Econ 432 Homework 1

Richard Grigorian (UID: 505-088-797)

1/20/23

Contents

1 Part I: Review Questions

1.1 Question 1

Consider the (actual) monthly adjusted closing price data for Starbucks over the period December 2011 through December 2012:

Date	Price
December, 2011	\$44.89
January, 2012	\$46.76
February, 2012	\$47.55
March, 2012	\$54.73
April, 2012	\$56.17
May, 2012	\$53.91
June, 2012	\$52.37
July, 2012	\$44.47
August, 2012	\$48.91
September, 2012	\$50.00
October, 2012	\$45.26
November, 2012	\$51.36
December, 2012	\$53.10

Part (a)

Using the data in the table, what is the simple monthly return between the end of December, 2011 and the end of January 2012? If you invested \$10,000 in Starbucks at the end of December 2011, how much would the investment be worth at the end of January 2012?

Solution

One period simple return is given by the expression

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

Therefore, return for Starbucks stock in the stated period is

$$R_t = \frac{46.76}{44.89} - 1 = 0.04166$$

If we save \$10,000 in Starbucks stock over this period, we would receive

$$10,000(1 + 0.04166) = \$10,416.60$$

Part (b)

Using the data in the table, what is the continuously compounded monthly return between December, 2011 and January 2012? Convert this continuously compounded return to a simple return (you should get the same answer as in part (a).

Solution

The expression for single period continuously compounding return is given by the natural log of simple returns. Hence, we write

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) = \ln\left(\frac{46.76}{44.89}\right) = 0.0408$$

We can convert this back to simple returns by undoing the log such that

$$1 + R_t = e^{r_t} = 1.04166 \implies R_t = 0.04166$$

which is equivalent to the answer from part (a).

Part (c)

Assuming that the simple monthly return you computed in part 1 is the same for 12 months, what is the simple annual return with monthly compounding?

Solution

In constructing simple annual returns, we assume that our one period return from earlier holds true for each period (month). Hence, by the multiplicative property of returns

$$1 + R_A = (1 + R_m)^{12} \implies R_A = (1 + 0.04166)^{12} - 1 = 0.6319$$

where R_m is the monthly return we found from part (a) that we assume holds for every 12 periods.

Part (d)

Assuming that the continuously compounded monthly return you com- puted in part 2 is the same for 12 months, what is the continuously compounded annual return?

Solution

We can approach this problem in two different ways. We can eitehr transform the simple return we found in part (c) into a continuously compounded return by applying a log, or we can derive the continuously compounding return from its base components. For completeness, we present both. First, we easily convert the simple return such that

$$r_A = \ln(1 + R_A) = \ln(1.6319) = 0.4898$$

Alternatively, we show

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \implies r(2) = \ln\left(\frac{p_t}{p_{t-1}} \cdot \frac{p_{t-1}}{p_{t-2}}\right) = \ln\left(\frac{p_t}{p_{t-1}}\right) + \ln\left(\frac{p_{t-1}}{p_{t-2}}\right) = r_t + r_{t-1}$$

Whilst a more formal proof would involve induction on n periods, we say for our purposes this establishes the additive property of continuously compounding returns. Thus, an annual return implies the sum of 12 individual period returns. Since we assume that each period return is equal to the return in the first period, we can instead write

$$r_A = r_m + r_m + \cdots + r_m = 12r_m = 12(0.040813) = 0.4898$$

Notice, that these two methods deliver us an identical result.

Part (e)

Using the data in the table, compute the actual simple annual return between December 2011 and December 2012. If you invested $10,000 in Starbucks at the end of December 2011, how much would the in- vestment be worth at the end of December 2012? Compare with your result in part (c).

Solution

The actual simple return for the 12 periods (year) can be found using the expression for simple returns from before

$$1 + R_t = \frac{P_t}{P_{t-1}} \implies R_t = \frac{53.10}{44.89} - 1 = 0.1829$$

Notice that $0.1829 < 0.6319$ which means that our actual simple return is much lower than the stylized annual return we found previously. This is because the previous method assumed that period 1 return is true for every period. This is clearly not true thus our overall 1 year return is far lower than the before.

