# Local Weighting–Based Diagnostics for Bayesian Poststratification

Ryan Giordano, Alice Cima, Erin Hartman, Jared Murray, Avi Feller

**Berkeley BSTARS September 2025**

# Are US non-voters becoming more Republican?

**Blue Rose research says yes:**

"Politically disengaged voters have become much more Republican, And because less-engaged voters swung away from [Democrats], an expanded electorate meant a more Republican electorate."

[Blue Rose Research, 2024] (On Ezra Klein show, major professional pollsters)

***On Data and Democracy* says no:**

"Claims of a decisive pro-Republican shift among the overall non-voting population are not supported by the most reliable, large-scale post-election data currently available."

[Bonica et al., 2025] (Berkeley professor co–author, major professional researchers)

---

- The problem is very hard (it's difficult to accurately poll non–voters)
- Different data sources
- **Very different statistical methods:** ⋆
  - Blue Rose uses Bayesian hierarchical modeling (MrP)
  - The CES uses weighted averages (calibration weighting)

**Our contribution**

We provide a calibration weighting interpretation of MrP analyses that:

- Is easily computable from MCMC draws and standard software, and
- Defines MrP versions of key diagnostics that motivate calibration weighting.

We provide apples-to-apples comparisons between MrP and calibration weighting.

# Outline

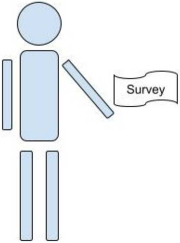
- Introduce the statisical problem and two methods (calibration weighting and MrP)
- Describe one of the classical calibration weighting diagnostics (covariate balance)
- Define MrPaw & state a key theorem
- Real-world results
- Future directions

# The basic problem

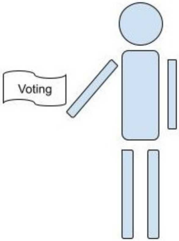
We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses $y$ (e.g. A binary response to “do you support policy such–and–such”)

We want the average response in a target population, in which we observe only covariates.




Observe $(\mathbf{x}_s, y_s)$ for $s = 1, \dots, S$



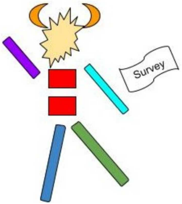

Observe $\mathbf{x}_t$ for $t = 1, \dots, T$

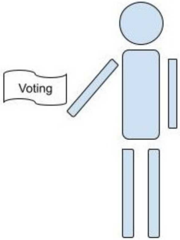
# The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses $y$ (e.g. A binary response to "do you support policy such–and–such")

We want the average response in a target population, in which we observe only covariates.




Observe $(\mathbf{x}_s, y_s)$ for $s = 1, \ldots, S$ | Observe $\mathbf{x}_t$ for $t = 1, \ldots, T$

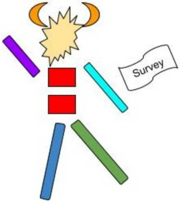
**The problem is that the populations are very different.**

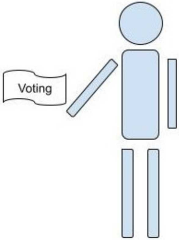
# The basic problem

We have a survey population, for whom we observe:

- Covariates $\mathbf{x}$ (e.g. race, gender, zip code, age, education level)
- Responses $y$ (e.g. A binary response to "do you support policy such–and–such")

We want the average response in a target population, in which we observe only covariates.




Observe $(\mathbf{x}_s, y_s)$ for $s = 1, \ldots, S$ | Observe $\mathbf{x}_t$ for $t = 1, \ldots, T$

**The problem is that the populations are very different.**

Our survey results may be biased.

How can we use the covariates to say something about the target responses?

