Natural Language Processing with Deep Learning

CS224N/Ling284

Lecture 8: Recurrent Neural Networks

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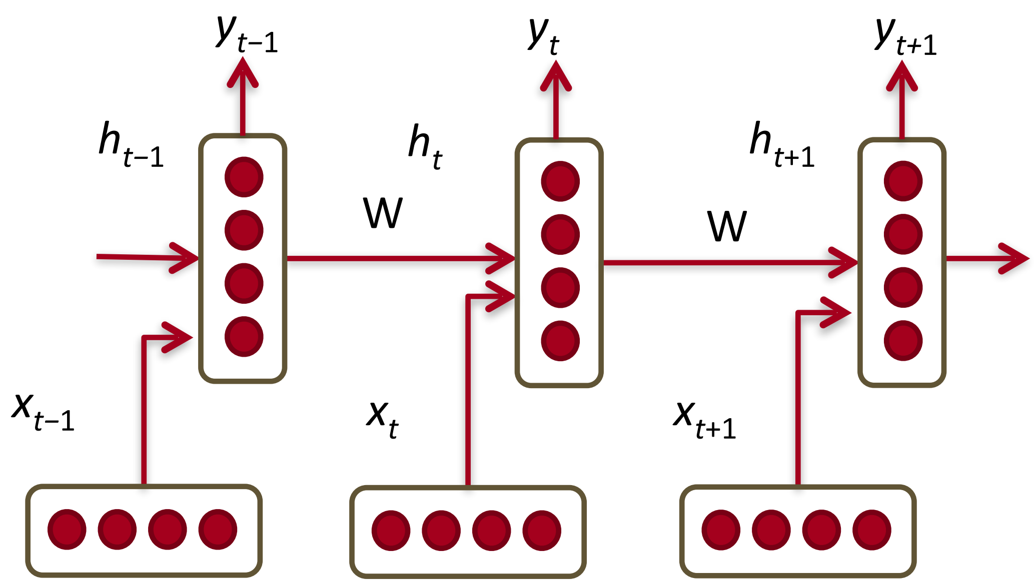
Recurrent Neural Networks!

- RNNs tie the weights at each time step
- Condition the neural network on all previous words
- RAM requirement only scales with number of words

