Locally Equivalent Weights for Bayesian MrP

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UT Austin Statistics Seminar
September 2025





Are US non-voters becoming more Republican?

Blue Rose research says yes:

"Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate."

(Blue Rose Research 2024)
(major professional pollsters)

On Data and Democracy **says no:**

"Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available."

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- Different data sources
- ⋆⋆⋆ **Different statistical methods**
    - Blue Rose uses Bayesian hierarchical modeling (MrP)
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Our contribution

We define "MrP local equivalent weights" (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key diagnostics that motivate calibration weighting.

⇒ **MrPlew provides direct comparisons between MrP and calibration weighting.**

Outline

- Introduce the statisical problem
  - Contrast CW and MrP
  - Prior work: Equivalent weights for linear models
  - Interlude: Approximate equivalent weights for some non–linear models
  - Our key idea: Locally equivalent weights for non–linear models

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  - Describe covariate balance
  - Define MrPlew weights and connect them to covariate balance
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  - Example of real-world results

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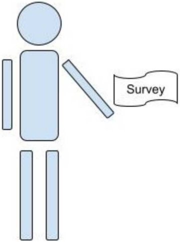
- Other uses of locally equivalent weights
  - Parital pooling
  - The meaning of negative weights
  - Frequentist variance estimation
- Future directions

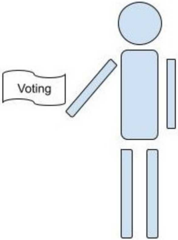
The basic problem

We have a survey population, for whom we observe:

- Covariates **x** (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to “do you support Trump”)

We want the average response in a target population, in which we observe only covariates.





Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$

Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

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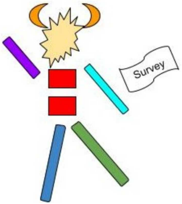

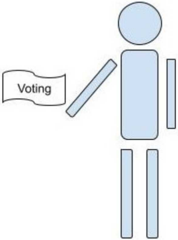

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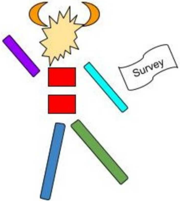

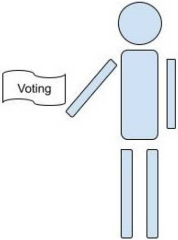

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The problem is that the populations may be very different.

Our survey results may be biased.

How can we use the covariates to say something about the target responses?

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

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Calibration weighting (CW)

▶ Choose "calibration weights" w_i using only the regressors $\mathbf{x}$ (e.g. raking weights)

Bayesian hierarchical modeling (MrP)

▶ Choose $\mathbb{E}\left[y|\mathbf{x}, \theta\right] = m(\theta^\intercal \mathbf{x})$, choose prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$ (e.g. Hierarchical logistic regression)

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- ▶ Weights give interpretable diagnostics:
  - Frequentist variability
  - Partial pooling
  - Regressor balance

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$\leftarrow$ We open this box, providing analogues of all these diagnostics

Prior work: Equivalent weights for linear models

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form $\hat{y}$:

$$\hat{\mu}_{\text{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T}\hat{y}_j = \frac{1}{N_T}\sum_{j=1}^{N_T}\underbrace{\mathbf{x}_j^\intercal\hat{\theta}}_{\text{Linear in } y_i}$$

Most existing literature on comparing CW and MrP focus on such linear models. [1]

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But what if you use a non–linear link function? Or a hierarchical model?

> *"It would also be desirable to use nonlinear methods ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods." — (Gelman 2007a)*

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Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \mathrm{Logistic}(\mathbf{x}^\intercal\theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}_{\mathrm{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$.

The map from $Y_{\mathcal{S}} = y_1, \ldots, y_{N_S} \mapsto m(\mathbf{x}_j^\intercal\hat{\theta})$ is *inherently nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal\hat{\theta})$ can be approximately linear.

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Additionally suppose $\mathbf{x} \in \mathcal{X}$ is discrete and saturated. **Then MrP is a CW estimator.**

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For $w_i^{\mathrm{MrP}} = \dfrac{N_T^c / N_T}{N_S^c / N_S}$ when $\mathbf{x}_i = c$.

