Local Weighting–Based Diagnostics for Bayesian Poststratification

Ryan Giordano, Alice Cima, Erin Hartman, Jared Murray, Avi Feller

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Are US non-voters becoming more Republican?

Blue Rose research says yes:

"Politically disengaged voters have become much more Republican, And because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate."

(**blueroseresearch:2024**)
(major professional pollsters)

On Data and Democracy **says no:**

"Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available."

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- The problem is very hard (it's difficult to accurately poll non–voters)
- Different data sources
- **Very different statistical methods:** ⋆
  - Blue Rose uses Bayesian hierarchical modeling (MrP)
  - The CES uses calibration weighting (CW)

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Our contribution

We define "MrP local equivalent weights" (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and

- Provide MrP versions of key diagnostics that motivate calibration weighting.

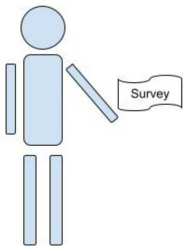⇒ **MrPlew provides direct comparisons between MrP and calibration weighting.**

- Introduce the statisical problem and two methods (CW and MrP)
- Describe covariate balance, one of the classical CW diagnostics
- Define MrPlew weights and connect them to covariate balance
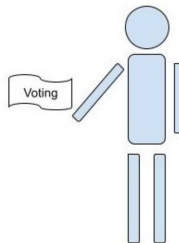- Example of real-world results
- Future directions

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support Trump")

We want the average response in a target population, in which we observe only covariates.

Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

The basic problem

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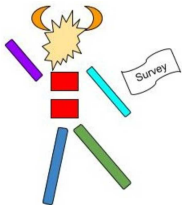Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

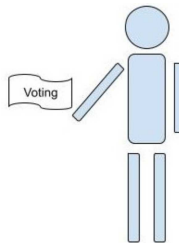**The problem is that the populations may be very different.**

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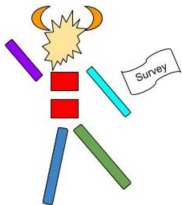Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

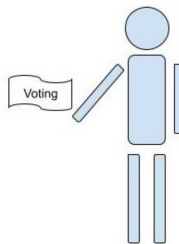**The problem is that the populations may be very different.**

Our survey results may be biased.

How can we use the covariates to say something about the target responses?

Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target population y_j.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

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Calibration weighting

▶ Choose "calibration weights" w_i
(e.g. raking weights)

Bayesian hierarchical modeling (MrP)

▶ Choose model $\mathbb{E}_{y|\mathbf{x}} [=] m(\theta^{\mathsf{T}}\mathbf{x})$ and
prior $\mathcal{P}(\theta)$
(e.g. Hierarchical logistic regression)

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▶ Take $\hat{y}_j = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})} [y|\mathbf{x}_j]$ and
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▶ Dependence on y_i is obvious (w_i typically chosen using only $\mathbf{x}$)	▶ Dependence on y_i very complicated (Typically via MCMC draws from $\mathcal{P}(\theta|\text{Survey data})$)

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▶ Weights give interpretable diagnostics:	▶ **Black box**

- Frequentist variability
- Partial pooling
- Regressor balance

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▶ **Black box**

← (We open this box, providing analogues of all these diagnostics)

Prior work

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But what if you use a non–linear link function? Or a hierarchical model?

"It would also be desirable to use nonlinear methods such as regression trees ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods." — *(**gelman:2007:strugglesrejoinder**)*

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Our approach

For nonlinear models, we will *define* $w_i^{\mathrm{MRP}} = \frac{\partial \hat{\mu}_{\mathrm{MRP}}}{\partial y_i}$.

Our primary task is then to **rigorously justify** such weights' use in common diagnostics.

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We want:

$$\text{Target average response} = \frac{1}{N_T} \sum_{j=1}^{N_T} y_j \approx \frac{1}{N_S} \sum_{i=1}^{N_S} w_i y_i = \text{Weighted survey average response}$$

We can't check this, because we don't observe y_j.

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Such weights satisfy "covariate balance" for $\mathbf{x}$.

You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

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You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

Raking calibration weights

"Raking" selects weights that

- Are as "close as possible" to some reference weights
- Under the constraint that they balance some selected regressors.

