Intensional infect proportion of newborn, finding eigenvalues

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$$\frac{\mathrm{d}S}{d\tau} = \epsilon(1-p) - \mathcal{R}_0 SI - \epsilon S \tag{1}$$

$$\frac{\mathrm{d}I}{d\tau} = \mathcal{R}_0 SI + \epsilon p - I \tag{2}$$

Here γ is mean infectious period, μ is birth/death rate, p is probability of intensional infection on newborns. $\epsilon = \frac{\mu}{\gamma+\mu}$,$\mathcal{R}_0$ is the basic reproduction number.

1 EE

So to find the E.E. Letting both equal to 0, solve get:

$$I = \frac{\epsilon(\mathcal{R}_0 - 1) + \epsilon\sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}}{2\mathcal{R}_0} \tag{3}$$

$$S = \frac{1}{\mathcal{R}_0} - \frac{2p}{(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}} \tag{4}$$

$$\mathcal{R}_0 I = \frac{\epsilon(\mathcal{R}_0 - 1) + \epsilon\sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}}{2} \tag{5}$$

$$\mathcal{R}_0 S = 1 - \frac{2p\mathcal{R}_0}{(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p}} \tag{6}$$

Jacobian is the following.

$$\mathcal{J} = \begin{bmatrix} -\mathcal{R}_0 I - \epsilon & -\mathcal{R}_0 S \\ \mathcal{R}_0 I & \mathcal{R}_0 S - 1 \end{bmatrix} \tag{7}$$

Now for simplicity, let $(\mathcal{R}_0 - 1) + \sqrt{(\mathcal{R}_0 - 1)^2 + 4\mathcal{R}_0 p} = K$.Notice, $K > 0$ if $q \neq 0$

So we get:

$$\mathcal{R}_0 I = \frac{\epsilon K}{2} \tag{8}$$

$$\mathcal{R}_0 S = 1 - \frac{2p\mathcal{R}_0}{K} \tag{9}$$

$$\mathcal{J} = \begin{bmatrix} -\frac{\epsilon K}{2} - \epsilon & -1 + \frac{2p\mathcal{R}_0}{K} \\ \frac{\epsilon K}{2} & -\frac{2p\mathcal{R}_0}{K} \end{bmatrix} \tag{10}$$

The eigenvalues of this Jacobian is:

$$\lambda = \frac{-(\epsilon K^2 + 2\epsilon K + 4p\mathcal{R}_0) \pm \sqrt{(\epsilon K^2 + 2\epsilon K + 4p\mathcal{R}_0)^2 - 4(2\epsilon K^3 + 8\epsilon K p \mathcal{R}_0)}}{4K} \tag{11}$$

Since $\sqrt{(\epsilon K^2 + 2\epsilon K + 4p\mathcal{R}_0)^2 - 4(2\epsilon K^3 + 8\epsilon K p\mathcal{R}_0)} \leq |\epsilon K^2 + 2\epsilon K + 4p\mathcal{R}_0|$, we can conclude that Re(λ)<0 for all $p \neq 0$

