

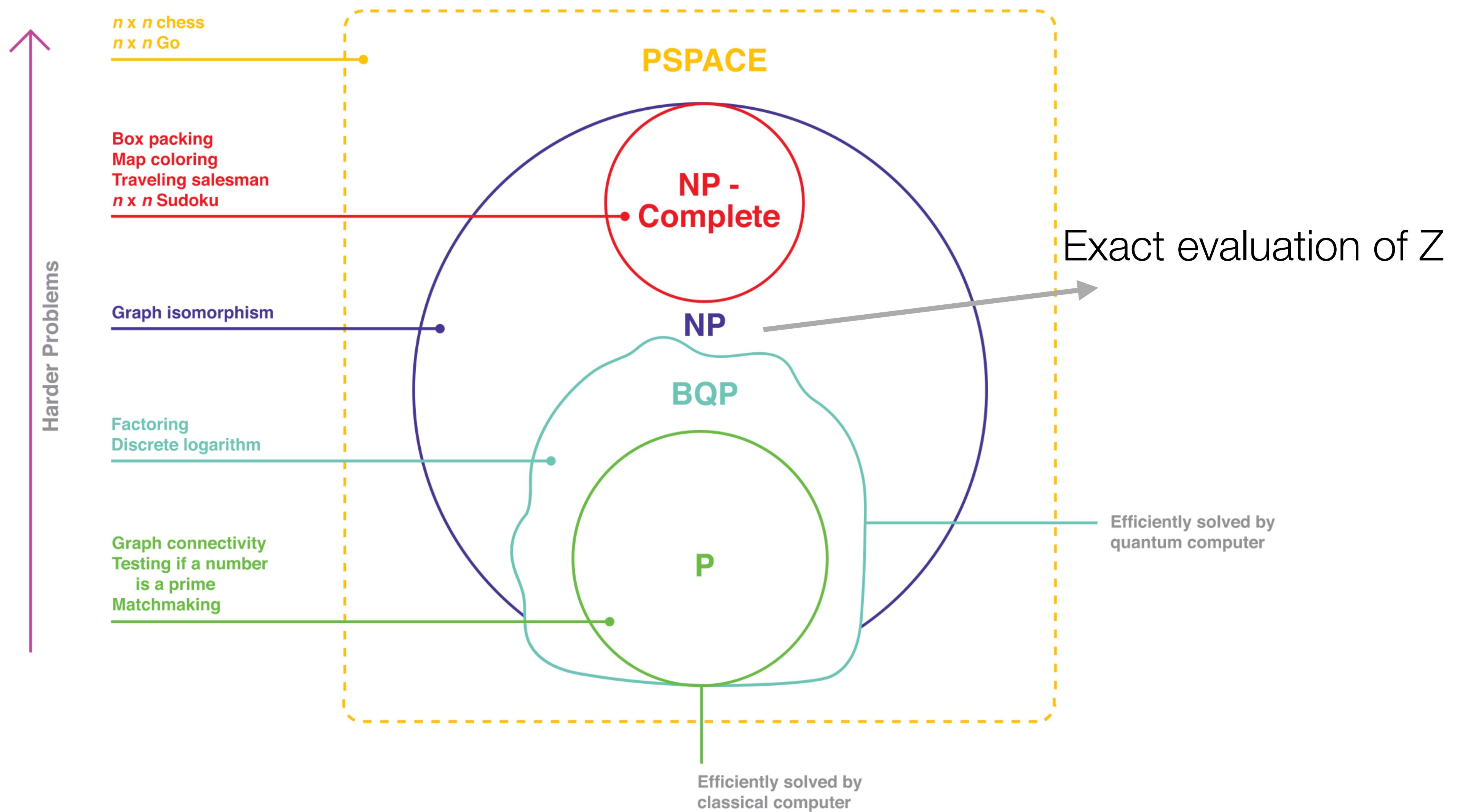
Approaches to universal quantum computing for spin and gauge models

Mostly based on work with S. Thompson, G. Siopsis, F. Ringer, Asad, B. Sambasivam
arXiv: 2308.06946, 2310.12512 [in press, Phys. Rev. A], 2311.17991 [in press, Phys. Rev. D]

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16 April 2024



Complexity



Can we touch the stars*?



The illustration features a scientist in a white lab coat standing on a blue sphere representing Earth. He is reaching up towards the top right corner. Below him, on the sphere, are colorful, swirling quantum particles. In front of the scientist is a vintage-style computer monitor displaying a green screen. The monitor sits atop a keyboard. To the left of the monitor is a yellow abacus. The background is a light yellow gradient.

THE LIMITS OF Quantum

By Scott Aaronson

Quantum computers would be exceptionally fast at a few specific tasks, but it appears that for most problems they would outclass today's computers only modestly. This realization may lead to a new fundamental physical principle

Humorous Physicists Develop 'Quantum Slacks,' " read a headline in the satirical weekly *The Onion*. By exploiting a bizarre "Schrödinger's Pants" duality, the article explained, these non-Newtonian pants could paradoxically behave like formal wear and casual wear at the same time. *Onion* writers were apparently spoofing the breathless articles about quantum computing that have filled the popular science press for a decade.

A common mistake—see for instance the February 15, 2007, issue of *The Economist*—is to claim that, in principle, quantum computers could rapidly solve a particularly difficult set of mathematical challenges called NP-complete problems, which even the best existing computers cannot solve quickly (so far as anyone knows). Quantum computers would supposedly achieve this feat not by being formal and casual at the same time but by having hardware capable of processing every possible answer simultaneously.

If we really could build a magic computer capable of solving an NP-complete problem in a snap, the world would be a very different place: we could ask our magic computer to look for whatever patterns might exist in stock-market data or in recordings of the weather or brain activity. Unlike with today's computers, finding these patterns would be completely routine and require no detailed understanding of the subject of the problem. The magic computer could also automate mathematical creativ-

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March 2008

Misconception: QC **can** solve all problems

- It turns out that for majority of problems, quantum computers would do no better than classical computers. A major research direction is to understand which problems can be solved efficiently by QCs.
- For example, we know that scattering in ϕ^4 in 1+1-dimensions can be solved efficiently by quantum computers.
- Class of problems which are best suited for quantum advantage belong to complexity class BQP. For ex: Shor's algorithm. Also Grover's algorithm but not as nice as Shor's (only polynomial speed-up).

Rough outline of this talk

- Approaches to universal quantum computing: Discrete and Continuous
- Some background on continuous variable quantum computing
- Application to Bose-Hubbard model and O(3) model
- Discrete variable approach to fermionic non-local SYK model - Complexity and Hardware results

Qubits and quantum modes (qumodes)

- In discrete approach, computation uses two-state quantum system which we refer to as qubit. However, this is not the only option as was realised in late 1990s. One can instead use the quantum modes of oscillator to encode information. This approach is called CV (continuous-variable) quantum computing and uses qumodes.
- We will give brief introduction to the CV states and CV gates similar to computational basis states or $|+\rangle$, $|-\rangle$ for qubits and unitary CV gates like qubit gates such as Hadamard and Pauli gates.

Comparison

		CV	Qubit
Basic element	Qumodes	Qubits	
Relevant operators	Quadrature operators \hat{x}, \hat{p} Mode operators \hat{a}, \hat{a}^\dagger		Pauli operators $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$
Common states	Coherent states $ \alpha\rangle$ Squeezed states $ z\rangle$ Number states $ n\rangle$		Pauli eigenstates $ 0/1\rangle, \pm\rangle, \pm i\rangle$
Common gates	Rotation, Displacement, Squeezing, Beamsplitter, Cubic Phase		Phase Shift, Hadamard, CNOT, T Gate

Fundamental difference between fermions and bosons

- Encoding bosonic degrees of freedom in qubits is possible but is not straightforward. Huge challenge. This is very different from classical computing where fermions are harder.
- The use of qumodes or CV d.o.f makes this natural. So, for scalar field theories, O(3) sigma model and other cases, it seems to be more useful.

Harmonic oscillator - Quick recap

- Hamiltonian is given by: $\hat{H} = \frac{\hat{p}^2}{2} + \frac{\hat{x}^2}{2}$
- Can define operators such as: $\hat{a} = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{p})$
- $\hat{x} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger), \quad \hat{p} = i\frac{1}{\sqrt{2}}(\hat{a} - \hat{a}^\dagger)$.
- $\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$ and $\hat{a} |n\rangle = \sqrt{n} |n-1\rangle$.

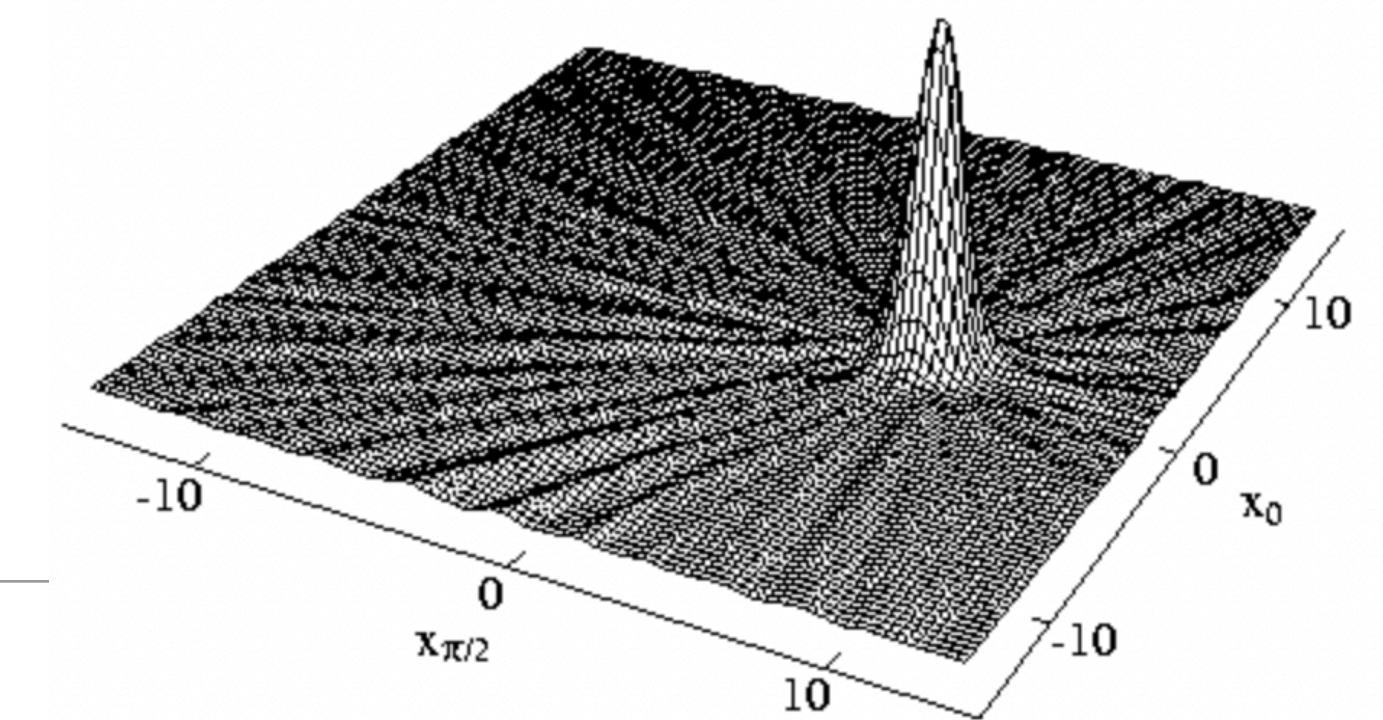
Example: CV gates and states

- Displacement operator (gate): $D(\alpha) = \exp\left(\alpha\hat{a}^\dagger - \bar{\alpha}\hat{a}\right)$. The set of all displacement operators form a Heisenberg-Weyl group similar to Pauli group for qubits.
- Squeezing operator: $S = \exp\left(\frac{1}{2}\left(z^*\hat{a}^2 - z\hat{a}^{\dagger 2}\right)\right)$
- Rotation operator: $R(\phi) = \exp\left(i\phi\hat{a}^\dagger\hat{a}\right)$
- Coherent state: Eigenstate of annihilation operator. States of photon whose exp. val corresponds to classical EM wave $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

