Locally Equivalent Weights for Bayesian MrP

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Are US non-voters becoming more Republican?

Blue Rose research says yes:

“Politically disengaged voters have become much more Republican, and because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate.”

(Blue Rose Research 2024)
(major professional pollsters)

***On Data and Democracy* says no:**

“Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available.”

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- The problem is very hard (it’s difficult to accurately poll non–voters)
- Different data sources
- ⋆⋆⋆ **Different statistical methods**
  - Blue Rose uses Bayesian hierarchical modeling (MrP)
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Our contribution

We define “MrP local equivalent weights” (MrPlew) that:

- Are easily computable from MCMC draws and standard software, and
- Provide MrP versions of key diagnostics that motivate calibration weighting.

⇒ **MrPlew provides direct comparisons between MrP and calibration weighting.**

Outline

- Introduce the statisical problem
  - Contrast CW and MrP
  - Prior work: Equivalent weights for linear models
  - Equivalent weights versus implicit weights for non–linear models
  - Our task: Rigorously justify using locally equivalent weights for diagnostics

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  - Describe classical covariate balance
  - Introduce a MrPlew “balance inspired sensitivity check”
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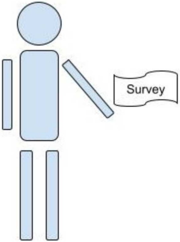
- Other directions
  - High–level restatement of the logic of our procedure
  - Local versions of other common diagnostics for linear estimators
  - Ongoing and future work

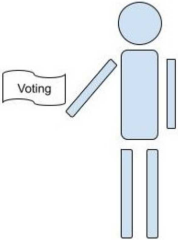
The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses y (e.g. A binary response to "do you support Trump")

We want the average response in a target population, in which we observe only covariates.





Observe $(\mathbf{x}_i, y_i)$ for $i = 1, \ldots, N_S$ | Observe $\mathbf{x}_j$ for $j = 1, \ldots, N_T$

[1]Photo copyright: Mark Taylor / `naturepl.com`

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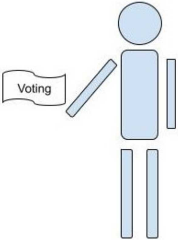

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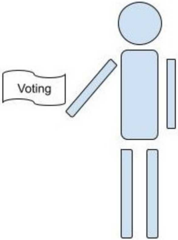

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How can we use the covariates to say something about the target responses?

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Two approaches

We want $\mu := \frac{1}{N_T} \sum_{j=1}^{N_T} y_j$, but don't observe target y_j. Let $Y_{\mathcal{S}} = \{y_1, \ldots, y_{N_S}\}$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
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Calibration weighting (CW)

▶ Choose "calibration weights" w_i
using only the regressors $\mathbf{x}$
(e.g. raking weights)

Bayesian hierarchical modeling (MrP)

▶ Choose $\mathbb{E}\left[y|\mathbf{x},\theta\right] = m(\theta^\intercal \mathbf{x})$,
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- ▶ Weights give interpretable diagnostics:
  - Frequentist variability
  - Regressor balance
  - Partial pooling

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$\leftarrow$ We open the box, providing analogues of all these diagnostics

Prior work: Equivalent weights for linear models

Gelman (2007b) observes that MrP is a CW estimator when one uses linear regression to form $\hat{y}$:

$$\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T}\sum_{j=1}^{N_T}\hat{y}_j = \frac{1}{N_T}\sum_{j=1}^{N_T}\underbrace{\mathbf{x}_j^{\intercal}\hat{\theta}}_{\text{Linear in } Y_{\mathcal{S}}}$$

Most existing literature on comparing CW and MrP focus on such linear models. [2]

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But what if you use a non–linear link function? Or a hierarchical model?

> *"It would also be desirable to use nonlinear methods ... but then it would seem difficult to construct even approximately equivalent weights. Weighting and fully nonlinear models would seem to be completely incompatible methods."* — *(Gelman 2007a)*

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Approximately equivalent weights for (some) logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$, with MLE $\hat{\theta}$.
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The map from $Y_{\mathcal{S}} \mapsto m(\mathbf{x}_j^\intercal \hat{\theta})$ is *inherently nonlinear*.

But *some sample averages* of $m(\mathbf{x}_j^\intercal \hat{\theta})$ can be approximately linear.

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Example

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

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[3] For MLEs, $\frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ is given by the implicit function theorem. (Krantz and Parks 2012; **G.**, Stephenson, et al. 2019)

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But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

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- MrP is $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta})$.

