

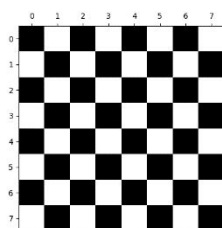
Mathematical Foundations

Student Name:
Subject Code:

Date:
Duration : 3 hrs

Section A (10 * 2 = 20 marks)

1. Check whether the vectors $\vec{a} = \{1; 1; 1\}$, $\vec{b} = \{-2; 3; 4\}$, $\vec{c} = \{4; 4; 4\}$, $\vec{d} = \{3; 4; 0\}$ are linearly independent. (2 marks)
2. Find the length of the vector $\vec{a} = \{4; 6\}$. (2 marks)
3. Are the vectors $a = \{1; -1\}$ and $b = \{2; 3\}$ orthogonal? (2 marks)
4. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ denotes the cost of manufacturing three commodities in two factories, and $B = \begin{pmatrix} 3 & 1 & 3 \\ 1 & 0 & 2 \end{pmatrix}$ is the cost of transportation, if the selling price is double that of cost price, then find the profit/ loss after selling the commodities. (2 marks)
5. How can you represent the following image as a matrix? (2 marks)



6. Find the equation of a curve passing through the point $(-2, 3)$, given that the slope of the tangent to the curve at any point (x, y) is $2x/y^2$. (2 marks)
7. Write down the properties of a continuous function. (2 marks)
8. Explain the geometrical meaning of derivative at a point. (2 marks)
9. Find the derivatives of :
10. What is a saddle point?

Section B (6 * 5 = 30 marks)

1. Solve the following set of equations using Gauss elimination method (5 marks)

set of equations

2. Daily Closing Prices of Two Stocks arranged as per returns. So calculate Covariance and tell if the returns of ABC increase/ decrease, what can you say about the returns of XYZ. (5 marks)

Day	ABC Returns	XYZ Returns
1	1.8	2.5
2	1.5	4.3
3	2.1	4.5
4	2.4	4.1
5	0.2	2.2

Use the following matrix 3*3 matrix for questions 4 and 5 below.

3. Find the eigen values (5 marks)
4. Find the eigen vectors (5 marks)

Graph the following functions:

5. $y = x^2 - 2|x|$
6. $y = |\log_e|x||$

Section C (3 * 10 = 30 marks)

1. (a) Perform 2D Convolution operation and give the final matrix (6 marks)
(b) Discuss the effect of the filter on the image matrix. (2 marks)
(c) Which mathematical operation can be used to detect edges in images? (2 marks)
2. Consider the data given below: How are the variables x_1 and x_2 related to y ? (2 marks)
Initialize the weights to -0.017, -0.048 and intercept to 0. Using the loss function $(\bar{y} - y)^2$
Update the weights such that the error is minimum using gradient descent. (8 marks)

x_1	x_2	y
4	1	2
3. Find the local minima and maxima of the function $f(x) = x^3 - 3x$

-----ALL THE BEST-----