# PHYS 229 - Ryan Kaufmann/Experiment 3/Cooling Prelab

SIGNED by Ryan Kaufmann Apr 10, 2018 @02:40 PM PDT

Ryan Kaufmann Apr 08, 2018 @02:26 PM PDT

## Cooling 0 Prelab Questions

Ryan Kaufmann Apr 08, 2018 @02:32 PM PDT

Let us first look at the convection equation of the rod. We can set up our differential equation as so:

Ryan Kaufmann Apr 09, 2018 @01:19 PM PDT

$$\frac{dQ}{dt} = -hA(T - T_0)$$

$$\frac{dQ}{dt} = -1.32A\frac{1}{D^{1/4}}(T - T_0)^{5/4}$$

$$\rho VC\frac{dT}{dt} = -1.32A\frac{1}{D^{1/4}}(T - T_0)^{5/4}$$

$$\frac{dT}{dt} = -1.32\frac{A}{\rho CV}\frac{1}{D^{1/4}}(T - T_0)^{5/4}$$

Ryan Kaufmann Apr 09, 2018 @07:49 PM PDT

We can then apply our constant conditions of the properties of the rod and the temperature of the lab. Our constants from the rod are set up such that we have the following:

Ryan Kaufmann Apr 09, 2018 @07:53 PM PDT

$$D = 0.0246m \pm 0.0001m$$

$$L = 0.305m \pm 0.001m$$

$$\rho = 2700kg/m^3 \pm 2kg/m^3$$

$$C = 904J/(kgK) \pm 2J/(kgK)$$

$$T_c = 293.0K \pm 0.5K$$

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Solving the differential equation gives us the following:

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$$\frac{dT}{dt} = -1.32\frac{A}{\rho CV}\frac{1}{D^{1/4}}(T-T_0)^{5/4}$$

$$\frac{dT}{dt} = -1.32\frac{0.02457}{2700*904*0.0001456}\frac{1}{0.02465^{1/4}}(T-293)^{5/4}$$

$$(T-293)^{-5/4}*dT = -0.0002304dt$$

$$\frac{4}{(T-293)^{1/4}} - \frac{4}{(T_c-293)^{1/4}} = -0.0002304t$$

Ryan Kaufmann Apr 08, 2018 @03:17 PM PDT

where Tc is the starting temperature of the rod (i.e. the temperature at t=0). We can then solve for both t when Tc is 363 and T is 303K and for the equation of T in terms of Tc and t:

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$$\frac{4}{(T-293)^{1/4}} - \frac{4}{(T_c-293)^{1/4}} = -0.0002304t$$

$$\frac{4}{(T-293)^{1/4}} = -0.0002304t - \frac{4}{(T_c-293)^{1/4}}$$

$$\frac{1}{(T-293)^{1/4}} = -0.00005761t - \frac{1}{(T_c-293)^{1/4}}$$

$$(T-293)^{1/4} = \frac{1}{-0.00005761t - \frac{1}{(T_c-293)^{1/4}}}$$

$$(T-293) = (\frac{1}{-0.00005761t - \frac{1}{(T_c-293)^{1/4}}})^4$$

$$T(t,T_c) = (\frac{1}{-0.00005761t - \frac{1}{(T_c-293)^{1/4}}})^4 + 293$$

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$$\frac{4}{(T-293)^{1/4}} + \frac{-4}{(T_c-293)^{1/4}} = -0.0002304t$$

$$\frac{4}{(363-293)^{1/4}} + \frac{-4}{(303-293)^{1/4}} = -0.0002304t$$

$$\frac{4}{(70)^{1/4}} + \frac{-4}{(10)^{1/4}} = -0.0002304t$$

$$\frac{4}{2.893} + \frac{-4}{1.778} = -0.0002304t$$

$$1.383 - 2.250 = -0.0002304t$$

$$-0.867 = -0.0002304t$$

$$t = 3762.57s$$

Ryan Kaufmann Apr 09, 2018 @08:06 PM PDT

Thus we have that it would take around 3,760 seconds for the rod to cool using convection.

