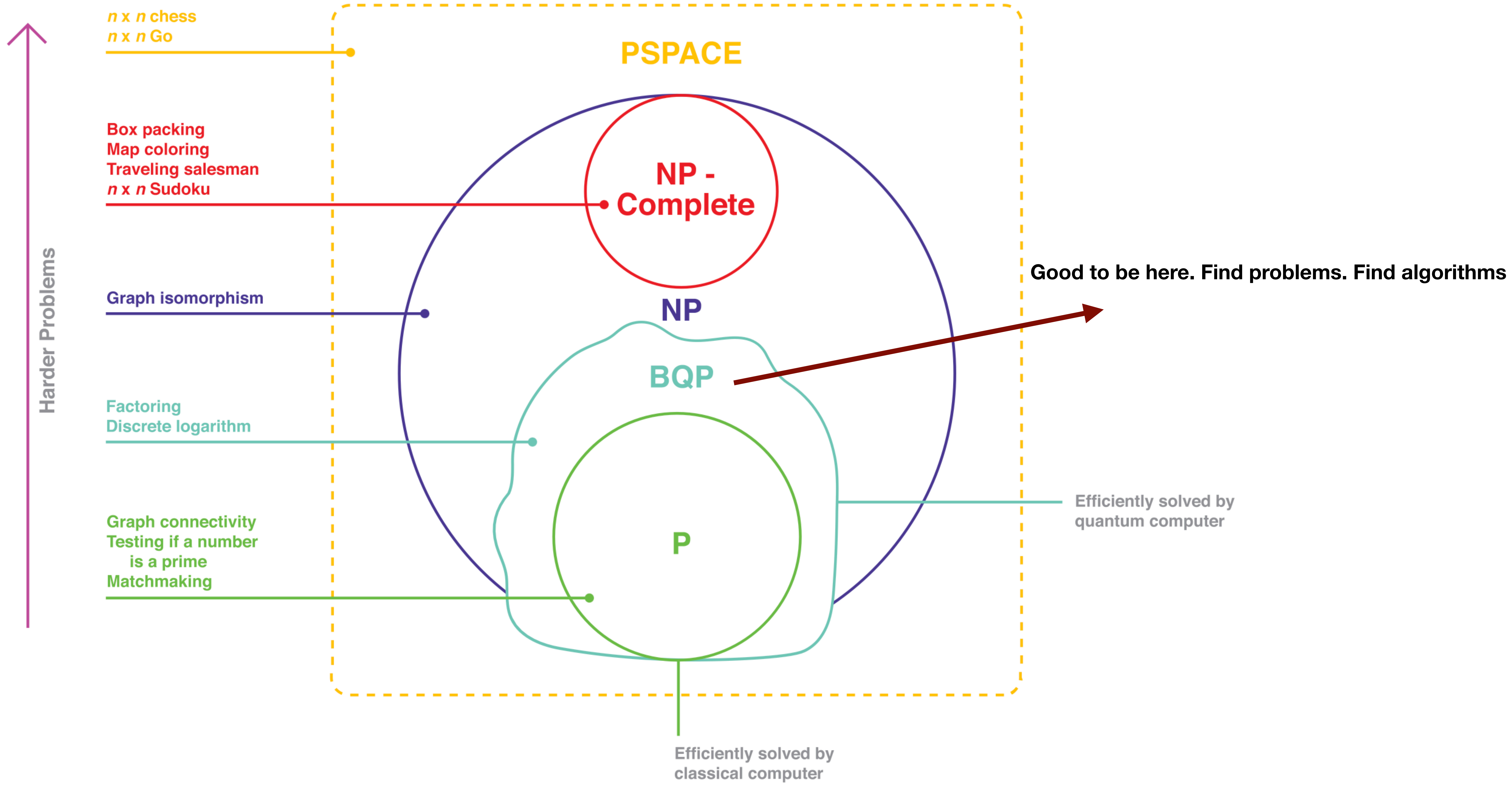
Quantum computing for quantum many-body systems

based on 2310. 12512 (to appear, PRA) & 2311.17991 (to appear, PRD) & upcoming work!

Talk at William & Mary

Raghav G. Jha
Jefferson Lab
17 April 2024

Complexity



n x n chess
n x n Go
Box packing
Map coloring
Traveling salesman
n x n Sudoku
Graph isomorphism
Factoring
Discrete logarithm
Graph connectivity
Testing if a number is a prime
Matchmaking
Harder Problems
PSPACE
NP - Complete
NP
BQP
P
Good to be here. Find problems. Find algorithms
Efficiently solved by quantum computer
Efficiently solved by classical computer

Can we touch the stars*?

THE LIMITS OF Quantum

By Scott Aaronson

Quantum computers would be exceptionally fast at a few specific tasks, but it appears that for most problems they would outclass today's computers only modestly. This realization may lead to a new fundamental physical principle

"Haggar Physicists Develop 'Quantum Slacks,' " read a headline in the satirical weekly the *Onion*. By exploiting a bizarre "Schrödinger's Pants" duality, the article explained, these non-Newtonian pants could paradoxically behave like formal wear and casual wear at the same time. *Onion* writers were apparently spoofing the breathless articles about quantum computing that have filled the popular science press for a decade.

A common mistake—see for instance the February 15, 2007, issue of the *Economist*—is to claim that, in principle, quantum computers could rapidly solve a particularly difficult set of mathematical challenges called NP-complete problems, which even the best existing computers cannot solve quickly (so far as anyone knows). Quantum computers would supposedly achieve this feat not by being formal and casual at the same time but by having hardware capable of processing every possible answer simultaneously.

If we really could build a magic computer capable of solving an NP-complete problem in a snap, the world would be a very different place: we could ask our magic computer to look for whatever patterns might exist in stock-market data or in recordings of the weather or brain activity. Unlike with today's computers, finding these patterns would be completely routine and require no detailed understanding of the subject of the problem. The magic computer could also automate mathematical creativ-

ILLUSTRATIONS BY DUŠAN PETRIČIĆ

Misconception: QC **can** solve all problems

- It turns out that for majority of problems, quantum computers would do no better than classical computers. A major research direction is to understand which problems can be solved efficiently by QCs.

- For example, we know that scattering in ϕ^4 in 1+1-dimensions can be solved efficiently by quantum computers.

- Class of problems which are best suited for quantum advantage belong to complexity class BQP. For ex: Shor's algorithm. Also Grover's algorithm but not as nice as Shor's (only polynomial speed-up).

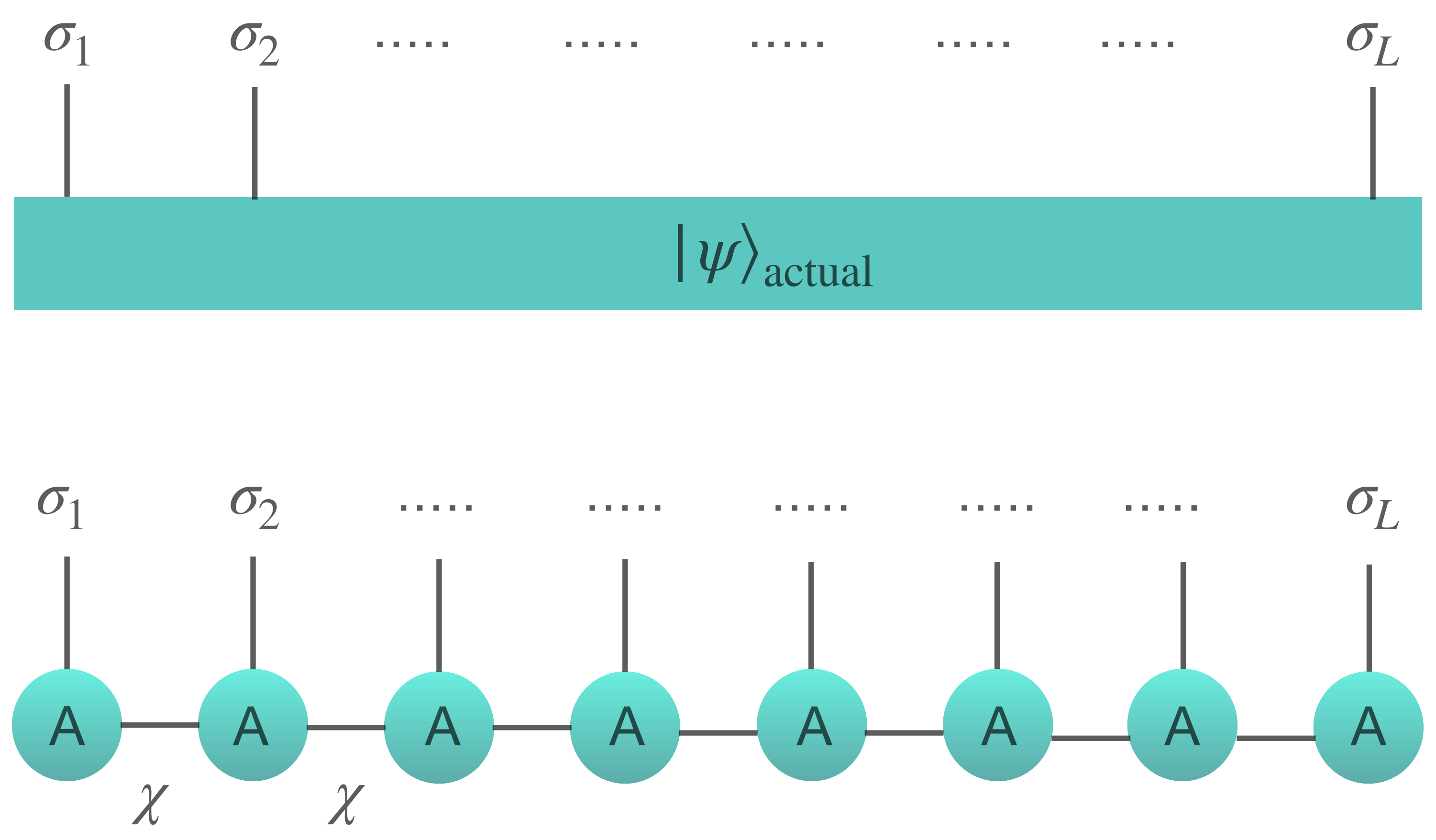
Outline of the talk

- Effectiveness of tensor networks for local Hamiltonians - MPS approximation and beyond
- Quantum gates and real-time evolution using quantum circuits
- Holographic model: SYK model with $N = 6, 8$ Majorana fermions on IBM quantum computers with error mitigation
- Summary and future directions

Tensor networks [Hamiltonian picture]

- The most efficient classical method of studying the properties of lower-dimensional systems is tensor networks. The idea is based on the fact that if the Hamiltonian is sufficiently local and gapped, then the relevant sector of the entire Hilbert space is a tiny region which satisfies area-law entanglement i.e., they are less entangled.

