

# Locally Equivalent Weights for Bayesian MrP

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**UT Austin Statistics Seminar**

**September 2025**



# Are US non-voters becoming more Republican?

## **Blue Rose research says yes:**

“Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate.”

(Blue Rose Research 2024)  
(major professional pollsters)

## ***On Data and Democracy says no:***

“Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available.”

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  - Different data sources
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## Our contribution

We define “MrP local equivalent weights” (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key diagnostics that motivate calibration weighting.

⇒ **MrPlew provides direct comparisons between MrP and calibration weighting.**

- Introduce the statistical problem
  - Contrast CW and MrP
  - Prior work: Equivalent weights for linear models
  - Interlude: Approximate equivalent weights for some non-linear models
  - Our key idea: Locally equivalent weights for non-linear models

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- Locally equivalent weights for covariate balance
  - Describe covariate balance
  - Define MrPlew weights and connect them to covariate balance
  - Theoretical support
  - Example of real-world results

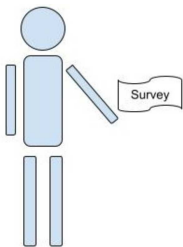
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- Other uses of locally equivalent weights
  - Partial pooling
  - The meaning of negative weights
  - Frequentist variance estimation
- Future directions

# The basic problem

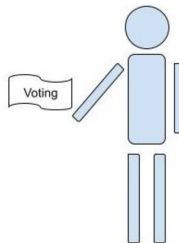
We have a survey population, for whom we observe:

- Covariates  $\mathbf{x}$  (e.g. race, gender, zip code, age, education level)
- Responses  $y$  (e.g. A binary response to “do you support Trump”)

We want the average response in a target population, in which we observe only covariates.



Observe  $(\mathbf{x}_i, y_i)$  for  $i = 1, \dots, N_S$



Observe  $\mathbf{x}_j$  for  $j = 1, \dots, N_T$

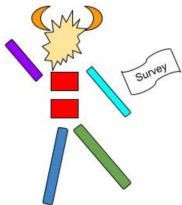


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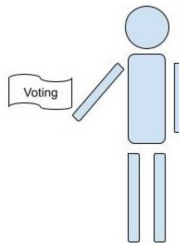
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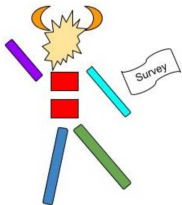
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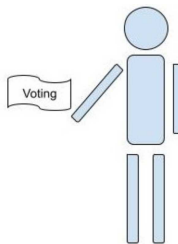
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**The problem is that the populations may be very different.**

Our survey results may be biased.

How can we use the covariates to say something about the target responses?

## Two approaches

We want  $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$ , but don't observe target population  $y_j$ . Let  $Y_S = \{y_1, \dots, y_{N_S}\}$ .

- Assume  $p(y|\mathbf{x})$  is the same in both populations,
- But the distribution of  $\mathbf{x}$  may be different in the survey and target.

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### Calibration weighting (CW)

- Choose “calibration weights”  $w_i$   
using only the regressors  $\mathbf{x}$   
(e.g. raking weights)

### Bayesian hierarchical modeling (MrP)

- Choose  $\mathbb{E}[y|\mathbf{x}, \theta] = m(\theta^\top \mathbf{x})$ ,  
choose prior  $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$   
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- ▶ Weights give interpretable diagnostics:
  - Frequentist variability
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  - ← We open this box, providing analogues of all these diagnostics



## Prior work: Equivalent weights for linear models

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form  $\hat{y}$ :

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But what if you use a non-linear link function? Or a hierarchical model?

*“It would also be desirable to use nonlinear methods ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods.” — (Gelman 2007a)*

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## Equivalent weights for (some) logistic regression MrP

- Suppose the model is  $m(\mathbf{x}^\top \theta) = \text{Logistic}(\mathbf{x}^\top \theta)$ , with MLE  $\hat{\theta}$ .
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The map from  $Y_S \mapsto m(\mathbf{x}_j^\top \hat{\theta})$  is *inherently nonlinear*.

