Variational Methods for Latent Variable Problems

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Outline

Outline for today:

- Some examples of latent variable models
- A template: The Neyman-Scott "paradox" and marginalization
- Bayesian versus frequentist approaches to marginalization
- The classical EM algorithm (in brief)

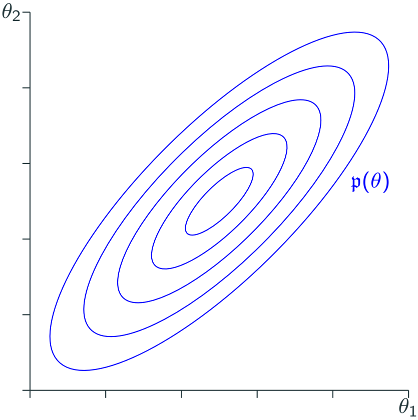
Next week, we will build on these ideas to present more general variational inference.

KL divergence exercises

$$\mathfrak{p}(\theta) = \text{Correlated bivariate normal}$$

$$\mathcal{Q} = \{\text{All bivariate normals}\}$$

What is $\mathfrak{q}^*(\theta) = \underset{\mathfrak{q} \in \mathcal{Q}}{\operatorname{argmin}} \, \mathrm{KL}\left(\mathfrak{q}(\theta) || \mathfrak{p}(\theta)\right)$?



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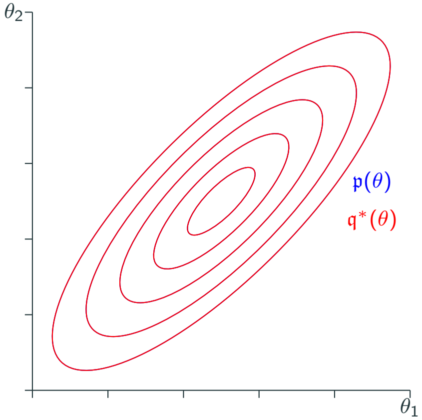

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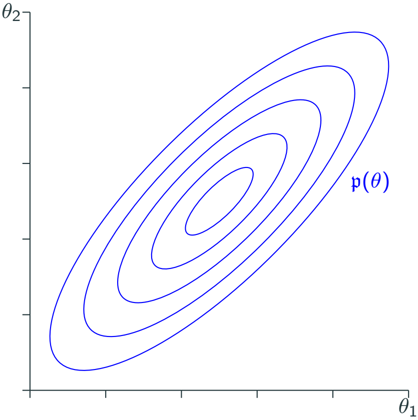

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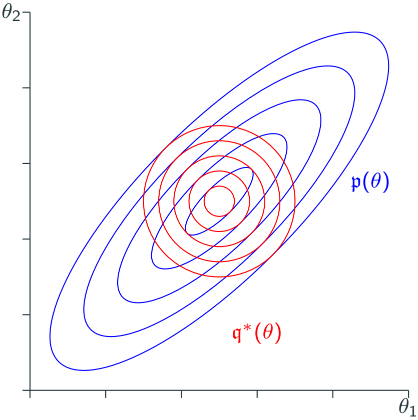

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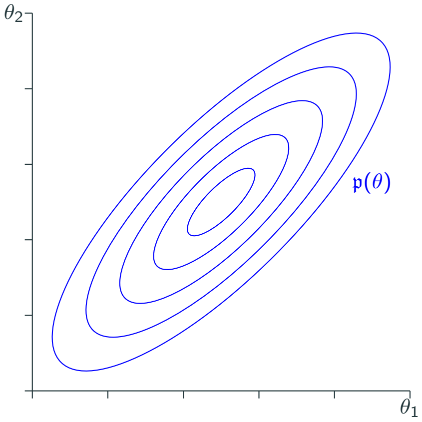

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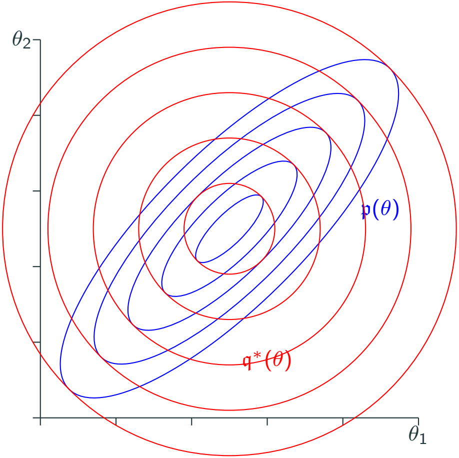

KL divergence exercises

Recall that

$$\mathrm{KL}\left(\mathfrak{q}(\theta)||\mathfrak{p}(\theta)\right) =$$
$$- \underset{\mathfrak{q}(\theta)}{\mathbb{E}}\left[\log \mathfrak{p}(\theta)\right] + \underset{\mathfrak{q}(\theta)}{\mathbb{E}}\left[\log \mathfrak{q}(\theta)\right]$$

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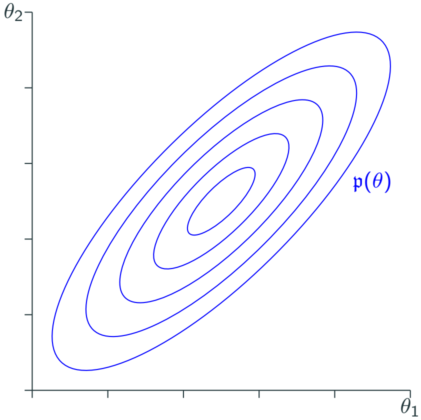

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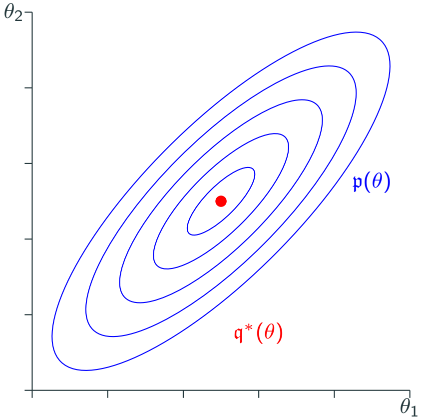

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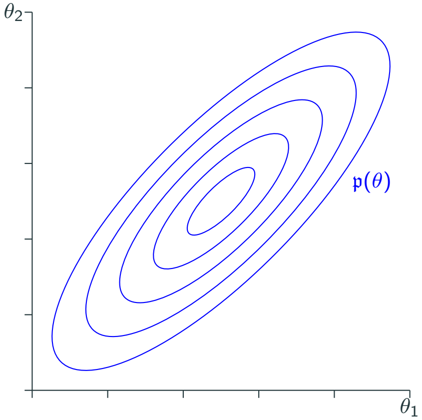

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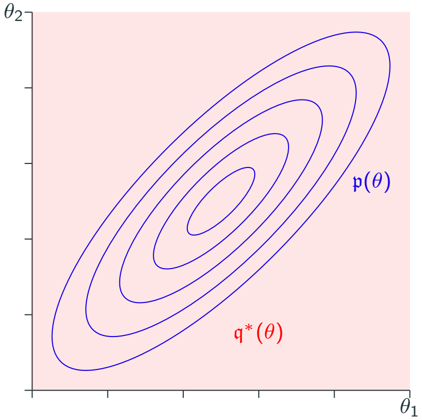

Conclusions