Multi-Field Inflation Overcomes the Swampland Distance Conjecture

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The swampland criteria

On The Cosmological Implications of the String Swampland

Agrawal, Obied, Steinhardt & Vafa (2018)

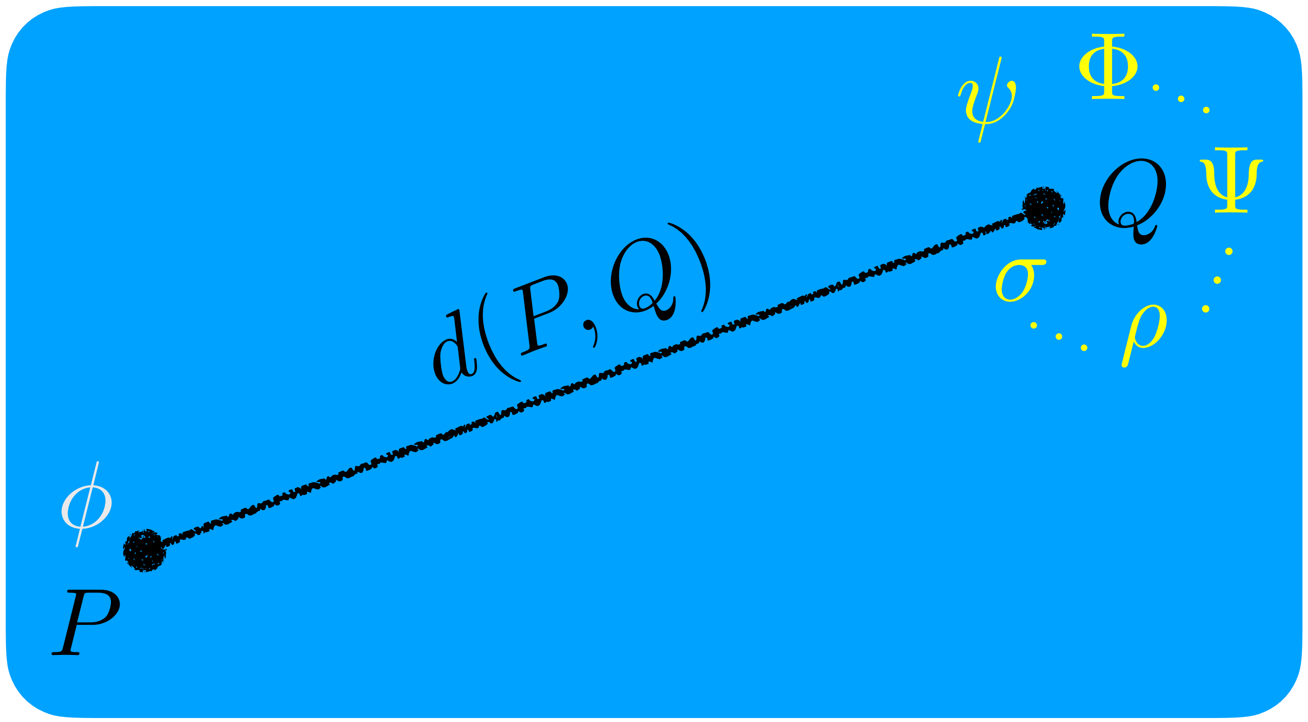
☑ **Criterion 1**: *The range traversed by scalar fields in field space is bounded by* $\Delta \sim \mathcal{O}(1) M_{\rm Pl}$

$$\Delta\phi < \mathcal{O}(1) M_{\rm Pl}$$

Ooguri & Vafa (2007)
Grimm, Palti & Valenzuela (2018)
Heidenreich, Reece & Rudelius (2018)
Blumenhagen (2018)

The distance conjecture

From string compactifications it is possible to find a tower of massless states at finite distances



Vafa (2005)

$$M(Q) = M(P)e^{-\nu d(Q,P)}$$

$$\Updownarrow \; ?$$

Weak **G**ravity **C**onjecture

Arkani-Hamed, Motl, Nicolis & Vafa (2006)
Klaewer & Palti (2016)