Example

Suppose $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})} \approx \alpha^\intercal \mathbf{x}$ for some α. **Then MrP is a *approximately* a CW estimator.**

$$\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) = \frac{1}{N_T} \sum_{j=1}^{N_T} m(\mathbf{x}_j^\intercal \hat{\theta}) = \frac{1}{N_S} \sum_{i=1}^{N_S} \underbrace{w_i^{\text{MrP}}}_{\alpha^\intercal \mathbf{x}_i} y_i + \text{Small error}$$

But what are the weights? We don't observe $\frac{\mathcal{P}_T(\mathbf{x})}{\mathcal{P}_S(\mathbf{x})}$, so can't estimate α directly.

Key idea (informal)

If $\hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ is approximately linear, then[3] $w_i^{\text{MrP}} \approx N_S \frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$.

[3] For MLEs, $\frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ is given by the implicit function theorem. (Krantz and Parks 2012; **G.**, Stephenson, et al. 2019)

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Note: The derivatives w_i^{MrP} are both *implicit* and *equivalent* weights.

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Locally equivalent weights for hierarchical logistic regression MrP

- Suppose the model is $m(\mathbf{x}^\intercal \theta) = \mathrm{Logistic}(\mathbf{x}^\intercal \theta)$.
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No reason to think $Y_{\mathcal{S}} \mapsto \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})$ is even approximately **globally** linear.

[4] Diaconis and Freedman 1986; Gustafson 1996; Efron 2015; **G.**, Broderick, and Jordan 2018.

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But can still compute and analyze $w_i^{\text{MrP}} := N_S \frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i}$ using Bayesian sensitivity analysis![4]

MrP weights for MCMC

$$w_i^{\text{MrP}} := N_S \frac{\partial \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})}{\partial y_i} = N_S \frac{1}{N_T}\sum_{j=1}^{N_T} \underbrace{\text{Cov}_{\mathcal{P}(\theta|\text{Survey data})}\left(m(\mathbf{x}_j^\intercal\theta), \theta^\intercal\mathbf{x}_i\right)}_{\text{Can estimate without rerunning MCMC!}}$$

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What do these weights mean? There are now two distinct possibilities:

- "Locally implicit weights"
  - An estimator of $\mathcal{P}_T(\mathbf{x})/\mathcal{P}_S(\mathbf{x})$ (via Riesz regression applied to the Gateaux derivative)
- "Locally equivalent weights"
  - A characterization of $Y_{\mathcal{S}} \mapsto \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ for diagnostics and interpretation

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- "Locally equivalent weights" $\leftarrow$ **The present talk will focus on this interpretation**
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MrP locally equivalent weights (MrPlew)

For new data $\tilde{Y}_{\mathcal{S}}$, form a **MrP locally equivalent weighting**:

$$\hat{\mu}^{\text{MrP}}(\tilde{Y}_{\mathcal{S}}) \approx \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) + \sum_{i=1}^{N_{\mathcal{S}}} w_i^{\text{MrP}} (\tilde{y}_i - y_i)$$

Our task is to rigorously show that even such local weights can be meaningfully used diagnostically in the same ways we use global weights.

The weights can look very different!

Does this mean anything?

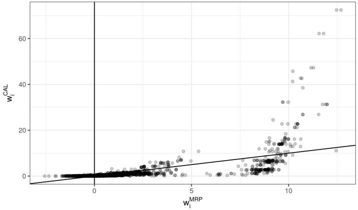

Figure 1: Comparison between raking and MrPlew weights for the Name Change dataset

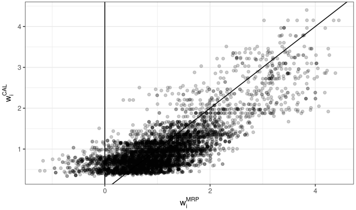

Figure 2: Comparison between raking and MrPlew weights for the Gay Marriage dataset

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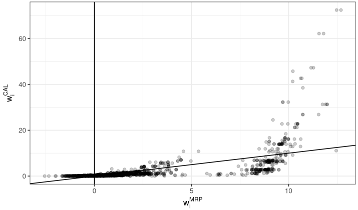
Does this mean anything?

Does the spread relate to frequentist variance?



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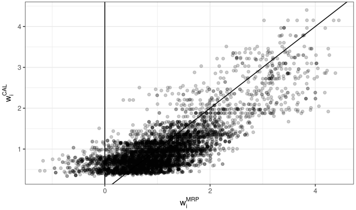

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Frequentist variance estimation

Let $\hat{\mathrm{Var}}(\cdot)$ denote the sample variance.

