Supervised Learning Classification

Agenda

- Understanding terminologies
- Entropy,Shannons entropy,Conditional entropy
- Information gain
- Decision Tree
- Measure of purity of node
- Gini index
- Classification error
- Construction of Decision Tree
- Decision Tree Algorithm

Information Theory

Information theory

Information theory is based on the intuition that

Event	Information Gain	Example
Most likely event	No information	The sun rose this morning
Likely event	Little information	The sun rose at 6:30 a.m. this morning
Unlikely event	Maximum information	There was a solar eclipse this morning

Reference: Goodfellow, Ian, Yoshua Bengio, and Aaron Courville. *Deep Learning*. MIT press, 2016.

Information theory

- Let I(x) denote the information of an event X
- It is the self information of an event X at x

$$I(x) = -\ln P(x)$$

- Since we have considered natural log its units is nat
- For log with base 2, we use units called bits or shannons

Understanding Terminologies

Shannon's entropy

- Entropy is the measure of information for classification problem, i.e. it measures the heterogeneity of a feature

- The entropy of a feature is calculated as

$$E = -\sum_{i=1}^{c} p_c \log_2 p_c$$

where p_c is the probability of occurrence of the class

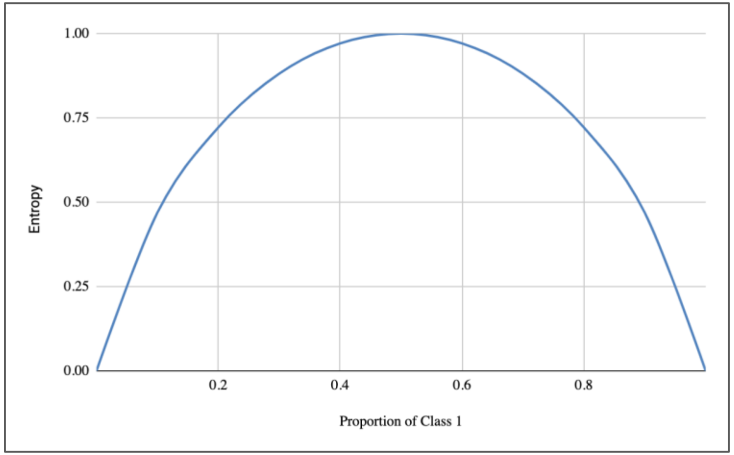
- A lower entropy is always prefered

- Entropy is always non-negative

Shannon's entropy

Consider a feature with two class the entropy for various proportion of classes is given below

Class 1	0	0.1	0.3	0.5	0.7	0.9	1
Class 2	1	0.9	0.7	0.5	0.3	0.1	0
Entropy	0	0.46	0.88	1	0.88	0.46	0



Entropy

Obtain the entropy of the given data

Entropy(Obesity) = - P(Not-obese) $\log_2$ (P(Not-obese))

-P(Obese) $\log_2$ (P(Obese))

Obesity		Total
Not-Obese	Obese	
20	15	35

Entropy(Obesity) = - 20/35 $\log_2$ (20/35) - 15/35 $\log_2$ (15/35)

Entropy(Obesity) = 0.985

Conditional entropy

The conditional entropy of one feature given other is calculated as from the contingency table of the two features.

$$E(T|X) = \sum_{x \in X} P(c)E(c)$$

It is the sum if the of the product of the probability of occurrence of the each class and the entropy of it.