The requirement $d(Q,P) < M_{\mathrm{Pl}}$ applies when the distance is a geodesic

$$\Delta\phi_{\mathrm{G}} < \mathcal{O}(1) M_{\mathrm{Pl}}$$

The Lyth bound

$$\frac{\Delta\phi_r}{M_{\rm Pl}} = \Delta N \sqrt{\frac{r}{8}}$$

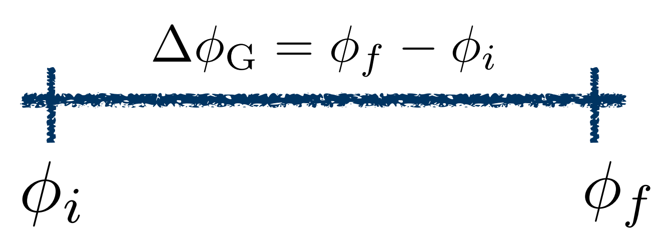
Lyth (1997)

$$\Delta N = 50 - 60 \qquad\qquad r \sim 0.01$$

near future CMB experiments

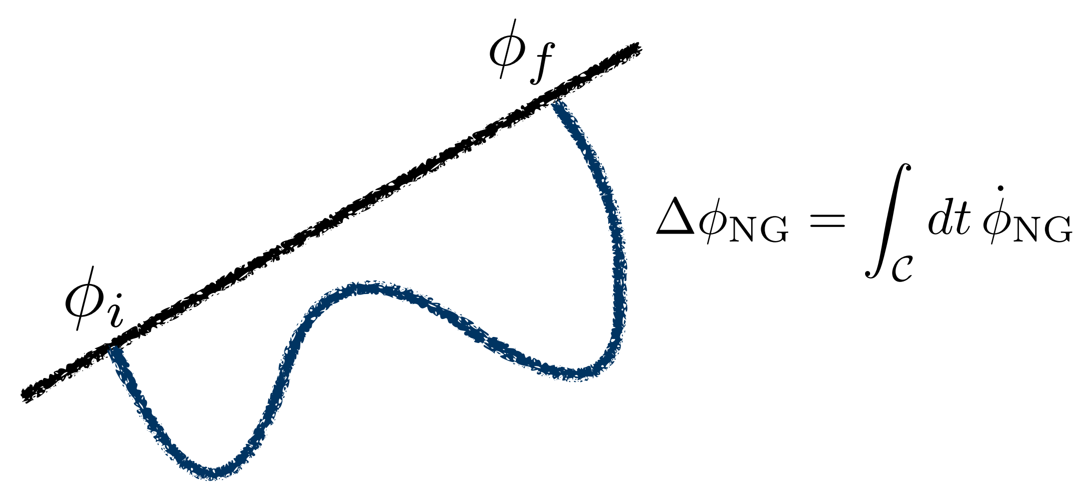
$$\boxed{\Delta\phi_r \gtrsim \mathcal{O}(1) M_{\rm Pl}}$$

Single-field inflation: $\Delta\phi_r = \Delta\phi_{\rm G}$



Multi-field Inflation: $\Delta\phi_r = \sqrt{\beta}\,\Delta\phi_{\rm NG}$

where does it come from?



The problem and the opportunity

If primordial gravitational waves are detected in the near future, then the inflaton necessarily had super-Planckian displacements.

$$\Delta\phi_r \gtrsim \mathcal{O}(1) M_{\mathrm{Pl}} \qquad\qquad \Delta\phi_{\mathrm{G}} < \mathcal{O}(1) M_{\mathrm{Pl}}$$

The Lyth bound and the SDC only refer to the same field displacement in the single-field case

$$\Delta\phi_r = \Delta\phi_{\mathrm{G}}$$

However, in multi-field inflation the trajectories in field space are non-geodesics, how can we include this fact in the SDC?

$$\Delta\phi_{\mathrm{G}} = f\left(\Delta\phi_{\mathrm{NG}}\right)$$

Multi-Scalar Field Theories

Multi-scalar field theories

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \gamma_{ab}(\phi) \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right] + \Delta S_\Lambda$$