- In this case, the vector space of dimension d^N can be described by $O(d\chi^2)$ where χ is the bond dimension of the MPS. This prescription fails for gapless systems and has to be replaced by more complicated network such as MERA.



approximation

Tensor networks [Renormalization group aka TRG, Action picture]

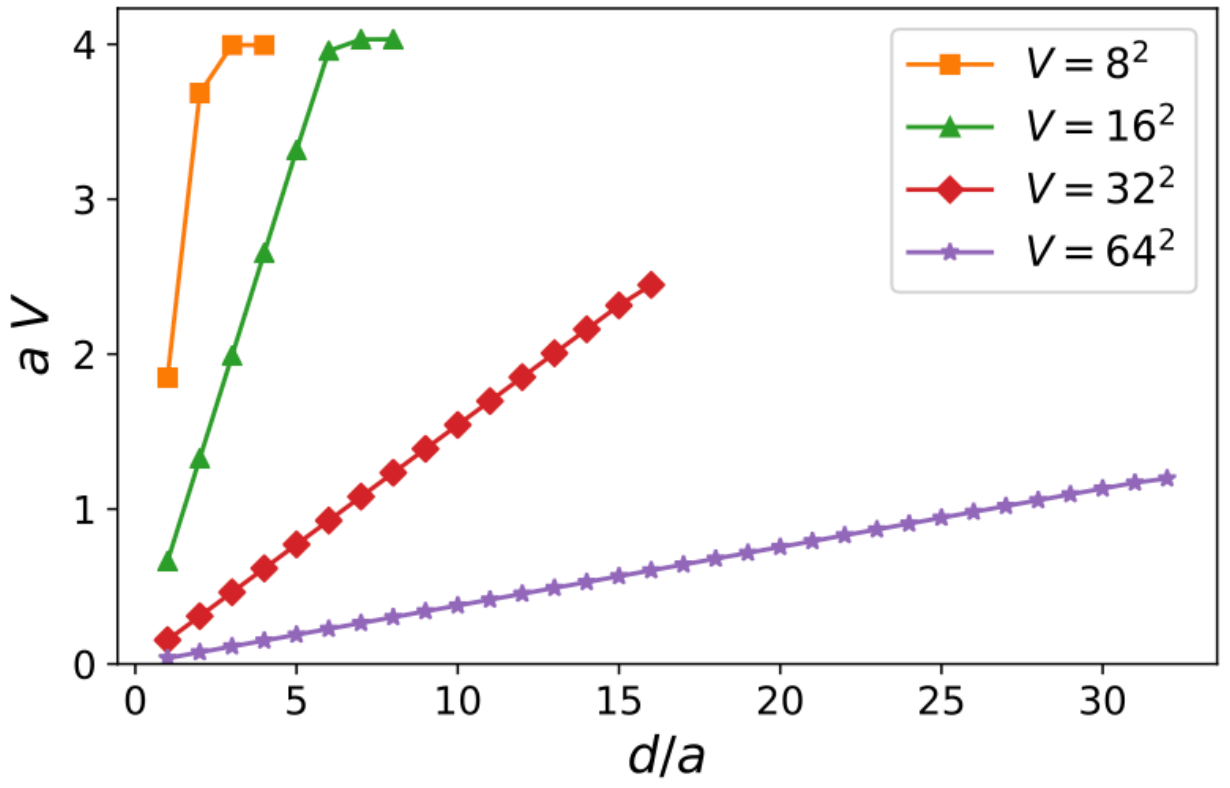
- Some good progress has been made using tensor networks to study 2d SU(2) Wilson lattice gauge theory with matter, 3d O(2) model at finite chemical potential, 3d SU(2) principal chiral model which is in same universality class as 3d O(4) Heisenberg model and is useful as a toy model for QCD (chiral breaking pattern) [Akiyama, Jha, Unmuth-Yockey, in progress 2024]

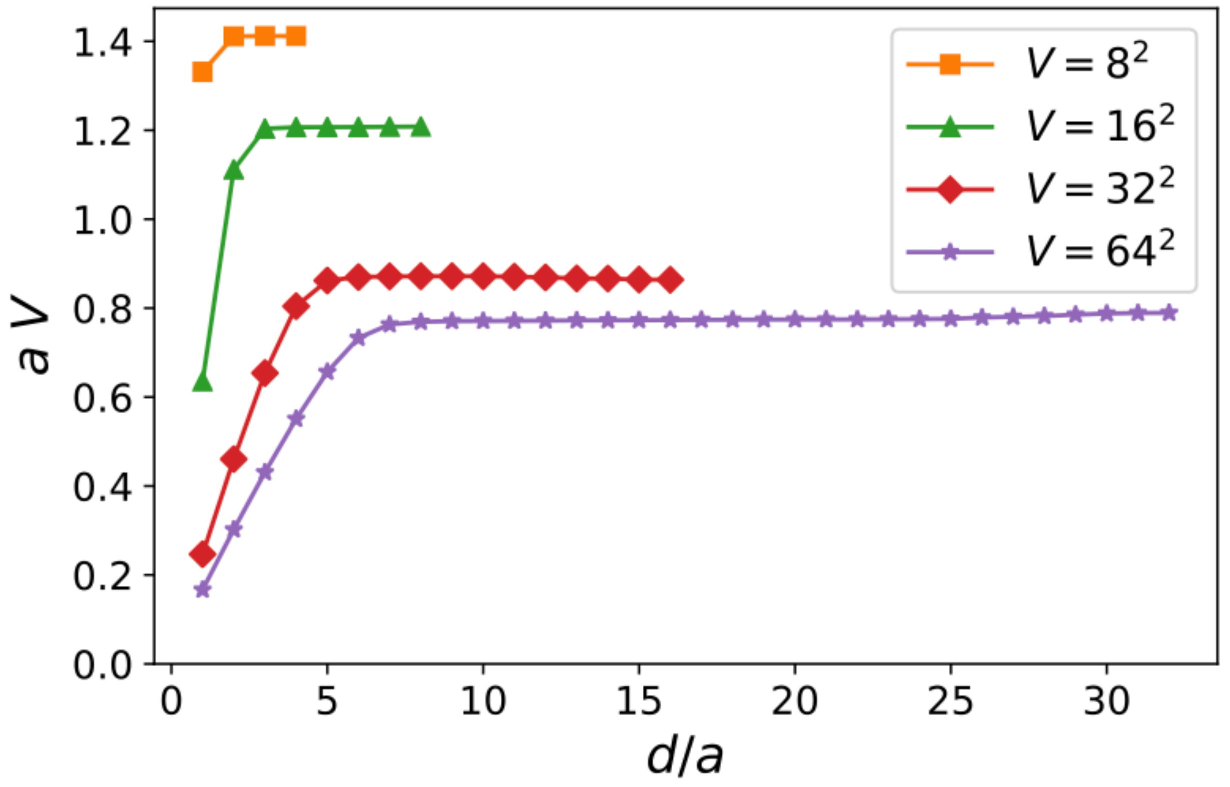
- Basic idea: Use the strong coupling (or character) expansion and express the degrees of freedom (such as gauge links, classical spins) as a tensor and then perform coarse-graining to obtain the continuum limit. Can also work for complex actions where MC fails.

- Hope to understand QCD phase diagram one day?

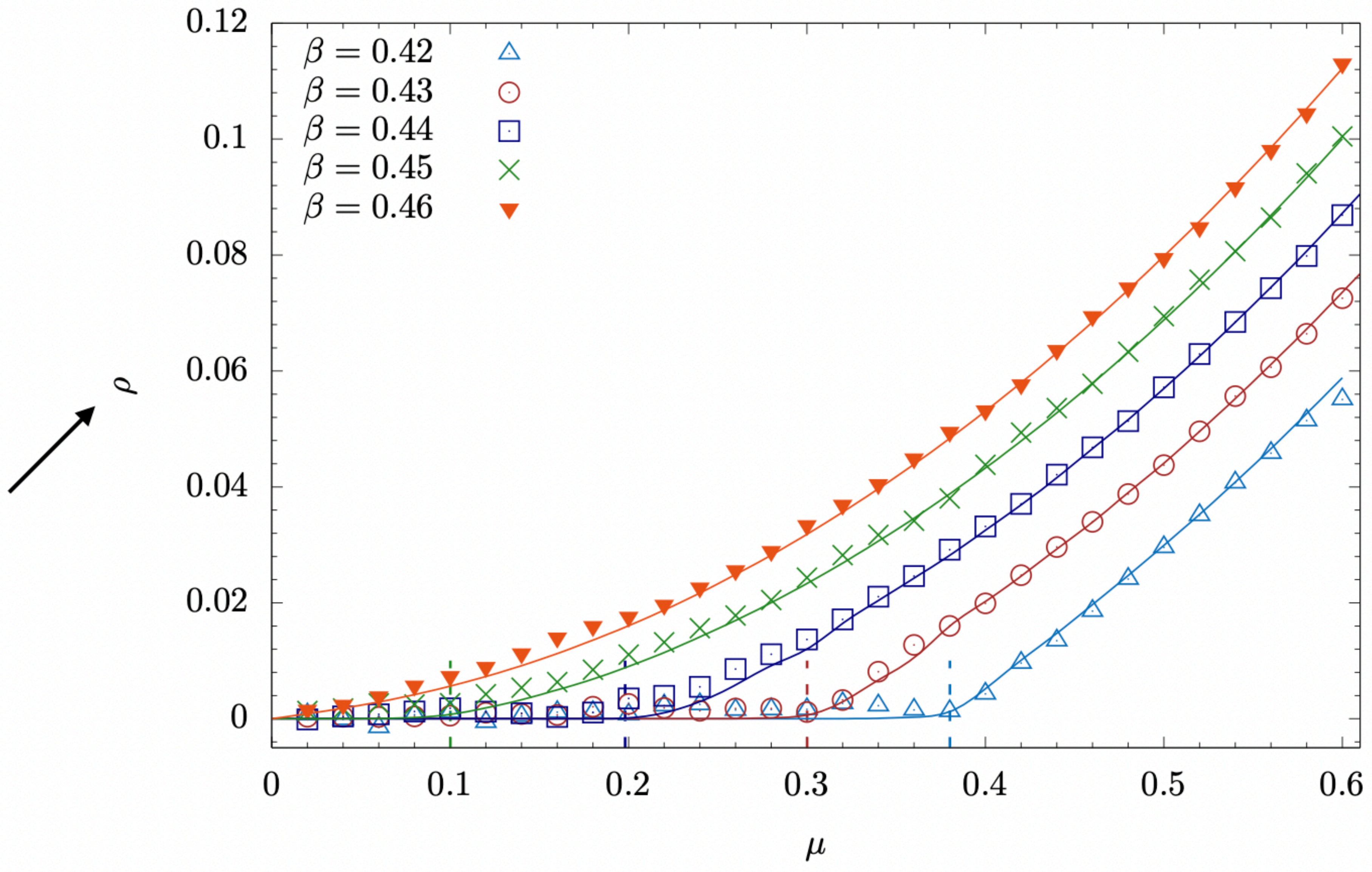
Some past works: 2d SU(2) and 3d O(2) model

arXiv: 1901.11443
arXiv: 2105.08066





$$\frac{\partial \ln Z}{\partial \mu}$$



Silver Blaze

Classical to Quantum

- Tensor networks can help sometimes but they have their own limitations. We need new tools to understand real-time dynamics of interacting field theories or quantum many-body systems.