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- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \text{Logistic}(\mathbf{x}^\intercal\theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal\mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$
\begin{aligned}
\hat{\mu}_{\text{MrP}} &= \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta}) \\
&\approx \int m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_T(\mathbf{x})d\mathbf{x} && \text{(Law of large numbers)} \\
&= \int \frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_S(\mathbf{x})d\mathbf{x} && \text{(Multiply by } \mathcal{P}_S(\mathbf{x})/\mathcal{P}_S(\mathbf{x})\text{)} \\
&\approx \int (\alpha^\intercal\mathbf{x})\, m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_S(\mathbf{x})d\mathbf{x} && \text{(By assumption)} \\
&\approx \alpha^\intercal \frac{1}{N_S}\sum_{i=1}^{N_S} \mathbf{x}_i m(\mathbf{x}_i^\intercal\hat{\theta}) && \text{(Law of large numbers)} \\
&= \alpha^\intercal \frac{1}{N_S}\sum_{i=1}^{N_S} \mathbf{x}_i y_i && \text{(Property of exponential family MLEs)}
\end{aligned}
$$

Nearly equivalent weights for (some) logistic regression MrP

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$$\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) = \frac{1}{N_S} \sum_{i=1}^{N_S} \underbrace{w_i^{\text{MrP}}}_{\alpha^\intercal \mathbf{x}_i} y_i + \text{Small error}$$

But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

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Key idea (informal)

If $\hat{\mu}_{\text{MrP}}$ is approximately linear, then $w_i^{\text{MrP}} \approx \frac{\partial\hat{\mu}_{\text{MrP}}}{\partial y_i}$.

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For logistic regression, could compute and analyze $\frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$ using the implicit function theorem.[2]

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \text{Logistic}(\mathbf{x}^\intercal\theta)$.
- Set a hierarchical prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$, use MCMC to draw from $\mathcal{P}(\theta|\text{Survey data})$.
- MrP is $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T} \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_j^\intercal\theta)\right]$.

No reason to think $Y_{\mathcal{S}} \mapsto \hat{\mu}_{\text{MrP}}(Y_{\mathcal{S}})$ is even approximately linear.

Butg we can still compute and analyze $w_i^{\text{MrP}} := \frac{\partial\hat{\mu}_{\text{MrP}}}{\partial y_i}$ using Bayesian sensitivity analysis![3]

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MrP locally equivalent weights (MrPlew)

For new data $\tilde{Y}_S$, form a series expansion

$$\hat{\mu}_{\mathrm{MrP}}(\tilde{Y}_S) \approx \hat{\mu}_{\mathrm{MrP}}(Y_S) + \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} (\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\mathrm{MrP}} := \frac{\partial \hat{\mu}_{\mathrm{MrP}}}{\partial y_i}.$$

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Our task is to rigorously show that even such local weights can be used diagnostically.

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

The weights can look very different!

Does this mean anything? Are the differences important?

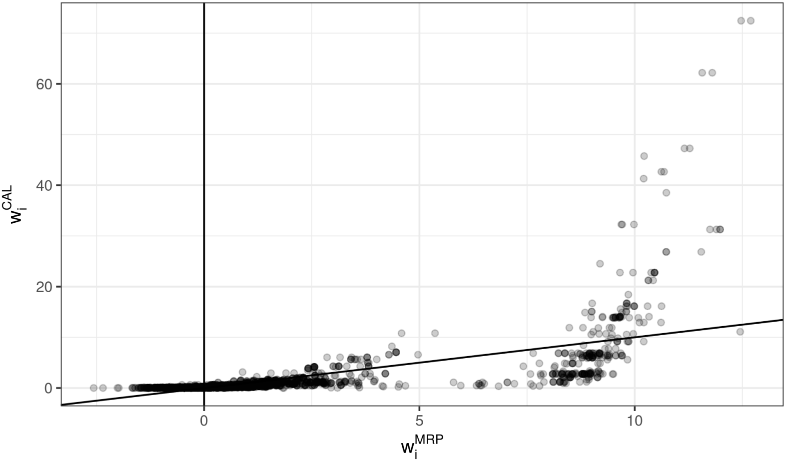

Figure 1: Comparison between raking and MrPlew weights for the Name Change dataset

What are we weighting for?[4]

$$\text{Target average response} = \frac{1}{N_T}\sum_{j=1}^{N_T} y_j \approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i = \text{Weighted survey average response}$$

We can't check this, because we don't observe y_j.

