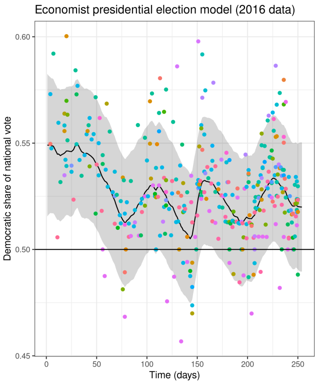
Approximate data deletion and replication with the Bayesian influence function

Ryan Giordano (`rgiordano@berkeley.edu`, UC Berkeley), Tamara Broderick (MIT)

2024 ISBA World Meeting

Economist 2016 Election Model [Gelman and Heidemanns, 2020]



A time series model to predict the 2016 US presidential election outcome from polling data.

Model:

- $X = x_1, \ldots, x_N =$ Polling data ($N = 361$).
- $\theta =$ Lots of random effects (day, pollster, etc.)
- $f(\theta) =$ Democratic % of vote on election day

We want to know $\mathop{\mathbb{E}}_{p(\theta|X)}[f(\theta)]$.

Typically, we compute Markov chain Monte Carlo (MCMC) draws from the posterior $p(\theta|X)$.

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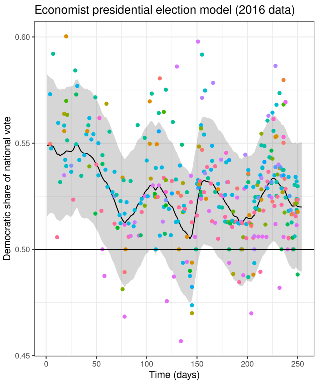

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If we had selected a different random sample, how much would our estimate have changed?

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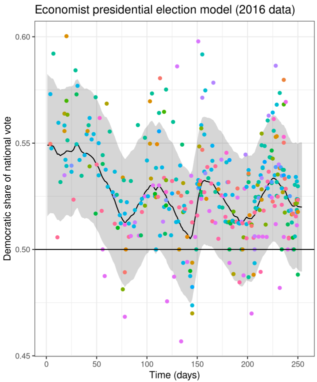

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Idea: Re-fit with bootstrap samples of data [Huggins and Miller, 2023]

Problem: Each MCMC run takes about 10 hours (Stan, six cores).

Results

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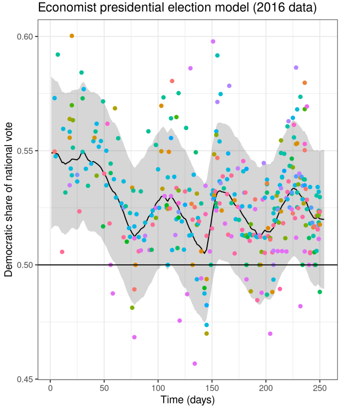

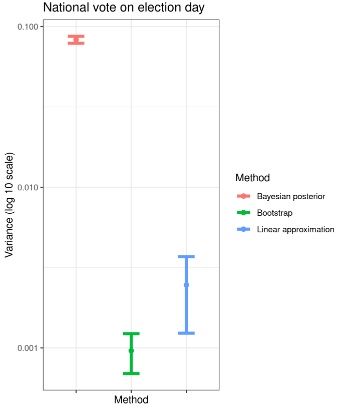

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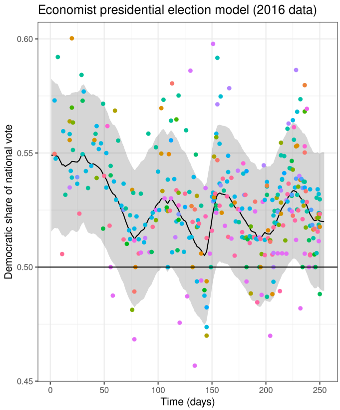

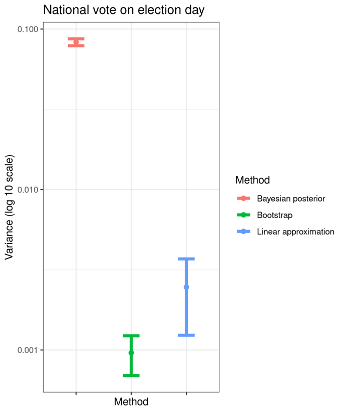

Compute time for 100 bootstraps: 51 days

Compute time for the linear approximation: Seconds
(But note the approximation has some error)

Data re-weighting.