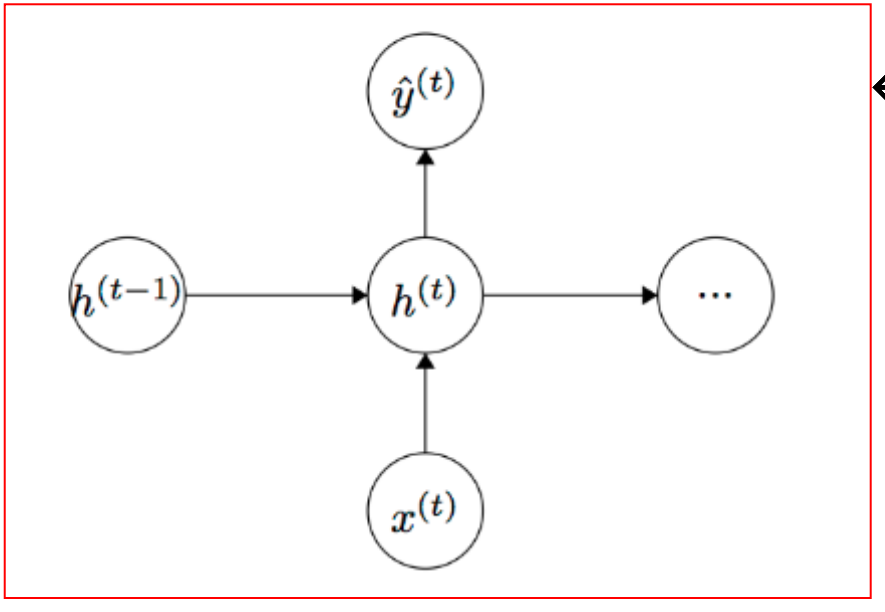


Recurrent Neural Network language model

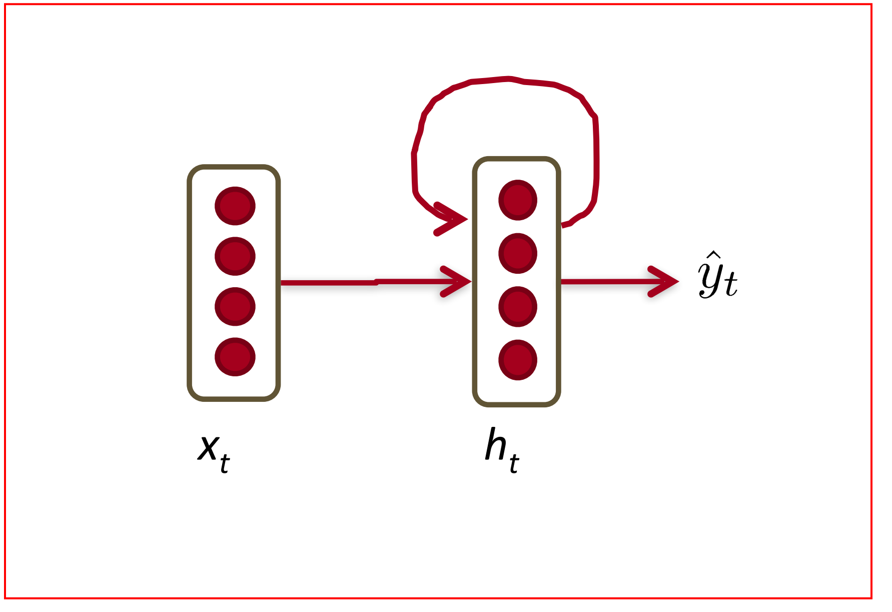
Given list of word **vectors**: $x_1, \ldots, x_{t-1}, x_t, x_{t+1}, \ldots, x_T$

At a single time step:

$$
\begin{aligned}
h_t &= \sigma\left(W^{(hh)} h_{t-1} + W^{(hx)} x_{[t]}\right) \\
\hat{y}_t &= \text{softmax}\left(W^{(S)} h_t\right) \\
\hat{P}(x_{t+1} = v_j \mid x_t, \ldots, x_1) &= \hat{y}_{t,j}
\end{aligned}
$$



$\leftarrow\rightarrow$



Recurrent Neural Network language model

Main idea: we use the same set of W weights at all time steps!

Everything else is the same:

$$\begin{aligned} h_t &= \sigma\left(W^{(hh)} h_{t-1} + W^{(hx)} x_{[t]}\right) \\ \hat{y}_t &= \mathrm{softmax}\left(W^{(S)} h_t\right) \\ \hat{P}(x_{t+1} = v_j \mid x_t, \ldots, x_1) &= \hat{y}_{t,j} \end{aligned}$$

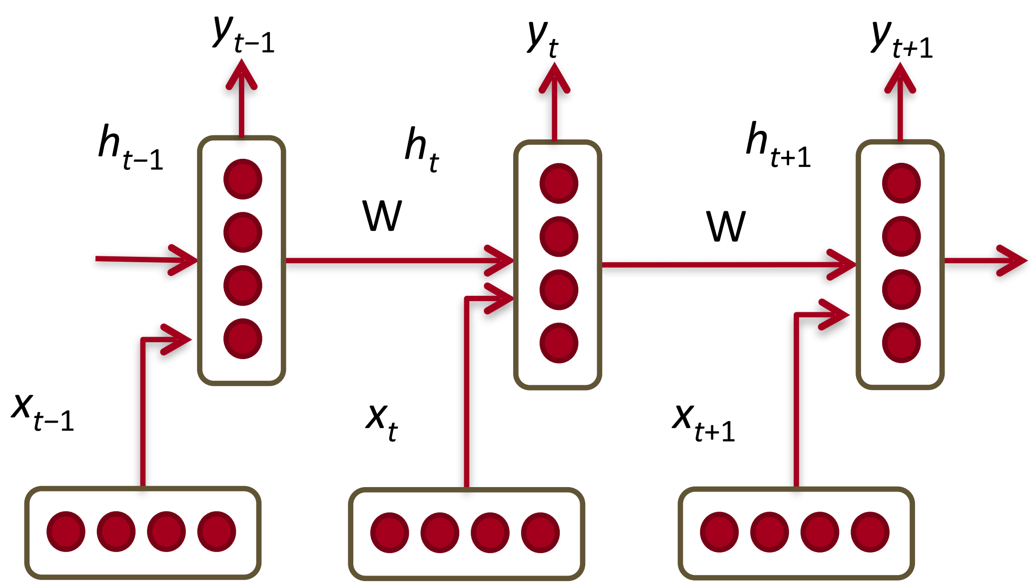
$h_0 \in \mathbb{R}^{D_h}$ is some initialization vector for the hidden layer at time step 0

