Quantum computation of random Hamiltonians

Based on Phys. Rev. D **109**, 105002 (2024) with Asad and B. Sambasivam
+ upcoming work

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Outline

- Sachdev-Ye-Kitaev (SYK) model of holography — an example of random and all-to-all interaction
- Quantum gates and real-time evolution using quantum circuits
- SYK model with N = 6, 8 Majorana fermions on quantum hardware with error mitigation - real-time dynamics. System size: 3 and 4 qubits but very deep Trotter circuits!
- Upcoming work related to ground state preparation of the volume law entangled state
- Summary and future directions

Quantum computing **cannot** solve all problems

- It turns out that for majority of problems, quantum computers would do no better than classical computers. A major research direction is to understand which problems can be solved efficiently by QCs.

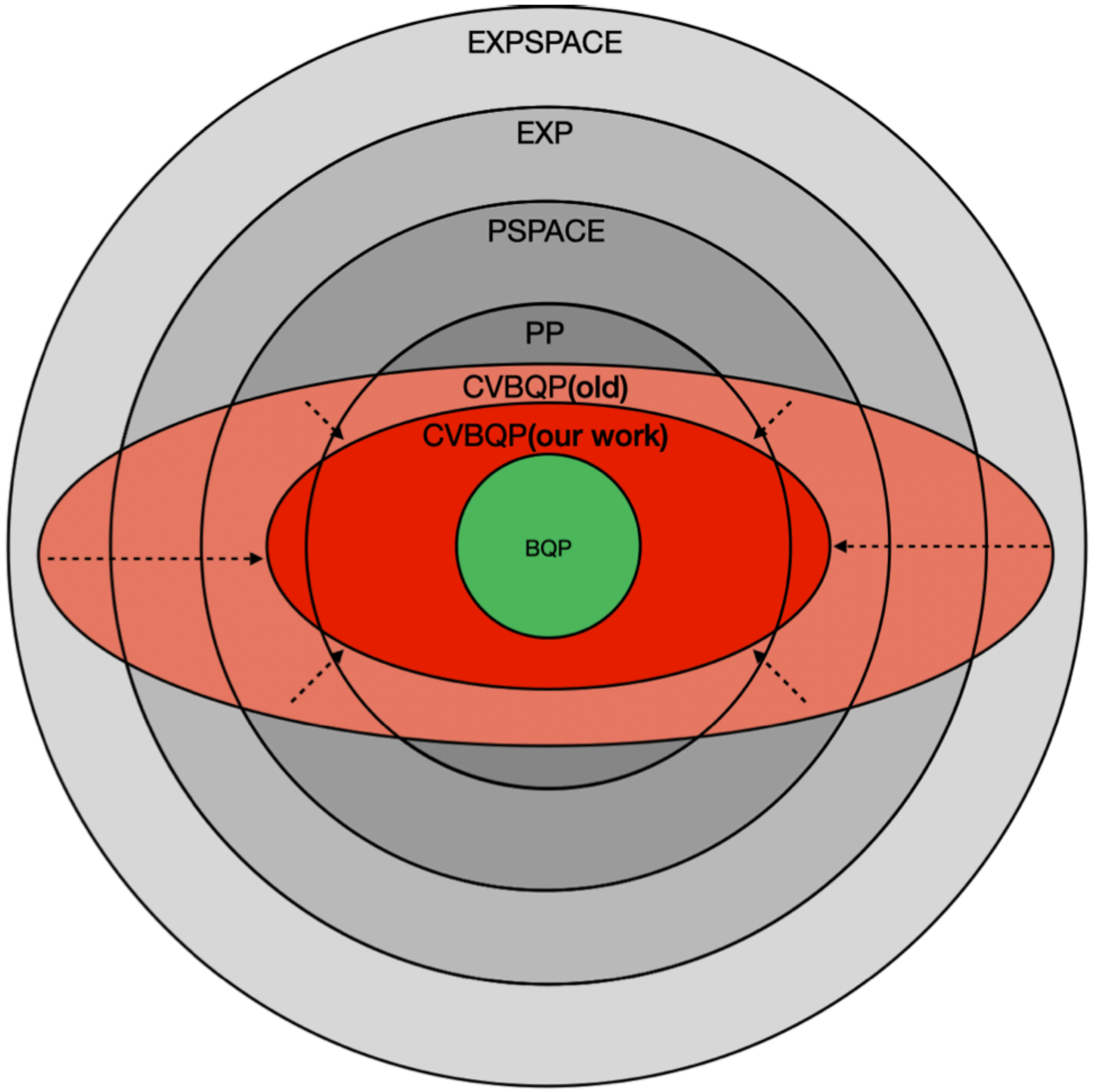
- For example, we know that scattering in ϕ^4 field theory can be solved efficiently by quantum computers [Jordan, Krovi, Lee, Preskill: arXiv: 1703.00454]. It is BQP-complete. Class of hardest type of problem within the BQP class, meaning it is in BQP + any other problem in BQP can be efficiently reduced to it using classical probabilistic algorithms.

- Class of problems which are best suited for quantum advantage belong to complexity class BQP. For ex: Shor's algorithm. Also Grover's algorithm but not as nice as Shor's (only polynomial speed-up).

- Goal is to understand how well can it do for random fermionic and bosonic Hamiltonians. This talk would be about random fermionic part. Can one study a mixed fermionic/bosonic random Hamiltonian efficiently using CV/DV approach?

