CMPT 310 Assignment 3.

Ques 1(a) $KB = \{C(a), C(b), C(c), \forall x((C(x) \wedge \neg S(x)) \supset M(x)), \forall x(\neg L(x, s) \supset \neg S(x)), \forall x(M(x) \supset \neg L(x, r)), \forall y(L(a,y) \supset \neg L(b,y)), \forall y(\neg L(a,y) \supset L(b,y)), L(a,r), L(a,s)\}$

Query $= \alpha$: $\exists x(C(x) \wedge M(x) \wedge \neg S(x))$

Clausal form:

① $[C(a)]$ ② $[C(b)]$ ③ $[C(c)]$

④ $[\sim C(x), S(x), M(x)]$ ⑤ $[\sim M(x), \sim L(x, r)]$

⑥ $[L(x, s), \sim S(x)]$ ⑦ $[\sim L(a, y), \sim L(b, y)]$

⑧ $[L(a,y), L(b,y)]$ ⑨ $[L(a, r)]$ ⑩ $[L(a, s)]$

⑪ $[\sim C(x), \sim M(x), S(x), A(x)]$ (negation of query)

Using Answer extraction:

⑪ $[\sim C(x), \sim M(x), S(x), A(x)]$ ④ $[\sim C(x), S(x), M(x)]$

x/b x/b

$[\sim C(b), S(b), A(b)]$ ② $[C(b)]$

$[S(b), A(b)]$ ⑥ $[L(x, s), \sim S(x)]$

x/b

$[A(b), L(b,s)]$ ⑩ $[L(a,s)]$

$[A(b), L(b,s), L(a,s)]$ ⑦ $[\sim L(a,y), \sim L(b,y)]$

y/s

$[A(b)]$

Bob is a member of Alpine club who is a mountain climber but not a skier.

Ques 1 (b)

The new knowledge base satisfies that:

Bob likes whatever Alice dislikes,

but not that:

Bob dislikes whatever Alice dislikes.

The new knowledge base is:

$KB' = \{C(a), C(b), C(c), \forall x((C(x) \wedge \neg S(x)) \supset M(x)), \forall x(M(x) \supset \neg L(x,y)), \forall x(\neg L(x,s) \supset \neg S(x)), \forall y(\neg L(a,y) \supset L(b,y)), L(a,r), L(a,s)\}$

$\alpha = \exists x(C(x) \wedge M(x) \wedge \neg S(x))$

Let $I = \langle D, \phi \rangle$ is an interpretation where D is domain,

$D = \{a, b, c, r, s\}$ such that $\phi(\text{Alice}) = a$, $\phi(\text{Bob}) = b$, $\phi(\text{Christine}) = c$, $\phi(\text{rain}) = r$, $\phi(\text{snow}) = s$, $\phi(\text{member of alpine club}) = \{a, b, c\}$, $\phi(\text{mountain climber}) = \{\}$, $\phi(\text{skier}) = \{a, b, c\}$,

ϕ likes $= \{(a,r), (b,r), (c,r), (a,s), (b,s), (c,s)\}$.

In this, Alice like rain and snow,
Bob likes rain and snow,
Christine likes rain and Snow

The interpretation staisfies all the clausal statement excluding ⑦ but it does not satisfy α as:

①, ②, ③ = true (directly given), ④ = true (no member in Alpine club that dislike skiing), ⑤ = true (No mountain climber), ⑥ = true (all members like snow and these are skier)

⑧ = true (Bob likes everything); ⑨, ⑩ = true (given directly in ϕ likes)

α = false (there is no mountain climber in Alpine club).

$\therefore I \models KB'$ but $I \not\models \alpha$.