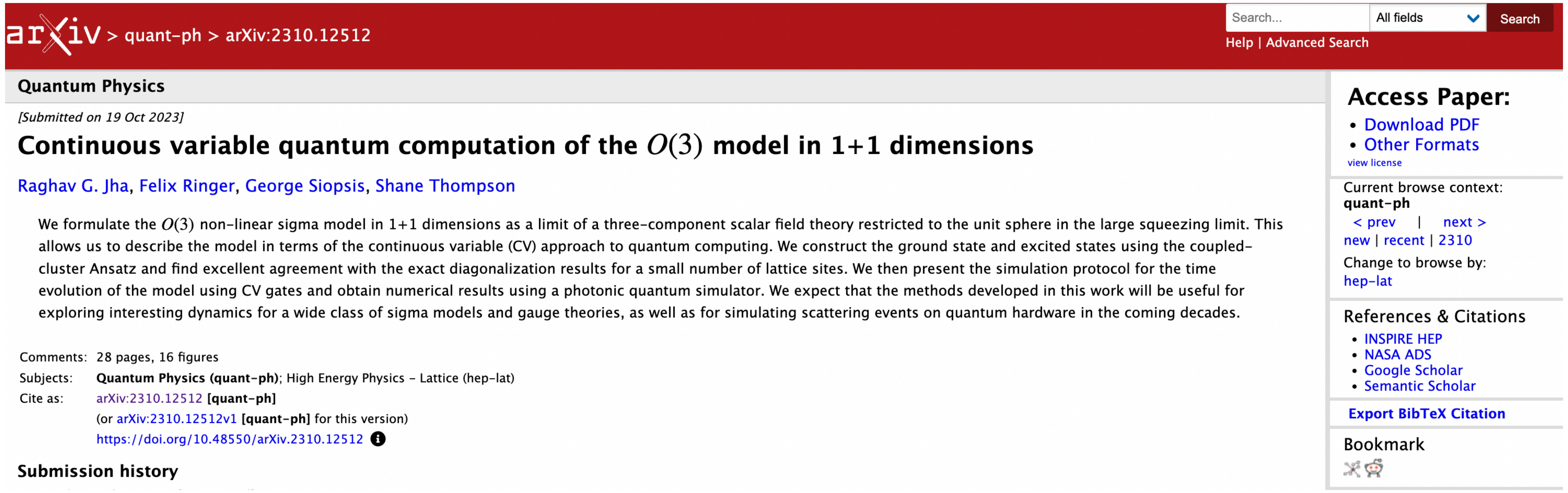
- We require fundamentally *new* idea of computing [Manin, Benioff, Feynman et al., circa 1978] such that we can compute exp(-iHt) for a given H in terms of circuits exploiting features of QM more efficiently than classical computers.

Approaches to universal quantum computing

- Qubit approach — Manipulate and utilise the two-state quantum system. More than dozen approaches. Two most popular — Superconducting and Trapped Ion.

- Qumodes approach — Use photons (quantum harmonic oscillator), infinite-dimensional HS. Not as popular as qubit approach. Error correction not that well-developed.

- This talk will discuss the qubit approach, however, other approach might be better suited for bosonic d.o.f as explored for NLSM model (see 2310.12512). Now extending the “CV” approach to SU(2) gauge theory [Kogut-Susskind Hamiltonian]


arXiv > quant-ph > arXiv:2310.12512

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Quantum Physics

[Submitted on 19 Oct 2023]

Continuous variable quantum computation of the $O(3)$ model in 1+1 dimensions

Raghav G. Jha, Felix Ringer, George Siopsis, Shane Thompson

We formulate the $O(3)$ non-linear sigma model in 1+1 dimensions as a limit of a three-component scalar field theory restricted to the unit sphere in the large squeezing limit. This allows us to describe the model in terms of the continuous variable (CV) approach to quantum computing. We construct the ground state and excited states using the coupled-cluster Ansatz and find excellent agreement with the exact diagonalization results for a small number of lattice sites. We then present the simulation protocol for the time evolution of the model using CV gates and obtain numerical results using a photonic quantum simulator. We expect that the methods developed in this work will be useful for exploring interesting dynamics for a wide class of sigma models and gauge theories, as well as for simulating scattering events on quantum hardware in the coming decades.

Comments: 28 pages, 16 figures
Subjects: **Quantum Physics (quant-ph)**; High Energy Physics - Lattice (hep-lat)
Cite as: arXiv:2310.12512 **[quant-ph]**
(or arXiv:2310.12512v1 **[quant-ph]** for this version)
https://doi.org/10.48550/arXiv.2310.12512

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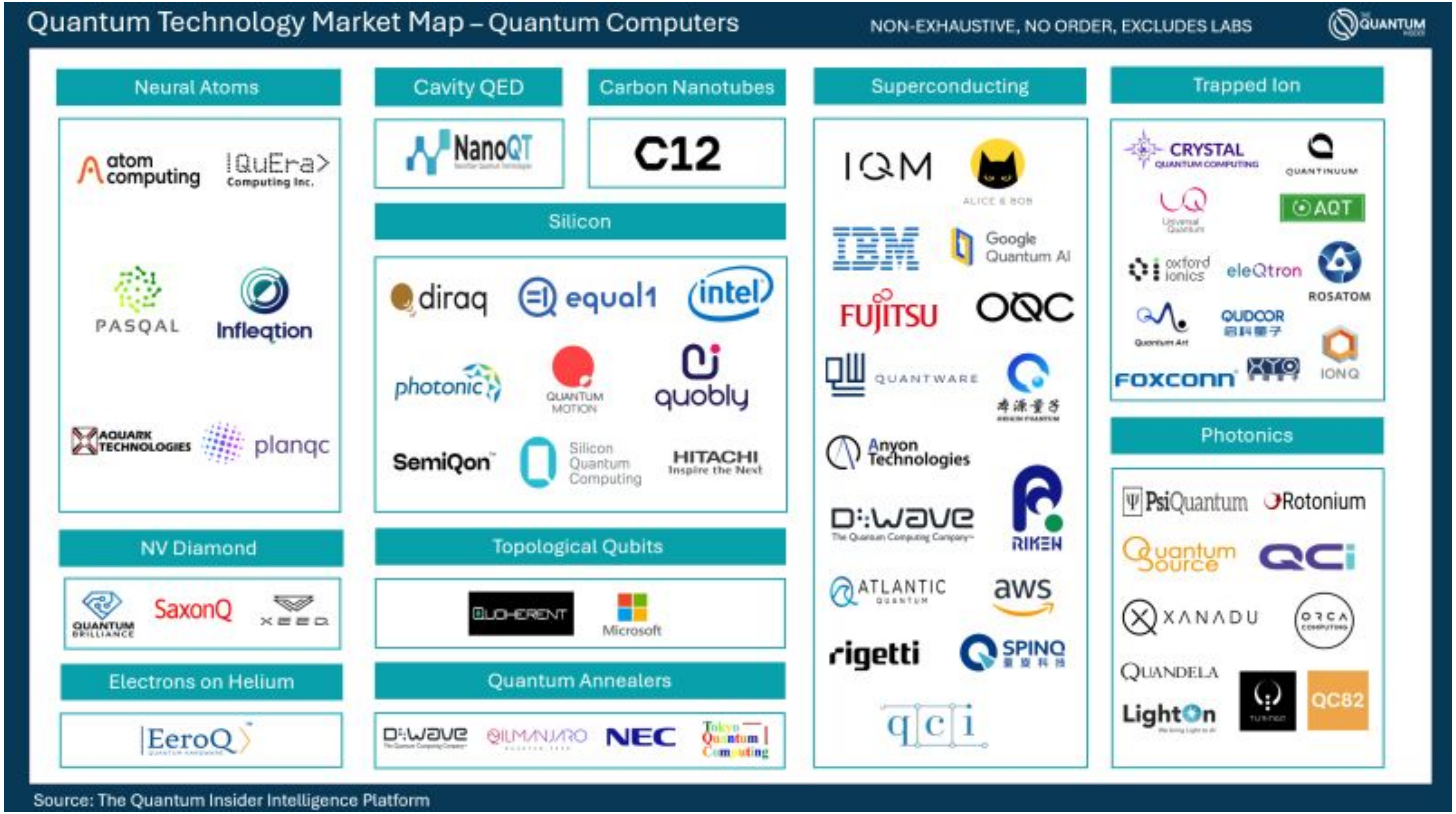
Universal quantum computing efforts

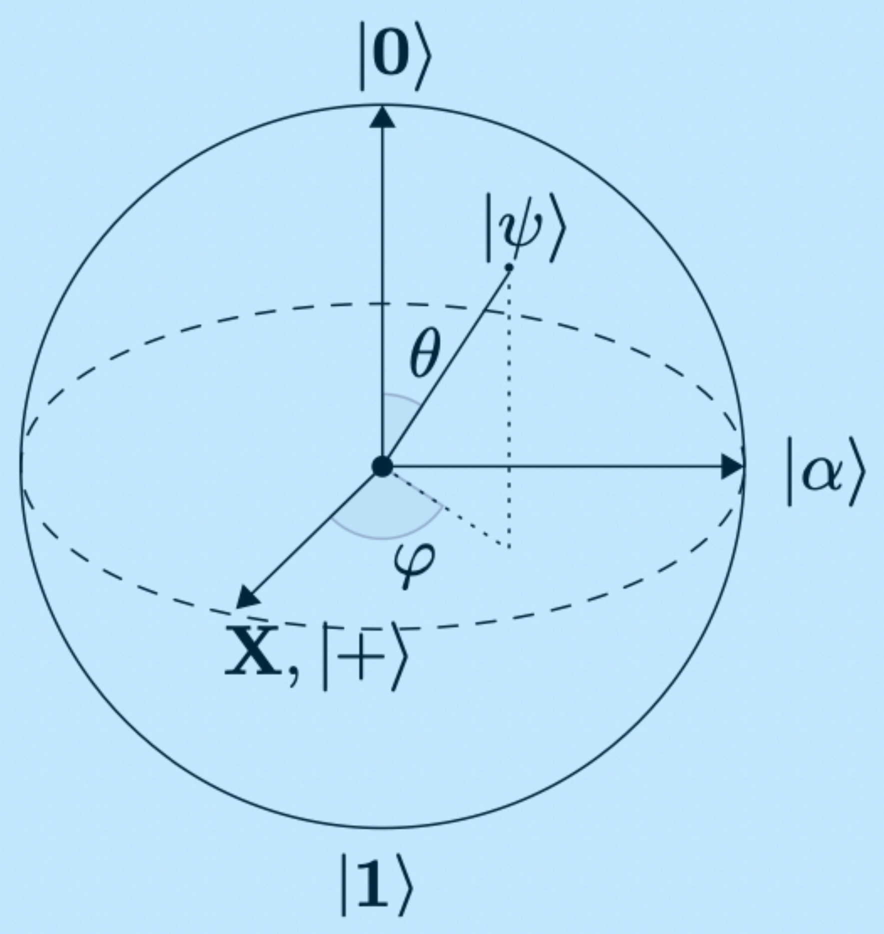


Qubits vs. Qumodes

	CV	Qubit
Basic element	Qumodes	Qubits
Relevant operators	Quadrature operators $\hat{x}, \hat{p}$ Mode operators $\hat{a}, \hat{a}^\dagger$	Pauli operators $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$
Common states	Coherent states $\|\alpha\rangle$ Squeezed states $\|z\rangle$ Number states $\|n\rangle$	Pauli eigenstates $\|0/1\rangle, \|\pm\rangle, \|\pm i\rangle$
Common gates	Rotation, Displacement, Squeezing, Beamsplitter, Cubic Phase	Phase Shift, Hadamard, CNOT, T Gate