Complexity approach to utility

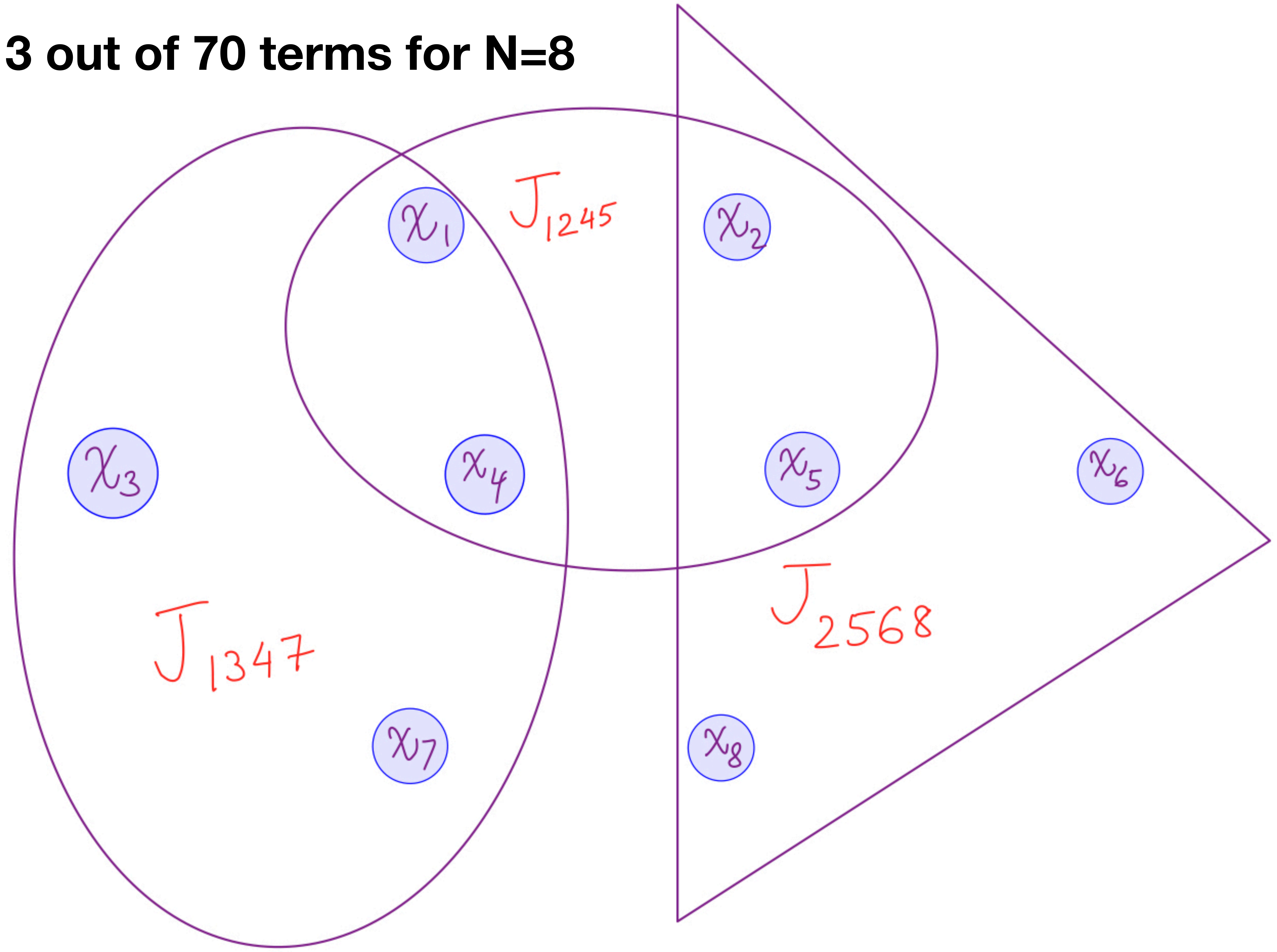
arXiv: 2503.03600
Chabaud et al.



SYK model

$$H = \frac{(i)^{q/2}}{q!} \sum_{i,j,k,\cdots,q=1}^{N} J_{ijk\cdots q} \quad \chi_i \chi_j \chi_k \cdots \chi_q,$$

3 out of 70 terms for N=8



- Model of N Majorana fermions with q-interaction terms with random coupling taken from a Gaussian distribution with $\overline{J_{\dots}} = 0, \ \overline{J_{\dots}^2} = \frac{q!J^2}{N^{q-1}}$.

- The fermions χ satisfy, $\chi_i \chi_j + \chi_j \chi_i = \delta_{ij}$. We will set $J = 1$. Note that it has units of energy and inverse time.

- In the limit of large number of fermions with $N \gg \beta J \gg 1$, the model has several interesting features such as maximal Lyapunov exponent.

Mapping fermions to qubits

$$\chi_{2k-1} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) X_k \mathbb{1}^{\otimes (N-2k)/2} \quad , \quad \chi_{2k} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) Y_k \mathbb{1}^{\otimes (N-2k)/2}$$

- N Majorana fermions requires N/2 qubits. We use the standard Jordan-Wigner mapping to write χ in terms of Pauli matrices X, Y, Z, and Identity.
- The SYK Hamiltonian is then written as sum of Pauli strings. The number of strings is $\binom{N}{q}$ and grows like $\sim N^q$. Simplest non-trivial case for is $N = q$ with one term. We restrict to $q = 4$.