Calibration weighting standard errors sketch: [5]

Suppose we have $\hat{\mu}^{\mathrm{CW}}(Y_{\mathcal{S}}) = \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i$ and a consistent residual estimate ε_i.

Then $\hat{\mathrm{Var}}(w_i \varepsilon_i) \approx \mathrm{Var}\left(\sqrt{N_S}\hat{\mu}^{\mathrm{CW}}(Y_{\mathcal{S}})\right)$.

[5] E.g. , Deville, Särndal, and Sautory (1993) and Fuller (2011).

[6] This is essentially a corollary of our earlier work on the Bayesian infinitesimal jackknife. (**G.** and Broderick 2024)

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MrPlew Standard error consistency theorem sketch (Our contribution):[6]

For Bayesian hierarchical logictic regression, define $\varepsilon_i = y_i - \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[m(\mathbf{x}_i^\intercal \theta)\right]$.

We state mild conditions under which, as $N_S \to \infty$, for some μ_∞ and variance V,

$$\sqrt{N_S}\left(\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) - \mu_\infty\right) \to \mathcal{N}\left(0, V\right) \quad \text{and} \quad \hat{\mathrm{Var}}\left(w_i^{\mathrm{MrP}} \varepsilon_i\right) \to V.$$

The use of w_i^{MrP} is analogous to the use of w_i for frequentist variance estimation.

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Standard error estimation

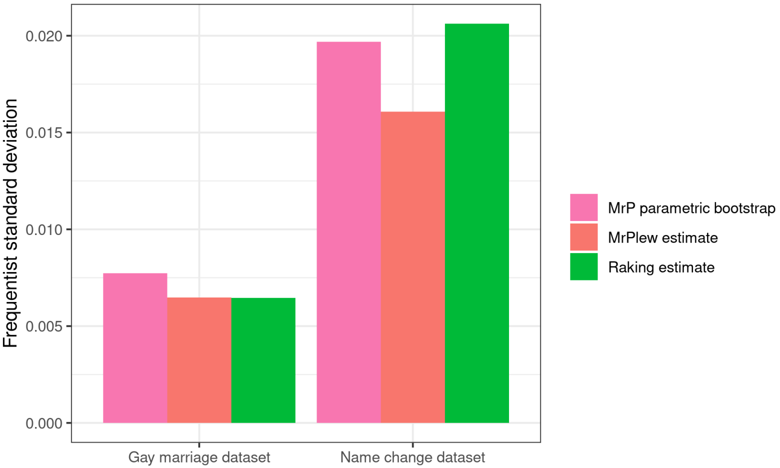


Figure 3: Frequentist standard deviation estimates

Other uses

Does this mean anything?

Yes: The “spread” relates to frequentist variance just as in calibration weighting.

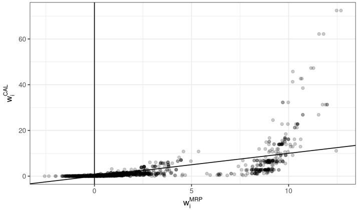
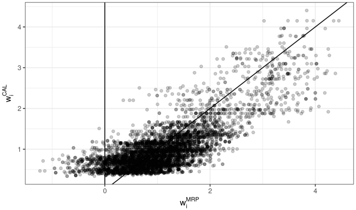


Figure 4: Comparison between raking and MrPlew weights for the Name Change dataset



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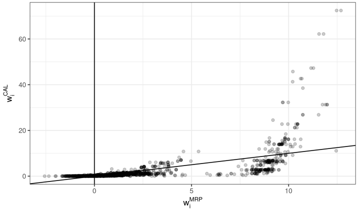
What about covariate balance?



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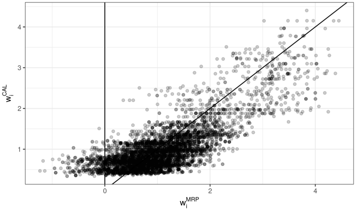

Figure 5: Comparison between raking and MrPlew weights for the Gay Marriage dataset

What are we weighting for?[7]

$$\text{Target average response} = \frac{1}{N_T}\sum_{j=1}^{N_T} y_j \approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i = \text{Weighted survey average response}$$

We can't check this, because we don't observe y_j.

