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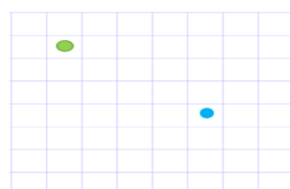
	PES University, Bengaluru (Established under Karnataka Act No. 16 of 2013)	**UE20CS904**
April 2022: END SEMESTER ASSESSMENT (ESA) **M TECH DATA SCIENCE AND MACHINE LEARNING_ SEMESTER I** **UE20CS904 - Mathematical Foundation**		
Time: 3 Hrs	Answer All Questions	Max Marks: 80

		Section A (20 marks)	
1	a)	Calculate $\|A^{-1}\|$ for $A = \begin{bmatrix} 2 & 4 & 5 \\ 6 & 1 & 3 \\ 4 & 0 & 7 \end{bmatrix}$	2
	b)	Which distance metric is suitable for calculating the least number of squares moved between the starting position (Green Point) and ending position (Blue point) on the chessboard (each square of unit length) for the Queen (Queen can move either diagonally or vertically or horizontally)? Give formula for the same. 	2
	c)	The total revenue in Rupees received from the sale of x units of a product is given by R(x) = 13x2 + 26x + 15. Find the marginal revenue when x = 7.	2
	d)	Find out whether the function is concave or convex , $f(x) = -8x^2 + 15$	2
	e)	For any two matrices $\boldsymbol{A}$ & $\boldsymbol{B}$,$\boldsymbol{A^T B^T} = (\boldsymbol{BA})^{\boldsymbol{T}}$ Check whether the statement is True for the following matrices or not $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{4} & \mathbf{3} \end{bmatrix}$ $\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{2} & \mathbf{-1} \end{bmatrix}$	2
2	a)	Calculate the Jacobian matrix for the following function $f_1(x,y) = x^3 y$ $f_2(x,y) = \frac{x^2}{y} + y^2$	2

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b) Define the following — 2

1) Linearly independent vector
2) Orthogonal Vector
3) Orthonormal vector
4) Basis Vector

c) If the RGB value of a pixel is given as $\{255, 20, 20\}$ what will be the color shade of the pixel? Explain the same. — 2

d) What is the effect of higher learning rate in Gradient descent algorithm? — 2

e) Given an image in 2D Translate it to $[1,1]$ from origin then rotate it by clockwise 45^0 write the corresponding combine matrix of both transformation — 2

Section B (30 Marks)

3 a) Match each linear transformation with its matrix — 5

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
2. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
3. $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$
4. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
5. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
6. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

A. Reflection in the y-axis
B. Projection onto the x-axis
C. Reflection in the line $y = x$
D. Identity transformation
E. Reflection in the origin
F. Contraction by a factor of 2

b) Let $f(x, y) = x^2 + y^2$, where $x = 3w_1 + 2w_2\,; y = 5w_1 + 6w_2$ calculate $\frac{\partial f}{\partial w_1}$ and $\frac{\partial f}{\partial w_2}$ using chain rule at point $(1,1)$ — 5

c) Compute the following convolution, what kind of output the following convolution will have on an image? — 5

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

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1	0	-1
1	0	-1
1	0	-1

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d) Find Eigen values and eigen vectors of — 5

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{bmatrix}$$

e) The Following table lists the weight and heights of 5 boys Find the covariance matrix for the data. — 5

Boy	1	2	3	4	5

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		<table><tr><td>Weight(lb)</td><td>120</td><td>125</td><td>125</td><td>135</td><td>145</td></tr><tr><td>Height(in.)</td><td>61</td><td>60</td><td>64</td><td>68</td><td>72</td></tr></table>	
	f)	Explain steps involved in gradient descent for fitting straight line to any data	5
		Section C (30 Marks)	
4	a)	A headphone manufacturer determines that in order to sell x units of a new headphone, the price per unit, in dollars, must be p(x) = 1000 - x. The manufacturer also determines that the total cost of producing x units is given by C(x)= 3000 + 20x. i) Find the total revenue R(x) ii) Find the total profit P(x). iii) How many units must the company produce and sell in order to maximize profit? iv) What is the maximum profit? v) What price per unit must be charged in order to make this maximum profit?	10
	b)	$u = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$ $w = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ $x = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$ Compute the following 1. $u.u, v.u, and \frac{v.u}{u.u}$ 2. $w.w, x.w, and \frac{(x.w)}{w.w}$ 3. $\frac{1}{w.w} w$ 4. $\frac{u.v}{v.v} v$ 5. $\|\|w - x\|\|$	10
	c)	Find singular Value decomposition of $A = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$	10