Phys219_2017 - Ryan Kaufmann/Exp. 1 (RC Circuit)/Exp 1 RC Circuit

SIGNED by Ryan Kaufmann Oct 03, 2017 @01:15 PM PDT

Ryan Kaufmann Oct 01, 2017 @11:50 AM PDT

Experiment #1: The RC Circuit and Time Constant

Partner: Eric Brock

Ryan Kaufmann Oct 01, 2017 @11:44 AM PDT

1.1 Objective

Ryan Kaufmann Oct 01, 2017 @12:17 PM PDT

Now that we understand the basics of how to build a circuit and navigate our way through the devices, we can start to analyze circuits that we have built. One of these basic circuits is called an RC circuit, which contains a power source, resistor, and a capacitor. The RC circuit has components of it that fluctuates with time. In this lab we will analyze the voltage across the capacitor in both a square and sinusoidal wave. Using these graphs and functions, we can find the time constant of the circuit and then the capacitance.

Ryan Kaufmann Oct 01, 2017 @11:45 AM PDT

1.2 Introduction

Ryan Kaufmann Oct 01, 2017 @12:58 PM PDT

RC circuits behave in a particular way. When charging, the capacitor's voltage follows a graph of V(1-e^-t/T). When discharging, the capacitor's voltage follows a graph of V(e^-t/T). In both of these cases, T is the time constant tau which is equal to R*C and V is the voltage amplitude of the source. That is V=Vin. If we graph the voltage across the capacitor, we can find the time constant tau. Since tau is equal to R*C, we can measure R to find the capacitance.

In this lab we will measure the time constant tau in two different ways. One in the time domain using a square wave and the other in the frequency domain using a sinusoidal wave. Then we will compare these answers and see if we can conclude on a single value of the capacitance.

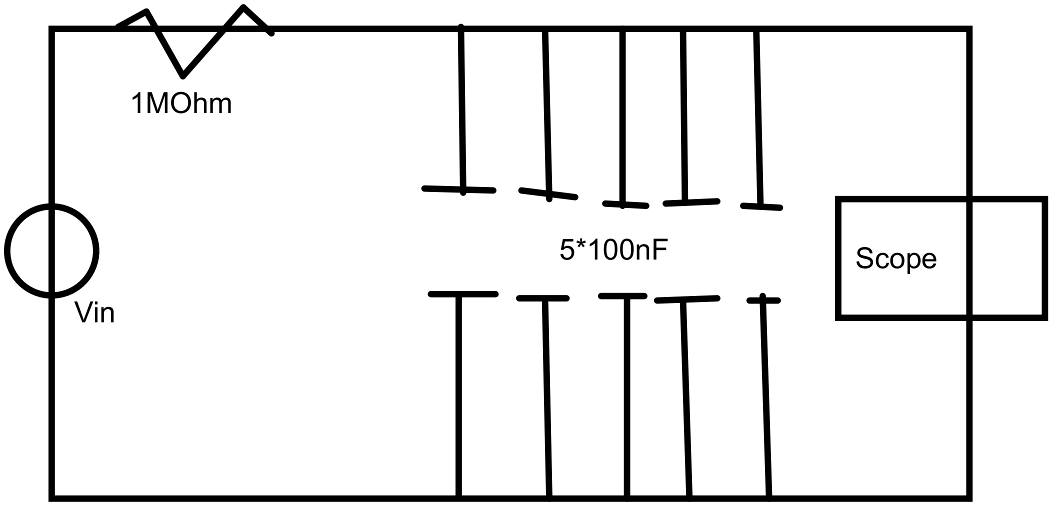
Ryan Kaufmann Oct 01, 2017 @11:46 AM PDT

1.3 The RC Time Constant in the Time Domain

Ryan Kaufmann Oct 01, 2017 @01:27 PM PDT

Let us first set up our circuit first. Since we don't have a 500 nF capacitor, we need to make one ourselves. Capacitors add in parallel and add in inverse in series. We have several 100 nF (0.1 microFarads) capacitors instead of the 500 nF capacitor. In order to set up our RC circuit, we put 5 100nF capacitors in parallel along with the oscilloscope. Then in series is the voltage source and the resistor.

Ryan Kaufmann Oct 01, 2017 @01:31 PM PDT



Ryan Kaufmann Oct 01, 2017 @02:11 PM PDT

Now that our circuit is set up, we can set our function generator and oscilloscope. The function generator is set to be a square wave with a peak to peak amplitude of 1 Volt from +1 Volt to 0 Volts. That is, the square wave is set to have a peak to peak amplitude of 1 Volts, an offset of 500 mVolts, and a period of 10/3 seconds. We set this period of 10/3 seconds based on our prelab. We estimated a time of 2.3 seconds for the Vin to drop 99%. Since we want to measure the complete drop in Vin, we want to extend this to be longer than that 99% so we can make sure we capture the complete picture.

