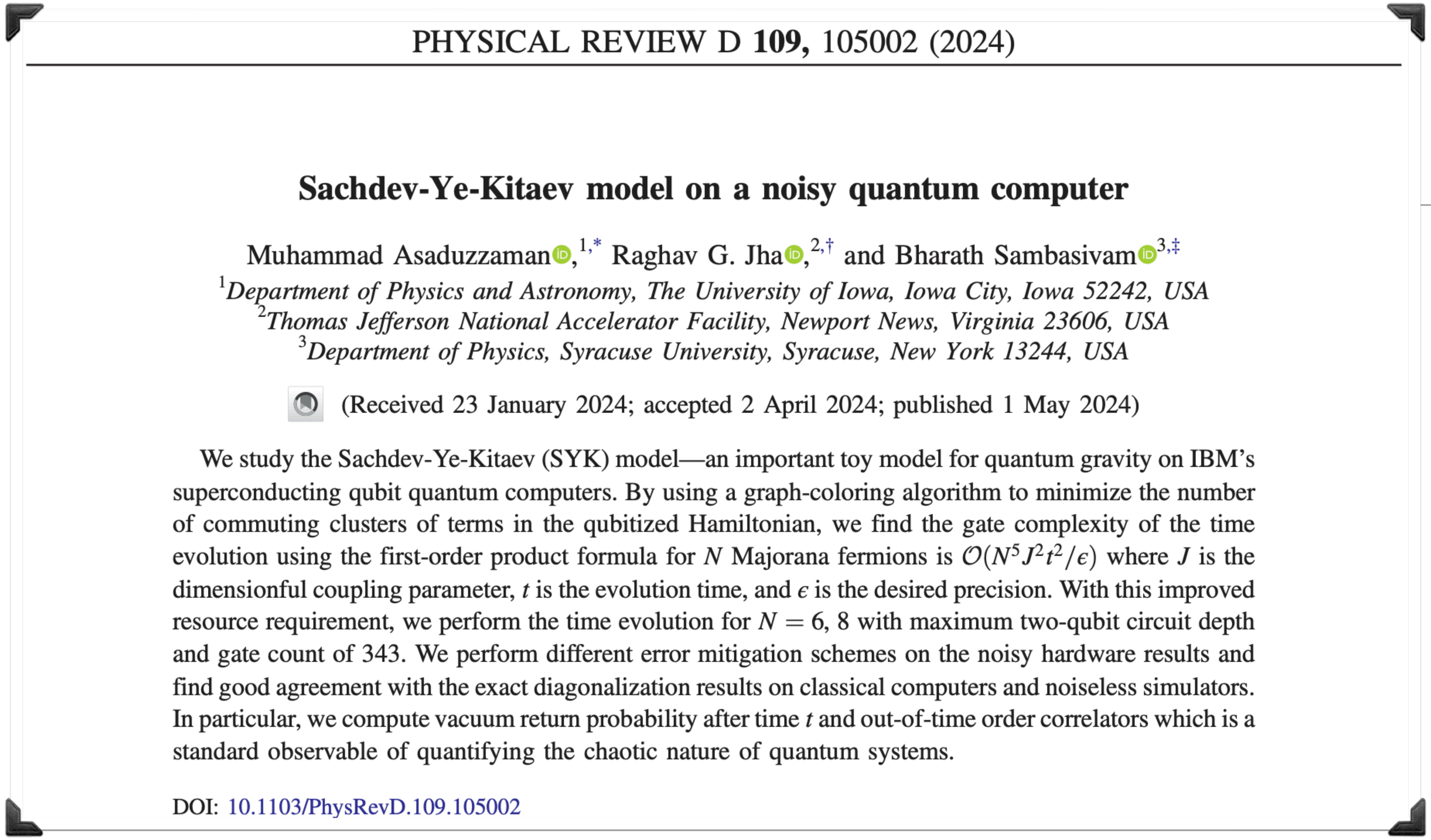
Thermal state preparation and dynamics of random all-to-all fermionic model

arXiv: 2311.17991 (Phys.Rev. D 109, 105002) [with Asad, B. Sambasivam]
arXiv: 2404.14784
arXiv: 2406.15545 [with Jack Araz, F. Ringer, B. Sambasivam]


PHYSICAL REVIEW D **109,** 105002 (2024)

Sachdev-Ye-Kitaev model on a noisy quantum computer

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We study the Sachdev-Ye-Kitaev (SYK) model—an important toy model for quantum gravity on IBM's superconducting qubit quantum computers. By using a graph-coloring algorithm to minimize the number of commuting clusters of terms in the qubitized Hamiltonian, we find the gate complexity of the time evolution using the first-order product formula for N Majorana fermions is $\mathcal{O}(N^5 J^2 t^2/\epsilon)$ where J is the dimensionful coupling parameter, t is the evolution time, and ϵ is the desired precision. With this improved resource requirement, we perform the time evolution for $N = 6, 8$ with maximum two-qubit circuit depth and gate count of 343. We perform different error mitigation schemes on the noisy hardware results and find good agreement with the exact diagonalization results on classical computers and noiseless simulators. In particular, we compute vacuum return probability after time t and out-of-time order correlators which is a standard observable of quantifying the chaotic nature of quantum systems.