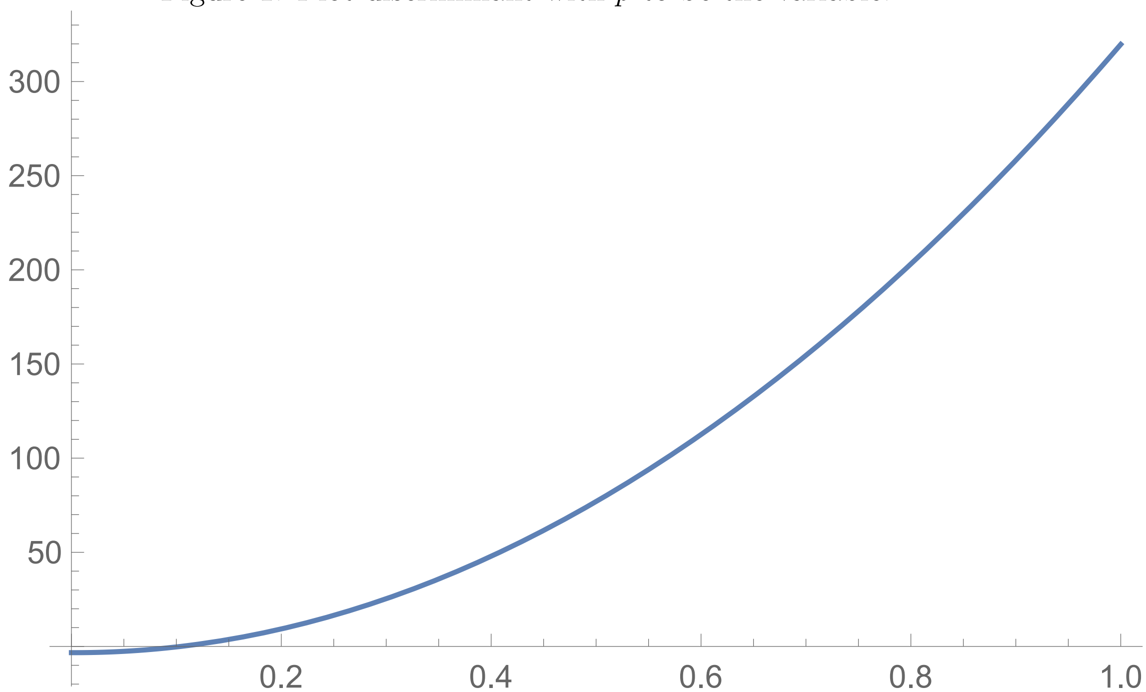
We are interested in whether this is a damped oscillation or not, so we look into the discriminant.

1.1 Take p as the variable

The following values are used: $\mu = \frac{1}{50*365}$, $\gamma = \frac{1}{22}$, $\mathcal{R}_0 = 4.5$. Also, $\epsilon = \frac{\mu}{\mu+\gamma} = 0.0012$

I plotted the discriminant with Mathematica, domain of p is $[0, 1]$.

Figure 1: Plot discriminant with p to be the variable.



The p-intercept of this graph is $p = 0.103995$. Meaning, with a proportion of 0.103995 or less, there is going to be a damped oscillation.

1.2 Take $\mathcal{R}_0$ as the variable

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This time, I take $\mu = \frac{1}{80*365}$, $\gamma = \frac{1}{22}$, $\epsilon = \frac{\mu}{\mu+\gamma} = 0.000753$.

I made a sequence of plots by taking $p = 0.1, 0.2, 0.3...$

Figure 2: Plot discriminant with $\mathcal{R}_0$ to be the variable.$p = 0.1$, the $\mathcal{R}_0$ -intercept is $\mathcal{R}_0 = 0, 5.74966$

