Weighting-Like Diagnostics for Nonlinear Bayesian Hierarchical Models

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Are US non-voters becoming more Republican?

Blue Rose research says yes:

"Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate."

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

"Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available."

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- Different data sources
- ⋆⋆⋆ **Different statistical methods**
  - Blue Rose uses Bayesian hierarchical modeling (MrP)
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Our contribution

We define "MrP local equivalent weights" (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key weighting estimator diagnostics.

$\Rightarrow$ **MrPlew provides direct comparisons between MrP and calibration weighting.**

Outline

Weighting (linear) estimators are great — they come with easy-to-understand diagnostics.

This talk is about making versions of such diagnostics for **complicated non-linear models**.

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The key idea is to convert the diagnostic into a *local sensitivity analysis* of this form:

1. Assume your initial model was accurate
2. Select some perturbation your model should be able to capture
3. Use local sensitivity to detect whether the change is what you expect
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I'll do this carefully for covariate balance and MCMC.

But many other variants are possible!

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- Introduce the statisical problem
  - Contrast calibration weighting and MrP
  - Prior work: Equivalent weights for linear models
  - Equivalent weights and implicit weights for non–linear models
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- Locally equivalent weights for covariate balance
  - Describe classical covariate balance
  - Introduce a MrPlew “local empirical consistency check”
  - Theoretical support
  - Examples of real-world results

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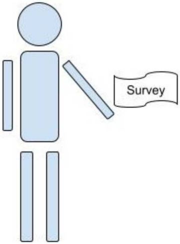
- Other directions
  - High–level restatement of the logic of our procedure
  - Local versions of other common diagnostics for linear estimators
  - Ongoing and future work

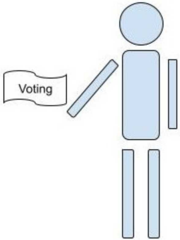
The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support candidate Z")

We want the average response in a target population, in which we observe only covariates.





Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

[1]Photo copyright: Mark Taylor / `naturepl.com`

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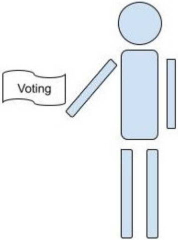

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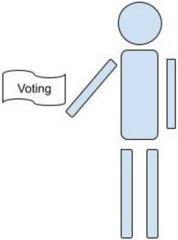

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How can we use the covariates to say something about the target responses?

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Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target y_j. Let $Y_{\mathcal{S}} = \{y_1, \ldots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
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▶ Choose "calibration weights" w_i
using only the regressors $\mathbf{x}$
(e.g. raking weights)

Bayesian hierarchical modeling (MrP)

▶ Choose $\mathbb{E}\left[y|\mathbf{x}, \theta\right] = m(\theta^\intercal \mathbf{x})$,
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  - Regressor balance
  - Partial pooling

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$\leftarrow$ Today, we'll open the box and provide MrP analogues of all these diagnostics

Prior work: Equivalent weights for linear models

Gelman (2007b) observes that MrP is a weighting estimator when $\hat{y}$ is computed with OLS:

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But what if you use a non–linear link function? Or a hierarchical model?

> *"It would also be desirable to use nonlinear methods ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods." — (Gelman 2007a)*

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Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.

The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *typically nonlinear*.

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Example: $x_i \sim \mathrm{Unif}[-0.5, 0.5]$, $y_i \overset{iid}{\sim} \mathrm{Binomial}(1/2)$. Let $\tilde{y}_i(\delta) = y_i + \delta \mathbb{I}\left(x_i > 2\right)$.

Each δ gives a different OLS fit $\hat{\beta}(\delta)$ and logistic regression coefficient $\hat{\theta}(\delta)$.

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For OLS, $\delta \mapsto \hat{\beta}(\delta) x_j$ is linear. For logistic regression $\delta \mapsto m(\hat{\theta}(\delta) x_j)$ is non-linear.

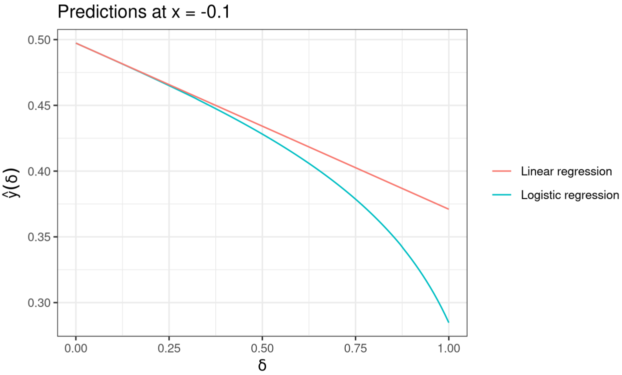

Figure 1: Simulated path through the space of responses

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The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *typically nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal \hat{\theta})$ can be approximately linear.

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- MrP is $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$.