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Raghav G. Jha
Jefferson Lab
17 July 2024

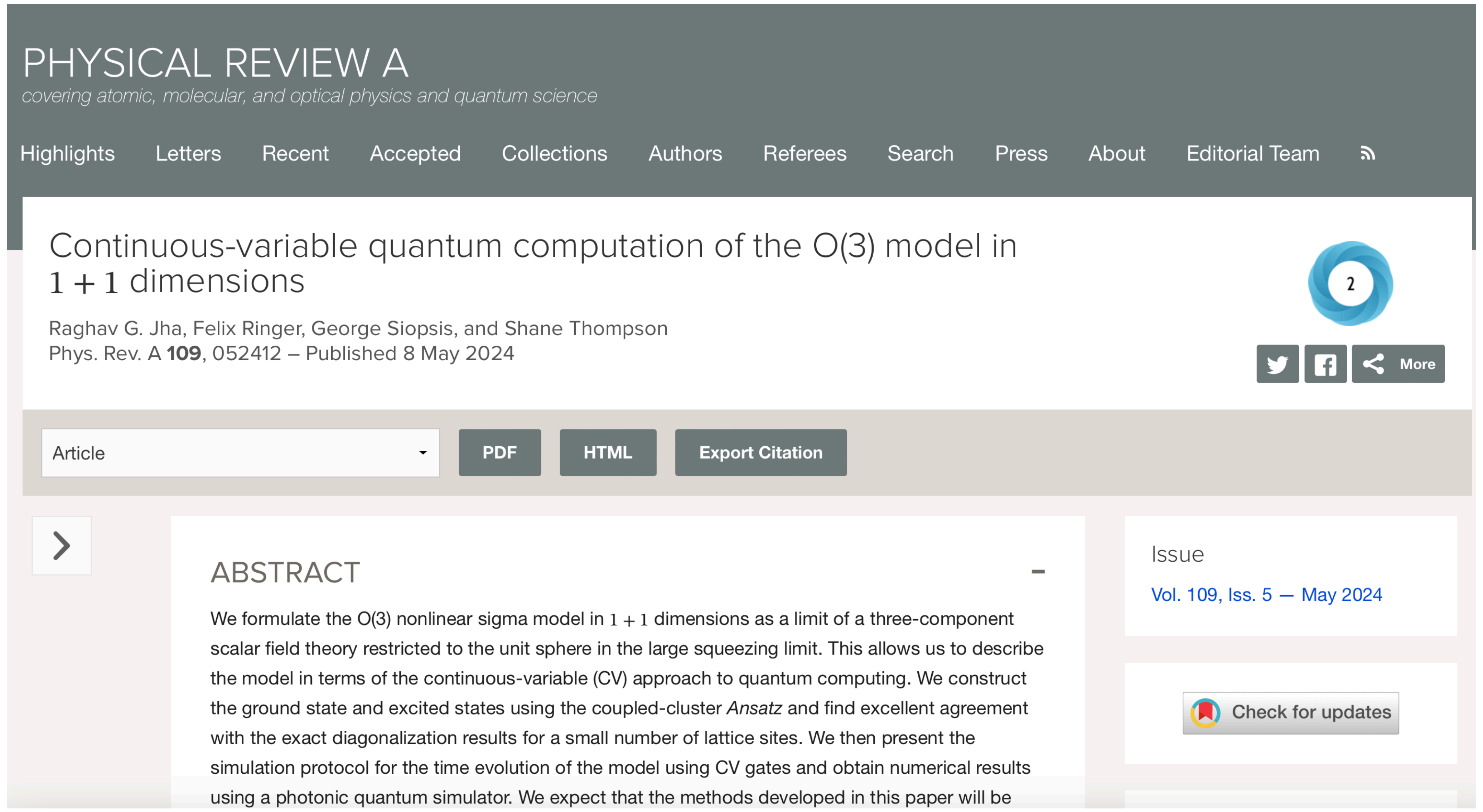
Co-design Center for Quantum Advantage

Talk: What and What not

- Exploring models of quantum gravity using quantum computers via quantum many-body systems.

- Holographic model: Dynamics of SYK model with N = 6, 8 Majorana fermions with error mitigation and thermal state preparation (variational). Both papers utilized IBM devices available through BNL in the past year.

- Will not discuss papers from 2023 exploring O(3) non-linear sigma models and upcoming paper on SU(2) lattice gauge theory in 1+1 and 2+1 dimensions using CV [in progress with Nora B., Victor Ale, S. Thompson, Siopsis, Ringer et. al]. Both of these are non-Abelian and more complicated than U(1) or Z2.


PHYSICAL REVIEW A
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Continuous-variable quantum computation of the O(3) model in $1+1$ dimensions

Raghav G. Jha, Felix Ringer, George Siopsis, and Shane Thompson
Phys. Rev. A **109**, 052412 – Published 8 May 2024

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ABSTRACT

We formulate the O(3) nonlinear sigma model in $1+1$ dimensions as a limit of a three-component scalar field theory restricted to the unit sphere in the large squeezing limit. This allows us to describe the model in terms of the continuous-variable (CV) approach to quantum computing. We construct the ground state and excited states using the coupled-cluster *Ansatz* and find excellent agreement with the exact diagonalization results for a small number of lattice sites. We then present the simulation protocol for the time evolution of the model using CV gates and obtain numerical results using a photonic quantum simulator. We expect that the methods developed in this paper will be