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$$\frac{1}{N_T}\sum_{j=1}^{N_T} \mathbf{x}_j = \frac{1}{N_S}\sum_{i=1}^{N_S} w_i \mathbf{x}_i$$

Such weights satisfy "covariate balance" for $\mathbf{x}$.

You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

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You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

Raking calibration weights

"Raking" selects weights that

- Are as "close as possible" to some reference weights
- Under the constraint that they balance some selected regressors.

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Balance checks as sensitivity analysis

One reason to balance $f(\mathbf{x})$ is because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

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Balance–informed sensitivity check (BISC) (informal)

Pick a small $\delta > 0$ and an $f(\cdot)$. Define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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$$\mathbb{E}\left[\mu(\tilde{y}) - \mu(y)|\mathbf{x}\right] = \frac{1}{N_T}\sum_{j=1}^{N_T}\left(\mathbb{E}\left[\tilde{y}|\mathbf{x}_p\right] - \mathbb{E}\left[y|\mathbf{x}_p\right]\right) = \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change for observed $\tilde{Y}_{\mathcal{S}}, Y_{\mathcal{S}}$:

$$\underbrace{\hat{\mu}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}(Y_{\mathcal{S}})}_{\text{Replace weighted averages with changes in an estimator}} \overset{\textbf{check}}{\approx} \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

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When $\hat{\mu}(\cdot) = \hat{\mu}^{\mathrm{CW}}(\cdot)$, BISC recovers the standard covariate balance check.

We will study $\hat{\mu}(\cdot) = \hat{\mu}^{\mathrm{MrP}}(\cdot)$.

BISC for MrP

Suppose I have $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.
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Problem: $\hat{\mu}^{\text{MrP}}(\cdot)$ is computed with MCMC.

- Each MCMC run typically takes hours, and
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Solution: Use our local approximation, MrPlew!

Balance informed sensitivity check with MrPlew:

For a wide set of judiciously chosen $f(\cdot)$, check

$$\begin{aligned}
\hat{\mu}^{\text{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}}) &\approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i^{\text{MrP}}(\tilde{y}_i - y_i) \\
&\approx \underbrace{\delta \frac{1}{N_S}\sum_{i=1}^{N_S} w_i^{\text{MrP}} f(\mathbf{x}_i) \overset{\textbf{check}}{\approx} \delta \frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j).}_{\text{What you actually check}}
\end{aligned}$$

Generating $\tilde{y}$

- We have defined BISC in terms of $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$
- We have approximated $\hat{\mu}^{\text{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\text{MrP}}(Y_{\mathcal{S}})$ for $\tilde{y} \approx y$

How to get such a $\tilde{y}$? **Recall y is binary!**

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Option 2: Allow $\tilde{y}$ to take generic values.

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1. Make *some* guess $\hat{m}(\mathbf{x}) \approx \mathbb{E}\left[y|\mathbf{x}\right]$
    - E.g. Posterior mean, or
    - Shrunken posterior mean, or
    - Some values that gives the same posterior
2. Take $u_i \overset{iid}{\sim} \mathrm{Unif}(0, 1)$
3. Assume $y_i = \mathbb{I}\left(u_i \le \hat{m}(\mathbf{x}_i)\right)$
4. Draw $u_n | y_n$
5. Set $\tilde{y}_i = \mathbb{I}\left(u_i \le \hat{m}(\mathbf{x}_i) + \delta \mathbf{x}_i\right)$

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1. Make *some* guess $\hat{m}(\mathbf{x}) \approx \mathbb{E}\left[y|\mathbf{x}\right]$
   - E.g. Posterior mean, or
   - Shrunken posterior mean, or
   - Some values that gives the same posterior
2. Take $u_i \overset{iid}{\sim} \text{Unif}(0, 1)$
3. Assume $y_i = \mathbb{I}\left(u_i \leq \hat{m}(\mathbf{x}_i)\right)$
4. Draw $u_n | y_n$
5. Set $\tilde{y}_i = \mathbb{I}\left(u_i \leq \hat{m}(\mathbf{x}_i) + \delta \mathbf{x}_i\right)$

Option 2: Allow $\tilde{y}$ to take generic values.

1. Set $\tilde{y}_i = y_i + \delta f(\mathbf{x}_i)$.
2. Then you're done.
3. There is nothing else to do.
4. This space deliberately left blank.