Conditional entropy

To obtain the conditional entropy of Obesity given the person is a smoker

Entropy(Obesity|Smoker) = P(Not-obese) E(Not-obese|Smoker=Yes,No)

+ P(Obese) E(obese|Smoker=Yes,No)

Entropy(Obesity|Smoker) = (20/35) Entropy(15,5) + (15/35) Entropy(7,8)

Entropy(Obesity|Smoker) = (0.571) (0.811)+ (0.428) (0.997)

Entropy(Obesity|Smoker) = 0.890

		Obesity	
		Not - Obese	Obese
Smoker	Yes	15	7
	No	5	8
Total		20	15

Information gain

- Information gain is the decrease in entropy at a node

$$\text{Information Gain }(T, X) = \text{Entropy }(T) - \text{Entropy }(T|X)$$

- To construct the decision tree, the feature with highest information gain is chosen
- Information gain is always positive

Information Gain

The information gain in the feature Obesity due to Smoker is

Information Gain (Obesity, Smoker) = Entropy(Obesity) - Entropy(Obesity|Smoker)

...from slides 8 and 10

= 0.985 - 0.890

= 0.095

Can Information Gain be negative?

After the split of data, the purity of data will be higher as a result the entropy will always be lower. Thus, the information gain is always positive.

Decision Trees for Classification

Business problem: loan approval

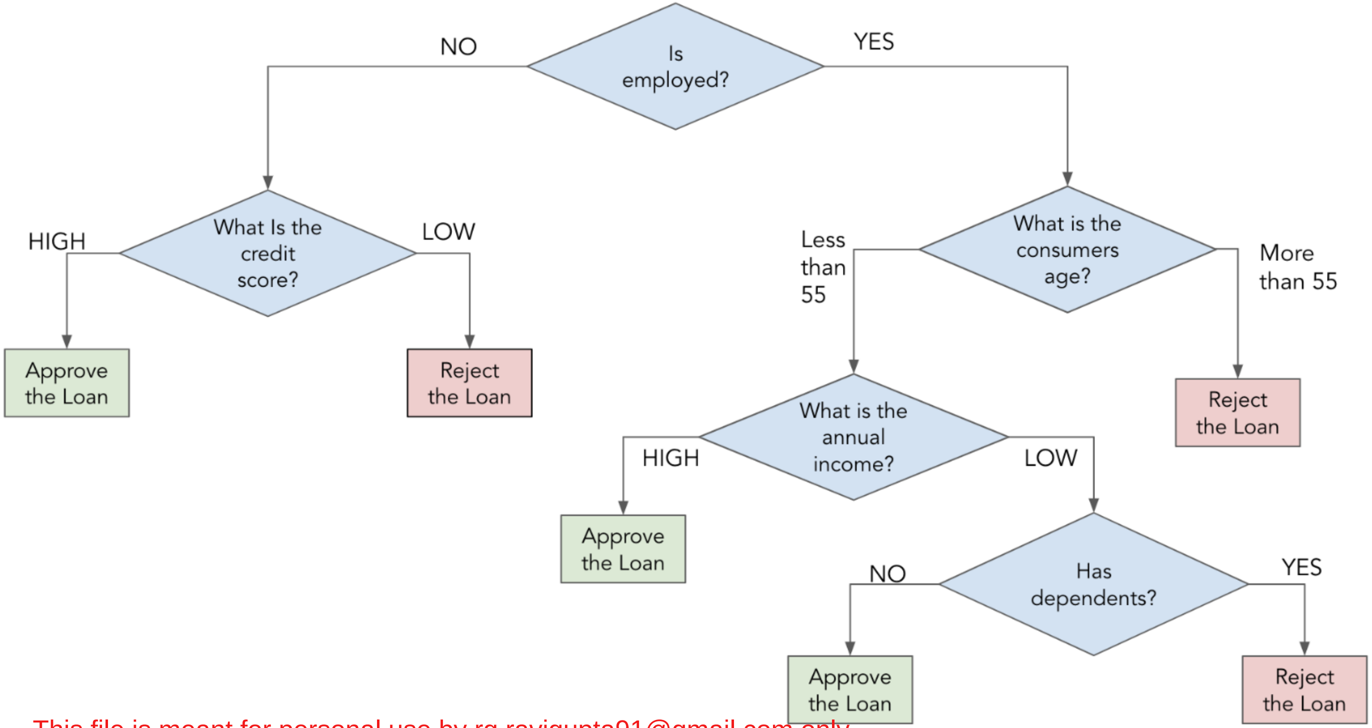
It is important to know the credibility of a consumer before lending a loan. It can be achieved by knowing answers to questions such as

- Is he employed?
- Is he nearing retirement?
- What his his annual income?
- Does he have any dependents?
- What is his age?
- What is his credit score?