Simplest case: N=4

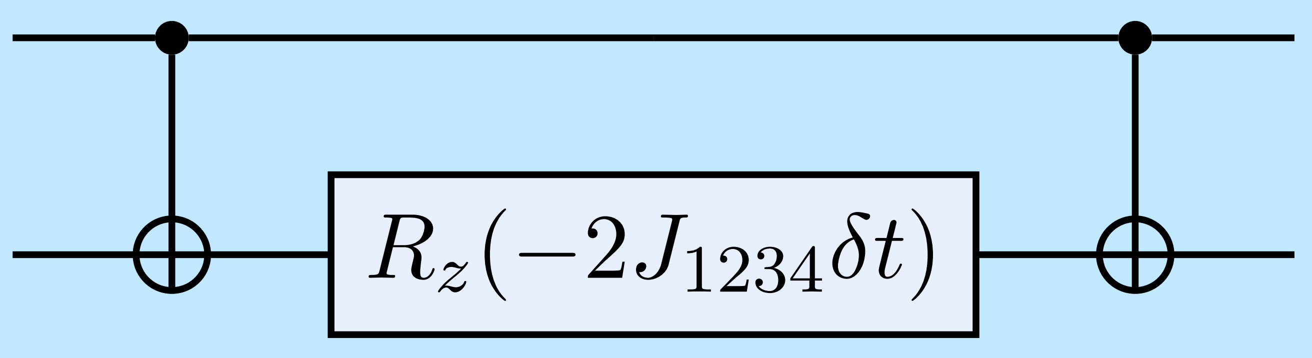
$$H = J_{1234}\chi_1\chi_2\chi_3\chi_4$$

$$\chi_1 = X\mathbb{1},\ \chi_2 = Y\mathbb{1},\ \chi_3 = ZX,\ \chi_4 = ZY$$

$$H = J_{1234}(X\mathbb{1}) \cdot (Y\mathbb{1}) \cdot (ZX) \cdot (ZY) = -J_{1234}ZZ$$

- The goal of quantum computation is to construct a unitary operator corresponding to this Hamiltonian. So, for this case we have $\exp(-iHt) = \exp(iJ_{1234}ZZt)$.

- This circuit is simple to construct and just needs 2 CNOTs and 1 rotation gate.



Circuit complexity

Definition: How many 2q-gates do we need to simulate the SYK model?

- Different approaches can be used to do the Hamiltonian simulation (aka time evolution). A popular method is Trotter method. It is based on Lie-Suzuki-Trotter product formula* (writing $H = \sum_{j=1}^{m} H_j$, $m \sim N^4$)

$$e^{-iHt} = \left(\prod_{j=1}^{m} e^{-iH_j t/r}\right)^r + \mathcal{O}\left(\sum_{j<k} \left|\left|[H_j, H_k]\right|\right| \frac{t^2}{r}\right),$$

- Depending on what error (ϵ) we desire in the time-evolution from the second term, we can compute the number of slices (r) we need to take. So, the complexity reduces to finding number of 2q-gates for each Trotter step. Recall that $N = 4$ needed just 2 2q-gates for each Trotter step.

* Corollary of Zassenhaus formula i.e., exp(t(X+Y)) = exp(tX) exp(tY) + O(t^2) (also known as dual of BCH formula).

Old work(s)

$\mathscr{C} = \mathcal{O}(N^{10}t^2/\epsilon)$ L. García-Álvarez et al., PRL 119, 040501 (2017)

$\mathscr{C} = \mathcal{O}(N^{8}t^2/\epsilon)$ Susskind, Swingle et al., arXiv: 2008.02303 (2020)

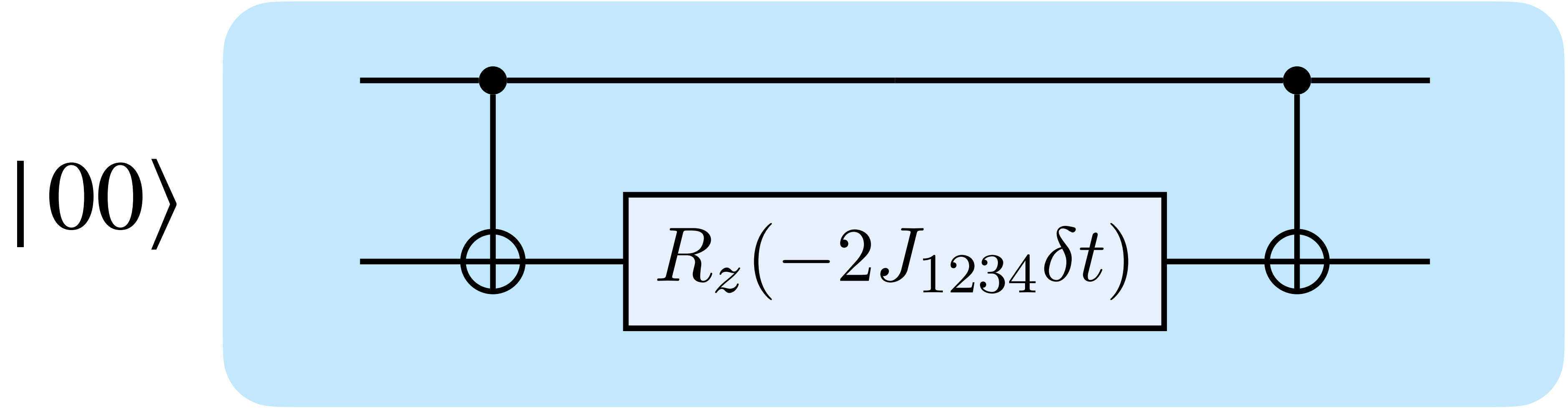
- In our paper, we improved the complexity to $\mathscr{C} = \mathcal{O}(N^5t^2/\epsilon)$ using first-order PFs. Some follow up have since improved our estimate. Current best estimate is $\mathscr{C} = \mathcal{O}(N^{4.5}t^{1+o(1)})$ using higher-order PF [Chen et al., 2502.18420].