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We can't check this, because we don't observe y_j. But we can check whether:

$$\frac{1}{N_T}\sum_{j=1}^{N_T} \mathbf{x}_j = \frac{1}{N_S}\sum_{i=1}^{N_S} w_i \mathbf{x}_i$$

Such weights satisfy "covariate balance" for **x**.

You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

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Such weights satisfy "covariate balance" for $\mathbf{x}$.

You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

Raking calibration weights

"Raking" selects weights that

- Are as "close as possible" to some reference weights
- Under the constraint that they balance some selected regressors.

[4] Pun attributable to Solon, Haider, and Wooldridge (2015)

Balance checks as sensitivity analysis

One reason to balance $f(\mathbf{x})$ is because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

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Balance–informed sensitivity check (BISC) (informal)

Pick a small $\delta > 0$ and an $f(\cdot)$. Define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}\left[\mu(\tilde{y}) - \mu(y)|\mathbf{x}\right] = \frac{1}{N_T}\sum_{j=1}^{N_T}\left(\mathbb{E}\left[\tilde{y}|\mathbf{x}_p\right] - \mathbb{E}\left[y|\mathbf{x}_p\right]\right) = \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change for observed $\tilde{y}, y$:

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\substack{\text{Replace weighted averages} \\ \text{with changes in an estimator}}} \overset{\textbf{check}}{\approx} \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

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When $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{CW}}(\cdot)$, BISC recovers the standard covariate balance check.

We will use $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{MrP}}(\cdot)$.

BISC for MrP

Suppose I have $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.
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Problem: $\hat{\mu}_{\mathrm{MrP}}(\cdot)$ is computed with MCMC.

- Each MCMC run typically takes hours, and
- Output is noisy, and $\hat{\mu}_{\mathrm{MrP}}(\tilde{y}) - \hat{\mu}_{\mathrm{MrP}}(y)$ may be small.

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Form the first-order Taylor series approximation

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Computation: The weights are given by weighted averages of posterior covariances[5].

They can be easily computed with standard software[6] **without re–running MCMC**.

[5] **G.**, Broderick, and Jordan 2018.
[6] We use `brms` (Bürkner 2017).

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Use in BISC: For a wide set of judiciously chosen $f(\cdot)$, check

$$\delta \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} f(\mathbf{x}_i) \overset{\text{check}}{\approx} \delta \frac{1}{N_T} \sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

Generating $\tilde{y}$

- We have defined BISC in terms of $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$
- We have approximated $\hat{\mu}_{\text{MrP}}(\tilde{y}) - \hat{\mu}_{\text{MrP}}(y)$ for $\tilde{y} \approx y$

How to get such a $\tilde{y}$? **Recall y is binary!** Two approaches:

Option 1: Force $\tilde{y}$ to be binary.

1. Make *some* guess $\hat{m}(\mathbf{x}) \approx \mathbb{E}\left[y|\mathbf{x}\right]$
   - E.g. Posterior mean, or
   - Shrunken posterior mean, or
   - Some values that gives the same posterior
2. Take $u_n \overset{iid}{\sim} \text{Unif}(0, 1)$
3. Assume $y_n = \mathbb{I}\left(u_n \le \hat{m}(\mathbf{x}_n)\right)$
4. Draw $u_n | y_n$
5. Set $\tilde{y}_n = \mathbb{I}\left(u_n \le \hat{m}(\mathbf{x}_n) + \delta \mathbf{x}_n\right)$

Option 2: Allow $\tilde{y}$ to take generic values.

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- Realistic
- Have to pick $\hat{m}(\mathbf{x})$
- $\tilde{Y} - Y_{\mathcal{S}}$ not infinitesimally small
- **Sanity check for theory**

Option 2: Allow $\tilde{y}$ to take generic values.

