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   Phys. Rev. Res. 7 (2025) 2, 023266

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9. Real-time scattering in Ising field theory using matrix product states
   Phys. Rev. D. 97 (2018) 5, 054504

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10. Continuous-variable quantum computation of the $O(3)$ model in dimensions
    Phys. Rev. A 109, 052412 (2024)

Tensor renormalization group study of the non-Abelian Higgs model in two dimensions

Alexei Bazavov,[1] Simon Catterall,[2] Raghav G. Jha,[2] and Judah Unmuth-Yockey[2,*]

[1]*Department of Computational Mathematics, Science and Engineering, and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*

[2]*Department of Physics, Syracuse University, Syracuse, New York 13244, USA*

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We study the $SU(2)$ gauge-Higgs model in two Euclidean dimensions using the tensor renormalization group (TRG) approach. We derive a tensor formulation for this model in the unitary gauge and compare the expectation values of different observables between TRG and Monte Carlo simulations finding excellent agreement between the two methods. In practice we find the TRG method to be far superior to Monte Carlo simulation for calculations of the Polyakov loop correlation function which is used to extract the static quark potential.

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I. INTRODUCTION

It is usually very difficult to extract the emergent, long distance properties of quantum field theories or many-body systems from the underlying partition function. A powerful approach pioneered by Wilson known as the real-space renormalization group attempts to replace the elementary degrees of freedom by new averaged–or block–variables at larger scales. In order to maintain the correct long distance behavior under one such blocking requires a change in the effective coupling constants of the theory. If this procedure is applied recursively one generates a description of the theory at longer and longer length scales accompanied by a corresponding flow in the effective couplings.

In the original scheme due to Wilson and Kadanoff, this coarse graining procedure was carried out on the original fields and their corresponding Hamiltonian or action. However, in recent years it has been appreciated that it is sometimes more efficient to carry out this operation on alternative representations of the partition function called tensor networks. Algorithms that attempt to compute the partition function (or low lying states in a Hamiltonian formulation) by a recursive blocking of these tensors are called tensor renormalization group (TRG) methods. In the last decade there have been many such proposals [1–4] and intriguing connections have been drawn between tensor networks such as MERA (multiscale entanglement renormalization ansatz) [5] which was designed to capture the behavior of critical systems and the AdS/CFT correspondence [6,7]. Tensor networks have also been used to study gauge theories and their real-time dynamics [8–12]. The implication of gauge symmetry in one such tensor network—the matrix product state (MPS)—applied to the Schwinger model was discussed and used to calculate the confining potential in Refs. [13,14]. In addition, the mass gap of a continuum theory was computed in the Hamiltonian formulation for several models—the Schwinger model in Refs. [13,15] and the $SU(2)$ model in $1+1$ dimensions in Ref. [16]. General gauge invariant formulations of tensor networks in two dimensions have been introduced in Refs. [17,18]. The Hamiltonian formulation and its connection to a possible quantum simulation, has also been discussed for some models, including fermionic matter coupled to a $SU(2)$ gauge field [19,20].

In this paper, we will derive an explicit tensor network representation of a two-dimensional non-Abelian gauge theory coupled to matter and show how a particular TRG method—the higher-order tensor renormalization group (HOTRG) algorithm [3]—can be used to efficiently calculate the free energy and other observables in the theory. We will compare these results with conventional Monte Carlo (MC) calculations to test the validity of our tensor renormalization group procedure. A similar study was carried out for the Abelian version of this model in Refs. [21,22], along with a study of the Schwinger model in Ref. [23]. Reformulations of models using similar discrete variables offer an alternative starting point for other sampling methods such as worm algorithms, e.g., Refs. [24,25].

Our results indicate that the tensor methods are typically much more efficient than the Monte Carlo method for the computation of observables in such theories. As an

*jfunmuthyockey@gmail.com

From this approximate transfer matrix, the relative energy eigenvalues can be obtained. The crossover seen in the thermodynamics and static potential is also visible in the mass gap which exhibits different behaviors as κ is varied.

One additional, important point to emphasize is that the TRG algorithm is able to calculate quantities like the static quark potential that are essentially impossible to calculate straightforwardly with Monte Carlo methods over large ranges of the parameter space because of an exponentially small signal which is swamped by noise. This problem is reminiscent of theories with a sign problem and indeed one of the principle advantages of tensor network methods is their ability to completely avoid sign problems.

Recasting the non-Abelian gauge-Higgs model in terms of the irreducible representations of the gauge group could allow a rotor formulation of the Hamiltonian of this model in the continuous-time limit. There are already indications of the final form in Eq. (38) for the kinetic term, although further work is needed to describe the matter-gauge coupling term. Such a formulation could provide a straightforward, gauge-invariant means to formulate the model for quantum simulations in the future.

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Testing holography using lattice super-Yang-Mills theory on a 2-torus

Simon Catterall,[1,*] Raghav G. Jha,[1,†] David Schaich,[1,2,‡] and Toby Wiseman[3,§]
[1]Department of Physics, Syracuse University, Syracuse, New York 13244, USA
[2]AEC Institute for Theoretical Physics, University of Bern, 3012 Bern, Switzerland
[3]Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, United Kingdom

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We consider maximally supersymmetric SU(N) Yang-Mills theory in Euclidean signature compactified on a flat two-dimensional torus with antiperiodic ("thermal") fermion boundary conditions imposed on one cycle. At large N, holography predicts that this theory describes certain black hole solutions in type IIA and IIB supergravity, and we use lattice gauge theory to test this. Unlike the one-dimensional quantum mechanics case where there is only the dimensionless temperature to vary, here we emphasize there are two more parameters which determine the shape of the flat torus. While a rectangular Euclidean torus yields a thermal interpretation, allowing for skewed tori modifies the holographic dual black hole predictions and results in another direction to test holography. Our lattice calculations are based on a supersymmetric formulation naturally adapted to a particular skewing. Using this we perform simulations up to $N = 16$ with several lattice spacings for both skewed and rectangular tori. We observe the two expected black hole phases with their predicted behavior, with a transition between them that is consistent with the gravity prediction based on the Gregory-Laflamme transition.

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I. INTRODUCTION

Maximally supersymmetric Yang-Mills (SYM) theory in $p+1$ dimensions has been conjectured to provide a holographic description of string theories containing Dp-branes. Specifically, this gauge/gravity duality states that $(p+1)$-dimensional SYM with gauge group SU(N) is dual to a type IIA (even p) or type IIB (odd p) superstring containing N coincident Dp-branes in the "decoupling" limit [1,2]. The $p = 3$ case corresponds to superconformal $\mathcal{N} = 4$ SYM in four dimensions and yields the original AdS/CFT correspondence [3]. In this paper we focus on the maximally supersymmetric Yang-Mills in two dimensions at finite temperature, with the spatial circle direction compactified with periodic fermion boundary conditions (BCs) about it.

In this context, at large N and low temperatures, the dual string theory is well described by supergravities whose dynamics are given by certain charged black holes. Two classes of black hole are required to describe these dynamics: those that wrap the spatial circle (so-called "homogeneous black strings") and those that are localized on it ("localized black holes") [4–9]. Indeed this system of black hole solutions is related by a simple transform to the static uncharged black holes arising in pure gravity in ten dimensions with one spatial dimension wrapped into a circle, i.e., ten-dimensional Kaluza-Klein theory [8,10] (for a review of black holes in Kaluza-Klein theory see Ref. [11]). The two classes have different thermodynamic behaviors, and there is a first-order Gregory-Laflamme [12] phase transition between them in the gravity dual. According to holography, all this should be reproduced by the thermal physics of the SYM. In particular, the phase transition is a deconfinement transition associated to the spatial circle, with the magnitude of the spatial Wilson line giving an order parameter. It is thought that this transition extends to high temperatures where an intricate phase structure has been revealed from numerical and analytic treatments [8,13,14].

