Locally Equivalent Weights for Bayesian MrP

Ryan Giordano, Alice Cima, Erin Hartman, Jared Murray, Avi Feller

UT Austin Statistics Seminar
September 2025

Are US non-voters becoming more Republican?

Blue Rose research says yes:

“Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate.”

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

“Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available.”

(Bonica et al. 2025)
(major professional researchers)

Are US non-voters becoming more Republican?

Blue Rose research says yes:

"Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate."

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

"Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available."

(Bonica et al. 2025)
(major professional researchers)

- The problem is very hard (it's difficult to accurately poll non–voters)
- Different data sources
- ⋆⋆⋆ **Different statistical methods**
  - Blue Rose uses Bayesian hierarchical modeling (MrP)
  - On Data and Democracy is using calibration weighting (CW)

Are US non-voters becoming more Republican?

Blue Rose research says yes:

"Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate."

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

"Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available."

(Bonica et al. 2025)
(major professional researchers)

- The problem is very hard (it's difficult to accurately poll non–voters)
- Different data sources
- ⋆⋆⋆ **Different statistical methods**
  - Blue Rose uses Bayesian hierarchical modeling (MrP)
  - On Data and Democracy is using calibration weighting (CW)

Our contribution

We define "MrP local equivalent weights" (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key diagnostics that motivate calibration weighting.

⇒ **MrPlew provides direct comparisons between MrP and calibration weighting.**

Outline

- Introduce the statisical problem
    - Contrast CW and MrP
    - Prior work: Equivalent weights for linear models
    - Interlude: Approximate equivalent weights for some non–linear models
    - Our key idea: Locally equivalent weights for non–linear models

Outline

- Introduce the statisical problem
  - Contrast CW and MrP
  - Prior work: Equivalent weights for linear models
  - Interlude: Approximate equivalent weights for some non–linear models
  - Our key idea: Locally equivalent weights for non–linear models
- Locally equivalent weights for covariate balance
  - Describe covariate balance
  - Define MrPlew weights and connect them to covariate balance
  - Theoretical support
  - Example of real-world results

Outline

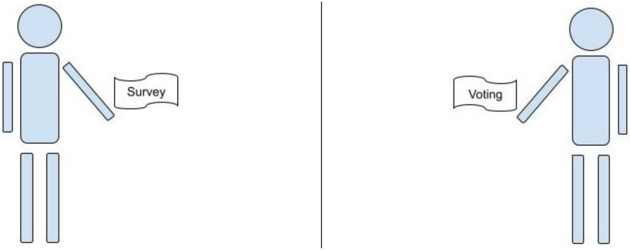
- Introduce the statisical problem
  - Contrast CW and MrP
  - Prior work: Equivalent weights for linear models
  - Interlude: Approximate equivalent weights for some non–linear models
  - Our key idea: Locally equivalent weights for non–linear models
- Locally equivalent weights for covariate balance
  - Describe covariate balance
  - Define MrPlew weights and connect them to covariate balance
  - Theoretical support
  - Example of real-world results
- Other uses of locally equivalent weights
  - Parital pooling
  - The meaning of negative weights
  - Frequentist variance estimation
- Future directions

The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to “do you support Trump”)

We want the average response in a target population, in which we observe only covariates.



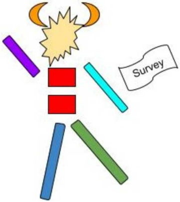
Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

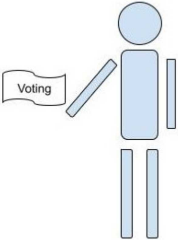
The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support Trump")

We want the average response in a target population, in which we observe only covariates.





Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

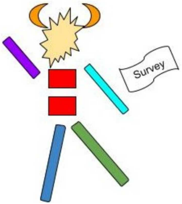
The problem is that the populations may be very different.

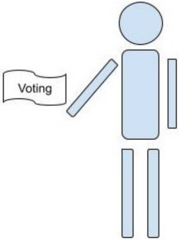
The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support Trump")

We want the average response in a target population, in which we observe only covariates.





Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

The problem is that the populations may be very different.

Our survey results may be biased.

How can we use the covariates to say something about the target responses?

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j. Let $Y_{\mathcal{S}} = \{y_1, \ldots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of **x** may be different in the survey and target.