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$$\text{For } w_i^{\text{MrP}} = \frac{N_T^c / N_T}{N_S^c / N_S} \text{ when } \mathbf{x}_i = c.$$



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$$\begin{aligned}\hat{\mu}_{\text{MrP}} &= \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\top \hat{\theta}) \\ &\approx \int m(\mathbf{x}^\top \hat{\theta}) \mathcal{P}_T(\mathbf{x}) d\mathbf{x} && \text{(Law of large numbers)} \\ &= \int \frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} m(\mathbf{x}^\top \hat{\theta}) \mathcal{P}_S(\mathbf{x}) d\mathbf{x} && \text{(Multiply by } \mathcal{P}_S(\mathbf{x})/\mathcal{P}_S(\mathbf{x}) \text{)} \\ &\approx \int (\alpha^\top \mathbf{x}) m(\mathbf{x}^\top \hat{\theta}) \mathcal{P}_S(\mathbf{x}) d\mathbf{x} && \text{(By assumption)} \\ &\approx \alpha^\top \frac{1}{N_S} \sum_{i=1}^{N_S} \mathbf{x}_i m(\mathbf{x}_i^\top \hat{\theta}) && \text{(Law of large numbers)}\end{aligned}$$

## Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is  $m(\mathbf{x}^\top \theta) = \text{Logistic}(\mathbf{x}^\top \theta)$ , with MLE  $\hat{\theta}$ .
- MrP is  $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\top \hat{\theta})$ .

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## Nearly equivalent weights for (some) logistic regression MrP

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$$\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\top \hat{\theta}) = \frac{1}{N_S} \sum_{i=1}^{N_S} \underbrace{w_i^{\text{MrP}}}_{\alpha^\top \mathbf{x}_i} y_i + \text{Small error}$$

But what are the weights? We don't observe  $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$ , so can't estimate  $\alpha$  directly.

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<sup>2</sup>Krantz and Parks 2012; G., Stephenson, et al. 2019.



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### Key idea (informal)

If  $\hat{\mu}_{\text{MrP}}$  is approximately linear, then  $w_i^{\text{MrP}} \approx \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$ .

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For logistic regression, could compute and analyze  $\frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$  using the implicit function theorem.<sup>2</sup>

---

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## Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is  $m(\mathbf{x}^\top \theta) = \text{Logistic}(\mathbf{x}^\top \theta)$ .
- Set a hierarchical prior  $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$ , use MCMC to draw from  $\mathcal{P}(\theta|\text{Survey data})$ .
- MrP is  $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})} \left[ m(\mathbf{x}_j^\top \theta) \right]$ .

No reason to think  $Y_{\mathcal{S}} \mapsto \hat{\mu}_{\text{MrP}}(Y_{\mathcal{S}})$  is even approximately linear.

But we can still compute and analyze  $w_i^{\text{MrP}} := \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$  using Bayesian sensitivity analysis!<sup>3</sup>

---

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For new data  $\tilde{Y}_S$ , form a series expansion

$$\hat{\mu}_{\text{MrP}}(\tilde{Y}_S) \approx \hat{\mu}_{\text{MrP}}(Y_S) + \sum_{i=1}^{N_S} w_i^{\text{MrP}} (\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MrP}} := \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}.$$

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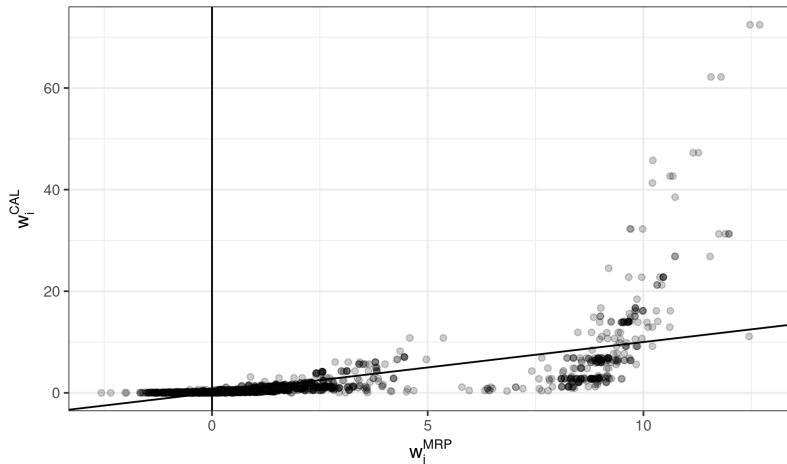
**Our task is to rigorously show that even such local weights can be used diagnostically.**

---

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# The weights can look very different!

Does this mean anything? Are the differences important?



**Figure 1:** Comparison between raking and MrPlew weights for the Name Change dataset

## What are we weighting for?<sup>4</sup>

$$\text{Target average response} = \frac{1}{N_T} \sum_{j=1}^{N_T} y_j \approx \frac{1}{N_S} \sum_{i=1}^{N_S} w_i y_i = \text{Weighted survey average response}$$

We can't check this, because we don't observe  $y_j$ .