The remarkably subtle nature of gauge/gravity duality has meant that while SYM thus provides a fundamental and microscopic quantum description of certain gravity systems, there still is no "proof" or derivation of this black hole thermodynamics from $(p+1)$-dimensional SYM directly. Indeed even understanding the local structure of the dual ten-dimensional spacetime which emerges from the strongly coupled SYM theory remains a mystery. While there has been some heuristic analytic treatment for general p that hints how certain aspects of black hole

*smcatter@syr.edu
†rgjha@syr.edu
‡schaich@itp.unibe.ch
§t.wiseman@imperial.ac.uk

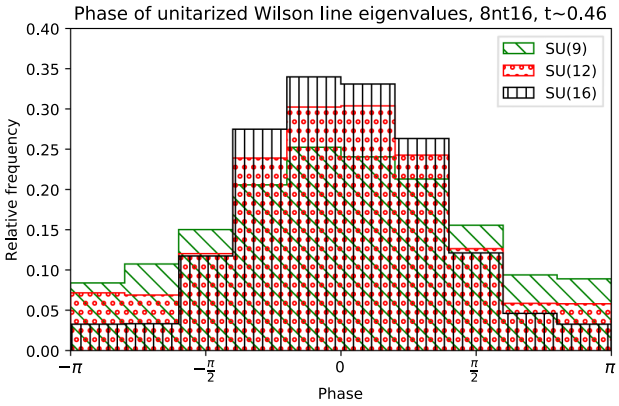

FIG. 13. Distributions of Wilson line eigenvalue phases, as in Fig. 5, for 8×16 lattices at $t \approx 0.46$ with $\mu^2 \approx 0.004$. The intermediate distributions, which become more compact as N increases, are consistent with expectations from the D0 phase of the gravity dual.

shown are results of $\mu^2 \to 0$ extrapolations. From Fig. 8 we expect the system to be spatially deconfined for the low temperature range $0.25 \lesssim t \lesssim 0.5$ shown here. Eventually at very low temperatures, presumably around $t \simeq 0.12$, it should confine, but we are not yet able to probe such a low-temperature regime. The dashed curve is the low-temperature gravity prediction from the (spatially deconfined) D0 phase, Eq. (38), which is indeed consistent with the data for $t \lesssim 0.35$. Figure 13 shows intermediate distributions for the Wilson line eigenvalue phases on 8×16 lattices at $t \approx 0.46$ (with $\mu^2 \approx 0.004$), which become more compact as N increases. This behavior supports our conclusion that the system is spatially deconfined in this region of the phase diagram, consistent with the dual gravity approaching the D0 phase in the large-N limit over this temperature range.

VI. CONCLUSIONS

We have studied two-dimensional SYM with maximal supersymmetry compactified on a flat but skewed torus in which an antiperiodic boundary condition is imposed on the fermion fields wrapping one of the cycles. The theory contains three dimensionless parameters: r_L, $r_\beta = 1/t$ and the skewing angle $\cos\theta = \gamma$. From the holographic conjecture, at low "generalized temperature" $t \ll 1$ this theory should give a description of a dual gravitational system containing various types of black holes arising in type IIA and IIB supergravity. The phase diagram of the gravitational system is expected to contain a region where homogeneous D1 (black string) solutions dominate and another in which localized D0 (black hole) solutions dominate. The critical line separating these two regions in the dual gravitational system is conjectured to be dual to a first-order deconfinement transition with the spatial Wilson loop magnitude P_L serving as an order parameter.

We used lattice gauge theory to explore and test this holographic conjecture using a recently constructed lattice action based on a formalism that maintains an exact supersymmetry at nonzero lattice spacing. The construction singles out a particular skewing angle $\gamma = -1/2$, which allowed us to test holography both for the usual rectangular tori and also—for the first time—for skewed tori as well.

We have mapped out the phase diagram of the SYM system and indeed found a line of transitions separating a spatially confined phase from a deconfined one. The parametric form of this phase boundary agrees with the results from the gravity dual. Furthermore, the action density computed in either phase is consistent at low temperatures with the corresponding black hole thermodynamics. Our results can be seen as the first step in checking the predictions of gauge/gravity duality in a situation that is distinct from SYM quantum mechanics and has a more subtle phase structure. However we expect it will be a considerable technical challenge to reach for this two-dimensional theory the degree of precision that is now the state of the art in the quantum-mechanical case, featuring fully quantified systematic uncertainties and controlled extrapolations to the large-N continuum limit.

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APPENDIX A: MODULAR GROUP AND FUNDAMENTAL DOMAIN

Let us recall some facts about the usual modular group $G_{\rm std}$ and its action on the complex torus parameter $\tau \in H$, where H is the upper half complex plane excluding the real line [so that $\mathrm{Im}(\tau) > 0$]. The action is given by

$$\tau' = \frac{a\tau + b}{c\tau + d} \tag{A1}$$

where $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$, which corresponds to an element $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{SL}(2, \mathbb{Z})$. This group is generated by

Sachdev-Ye-Kitaev model on a noisy quantum computer

Muhammad Asaduzzaman,[1,*] Raghav G. Jha,[2,†] and Bharath Sambasivam[3,‡]
[1]*Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242, USA*
[2]*Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*
[3]*Department of Physics, Syracuse University, Syracuse, New York 13244, USA*

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We study the Sachdev-Ye-Kitaev (SYK) model—an important toy model for quantum gravity on IBM's superconducting qubit quantum computers. By using a graph-coloring algorithm to minimize the number of commuting clusters of terms in the qubitized Hamiltonian, we find the gate complexity of the time evolution using the first-order product formula for N Majorana fermions is $\mathcal{O}(N^5J^2t^2/\epsilon)$ where J is the dimensionful coupling parameter, t is the evolution time, and ϵ is the desired precision. With this improved resource requirement, we perform the time evolution for $N = 6$, 8 with maximum two-qubit circuit depth and gate count of 343. We perform different error mitigation schemes on the noisy hardware results and find good agreement with the exact diagonalization results on classical computers and noiseless simulators. In particular, we compute vacuum return probability after time t and out-of-time order correlators which is a standard observable of quantifying the chaotic nature of quantum systems.

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I. INTRODUCTION

The holographic duality [1] relates a special class of quantum field theories in d dimensions and quantum gravity in $d + 1$ dimensions. This strong/weak duality enables one to study the properties of strongly coupled field theory using classical supergravity and vice versa. However, there are no cases where both sides of the duality can be studied analytically at the same time. Several attempts have been made on the lattice using Monte Carlo [2,3] to study these theories but they have their limitations. Therefore, it is often of interest to find simpler models that have holographic properties and can be studied in the strong coupling limit. One such model is the Sachdev-Ye-Kitaev (SYK) model [4–7] consisting of N Majorana fermions in $0 + 1$ dimensions with random couplings between q fermions at a time chosen from a Gaussian distribution with zero mean and variance proportional to J^2/N^{q-1}.

An interesting feature of the SYK model is that it develops an approximate conformal symmetry in the large N, low-temperature limit i.e., $N \gg \beta J \gg 1$ (β is the inverse temperature), where it is related to near extremal black holes that develop the nAdS_2 (near AdS_2) geometry. It was shown to saturate the chaos bound [8], a feature that is associated with holographic behavior. Since this is a $0 + 1$-dimensional model, it is computationally tractable and has been studied up to 60 Majorana fermions [9,10].

As a toy model for quantum gravity, it is, therefore, crucial to study the real-time dynamics of this model beyond methods accessible by classical computing. This direction has already been explored starting with Ref. [11]. In another work [12], the authors studied a generalized SYK model using a four-qubit nuclear magnetic resonance quantum simulator and computed bosonic correlation functions.

We put forth an improved circuit complexity[1] and study the SYK model on noisy superconducting quantum computers for the first time to our knowledge. Specifically, we find an improved complexity from earlier proposals of $\mathcal{O}(N^{10}J^2t^2/\epsilon)$ [11] and $\mathcal{O}(N^8J^2t^2/\epsilon)$ [13] to $\mathcal{O}(N^5J^2t^2/\epsilon)$ for the Lie-Trotter-based algorithm [14]. Using this improvement, we study the time evolution up to *eight* Trotter steps on quantum hardware available through IBM and compute the return probability and four-point out-of-time-ordered correlators.

II. SYK HAMILTONIAN

The Hamiltonian for the SYK model with N Majorana fermions and q-fermion interaction terms is

$$H = \frac{(i)^{q/2}}{q!} \sum_{i,j,k,\ldots,q=1}^{N} J_{ijk\cdots q}\chi_i\chi_j\chi_k\cdots\chi_q, \tag{1}$$

*muhammad-asaduzzaman@uiowa.edu
†raghav.govind.jha@gmail.com
‡bsambasi@syr.edu

[1]The complexity is defined as the least number of two-qubit gates in the circuit that implements the time evolution of the Hamiltonian H.

evolution of the operator W in the Heisenberg representation is $W(t) = e^{iHt}W(0)e^{-iHt}$ and $\langle\cdot\rangle_\beta = \mathrm{Tr}\{\rho\cdot\}$ denotes the thermal average at inverse temperature β. A common choice for ρ is to just use the $T \to \infty$ limit given by the normalized identity matrix, $\mathbb{1}/\dim(\mathcal{H})$ [38]. For the W and V we can either take Pauli matrices or Majorana fermions such that $W = \chi_i$ and $V = \chi_j$ with $i \neq j$ and average over the different pairs (i, j). We used the simplest case of a single Pauli matrix, i.e., $W = V = Z$. We denote $1 - F(t)$ by $1 - F(t)_{ij} = O_{ij}(t)$ where i and j denote the qubit location of the single-qubit operator W and V respectively and compute this on the hardware. The choice of these operators does not change the basic features of exponential growth and saturation. The SYK model saturates the chaos bound [8] at low temperatures and they have been extensively studied using classical computing methods [39,40]. The current resources do not allow us to access very large values of N, but we take a first step at computing in the simplest setting on the quantum computer.