$x_{[t]}$ is the column vector of L at index [t] at time step t

$W^{(hh)} \in \mathbb{R}^{D_h \times D_h}$ $\quad W^{(hx)} \in \mathbb{R}^{D_h \times d}$ $\quad W^{(S)} \in \mathbb{R}^{|V| \times D_h}$

Training RNNs is hard

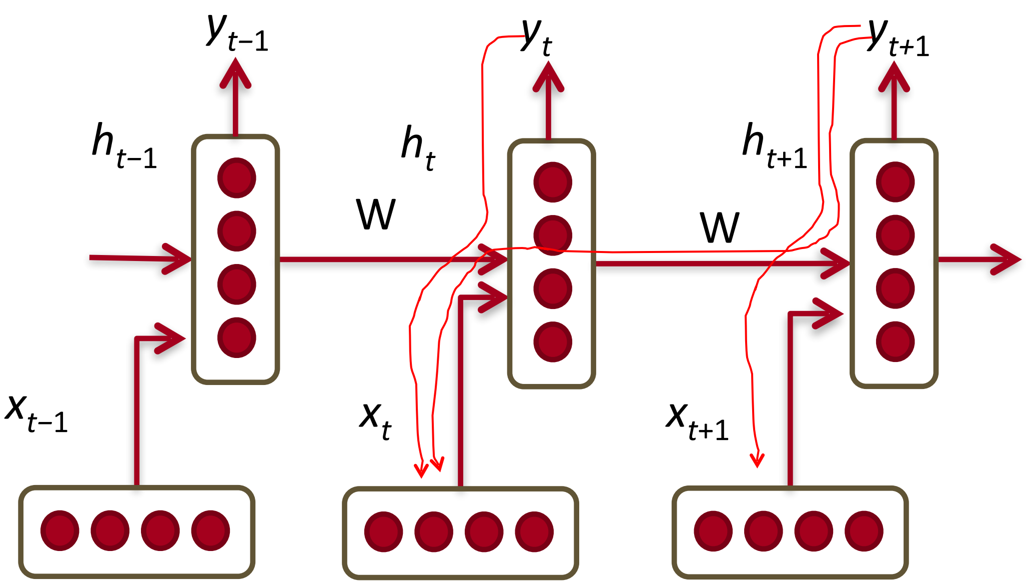
- Multiply the same matrix at each time step during forward prop



- Ideally inputs from many time steps ago can modify output *y*
- Take $\frac{\partial E_2}{\partial W}$ for an example RNN with 2 time steps! Insightful!

The vanishing/exploding gradient problem

- Multiply the same matrix at each time step during backprop



The vanishing gradient problem - Details

- Similar but simpler RNN formulation:

$$\begin{aligned} h_t &= W f(h_{t-1}) + W^{(hx)} x_{[t]} \\ \hat{y}_t &= W^{(S)} f(h_t) \end{aligned}$$

- Total error is the sum of each error at time steps t

$$\frac{\partial E}{\partial W} = \sum_{t=1}^{T} \frac{\partial E_t}{\partial W}$$

- Hardcore chain rule application:

$$\frac{\partial E_t}{\partial W} = \sum_{k=1}^{t} \frac{\partial E_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W}$$

The vanishing gradient problem - Details

- Similar to backprop but less efficient formulation
- Useful for analysis we'll look at:

$$\frac{\partial E_t}{\partial W} = \sum_{k=1}^{t} \frac{\partial E_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \boxed{\frac{\partial h_t}{\partial h_k}} \frac{\partial h_k}{\partial W}$$

- Remember: $h_t = W f(h_{t-1}) + W^{(hx)} x_{[t]}$
- More chain rule, remember:

$$\frac{\partial h_t}{\partial h_k} = \prod_{j=k+1}^{t} \frac{\partial h_j}{\partial h_{j-1}}$$

- Each partial is a Jacobian:

$$\frac{d\mathbf{f}}{d\mathbf{x}} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial x_1} & \cdots & \frac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

The vanishing gradient problem - Details

- Analyzing the norms of the Jacobians, yields:

$$\left\| \frac{\partial h_j}{\partial h_{j-1}} \right\| \leq \|W^T\| \|\text{diag}[f'(h_{j-1})]\| \leq \beta_W \beta_h$$

