# An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Make a Big Difference?

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January 2022

[1] With coauthors Rachael Meager (LSE) and Tamara Broderick (MIT)

# Dropping data: Motivation

More data & cheaper computation $\Rightarrow$
Statistical analyses are playing larger roles in decision making.

Decisions are important: We want **trustworthy** conclusions.
Data / models not always perfect: We want **robust** conclusions.

Would you be concerned if you could **reverse your conclusion** by removing a **small proportion** (say, 0.1%) of your data?

**Running example:** Angelucci et al. [2015], a randomized controlled trial study of the efficacy of microcredit based on 16,560 data points.
We can reverse the studies qualitative conclusions by removing 15 observations ($< 0.1\%$ of the data).

**How do we find sets of influential points?** Difficult in general!

We provide a **automatic approximation** with finite-sample guarantees.

Studying the approximation reveals the causes of non-robustness.

# Dropping data: Mexico Microcredit

Consider Angelucci et al. [2015], a randomized controlled trial study of the efficacy of microcredit in Mexico based on 16,560 data points. The variable "Beta" estimates the effect of microcredit in US dollars.

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**The culprit is signal to noise ratio.**
By the end of the talk, we will see that the sensitivity is due to

- High variability of the outcome (hosehold profit) relative to
- A small signal driving the conclusion (statistical significance)

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Would you be concerned if you could **reverse your conclusion** by removing a **small proportion** (say, 0.1%) of your data?

Not always! But sometimes, surely yes.

Thinking without random noise can be helpful.

Suppose you have a farm, and want to know whether your average yield is greater than 170 bushels per acre. At harvest, you measure 200 bushels per acre.

- Scenario one: If your yield is greater than 170 bushels per acre, you make a profit.
  - Don't care about sensitivity to small subsets
- Scenario two: You want to recommend your farming methods to a friend across the valley.
  - Might care about sensitivity to small subsets

For example, often in economics:

- Small fractions of data are missing not-at-random,
- Policy population is different from analyzed population,
- We report a convenient summary (e.g. mean) of a complex effect,
- Models are stylized proxies of reality.

**Question 1:**
**How do we find influential datapoints?**

# Which estimators do we study?

**Z-estimators.** Suppose we have $N$ data points $\vec{d} = d_1, \ldots, d_N$. Then:

$$\hat{\theta} := \vec{\theta} \text{ such that } \sum_{n=1}^{N} G(\vec{\theta}, d_n) = 0_P.$$

**Examples:** MLE, OLS, VB, &c (all minimizers of smooth empirical loss).

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**Function of interest.** Qualitative decision based on $\phi(\hat{\theta}) \in \mathbb{R}$. E.g.:

- A particular component: $\phi(\theta) = \theta_d$
- The end of a confidence interval: $\phi(\theta) = \theta_d + \frac{1.96}{\sqrt{N}} \hat{\sigma}(\hat{\theta})$

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Fix a proportion $0 < \alpha \ll 1$ of points to drop and find a set $\mathcal{S} \subset \{1, \ldots N\}$ with $|\mathcal{S}| \leq \lfloor \alpha N \rfloor$ that extremizes $\phi(\hat{\theta})$ when dropped.

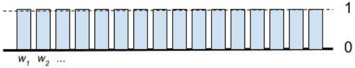
- **Problem:** There are many sets with $|\mathcal{S}| \leq \lfloor \alpha N \rfloor$.
  - E.g., in Angelucci et al. [2015], $\binom{16{,}560}{15} \approx 1.5 \cdot 10^{51}$
- **Problem:** Evaluating $\phi(\hat{\theta}(\vec{d}_{-\mathcal{S}}))$ requires an estimation problem.
  - E.g., in Angelucci et al. [2015] computing the OLS estimator.
  - Other examples are even harder (VB, machine learning)

**An approximation is needed!**

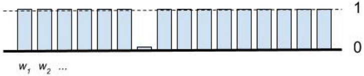
# Which estimators do we study?

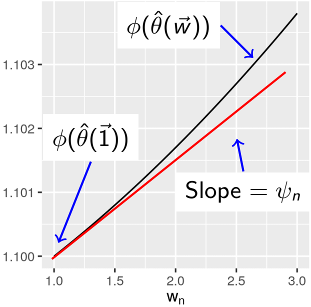
$$\hat{\theta} := \vec{\theta} \ \text{ such that } \ \sum_{n=1}^{N} G(\vec{\theta}, d_n) = 0_P.$$

Original weights: $\vec{1} = (1, \ldots, 1)$




Leave points out by setting their elements of $\vec{w}$ to zero.