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Augment the problem with *data weights* $w_1, \ldots, w_N$.

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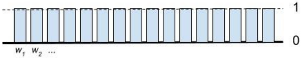

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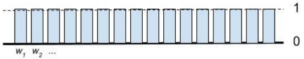

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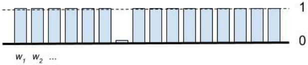

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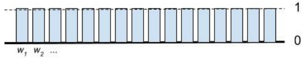

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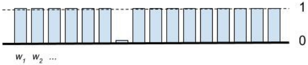

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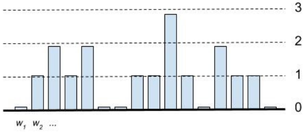

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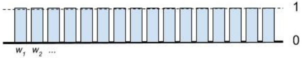

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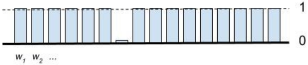

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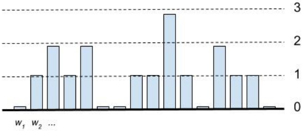

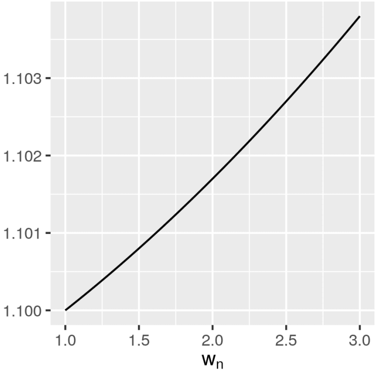

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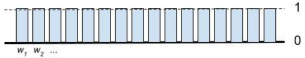

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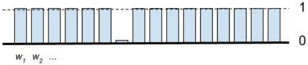

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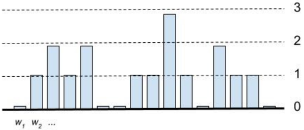

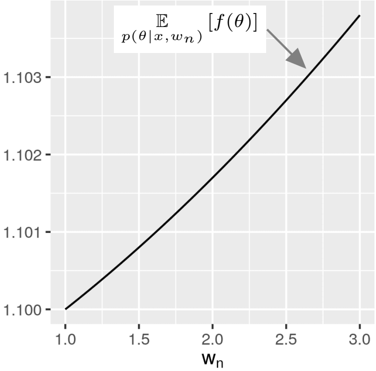

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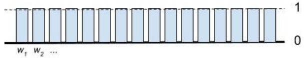

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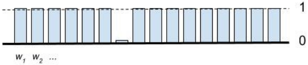

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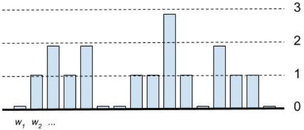

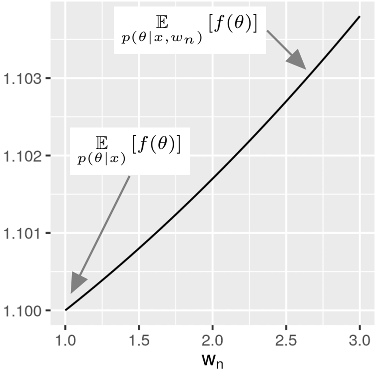

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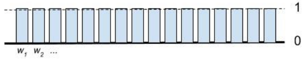

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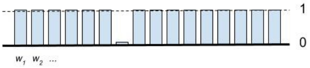

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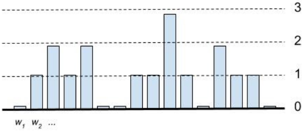

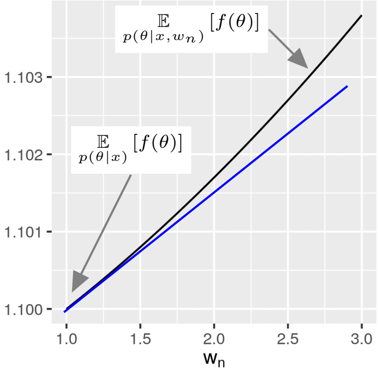

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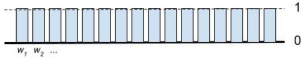