Return probability

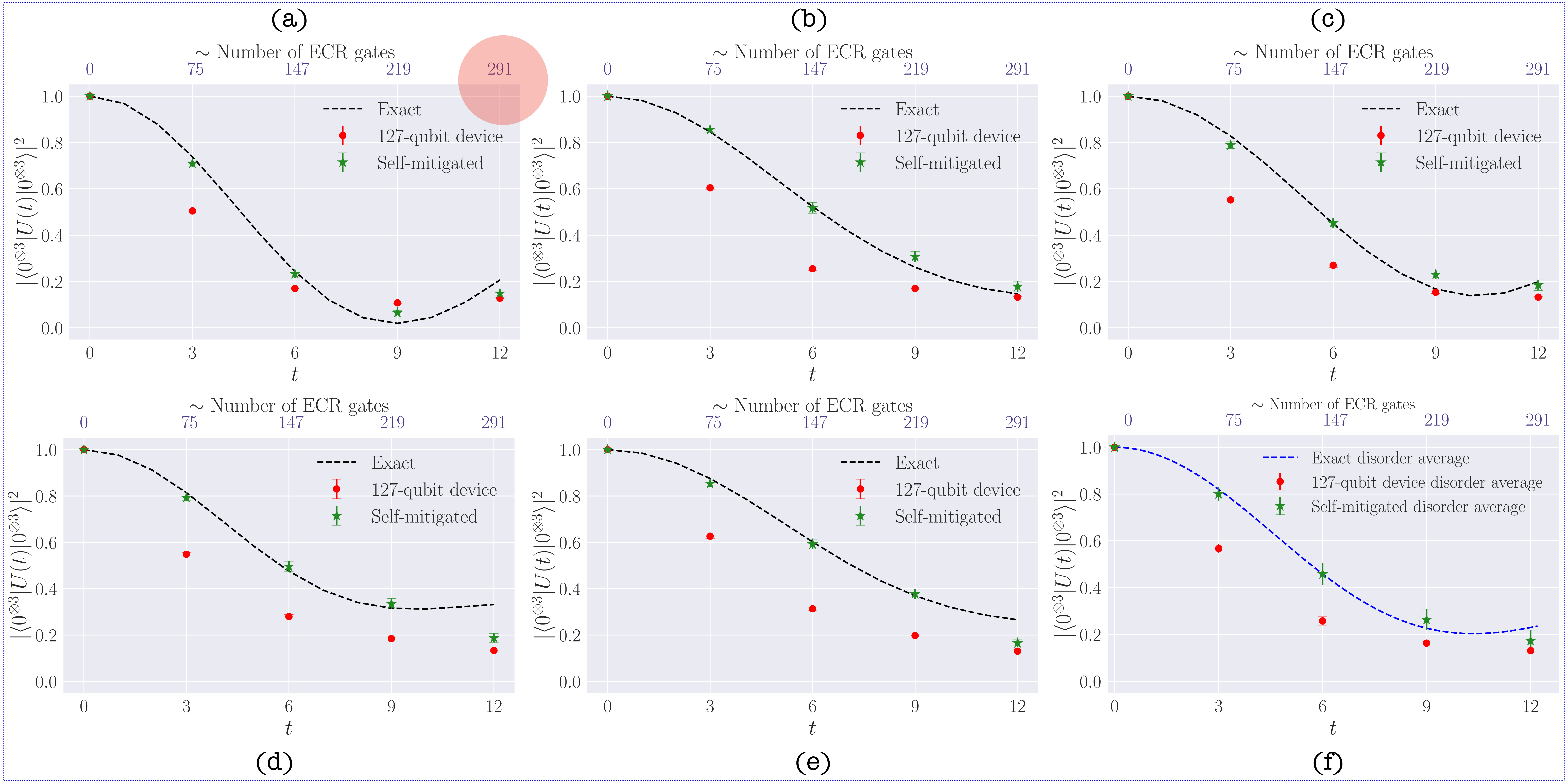
- A simple observable we can compute is the probability that we return to the same initial state after some evolution time t i.e., $\mathscr{P}_0 = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|^2$. For initial state, we take $|0\rangle^{\otimes N/2}$.

- For approximating the unitary, we use the first-order product formula and construct the corresponding quantum circuit.

- For $N = 4$, we have a simple circuit of only two 2Q gates, so the entire circuit for return prob. is straightforward. For $N = 6$, there are 30 2Q gates per step which we cannot show here.



Return probability - IBM Results



Error Mitigation

- The results from the 127-qubit device (red) agrees slightly less than those with self-mitigation (green). The red points have been read from some fixed number of measurements/shots and post-processed with mild mitigation including M3 to correct read-out errors and DD to increase coherence time of qubits.

