Locally Equivalent Weights for Bayesian MrP

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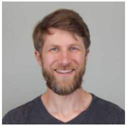

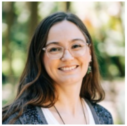
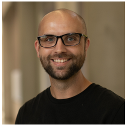


What are we weighting for?[1]

$$\text{Target average response} = \frac{1}{N_T}\sum_{j=1}^{N_T} y_j \approx \frac{1}{N_S}\sum_{i=1}^{N_S} w_i y_i = \text{Weighted survey average response}$$

We can't check this, because we don't observe y_j.

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$$\frac{1}{N_T}\sum_{j=1}^{N_T} \mathbf{x}_j = \frac{1}{N_S}\sum_{i=1}^{N_S} w_i \mathbf{x}_i$$

Such weights satisfy "covariate balance" for $\mathbf{x}$.

You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

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You can check covariate balance for any calibration weighting estimator, and any function $f(\mathbf{x})$.

Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

Raking calibration weights

"Raking" selects weights that

- Are as "close as possible" to some reference weights
- Under the constraint that they balance some selected regressors.

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Balance checks as sensitivity analysis

One reason to balance $f(\mathbf{x})$ is because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

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Balance–informed sensitivity check (BISC) (informal)

Pick a small $\delta > 0$ and an $f(\cdot)$. Define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

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We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}\left[\mu(\tilde{y}) - \mu(y)|\mathbf{x}\right] = \frac{1}{N_T}\sum_{j=1}^{N_T}\left(\mathbb{E}\left[\tilde{y}|\mathbf{x}_p\right] - \mathbb{E}\left[y|\mathbf{x}_p\right]\right) = \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change for observed $\tilde{y}, y$:

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\substack{\text{Replace weighted averages} \\ \text{with changes in an estimator}}} \overset{\text{check}}{\approx} \delta\frac{1}{N_T}\sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

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When $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{CW}}(\cdot)$, BISC recovers the standard covariate balance check.

We will use $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{MrP}}(\cdot)$.

BISC for MrP

Suppose I have $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.
Now I need to evaluate $\hat{\mu}_{\text{MrP}}(\tilde{y}) - \hat{\mu}_{\text{MrP}}(y)$.

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Problem: $\hat{\mu}_{\text{MrP}}(\cdot)$ is computed with MCMC.

- Each MCMC run typically takes hours, and
- Output is noisy, and $\hat{\mu}_{\text{MrP}}(\tilde{y}) - \hat{\mu}_{\text{MrP}}(y)$ may be small.

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MrP Local Equivalent Weights (MrPlew)

Form the first-order Taylor series approximation

$$\hat{\mu}_{\text{MrP}}(\tilde{y}) - \hat{\mu}_{\text{MrP}}(y) \approx \sum_{i=1}^{N_S} w_i^{\text{MrP}}(\tilde{y}_i - y_i) \quad \text{where} \quad w_i^{\text{MrP}} := \frac{d}{dy_i}\hat{\mu}_{\text{MrP}}(y).$$

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Computation: The weights are given by weighted averages of posterior covariances[2].

They can be easily computed with standard software[3] **without re–running MCMC**.

[2] **G.**, Broderick, and Jordan 2018.
[3] We use `brms` (Bürkner 2017).

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Use in BISC: For a wide set of judiciously chosen $f(\cdot)$, check

$$\delta \sum_{i=1}^{N_S} w_i^{\mathrm{MrP}} f(\mathbf{x}_i) \overset{\text{check}}{\approx} \delta \frac{1}{N_T} \sum_{j=1}^{N_T} f(\mathbf{x}_j).$$

Theory

BISC Theorem: (sketch)

We state conditions for Bayesian hierarchical logistic regression under which

$$\left|\hat{\mu}_{\text{MrP}}(\tilde{y}) - \hat{\mu}_{\text{MrP}}(y) - \delta \sum_{i=1}^{N_S} w_i^{\text{MrP}} f(\mathbf{x}_i)\right| = \text{Small?}$$

[2] Donsker with uniformly bounded $\mathbb{E}\left[\mathbf{x} f(\mathbf{x})\right]$.

[3] **G.** and Broderick 2024; Kasprzak, **G.**, and Broderick 2025.

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For a very broad class[2] of $\mathcal{F}$.

Uniformity justifies searching for "imabalanced" f.

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The uniformity result builds on our earlier work on uniform and finite–sample error bounds for Bernstein–von Mises theorem–like results[3].

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Future work

Note that there was no talk of correct specification for the data you have.

That was a foregone conclusion when we started looking at equivalent weights!

How do you peform model checking with sensitivity analysis?

Existing methods evaluate whether the analysis changes “a lot” when you:

- Parametrically perturb the model (e.g. fit a richer model class)
- Non–parameterically perturb the data (e.g. produce gross outliers)

The problem is:

- How much is “a lot”?
- Non–parametric data perturbations are hard to reason about
- It’s hard to say whether parametric model changes are enough

Instead, we

- Parametrically perturb the data
- Observe whether our model could detect the change

- Know exactly the expected change (don’t have to decide on what “a lot” means)
- Easy to reason about whether the data perturbation is reasonable
- Don’t need to propose an alternative model, instead study the model you have

Related and future work

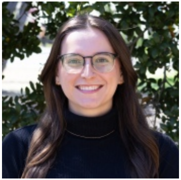
Student contributions and future work:

- **Alice Cima** contributed significantly to this work
- **Vladimir Palmin** is working on extending MrPlew to `lme4`
- **Sequoia Andrade** is working on generalizing to other local sensitivity checks
- **Lucas Schwengber** is working on novel flow–based techniques for local sensitivity

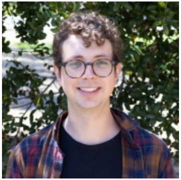
Alice Cima

No picture!

Vladimir Palmin



Sequoia Andrade



Lucas Schwengber

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