

# **An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Make a Big Difference?**

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# Dropping data: Motivation

More data & cheaper computation  $\Rightarrow$

Statistical analyses are playing larger roles in decision making.

Decisions are important: We want **trustworthy** conclusions.

Data / models not always perfect: We want **robust** conclusions.

Would you be concerned if you could **reverse your conclusion** by removing a **small proportion** (say, 0.1%) of your data?

**Running example:** Angelucci et al. [2015], a randomized controlled trial study of the efficacy of microcredit based on 16,560 data points.

We can reverse the studies qualitative conclusions by removing 15 observations ( $< 0.1\%$  of the data).

**How do we find sets of influential points?** Difficult in general!

We provide a **automatic approximation** with finite-sample guarantees.

Studying the approximation reveals the causes of non-robustness.

# Dropping data: Mexico Microcredit

Consider Angelucci et al. [2015], a randomized controlled trial study of the efficacy of microcredit in Mexico based on 16,560 data points.

The variable “Beta” estimates the effect of microcredit in US dollars.

	Beta (SE)
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## The culprit is signal to noise ratio.

By the end of the talk, we will see that the sensitivity is due to

- High variability of the outcome (household profit) relative to
- A small signal driving the conclusion (statistical significance)



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Would you be concerned if you could **reverse your conclusion** by removing a **small proportion** (say, 0.1%) of your data?

Not always! But sometimes, surely yes.

Thinking without random noise can be helpful.

Suppose you have a farm, and want to know whether your average yield is greater than 170 bushels per acre. At harvest, you measure 200 bushels per acre.

- Scenario one: If your yield is greater than 170 bushels per acre, you make a profit.
  - Don't care about sensitivity to small subsets
- Scenario two: You want to recommend your farming methods to a friend across the valley.
  - Might care about sensitivity to small subsets

For example, often in economics:

- Small fractions of data are missing not-at-random,
- Policy population is different from analyzed population,
- We report a convenient summary (e.g. mean) of a complex effect,
- Models are stylized proxies of reality.

# Which estimators do we study?

**Z-estimators.** Suppose we have  $N$  data points  $\vec{d} = d_1, \dots, d_N$ . Then:

$$\hat{\theta} := \vec{\theta} \text{ such that } \sum_{n=1}^N G(\vec{\theta}, d_n) = 0_P.$$

**Examples:** MLE, OLS, VB, &c (all minimizers of smooth empirical loss).

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**Function of interest.** Qualitative decision based on  $\phi(\hat{\theta}) \in \mathbb{R}$ . E.g.:

- A particular component:  $\phi(\theta) = \theta_d$
- The end of a confidence interval:  $\phi(\theta) = \theta_d + \frac{1.96}{\sqrt{N}} \hat{\sigma}(\hat{\theta})$

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Fix a proportion  $0 < \alpha \ll 1$  of points to drop and find a set  $\mathcal{S} \subset \{1, \dots, N\}$  with  $|\mathcal{S}| \leq \lfloor \alpha N \rfloor$  that extremizes  $\phi(\hat{\theta})$  when dropped.

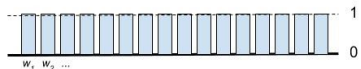
- **Problem:** There are many sets with  $|\mathcal{S}| \leq \lfloor \alpha N \rfloor$ .
  - E.g., in Angelucci et al. [2015],  $\binom{16,560}{15} \approx 1.5 \cdot 10^{51}$
- **Problem:** Evaluating  $\phi(\hat{\theta}(\vec{d}_{-\mathcal{S}}))$  requires an estimation problem.
  - E.g., in Angelucci et al. [2015] computing the OLS estimator.
  - Other examples are even harder (VB, machine learning)

**An approximation is needed!**

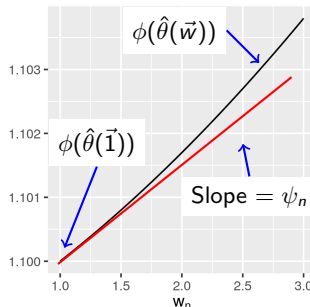
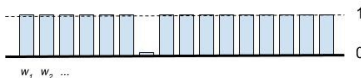
# Which estimators do we study?

$$\hat{\theta} := \vec{\theta} \text{ such that } \sum_{n=1}^N G(\vec{\theta}, d_n) = 0_P.$$

Original weights:  $\vec{1} = (1, \dots, 1)$



Leave points out by setting their elements of  $\vec{w}$  to zero.



