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PES UNIVERSITY	**PES University, Bengaluru** (Established under Karnataka Act No. 16 of 2013)	**UE20CS904**

May 2022: END SEMESTER ASSESSMENT (ESA)
M TECH DATA SCIENCE AND MACHINE LEARNING_ SEMESTER I

UE20CS904 - Mathematical Foundation

Time: 3 Hrs	Answer All Questions	Max Marks: 80

Instructions:
- Answer all the questions in the answer sheet provided.
- Marks will be allotted only if all the steps are shown.

		Section A (20 marks)	
1	a)	Find x if $\det\begin{pmatrix}2 & 4\\ 5 & 1\end{pmatrix} = det\begin{pmatrix}2x & 4\\ 6 & x\end{pmatrix}$	2
	b)	Consider the slope function $f'(x) = \begin{cases}0, & x < 0\\ 1, & x \geq 0\end{cases}$ which of the following is possible function for f a) $f(x) = x\ , x \geq 0$ b) $f'(x) = \begin{cases} x, & x < 0\\ \frac{x^2}{2}, & x \geq 0\end{cases}$ c) $f'(x) = \begin{cases} 1, & x < 0\\ x + 1, & x \geq 0\end{cases}$ d) $f'(x) = \begin{cases}0, & x < 0\\ 1, & x \geq 0\end{cases}$	2
	c)	If the total revenue function of a good is given by $100Q - Q^2$ what is marginal revenue function if the current demand is 60?	2
	d)	Find out whether the function is concave or convex, $f(x) = -8x^2 + 15$	2
	e)	A Linear transformation $A = \begin{bmatrix}6.4 & -7.2\\ -7.2 & 10.6\end{bmatrix}$maps the general point (x , y) to the general point by (x_{new}, y_{new}) then the area of transformed region is	2
2	a)	Calculate the Jacobian matrix for the following function $f_1(x, y) = x^3y$ $f_2(x, y) = \frac{x^2}{y} + y^2$	2

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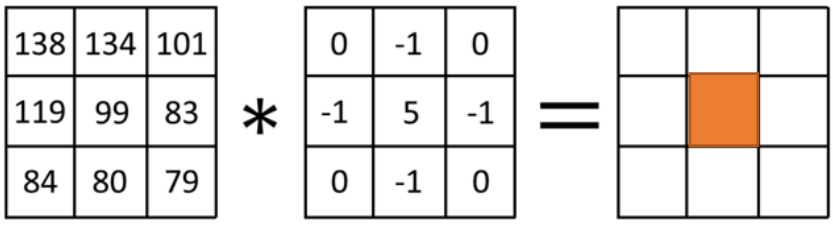
	b)	Suppose for vector $\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ if $softmax(\boldsymbol{x}) = \left(\frac{e^{x_1}}{e^{x_1}+e^{x_1}}, \frac{e^{x_2}}{e^{x_1}+e^{x_1}}\right)^T$ then find $\boldsymbol{softmax}(\begin{pmatrix} log2 \\ 0 \end{pmatrix})$	2
	c)	If the RGB value of a pixel is given as $\{20, 255, 20\}$ what will be the color shade of the pixel? Explain the same.	2
	d)	In simple linear regression for a single data point (x_1, y_1) we define loss L as $L(w_0, w_1) = \left(\widehat{y_1} - (w_0 + w_1 x_1)\right)^2$ where $\widehat{y_1}$ is predicted value for y_1 find $\frac{\partial L}{\partial w_0}$ and $\frac{\partial L}{\partial w_1}$	2
	e)	Given an image in 2D Translate it to $[1,1]$ from origin then rotate it by clockwise 45^0 write the corresponding combine matrix of both transformation	2
		Section B (30 marks)	
3	a)	The 2× 2 matrix B is given by $B = \begin{bmatrix} a & 2 \\ b & 3 \end{bmatrix}$ where a, b are scalars. The vector $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ is mapped by B on to the vector $\begin{pmatrix} 5 \\ 13 \end{pmatrix}$ Determine the value of a and the value of b. Find the corresponding region of unit square after transformation	5
	b)	Let $f(x, y) = x^2 + y^2$, where $x = 3w_1 + 2w_2$; $y = 5w_1 + 6w_2$ calculate $\frac{\partial f}{\partial w_1}$ and $\frac{\partial f}{\partial w_2}$ using chain rule at point $(1,1)$	5
	c)	Compute the following convolution for colored cell, what kind of output the following convolution will have on an image? 	5
	d)	Find Eigen values and eigen vectors of $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{bmatrix}$	5
	e)	The Following table lists the weight and heights of 5 boys Find the covariance matrix for the data.	5

Boy	1	2	3	4	5
Weight(lb)	120	125	125	135	145
Height(in.)	61	60	64	68	72

	f)	Explain steps involved in gradient descent for fitting straight line to any data	5
		Section C (30 marks)	
4	a)	A headphone manufacturer determines that in order to sell x units of a new headphone, the price per unit, in dollars, must be p(x) = 1000 - x. The manufacturer also determines that the total cost of producing x units is given by C(x)= 3000 + 20x. i) Find the total revenue R(x)	10

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		ii) Find the total profit P(x). iii) How many units must the company produce and sell in order to maximize profit? iv) What is the maximum profit? v) What price per unit must be charged in order to make this maximum profit?	
	b)	$u = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$ $w = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ $x = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$ Compute the following 1. $u.u, v.u, and \frac{v.u}{u.u}$ 2. $w.w, x.w, and \frac{(x.w)}{w.w}$ 3. $\frac{1}{w.w}w$ 4. $\frac{u.v}{v.v}v$ 5. $\|w - x\|$	10
	c)	Find singular Value decomposition of $A = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$	10