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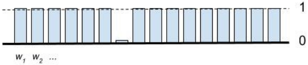

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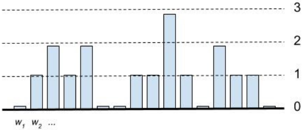

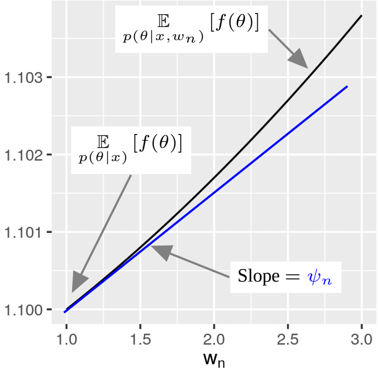

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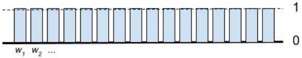

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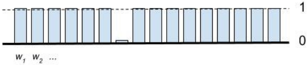

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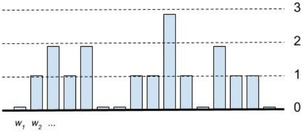

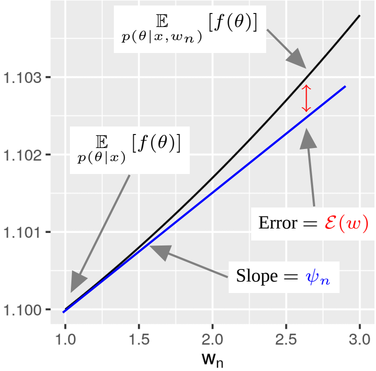

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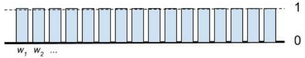

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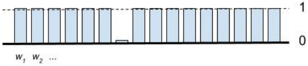

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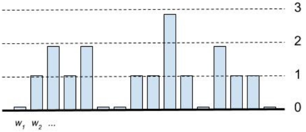

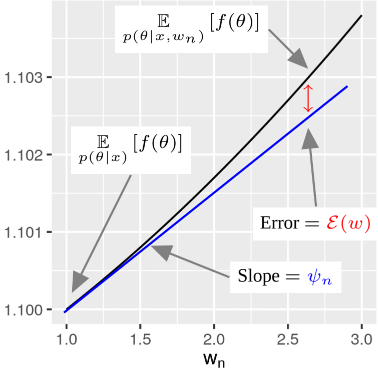

The re-scaled slope $N\psi_n$ is known as the "influence function" at data point x_n.

$$\underset{p(\theta|X,w)}{\mathbb{E}}[f(\theta)] - \underset{p(\theta|X)}{\mathbb{E}}[f(\theta)] = \sum_{n=1}^{N} \psi_n (w_n - 1) + \mathcal{E}(w).$$

Data re-weighting.

How can we use the approximation?

Example: Approximate bootstrap.

Draw bootstrap weights $w \sim p(w) = \text{Multinomial}(N, N^{-1})$.

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The final line is also known as the "infinitesimal jackknife" variance approximation.

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Other examples: Cross validation, conformal inference, outlier identification, etc.

Expressions for the slope and error

How to compute the slopes ψ_n? How can we analyze the error $\mathcal{E}(w)$?

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Furthermore, by the mean value theorem, for some $\tilde{w}$,

$$\mathcal{E}(w) = \frac{1}{2} \sum_{n=1}^{N} \sum_{n'=1}^{N} \underbrace{\underset{p(\theta|X,\tilde{w})}{\mathbb{E}}\left[\bar{f}(\theta)\bar{\ell}_n(\theta)\bar{\ell}_{n'}(\theta)\right]}_{\substack{\text{Cannot compute directly!} \\ \text{(we don't know the intermediate value theorem's } \tilde{w}\text{)}.}} (w_n - 1)(w_{n'} - 1)$$

But we can analyze it.

Here, an overbar denotes "posterior–mean zero." For example, $\bar{f}(\theta) := f(\theta) - \underset{p(\theta|X)}{\mathbb{E}}[f(\theta)]$.

