SUMMER SCHOOL @ RPI

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Lecture 1: Basics and Setting up (possibly) QISKIT

Lecture 2: Some basic algorithms & programs

We'll start from Classical computation which is based on transistors & in turn on classical logic gates.

A	B
0	0
0	1
1	0
1	1

$\longrightarrow$ $\underbrace{\text{AND gate}}_{\text{Classical logic gate}}$ $\longrightarrow$ transistors $\rightarrow$ Laptop

0
0
0
1

$\longrightarrow$ "non-reversible"

①

Notation (due to Dirac)

bra-ket notation

$$|\alpha\rangle = \begin{pmatrix} \alpha_1 \\ \vdots \\ \vdots \\ \alpha_N \end{pmatrix}$$

"bra"

$$\langle\beta| = \begin{pmatrix} \beta_1 & \dots & \dots & \beta_N \end{pmatrix}$$

"ket"

Quantum Computation

$|0\rangle$ $|1\rangle$ $\langle 0|$ $\langle 1|$

(No. of inputs = No. of outputs)

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

"bra" "state of some system in H"

$\frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

one-qubit

$2^1 = 2$ states.

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Two-qubits, $2^2 = 4$ states

$|00\rangle, \ |01\rangle, \ |10\rangle, \ |11\rangle$

$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$; $|11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$\underline{|00\rangle = |0\rangle \otimes |0\rangle}$

One of the most important Quantum gates (unitary). $U^\dagger U = \mathbb{1} \Rightarrow \underline{U^\dagger = U^{-1}}$.

Quantum Mechanics $\rightarrow$ Superposition

$|0\rangle \qquad\qquad |1\rangle$

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} = |\alpha_{new}\rangle$$

Hadamard gate (H)

$$\left.\begin{aligned} |0\rangle &\xrightarrow{H} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\ |1\rangle &\xrightarrow{H} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \end{aligned}\right]$$

Matrix representation

$$\left. H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right\} \underline{\text{QISKIT}}$$

NOT gate $\rightarrow$ Quantum gate

$\rightarrow$ Unitary.

"Control-NOT gate" $\rightsquigarrow$ 2 qubits

$\rightarrow$ Control-X, CX gate

Control

(CNOT gate)

$|00\rangle \longrightarrow |00\rangle$

$|01\rangle \longrightarrow |01\rangle$

$|10\rangle \longrightarrow |11\rangle$

$|11\rangle \longrightarrow |10\rangle$

What is the matrix representation of CNOT ??

$\rightarrow$ 4x4 matrix

$$\text{CNOT} := \begin{pmatrix} 1 & & \text{zero} & \\ & 1 & & \\ \text{zero} & & 0 & 1 \\ & & 1 & 0 \end{pmatrix}$$

CCNOT, CCX, controlled-controlled NOT gate.

$|000\rangle \rightarrow |000\rangle$

$|001\rangle$

$|010\rangle$

$|011\rangle$

$|100\rangle$

$|101\rangle$

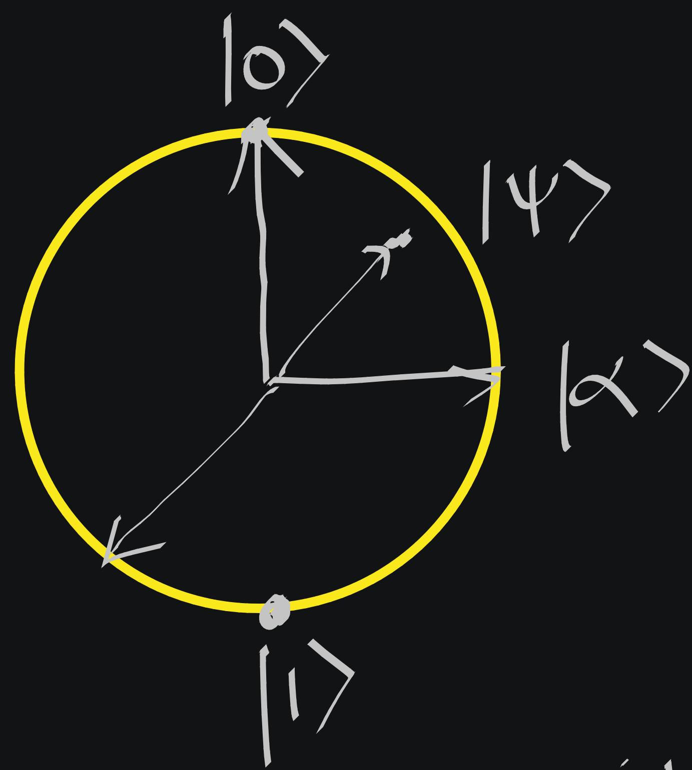
unchanged.