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j. Let $Y_{\mathcal{S}} = \{y_1, \dots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

Calibration weighting (CW)

▶ Choose "calibration weights" w_i
using only the regressors $\mathbf{x}$
(e.g. raking weights)

Bayesian hierarchical modeling (MrP)

▶ Choose $\mathbb{E}\left[y|\mathbf{x}, \theta\right] = m(\theta^\intercal \mathbf{x})$,
choose prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$
(e.g. Hierarchical logistic regression)

Two approaches

We want $\mu := \frac{1}{N_T}\sum_{j=1}^{N_T} y_j$, but don't observe target population y_j. Let $Y_{\mathcal{S}} = \{y_1, \dots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

Calibration weighting (CW)

- ▶ Choose "calibration weights" w_i using only the regressors $\mathbf{x}$ (e.g. raking weights)
- ▶ Take $\hat{\mu}^{\text{CW}}(Y_{\mathcal{S}}) = \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i$

Bayesian hierarchical modeling (MrP)

- ▶ Choose $\mathbb{E}\left[y|\mathbf{x},\theta\right] = m(\theta^\intercal \mathbf{x})$, choose prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$ (e.g. Hierarchical logistic regression)
- ▶ Take $\hat{y}_j = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[y|\mathbf{x}_j\right]$ and $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T}\hat{y}_j$

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j. Let $Y_{\mathcal{S}} = \{y_1, \ldots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

Calibration weighting (CW)

- Choose "calibration weights" w_i using only the regressors $\mathbf{x}$ (e.g. raking weights)
- Take $\hat{\mu}^{\mathrm{CW}}(Y_{\mathcal{S}}) = \frac{1}{N_S} \sum_{i=1}^{N_S} w_i y_i$
- Dependence on y_i is clear

Bayesian hierarchical modeling (MrP)

- Choose $\mathbb{E}\left[y|\mathbf{x}, \theta\right] = m(\theta^\intercal \mathbf{x})$, choose prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$ (e.g. Hierarchical logistic regression)
- Take $\hat{y}_j = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[y|\mathbf{x}_j\right]$ and $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} \hat{y}_j$
- Dependence on y_i very complicated (Typically via MCMC draws from $\mathcal{P}(\theta|\text{Survey data})$)

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j. Let $Y_{\mathcal{S}} = \{y_1, \ldots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

Calibration weighting (CW)

- ▶ Choose "calibration weights" w_i using only the regressors $\mathbf{x}$ (e.g. raking weights)
- ▶ Take $\hat{\mu}^{\text{CW}}(Y_{\mathcal{S}}) = \frac{1}{N_S} \sum_{i=1}^{N_S} w_i y_i$
- ▶ Dependence on y_i is clear
- ▶ Weights give interpretable diagnostics:
  - Frequentist variability
  - Partial pooling
  - Regressor balance

Bayesian hierarchical modeling (MrP)

- ▶ Choose $\mathbb{E}\left[y|\mathbf{x}, \theta\right] = m(\theta^\intercal \mathbf{x})$, choose prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$ (e.g. Hierarchical logistic regression)
- ▶ Take $\hat{y}_j = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[y|\mathbf{x}_j\right]$ and $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} \hat{y}_j$
- ▶ Dependence on y_i very complicated (Typically via MCMC draws from $\mathcal{P}(\theta|\text{Survey data})$)
- ▶ **Black box**

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j. Let $Y_{\mathcal{S}} = \{y_1, \ldots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

Calibration weighting (CW)

- Choose "calibration weights" w_i using only the regressors $\mathbf{x}$ (e.g. raking weights)
- Take $\hat{\mu}^{\text{CW}}(Y_{\mathcal{S}}) = \frac{1}{N_S} \sum_{i=1}^{N_S} w_i y_i$
- Dependence on y_i is clear
- Weights give interpretable diagnostics:
  - Frequentist variability
  - Partial pooling
  - Regressor balance

Bayesian hierarchical modeling (MrP)

- Choose $\mathbb{E}\left[y|\mathbf{x}, \theta\right] = m(\theta^\intercal \mathbf{x})$, choose prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$ (e.g. Hierarchical logistic regression)
- Take $\hat{y}_j = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[y|\mathbf{x}_j\right]$ and $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} \hat{y}_j$
- Dependence on y_i very complicated (Typically via MCMC draws from $\mathcal{P}(\theta|\text{Survey data})$)
- **Black box**

$\leftarrow$ We open this box, providing analogues of all these diagnostics

Prior work: Equivalent weights for linear models

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form $\hat{y}$:

$$\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} \hat{y}_j = \frac{1}{N_T}\sum_{j=1}^{N_T} \underbrace{\mathbf{x}_j^\intercal \hat{\theta}}_{\text{Linear in } Y_{\mathcal{S}}}$$

Most existing literature on comparing CW and MrP focus on such linear models. [1]

[1] For example, Gelman (2007b), B., F., and H. (2021), and Chattopadhyay and Zubizarreta (2023).