# Two approaches

We want $\mu := \frac{1}{T} \sum_{t=1}^{T} y_t$, but don't observe target population $y_t$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

# Two approaches

We want $\mu := \frac{1}{T}\sum_{t=1}^{T} y_t$, but don't observe target population $y_t$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

**Calibration weighting**

► Choose "calibration weights" $w_s$
(e.g. raking weights)

**Bayesian hierarchical modeling (MrP)**

► Choose model $\mathcal{P}(y|x, \theta)$ and prior $\mathcal{P}(\theta)$
(e.g. Hierarchical logitstic regression)

# Two approaches

We want $\mu := \frac{1}{T}\sum_{t=1}^{T} y_t$, but don't observe target population $y_t$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

**Calibration weighting**

- ▶ Choose "calibration weights" $w_s$ (e.g. raking weights)
- ▶ Take $\hat{\mu}_{\text{CAL}} = \frac{1}{S}\sum_{s=1}^{S} w_s y_s$

**Bayesian hierarchical modeling (MrP)**

- ▶ Choose model $\mathcal{P}(y|x,\theta)$ and prior $\mathcal{P}(\theta)$ (e.g. Hierarchical logitstic regression)
- ▶ Take $\hat{y}_t = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}[y|\mathbf{x}_t]$ and $\hat{\mu}_{\text{MRP}} = \frac{1}{T}\sum_{t=1}^{T} \hat{y}_t$

# Two approaches

We want $\mu := \frac{1}{T}\sum_{t=1}^{T} y_t$, but don't observe target population $y_t$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of $\mathbf{x}$ may be different in the survey and target.

**Calibration weighting**

▶ Choose "calibration weights" $w_s$ (e.g. raking weights)

▶ Take $\hat{\mu}_{\text{CAL}} = \frac{1}{S}\sum_{s=1}^{S} w_s y_s$

▶ Dependence on $y_s$ is obvious ($w_s$ typically chosen using only $\mathbf{x}$)

**Bayesian hierarchical modeling (MrP)**

▶ Choose model $\mathcal{P}(y|x,\theta)$ and prior $\mathcal{P}(\theta)$ (e.g. Hierarchical logitstic regression)

▶ Take $\hat{y}_t = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[y|\mathbf{x}_t\right]$ and $\hat{\mu}_{\text{MRP}} = \frac{1}{T}\sum_{t=1}^{T}\hat{y}_t$

▶ Dependence on $y_s$ very complicated (Typically via MCMC draws from $\mathcal{P}(\theta|\text{Survey data})$)

## Two approaches

We want $\mu := \frac{1}{T}\sum_{t=1}^{T} y_t$, but don't observe target population $y_t$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of **x** may be different in the survey and target.

**Calibration weighting**

- ▶ Choose "calibration weights" $w_s$ (e.g. raking weights)
- ▶ Take $\hat{\mu}_{\text{CAL}} = \frac{1}{S}\sum_{s=1}^{S} w_s y_s$
- ▶ Dependence on $y_s$ is obvious ($w_s$ typically chosen using only **x**)
- ▶ Weights give interpretable diagnostics:
  - Frequentist variability
  - Partial pooling
  - Regressor balance

**Bayesian hierarchical modeling (MrP)**

- ▶ Choose model $\mathcal{P}(y|x,\theta)$ and prior $\mathcal{P}(\theta)$ (e.g. Hierarchical logitstic regression)
- ▶ Take $\hat{y}_t = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}\left[y|\mathbf{x}_t\right]$ and $\hat{\mu}_{\text{MRP}} = \frac{1}{T}\sum_{t=1}^{T} \hat{y}_t$
- ▶ Dependence on $y_s$ very complicated (Typically via MCMC draws from $\mathcal{P}(\theta|\text{Survey data})$)
- ▶ **Black box**

# Two approaches

We want $\mu := \frac{1}{T}\sum_{t=1}^{T} y_t$, but don't observe target population $y_t$.

- Assume $p(y|\mathbf{x})$ is the same in both populations,
- But the distribution of **x** may be different in the survey and target.

**Calibration weighting**

▶ Choose "calibration weights" $w_s$ (e.g. raking weights)

▶ Take $\hat{\mu}_{\text{CAL}} = \frac{1}{S}\sum_{s=1}^{S} w_s y_s$

▶ Dependence on $y_s$ is obvious ($w_s$ typically chosen using only **x**)

▶ Weights give interpretable diagnostics:

- Frequentist variability
- Partial pooling
- Regressor balance

**Bayesian hierarchical modeling (MrP)**

▶ Choose model $\mathcal{P}(y|x,\theta)$ and prior $\mathcal{P}(\theta)$ (e.g. Hierarchical logitstic regression)

