Locally Equivalent Weights for Bayesian MrP

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Are US non-voters becoming more Republican?

Blue Rose research says yes:

“Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate.”

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

“Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available.”

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- Different data sources
- ⋆⋆⋆ **Different statistical methods**
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Our contribution

We define "MrP local equivalent weights" (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key diagnostics that motivate calibration weighting.

$\Rightarrow$ **MrPlew provides direct comparisons between MrP and calibration weighting.**

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- Introduce the statisical problem
    - Contrast CW and MrP
    - Prior work: Equivalent weights for linear models
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  - Define MrPlew weights and connect them to covariate balance
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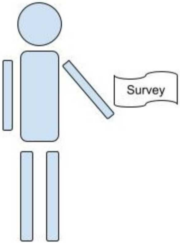
- Other uses of locally equivalent weights
  - Parital pooling
  - The meaning of negative weights
  - Frequentist variance estimation
- Future directions

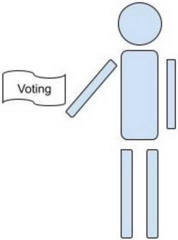
The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support Trump")

We want the average response in a target population, in which we observe only covariates.





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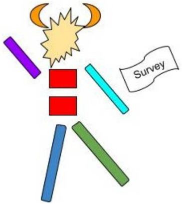

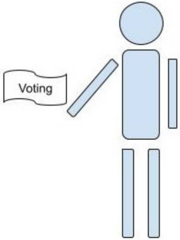

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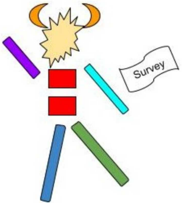

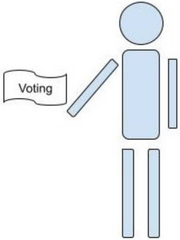

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The problem is that the populations may be very different.

Our survey results may be biased.

How can we use the covariates to say something about the target responses?

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

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Calibration weighting (CW)

▶ Choose "calibration weights" w_i
using only the regressors $\mathbf{x}$
(e.g. raking weights)

Bayesian hierarchical modeling (MrP)

▶ Choose $\mathbb{E}\left[y|\mathbf{x},\theta\right] = m(\theta^\intercal \mathbf{x})$,
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  - Partial pooling
  - Regressor balance

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- **Black box**

$\leftarrow$ We open this box, providing analogues of all these diagnostics

Prior work

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form $\hat{y}$:

$$\hat{\mu}_{\mathrm{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T} \hat{y}_j = \frac{1}{N_T}\sum_{j=1}^{N_T} \underbrace{\mathbf{x}_j^{\intercal}\hat{\theta}}_{\text{Linear in } y_i}$$

Most existing literature on comparing CW and MrP focus on such linear models. [1]

Let's spend some time discussing why it is reasonable to even attempt such a thing as forming approximate equivalent weights for non–linear estimators.

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But what if you use a non–linear link function? Or a hierarchical model?

> *"It would also be desirable to use nonlinear methods ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods." — (Gelman 2007a)*

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Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.
- Suppose $\mathbf{x} \in \mathcal{X}$ is discrete and saturated.

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- Let $\overline{y}_S^c$ denote the survey average among $\mathbf{x} = c$ for $c \in \mathcal{X}$
- For $\mathbf{x} = c$, $m(\hat{\theta}^\intercal \mathbf{x}) = \overline{y}_S^c$
- Let N_S^c (or N_S^c) denote the # of survey (or target) observations with $\mathbf{x}_n = c$.

$$\hat{\mu}_{\mathrm{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) = \frac{1}{N_T} \sum_{c \in \mathcal{X}} \underbrace{N_T^c \overline{y}_S^c}_{\text{Linear in } y_i} = \frac{1}{N_S} \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} y_i$$

For $w_i^{\mathrm{MrP}} = \dfrac{N_T^c / N_T}{N_S^c / N_S}$ when $\mathbf{x}_i = c$.

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&= \alpha^\intercal \frac{1}{N_S}\sum_{i=1}^{N_S} \mathbf{x}_i y_i && \text{(Property of exponential family MLEs)}
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$$\hat{\mu}_{\text{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta}) = \frac{1}{N_S}\sum_{i=1}^{N_S} \alpha^\intercal\mathbf{x}_i y_i + \text{Small error}$$

We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so it's hard to estimate α directly.