Ques 2(a)

Start State: Op(Action: Start,
Effect: $Clear(A) \wedge Clear(B) \wedge Clear(C) \wedge On(A,T) \wedge On(B,T) \wedge On(C,T)$)

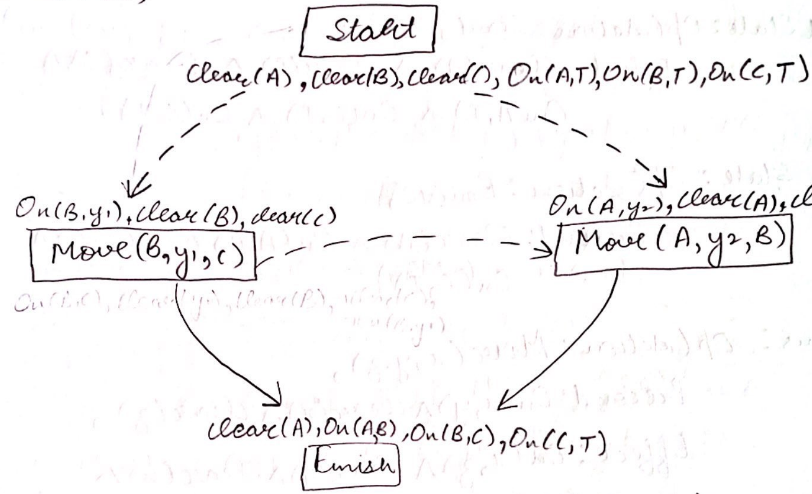
Goal State: Op(Action: Finish,
PreCond: $Clear(A) \wedge On(A,B) \wedge On(B,C) \wedge On(C,T)$)

Actions: Op(Action: Move(x, y, z),
Precond: $On(x,y) \wedge Clear(x) \wedge Clear(z)$,
Effect: $On(x,z) \wedge Clear(y) \wedge Clear(x) \wedge \neg Clear(z) \wedge \neg On(x,y)$)

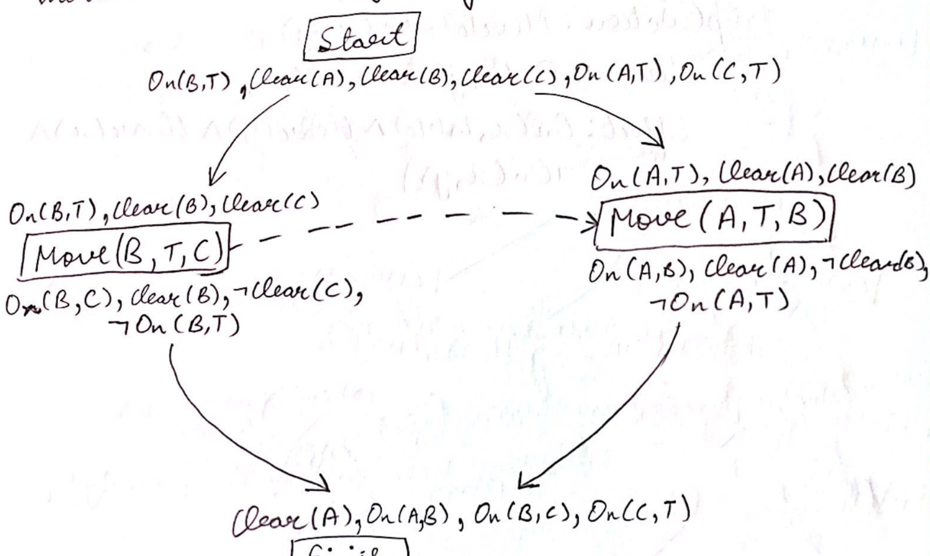
Op(Action: MoveToTable(x, y),
Precond: $On(x,y) \wedge Clear(x)$,
Effect: $On(x, table) \wedge Clear(y) \wedge Clear(x) \wedge \neg On(x,y)$)

Scanned with CamScanner

Ques 2(b)