If we save $10,000 in Starbucks stock, then at the end of 2012 we would have

$$10,000(1 + 0.1829) = \$11,829 \quad \text{vs.} \quad 10,000(1 + 0.6319) = \$16,319$$

From this quick comparison, we see again that if we use the actual annual simple return we end up making far less money from this savings.

Part (f)

Using the data in the table, compute the actual annual continuously compounded return between December 2011 and December 2012. Compare with your result in part d. Convert this continuously compounded return to a simple return (you should get the same answer as in part (e).

Solution

Below we calculate the actual annual continuously compounded return using the same expression as earlier

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) = \ln\left(\frac{53.10}{44.89}\right) = 0.16796 < 0.4898$$

Converted to simple returns this becomes

$$R_t = e^{r_t} - 1 = 0.1829$$

This is, indeed, equivalent to the simple return we found in part (e).

1.2 Question 2

Suppose we buy 100 shares of Starbucks and 100 shares of Amazon at the end of December, 2011. The monthly adjusted closing price of these two stocks are contained in the following table:

Date	Price of Starbucks	Price of Amazon
December, 2011	\$44.89	\$173.10
January, 2012	\$46.76	\$194.44
February, 2012	\$47.55	\$179.69

Part (a)

What is the portfolio we are holding in the January, 2012? What is the monthly simple return of this portfolio in the January, 2012?

Solution

Since we are buying 100 shares of each stock, the total amount of money saved is

$$100(44.89) + 100(173.10) = 21,799$$

To find portfolio shares, we divide the money saved into each stock by the total money saved

$$\frac{4489}{21799} = 0.206 \quad \text{and} \quad \frac{17310}{21799} = 0.794$$

We can now consider the portfolio as a column vector

$$\begin{bmatrix} x_A \\ x_B \end{bmatrix} = \begin{bmatrix} 0.206 \\ 0.794 \end{bmatrix}$$

where $A \equiv$ Starbucks and $B \equiv$ Amazon. To find simple portfolio returns, we simply take the share weighted average of the simple returns on the individual assets

$$\begin{aligned} R_{p,t} = x_A R_{A,t} + x_B R_{B,t} &= 0.206 \left(\frac{46.76}{44.89} - 1 \right) + 0.794 \left(\frac{194.44}{173.10} - 1 \right) \\ &= 0.206(0.04166) + 0.794(0.1233) \\ &= 0.1065 \end{aligned}$$

Part (b)

Suppose our investment strategy is to hold the portfolio constructed above for two month and sell it at the end of February, 2012. What is the two-month simple return? What is the average monthly simple return of this investment strategy?

Solution

We first compute the simple two-month return using the same method as above except with a 2 period gap rather than a 1 period gap

$$R_{p,t}(2) = 0.206 \left(\frac{47.55}{44.89} - 1 \right) + 0.794 \left(\frac{179.69}{173.10} - 1 \right) = 0.0424$$

To compute average monthly simple returns, we take the geometric mean of the 2 individual monthly simple returns to get

$$\bar{R} = [(1 + R_1)(1 + R_2)]^{1/2} - 1$$

Thus we must first compute the second period return

$$R_2 = 0.206 \left(\frac{47.55}{46.76} - 1 \right) + 0.794 \left(\frac{179.69}{194.44} - 1 \right) = -0.0568$$

Therefore, we can compute the average monthly return as

$$\bar{R} = [(1 + 0.1065)(1 - 0.0568)]^{1/2} = 0.0216$$

Part (c)

Suppose that we "believe" the portfolio we constructed at the end of December, 2011 is "optimal", and we will make sure (by selling one stock and buying the other) we are holding this portfolio the end of each month. How do we implement this strategy? What is the average monthly simple return of this investment strategy? Is this new strategy turns out to be better than the strategy in question (b)?

