Outline

- Logic
- Deduction and induction
- Classes of inductive questions (from philosophical induction to probability)
- Extreme resolutions: “Bayesian,” “falsificationist,” “conventionalist”

Logic

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An argument is valid if it is logically sound.

A proposition is a statement which is either true or false.

Example:
If James wants a job, then he will get a haircut tomorrow.
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Questions:
Which argument is valid?
What are the propositions?
Which propositions are true?

Which of these arguments are valid? (Hacking 2001, Ch.1 Question 7)

- I follow three major league teams. Most of their top hitters chew tobacco at the plate.
  $\Rightarrow$ Chewing tobacco improves batting average.
- The top six hitters in the National League chew tobacco at the plate.
  $\Rightarrow$ Chewing tobacco improves batting average.
- A study by the American Dental Association of 158 players on seven major league teams during the 1988 season showed that the mean batting average for chewers was 0.238 compared to 0.248 for non-users. Abstainers also had a higher fielding average.
  $\Rightarrow$ Chewing tobacco does not improve batting average.
- In 1921, every major league pitcher who chewed tobacco when up to bat had a higher batting average than any major league pitcher who did not.
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None of them are valid.

But some are better than others. In what sense? In any logical sense?

Logic

The Stoics identify the following syllogisms, purported patterns of valid inference:

- *Modus Ponens*: If A, then B. A. Therefore, B.
- *Modus Tollens*: If A, then B. Not B. Therefore, not A.
- The Hypothetical Syllogism: If A, then B. If B, then C. Therefore, if A, then C.
- The Conjunctive Syllogism: Not both A and B. A. Therefore, not B.
- The Dilemma: If A, then B. If C, then B. A or C. Therefore, B.
- The Disjunctive Syllogism: A or B. But not A. Therefore, B.

Proposition: If we begin with true propositions, and combine them according to the above syllogisms, we necessarily reach a true conclusion.

Reasoning in this way is called **deductive reasoning**.

Example

Set theory can provide a means to visualize and analyze logical reasoning. Here is an example that I think may be useful.[1]

Suppose a bag contains three coins: one regular coin (HT), one with both faces tails (TT), and one with both faces heads (HH). The coin is flipped, and either the first or second side comes up.

Exactly one possible outcome of coin $\times$ side occurs; call this the "truth."

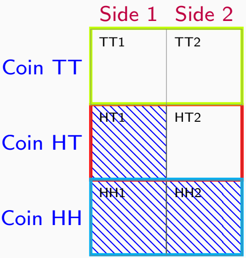
	Side 1	Side 2
Coin TT	TT1	TT2
Coin HT	HT1	HT2
Coin HH	HH1	HH2 ×

Example:
Coin HH was picked and the second side came up (HH2).

In this context, classical propositional logic can be represented as set operations.

[1] I cooked this up and have no idea how standard this is. I realized preparing for this how little I know about logic.

Example



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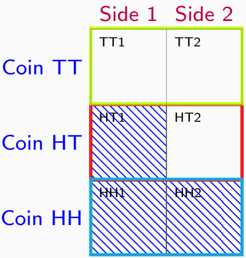

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Questions:

Suppose we observe heads (so we know $\times \in$ ).

What can we deduce about whether we drew the TT coin?

What can we deduce about whether we drew the HT coin?

What can we deduce about whether we drew the HH coin?

Hint: Note that  $\subseteq$ ^{c}.

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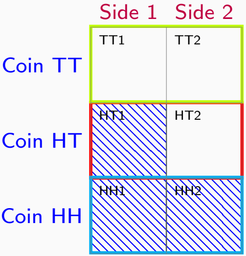

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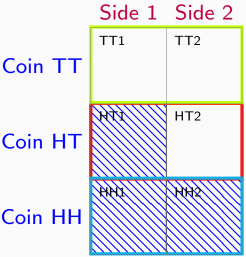
In general: Set inclusion is the same as deductive entailment:

A is true and $A \subseteq B$ implies that B is true. This is the set version of *modus ponens*.