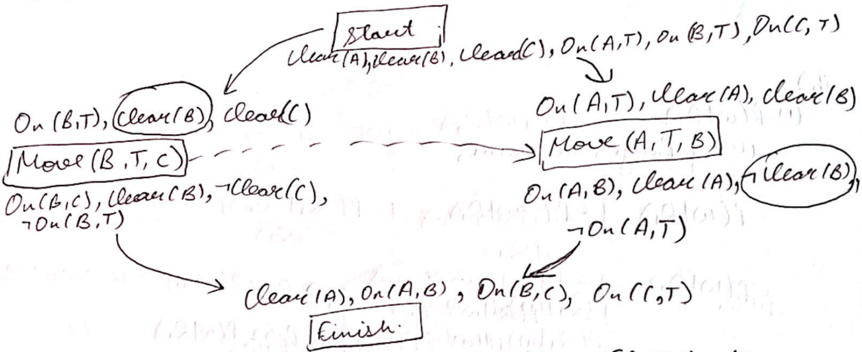
At start, all blocks are on table and causal links are made when variables y_1 and y_2 are substituted with T.



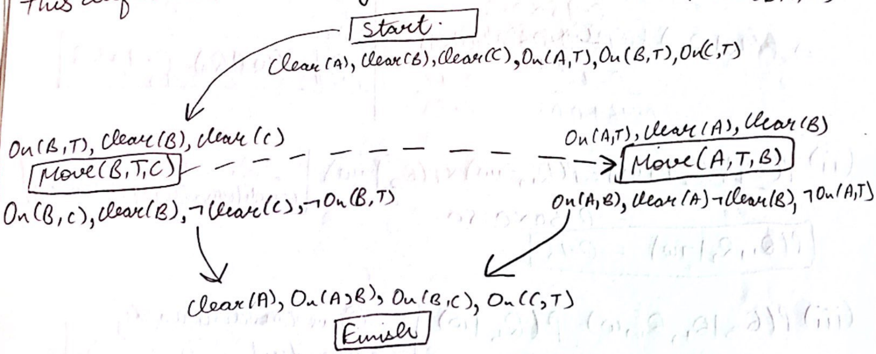
Ques 2

(C) The precondition/effect pair that causes threat is Move(B,T,C) and Move(A,T,B)

The effect of Move(A,T,B) that is ¬Clear(B) violates the pre condition of Move(B,T,C) that is Clear(B), hence causing a threat.



This conflict is resolved by promoting Move(A,T,B) after Move(B,T,C)



Yes, this plan is consistent and complete.

Ques 3

(a) $P(Q_1|W) = 0.95$ $\quad$ $P(Q_1|\neg w) = 0.30$

$P(Q_2|W) = 0.95$ $\quad$ $P(Q_2|\neg w) = 0.50$

$P(Q_3|W) = 0.95$ $\quad$ $P(Q_3|\neg w) = 0.10$

$P(w) = \frac{4}{5} = 0.80$

(b)

(i) $P(w|Q_1)$

using Bayes Theorem,

$$P(w|Q_1) = 1 - P(\neg w|Q_1) = 1 - \frac{P(Q_1|\neg w)P(\neg w)}{P(Q_1)}$$

$$\Rightarrow P(w|Q_1) = 1 - \frac{P(Q_1|\neg w)P(\neg w)}{\frac{P(Q_1|w)P(w)}{P(w|Q_1)}}$$

$$\Rightarrow P(w|Q_1) = 1 - \frac{0.3 \times 0.2 P(w|Q_1)}{0.95 \times 0.8}$$

$$\Rightarrow P(w|Q_1) = 1 - \frac{0.06}{0.76} P(w|Q_1)$$

$$\Rightarrow 0.76 P(w|Q_1) = 0.76 - 0.06 P(w|Q_1)$$

$$\Rightarrow 0.82 P(w|Q_1) = 0.76$$

$$\Rightarrow P(w|Q_1) = \frac{0.76}{0.82}$$

$$\Rightarrow \boxed{P(w|Q_1) = 0.9268}$$

(ii) $P(Q_1, Q_2|\neg w) = P(Q_1, \neg w) \times P(Q_2|\neg w)$ [$\because$ Q_1 and Q_2 are conditionally independent]