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Generalized covariate balance checks

We want to balance $f(\mathbf{x})$ because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

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Balance–informed sensitivity check (BISC) (informal)

Pick a small δ, and define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}[\tilde{y}|\mathbf{x}] = \mathbb{E}[y|\mathbf{x}] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}\left[\mu(\tilde{y}) - \mu(y)|\mathbf{x}\right] = \frac{1}{N_T}\sum_{j=1}^{N_T}\left(\mathbb{E}\left[\tilde{y}|\mathbf{x}_p\right] - \mathbb{E}\left[y|\mathbf{x}_p\right]\right) = \delta\frac{1}{N_T}\sum_{j=1}^{N_T}f(\mathbf{x}_j)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change for observed $\tilde{y}, y$:

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\substack{\text{Replace weighted averages} \\ \text{with changes in an estimator}}} \overset{\text{check}}{\approx} \delta\frac{1}{N_T}\sum_{j=1}^{N_T}f(\mathbf{x}_j).$$

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When $\hat{\mu}(\cdot) = \hat{\mu}_{\text{CAL}}(\cdot)$, BISC recovers the standard covariate balance check.

When $\hat{\mu}(\cdot) = \hat{\mu}_{\text{MRP}}(\cdot)$ and δ is small, BISC recovers our proposal.

Generalized covariate balance for MrP

Step one: Construct $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.

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Two possibilities:

- Allow $\tilde{y}$ to take values other than $\{0, 1\}$ and set $\tilde{y} = y + \delta f(\mathbf{x})$, or
- Use an estimate of $\mathbb{E}\left[y|\mathbf{x}\right]$ to draw new binary $\tilde{y}$.

Our approach:

- Use $\tilde{y} = y + \delta f(\mathbf{x})$ to identify problematic "imbalanced" $f(\mathbf{x})$
- Sanity check by generating binary $\tilde{y}$ using $f(\mathbf{x})$ (which is fast and easy)

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Problem: $\hat{\mu}_{\text{MRP}}(\cdot)$ is computed with MCMC.

- Each MCMC run typically takes hours, and
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MrP Local Equivalent Weights (MrPlew)

Form the first-order Taylor series approximation

$$\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \hat{\mu}_{\mathrm{MRP}}(y) \approx \sum_{i=1}^{N_S} w_i^{\mathrm{MRP}}(\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\mathrm{MRP}} := \frac{d}{dy_i}\hat{\mu}_{\mathrm{MRP}}(y).$$

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Use in BISC: For a wide set of judiciously chosen $f(\cdot)$, check

$$\underbrace{\delta \sum_{i=1}^{N_S} w_i^{\text{MRP}} f(\mathbf{x}_i)}_{\approx \hat{\mu}_{\text{MRP}}(\tilde{y}) - \hat{\mu}_{\text{MRP}}(y)} \overset{\text{check}}{\approx} \delta \frac{1}{N_T} \sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

This a **sensitivity analysis** that formally coincides with a **balance check**.

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Computation: The weights are given by weighted averages of posterior covariances[3].

They can be easily computed with standard software[4] **without re–running MCMC**.

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[3] **giordano:2018:covariances**.

[4] We use `brms` (**brms**).

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Theory: We state conditions under which, as $\delta \to 0$, and $N \to \infty$,

- The residual is of lower order than the MrPlew term,
- *Uniformly* over a very wide class of $f(\cdot)$.

Uniformity is the hard part, but this justifies using MrPlew to *identify* problematic $f(\cdot)$.

Builds on earlier work on uniform error bounds for Bernstein–von Mises theorem(–ish) results[3].

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[3]**giordano:2024:bayesij**; **kasprzak:2025:laplace**.

Real Data

Analysis of changing names after marriage (based on **alexander:2019:namechange**).

- **Target population:** ACS survey of US population 2017–2022[4]
- **Survey population:** Marital Name Change Survey[5]
- **Respose:** Did the female partner keep their name after marriage?
- For regressors, use bins of age, education, state, and decade married.

$$
\begin{array}{ll}
\text{Survey observations:} & N_S = 4,364 \\
\text{Target observations (rows):} & N_T = 4,085,282
\end{array}
$$

$$
\begin{array}{ll}
\text{Uncorrected survey mean:} & \dfrac{1}{N_S} \sum_{i=1}^{N_S} y_i = 0.462 \\
\text{Raking:} & \hat{\mu}_{\text{CAL}} = 0.263 \\
\text{MrP:} & \hat{\mu}_{\text{MRP}} = 0.288 \quad \text{(Post. sd} = 0.0169)
\end{array}
$$

---

[4]**ipumsusa**.

[5]**cohen:2019:namechange**.