- To get closer to the exact results, we found that an idea similar to CNOT only mitigation (known as self-mitigation) seems to help drastically. Basic idea introduced in Urbanek et al. arXiv: 2103.08591 and extended in Rahman et al. arXiv:2205.09247

M3 is a matrix measurement mitigation (MMM or M3) technique that solves for corrected measurement probabilities using a dimensionality reduction step

DD (dynamical decoupling) — a series of strong fast pulses are applied on the system which on average increases the lifetime of qubits and delays decoherence (or effect of interactions with environment)

SYK model - Bound on chaos

- SYK model famously saturated the Lyapunov exponent i.e., $\lambda = 2\pi T$ for $J/T \gg 1$ when N is large.

- One considers $C(t) = -\langle [W(t), V(0)]\,[W(t), V(0)] \rangle$ and the expansion of the commutator gives OTOC := $\langle W(t)V(0)W(t)V(0) \rangle_\beta =$ Tr$(\rho W(t)V(0)W(t)V(0))$ which characterizes quantum chaos.

- Suppose one starts at $t = 0$, and computes also the two-pt correlator given by $\langle W(t)W(0) \rangle$, the time scales at which the lower order correlators decay is called 'dissipation time'. After this time, the OTOC grows as $\exp(\lambda t)$ and saturates beyond $t_\star$ known as scrambling time. Black holes are fastest scramblers!

- These correlators have been computed up to $N = 60$ numerically i.e., H has ~million terms and matrix has size ~billion. Hard for classical computers.

Out-of-time correlators (OTOC)

- So the goal is to compute $\langle W(t)V(0)W(t)V(0)\rangle_\beta$ on a quantum computer. Thermal correlators are not easy to compute! One simplification we can make is consider the $\beta \to 0$ limit of OTOC. This is not at all interesting for holography, but this is where we must start. Hence, the density matrix is just $\rho \propto \mathbb{I}$.

- The unusual time-ordering of OTOC is also hard for quantum computers which often mean carrying out forward and backward evolution. We use a protocol (next slide) which uses only forward evolution to compute OTOC on quantum hardware.

Randomised Protocol

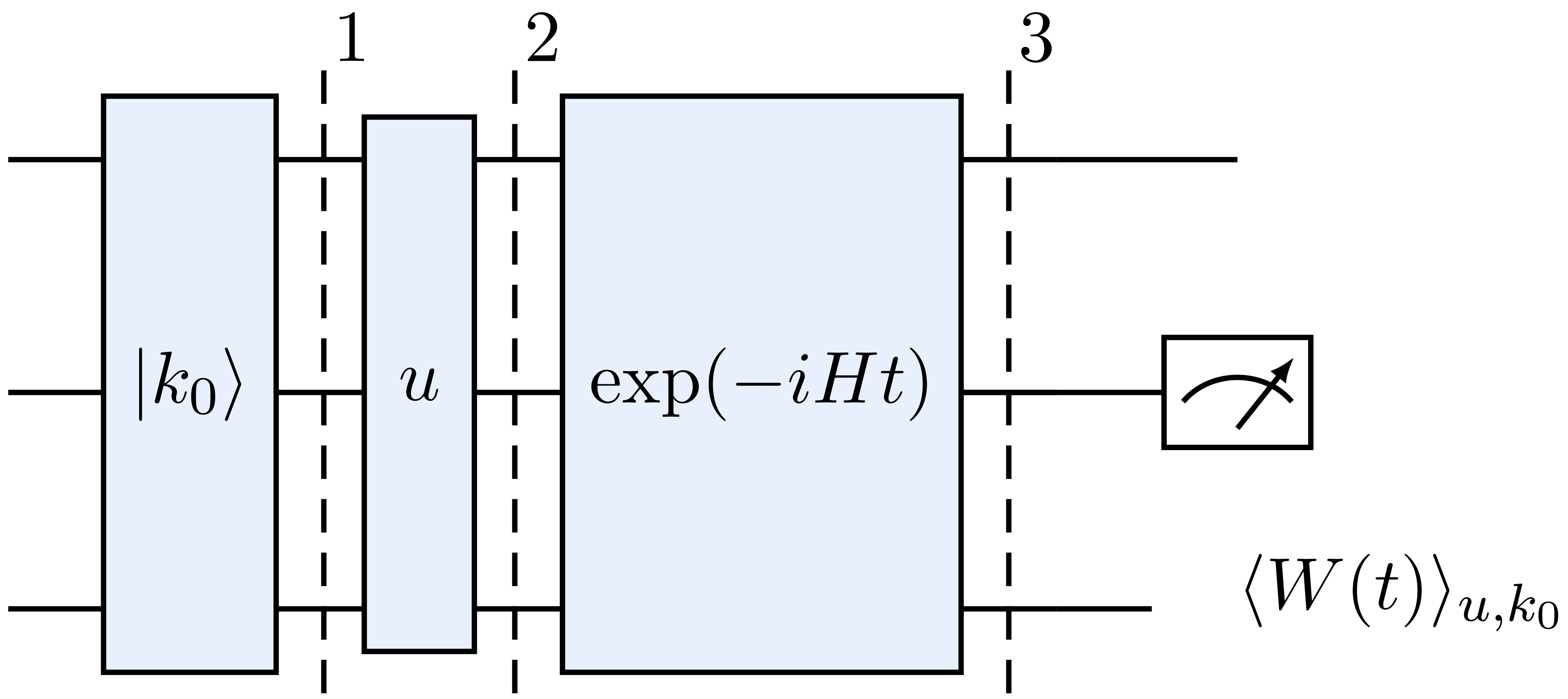
- There are various protocols to measure OTOC on quantum computers, see Swingle 2202.07060 for review.