▶ Take $\hat{y}_t = \mathbb{E}_{\mathcal{P}(\theta|\text{Survey data})}[y|\mathbf{x}_t]$ and $\hat{\mu}_{\text{MRP}} = \frac{1}{T}\sum_{t=1}^{T} \hat{y}_t$

▶ Dependence on $y_s$ very complicated (Typically via MCMC draws from $\mathcal{P}(\theta|\text{Survey data})$)

▶ **Black box**

$\leftarrow$ (We open this box, providing analogues of all these diagnostics)

# What are we weighting for?[1]

We want:

$$\text{Target average response} = \frac{1}{T}\sum_{t=1}^{T} y_p \approx \frac{1}{S}\sum_{s=1}^{S} w_s y_s = \text{Weighted survey average response}$$

We can't check this, because we don't observe $y_p$.

[1] Pun attributable to Solon et al. [2015]

# What are we weighting for?[1]

We want:

$$\text{Target average response} = \frac{1}{T}\sum_{t=1}^{T} y_p \approx \frac{1}{S}\sum_{s=1}^{S} w_s y_s = \text{Weighted survey average response}$$

We can't check this, because we don't observe $y_p$. But we can check whether:

$$\frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_p = \frac{1}{S}\sum_{s=1}^{S} w_s \mathbf{x}_s$$

Such weights satisfy "covariate balance" for $\mathbf{x}$.

You can check covariate balance for any calibration weighting estimator.

[1] Pun attributable to Solon et al. [2015]

# What are we weighting for?[1]

We want:

$$\text{Target average response} = \frac{1}{T}\sum_{t=1}^{T} y_p \approx \frac{1}{S}\sum_{s=1}^{S} w_s y_s = \text{Weighted survey average response}$$

We can't check this, because we don't observe $y_p$. But we can check whether:

$$\frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_p = \frac{1}{S}\sum_{s=1}^{S} w_s \mathbf{x}_s$$

Such weights satisfy "covariate balance" for $\mathbf{x}$.

You can check covariate balance for any calibration weighting estimator.

Even more, covariate balance is the criterion for a popular class of calibration weight estimators:

**Raking calibration weights**

"Raking" selects weights that

- Are as "close as possible" to some reference weights
- Under the constraint that they balance some selected regressors.

[1] Pun attributable to Solon et al. [2015]

# Generalized covariate balance checks

We want to balance $f(\mathbf{x})$ because we think $\mathbb{E}[y|\mathbf{x}]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

This motivates the following **generalized covariate balance check**:

# Generalized covariate balance checks

We want to balance $f(\mathbf{x})$ because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

This motivates the following **generalized covariate balance check**:

**Generalized covariate balance (GCB) (informal)**

Pick a small $\delta$, and define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the change this is supposed to induce in the target population.

Covariate balance checks whether our estimators produce the same change.

# Generalized covariate balance checks

We want to balance $f(\mathbf{x})$ because we think $\mathbb{E}[y|\mathbf{x}]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

This motivates the following **generalized covariate balance check**:

**Generalized covariate balance (GCB) (formal)**

Pick a small $\delta$, and define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}[\tilde{y}|\mathbf{x}] = \mathbb{E}[y|\mathbf{x}] + \delta f(\mathbf{x}).$$

We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}[\mu(\tilde{y}) - \mu(y)|\mathbf{x}] = \frac{1}{T}\sum_{t=1}^{T}\left(\mathbb{E}[\tilde{y}|\mathbf{x}_p] - \mathbb{E}[y|\mathbf{x}_p]\right) = \delta\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{x}_p)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change:

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\substack{\text{Replace weighted averages} \\ \text{with changes in an estimator}}} \overset{\text{check}}{=} \delta\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{x}_p).$$

# Generalized covariate balance checks

We want to balance $f(\mathbf{x})$ because we think $\mathbb{E}\left[y|\mathbf{x}\right]$ might plausibly vary $\propto f(\mathbf{x})$, and want to check whether our estimator can capture this variability.