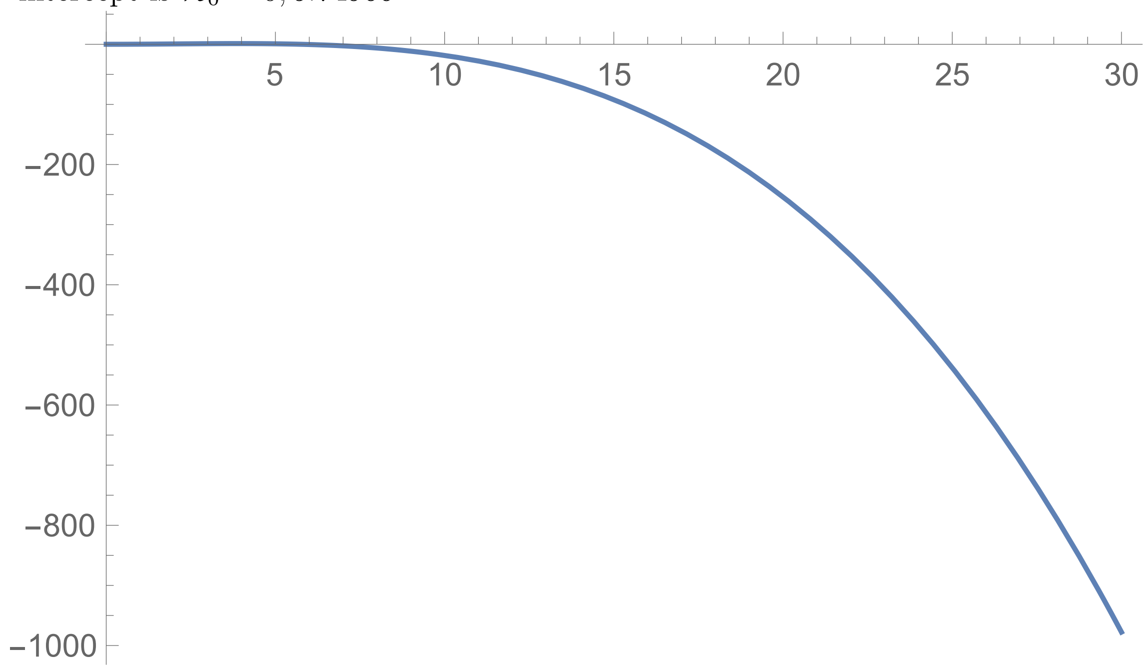


Figure 3: Plot discriminant with $\mathcal{R}_0$ to be the variable.$p = 0.2$, the $\mathcal{R}_0$ -intercept is $\mathcal{R}_0 = 0, 17.0518$

