Intensional infect proportion of newborn, finding eigenvalues

May 2, 2018

$$\frac{\mathrm{d}S}{d\tau} = \epsilon(1-p) - \mathcal{R}_0 SI - \epsilon S \tag{1}$$

$$\frac{\mathrm{d}I}{d\tau} = \mathcal{R}_0 SI + \epsilon p - I \tag{2}$$

Here γ is mean infectious period, μ is birth/death rate, p is probability of intensional infection on newborns. $\epsilon = \frac{\mu}{\gamma+\mu}$,$\mathcal{R}_0$ is the basic reproduction number.

1 EE

So to find the E.E. Letting both equal to 0, solve get:

$$I = \frac{\epsilon(\mathcal{R}_0 - 1) + \epsilon\sqrt{(\mathcal{R}_0-1)^2 + 4\mathcal{R}_0 p}}{2\mathcal{R}_0} \tag{3}$$

$$S = \frac{1}{\mathcal{R}_0} - \frac{2p}{(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0-1)^2 + 4\mathcal{R}_0 p}} \tag{4}$$

$$\mathcal{R}_0 I = \frac{\epsilon(\mathcal{R}_0 - 1) + \epsilon\sqrt{(\mathcal{R}_0-1)^2 + 4\mathcal{R}_0 p}}{2} \tag{5}$$

$$\mathcal{R}_0 S = 1 - \frac{2p\mathcal{R}_0}{(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0-1)^2 + 4\mathcal{R}_0 p}} \tag{6}$$

Jacobian is the following.

$$\mathcal{J} = \begin{bmatrix} -\mathcal{R}_0 I - \epsilon & -\mathcal{R}_0 S \\ \mathcal{R}_0 I & \mathcal{R}_0 S - 1 \end{bmatrix} \tag{7}$$

Now for simplicity, let $(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p} = K$
So we get:

$$\mathcal{R}_0 I = \frac{\epsilon K}{2} \tag{8}$$

$$\mathcal{R}_0 S = 1 - \frac{2p\mathcal{R}_0}{K} \tag{9}$$

$$\mathcal{J} = \begin{bmatrix} -\frac{\epsilon K}{2} - \epsilon & -1 + \frac{2p\mathcal{R}_0}{K} \\ \frac{\epsilon K}{2} & -\frac{2p\mathcal{R}_0}{K} \end{bmatrix} \tag{10}$$

The eigenvalues of this Jacobian is:

$$\lambda = \frac{-(\epsilon K^2 + 2\epsilon K + 4p\mathcal{R}_0) \pm \sqrt{(\epsilon K^2 + 2\epsilon K + 4p\mathcal{R}_0)^2 - 4(2\epsilon K^3 + 8\epsilon K p \mathcal{R}_0)}}{4K} \tag{11}$$

Reminder: $K = (\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}$

2 DFE

Certainly at DFE, $S = 1$ and $I = 0$. By using the same Jacobian, we get the following:

$$\mathcal{J} = \begin{bmatrix} -\epsilon & -\mathcal{R}_0 \\ 0 & \mathcal{R}_0 - 1 \end{bmatrix} \tag{12}$$

So eigenvalues are just the entries on the diagonal:

$$\lambda_1 = -\epsilon \tag{13}$$

$$\lambda_2 = \mathcal{R}_0 - 1 \tag{14}$$

This means, DFE is stable iff $\mathcal{R}_0 < 1$