Ryan Kaufmann Oct 01, 2017 @03:15 PM PDT

So we then take our data from a couple of oscillations and plot our data. We notice that Vin behaves as we would expect with on part at 1 Volt and the other a 0 Volts. The voltage over the capacitor is a little more interesting. It oscillates between an exponential and a logarithmic. The data gives us the following plots:

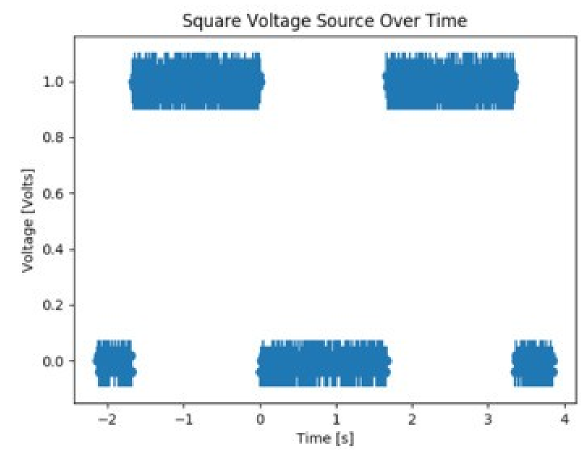
Ryan Kaufmann Oct 03, 2017 @12:50 PM PDT

RCSquareSourceVoltage1MOhm.csv(11.5 KB)

Ryan Kaufmann Oct 03, 2017 @12:50 PM PDT

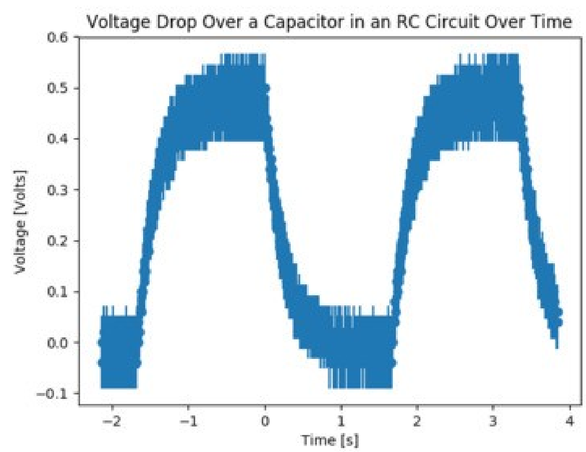
RCSquareCapacitorVoltage1MOhm.csv(11.4 KB)

Ryan Kaufmann Oct 02, 2017 @09:21 PM PDT



RCSquareSource1MOhm.png(18.3 KB)

Ryan Kaufmann Oct 02, 2017 @09:22 PM PDT



RCSquareCapacitor1MOhm.png(23.3 KB)

Ryan Kaufmann Oct 01, 2017 @04:12 PM PDT

We can then estimate our tau from measuring how long it takes for Vc to drop by a factor of 1/e. Using the cursor functions on the oscilloscope, we set our scales to be 50mV in the vertical and 50ms in the horizontal. We set this so that we could see the entire drop in one frame but not much outside of it. That is, we can get the most detailed view of the area we want to examine without having to scroll through the plot.

We measured a starting voltage of 0.5 Volts. Thus our next value we need to measure should be around 0.1839 Volts. We then measured at 0.184 Volts. The time we measured between these two voltages is 246ms. We added some random error and instrument error as well, which we calculated either by hand or by using the error calculations in the booklet. Thus we get the following numbers:

Ryan Kaufmann Oct 01, 2017 @06:03 PM PDT

$$V_{cint} = 0.5 \pm (0.03 * 0.5 + 0.05 * 0.05)V = 0.5 \pm 0.0175V$$

$$V_{cfin} = 0.184 \pm (0.03 * 0.5 + 0.05 * 0.05)V = 0.184 \pm 0.0175V$$

$$\tau = 246.0 \pm \sqrt{1^2 + (50 + 0.0001 * 246 + 0.0000004)^2} ms = 246.0 \pm 50ms$$

Ryan Kaufmann Oct 02, 2017 @10:59 AM PDT

Using the python curve fit, we got some better numbers than before. We reduce our data down to a single drop of the capacitor, so that we can get a clean fit. Then using the exponential with the offset, we add in our data and try to get our fit. The fit is not the best. Although our chi squared is 0.815, our residuals shows a pattern towards the latter half of the graph. We tried to fit the graph by changing the initial guess of the program, but nothing seemed to cause much change in the residuals. The fit gives us the following equation and graphs:

Ryan Kaufmann Oct 01, 2017 @05:46 PM PDT

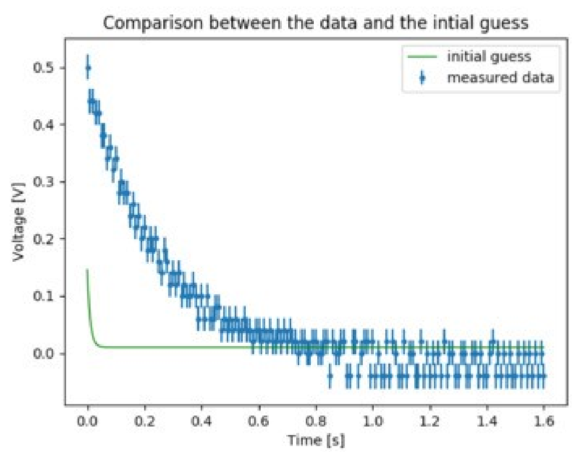
$$V_c = V_{cmax} * e^{-t/\tau} + V_{offset}$$

$$V_{cmax} = 0.5336 \pm 0.009572V$$

$$\tau = 0.2620 \pm 0.008488 seconds$$

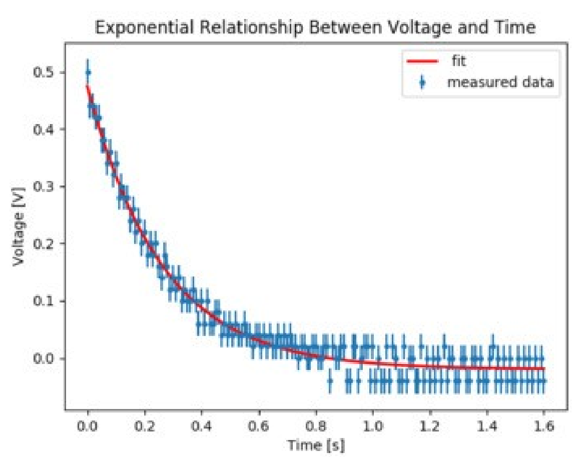
$$V_{offset} = -0.01995 \pm 0.00283V$$

Ryan Kaufmann Oct 02, 2017 @11:00 AM PDT



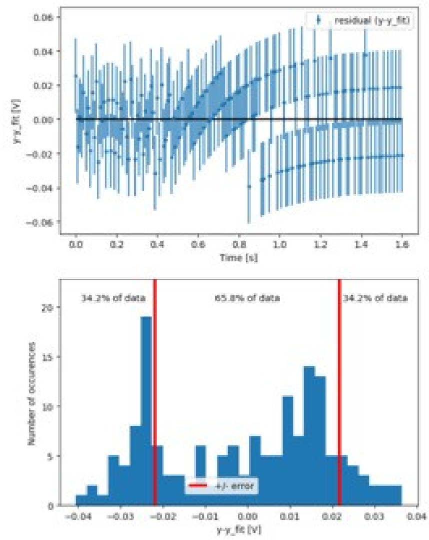
RCSquareCapacitor1MOhmFirstGuess.png(26.8 KB)

Ryan Kaufmann Oct 02, 2017 @11:00 AM PDT



RCSquareCapacitor1MOhmChiSquareFit.png(32.5 KB)

Ryan Kaufmann   Oct 02, 2017 @11:00 AM PDT



RCSquareCapacitor1MOhmResids.png(38.9 KB)

Ryan Kaufmann   Oct 01, 2017 @06:03 PM PDT

We can compare these two values for tau with a t-score. The result gives us the following:

Ryan Kaufmann   Oct 01, 2017 @06:12 PM PDT

$$T_{score} = \frac{\tau_{fitted} - \tau_{cursor}}{\sqrt{\sigma^2_{\tau_{fitted}} + \sigma^2_{\tau_{cursor}}}}$$

$$T_{score} = \frac{262.0 - 246.0}{\sqrt{8.5^2 + 50^2}}$$

$$T_{score} = 0.3155$$

Ryan Kaufmann   Oct 01, 2017 @07:16 PM PDT

Given the t-score that we received, we could assume that there is not a difference between the two numbers.

Now that we have analyzed the 1MOhm resistor, let us try a different resistor to get a different measurement of tau that we can compare two. In our figure above, let us replace the 1MOhm resistor with a 10kOhm one. Now we have a different time constant. We adjusted our frequency generator accordingly. We estimated a 23.03 millisecond time drop when we approximated the 99% drop in the voltage. Thus in order to see the entire drop we set the our frequency generator to a period of 63 1/3 milliseconds.