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Holography

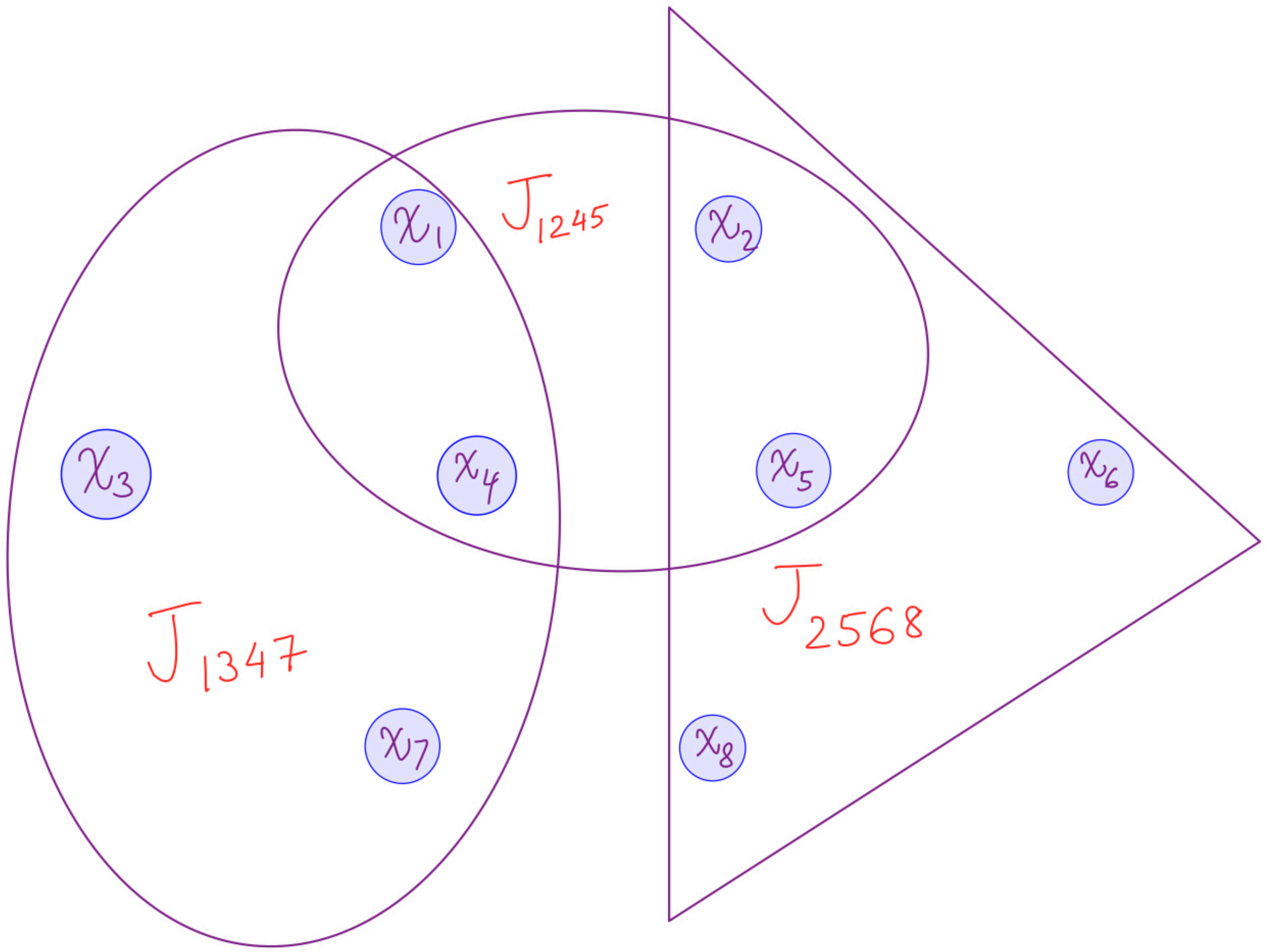
- Insights into quantum gravity through field theories and quantum many-body systems.

- Famous example: AdS/CFT, a version of it was soon also extended to super Yang-Mills (SYM) in p+1-dimensions for $p < 3$ [Maldacena et al., PRD 58 046004(1998)]. We spent years doing qualitative checks of holography in Euclidean space with Monte Carlo.

- We have limited tools to study real-time dynamics of such strongly coupled models. We consider the simplest system which has holographic behaviour i.e., Sachdev-Ye-Kitaev (SYK) model using quantum computing methods.

SYK model

$$H = \frac{(i)^{q/2}}{q!} \sum_{i,j,k,\cdots,q=1}^{N} J_{ijk\cdots q} \ \ \chi_i \chi_j \chi_k \cdots \chi_q,$$



- Model of N Majorana fermions with q-interaction terms with random coupling taken from a Gaussian distribution with $\overline{J_{\ldots}} = 0, \ \overline{J_{\ldots}^2} = \frac{q! J^2}{N^{q-1}}$.

- The fermions χ satisfy, $\chi_i \chi_j + \chi_j \chi_i = \delta_{ij}$. We will set $J = 1$. Note that it has units of energy and inverse time.

- In the limit of large number of fermions with $N \gg \beta J \gg 1$, the model has several interesting features such as maximal Lyapunov exponent.

Mapping fermions to qubits

$$\chi_{2k-1} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) X_k \mathbb{1}^{\otimes (N-2k)/2} \quad , \quad \chi_{2k} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) Y_k \mathbb{1}^{\otimes (N-2k)/2}$$

- N fermions requires N/2 qubits. We use the standard Jordan-Wigner mapping to write χ in terms of Pauli matrices X, Y, Z, and Identity.

- The SYK Hamiltonian is then written as sum of Pauli strings. The number of strings is $\binom{N}{q}$ and grows like $\sim N^q$. Simplest non-trivial case for is $N = q$ with one term. We restrict to $q = 4$.

