# An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Make a Big Difference?

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# Dropping data: Motivation

More data & cheaper computation  $\Rightarrow$  Statistical analyses are playing larger roles in decision making.

Decisions are important: We want **trustworthy** conclusions. Data / models not always perfect: We want **robust** conclusions.

Would you be concerned if you could **reverse your conclusion** by removing a **small proportion** (say, 0.1%) of your data?

**Running example:** Angelucci et al. [2015], a randomized controlled trial study of the efficacy of microcredit based on 16,560 data points. We can reverse the studies qualitative conclusions by removing 15 observations (< 0.1% of the data).

How do we find sets of influential points? Difficult in general!

We provide a **automatic approximation** with finite-sample guarantees.

The approximation gives the causes of sensitivity to data dropping.

Consider Angelucci et al. [2015], a randomized controlled trial study of the efficacy of microcredit in Mexico based on 16,560 data points. The variable "Beta" estimates the effect of microcredit in US dollars.

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### The culprit is signal to noise ratio.

By the end of the talk, we will see that the sensitivity is due to

- High variability of the outcome (hosehold profit) relative to
- A small signal driving the conclusion (statistical significance)

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Would you be concerned if you could **reverse your conclusion** by removing a **small proportion** (say, 0.1%) of your data? Not always! But sometimes, surely yes.

Thinking without random noise can be helpful.

Suppose you have a farm, and want to know whether your average yield is greater than 170 bushels per acre. At harvest, you measure 200 bushels per acre.

- Scenario one: If your yield is greater than 170 bushels per acre, you
  make a profit.
  - Don't care about sensitivity to small subsets
- Scenario two: You want to recommend your farming methods to a friend across the valley.
  - Might care about sensitivity to small subsets

### For example, often in economics:

- Small fractions of data are missing not-at-random,
- Policy population is different from analyzed population,
- We report a convenient summary (e.g. mean) of a complex effect,
- Models are stylized proxies of reality.

# Which estimators do we study?

**Z-estimators.** Suppose we have N data points  $\vec{d} = d_1, \dots, d_N$ . Then:

$$\hat{\theta} := \vec{\theta}$$
 such that  $\sum_{n=1}^{N} G(\vec{\theta}, d_n) = 0_P$ .

Examples: MLE, OLS, VB, &c (all minimizers of smooth empirical loss).

**Function of interest.** Qualitative decision based on  $\phi(\hat{\theta}) \in \mathbb{R}$ . E.g.:

- A particular component:  $\phi(\theta) = \theta_d$
- The end of a confidence interval:  $\phi(\theta) = \theta_d + \frac{1.96}{\sqrt{N}}\hat{\sigma}(\hat{\theta})$

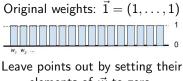
Fix a proportion  $0 < \alpha \ll 1$  of points to drop and find a set  $\mathcal{S} \subset \{1, \dots N\}$  with  $|\mathcal{S}| \leq \lfloor \alpha N \rfloor$  that extremizes  $\phi(\hat{\theta})$  when dropped.

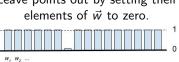
- **Problem:** There are many sets with  $|\mathcal{S}| \leq \lfloor \alpha N \rfloor$ . • E.g., in Angelucci et al. [2015],  $\binom{16,560}{15} \approx 1.5 \cdot 10^{51}$
- ullet Problem: Evaluating  $\phi(\hat{ heta}(\vec{d}_{-\mathcal{S}}))$  requires an estimation problem.
  - E.g., in Angelucci et al. [2015] computing the OLS estimator.
  - Other examples are even harder (VB, machine learning)

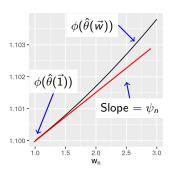
### An approximation is needed!

# Which estimators do we study?

$$\hat{\theta} := \vec{\theta} \text{ such that } \sum_{n=1}^{N} G(\vec{\theta}, d_n) = 0_P.$$





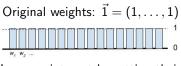


The slopes  $\psi_n := \frac{\partial \phi(\hat{\theta}(\vec{w}))}{\partial \vec{w}_n} \Big|_{\vec{1}}$  are values of the **empirical influence** function [Hampel, 1986]. We call them "influence scores."

Second-order derivatives control the error of the linear approximation.

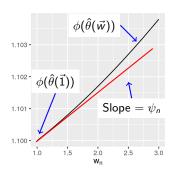
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$$\hat{\theta}(\vec{w}) := \vec{\theta}$$
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Leave points out by setting their elements of  $\vec{w}$  to zero.





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**Problem:** How large can you make  $\phi(\hat{\theta}(\vec{w}))$  leaving out no more than  $\lfloor \alpha N \rfloor$  points? **Combinatorially hard!** 

To simplify the search over  $\vec{w}$ , we form the Taylor series approximation:

$$\phi(\hat{\theta}(\vec{w})) \approx \phi^{\text{lin}}(\vec{w}) := \phi(\hat{\theta}(\vec{1})) + \sum_{n=1}^{N} \psi_n(\vec{w}_n - 1)$$

**Approximate solution:** How large can you make  $\phi^{\text{lin}}(\vec{w})$  leaving out no more than  $|\alpha N|$  points? **Easy!** 

The most influential points for  $\phi^{\text{lin}}(\vec{w})$  have the most negative  $\psi_n$ .