1. Set $\tilde{y}_n = y_n + \delta f(\mathbf{x}_n)$.

- Not realistic
- No additional assumptions
- $\tilde{Y} - Y_{\mathcal{S}}$ may be infinitesimally small
- **Use for theory**

Theory

BISC Theorem: (sketch)

Take $\tilde{y}_n = y_n + \delta f(\mathbf{x}_n)$.

We state conditions for Bayesian hierarchical logistic regression under which

$$\left| \hat{\mu}_{\mathrm{MrP}}(Y_{\mathcal{S}}) - \hat{\mu}_{\mathrm{MrP}}(Y_{\mathcal{S}}) - \delta \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} f(\mathbf{x}_i) \right| = \text{Small?}$$

[5] Measurable functions with uniformly bounded $\mathbb{E}\left[\mathbf{x} f(\mathbf{x})\right]$.

[6] **G.** and Broderick 2024; Kasprzak, **G.**, and Broderick 2025.

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For a very broad class[5] of $\mathcal{F}$.

Uniformity justifies searching for "imabalanced" f.

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The uniformity result builds on our earlier work on uniform and finite–sample error bounds for Bernstein–von Mises theorem–like results[6].

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[6] **G.** and Broderick 2024; Kasprzak, **G.**, and Broderick 2025.

Real Data: Marital Name Change Survey

Analysis of changing names after marriage[7].

- **Target population:** ACS survey of US population 2017–2022[8]
- **Survey population:** Marital Name Change Survey (from Twitter)[9]
- **Respose:** Did the female partner keep their name after marriage?
- For regressors, use bins of age, education, state, and decade married.

Survey observations:	$N_S = 4,364$
Target observations (rows):	$N_T = 4,085,282$

Uncorrected survey mean:	$\frac{1}{N_S}\sum_{i=1}^{N_S} y_i = 0.462$
Raking:	$\hat{\mu}_{\text{CW}} = 0.263$
MrP:	$\hat{\mu}_{\text{MrP}} = 0.288 \quad (\text{Post. sd} = 0.0169)$

[7] Based on Alexander (2019).

[8] Ruggles et al. 2024.

[9] Cohen 2019.