Example

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal\mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta}) = \frac{1}{N_S}\sum_{i=1}^{N_S} \underbrace{w_i^{\mathrm{MrP}}}_{\alpha^\intercal\mathbf{x}_i} y_i + \text{Small error}$$

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But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

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Key idea (informal)

If $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})$ is approximately linear, then[3] $w_i^{\mathrm{MrP}} \approx N_S \frac{\partial \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$.

[3] For MLEs, $\frac{\partial \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ is given by the implicit function theorem. (Krantz and Parks 2012; **G.**, Stephenson, et al. 2019)

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Note: The derivatives w_i^{MrP} now have two potentially distinct interpretations:

- **Equivalent weights:** A characterization of $Y_{\mathcal{S}} \mapsto \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})$ for diagnostics
- **Implicit weights:** An estimate of $\mathcal{P}_T(\mathbf{x})/\mathcal{P}_S(\mathbf{x})$

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Local weights for nonlinear hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$.
- Set a hierarchical prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$, use MCMC to draw from $\mathcal{P}(\theta|\text{Survey data})$.
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No reason to think $Y_{\mathcal{S}} \mapsto \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})$ is even approximately **globally** linear.

[4] Diaconis and Freedman 1986; Gustafson 1996; Efron 2015; **G.**, Broderick, and Jordan 2018.

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But can still compute and analyze $w_i^{\mathrm{MrP}} := N_S \frac{\partial \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ using Bayesian sensitivity analysis![4]

MrP weights for MCMC

$$w_i^{\mathrm{MrP}} := N_S \frac{\partial \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})}{\partial y_i} = N_S \frac{1}{N_T} \sum_{j=1}^{N_T} \underbrace{\mathrm{Cov}_{\mathcal{P}(\theta|\text{Survey data})} \left(m(\mathbf{x}_j^\intercal \theta), \theta^\intercal \mathbf{x}_i \right)}_{\text{Can estimate without rerunning MCMC!}}$$

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The derivatives w_i^{MrP} *again* have two potentially distinct interpretations:

- **Locally equivalent weights:** A characterization of $Y_{\mathcal{S}} \mapsto \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ for diagnostics
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This talk will focus only on locally equivalent weights. (Implicit weights is ongoing work!)

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Locally equivalent weights for hierarchical logistic regression MrP

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MrP locally equivalent weights (MrPlew)

For new data $\tilde{Y}_{\mathcal{S}}$, form a **MrP locally equivalent weighting**:

$$\hat{\mu}^{\text{MrP}}(\tilde{Y}_{\mathcal{S}}) \approx \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) + \sum_{i=1}^{N_{\mathcal{S}}} w_i^{\text{MrP}} (\tilde{y}_i - y_i)$$

Our task is to rigorously show that even such local weights can be meaningfully used diagnostically in the same ways we use global weights.

Real Data: Marital Name Change Survey

Analysis of changing names after marriage[5].

- **Target population:** ACS survey of US population 2017–2022
- **Survey population:** Marital Name Change Survey (from Twitter)
- **Respose:** Did the female partner keep their name after marriage?
- For regressors, use bins of age, education, state, and decade married.

MrP computed with `brms` (Bürkner 2017):

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kept_name ~ (1 | age_group) + (1 | educ_group) + (1 | state_name) + (1 | decade_married)
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CW used raking on coarsened regressor marginals (`survey::calibrate` from Lumley (2024))

$$N_S = 4,364 \qquad N_T = 4,085,282$$

Uncorrected survey mean: $$\frac{1}{N_S}\sum_{i=1}^{N_S} y_i = 0.462$$

Raking: $$\hat{\mu}^{\mathrm{WGT}}(Y_{\mathcal{S}}) = 0.263$$

MrP: $$\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = 0.288 \quad (\text{Post. sd} = 0.0169)$$

[5] Based on Alexander (2019), Cohen (2019), and Ruggles et al. (2024).

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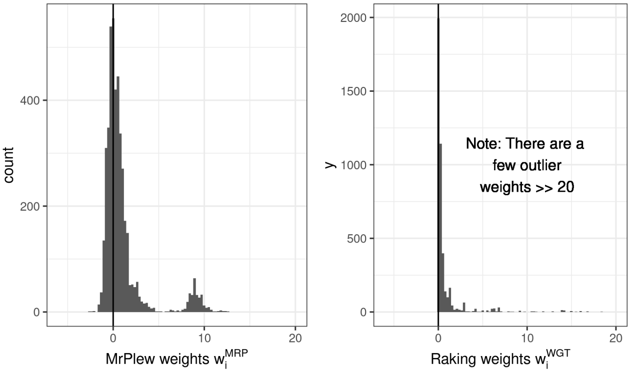