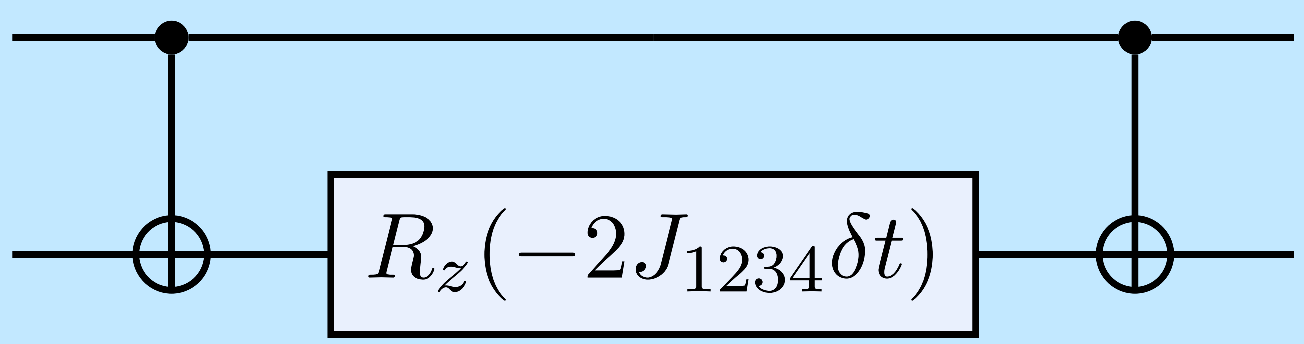
Simplest case: N=4

$$H = J_{1234}\chi_1\chi_2\chi_3\chi_4$$

$$\chi_1 = X\mathbb{1},\ \chi_2 = Y\mathbb{1},\ \chi_3 = ZX,\ \chi_4 = ZY$$

$$H = J_{1234}(X\mathbb{1})\cdot(Y\mathbb{1})\cdot(ZX)\cdot(ZY) = -J_{1234}ZZ$$

- The goal of quantum computation is to construct a unitary operator corresponding to this Hamiltonian. So, for this case we have $\exp(-iHt) = \exp(iJ_{1234}ZZt)$.
- This circuit is simple to construct and just needs 2 CNOTs and 1 rotation gate.



Old work(s)

$$\mathscr{C} = \mathcal{O}(N^{10}t^2/\epsilon)$$ L. García-Álvarez et al., PRL 119, 040501 (2017)

$$\mathscr{C} = \tilde{\mathcal{O}}(N^{7/2}t)$$ Babbush et al., Phys. Rev. A 99, 040301 (2019)

- Using the Trotter methods, the best seems to be $\sim N^{10}$. In our paper, we improved the complexity to $\mathscr{C} = \mathcal{O}(N^5 t^2/\epsilon)$. In a follow-up paper, we simplified the model and found the resources required in that case as well.

Simplest holographic model for quantum computer?

- In fact, there is another version of the SYK model where we remove majority of the terms and still find holographic behaviour. This was first studied in arXiv: 2008.02303 and arXiv: 2007.13837. Recently this was studied Orman, Gharibyan, and Preskill in arXiv: 2403.13884

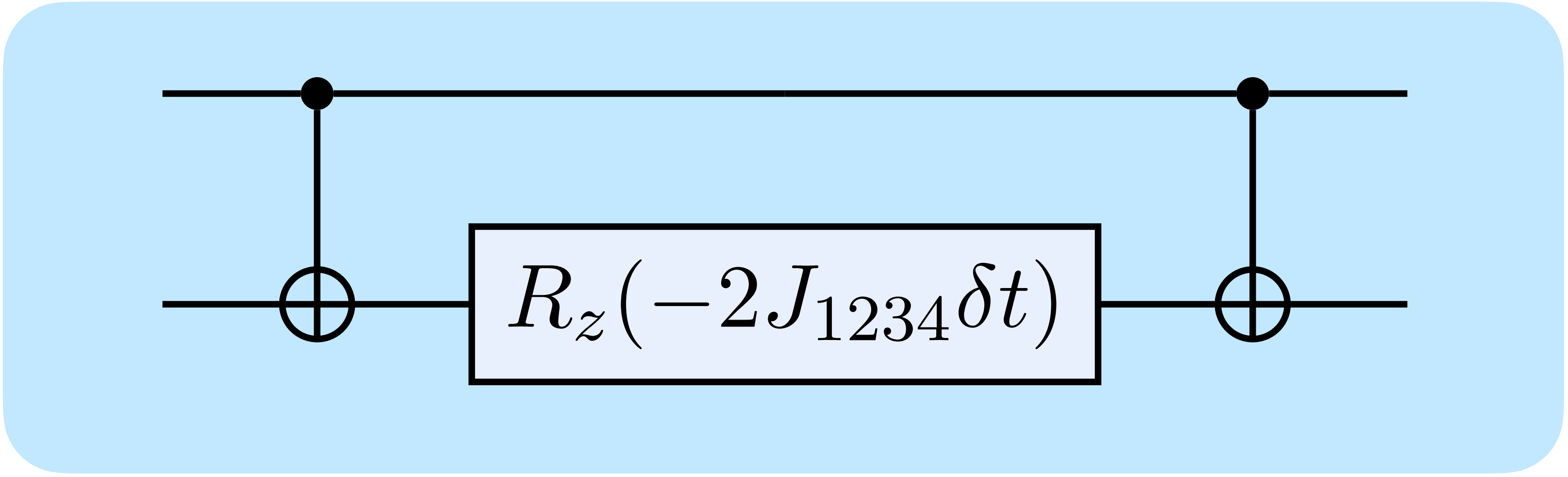
- In arXiv: 2404.14784 (in review, NPB) we considered the simplest model which can be studied on quantum computer in coming decades and has holographic properties. The complexity of quantum circuits goes like $\mathcal{O}(k^p N^{3/2} \log N \ t^{3/2}/\sqrt{\epsilon})$ where k determines how sparse the model is and $p < 1$. To beat Krylov subspace time evolutions methods, need less than 100,000 2q gates per time step and less than 100 logical qubits. Note that unlike SYK model where #terms $\sim N^4$, here #terms $\lesssim N^2$