$\mathcal{O}(1)$ Wilson coefficients

$$\Delta S_\Lambda \supset -\frac{1}{4} \int d^4x \sqrt{-g} g^{\mu\nu} \frac{f_{abcd}}{\Lambda^2} \Delta\phi^c \Delta\phi^d \partial_\mu \phi^a \partial_\nu \phi^b$$

EFT cut-off

Effective field-space metric

$$\gamma_{ab}^{\Lambda}(\phi) \equiv \gamma_{ab} + \frac{f_{abcd}}{2\Lambda^2} \Delta\phi^c \Delta\phi^d + \ldots$$

Riemann Normal Coordinates

$$\gamma_{ab}^{\Lambda}(\phi) = \delta_{ab} - \frac{1}{3} \mathbb{R}^{\Lambda}_{acbd}(\phi_\star) \phi^c \phi^d + \ldots$$

Multi-scalar field theories

The Riemann tensor

$$\mathbb{R}^{\Lambda}_{abcd} = \mathbb{R}_{abcd} + \frac{1}{\Lambda^2} g_{abcd}(f) + \ldots$$

Riemann-symmetrized linear combination of f *

Characteristic mass scale curvature $R_0 \longrightarrow \mathbb{R} \sim R_0^{-2}$

- [ ] $R_0 > \Lambda$: The theory is indistinguishable from a theory with flat geometry $\gamma_{ab} = \delta_{ab}$
- [x] $R_0 < \Lambda$: It is possible study genuine non-trivial effects from γ_{ab}

For our purposes

$$\Lambda = M_{\rm Pl}$$

*$g_{abcd}(f) = \frac{1}{2}(f_{adbc} - f_{dbac} - f_{acbd} + f_{cbad})$

Two-field inflation with constant turning rates

Background

$$D_t\dot{\phi}_0^a + 3H\dot{\phi}_0^a + \gamma^{ab}V_b = 0 \qquad\qquad \gamma_{ab} = \gamma_{ab}(\mathcal{X},\mathcal{Y})$$

It is always possible to adapt the field-coordinate system around an inflationary trajectory such that

$$\dot{\mathcal{Y}} = 0$$

This particular choice, allows us immediately to write the tangent and normal vectors

$$D_t T^a \equiv -\Omega N^a$$

With the previous assumptions, it is straightforward to find

$$\dot{\mathcal{X}} = -\frac{\gamma_{\mathcal{X}\mathcal{X}}}{\sqrt{\det(\gamma_{ab})}}\frac{\Omega}{\Gamma^{\mathcal{Y}}{}_{\mathcal{X}\mathcal{X}}}$$

Then, the non-geodesic evolution and proper distances are given by

$$\dot{\phi}_{\rm NG} = \sqrt{\gamma_{\mathcal{X}\mathcal{X}}}\,\dot{\mathcal{X}}, \qquad \Delta\phi_{\rm NG} = \int_{\mathcal{C}} dt\,\sqrt{\gamma_{\mathcal{X}\mathcal{X}}}\,\dot{\mathcal{X}}$$

Two-field inflation with constant turning rates

Perturbations

$$ds^2 = -N^2 dt^2 + a^2(t) e^{2\mathcal{R}(\mathbf{x},t)} \delta_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

isocurvature perturbation

$$\delta\phi^a(\mathbf{x},t) = T^a \underbrace{\cancelto{0}{\delta\phi_{||}}}_{\text{comoving gauge}}(\mathbf{x},t) + N^a \sigma(\mathbf{x},t)$$

The Second order action

$$S^{(2)} = \int d^4x\, a^3 \left[\epsilon \left(\dot{\mathcal{R}} - \lambda \frac{H}{\sqrt{2\epsilon}} \sigma \right) - \epsilon \frac{(\nabla\mathcal{R})^2}{a^2} + \frac{1}{2} \left(\dot{\sigma}^2 - \frac{(\nabla\sigma)^2}{a^2} - \frac{1}{2}\mu^2\sigma^2 \right) \right]$$

We have defined

Coupling $\longrightarrow$ $\lambda \equiv -\dfrac{2\Omega}{H}$

Entropy mass of σ $\longrightarrow$ $\mu^2 \equiv N^a N^b (V_a - \Gamma^c_{ab} V_c) + \epsilon H^2 \mathbb{R} + 3\Omega^2$

