Variational Methods for Latent Variable Problems

Ryan Giordano (for Johns Hopkins Biostats BLAST working group)

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Massachusetts Institute of Technology

Outline

Outline for today:

- Some examples of latent variable models
- A template: The Neyman-Scott "paradox" and marginalization
- Bayesian versus frequentist approaches to marginalization
- The classical EM algorithm (in brief)

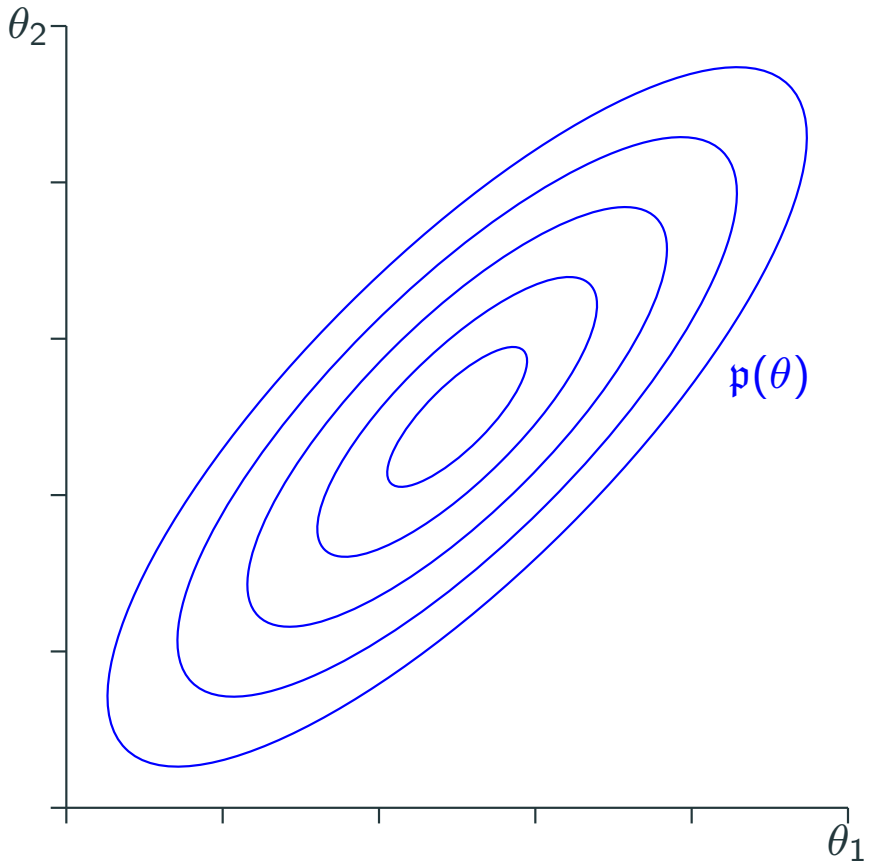
Next week, we will build on these ideas to present more general variational inference.

KL divergence exercises

$\mathfrak{p}(\theta) =$ Correlated bivariate normal

$\mathcal{Q} = \{\text{All bivariate normals}\}$

What is $\mathfrak{q}^*(\theta) = \underset{\mathfrak{q} \in \mathcal{Q}}{\operatorname{argmin}} \operatorname{KL}(\mathfrak{q}(\theta) || \mathfrak{p}(\theta))$?