DV gate set

$$|+\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$$



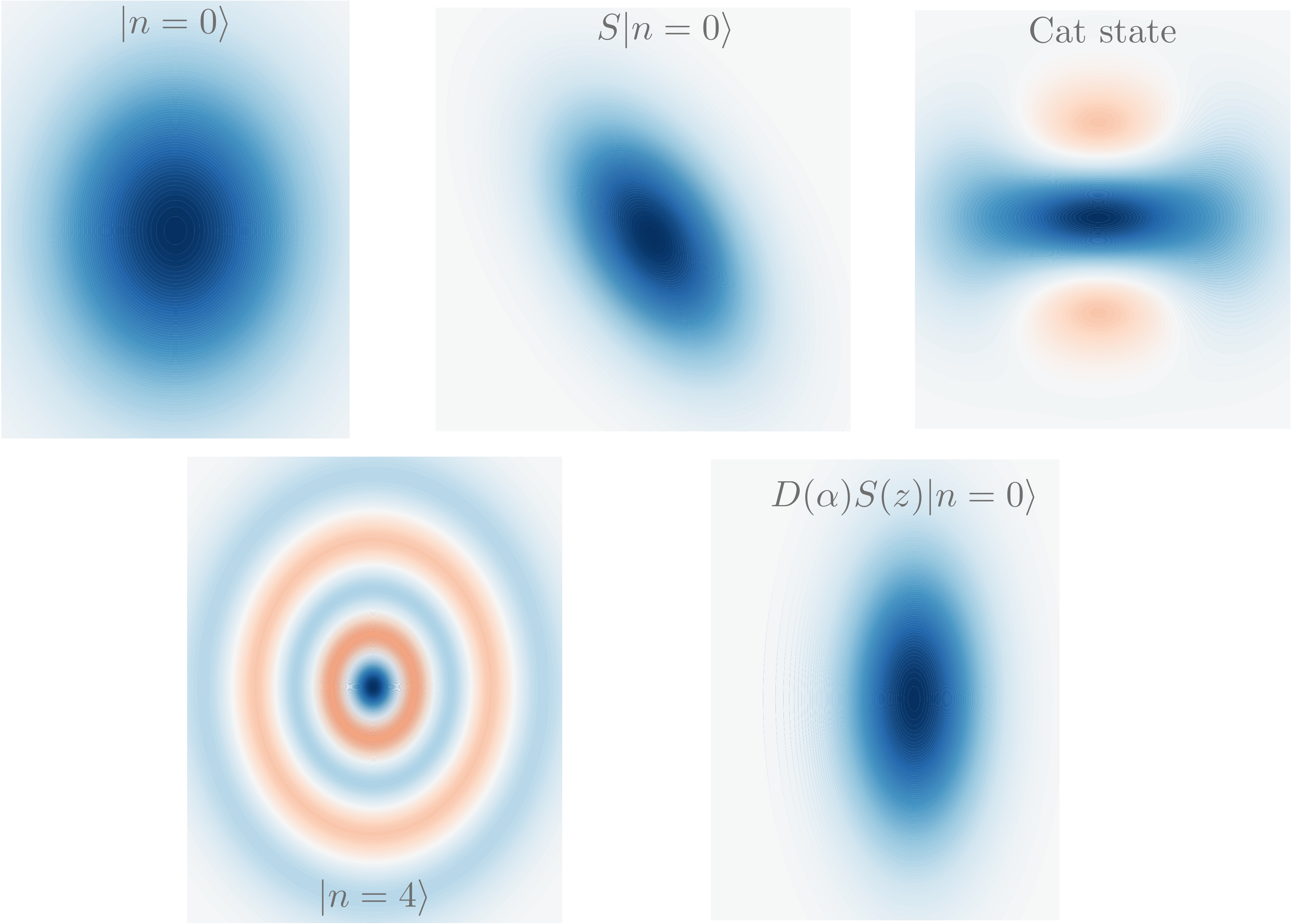
Operator	Gate(s)	Matrix
Pauli-X (X)	$\boxed{\mathbf{X}}$	$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$
Pauli-Y (Y)	$\boxed{\mathbf{Y}}$	$\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$
Pauli-Z (Z)	$\boxed{\mathbf{Z}}$	$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$
Hadamard (H)	$\boxed{\mathbf{H}}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
Phase (S, P)	$\boxed{\mathbf{S}}$	$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$
$\pi/8$ (T)	$\boxed{\mathbf{T}}$	$\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$
Controlled Z (CZ)	$\boxed{\mathbf{Z}}$	$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}$
SWAP		$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$

Universal CV gate set

- Displacement operator (gate): $D(\alpha) = \exp\left(\alpha\hat{a}^\dagger - \bar{\alpha}\hat{a}\right)$. The set of all displacement operators form a Heisenberg-Weyl group.
- Squeezing operator: $S = \exp\left(\frac{1}{2}\left(z^*\hat{a}^2 - z\hat{a}^{\dagger 2}\right)\right)$
- Controlled gates: $CX_{ab}(s) = \exp(-isq_a p_b)$ and $CZ_{ab}(s) = \exp(isq_a q_b)$ related by Fourier gate.
- Fourier gate: $F = \exp(i\phi(p^2 + q^2))$ and $CZ_{ab} = F_b^\dagger \cdot CX_{ab} \cdot F_b$
- Beam-splitter gate: $BS_{i,j}(\theta, \phi) = \exp(\theta(e^{i\phi}a_i^\dagger a_j - e^{-i\phi}a_i a_j^\dagger))$
- Universal set: {D,S,F,BS} + *non-Gaussian gate!*

Wigner (quasi-probability) functions

- Most natural way to visualise these states etc. (like say Bloch sphere for qubit approach) is to use Wigner functions.



Gottesmann-Knill theorem

- Any quantum circuit consisting of H, S, and CNOT can be simulated classically. However, if we just add the T-gate, it becomes universal gate set (quantum computing).
- There is an equivalent statement in CV language too: Need to add non-Gaussian gate to achieve universal computing. Achieved either by cubic phase gate, or photon measurement (detection). Photon detection is a non-Gaussian operation since it localises on the # of photons in the cavity. It changes a Gaussian state to non-Gaussian. Was shown to be sufficient for universality.

Questions?

Holographic duality

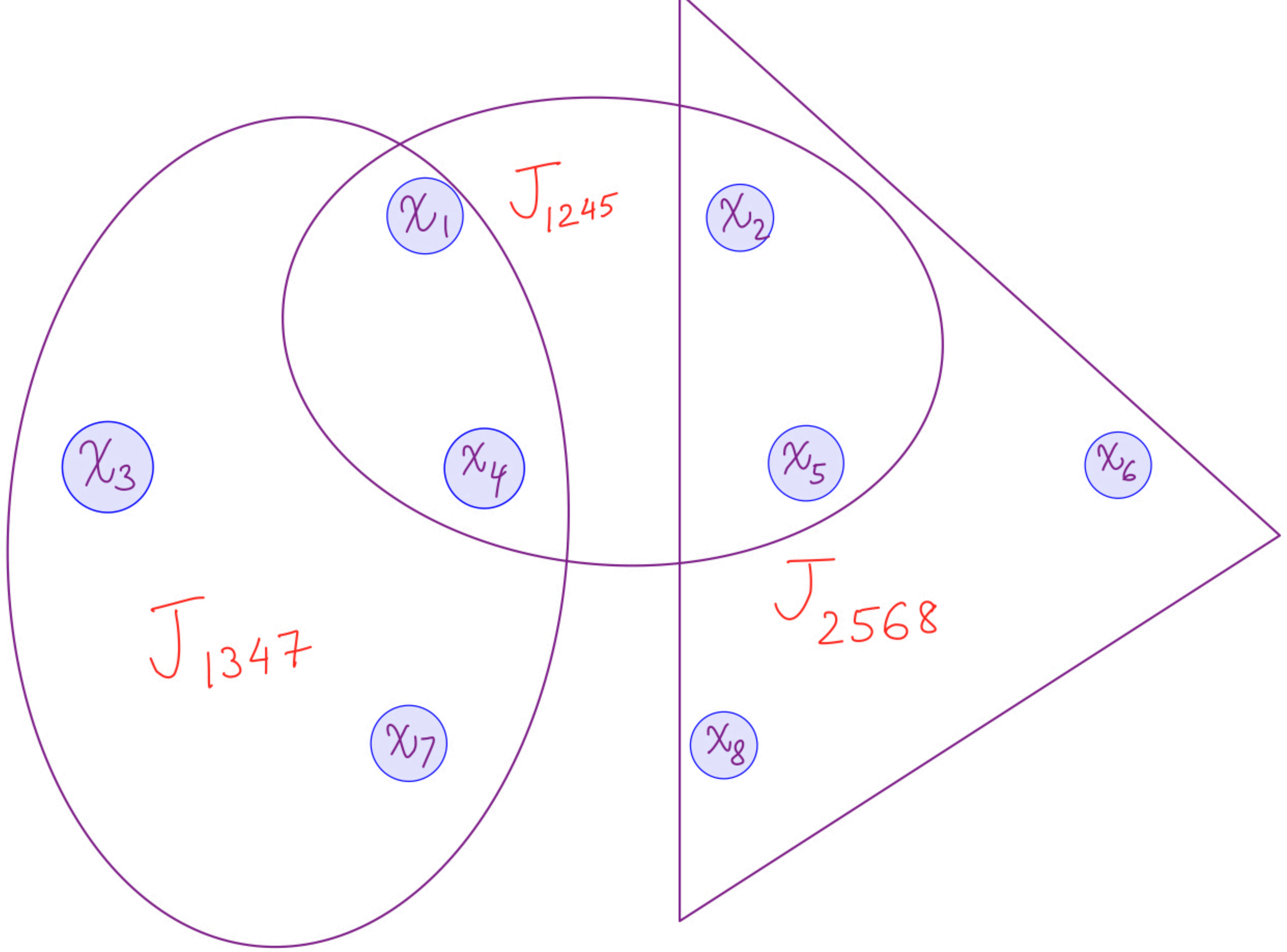
- Certain supersymmetric (maximal) gauge theories are dual to Type IIA/B supergravity at strong couplings in the large N (planar) limit.

- Insights into quantum gravity through field theories and quantum many-body systems.

- Famous example: AdS/CFT, a version of it was soon also extended to super Yang-Mills (SYM) in p+1-dimensions for $p < 3$ [Maldacena et al., PRD 58 046004(1998)]

- We have limited tools to study real-time dynamics of such strongly coupled models. One leading candidate is tensor networks. We consider the simplest system which has holographic behaviour.

SYK model



$$H = \frac{(i)^{q/2}}{q!} \sum_{i,j,k,\cdots,q=1}^{N} J_{ijk\cdots q} \quad \chi_i \chi_j \chi_k \cdots \chi_q,$$

- Model of N Majorana fermions with q-interaction terms with random coupling taken from a Gaussian distribution with $\overline{J_{\ldots}} = 0, \ \overline{J_{\ldots}^2} = \frac{q!J^2}{N^{q-1}}$.
- The fermions χ satisfy, $\chi_i \chi_j + \chi_j \chi_i = \delta_{ij}$. We will set $J = 1$. Note that it has units of energy and inverse time.
- In the limit of large number of fermions with $N \gg \beta J \gg 1$, the model has several interesting features such as maximal Lyapunov exponent.

Mapping fermions to qubits

$$\chi_{2k-1} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) X_k \mathbb{I}^{\otimes (N-2k)/2} \quad , \quad \chi_{2k} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) Y_k \mathbb{I}^{\otimes (N-2k)/2}$$

- N fermions requires N/2 qubits. We use the standard Jordan-Wigner mapping to write χ in terms of Pauli matrices X, Y, Z, and Identity.