Two-field inflation with constant turning rates

Perturbations: canonical quantization $\mathcal{R}_c \equiv \sqrt{2\epsilon}\,\mathcal{R} \longrightarrow$ canonical field

$$\mathcal{R}_c(\boldsymbol{x},t) = \int \frac{d^3k}{(2\pi)^3} \hat{\mathcal{R}}_c(\boldsymbol{k},t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \qquad \sigma(\boldsymbol{x},t) = \int \frac{d^3k}{(2\pi)^3} \hat{\sigma}(\boldsymbol{k},t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\hat{\mathcal{R}}_c(\boldsymbol{k},t) = \frac{1}{a}\sum_{\alpha=1}^{2}\left[\hat{a}_\alpha(\boldsymbol{k}) u_\alpha(k,t) + \hat{a}^\dagger_\alpha(-\boldsymbol{k}) u^*_\alpha(k,t)\right]$$

$$\hat{\sigma}(\boldsymbol{k},t) = \frac{1}{a}\sum_{\alpha=1}^{2}\left[\hat{a}_\alpha(\boldsymbol{k}) v_\alpha(k,t) + \hat{a}^\dagger_\alpha(-\boldsymbol{k}) v^*_\alpha(k,t)\right]$$

The power spectrum in absence of coupling: $\lambda = 0$

$$\mathcal{P}_{\mathcal{R}_c} = \frac{H^2}{4\pi^2}$$

In terms of the original field

$$\mathcal{P}_{\mathcal{R}} = \frac{H^2}{8\pi^2\epsilon}$$

Two-field inflation with constant turning rates

Perturbations: the equations of motion for $\lambda \neq 0$

$$\boxed{\begin{aligned} u''_\alpha - \frac{2}{\tau^2}\,u_\alpha + k^2\,u_\alpha + \frac{\lambda}{\tau}\,v'_\alpha - \frac{2\lambda}{\tau^2}\,v_\alpha = 0 \\ v''_\alpha - \frac{2}{\tau^2}\,v_\alpha + k^2\,v_\alpha + \frac{\tilde{\mu}}{\tau^2}\,v_\alpha - \frac{\lambda}{\tau}\left(u'_\alpha + \frac{1}{\tau}\,u_\alpha + \frac{\lambda}{\tau}\,v_\alpha\right) = 0 \end{aligned}}$$

dimensionless variable $\tilde{\mu} \equiv \frac{\mu}{H}$

The power spectrum:

$$\mathcal{P}_\mathcal{R} \equiv \frac{k^3}{2\pi^2}\left(|\mathcal{R}_1|^2 + |\mathcal{R}_2|^2\right)$$