$|110\rangle \longrightarrow |111\rangle$

$|111\rangle \longrightarrow |110\rangle$

8 x 8 matrix

Representation of states on Bloch Sphere



$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

↓

general state on a Bloch sphere.

Rotation gates ⟶ Quantum gate

RX, RY, RZ

$$|\alpha\rangle = \frac{|0\rangle}{\sqrt{2}} + \frac{i}{\sqrt{2}}|1\rangle$$

$X \longrightarrow$ Pauli matrices / gates

$X, Y, Z \xrightarrow{I}$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

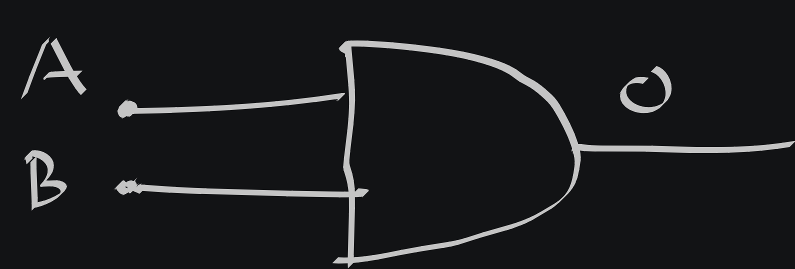
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\{H, \text{CNOT}, \text{CCNOT}, X, Y, Z\}$$

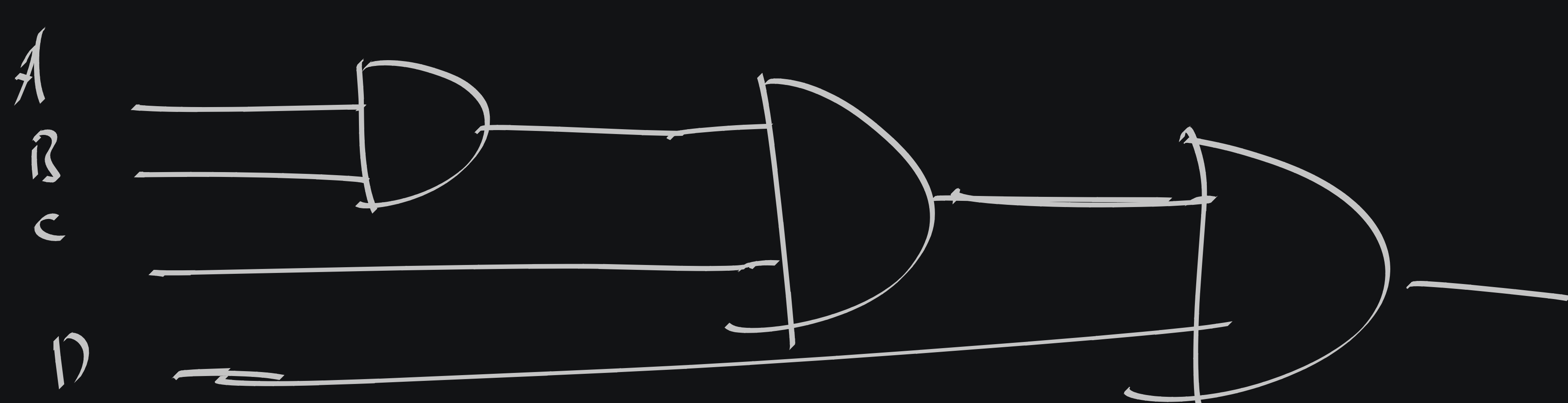
Circuit Diagrams

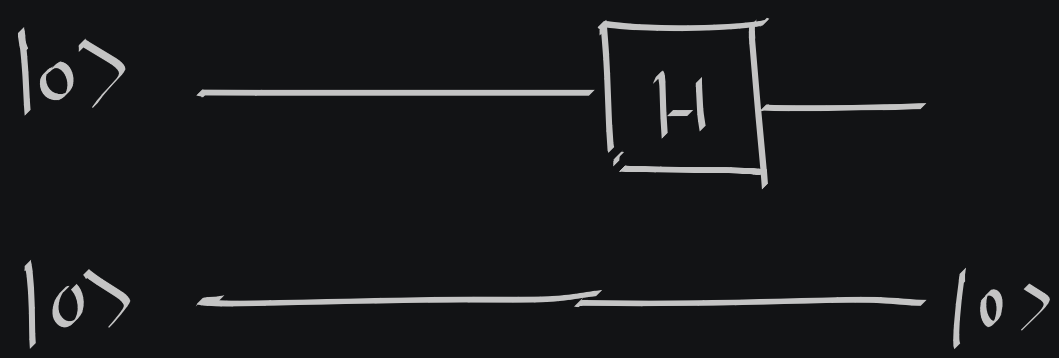
AND gate.