Covariate balance for primary effects

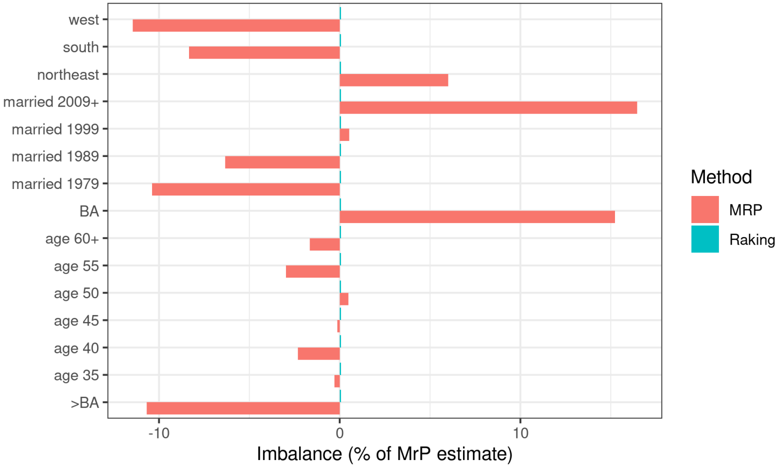


Figure 2: Imbalance plot for primary effects in the Name Change dataset

Covariate balance for interaction effects

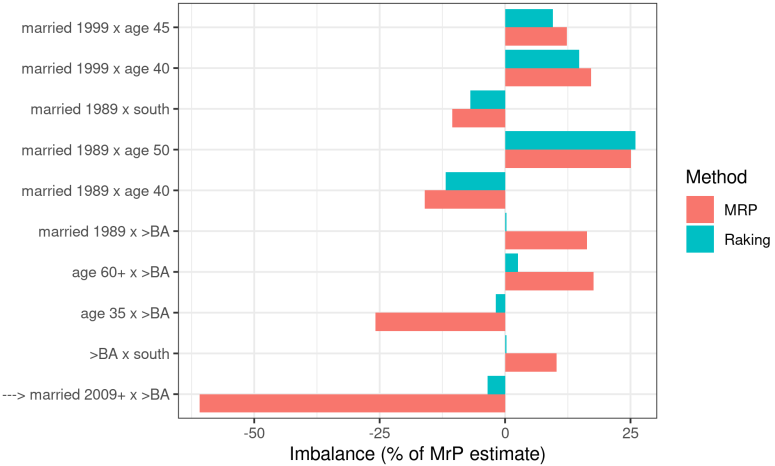


Figure 3: Imbalance plot for select interaction effects in the Name Change dataset

Predictions

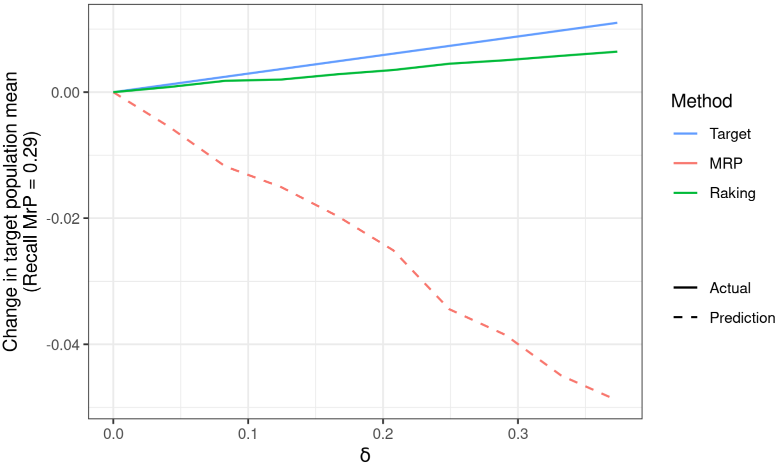


Figure 4: Predictions on binary data for the Name Change dataset

Predictions and actual MCMC results

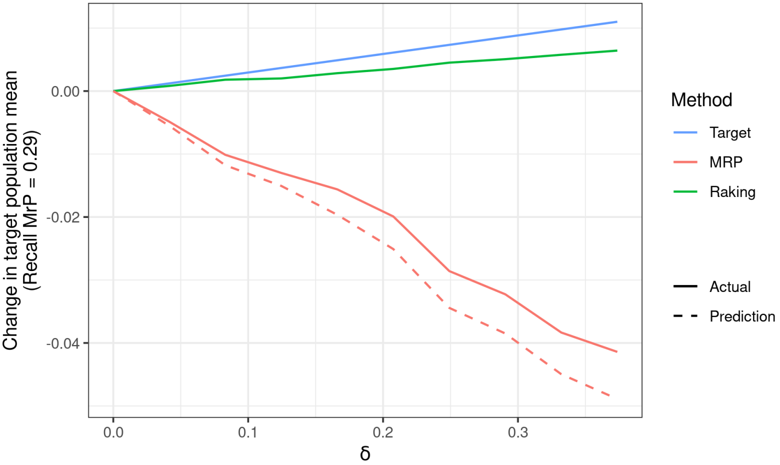


Figure 5: Predictions and refit on binary data for the Name Change dataset

Running ten MCMC refits: 10 hours Computing approximate weights: 16 seconds

Real Data: Lax Philips

Analysis of national support for gay marriage.[10]

- **Target population:** US Census Public Use Microdata Sample 2000
- **Survey population:** Combined national-level polls from 2004
- **Respose:** "Do you favor allowing gay and lesbian couples to marry legally?"
- For regressors, use race, gender, age, education, state, region, and continuous statewide religion and political characteristics, including some analyst–selected interactions.

