Correlations and entanglement in the 2D RVB wavefunction

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1 Introduction

1. Motivation behind RVB wavefunction
2. Results on known limits?

2 Simulations

Using Monte Carlo simulations in the valence bond basis, we study the SU(N) RVB wavefunction. The RVB wavefunction is an equal-amplitude superposition of all nearest-neighbor valence-bond states, $|\Psi\rangle = \sum_{\langle ij\rangle} |\phi\rangle_{ij}$, where

$$|\phi\rangle_{ij} = \frac{1}{2S+1} \sum_{m=-S}^{S} (-1)^{m-S} |m\rangle_i \otimes |-m\rangle_j, \tag{1}$$

in the S^z basis ($2S+1 \equiv N$). Here, i, j are nearest neighbor sites on the square lattice. Because the wavefunction is inversely proportional to $N \equiv 2S+1$, as $N \to \infty$, the RVB wavefunction approaches the limit of classical dimers, where the states ϕ are orthogonal. The RVB Monte Carlo sampling algorithm does a random walk through the possible states by creating a defect at some spatial point and propagating it through the system (thereby rearranging the nearest-neighbor bonds) until the defect reaches the initial point and its path forms a closed loop.

We give the estimators for the correlation functions. Details can be found in Beach's paper. We wish to compute the dimer-dimer correlation function

$$\langle D_i(r)\, D_j(r')\rangle \sim \langle (S_r \cdot S_{r+\hat{e}_i})(S_{r'} \cdot S_{r'+\hat{e}_j})\rangle - \langle S_r \cdot S_{r+\hat{e}_i}\rangle \langle S_{r'} \cdot S_{r'+\hat{e}_j}\rangle. \tag{2}$$

The spin-spin correlations:

$$\langle S_r \cdot S_{r+\hat{e}_i}\rangle = \epsilon_{r,r'}\, S(S+1), \tag{3}$$

where $\epsilon_{r,r'} = 1$ if r, r' are on the same sublattice and -1 otherwise. In the simulations, we compute only parallel dimer-dimer correlations, i.e. $D_y(r)\, D_y(r+n\hat{e}_x)$, with $n = 0, 1, ...$ The estimators are

$$D_y(r)\, D_y(r') = \begin{cases} S^2(S+1)^2 & \text{if} \quad \text{all dimers are in the same loop} \\ \frac{1}{3}S^2(S+1)^2 & \text{if} \quad r, r' \text{ are in the same loop and their dimer neighbors are in another loop} \end{cases} \tag{4}$$

3 Entropy scaling of the SU(N) wavefunction

4 Jean-Marie's calculations?

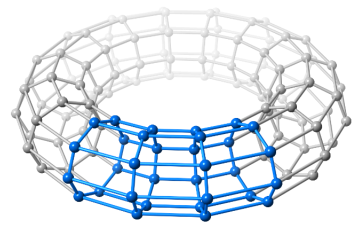

Figure 1: A 8×16 torus. Compute entanglement entropy using the SWAP operator and the ratio trick

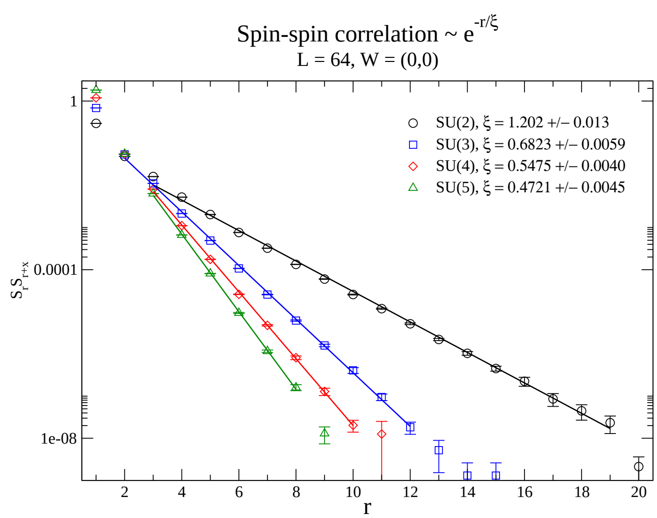

Figure 2: Spin-spin correlations are exponentially decaying and the correlation length is inversely proportional to N.