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$$\frac{1}{N_T} \sum_{j=1}^{N_T} \mathbf{x}_j = \frac{1}{N_S} \sum_{i=1}^{N_S} w_i \mathbf{x}_i$$

Such weights satisfy “covariate balance” for  $\mathbf{x}$ .

You can check covariate balance for any calibration weighting estimator, and any function  $f(\mathbf{x})$ .

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You can check covariate balance for any calibration weighting estimator, and any function  $f(\mathbf{x})$ .

Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

### Raking calibration weights

“Raking” selects weights that

- Are as “close as possible” to some reference weights
- Under the constraint that they balance some selected regressors.

---

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## Balance checks as sensitivity analysis

One reason to balance  $f(\mathbf{x})$  is because we think  $\mathbb{E}[y|\mathbf{x}]$  might plausibly vary  $\propto f(\mathbf{x})$ , and want to check whether our estimator can capture this variability.

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### Balance-informed sensitivity check (BISC) (informal)

Pick a small  $\delta > 0$  and an  $f(\cdot)$ . Define a *new response variable*  $\tilde{y}$  such that

$$\mathbb{E}[\tilde{y}|\mathbf{x}] = \mathbb{E}[y|\mathbf{x}] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}[\mu(\tilde{y}) - \mu(y)|\mathbf{x}] = \frac{1}{N_T} \sum_{j=1}^{N_T} (\mathbb{E}[\tilde{y}|\mathbf{x}_j] - \mathbb{E}[y|\mathbf{x}_j]) = \delta \frac{1}{N_T} \sum_{j=1}^{N_T} f(\mathbf{x}_j)$$

Then, check whether your estimator  $\hat{\mu}(\cdot)$  produces the same change for observed  $\tilde{y}, y$ :

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\text{Replace weighted averages with changes in an estimator}} \stackrel{\text{check}}{\approx} \delta \frac{1}{N_T} \sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

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When  $\hat{\mu}(\cdot) = \hat{\mu}_{\text{CW}}(\cdot)$ , BISC recovers the standard covariate balance check.

We will use  $\hat{\mu}(\cdot) = \hat{\mu}_{\text{MRP}}(\cdot)$ .

Suppose I have  $\tilde{y}$  such that  $\mathbb{E} [\tilde{y}|\mathbf{x}] = \mathbb{E} [y|\mathbf{x}] + \delta f(\mathbf{x})$ .

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**Problem:**  $\hat{\mu}_{\text{MrP}}(\cdot)$  is computed with MCMC.

- Each MCMC run typically takes hours, and
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## MrP Local Equivalent Weights (MrPlew)

Form the first-order Taylor series approximation

$$\hat{\mu}_{\text{MrP}}(\tilde{y}) - \hat{\mu}_{\text{MrP}}(y) \approx \sum_{i=1}^{N_S} w_i^{\text{MrP}} (\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MrP}} := \frac{d}{dy_i} \hat{\mu}_{\text{MrP}}(y).$$



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**Computation:** The weights are given by weighted averages of posterior covariances<sup>5</sup>.

They can be easily computed with standard software<sup>6</sup> **without re-running MCMC**.

---

<sup>5</sup>G., Broderick, and Jordan 2018.

<sup>6</sup>We use `brms` (Bürkner 2017).

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**Use in BISC:** For a wide set of judiciously chosen  $f(\cdot)$ , check

$$\delta \sum_{i=1}^{N_S} w_i^{\text{MrP}} f(\mathbf{x}_i) \stackrel{\text{check}}{\approx} \delta \frac{1}{N_T} \sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

- We have defined BISC in terms of  $\tilde{y}$  such that  $\mathbb{E} [\tilde{y}|\mathbf{x}] = \mathbb{E} [y|\mathbf{x}] + \delta f(\mathbf{x})$
- We have approximated  $\hat{\mu}_{\text{MrP}}(\tilde{y}) - \hat{\mu}_{\text{MrP}}(y)$  for  $\tilde{y} \approx y$

How to get such a  $\tilde{y}$ ? **Recall  $y$  is binary!** Two approaches:

**Option 1:** Force  $\tilde{y}$  to be binary.

1. Make *some* guess  $\hat{m}(\mathbf{x}) \approx \mathbb{E} [y|\mathbf{x}]$ 
  - E.g. Posterior mean, or
  - Shrunk posterior mean, or
  - Some values that gives the same posterior
2. Take  $u_n \stackrel{iid}{\sim} \text{Unif}(0, 1)$
3. Assume  $y_n = \mathbb{I}(u_n \leq \hat{m}(\mathbf{x}_n))$
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**Option 2:** Allow  $\tilde{y}$  to take generic values.