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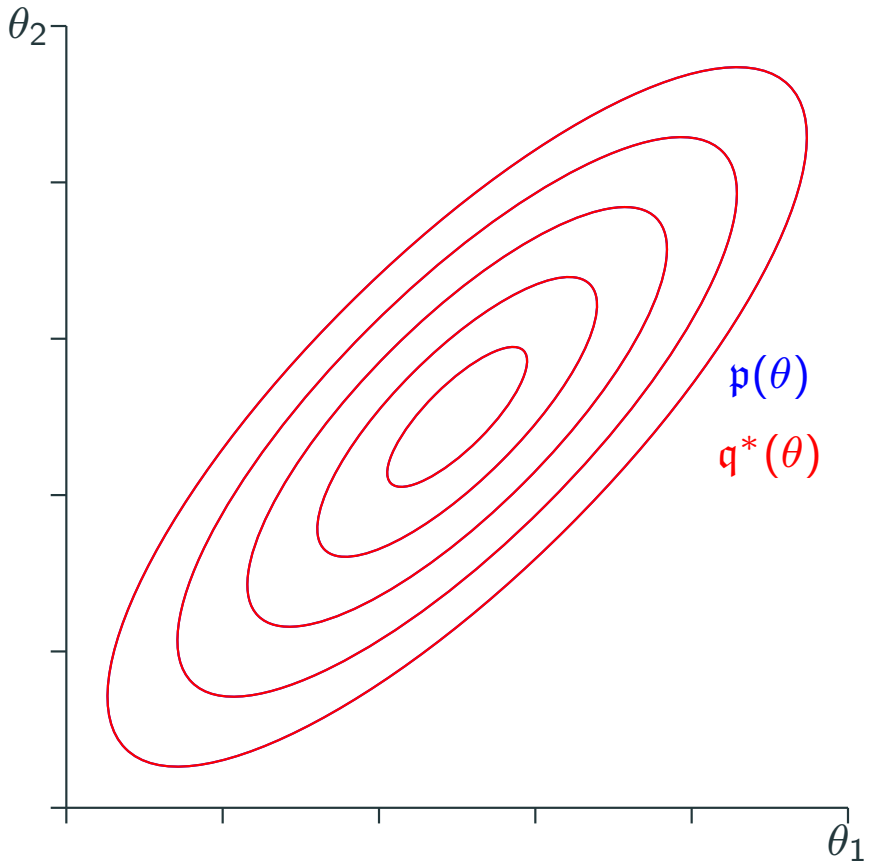

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$\mathfrak{p}(\theta) =$ Correlated bivariate normal

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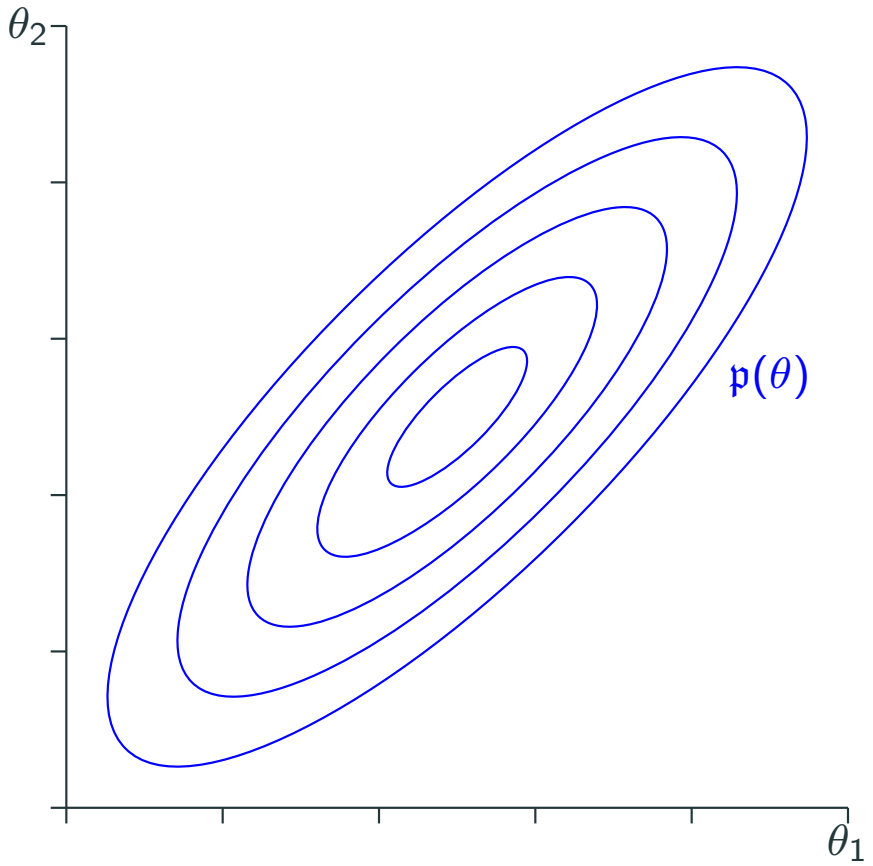