The slopes $\psi_n := \left. \frac{\partial \phi(\hat{\theta}(\vec{w}))}{\partial \vec{w}_n} \right|_{\vec{1}}$ are values of the **empirical influence function** [Hampel, 1986]. We call them "influence scores."

Second-order derivatives control the error of the linear approximation.

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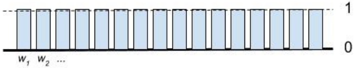


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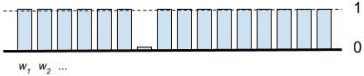


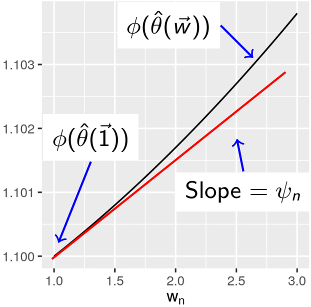


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Second-order derivatives control the error of the linear approximation.

# Taylor series approximation.

**Problem:** How large can you make $\phi(\hat{\theta}(\vec{w}))$ leaving out no more than $\lfloor \alpha N \rfloor$ points? **Combinatorially hard!**

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To simplify the search over $\vec{w}$, we form the Taylor series approximation:

$$\phi(\hat{\theta}(\vec{w})) \approx \phi^{\mathrm{lin}}(\vec{w}) := \phi(\hat{\theta}(\vec{1})) + \sum_{n=1}^{N} \psi_n (\vec{w}_n - 1)$$

**Approximate solution:** How large can you make $\phi^{\mathrm{lin}}(\vec{w})$ leaving out no more than $\lfloor \alpha N \rfloor$ points? **Easy!**

The most influential points for $\phi^{\mathrm{lin}}(\vec{w})$ have the most negative $\psi_n$.

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We provide **finite-sample theory** showing that
$\left| \phi(\hat{\theta}(\vec{w})) - \phi^{\mathrm{lin}}(\vec{w}) \right| = O\left( \left\| \frac{1}{N}(\vec{w} - \vec{1}) \right\|_2^2 \right) = O(\alpha)$ as $\alpha \to 0$.

# Taylor series approximation.

**How to compute the influence scores $\psi_n$?**
By the chain rule, $\psi_n = \left.\frac{\partial \phi(\hat{\theta}(\vec{w}))}{\partial \vec{w}_n}\right|_{\vec{1}} = \left.\frac{\mathrm{d}\phi(\theta)}{\mathrm{d}\theta^T}\right|_{\hat{\theta}} \left.\frac{\partial \hat{\theta}(\vec{w})}{\partial \vec{w}_n}\right|_{\vec{1}}$.
Recall that $\hat{\theta}(\vec{w}) := \vec{\theta} \text{ such that } \sum_{n=1}^{N} \vec{w}_n G(\vec{\theta}, d_n) = 0_P$.
The **implicit function theorem** expresses $\left.\frac{\partial \hat{\theta}(\vec{w})}{\partial \vec{w}_n}\right|_{\vec{1}}$ as a linear system.

Computation of $\psi_n$ is fully automatible from a software implemenation of $G(\cdot, \cdot)$ and $\phi(\cdot)$ with **automatic differentiation** [Baydin et al., 2017].

We have an `R` package, `rgiordan/zaminfluence`, for OLS and IV.

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6. **Optional:** Compute $\hat{\theta}(\vec{w}^*)$, and verify that $\phi(\hat{\theta}(\vec{w}^*)) - \phi(\hat{\theta}) \geq \Delta$.

**Question 2:**
**How does it work in practice?**

# The linear approximation.

For $N = 5,000$ data points, compute the OLS estimator from:

| Regressors | Residuals | Responses |
|---|---|---|
| $x_n \sim \mathcal{N}(0, \sigma_x^2)$ | $\varepsilon_n \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ | $y_n = \theta_0 x_n + \varepsilon_n$ |