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# Generating $\tilde{y}$

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**Pros and cons:**

- Realistic
- Have to pick  $\hat{m}(\mathbf{x})$
- $\tilde{Y} - Y_{\mathcal{S}}$  not infinitesimally small
- **Sanity check for theory**

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3. There is nothing else to do.

**Pros and cons:**

- Not realistic
- No additional assumptions
- $\tilde{Y} - Y_{\mathcal{S}}$  may be infinitesimally small
- **Use for theory**

## BISC Theorem: (sketch)

Take  $\tilde{y}_n = y_n + \delta f(\mathbf{x}_n)$ .

We state conditions for Bayesian hierarchical logistic regression under which

$$\left| \hat{\mu}_{\text{MrP}}(Y_S) - \hat{\mu}_{\text{MrP}}(Y_S) - \delta \sum_{i=1}^{N_S} w_i^{\text{MrP}} f(\mathbf{x}_i) \right| = \text{Small?}$$

---

<sup>5</sup>Measurable functions with uniformly bounded  $\mathbb{E} [f(\mathbf{x})]$ .

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$$\left| \hat{\mu}_{\text{MrP}}(Y_S) - \hat{\mu}_{\text{MrP}}(Y_S) - \delta \sum_{i=1}^{N_S} w_i^{\text{MrP}} f(\mathbf{x}_i) \right| = O(\delta^2) \text{ as } N \rightarrow \infty$$

---

<sup>5</sup>Measurable functions with uniformly bounded  $\mathbb{E} [ \mathbf{x} f(\mathbf{x}) ]$ .

<sup>6</sup>G. and Broderick 2024; Kasprzak, G., and Broderick 2025.



## BISC Theorem: (sketch)

Take  $\tilde{y}_n = y_n + \delta f(\mathbf{x}_n)$ .

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For a very broad class<sup>5</sup> of  $\mathcal{F}$ .

**Uniformity justifies searching for “imbalanced”  $f$ .**

---

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## Uniformity justifies searching for “imbalanced” $f$ .

The uniformity result builds on our earlier work on uniform and finite-sample error bounds for Bernstein–von Mises theorem–like results<sup>6</sup>.

---

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# Real Data: Marital Name Change Survey

Analysis of changing names after marriage<sup>7</sup>.

- **Target population:** ACS survey of US population 2017–2022<sup>8</sup>
- **Survey population:** Marital Name Change Survey (from Twitter)<sup>9</sup>
- **Respose:** Did the female partner keep their name after marriage?
- For regressors, use bins of age, education, state, and decade married.

Survey observations:  $N_S = 4,364$

Target observations (rows):  $N_T = 4,085,282$

Uncorrected survey mean:  $\frac{1}{N_S} \sum_{i=1}^{N_S} y_i = 0.462$

Raking:  $\hat{\mu}_{CW} = 0.263$

MrP:  $\hat{\mu}_{MrP} = 0.288$  (Post. sd = 0.0169)

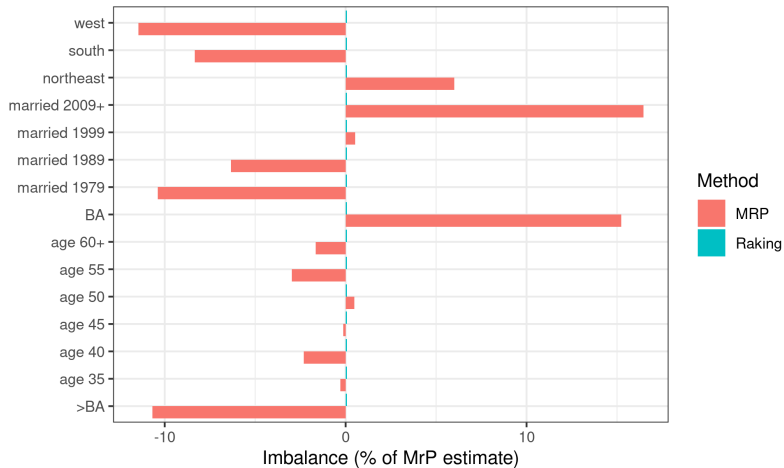
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<sup>7</sup>Based on Alexander (2019).

<sup>8</sup>Ruggles et al. 2024.