- The SYK Hamiltonian is then written as sum of Pauli strings. The number of strings is $\binom{N}{q}$ and grows like $\sim N^q$. Simplest non-trivial case for is $N = q$ with one term. We restrict to $q = 4$.

Simplest case: N=4

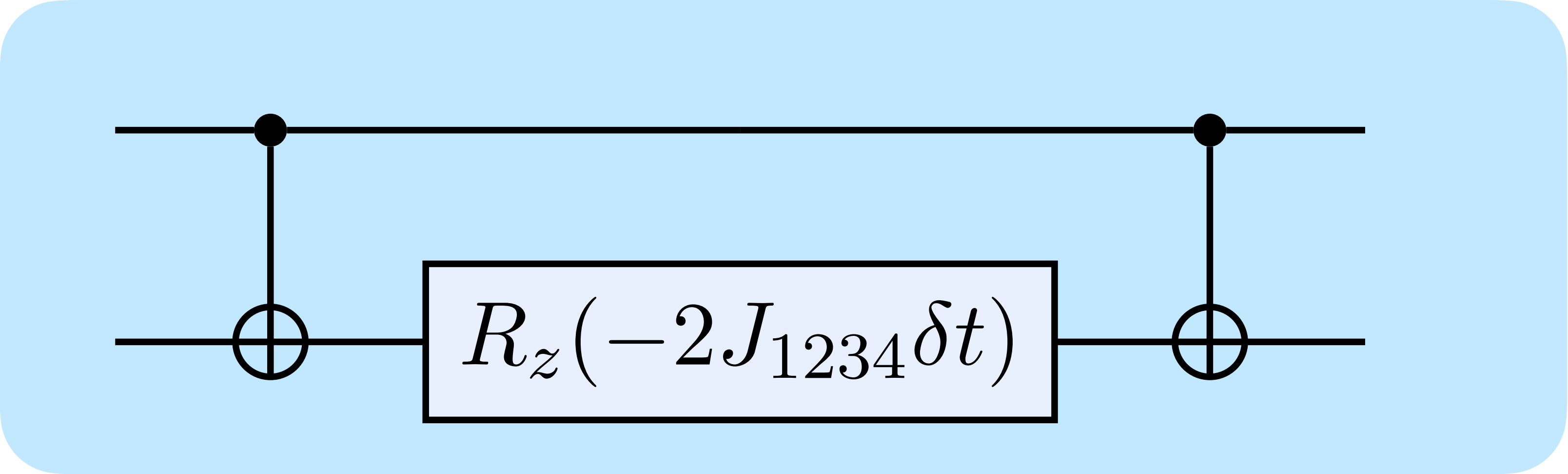
$$H = J_{1234}\chi_1\chi_2\chi_3\chi_4$$

$$\chi_1 = X\mathbb{1},\ \chi_2 = Y\mathbb{1},\ \chi_3 = ZX,\ \chi_4 = ZY$$

$$H = J_{1234}(X\mathbb{1})\cdot(Y\mathbb{1})\cdot(ZX)\cdot(ZY) = -J_{1234}ZZ$$

- The goal of quantum computation is to construct a unitary operator corresponding to this Hamiltonian. So, for this case we have $\exp(-iHt) = \exp(iJ_{1234}ZZt)$.

- This circuit is simple to construct and just needs 2 CNOTs and 1 rotation gate.



Circuit complexity

Definition: How many 2q-gates do we need to simulate the SYK model?

- Different approaches can be used to do the Hamiltonian simulation (aka time evolution). A popular method is Trotter method. It is based on Lie-Suzuki-Trotter product formula* (writing $H = \sum_{j=1}^{m} H_j$, $m \sim N^4$)

$$e^{-iHt} = \left(\prod_{j=1}^{m} e^{-iH_j t/r}\right)^r + \mathcal{O}\left(\sum_{j<k} \left|\left|[H_j, H_k]\right|\right| \frac{t^2}{r}\right),$$

- Depending on what error (ϵ) we desire in the time-evolution from the second term, we can compute the number of slices (r) we need to take. So, the complexity reduces to finding number of 2q-gates for each Trotter step. Recall that $N = 4$ needed just 2 2q-gates for each Trotter step.

* Corollary of Zassenhaus formula i.e., exp(t(X+Y)) = exp(tX) exp(tY) + O(t^2) (also known as dual of BCH formula).

Old work(s)

$\mathscr{C} = \mathcal{O}(N^{10}t^2/\epsilon)$	L. García-Álvarez et al., PRL 119, 040501 (2017)
$\mathscr{C} = \mathcal{O}(N^{8}t^2/\epsilon)$	Susskind, Swingle et al., arXiv: 2008.02303 (2020)
$\mathscr{C} = \tilde{\mathcal{O}}(N^{7/2}t)$	Babbush et al., Phys. Rev. A 99, 040301 (2019)

- The last one clearly is the most efficient, however, in the noisy-era implementing this is not feasible. It requires fault-tolerant quantum resources + ancillas since it is based on the basic idea of embedding H in a bigger vector space.

- Using the Trotter methods, the best seems to be $\sim N^8$. In our paper, we improved the complexity to $\mathscr{C} = \mathcal{O}(N^5 t^2/\epsilon)$ which we now discuss.

Commuting terms

The costs can be simplified if we are little careful in splitting the SYK Hamiltonian.

The number of terms grows like $\sim N^4$, however, a large fraction of them commute with one another and can be collected together and then time-evolved more efficiently. We can find diagonalising circuit for each cluster and then apply time-evolution operator.

Finding optimal number of such clusters is a well-studied computer science problem. This is in general a NP-hard problem but various approx. algorithms exits.

Estimate based on general commutivity

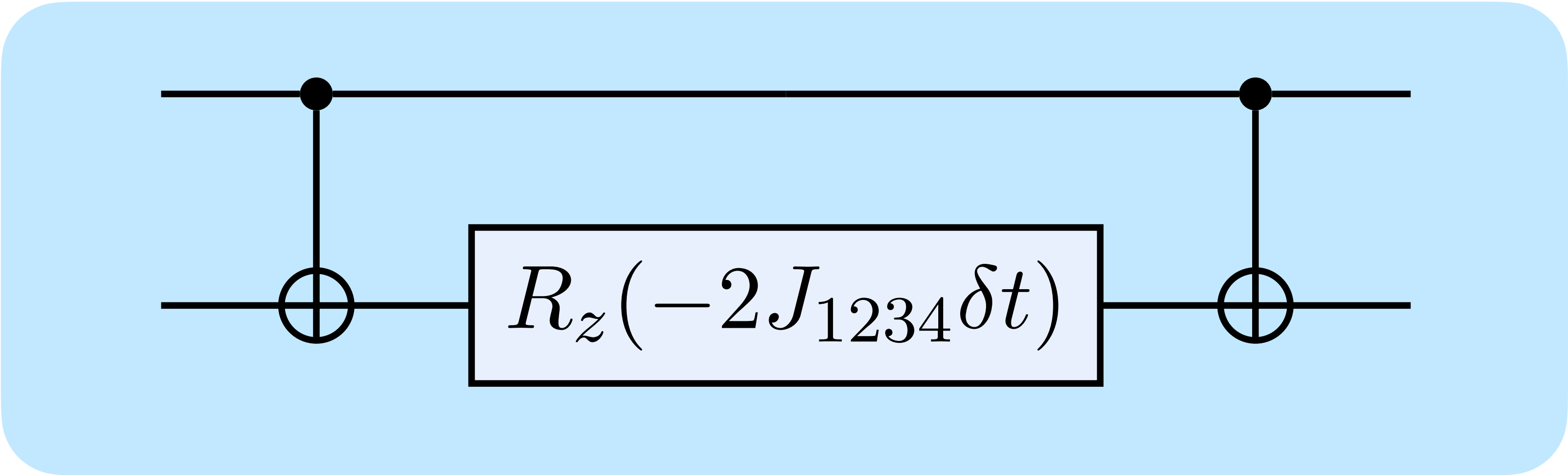
N	Pauli strings	Clusters	Two-qubit gates
4	1	1	2
6	15	5	30
8	70	6	110
10	210	23	498
12	495	57	1504
14	1001	92	3560
16	1820	116	6812
18	3060	175	11962
20	4845	246	19984

Return probability

- A simple observable we can compute is the probability that we return to the same initial state after some evolution time t i.e., $\mathscr{P}_0 = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|^2$. For initial state, we take $|0\rangle^{\otimes N/2}$.

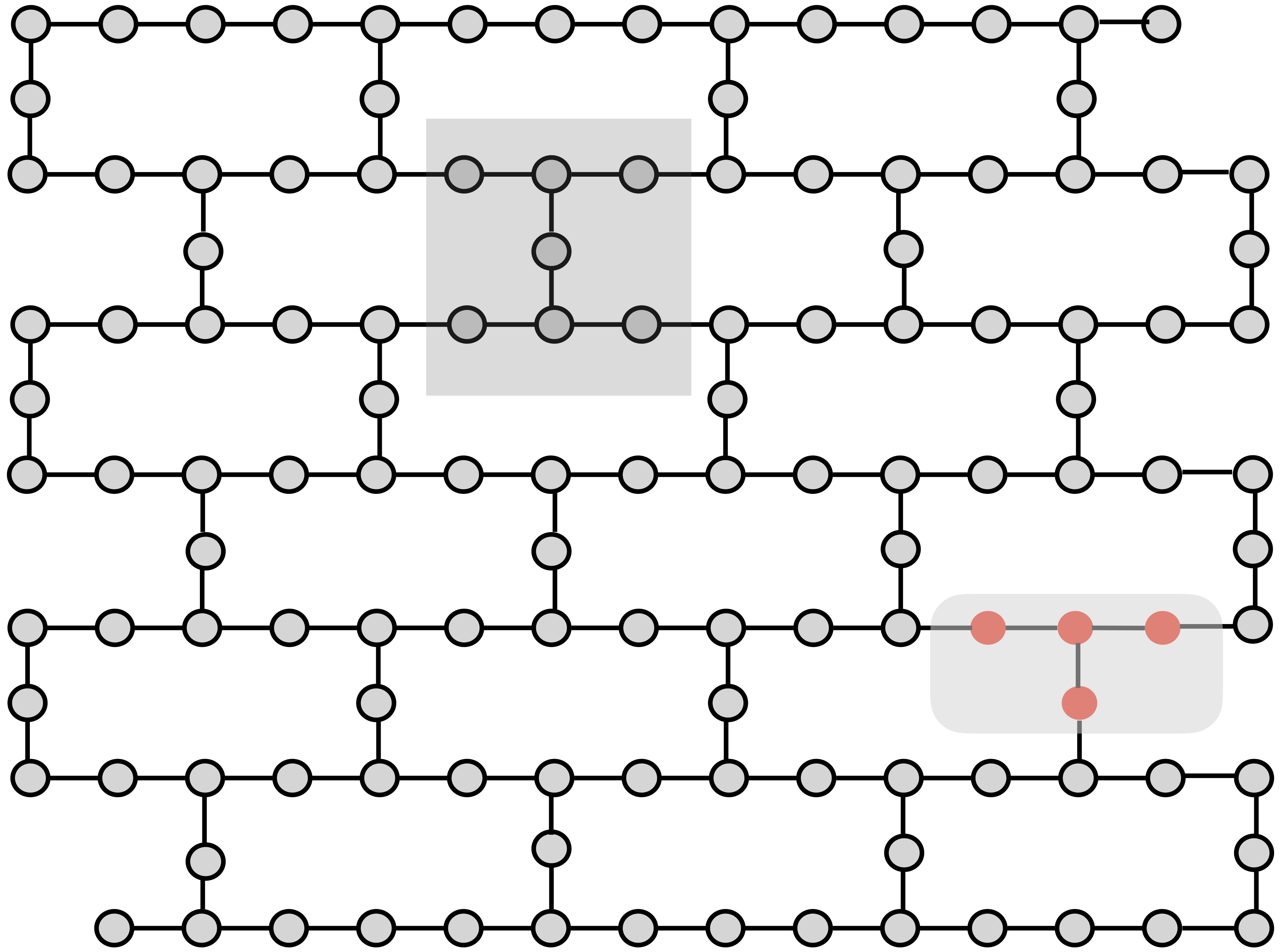
- For approximating the unitary, we use the first-order product formula and construct the corresponding quantum circuit.