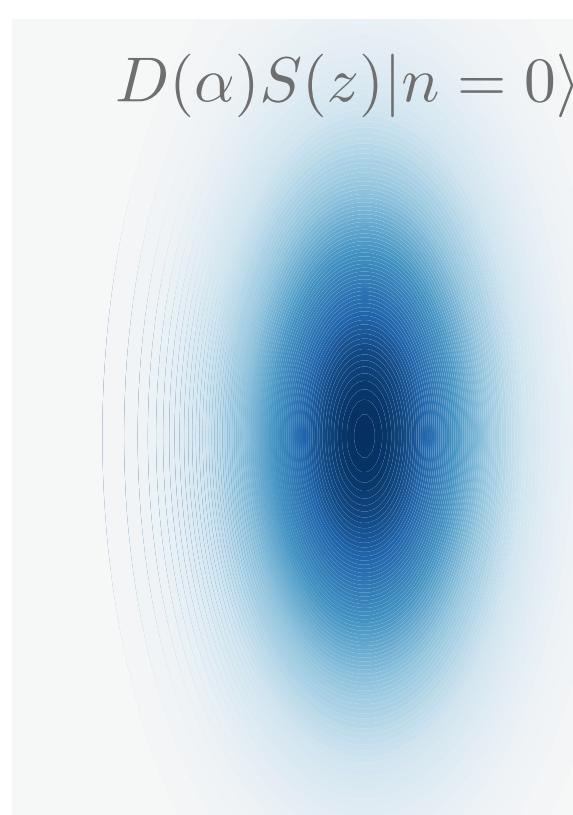
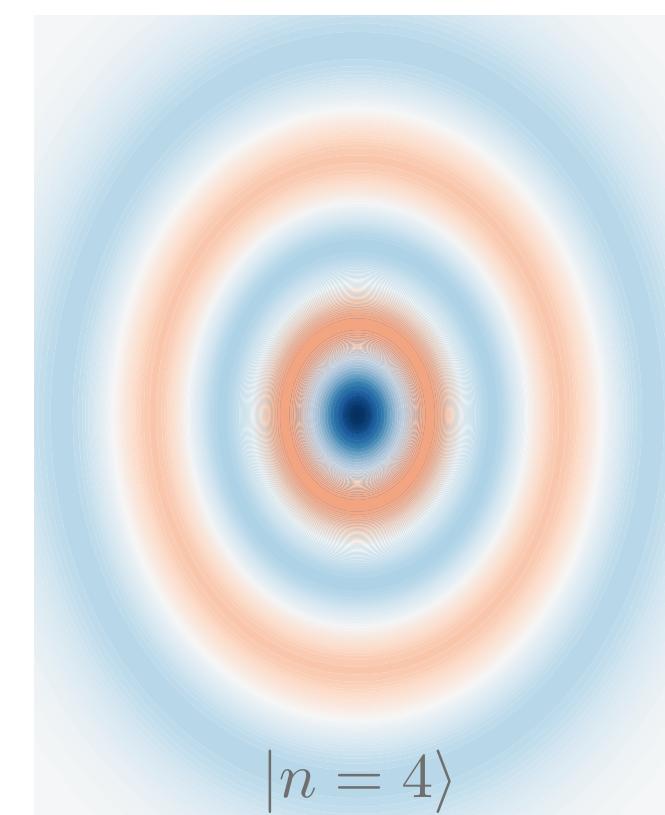
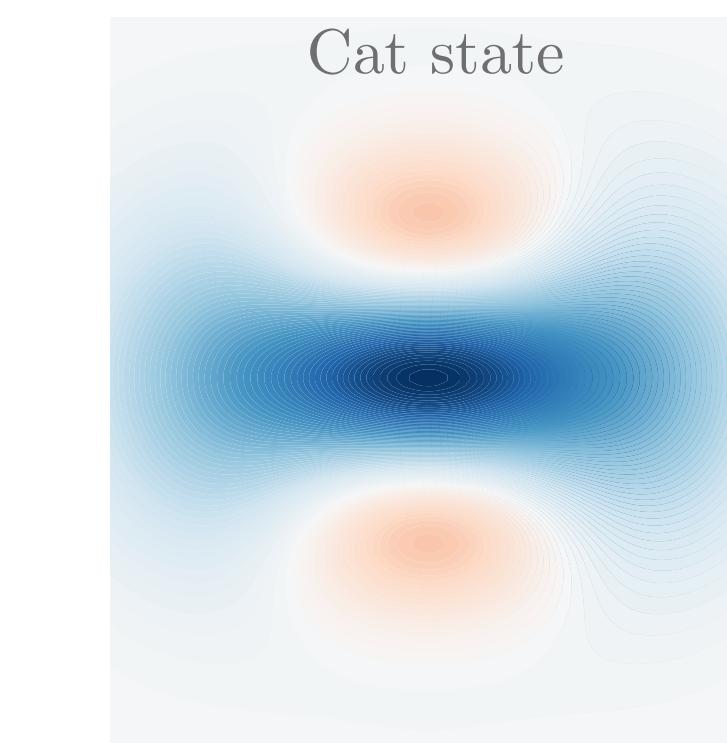
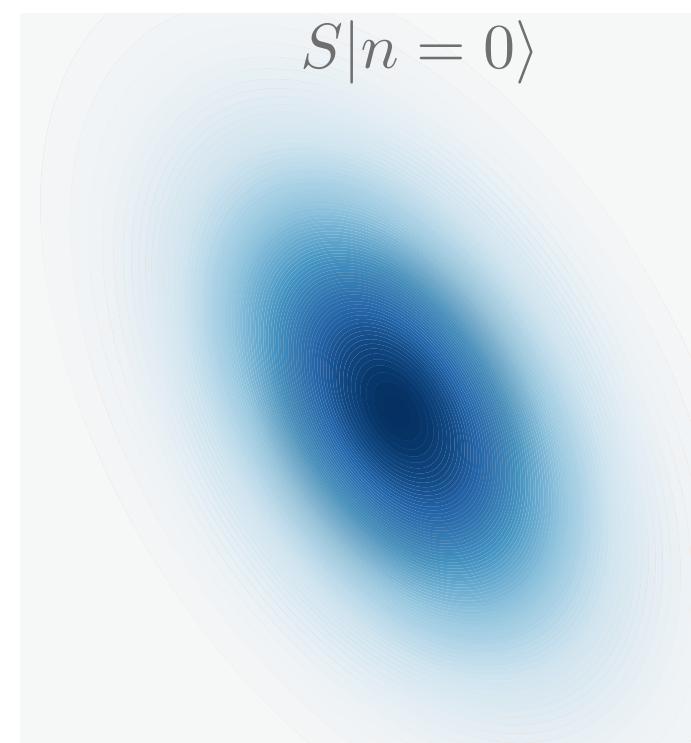
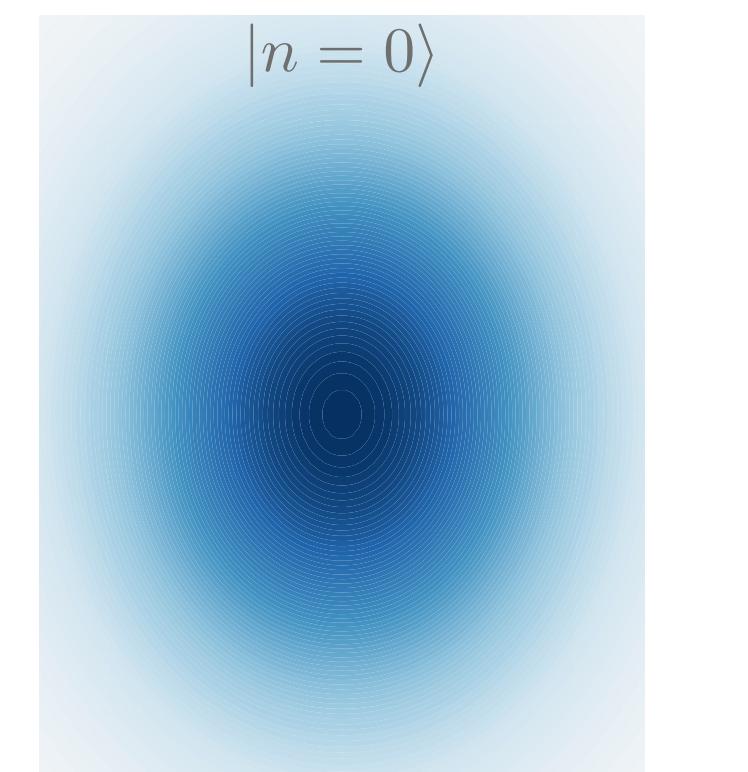
Universal CV gate set

- Controlled gates: $CX_{ab}(s) = \exp(-isq_a p_b)$ and $CZ_{ab}(s) = \exp(isq_a q_b)$ related by Fourier gate.
- Fourier gate: $F = R(\pi/2) = \exp(i(\pi/2)a^\dagger a)$ and $CZ_{ab} = F_b^\dagger \cdot CX_{ab} \cdot F_b$
- Beam-splitter gate: $BS_{ij}(\theta, \phi) = \exp(\theta(e^{i\phi}a_i a_j^\dagger - e^{-i\phi}a_i^\dagger a_j))$
- Kerr-gate (non-Gaussian): $K(\kappa) = e^{i\kappa \hat{n}^2}$
- Universal set: {D,S,F,BS} + *non-Gaussian gate!*

Wigner (quasi-probability) functions



- Most natural way to visualise these states etc. (like say Bloch sphere for qubit approach) is to use Wigner functions.



arXiv: 2301.09679
Made using [qutip](#)

Gottesmann-Knill theorem

- Any quantum circuit consisting of H, S, and CNOT can be simulated classically (in polynomial time). However, if we just add the T -gate, it becomes universal gate set (quantum computing) hence not always solvable in poly. time
- There is an equivalent statement in CV language too: Need to add non-Gaussian gate to achieve universal computing. Achieved either by cubic phase gate, or photon measurement (detection). Just having Gaussian gates or states are not enough.

