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- 3. Try to capture important properties of p with simple  $\mathcal Q$ 
  - Begins with understanding how things go wrong (this paper!)
  - Hope to have our cake and eat it too (e.g. marginals and easy computation)
  - Much harder! But important, with big potential benefits

I would love to see more work like this!

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Why is n the right scaling? Why do we care about the numerical entropy gap anyway?

It's clear why variance matters. Less so the entropy gap, especially as n changes.