PHYS 229 - Ryan Kaufmann/Experiment 3/Cooling Prelab

SIGNED by Ryan Kaufmann Apr 10, 2018 @02:40 PM PDT

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Cooling 0 Prelab Questions

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Let us first look at the convection equation of the rod. We can set up our differential equation as so:

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$$\frac{dQ}{dt} = -hA(T - T_0)$$

$$\frac{dQ}{dt} = -1.32A\frac{1}{D^{1/4}}(T - T_0)^{5/4}$$

$$\rho VC\frac{dT}{dt} = -1.32A\frac{1}{D^{1/4}}(T - T_0)^{5/4}$$

$$\frac{dT}{dt} = -1.32\frac{A}{\rho CV}\frac{1}{D^{1/4}}(T - T_0)^{5/4}$$

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We can then apply our constant conditions of the properties of the rod and the temperature of the lab. Our constants from the rod are set up such that we have the following:

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$$D = 0.0246m \pm 0.0001m$$

$$L = 0.305m \pm 0.001m$$

$$\rho = 2700kg/m^3 \pm 2kg/m^3$$

$$C = 904J/(kgK) \pm 2J/(kgK)$$

$$T_c = 293.0K \pm 0.5K$$

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Solving the differential equation gives us the following:

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$$\frac{dT}{dt} = -1.32\frac{A}{\rho CV}\frac{1}{D^{1/4}}(T-T_0)^{5/4}$$

$$\frac{dT}{dt} = -1.32\frac{0.02457}{2700*904*0.0001456}\frac{1}{0.02465^{1/4}}(T-293)^{5/4}$$

$$(T-293)^{-5/4}*dT = -0.0002304dt$$

$$\frac{4}{(T-293)^{1/4}} - \frac{4}{(T_c-293)^{1/4}} = -0.0002304t$$

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where Tc is the starting temperature of the rod (i.e. the temperature at t=0). We can then solve for both t when Tc is 363 and T is 303K and for the equation of T in terms of Tc and t:

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$$\frac{4}{(T-293)^{1/4}} - \frac{4}{(T_c-293)^{1/4}} = -0.0002304t$$

$$\frac{4}{(T-293)^{1/4}} = -0.0002304t - \frac{4}{(T_c-293)^{1/4}}$$

$$\frac{1}{(T-293)^{1/4}} = -0.00005761t - \frac{1}{(T_c-293)^{1/4}}$$

$$(T-293)^{1/4} = \frac{1}{-0.00005761t - \frac{1}{(T_c-293)^{1/4}}}$$

$$(T-293) = (\frac{1}{-0.00005761t - \frac{1}{(T_c-293)^{1/4}}})^4$$

$$T(t,T_c) = (\frac{1}{-0.00005761t - \frac{1}{(T_c-293)^{1/4}}})^4 + 293$$

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$$\frac{4}{(T-293)^{1/4}}+\frac{-4}{(T_c-293)^{1/4}}=-0.0002304t$$
$$\frac{4}{(363-293)^{1/4}}+\frac{-4}{(303-293)^{1/4}}=-0.0002304t$$
$$\frac{4}{(70)^{1/4}}+\frac{-4}{(10)^{1/4}}=-0.0002304t$$
$$\frac{4}{2.893}+\frac{-4}{1.778}=-0.0002304t$$
$$1.383-2.250=-0.0002304t$$
$$-0.867=-0.0002304t$$
$$t=3762.57s$$

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Thus we have that it would take around 3,760 seconds for the rod to cool using convection.

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Applying the same differential equation techniques to the radiation formula gives us the following set up:

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$$\frac{dQ}{dt}=Ae\sigma T_0^4-Ae\sigma T^4$$
$$\rho VC\frac{dT}{dt}=Ae\sigma T_0^4-Ae\sigma T^4$$
$$\frac{dT}{dt}=\frac{Ae\sigma}{\rho VC}T_0^4-\frac{Ae\sigma}{\rho VC}T^4$$

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Then applying our constants, we have:

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$$\frac{dT}{dt} = \frac{0.02457\sigma}{2700 * 904 * 0.0001456} * (293)^4 - \frac{0.02457\sigma}{2700 * 904 * 0.0001456} * T^4$$

$$\frac{1}{(293^4 - T^4)} \frac{dT}{dt} = 9.899 * 10^{-12}$$

$$\ln(293 + T_c) - 2\arctan(293/T_c) - \ln(293 + T) + 2\arctan(293/T) = 0.0009960t$$

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While this is unsolvable for a analytical solution of T in terms of Tc and t, we can plug in values of T and Tc to find the cooling time:

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$$\ln(293 + 363) - 2\arctan(293/363) - \ln(293 + 303) + 2\arctan(293/303) = 0.0009960t$$

$$1.6709 = 0.0009960t$$

$$t = 1677.7s$$

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Which gives us a shorter around 1,678 seconds for the rod to cool using radiation.