Bose-Hubbard model

- Consider the Bose-Hubbard model where the H is given by:

$$H = J \sum_{\langle ij \rangle} a_i^\dagger a_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

where we have used create /annihilation operators and the number operators. The first term denotes the hopping of bosons between neighbouring sites and second term is the on-site potential term.

Two-site Bose-Hubbard model

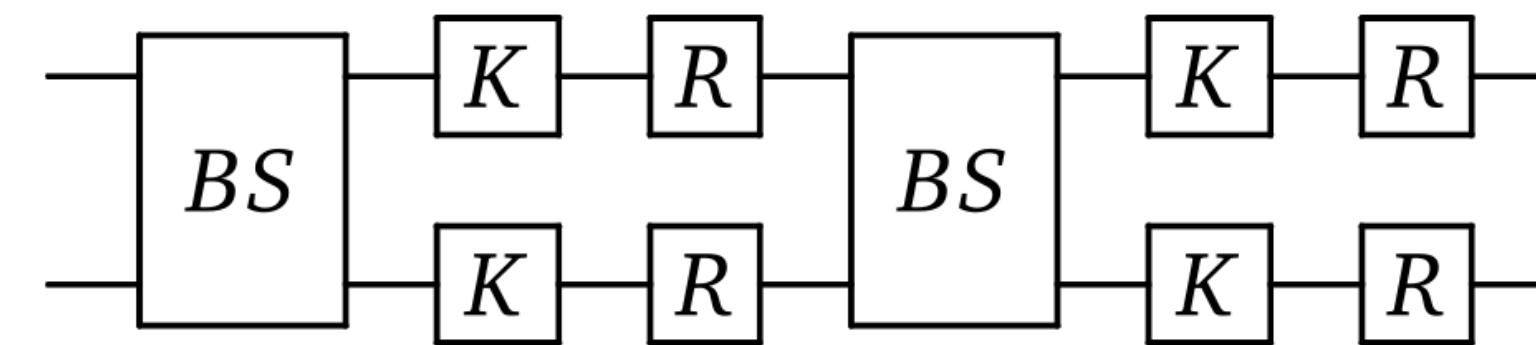
- We can write the time-evolution operator as:

$$e^{iHt} = \left[BS(\theta, \phi) (K(r)R(-r) \otimes K(r)R(-r)) \right]^N + \mathcal{O}(t^2/N) \quad ; \theta = -Jt/N, \phi = \pi/2, r = -Ut/2N$$

where BS is the beam-splitter gate, K is the Kerr gate (non-Gaussian), and R is the rotation gate. These gates are qumodes equivalent of the qubit gates we saw before. For example, $K(\kappa) = \exp(i\kappa\hat{n}^2)$. Constructing these gates are major undertaking in quantum photonics labs where the photon is modelled as an oscillator.

Trotterized evolution

- One Trotter step needs 1 BS, 2 Kerr, and 2 rotations gates. Shown below is two Trotter steps as an example.



$$H = \underbrace{J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)}_{\text{hopping}} + \frac{U}{2} (\hat{n}_1^2 - \hat{n}_1 + \hat{n}_2^2 - \hat{n}_2)$$

Use Lie-Product formula:

$$e^{A+B} = \lim_{N \rightarrow \infty} \left(e^{\frac{A}{N}} e^{\frac{B}{N}} \right)^N$$

We can write

$$e^{-iHt} = \underbrace{e^{-\frac{-iJt}{K}(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)}}_{\text{BS}_{12}} \underbrace{e^{-\frac{-iUt}{2K}\hat{n}_1^2}}_{\substack{K_1 \\ R_1, R_2}} \underbrace{e^{-(-)-(-)-(-)}}_{\substack{K_2 \\ R_1, R_2}} + \mathcal{O}(t^2/K)$$

O(3) model

$$\widehat{H} = \frac{1}{2\beta} \sum_{j=1}^N \mathsf{L}_j^2 - \beta \sum_{\langle jk \rangle} \mathbf{n}_j \cdot \mathbf{n}_k,$$

Hamer-Kogut-Susskind, 1978

- In the Bose-Hubbard example we considered, we had one set of creation and annihilation operator i.e., a, a^\dagger at each lattice site.
- Consider that there are two sets of Bose operators at each site. Let us denote them by a, b respectively. In this case, we then have $\hat{n} = a^\dagger a + b^\dagger b$. It turns out that we can define operators at each site such as:

$$K_+ := a^\dagger b^\dagger, K_- := ba, K_3 := \frac{1}{2}(\hat{n} + \mathbb{I})$$

- These operators form representation theory of $\mathfrak{su}(1,1)$ algebra. So, the O(3) model can be beautifully be written in terms of two oscillators.

O(3) model

- We have to express the rotor Hamiltonian in terms of oscillators. This can be done using work due to [Schwinger 1952]. It turns out we need two modes per site (i.e., two oscillators). Note that for Bose-Hubbard, we needed just one per site!
- We make use of the relation:

$$|l, m\rangle = \frac{(a^\dagger)^{l+m}(b^\dagger)^{l-m}}{\sqrt{(l+m)!(l-m)!}} |0,0\rangle$$

- The kinetic term at each site becomes:

$$\mathcal{L}_i^2 = \frac{n_i}{2} \left(\frac{n_i}{2} + 1 \right)$$

where $n_i = n_a + n_b$ with **a** and **b** being two oscillators at each site.

- The interaction term needs more work but it can also be written in terms of $a^\dagger, a, b^\dagger, b$. I will spare the details.

O(3) model

- The number operator at each site i.e., $n_i = n_a + n_b$ is related to the truncation over the angular momentum states in the rotor Hamiltonian as $n = 2l_{\max}$. at each site.
- The correct continuum limit is usually interpreted by computing the mass gap and observing its scaling with β . Tensor network computations using MPS methods [[Bruckmann, 2018](#)] have shown that $\beta \sim 1.3$ with $l_{\max} \sim 4,5$ can reproduce the continuum Physics reliably well.
- This is good because we do not need to consider very high photon number states and $|n = 2 \times 5 = 10\rangle$ should suffice.
- The current state of the art methods in photonics quantum experiments have create Fock states up to $|n = 15\rangle$ and is within limit of resources needed. However, the total number of modes (which depends on number of sites in O(3) model) would be a challenge. Implementation of time-evolution of this formulation is work in progress. State preparation in terms of techniques from quantum chemistry/nuclear physics.

$O(3)$ model

- In arXiv: 2308.06946 we give the Hamiltonian in terms of two oscillators. We also studied another formulation in terms of triplet of scalar fields with the constraint implement by making the potential deep See [2310.12512](#) for details.
- We constructed the ground state using the coupled-cluster ansatz (non-unitary) which is inspired by Hartee-Fock ansatz in quantum chemistry. In 1990s, it was shown to work well for sigma models in any dimensions.

The screenshot shows a detailed view of an arXiv paper page. At the top, there's a red header bar with the arXiv logo, the category 'quant-ph', and the identifier 'arXiv:2310.12512'. To the right of the header are search fields and links for 'Help | Advanced Search' and 'Search'.

The main content area has a white background. It starts with a section titled 'Quantum Physics' and a note 'Submitted on 19 Oct 2023'. Below this is the title 'Continuous variable quantum computation of the $O(3)$ model in 1+1 dimensions'. The authors listed are Raghav G. Jha, Felix Ringer, George Siopsis, and Shane Thompson.

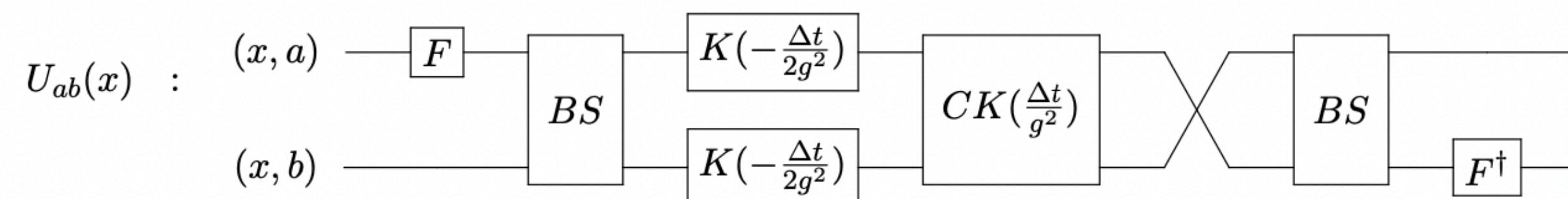
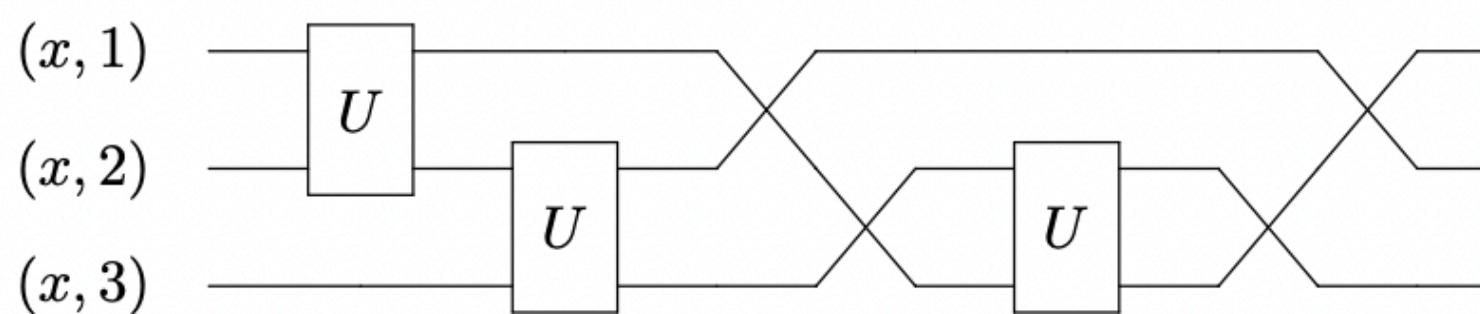
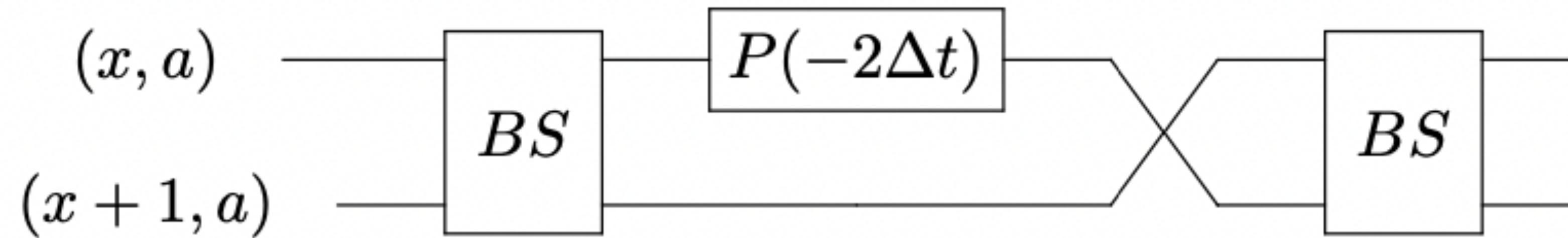
The abstract begins with: 'We formulate the $O(3)$ non-linear sigma model in 1+1 dimensions as a limit of a three-component scalar field theory restricted to the unit sphere in the large squeezing limit. This allows us to describe the model in terms of the continuous variable (CV) approach to quantum computing. We construct the ground state and excited states using the coupled-cluster Ansatz and find excellent agreement with the exact diagonalization results for a small number of lattice sites. We then present the simulation protocol for the time evolution of the model using CV gates and obtain numerical results using a photonic quantum simulator. We expect that the methods developed in this work will be useful for exploring interesting dynamics for a wide class of sigma models and gauge theories, as well as for simulating scattering events on quantum hardware in the coming decades.'