Once again, we can take initial measurements using the cursor function on the oscilloscope. We got our initial measurement at 0.96 volts. Then the voltage drop we are looking for is 0.353 volts. However, the closest we could come to the 1/e factor is 360 millivolts. This gave us a measurement of tau of 4.84 milliseconds. With the uncertainty measurements, we get the following:

Ryan Kaufmann Oct 01, 2017 @08:34 PM PDT

$$V_{cint} = 0.960 \pm (0.03 * 0.960 + 0.05 * 0.1)V = 0.960 \pm 0.0338V$$

$$V_{cfin} = 0.360 \pm (0.03 * 0.360 + 0.05 * 0.1)V = 0.360 \pm 0.0158V$$

$$\tau = 4.84 \pm (1 + 0.0001 * 4.84 + 0.0000004)ms = 4.84 \pm 1.000ms$$

Ryan Kaufmann Oct 01, 2017 @07:35 PM PDT

We can then analyze the data from the oscilloscope. First we view the plotted points, which can be shown as the following graphs:

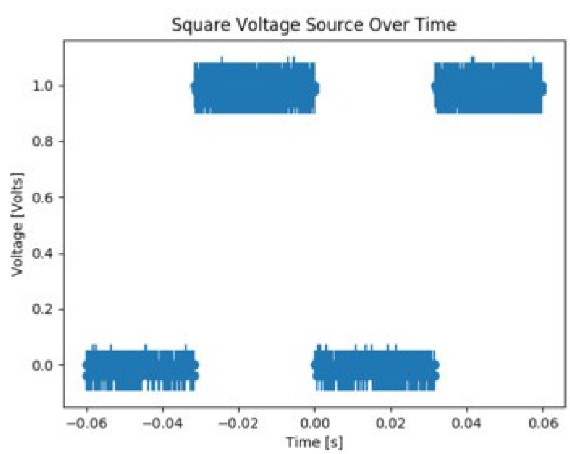
Ryan Kaufmann Oct 03, 2017 @12:50 PM PDT

RCSquareSourceVoltage10kOhm.csv(11.6 KB)

Ryan Kaufmann Oct 03, 2017 @12:50 PM PDT

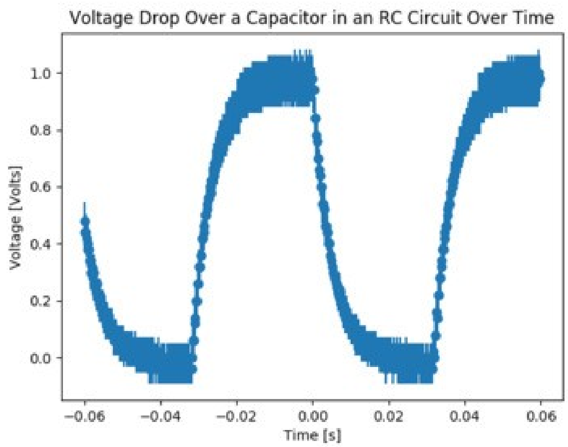
RCSquareCapacitorVoltage10kOhm.csv(16.9 KB)

Ryan Kaufmann Oct 02, 2017 @09:22 PM PDT



RCSquareSource10kOhm.png(18.9 KB)

Ryan Kaufmann Oct 02, 2017 @09:22 PM PDT



RCSquareCapacitor10kOhm.png(26.1 KB)

Ryan Kaufmann Oct 02, 2017 @10:53 AM PDT

Once again using the curve fit function of python, we can find a more accurate reading for tau. Just like the other time, we don't have the best fit. Although it has a chi squared of 1.054, it once again had a pattern towards the latter half of the graph. This pattern did not go away when we changed the initial guess. The fit gave us the following equation and graphs:

Ryan Kaufmann Oct 01, 2017 @08:33 PM PDT

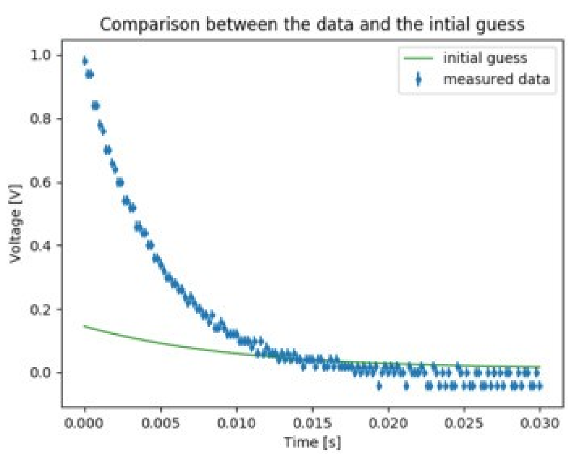
$$V_c = V_{cmax} * e^{-t/\tau} + V_{offset}$$