Figure 2: Weight comparison for the Name Change dataset

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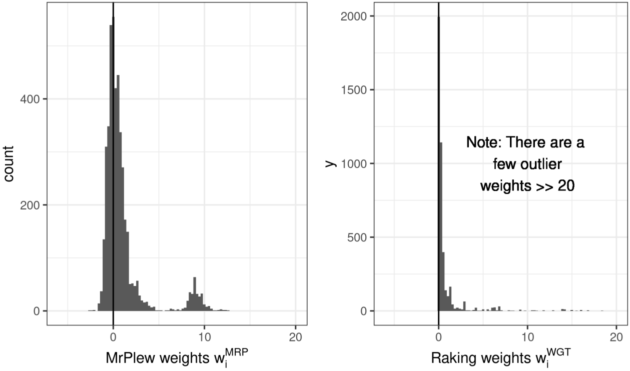
This is essentially a corollary of our earlier work on the Bayesian infinitesimal jackknife.[6]



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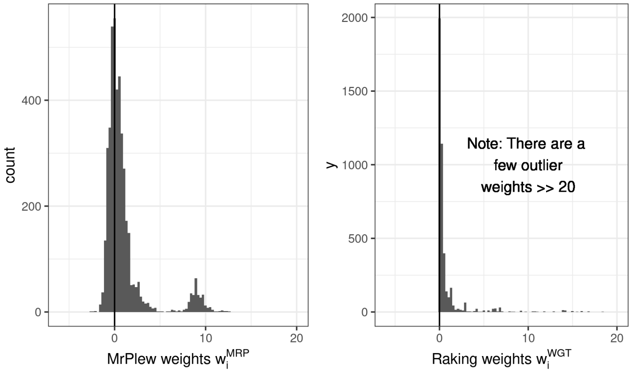
What about covariate balance?



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Introduction to covariate balance: What are we weighting for?[7]

$$\text{Target average response} = \frac{1}{N_T}\sum_{j=1}^{N_T} y_j \approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i = \text{Weighted survey average response}$$

We can't check this, because we don't observe y_j.

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$$\frac{1}{N_T}\sum_{j=1}^{N_T} \mathbf{x}_j \overset{\text{check}}{=} \frac{1}{N_S}\sum_{i=1}^{N_S} w_i \mathbf{x}_i$$

Weights that pass this check satisfy "covariate balance" for **x**.

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Weights that pass this check satisfy "covariate balance" for $\mathbf{x}$.

You can check covariate balance for any weighting estimator, and any function $f(\mathbf{x})$.

Recall that **raking calibration weights** aim to exactly balance some set of regressors.

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Balance checks as local sensitivity

One reason to balance $f(\mathbf{x})$ is because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

Key idea: Define a data perturbation that captures this intuition.

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One reason to balance $f(\mathbf{x})$ is because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

Balance–informed sensitivity check (BISC) (informal)

Pick a small $\delta > 0$ and an $f(\cdot)$. Define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}[\mu(\tilde{y}) - \mu(y)|\mathbf{x}] = \frac{1}{N_T}\sum_{j=1}^{N_T}(\mathbb{E}[\tilde{y}|\mathbf{x}_p] - \mathbb{E}[y|\mathbf{x}_p]) = \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change for observed $\tilde{Y}_{\mathcal{S}}, Y_{\mathcal{S}}$:

$$\underbrace{\hat{\mu}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}(Y_{\mathcal{S}})}_{\text{Replace weighted averages with changes in an estimator}} \overset{\textbf{check}}{\approx} \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

Balance checks as local sensitivity

When $\hat{\mu}(\cdot) = \hat{\mu}^{\text{WGT}}(\cdot)$, BISC recovers the standard covariate balance check.

$$\underbrace{\hat{\mu}^{\text{WGT}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\text{WGT}}(Y_{\mathcal{S}})}_{\substack{\text{Replace weighted averages} \\ \text{with changes in an estimator}}} = \frac{1}{N_S}\sum_{i=1}^{N_S} w_i \tilde{y}_i - \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i$$

$$= \frac{1}{N_S}\sum_{i=1}^{N_S} w_i (y_i + f(\mathbf{x}_i)) - \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i$$

$$= \frac{1}{N_S}\sum_{i=1}^{N_S} w_i f(\mathbf{x}_i)$$

$$\overset{\text{check}}{=} \delta \frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

We will study $\hat{\mu}(\cdot) = \hat{\mu}^{\text{MrP}}(\cdot)$.

BISC for MrP

Suppose I have $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.
Now I need to evaluate $\hat{\mu}^{\mathrm{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})$.

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Problem: $\hat{\mu}^{\text{MrP}}(\cdot)$ is computed with MCMC.

- Each MCMC run typically takes hours, and
- MCMC output is noisy, and $\hat{\mu}^{\text{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ may be small.

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Solution: Use our local approximation, MrPlew!

Balance informed sensitivity check with MrPlew:

For a wide set of judiciously chosen $f(\cdot)$, check

$$\begin{aligned}\hat{\mu}^{\mathrm{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) &\approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i^{\mathrm{MrP}}(\tilde{y}_i - y_i) \\ &\approx \underbrace{\delta \frac{1}{N_S}\sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} f(\mathbf{x}_i) \overset{\textbf{check}}{\approx} \delta \frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j)}_{\text{What you actually check}}.\end{aligned}$$

Generating $\tilde{y}$

- We have defined BISC in terms of $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$
- We have approximated $\hat{\mu}^{\text{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ for $\tilde{y} \approx y$

How to get such a $\tilde{y}$? **Recall y is binary!**

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Option 2: Allow $\tilde{y}$ to take generic values.