A. Results for OTOC

Even though the OTOC seems simple to compute, the experimental/quantum computer measurement of the OTOC is very challenging because of the unusual time ordering. In a preliminary investigation, Ref. [41] studied OTOC of local operators on a nuclear magnetic resonance quantum simulator followed soon on trapped-ion [42]. In order to compute OTOC on quantum hardware, we need to define a protocol for the measurement. Several proposals have been put forth [43–46] and we use the protocol proposed in Ref. [45] which computes OTOC only by considering forward time evolution and exploiting the correlations between randomized measurements [47]. We discuss the details of the global protocol based on randomized measurements in the SM [16].

The results for OTOC from `ibm_cusco` and `ibm_kyoto` for three instances of $N = 6$ and the disorder average over them is shown in Fig. 4. For this computation, we used the M3 readout error mitigation protocol and dynamical decoupling. Even without self-mitigation, we find good agreement with the exact results.

V. SUMMARY AND DISCUSSION

We have proposed circuit complexity of $\mathcal{O}(N^5t^2/\epsilon)$ for Hamiltonian simulation of the SYK model with N Majorana fermions, a substantial improvement over existing results and performed quantum simulations on noisy 127-qubit quantum computers. We studied the return probability for $N = 6, 8$ Majorana fermions and computed the out-of-time order correlators for $N = 6$, a diagnostic of the chaotic behavior of quantum many-body systems. Due to the noisy devices currently available, we applied advanced mitigation methods to the hardware result and showed that it agrees well with the exact time evolution.

It might appear that the superconducting platform is not the best method to study the quantum simulation of this model as we could have applied the ion-based approaches to quantum simulation. The advantage of superconducting platforms is the low gate times, but the limitation is the qubit connectivity. With ion-based platforms, this is the opposite—there is more freedom with connectivity, but the gate times are much longer. For the dense SYK model and other dense random Hamiltonians, both of these things are important. In this work, we take a step toward identifying which of these is more important by pushing the superconducting platform to push the limits with limited connectivity. We hope to extend this work with hardware admitting all-to-all connectivity.

Though we cannot study the strict holographic limit and see signs of saturation of chaos bound on current devices, we believe that our work will be useful in future explorations of this model. In this regard, it might also be useful to consider simplified models similar to SYK [13,48–50] that are conjectured to have the same holographic behavior as the pure SYK model considered here. Another direction is to consider $q > 4$ and explore the resource requirements and time evolution. It would be useful to study the dynamics of the model over different timescales for the return probability at finite β. These interesting problems would require resources that are beyond the contemporary hardware era. We leave these questions for future work. The use of quantum computers for models such as the SYK model in coming decades will not only provide new insights into the holographic principle but also into the interesting world of strange metals and quantum many-body systems [51].

The $N = 6, 8$ SYK Hamiltonian realizations, the Pauli decomposition, and the time evolution circuit as OPEN QASM 2.0 files for the single Trotter step can be obtained from Ref. [52].

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PAPER: Classical statistical mechanics, equilibrium and non-equilibrium

Critical analysis of two-dimensional classical XY model

Raghav G Jha

Perimeter Institute for Theoretical Physics, 31 Caroline Street North Waterloo, ON, Canada

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Abstract. We consider the two-dimensional classical XY model on a square lattice in the thermodynamic limit using tensor renormalization group and precisely determine the critical temperature corresponding to the Berezinskii–Kosterlitz–Thouless (BKT) phase transition to be 0.89290(5) which is an improvement compared to earlier studies using tensor network methods.

Keywords: classical phase transitions, numerical simulations, tensor network simulations

Contents

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here and extract the critical exponents precisely but it would be challenging. In the future, it will be interesting to explore phase transitions in *frustated* XY model and other spin models such as J-Q model exhibiting continuous symmetry using these methods.

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Appendix A. Numerical details

In this appendix, we elaborate on some numerical details and steps to determine the critical temperature. We normalize the tensor $\mathtt{T}_\nu$ at each coarse-graining step (denoted by ν) by the maximum element of the tensor having a total of χ^4 elements which can be implemented in Python using NumPy library as $\mathtt{norm}_\nu = \mathtt{numpy.max}(\mathtt{T}_\nu)$. We can then calculate the free energy density (f) from these normalization factors, see for instance [29]:

$$f = -\frac{1}{\beta}\left(\sum_{\nu=0}^{N}\frac{\log(\mathtt{norm}_\nu)}{4^\nu} + \frac{\log(Z_N)}{4^N}\right) = -\frac{1}{\beta 4^N}\left(\sum_{\nu=0}^{N}\log(\mathtt{norm}_\nu)4^{N-\nu} + \log\ Z_N\right), \tag{12}$$

where $Z_N = \mathbf{t}\,\mathbf{Tr}(\mathtt{T}_\mathtt{N})$ is calculated from the tensor contraction after the last step of CG and 4^N is the lattice volume and N is the number of times we do coarse-graining along $\mathtt{x}+\mathtt{y}$ direction. To compute expectation values, one needs to insert the appropriate 'observable' tensor in the network. By inserting $\tilde{\mathtt{T}}$, which is just the derivative of $\mathtt{T}$ with respect to h, we evaluated the magnetization for a range of magnetic fields at various temperatures. The tensor, $\tilde{\mathtt{T}}_\nu$, is normalized by $\mathtt{norm}_\nu$ as well and is calculated as $M = \frac{\mathrm{tTr}(\tilde{\mathtt{T}}_\nu)}{\mathrm{tTr}(\mathtt{T}_\nu)}$ at each step. The diagram representing this contraction is shown in figure 1. Though the free energy density converges rather quickly, we really need to work in the thermodynamic limit with as large χ as possible to precisely evaluate magnetization. In order to compute the magnetization for $\mathtt{10}^{-15} \leqslant h < \mathtt{10}^{-9}$, we used single-precision float ($\mathtt{np.float32}$) rather than the default double precision. This enabled us to go up to $\chi = \mathtt{53}$ compared to $\chi = \mathtt{47}$ for $h \geqslant \mathtt{10}^{-9}$. Once the magnetization M is computed for different h, we evaluate $\Delta M/\Delta h|_{T,h_{\mathrm{mid}}}$ and plot it against temperature and determine the critical temperature corresponding to the peak of the susceptibility. The data for

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Tensor renormalization group study of the three-dimensional $O(2)$ model

Jacques Bloch,[1,*] Raghav G. Jha,[2,†] Robert Lohmayer,[1,3,‡] and Maximilian Meister[1,§]

[1]*Institute for Theoretical Physics, University of Regensburg, 93040 Regensburg, Germany*
[2]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
[3]*RCI Regensburg Center for Interventional Immunology, 93053 Regensburg, Germany*

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We calculate thermodynamic potentials and their derivatives for the three-dimensional $O(2)$ model using tensor-network methods to investigate the well-known second-order phase transition. We also consider the model at nonzero chemical potential to study the Silver Blaze phenomenon, which is related to the particle number density at zero temperature. Furthermore, the temperature dependence of the number density is explored using asymmetric lattices. Our results for both zero and nonzero magnetic field, temperature, and chemical potential are consistent with those obtained using other methods.

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I. INTRODUCTION

Our understanding of quantum many-body systems has considerably improved in the past two decades mainly due to the refined understanding of the entangled ground state structure of systems with local Hamiltonians. Successful methods using these entanglement properties are based on the idea of tensor-network states such as matrix product states (MPS) [1–4]. They provide an efficient description of the ground states of local, gapped Hamiltonians, which exhibit an area-law behavior. MPS have been applied to a wide range of problems in different fields. These ideas have also been extended to two spatial dimensions (i.e., $2+1$-dimensional quantum systems) using the generalization of MPS known as projected entangled pair states (PEPS), but the success has been limited.

In addition to these methods for the continuous-time approach, an alternate method based on the idea of the tensor renormalization group (TRG) in discretized Euclidean space has also been very successful. This started with the pioneering work of Levin and Nave in two dimensions [5].

*jacques.bloch@ur.de
†raghav.govind.jha@gmail.com
‡robert.lohmayer@ur.de
§maximilian.meister@ur.de

Both approaches have resulted in a better understanding of spin systems and some simple gauge theories [6–10] and have been a fruitful avenue where good progress has been made. Though this success is impressive, it has mostly been restricted to two-dimensional classical or $1+1$-dimensional quantum systems.