Figure
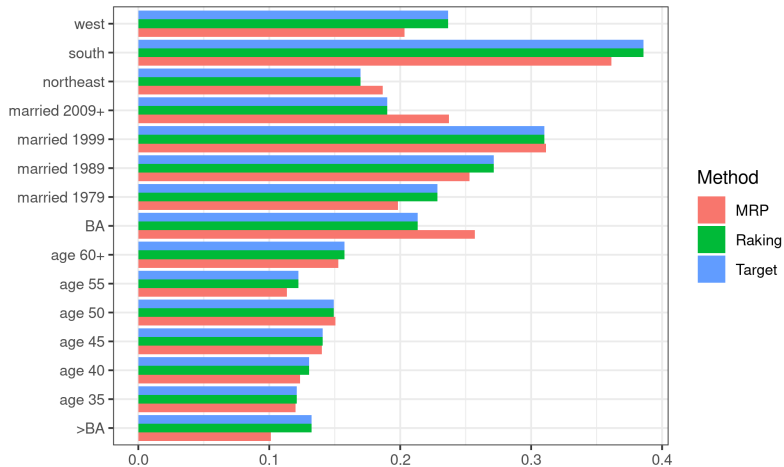
Figure 1: Imbalance plot for primary effects

Figure
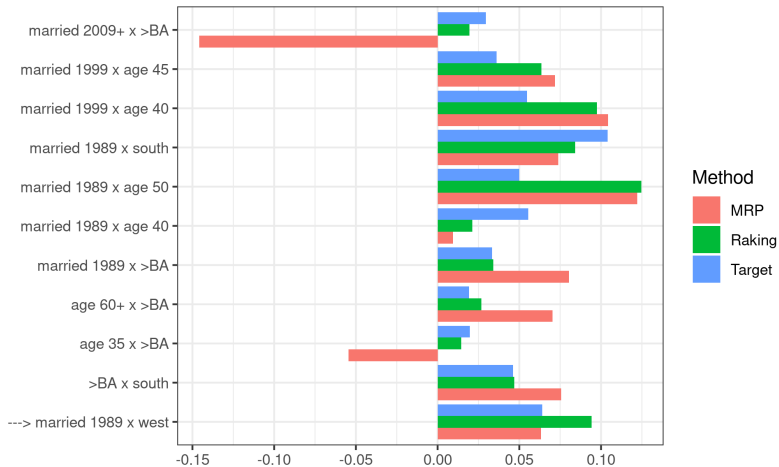
Figure 2: Imbalance plot for select interaction effects
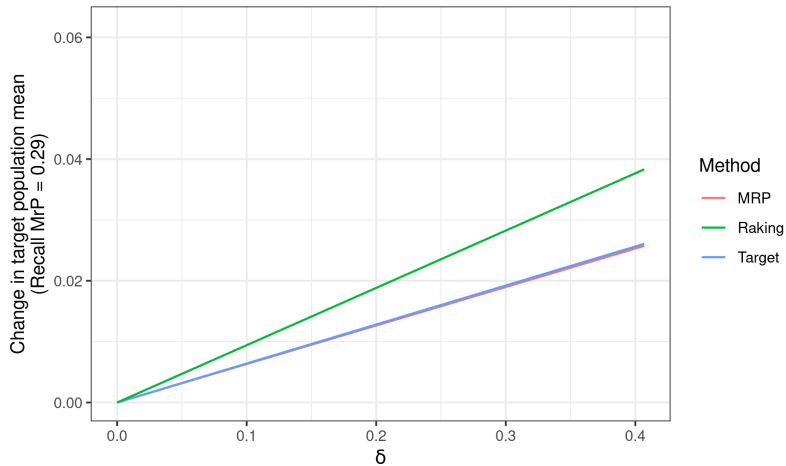
Figure