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   - Some values that gives the same posterior
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4. Draw $u_n | y_n$
5. Set $\tilde{y}_i = \mathbb{I}\left(u_i \leq \hat{m}(\mathbf{x}_i) + \delta \mathbf{x}_i\right)$

Option 2: Allow $\tilde{y}$ to take generic values.

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5. Set $\tilde{y}_i = \mathbb{I}\left(u_i \leq \hat{m}(\mathbf{x}_i) + \delta \mathbf{x}_i\right)$

Option 2: Allow $\tilde{y}$ to take generic values.

1. Set $\tilde{y}_i = y_i + \delta f(\mathbf{x}_i)$.
2. Then you're done.
3. There is nothing else to do.
4. This space deliberately left blank.

Generating $\tilde{y}$

- We have defined BISC in terms of $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$
- We have approximated $\hat{\mu}^{\mathrm{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})$ for $\tilde{y} \approx y$

How to get such a $\tilde{y}$? **Recall y is binary! Two solutions, with their own pros and cons:**

Option 1: Force $\tilde{y}$ to be binary.

1. Make *some* guess $\hat{m}(\mathbf{x}) \approx \mathbb{E}\left[y|\mathbf{x}\right]$
   - E.g. Posterior mean, or
   - Shrunken posterior mean, or
   - Some values that gives the same posterior
2. Take $u_i \overset{iid}{\sim} \mathrm{Unif}(0,1)$
3. Assume $y_i = \mathbb{I}\left(u_i \le \hat{m}(\mathbf{x}_i)\right)$
4. Draw $u_n | y_n$
5. Set $\tilde{y}_i = \mathbb{I}\left(u_i \le \hat{m}(\mathbf{x}_i) + \delta \mathbf{x}_i\right)$

Pros and cons:

- Realistic
- Have to pick $\hat{m}(\mathbf{x})$
- $\tilde{Y}_{\mathcal{S}} - Y_{\mathcal{S}}$ not infinitesimally small
- **Use for checks & experiments**

Option 2: Allow $\tilde{y}$ to take generic values.

1. Set $\tilde{y}_i = y_i + \delta f(\mathbf{x}_i)$.
2. Then you're done.
3. There is nothing else to do.
4. This space deliberately left blank.

Pros and cons:

- Not realistic
- No additional assumptions
- $\tilde{Y}_{\mathcal{S}} - Y_{\mathcal{S}}$ may be infinitesimally small
- **Use for theory**

When is the local approximation accurate?

BISC Theorem: (sketch)

Take $\tilde{y}_i = y_i + \delta f(\mathbf{x}_i)$.

We state conditions for Bayesian hierarchical logistic regression under which

$$\left| \hat{\mu}^{\mathrm{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) - \delta \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} f(\mathbf{x}_i) \right| = \text{Small}$$

[8] $\mathcal{F}$ can be any Donsker class of measurable functions with uniformly bounded $\mathbb{E}\left[\mathbf{x} f(\mathbf{x})\right]$.

[9] **G.** and Broderick 2024; Kasprzak, **G.**, and Broderick 2025.

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Theory

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...for a very broad class of $\mathcal{F}$. [8]

Uniformity justifies searching for "imbalanced" f.

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The uniformity result builds on our earlier work on uniform and finite–sample error bounds for Bernstein–von Mises theorem–like results[9].

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Covariate balance for primary effects

