

TOY MODELS IN THEORETICAL PHYSICS

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ISING MODEL WITH FIELDS

$$\mathcal{H} = -J \sum_i \sigma_i^z \sigma_{i+1}^z - h \sum_i \sigma_i^x - g \sum_i \sigma_i^z \quad (1)$$

q-STATE POTTS MODEL (CLASSICAL)

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \delta_{\sigma_i, \sigma_j}, \quad \sigma_i \in 1, \dots, q \quad (2)$$

SHERRINGTON-KIRKPATRICK MODEL (SK)

$$\mathcal{H} = - \sum_{i < j \leq N} J_{ij} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x, \quad J_{ij} \in \mathcal{N}(0, J^2) \quad (3)$$

KOGUT-SUSSKIND HAMILTONIAN (PURE GAUGE)

$$\mathcal{H} = \frac{g^2}{2} \sum_{\ell} E_{\ell}^2 - \frac{1}{2g^2} \sum_{\square} \text{Tr}(U_{\square} + U_{\square}^{\dagger}) \quad (4)$$

AKLT (AFFLECK-KENNEDY-LIEB-TASAKI) HAMILTONIAN

$$\mathcal{H} = \sum_{\langle i,j \rangle} \left(\vec{S}_i \cdot \vec{S}_j + \frac{1}{3} \left(\vec{S}_i \cdot \vec{S}_j \right)^2 \right) \quad (5)$$

where $\vec{S} = (S_1, S_2, S_3)$

$$S_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (6)$$

Notes: It has the famous Haldane gap.

★ $\mathcal{N} = 4$ SYM - (3+1)-DIMENSIONS

$$\mathcal{L} = \text{Tr} \left[F^{\mu\nu} F_{\mu\nu} + (D_{\mu} X_i)^2 - \frac{1}{2} [X_i, X_j]^2 + \Psi^T \not{D} \Psi + \Psi^T \gamma_i [X, \Psi] \right] \quad (7)$$

BFSS (0+1)-DIMENSIONS

$$\mathcal{L} = \text{Tr} \left((D_t X_i)^2 - [X_i, X_j]^2 \right) + \Psi^T \not{D} \Psi + \Psi^T \gamma_i [X, \Psi] \quad (8)$$

BMN/PWMM - (0+1)-DIMENSIONS WITH I,J,K = 1...3 AND M = 4...9

$$\mathcal{L} = \mathcal{L}_{\text{BFSS}} + \text{Tr} \left[\left(\frac{\mu}{3} X_I \right)^2 + \left(\frac{\mu}{6} X_M \right)^2 + \frac{\mu}{4} \Psi_\alpha^T \gamma_{\alpha\beta}^{123} \Psi_\beta + \frac{\sqrt{2}\mu}{3} \epsilon_{IJK} X_I X_J X_K \right]. \quad (9)$$

SYK IN (0+1)

$$\mathcal{H} = \frac{1}{4!} \sum_{i,j,k,l=1}^N J_{ijkl} \chi_i \chi_j \chi_k \chi_l \quad (10)$$

IKKT (0+0)-DIMENSIONS, WITH I,J = 1...10

$$\mathcal{L} = \text{Tr} \left([X_I, X_J]^2 \right) + \Psi^T \not{D} \Psi + \Psi^T \gamma_I [X, \Psi] \quad (11)$$

EINSTEIN-HILBERT ACTION

$$S = \frac{c^4}{16\pi G} \int d^4x (R - 2\Lambda) \quad (12)$$

NAMBU-GOTO (NG) ACTION

$$S = -T \int d^2\sigma \sqrt{-(\dot{X}^2)(X')^2 + (\dot{X} \cdot X')^2} \quad (13)$$

$$\dot{X}^\mu = \partial X^\mu / \partial \tau, (X')^\mu = \partial X^\mu / \partial \sigma$$

POLYAKOV ACTION

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \quad (14)$$

CHERN-SIMONS ACTION

$$S = \frac{k}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \text{Tr} \left(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right) \quad (15)$$

PRINCIPAL CHIRAL FIELD

$$\mathcal{L} = \frac{\beta}{2} \text{Tr} (\partial_\mu g^{-1} \partial_\mu g) \text{ where } g \in SU(N) \quad (16)$$

MASSLESS SCHWINGER (1+1)

$$\mathcal{L} = \frac{1}{2} (\epsilon^{\mu\nu} \partial_\nu A_\mu)^2 - e j^\mu A_\mu + \bar{\Psi} \not{\partial} \Psi \quad (17)$$