$$V_{cmax} = 56.38 \pm 2.97V$$

$$\tau = 0.004949 \pm 0.00006056s$$

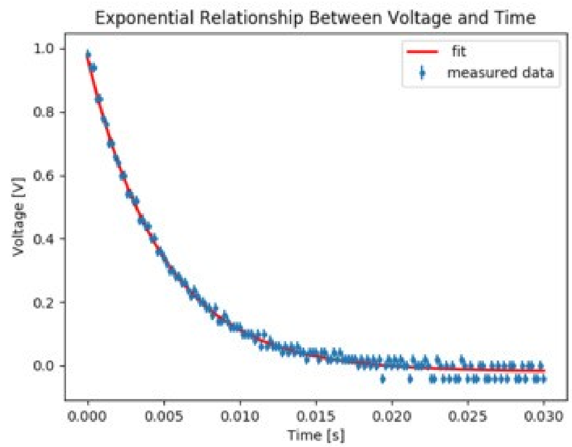
$$V_{offset} = -0.01897 \pm 0.002156V$$

Ryan Kaufmann Oct 02, 2017 @10:56 AM PDT



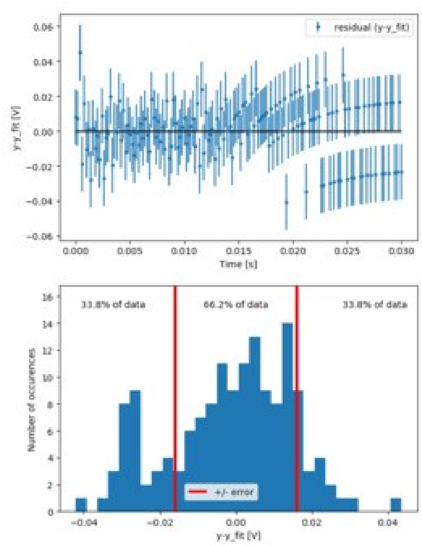
RCSquareCapacitor10kOhmFirstGuess.png(28.6 KB)

Ryan Kaufmann Oct 02, 2017 @10:56 AM PDT



RCSquareCapacitor10kOhmChiSquareFit.png(30.5 KB)

Ryan Kaufmann Oct 02, 2017 @10:56 AM PDT



RCSquareCapacitor10kOhmResids.png(39.1 KB)

Ryan Kaufmann Oct 01, 2017 @08:28 PM PDT

We can once again compare these two taus using a t-score, which gives us the following:

Ryan Kaufmann Oct 01, 2017 @08:32 PM PDT

$$T_{score} = \frac{\tau_{fitted} - \tau_{cursor}}{\sqrt{\sigma^2_{\tau_{fitted}} + \sigma^2_{\tau_{cursor}}}}$$

$$T_{score} = \frac{0.004949 - 0.00484}{\sqrt{0.00006056^2 + 0.001^2}}$$

$$T_{score} = 0.1088$$

Ryan Kaufmann Oct 01, 2017 @08:47 PM PDT

Now that we have some reasonable taus from both resistors, we can use the resistances we get from checking the resistors with the DMM to find the capacitance of the system. We get the following calculations:

Ryan Kaufmann Oct 01, 2017 @09:27 PM PDT

$$R_{1M} = 1.00380 \pm 0.00015 M\,\Omega$$

$$R_{10k} = 9.8525 \pm 0.0005 k\,\Omega$$

$$\tau = R * C$$

$$C = \frac{\tau}{R} = \frac{\tau_{1M}}{R_{1M}} = \frac{\tau_{10k}}{R_{10k}}$$

$$C_{1M} = \frac{262.0 * 10^{-3}}{1.00380 * 10^{6}} = 261nF$$

$$\sigma_{C1M} = C_{1M} * \sqrt{(\frac{\sigma_\tau}{\tau})^2 + (\frac{\sigma_{R1M}}{R_{1M}})^2} = 261 * \sqrt{(\frac{0.008488}{0.2620})^2 + (\frac{0.00015 * 10^6}{1.00380 * 10^6})^2} = 8.456nF$$

$$C_{10k} = \frac{0.004949}{9.8525 * 10^{3}} = 502.3nF$$

$$\sigma_{C10k} = C_{10k} * \sqrt{(\frac{\sigma_\tau}{\tau})^2 + (\frac{\sigma_{R10k}}{R_{10k}})^2} = 502.3 * \sqrt{(\frac{0.00006056}{0.004949})^2 + (\frac{0.0005 * 10^3}{9.8525 * 10^3})^2} = 6.147nF$$

Ryan Kaufmann Oct 01, 2017 @10:19 PM PDT

We now have two different values of the capacitance. We can compare these values of capacitance to see if there is a difference between them. Thus we calculate the following t-score:

Ryan Kaufmann Oct 01, 2017 @10:26 PM PDT

$$T_{score} = \frac{C_{10k} - C_{1M}}{\sqrt{(\sigma_{C10k})^2 + (\sigma_{C1M})^2}}$$

$$T_{score} = \frac{502.3 - 261}{\sqrt{(6.147)^2 + (8.456)^2}}$$

$$T_{score} = 23.08$$

Ryan Kaufmann Oct 01, 2017 @11:58 PM PDT

We receive a t-score of 23.08, which we could have predicted to some extent from the difference between our two capacitance. There must be something outside our circuit or some resistance that we are not taking into account that causes this difference. We noticed that the tau measured in the 1 MOhm resistor was half of what we expected. Furthermore, the capacitors were charging up to half of the voltage we expected.