Theoretical results

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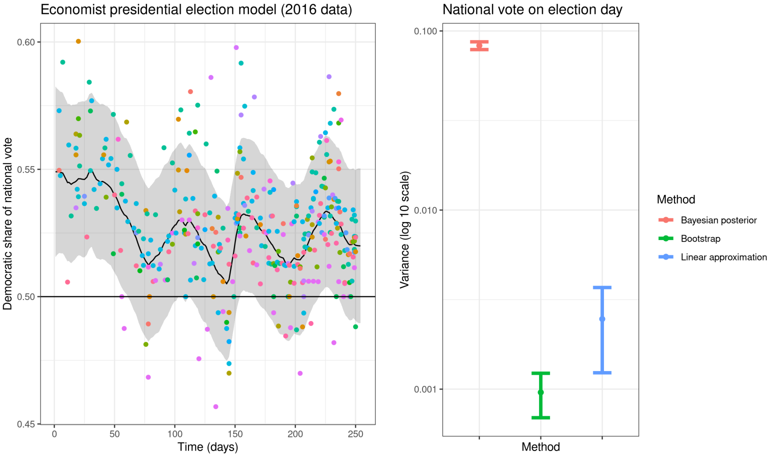
Theorem 4 of Giordano and Broderick [2023] (paraphrase):
In a flexible class of high–dimensional exponential family models,
even when $p\left(f(\theta)|X\right)$ **obeys a BVM marginally (!),**

- $\sqrt{N}\mathcal{E}(w)$ does not converge to zero (so the IJ covariance is inconsistent), but...
- $\sqrt{N}\mathcal{E}(w) = \tilde{O}_p\left(1\right)$, and proportional to the nuisance parameters' posterior covariance

- Proofs use the von Mises expansion to accomodate high–dimensional θ [von Mises, 1947].

$\Rightarrow$ **Proofs (and experiments) strongly suggest the bootstrap is inconsistent as well.**

Observations and consequences



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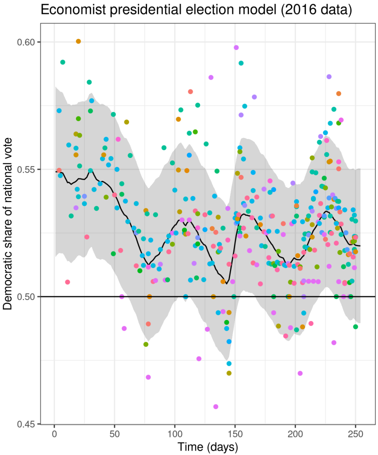

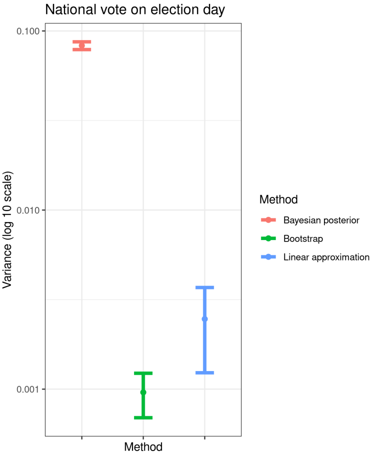

Preprint: Giordano and Broderick [2023] (`arXiv:2305.06466`)

- Detailed proofs
- Simple analytical examples
- Simulated and real–world experiments

Arxiv link

References

A. Gelman and M. Heidemanns. The Economist: Forecasting the US elections., 2020. URL `https://projects.economist.com/us-2020-forecast/president`. Data and model accessed Oct., 2020.

R. Giordano and T. Broderick. The Bayesian infinitesimal jackknife for variance. *arXiv preprint arXiv:2305.06466*, 2023.

J. Huggins and J. Miller. Reproducible model selection using bagged posteriors. *Bayesian Analysis*, 18(1):79–104, 2023.

R. von Mises. On the asymptotic distribution of differentiable statistical functions. *The Annals of Mathematical Statistics*, 18 (3):309–348, 1947.

Data re-weighting.

How can we use the approximation?