Solution

In order to keep the exact portfolio we constructed at the end of December 2011, we will need to sell one stock and by the other at the end of each month to bring us back to the wealth share split we had earlier (roughly a 20:80 split). At the end of January 2012, our the stock price appreciation leads our 100 shares of each stock to be worth

$$100(46.76) + 100(194.44) = 24,120$$

Where the individual portfolio shares become

$$x'_A = \frac{4676}{24120} = 0.194 \quad \text{and} \quad x'_B = \frac{19444}{24120} = 0.806$$

Hence we to regain our original split, we need to adjust the value holdings of our portfolio to

$$V_i = x_i V'$$

for $i \in \{A, B\}$ For asset A (Starbucks):

$$0.206(24120) = 4968.72$$

which implies that we need to buy shares until we have

$$\frac{4968.72}{46.76} = 106.26 \quad \text{shares of A}$$

and for asset B (Amazon)

$$0.794(24120) = 19151.28$$

which implies we need to buy shares until we have

$$\frac{19151.28}{194.44} = 98.49 \quad \text{shares of B}$$

We would then need to repeat this strategy at the end of each month – buying and selling stocks of each company to ensure that our portfolio shares stay constant.

In fact, the average monthly simple return of this investment strategy will be identical to the one in question (b). This is because we will be keeping our portfolio shares constant and thereby using the same original x_A, x_B values as before. No other part of the computation changes. Namely, this matches our part (b) result because the typical portfolio returns formual does not subscript portfolio shares with a time t. Therefore, it implicitly assumes that the portfolio shares are held constant across time just as we did above.

2 Part II: Python Exercises

2.1 Question 1

Import the data in the file sbuxPrices.csv using pandas.read_csv() into the data.frame object sbux_df. Explain what you will get from the following lines of codes.

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

```
# Import data
sbux_df = pd.read_csv('sbuxPrices.csv')
sbux_df.head()
```

	Date	Adj Close
0	4/1/1998	2.415702
1	5/1/1998	2.409427
2	6/1/1998	2.682370
3	7/1/1998	2.101974
4	8/1/1998	1.584324

The `.head()` function displays the first 5 rows of the dataframe.

```
sbux_df.tail()
```

	Date	Adj Close
175	11/1/2012	21.659863
176	12/1/2012	22.487991
177	1/1/2013	23.532091
178	2/1/2013	22.999552
179	3/1/2013	23.969866

The `.tail()` function displays the last 5 rows of the dataframe.

```
sbuxPrices_df = sbux_df.set_index('Date')
```

	Adj Close
Date	
4/1/1998	2.415702
5/1/1998	2.409427
6/1/1998	2.682370
7/1/1998	2.101974
8/1/1998	1.584324
...	...
11/1/2012	21.659863
12/1/2012	22.487991
1/1/2013	23.532091
2/1/2013	22.999552
3/1/2013	23.969866

The `.set_index()` command allows us to change the index of the pandas dataframe. For instance, we can create a new dataframe and pass through the old one with a new index. In this case, we are indexing the dataframe by the date rather than from 0 to n.

```
sbux_df.columns
```

```
Index(['Date', 'Adj Close'], dtype='object')
```

The `.columns` attribute gives us an object associated with our dataframe object. In particular, it gives us the column names of our dataframe.

```
sbuxPrices_df.head()
```

	Adj Close
Date	
4/1/1998	2.415702
5/1/1998	2.409427
6/1/1998	2.682370
7/1/1998	2.101974
8/1/1998	1.584324

Once again the `.head()` method displays the first 5 rows of the dataframe. In this case, it is showing us the first 5 months of our newly indexed Starbucks stock adjusted closing price data.

```
sbux_df.iloc[100:132]
```

