1 Model Definition

1.1 State Space

We consider a universe of n binary variables ("bits"):

$$i \in \{1, \dots, n\}.$$

A population of N agents is represented as bitstrings

$$x^{(a)} \in \{0,1\}^n, \qquad a = 1, \dots, N,$$

assembled into a matrix

$$X \in \{0,1\}^{N \times n}.$$

1.2 Constraints (Couplings)

A constraint is a triple

$$c = (\mathcal{S}, f, s),$$

where:

- $\mathcal{S} \subseteq \{1, \dots, n\}$ is the scope,
- $f \in \{0, \dots, 2^{|\mathcal{S}|} - 1\}$ is the forbidden pattern index,
- $s \in \mathbb{R}_+$ is the constraint strength.

For an agent a, the local pattern on $\mathcal{S}$ is encoded as

$$I^{(a)}(\mathcal{S}) = \sum_{k=1}^{|\mathcal{S}|} w_{\mathcal{S},k}\, x^{(a)}_{\mathcal{S},k},$$

where $w_{\mathcal{S}}$ is a fixed binary-weight vector (e.g. $(2,1)$ or $(4,2,1)$).
A violation occurs when

$$I^{(a)}(\mathcal{S}) = f.$$

1.3 Entropy

Let p_i be the empirical frequency of 1 at bit i:

$$p_i = \frac{1}{N} \sum_{a=1}^{N} x_i^{(a)}.$$

The per-bit Shannon entropy is

$$H_i = -p_i \log_2 p_i - (1 - p_i) \log_2 (1 - p_i),$$

and the global entropy is

$$H = \frac{1}{n} \sum_{i=1}^{n} H_i.$$

1.4 Emergent Geometry

Define a weighted adjacency matrix $A \in \mathbb{R}^{n\times n}$:

$$A_{ij} = \sum_{c\in\mathcal{C}} s_c \,\mathbf{1}[i, j \in \mathcal{S}_c,\ i \neq j].$$

A decay step is applied:

$$A \leftarrow \lambda A, \qquad 0 < \lambda < 1.$$

The emergent geometric distance is

$$d_{\text{geom}}(i, j) = \frac{1}{1 + A_{ij}}.$$

1.5 Emergent Mass

The mass of bit i is defined as the total constraint strength involving it:

$$m_i = \sum_{c\in\mathcal{C}} s_c \,\mathbf{1}[i \in \mathcal{S}_c].$$

1.6 Saturated Gravitational Attraction

Define the effective attraction

$$a_i = (a_{\min} + m_i)\,\frac{1}{1 + m_i/M_0},$$

where $a_{\min} > 0$ and $M_0 > 0$ are constants.

Normalize to obtain a probability distribution for selecting the first bit of a new scope:

$$P_{\text{global}}(i) = \frac{a_i}{\sum_{k=1}^{n} a_k}.$$

1.7 Scope Selection (Gravity + Locality)

To form a scope of size s:

First bit.

$$\mathbb{P}(i_1 = i) = P_{\text{global}}(i).$$

Subsequent bits. For $k = 2, \ldots, s$, define for each candidate j:

$$\bar{d}(j) = \frac{1}{k-1}\sum_{\ell=1}^{k-1} d_{\text{geom}}(j, i_\ell),$$

$$L(j) = \frac{1}{\bar{d}(j) + \varepsilon},$$

$$w(j) = a_j \cdot L(j),$$

$$\mathbb{P}(i_k = j) = \frac{w(j)}{\sum_{j'} w(j')}.$$

1.8 Relaxation Dynamics

For each constraint $c = (\mathcal{S}, f, s)$, violators satisfy

$$I^{(a)}(\mathcal{S}) = f.$$

Each violator flips a random bit in $\mathcal{S}$ with probability s.

1.9 Hebbian Decay and Hardening

Each step:

$$s \leftarrow \gamma s, \qquad 0 < \gamma < 1.$$

Let

$$q_c = \mathbb{P}(I^{(a)}(\mathcal{S}) \neq f).$$

If $q_c > q_{\text{thr}}$,

$$s \leftarrow s + \Delta.$$

Constraints with $s < s_{\min}$ are removed.

1.10 Emergent Force Field

On the largest connected component H with graph distance $d_H(i,j)$:

$$F(i) = \sum_{\substack{j \in H \\ j \neq i}} \frac{m_j}{(d_H(i,j) + \varepsilon)^2}.$$

Empirically, the force law satisfies

$$F \propto \left(\frac{1}{r}\right)^{\alpha},$$

with exponent $\alpha \approx 0.8$ obtained from a log–log fit:

$$\log F \approx \alpha \log\left(\frac{1}{r}\right) + \text{const}.$$

1.11 Mass–Geometry Correlations

Define average graph distance

$$\bar{d}_i = \frac{1}{|H| - 1} \sum_{\substack{j \in H \\ j \neq i}} d_H(i,j).$$

Empirically:

$$\text{corr}(m_i,\, 1/\bar{d}_i) \approx 0.87\text{–}0.88.$$

A curvature proxy is given by the clustering coefficient C_i:

$$\kappa_i \approx C_i.$$

Mass–curvature correlation is small:

$$\text{corr}(m_i,\, \kappa_i) \approx 0.$$