Quantum Simulation of $O(3)$ model.

Raghav G. Jha
Jefferson Lab

① New York

30^{th} May, 2023

WISH LIST

↳ Real-time simulation of interacting QFT's (quantum field theories)

↳ Real-time simulation of QCD (at finite density?)

Lower the Expectations

We'll consider 1+1-dimensional O(3) model. This model has several features similar to QCD.

- asymptotic freedom
- existence of instantons
- mass-gap

3

O(3) model ..

Some work done in literature:

1) Ground state prep.

2) VQE upto some truncation using Pennylane/QISKIT etc.

We want to go → CV approach ...

Note: Even qubit approach has several interesting open questions for $\theta = 0$ & π.

Disclaimer

We are working on this model from two different viewpoints (Physics wise).

PART 1 : Limiting case of another model

PART 2 : Directly in terms of rotor models using Schwinger boson approach

O(3) model

$$S = \frac{1}{2g^2}\int d^2x \;\; \partial_\mu \vec{n} \cdot \partial^\mu \vec{n}$$

where $\vec{n} \in S^2$

and is a unit 3-vector

i.e $\boxed{n \cdot n = 1}$

We would like a Hamiltonian ..

↳ (1979) Hamer-Kogut-Susskind

6

$$\hat{\mathcal{H}}_{O(3)} = \frac{1}{2\beta}\sum_i L_i^2 - \beta \sum_{\langle ij \rangle} \vec{n}_i \cdot \vec{n}_j$$

Can add changes
But here only $\boxed{\theta = 0}$

Coupled Quantum rotors ...

$$[L^\alpha, L^\beta] = i\epsilon^{\alpha\beta\gamma} L^\gamma$$

$$[n^\alpha, n^\beta] = 0$$

$$[L^\alpha, n^\beta] = i\epsilon^{\alpha\beta\gamma} n^\gamma$$

$$\alpha, \beta, \gamma \in \{x, y, z\}$$

7

Suitable basis often used

$$\vec{L}^2 |\ell m\rangle = \ell(\ell+1)|\ell m\rangle$$

$$\ell \in \mathbb{N}^+$$

$$L^z |\ell m\rangle = m |\ell m\rangle$$

$$m \in [-\ell, \ell]$$

Now, we want to rewrite the H in terms of Bose operators to bring closer to CV language.

Schwinger in 50s

ON ANGULAR MOMENTUM

Julian Schwinger

The commutation relations of an arbitrary angular momentum vector can be reduced to those of the harmonic oscillator. This provides a powerful method for constructing and developing the properties of angular momentum eigenvectors. In this paper many known theorems are derived in this way, and some new results obtained. Among the topics treated are the properties of the rotation matrices; the addition of two, three, and four angular momenta; and the theory of tensor operators.

Schwinger (1952)

Map between angular momentum basis to oscillator basis.

In this case, we need two bosons describing two oscillators at each site to rewrite the O(3) Hamiltonian.

$$|l\,m\rangle = \frac{(a^{\dagger})^{l-m}\,(b^{\dagger})^{l-m}\,|0,0\rangle}{\sqrt{(l+m)!\,(l-m)!}}$$

a, b are two oscillators (qumodes) at each site.

$$n = a^{\dagger}a + b^{\dagger}b$$
$$= n_a + n_b$$

Kinetic term is easy !!

$$\vec{L}_i^{\,2} = \frac{n_i}{2}\left(\frac{n_i}{2} + 1\right)$$

where $n_i = n_a + n_b$

However, $\vec{n}_i \cdot \vec{n}_j$ term needs more work..

Note that

$$\vec{n}_i \cdot \vec{n}_j = \cos\theta_i \cos\theta_j + \frac{1}{2}\left[\sin\theta_i e^{i\phi_i} \sin\theta_j e^{-i\phi_j} + h.c\right]$$

Using relation with Spherical harmonics etc., one can write this as:

$$\vec{n}_i \cdot \vec{n}_j \propto \left(a_i^{\dagger} b_i^{\dagger} + a_i b_i\right)\left(a_j^{\dagger} b_j^{\dagger} + a_j b_j\right) + \cdots\cdots$$

Can we implement* this even for a 2-site model.??

(non-linear, quartic in bosonic op.) ↳ PART-2.

For ex:

Consider the Bose-Hubbard model

$$\mathcal{H} = -J \sum a_i a_{i+1}^{\dagger} + h.c + \frac{U}{2} \hat{n}_i (\hat{n}_i - 1)$$

14

Seems like a $\sqrt{}$ version of O(3) model? For the BH model, the Hamiltonian simulation is known in terms of BS, K, and rotation gates.

$$e^{-i\hat{H}t} = \Big[\overbrace{BS}^{\text{two-mode (hopping)}} \big[K \cdot R \otimes K \cdot R \big] \Big]^N + O\left(t^2/N\right)$$

$\boxed{\text{2-site version}}$.

15

Summary

We outlined our goal (WIP) to understand the simplest 1+1-d model with features similar to QCD using the CV formulation. Even a 2-site circuit to compute overlaps, such as $\langle \psi(t) | \psi(0) \rangle$ would be useful. Scattering in O(3) model and beyond is the eventual goal.

Thank You..

$$\mathcal{H} = \sum \frac{l(l+1)}{2\beta} - \beta \sum \left(n_+^i n_-^{i+1} + n_-^i n_+^{i+1} + n_z^i n_z^{i+1} \right)$$

Now, consider $l_{max} = \frac{1}{2}$

Size of H for N sites is :

$$(2l+1)^N \otimes (2l+1)^N$$

$$= 2^N \otimes 2^N$$

$$n^{\pm} = \frac{-1}{3\sqrt{2}} \left(\sigma_x \pm i \sigma_y \right), \quad n_z = \frac{1}{3} \sigma_z$$

$$D_\mu F_{\mu\nu} = 0 \longleftarrow$$ finite action solution to these are instantons

$$\downarrow$$

$$A_\mu \xrightarrow[\text{pure gauge}]{} U^+ \partial_\mu U$$

Otherwise $S \sim \int d^4x \left(\frac{1}{r^2}\right)^2$

$$\sim \log(r) \longrightarrow \infty$$

no finite action

$$F \propto \partial_\mu A$$

$$\propto \frac{1}{r^3} \Longrightarrow \boxed{A \propto \frac{1}{r^2}}$$

But what if $\underline{\underline{F=0}}$

(possible when A is some GT of zero). Such a configuration is called pure gauge.

$$A_\mu = V \partial_\mu V^+$$

$$
\begin{aligned}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \\
&= \partial_\mu V \partial_\nu V^+ + V \partial_\mu \partial_\nu (V^+) \\
&\quad - \partial_\nu V \partial_\mu V^+ \\
&\quad - V
\end{aligned}
$$

zero

Perform GT i.e.

$$A_\mu \to U A_\mu U^{-1} + U \partial_\mu U^{-1}$$

on pure gauge i.e $V \partial_\mu V^{-1}$

$$V \partial_\mu V^{-1} = U V \partial_\mu V^{-1} U^{-1} + U \partial_\mu U^{-1}$$

$$= U V \partial_\mu (V^{-1} U^{-1}) - U V V^{-1} \partial_\mu U^{-1} + U \partial_\mu U^{-1}$$

$$= U V \partial_\mu (U V)^{-1}$$

✓