Upon further inspection, we noticed that the oscilloscope had a 1MOhm internal resistance. This resistance would be in parallel with the capacitors. With a 10 kilohm resistance, most of the current would not flow through the 1MOhm branch of the circuit. Instead, it will flow through the capacitors, filling them up completely. However, since the internal resistance is comparable when we raise the outside resistance to 1 MOhm, it changes the maximum voltage of the capacitor significantly. This is because the current begins to flow through oscilloscope rather than the capacitors.

This also affects the time constant as well though. The resistance of the circuit changes when we take into account this extra resistance. Thus the time constant changes. In fact, the two resistors should be in parallel when we view the discharge circuit. Thus when we calculate the equivalent resistance it drops by a factor of two. If we calculate using the new resistance of the circuit, we get the following:

Ryan Kaufmann Oct 02, 2017 @12:19 AM PDT

$$R_{1MNew} = \frac{1}{2} * R_{1M} = \frac{1}{2} * 1.00380MV = 0.5019MV$$

$$\sigma_{R1MNew} = R_{1MNew} * \frac{\sigma_{R1M}}{R_{1M}} = 0.5019 * \frac{0.00015}{1.00380} MV = 0.000075MV$$

$$C_{1M} = \frac{\tau_{1M}}{R_{1M}} = \frac{262.0 * 10^{-3}}{0.5019 * 10^{6}} = 522nF$$

$$\sigma_{C1M} = C_{1M} * \sqrt{(\frac{\sigma_\tau}{\tau})^2 + (\frac{\sigma_{R1M}}{R_{1M}})^2} = 522 * \sqrt{(\frac{0.008488}{0.2620})^2 + (\frac{0.000075 * 10^6}{0.5019 * 10^6})^2} = 16.91nF$$

$$T_{score} = \frac{522 - 502.3}{\sqrt{(16.91)^2 + (6.147)^2}} = 1.0949$$

Ryan Kaufmann Oct 02, 2017 @12:23 AM PDT

Now our t-score is 1.09 rather than 23, which is much better. Now it seems that there is little to no difference between our values. Moreover, these values coincide with our though about what the actual value of the capacitor is.

Ryan Kaufmann Oct 01, 2017 @11:46 AM PDT

1.4 The RC Time Constant in the Frequency Domain

Ryan Kaufmann Oct 02, 2017 @09:05 PM PDT

There are other ways we can estimate our capacitance. One of these ways is by using a sinusoidal output from the function generator and analyzing the behavior of the capacitor. For this, we use the same set up we had in the previous section but with a 10 kilohm resistor. We don't want the resistance of the oscilloscope to become comparable to the resistor so that we measure the same tau and capacitance as before. In addition, we want the capacitors to fully charge up so that we can see the smooth curve. Then we set up our function generator. This time, we change the generator to supply a sine wave with a peak to peak amplitude of 2V and an offset of 0V. We keep the frequency open for now so that we can change it as we need. We want to vary the frequency with time to see how both the amplitude and phase shift of the voltage across the capacitor change. Since these functions also include a component based on tau, we can curve fit over several frequencies to try to find a value for tau. However, we only have a good fit for amplitude, because the functions are as follows:

Ryan Kaufmann Oct 02, 2017 @09:07 PM PDT

$$\phi(\omega) = \tan^{-1}(-\omega\tau)$$

$$V_C(\omega) = \frac{V_{in}^0}{\sqrt{1 + (\omega\tau)^2}}$$

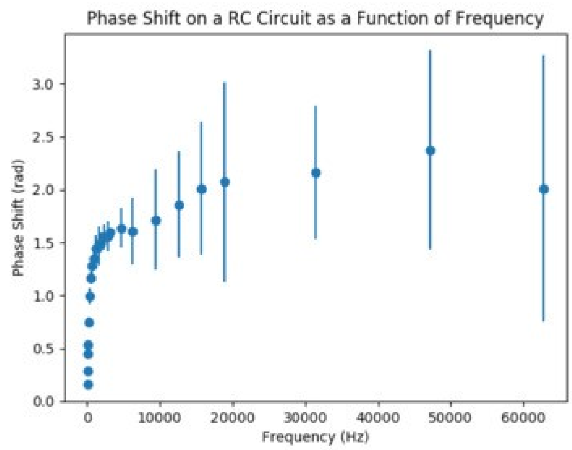
Ryan Kaufmann Oct 02, 2017 @09:10 PM PDT

where V^0_{in} is 1 Volt. There is no good fit procedure to find an inverse tangent so we will only look at V_C. However we did take data for the phase shift, which, when plotted, looks like the following:

Ryan Kaufmann Oct 03, 2017 @12:50 PM PDT

RCSinePhaseShiftOnly.csv(944 Bytes)

Ryan Kaufmann Oct 02, 2017 @09:10 PM PDT



PhaseShift.png(23.5 KB)

Ryan Kaufmann Oct 02, 2017 @11:04 PM PDT

We took the data by hand by lining up the points where each sine wave (the source and the capacitor) dropped passed 0V. We recognized that it had to be when it dropped otherwise we the phase shifts wouldn't line up correctly, since the phase shift describes the difference in time between two like behaviors occur in sine waves. Furthermore, 0V was an easier point to take data from as it had the steepest slope and therefore the least error in measuring it. That is, there was little to no disputing where the sine wave crossed 0V, especially compared to, for example, where the sine wave reached their peaks. However, as we increased the frequency we encountered a large amount of noise, giving us larger and larger uncertainties in the phase shift. We estimated the uncertainties by observing how wide the line crossing 0V was and making a random error guess based on the number of divisions it took up.