Generating $\tilde{y}$

- We have defined BISC in terms of $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$
- We have approximated $\hat{\mu}^{\mathrm{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}})$ for $\tilde{y} \approx y$

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Pros and cons:

- Realistic
- Have to pick $\hat{m}(\mathbf{x})$
- $\tilde{Y}_{\mathcal{S}} - Y_{\mathcal{S}}$ not infinitesimally small
- **Use for checks & experiments**

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3. There is nothing else to do.
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Pros and cons:

- Not realistic
- No additional assumptions
- $\tilde{Y}_{\mathcal{S}} - Y_{\mathcal{S}}$ may be infinitesimally small
- **Use for theory**

Theory

When is the local approximation accurate?

BISC Theorem: (sketch)

Take $\tilde{y}_i = y_i + \delta f(\mathbf{x}_i)$.

We state conditions for Bayesian hierarchical logistic regression under which

$$\left| \hat{\mu}^{\mathrm{MrP}}(\tilde{Y}_{\mathcal{S}}) - \hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) - \delta \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} f(\mathbf{x}_i) \right| = \text{Small}$$

[8] $\mathcal{F}$ can be any Donsker class of measurable functions with uniformly bounded $\mathbb{E}\left[\mathbf{x} f(\mathbf{x})\right]$.

[9] **G.** and Broderick 2024; Kasprzak, **G.**, and Broderick 2025.

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Uniformity justifies searching for "imbalanced" f.

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Uniformity justifies searching for "imbalanced" f.

The uniformity result builds on our earlier work on uniform and finite–sample error bounds for Bernstein–von Mises theorem–like results[9].

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Real Data: Marital Name Change Survey

Analysis of changing names after marriage[10].

- **Target population:** ACS survey of US population 2017–2022[11]
- **Survey population:** Marital Name Change Survey (from Twitter)[12]
- **Respose:** Did the female partner keep their name after marriage?
- For regressors, use bins of age, education, state, and decade married.

Survey observations: $N_S = 4,364$

Target observations (rows): $N_T = 4,085,282$

Uncorrected survey mean: $\frac{1}{N_S}\sum_{i=1}^{N_S} y_i = 0.462$

Raking: $\hat{\mu}^{\mathrm{CW}}(Y_{\mathcal{S}}) = 0.263$

MrP: $\hat{\mu}^{\mathrm{MrP}}(Y_{\mathcal{S}}) = 0.288 \quad (\text{Post. sd} = 0.0169)$

[10] Based on Alexander (2019).

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Covariate balance for primary effects

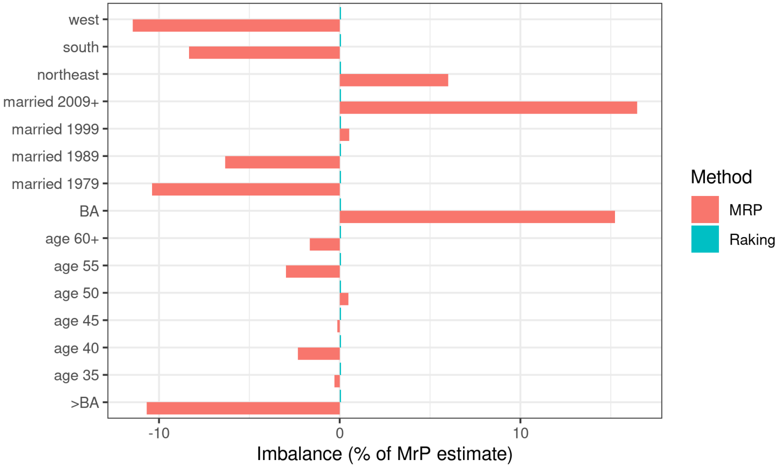

Figure 6: Imbalance plot for primary effects in the Name Change dataset

Covariate balance for interaction effects

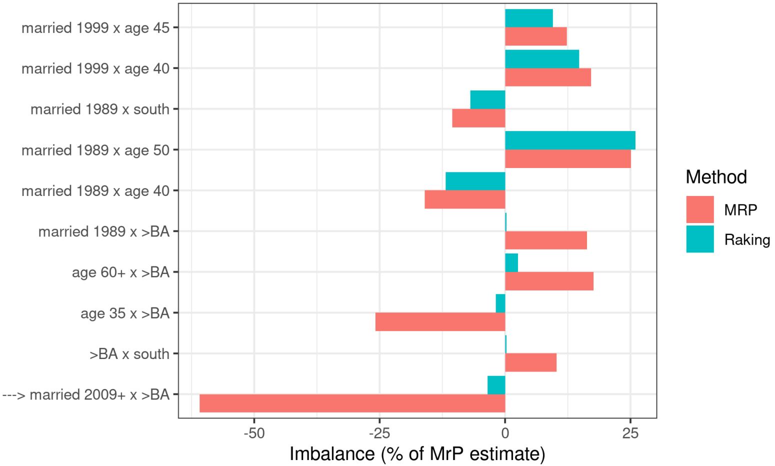

Figure 7: Imbalance plot for select interaction effects in the Name Change dataset