▲ MASSIVE THIRRING MODEL

$$\mathcal{L} = \bar{\Psi} i \not{\partial} \Psi - m_F \bar{\Psi} \Psi - \frac{g}{2} (\bar{\Psi} \gamma^\mu \Psi)^2; \quad \not{\partial} = \gamma^\mu \partial_\mu \quad (18)$$

CLASSICAL XY MODEL

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) - h \sum_i \cos \theta_i, \quad \theta \in [0, 2\pi) \quad (19)$$

$O(N)$ NON-LINEAR σ IN 1+1

$$\mathcal{L} = \frac{1}{2g} \sum_{i=1}^N (\partial^\mu \hat{n}_i)^2 \quad (20)$$

▲ SINE-GORDON

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^4}{\lambda} \left(1 - \cos \left(\frac{\sqrt{\lambda} \phi}{m} \right) \right) \quad (21)$$

★ HEISENBERG MODEL [1928, SOLVED BY BETHE (1931)]

$$\mathcal{H}_{XXX} = \frac{J}{2} \sum_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z \right) \quad (22)$$

FERMI-HUBBARD MODEL

$$\mathcal{H} = -t \sum_{\langle i,j \rangle, \sigma} \left(c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}, \quad ; \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij} \delta_{\alpha\beta}, \quad ; \alpha, \beta = \uparrow, \downarrow \quad (23)$$

BOSE-HUBBARD MODEL

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} \left(b_i^\dagger b_j + b_j^\dagger b_i \right) + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i \quad ; \quad [b_i, b_j^\dagger] = \delta_{ij} \quad (24)$$

BLUME-CAPEL MODEL

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} Z_i Z_j + \Delta \sum_i Z_i^2 - h \sum_i Z_i \quad ; \quad Z = \text{Pauli}Z = \text{diag}(1, -1) \quad (25)$$

O'BRIEN-FENDLEY MODEL (TRI-CRITICAL ISING)

$$\mathcal{H} = - \sum_{j=1}^{N-1} Z_j Z_{j+1} - g \sum_{j=1}^N X_j - h \sum_{j=1}^N Z_j + \lambda \underbrace{\sum_{j=1}^{N-2} (X_j Z_{j+1} Z_{j+2} + Z_j Z_{j+1} X_{j+2})}_{\text{Staggered term}} \quad (26)$$

NAMBU-JONA-LASINIO MODEL

$$\mathcal{L} = \bar{\Psi} i \not{\partial} \Psi + \frac{\lambda}{4} [(\bar{\Psi} \Psi)(\bar{\Psi} \Psi) - (\bar{\Psi} \gamma^5 \Psi)(\bar{\Psi} \gamma^5 \Psi)] \quad ; \quad \bar{\Psi} = \Psi^\dagger \gamma^0 \quad (27)$$

GROSS-NEVEU MODEL

$$\mathcal{L} = \bar{\Psi}_k (i \not{\partial} + m) \Psi_k - \frac{g}{2} (\bar{\Psi}_k \Psi_k)^2 \quad ; \quad k = 1, \dots, N(\text{flavors}) \quad (28)$$

SU-SCHRIEFFER-HEEGER (SSH) MODEL

$$\mathcal{H} = \sum_i \left[t_1 (c_{Ai}^\dagger c_{Bi} + \text{H.c.}) - t_2 (c_{Bi}^\dagger c_{A,i+1} + \text{H.c.}) \right] \quad ; \quad A, B \equiv \text{flavors}, \quad (29)$$

KITAEV CHAIN MODEL

$$\mathcal{H} = \sum_j \left[-t(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - \mu c_j^\dagger c_j + \underbrace{\Delta}_{\text{p-wave SC pairing}} (c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger) \right] \quad (30)$$

JAYNES-CUMMINGS-HUBBARD MODEL

$$\mathcal{H} = \sum_i \left[\omega_c a_i^\dagger a_i + \omega_a \sigma_i^+ \sigma_i^- + g(a_i^\dagger \sigma_i^- + a_i \sigma_i^+) \right] - J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) \quad (31)$$

DICKE MODEL

$$\mathcal{H} = \omega_c a^\dagger a + \omega \sum_j Z_j + \frac{\lambda}{\sqrt{N}} \sum_j \underbrace{(a^\dagger \sigma_j^- + a \sigma_j^+)}_{\text{co-rotating}} + \frac{\lambda}{\sqrt{N}} \sum_j \underbrace{(a \sigma_j^- + a^\dagger \sigma_j^+)}_{\text{counter-rotating}} \quad (32)$$

KITAEV TORIC CODE

$$\mathcal{H} = - \sum_s A_s - \sum_{\square} B_p = - \prod_{i \in +} X_i - \prod_{i \in \square} Z_i \quad (33)$$