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Cross validation. Let $w_{(-n)}$ leave out point n, and loss $f(\theta) = -\ell(x_n|\theta)$.

$$\text{LOO CV loss at point } n = \mathop{\mathbb{E}}_{p(\theta|x,w_{(-n)})}[f(\theta)] \approx \mathop{\mathbb{E}}_{p(\theta|x)}[f(\theta)] - \psi_n$$

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Example: Approximate bootstrap.

Draw bootstrap weights $w \sim p(w) = \text{Multinomial}(N, N^{-1})$.

$$\begin{aligned}
\text{Bootstrap variance} &= \underset{p(w)}{\text{Var}}\left(\underset{p(\theta|x,w)}{\mathbb{E}}[f(\theta)]\right) \\
&\approx \underset{p(w)}{\text{Var}}\left(\underset{p(\theta|x)}{\mathbb{E}}[f(\theta)] + \psi_n(w_n - 1)\right) \\
&= \sum_{n=1}^{N}\left(\psi_n - \overline{\psi}\right)^2.
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Influential subsets: Approximate maximum influence perturbation (AMIP).

Let $W_{(-K)}$ denote weights leaving out K points.

$$\max_{w \in W_{(-K)}}\left(\mathop{\mathbb{E}}_{p(\theta|x,w)}[f(\theta)] - \mathop{\mathbb{E}}_{p(\theta|x)}[f(\theta)]\right) \approx -\sum_{n=1}^{K}\psi_{(n)}.$$

Example: A negative binomial model

Consider $p(X|\gamma) = \prod_{n=1}^{N} \text{NegativeBinomial}(x_n|\gamma)$. Here, $\theta = \gamma$ is a scalar.

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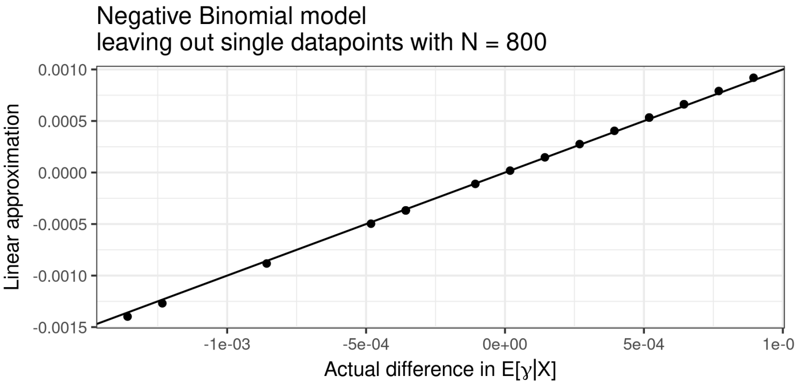

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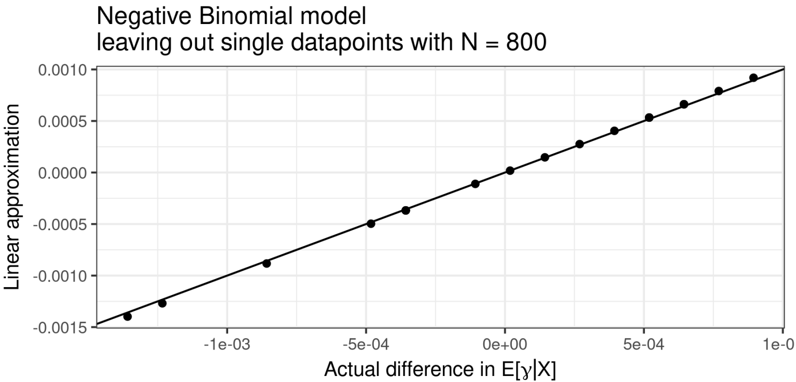
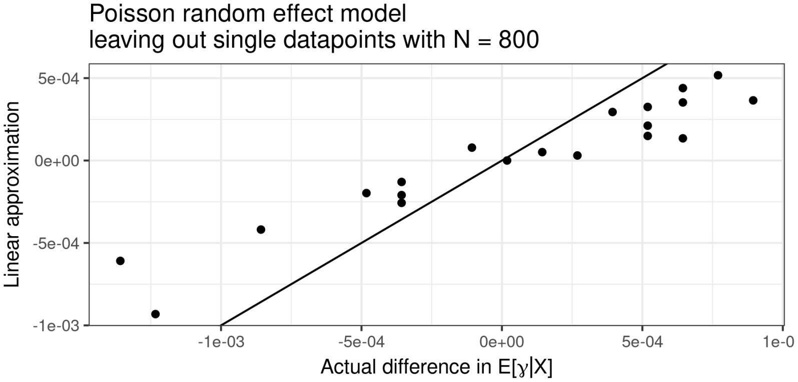


Problem: Most computationally hard Bayesian problems don't concentrate.