Prior work: Equivalent weights for linear models

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form $\hat{y}$:

$$\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} \hat{y}_j = \frac{1}{N_T}\sum_{j=1}^{N_T} \underbrace{\mathbf{x}_j^\intercal \hat{\theta}}_{\text{Linear in } Y_{\mathcal{S}}}$$

Most existing literature on comparing CW and MrP focus on such linear models. [1]

But what if you use a non–linear link function? Or a hierarchical model?

> *"It would also be desirable to use nonlinear methods ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods." — (Gelman 2007a)*

[1] For example, Gelman (2007b), B., F., and H. (2021), and Chattopadhyay and Zubizarreta (2023).

Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *inherently nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal \hat{\theta})$ can be approximately linear.

Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *inherently nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal \hat{\theta})$ can be approximately linear.

Example #1

Additionally suppose $\mathbf{x} \in \mathcal{X}$ is discrete and saturated. **Then MrP is a CW estimator.**

Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \text{Logistic}(\mathbf{x}^\intercal\theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$.

The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal\hat{\theta})$ is *inherently nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal\hat{\theta})$ can be approximately linear.

Example #1

Additionally suppose $\mathbf{x} \in \mathcal{X}$ is discrete and saturated. **Then MrP is a CW estimator.**

- Let $\overline{y}_{\mathcal{S}}^{c}$ denote the survey average among $\mathbf{x} = c$ for $c \in \mathcal{X}$

Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *inherently nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal \hat{\theta})$ can be approximately linear.

Example #1

Additionally suppose $\mathbf{x} \in \mathcal{X}$ is discrete and saturated. **Then MrP is a CW estimator.**

- Let $\overline{y}_S^c$ denote the survey average among $\mathbf{x} = c$ for $c \in \mathcal{X}$
- For $\mathbf{x} = c$, the MLE satisfies $m(\hat{\theta}^\intercal \mathbf{x}) = \overline{y}_S^c$

Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *inherently nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal \hat{\theta})$ can be approximately linear.

Example #1

Additionally suppose $\mathbf{x} \in \mathcal{X}$ is discrete and saturated. **Then MrP is a CW estimator.**

- Let $\overline{y}_S^c$ denote the survey average among $\mathbf{x} = c$ for $c \in \mathcal{X}$
- For $\mathbf{x} = c$, the MLE satisfies $m(\hat{\theta}^\intercal \mathbf{x}) = \overline{y}_S^c$
- Let N_S^c (or N_T^c) denote the # of survey (or target) observations with $\mathbf{x}_n = c$.

Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *inherently nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal \hat{\theta})$ can be approximately linear.

Example #1

Additionally suppose $\mathbf{x} \in \mathcal{X}$ is discrete and saturated. **Then MrP is a CW estimator.**

- Let $\overline{y}_S^c$ denote the survey average among $\mathbf{x} = c$ for $c \in \mathcal{X}$
- For $\mathbf{x} = c$, the MLE satisfies $m(\hat{\theta}^\intercal \mathbf{x}) = \overline{y}_S^c$
- Let N_S^c (or N_T^c) denote the # of survey (or target) observations with $\mathbf{x}_n = c$.

$$\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) = \frac{1}{N_T} \sum_{c \in \mathcal{X}} \underbrace{N_T^c \overline{y}_S^c}_{\text{Linear in } Y_{\mathcal{S}}} = \frac{1}{N_S} \sum_{i=1}^{N_S} w_i^{\text{MrP}} y_i$$

For $w_i^{\text{MrP}} = \dfrac{N_T^c / N_T}{N_S^c / N_S}$ when $\mathbf{x}_i = c$.

Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \mathrm{Logistic}(\mathbf{x}^\intercal\theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal\mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$$

Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$$

$$\approx \int m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_T(\mathbf{x}) d\mathbf{x} \qquad \text{(Law of large numbers)}$$

Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$
\begin{aligned}
\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) &= \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) \\
&\approx \int m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_T(\mathbf{x}) d\mathbf{x} && \text{(Law of large numbers)} \\
&= \int \frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_S(\mathbf{x}) d\mathbf{x} && \text{(Multiply by } \mathcal{P}_S(\mathbf{x})/\mathcal{P}_S(\mathbf{x})\text{)}
\end{aligned}
$$

Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$
\begin{aligned}
\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) &= \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) \\
&\approx \int m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_T(\mathbf{x}) d\mathbf{x} && \text{(Law of large numbers)} \\
&= \int \frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_S(\mathbf{x}) d\mathbf{x} && \text{(Multiply by } \mathcal{P}_S(\mathbf{x})/\mathcal{P}_S(\mathbf{x})\text{)} \\
&\approx \int (\alpha^\intercal \mathbf{x})\, m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_S(\mathbf{x}) d\mathbf{x} && \text{(By assumption)}
\end{aligned}
$$

Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$
\begin{aligned}
\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) &= \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) \\
&\approx \int m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_T(\mathbf{x}) d\mathbf{x} && \text{(Law of large numbers)} \\
&= \int \frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_S(\mathbf{x}) d\mathbf{x} && \text{(Multiply by } \mathcal{P}_S(\mathbf{x})/\mathcal{P}_S(\mathbf{x})\text{)} \\
&\approx \int (\alpha^\intercal \mathbf{x})\, m(\mathbf{x}^\intercal \hat{\theta}) \mathcal{P}_S(\mathbf{x}) d\mathbf{x} && \text{(By assumption)} \\
&\approx \alpha^\intercal \frac{1}{N_S} \sum_{i=1}^{N_S} \mathbf{x}_i m(\mathbf{x}_i^\intercal \hat{\theta}) && \text{(Law of large numbers)}
\end{aligned}
$$

Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \text{Logistic}(\mathbf{x}^\intercal\theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_\mathcal{S}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal\mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$
\begin{aligned}
\hat{\mu}^{\text{MrP}}(Y_\mathcal{S}) &= \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta}) \\
&\approx \int m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_T(\mathbf{x})d\mathbf{x} && \text{(Law of large numbers)} \\
&= \int \frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_S(\mathbf{x})d\mathbf{x} && \text{(Multiply by } \mathcal{P}_S(\mathbf{x})/\mathcal{P}_S(\mathbf{x})\text{)} \\
&\approx \int (\alpha^\intercal\mathbf{x})\, m(\mathbf{x}^\intercal\hat{\theta})\mathcal{P}_S(\mathbf{x})d\mathbf{x} && \text{(By assumption)} \\
&\approx \alpha^\intercal \frac{1}{N_S}\sum_{i=1}^{N_S} \mathbf{x}_i m(\mathbf{x}_i^\intercal\hat{\theta}) && \text{(Law of large numbers)} \\
&= \alpha^\intercal \frac{1}{N_S}\sum_{i=1}^{N_S} \mathbf{x}_i y_i && \text{(Property of exponential family MLEs)}
\end{aligned}
$$

Nearly equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \text{Logistic}(\mathbf{x}^\intercal\theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal\mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal\hat{\theta}) = \frac{1}{N_S}\sum_{i=1}^{N_S} \underbrace{w_i^{\text{MrP}}}_{\alpha^\intercal\mathbf{x}_i} y_i + \text{Small error}$$

But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

Nearly equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) = \frac{1}{N_S} \sum_{i=1}^{N_S} \underbrace{w_i^{\text{MrP}}}_{\alpha^\intercal \mathbf{x}_i} y_i + \text{Small error}$$

But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

Key idea (informal)

If $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ is approximately linear, then $w_i^{\text{MrP}} \approx \frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$.

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

Nearly equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

Example #2

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) = \frac{1}{N_S} \sum_{i=1}^{N_S} \underbrace{w_i^{\text{MrP}}}_{\alpha^\intercal \mathbf{x}_i} y_i + \text{Small error}$$

But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

Key idea (informal)

If $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ is approximately linear, then $w_i^{\text{MrP}} \approx \frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$.

For logistic regression, compute and analyze $\frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ using the implicit function theorem.[2]

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \mathrm{Logistic}(\mathbf{x}^\intercal\theta)$.
- Set a hierarchical prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$, use MCMC to draw from $\mathcal{P}(\theta|\text{Survey data})$.
- MrP is $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T}\mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_j^\intercal\theta)\right]$.

No reason to think $Y_{\mathcal{S}} \mapsto \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})$ is even approximately **globally** linear.

But we can still compute and analyze $w_i^{\mathrm{MrP}} := \frac{\partial \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ using Bayesian sensitivity analysis![3]

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$.
- Set a hierarchical prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$, use MCMC to draw from $\mathcal{P}(\theta|\text{Survey data})$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T} \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_j^\intercal \theta)\right]$.