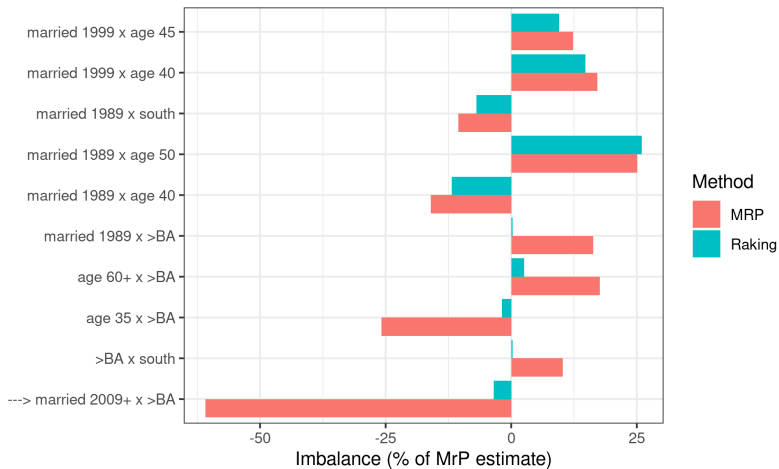
<sup>9</sup>Cohen 2019.

## Covariate balance for primary effects

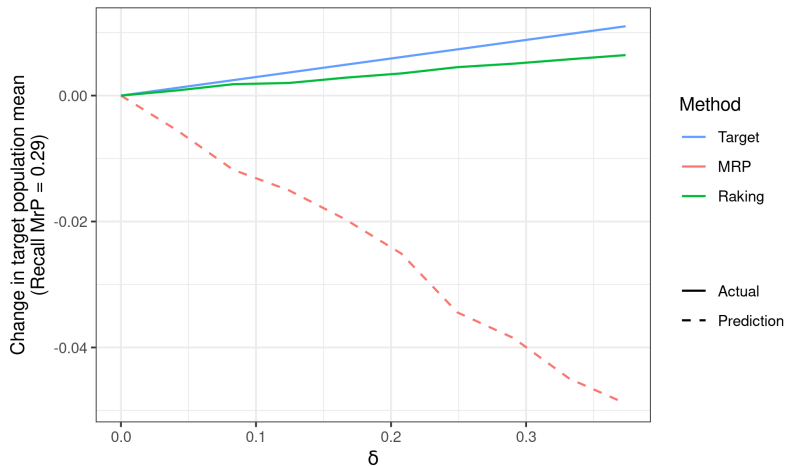


**Figure 2:** Imbalance plot for primary effects in the Name Change dataset

## Covariate balance for interaction effects

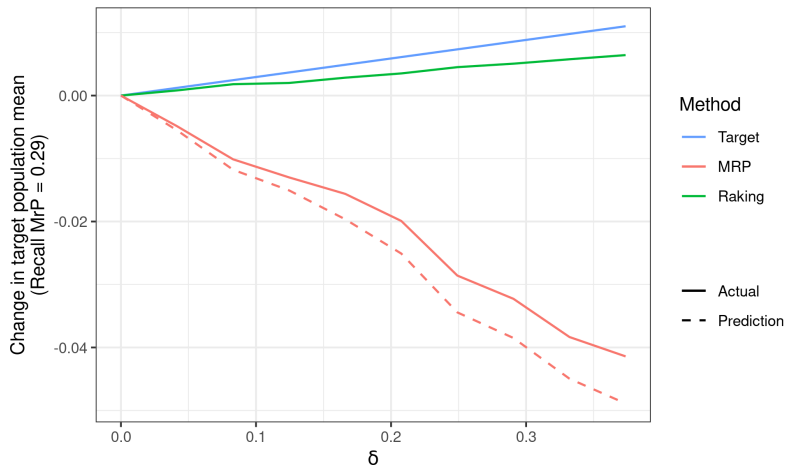


**Figure 3:** Imbalance plot for select interaction effects in the Name Change dataset



**Figure 4:** Predictions on binary data for the Name Change dataset

## Predictions and actual MCMC results



**Figure 5:** Predictions and refit on binary data for the Name Change dataset

Running ten MCMC refits: 10 hours    Computing approximate weights: 16 seconds

Analysis of national support for gay marriage.<sup>10</sup>

- **Target population:** US Census Public Use Microdata Sample 2000
- **Survey population:** Combined national-level polls from 2004
- **Response:** “Do you favor allowing gay and lesbian couples to marry legally?”
- For regressors, use race, gender, age, education, state, region, and continuous statewide religion and political characteristics, including some analyst–selected interactions.