Return probability

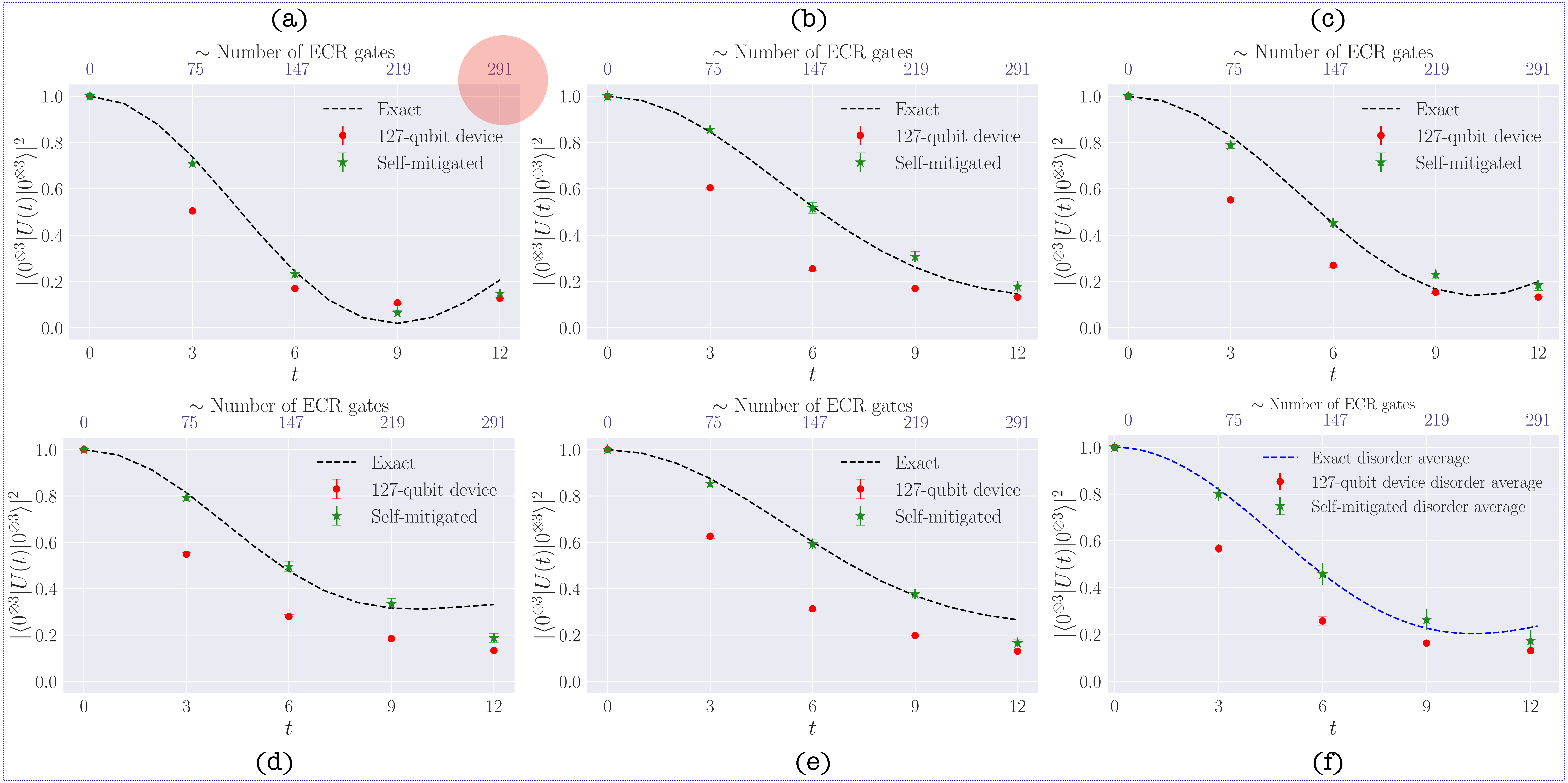
- A simple observable we can compute is the probability that we return to the same initial state after some evolution time t i.e., $\mathscr{P}_0 = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|^2$. For initial state, we take $|0\rangle^{\otimes N/2}$.

- For $N = 4$, we have a simple circuit of only two 2Q gates, so the entire circuit for return prob. is straightforward. For $N = 6$, there are 30 2q gates per step which we cannot show here.

$|00\rangle$



Return probability - IBM Quantum Results



Error Mitigation

- The results from the 127-qubit device (red) agrees slightly less than those with self-mitigation (green). The red points have been read from some fixed number of measurements/shots and post-processed with mild mitigation including M3 to correct read-out errors and DD to increase coherence time of qubits.

- To get closer to the exact results, we found that an idea similar to CNOT only mitigation (known as self-mitigation) seems to help drastically. Basic idea introduced in Urbanek et al. arXiv: 2103.08591

M3 is a matrix measurement mitigation (MMM or M3) technique that solves for corrected measurement probabilities using a dimensionality reduction step

DD (dynamical decoupling) — a series of strong fast pulses are applied on the system which on average increases the lifetime of qubits and delays decoherence (or effect of interactions with environment)

CNOT-only and self-mitigation

- If the input state is $|0\rangle^{\otimes n}$, then applying any of CNOT will still result in the same input state. However, in practice, the errors of 2q gates (CNOT) is the dominant source of gate error in current devices.