This motivates the following **generalized covariate balance check**:

**Generalized covariate balance (GCB) (formal)**

Pick a small $\delta$, and define a *new response variable* $\tilde{y}$ such that

$$\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x}).$$

We know the expected change this perturbation produces in the target distribution:

$$\mathbb{E}\left[\mu(\tilde{y}) - \mu(y)|\mathbf{x}\right] = \frac{1}{T}\sum_{t=1}^{T}\left(\mathbb{E}\left[\tilde{y}|\mathbf{x}_p\right] - \mathbb{E}\left[y|\mathbf{x}_p\right]\right) = \delta\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{x}_p)$$

Then, check whether your estimator $\hat{\mu}(\cdot)$ produces the same change:

$$\underbrace{\hat{\mu}(\tilde{y}) - \hat{\mu}(y)}_{\substack{\text{Replace weighted averages}\\ \text{with changes in an estimator}}} \overset{\text{check}}{=} \delta\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{x}_p).$$

When $\hat{\mu}(\cdot) = \hat{\mu}_{\mathrm{CAL}}(\cdot)$, GCB recovers the standard covariate balance check.

# Generalized covariate balance for MrP

**Step one:** Construct $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.

# Generalized covariate balance for MrP

**Step one:** Construct $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.

**Problem:** Our $y$ is binary! (We're motivated by hierarchical linear regression.)

# Generalized covariate balance for MrP

**Step one:** Construct $\tilde{y}$ such that $\mathbb{E}\left[\tilde{y}|\mathbf{x}\right] = \mathbb{E}\left[y|\mathbf{x}\right] + \delta f(\mathbf{x})$.

**Problem:** Our $y$ is binary! (We're motivated by hierarchical linear regression.)

Two possibilities:

- Allow $\tilde{y}$ to take values other than $\{0, 1\}$ and set $\tilde{y} = y + \delta f(\mathbf{x})$, or
- Use an estimate of $\mathbb{E}\left[y|\mathbf{x}\right]$ to draw new binary $\tilde{y}$.

We define theory and methods for the first, and provide tools for generating data using the second method for potentially problematic regressors.

# Generalized covariate balance for MrP

**Step one:** Take $\tilde{y} = y + \delta f(\mathbf{x})$.
**Step two:** Evaluate $\hat{\mu}_{\text{MRP}}(\tilde{y}) - \hat{\mu}(y)$.

# Generalized covariate balance for MrP

**Step one:** Take $\tilde{y} = y + \delta f(\mathbf{x})$.
**Step two:** Evaluate $\hat{\mu}_{\text{MRP}}(\tilde{y}) - \hat{\mu}(y)$.

**Problem:** $\hat{\mu}_{\text{MRP}}(\cdot)$ is computed with MCMC.

- Takes hours to re-run, and
- Output is noisy, and $\hat{\mu}_{\text{MRP}}(\tilde{y}) - \hat{\mu}(y)$ may be small.

# Generalized covariate balance for MrP

**Step one:** Take $\tilde{y} = y + \delta f(\mathbf{x})$.
**Step two:** Evaluate $\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \hat{\mu}(y)$.

**Problem:** $\hat{\mu}_{\mathrm{MRP}}(\cdot)$ is computed with MCMC.

- Takes hours to re-run, and
- Output is noisy, and $\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \hat{\mu}(y)$ may be small.

**Taylor series**

Form the approximation

$$\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) = \sum_{s=1}^{S} w_s^{\mathrm{MRP}}(\tilde{y}_s - y_s) + \text{Residual} \quad \text{where} \quad w_s^{\mathrm{MRP}} := \frac{d}{dy_s}\hat{\mu}_{\mathrm{MRP}}(y).$$

If MrP were linear (e.g. if you use OLS instead of hierarchical logistic regression), then

- The residual is zero,
- $\hat{\mu}_{\mathrm{MRP}}(y) = \sum_{s=1}^{S} w_s^{\mathrm{MRP}} y_s$, and so
- $\hat{\mu}_{\mathrm{MRP}}(\tilde{y})$ is a calibration weighting estimator, and $w_s^{\mathrm{MRP}}$ are its weights. (Cite Gelman)

In general, MrP is truly nonlinear. The residual is only small when $\tilde{y} \approx y$ (i.e., when $\delta \ll 1$).

# Generalized covariate balance for MrP

**Step one:** Take $\tilde{y} = y + \delta f(\mathbf{x})$.
**Step two:** Evaluate $\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \hat{\mu}(y)$.

**Problem:** $\hat{\mu}_{\mathrm{MRP}}(\cdot)$ is computed with MCMC.

- Takes hours to re-run, and
- Output is noisy, and $\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \hat{\mu}(y)$ may be small.

