Toy models for physicists

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Transverse Field Ising Model (TFIM)

$$\mathcal{H} = -J\sum_i \sigma_i^z\sigma_{i+1}^z - h\sum_i \sigma_i^x - g\sum_i \sigma_i^z \tag{1}$$

Sherrington-Kirpatrick Model (SK)

$$\mathcal{H} = -\sum_{i<j\leq N} J_{ij}\sigma_i^z\sigma_j^z - h\sum_i \sigma_i^x, \quad J_{ij}\in\mathcal{N}(0,J^2) \tag{2}$$

Kogut-Susskind Hamiltonian (pure gauge)

$$\mathcal{H} = \frac{g^2}{2}\sum_\ell E_\ell^2 - \frac{1}{2g^2}\sum_\square \mathrm{Tr}(U_\square + U_\square^\dagger) \tag{3}$$

AKLT (Affleck-Kennedy-Lieb-Tasaki) Hamiltonian

$$\mathcal{H} = \sum_{\langle i,j\rangle}\left(\vec{S}_i\cdot\vec{S}_j + \frac{1}{3}\left(\vec{S}_i\cdot\vec{S}_j\right)^2\right) \tag{4}$$

★ $\mathcal{N}=4$ SYM - (3+1)-dimensions

$$\mathcal{L} = \mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu} + (D_\mu X_i)^2 - \frac{1}{2}[X_i,X_j]^2 + \Psi^T\not{D}\Psi + \Psi^T\gamma_i[X,\Psi]\right] \tag{5}$$

BFSS (0+1)-dimensions

$$\mathcal{L} = \mathrm{Tr}\left((D_t X_i)^2 - [X_i,X_j]^2\right) + \Psi^T\not{D}\Psi + \Psi^T\gamma_i[X,\Psi] \tag{6}$$

BMN/PWMM - (0+1)-dimensions with i,j,k $= 1\cdots3$ and $M = 4\cdots9$

$$\mathcal{L} = \mathcal{L}_{\text{BFSS}} + \mathrm{Tr}\left[\left(\frac{\mu}{3}X_I\right)^2 + \left(\frac{\mu}{6}X_M\right)^2 + \frac{\mu}{4}\Psi_\alpha^T\gamma^{123}_{\alpha\beta}\Psi_\beta + \frac{\sqrt{2}\mu}{3}\epsilon_{IJK}X_IX_JX_K\right]. \tag{7}$$

SYK in (0+1)

$$\mathcal{H} = \frac{1}{4!}\sum_{i,j,k,l=1}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l \tag{8}$$

IKKT (0+0)-dimensions, with i,j $= 1\cdots 10$

$$\mathcal{L} = \mathrm{Tr}\Big([X_I, X_J]^2\Big) + \Psi^T \not{D}\Psi + \Psi^T\gamma_I[X,\Psi] \tag{9}$$

Einstein-Hilbert action

$$S = \frac{c^4}{16\pi G}\int d^4x\,(R - 2\Lambda) \tag{10}$$

Nambu-Goto (NG) action

$$S = -T\int d^2\sigma\sqrt{-(\dot{X}^2)(X')^2 + (\dot{X}\cdot X')^2} \tag{11}$$

$\dot{X}^\mu = \partial X^\mu/\partial\tau$, $(X')^\mu = \partial X^\mu/\partial\sigma$

Polyakov action

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu} \tag{12}$$

Chern-Simons action

$$S = \frac{k}{4\pi}\int d^3x\;\epsilon^{\mu\nu\rho}\;\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right) \tag{13}$$

Principal Chiral Field

$$\mathcal{L} = \frac{\beta}{2}\mathrm{Tr}\left(\partial_\mu g^{-1}\partial_\mu g\right) \text{ where } g \in SU(N) \tag{14}$$

Massless Schwinger (1+1)

$$\mathcal{L} = \frac{1}{2}(\epsilon^{\mu\nu}\partial_\nu A_\mu)^2 - ej^\mu A_\mu + \overline{\Psi}\not{\partial}\Psi \tag{15}$$

▲ Massive Thirring

$$\mathcal{L} = \overline{\Psi}\partial\!\!\!/\Psi - m_F\overline{\Psi}\Psi - \frac{g}{2}(\overline{\Psi}\gamma^\mu\Psi)^2 \tag{16}$$

O(N) non-linear σ in 1+1

$$\mathcal{L} = \frac{1}{2g}\sum_{i=1}^{N}(\partial^\mu \hat{n_i})^2 \tag{17}$$

Ising

$$\mathcal{H} = -\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_j h_j\sigma_j \tag{18}$$

▲ Sine-Gordon

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^4}{\lambda}\left(1 - \cos\left(\frac{\sqrt{\lambda}\phi}{m}\right)\right) \tag{19}$$

★ Heisenberg model [1928, Solved by Bethe (1931)]

$$\mathcal{H}_{XXX} = \frac{J}{2}\sum_i \left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + \sigma_i^z\sigma_{i+1}^z\right) \tag{20}$$

Fermi-Hubbard model

$$\mathcal{H} = -t\sum_{\langle i,j\rangle,\sigma}\left(c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}\right) + U\sum_i n_{i\uparrow}n_{i\downarrow}, \tag{21}$$

Bose-Hubbard model

$$\mathcal{H} = -t\sum_{\langle i,j\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i(n_i-1) - \mu\sum_i n_i \quad ; \quad [b_i, b_j^\dagger] = \delta_{ij} \tag{22}$$

Blume-Capel model

$$\mathcal{H} = -J\sum_{\langle i,j\rangle} Z_iZ_j + \Delta\sum_i Z_i^2 - h\sum_i Z_i \quad ; \quad Z = \text{Pauli}Z = \text{diag}(1,-1) \tag{23}$$

O'Brien-Fendley model (tri-critical Ising)

$$\mathcal{H} = -\sum_{j=1}^{N-1} Z_j Z_{j+1} - g\sum_{j=1}^{N} X_j - h\sum_{j=1}^{N} Z_j + \lambda\sum_{j=1}^{N-2}(X_j Z_{j+1} Z_{j+2} + Z_j Z_{j+1} X_{j+2}) \tag{24}$$

NJL model

$$\mathcal{L} = -i\,\bar{\psi}\,\partial\!\!\!/\,\psi + \frac{\lambda}{4}\left[(\bar{\psi}\psi)(\bar{\psi}\psi) - (\bar{\psi}\gamma^5\psi)(\bar{\psi}\gamma^5\psi)\right] \quad ; \quad \bar{\psi} = \psi^\dagger\gamma^0 \tag{25}$$

Gross-Neveu model

$$\mathcal{H} = -i\bar{\psi}(\gamma^j\partial_j + m)\psi - \frac{g}{2}(\bar{\psi}\psi)^2 \tag{26}$$