Survey observations:  $N_S = 6,341$

Target observations (rows):  $N_T = 9,694,541$

Uncorrected survey mean:  $\frac{1}{N_S} \sum_{i=1}^{N_S} y_i = 0.333$

Raking:  $\hat{\mu}_{\text{CW}} = 0.33$

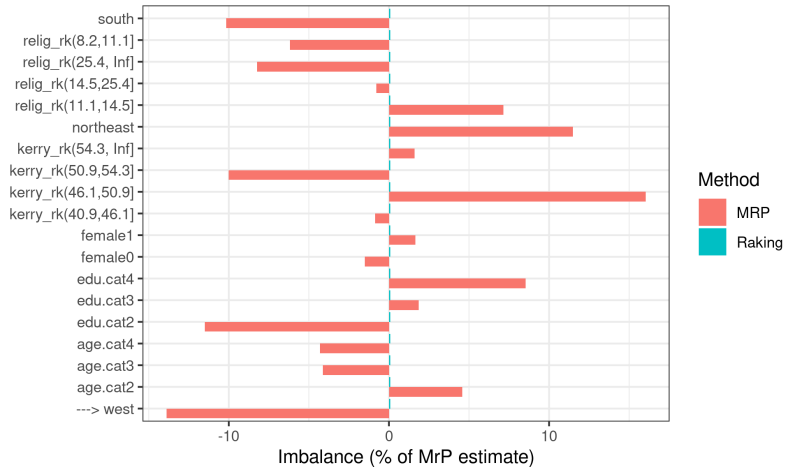
MrP:  $\hat{\mu}_{\text{MrP}} = 0.337$  (Post. sd = 0.039)

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<sup>10</sup>Based on Kastellec, Lax, and Phillips (2010), see also Lax and Phillips (2009).