Below the abstract, there are sections for 'Comments', 'Subjects', 'Cite as', and a 'Submission history' link. The 'Comments' section notes '28 pages, 16 figures'. The 'Subjects' section lists 'Quantum Physics (quant-ph); High Energy Physics – Lattice (hep-lat)'. The 'Cite as' section provides the arXiv ID 'arXiv:2310.12512 [quant-ph]' and a link to the DOI 'https://doi.org/10.48550/arXiv.2310.12512'.

To the right of the main content, there's a sidebar with sections for 'Access Paper:', 'References & Citations', and 'Bookmark'. The 'Access Paper:' section includes links for 'Download PDF', 'Other Formats', and 'view license'. The 'References & Citations' section lists links to INSPIRE HEP, NASA ADS, Google Scholar, and Semantic Scholar. The 'Bookmark' section features icons for various social media and bookmarking services.

O(3) model

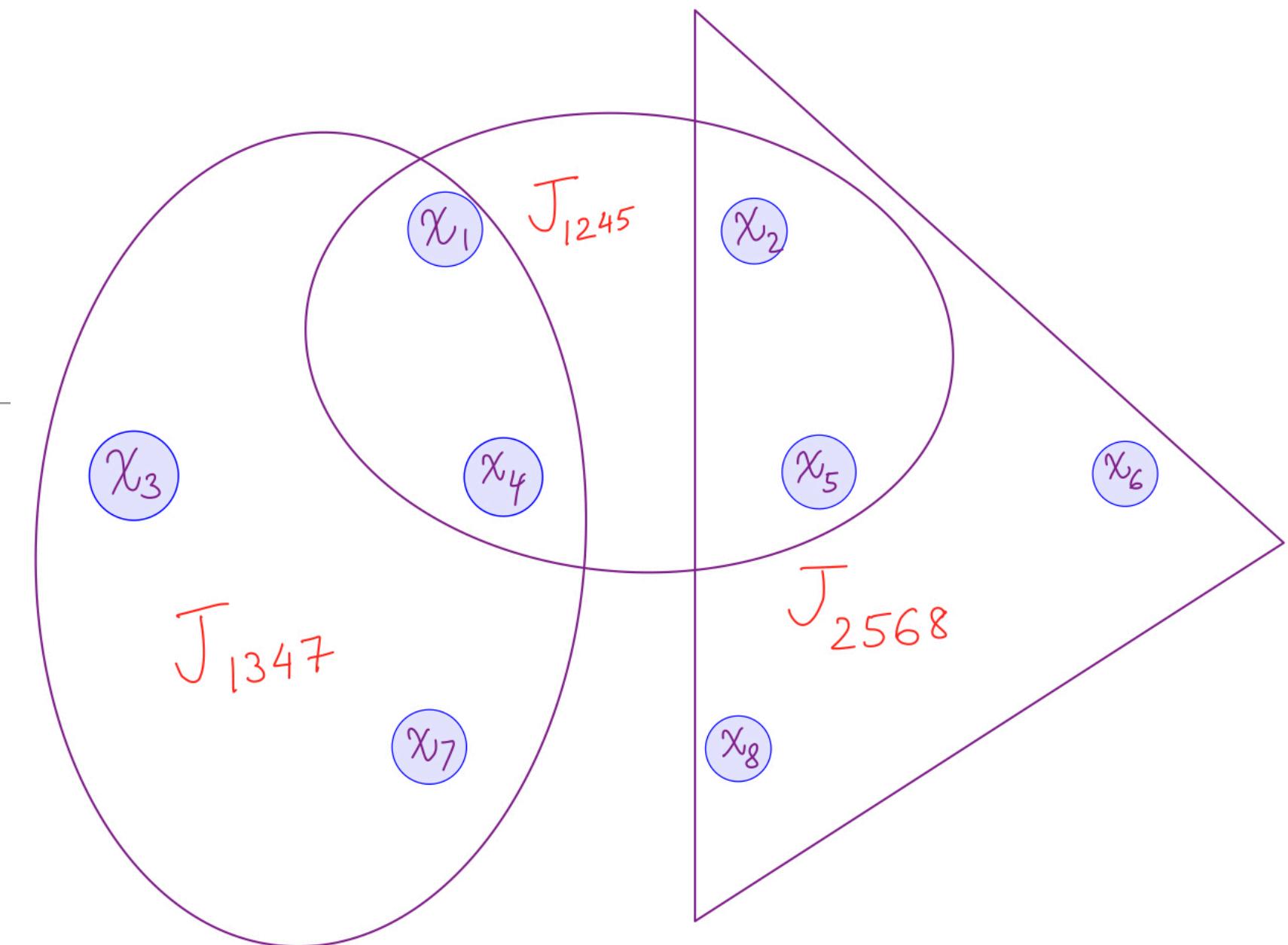
- Time-evolution circuits for kinetic (on-site) and interaction term was given in terms of CV gates.



Questions?

SYK model

$$H = \frac{(i)^{q/2}}{q!} \sum_{i,j,k,\dots,q=1}^N J_{ijk\dots q} \chi_i \chi_j \chi_k \dots \chi_q,$$



3 out of 70 terms in H

- Model of N Majorana fermions with q -interaction terms with random coupling taken from a Gaussian distribution with $\overline{J_{\dots}} = 0$, $\overline{J_{\dots}^2} = \frac{q!J^2}{N^{q-1}}$.
- The fermions χ satisfy, $\chi_i \chi_j + \chi_j \chi_i = \delta_{ij}$. We will set $J = 1$. Note that it has units of energy and inverse time.
- In the limit of large number of fermions with $N \gg \beta J \gg 1$, the model has several interesting features such as maximal Lyapunov exponent.

Mapping fermions to qubits

$$\chi_{2k-1} = \frac{1}{\sqrt{2}} \left(\prod_{j=1}^{k-1} Z_j \right) X_k \mathbb{I}^{\otimes(N-2k)/2} \quad , \quad \chi_{2k} = \frac{1}{\sqrt{2}} \left(\prod_{j=1}^{k-1} Z_j \right) Y_k \mathbb{I}^{\otimes(N-2k)/2}$$

- N Majorana fermions requires N/2 qubits. We use the standard Jordan-Wigner mapping to write χ in terms of Pauli matrices X, Y, Z, and Identity.
- The SYK Hamiltonian is then written as sum of Pauli strings. The number of strings is $\binom{N}{q}$ and grows like $\sim N^q$. Simplest non-trivial case for is $N = q$ with one term. We restrict to $q = 4$.

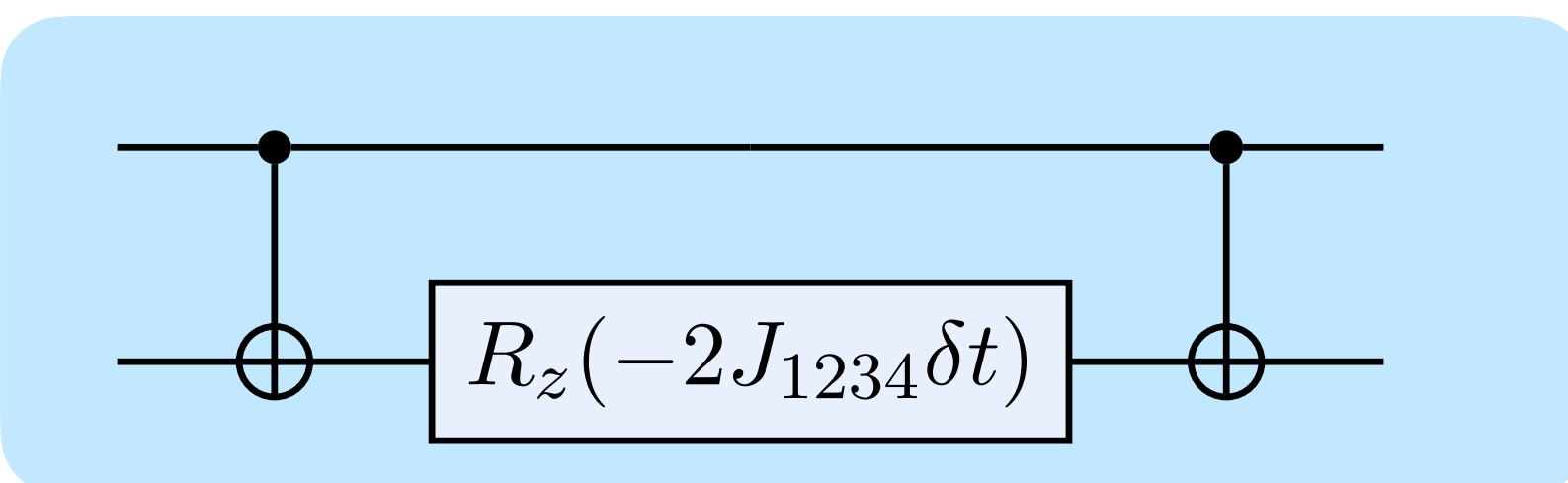
Simplest case: N=4

$$H = J_{1234} \chi_1 \chi_2 \chi_3 \chi_4$$

$$\chi_1 = X\mathbb{I}, \chi_2 = Y\mathbb{I}, \chi_3 = ZX, \chi_4 = ZY$$

$$H = J_{1234} (X\mathbb{I}) \cdot (Y\mathbb{I}) \cdot (ZX) \cdot (ZY) = -J_{1234} ZZ$$

- The goal of quantum computation is to construct a unitary operator corresponding to this Hamiltonian. So, for this case we have $\exp(-iHt) = \exp(iJ_{1234}ZZt)$.
- This circuit is simple to construct and just needs 2 CNOTs and 1 rotation gate.