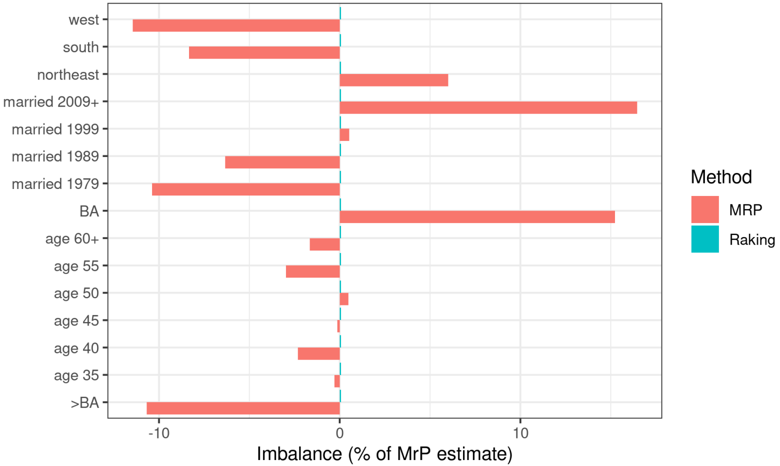

Figure 3: Imbalance plot for primary effects in the Name Change dataset

Covariate balance for interaction effects

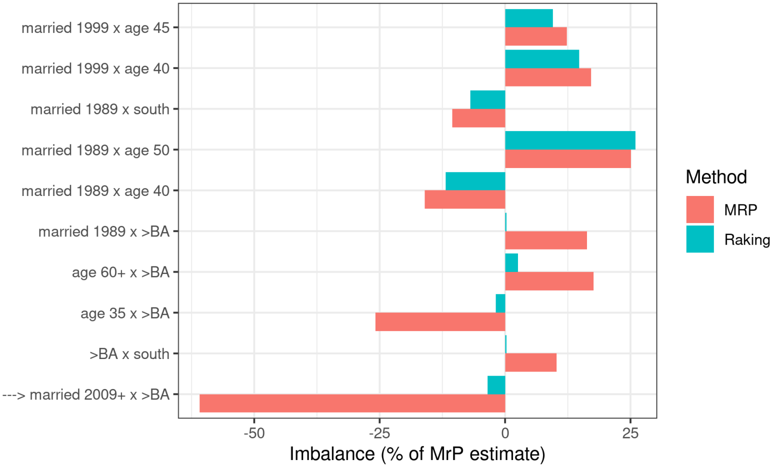

Figure 4: Imbalance plot for select interaction effects in the Name Change dataset

Predictions

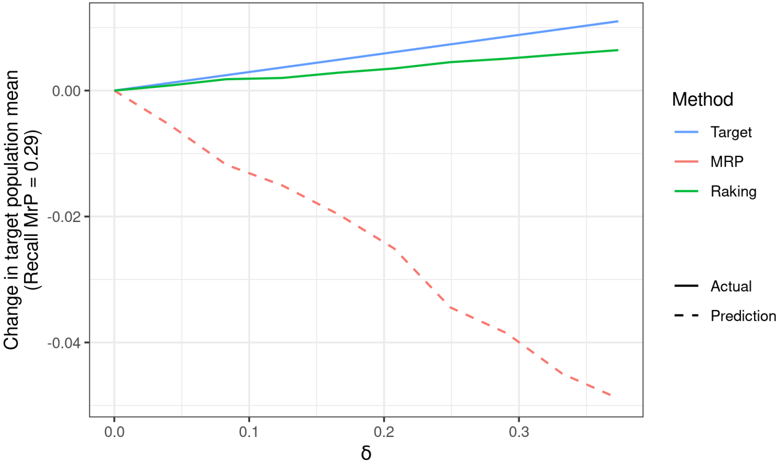

Figure 5: Predictions on binary data for the Name Change dataset

Predictions and actual MCMC results

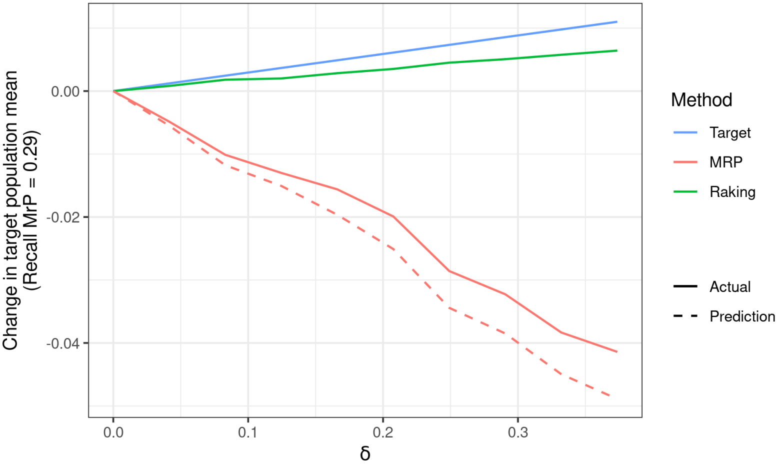

Figure 6: Predictions and refit on binary data for the Name Change dataset

Running ten MCMC refits: 10 hours Computing approximate weights: 16 seconds

Partial Pooling

By applying the same idea to subsets of the target population, you can measure *MrP partial pooling*.

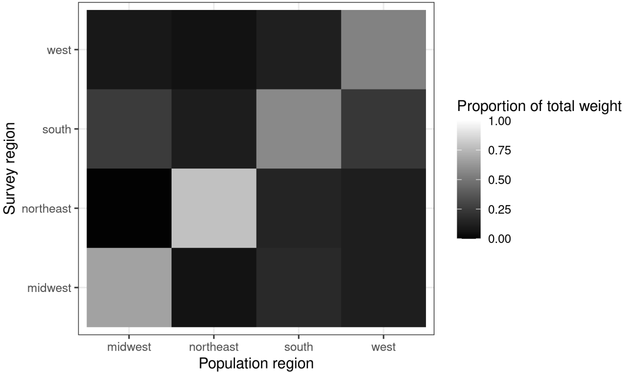

Figure 7: Region partial pooling for the Name Change dataset

Future work

Notice that there was no discussion of misspecification!

