Intensional infect proportion of newborn, finding eigenvalues

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$$\frac{\mathrm{d}S}{d\tau} = \epsilon(1-p) - \mathcal{R}_0 SI - \epsilon S \tag{1}$$

$$\frac{\mathrm{d}I}{d\tau} = \mathcal{R}_0 SI + \epsilon p - I \tag{2}$$

So to find the E.E. Letting both equal to 0, solve get:

$$I = \frac{\epsilon(\mathcal{R}_0 - 1) + \sqrt{(\epsilon\mathcal{R}_0 - \epsilon)^2 + 4\mathcal{R}_0\epsilon^2 p}}{2\mathcal{R}_0} \tag{3}$$

$$S = \frac{2(\frac{\epsilon(\mathcal{R}_0-1)+\sqrt{(\epsilon\mathcal{R}_0-\epsilon)^2+4\mathcal{R}_0\epsilon^2 p}}{2\mathcal{R}_0} - \epsilon p)}{\epsilon(\mathcal{R}_0 - 1) + \sqrt{(\epsilon\mathcal{R}_0 - \epsilon)^2 + 4\mathcal{R}_0\epsilon^2 p}} \tag{4}$$

Jacobian is the following.

$$\mathcal{J} = \begin{bmatrix} -\mathcal{R}_0 I - \epsilon & -\mathcal{R}_0 S \\ \mathcal{R}_0 I & \mathcal{R}_0 S - 1 \end{bmatrix} \tag{5}$$

The eigenvalues of this Jacobian is:

$$\lambda = \frac{-(1-\epsilon+\mathcal{R}_0 I - \mathcal{R}_0 S) \pm \sqrt{(1-\epsilon+\mathcal{R}_0 I - \mathcal{R}_0 S)^2 - 4(\mathcal{R}_0 I - \epsilon\mathcal{R}_0 S + \epsilon)}}{2} \tag{6}$$

Since the value of ϵ is small, we can claim that the term $4(\mathcal{R}_0 I - \epsilon\mathcal{R}_0 S + \epsilon)$ is positive.

Thus, from (5), it is sufficient to conclude that both eigenvalues are negative, so E.E. is stable. But to find the relative parameters, i.e. e-folding time, etc. I need a computational software to do the rest of the job.