Predictions

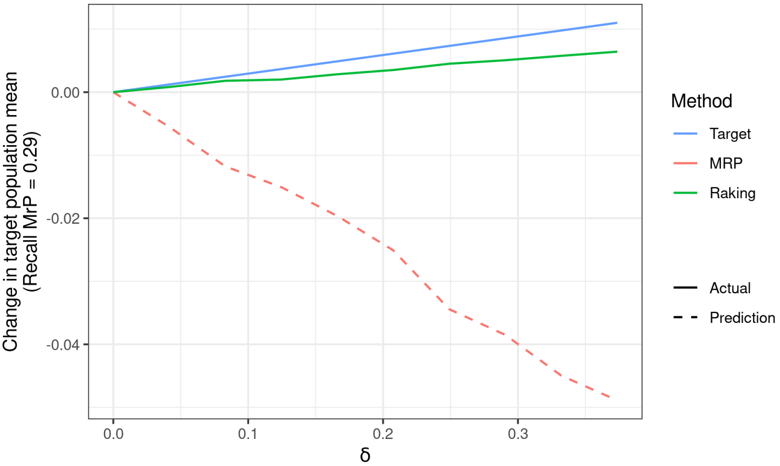

Figure 8: Predictions on binary data for the Name Change dataset

Predictions and actual MCMC results

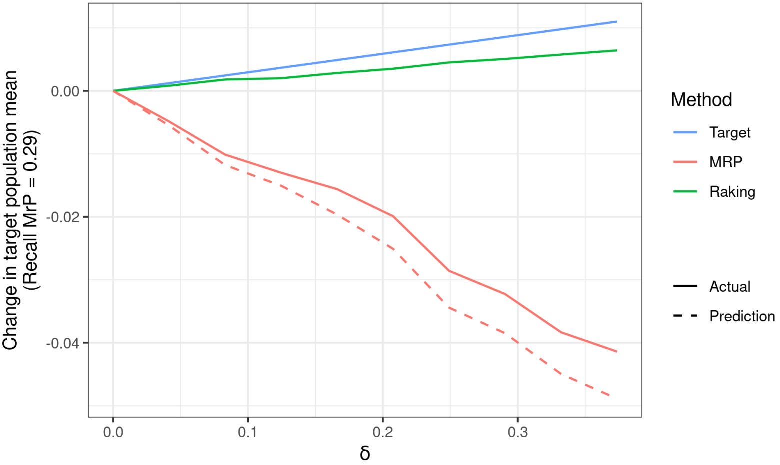

Figure 9: Predictions and refit on binary data for the Name Change dataset

Running ten MCMC refits: 10 hours Computing approximate weights: 16 seconds

Real Data: Lax Philips

Analysis of national support for gay marriage.[13]

- **Target population:** US Census Public Use Microdata Sample 2000
- **Survey population:** Combined national-level polls from 2004
- **Respose:** “Do you favor allowing gay and lesbian couples to marry legally?”
- For regressors, use race, gender, age, education, state, region, and continuous statewide religion and political characteristics, including some analyst–selected interactions.

Survey observations: $N_S = 6,341$

Target observations (rows): $N_T = 9,694,541$

Uncorrected survey mean: $\frac{1}{N_S} \sum_{i=1}^{N_S} y_i = 0.333$

Raking: $\hat{\mu}_{\mathrm{CW}} = 0.33$

MrP: $\hat{\mu}_{\mathrm{MrP}} = 0.337 \quad (\text{Post. sd} = 0.039)$

[13] Based on Kastellec, Lax, and Phillips (2010), see also Lax and Phillips (2009).