- For $N = 4$, we have a simple circuit of only two 2Q gates, so the entire circuit for return prob. is straightforward. For $N = 6$, there are 30 2Q gates per step which we cannot show here.

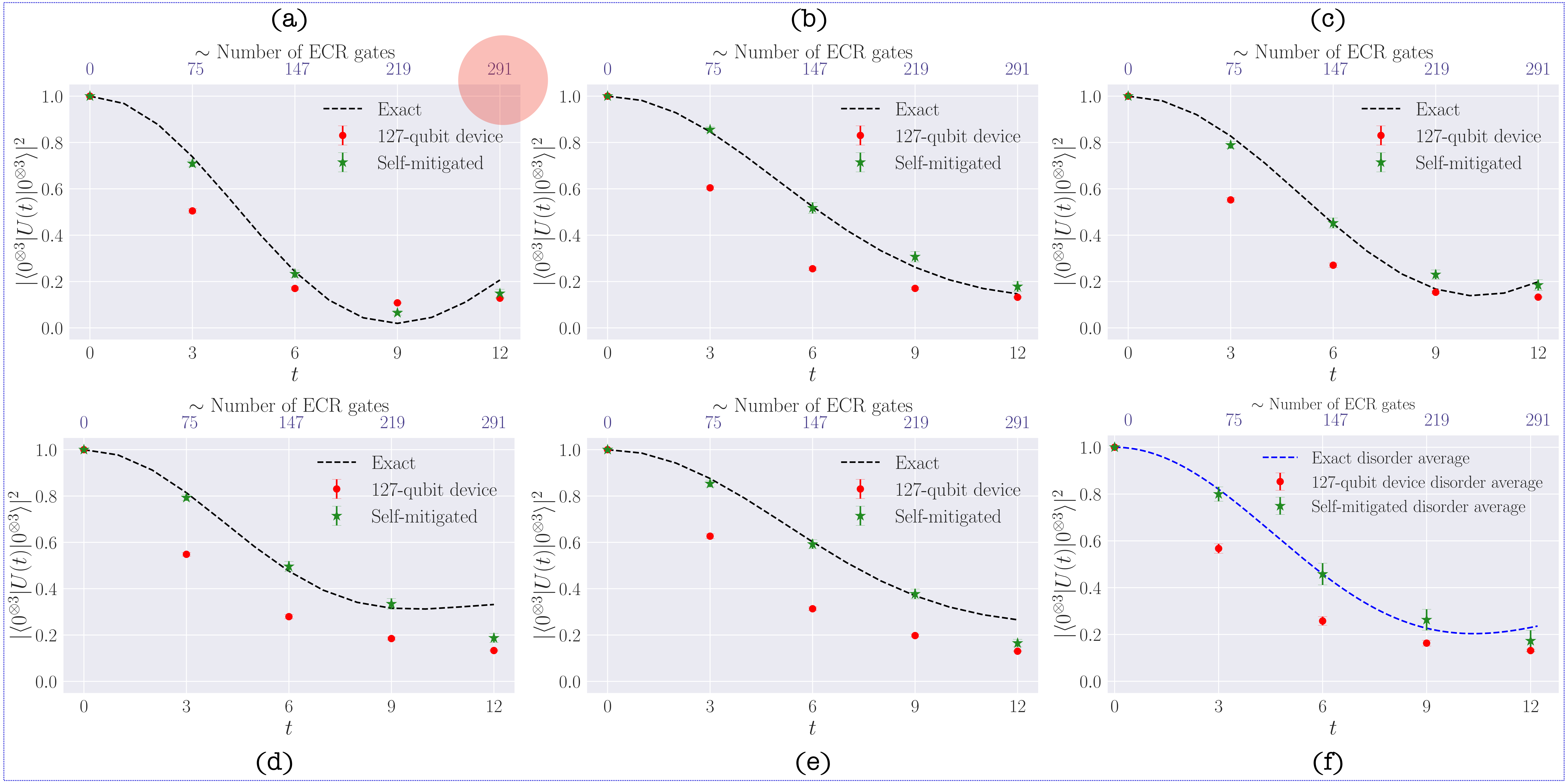
$|00\rangle$ 

IBM chip topology

- We used the quantum computers available through IBM to simulate the SYK model. The topology of the processor is shown below. In practice, we need more gates than necessary. For example, we show a combination of qubits we used for $N = 8$. This chip topology is 'heavy-hex'.



Return probability - IBM Results



Error Mitigation

- The results from the 127-qubit device (red) agrees slightly less than those with self-mitigation (green). The red points have been read from some fixed number of measurements/shots and post-processed with mild mitigation including M3 to correct read-out errors and DD to increase coherence time of qubits.

- To get closer to the exact results, we found that an idea similar to CNOT only mitigation (known as self-mitigation) seems to help drastically. Basic idea introduced in Urbanek et al. arXiv: 2103.08591 and extended to SU(2) work of Rahman et al. arXiv:2205.09247

M3 is a matrix measurement mitigation (MMM or M3) technique that solves for corrected measurement probabilities using a dimensionality reduction step

DD (dynamical decoupling) — a series of strong fast pulses are applied on the system which on average increases the lifetime of qubits and delays decoherence (or effect of interactions with environment)

CNOT-only and self-mitigation

- We saw previously that if the input state is $|0\rangle^{\otimes n}$, then applying any of CNOT will still result in the same input state. However, in practice, the errors of 2q gates (CNOT) is the dominant source of gate error in current devices.

- This can be used to quantify the errors occurring in the time-evolution circuit. Remove all the single-qubit gates from $\exp(-iHt)$ and apply it on the $|0\rangle^{\otimes n}$ state. Measure the output. The deviation from $|0\rangle^{\otimes n}$ is a measure of the probability of error and used to correct the expectation value of the observable. This is CNOT-only mitigation.

- However, this underestimates the error. Self-mitigation argues to not remove any gates from $\exp(-iHt)$. One constructs two circuits: Physics (P) and Self-Mitigated (SM) circuits and then run the P circuits for N Trotter steps and the SM mitigation circuit for $N/2$ Trotter steps with dt and the other $N/2$ with -dt. Note the error from SM circuits, use it to correct exp. value of P circuits.

Noise model: Quantum depolarizing channel

- An efficient way to model decoherence of qubit is to use a depolarising quantum channel which is a CPTP (completely-positive trace preserving, (Tr $\mathscr{E}(\rho) =$ Tr $\rho = 1$ and $\mathscr{E}(\rho) > 0$) map:

$$\mathscr{E}(\rho) = (1-p)\rho + p\mathbb{I}/2^n,$$

- If the quantum channel is free of noise, then the depolarising parameter (error rate) is $p = 0$.

- Once the error rate is determined from self-mitigation, we use it to correct the expectation value of the observable using $\langle O_n \rangle = (1-p)\langle O_c \rangle + (p/2^n)\mathrm{Tr}(\mathbb{I})$ where n and c are noisy and corrected value.

SYK model - Bound on chaos

- SYK model famously saturated the Lyapunov exponent i.e., $\lambda = 2\pi T$ for $J/T \gg 1$ when N is large.
- One considers $C(t) = -\langle [W(t), V(0)]\,[W(t), V(0)]\rangle$ and the expansion of the commutator gives OTOC := $\langle W(t)V(0)W(t)V(0)\rangle_\beta =$ Tr$(\rho W(t)V(0)W(t)V(0))$ which characterizes quantum chaos.
- Suppose one starts at $t = 0$, and computes also the two-pt correlator given by $\langle W(t)W(0)\rangle$, the time scales at which the lower order correlators decay is called 'dissipation time'. After this time, the OTOC grows as $\exp(\lambda t)$ and saturates beyond $t_\star$ known as scrambling time. Black holes are fastest scramblers!
- These correlators have been computed up to $N = 60$ numerically i.e., H has ~million terms and matrix has size ~billion. Hard for classical computers.

Out-of-time correlators (OTOC)

- So the goal is to compute $\langle W(t)V(0)W(t)V(0)\rangle_\beta$ on a quantum computer. Thermal correlators are currently not easy to compute due to limited resources. One simplification we can make is consider the $\beta \to 0$ limit of OTOC. This is not at all interesting for holography, but this is where we must start. Hence, the density matrix is just $\rho \propto \mathbb{I}$. As $\beta \to \infty$, we have the ground state (zero temp.) but as β is decreased, the state becomes mixed and it becomes maximally mixed at $\beta \to 0$.

- The unusual time-ordering of OTOC is also hard for quantum computers which often mean carrying out forward and backward evolution. We use a protocol (next slide) which uses only forward evolution to compute OTOC on quantum hardware.