- We use the one proposed in 1807.09087 now known as 'randomised protocol' since it computes OTOC through statistical correlations of observables measured on random states generated from a given matrix ensemble (CUE).

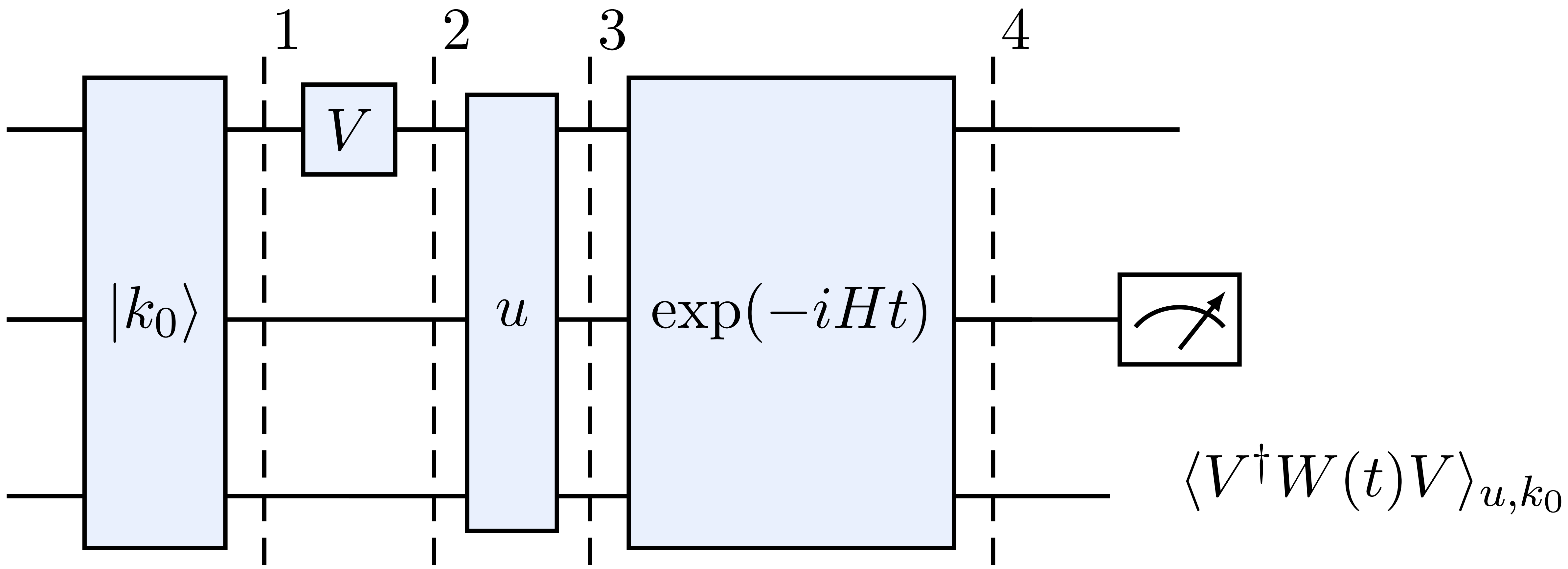
- Infinite-temp OTOC is given by Tr$(W(t)V^\dagger W(t)V) \propto \overline{\langle W(t)\rangle_u \langle V^\dagger W(t) V\rangle_u}$ where the average is over different random states $|\psi_u\rangle$ prepared by acting with random unitary on arbitrary state say $|0\rangle^{\otimes n}$. Note that this protocol works when W is traceless operator.

Randomised Protocol

- We need two measurements (between which we compute the statistical correlation) and it is shown below. This is the global version of the protocol (since u has support over all qubits). There is also a local version of the protocol. Note that cost of decomposing arbitrary u increases exponentially, one can instead use unitary from a subset of Haar measure. They are called unitary t-designs* in literature or, we can use pseudorandom unitaries (PRUs)



(a)