**Taylor series**

Form the approximation

$$\hat{\mu}_{\mathrm{MRP}}(\tilde{y}) = \sum_{s=1}^{S} w_s^{\mathrm{MRP}} (\tilde{y}_s - y_s) + \text{Residual} \quad \text{where} \quad w_s^{\mathrm{MRP}} := \frac{d}{dy_s} \hat{\mu}_{\mathrm{MRP}}(y).$$

It happens that the needed derivatives are given by simple a posterior covariances involving only the inverse link function $m(\mathbf{x}; \theta)$ and log likelihood [Giordano et al., 2018].

These can be computed using standard MCMC software (e.g. Bürkner [2017]).

# Covariate balance

**Theorem**

- Let $\tilde{y} = y + \delta f(\mathbf{x})$,
- $\hat{\mu}_{\mathrm{MRP}}$ be a hierachical logistic regression posterior expectation, and
- $\mathcal{F}$ be a Donsker class of uniformly bounded functions on $\mathbf{x}$.

Then, with probability approaching one, as $N \to \infty$,

$$\sup_{f \in \mathcal{F}} \left( \hat{\mu}_{\mathrm{MRP}}(\tilde{y}) - \left( \hat{\mu}_{\mathrm{MRP}}(y) + \sum_{s=1}^{S} w_s^{\mathrm{MRP}} \delta f(\mathbf{x}_s) \right) \right) = O(\delta^2) \quad \text{as } \delta \to 0$$

The supremum over $\mathcal{F}$ is the primary technical contribution! It means we are justified in searching over regressors to find imbalance.

Draws on our prior work on uniform and finite–sample error bounds for Bernstein–von Mises theorem–like results [Giordano and Broderick, 2024, Kasprzak et al., 2025].

# Real Data

Analysis of changing names after marriage (based on Alexander [2019])

- **Target population:** ACS survey of US population 2017–2022 [Ruggles et al., 2024])
- **Survey population:** Marital Name Change Survey [Cohen, 2019]
- **Respose:** Did the female partner keep their name after marriage?
- For regressors, use bins of age, education, state, and decade married.

| | |
|---|---|
| Survey observations: | $S = 4,364$ |
| Target observations (rows): | $T = 4,085,282$ |

| | |
|---|---|
| Uncorrected survey mean: | $\frac{1}{S}\sum_{s=1}^{S} y_n = 0.462$ |
| Raking: | $\hat{\mu}_{\text{CAL}} = 0.263$ |
| MrP: | $\hat{\mu}_{\text{MRP}} = 0.288 \quad (\text{Post. sd} = 0.0169)$ |

