# Local Weighting–Based Diagnostics for Bayesian Poststratification

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# Are US non-voters becoming more Republican?

**Blue Rose research says yes:**

"Politically disengaged voters have become much more Republican, And because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate."

(**?**) (On Ezra Klein show, major professional pollsters)

***On Data and Democracy* says no:**

"Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available."

(**?**) (Berkeley professor co–author, major professional researchers)

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- The problem is very hard (it's difficult to accurately poll non–voters)
- Different data sources
- **Very different statistical methods:** ⋆
  - Blue Rose uses Bayesian hierarchical modeling (MrP)
  - The CES uses weighted averages (calibration weighting)

**Our contribution**

We provide a calibration weighting interpretation of MrP analyses that:

- Is easily computable from MCMC draws and standard software, and
- Defines MrP versions of key diagnostics that motivate calibration weighting.

We provide apples-to-apples comparisons between MrP and calibration weighting.

# Outline

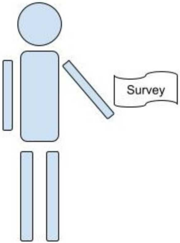
- Introduce the statisical problem and two methods (calibration weighting and MrP)
- Describe one of the classical calibration weighting diagnostics (covariate balance)
- Define MrPaw & state a key theorem
- Real-world results
- Future directions

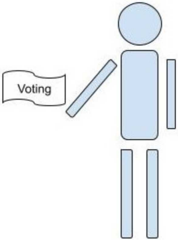
# The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses $y$ (e.g. A binary response to "do you support policy such–and–such")

We want the average response in a target population, in which we observe only covariates.

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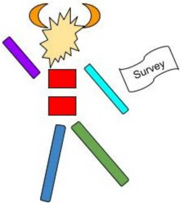


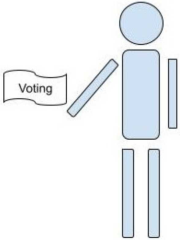


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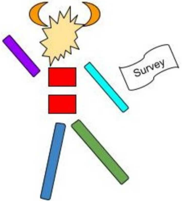


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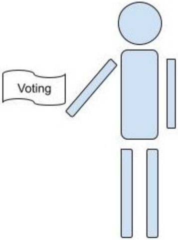


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**The problem is that the populations are very different.**

Our survey results may be biased.

How can we use the covariates to say something about the target responses?

# Two approaches

We want $\mu := \frac{1}{T}\sum_{t=1}^{T} y_t$, but don't observe target population $y_t$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

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► Choose "calibration weights" $w_s$
(e.g. raking weights)

**Bayesian hierarchical modeling (MrP)**

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  - Partial pooling
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$\leftarrow$ (We open this box, providing analogues
of all these diagnostics)

# What are we weighting for?[1]

We want:

$$\text{Target average response} = \frac{1}{T}\sum_{t=1}^{T} y_p \approx \frac{1}{S}\sum_{s=1}^{S} w_s y_s = \text{Weighted survey average response}$$

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You can check covariate balance for any calibration weighting estimator.

Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

**Raking calibration weights**

"Raking" selects weights that

- Are as "close as possible" to some reference weights
- Under the constraint that they balance some selected regressors.

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# Generalized covariate balance checks

We want to balance $f(\mathbf{x})$ because we think $\mathbb{E}[y|\mathbf{x}]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

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**Generalized covariate balance (GCB) (informal)**

Pick a small $\delta$, and define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}\left[\mu(\tilde{y}) - \mu(y)|\mathbf{x}\right] = \frac{1}{T}\sum_{t=1}^{T}\left(\mathbb{E}\left[\tilde{y}|\mathbf{x}_p\right] - \mathbb{E}\left[y|\mathbf{x}_p\right]\right) = \delta\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{x}_p)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change:

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\substack{\text{Replace weighted averages}\\ \text{with changes in an estimator}}} \overset{\text{check}}{=} \delta\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{x}_p).$$

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When $\hat{\mu}(\cdot) = \hat{\mu}_{\text{CAL}}(\cdot)$, GCB recovers the standard covariate balance check.

# Generalized covariate balance for MrP

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Two possibilities:

- Allow $\tilde{y}$ to take values other than $\{0, 1\}$ and set $\tilde{y} = y + \delta f(\mathbf{x})$, or
- Use an estimate of $\mathbb{E}\left[y|\mathbf{x}\right]$ to draw new binary $\tilde{y}$.