However, the higher-order tensor renormalization group (HOTRG) method [11], a Euclidean-space coarse-graining tensor method based on the higher-order singular value decomposition (HOSVD) [12], is also applicable to higher-dimensional models. It was successfully employed to determine the critical temperature of the three-dimensional Ising model on a cubic lattice. Recently this method was used to investigate the critical behavior of the four-dimensional Ising model [13]. The HOTRG method was also applied to study spin models with larger discrete symmetry groups such as the q-state Potts models and those with continuous global symmetries, like the classical $O(2)$ model in two dimensions [14], the $1+1$-dimensional $O(2)$ model with chemical potential [15,16], and even gauge theories [17–19]. For a review of the tensor approach to spin systems and field theory, we refer the reader to [20].

A major drawback of the HOTRG approach is that it is very expensive in dimensions $d \geq 3$ as the computational cost naively scales as $\mathcal{O}(D^{4d-1})$ with memory complexity of $\mathcal{O}(D^{2d})$ for a bond dimension D. In order to overcome this problem, new higher-dimensional tensor coarse-graining schemes, like the anisotropic TRG (ATRG) [21] and the triad TRG (TTRG) [22], were recently developed. For the ATRG the computational and storage complexity is $\mathcal{O}(D^{2d+1})$ and $\mathcal{O}(D^{d+1})$, respectively. In this work we will use the triad method, for which the computational cost scales like $\mathcal{O}(D^{d+3})$ and the memory

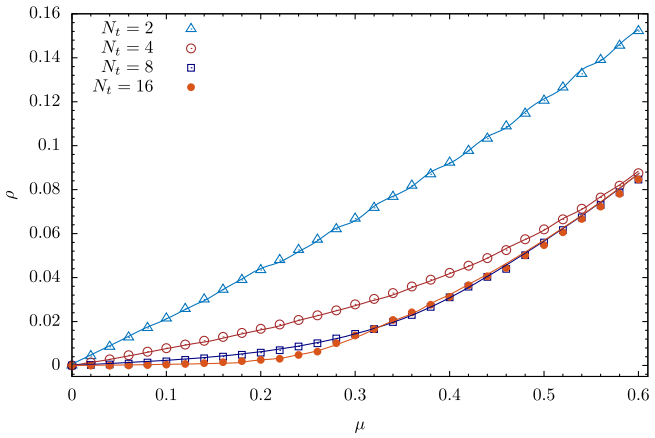

FIG. 6. We use the HOTRG with $D = 13$, improved contraction order, and stabilized finite differences to compute the particle density ρ for a $64^2 \times N_t$ lattice with $N_t = 2, 4, 8, 16$ (symbols) at $\beta = 0.44$ and compare to the results obtained using the worm algorithm (smooth lines). There is clear indication that as we move towards zero temperature, the behavior we see in Fig. 5 starts to emerge.

To validate the nonzero temperature tensor results, we used the worm algorithm [32] at nonzero μ and found good agreement. This is illustrated in Fig. 6 where we show the temperature dependence of the three-dimensional $O(2)$ model by studying the system on a $64^2 \times N_t$ lattice for $N_t = 2, 4, 8, 16$. The tensor results were obtained using the ICO enhanced HOTRG method with $D = 13$. The particle number density was computed using a stabilized finite-difference scheme (see Sec. III A), and tensor manipulations were performed using the TBLIS library [37].

IV. SUMMARY AND DISCUSSION

In this work, we have carried out the first tensor-network study of the three-dimensional classical $O(2)$ model at both zero and nonzero magnetic field, chemical potential, and temperature. The results obtained for the internal energy and the specific heat are consistent with MC data. However, our determination of the critical coupling is several orders of magnitude less precise than state-of-the-art MC results. We calculated the magnetization in the presence of a small magnetic field by inserting an impure tensor. At nonzero chemical potential, we were able to reproduce the Silver Blaze phenomenon at zero temperature. We considered nonzero temperature by varying the temporal extent of the lattice and computed the particle density at nonzero chemical potential. Our results agree with those obtained with the worm algorithm.

In the Appendix, we discuss the convergence of $\ln Z/V$ with the bond dimension D. We expect that this convergence will play a key role in a more precise determination of β_c and in exploring the corresponding field-theory limit in the future. To this end, improved coarse-graining schemes will have to be developed. Such improvements will also be useful to explore other interesting spin models in the future.

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APPENDIX: CONVERGENCE OF $\ln(Z)$ WITH D

It is a known problem that the truncations used in tensor-network methods sometimes lead to drastic modifications of the properties of the model whose thermodynamic behavior one intends to study. In this appendix, we investigate the convergence of $\ln Z/V$ with the local bond dimension D in the triad approximation of the HOTRG method, in the large-volume limit for the three-dimensional cubic Ising and $O(2)$ models. We tune the couplings close to their critical values to make the dependence on D prominent. This is illustrated in Figs. 7 and 8. The shaded areas enclose the various fits to the data (corresponding to various fit ranges and different fit formulas, including the ansatz $a + bD^{-c}$). The extrapolation to $D \to \infty$ can be read off from the intercept with the vertical axis. The convergence for the Ising model is faster than for the $O(2)$ model, which may hint to a different efficiency of tensor methods for systems with

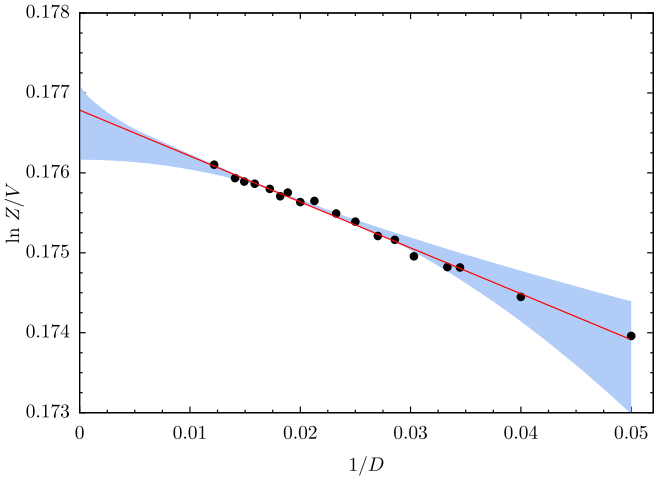

FIG. 7. Dependence of $\ln Z/V$ on the bond dimension D on a lattice of volume $(2^{15})^3$ at $\beta = 0.45417$ for the three-dimensional $O(2)$ model obtained using the triad method. The red line shows the result of a linear fit using all data points.

Positive geometries for all scalar theories from twisted intersection theory

Nikhil Kalyanapuram[1,2,*] and Raghav G. Jha[1,†]

[1]*Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario, Canada N2L 2Y5*

[2]*Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1*

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We show that accordiohedra furnish polytopes which encode amplitudes for all massive scalar field theories with generic interactions. This is done by deriving integral formulas for the Feynman diagrams at the tree level and integrands at the one-loop level in the planar limit using the twisted intersection theory of convex realizations of the accordiohedron polytopes.

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I. INTRODUCTION

The study of scattering amplitudes in the past decade has revealed a number of surprising connections with mathematics. Crucially, deep ties to geometry, topology, and combinatorics [1–36] have been established, which have led to the discovery of new ways of computing these amplitudes.

In this work we focus on building upon the seminal developments in the past few years, namely, the positive geometry program due to Arkani-Hamed *et al.* [7] and the twisted intersection theory of Mizera [2]. In these works it was seen that for a wide class of theories built out of trivalent vertices, the planar Feynman diagrams are encoded by the geometry of a polytope known as the associahedron. This was extended to massless scalar theories with generic interactions in [17], in which a polytope known as the accordiohedron was introduced. In this paper we propose a broad generalization of this line of research by applying the technology of intersection theory to the accordiohedron polytopes.

We seek to address two open questions in the literature. These are as follows. So far, attention has been restricted to the handling of massless interacting particles. The reason for this is the specific realization of the associahedra as convex polytopes, which puts severe restrictions on the masses of the interacting particles. Here we extend the positive geometry program to all scalar theories while utilizing a convex realization of accordiohedra that removes this restriction on the mass and are thus able to treat without any difficulty the interactions between particles of arbitrary mass.

As far as the positive geometry program is concerned, loop effects have been difficult to incorporate. Technical restrictions have forced us to only deal with ϕ^3 interactions among massless particles at the one-loop level. We rectify this by proposing a class of accordiohedra which describes interactions between particles in any scalar theory at the one-loop level, in the planar limit. Our construction also allows us to handle different kinds of Feynman diagrams separately, for example, allowing us to treat tadpoles and bubble diagrams distinctly.