- This can be used to quantify the errors occurring in the time-evolution circuit. Remove all the single-qubit gates from $\exp(-iHt)$ and apply it on the $|0\rangle^{\otimes n}$ state. Measure the output. The deviation from $|0\rangle^{\otimes n}$ is a measure of the probability of error and used to correct the expectation value of the observable. This is CNOT-only mitigation.

- However, this underestimates the error. Self-mitigation argues to not remove any gates from $\exp(-iHt)$. One constructs two circuits: Physics (P) and Self-Mitigated (SM) circuits and then run the P circuits for N Trotter steps and the SM mitigation circuit for $N/2$ Trotter steps with dt and the other $N/2$ with -dt. Note the error from SM circuits, use it to correct exp. value of P circuits.

Noise model: Quantum depolarizing channel

- An efficient way to model decoherence of qubit is to use a depolarising quantum channel which is a CPTP (completely-positive trace preserving, (Tr $\mathscr{E}(\rho) =$ Tr $\rho = 1$ and $\mathscr{E}(\rho) > 0$) map:

$$\mathscr{E}(\rho) = (1 - p)\rho + p\mathbb{I}/2^n,$$

- If the quantum channel is free of noise, then the depolarising parameter (error rate) is p = 0.

- Once the error rate is determined from self-mitigation, we use it to correct the expectation value of the observable using $\langle O_n \rangle = (1 - p)\langle O_c \rangle + (p/2^n)\mathrm{Tr}(\mathbb{I})$ where n and c are noisy and corrected value.

SYK model - Bound on chaos

- SYK model famously saturated the Lyapunov exponent i.e., $\lambda = 2\pi T$ for $J/T \gg 1$ when N is large.

- One considers $C(t) = -\langle [W(t), V(0)]\,[W(t), V(0)] \rangle$ and the expansion of the commutator gives OTOC := $\langle W(t)V(0)W(t)V(0) \rangle_\beta =$ Tr$(\rho W(t)V(0)W(t)V(0))$ which characterizes quantum chaos.

- So the goal is to compute $\langle W(t)V(0)W(t)V(0) \rangle_\beta$ on a quantum computer. Thermal correlators are currently not easy to compute due to limited resources. One simplification we can make is consider the $\beta \to 0$ limit of OTOC. This is not at all interesting for holography, but this is where we must start. Hence, the density matrix is just $\rho \propto \mathbb{I}$. As $\beta \to \infty$, we have the ground state (zero temp.) but as β is decreased, the state becomes mixed and it becomes maximally mixed at $\beta \to 0$.

- The unusual time-ordering of OTOC is also hard for quantum computers which often mean carrying out forward and backward evolution. We use a protocol (next slide) which uses only forward evolution to compute OTOC on quantum hardware.

Randomised Protocol

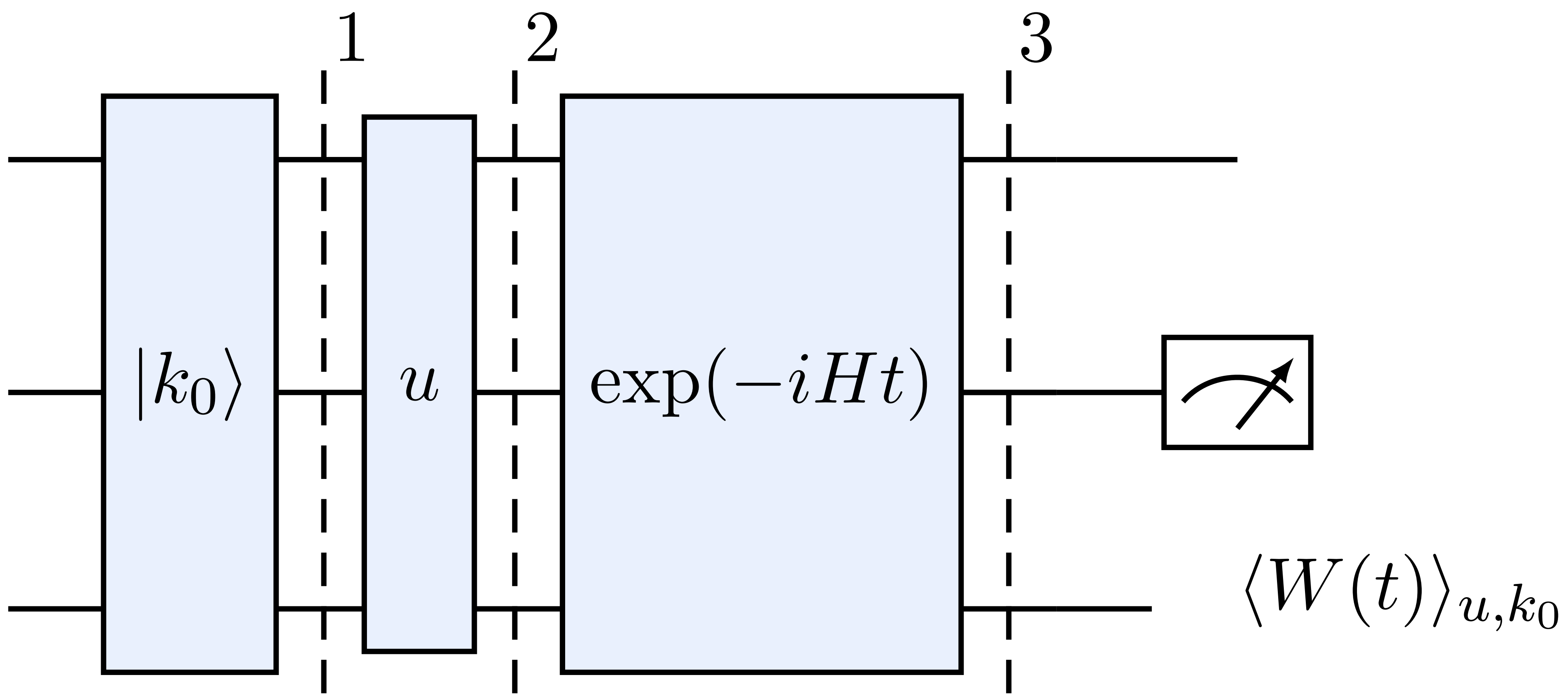
- There are various protocols to measure OTOC on quantum computers, see Swingle 2202.07060 for review.