Survey observations:	$N_S = 6,341$
Target observations (rows):	$N_T = 9,694,541$

Uncorrected survey mean:	$\frac{1}{N_S} \sum_{i=1}^{N_S} y_i = 0.333$
Raking:	$\hat{\mu}_{\mathrm{CW}} = 0.33$
MrP:	$\hat{\mu}_{\mathrm{MrP}} = 0.337 \quad (\text{Post. sd} = 0.039)$

[10] Based on Kastellec, Lax, and Phillips (2010), see also Lax and Phillips (2009).

Covariate balance for primary effects

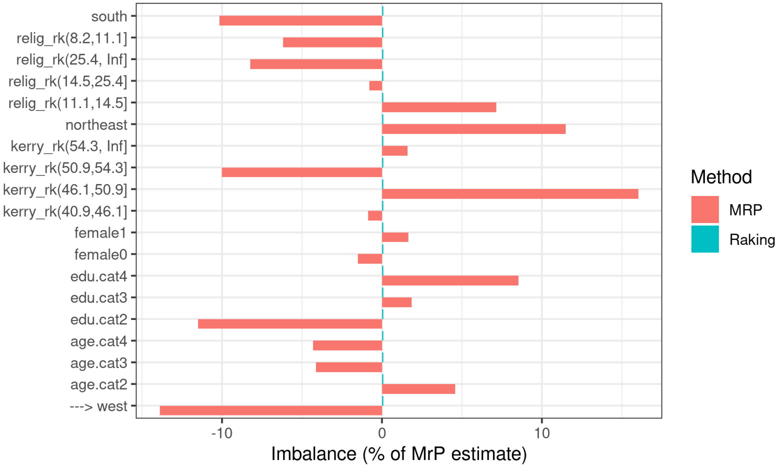


Figure 6: Imbalance plot for primary effects in the Gay Marriage dataset

Covariate balance for interaction effects

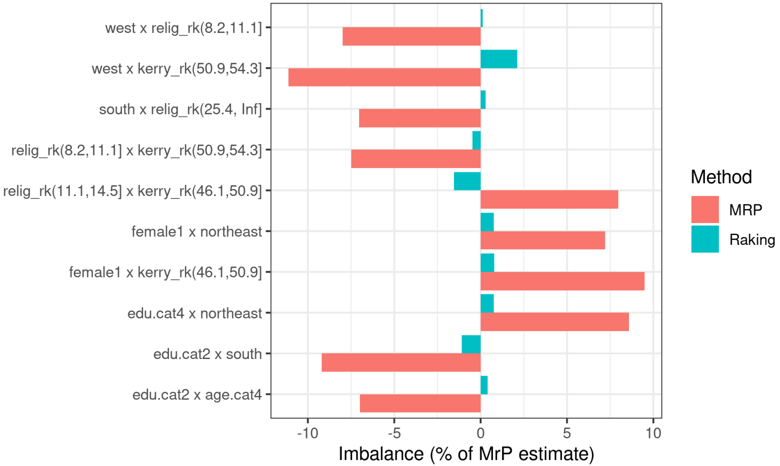


Figure 7: Imbalance plot for select interaction effects in the Gay Marriage dataset