# Figure

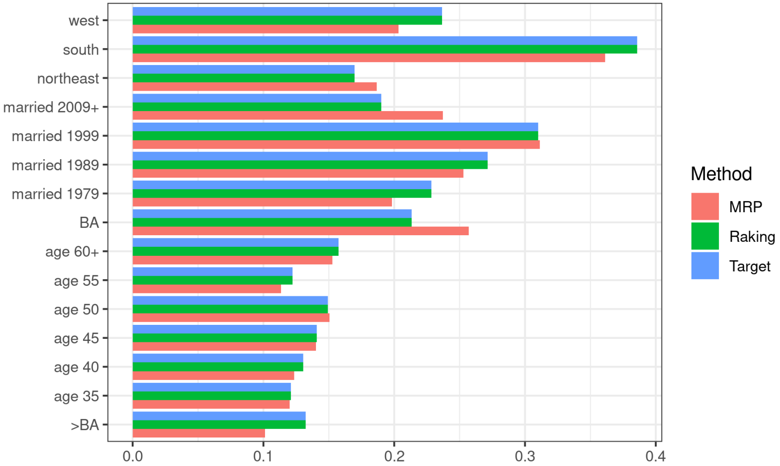



**Figure 1:** Imbalance plot for primary effects

# Figure

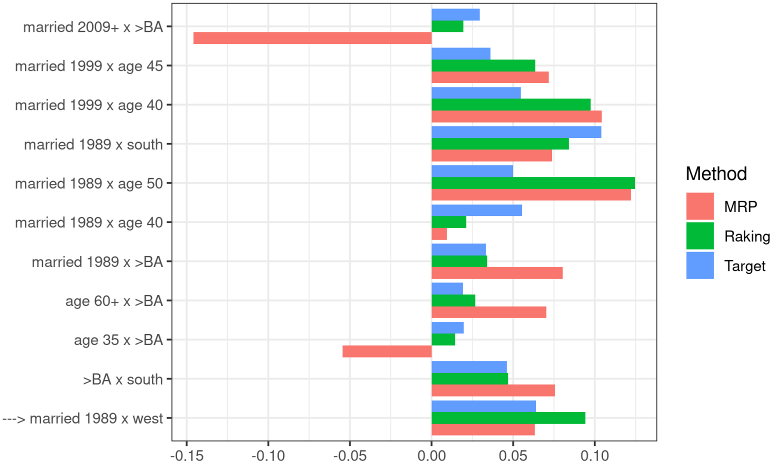



**Figure 2:** Imbalance plot for select interaction effects

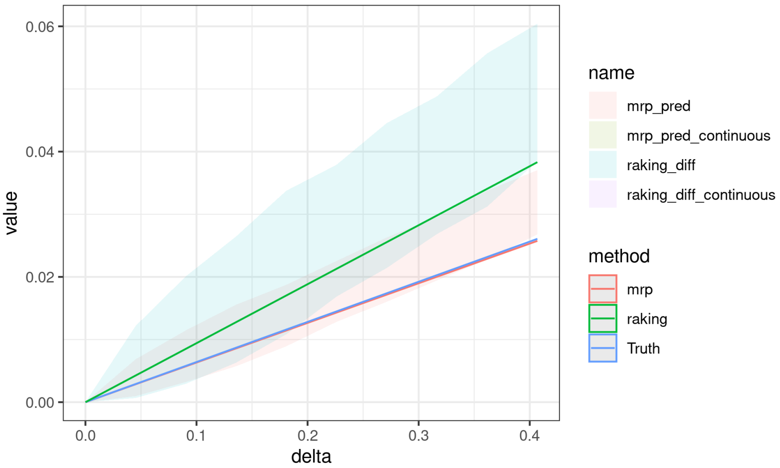



**Figure 3:** Balance checks for Alexander

# Future work and generalizations

- Instance of a very general class of local consistency checks that generalize classical regression checks (work with Sequoia)
- Versions for GLMMs (work with Vladimir)
- Going beyond classical Bayesian sensitivity (work with Lucas)

# References


M. Alexander. Analyzing name changes after marriage using a non-representative survey, 2019. URL https://www.monicaalexander.com/posts/2019-08-07-mrp/.

Blue Rose Research. 2024 Election Retrospective Presentation. https://data.blueroseresearch.org/2024retro-download, 2024. Accessed on 2024-10-26.

A. Bonica, R. Fordham, J. Grumbach, and E. Tiburcio. Did non-voters really flip Republican in 2024? The evidence says no. https://data4democracy.substack.com/p/did-non-voters-really-flip-republican, April 2025.

Paul-Christian Bürkner. brms: An R package for Bayesian multilevel models using Stan. *Journal of Statistical Software*, 80(1): 1–28, 2017. doi: 10.18637/jss.v080.i01.

P. Cohen. Marital name change survey, Apr 2019. URL osf.io/uzqdn.

R. Giordano and T. Broderick. The Bayesian infinitesimal jackknife for variance, 2024. URL https://arxiv.org/abs/2305.06466.

R. Giordano, T. Broderick, and M. I. Jordan. Covariances, robustness and variational bayes. *Journal of machine learning research*, 19(51), 2018.

M. Kasprzak, R. Giordano, and T. Broderick. How good is your Laplace approximation of the bayesian posterior? Finite-sample computable error bounds for a variety of useful divergences, 2025. URL https://arxiv.org/abs/2209.14992.

S. Ruggles, S. Flood, M. Sobek, D. Backman, A. Chen, G. Cooper, S. Richards, R. Rodgers, and Megan S. IPUMS USA: Version 15.0 [dataset], 2024. URL https://usa.ipums.org.

G. Solon, S. Haider, and J. Wooldridge. What are we weighting for? *Journal of Human resources*, 50(2):301–316, 2015.