Observing the amplitude of the sine waves was a similar procedure. We attempted to find a medium when the graph was flattest on the oscilloscope. Then by observing how many voltage divisions the graph spanned when it was flattest, we calculated a random error in the amplitude which we included in our data. Once we had our data, we could input it into our curve fit function, which gave use the following function and fit:

Ryan Kaufmann Oct 03, 2017 @12:49 PM PDT

RCSineVoltageOnly.csv(625 Bytes)

Ryan Kaufmann Oct 02, 2017 @11:22 PM PDT

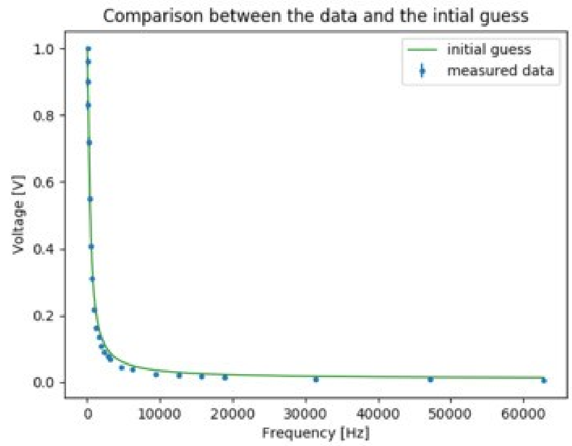
$$V_C(\omega) = \frac{V_{in}^0}{\sqrt{1 + (\omega\tau)^2}} + V_{offset}$$

$$V_{in}^0 = 1.007 \pm 0.002743V$$

$$\tau = 0.004914 \pm 0.00004405 seconds$$

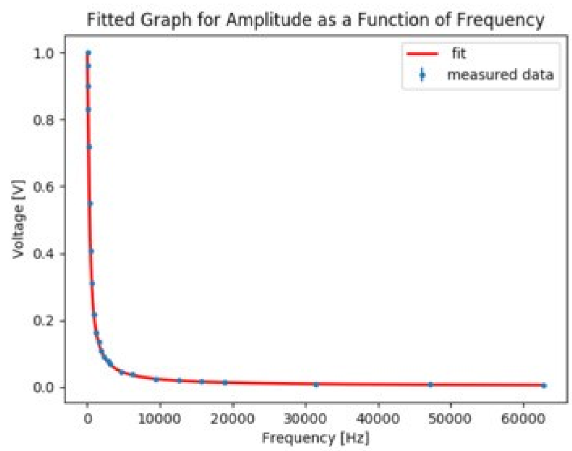
$$V_{offset} = 0.003279 \pm 0.0004813V$$

Ryan Kaufmann Oct 02, 2017 @11:23 PM PDT



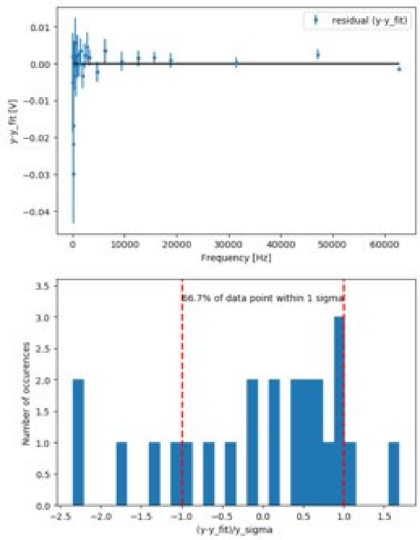
AmplitudeInitial.png(27.3 KB)

Ryan Kaufmann Oct 02, 2017 @11:23 PM PDT



AmplitudeFitted.png(26.2 KB)

Ryan Kaufmann Oct 02, 2017 @11:23 PM PDT



AmplitudeResids.png(37.7 KB)

Ryan Kaufmann Oct 03, 2017 @12:31 PM PDT

With this fit, we get a chi-squared of 1.278, which alludes to a good fit. Furthermore, the residual and the graphs agree. There doesn't seem to be much of a pattern in the residuals and they revolve around the zero of the graph. No residuals are much over 2 error bars away from the zero as well. Thus we believe this to be a good approximation of a function describing our data. We can then use our tau which we received from the curve fit to approximate our capacitance.