Ryan Kaufmann Apr 08, 2018 @03:59 PM PDT

Applying the same differential equation techniques to the radiation formula gives us the following set up:

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$$\frac{dQ}{dt} = Ae\sigma T_0^4 - Ae\sigma T^4$$

$$\rho VC\frac{dT}{dt} = Ae\sigma T_0^4 - Ae\sigma T^4$$

$$\frac{dT}{dt} = \frac{Ae\sigma}{\rho VC}T_0^4 - \frac{Ae\sigma}{\rho VC}T^4$$

Ryan Kaufmann Apr 08, 2018 @04:09 PM PDT

Then applying our constants, we have:

Ryan Kaufmann Apr 09, 2018 @08:27 PM PDT

$$\frac{dT}{dt} = \frac{0.02457\sigma}{2700 * 904 * 0.0001456} * (293)^4 - \frac{0.02457\sigma}{2700 * 904 * 0.0001456} * T^4$$

$$\frac{1}{(293^4 - T^4)} \frac{dT}{dt} = 9.899 * 10^{-12}$$

$$\ln(293 + T_c) - 2\arctan(293/T_c) - \ln(293 + T) + 2\arctan(293/T) = 0.0009960t$$

Ryan Kaufmann Apr 08, 2018 @11:14 PM PDT

While this is unsolvable for a analytical solution of T in terms of Tc and t, we can plug in values of T and Tc to find the cooling time:

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$$\ln(293 + 363) - 2\arctan(293/363) - \ln(293 + 303) + 2\arctan(293/303) = 0.0009960t$$

$$1.6709 = 0.0009960t$$

$$t = 1677.7s$$

Ryan Kaufmann Apr 09, 2018 @08:30 PM PDT

Which gives us a shorter around 1,678 seconds for the rod to cool using radiation.

# PHYS 229 - Ryan Kaufmann/Experiment 3/Cooling Lab

SIGNED by Ryan Kaufmann Apr 10, 2018 @02:40 PM PDT

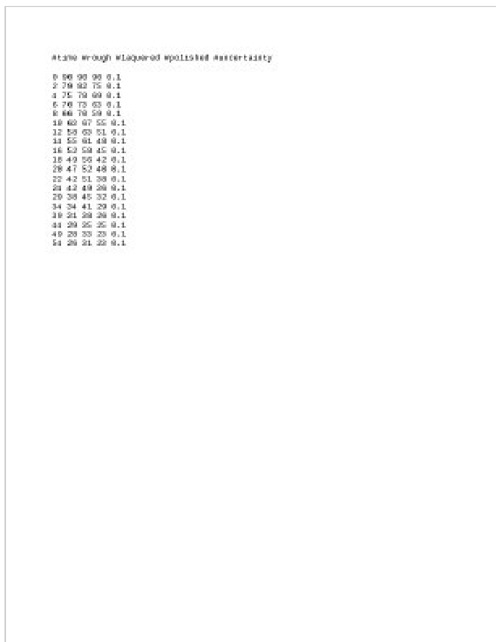
Ryan Kaufmann Apr 09, 2018 @08:37 PM PDT

## Cooling 1 - Comparison to Convection

Ryan Kaufmann Apr 09, 2018 @05:42 PM PDT

In the first section of this lab, we took data from the cooling of the rods. After placing the rods in boiling water and rising the temperature of each up to approximately 90 degrees Celsius, we placed the rods on a stand and let them cool down to 30 degrees Celsius. A thermometer was inserted in the top of each rod into a pre-cut hole to measure the temperature as it cooled. Measurements of the temperature of the rod were taken every 2 minutes, but were extended to 5 minutes when the rod wasn't cooling fast enough, or rather when the change in temperature was less than 1 degree. This continued until the rod reached 30 degrees Celsius. We got the following data sets:

Ryan Kaufmann Apr 09, 2018 @06:19 PM PDT



```
#time #rough #laquered #polished #uncertainty

0 90 90 90 0.1
2 78 82 75 0.1
4 75 78 69 0.1
6 70 73 63 0.1
8 66 70 59 0.1
10 62 67 55 0.1
12 58 63 51 0.1
14 55 61 48 0.1
16 52 58 45 0.1
18 49 56 42 0.1
20 47 52 40 0.1
22 42 51 38 0.1
24 42 49 36 0.1
29 38 45 32 0.1
34 34 41 29 0.1
39 31 38 26 0.1
44 29 35 25 0.1
49 28 33 23 0.1
54 26 31 22 0.1
```