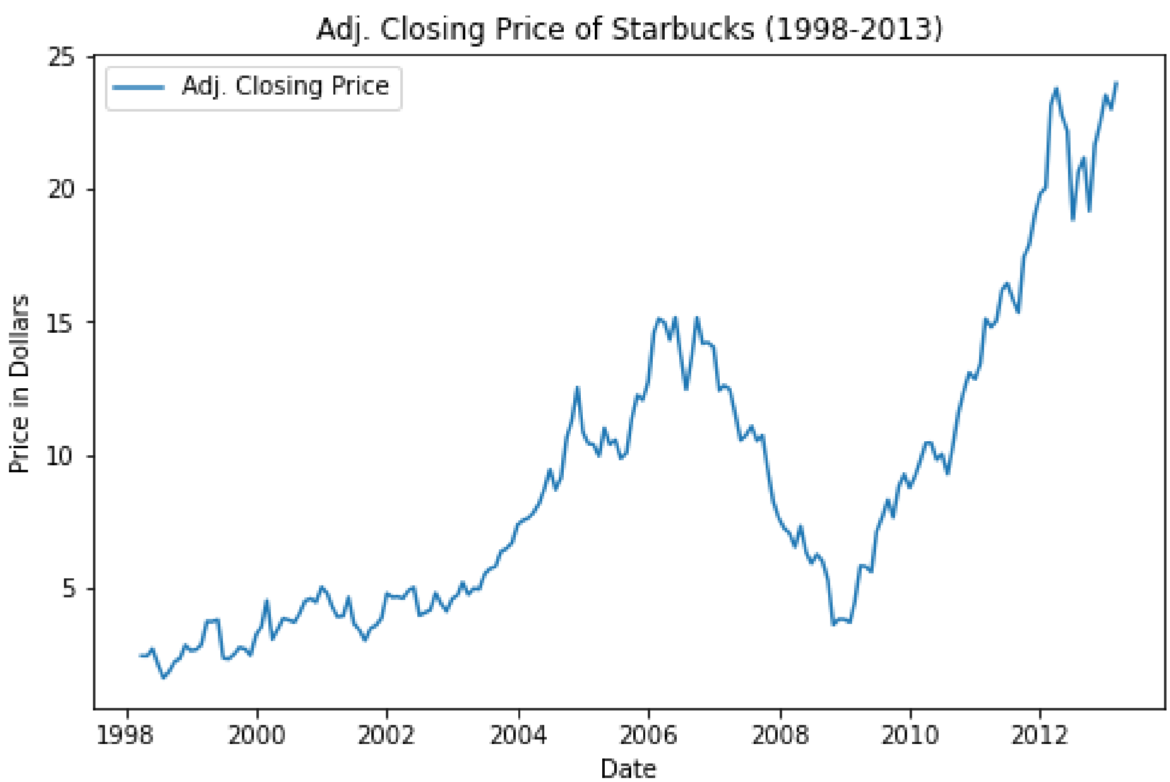
	Date	Adj Close
100	8/1/2006	12.452722
101	9/1/2006	13.673493
102	10/1/2006	15.159312
103	11/1/2006	14.171446
104	12/1/2006	14.223644
105	1/1/2007	14.030895
106	2/1/2007	12.408547
107	3/1/2007	12.593272
108	4/1/2007	12.456736
109	5/1/2007	11.569265
110	6/1/2007	10.537227
111	7/1/2007	10.713920
112	8/1/2007	11.063286
113	9/1/2007	10.521164
114	10/1/2007	10.713920
115	11/1/2007	9.392748
116	12/1/2007	8.220162
117	1/1/2008	7.593711
118	2/1/2008	7.220250
119	3/1/2008	7.027494
120	4/1/2008	6.517499
121	5/1/2008	7.304582
122	6/1/2008	6.320730
123	7/1/2008	5.899081
124	8/1/2008	6.248446
125	9/1/2008	5.971364
126	10/1/2008	5.272628
127	11/1/2008	3.586031
128	12/1/2008	3.798863
129	1/1/2009	3.790831
130	2/1/2009	3.674376
131	3/1/2009	4.461455

The `.iloc[]` method allows us to slice pandas dataframes by index. In particular, here we are pulling the rows of our original starbucks price dataframe that are indexed from 100 to 131 (note that in pandas, the first index is inclusive but the second index is exclusive).

2.2 Question 2

Plot the closing price data using the `matplotlib.pyplot.plot()` function with blue line. Add a title and a legend to your picture.