Randomised Protocol

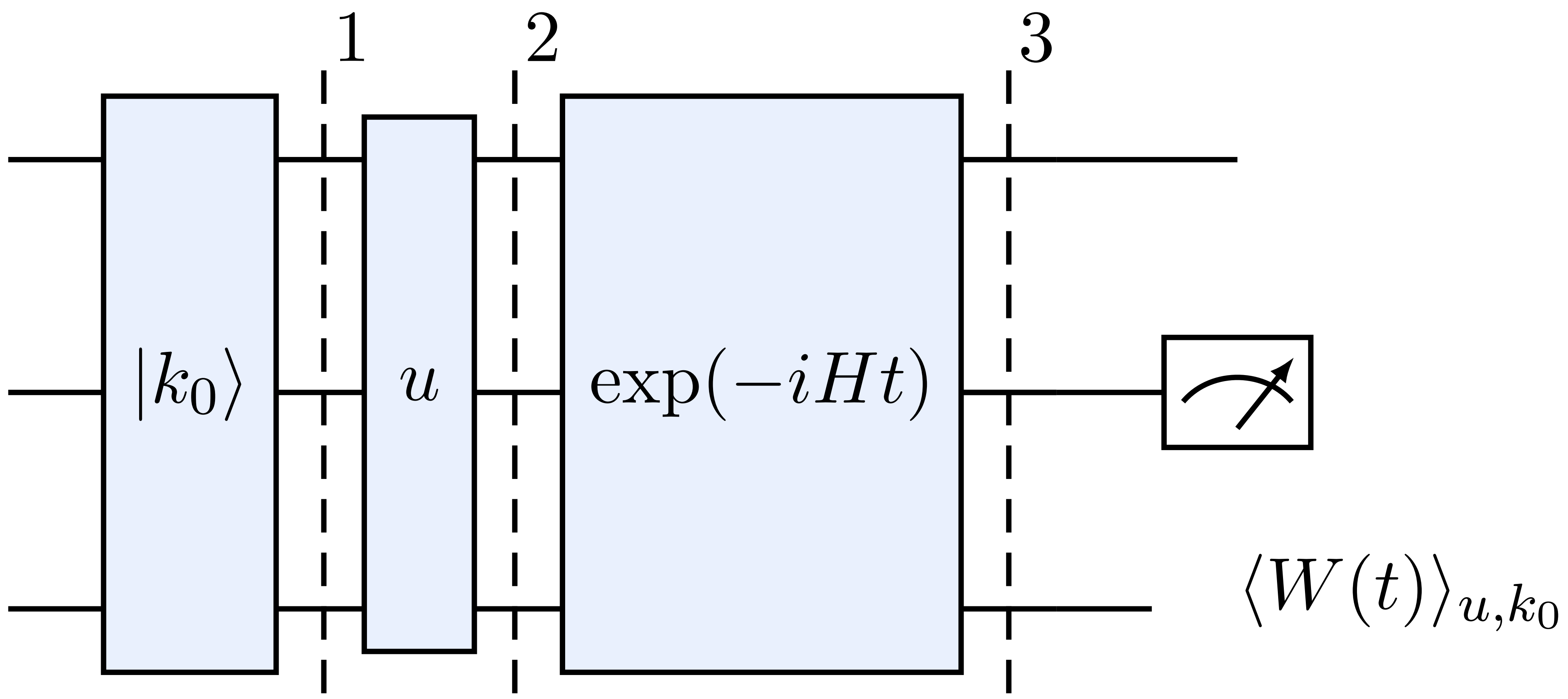
- There are various protocols to measure OTOC on quantum computers, see Swingle 2202.07060 for review.

- We use the one proposed in 1807.09087 now known as 'randomised protocol' since it computes OTOC through statistical correlations of observables measured on random states generated from a given matrix ensemble (CUE).

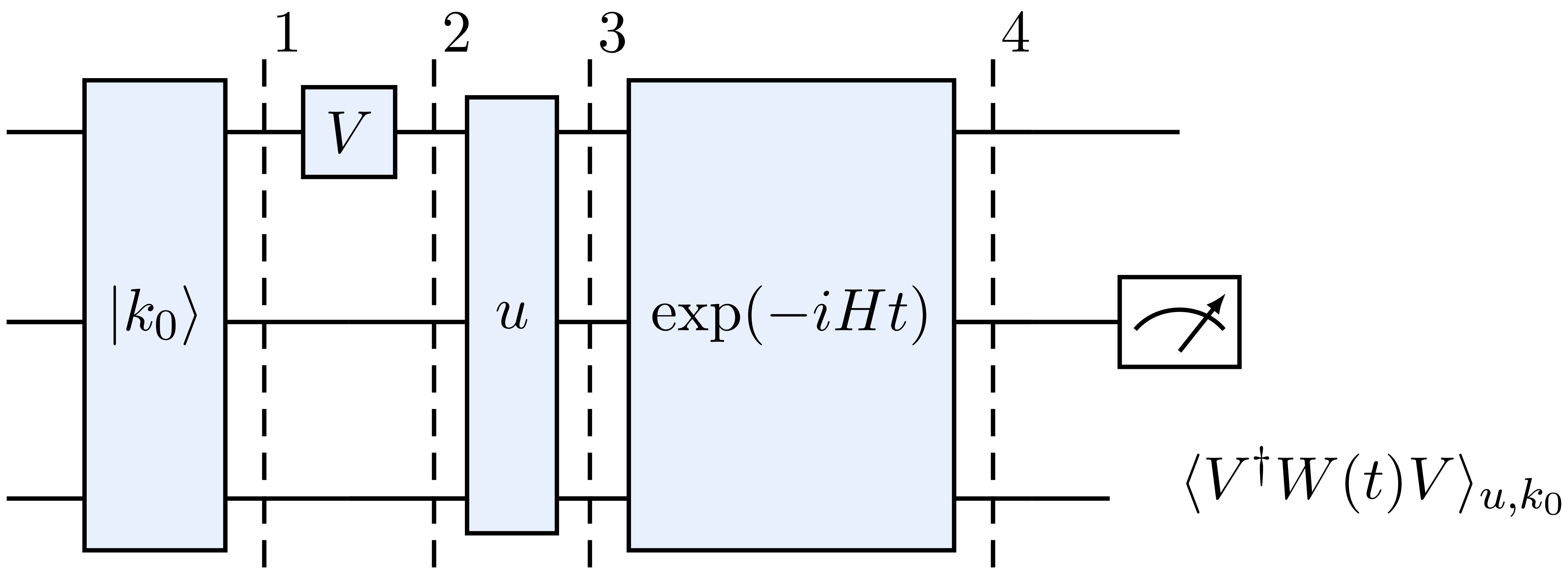
- Infinite-temp OTOC is given by $\mathrm{Tr}(W(t)V^\dagger W(t)V) \propto \overline{\langle W(t)\rangle_u \langle V^\dagger W(t)V\rangle_u}$ where the average is over different random states $|\psi_u\rangle$ prepared by acting with random unitary on arbitrary state say $|0\rangle^{\otimes n}$. Note that this protocol works when W is traceless operator.

Randomised Protocol

- We need two measurements (between which we compute the statistical correlation) and it is shown below. This is the global version of the protocol (since u has support over all qubits). There is also a local version of the protocol. Note that cost of decomposing arbitrary u increases exponentially, one can instead use unitary from a subset of Haar measure. They are called unitary t-designs* in literature.



(a)

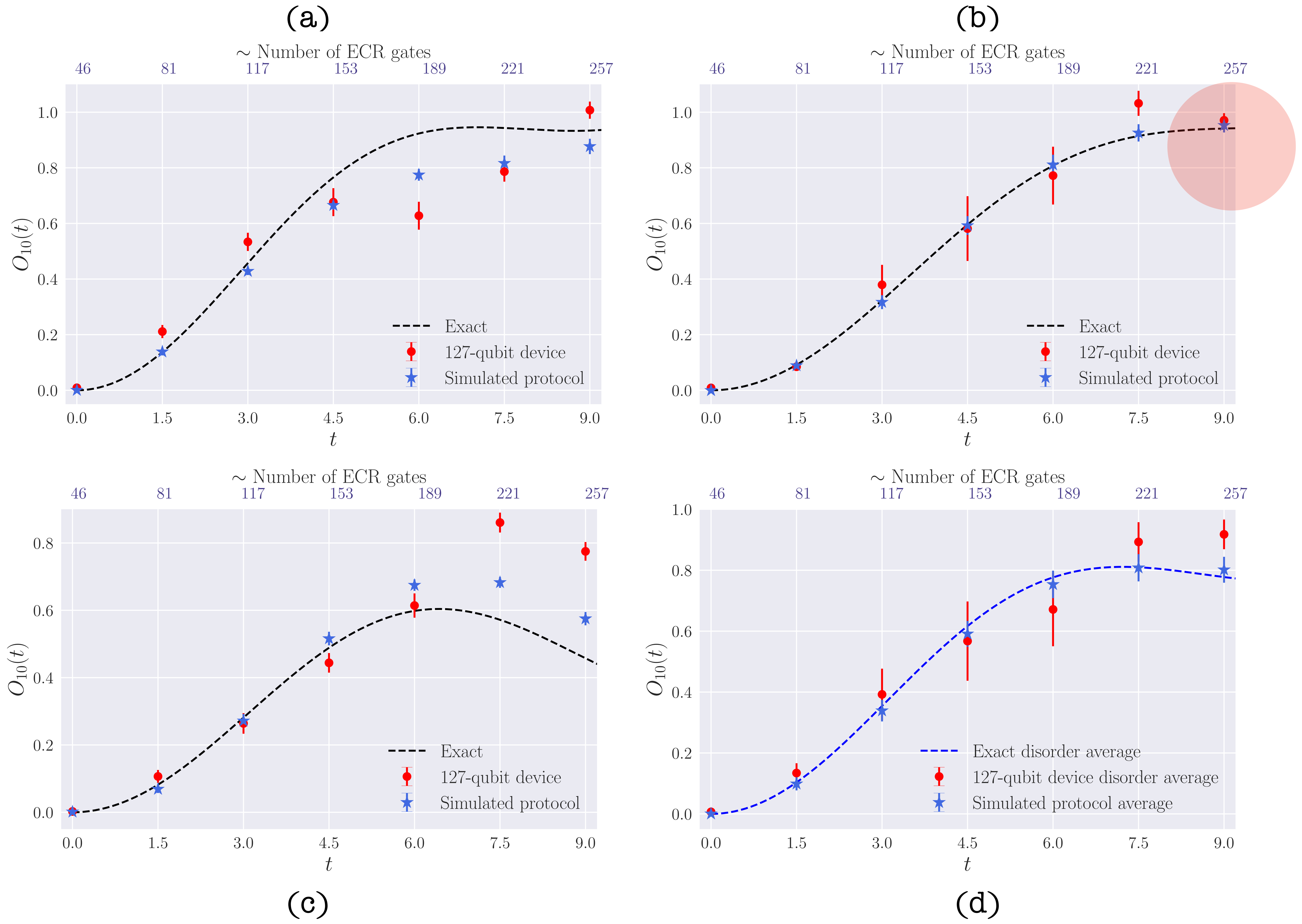


(b)

t-designs equivalent to first t moments of Haar group

OTOC Results

- We used ibm_cusco and ibm_nazca to obtain the results show for $N = 6$. We took simplest operators where both W and V were taken to be single Pauli. We see good agreement without need to do self-mitigation like we did for return probability.



Finite-temperature SYK model

- We considered OTOC measured over random states (maximally mixed) generated i.e., $\beta = 1/T = 0$. However, much of interesting Physics of the SYK happens in the region $\beta \gg 1$ and classical computations have argued that you need $\beta \sim 70$ to extract Lyapunov exponents close to the chaos bound.

- Finite-temperature OTOCs are difficult for quantum computers in general. No simple/general cost-effective protocol exists. To move towards this goal, we are studying the preparation of Gibbs (thermal) states on quantum computer for the SYK model.