Calibration weights (typically) do not depend on $Y_{\mathcal{S}}$.

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But the high level idea can be extended much more widely:

1. Assume your initial model was accurate
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3. Use local sensitivity to detect whether the change is what you expect
4. If the change is not what you expect, either (1) or (2) was wrong

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3. Use local sensitivity to detect whether the change is what you expect
4. If the change is not what you expect, either (1) or (2) was wrong

Checks of this form give generalized versions of many standard linear model diagnostics:

- Local “Fisher consistency” checks
- Checks for exogeneity of residuals (even without residuals)
- Checks for whether inverse Fisher information $\overset{\text{check}}{=}$ score covariance (even without scores)

Future work

Student contributions and ongoing work:

- **Vladimir Palmin** is working on extending MrPlew to `lme4`
- **Sequoia Andrade** is working on generalizing to other local sensitivity checks
- **Lucas Schwengber** is working on novel flow–based techniques for local sensitivity
- **(Currently recruiting!)** Doubly–robust Bayesian MrP (the “implicit weights” version)

Vladimir Palmin

Sequoia Andrade

Lucas Schwengber

Preprint and R package coming soon! 🙏

Extra slides

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Frequentist variance estimation

Let $\hat{\text{Var}}(\cdot)$ denote the sample variance.

Calibration weighting standard errors sketch: [10]

If we have $\hat{\mu}^{\text{WGT}}(Y_{\mathcal{S}}) = \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i$ and a consistent residual estimate ε_i, then

$$\hat{\text{Var}}(w_i \varepsilon_i) \approx \text{Var}\left(\sqrt{N_S}\hat{\mu}^{\text{WGT}}(Y_{\mathcal{S}})\right).$$

[10] E.g. , Deville, Särndal, and Sautory (1993) and Fuller (2011).

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MrPlew Standard error consistency theorem sketch (Our contribution): [11]

For Bayesian hierarchical logictic regression, define $\varepsilon_i = y_i - \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_i^\intercal \theta)\right]$.

We state mild conditions under which, as $N_S \to \infty$, for some μ_∞ and variance V,

$$\sqrt{N_S}\left(\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) - \mu_\infty\right) \to \mathcal{N}(0, V) \quad \text{and}$$
$$\hat{\mathrm{Var}}\left(w_i^{\mathrm{MrP}} \varepsilon_i\right) \to V.$$

The use of w_i^{MrP} is analogous to the use of w_i for frequentist variance estimation.

[10] E.g. , Deville, Särndal, and Sautory (1993) and Fuller (2011).

[11] This is essentially a corollary of our earlier work on the Bayesian infinitesimal jackknife. (**G.** and Broderick 2024)

Standard error estimation experiment

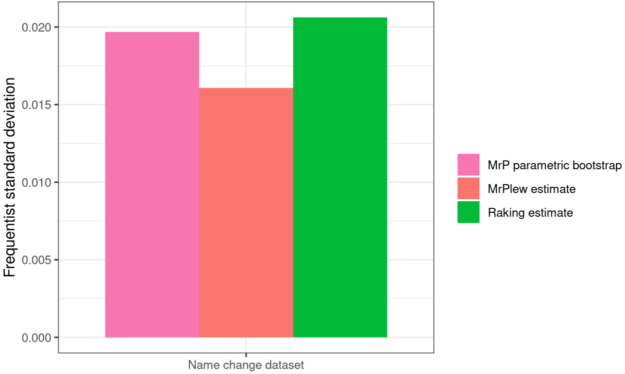

Figure 8: Frequentist standard deviation estimates

Standard error estimation experiment

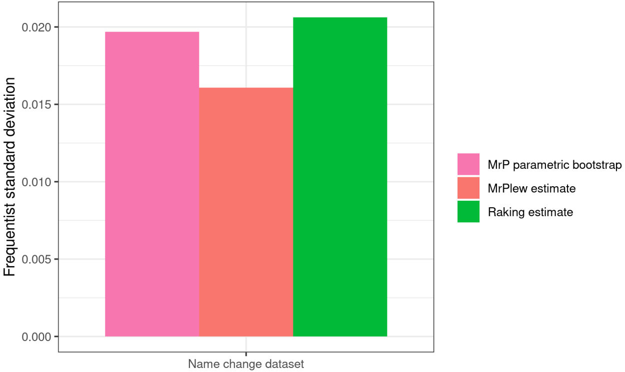

Figure 8: Frequentist standard deviation estimates

Running fifty MCMC parametric bootstraps: $\approx$ 79 hours

Computing approximate weights: 16 seconds

Real Data: Lax Philips

Analysis of national support for gay marriage.[12]

- **Target population:** US Census Public Use Microdata Sample 2000
- **Survey population:** Combined national-level polls from 2004
- **Respose:** “Do you favor allowing gay and lesbian couples to marry legally?”
- For regressors, use race, gender, age, education, state, region, and continuous statewide religion and political characteristics, including some analyst–selected interactions.