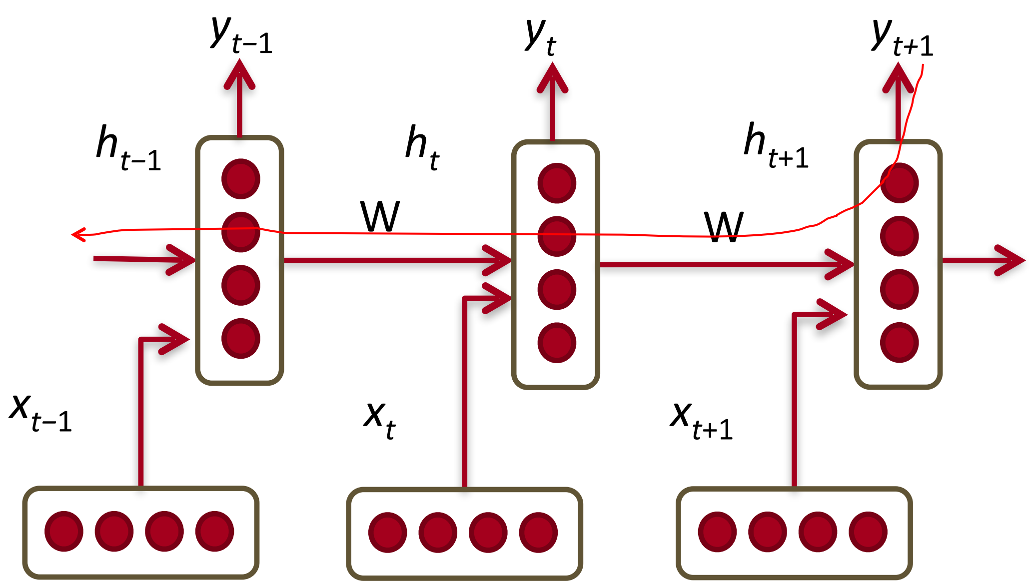
- Where we defined β's as upper bounds of the norms
- The gradient is a product of Jacobian matrices, each associated with a step in the forward computation.

$$\left\| \frac{\partial h_t}{\partial h_k} \right\| = \left\| \prod_{j=k+1}^{t} \frac{\partial h_j}{\partial h_{j-1}} \right\| \leq (\beta_W \beta_h)^{t-k}$$

- This can become very small or very large quickly [Bengio et al 1994], and the locality assumption of gradient descent breaks down. → **Vanishing or exploding gradient**

Why is the vanishing gradient a problem?

- The error at a time step ideally can tell a previous time step from many steps away to change during backprop



The vanishing gradient problem for language models

- In the case of language modeling or question answering words from time steps far away are not taken into consideration when training to predict the next word

- Example:

  Jane walked into the room. John walked in too. It was late in the day. Jane said hi to ____

Trick for exploding gradient: clipping trick

- The solution first introduced by Mikolov is to clip gradients to a maximum value.

Algorithm 1 Pseudo-code for norm clipping the gradients whenever they explode

---

$\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$

if $\|\hat{\mathbf{g}}\| \geq threshold$ **then**

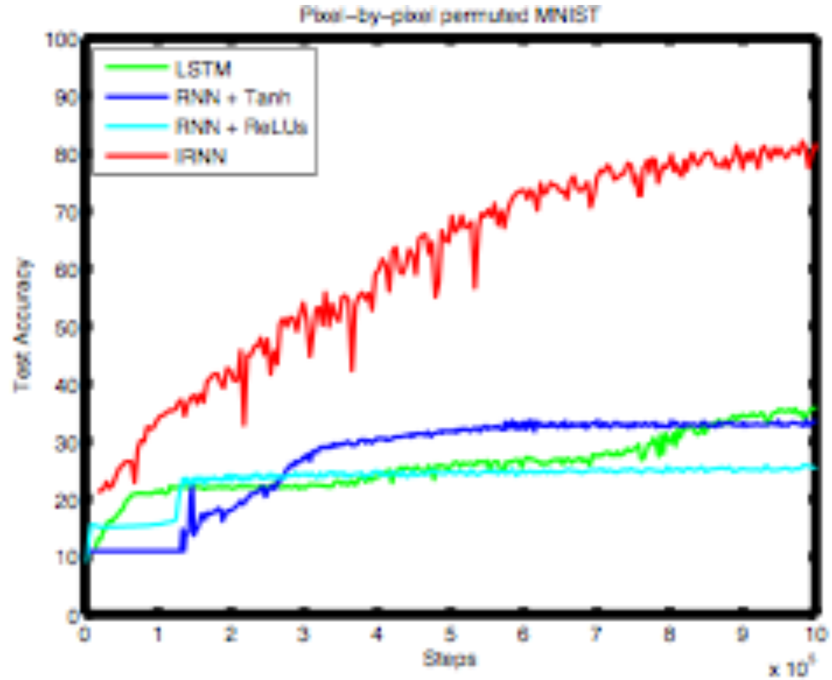
  $\hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}}$

end if

- Makes a big difference in RNNs.