We define theory and methods for the first, and provide tools for generating data using the second method for potentially problematic regressors.

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**Taylor series**

Form the approximation

$$\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) = \sum_{s=1}^{S} w_s^{\mathrm{MRP}} (\tilde{y}_s - y_s) + \text{Residual} \quad \text{where} \quad w_s^{\mathrm{MRP}} := \frac{d}{dy_s} \hat{\mu}_{\mathrm{MRP}}(y).$$

If MrP were linear (e.g. if you use OLS instead of hierarchical logistic regression), then

- The residual is zero,
- $\hat{\mu}_{\mathrm{MRP}}(y) = \sum_{s=1}^{S} w_s^{\mathrm{MRP}} y_s$, and so
- $\hat{\mu}_{\mathrm{MRP}}(\tilde{y})$ is a calibration weighting estimator, and $w_s^{\mathrm{MRP}}$ are its weights. (Cite Gelman)

In general, MrP is truly nonlinear. The residual is only small when $\tilde{y} \approx y$ (i.e., when $\delta \ll 1$).

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It happens that the needed derivatives are given by simple a posterior covariances involving only the inverse link function $m(\mathbf{x}; \theta)$ and log likelihood (**?**).

These can be computed using standard MCMC software (e.g. **?**).

# Covariate balance

**Theorem**

- Let $\tilde{y} = y + \delta f(\mathbf{x})$,
- $\hat{\mu}_{\mathrm{MRP}}$ be a hierachical logistic regression posterior expectation, and
- $\mathcal{F}$ be a Donsker class of uniformly bounded functions on $\mathbf{x}$.

Then, with probability approaching one, as $N \to \infty$,

$$\sup_{f \in \mathcal{F}} \left( \hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \left( \hat{\mu}_{\mathrm{MRP}}(y) + \sum_{s=1}^{S} w_s^{\mathrm{MRP}} \delta f(\mathbf{x}_s) \right) \right) = O(\delta^2) \quad \text{as } \delta \to 0$$

The supremum over $\mathcal{F}$ is the primary technical contribution! It means we are justified in searching over regressors to find imbalance.

Draws on our prior work on uniform and finite–sample error bounds for Bernstein–von Mises theorem–like results (**??**).

# Real Data

Textbook example from Chapter 16 of “Telling Stories With Data.” (**?**)

- **Target population:** US Population Aged 18+ (ACS) (**?**)
- **Survey population:** The Nationscape survey June–Oct 2020 (**?**)
- **Respose:** “If the election for president were going to be held now and the Democratic nominee was Joe Biden and the Republican nominee was Donald Trump, would you vote for …” (Biden = 1, Trump = 0, other answers excluded)

| | |
|---|---|
| Survey observations | $S = 4,364$ |
| Target observations (rows) | $T = 4,085,282$ |
| Uncorrected survey mean | $\frac{1}{S}\sum_{s=1}^{S} y_n = 0.462$ |

# Figure

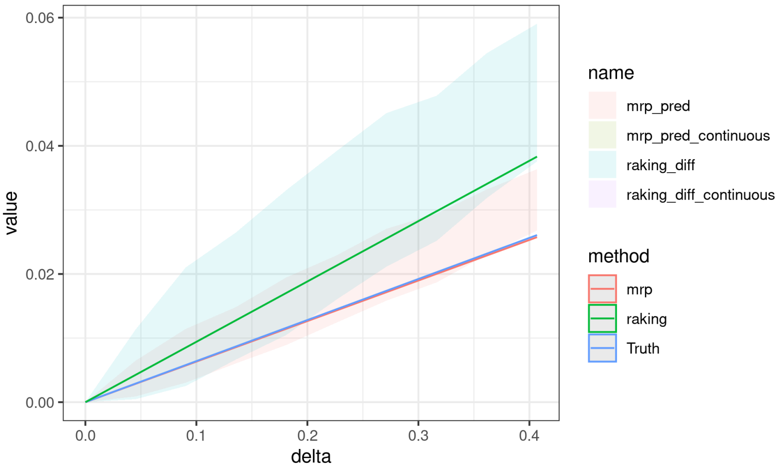


**Figure 1:** Balance checks for Alexander

# Future work and generalizations

- Instance of a very general class of local consistency checks that generalize classical regression checks (work with Sequoia)
- Versions for GLMMs (work with Vladimir)
- Going beyond classical Bayesian sensitivity (work with Lucas)

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