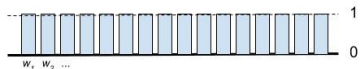
The slopes  $\psi_n := \left. \frac{\partial \phi(\hat{\theta}(\vec{w}))}{\partial w_n} \right|_{\vec{1}}$  are values of the **empirical influence function** [Hampel, 1986]. We call them “influence scores.”

Second-order derivatives control the error of the linear approximation.

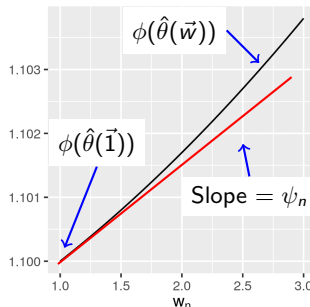
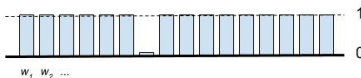
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$$\hat{\theta}(\vec{w}) := \vec{\theta} \text{ such that } \sum_{n=1}^N \vec{w}_n G(\vec{\theta}, d_n) = 0_P.$$

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# Taylor series approximation.

**Problem:** How large can you make  $\phi(\hat{\theta}(\vec{w}))$  leaving out no more than  $\lfloor \alpha N \rfloor$  points? **Combinatorially hard!**

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To simplify the search over  $\vec{w}$ , we form the Taylor series approximation:

$$\phi(\hat{\theta}(\vec{w})) \approx \phi^{\text{lin}}(\vec{w}) := \phi(\hat{\theta}(\vec{1})) + \sum_{n=1}^N \psi_n(\vec{w}_n - 1)$$

**Approximate solution:** How large can you make  $\phi^{\text{lin}}(\vec{w})$  leaving out no more than  $\lfloor \alpha N \rfloor$  points? **Easy!**

The most influential points for  $\phi^{\text{lin}}(\vec{w})$  have the most negative  $\psi_n$ .

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We provide **finite-sample theory** showing that

$$\left| \phi(\hat{\theta}(\vec{w})) - \phi^{\text{lin}}(\vec{w}) \right| = O \left( \left\| \frac{1}{N}(\vec{w} - \vec{1}) \right\|_2^2 \right) = O(\alpha) \text{ as } \alpha \rightarrow 0.$$

## How to compute the influence scores $\psi_n$ ?

By the chain rule,  $\psi_n = \left. \frac{\partial \phi(\hat{\theta}(\vec{w}))}{\partial \vec{w}_n} \right|_{\vec{1}} = \left. \frac{d\phi(\theta)}{d\theta^T} \right|_{\hat{\theta}} \left. \frac{\partial \hat{\theta}(\vec{w})}{\partial \vec{w}_n} \right|_{\vec{1}}.$

Recall that  $\hat{\theta}(\vec{w}) := \vec{\theta}$  such that  $\sum_{n=1}^N \vec{w}_n G(\vec{\theta}, d_n) = 0_P.$

The **implicit function theorem** expresses  $\left. \frac{\partial \hat{\theta}(\vec{w})}{\partial \vec{w}_n} \right|_{\vec{1}}$  as a linear system.

Computation of  $\psi_n$  is fully automatable from a software implementation of  $G(\cdot, \cdot)$  and  $\phi(\cdot)$  with **automatic differentiation** [Baydin et al., 2017].