(**Optional exercise:** Rewrite the other logical syllogisms as set operations.)

If A is true, and A overlaps partially with B, nothing about B follows deductively, because **you might be wrong**.

Example



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And yet it seems silly to claim that observing a heads tells you nothing about whether you have chosen the HH coin.

After all, the "prior" $p(\mathrm{HH\ coin}) = 1/3$, but the "posterior" $p(\mathrm{HH\ coin}|\mathrm{heads}) = 2/3$.

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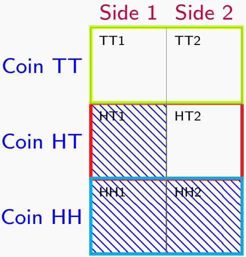

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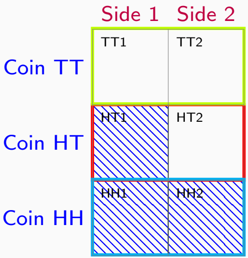

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Questions:

Is induction even possible? If so, how?
As we will see, early statisticians were extremely preoccupied with this question.[2]

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The European Enlightenment was an exciting time

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We have asked this question in the setting when it has, arguably, the best chance of being possible (a well-defined probability setup).

Hume asked it in much, much greater generality.

Hume

Is induction even possible? If so, how?

1. All questions are "Relations of Ideas" or "Matters of Fact."
2. "Relations of Ideas" are deductive, certain, mathematical.
3. "Matters of Fact" are sensory, experiental, contingent, local in time and space.
4. The content of ideas is entirely matters of fact.

Here, "matters of fact" correspond to propositions, and "relations of ideas" to deductive logic.

Hume's question: We believe much more about the world than we experience directly. What is the foundation of this belief? Is it experience? Is it deduction? What combination of the two?

Cause and Effect

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1. Resemblance
2. Co-occurence
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Question: What is the difference between Hume's cause and effect and the counterfactulas of the Neyman-Rubin framework?

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Question: What is Hume saying about inductive reasoning?

Question: In Hume's view, what classes of questions fall under the category of inductive reasoning?

Hume on science

Hume writes:

> "Elasticity, gravity, cohesion of parts, communication of motion by impulse; these are probably the ultimate causes and principles which we shall ever discover in nature." (26)

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Question: These observations have not held up well. What consequences does this have for Hume's view of science, if any?

Hume on everyday life

Question: In Hume's view, what consequences do his skepticism have for real life?

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Hume writes:

> "And though none but a fool or madman will ever pretend to dispute the authority of experience, or to reject that great guid of human life, it may surely be allowed a philosopher to have so much curiosity at least as to examine the principle of human nature, which gives this mighty authority to experience, and makes us draw advantage from the similarity which nature has placed among different objects." (31)

> "My practice, you say, refutes my doubts. But you mistake the purport of my question. As an agent, I ma quite satisfied in the point; but as a philosopher, who has some share of curiosity, I will not say scepticism, I want to learn the foundation of this inference." (32)

A way forward?

"There is certainly a probability, which arises from a superiority of chances on any side; and acoording as this superiority encreases, and surpasses the opposite chances, the probability receive a proportional increase, and begets still a higher degree of belief or assent to that side, in which we discover the superiority." (46)

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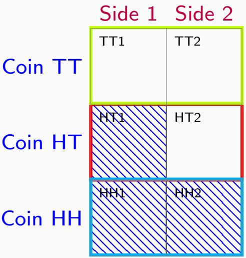
Question: Is Hume putting this forward as a potential solution?

A different way forward?

Karl Popper answers Hume using a very different technique than probability.



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Question: Recall our coin example. Suppose we have two scientific theories, corresponding to "coin HH was chosen" and "coin TT was chosen." We observe heads. What does Popper say we can conclude? How does this relate to Hume's question of induction?

References

Hacking, I. (2001). *An introduction to probability and inductive logic*. Cambridge university press.

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Shenefelt, M. and H. White (2013). *If A, then B: How the world discovered logic*. Columbia University Press.