

# Intensional infect proportion of newborn, finding eigenvalues

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$$\frac{dS}{d\tau} = \epsilon(1 - p) - \mathcal{R}_0 SI - \epsilon S \quad (1)$$

$$\frac{dI}{d\tau} = \mathcal{R}_0 SI + \epsilon p - I \quad (2)$$

Here  $\gamma$  is mean infectious period,  $\mu$  is birth/death rate,  $p$  is probability of intensional infection on newborns.  $\epsilon = \frac{\mu}{\gamma + \mu}$ ,  $\mathcal{R}_0$  is the basic reproduction number.

So to find the E.E. Letting both equal to 0, solve get:

$$I = \frac{\epsilon(\mathcal{R}_0 - 1) + \epsilon\sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}}{2\mathcal{R}_0} \quad (3)$$

$$S = \frac{1}{\mathcal{R}_0} - \frac{2p}{(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}} \quad (4)$$

$$\mathcal{R}_0 I = \frac{\epsilon(\mathcal{R}_0 - 1) + \epsilon\sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}}{2} \quad (5)$$

$$\mathcal{R}_0 S = 1 - \frac{2p\mathcal{R}_0}{(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}} \quad (6)$$

Jacobian is the following.

$$\mathcal{J} = \begin{bmatrix} -\mathcal{R}_0 I - \epsilon & -\mathcal{R}_0 S \\ \mathcal{R}_0 I & \mathcal{R}_0 S - 1 \end{bmatrix} \quad (7)$$

Now for simplicity, let  $(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p} = K$

So we get:

$$\mathcal{R}_0 I = \frac{\epsilon K}{2} \quad (8)$$

$$\mathcal{R}_0 S = 1 - \frac{2p\mathcal{R}_0}{K} \quad (9)$$

$$\mathcal{J} = \begin{bmatrix} -\frac{\epsilon K}{2} - \epsilon & -1 + \frac{2p\mathcal{R}_0}{K} \\ \frac{\epsilon K}{2} & -\frac{2p\mathcal{R}_0}{K} \end{bmatrix} \quad (10)$$

The eigenvalues of this Jacobian is:

$$\lambda = \frac{-\epsilon K^2 - 2\epsilon K - 4p\mathcal{R}_0 \pm \sqrt{(\epsilon K^2 + 2\epsilon K + 4p\mathcal{R}_0)^2 - 4(2\epsilon K^3 + 8\epsilon K p \mathcal{R}_0)}}{4K} \quad (11)$$

Reminder:  $K = (\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}$