- We use the one proposed in 1807.09087 now known as 'randomised protocol' since it computes OTOC through statistical correlations of observables measured on random states generated from a given matrix ensemble (CUE).

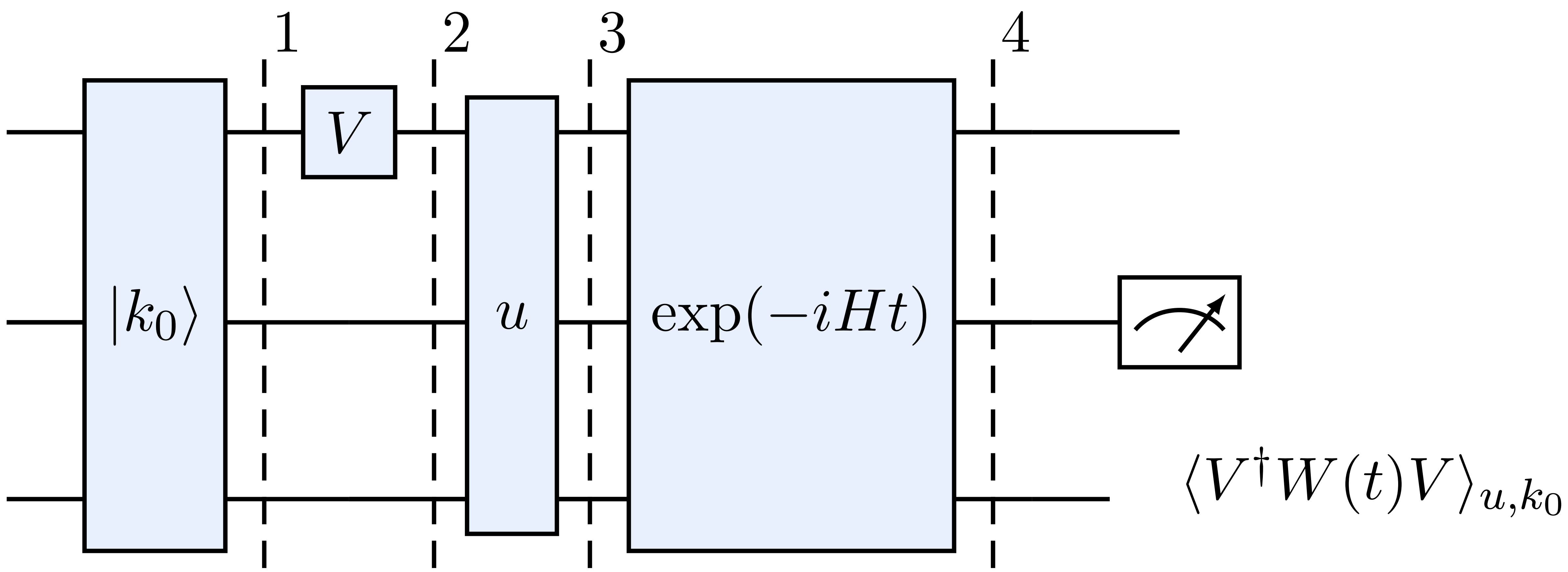
- Infinite-temp OTOC is given by Tr$(W(t)V^\dagger W(t)V) \propto \overline{\langle W(t)\rangle_u \langle V^\dagger W(t) V\rangle_u}$ where the average is over different random states $|\psi_u\rangle$ prepared by acting with random unitary on arbitrary state say $|0\rangle^{\otimes n}$. Note that this protocol works when W is traceless operator.

Randomised Protocol

- We need two measurements (between which we compute the statistical correlation) and it is shown below. This is the global version of the protocol (since u has support over all qubits). There is also a local version of the protocol. Note that cost of decomposing arbitrary u increases exponentially, one can instead use unitary from a subset of Haar measure. They are called unitary t-designs* in literature.



(a)



(b)

t-designs equivalent to first t moments of Haar group

Finite-temperature SYK model

- We considered OTOC measured over random states (maximally mixed) generated i.e., $\beta = 1/T = 0$. However, much of interesting Physics of the SYK happens in the region $\beta \gg 1$ and classical computations have argued that you need $\beta \sim 70$ to extract Lyapunov exponents close to the chaos bound.

- Finite-temperature OTOCs are difficult for quantum computers in general. No simple/general cost-effective protocol exists. To move towards this goal, we are studying the preparation of Gibbs (thermal) states on quantum computer for the SYK model.

