

Scaling of entanglement entropy in the RVB wavefunction

February 7, 2012

1 Simulations

Using Monte Carlo simulations in the valence bond basis, we study the $SU(N)$ RVB wavefunction. The RVB wavefunction is an equal-amplitude superposition of all nearest-neighbor valence-bond states, $|\Psi\rangle = \sum_{\langle ij \rangle} |\phi\rangle_{ij}$, where

$$|\phi\rangle_{ij} = \frac{1}{2S+1} \sum_{m=-S}^S (-1)^{m-S} |m\rangle_i \otimes |-m\rangle_j, \quad (1)$$

in the S^z basis ($2S+1 \equiv N$). Here, i, j are nearest neighbor sites on the square lattice. Because the wavefunction is inversely proportional to $N \equiv 2S+1$, as $N \rightarrow \infty$, the RVB wavefunction approaches the limit of classical dimers, where the states ϕ are orthogonal. The RVB Monte Carlo sampling algorithm does a random walk through the possible states by creating a defect at some spatial point and propagating it through the system (thereby rearranging the nearest-neighbor bonds) until the defect reaches the initial point and its path forms a closed loop.

We give the estimators for the correlation functions. Details can be found in Beach's paper. We wish to compute the dimer-dimer correlation function

$$\langle D_i(r) D_j(r') \rangle \sim \langle (S_r \cdot S_{r+\hat{e}_i})(S_{r'} \cdot S_{r'+\hat{e}_j}) \rangle - \langle S_r \cdot S_{r+\hat{e}_i} \rangle \langle S_{r'} \cdot S_{r'+\hat{e}_j} \rangle. \quad (2)$$

The spin-spin correlations:

$$\langle S_r \cdot S_{r+\hat{e}_i} \rangle = \epsilon_{r,r'} S(S+1), \quad (3)$$

where $\epsilon_{r,r'} = 1$ if r, r' are on the same sublattice and -1 otherwise. In the simulations, we compute only parallel dimer-dimer correlations, i.e. $D_y(r) D_y(r + n\hat{e}_x)$, with $n = 0, 1, \dots$. The estimators are

$$D_y(r) D_y(r') = \begin{cases} S^2(S+1)^2 & \text{if all dimers are in the same loop} \\ \frac{1}{3} S^2(S+1)^2 & \text{if } r, r' \text{ are in the same loop and their dimer neighbors are in another loop} \end{cases} \quad (4)$$

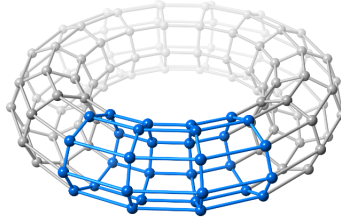


Figure 1: A 8×16 torus. Compute entanglement entropy using the SWAP operator and the ratio trick

2 Entropy scaling of the $SU(N)$ wavefunction

3 Jean-Marie's calculations?

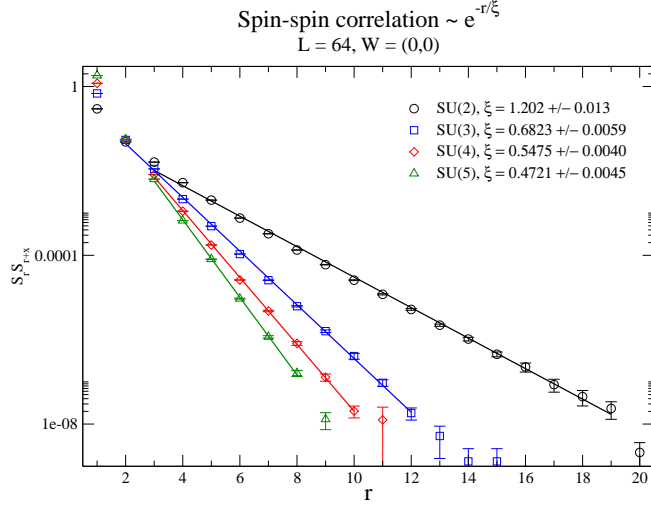


Figure 2: Spin-spin correlations are exponentially decaying and the correlation length is inversely proportional to N .