A	B	O
0	0	0
0	1	0
1	0	0
1	1	1



AND gate.





$|\psi_{out}\rangle$

$|0\rangle \otimes |0\rangle$

$= |00\rangle$

What is $|\psi_{out}\rangle$??

$$|\psi_{out}\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |0\rangle$$

$$= \frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

$$\boxed{P} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

$$\boxed{i = e^{i\pi/2}}$$

$$\boxed{S} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\boxed{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \qquad T\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = ?$$

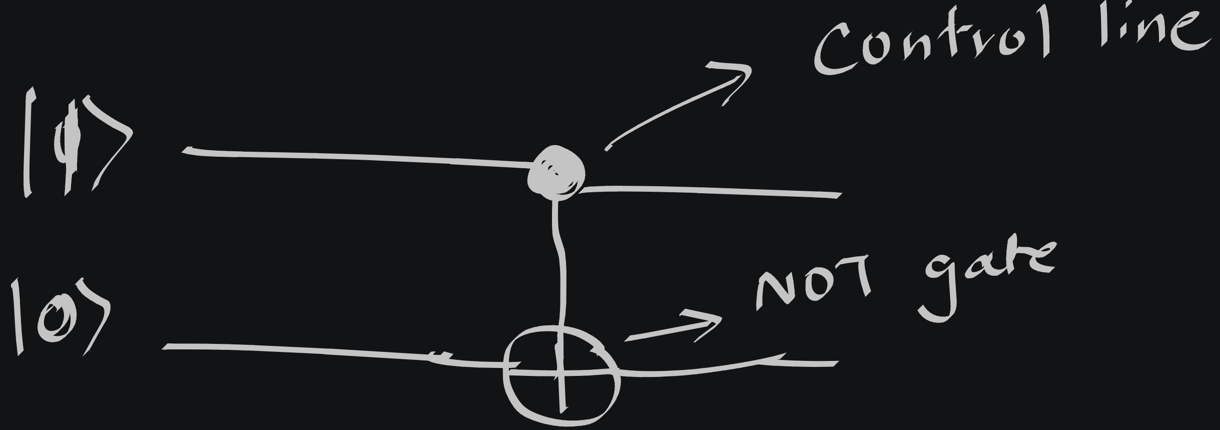
$$S|?\rangle = |??\rangle$$

Calculate what is $H|??\rangle = |Ans\rangle$

$$H \cdot S \cdot T \cdot H \, |0\rangle =$$

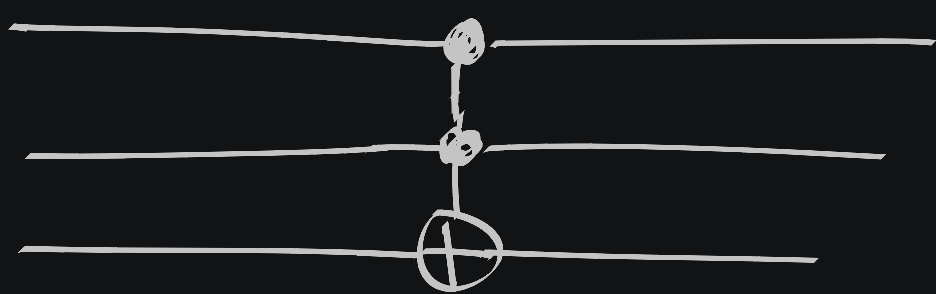
$$\frac{1}{2}\left[\left(1 + e^{\frac{i3\pi}{4}}\right)|0\rangle + \left(1 - e^{\frac{i3\pi}{4}}\right)|1\rangle\right]$$