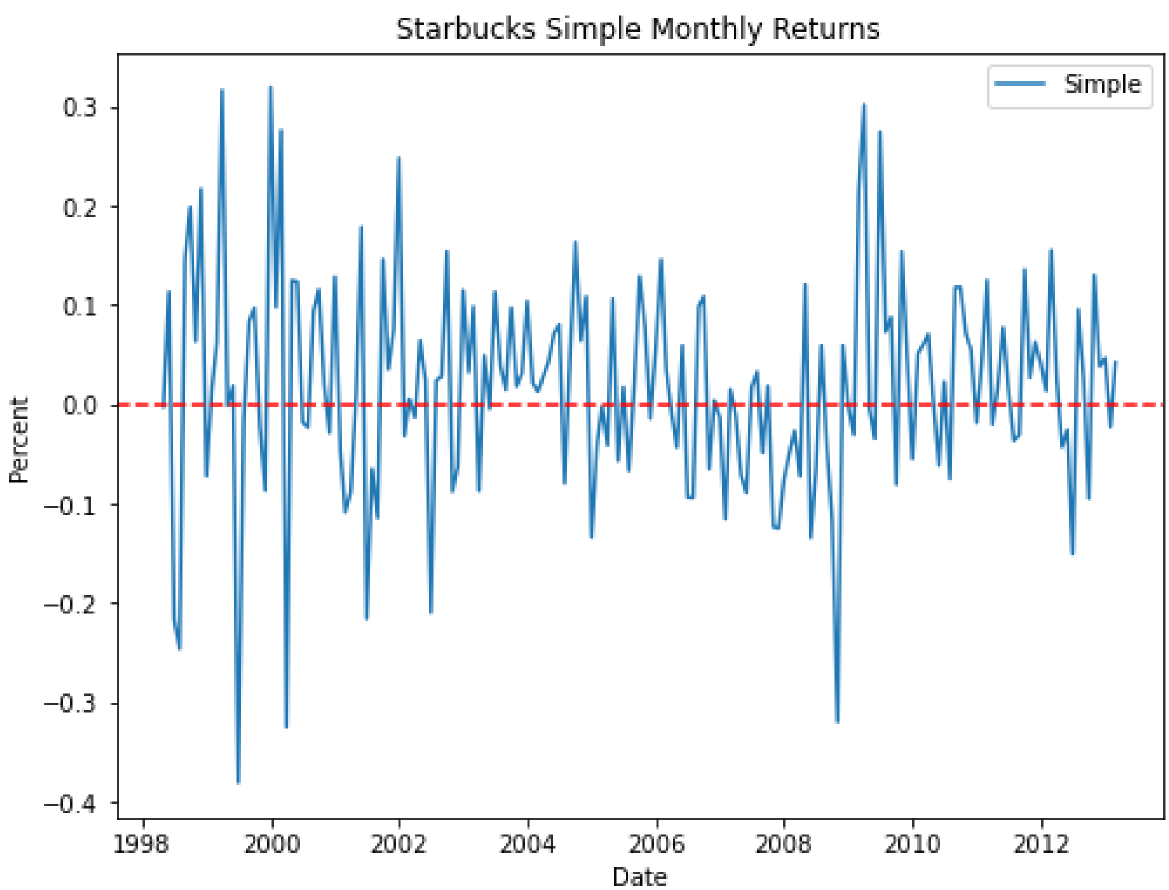
```
sbux_df["Date"] = pd.to_datetime(sbux_df["Date"])
date = sbux_df["Date"]
price = sbux_df["Adj Close"]

plt.figure(figsize=(8,5))
plt.plot(date,price, label='Adj. Closing Price')
plt.title("Adj. Closing Price of Starbucks (1998-2013)")
plt.xlabel("Date")
plt.ylabel("Price in Dollars")
plt.legend()
plt.show()
```