Let us briefly discuss what has is done in the paper and the organization of the text. What is accomplished is a generalization of the positive geometry framework to take care of massive particles as well. This is done in Sec. II. Following this, in Sec. III we also describe a simple example indicating that the story can be pushed to at least one-loop order in arbitrary theories and point out the problems involved in higher-loop cases. In doing so we rectify a problem which has been ignored in the literature, namely, the handling of symmetry factors in Feynman diagrams.

II. MASSIVE $\phi^3 + \phi^4$ SCALAR THEORIES

In this section we describe how the twisted intersection theory of accordiohedra can be used to compute scattering amplitudes for generic scalar theories involving massive particles.

Much of the work on positive geometries for scalar theories beyond ϕ^3 has been done quite recently. For the case of ϕ^4 and ϕ^p interactions, the relevant papers are [12,16], respectively. The formalism for studying generic theories was worked out in [17]. Conspicuously, the analysis in these papers worked specifically for massless particles.

In this section we illustrate how the positive geometry formalism can accommodate massive particles through a development of the intersection theory governing amplitudes in massive scalar theories with $\phi^3 + \phi^4$ interactions. It will turn out that this is the right arena to generalize the study of polytopes controlling these amplitudes for massless particles to massive ones. To do this, we make use of the accordiohedron data first presented in [17] and the method of realizing these as convex polytopes reviewed in [37]. To keep the discussion simple, let us restrict ourselves to the case of six-particle scattering. This particular process gives rise to two classes of accordiohedra, namely, squares and pentagons. Let us begin with the square, which is obtained from the

*nkalyanapuram@perimeterinstitute.ca

†rjha1@perimeterinstitute.ca

allows for arbitrary choices of mass as well as moving off the mass shell. Among other things, this allows us to treat tadpoles and bubble diagrams with relative ease. Furthermore, we have been able to bring loop amplitudes, at least up to the one-loop level, into the discussion as well while taking care of symmetry factors.

It seems that there are some aspects of this work which can be easily extended. First, in order to keep track of symmetry factors at the loop level, we have by hand been restricted to specific subsets of dissections giving rise to loop diagrams according to the nature of renormalization (e.g., mass renormalization and coupling constant renormalization in ϕ^4 are treated separately). It remains to be seen whether the symmetry factors and all loop diagrams can be consistently reconciled with one another in the polytopes picture. This seems unlikely, but will surely constitute an interesting future investigation.

Second, it may be interesting to extend our analysis past the realm of scalar theories into richer domains, such as effective field theories (EFTs). Historically, the Cachazo-He-Yuan formalism has provided ample insight into EFTs which can be obtained by dimensional reduction of gravity and Yang-Mills theory. Now the technology developed here to understand more generic vertices might give us room to look at more exotic EFTs. This is a long-term goal that we hope to pursue in the future.

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Three-dimensional super-Yang-Mills theory on the lattice and dual black branes

Simon Catterall,[1,*] Joel Giedt,[2,†] Raghav G. Jha,[3,‡] David Schaich,[4,§] and Toby Wiseman[5,‖]

[1]*Department of Physics, Syracuse University, Syracuse, New York 13244, USA*
[2]*Department of Physics, Applied Physics and Astronomy, Rensselaer Polytechnic Institute, 110 8th Street, Troy, New York 12065, USA*
[3]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
[4]*Department of Mathematical Sciences, University of Liverpool, Liverpool L69 7ZL, United Kingdom*
[5]*Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, United Kingdom*

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In the large-N and strong-coupling limit, maximally supersymmetric SU(N) Yang-Mills theory in $(2+1)$ dimensions is conjectured to be dual to the decoupling limit of a stack of N D2-branes, which may be described by IIA supergravity. We study this conjecture in the Euclidean setting using nonperturbative lattice gauge theory calculations. Our supersymmetric lattice construction naturally puts the theory on a skewed Euclidean 3-torus. Taking one cycle to have antiperiodic fermion boundary conditions, the large-torus limit is described by certain Euclidean black holes. We compute the bosonic action—the variation of the partition function—and compare our numerical results to the supergravity prediction as the size of the torus is changed, keeping its shape fixed. Our lattice calculations primarily utilize $N=8$ with extrapolations to the continuum limit, and our results are consistent with the expected gravity behavior in the appropriate large-torus limit.

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I. INTRODUCTION

It has been conjectured [1–4] that the large-N limits of maximally supersymmetric Yang-Mills (SYM) theories, obtained from the dimensional reduction of $\mathcal{N}=1$ SYM in ten dimensions down to $(p+1)$ dimensions, are dual to string theories containing Dp-branes. In the large-N and strong-coupling limit this relates properties of gauge theories to the dual properties of Dp-brane solutions in supergravity. The $p=3$ case is the AdS/CFT correspondence, which has received much attention, in part due to its additional conformal symmetries. For direct numerical tests of holographic duality the $p<3$ cases are more attractive to consider, as they feature more tractable gauge theories [5].

For example, the D0-brane or $p=0$ case is a quantum-mechanical description well-known as the Banks-Fischler-Shenker-Susskind (BFSS) model [6,7]. One of the earliest efforts to understand holographic duality in the quantum-mechanical case directly from non-perturbative gauge theory was described in Refs. [8–10]. In recent years, good agreement has been obtained for the case of $p=0$ in the Euclidean setting using numerical Monte Carlo calculations. These efforts started with Refs. [11–16], and more sophisticated recent lattice analyses give convincing agreement with dual-gravity black hole predictions in the large-N low-temperature limit [17–20]. In addition to the BFSS quantum mechanics, a maximally supersymmetric deformation of it known as the Berenstein-Maldacena-Nastase (BMN) model [21], which may also be dual to black holes at low temperatures [22], is now also starting to be studied on the lattice [23–25].

This Euclidean lattice approach was extended to the higher-dimensional D1-brane case in Refs. [26–29]. To allow numerical lattice calculations, one must compactify the spatial direction. In the continuum this corresponds to placing the dual theory on a Euclidean torus with all bosonic fields subject to periodic boundary conditions along all directions. With periodic fermion boundary conditions along all directions, supersymmetry is unbroken and the partition function is independent of the size and shape of this torus. In order to study more interesting behavior, we take one cycle to be anti-periodic for fermions.

As discussed in Refs. [28,29], a conventional thermodynamic interpretation would require the gauge theory to

*smcatter@syr.edu
†giedtj@rpi.edu
‡rjha1@perimeterinstitute.ca
§david.schaich@liverpool.ac.uk
‖t.wiseman@imperial.ac.uk

$N = 8$ results for this same $t \approx 0.31$ and $\zeta^2 = 0.09$. Thus we confirm that our small-t results do indeed appear to be in the large-volume regime and consistent with the dual supergravity predictions. Presumably there is a large-N phase transition separating the small- and large-volume regimes, although such a transition is difficult to see in our $N = 8$ data on the lattice sizes we consider here.

V. CONCLUSIONS AND NEXT STEPS

We have presented the first numerical lattice gauge theory studies of three-dimensional maximally supersymmetric Yang-Mills theory, advancing our program of nonperturbatively testing holography. Such tests provide direct first-principles checks of holographic duality at finite temperatures and in nonconformal settings, where tools such as integrability and supersymmetric localization are not available.

Already at modest $N = 8$ our results indicate that the large-N predictions of the dual-gravity black holes can emerge for large tori. We have seen that the bosonic action density interpolates rather smoothly between the small-volume regime and the large-volume supergravity regime, similar to results for lower-dimensional cases [14,15,19,20,27–29]. We are able to see qualitative agreement with the supergravity prediction derived from the dual black hole action density, and continuum extrapolations indicate no significant discretization artifacts for $t \geq 0.33$. We also see that the Wilson lines about the spatial directions of the torus are consistent with a transition from a localized angular eigenvalue distribution at small volumes to the expected homogeneous distribution at large volumes, presumably with a large-N phase transition at an intermediate torus size.

In the future, we plan to look at the Maldacena-Wilson loop and compare it to the results obtained from the dual-gravity computations. In addition, similar to our previous study [28,29], we can also change the aspect ratios of the torus cycle sizes to study phase transitions from the homogeneous D2-phase we consider here to D1-phases or even localized D0-phases. It will also be interesting to understand the nature of the large-N phase transition at intermediate volumes, although this has proved difficult to study even in simpler settings [65].

Though our results approach the supergravity predictions in the appropriate regime, even larger N would help to better satisfy the conditions on the validity of the classical supergravity description. Numerical calculations at larger N are certainly possible, but would require much more substantial computational resources due to computational costs increasing more rapidly than N^3 [55]. Our current results in this paper nevertheless show the approach to this regime in detail and are certainly consistent with the supergravity results.