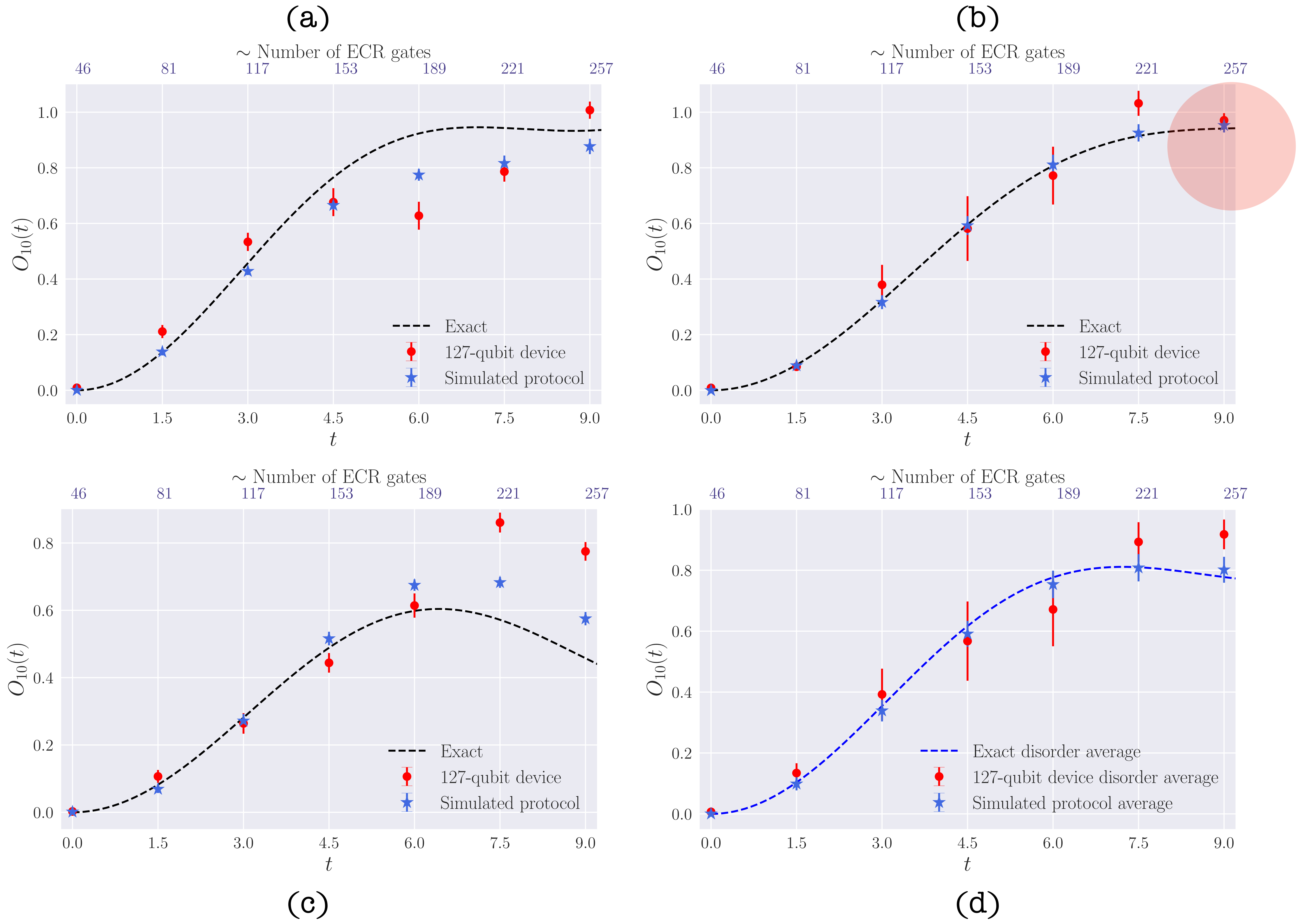


(b)

t-designs equivalent to first t moments of Haar group

OTOC Results

- We used ibm_cusco and ibm_nazca to obtain the results show for $N = 6$. We took simplest operators where both W and V were taken to be single Pauli. We see good agreement without need to do self-mitigation like we did for return probability.



Finite-temperature SYK model

- We considered OTOC measured over random states (maximally mixed) generated i.e., $\beta = 1/T = 0$. However, much of interesting Physics of the SYK happens in the region $\beta \gg 1$ and classical computations have argued that you need $\beta \sim 70$ to extract Lyapunov exponents close to the chaos bound.

- Finite-temperature OTOCs are difficult for quantum computers in general. No simple/general cost-effective protocol exists.

- In addition to the thermal state (mixed) of the SYK model, one can also consider a purification of this known as thermo-field double state (TFD). TFD state is a pure state (up to unitary trans.) of some other system (for ex: coupled SYK model) and when we perform partial trace over either system, we recover thermal state on the other one.

Ground state of the SYK model

Upcoming work

- While we were the first to study time dynamics of the SYK on a quantum computer, since then there have been other papers on real-time dynamics, however, a quantum computational approach to ground state preparation is still lacking.