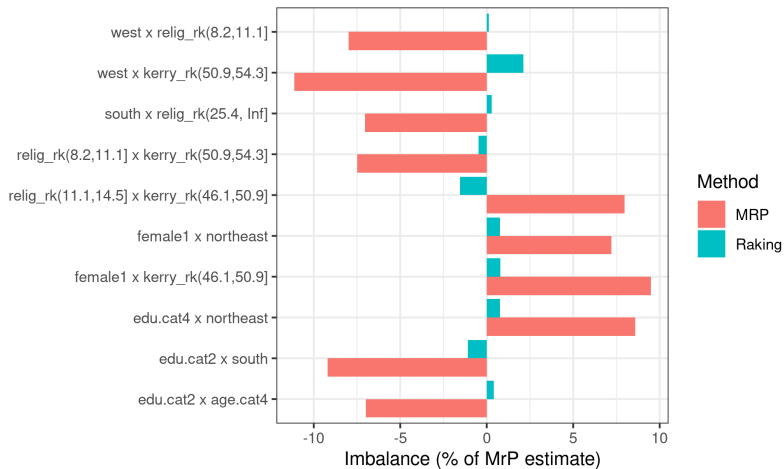


## Covariate balance for primary effects

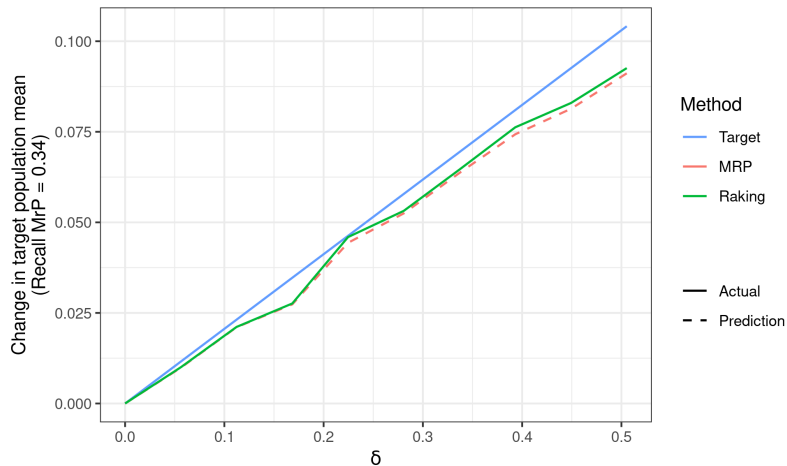


**Figure 6:** Imbalance plot for primary effects in the Gay Marriage dataset

## Covariate balance for interaction effects

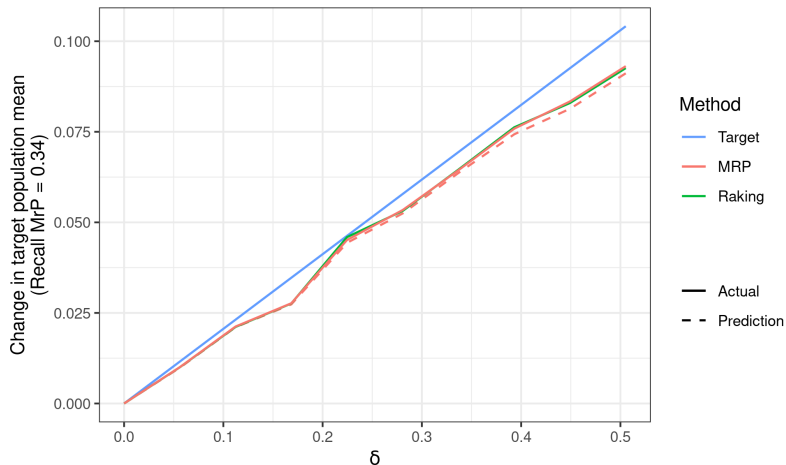


**Figure 7:** Imbalance plot for select interaction effects in the Gay Marriage dataset



**Figure 8:** Predictions on binary data for the Gay Marriage dataset

## Predictions and actual MCMC results



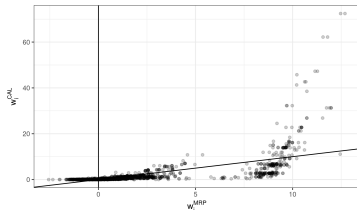
**Figure 9:** Predictions and refit on binary data for the Gay Marriage dataset

Running ten MCMC refits: 11 hours    Computing approximate weights: 23 seconds

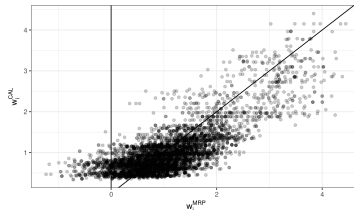
Does this mean anything?

**Yes:** We can meaningfully sum these weights against regressors.

What else might it mean?



**Figure 10:** Comparison between raking and MrPlew weights for the Name Change dataset



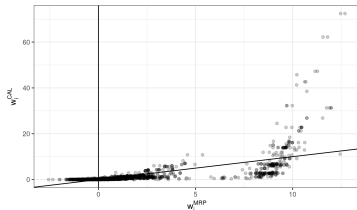
**Figure 11:** Comparison between raking and MrPlew weights for the Gay Marriage dataset

Does this mean anything?

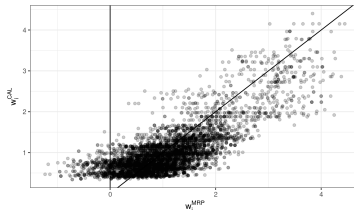
**Yes:** We can meaningfully sum these weights against regressors.

What else might it mean?

**Does the spread relate to frequentist variance?**



**Figure 10:** Comparison between raking and MrPlew weights for the Name Change dataset



**Figure 11:** Comparison between raking and MrPlew weights for the Gay Marriage dataset

## Standard error consistency theorem: (sketch)

For Bayesian hierarchical logistic regression, define

$$\varepsilon_n = y_n - \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})} [m(\mathbf{x}_n^\top \theta)] \quad \text{and} \quad \psi_n := N_S w_n^{\text{MrP}} \varepsilon_n.$$

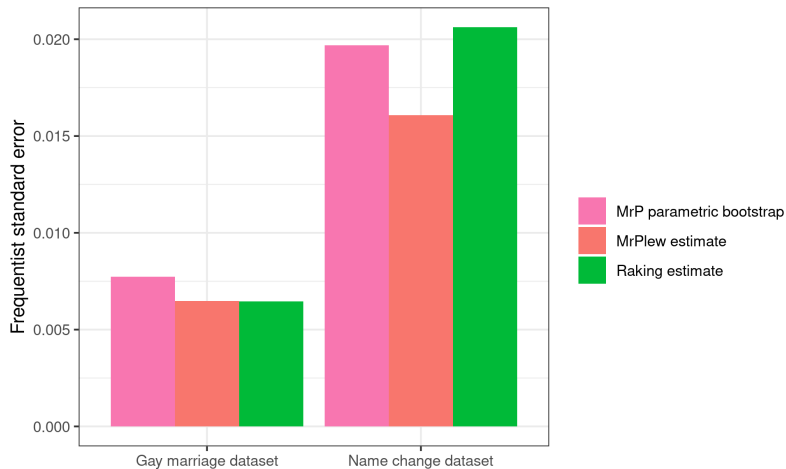
We state mild conditions under which, as  $N \rightarrow \infty$ ,

$$\begin{aligned} \sqrt{N} (\hat{\mu}_{\text{MrP}} - \mu_\infty) &\rightarrow \mathcal{N}(0, V) \quad \text{for some } \mu_\infty \text{ and variance } V, \text{ and} \\ \frac{1}{N_S} \sum_{i=1}^{N_S} (\psi_n - \bar{\psi})^2 &\rightarrow V. \end{aligned}$$

The use of  $w_n^{\text{MrP}}$  is exactly analogous to the use of raking weights for standard error estimation. This builds on our earlier work on the Bayesian infinitesimal jackknife<sup>11</sup>.