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

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Key idea (informal)

If $\hat{\mu}_{\text{MrP}} \approx \frac{1}{N_S} \sum_{i=1}^{N_S} w_i^{\text{MrP}} y_i$ for some w_i^{MrP}, then

$$
w_i^{\text{MrP}} \approx \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}
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Nearly equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.
- Suppose there exists α such that $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$.

$$\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) = \frac{1}{N_S} \sum_{i=1}^{N_S} \alpha^\intercal \mathbf{x}_i y_i + \text{Small error}$$

We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so it's hard to estimate α directly.

Key idea (informal)

If $\hat{\mu}_{\text{MrP}} \approx \frac{1}{N_S} \sum_{i=1}^{N_S} w_i^{\text{MrP}} y_i$ for some w_i^{MrP}, then

$$w_i^{\text{MrP}} \approx \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$$

For logistic regression, could compute and analyze $\frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$ using the implicit function theorem.[2]

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$.
- Set a hierarchical prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$, use MCMC to draw from $\mathcal{P}(\theta|\text{Survey data})$.
- MrP is $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})} \left[m(\mathbf{x}_j^\intercal \theta) \right]$.

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

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MrP locally equivalent weights (MrPlew)

For new data $\tilde{Y}_S$, form a series expansion

$$\hat{\mu}_{\text{MrP}}(\tilde{Y}_S) \approx \hat{\mu}_{\text{MrP}}(Y_S) + \sum_{i=1}^{N_S} w_i^{\text{MrP}}(\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MrP}} := \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}.$$

Our task is to rigorously show that even such local weights can be used diagnostically.

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

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Our task is to rigorously show that even such local weights can be used diagnostically.

- If $\hat{\mu}_{\text{MrP}}$ is linear in Y_S, this definition recovers the true weights
- Whether or not $\hat{\mu}_{\text{MrP}}$ is linear in Y_S, this definition is valid “locally” for $\tilde{Y}_S$ “nearby” Y_S.
- For MCMC, compute and analyze $\frac{\partial\hat{\mu}_{\text{MrP}}}{\partial y_i}$ using Bayesian sensitivity analysis.[3]

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

The weights can look very different!

Does this mean anything? Are the differences important?

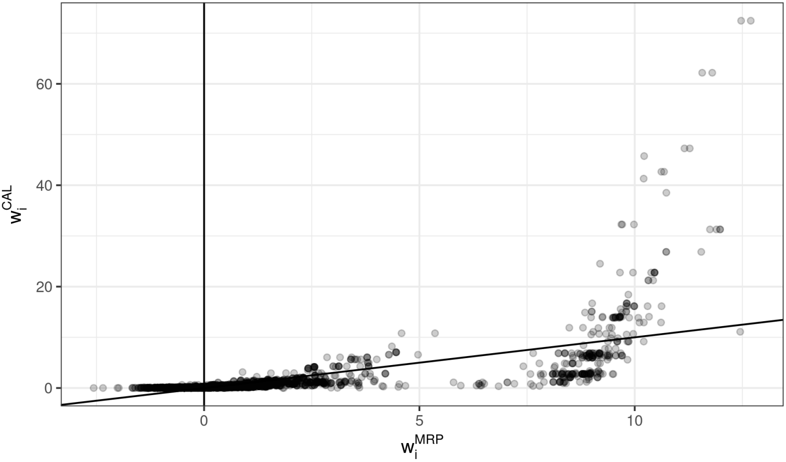

Figure 1: Comparison between raking and MrPlew weights for a particular example

Future work

Note that there was no talk of correct specification for the data you have.

That was a foregone conclusion when we started looking at equivalent weights!

How do you peform model checking with sensitivity analysis?

Existing methods evaluate whether the analysis changes “a lot” when you:

- Parametrically perturb the model (e.g. fit a richer model class)
- Non–parameterically perturb the data (e.g. produce gross outliers)

The problem is:

- How much is “a lot”?
- Non–parametric data perturbations are hard to reason about
- It’s hard to say whether parametric model changes are enough

Instead, we

- Parametrically perturb the data
- Observe whether our model could detect the change

- Know exactly the expected change (don’t have to decide on what “a lot” means)
- Easy to reason about whether the data perturbation is reasonable
- Don’t need to propose an alternative model, instead study the model you have

Related and future work

Student contributions and future work:

- **Alice Cima** contributed significantly to this work
- **Vladimir Palmin** is working on extending MrPlew to `lme4`
- **Sequoia Andrade** is working on generalizing to other local sensitivity checks
- **Lucas Schwengber** is working on novel flow–based techniques for local sensitivity

Alice Cima

No picture!
Vladimir Palmin

Sequoia Andrade

Lucas Schwengber

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