Predictions

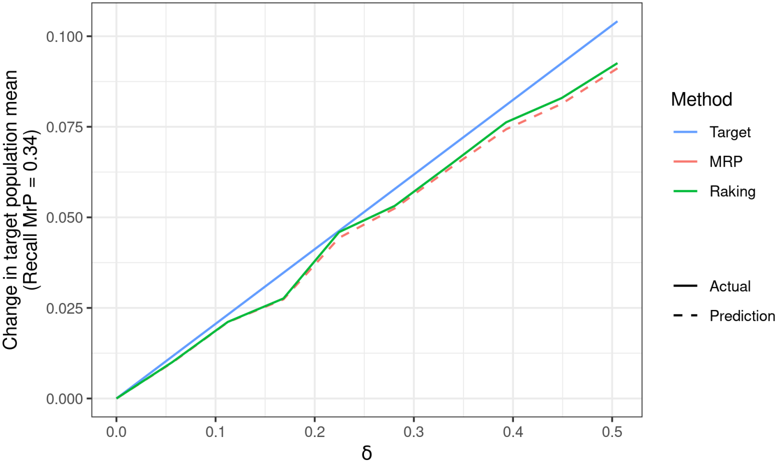


Figure 8: Predictions on binary data for the Gay Marriage dataset

Predictions and actual MCMC results

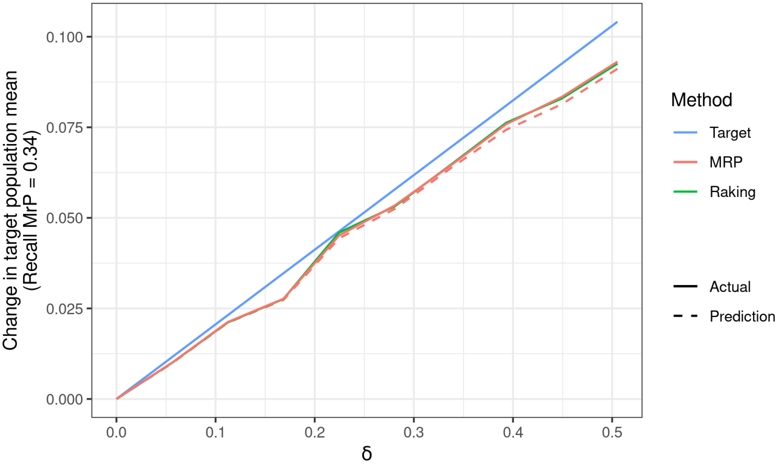


Figure 9: Predictions and refit on binary data for the Gay Marriage dataset

Running ten MCMC refits: 11 hours Computing approximate weights: 23 seconds

Standard error estimation

Does this mean anything?

Yes: We can meaningful sum these weights against regressors.

What else might it mean?

Does the spread relate to frequentist variance?

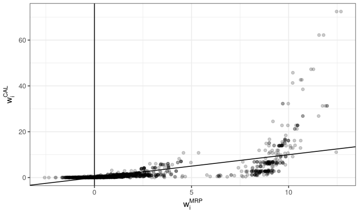


Figure 10: Comparison between raking and MrPlew weights for the Name Change dataset

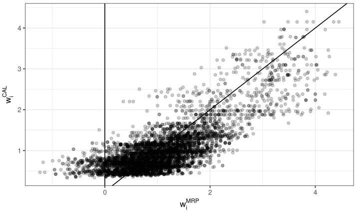


Figure 11: Comparison between raking and MrPlew weights for the Gay Marriage dataset

Standard error estimation

Standard error consistency theorm: (sketch)

For Bayesian hierarchical logictic regression, define

$$\varepsilon_n = y_n - \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_n^\intercal \theta)\right] \quad \text{and} \quad \psi_n := N_S w_n^{\mathrm{MrP}} \varepsilon_n.$$

We state mild conditions under which, as $N \to \infty$,

$$\sqrt{N}\left(\hat{\mu}_{\mathrm{MrP}} - \mu_\infty\right) \to \mathcal{N}\left(0, V\right) \quad \text{for some } \mu_\infty \text{ and variance } V \text{, and}$$

$$\frac{1}{N_S}\sum_{i=1}^{N_S}(\psi_n - \overline{\psi})^2 \to V.$$

The use of w_n^{MrP} is exactly analogous to the use of raking weights for standard error estimation.

This builds on our earlier work on the Bayesian infinitesimal jackknife[11].

[11] **G.** and Broderick 2024.