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APPENDIX: LATTICE ACTION AND COMPUTATION OF THE BOSONIC ACTION

Our lattice formulation of maximally supersymmetric Yang-Mills theory in $d < 4$ dimensions discretized on the A_d^* lattice is obtained by classical dimensional reduction from the parent four-dimensional theory. The lattice action for topologically twisted $\mathcal{N} = 4$ SYM in $d = 4$ dimensions is the sum of the following $\mathcal{Q}$-exact and $\mathcal{Q}$-closed terms [44–52]:

$$S_{\text{exact}} = \frac{N}{4\lambda_{\text{lat}}} \sum_{\mathbf{n}} \text{Tr}\left[-\bar{\mathcal{F}}_{ab}(\mathbf{n})\mathcal{F}_{ab}(\mathbf{n}) - \chi_{ab}(\mathbf{n})\mathcal{D}^{(+)}_{[a}\psi_{b]}(\mathbf{n}) - \eta(\mathbf{n})\bar{\mathcal{D}}^{(-)}_{a}\psi_a(\mathbf{n}) + \frac{1}{2}\left(\bar{\mathcal{D}}^{(-)}_{a}\mathcal{U}_a(\mathbf{n})\right)^2 \right], \tag{A1}$$

$$S_{\text{closed}} = -\frac{N}{16\lambda_{\text{lat}}} \sum_{\mathbf{n}} \text{Tr}[\epsilon_{abcde}\chi_{de}(\mathbf{n} + \hat{\boldsymbol{\mu}}_a + \hat{\boldsymbol{\mu}}_b + \hat{\boldsymbol{\mu}}_c)\bar{\mathcal{D}}^{(-)}_{c}\chi_{ab}(\mathbf{n})], \tag{A2}$$

where λ_{lat} is the dimensionless 't Hooft coupling defined by $r_{\tau,\text{lattice}} = \lambda_{\text{lat}} N_\tau^{4-d}$. The indices run from $1, \ldots, 5$, spanning the basis vectors of the A_4^* lattice, and $\sum_{\mathbf{n}}$ is over all lattice sites. The $1 + 5 + 10$ fermion fields η, ψ_a, and $\chi_{ab} = -\chi_{ba}$ transform in representations of the S_5 point group symmetry, as do the five complexified gauge links $\mathcal{U}_a$ and $\bar{\mathcal{U}}_a$

Nonperturbative study of dynamical SUSY breaking in $\mathcal{N} = (2,2)$ Yang-Mills theory

Simon Catterall,[1,*] Raghav G. Jha,[1,†] and Anosh Joseph[2,‡]

[1]*Department of Physics, Syracuse University, Syracuse, New York 13244, USA*
[2]*International Centre for Theoretical Sciences (ICTS-TIFR), Tata Institute of Fundamental Research, Bangalore 560089, India*

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We examine the possibility of dynamical supersymmetry breaking in two-dimensional $\mathcal{N} = (2, 2)$ supersymmetric Yang-Mills theory. The theory is discretized on a Euclidean spacetime lattice using a supersymmetric lattice action. We compute the vacuum energy of the theory at finite temperature and take the zero-temperature limit. Supersymmetry will be spontaneously broken in this theory if the measured ground-state energy is nonzero. By performing simulations on a range of lattices up to 96×96 we are able to perform a careful extrapolation to the continuum limit for a wide range of temperatures. Subsequent extrapolations to the zero-temperature limit yield an upper bound on the ground-state energy density. We find the energy density to be statistically consistent with zero in agreement with the absence of dynamical supersymmetry breaking in this theory.

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I. INTRODUCTION

The investigations of supersymmetric gauge theories on a spacetime lattice are important for understanding the nonperturbative structure of such theories and in particular they can address the question of whether dynamical supersymmetry (SUSY) breaking takes place in such theories. This is a crucial question for efforts to construct supersymmetric theories which go beyond the Standard Model since the low-energy world is clearly not supersymmetric while nonrenormalization theorems typically ensure that supersymmetry cannot break in perturbation theory [1].

Unfortunately, there are a plethora of problems to overcome for lattice formulations of supersymmetric theories. Supersymmetry is a spacetime symmetry, which is generically broken by the lattice regularization procedure. Hence, the effective action of the lattice theory typically contains relevant supersymmetry-breaking interactions. To achieve a supersymmetric continuum limit it is necessary to fine-tune the lattice couplings to these terms as the lattice spacing is reduced. Since generically there are very many such terms this is in practice impossible. One exception to this is $\mathcal{N} = 1$ super Yang-Mills (SYM) where only a single coupling, the gluino mass, must be tuned. In addition, it has also been shown that fine-tuning to a supersymmetric continuum limit is also possible for $\mathcal{N} = (2, 2)$ in two dimensions. Using Wilson fermions, the only relevant parameter that has to be fine-tuned is the scalar mass since the bare gluino mass is an irrelevant parameter. The continuum value for the critical scalar mass is known up to one-loop order in lattice perturbation theory and that has already been employed in the numerical simulations. See Refs. [2–4] for discussions and references therein.

The attempt to formulate supersymmetric theories on the lattice has a long history starting in Refs. [5–10]. Recent approaches to this problem have focused on preserving a subalgebra of the full supersymmetry algebra which can protect the theory from some of these dangerous supersymmetry-violating terms; for a review, see Ref. [11]. For supersymmetric theories with extended supersymmetry various supersymmetric lattice formulations exist. One approach that was pioneered by Cohen, Kaplan, Katz and Ünsal in Refs. [12–14] is based on orbifolding and deconstruction of a supersymmetric matrix model. A second approach uses the idea of topological twisting to isolate appropriate nilpotent scalar supersymmetries that can be transferred to the lattice. Two independent discretization schemes have been proposed in this approach: that proposed by Sugino in Refs. [15,16] where the fermions are associated with sites and a geometrical approach in which fermions are generically

*smcatter@syr.edu
†rgjha@syr.edu
‡anosh.joseph@icts.res.in

TABLE I. Numerical results showing that our action is effectively $\mathcal{O}(a)$ improved. We measure the deviation of the bosonic action/site from its supersymmetric value of $\frac{3}{2}N^2$ and fit it to a power law. The first column shows the soft-mass parameter, ζ, we use to regulate the flat directions. The second column is the obtained value of the power, p, when constraining the intercept to vanish, while the third is the obtained value of the power, p, *without* constraining the intercept. We quote results from one of the couplings used in this work, $r_\tau = 6$. On the top, we show the results with $U(3)$ and with $U(2)$ at the bottom. The fits are very good with a maximum $\chi^2/\text{d.o.f.} = 2.80$.

ζ	$\propto (a/L)^p$	$\propto (a/L)^p + c$
0.40	1.86(9)	1.76(22)
0.50	1.76(6)	1.60(15)
0.55	1.79(5)	1.90(11)
0.60	1.74(4)	1.70(11)

ζ	$\propto (a/L)^p$	$\propto (a/L)^p + c$
0.40	1.73(10)	1.58(24)
0.50	1.71(7)	1.74(17)
0.55	1.69(6)	1.57(14)
0.60	1.78(5)	1.98(12)

$$\frac{\mathcal{E}_{\text{VAC}}}{N^2\lambda} = \begin{cases} 0.05(2), & \chi^2/\text{d.o.f.} = 0.11\text{: power-law fit,} \\ 0.04(4), & \chi^2/\text{d.o.f.} = 0.11\text{:exponential fit,} \\ 0.05(2), & \chi^2/\text{d.o.f.} = 0.06\text{:constant fit.} \end{cases} \tag{20}$$

We note that the errors in our results do not allow us to make conclusive statements about the exact form of the energy dependence on the temperature. Both power, exponential and constant fitting functions yield comparable results consistent with vanishing ground-state energy. Our calculation puts an upper bound on the dimensionless energy density using the constant fit at $\frac{\mathcal{E}_{\text{VAC}}}{N^2\lambda} = 0.08(2)$ for $U(2)$ and $\frac{\mathcal{E}_{\text{VAC}}}{N^2\lambda} = 0.05(2)$ for $U(3)$.

While this work was in progress results were presented on the tree-level $\mathcal{O}(a)$ improvement of Sugino's lattice action for two-dimensional $\mathcal{N} = (2,2)$ SYM [47]. We note that our lattice formulation already possesses this improvement which we see in Fig. 3 and in Table I.