For vanishing gradients: Initialization + ReLus!

- Initialize $W^{(*)}$‘s to identity matrix I
  and
  f(z) = $\mathrm{rect}(z) = \max(z, 0)$
- → Huge difference!



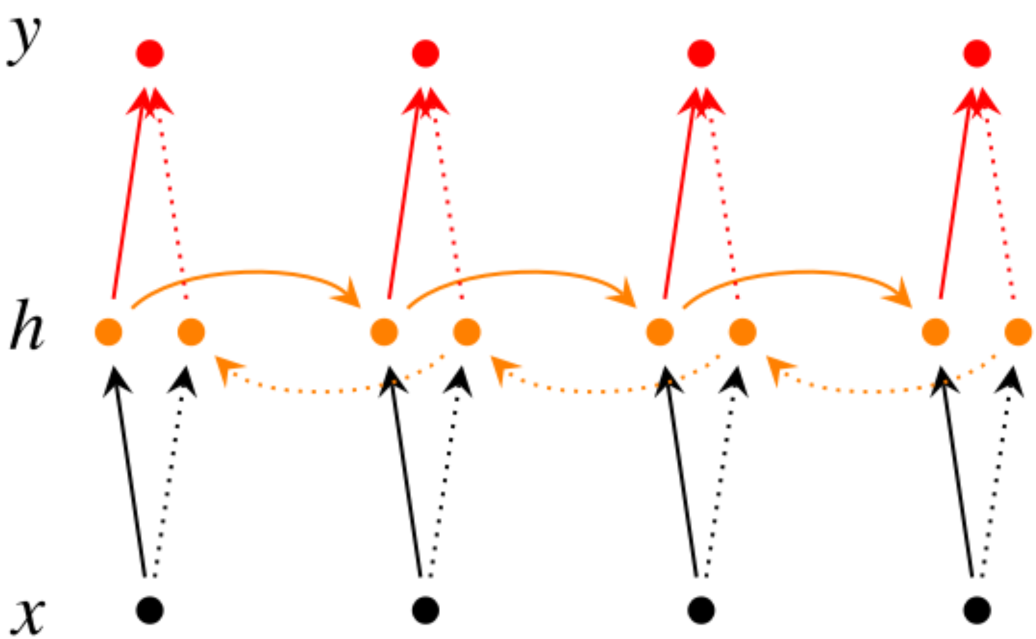
- Initialization idea first introduced in *Parsing with Compositional Vector Grammars*, Socher et al. 2013
- New experiments with recurrent neural nets in *A Simple Way to Initialize Recurrent Networks of Rectified Linear* Units, Le et al. 2015

One last implementation trick

- You only need to pass backwards through your sequence once and accumulate all the deltas from each E_t

Bidirectional RNNs

Problem: For classification you want to incorporate information from words both preceding and following



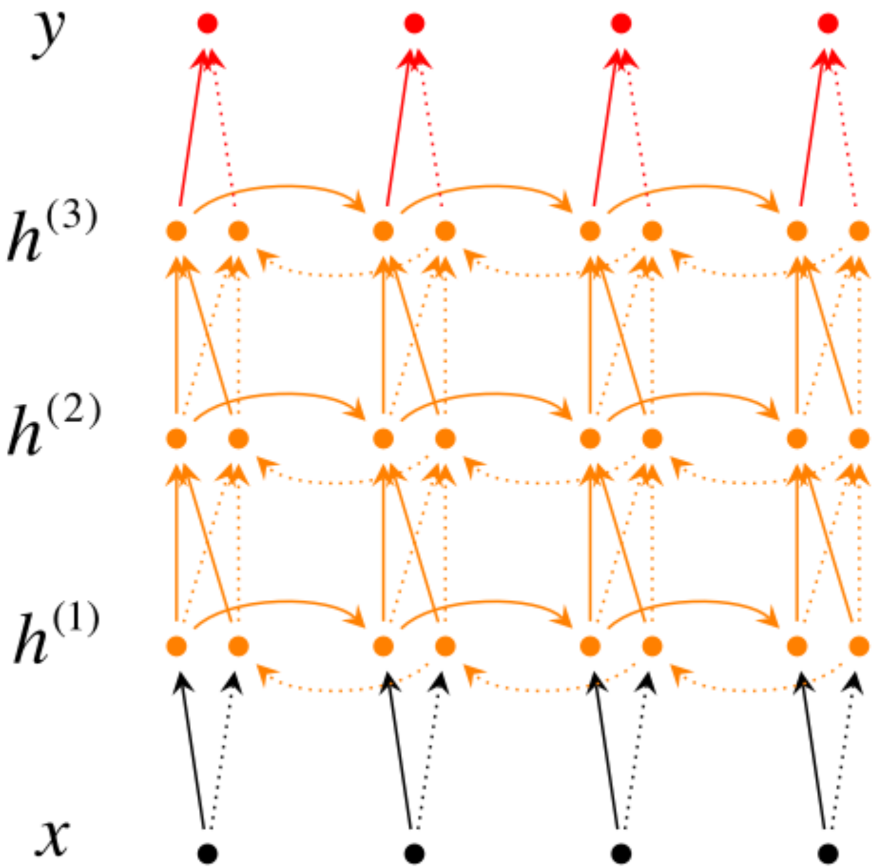
$$\overrightarrow{h}_t = f(\overrightarrow{W}x_t + \overrightarrow{V}\overrightarrow{h}_{t-1} + \overrightarrow{b})$$