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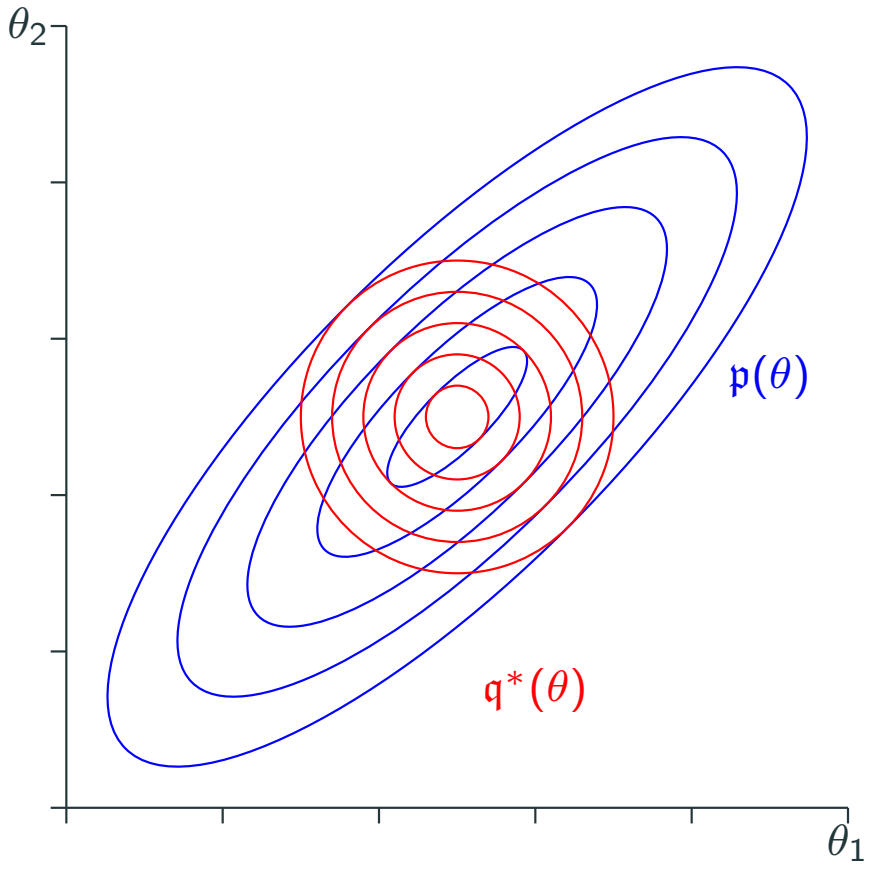

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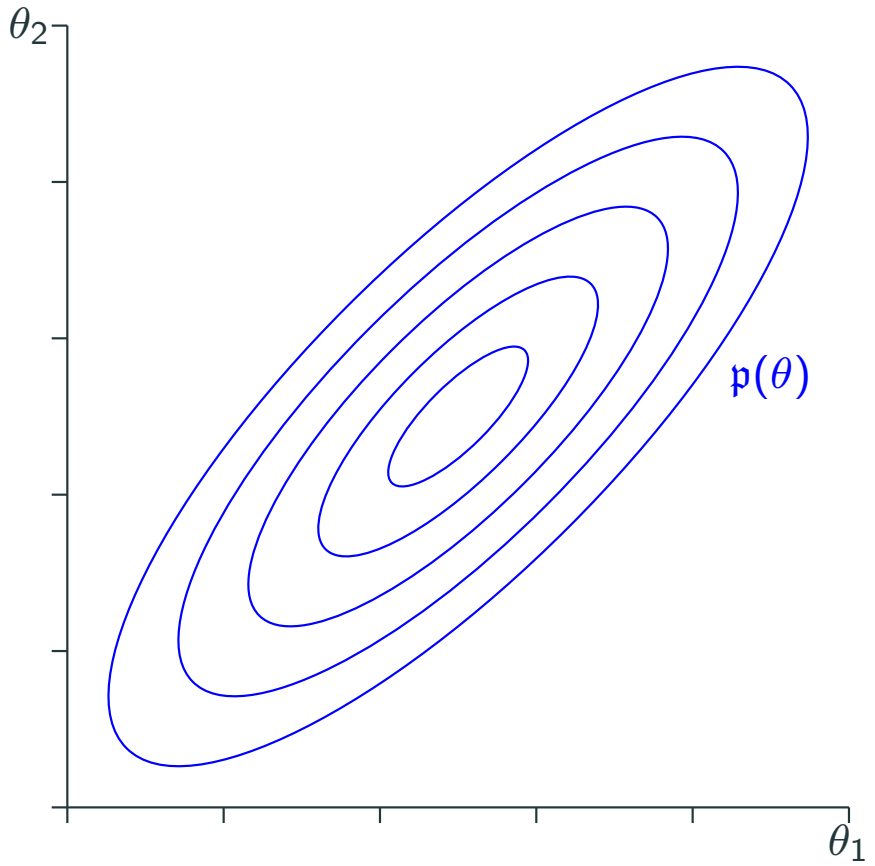