Circuit complexity

Definition: How many 2q-gates do we need to simulate the SYK model?

- Different approaches can be used to do the Hamiltonian simulation (aka time evolution). A popular method is Trotter method. It is based on Lie-Suzuki-Trotter product formula* (writing $H = \sum_{j=1}^m H_j$, $m \sim N^4$)

$$e^{-iHt} = \left(\prod_{j=1}^m e^{-iH_j t/r} \right)^r + \mathcal{O}\left(\sum_{j < k} \left\| [H_j, H_k] \right\| \frac{t^2}{r} \right),$$

- Depending on what error (ϵ) we desire in the time-evolution from the second term, we can compute the number of slices (r) we need to take. So, the complexity reduces to finding number of 2q-gates for each Trotter step. Recall that $N = 4$ needed just 2 2q-gates for each Trotter step.

* Corollary of Zassenhaus formula i.e., $\exp(t(X+Y)) = \exp(tX) \exp(tY) + O(t^2)$ (also known as dual of BCH formula).

Old work(s)

$$\mathcal{C} = \mathcal{O}(N^{10}t^2/\epsilon)$$

L. García-Álvarez et al., [PRL 119, 040501 \(2017\)](#)

$$\mathcal{C} = \mathcal{O}(N^8t^2/\epsilon)$$

Susskind, Swingle et al., [arXiv: 2008.02303 \(2020\)](#)

$$\mathcal{C} = \tilde{\mathcal{O}}(N^{7/2}t)$$

Babbush et al., [Phys. Rev. A 99, 040301 \(2019\)](#)

- The last one clearly is the most efficient, however, in the noisy-era implementing this is not feasible. It requires fault-tolerant quantum resources + ancillas since it is based on the basic idea of embedding H in a bigger vector space.
- Using the Trotter methods, the best seems to be $\sim N^8$. We improved the complexity to $\mathcal{C} = \mathcal{O}(N^5t^2/\epsilon)$.

Commuting terms

The costs can be simplified if we are little careful in splitting the SYK Hamiltonian.

The number of terms grows like $\sim N^4$, however, a large fraction of them commute with one another and can be collected together and then time-evolved more efficiently. We can find diagonalising circuit for each cluster and then apply time-evolution operator.

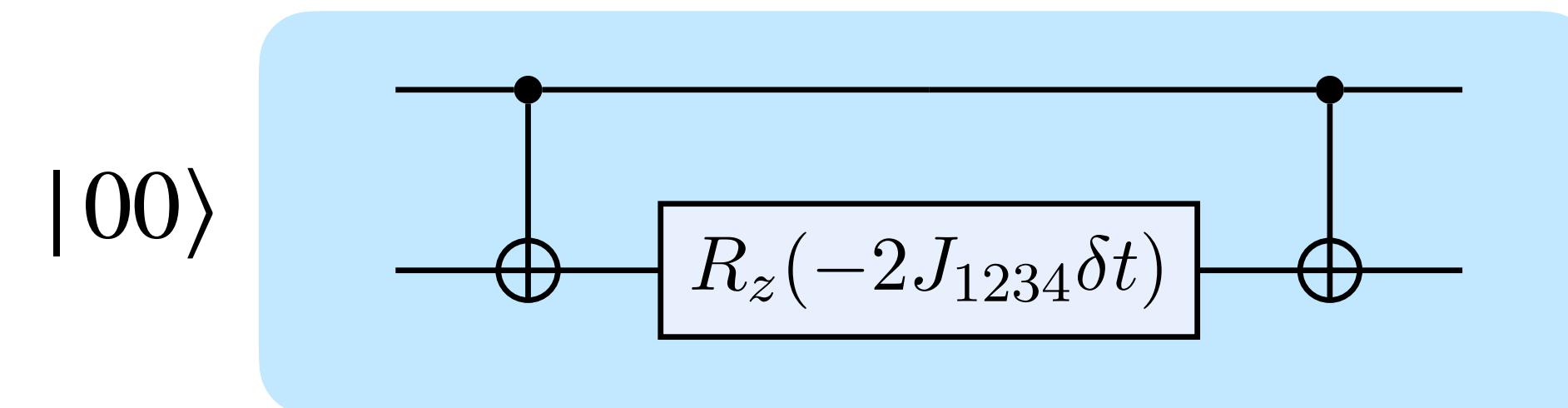
Finding optimal number of such clusters is a well-studied computer science problem. This is in general a NP-hard problem but various approx. algorithms exits.

Estimate based on general commutivity

N	Pauli strings	Clusters	Two-qubit gates
4	1	1	2
6	15	5	30
8	70	6	110
10	210	23	498
12	495	57	1504
14	1001	92	3560
16	1820	116	6812
18	3060	175	11962
20	4845	246	19984

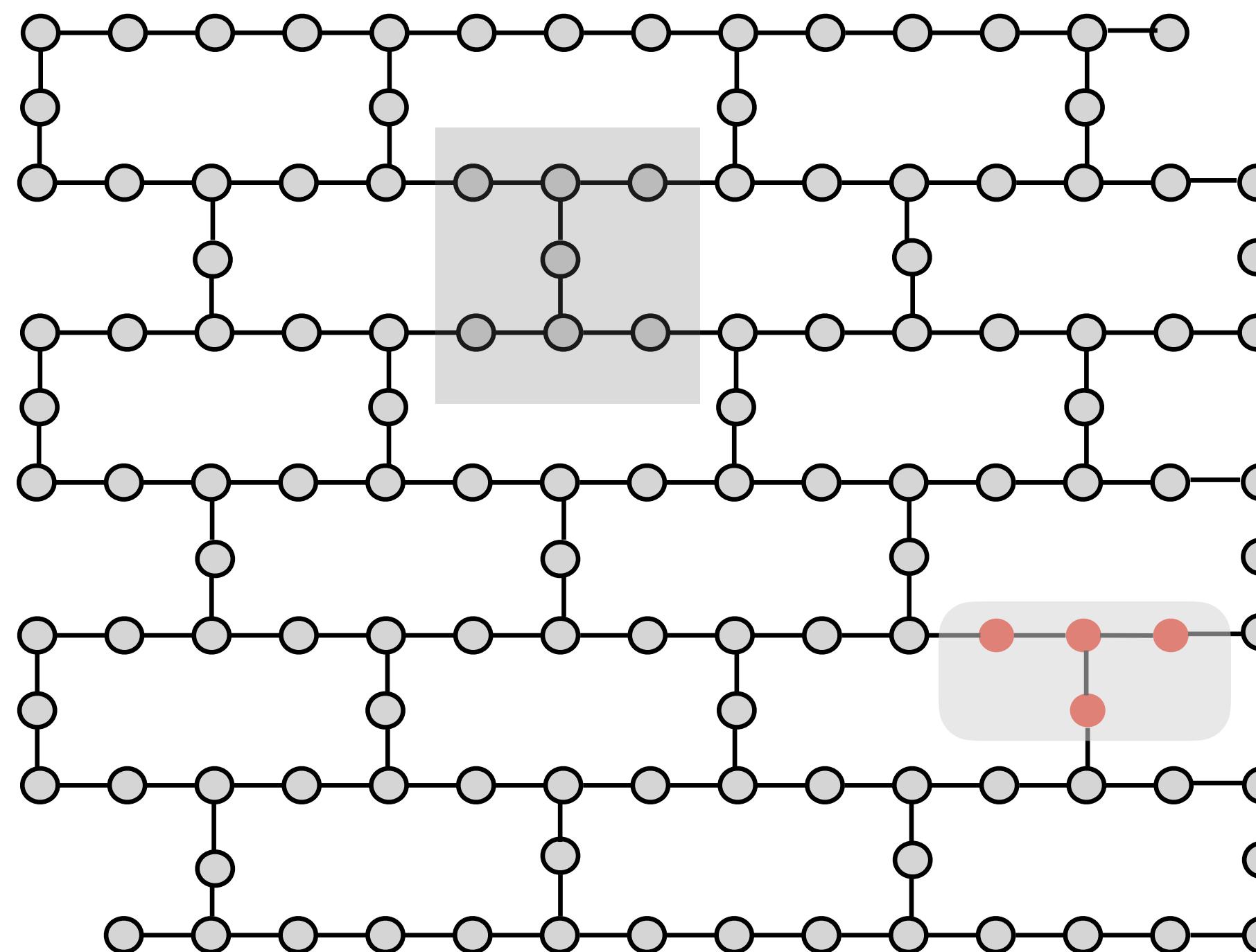
Return probability

- A simple observable we can compute is the probability that we return to the same initial state after some evolution time t i.e., $\mathcal{P}_0 = |\langle \psi_0 | e^{-iHt} | \psi_0 \rangle|^2$. For initial state, we take $|0\rangle^{\otimes N/2}$.
- For approximating the unitary, we use the first-order product formula and construct the corresponding quantum circuit.
- For $N = 4$, we have a simple circuit of only two 2Q gates, so the entire circuit for return prob. is straightforward. For $N = 6$, there are 30 2Q gates per step.