Covariate balance for primary effects

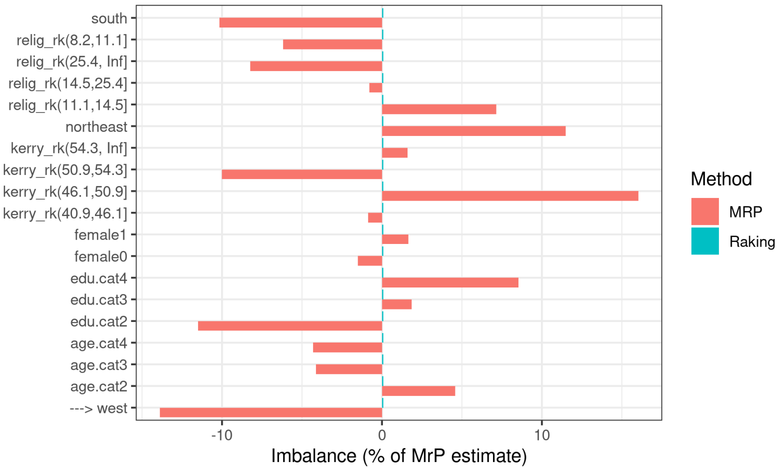

Figure 10: Imbalance plot for primary effects in the Gay Marriage dataset

Covariate balance for interaction effects

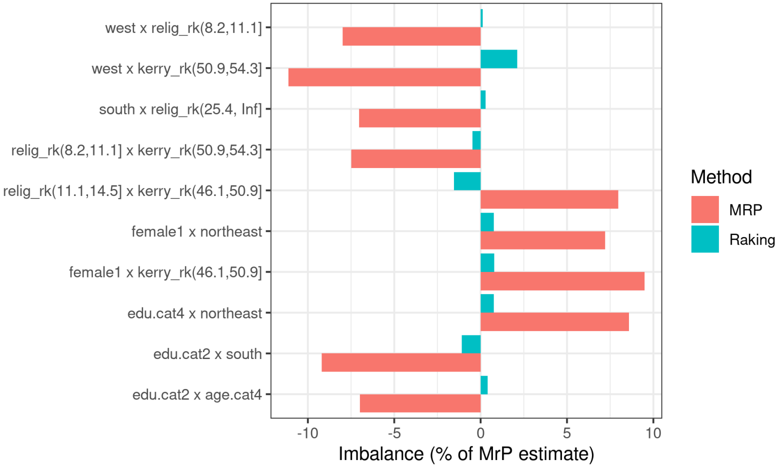

Figure 11: Imbalance plot for select interaction effects in the Gay Marriage dataset

Predictions

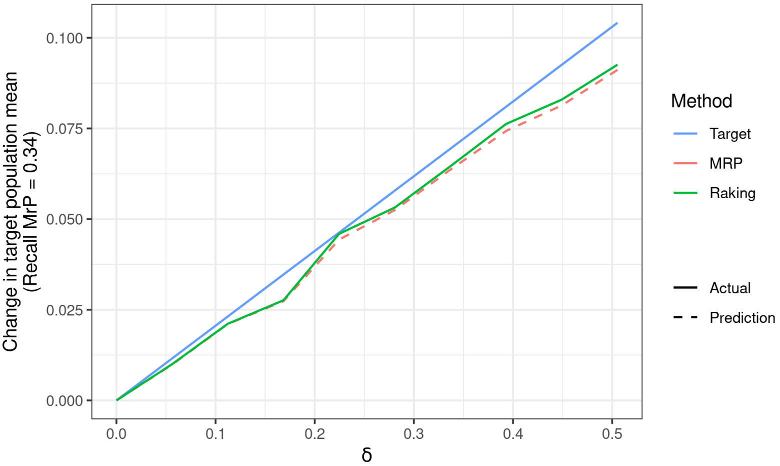

Figure 12: Predictions on binary data for the Gay Marriage dataset

Predictions and actual MCMC results

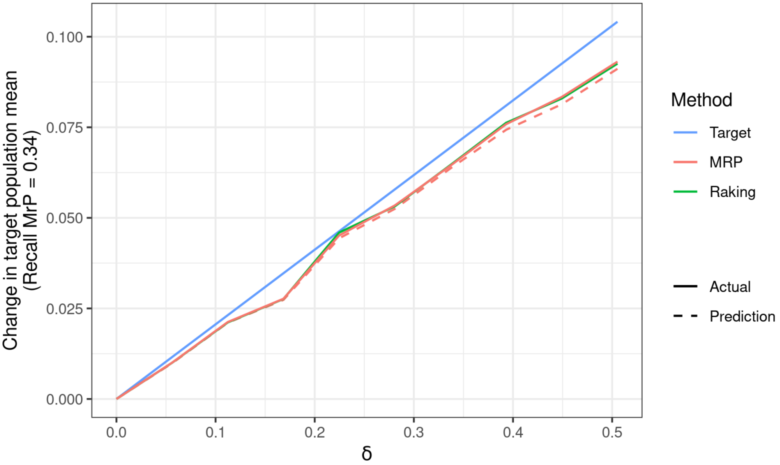

Figure 13: Predictions and refit on binary data for the Gay Marriage dataset

Running ten MCMC refits: 11 hours Computing approximate weights: 23 seconds

Future work

Notice that there was no discussion of misspecification!

