Intensional infect proportion of susceptible

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Here is the system we are investigating:

$$\frac{\mathrm{d}S}{\mathrm{d}\tau} = \epsilon - \mathcal{R}_0 SI - \frac{r}{\gamma+\mu}S - \epsilon S \tag{1}$$
$$\frac{\mathrm{d}I}{\mathrm{d}\tau} = \frac{r}{\gamma+\mu}S + \mathcal{R}_0 SI - I \tag{2}$$
$$\frac{\mathrm{d}R}{\mathrm{d}\tau} = (1-\epsilon)I - \epsilon R \tag{3}$$

Here γ is mean infectious period, μ is birth/death rate, r is rate of intensional infection. $\epsilon = \frac{\mu}{\gamma+\mu}$,$\mathcal{R}_0$ is the basic reproduction number.

Since last time we discussed that, it is not very meaningful to divide I into I_T and I_N. Thus, I just used I this time to investigate the system's equilibrium, stability and other properties.

First of all, for simplicity reasons, let $L = \frac{r}{\gamma+\mu}$.

Thus, the Endemic equilibrium is the following:

$$I = \frac{-(L + \epsilon - \epsilon\mathcal{R}_0) + \sqrt{(L + \epsilon - \epsilon\mathcal{R}_0)^2 + 4\mathcal{R}_0\epsilon L}}{2\mathcal{R}_0} \tag{4}$$

$$S = \frac{1}{\mathcal{R}_0} - \frac{2L}{\mathcal{R}_0(-(L + \epsilon - \epsilon\mathcal{R}_0) + \sqrt{(L + \epsilon - \epsilon\mathcal{R}_0)^2 + 4\mathcal{R}_0\epsilon L} + 2L)} \tag{5}$$

Jacobian is the following.

$$\mathcal{J} = \begin{bmatrix} -\mathcal{R}_0 I - L - \epsilon & -\mathcal{R}_0 S \\ L + \mathcal{R}_0 I & \mathcal{R}_0 S - 1 \end{bmatrix} \tag{6}$$

Again, for simplicity. Let $G = -(L + \epsilon - \epsilon\mathcal{R}_0) + \sqrt{(L + \epsilon - \epsilon\mathcal{R}_0)^2 + 4\mathcal{R}_0\epsilon L}$.

So Jacobian at E.E. is:

$$\mathcal{J} = \begin{bmatrix} \frac{G}{2} - L - \epsilon & -1 + \frac{2L}{G+2L} \\ L + \frac{G}{2} & -\frac{2L}{G+2L} \end{bmatrix} \tag{7}$$

I then tried to use Wolfram Alpha to compute the eigenvalues for me and I got the following:

Figure 1: Eigenvalue computations from Wolfram

Input:

eigenvalues	$\begin{pmatrix} -\frac{G}{2} - L - \epsilon & -1 + 2 \times \frac{L}{G+2L} \\ L + \frac{G}{2} & -2 \times \frac{L}{G+2L} \end{pmatrix}$

Results:

$$\lambda_1 = \frac{1}{4(G+2L)} \left(-G^2 - \sqrt{\left((G^2 + 4GL + 2G\epsilon + 4L^2 + 4L\epsilon + 4L)^2 - 4(2G^3 + 12G^2L + 24GL^2 + 8GL\epsilon + 16L^3 + 16L^2\epsilon)\right)} - 4GL - 2G\epsilon - 4L^2 - 4L\epsilon - 4L\right)$$

$$\lambda_2 = \frac{1}{4(G+2L)} \left(-G^2 + \sqrt{\left((G^2 + 4GL + 2G\epsilon + 4L^2 + 4L\epsilon + 4L)^2 - 4(2G^3 + 12G^2L + 24GL^2 + 8GL\epsilon + 16L^3 + 16L^2\epsilon)\right)} - 4GL - 2G\epsilon - 4L^2 - 4L\epsilon - 4L\right)$$