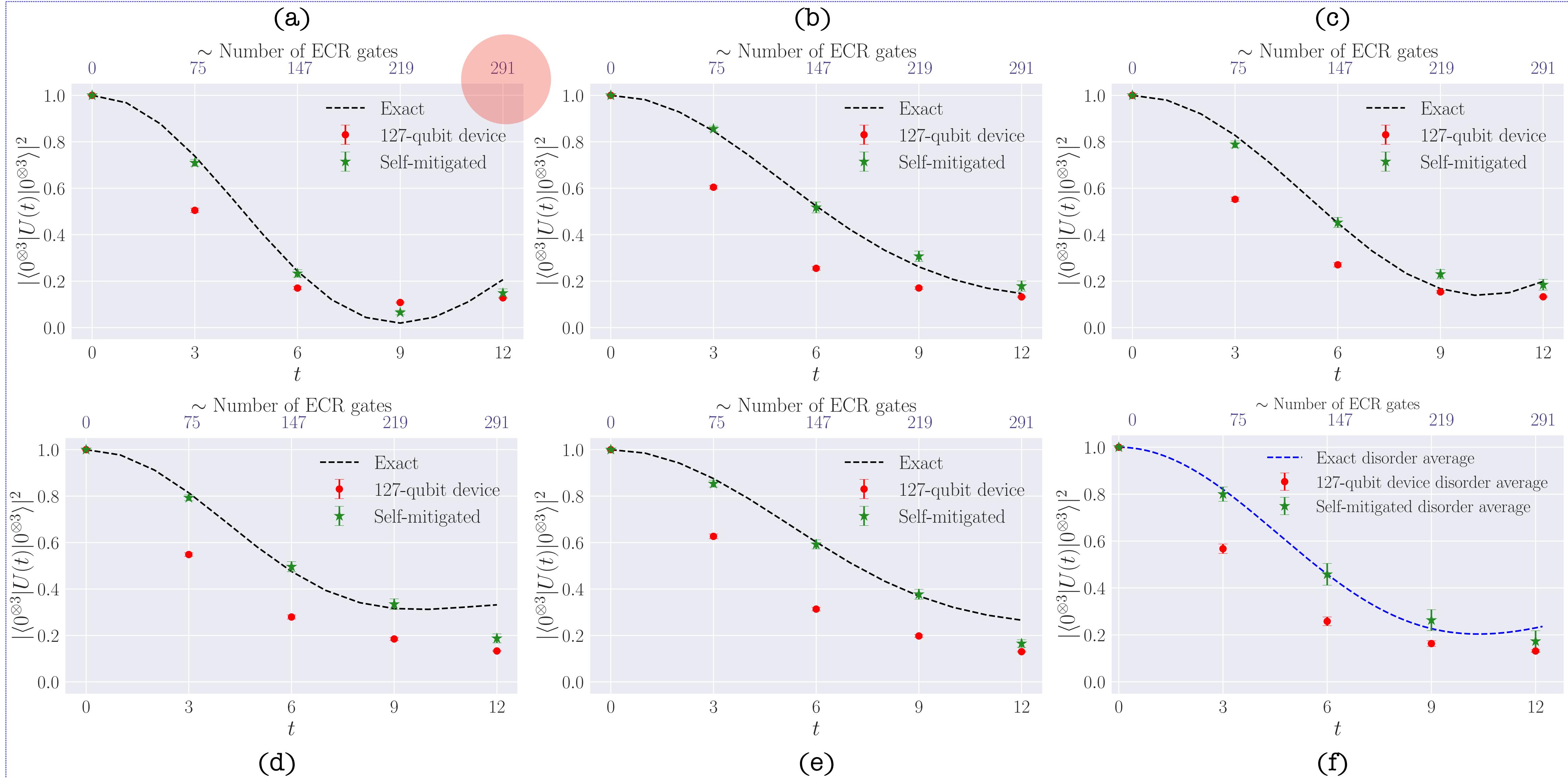


IBM chip topology

- We used the quantum computers available through IBM to simulate the SYK model. The topology of the processor is shown below. In practice, we need more gates than necessary. For example, we show a combination of qubits we used for $N = 8$. This chip topology is ‘heavy-hex’.



Return probability - IBM Results



CNOT-only and self-mitigation

- We saw previously that if the input state is $|0\rangle^{\otimes n}$, then applying any of CNOT will still result in the same input state. However, in practice, the errors of 2q gates (CNOT) is the dominant source of gate error in current devices.
- This can be used to quantify the errors occurring in the time-evolution circuit. Remove all the single-qubit gates from $\exp(-iHt)$ and apply it on the $|0\rangle^{\otimes n}$ state. Measure the output. The deviation from $|0\rangle^{\otimes n}$ is a measure of the probability of error and used to correct the expectation value of the observable. This is CNOT-only mitigation.
- However, this underestimates the error. Self-mitigation argues to not remove any gates from $\exp(-iHt)$. One constructs two circuits: Physics (P) and Self-Mitigated (SM) circuits and then run the P circuits for N Trotter steps and the SM mitigation circuit for $N/2$ Trotter steps with dt and the other $N/2$ with $-dt$. Note the error from SM circuits, use it to correct exp. value of P circuits.

Noise model: Quantum depolarizing channel

- An efficient way to model decoherence of qubit is to use a depolarising quantum channel which is a CPTP (completely-positive trace preserving, $\text{Tr } \mathcal{E}(\rho) = \text{Tr } \rho = 1$ and $\mathcal{E}(\rho) > 0$) map:

$$\mathcal{E}(\rho) = (1 - p)\rho + p\mathbb{I}/2^n,$$

- If the quantum channel is free of noise, then the depolarising parameter (error rate) is $p = 0$.
- Once the error rate is determined from self-mitigation, we use it to correct the expectation value of the observable using $\langle O_n \rangle = (1 - p)\langle O_c \rangle + (p/2^n)\text{Tr}(\mathbb{I})$ where n and c are noisy and corrected value.

SYK model - Bound on chaos

- SYK model famously saturated the Lyapunov exponent i.e., $\lambda = 2\pi T$ for $J/T \gg 1$ when N is large.
- One considers $C(t) = -\langle [W(t), V(0)] [W(t), V(0)] \rangle$ and the expansion of the commutator gives OTOC := $\langle W(t)V(0)W(t)V(0) \rangle_\beta = \text{Tr}(\rho W(t)V(0)W(t)V(0))$ which characterizes quantum chaos.
- Suppose one starts at $t = 0$, and computes also the two-pt correlator given by $\langle W(t)W(0) \rangle$, the time scales at which the lower order correlators decay is called ‘dissipation time’. After this time, the OTOC grows as $\exp(\lambda t)$ and saturates beyond t_\star known as scrambling time. Black holes are fastest scramblers!
- These correlators have been computed up to $N = 60$ numerically i.e., H has ~million terms and matrix has size ~billion. Hard for classical computers.

Out-of-time correlators (OTOC)

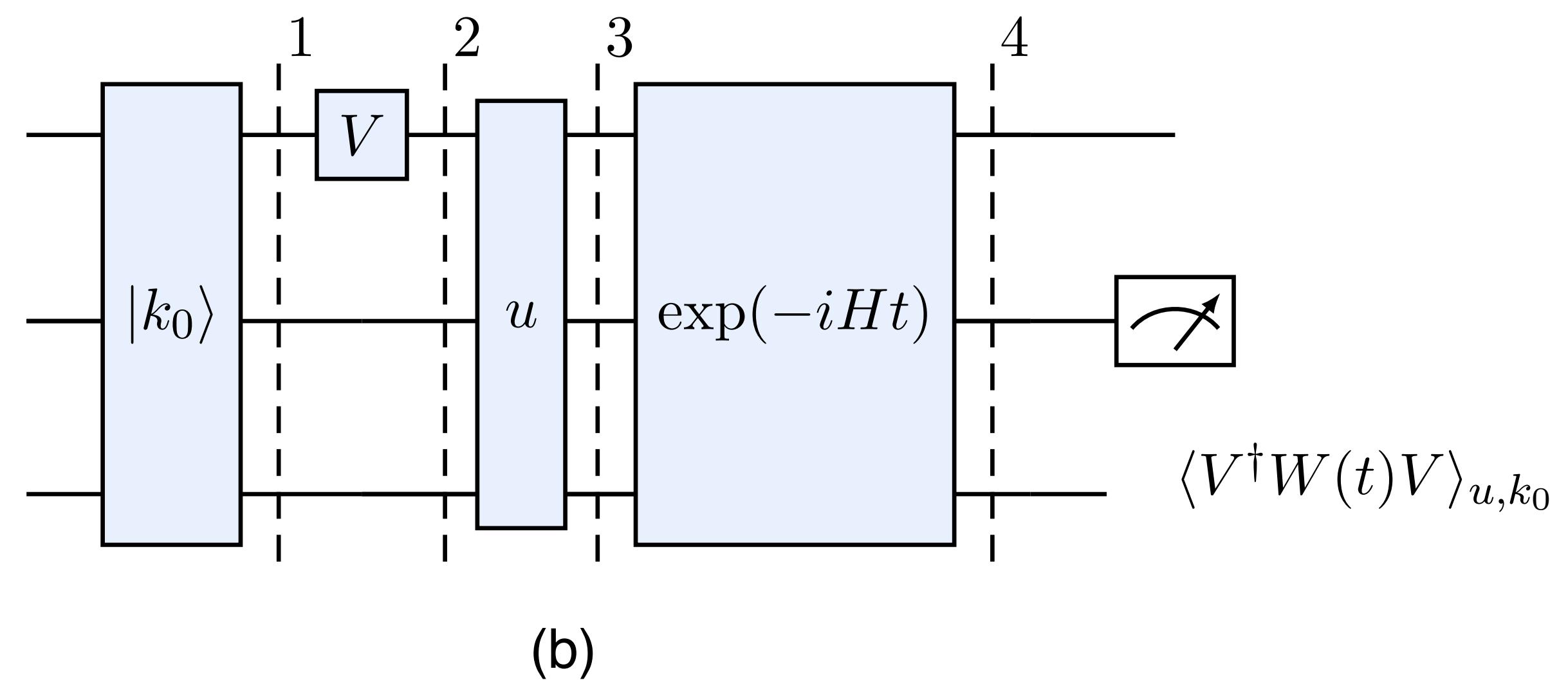
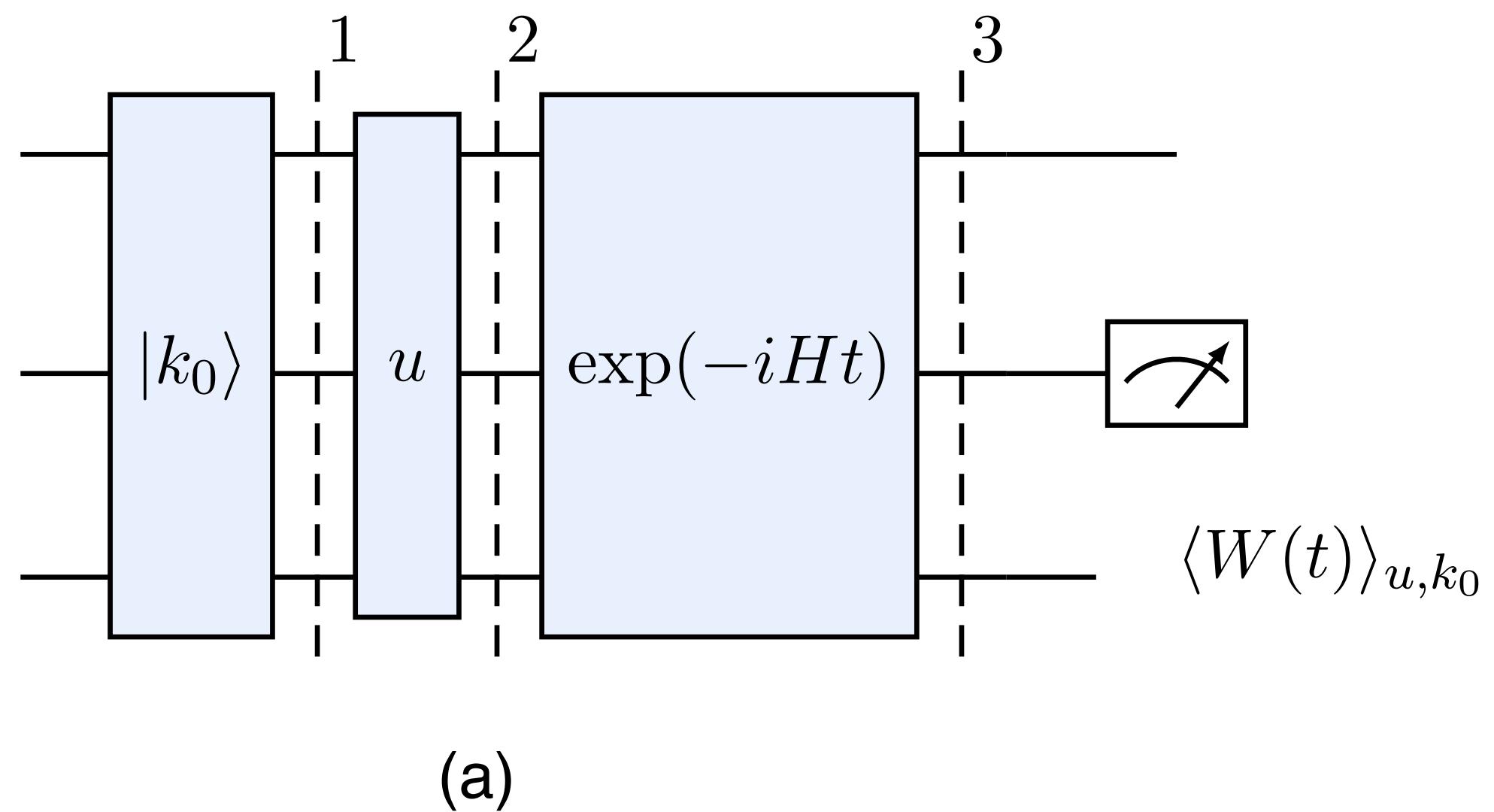
- So the goal is to compute $\langle W(t)V(0)W(t)V(0) \rangle_\beta$ on a quantum computer. Thermal correlators are currently not easy to compute due to limited resources. One simplification we can make is consider the $\beta \rightarrow 0$ limit of OTOC. This is not at all interesting for holography, but this is where we must start. Hence, the density matrix is just $\rho \propto \mathbb{I}$.
- The unusual time-ordering of OTOC is also hard for quantum computers which often mean carrying out forward and backward evolution. We use a protocol (next slide) which uses only forward evolution to compute OTOC on quantum hardware.

