression

General models

Consistency / Unbiased

$$\begin{aligned} y &= \theta^\mathsf{T} \mathbf{x} + \varepsilon \\ \tilde{y} &= (\theta + \delta)^\mathsf{T} \mathbf{x} + \varepsilon \\ \hat{\theta}(\tilde{y}) &\stackrel{\mathsf{check}}{=} \hat{\theta}(y) + \delta \end{aligned}$$

$$\begin{split} y &= f(\mathbf{x}, \varepsilon, \theta) \\ \tilde{y} &= f(\mathbf{x}, \varepsilon, \theta + \delta) \\ \hat{\theta}(\tilde{y}) &\stackrel{\text{check}}{=} \hat{\theta}(y) + \delta \end{split}$$

Exogonous residuals

$$y = \theta^{\mathsf{T}} \mathbf{x} + \varepsilon$$
$$\tilde{y} = y + \varepsilon z$$
$$\hat{\theta}(\tilde{y}) \stackrel{\mathsf{check}}{=} \hat{\theta}(y)$$

$$y \sim \mathcal{P}(y|\mathbf{x}) \text{ and } \mathcal{P}(\mathbf{x}) = w$$

$$\tilde{w} = w + \delta z$$

$$\hat{\theta}(\tilde{w}) \overset{\text{check}}{=} \hat{\theta}(w)$$

Fisher information

$$\mathcal{I}:=$$
 Fisher information $\Sigma:=$ Score covariance

$$\mathcal{I}^{-1} \overset{\mathrm{check}}{=} \Sigma$$

Exogonous

Consistency / Unbiased	$y = \theta^\intercal \mathbf{x} + \varepsilon$
	$\tilde{y} = (\theta + \delta)^T \mathbf{x} + \varepsilon$
	$\hat{\theta}(\tilde{y}) \stackrel{\mathrm{check}}{=} \hat{\theta}(y) + \delta$

Regression

 $y = \theta^{\mathsf{T}} \mathbf{x} + \varepsilon$

General models

$$\begin{split} y &= f(\mathbf{x}, \varepsilon, \theta) \\ \tilde{y} &= f(\mathbf{x}, \varepsilon, \theta + \delta) \\ \hat{\theta}(\tilde{y}) &\stackrel{\text{check}}{=} \hat{\theta}(y) + \delta \end{split}$$

 $y \sim \mathcal{P}(y|\mathbf{x})$ and $\mathcal{P}(\mathbf{x}) = w$

residuals
$$\tilde{y} = y + \varepsilon z$$

$$\hat{\theta}(\tilde{y}) \stackrel{\mathrm{check}}{=} \hat{\theta}(y)$$

$$\tilde{w} = w + \delta z$$

$$\hat{\theta}(\tilde{w}) \stackrel{\text{check}}{=} \hat{\theta}(w)$$

Fisher
$$\mathcal{I}:= \text{Fisher information}$$
 information
$$\Sigma:= \text{Score covariance}$$

$$\mathcal{I}^{-1} \overset{\text{check}}{=} \Sigma$$

$$y \sim \mathcal{P}(y|\theta)$$
 $ilde{y} \sim ext{Importance sample } y$ using $ilde{w} = rac{\mathcal{P}(y|\hat{\theta}+\delta)}{\mathcal{P}(y|\hat{\theta})}$ $\hat{\theta}(ilde{w}) \stackrel{\text{check}}{=} \hat{\theta}(1) + \delta$

Fisher

information

	11091001011	General models
Consistency / Unbiased	$\begin{split} y &= \theta^T \mathbf{x} + \varepsilon \\ \tilde{y} &= (\theta + \delta)^T \mathbf{x} + \varepsilon \\ \hat{\theta}(\tilde{y}) &\stackrel{check}{=} \hat{\theta}(y) + \delta \end{split}$	$\begin{split} y &= f(\mathbf{x}, \varepsilon, \theta) \\ \tilde{y} &= f(\mathbf{x}, \varepsilon, \theta + \delta) \\ \hat{\theta}(\tilde{y}) &\stackrel{check}{=} \hat{\theta}(y) + \delta \end{split}$
Exogonous residuals	$y = \theta^{T} \mathbf{x} + \varepsilon$ $\tilde{y} = y + \varepsilon z$	$y \sim \mathcal{P}(y \mathbf{x})$ and $\mathcal{P}(\mathbf{x}) = w$ $\tilde{w} = w + \delta z$

Regression

General models

$$\begin{aligned} y &= f(\mathbf{x}, \varepsilon, \theta) \\ \tilde{y} &= f(\mathbf{x}, \varepsilon, \theta + \delta) \\ (\tilde{y}) &\stackrel{\text{check}}{=} \hat{\theta}(y) + \delta \end{aligned}$$

$$\mathcal{I}:=$$
 Fisher information

$$y \sim \mathcal{P}(y| heta)$$
 $ilde{y} \sim ext{Importance sample } y$ using $ilde{w} = rac{\mathcal{P}(y|\hat{ heta} + \delta)}{\mathcal{P}(y|\hat{ heta})}$

 $\hat{\theta}(\tilde{w}) \stackrel{\text{check}}{=} \hat{\theta}(w)$

 $\hat{\theta}(\tilde{w}) \stackrel{\text{check}}{=} \hat{\theta}(1) + \delta$

$$\Sigma :=$$
 Score covariance $\mathcal{I}^{-1} \overset{\mathrm{check}}{=} \Sigma$

 $\hat{\theta}(\tilde{y}) \stackrel{\text{check}}{=} \hat{\theta}(y)$

Student contributions and ongoing work:

- · Vladimir Palmin is working on extending MrPlew to lme4
- Sequoia Andrade is working on generalizing to other local sensitivity checks
- · Lucas Schwengber is working on novel flow-based techniques for local sensitivity
- (Currently recruiting!) Doubly–robust Bayesian Hierarchical MrP



Vladimir Palmin



Seguoia Andrade



Lucas Schwengber

Preprint and R package (hopefully) coming soon!

References i