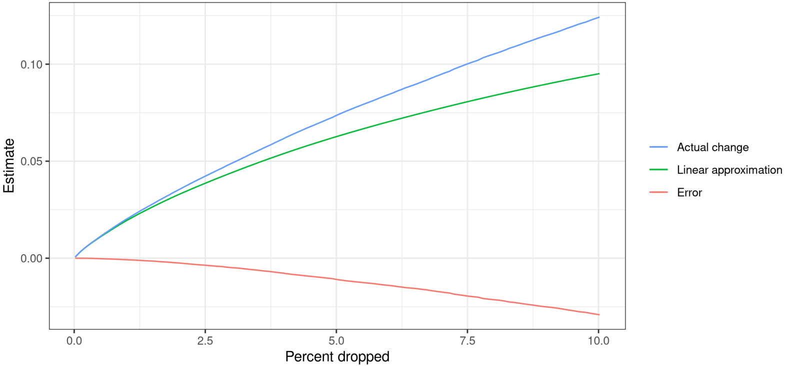
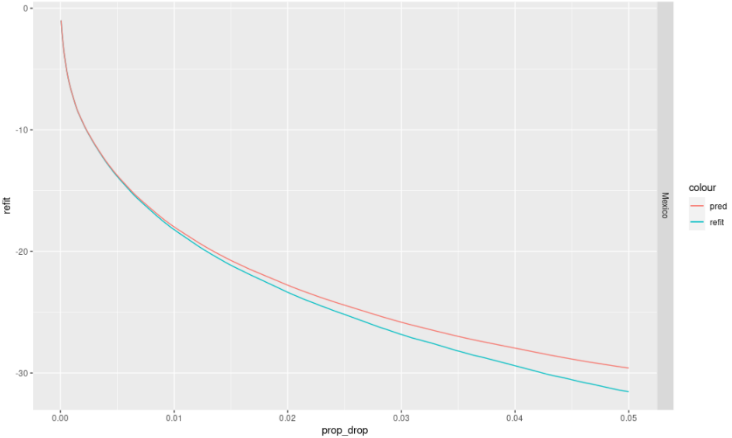



Figure: The actual change, linear approximation to the change, and approximation error.Here, $\sigma_x = 2$, $\sigma_\varepsilon = 1$, and $\theta_0 = 0.5$.

Mexico example:



# Microcredit.

| Study case | Original estimate | Target change | Refit estimate | Observations dropped |
|---|---|---|---|---|
| Bosnia | 37.534 (19.780) | Sign change | **-2.226 (15.628)** | 14 = 1.17% |
| | | Significance change | **43.732 (18.889)*** | 1 = 0.08% |
| | | Significant sign change | **-34.929 (14.323)*** | 40 = 3.35% |
| Ethiopia | 7.289 (7.893) | Sign change | **-0.053 (2.513)** | 1 = 0.03% |
| | | Significance change | **15.356 (7.763)*** | 45 = 1.45% |
| | | Significant sign change | **-8.755 (1.852)*** | 66 = 2.12% |
| India | 16.722 (11.830) | Sign change | **-0.501 (8.221)** | 6 = 0.09% |
| | | Significance change | **22.895 (10.267)*** | 1 = 0.01% |
| | | Significant sign change | **-16.638 (7.537)*** | 32 = 0.47% |
| Mexico | -4.549 (5.879) | Sign change | **0.398 (3.194)** | 1 = 0.01% |
| | | Significance change | **-10.962 (5.565)*** | 14 = 0.08% |
| | | Significant sign change | **7.030 (2.549)*** | 15 = 0.09% |
| Mongolia | -0.341 (0.223) | Sign change | **0.021 (0.184)** | 16 = 1.66% |
| | | Significance change | **-0.436 (0.220)*** | 2 = 0.21% |
| | | Significant sign change | **0.361 (0.147)*** | 38 = 3.95% |
| Morocco | 17.544 (11.401) | Sign change | **-0.569 (9.920)** | 11 = 0.20% |
| | | Significance change | **21.720 (11.003)*** | 2 = 0.04% |
| | | Significant sign change | **-18.847 (9.007)*** | 30 = 0.55% |
| Philippines | 66.564 (78.127) | Sign change | **-4.014 (57.204)** | 9 = 0.81% |
| | | Significance change | **138.929 (66.880)*** | 4 = 0.36% |
| | | Significant sign change | **-122.494 (49.409)*** | 58 = 5.21% |

Table: Microcredit regressions for the profit outcome. The "Refit estimate" column shows the result of re-fitting the model removing the Approximate Most Influential Set. Stars indicate significance at the 5% level. Refits that achieved the desired change are bolded.

