#### Measuring of Entanglement Entropy in Valence Bond Quantum Monte Carlo Simulations

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

#### Abstract

In this thesis we present methods for measuring entanglement entropy in spin-1/2 Heisenberg systems using quantum Monte Carlo in the valence bond basis. We first directly compare the recently proposed valence bond entanglement entropy to the standard definition of entanglement entropy: the von Neumann entanglement entropy. We find both cases in which SHUT UP, THESIS

We explain VB QMC techniques:

- single projector
- double projector
- loop algorithm Look at VB EE compared to vN Look at Renyi EE Area Laws

#### Acknowledgements

I would like to thank all the people who made this possible.

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#### Introduction

Explain entanglement
Explain measures of entanglement
Talk about entanglement entropy
Area Law
Corrections to Area Law

- 1.1 Entanglement
- 1.2 Measures of Entanglement
- 1.3 The von Neumann Entanglement Entropy
- 1.4 The Area Law

# Quantum Monte Carlo in the Valence Bond Basis

- 2.1 Single Projector
- 2.2 Double Projector
- 2.3 Loop Moves

#### Valence Bond Entanglement Entropy

- 3.1 One Dimensional Systems
- 3.2 Approaching Two Dimensions
- 3.3 The Area Law

### Measuring Rényi Entanglement Entropy

- 4.1 The Swap Operator
- 4.2 1D Results
- 4.3 The Ratio Operator
- 4.4 2D Results
- 4.5 The Area Law

Prospects for Future Research

Conclusions

#### **APPENDICES**

### Bibliography

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