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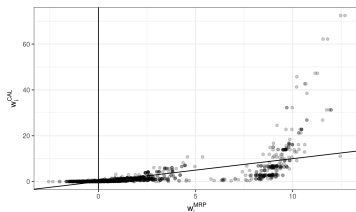
<sup>11</sup>G. and Broderick 2024.



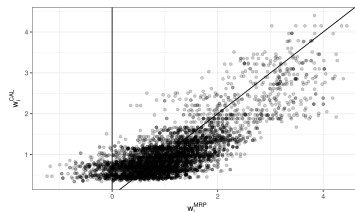
**Figure 12:** Frequentist standard deviation estimates



**Covariate balance** corresponds by BISC.  
**Weight spread** measures frequentist standard errors.



**Figure 13:** Comparison between raking and MrPlew weights for the Name Change dataset



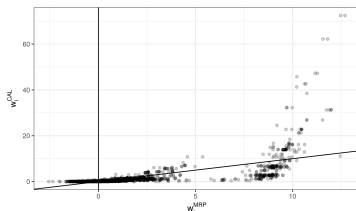
**Figure 14:** Comparison between raking and MrPlew weights for the Gay Marriage dataset

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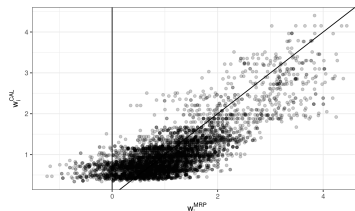
**Weight spread** measures frequentist standard errors.

**Partial pooling** is BISC with different targets (e.g. sub-populations).

**Negative weights** indicate *non-monotonicity* of  $Y_S \mapsto \hat{\mu}_{\text{MrP}}(Y_S)$ .



**Figure 13:** Comparison between raking and MrPlew weights for the Name Change dataset



**Figure 14:** Comparison between raking and MrPlew weights for the Gay Marriage dataset

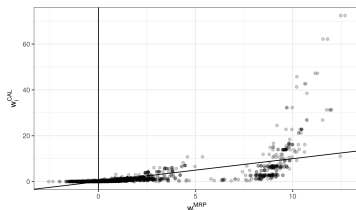
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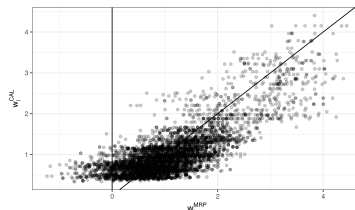
**Partial pooling** is BISC with different targets (e.g. sub-populations).

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### Other checks?



**Figure 13:** Comparison between raking and MrPlew weights for the Name Change dataset



**Figure 14:** Comparison between raking and MrPlew weights for the Gay Marriage dataset

## Future work

Notice that there was no discussion of misspecification!

*Calibration weights (typically) do not depend on  $Y_S$ .*

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But the high level idea can be extended much more widely:

1. Assume your initial model was accurate
2. Select some perturbation your model should be able to capture
3. Use local sensitivity to detect whether the change is what you expect
4. If the change is not what you expect, either (1) or (2) was wrong

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Such checks recover generalized versions of many standard diagnostics for linear models.

Examples:

- Additive parameter shifts  $\leftrightarrow$  Unbiasedness
- Invariance to survey data weighting  $\leftrightarrow$  Regressor + residual orthogonality
- Importance sampling  $\leftrightarrow$  Sandwich covariance  $\stackrel{?}{=}$  Inverse Fisher information

Student contributions and ongoing work:

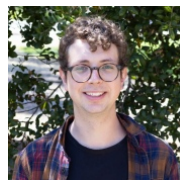
- **Vladimir Palmin** is working on extending MrPlew to `lme4`
- **Sequoia Andrade** is working on generalizing to other local sensitivity checks
- **Lucas Schwengber** is working on novel flow-based techniques for local sensitivity
- **(Currently recruiting!)** Doubly-robust Bayesian Hierarchical MrP



Vladimir Palmin



Sequoia Andrade



Lucas Schwengber

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**Preprint and R package (hopefully) coming soon!**



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