# Cash transfers.

| Study case | Original estimate | Target change | Refit estimate | Observations dropped |
|---|---|---|---|---|
| Poor, period 10 | 33.861 (4.468)* | Sign change | -**2.559 (3.541)** | $697 = 6.63\%$ |
| | | Significance change | **4.806 (3.684)** | $435 = 4.14\%$ |
| | | Significant sign change | -**9.416 (3.296)*** | $986 = 9.37\%$ |
| Non-poor, period 10 | 21.493 (9.405)* | Sign change | -**0.573 (6.750)** | $30 = 0.70\%$ |
| | | Significance change | **16.262 (8.927)** | $3 = 0.07\%$ |
| | | Significant sign change | -10.845 (6.467) | $92 = 2.16\%$ |

Table: Cash transfers results for the final study period. The "Refit estimate" column shows the result of re-fitting the model removing the Approximate Most Influential Set. Stars indicate significance at the 5% level. Refits that achieved the desired change are bolded.

## Question 3:
## What makes an estimator non-robust?

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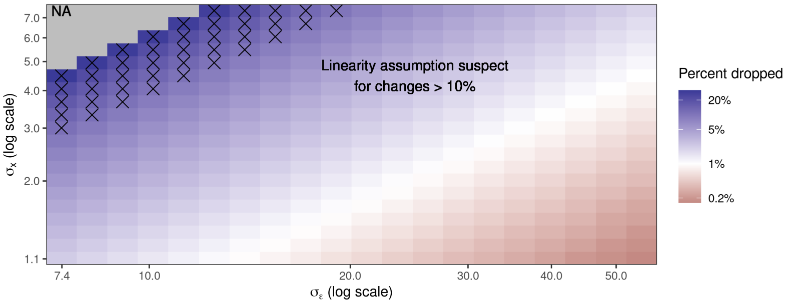



Figure: The approximate perturbation inducing proportion at differing values of $\sigma_x$ and $\sigma_\varepsilon$. Red colors indicate datasets whose sign can is predicted to change when dropping less than 1% of datapoints. The grey areas indicate $\hat{\Psi}_\alpha = \mathtt{NA}$, a failure of the linear approximation to locate any way to change the sign.

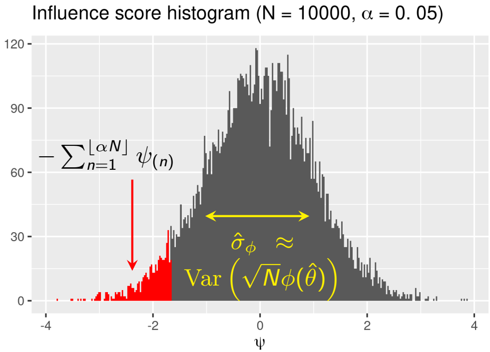
# What makes an estimator non-robust? A tail sum.

Report non-robustness if:

$$\Delta \leq \phi^{\mathrm{lin}}(\vec{w}^*) - \phi(\hat{\theta}) = -\sum_{n=1}^{\lfloor \alpha N \rfloor} \psi_{(n)} =: \hat{\sigma}_\phi \hat{\mathscr{T}}_\alpha$$

We will show that:

- The "noise" $\hat{\sigma}_\phi^2 \to \mathrm{Var}(\sqrt{N}\phi)$ [Hampel, 1986]
- The "shape" $\hat{\mathscr{T}}_\alpha \leq \sqrt{\alpha(1-\alpha)}$ and converges to a nonzero constant



# Corollaries.

Report non-robustness if:

$$\Delta \le \phi^{\mathrm{lin}}(\vec{w}^*) - \phi(\hat{\theta}) = \hat{\sigma}_\phi \hat{\mathscr{T}}_\alpha \qquad \Leftrightarrow \qquad \frac{\Delta}{\hat{\sigma}_\phi} \le \hat{\mathscr{T}}_\alpha.$$

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**Corollary: Insignificance is always non-robust.**
Take $\Delta = \frac{1.96 \hat{\sigma}_\phi}{\sqrt{N}} \to 0 \le \hat{\mathscr{T}}_\alpha$.

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**Corollary: Insignificance is always non-robust.**
Take $\Delta = \frac{1.96 \hat{\sigma}_\phi}{\sqrt{N}} \to 0 \leq \hat{\mathscr{T}}_\alpha$.

**Corollary: Gross outliers primarily affect robustness through $\hat{\sigma}_\phi$.**
Cauchy-Schwartz is tight when all the influence scores are the same.

# Conclusion:
Related work and future directions

# Links and references

Tamara Broderick, Ryan Giordano, Rachael Meager (alphabetical authors)
"An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Change Conclusions?"
https://arxiv.org/abs/2011.14999

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