Locally Equivalent Weights for Bayesian MrP

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Are US non-voters becoming more Republican?

Blue Rose research says yes:

"Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate."

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

"Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available."

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- The problem is very hard (it’s difficult to accurately poll non–voters)
- Different data sources
- ⋆⋆⋆ **Different statistical methods**
  - Blue Rose uses Bayesian hierarchical modeling (MrP)
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Our contribution

We define "MrP local equivalent weights" (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key diagnostics that motivate calibration weighting.

$\Rightarrow$ **MrPlew provides direct comparisons between MrP and calibration weighting.**

Outline

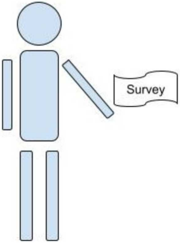
- Introduce the statisical problem and two methods (CW and MrP)
- Describe covariate balance, one of the classical CW diagnostics
- Define MrPlew weights and connect them to covariate balance
- Example of real-world results
- Future directions

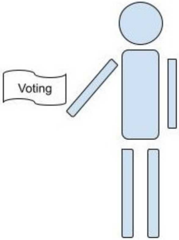
The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support Trump")

We want the average response in a target population, in which we observe only covariates.





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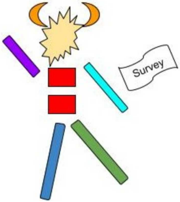

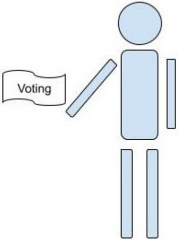

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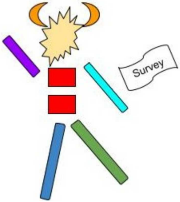

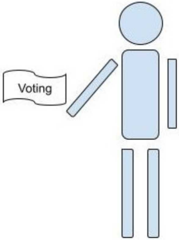

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Our survey results may be biased.

How can we use the covariates to say something about the target responses?

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

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Calibration weighting (CW)

▶ Choose "calibration weights" w_i
using only the regressors $\mathbf{x}$
(e.g. raking weights)

Bayesian hierarchical modeling (MrP)

▶ Choose $\mathbb{E}\left[y|\mathbf{x}, \theta\right] = m(\theta^\intercal \mathbf{x})$,
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- ▶ Dependence on y_i is clear
- ▶ Weights give interpretable diagnostics:
  - Frequentist variability
  - Partial pooling
  - Regressor balance

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- **Black box**

$\leftarrow$ We open this box, providing analogues of all these diagnostics

Prior work

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form $\hat{y}$:

$$\hat{\mu}_{\mathrm{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} \hat{y}_j = \frac{1}{N_T} \sum_{j=1}^{N_T} \underbrace{\mathbf{x}_j^\intercal \hat{\theta}}_{\text{Linear in } y_i}$$

Most existing literature on comparing CW and MrP focus on such linear models. [1]

Let's spend some time discussing why it is reasonable to even attempt such a thing as forming approximate equivalent weights for non–linear estimators.

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But what if you use a non–linear link function? Or a hierarchical model?

> *"It would also be desirable to use nonlinear methods ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods."* — *(Gelman 2007a)*

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Equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \text{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
- MrP is $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.
- Suppose $\mathbf{x} \in \mathcal{X}$ is discrete and saturated.

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- Let $\overline{y}_S^c$ denote the survey average among $\mathbf{x} = c$ for $c \in \mathcal{X}$
- For $\mathbf{x} = c$, $m(\hat{\theta}^\intercal \mathbf{x}) = \overline{y}_S^c$
- Let N_S^c (or N_S^c) denote the # of survey (or target) observations with $\mathbf{x}_n = c$.

$$\hat{\mu}_{\mathrm{MrP}} = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) = \frac{1}{N_T} \sum_{c \in \mathcal{X}} \underbrace{N_T^c \overline{y}_S^c}_{\text{Linear in } y_i} = \frac{1}{N_S} \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} y_i$$

For $w_i^{\mathrm{MrP}} = \dfrac{N_T^c / N_T}{N_S^c / N_S}$ when $\mathbf{x}_i = c$.

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We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so it's hard to estimate α directly.

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

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Key idea (informal)

If $\hat{\mu}_{\text{MrP}} \approx \frac{1}{N_S} \sum_{i=1}^{N_S} w_i^{\text{MrP}} y_i$ for some w_i^{MrP}, then

$$w_i^{\text{MrP}} \approx \frac{\partial \hat{\mu}_{\text{MrP}}}{\partial y_i}$$

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We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so it's hard to estimate α directly.