These series of questions can be organised in a hierarchical structure.

Business problem: loan approval

We create a flowchart like hierarchical structure to decide whether to approve the loan.



Decision Tree

- Decision tree is a classifier that results in flowchart-like structure with nodes and edges

- Each node denotes a condition on an attribute value
(Condition: High or Low for the credit score)

- Each branch represents the outcome of the condition

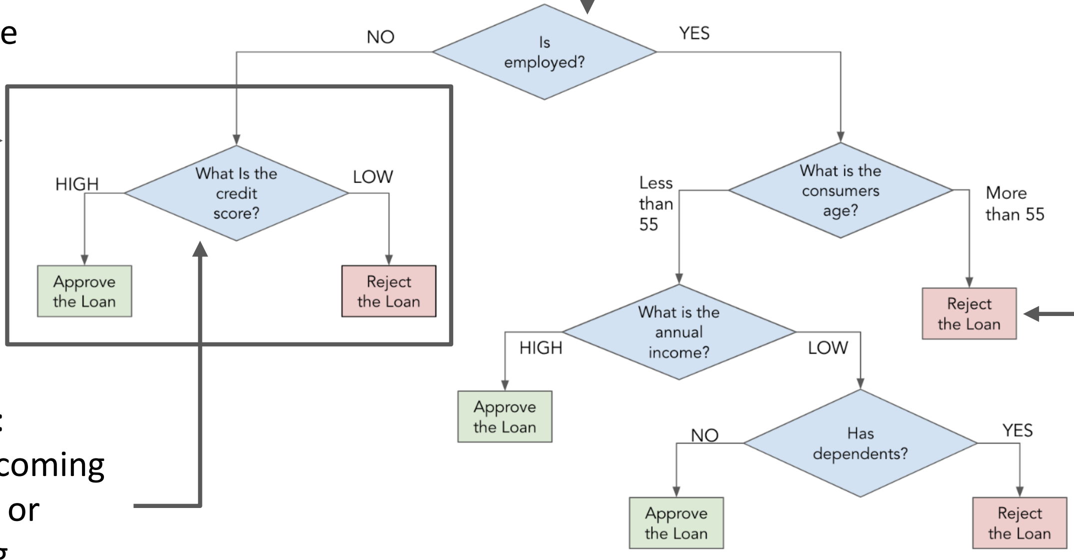
- The outcome nodes are called the child nodes

(Outcome: Approve or Reject the loan)

Terminologies

Root Node: no incoming edge and zero or more outgoing edges.

Branch/subtree: a subsection of the entire decision tree



Internal Node: exactly one incoming edge and zero or more outgoing edges.

Leaf/Terminal Node: exactly one incoming edge and no outgoing edge.

Measure of Purity of Node

Pure nodes

Consider the feature Credit Score, observe that for Credit Score = High, the loan proposal is approved

Credit Score = High	
Approved	Rejected
4	0

Employed	Credit Score	Income	Dependents	Loan
Yes	Low	Low	No	Rejected
Yes	Low	High	No	Rejected
Yes	Low	Low	Yes	Rejected
No	Low	Low	No	Rejected
Yes	Low	High	Yes	Approved
Yes	Low	Low	Yes	Rejected
No	Low	Low	No	Rejected
No	Low	High	Yes	Rejected
Yes	High	Low	Yes	Approved
No	Low	Low	No	Rejected
Yes	High	High	Yes	Approved
Yes	Low	Low	No	Rejected
No	High	High	No	Approved
Yes	High	High	No	Approved
No	Low	Low	No	Rejected

That to say the child nodes are pure or homogeneous.