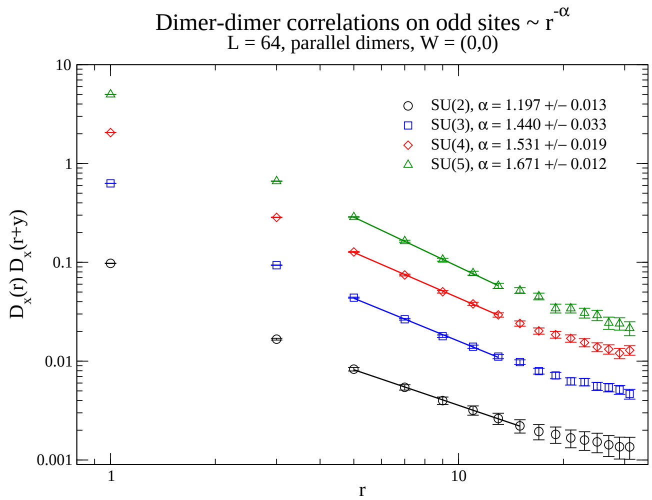

Figure 3: Critical dimer-dimer correlations for all N. We know that the classical value, $N \to \infty$, is $\alpha = 2$. This seems to be saying that SU(3) is not close to being classical.

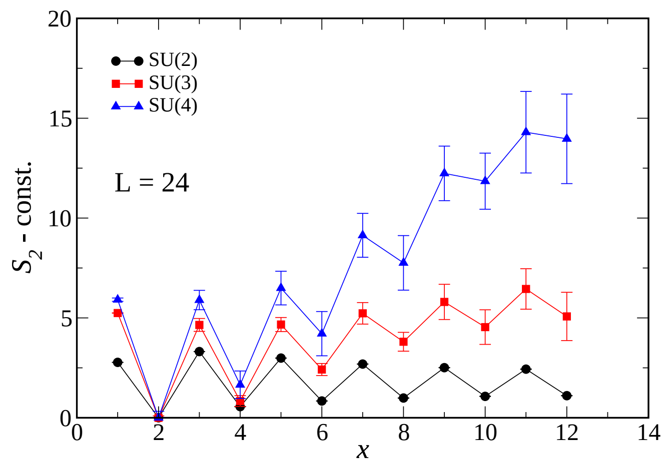

Figure 4: Prelim figure. Shows the behavior of the entanglement entropy as a function of N. Still trying to converge the SU(3) and SU(4) data. This alone is probably not enough to give any conclusive statements about larger N, but maybe we can use the dimer-dimer correlations.

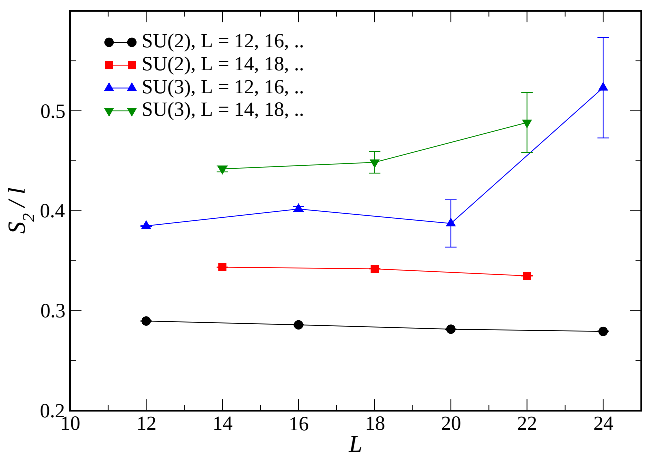

Figure 5: Entanglement entropy taken at $x = L/2$ for those that end on even/odd branch. This is probably not necessary. One thing that might be helpful though is to fit the SU(3 or 4) data to $\ln[\sin(\pi x/L)]$ and fit its coefficient.