Figure 3: Continuous predictions Alexander
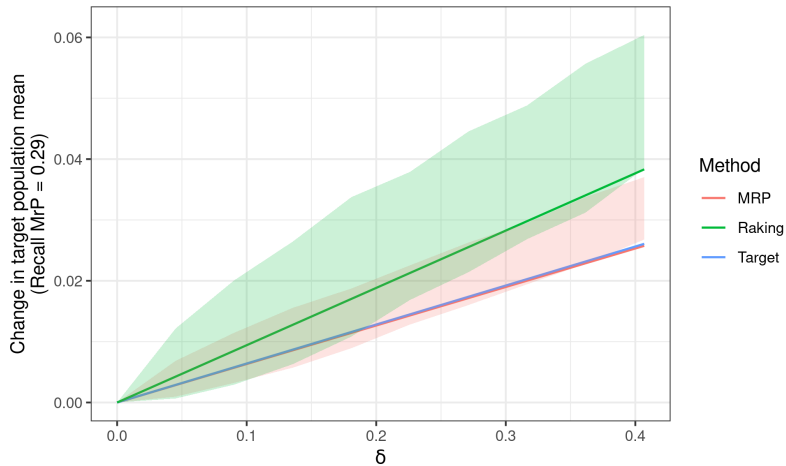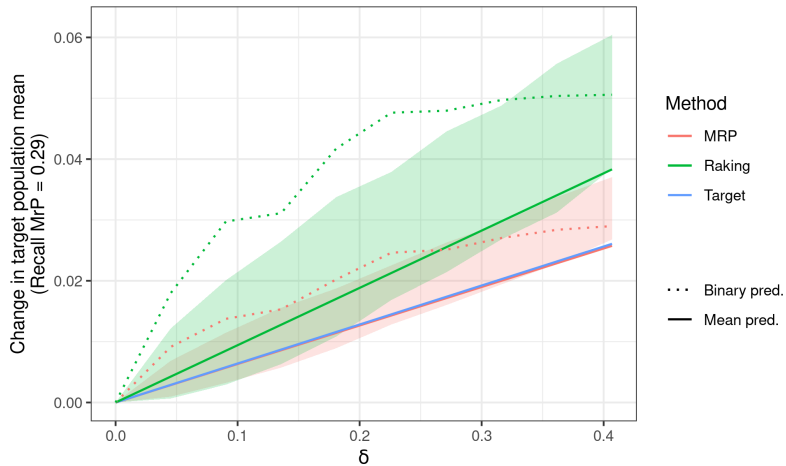
Figure

Figure 4: Continuous predictions Alexander

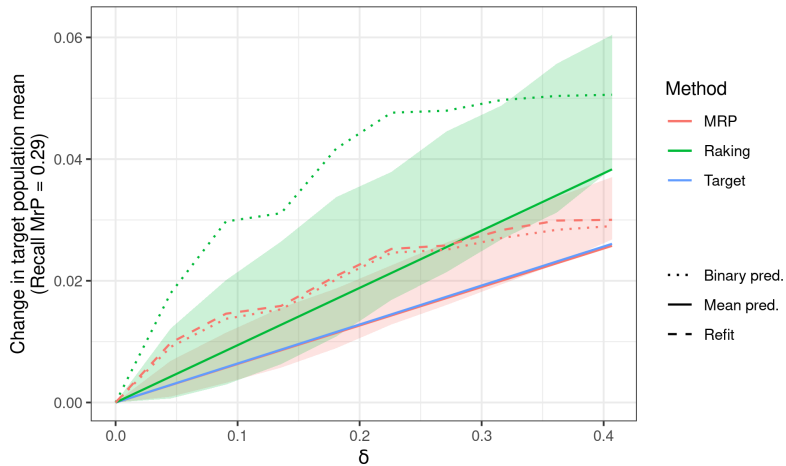**Figure 5:** Continuous predictions Alexander

Figure

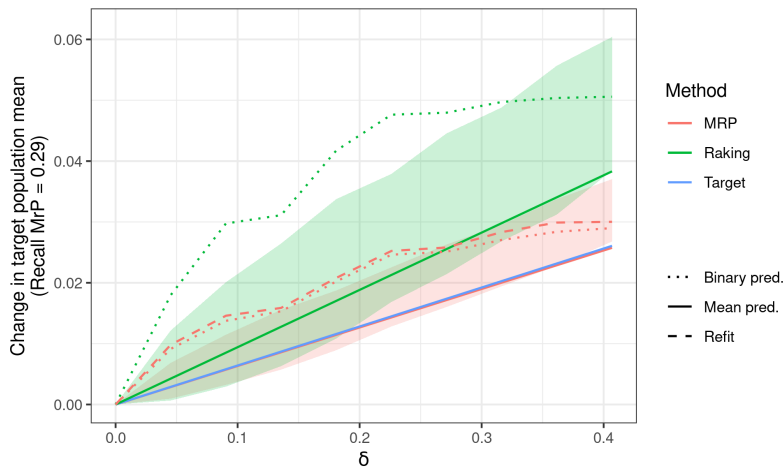**Figure 6:** Continuous predictions Alexander

Figure

Figure 6: Continuous predictions Alexander

Running ten MCMC refits: 28 hours   Computing approximate weights: 27 seconds

Today, I focused on covariate balance. In this work, we also provide rigorous justification for

- Frequentist covariance estimation
- Parital pooling
- Negative weights (extrapolation)

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- **Alice Cima** contributed significantly to this work
- **Vladimir Palmin** is working on extending MrPlew to `lme4`
- **Sequoia Andrade** is working on generalizing to other local sensitivity checks
- **Lucas Schwengber** is working on novel flow–based techniques for local sensitivity