$= 0.30 \times 0.50$

$\boxed{P(Q_1, Q_2|\neg w) = 0.15}$

(iii) $P(Q_3|Q_1, Q_2, w) = P(Q_3|w)$ [$\because$ Q_3 is conditionally independent of Q_1 and Q_2]

$\boxed{P(Q_3|Q_1, Q_2, w) = 0.95}$

Ques 3 (c)

When Q_1, Q_2 and Q_3 are conditionally independent

$$P(\omega|Q_1,Q_2,\neg Q_3) = \alpha \cdot P(\omega) \cdot P(Q_1|\omega) P(Q_2|\omega) P(\neg Q_3|\omega)$$

$$= \alpha \cdot P(\omega) \cdot P(Q_1|\omega) \cdot P(Q_2|\omega) \cdot (1 - P(Q_3|\omega))$$

$$= \alpha \times 0.8 \times 0.95 \times 0.95 \times (1-0.95)$$

$$= \alpha \times 0.8 \times 0.95 \times 0.95 \times 0.05$$

$$= 0.0361\,\alpha$$

To calculate α, we need to find $P(\neg\omega|Q_1,Q_2,\neg Q_3)$:

$$P(\neg\omega|Q_1,Q_2,\neg Q_3) = \alpha \cdot P(\neg\omega) \cdot P(Q_1|\neg\omega) \cdot P(Q_2|\neg\omega) \cdot P(\neg Q_3|\neg\omega)$$

$$= \alpha \cdot P(\neg\omega) \cdot P(Q_1|\neg\omega) \cdot P(Q_2|\neg\omega) \cdot (1 - P(Q_3|\neg\omega))$$

$$= \alpha \times 0.2 \times 0.3 \times 0.50 \times (1-0.10)$$

$$= \alpha \times 0.2 \times 0.3 \times 0.5 \times 0.9$$

$$= 0.027\,\alpha$$

Since $P(\omega|Q_1,Q_2,\neg Q_3) + P(\neg\omega|Q_1,Q_2,\neg Q_3) = 1$

$$0.0361\alpha + 0.027\alpha = 1$$

$$0.0631\alpha = 1 \Rightarrow \alpha = \frac{1}{0.0631}$$

$$P(\omega|Q_1,Q_2,\neg Q_3) = 0.0361\,\alpha$$

$$= \frac{0.0361}{0.0631} = 0.5721$$

$$\boxed{P(\omega|Q_1,Q_2,\neg Q_3) = 0.5721}$$

Ques 3(d)

$$P(\omega \mid Q_1, \neg Q_2, \neg Q_3) = \alpha \cdot P(\omega) \cdot P(Q_1|\omega) \cdot P(\neg Q_2|\omega) \cdot P(\neg Q_3|\omega)$$

$$= \alpha \cdot P(\omega) \cdot P(Q_1|\omega) \cdot (1 - P(Q_2|\omega)) \cdot (1 - P(Q_3|\omega))$$

$$= \alpha \cdot 0.8 \times 0.95 \times (1 - 0.95) \times (1 - 0.95)$$

$$= \alpha \times 0.8 \times 0.95 \times 0.05 \times 0.05$$

$$= 0.0019\alpha$$

To calculate α, we need to calculate $P(\neg\omega \mid Q_1, \neg Q_2, \neg Q_3)$.

$$P(\neg\omega \mid Q_1, \neg Q_2, \neg Q_3) = \alpha \cdot P(\neg\omega) \cdot P(Q_1|\neg\omega) \cdot P(\neg Q_2|\neg\omega) \cdot P(\neg Q_3|\neg\omega)$$

$$= \alpha \cdot P(\neg\omega) \cdot P(Q_1|\neg\omega) \cdot (1 - P(Q_2|\neg\omega)) \cdot (1 - P(Q_3|\neg\omega))$$