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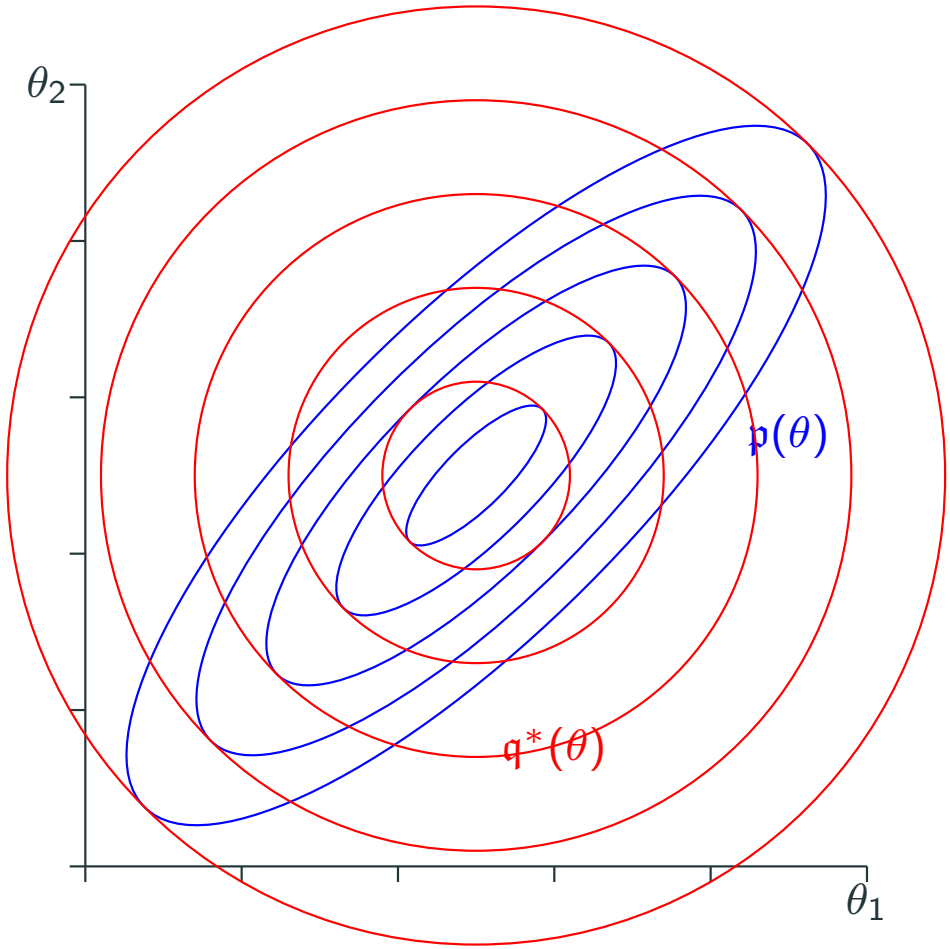

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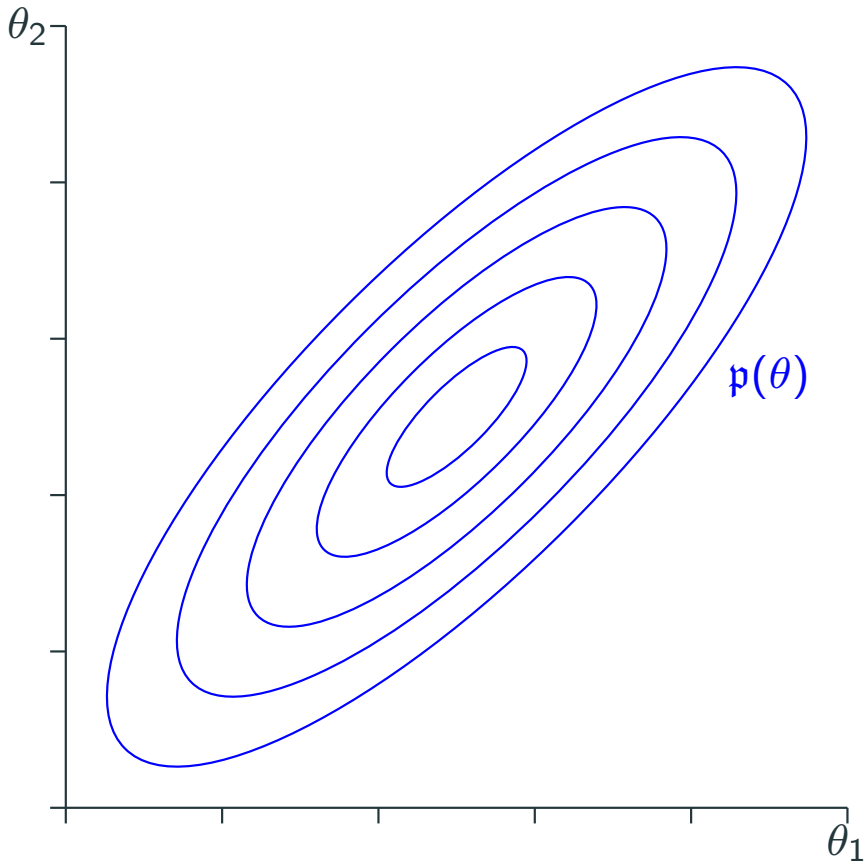