Survey observations: $N_S = 6,341$

Target observations (rows): $N_T = 9,694,541$

Uncorrected survey mean: $\frac{1}{N_S}\sum_{i=1}^{N_S} y_i = 0.333$

Raking: $\hat{\mu}_{\text{WGT}} = 0.33$

MrP: $\hat{\mu}_{\text{MrP}} = 0.337 \quad (\text{Post. sd} = 0.039)$

[12]Based on Kastellec, Lax, and Phillips (2010), see also Lax and Phillips (2009).

Covariate balance for primary effects

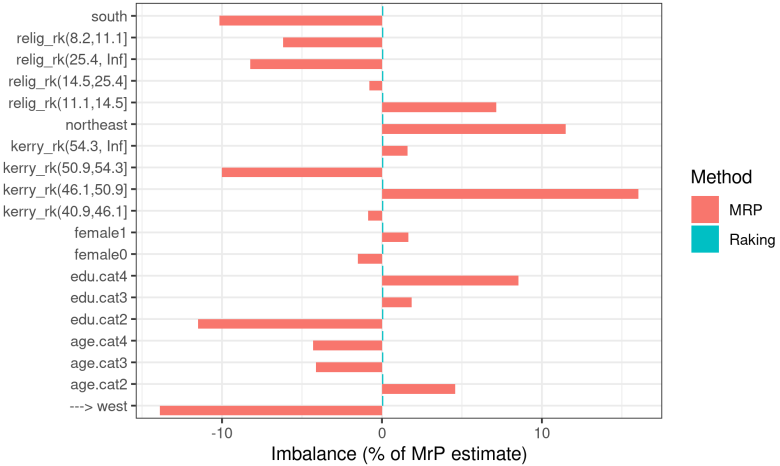

Figure 9: Imbalance plot for primary effects in the Gay Marriage dataset

Covariate balance for interaction effects

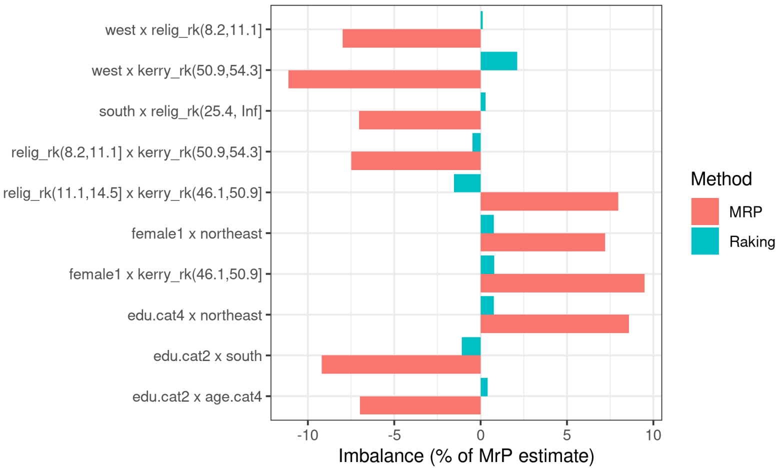

Figure 10: Imbalance plot for select interaction effects in the Gay Marriage dataset