We remeasured our resistor using the DMM to get an accurate reading of the resistance. Since tau is equal to resistance times capacitance, we can calculate tau using the values we have now. Once we have a value of tau, we can compare with our previous values using a t-score. The derivation is as follows:

Ryan Kaufmann Oct 03, 2017 @12:41 PM PDT

$$\tau = 0.004914 \pm 0.00004405 seconds$$

$$R_{10k} = 9.8555 \pm 0.0020 k\,\Omega = 9855.5 \pm 2.0\Omega$$

$$C = \frac{\tau}{R_{10k}} = \frac{0.004914}{9855.5} F = 0.0000004986F = 498.6nF$$

$$\sigma_C = C * \sqrt{(\frac{\sigma_\tau}{\tau})^2 + (\frac{\sigma_{R10k}}{R_{10k}})^2} = 498.6 * \sqrt{(\frac{0.00004405}{0.004914})^2 + (\frac{2.0}{9855.5})^2} = 4.47nF$$

$$T_{score1M} = \frac{C_{1M} - C}{\sqrt{\sigma_{C1M}^2 + \sigma_C^2}} = \frac{522 - 498.6}{\sqrt{16.91^2 + 4.47^2}} = 1.338$$

$$T_{score10k} = \frac{C_{10k} - C}{\sqrt{\sigma_{C10k}^2 + \sigma_C^2}} = \frac{502.3 - 498.6}{\sqrt{6.147^2 + 4.47^2}} = 0.4868$$

Ryan Kaufmann Oct 03, 2017 @12:56 PM PDT

We get t-scores that show very little difference. The t-score between our frequency value of C and the 10k value of C in the time domain is 0.4868, which is much less than 1. From this we can conclude that there is no difference between the two numbers. The actual value for capacitance probably lies within the ranges of these two numbers. However, compared to the 1M value of capacitance, we received a t-score of 1.338. Since it is above 1, we cannot conclude that there is a difference between the two numbers. We could attempt to get a better fit for the capacitance by measuring the resistance in the oscilloscope accurately. Furthermore, we can refine our frequency approximation of capacitance by taking more points, especially around the dip to try and get a better fit, or by averaging the signal over time to try to get more accurate numbers. Otherwise, our numbers seemed to conclude that our capacitance is somewhere around 500 nanoFarads, which is what we expected.

Another interesting application can be shown by looking at the graph for the amplitude. The amplitude is highest at the lower frequencies, the ones we measured at 10, 15, 20, etc. Then it dropped and was lowest at the higher frequencies, especially 5000, 7500, 10000, etc. This could play an important role in sound systems. If an input had an audio varying in frequencies or a signal with multiple frequencies, higher frequencies can be filtered out by using different combinations of resistors and capacitors. A system that has a high tau will let higher frequencies flow through. That is a tau of 100 seconds will let frequencies of 100 Hertz have a higher amplitude than a tau of 10 seconds.

Ryan Kaufmann Oct 01, 2017 @11:48 AM PDT

1.5 Conclusion

Ryan Kaufmann Oct 03, 2017 @01:15 PM PDT

This lab focused on using the oscilloscope to analyze a circuit we built with a oscillating behavior. The circuit consisted of five capacitors, equating to a capacitance of supposedly 500nF, and a resistor, which switched between 1MOhm and 10kOhms. In the lab, we took measurements of the voltage behavior when a square wave was applied to the circuit. We then analyzed this behavior to see how the data fell as a function and approximated the time constant. Using this we calculated two values for capacitance, each with a different resistor. However, in the second part, we took a different approach. Using a sinusoidal wave this time, we looked at how the amplitude of the voltage across the capacitor changed with respect to the frequency. After taking several points for the amplitude, we fit the data to a curve relating the amplitude to the frequency and approximated the time constant again, which we used to find the capacitance. The results of our analysis are as follows:

Measurement Type	Capacitance	Uncertainty	t-score with one above	t-score with one below	sigmas until 500 nF
Time Domain with 1M	522 nF	16.91 nF	1.338	1.0949	2
Time Domain with 10k	502.3 nF	6.147 nF	1.0949	0.4868	1
Frequency Domain	498.6 nF	4.47 nF	0.4868	1.338	1

Judging from this data, it may seem that the capacitance is a bit lower than we expected, closer to that we see in the frequency domain of 498.6 nF. We understand that the capacitor is most likely not exactly 500nF, especially since we used five 100nF capacitors to create it, and that there is some error in this number. Furthermore, the actual capacitance is most likely closer to the numbers that have a lower uncertainty. Thus we placed our guess for the capacitance between 500nF and 498.6nF, although we are not sure exactly where it is.

This lab has given us more hands on experience with creating circuits and analyzing them. Along with reminding us of how to add components in series and in parallel, we got a better look at the functions the oscilloscope and function generator performed. Finally, the lab reacquainted us with the procedures needed to analyze and compare data, including chi-squared and t-scores.