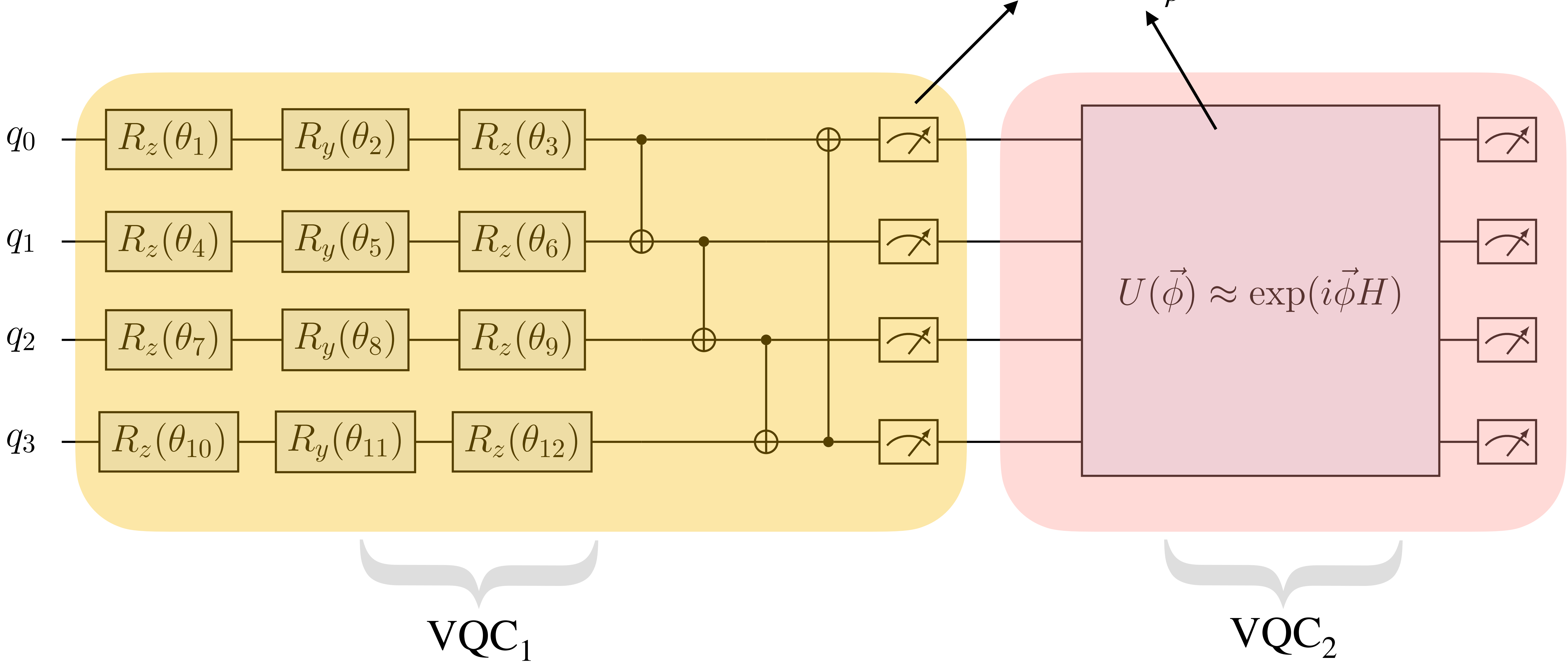
- In addition to the thermal state (mixed) of the SYK model, one can also consider a purification of this known as thermo-field double state (TFD). TFD state is a pure state (up to unitary trans.) of some other system (for ex: coupled SYK model) and when we perform partial trace over either system, we recover thermal state on the other one.

VQE for finite temperatures

arXiv: 2406.15545

- Finite-temperature VQE methods are still an active area of research. Many proposals exist. The cost function is no longer E but rather $E - TS$ (free energy) which can be hard to compute on QC.

$$\mathrm{F} = -\beta^{-1}\mathrm{S} + \langle \mathrm{H} \rangle_\beta$$



Please see paper for details

Selisko et al., 2208.07621

Thank you

Bosonic SYK anyone?

Term H_i	**Description**	**Synthesis of $e^{-iH_i t}$**	**Example Models**
$a_i^\dagger a_i$	Mode energy, Chemical potential	$R(\theta)$	Jaynes-Cummings [503], Bose-Hubbard [504]
$\hat{n}_i\,(\hat{n}_i - 1)$	On-site interactions	SNAP($\vec{\varphi}$)	Bose-Hubbard [504]
$e^{i\phi}\hat{a}_i^\dagger \hat{a}_j + \text{h.c.}$	Hopping	BS(θ, φ)	Bose-Hubbard [504], Su–Schrieffer–Heeger [505]
$Z_i(a + a^\dagger)$	Spin-dependent displacement	CD(α)	Rabi [506], Dicke [507], Hubbard-Holstein [508]
$Z_i Z_j(a + a^\dagger)$	Spin-spin-dependent displacement	CD(α), SWAP, CP (Sec. V C 1)	Photosynthetic complex [509]
$Z_i(e^{i\varphi} a_i^\dagger a_j + \text{h.c.})$	Spin-dependent hopping	CBS(θ, φ)	Bosonic $\mathbb{Z}_2$ LGT [510]
$a_i^\dagger a_j a_k$, $a_i^\dagger a_j^\dagger a_k a_l$, etc.	Higher-order terms	$R(\theta)$, $D(\alpha)$, TMS(r, φ), BS(θ, φ) + Approx.	Bosonic SYK [511, 512]
$S_i^z S_j^z$, $(\vec{S}_i \cdot \vec{S}_j)^k$, etc.	(Large)-spin interactions	$R(\theta)$, BS(θ, φ) + Approx.	AKLT [513]
$f_i^\dagger f_j$, $f_i^\dagger f_j^\dagger f_k f_l$, etc.	Fermionic matter	Multi-qubit gates (Sec. V C 2)	Electronic structure [514], Hubbard-Holstein [508] Fermi-Hubbard [515]