Predictions

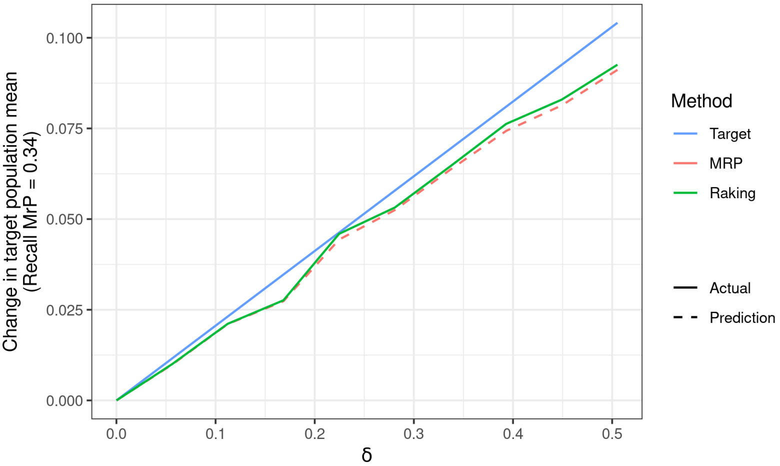

Figure 11: Predictions on binary data for the Gay Marriage dataset

Predictions and actual MCMC results

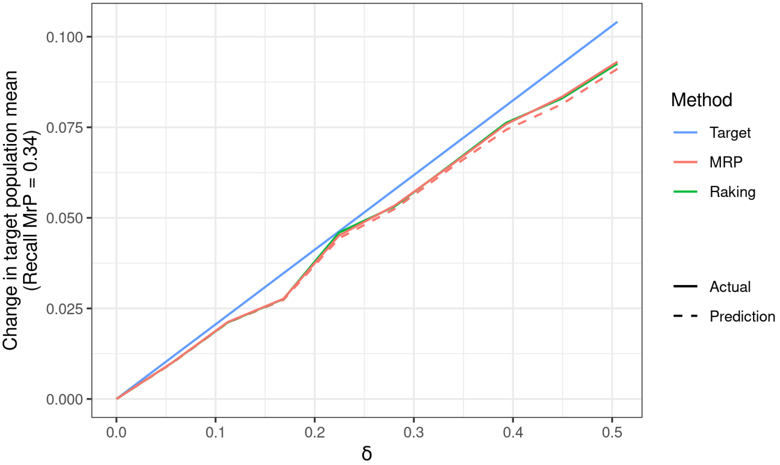

Figure 12: Predictions and refit on binary data for the Gay Marriage dataset

Running ten MCMC refits: 11 hours Computing approximate weights: 23 seconds

Some generalized diagnostics

Regression

Some generalized diagnostics

Regression	General models

Some generalized diagnostics

	Regression	General models
Consistency / Unbiased	$y = \theta^\intercal \mathbf{x} + \varepsilon$ $\tilde{y} = (\theta + \delta)^\intercal \mathbf{x} + \varepsilon$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$	

Some generalized diagnostics

	Regression	General models
Consistency / Unbiased	$y = \theta^\intercal \mathbf{x} + \varepsilon$	$y = f(\mathbf{x}, \varepsilon, \theta)$
	$\tilde{y} = (\theta + \delta)^\intercal \mathbf{x} + \varepsilon$	$\tilde{y} = f(\mathbf{x}, \varepsilon, \theta + \delta)$
	$\hat{\theta}(\tilde{y}) \stackrel{\text{check}}{=} \hat{\theta}(y) + \delta$	$\hat{\theta}(\tilde{y}) \stackrel{\text{check}}{=} \hat{\theta}(y) + \delta$

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Exogonous residuals	$y = \theta^\intercal \mathbf{x} + \varepsilon$ $\tilde{y} = y + \varepsilon z$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y)$	

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Consistency / Unbiased	$y = \theta^\intercal \mathbf{x} + \varepsilon$ $\tilde{y} = (\theta + \delta)^\intercal \mathbf{x} + \varepsilon$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$	$y = f(\mathbf{x}, \varepsilon, \theta)$ $\tilde{y} = f(\mathbf{x}, \varepsilon, \theta + \delta)$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$
Exogonous residuals	$y = \theta^\intercal \mathbf{x} + \varepsilon$ $\tilde{y} = y + \varepsilon z$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y)$	$y \sim \mathcal{P}(y\|\mathbf{x})$ and $\mathcal{P}(\mathbf{x}) = w$ $\tilde{w} = w + \delta z$ $\hat{\theta}(\tilde{w}) \overset{\text{check}}{=} \hat{\theta}(w)$

Some generalized diagnostics

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Consistency / Unbiased	$y = \theta^\intercal \mathbf{x} + \varepsilon$ $\tilde{y} = (\theta + \delta)^\intercal \mathbf{x} + \varepsilon$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$	$y = f(\mathbf{x}, \varepsilon, \theta)$ $\tilde{y} = f(\mathbf{x}, \varepsilon, \theta + \delta)$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$
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Fisher information	$\mathcal{I} :=$ Fisher information $\Sigma :=$ Score covariance $\mathcal{I}^{-1} \overset{\text{check}}{=} \Sigma$	

Some generalized diagnostics

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Fisher information	$\mathcal{I} :=$ Fisher information $\Sigma :=$ Score covariance $\mathcal{I}^{-1} \overset{\text{check}}{=} \Sigma$	$y \sim \mathcal{P}(y \vert \theta)$ $\tilde{y} \sim$ Importance sample y using $\tilde{w} = \dfrac{\mathcal{P}(y \vert \hat{\theta} + \delta)}{\mathcal{P}(y \vert \hat{\theta})}$ $\hat{\theta}(\tilde{w}) \overset{\text{check}}{=} \hat{\theta}(1) + \delta$

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Exogonous residuals	$y = \theta^\intercal \mathbf{x} + \varepsilon$ $\tilde{y} = y + \varepsilon z$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y)$	$y \sim \mathcal{P}(y\|\mathbf{x})$ and $\mathcal{P}(\mathbf{x}) = w$ $\tilde{w} = w + \delta z$ $\hat{\theta}(\tilde{w}) \overset{\text{check}}{=} \hat{\theta}(w)$
Fisher information	$\mathcal{I} :=$ Fisher information $\Sigma :=$ Score covariance $\mathcal{I}^{-1} \overset{\text{check}}{=} \Sigma$	$y \sim \mathcal{P}(y\|\theta)$ $\tilde{y} \sim$ Importance sample y using $\tilde{w} = \frac{\mathcal{P}(y\|\hat{\theta} + \delta)}{\mathcal{P}(y\|\hat{\theta})}$ $\hat{\theta}(\tilde{w}) \overset{\text{check}}{=} \hat{\theta}(1) + \delta$