No reason to think $Y_{\mathcal{S}} \mapsto \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ is even approximately **globally** linear.

But we can still compute and analyze $w_i^{\text{MrP}} := \frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ using Bayesian sensitivity analysis![3]

MrP locally equivalent weights (MrPlew)

For new data $\tilde{Y}_{\mathcal{S}}$, form a **local equivalent weighting**

$$\hat{\mu}^{\text{MrP}}(\tilde{Y}_{\mathcal{S}}) \approx \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) + \sum_{i=1}^{N_S} w_i^{\text{MrP}}(\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MrP}} := \frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}.$$

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \text{Logistic}(\mathbf{x}^\intercal\theta)$.
- Set a hierarchical prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$, use MCMC to draw from $\mathcal{P}(\theta|\text{Survey data})$.
- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T}\mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_j^\intercal\theta)\right]$.

No reason to think $Y_{\mathcal{S}} \mapsto \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ is even approximately **globally** linear.

But we can still compute and analyze $w_i^{\text{MrP}} := \frac{\partial\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ using Bayesian sensitivity analysis![3]

MrP locally equivalent weights (MrPlew)

For new data $\tilde{Y}_{\mathcal{S}}$, form a **local equivalent weighting**

$$\hat{\mu}^{\text{MrP}}(\tilde{Y}_{\mathcal{S}}) \approx \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) + \sum_{i=1}^{N_S} w_i^{\text{MrP}}(\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MrP}} := \frac{\partial\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}.$$

Our task is to rigorously show that even such local weights can be used diagnostically.

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

The weights can look very different!

Does this mean anything? Are the differences important?

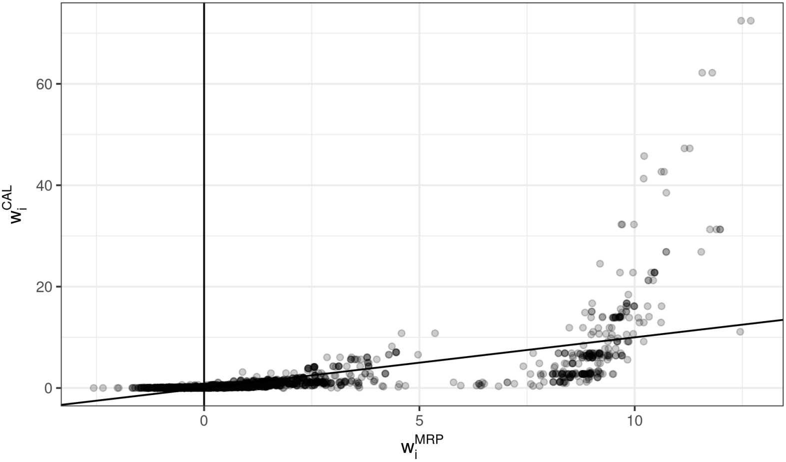


Figure 1: Comparison between raking and MrPlew weights for the Name Change dataset

References i

B., Eli, Avi F., and Erin H. (2021). *Multilevel calibration weighting for survey data*. arXiv: 2102.09052 [stat.ME].

Blue Rose Research (2024). *2024 Election Retrospective Presentation*. https://data.blueroseresearch.org/2024retro-download. Accessed on 2024-10-26.

Bonica, A. et al. (Apr. 2025). *Did Non-Voters Really Flip Republican in 2024? The Evidence Says No.* https://data4democracy.substack.com/p/did-non-voters-really-flip-republican.

Chattopadhyay, A. and J. Zubizarreta (2023). "On the implied weights of linear regression for causal inference". In: *Biometrika* 110.3, pp. 615–629.

G., T. Broderick, and M. I. Jordan (2018). "Covariances, robustness and variational bayes". In: *Journal of machine learning research* 19.51.

G., W. Stephenson, et al. (2019). "A swiss army infinitesimal jackknife". In: *The 22nd International Conference on Artificial Intelligence and Statistics*. PMLR, pp. 1139–1147.

Gelman, A. (2007a). "Rejoinder: Struggles with survey weighting and regression modelling". In: *Statistical Science* 22.2, pp. 184–188.

— (2007b). "Struggles with survey weighting and regression modeling". In.

Gustafson, P. (1996). "Local sensitivity of posterior expectations". In: *The Annals of Statistics* 24.1, pp. 174–195.

Krantz, S. and H. Parks (2012). *The Implicit Function Theorem: History, Theory, and Applications*. Springer Science & Business Media.