(Homogeneous values in the target variable are all same)

Pure nodes

Is there any feature, other than Credit Score, which can be grouped to get pure nodes?

Employed	Credit Score	Income	Dependents	Loan
Yes	Low	Low	No	Rejected
Yes	Low	High	No	Rejected
Yes	Low	Low	Yes	Rejected
No	Low	Low	No	Rejected
Yes	Low	High	Yes	Approved
Yes	Low	Low	Yes	Rejected
No	Low	Low	No	Rejected
No	Low	High	Yes	Rejected
Yes	High	Low	Yes	Approved
No	Low	Low	No	Rejected
Yes	High	High	Yes	Approved
Yes	Low	Low	No	Rejected
No	High	High	No	Approved
Yes	High	High	No	Approved
No	Low	Low	No	Rejected

Measures of Purity of a node

- Entropy
- Gini Index
- Classification error

Measures of Purity of a node

- Entropy
- Gini Index
- Classification error

Entropy

- The entropy of a variable is calculated as

$$E = -\sum_{i=1}^{c} p_c \log_2 p_c$$ where p_c: probability of occurrence of the class

- The entropy of two variables is calculated as from the contingency table of the two variables

$$E(T, X) = \sum_{x \in X} P(c) E(c)$$

It is the sum if the of the product of the probability of occurrence of the class and its entropy.

Measures of Purity of a node

- Entropy
- $\boxed{\text{Gini Index}}$
- Classification error

Gini index

- The gini index of a variable is calculated as

$$Gini = 1 - \sum_{c=1}^{n} p_c^2$$

where p_c: probability of occurrence of the class

- For samples belonging to one class, the gini index is 0 and for equally distributed samples, the gini index is also 0

Gini index

Obtain the gini index of the given data

Obesity		Total
Not-Obese	Obese	
20	15	35

$$\text{Gini(Obesity)} = 1 - [P(\text{Not-obese})^2 + P(\text{Obese})^2]$$

$$\text{Gini(Obesity)} = 1 - [(20/35)^2 + (15/35)^2]$$

$$\text{Gini(Obesity)} = 0.306$$

Information gain using gini index

- It is similar to that of the information gain using entropy

- Information gain is the reduction in gini index

$$\text{Information Gain }(T, X) = \text{Gini index}(T) - \text{Gini index}(T|X)$$

Measures of Purity of a node

- Entropy
- Gini Index
- Classification error

Classification Error

- The classification error of a variable is calculated as

$$Error = 1 - \max p_c^2$$

where p_c: probability of occurrence of the class

- For samples belonging to one class, the classification error is 0 and for equally distributed samples, the classification error is 0.5

Construction of Decision Tree

Construction of decision tree

- A decision tree is built from top to bottom. That is we begin with the root node

- While constructing a decision tree we try to achieve pure nodes

- A node is considered to be pure when all the data points belong to the same class

- This purity of nodes is determined using the entropy value

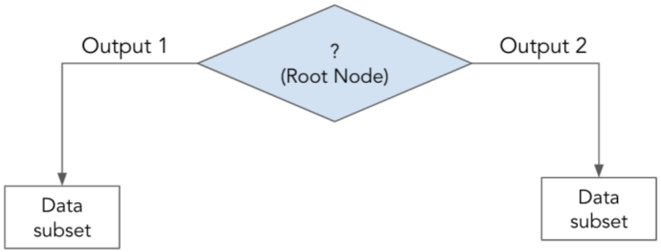
Construction of a decision tree

- Now we create the subset of the data
- Consider the remaining variables for the next iteration
- The child node which has Credit Score = High is pure, so the process terminates for that node
- It is the leaf node