We have an R package, `rgiordan/zaminfluence`, for OLS and IV.

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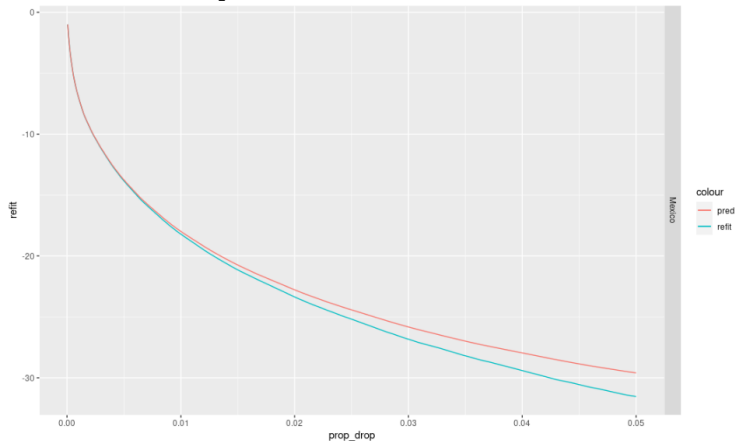
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- 6 **Optional:** Compute  $\hat{\theta}(\vec{w}^*)$ , and verify that  $\phi(\hat{\theta}(\vec{w}^*)) - \phi(\hat{\theta}) \geq \Delta$ .

Mexico example:

See `microcredit_profit_sandbox.R`.



# Selected experimental results.

Study case	Original estimate (SE)	Target change	Refit estimate	Observations dropped
Mexico	-4.549 (5.879)	Sign change	<b>0.398 (3.194)</b>	1 = 0.01%
		Significance change	<b>-10.962 (5.565)*</b>	14 = 0.08%
		Significant sign change	<b>7.030 (2.549)*</b>	15 = 0.09%

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		Significance change	<b>4.806 (3.684)</b>	435 = 4.14%
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Table: Medicaid profit results [Finkelstein et al., 2012]

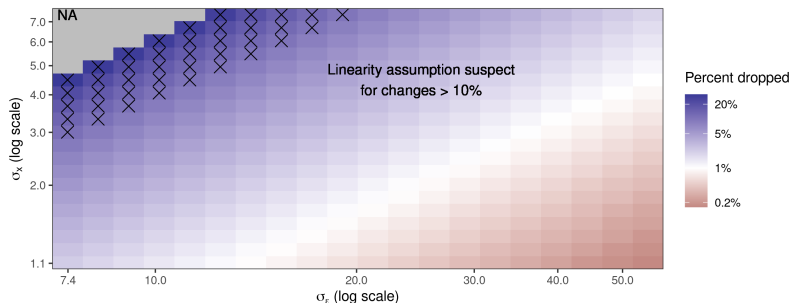
# A simulation

For  $N = 5,000$  data points, compute the OLS estimator from:

Regressors  
 $x_n \sim \mathcal{N}(0, \sigma_x^2)$

Residuals  
 $\varepsilon_n \sim \mathcal{N}(0, \sigma_\varepsilon^2)$

Responses  
 $y_n = 0.5x_n + \varepsilon_n$



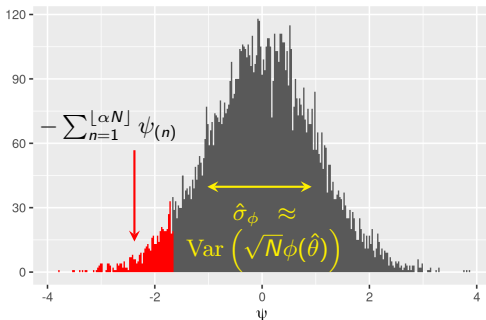
**Figure:** The approximate perturbation inducing proportion at differing values of  $\sigma_x$  and  $\sigma_\varepsilon$ . Red colors indicate datasets whose sign can be predicted to change when dropping less than 1% of datapoints. The grey areas indicate  $\hat{\Psi}_\alpha = \text{NA}$ , a failure of the linear approximation to locate any way to change the sign.