IV. CONCLUSIONS

In this paper we have examined the possibility of dynamical supersymmetry breaking in two-dimensional $\mathcal{N} = (2,2)$ SYM through lattice simulations. The lattice theory is exactly supersymmetric, gauge-invariant, local, and doubler free. We found an upper bound on the vacuum energy density of $\frac{\mathcal{E}_{\text{VAC}}}{N^2\lambda} = 0.08(2)$ and $\frac{\mathcal{E}_{\text{VAC}}}{N^2\lambda} = 0.05(2)$ for $U(2)$ and $U(3)$ respectively. The energy density is statistically consistent with zero and hence with the absence of dynamical supersymmetry breaking. It would be interesting to examine the spectrum in future work to confirm the absence of spontaneous supersymmetry breaking perhaps by searching for signals of a Goldstino as was done in Ref. [31]. We have also measured the phase of the Pfaffian on all our ensembles and found that while the average phase grows with the coupling it decreases as we take the continuum limit in agreement with theoretical expectations. In practice, it is numerically small for all our ensembles. The question of supersymmetry breaking in this model was addressed before in Ref. [29]. Our current work, in addition to using a different lattice action, has employed stronger couplings (and hence lower temperatures) and much smaller lattice spacings. For example, the lowest temperature used in the earlier work was $t = 1/6$ as compared to $t = 1/9$ in this work while the largest lattice used here is 96×96 as compared to 30×12 in the earlier study.

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Real-time scattering in Ising field theory using matrix product states

Raghav G. Jha [1], Ashley Milsted [2], Dominik Neuenfeld [3], John Preskill,[2,4] and Pedro Vieira[5,6]

[1] Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA
[2] AWS Center for Quantum Computing, Pasadena, California 91125, USA
[3] Institute for Theoretical Physics and Astrophysics, Julius-Maximilians-Universität Würzburg, Am Hubland, 97074 Würzburg, Germany
[4] Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California 91125, USA
[5] Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada N2L 2Y5
[6] ICTP South American Institute for Fundamental Research, IFT-UNESP, São Paulo, São Paulo 01440-070, Brazil

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We study scattering in Ising field theory (IFT) using matrix product states and the time-dependent variational principle. IFT is a one-parameter family of strongly coupled nonintegrable quantum field theories in $1+1$ dimensions, interpolating between massive free fermion theory and Zamolodchikov's integrable massive E_8 theory. Particles in IFT may scatter either elastically or inelastically. In the postcollision wave function, particle tracks from all final-state channels occur in superposition; processes of interest can be isolated by projecting the wave function onto definite particle sectors, or by evaluating energy density correlation functions. Using numerical simulations we determine the time delay of elastic scattering and the probability of inelastic particle production as a function of collision energy. We also study the mass and width of the lightest resonance near the E_8 point in detail. Close to both the free fermion and E_8 theories, our results for both elastic and inelastic scattering are in good agreement with expectations from form-factor perturbation theory. Using numerical computations to go beyond the regime accessible by perturbation theory, we find that the high-energy behavior of the two-to-two particle scattering probability in IFT is consistent with a conjecture of Zamolodchikov. Our results demonstrate the efficacy of tensor-network methods for simulating the real-time dynamics of strongly coupled quantum field theories in $1+1$ dimensions.

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I. INTRODUCTION

Much of what we know about fundamental particle physics has been inferred from collider experiments in which high-energy collisions produce many outgoing particles. However, our ability to analyze these processes from fundamental principles is currently restricted due to the lack of theoretical tools. Feynman diagram methods can be effective in the weak-coupling regime of a quantum field theory, and in some special cases scattering data can be extracted from Euclidean lattice QCD computations [1,2], but real-time analysis of particle production at strong coupling and high energy is largely beyond the reach of existing methods. For example, there is no known technique for computing the high-energy behavior of glueball-glueball scattering in pure Yang-Mills theory.

The situation is less dire for strongly coupled quantum field theory in $1+1$ spacetime dimensions. We can succinctly represent the state of a quantum spin chain as a matrix product state (MPS), and if the state does not become too highly entangled, then we can efficiently simulate its evolution under a local Hamiltonian. Furthermore, relatively simple spin chains close to criticality can approximate a variety of nonintegrable continuum quantum field theories. First steps in this direction have already been pursued in Refs. [3–7], however not in the continuum limit needed to learn about quantum field theories. By studying elastic and inelastic scattering processes in such field theories using the MPS method for sufficiently large correlation lengths and wave-packet sizes, we can hope to address some longstanding open questions about these processes.

In this work, we demonstrate the feasibility of this strategy by using MPS methods to study Ising field theory (IFT), a parameterized family of massive continuum theories obtained by deforming the critical two-dimensional Ising model. The action of IFT is

$$I_{\mathrm{IFT}} = I_{\mathrm{ICFT}} - \tau \int d^2x\, \epsilon(x) - h \int d^2x\, \sigma(x), \qquad (1)$$

where I_{ICFT} is the action of the Ising conformal field theory (ICFT), ϵ and σ are the relevant energy and spin operators with scaling dimensions $\Delta_\epsilon = 1$ and $\Delta_\sigma = \frac{1}{8}$, respectively, and τ and h are the thermal and magnetic deformation parameters. The term in Eq. (1) that contains the energy operator ϵ is called the thermal deformation of ICFT because it arises from perturbing the temperature in the Ising model away from the critical temperature. The term that contains the spin operator σ is called the magnetic deformation of ICFT because it arises from a $\mathbb{Z}_2$-symmetry-breaking external magnetic field in the Ising model. Because τ has scale dimension 1 (in units of mass) and h has scale dimension $15/8$, we characterize the relative weight of the two perturbations using the dimension-

elastic at high energies, while near the free fermion point, scattering should be completely inelastic. By extrapolating scattering probabilities at moderate energies we found support for this conjecture and were able to locate a possible region far inside the nonperturbative region of the parameter space where a crossover from trivial to completely inelastic scattering at high energies might occur.

The S-matrix is an analytic function of energy E on a multisheeted Riemann surface which encodes the spectrum and couplings of the theory. Features like resonances and antibound states are encoded in poles that are a finite distance away from the physical scattering region $4m_1^2 \leqslant E \in \mathbb{R}$. We have demonstrated that we can extract masses and decay rates of resonance poles from our real-time simulations, and hence infer features of the S-matrix's analytic structure. Our work demonstrates how real-time simulations of elastic and inelastic scattering phenomena provide an instructive nonperturbative tool for exploring the space of two-dimensional quantum field theories that arise as scaling limits of quantum spin chains.

However, to extract more precise data further work is needed to improve our methods. For example, when extracting the time delays near the free fermion theory, we leveraged our knowledge that time delays vanish in the free theory to subtract a systematic error and hence obtain accurate results. Far from the integrable points, however, we know little about IFT and alternative schemes will be needed; hence, we have not yet attempted to predict the time delay far away from integrability. One might attain more accurate estimates of scattering phase shifts in this regime by increasing the computational resources and/or modifying the algorithms used for time evolution or extraction of the phase shift.

Already in IFT there are many more open questions worth pursuing such as numerical studies of IFT in Euclidean time near the Yang-Lee point (see, e.g., Ref. [39]). Moreover, our method based on real-time simulation could be extended to hunt for resonances in the higher-energy region where inelastic scattering can occur. By extending our analysis far into the inelastic regime one might track the location of resonances as a function of the deformation parameter η, which could provide, e.g., further insight into the role of m_3 for the high-energy behavior of the scattering amplitude. Our methods can also be applied to other theories beyond IFT, such as scalar ϕ^4 theory [28], the $O(3)$ sigma model [40], the three-state Potts model [41], and the Schwinger model [42].

Once precision data is obtained one might use numerical results from real-time simulations as an input in the S-matrix bootstrap [43–45], to further narrow down the space of allowed S-matrices of nonintegrable, (1+1)-dimensional quantum field theories. (Attempts to bootstrap the IFT include [43,45–47].) Injecting approximate inelastic scattering probabilities into the bootstrap program is expected to yield $O(1)$ improvements in bounds constraining the S-matrix [43,48–52]. Even qualitative estimates indicating whether high-energy amplitudes are mostly elastic or mostly inelastic could be very useful. For example, the scattering of the excitations of the one-dimensional QCD flux tube was introduced in Ref. [53] and bootstrapped in Refs. [54–56]. We know little about how this S-matrix behaves at high energy. Simulations akin to our studies of IFT might help, starting with two-dimensional adjoint QCD recently revisited in Ref. [57].