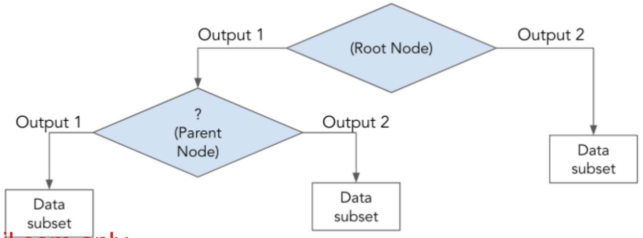
Credit Score	Employed	Income	Dependents	Loan
Low	Yes	Low	No	Approved
	Yes	Low	No	Approved
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	No	Low	Yes	Rejected
	No	High	No	Approved
High	Yes	Low	Yes	Approved
	No	Low	No	Approved
	Yes	High	Yes	Approved
	Yes	Low	No	Approved
	No	High	No	Approved
	Yes	High	No	Approved
	No	Low	No	Approved

Construction of a decision tree - Procedure

- It is a recursive procedure
- To select the root node, from k features, select the feature with the highest information gain



- Split the data on this feature
- At the next node, from (k-1) features, select the feature with the highest information gain



- Split the data on this feature
- Continue the process till you exhaust all features

Construction of a decision tree

- Consider the adjacent data. We have 4 categorical features
- We shall first find the root node
- To do so, calculate the information gain on each feature

Employed	Credit Score	Income	Dependents	Loan
Yes	Low	Low	No	Approved
Yes	Low	Low	No	Approved
Yes	Low	Low	Yes	Rejected
Yes	Low	Low	Yes	Rejected
Yes	Low	Low	Yes	Rejected
Yes	Low	Low	Yes	Rejected
No	Low	Low	Yes	Rejected
No	Low	High	No	Approved
Yes	High	Low	Yes	Approved
No	High	Low	No	Approved
Yes	High	High	Yes	Approved
Yes	High	Low	No	Approved
No	High	High	No	Approved
Yes	High	High	No	Approved
No	High	Low	No	Approved

Construction of a decision tree

Consider the categorical features and calculate the information gain.

Employed	Credit Score	Income	Dependents	Loan
Yes	Low	Low	No	Approved
Yes	Low	Low	No	Approved
Yes	Low	Low	Yes	Rejected
Yes	Low	Low	Yes	Rejected
Yes	Low	Low	Yes	Rejected
Yes	Low	Low	Yes	Rejected
No	Low	Low	Yes	Rejected
No	Low	High	No	Approved
Yes	High	Low	Yes	Approved
No	High	Low	No	Approved
Yes	High	High	Yes	Approved
Yes	High	Low	No	Approved
No	High	High	No	Approved
Yes	High	High	No	Approved
No	High	Low	No	Approved

Variable	Employed	Credit Score	Income	Dependents
Information gain	0.030	0.331	0.185	0.282

Refer 8-13 for information gain calculation

Construction of a decision tree

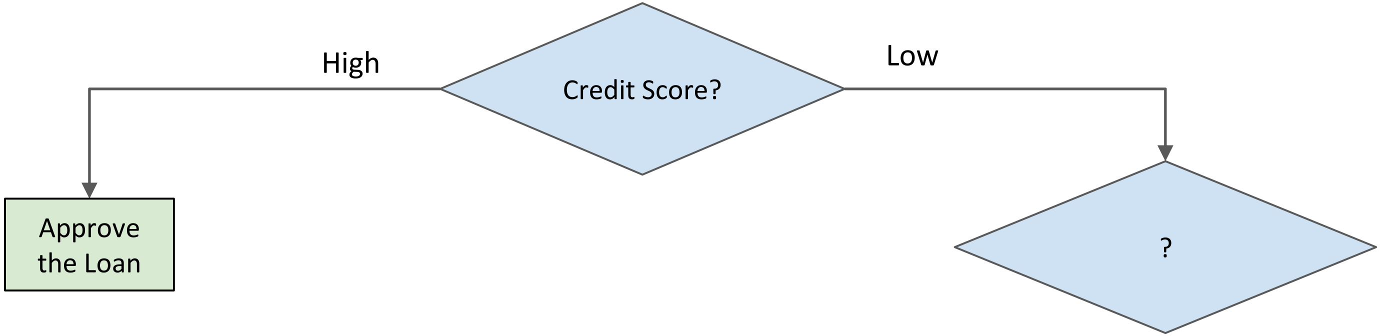
Thus, we have information gain values as