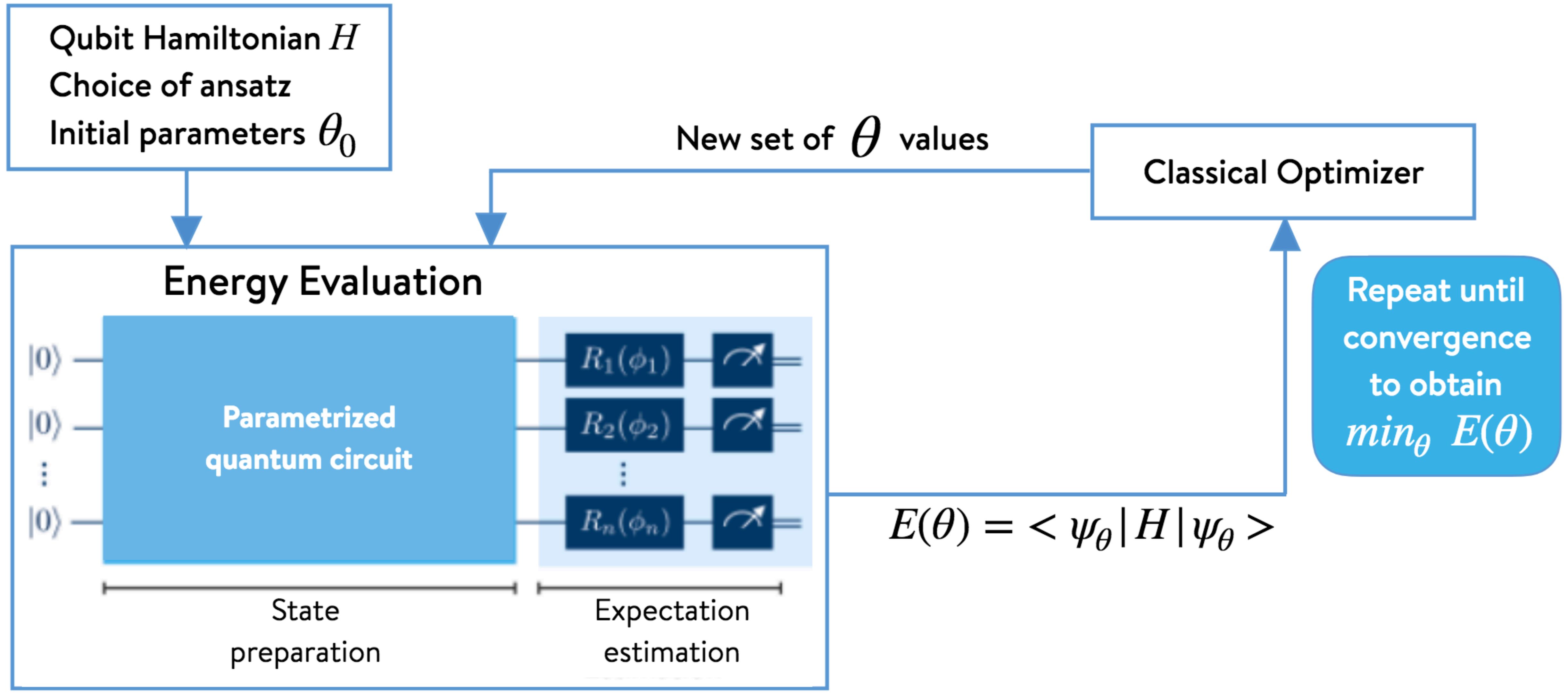
- In addition to the thermal state (mixed) of the SYK model, one can also consider a purification of this known as thermo-field double state (TFD). TFD state is a pure state (up to unitary trans.) of some other system (for ex: coupled SYK model) and when we perform partial trace over either system, we recover thermal state on the other one.

VQE algorithm

- Before we move to preparation of Gibbs state, let's us look at popular algorithm for preparing (approximate) ground states on QC.

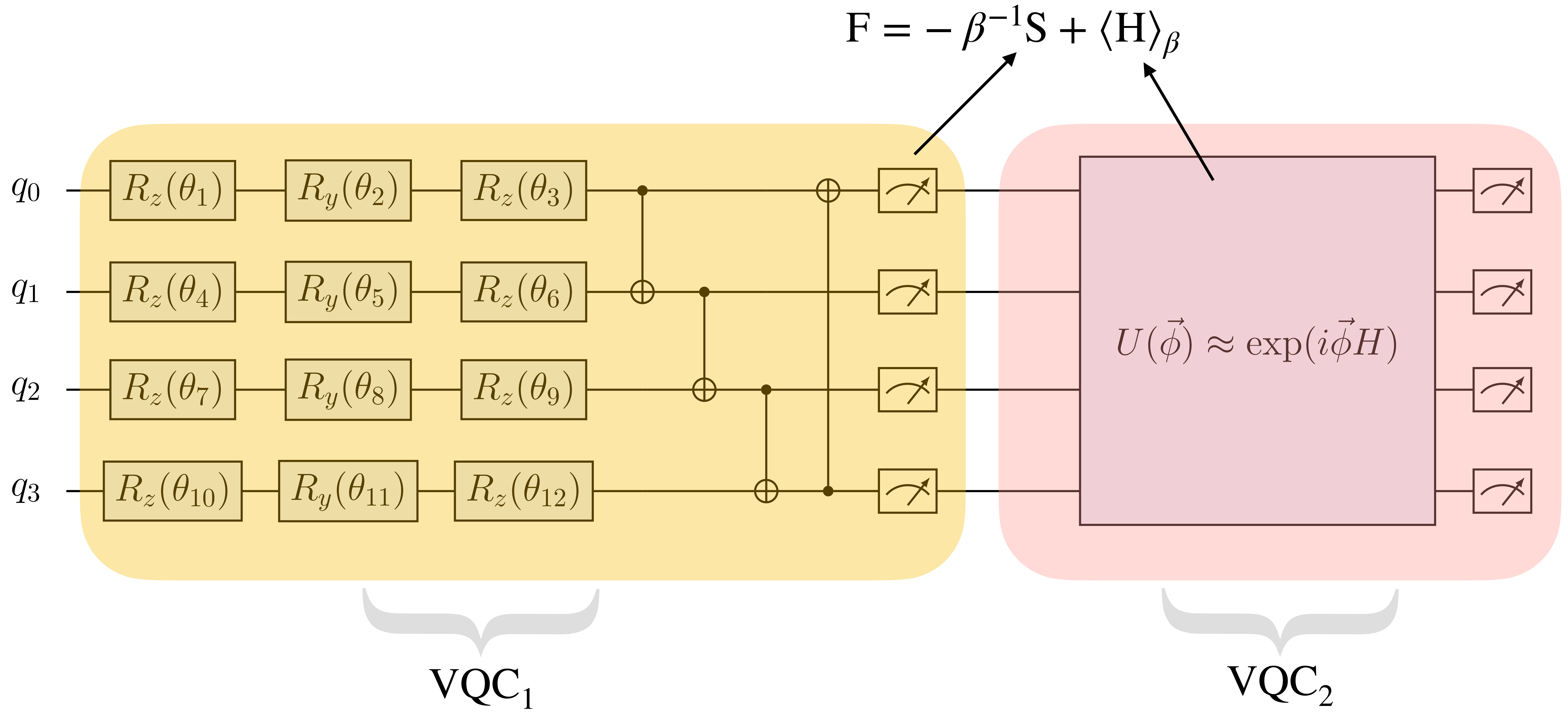
- Hybrid classical/quantum algorithm to find the ground state problem of a given Hamiltonian by finding the parameters of a quantum circuit ansatz that minimizes the Hamiltonian expectation value.

- It primarily consists of three steps: 1) Prepare initial ansatz on QC i.e., $|\psi(\vec{\Theta})\rangle$, 2) Measure energy on QC and optimise the parameters Θ using classical optimisers and 3) Repeat until desired convergence is achieved.



VQE for finite temperatures

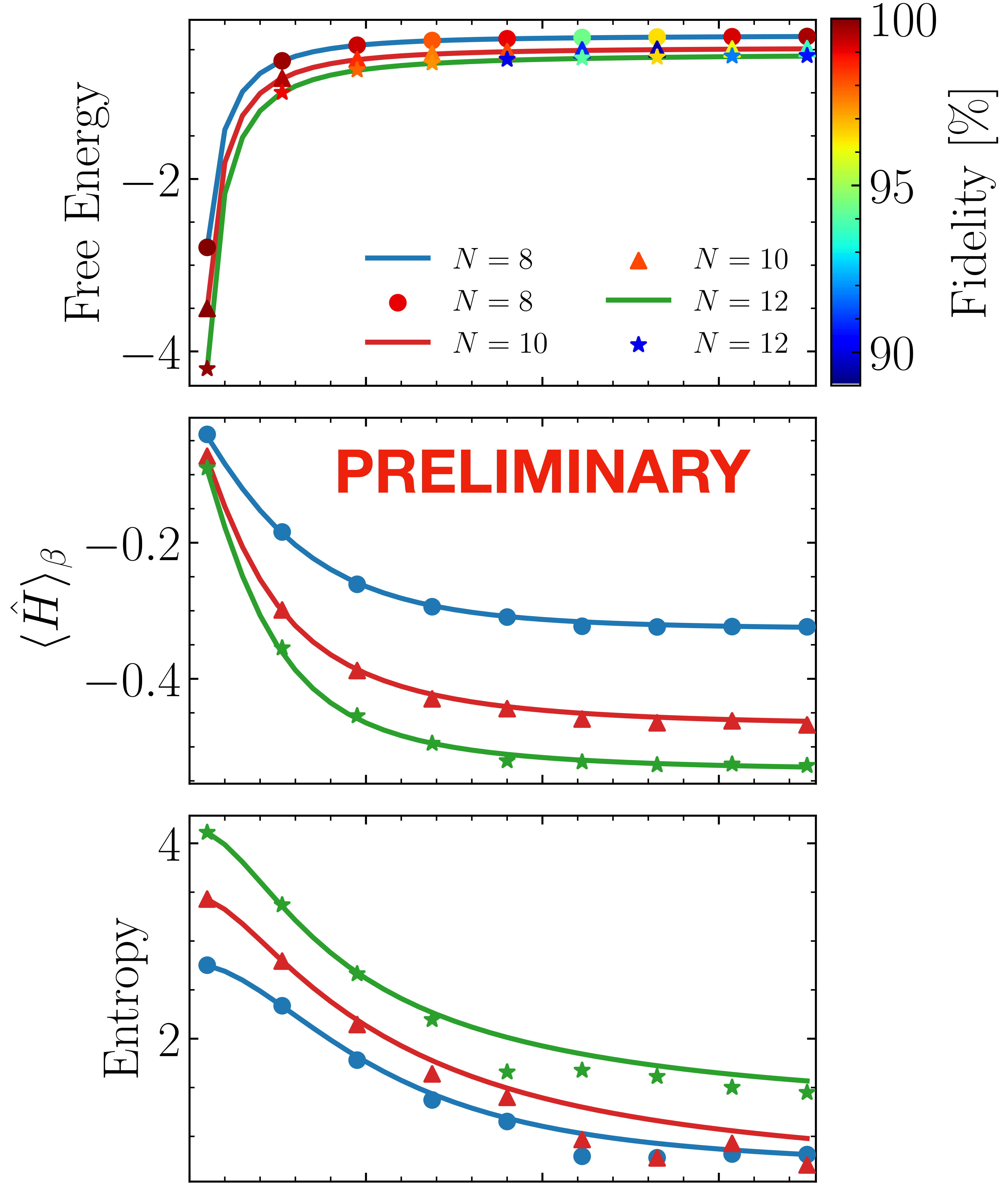
- Finite-temperature VQE methods are still an active area of research. Many proposals exist. The cost function is no longer E but rather $E - TS$ (free energy) which can be hard to compute on QC.



Selisko et al., 2208.07621

Finite-temperature SYK model

[upcoming work with J. Araz, B. Sambasivam, and F. Ringer]



Summary + Future directions

- We are entering an era where we can compute few things (even if they can be done quickly) using our laptops. Exploring these toy models will hopefully teach us better algorithms/methods.
- It is instructive to see that if we can characterise the noise in these quantum devices, we can mitigate and get reasonable results!
- However, there is nothing which we can do today on QCs which can't be done on laptops.
- Use of hybrid discrete/CV systems to model physical models is another potential direction
- Finding problems where quantum computers would really have the "edge" is a rewarding research area.

Thank you