Experiments

Example: **Poisson model with random effects (REs) λ and fixed effect γ.**



A contradiction?

Negative binomial observations.

Asymptotically linear in w.

Poisson observations with random effects.

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With a constant regressor, Gamma REs, and one RE per observation, these are the same model, with the same $p(\gamma|X)$.

Is $\mathop{\mathbb{E}}_{p(\gamma|X,w)}[\gamma]$ linear in the data weights or not?

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$$\log p(X|\gamma, w^m) = \sum_{n=1}^{N} w_n^m \log p(x_n|\gamma)$$

Poisson observations with random effects.

Asymptotically non-linear in w.

$$\log p(X|\gamma, \lambda, w^c) = \sum_{n=1}^{N} w_n^c \log p(x_n|\lambda, \gamma)$$

With a constant regressor, Gamma REs, and one RE per observation, these are the same model, with the same $p(\gamma|X)$.

Is $\underset{p(\gamma|X,w)}{\mathbb{E}}[\gamma]$ linear in the data weights or not?

Trick question! We weight a log likelihood contribution, not a datapoint.

The two weightings are not equivalent in general.

Experimental results

Our results were actually computed on **identical datasets** with $G = N$ and $g_n = n$.

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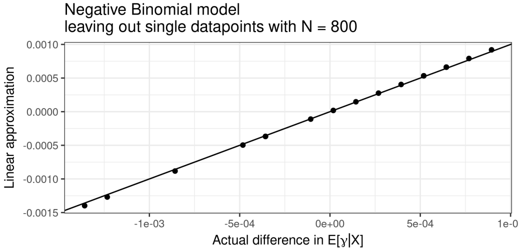
$$\psi_n = \mathop{\mathbb{E}}_{p(\gamma|X)} \left[\bar{\gamma}\bar{\ell}_n(\gamma)\right]$$



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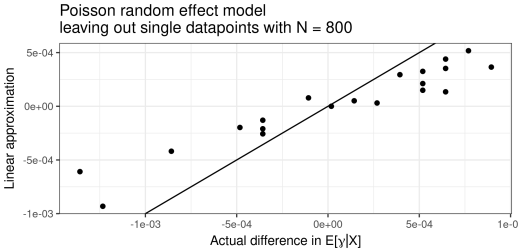

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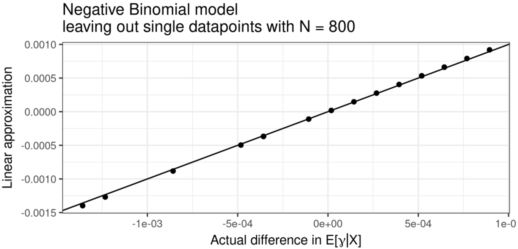

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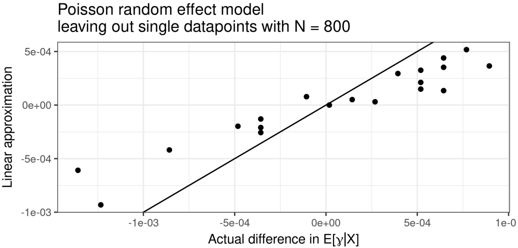

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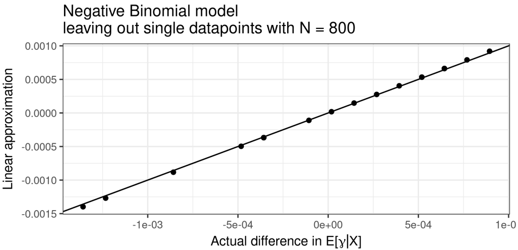

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May still be useful when $p(\lambda|X)$ is *somewhat* concentrated.