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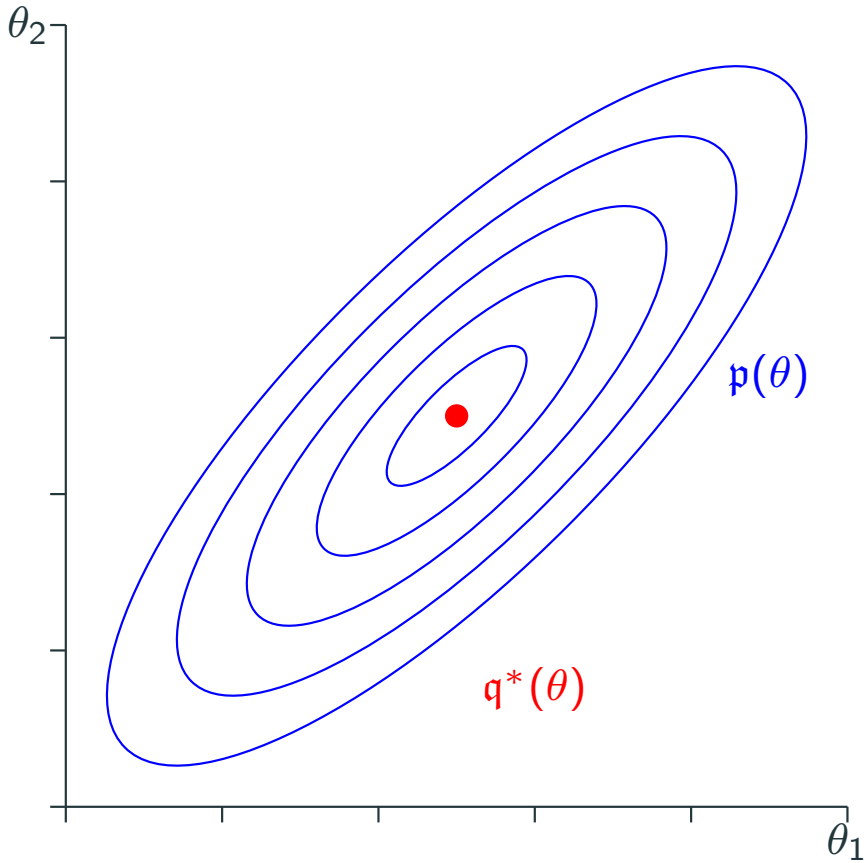

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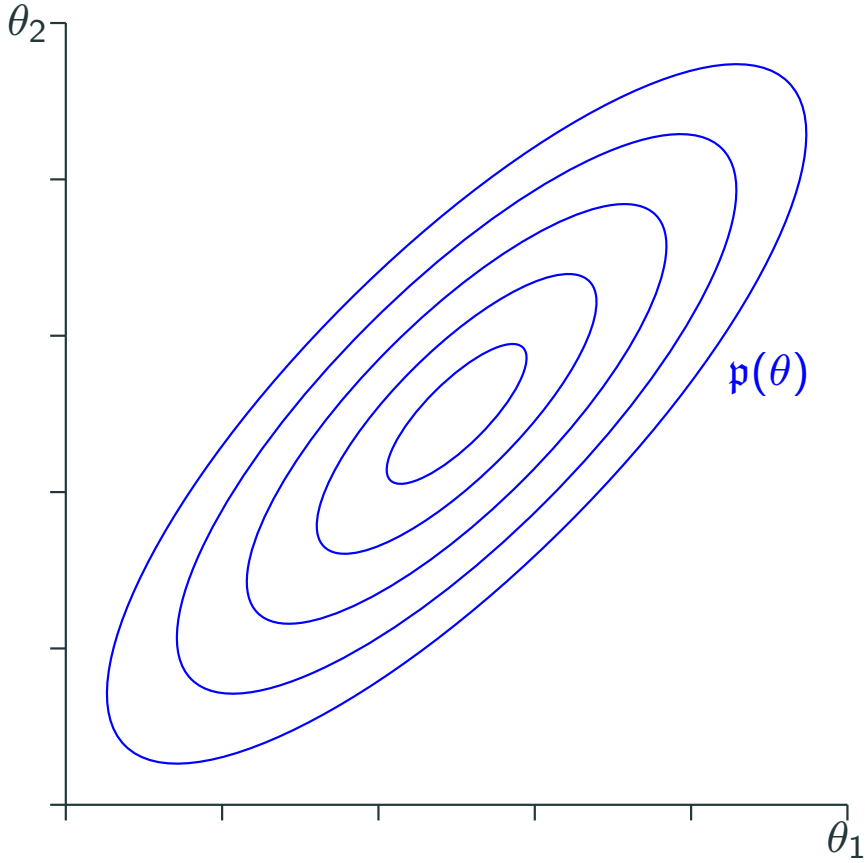