$$\overleftarrow{h}_t = f(\overleftarrow{W}x_t + \overleftarrow{V}\overleftarrow{h}_{t+1} + \overleftarrow{b})$$

$$y_t = g(U[\overrightarrow{h}_t;\overleftarrow{h}_t] + c)$$

$h = [\overrightarrow{h};\overleftarrow{h}]$ now represents (summarizes) the past and future around a single token.

Deep Bidirectional RNNs



$$\overrightarrow{h}_t^{(i)} = f(\overrightarrow{W}^{(i)} h_t^{(i-1)} + \overrightarrow{V}^{(i)} \overrightarrow{h}_{t-1}^{(i)} + \overrightarrow{b}^{(i)})$$

$$\overleftarrow{h}_t^{(i)} = f(\overleftarrow{W}^{(i)} h_t^{(i-1)} + \overleftarrow{V}^{(i)} \overleftarrow{h}_{t+1}^{(i)} + \overleftarrow{b}^{(i)})$$

$$y_t = g(U[\overrightarrow{h}_t^{(L)}; \overleftarrow{h}_t^{(L)}] + c)$$

Each memory layer passes an intermediate sequential representation to the next.

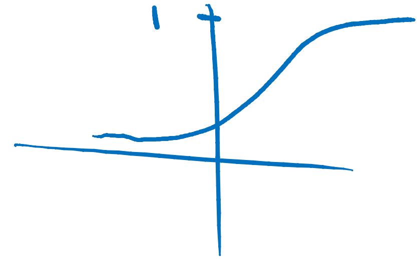
Recap

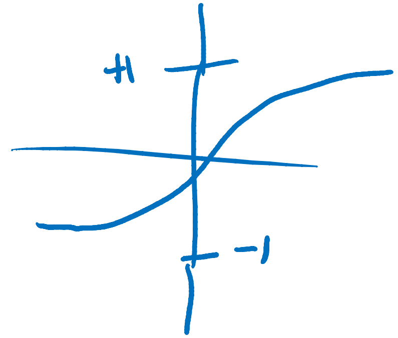
- Recurrent Neural Network is one of the best deepNLP model families
- Training them is hard because of vanishing and exploding gradient problems
- They can be extended in many ways and their training improved with many tricks (more to come)
- Next week: Most important and powerful RNN extensions with LSTMs and GRUs