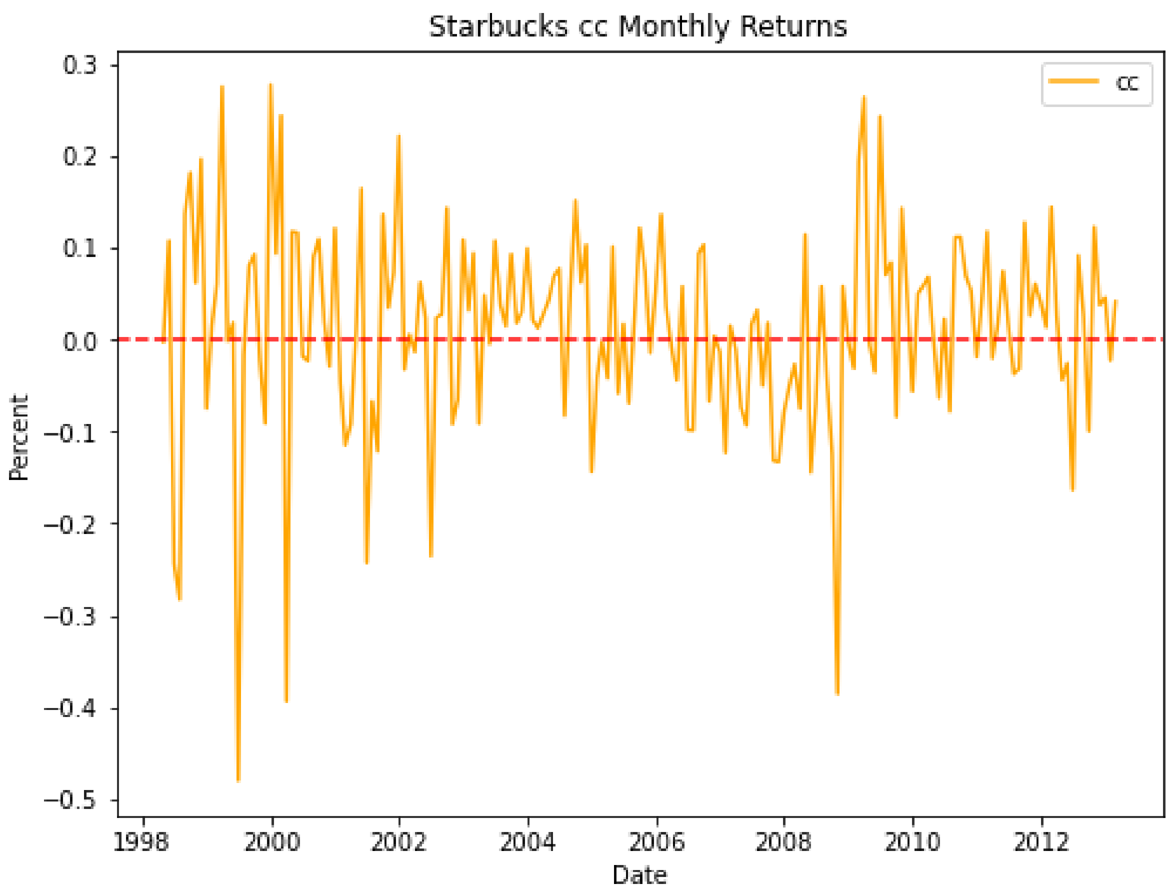
2.3 Question 3

Compute monthly simple and continuously compounded returns. Plot these returns separately first. Then also plot on the same graph.

```
# Simple Returns
sbux_monthly_simplereturns = sbuxPrices_df['Adj Close'].pct_change()
plt.figure(figsize=(8,6))
plt.plot(date,sbux_monthly_simplereturns, label='Simple')
plt.axhline(y=0, color='r', linestyle='--')
plt.xlabel("Date")
plt.ylabel("Percent")
plt.title("Starbucks Simple Monthly Returns")
plt.legend()
plt.show()
```