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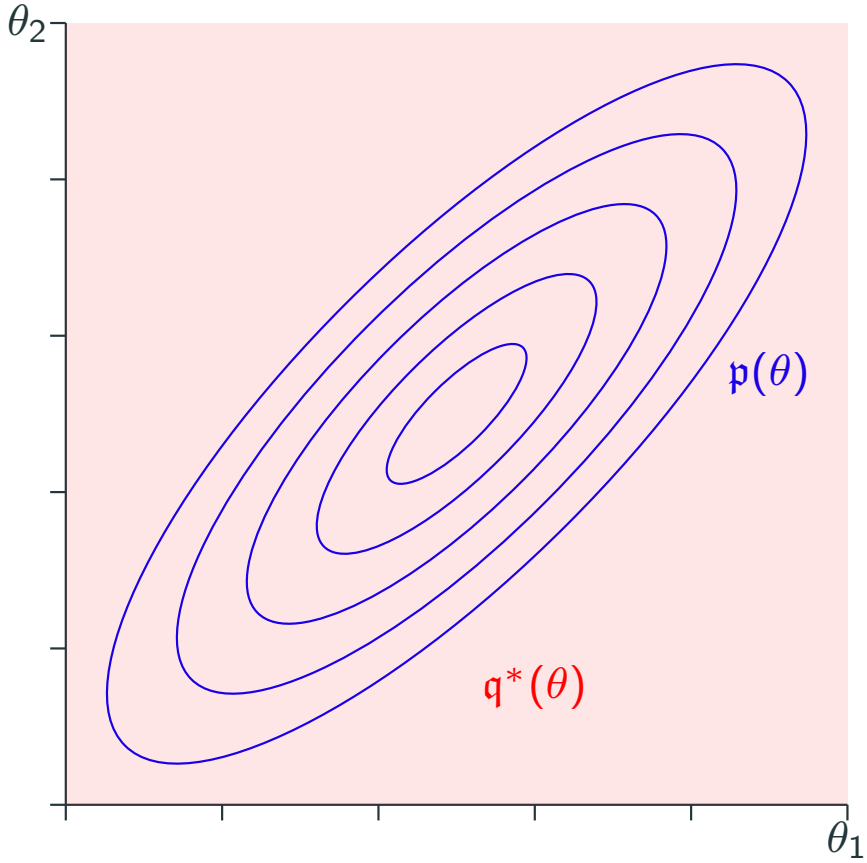