# What makes an estimator non-robust? A tail sum.

We show that  $\phi^{\text{lin}}(\vec{w}^*) - \phi(\hat{\theta}) = -\sum_{n=1}^{\lfloor \alpha N \rfloor} \psi_{(n)} =: \hat{\sigma}_{\phi} \hat{\mathcal{T}}_{\alpha}$  where

- The “noise”  $\hat{\sigma}_{\phi}^2 \rightarrow \text{Var}(\sqrt{N}\phi)$ 
  - $\hat{\sigma}_{\phi}^2$  is the robust “sandwich” variance estimator [Hampel, 1986]
- The “shape”  $\hat{\mathcal{T}}_{\alpha} \leq \sqrt{\alpha(1-\alpha)}$  determined by  $\psi_n$  distribution
  - $\hat{\mathcal{T}}_{\alpha}$  converges to a nonzero constant

Influence score histogram (N = 10000,  $\alpha = 0.05$ )



# Example.

Report non-robustness if:

$$\phi^{\text{lin}}(\vec{w}^*) - \phi(\hat{\theta}) = \hat{\sigma}_{\phi} \hat{\mathcal{T}}_{\alpha} \geq \Delta \quad \Leftrightarrow \quad \frac{\Delta}{\hat{\sigma}_{\phi}} \leq \hat{\mathcal{T}}_{\alpha}.$$

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Let's analyze with  $\alpha = 0.01 = 1\%$ .

$$\begin{array}{llll}
 \phi(\hat{\theta}) = & -0.029 & (\text{Increase QOI by defn}) & \Delta = 0.029 \\
 \hat{\sigma}_{\phi} = & 0.766 & (\text{Noise}) & \frac{1}{\sqrt{N}} \hat{\sigma}_{\phi} = 0.005 \quad (\text{SE}) \\
 \hat{\mathcal{T}}_{\alpha} = & 0.046 & (\text{Shape}) & \frac{1.96}{\sqrt{N}} = 0.0128 \rightarrow 0 \text{ as } N \rightarrow \infty \\
 \hat{\mathcal{T}}_{\alpha} \hat{\sigma}_{\phi} = & 0.035 & (\text{Data dropping sensitivity}) & \frac{1.96}{\sqrt{N}} \hat{\sigma}_{\phi} = 0.010 \quad (\text{SE sensitivity})
 \end{array}$$

The noise is much larger than the signal  $\Rightarrow$  Sensitive to data dropping.

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**Corollary: Leave- $\lfloor \alpha N \rfloor$ -out is different from standard errors.**

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**Corollary: Gross outliers primarily affect robustness through  $\hat{\sigma}_{\phi}$ .**

See paper for more details.

# Other forms of robustness

We proceeded as follows:

- 1 Took presence of datapoints as a model input,
- 2 Formed an automatically-computable differential approximation,
- 3 Provided theory by analyzing higher-order derivatives,
- 4 Studied its effectiveness in problems with open-access data.

## **Presence of datapoints is only one model input of many!**

- Prior sensitivity in Bayesian nonparametrics [Giordano et al., 2021]
- Model sensitivity of MCMC output [Gustafson, 2000, Giordano et al., 2018]
- Cross-validation [Giordano et al., 2019, Wilson et al., 2020]
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- Frequentist variances of MCMC posteriors (in progress)

Tamara Broderick, Ryan Giordano, Rachael Meager (alphabetical authors)  
“An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Change Conclusions?”

<https://arxiv.org/abs/2011.14999>

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Open-source software:

R package `zaminfluence`    <https://github.com/rgiordan/zaminfluence>

Python package `vittles`    <https://github.com/rgiordan/vittles>

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Some related content can be found on my blog:

<https://rgiordan.github.io/>

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