Variable	Employed	Credit Score	Income	Dependents
Information gain	0.030	0.331	0.185	0.282

Hence, we conclude Credit Score has the highest information gain.

Construction of a decision tree

The resultant tree at this stage is:



Construction of a decision tree

- The decision tree will now grow on the child node with Credit Score = Low

- Note that we now use only the subset of the data. So the entropy of the target variable needs to be computed again

- Compute the information gain for the remaining variables - Employed, Income and Dependents

Credit Score	Employed	Income	Dependents	Loan
Low	Yes	Low	No	Approved
	Yes	Low	No	Approved
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	No	Low	Yes	Rejected
	No	High	No	Approved

Construction of a decision tree

Thus, we have information gain values as

Variable	Employed	Income	Dependents
Information gain	0.159	0.610	0.954

Hence, we conclude Dependents has the highest information gain.

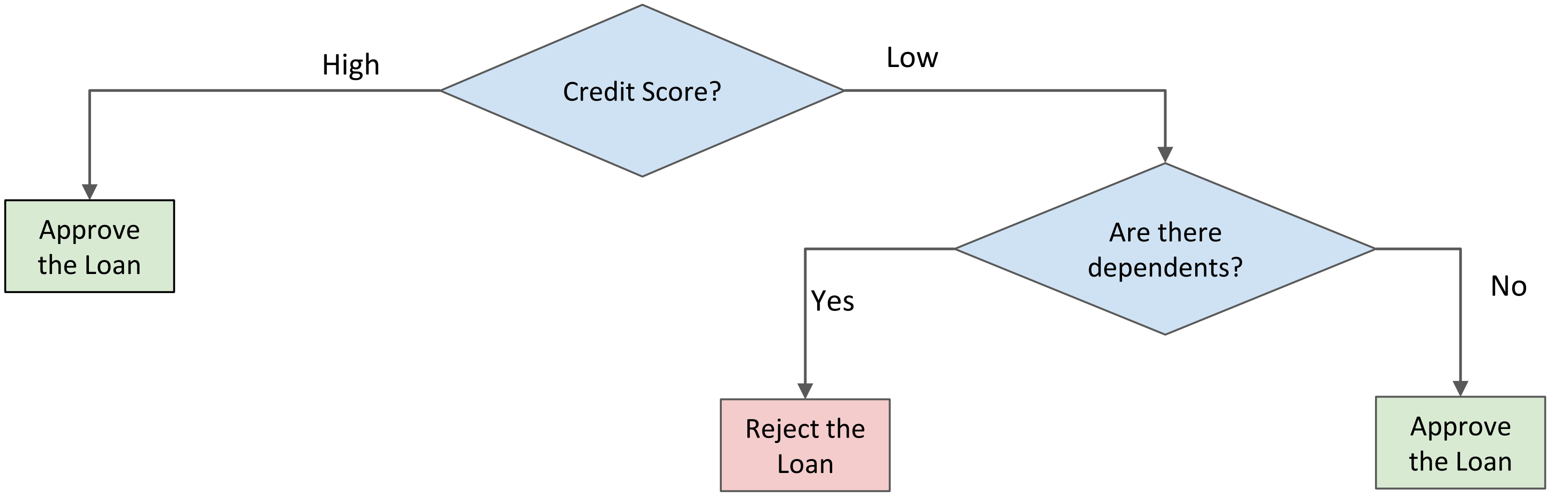
Construction of a decision tree

- Now we create the subset of the data
- Consider the remaining variables for the next iteration