Calibration weights (typically) do not depend on $Y_{\mathcal{S}}$.

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But the high level idea can be extended much more widely:

1. Assume your initial model was accurate
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4. If the change is not what you expect, either (1) or (2) was wrong

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Checks of this form give generalized versions of many standard linear model diagnostics.

Some generalized diagnostics

Regression

Some generalized diagnostics

	Regression	General models

Some generalized diagnostics

	Regression	General models
Consistency / Unbiased	$y = \theta^\intercal \mathbf{x} + \varepsilon$ $\tilde{y} = (\theta + \delta)^\intercal \mathbf{x} + \varepsilon$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$	

Some generalized diagnostics

	Regression	General models
Consistency / Unbiased	$y = \theta^\intercal \mathbf{x} + \varepsilon$	$y = f(\mathbf{x}, \varepsilon, \theta)$
	$\tilde{y} = (\theta + \delta)^\intercal \mathbf{x} + \varepsilon$	$\tilde{y} = f(\mathbf{x}, \varepsilon, \theta + \delta)$
	$\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$	$\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$

Some generalized diagnostics

	Regression	General models
Consistency / Unbiased	$y = \theta^\mathsf{T}\mathbf{x} + \varepsilon$ $\tilde{y} = (\theta + \delta)^\mathsf{T}\mathbf{x} + \varepsilon$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$	$y = f(\mathbf{x}, \varepsilon, \theta)$ $\tilde{y} = f(\mathbf{x}, \varepsilon, \theta + \delta)$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y) + \delta$
Exogonous residuals	$y = \theta^\mathsf{T}\mathbf{x} + \varepsilon$ $\tilde{y} = y + \varepsilon z$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y)$	

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Exogonous residuals	$y = \theta^\mathsf{T}\mathbf{x} + \varepsilon$ $\tilde{y} = y + \varepsilon z$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y)$	$y \sim \mathcal{P}(y\|\mathbf{x})$ and $\mathcal{P}(\mathbf{x}) = w$ $\tilde{w} = w + \delta z$ $\hat{\theta}(\tilde{w}) \overset{\text{check}}{=} \hat{\theta}(w)$

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Fisher information	$\mathcal{I} :=$ Fisher information $\Sigma :=$ Score covariance $\mathcal{I}^{-1} \overset{\text{check}}{=} \Sigma$	

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Fisher information	$\mathcal{I} :=$ Fisher information $\Sigma :=$ Score covariance $\mathcal{I}^{-1} \overset{\text{check}}{=} \Sigma$	$y \sim \mathcal{P}(y \vert \theta)$ $\tilde{y} \sim$ Importance sample y using $\tilde{w} = \frac{\mathcal{P}(y \vert \hat{\theta} + \delta)}{\mathcal{P}(y \vert \hat{\theta})}$ $\hat{\theta}(\tilde{w}) \overset{\text{check}}{=} \hat{\theta}(1) + \delta$

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Exogonous residuals	$y = \theta^\intercal \mathbf{x} + \varepsilon$ $\tilde{y} = y + \varepsilon z$ $\hat{\theta}(\tilde{y}) \overset{\text{check}}{=} \hat{\theta}(y)$	$y \sim \mathcal{P}(y\vert\mathbf{x})$ and $\mathcal{P}(\mathbf{x}) = w$ $\tilde{w} = w + \delta z$ $\hat{\theta}(\tilde{w}) \overset{\text{check}}{=} \hat{\theta}(w)$
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Future work

Student contributions and ongoing work:

- **Vladimir Palmin** is working on extending MrPlew to `lme4`
- **Sequoia Andrade** is working on generalizing to other local sensitivity checks
- **Lucas Schwengber** is working on novel flow–based techniques for local sensitivity
- **(Currently recruiting!)** Doubly–robust Bayesian MrP (the "implicit weights" path)

Vladimir Palmin

Sequoia Andrade

Lucas Schwengber

Preprint and R package coming soon! 🙏

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