

# Measuring of Entanglement Entropy in Valence Bond Quantum Monte Carlo Simulations

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

## Abstract

In this thesis we present methods for measuring entanglement entropy in spin-1/2 Heisenberg systems using quantum Monte Carlo in the valence bond basis. We first directly compare the recently proposed valence bond entanglement entropy to the standard definition of entanglement entropy: the von Neumann entanglement entropy. We find both cases in which SHUT UP, THESIS

We explain VB QMC techniques:

- single projector
- double projector
- loop algorithm

Look at VB EE compared to vN

Look at Renyi EE

Area Laws

## **Acknowledgements**

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# Chapter 1

## Introduction

Explain entanglement  
Explain measures of entanglement  
Talk about entanglement entropy  
Area Law  
Corrections to Area Law

### 1.1 Entanglement

### 1.2 Measures of Entanglement

### 1.3 The von Neumann Entanglement Entropy

### 1.4 The Area Law

## Chapter 2

# Quantum Monte Carlo in the Valence Bond Basis

### 2.1 The Valence Bond Basis

### 2.2 Ground State Projection

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### 2.4 Single Projector

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### 2.5 Double Projector

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# Chapter 3

## Valence Bond Entanglement Entropy

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### 4.2 1D Results

### 4.3 The Ratio Operator

### 4.4 2D Results

### 4.5 The Area Law

## Chapter 5

# Prospects for Future Research

# Chapter 6

## Conclusions

# APPENDICES

# Bibliography

- [1] Anders W. Sandvik. Ground state projection of quantum spin systems in the valence-bond basis. *Phys. Rev. Lett.*, 95(20):207203, Nov 2005.