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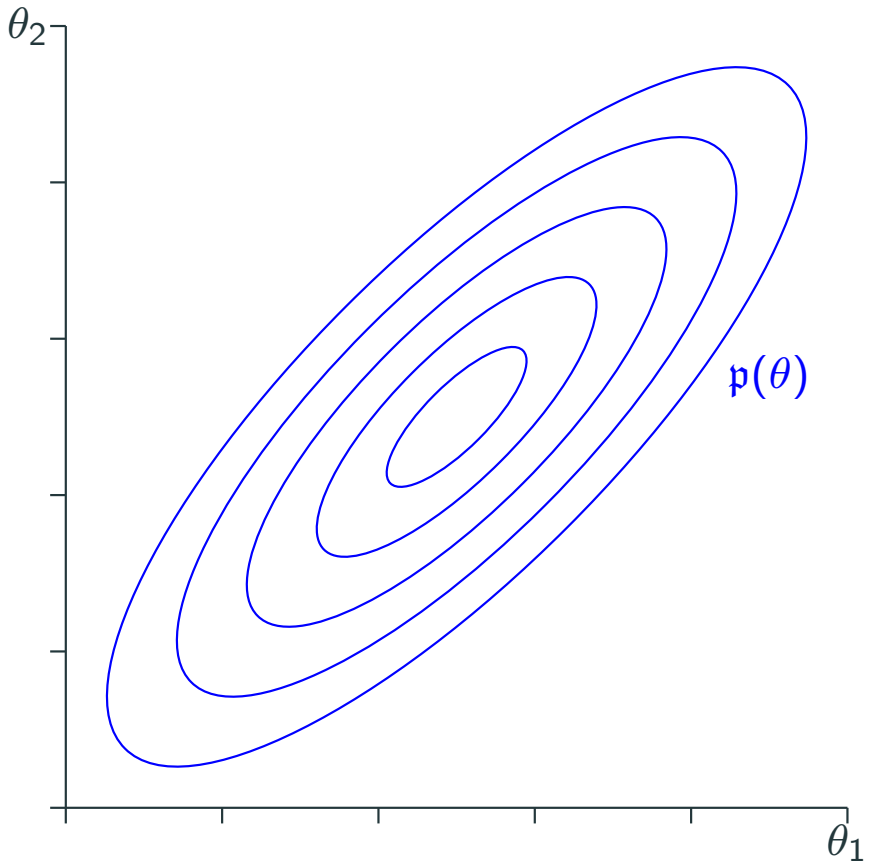

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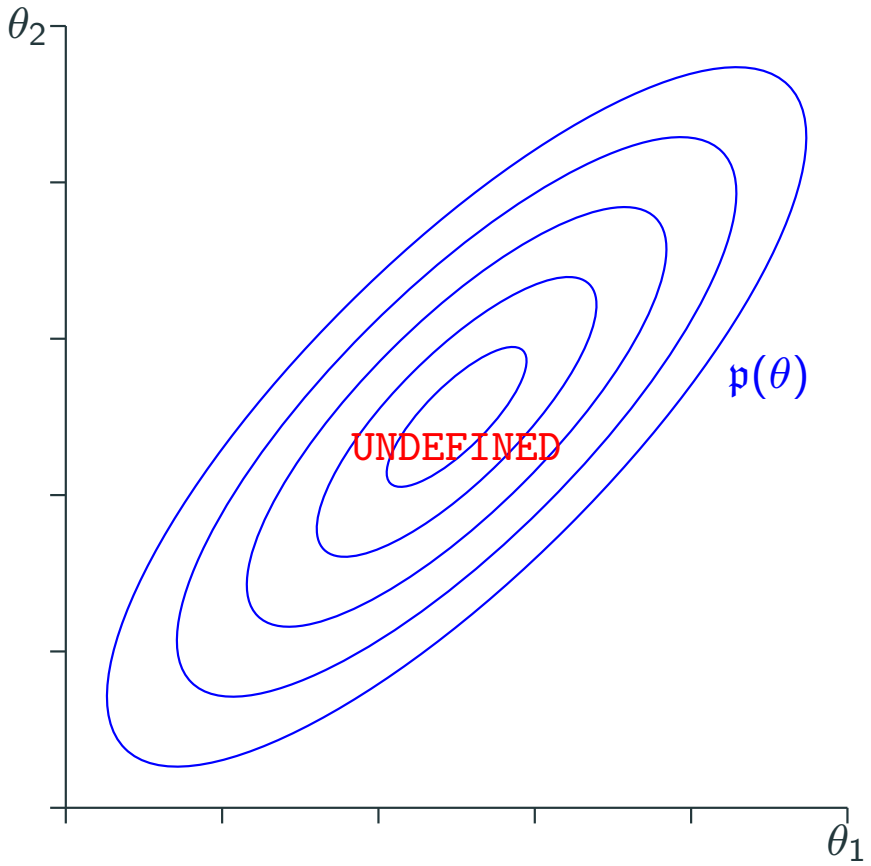