$$= \alpha \times 0.2 \times 0.3 \times (1 - 0.5) \times (1 - 0.10)$$

$$= \alpha \times 0.2 \times 0.3 \times 0.5 \times 0.9$$

$$= 0.027\alpha$$

Since $P(\omega \mid Q_1, \neg Q_2, \neg Q_3) + P(\neg\omega \mid Q_1, \neg Q_2, \neg Q_3) = 1$

$$0.0019\alpha + 0.027\alpha = 1$$

$$0.0289\alpha = 1$$

$$\alpha = \frac{1}{0.0289}$$

$$P(\omega \mid Q_1, \neg Q_2, \neg Q_3) = 0.0019\alpha$$

$$= \frac{0.0019}{0.0289} = 0.0657$$

$$\boxed{P(\omega \mid Q_1, \neg Q_2, \neg Q_3) = 0.0657}$$

Ques 3 (e)

Answering some questions correctly or incorrectly, neither increases nor decreases the chance for answering another question correctly.

This statement is important as if questions are dependent then it will be hard for students to score they do not know the solution of just single solution. In that scenario, a student can either get full marks by answering all answers correctly or get zero marks by answering one question wrong. So It is important that correct or incorrect answers to some questions do not influence the chance for answering another questions correctly.

Ques 4:

(a) $P(W, \neg L, R, S = \text{Spring}) =$

$= P(W \mid \neg L \wedge R \wedge S = \text{spring}) \times P(\neg L \mid R \wedge S = \text{Spring}) \times P(R \mid S = \text{spring}) P(S = \text{spring})$

$= P(W \mid \neg L \wedge R) \times P(\neg L \mid S = \text{spring}) \, P(R \mid S = \text{spring}) \, P(S = \text{spring})$

$= 0.95 \times (1 - 0.15) \times 0.45 \times 0.25$

$= 0.95 \times 0.85 \times 0.45 \times 0.25$

$= 0.09084 = 0.091$

(b) $P(S = \text{Winter} \mid \neg R \wedge \neg L) = P(S = \text{winter} \mid \neg R) \cdot \dfrac{P(\neg L \mid S = \text{winter})}{P(\neg L \mid \neg R)}$

[Bayesian update]

$P(R) = P(R \mid \text{summer}) P(\text{summer}) + P(R \mid \text{autum}) P(\text{autum}) + P(R \mid \text{spring}) P(\text{spring}) + P(R \mid \text{winter}) P(\text{winter}) = (0.45 + 0.15 + 0.35 + 0.20)\, 0.25$

$= 1.15 \times 0.25 = 0.2875$

$P(\neg R) = 1 - 0.2875 = 0.7125$

$P(L) = P(L \mid \text{summer}) P(\text{summer}) + P(L \mid \text{autum}) P(\text{autum}) + P(L \mid \text{spring}) P(\text{spring}) + P(L \mid \text{winter}) P(\text{winter}) = (0.15 + 0.30 + 0.05 + 0.00) \times 0.25$

$= 0.125$

$P(\neg L) = 1 - P(L) = 1 - 0.125 = 0.875$

$P(\text{winter} \mid \neg R) = \dfrac{P(\neg R \mid \text{winter}) P(\text{winter})}{P(\neg R)} = \dfrac{(1 - 0.2) \times 0.25}{0.7125} = 0.2807$

$P(S = \text{winter} \mid \neg R \wedge \neg L) = \dfrac{P(\text{winter} \mid \neg R)(1 - P(L \mid \text{winter}))}{\dfrac{P(\neg L \wedge \neg R)}{P(\neg R)}}$

$$= \frac{0.2807 \times (1 - 0.00)}{\frac{0.875 \times \cancel{0.7125}}{\cancel{0.7125}}}$$

$$= \frac{0.2807 \times 1}{0.875} = 0.3208 = 0.321$$

(C) $P(R \mid W \wedge S = \text{summer}) = P(R \mid S = \text{summer})$ [$\because$ Wet sidewalk does not affect rain as R is parent of W. but rain affect Wet sidewalk.]

$= 0.15$