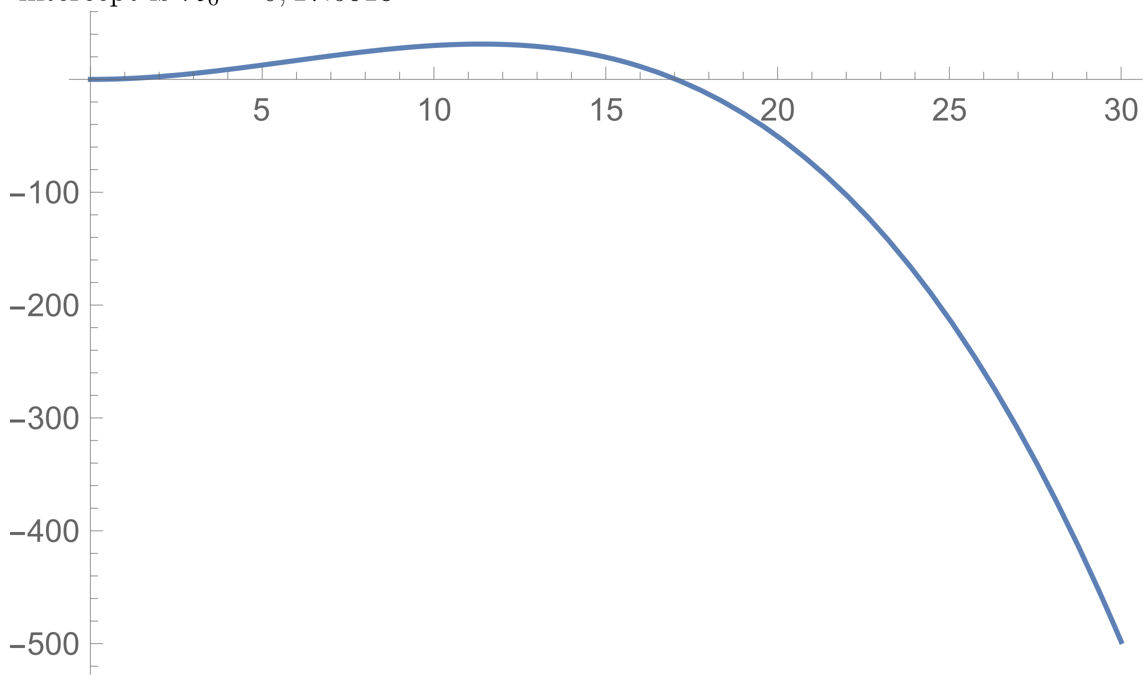
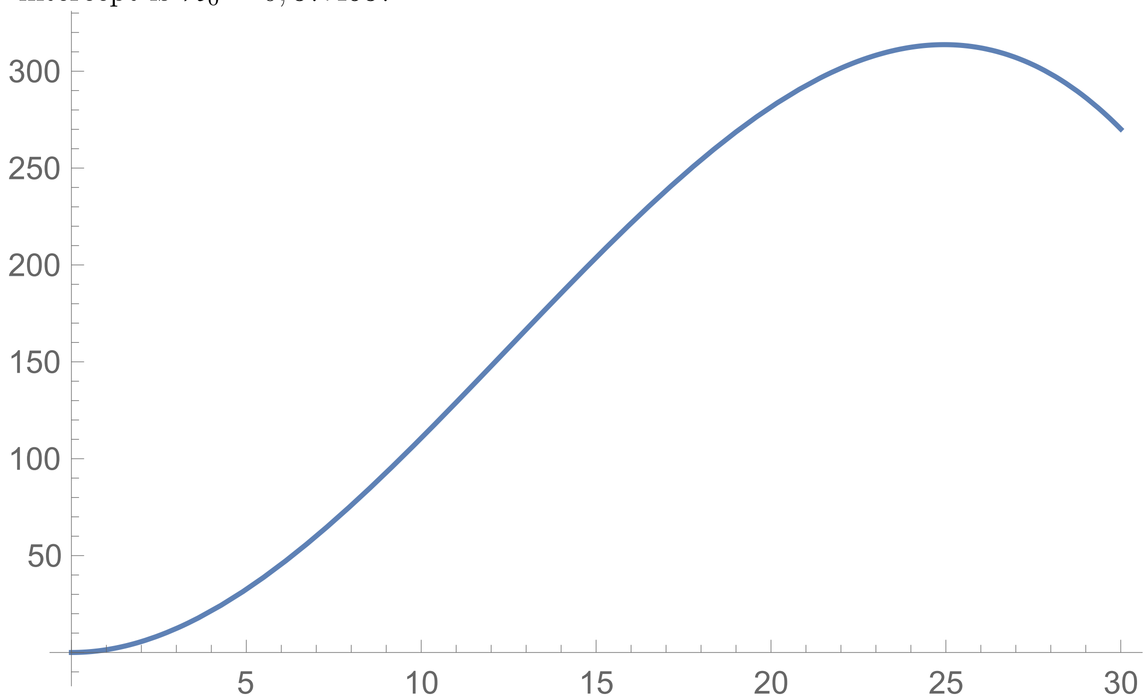


Figure 4: Plot discriminant with $\mathcal{R}_0$ to be the variable.$p = 0.3$, the $\mathcal{R}_0$ -intercept is $\mathcal{R}_0 = 0, 37.4537$



For $p = 0.4$ or higher, the graphs look similar, but with even higher value for the discriminant and larger value for $\mathcal{R}_0$-intercept.

2 DFE

Disease free equilibrium is the equilibrium when there is nobody infected in the system, which means, $I = 0$

Thus we have $\frac{\mathrm{d}I}{d\tau} = \mathcal{R}_0 SI + \epsilon p - I = \epsilon p = 0$

In our case, there is no DFE. When $I = 0$, we are still intensionally infecting susceptible, therefore, there is no equilibrium in which $I = 0$.

3 I at E.E.

Figure 5: Infected population at E.E. as a function of $\mathcal{R}_0$