**DataSet1.txt(364 Bytes)**

Ryan Kaufmann Apr 09, 2018 @05:36 PM PDT

```
#Time #Smooth(30.3cm) #Lacquered(30.5cm) #Rough(30.6cm) #Uncertainty
0 79 80.5 82 0.1
2 76 76 79 0.1
4 72 70 74 0.1
6 68 64 69 0.1
8 64 60 64 0.1
10 61 56 60 0.1
12 58 52 56 0.1
14 55 49.5 53 0.1
16 52 46.5 50.5 0.1
18 49.5 44 49 0.1
20 47 41.5 45.5 0.1
22 46 40 43.5 0.1
24 44 38 41 0.1
26 42 36 40 0.1
28 40 34 38 0.1
30 38.5 33 36.5 0.1
32 37 31.5 35 0.1
34 36 30.5 34 0.1
36 35 29.5 32 0.1
38 33.5 28.5 31.5 0.1
40 32 28 30.5 0.1
42 31.7 27.7 29.9 0.1
44 31 27.5 29.4 0.1
46 30 27.3 28.9 0.1
```

**DataSet2.txt(519 Bytes)**

Ryan Kaufmann Apr 09, 2018 @05:36 PM PDT

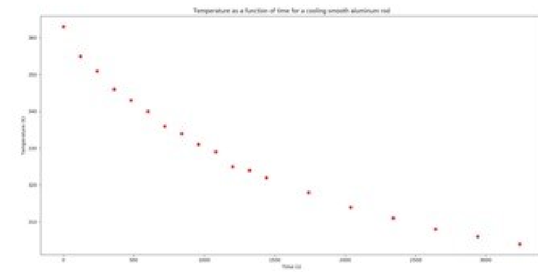
```
#Time #Smooth #Lacquered #Rough #Uncertainty
0 81.0 76.5 80.1 0.1
2 77.0 71.8 75.3 0.1
4 73.9 67.6 71.3 0.1
6 70.7 62.9 67.1 0.1
8 67.4 58.8 62.9 0.1
10 64.2 54.7 59.2 0.1
12 61.4 51.1 55.6 0.1
14 58.6 48.2 52.9 0.1
16 56.0 45.7 50.2 0.1
18 53.9 43.0 47.9 0.1
20 51.5 40.9 45.6 0.1
25 46.9 36.7 40.8 0.1
30 42.9 33.2 36.9 0.1
40 36.9 28.7 31.0 0.1
50 32.2 25.8 27.6 0.1
60 29.4 24.0 25.3 0.1
```

**DataSet3.txt(407 Bytes)**

Ryan Kaufmann Apr 09, 2018 @05:38 PM PDT

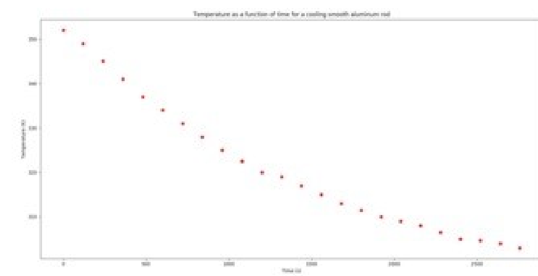
When can then plot each of the data sets separately and compare them all, eventually with the convection equation we derived in the prelab. For the smooth rods, we get the following data plots:

Ryan Kaufmann Apr 09, 2018 @08:37 PM PDT



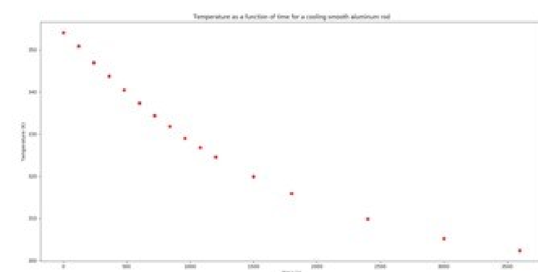
**SmoothSet1.png(32 KB)**

Ryan Kaufmann Apr 09, 2018 @08:37 PM PDT



**SmoothSet2.png(31.6 KB)**

Ryan Kaufmann Apr 09, 2018 @08:37 PM PDT



**SmoothSet3.png(31.4 KB)**

Ryan Kaufmann Apr 09, 2018 @06:20 PM PDT

And equally for the lacquered rods:

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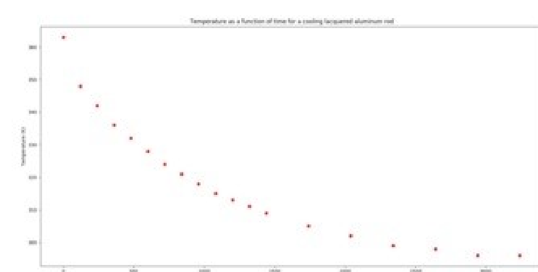
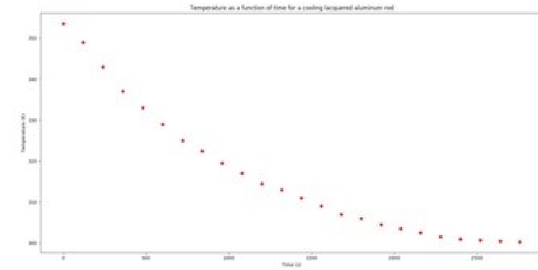


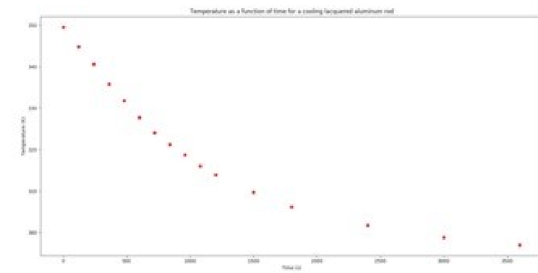
**LacqueredSet1.png(33.3 KB)**

Ryan Kaufmann Apr 09, 2018 @08:37 PM PDT




**LacqueredSet2.png(33 KB)**

Ryan Kaufmann Apr 09, 2018 @08:37 PM PDT

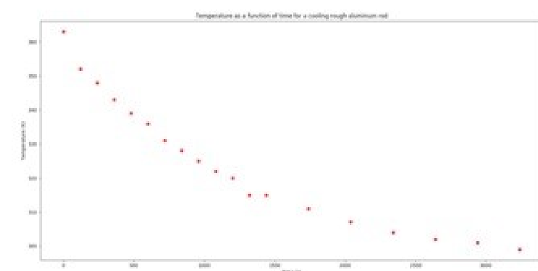



**LacqueredSet3.png(32.2 KB)**

Ryan Kaufmann Apr 09, 2018 @06:20 PM PDT

And rough rods:

Ryan Kaufmann Apr 09, 2018 @08:38 PM PDT




**RoughSet1.png(32.4 KB)**

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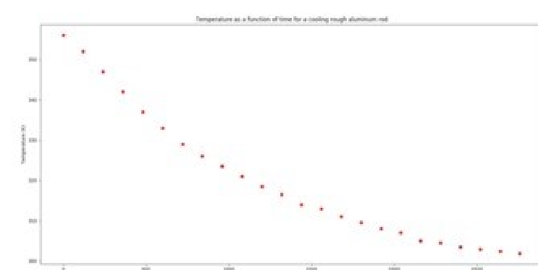
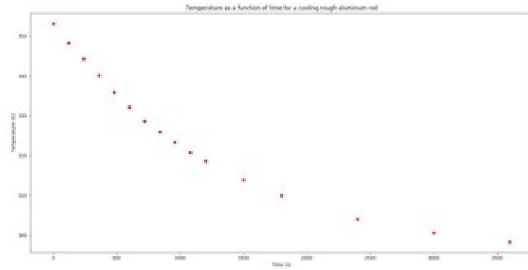