Credit Score	Dependents	Income	Employed	Loan
Low	No	Low	Yes	Approved
	No	Low	Yes	Approved
	No	High	No	Approved
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	No	Rejected
High	Yes	Low	Yes	Approved
	No	Low	No	Approved
	Yes	High	Yes	Approved
	No	Low	Yes	Approved
	No	High	No	Approved
	No	High	Yes	Approved
	No	Low	No	Approved

Construction of a decision tree

The resultant decision tree:



Construction of a decision tree

- The child nodes for Dependents are pure, so the process terminates here
- They are the leaf nodes

Credit Score	Dependents	Income	Employed	Loan
Low	No	Low	Yes	Approved
	No	Low	Yes	Approved
	No	High	No	Approved
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	Yes	Rejected
	Yes	Low	No	Rejected
High	Yes	Low	Yes	Approved
	No	Low	No	Approved
	Yes	High	Yes	Approved
	No	Low	Yes	Approved
	No	High	No	Approved
	No	High	Yes	Approved
	No	Low	No	Approved

Choosing between attributes with same information gain

- The first predictor found from left to right in a data set is considered
- Some decision tree algorithm implementations might consider each of the variables with same information gain at a time and check which model performs better
- This rule applies to all parent nodes

Decision Tree Algorithms

Decision tree algorithms

- The decision tree algorithms are:
  - ID3 (Iterative Dichotomiser)
  - C4.5
  - C5.0
- Hunt's algorithm forms the basic to many of the decision tree algorithms like ID3, C4.5, CART and so on
- It grows a decision tree in a recursive manner by partitioning the samples into successively purer subsets

Hunt's algorithm

Let S_n be the training samples associated with node n and y_c be the class labels

The algorithm is as follows

1. If all samples belong to the same class y_c, then node n is a leaf node with label y_c

2. If S_n has samples with more than one class, an attribute value is selected to partition the samples into smaller subsets such that the samples in the subsets belong to the same class

Decision tree algorithms

ID3 Algorithm

- Invented by Ross Quinlan
- Handles only categorical data
- May not converge to optimal decision tree
- May overfit

C4.5 Algorithm

- Extension to ID3 algorithm
- Handles both categorical and numeric data
- Handles the missing data marked by ‘?’

C5.0 Algorithm

- Works faster than C4.5 algorithm
- More memory efficient than C4.5 algorithm
- Creates smaller trees with similar efficiency

How is the entropy calculated for numeric features?

1. Sort the data in ascending order and compute the midpoints of the successive values. These midpoints act as the thresholds

2. For each of these threshold values, enumerate through all possible values to compute the information gain

3. Select the value which has the highest information gain

(Demonstration in the next slide)

Entropy for numeric feature

Sorted Data

Da

Age	Loan
45	Rejected
54	Approved
56	Rejected
58	Rejected
21	Approved
31	Rejected
45	Approved

Age	Loan
21	Approved
31	Rejected
45	Approved
45	Rejected
54	Approved
56	Rejected
58	Rejected

Midpoints

Age	Midpoints
21	-
31	26
45	38
54	49.5
56	55
58	57

For repeated values, we consider it only once (in this case 45).

Continued to next page...

Information gain for numeric feature

Midpoints

Age	Midpoints
21	-
31	26
45	38
54	49.5
56	55
58	57

For midpoint, m, the data is divided into two parts - data less than m and data more than m

These two part form the two branches in the tree

Information gain for all midpoints

Midpoints	Information Gain
-	
26	0.5916727786
38	0.1280852789
49.5	0.02024420715
55	0.4137995646
57	0.5216406363

We consider the threshold with maximum information gain. In this case, we consider Age < 26

Thank You