"Control-NOT gate" CNOT

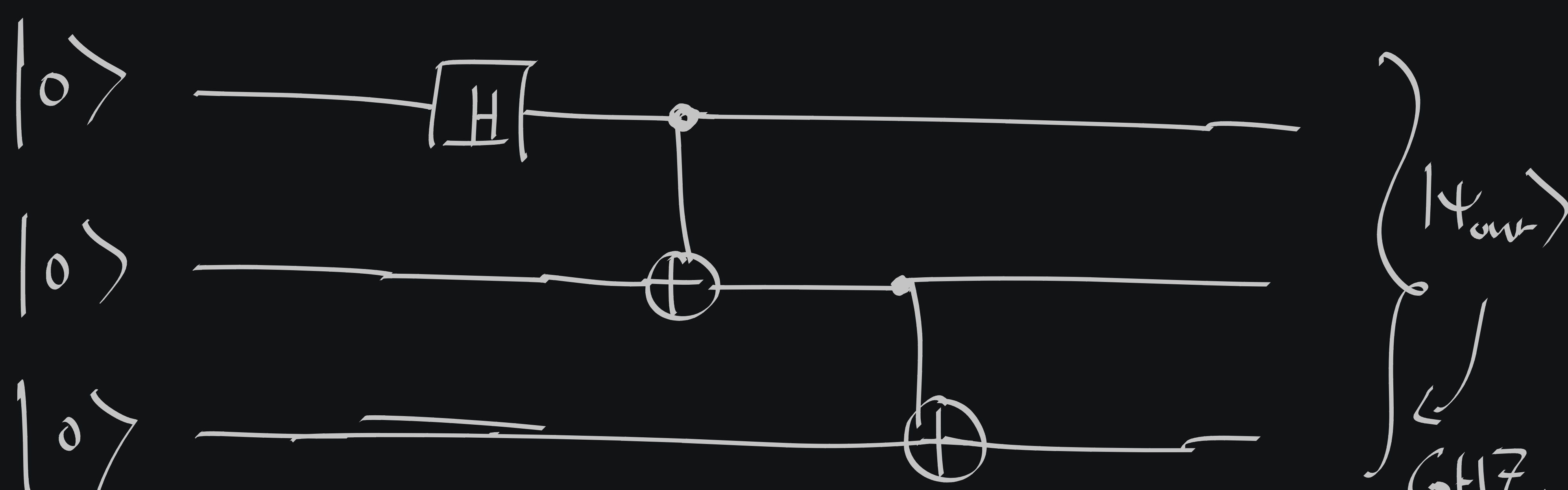


$|\phi\rangle$

$|\phi\rangle$



CCNOT,



GHZ state

3-qubit

$$\frac{1}{\sqrt{2}}|abc\rangle + \frac{1}{\sqrt{2}}|def\rangle$$

$$|000\rangle \longrightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}|00\rangle$$

$$\longrightarrow \frac{|000\rangle}{\sqrt{2}} + \frac{|100\rangle}{\sqrt{2}}$$

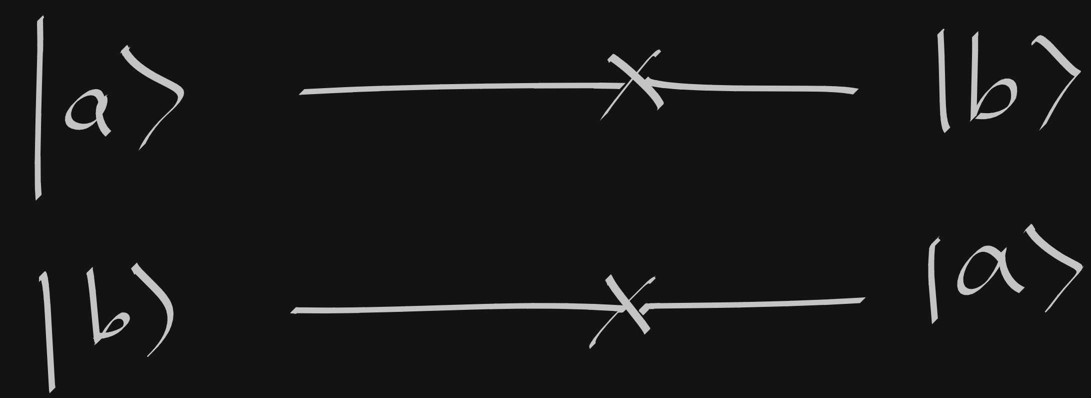
$$\xrightarrow{\text{CNOT}} \frac{|000\rangle}{\sqrt{2}} + \frac{|110\rangle}{\sqrt{2}}$$

$$\xrightarrow{\text{CNOT}} \underbrace{\frac{|000\rangle}{\sqrt{2}} + \frac{|111\rangle}{\sqrt{2}}}_{\text{entanglement of 3-qubits.}} \rightarrow \text{GHZ state.}$$