**RoughSet2.png(32.3 KB)**

Ryan Kaufmann Apr 09, 2018 @08:39 PM PDT

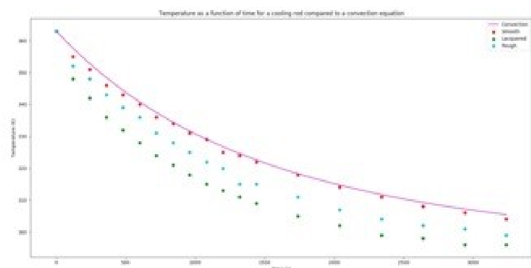


**RoughSet3.png(31.2 KB)**

Ryan Kaufmann Apr 09, 2018 @06:28 PM PDT

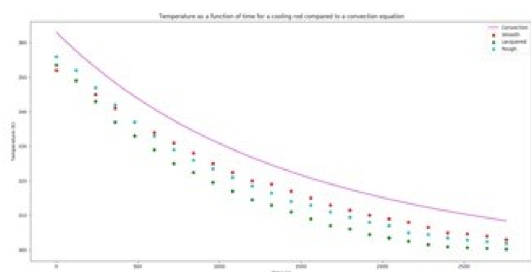
We can also see them compared to the convective heat loss equation. Comparing all three of these data sets to the convection equation (setting the maximum at 90 degrees Celsius) we derived gives us the following graphs:

Ryan Kaufmann Apr 09, 2018 @08:39 PM PDT



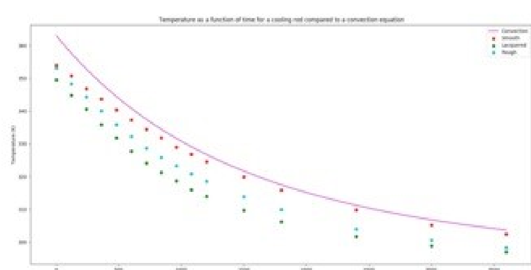
**AllSet1.png(68.2 KB)**

Ryan Kaufmann Apr 09, 2018 @08:39 PM PDT



**AllSet2.png(71.8 KB)**

Ryan Kaufmann Apr 09, 2018 @08:39 PM PDT



**AllSet3.png(68.1 KB)**

Ryan Kaufmann Apr 09, 2018 @08:41 PM PDT

We can see that there is some comparable form between the convection curves and the curve in the data sets, but this curve cannot be adjusted for anything except for the starting temperature. Instead, let us look at the radiation equation using numerical integration.

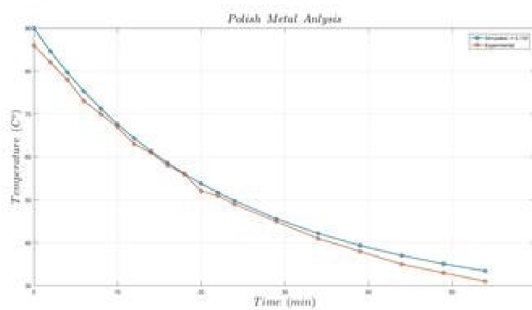
Ryan Kaufmann Apr 09, 2018 @08:38 PM PDT

## Cooling Process 2 - Radiation Equation

Ryan Kaufmann Apr 10, 2018 @02:22 PM PDT

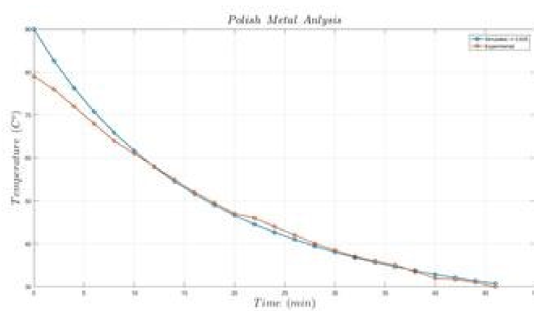
We can see that the convection equation doesn't quite work out, so we can also try to add radiation into the equation. When we do so we get more interesting results. Since the equation for radiation requires one more variable which we cannot measure directly, we can adjust our equation such that we find this variable. We are looking at the emissivity of the rod. The emissivity of the rod depends on various properties such as the roughness of the surface or the coating no the rod. We can use matlab codes to set up differential equations and numerically solve them until we get reasonable values for emissivity. We get the following graphs for the smooth rod:

Ryan Kaufmann Apr 10, 2018 @02:22 PM PDT



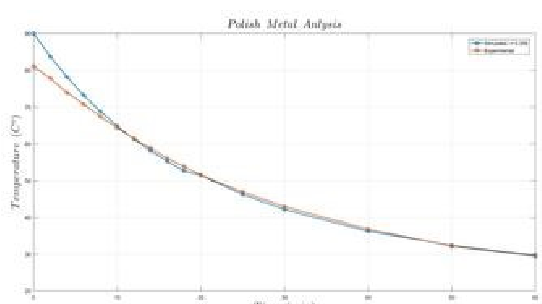

**SmoothSet1Radiation.jpg(81.2 KB)**

Ryan Kaufmann Apr 10, 2018 @02:22 PM PDT




**SmoothSet2Radiation.jpg(84.5 KB)**

Ryan Kaufmann Apr 10, 2018 @02:22 PM PDT




**SmoothSet3Radiation.jpg(80.1 KB)**

Ryan Kaufmann Apr 10, 2018 @02:23 PM PDT

and the lacquered:

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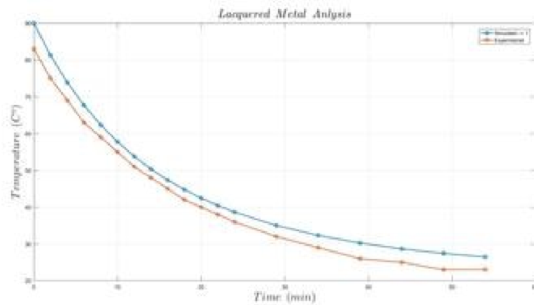
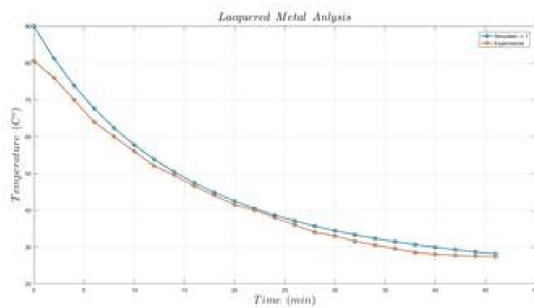


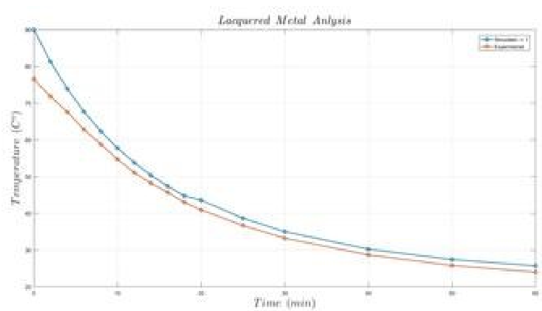

**LacqueredSet1Radiation.jpg(84.5 KB)**

Ryan Kaufmann Apr 10, 2018 @02:23 PM PDT




**LacqueredSet2Radiation.jpg(87.9 KB)**

Ryan Kaufmann Apr 10, 2018 @02:23 PM PDT




**LacqueredSet3Radiation.jpg(84.2 KB)**

Ryan Kaufmann Apr 10, 2018 @02:23 PM PDT

and rough rods:

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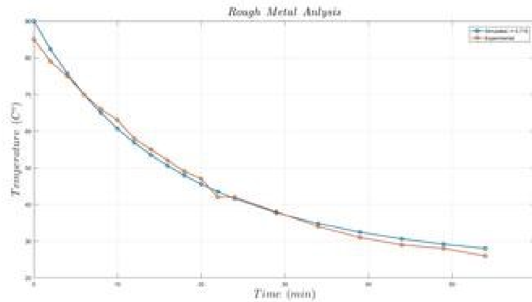
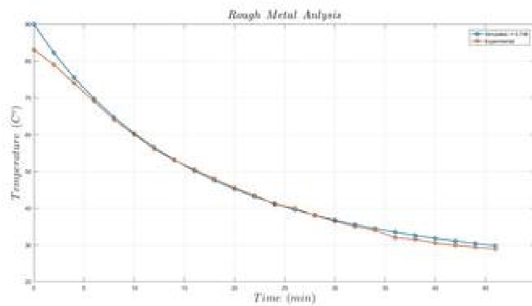