Key idea (informal)

If $\hat{\mu}_{\text{MrP}} \approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i^{\text{MrP}} y_i$ for some w_i^{MrP}, then

$$w_i^{\text{MrP}} \approx \frac{\partial\hat{\mu}_{\text{MrP}}}{\partial y_i}$$

For logistic regression, could compute and analyze $\frac{\partial\hat{\mu}_{\text{MrP}}}{\partial y_i}$ using the implicit function theorem.[2]

[2] Krantz and Parks 2012; **G.**, Stephenson, et al. 2019.

Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal\theta) = \mathrm{Logistic}(\mathbf{x}^\intercal\theta)$.
- Set a hierarchical prior $\mathcal{P}(\theta|\Sigma)\mathcal{P}(\Sigma)$, use MCMC to draw from $\mathcal{P}(\theta|\text{Survey data})$.
- MrP is $\hat{\mu}_{\text{MrP}} = \frac{1}{N_T}\sum_{j=1}^{N_T}\mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_j^\intercal\theta)\right]$.

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

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MrP locally equivalent weights (MrPlew)

For new data $\tilde{Y}_S$, form a series expansion

$$\hat{\mu}_{\mathrm{MrP}}(\tilde{Y}_S) \approx \hat{\mu}_{\mathrm{MrP}}(Y_S) + \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}}(\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\mathrm{MrP}} := \frac{\partial\hat{\mu}_{\mathrm{MrP}}}{\partial y_i}.$$

Our task is to rigorously show that even such local weights can be used diagnostically.

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Our task is to rigorously show that even such local weights can be used diagnostically.

- If $\hat{\mu}_{\text{MrP}}$ is linear in Y_S, this definition recovers the true weights
- Whether or not $\hat{\mu}_{\text{MrP}}$ is linear in Y_S, this definition is valid "locally" for $\tilde{Y}_S$ "nearby" Y_S.
- For MCMC, compute and analyze $\frac{\partial\hat{\mu}_{\text{MrP}}}{\partial y_i}$ using Bayesian sensitivity analysis.[3]

[3] Gustafson 1996; **G.**, Broderick, and Jordan 2018.

The weights can look very different!

Does this mean anything? Are the differences important?

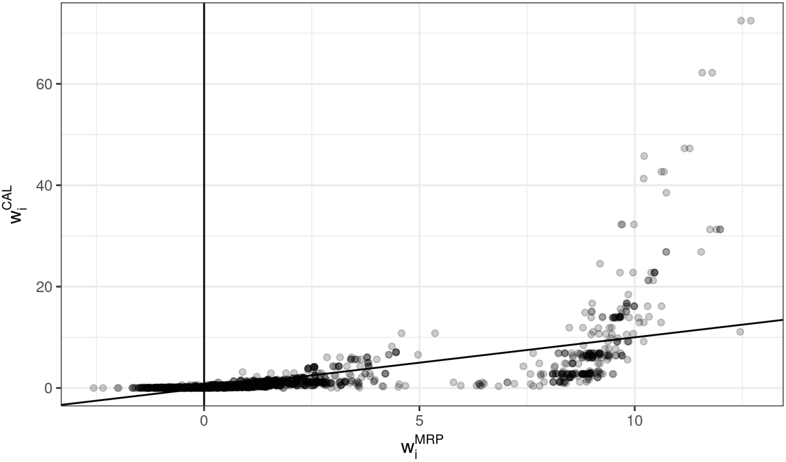

Figure 1: Comparison between raking and MrPlew weights for a particular example

What are we weighting for?[4]

We want:

$$\text{Target average response} = \frac{1}{N_T}\sum_{j=1}^{N_T} y_j \approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i = \text{Weighted survey average response}$$

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$$\frac{1}{N_T}\sum_{j=1}^{N_T} \mathbf{x}_j = \frac{1}{N_S}\sum_{i=1}^{N_S} w_i \mathbf{x}_i$$

Such weights satisfy "covariate balance" for $\mathbf{x}$.

You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

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You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

Raking calibration weights

"Raking" selects weights that

- Are as "close as possible" to some reference weights
- Under the constraint that they balance some selected regressors.

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Generalized covariate balance checks

We want to balance $f(\mathbf{x})$ because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

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Balance–informed sensitivity check (BISC) (informal)

Pick a small $\delta > 0$ and an $f(\cdot)$. Define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}\left[\mu(\tilde{y}) - \mu(y)|\mathbf{x}\right] = \frac{1}{N_T}\sum_{j=1}^{N_T}\left(\mathbb{E}\left[\tilde{y}|\mathbf{x}_p\right] - \mathbb{E}\left[y|\mathbf{x}_p\right]\right) = \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change for observed $\tilde{y}, y$:

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\substack{\text{Replace weighted averages} \\ \text{with changes in an estimator}}} \overset{\text{check}}{\approx} \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

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When $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{CW}}(\cdot)$, BISC recovers the standard covariate balance check.

When $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{MrP}}(\cdot)$ and δ is small, BISC recovers our proposal.

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