國 立 中 興 大 學

資 訊 工 程 學 系

CAD Homework #1

The Column Covering Problem

(due Nov. 9, 2022)

Problem Formulation

- **Column Cover**

Let X and Y be two sets, and R be a relation defined on $X \times Y$. We say that y covers x when xRy. The matrix associated with the column covering problem $<X, Y, R>$ has rows labeled with elements of X and columns labeled with elements of Y, such that the element $[x, y]$ of the matrix is equal to 1 iff xRy. A subset C of Y is a **column cover** of X iff for each element x in X, there exists some element y in C such that xRy.

- **Cost**

The cost of a column cover is

cost(C) = (|C|, weight(C)),

where |C| is the cardinality of the column cover C, weight(C) = $\sum_{y \in C}$weight(y), and weight(y) is the weight of an element y.

Given two column covers C_1 and C_2, cost(C_1) < cost(C_2) either if $|C_1| < |C_2|$ or if $|C_1| = |C_2|$ and weight(C_1)<weight(C_2).

- **Problem**

Given X, Y, R, and the weight associated with each element of Y, write a program to find the column cover of X with the minimum cost.

Benchmarks

Your program should allow input from a user-specified file and report the results. The following shows an example.

- **Input Format**

```
4  4                  // the cardinalities of X and Y
1  2  4  3           // the weights of elements of Y
1  1                  // R is described from this line, [x1, y1]=1
1  4                  // [x1, y4]=1
2  1                  // [x2, y1]=1
2  3                  // [x2, y3]=1
3  2                  // [x3, y2]=1
3  3                  // [x3, y3]=1
4  4                  // [x4, y4]=1
```

	1	2	4	3
	y_1	y_2	y_3	y_4
x_1	1			1
x_2	1		1	
x_3		1	1	
x_4				1

- **Output Format**

3 4 // the minimum column cover C, $\{y_3, y_4\}$
(2, 7) // cost(C)

Requirements

Your program must be able to be executed at **UNIX** or **Window** operation system in the following format.

% executable_file input_file

The document detailing the features of your approach and complexity reduction strategy is a must. Please send the compressed file of the source code, the executable file and the document to your teaching assistant. (Please specify your **student ID** in the subject line.) Performance will be evaluated by five instances (bench2.txt, bench3.txt, bench4.txt, bench5.txt, and bench6.txt). The benchmark circuits are posted along with this assignment.

Grading

Unique source code	20%
Show one minimum column cover	20%
Show all minimum column covers	15%
Documentation	10%
Performance	20%
Complexity reduction strategy	15%
(Bonus) Executable at UNIX system	10%