The  $\psi_n$  are automatically computable using the **implicit function** theorem and automatic differentiation.

We provide finite-sample theory showing that

$$\left|\phi(\hat{\theta}(\vec{w})) - \phi^{\mathrm{lin}}(\vec{w})\right| = O\left(\left\|\frac{1}{N}(\vec{w} - \vec{1})\right\|_{2}^{2}\right) = O\left(\alpha\right) \text{ as } \alpha \to 0.$$

#### Procedure:

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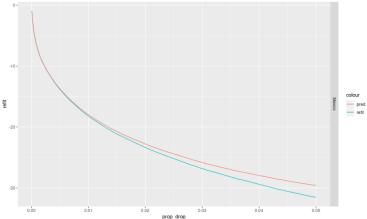
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- **Optional:** Compute  $\hat{\theta}(\vec{w}^*)$ , and verify that  $\phi(\hat{\theta}(\vec{w}^*)) \phi(\hat{\theta}) \geq \Delta$ .

### Mexico example:

See  $microcredit_profit_sandbox.R.$ 



# Selected experimental results.

Study case	Original estimate (SE)	Target change	Refit estimate	Observations dropped
Mexico	-4.549 (5.879)	Sign change Significance change Significant sign change	0.398 (3.194) -10.962 (5.565)* 7.030 (2.549)*	1 = 0.01% $14 = 0.08%$ $15 = 0.09%$

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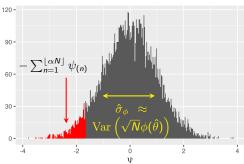
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# What makes an estimator non-robust? A tail sum.

We show that 
$$\phi^{\text{lin}}(\vec{w}^*) - \phi(\hat{\theta}) = -\sum_{n=1}^{\lfloor \alpha N \rfloor} \psi_{(n)} =: \hat{\sigma}_{\phi} \hat{\mathcal{T}}_{\alpha}$$
 where

- The "noise"  $\hat{\sigma}_{\phi}^2 o \mathrm{Var}(\sqrt{N}\phi)$ 
  - $\hat{\sigma}_{\phi}^2=$  is the robust "sandwich" variance estimator [Hampel, 1986]
- The "shape"  $\hat{\mathscr{T}}_{\alpha} \leq \sqrt{\alpha(1-\alpha)}$  determined by  $\psi_n$  distribution

Influence score histogram (N = 10000,  $\alpha$  = 0.05)



# Example.

Report non-robustness if:

$$\phi^{\mathrm{lin}}(\vec{w}^*) - \phi(\hat{\theta}) = \hat{\sigma}_{\phi} \hat{\mathscr{T}}_{\alpha} \geq \Delta \qquad \Leftrightarrow \qquad \frac{\Delta}{\hat{\sigma}_{\phi}} \leq \hat{\mathscr{T}}_{\alpha}.$$

The **signal to noise ratio**  $\frac{\Delta}{\hat{\sigma}_{\phi}}$  determines sensitivity to data dropping.

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Let's analyze with  $\alpha = 0.01 = 1\%$ .

$$\begin{array}{llll} \phi(\hat{\theta}) = & -0.029 & (\text{Increase QOI by defn}) & \Delta = & 0.029 \\ \hat{\sigma}_{\phi} = & 0.766 & (\text{Noise}) & \frac{1}{\sqrt{N}} \hat{\sigma}_{\phi} = & 0.005 & (\text{SE}) \\ & & & & \\ \hat{\mathcal{T}}_{\alpha} = & 0.046 & (\text{Shape}) & \frac{1.96}{\sqrt{N}} = & 0.0128 & \rightarrow 0 \text{ as } N \rightarrow \infty \\ & & & & \\ \hat{\mathcal{T}}_{\alpha} \hat{\sigma}_{\phi} = & 0.035 & (\text{Data dropping sensitivity}) & \frac{1.96}{\sqrt{N}} \hat{\sigma}_{\phi} = & 0.010 & (\text{SE sensitivity}) \end{array}$$

The noise is much larger than the signal  $\Rightarrow$  Sensitive to data dropping.

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Corollary: To robustify, reduce the noise or increase the signal.

# Other forms of robustness

### We proceeded as follows:

- Took presence of datapoints as a model input,
- Formed an automatically-computable differential approximation,
- Provided theory by analyzing higher-order derivatives,
- Studied its effectiveness in problems with open-access data.

### Presence of datapoints is only one model input of many!

- Prior sensitivity in Bayesian nonparametrics [Giordano et al., 2021]
- Model sensitivity of MCMC output [Gustafson, 2000, Giordano et al., 2018]
- Cross-validation [Giordano et al., 2019, Wilson et al., 2020]
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- Frequentist variances of MCMC posteriors (in progress)

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- We can quickly and automatically find an approximate influential set which is accurate for small sets.
- Robustness to removing small sets is principally determined by the signal to noise ratio.
- In the present work, we studied data dropping. But we provide a framework for studying many other robustness questions, both to data and model perturbations.

### Links and references

Tamara Broderick, Ryan Giordano, Rachael Meager (alphabetical authors) "An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Change Conclusions?"

https://arxiv.org/abs/2011.14999

Open-source software:

R package zaminfluence https://github.com/rgiordan/zaminfluence Python package vittles https://github.com/rgiordan/vittles

Some related content can be found on my blog: https://rgiordan.github.io/

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