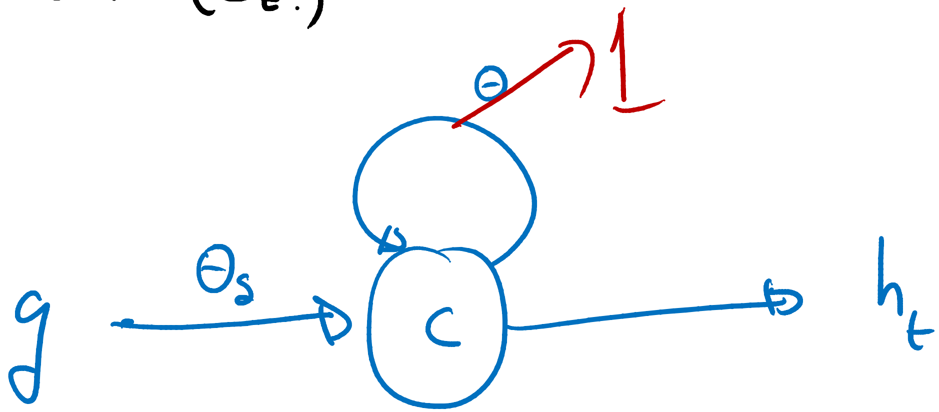
Simple solution

$$c_t = \overset{1}{\cancel{\theta}}\, c_{t-1} + \theta_s g_t$$

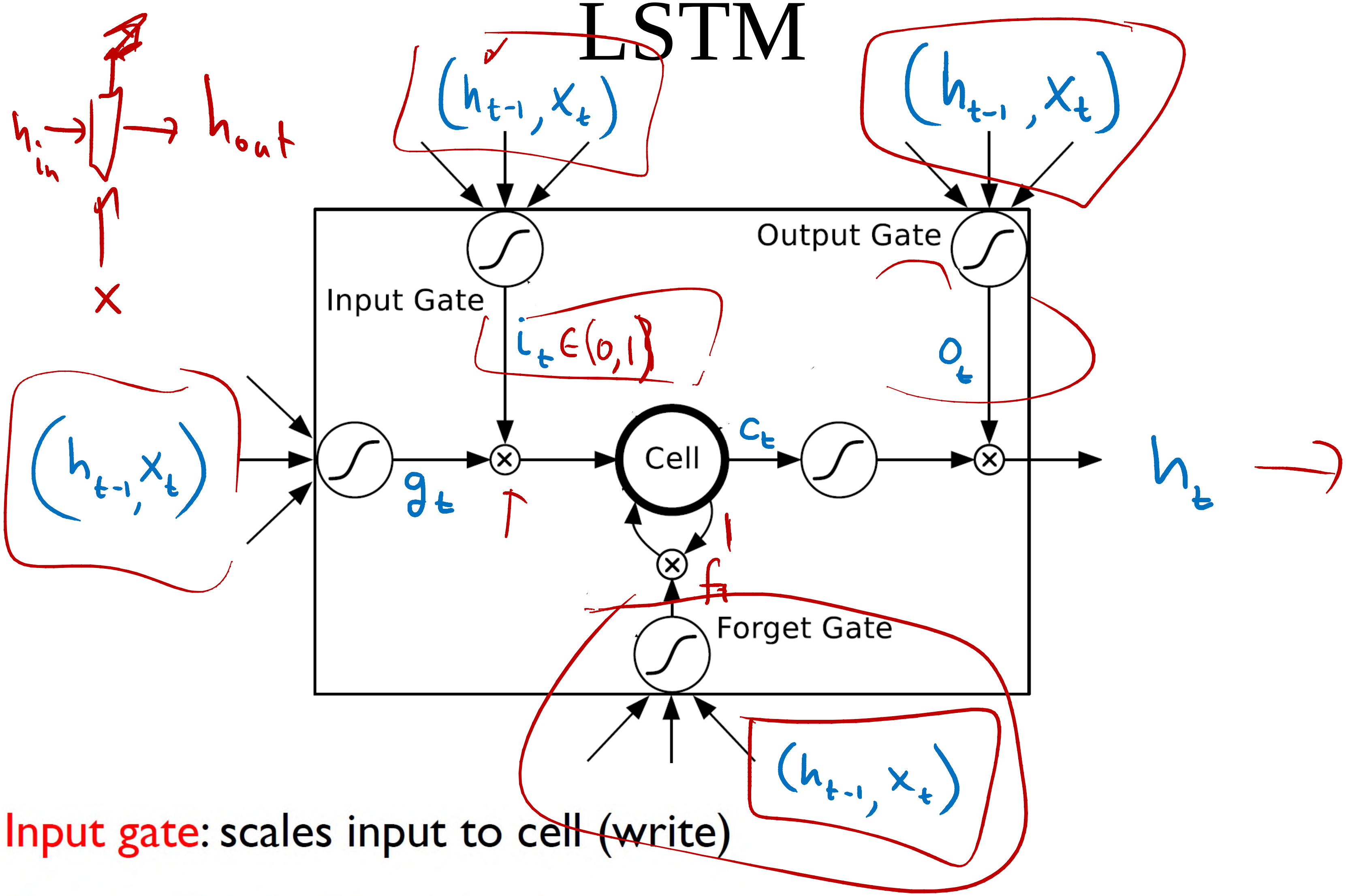
$$h_t = \text{Tanh}(c_t)$$







LSTM



Input gate: scales input to cell (write)