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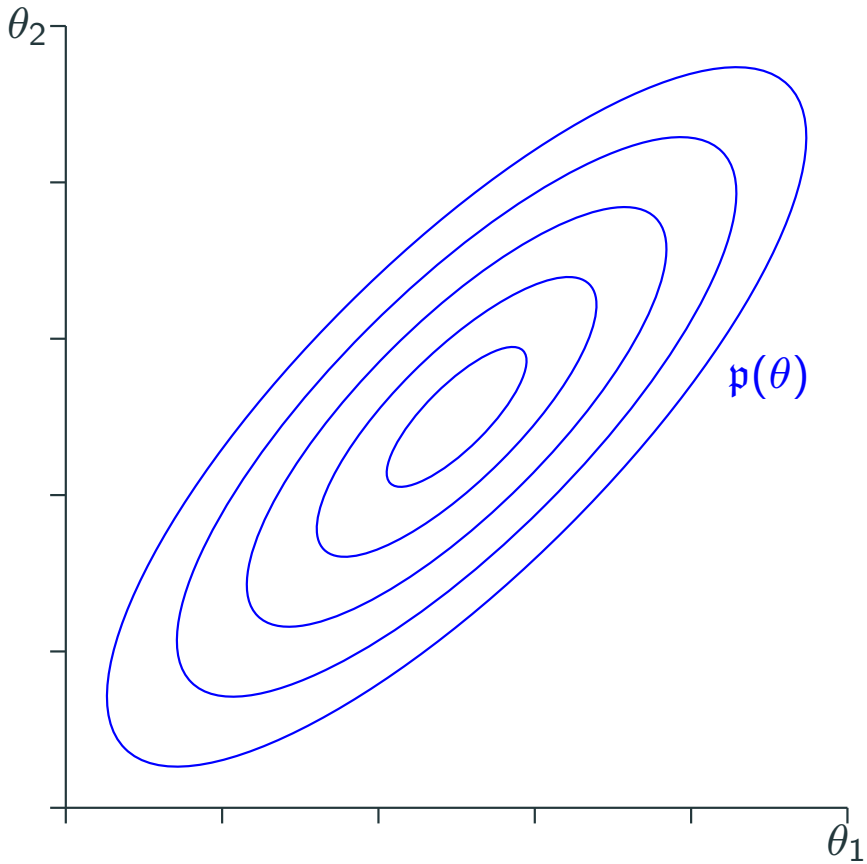

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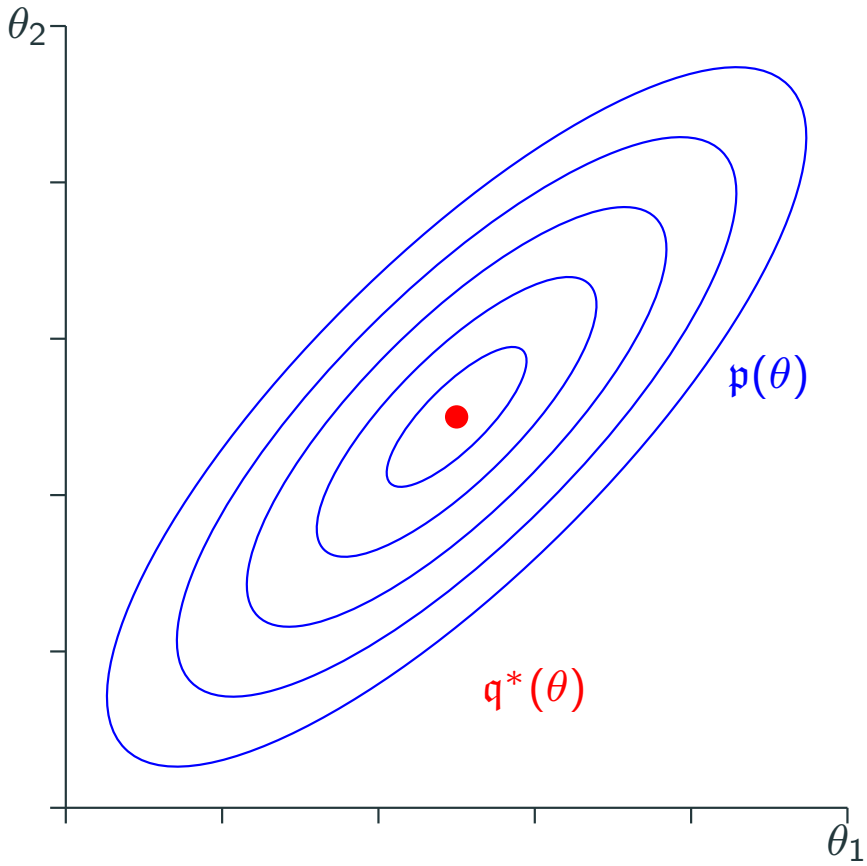

Conclusions