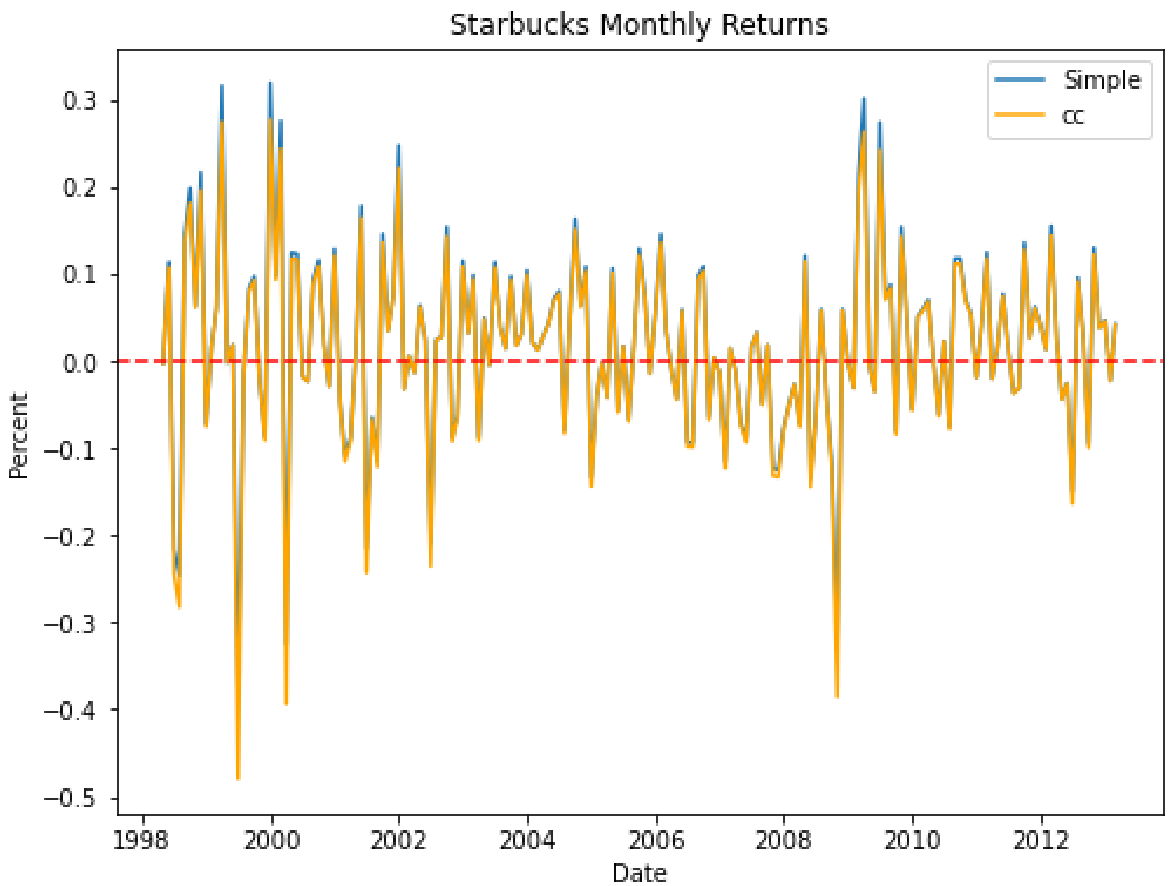


```python
# CC Returns
sbux_monthly_ccreturns = np.log(1 + sbux_monthly_simplereturns)

plt.figure(figsize=(8,6))
plt.plot(date,sbux_monthly_ccreturns, color='orange', label = 'cc')
plt.axhline(y=0, color='r', linestyle='--')
plt.xlabel("Date")
plt.ylabel("Percent")
plt.title("Starbucks cc Monthly Returns")
plt.legend()
plt.show()
```




```python
## Same Plot
plt.figure(figsize=(8,6))
```

```
plt.plot(date,sbux_monthly_simplereturns, label='Simple')
plt.plot(date,sbux_monthly_ccreturns, color='orange', label = 'cc')
plt.axhline(y=0, color='r', linestyle='--')
plt.xlabel("Date")
plt.ylabel("Percent")
plt.title("Starbucks Monthly Returns")
plt.legend()
plt.show()
```




```
# Scatterplot of Simple vs. cc Returns
plt.figure(figsize=(8,6))
plt.scatter(sbux_monthly_simplereturns,sbux_monthly_ccreturns)
plt.plot([-.5, .3], [-.5,.3], ls="--", color='r', label='reference')
#plt.axhline(y=0, color='r', linestyle='--')
```

```
plt.xlabel("Simple Returns")
plt.ylabel("cc Returns")
plt.title("Starbucks Monthly Returns")
plt.legend()
plt.grid()
```