Randomised Protocol

- There are various protocols to measure OTOC on quantum computers, see Swingle [2202.07060](#) for review.
- We use the one proposed in [1807.09087](#) now known as ‘randomised protocol’ since it computes OTOC through statistical correlations of observables measured on random states generated from a given matrix ensemble (CUE).
- Infinite-temp OTOC is given by $\text{Tr}(W(t)V^\dagger W(t)V) \propto \overline{\langle W(t) \rangle_u \langle V^\dagger W(t)V \rangle_u}$ where the average is over different random states $|\psi_u\rangle$ prepared by acting with random unitary on arbitrary state say $|0\rangle^{\otimes n}$. Note that this protocol works *only when* W is traceless operator.

Randomised Protocol

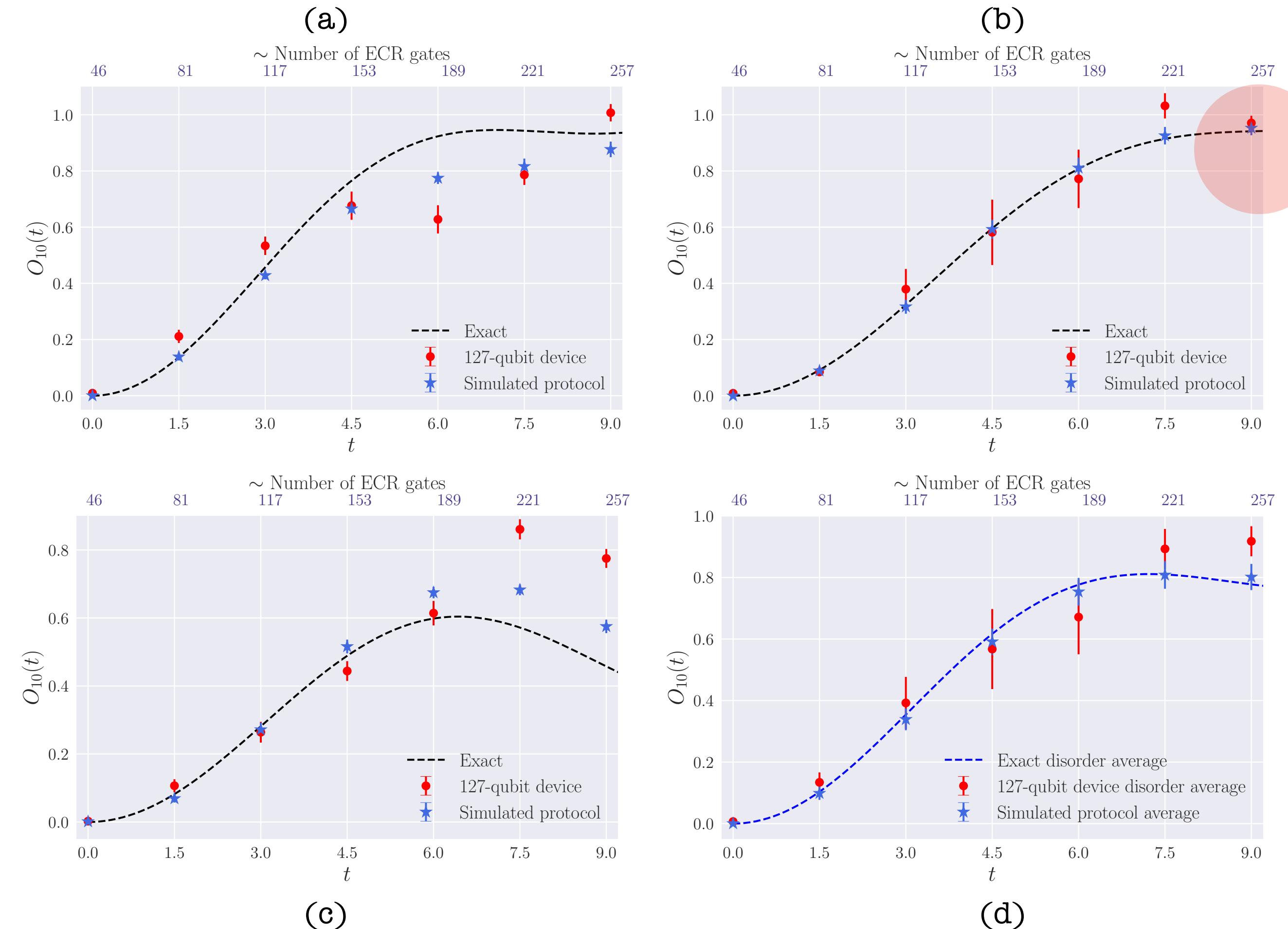
- We need two measurements (between which we compute the statistical correlation) and it is shown below. This is the global version of the protocol (since u has support over all qubits). There is also a local version of the protocol. Note that cost of decomposing arbitrary u increases exponentially, one can instead use unitary from a subset of Haar measure. They are called unitary t -designs* in literature.



t -designs equivalent to first t moments of Haar group

OTOC Results

- We used `ibm_cusco` and `ibm_nazca` to obtain the results show for $N = 6$. We took simplest operators where both W and V were taken to be single Pauli. We see good agreement without need to do self-mitigation like we did for return probability.

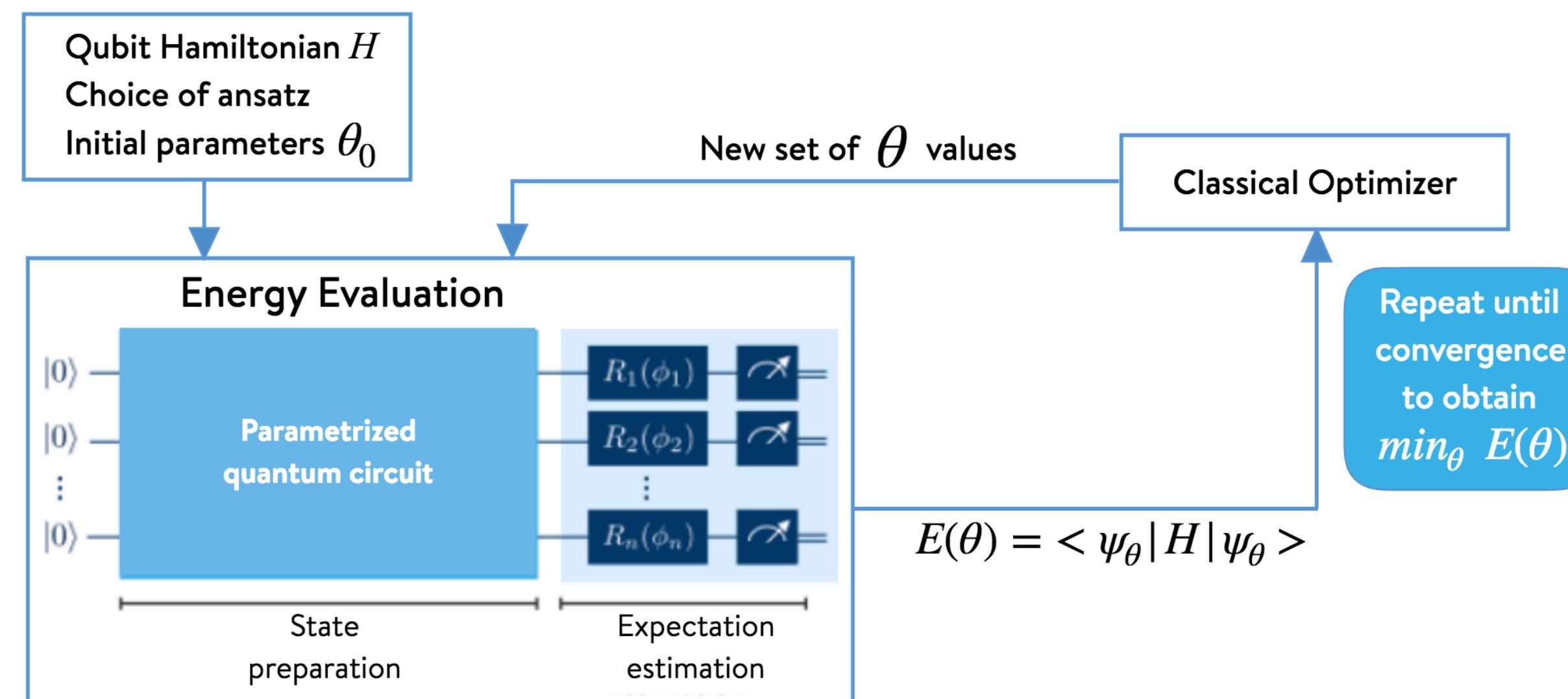


Finite-temperature SYK model

- We considered OTOC measured over random states (maximally mixed) generated i.e., $\beta = 1/T = 0$. However, much of interesting Physics of the SYK happens in the region $\beta \gg 1$ and classical computations have argued that you need $\beta \sim 70$ to extract Lyapunov exponents close to the chaos bound.
- Finite-temperature OTOCs are difficult for quantum computers in general. No simple/general cost-effective protocol exists. To move towards this goal, we are studying the preparation of Gibbs (thermal) states on quantum computer for the SYK model.
- In addition to the thermal state (mixed) of the SYK model, one can also consider a purification of this known as thermo-field double state (TFD). TFD state is a pure state (up to unitary trans.) of some other system (for ex: coupled SYK model) and when we perform partial trace over either system, we recover thermal state on the other one.