Another way: W state

$$|W\rangle = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$$

SWAP gate



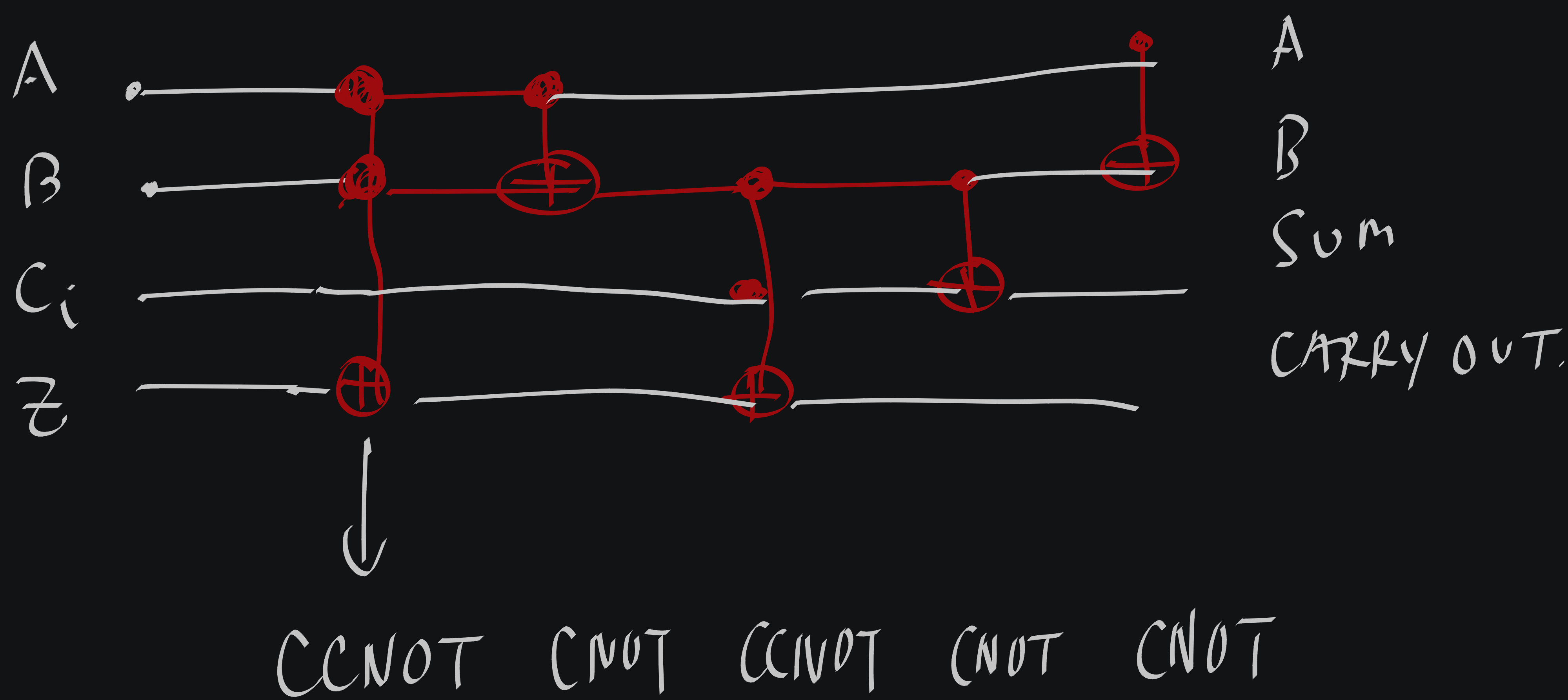
$$|a,b\rangle \longrightarrow |b,a\rangle$$

Digital Electronics $\longrightarrow$ Full-adder circuits

carry-in $\rightarrow$ C_i; carry-out $\rightarrow$ C_o

A	B	C_i	Z	C_o	S	A	B
0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	0	1	0	1
0	1	1	0	1	0	0	1
1	0	0	0	0	1	1	0
1	0	1	0	1	0	1	0
1	1	0	0	1	0	1	1
1	1	1	0	1	1	1	1

Circuit for Full-Adder



3 CNOT and 2 CCNOT gates

CCNOT := Toffoli gate $\rightarrow$

CCNOT $\longrightarrow$ 2 - qubit gates.