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Written HW 1

Question 1:

a. True

P(a|b,c) = (P(b|a,c)*P(a|c))/P(b|c).
If P(a|b,c) = P(b|a,c), then P(a|b,c) = (P(a|b,c)*P(a|c))/P(b|c).
Therefore, P(a|b,c)*P(b|c) = P(a|b,c)*P(a|c)
Therefore, P(b|c) = P(a|c)

b. False

Say P(a) = the probability that I eat dinner
Say P(b) = the probability that I yawn
Say P(c) = the probability that I slept for less than 5 hours last night

The probability that I eat dinner is independent of whether I yawn or how much I slept last night. Therefore, P(a|b,c) = P(a). However, P(b|c) != P(b), as whether I yawn is directly influenced by how much sleep I got last night. Therefore, even though P(a|b,c) = P(a), P(b|c) != P(b).

c. True

If P(a|b) = P(a), then P(a) is independent of P(b). Therefore, P(a|b,x,y,z) = P(a|x,y,z). Therefore, P(a|b,c) = P(a|c).

d. True

P(a|b) = P(a,b)/P(b)
Using chain rule, P(a,b) = P(a)*P(b|a)
Therefore, P(a|b) = (P(a)*P(b|a))/P(b)

Question 2:

a) We know that P(x,y) = P(y)*P(x|y). It makes sense, then, that adding a new hoop to jump through won't change this equation. For example, P(x,y|z) = P(y|z)*P(x|y,z). Therefore, P(x,y|*e*) = P(x|y,*e*)*P(y|*e*).

b) We know that P(y|x,e) = P(x,y,z)/P(x,e) and that P(x|y) = P(x,y)/P(y)
P(x|y,e) = P(x,y,e)/P(y,e)
P(y|e) = P(y,e)/P(e)

P(x|e) = P(x,e)/P(e)
Therefore, (P(x|y,e)*P(y|e))/P(x|e) = (P(x,y,e)*P(y,e)*P(e))/(P(y,e)*P(e)*P(x,e))
Therefore, (P(x|y,e)*P(y|e))/P(x|e) = P(x,y,e)/P(x,e)
We know that P(y|x,e) = P(x,y,z)/P(x,e), so P(y|x,e) = (P(x|y,e)*P(y|e))/P(x|e)

Question 3:

1. Add A: No parents.

2. Add B: If the alarm went off, it's more likely there was a burglary. Parents(B) = {A}

3. Add M: If the alarm went off, it's more likely that Mary called. Parents(M) = {A,B}

4. Add E: If the alarm went off or Mary called, it's more likely there was an earthquake. Parents(E) = {A,M}

5. Add J: If the alarm went off, there was an earthquake, or there was a burglary, it's more likely that John called. It's also more likely that John called if Mary called, as her call would indicate one of the above occurring. Parents(J) = {A,B,E,M}

Question 4:

a) Say A = {2,2,2,2,2,2} and B = {10,10,1,1,1,1}.

Because 4 of A's values are greater than 4 of B's values, P(A>B) = 4/6 = 2/3, which is greater than 1/2.

However, E(A) = 12/6 = 2, while E(B) = 24/6 = 4. So E(B) > E(A).

Therefore, it is possible that P(A>B) > 1/2 and E(B) > E(A).

b) Say A = {10,2,2,2,2,2} and B = {9,9,9,9,9,1}.

maxA = 10. maxB = 9. Therefore maxA > maxB.
minA = 2. minB = 1. Therefore minA > minB.

However, E(A) = 30/6 = 5, while E(B) = 46/6 = 7.67. So E(B) > E(A).

c) No. If A > B and B > C, C cannot be greater than A. That's just how the transitive property works.

d) No. Same as above. If A has more than 3 numbers larger than B and B has more than 3 numbers larger than C, C cannot have more than 3 numbers larger than A.