Standard error estimation

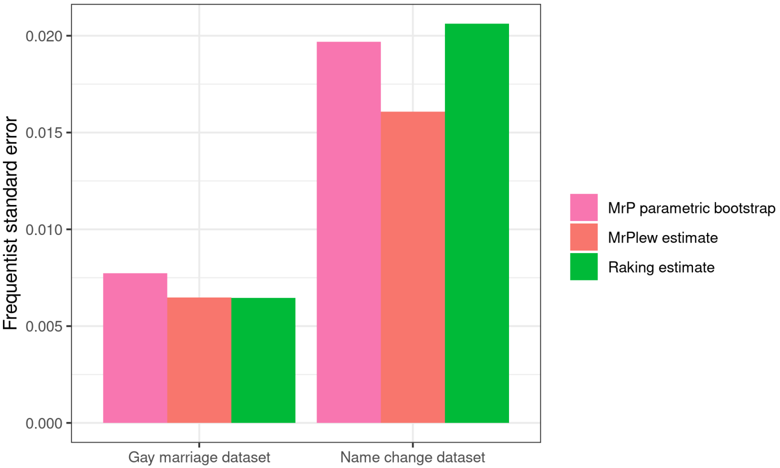


Figure 12: Frequentist standard deviation estimates

Covariate balance corresponds by BISC.
Weight spread measures frequentist standard errors.

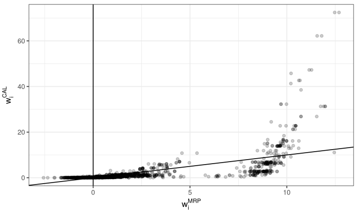


Figure 13: Comparison between raking and MrPlew weights for the Name Change dataset

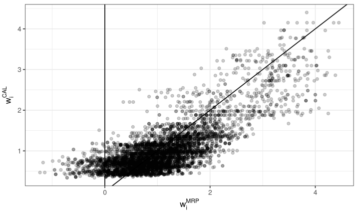
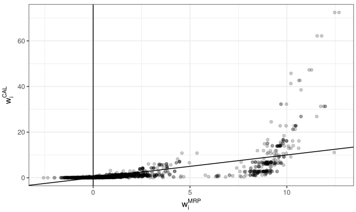


Figure 14: Comparison between raking and MrPlew weights for the Gay Marriage dataset

Other uses

Covariate balance corresponds by BISC.
Weight spread measures frequentist standard errors.

Partial pooling is BISC with different targets (e.g. sub–populations).
Negative weights indicate *non–monotonicity* of $Y_{\mathcal{S}} \mapsto \hat{\mu}_{\mathrm{MrP}}(Y_{\mathcal{S}})$.



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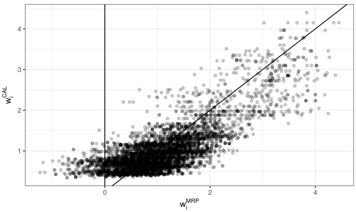

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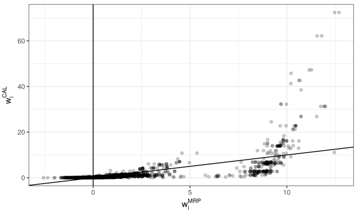
Other checks?



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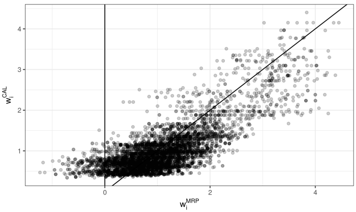

Figure 14: Comparison between raking and MrPlew weights for the Gay Marriage dataset

Future work

Notice that there was no discussion of misspecification!

Calibration weights (typically) do not depend on $Y_{\mathcal{S}}$.

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But the high level idea can be extended much more widely:

1. Assume your initial model was accurate
2. Select some perturbation your model should be able to capture
3. Use local sensitivity to detect whether the change is what you expect
4. If the change is not what you expect, either (1) or (2) was wrong

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Such checks recover generlized versions of many standard diagnostics for linear models.

Examples:

- Additive parameter shifts $\leftrightarrow$ Unbiasedness
- Invariance to survey data weighting $\leftrightarrow$ Regressor + residual orthogonality
- Importance sampling $\leftrightarrow$ Sandwich covariance $\stackrel{?}{=}$ Inverse Fisher information

Future work

Student contributions and ongoing work:

- **Vladimir Palmin** is working on extending MrPlew to `lme4`
- **Sequoia Andrade** is working on generalizing to other local sensitivity checks
- **Lucas Schwengber** is working on novel flow–based techniques for local sensitivity
- **(Currently recruiting!)** Doubly–robust Bayesian Hierarchical MrP

Vladimir Palmin

Sequoia Andrade

Lucas Schwengber

Preprint and R package (hopefully) coming soon!

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