	PES University, Bengaluru (Established under Karnataka Act No. 16 of 2013)	**UE20CS904**

March 2024: END SEMESTER ASSESSMENT (ESA)
M TECH DATA SCIENCE AND MACHINE LEARNING_ SEMESTER I

UE20CS904 - Mathematical Foundation

Time: 3 Hrs	Answer All Questions	Max Marks: 100

Instruction

1. Answer all questions in the answer booklet.

1	a	Consider the vectors $a = [1\ 2\ 3\ ]; b = [4\ 5\ 6\ ]$ i) Calculate $\|\|a-b\|\|_1, \|\|a-b\|\|_2$ (2 marks) ii) Find the angle θ between a and b (1 marks) iii) Find unit vector along $a+b$ (2 marks)	5
	b	$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 0 & 5 \end{bmatrix}$ find $trace(A)$, Also verify that $A^T + A$ is symmetric	5
	c	Find x for which $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$ has an eigenvalue $\lambda_1 = 4, \lambda_2 = 8$ find x, y?	5
	d	Find the points of local maxima, local minima of the function $f(x) = (x-1)(x+2)^2$. Also, find the corresponding local maximum and local minimum values.	5
2	a	Find out the inverse of the following matrix using adjoint method $A = \begin{bmatrix} 1 & 5 & 7 \\ 2 & 6 & 0 \\ 0 & 3 & 5 \end{bmatrix}$	6
	b	Verify that $[2\ 2\ 1\ ]; [-4\ 6\ 5\ ]; [1\ 0\ 0\ ]$ linearly independent	7
	c	If $f(x_1, x_2) = (w_0 - w_1x_1 - w_2x_2)^2$ find $\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_1}$ find $\nabla f\|_{[1,1]}$ also calculate $[3\ 4\ ] - \eta \times \nabla f\|_{[1,1]}$ if $w_0 = 1, w_1 = -1, w_2 = -2$ and $\eta = 1.2$	7
3	a	Find SVD of $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$	10

	b	i) The following box was Applied transformation $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ then calculate the distance between the new points that are obtained from [1,0] and [1,1]  ii) $u = x + 2y; v = 3x - y$ find Jacobian $J(u, v)$	5 5
4	a	Math Scores X: [60, 70, 80, 65, 75] English Scores Y: [62, 82, 78, 70, 80] Calculate the covariance between the Math scores X and the English scores Y. What does the value of the covariance signify? Is there a positive or negative relationship between Math and English scores?	4
	b	Consider a 3x3 image A with values $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$ and a 2x2 kernel matrix K with values $\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$ Perform the convolution operation.	3
	c	Find eigen value and eigen vector of $A = \begin{bmatrix}5 & 2 & 0\\2 & 5 & 0\\-3 & 4 & 6\end{bmatrix}$	6
	d	Transform the following basis into orthogonal basis using Gram-Schmidt Process. $u_1 = \begin{bmatrix}2\\1\\0\end{bmatrix}$ $u_2 = \begin{bmatrix}3\\2\\1\end{bmatrix}$ $u_3 = \begin{bmatrix}4\\1\\2\end{bmatrix}$	5
	e	For the interval $(-1,1)$ determine if $f(x) = x^4 - 6x^2$ is convex or concave.	2
5	a	Compute PCA on the following data	10

		Data Point	X1	X2 | 1	2	4 2	3	5 3	5	4 4	6	7 5	7	6 1) Centre the data 2) Calculate Covariance Matrix 3) Compute Eigen values and Eigen vectors 4) Select principal components 5) Transform the data	
	b	We have recorded the weekly average conversion rate of Dollar for over 6 consecutive weeks. Y shows the weekly average conversion rate and x shows the number of the week. Try to fit the best possible function 'f' to establish the relationship between the number of the day and conversion rate. (Applying Gradient descent) where $f(x) = y = a + b\ x$. x | y 1 | 10 2 | 14 3 | 18 4 | 22 5 | 25 6 | 33 The initial values of a & b are, a= 4.9 & b=4.401. The learning rate is mentioned as .05The error rate of a & b should be less than .01. Perform 5 epochs	10