Output gate: scales output from cell (read)

Forget gate: scales old cell value (reset)

[Alex Graves]

LSTM

$$\mathbf{i}_t = Sigm(\boldsymbol{\theta}_{xi}\underline{\mathbf{x}_t} + \boldsymbol{\theta}_{hi}\underline{\mathbf{h}_{t-1}} + \mathbf{b}_i)$$

$$\mathbf{f}_t = Sigm(\boldsymbol{\theta}_{xf}\mathbf{x}_t + \boldsymbol{\theta}_{hf}\mathbf{h}_{t-1} + \mathbf{b}_f)$$

$$\mathbf{o}_t = Sigm(\boldsymbol{\theta}_{xo}\mathbf{x}_t + \boldsymbol{\theta}_{ho}\mathbf{h}_{t-1} + \mathbf{b}_o)$$

$$\mathbf{g}_t = Tanh(\boldsymbol{\theta}_{xg}\mathbf{x}_t + \boldsymbol{\theta}_{hg}\mathbf{h}_{t-1} + \mathbf{b}_g)$$

$$\mathbf{c}_t = \mathbf{f}_t \odot \mathbf{c}_{t-1} \boxed{+} \mathbf{i}_t \odot \mathbf{g}_t$$

$$\mathbf{h}_t = \mathbf{o}_t \odot Tanh(\mathbf{c}_t)$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \odot \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 y_1 \\ x_2 y_2 \end{bmatrix}$$

6. Main Improvement: Better Units

- More complex hidden unit computation in recurrence!
- Gated Recurrent Units (GRU)
  introduced by Cho et al. 2014 (see reading list)
- Main ideas:
  - keep around memories to capture long distance dependencies
  - allow error messages to flow at different strengths depending on the inputs

GRUs

- Update gate $z_t = \sigma\left(W^{(z)}x_t + U^{(z)}h_{t-1}\right)$
- Reset gate $r_t = \sigma\left(W^{(r)}x_t + U^{(r)}h_{t-1}\right)$
- New memory content: $\tilde{h}_t = \tanh\left(Wx_t + r_t \circ Uh_{t-1}\right)$
  If reset gate unit is ~0, then this ignores previous memory and only stores the new word information
- Final memory at time step combines current and previous time steps: $h_t = z_t \circ h_{t-1} + (1 - z_t) \circ \tilde{h}_t$

GRU intuition

- If reset is close to 0, ignore previous hidden state → Allows model to drop information that is irrelevant in the future

$$z_t = \sigma\left(W^{(z)}x_t + U^{(z)}h_{t-1}\right)$$
$$r_t = \sigma\left(W^{(r)}x_t + U^{(r)}h_{t-1}\right)$$
$$\tilde{h}_t = \tanh\left(Wx_t + r_t \circ Uh_{t-1}\right)$$
$$h_t = z_t \circ h_{t-1} + (1 - z_t) \circ \tilde{h}_t$$

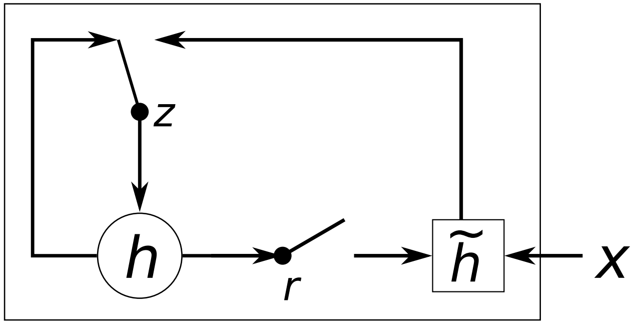
- Update gate z controls how much of past state should matter now.
  - If z close to 1, then we can copy information in that unit through many time steps! **Less vanishing gradient**!
- Units with short-term dependencies often have reset gates very active

GRU intuition

- Units with long term dependencies have active update gates z

$$z_t = \sigma\left(W^{(z)}x_t + U^{(z)}h_{t-1}\right)$$
$$r_t = \sigma\left(W^{(r)}x_t + U^{(r)}h_{t-1}\right)$$
$$\tilde{h}_t = \tanh\left(Wx_t + r_t \circ Uh_{t-1}\right)$$
$$h_t = z_t \circ h_{t-1} + (1 - z_t) \circ \tilde{h}_t$$

- Illustration:



- Derivative of $\frac{\partial}{\partial x_1} x_1 x_2$? → rest is same chain rule, but implement with **modularization** or automatic differentiation