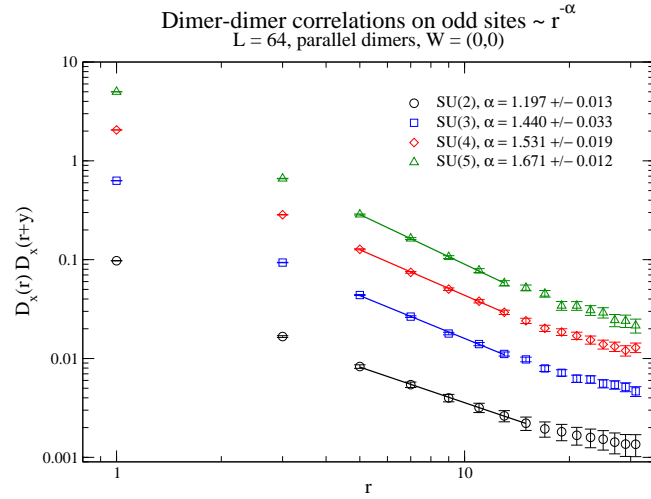


Figure 3: Critical dimer-dimer correlations for all N . We know that the classical value, $N \rightarrow \infty$, is $\alpha = 2$. This seems to be saying that SU(3) is not close to being classical.

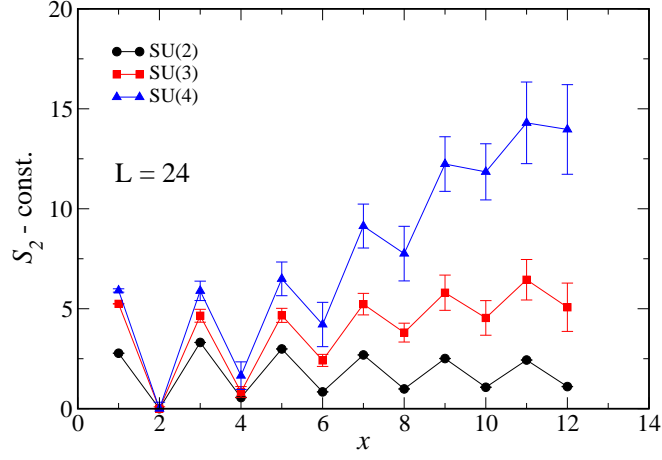


Figure 4: Prelim figure. Shows the behavior of the entanglement entropy as a function of N . Still trying to converge the SU(3) and SU(4) data. This alone is probably not enough to give any conclusive statements about larger N , but maybe we can use the dimer-dimer correlations.

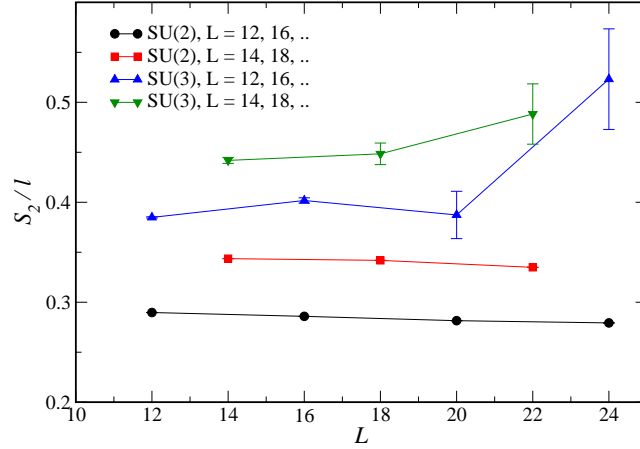


Figure 5: Entanglement entropy taken at $x = L/2$ for those that end on even/odd branch. This is probably not necessary. One thing that might be helpful though is to fit the SU(3 or 4) data to $\ln[\sin(\pi x/L)]$ and fit its coefficient.