**RoughSet1Radiation.jpg(81.6 KB)**

Ryan Kaufmann Apr 10, 2018 @02:23 PM PDT




**RoughSet2Radiation.jpg(84.5 KB)**

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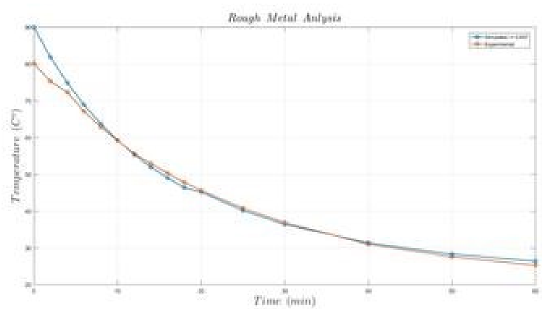


**RoughSet3Radiation.jpg(80.5 KB)**

Ryan Kaufmann Apr 10, 2018 @02:25 PM PDT

All of these data sets give slightly different values for emissivity which we can average to find the closest approximate value of emissivity. Between all three sets, the order of the highest to lowest emissivity is maintained so that we have a general idea of this order. From the fitting, we get the following values of emissivity:

Ryan Kaufmann Apr 10, 2018 @02:30 PM PDT

$$\epsilon_{s1} = 0.1330$$
$$\epsilon_{s2} = 0.6250$$
$$\epsilon_{s3} = 0.3550$$
$$\epsilon_{l1} = 1$$
$$\epsilon_{l2} = 1$$
$$\epsilon_{l3} = 1$$
$$\epsilon_{r1} = 0.7150$$
$$\epsilon_{r2} = 0.7480$$
$$\epsilon_{r3} = 0.8470$$

Ryan Kaufmann Apr 10, 2018 @02:31 PM PDT

We can then average these values to see the approximate emissivity of each of the type of rods. We notice a particularly large spread in the emissivity of the smooth rod which we believe is simply a property of the smooth rod and the other conditions of the room. We get the following averages:

Ryan Kaufmann Apr 10, 2018 @02:33 PM PDT

$$\epsilon_s = 0.3710$$
$$\epsilon_l = 1$$
$$\epsilon_r = 0.7700$$

Ryan Kaufmann Apr 10, 2018 @02:34 PM PDT

As we can see from the graphs as well, the fitted functions generally hold the shape of the data points as well compared to the convection equations, suggesting that the convection along with the radiation equation is the most accurate of the two methods for analyze the cooling of the rods, although it may require numerical solutions.

Ryan Kaufmann Apr 10, 2018 @02:34 PM PDT

## Cooling Process 3 - Conclusion

Ryan Kaufmann Apr 10, 2018 @02:40 PM PDT

This lab delved deep into using methods of differential equations to analyze physical properties of objects, namely a cooling rod. We see from the data that when we take into account the radiation of the object, we get a much better fit for the data then simply looking at the convection equation. While in this case only two types of cooling were considered, we can conclude that one is better than the other from our graphs. Although this answer isn't completely sure, as it doesn't take into account any residuals or chi-squares but simply the shape of the graph, which in some cases is not the best comparison, such as the lacquered rod. However, it does give some evidence that one method is better at analyzing than the other. Furthermore, we could make approximation about physical quantities of the object, such as emissivity. In this case, while it is not a fantastic method of measuring such a quantity, it gives us ballpark estimates of each of the rod, but more importantly the order from highest emissivity to lowest emisivity. Finally this lab shows, that while the physics or mathematics of a particular phenomenon might not be known, by recreating the process we can make some approximation as to how certain things can be order or an approximation of the values that are needed to make equations work.