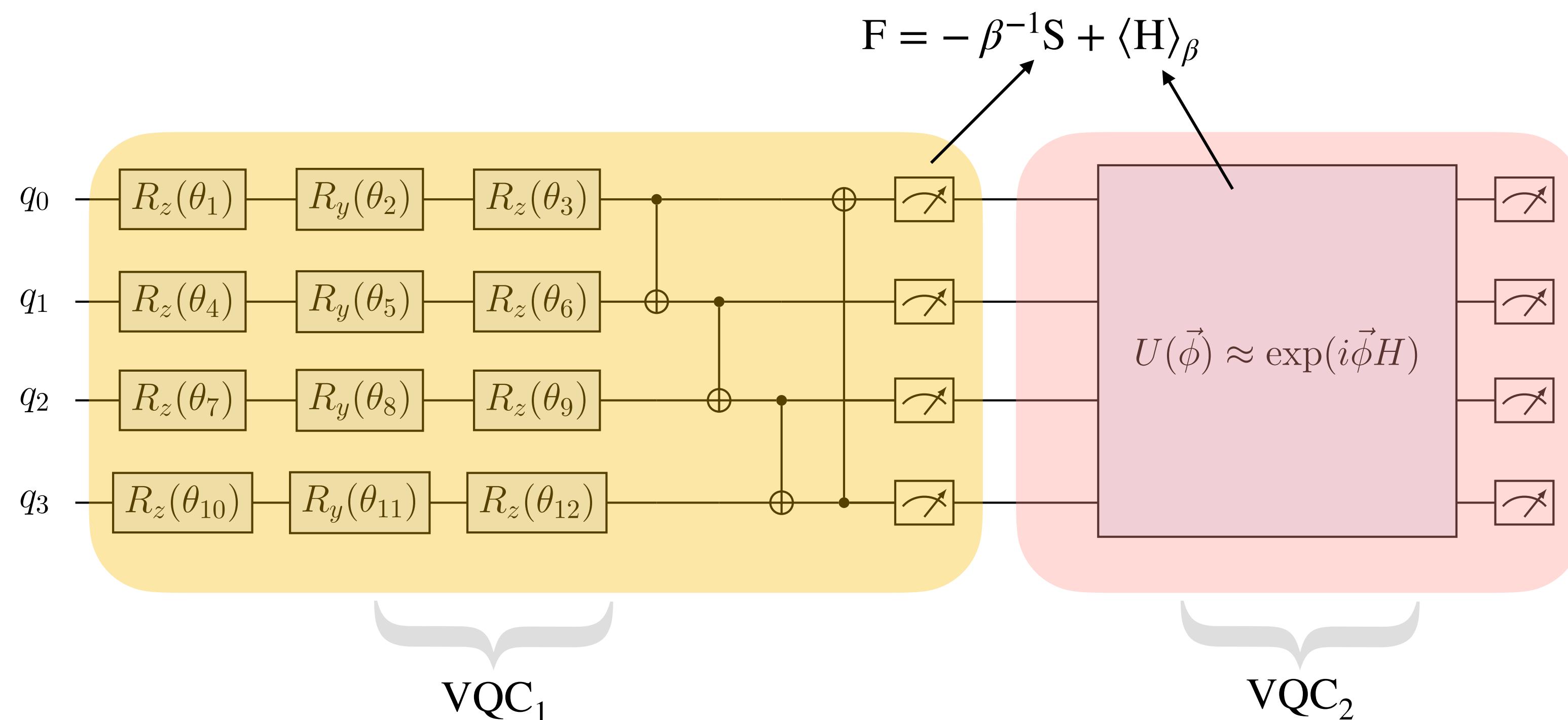
VQE algorithm

- Before we move to preparation of Gibbs state, let's us look at popular algorithm for preparing (approximate) ground states on QC.
- Hybrid classical/quantum algorithm to find the ground state problem of a given Hamiltonian by finding the parameters of a quantum circuit ansatz that minimizes the Hamiltonian expectation value.
- It primarily consists of three steps: 1) Prepare initial ansatz on QC i.e., $|\psi(\vec{\Theta})\rangle$, 2) Measure energy on QC and optimise the parameters Θ using classical optimisers and 3) Repeat until desired convergence is achieved.



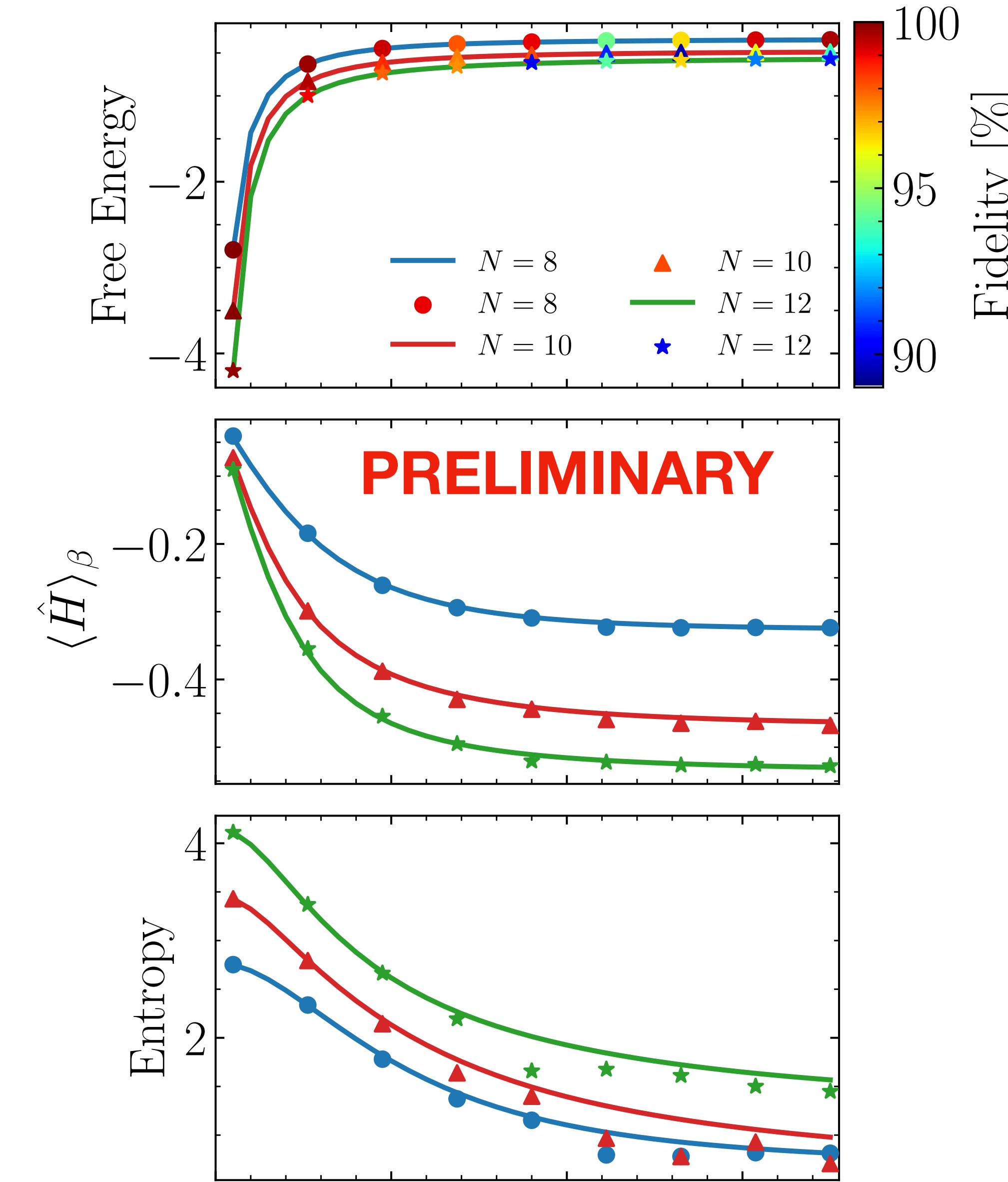
VQE for finite temperatures

- Finite-temperature VQE methods are still an active area of research. Many proposals exist. The cost function is no longer E but rather $E - TS$ (free energy) which can be hard to compute on QC.



Finite-temperature SYK model

[upcoming work with J. Araz, B. Sambasivam, and F. Ringer]



Summary

- We are entering an era where we can compute few things (even if they can be done quickly) using our laptops. Exploring these toy models will hopefully reveal to us better algorithms/methods.
- It is instructive to see that if we can characterise the noise in these quantum devices, we can mitigate and get reasonable results!
- In addition, we are looking at preparation of thermal states and its purifications known as TFD states.

Resources and Data Statement

Published November 25, 2023 | Version v1

Computational notebook [Open](#)

A model of quantum gravity on a noisy quantum computer -- code and circuit release

Asaduzzaman, Muhammad¹ ; Jha, Raghav G.² ; Sambasivam, Bharath³

Show affiliations

Additional resources for the arXiv article: <https://arxiv.org/abs/2311.17991> including the matrices and open qasm files. See the paper for details.

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Files

OTOC_N6.zip

OTOC_N6.zip

N=6

- H_N6_3.mtx 1.7 kB
- H_N6_4.mtx 1.7 kB
- H_N6_7.mtx 1.7 kB
- QC_N6_3.qasm 1.3 kB
- QC_N6_4.qasm 1.3 kB
- QC_N6_7.qasm 1.3 kB
- ham_paulis_N6_3.txt 380 Bytes

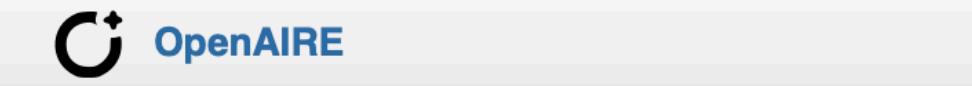
Versions

Version v1 Nov 25, 2023
10.5281/zenodo.10202045

Cite all versions? You can cite all versions by using the DOI [10.5281/zenodo.10202044](https://doi.org/10.5281/zenodo.10202044). This DOI represents all versions, and will always resolve to the latest one. [Read more](#).

External resources

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Both classical and quantum code available at: <https://github.com/rjha/SYKquantumcomp>