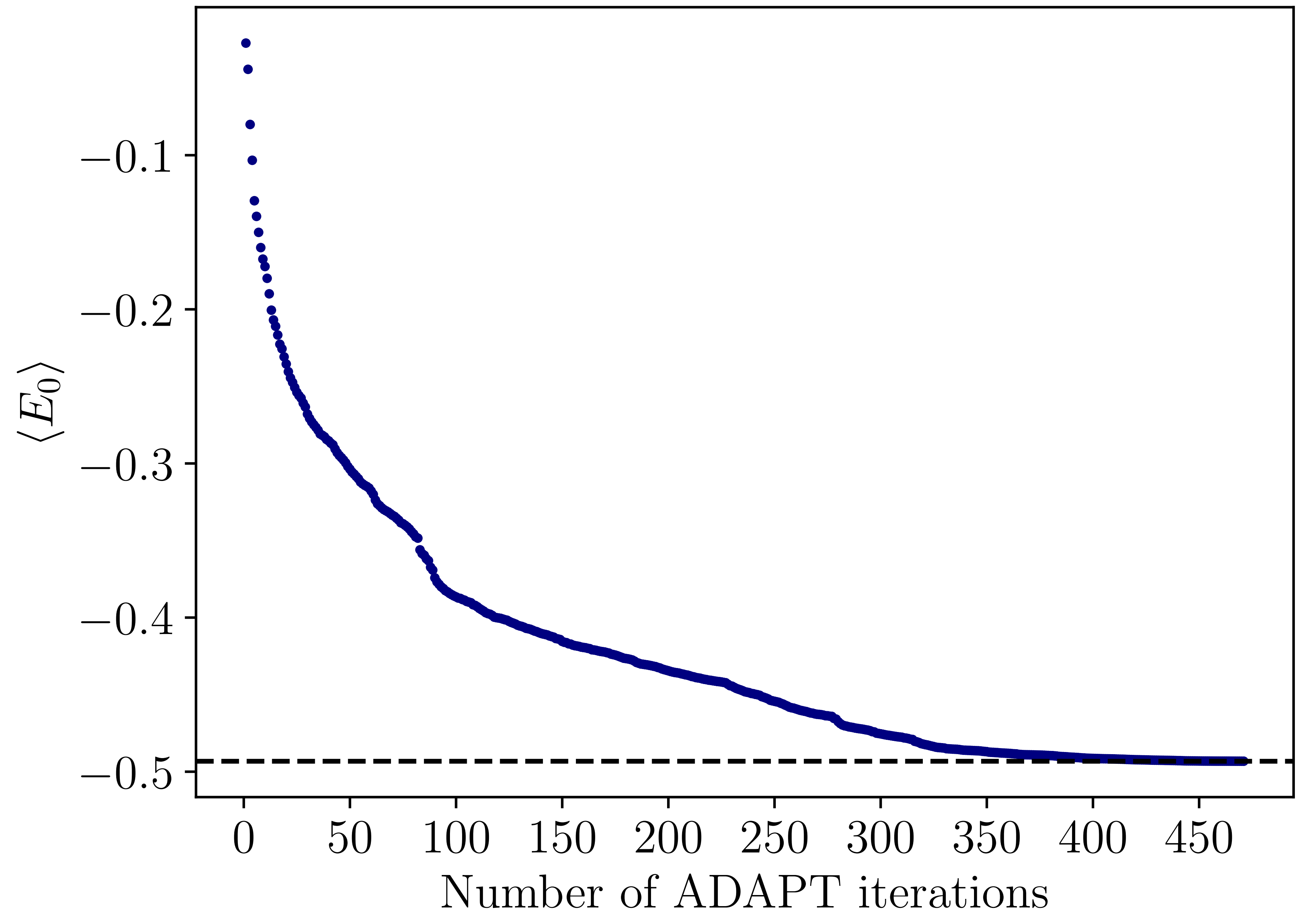
- Why is that so hard? Part of the reason is because the GS is very entangled. For ex: the bipartite vN entropy scales like $S \sim N$. And for $N = 20$ i.e., 10 qubits, we have 4845 terms in the Hamiltonian.

- However, the Hamiltonian has a parity symmetry, $[H, P] = 0$ where $P = \prod_i Z_i$ and we can use symmetry inspired pool with ADAPT-VQE and TETRIS algorithm [arXiv:2209.10562] to build an efficient and expressive circuit that can reproduce the ground state to good fidelity.

Ground state of the SYK model

Upcoming work

- We start with some initial state and the carry out k iterations to end with an ansatz i.e., $|\psi_{\text{ADAPT}}(\vec{\theta})\rangle^{(k)} = e^{\theta_k \hat{A}_k} \cdots e^{\theta_2 \hat{A}_2} e^{\theta_1 \hat{A}_1} |\psi_{\text{start}}\rangle$. For $N = 18$, we could reach good convergence with about $k \sim 450$ where the circuit is of CX depth ~ 810. We have been able to extend this to $N = 24$ but it becomes hard.



H has 3060 terms

Ground state of the SYK model

Upcoming work

- However, we know that this is likely to happen since we know that for SYK, the dynamical Lie algebra is $\dim(\mathfrak{g}) = 4^n$ for $n \gg 1$ and therefore one would run into the well-known barren plateau problems since variance vanishes exponentially!

- It is likely that ground state preparation of dense SYK is a QMA-hard problem. We don't know!

- We are also exploring the sparse version of the random Hamiltonian where rather than keeping $\sim N^4$ terms in the Hamiltonian, we just keep $O(N)$ terms. Should the training/QAOA be hard or easy? We find it to be the same!

- What happens for other volume-law entangled states? There has been no work to show whether quantum algorithms can surpass or match classical state-of-the-art (neural quantum states aka NQS)?

- Is there any quantum advantage to preparing ground states **in practice**?

Ground state of the SK model

Upcoming work

- There is another famous random, all-to-all Hamiltonian (in 1d) known as quantum Sherrington-Kirkpatrick (SK) model. This model was initially proposed to understand the spin glass systems.

- The Hamiltonian is: $H = -\sum_{i,j>i} J_{ij} Z_i Z_j - \Gamma \sum_i X_i$ and has volume law entanglement. Another class of hard models to study.

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Variational *Ansatz* for the Ground State of the Quantum Sherrington-Kirkpatrick Model

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We present an *Ansatz* for the ground states of the quantum Sherrington-Kirkpatrick model, a paradigmatic model for quantum spin glasses. Our *Ansatz*, based on the concept of generalized coherent states, very well captures the fundamental aspects of the model, including the ground state energy and the position of the spin glass phase transition. It further enables us to study some previously unexplored features, such as the nonvanishing longitudinal field regime and the entanglement structure of the ground states. We find that the ground state entanglement can be captured by a simple ensemble of weighted graph states with normally distributed phase gates, leading to a volume law entanglement, contrasting with predictions based on entanglement monogamy.

DOI: 10.1103/PhysRevLett.129.220401

Summary

- We are entering an era where we can do small computations reliably on quantum computers. Exploring these toy models will hopefully reveal to us better algorithms/methods. But, there is a long way to go using both DV, hybrid CV/DV based approaches.
- What we discussed here is one part of the story. The next step is to study random bosonic Hamiltonians using CV approach.
- Random Hamiltonians have deep connections to nuclear physics, quantum gravity, and spin glass physics, and understanding them with quantum computers would (hopefully) be useful.

Thank you