However, MPS methods like the ones discussed in the present paper are best suited for studies of one-dimensional systems, and even there are limited to simulations of processes that do not produce profoundly entangled final states. Eventually, simulations performed on quantum computers will open the door to compelling visualizations as well as quantitative studies of higher-energy scattering events producing many particles; see [36,58–63] for steps toward this goal. Until then, as we have demonstrated, real-time MPS simulations of scattering in (1+1)-dimensional quantum field theories provide a powerful tool to gain both intuition and quantitative results addressing many open questions.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

Continuous-variable quantum computation of the O(3) model in 1 + 1 dimensions

Raghav G. Jha,[1,*] Felix Ringer,[1,2,†] George Siopsis,[3,‡] and Shane Thompson[3,§]
[1]*Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*
[2]*Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA*
[3]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*

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We formulate the O(3) nonlinear sigma model in 1 + 1 dimensions as a limit of a three-component scalar field theory restricted to the unit sphere in the large squeezing limit. This allows us to describe the model in terms of the continuous-variable (CV) approach to quantum computing. We construct the ground state and excited states using the coupled-cluster *Ansatz* and find excellent agreement with the exact diagonalization results for a small number of lattice sites. We then present the simulation protocol for the time evolution of the model using CV gates and obtain numerical results using a photonic quantum simulator. We expect that the methods developed in this paper will be useful for exploring interesting dynamics for a wide class of sigma models and gauge theories, as well as for simulating scattering events on quantum hardware in the coming decades.

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I. INTRODUCTION

All fundamental interactions in nature except gravity have been successfully described within the framework of quantum field theory. A full understanding of the real-time dynamics of these interacting field theories is still an open problem. In lower dimensions, some progress has been made using classical computations based on matrix product state (MPS) and projected entangled-pair state tensor network methods [1]. However, for theories close to the critical point these methods are limited because they can only efficiently represent ground states of a special class of local Hamiltonians with a gapped spectrum due to their peculiar entanglement scaling. Another limitation is that even for systems with a gapped spectrum, the growth of entanglement during real-time evolution can render these methods ineffective. The limitations of these classical methods are expected to be overcome by quantum computers, which has led to an increased effort aimed at understanding various lattice models and lattice field theories using quantum computing algorithms.

From the perspective of fundamental nuclear and particle physics [2], the long-term goal is the study of real-time dynamics of quantum chromodynamics (QCD) in four dimensions, which is relevant for the description of inelastic-scattering processes at current and future collider experiments [3]. Due to current hardware limitations, lower-dimensional models have typically been considered that represent important stepping stones toward simulations of QCD. In this regard, the O(3) nonlinear sigma model in 1 + 1 dimensions is a particularly interesting test ground. This model exhibits a global non-Abelian symmetry group and shares several interesting properties with QCD, the most important of which is asymptotic freedom [4]. The O(3) model has a dynamical mass gap and also admits instanton solutions similar to four-dimensional QCD [5]. In addition, it is relevant for low-energy dynamics of pions in nuclear physics as discussed in more detail below. This makes it a preferred toy model [6] to explore fundamental questions related to nuclear and particle physics. Even though this model has been extensively studied with classical numerical methods, it has recently attracted increased interest because new classical and quantum computing methods have been developed. This model has been studied using tensor network techniques based on MPSs with and without a topological term (for $\theta = \pi, 0$) [7,8] and using higher-order tensor renormalization-group methods [9]. Different regularizations for lattice simulations of the O(3) model have been explored such as the fuzzy sphere qubitization, D-theory, the angular momentum basis, the Heisenberg comb, and the Schwinger boson formulation [10–18]. In Ref. [19], simulations and the preparation of ground states of the O(3) model were discussed in the context of cold atom quantum simulators using near-term quantum platforms. In order to regularize the theory and carry out numerical computations, recent efforts have mostly focused on the qubit approach to quantum computing [16,19–21]. Within the qubitization program, it has been argued that to reproduce the critical point of the continuum field theory in this model, only two qubits per site are required.

In this paper, we present an alternative method, which was been argued to be more natural than qubit-based quantum computing [22] in simulating, for example, bosonic models or lattice field theories such as ϕ^4 scalar field theory [23,24]. This approach is known as continuous-variable (CV) quantum computing [25], which we employ in this paper for simulations of the O(3) model. Instead of the qubit, the fundamental unit to carry out computations is the qumode, which can

*raghav.govind.jha@gmail.com
†fmringer@jlab.org
‡siopsis@tennessee.edu
§sthomp78@tennessee.edu

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APPENDIX: CONTINUOUS-VARIABLE GATES

In this Appendix, we review some of the CV gates used in this paper. Additional details and gates can be found in Ref. [31].

1. Single qumode gates

To create a squeezed state on the sphere, one must first apply a squeeze operation and then a displacement in position. Squeezing is implemented by

$$S(r) = e^{\frac{r}{2}(a^{\dagger 2} - a^2)}, \tag{A1}$$

where a and $a^\dagger$ are bosonic creation and annihilation operators with $[a, a^\dagger] = 1$. The position and conjugate momentum are written as

$$q = \frac{1}{\sqrt{2}}(a^\dagger + a), \quad p = \frac{i}{\sqrt{2}}(a^\dagger - a). \tag{A2}$$

Recall that in this paper the components of the field $\phi_i(x)$ at L sites give $3L$ independent position operators, all of which are accompanied by their conjugate momentum $\pi_i(x)$. It follows from (A1) and (A2) that

$$S^\dagger(r) q S(r) = e^{-r} q, \quad S^\dagger(r) p S(r) = e^r p. \tag{A3}$$

Note that we have

$$r = \ln \Lambda, \tag{A4}$$

where Λ is the cutoff parameter restricting the wave functions to the sphere. A displacement in position can be achieved using a displacement gate, which depends on a real-valued parameter $x \in \mathbb{R}$ as

$$e^{-ipx} = D(x/\sqrt{2}) = e^{\frac{x}{\sqrt{2}}(a^\dagger - a)}. \tag{A5}$$

Its action is given by

$$q \to q + x, \quad p \to p. \tag{A6}$$

This action gives us a way to compute the expectation value of q as

$$q = \frac{1}{2x}(e^{ipx} N e^{-ipx} - e^{-ipx} N e^{ipx}), \quad N = \frac{1}{2}(q^2 + p^2). \tag{A7}$$

Thus to compute $\langle\psi|\, q \,|\psi\rangle$, we compute the mean photon number in the two states

$$e^{-ipx}\,|\psi\rangle\,, \quad e^{ipx}\,|\psi\rangle \tag{A8}$$

and take the difference.

Next, the rotation gate is defined by

$$R(\theta) = e^{iN\theta}, \quad \theta \in \mathbb{R}. \tag{A9}$$

It rotates the position and momentum as

$$\begin{pmatrix} q \\ p \end{pmatrix} \to \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix}. \tag{A10}$$

Another useful gate we have used in the main text is the Fourier transform F. This gate is the continuous-variable version of the Hadamard gate. It is a special case of the rotation gate:

$$F = R\left(\frac{\pi}{2}\right) = e^{\frac{i\pi}{4}(p^2 + q^2)}. \tag{A11}$$

We make use of it in creating the state $|\Omega(\Lambda)\rangle$ [see Eq. (27)], and to help implement the angular momentum operator $\boldsymbol{L}$.

A particularly useful single-qumode gate is the quadratic phase gate,

$$P(s) = e^{isq^2/2}, \tag{A12}$$

which has the transformation properties

$$P^\dagger(s) q P(s) = q, \quad P^\dagger(s) p P(s) = p + sq. \tag{A13}$$

From this, we find

$$P^\dagger(s) N P(s) = P^\dagger(s) \tfrac{1}{2}(q^2 + p^2) P(s) = \tfrac{1}{2}[q^2 + (p + sq)^2], \tag{A14}$$

and thus

$$q^2 = \frac{1}{s^2}(P^\dagger(s) N P(s) + P^\dagger(-s) N P(-s) - 2P^\dagger(0) N P(0)). \tag{A15}$$

This means that we can compute the expectation value $\langle\psi|\, q^2 \,|\psi\rangle$ by instead computing

$$\langle\psi|\, P^\dagger(s) N P(s)\,|\psi\rangle\,, \tag{A16}$$

for three values of s. These are found simply by measuring the mean photon number in the state $P(s)\,|\psi\rangle$. Note that it is possible to express the P gate in terms of the more elementary rotation and single-mode squeeze gates [31].

2. Two qumode gates

The fundamental two qumode gate that is typically considered is the beam-splitter gate given by

$$(U_{\mathrm{BS}})_{ab}(\theta) = e^{\theta(ab^\dagger - a^\dagger b)} = (U_{\mathrm{BS}})^\dagger_{ba}(\theta), \ \theta \in \mathbb{R}. \tag{A17}$$

We use it in conjunction with F to implement the angular momentum operator $\boldsymbol{L}$. Its action is given by

$$\begin{pmatrix} a \\ b \end{pmatrix} \to \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \tag{A18}$$