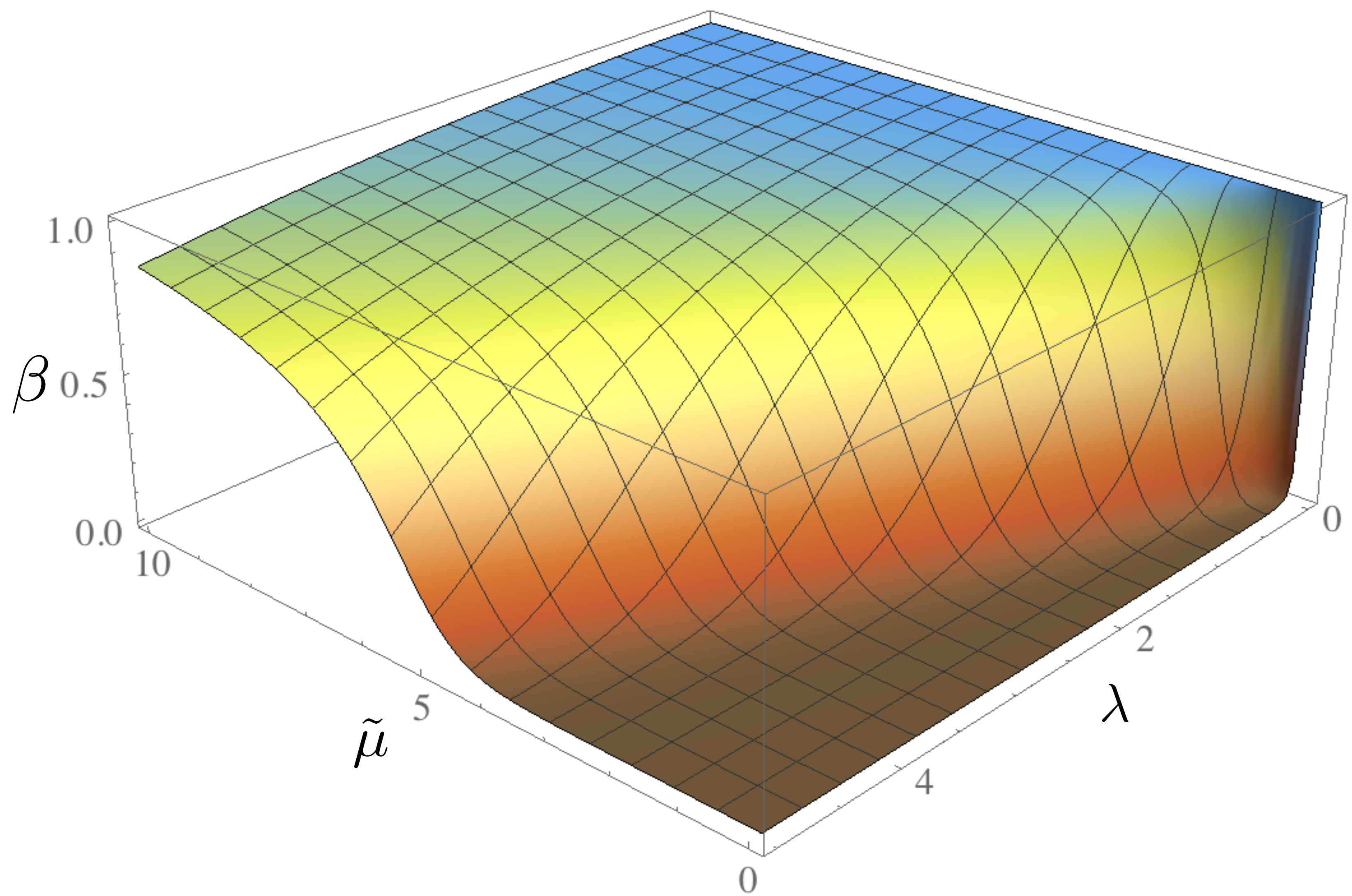
Necessarily has the form

$$\mathcal{P}_\mathcal{R} = \frac{H^2}{8\pi^2\epsilon\,\beta(\lambda, \tilde{\mu})}$$

dimensionless function

Two-field inflation with constant turning rates

$$\beta\left(\lambda, \tilde{\mu}\right) = \frac{H^2}{8\pi^2\,\epsilon\,\mathcal{P}_{\mathcal{R}}}$$



$0 < \beta \leq 1$

Single-field (geodesic) case $\lambda = 0 \longrightarrow \beta = 1$

The modified Lyth bound

Since the power spectrum is modified, the tensor-to-scalar ratio reads

$$r = \frac{\mathcal{P}_h}{\mathcal{P}_\mathcal{R}} = 16\epsilon\, \beta(\lambda, \tilde{\mu})$$

The modified Lyth bound

$$\frac{\Delta\phi_{\mathrm{NG}}}{M_{\mathrm{Pl}}} = \Delta N \sqrt{\frac{r}{8\beta(\lambda, \tilde{\mu})}}$$

More restrictive than the original one !

Observable primordial gravitational waves, necessarily implies:

$$\Delta\phi_{\mathrm{NG}} > M_{\mathrm{Pl}}$$

Example: Hyperbolic Spaces

Two-field inflation in hyperbolic spaces

Let us consider a particular parametrization of the hyperbolic space

$$\gamma_{ab} = \begin{pmatrix} e^{2\mathcal{Y}/R_0} & 0 \\ 0 & 1 \end{pmatrix} \longrightarrow \mathbb{R} = -\frac{2}{R_0^2}$$

The equations of motion:

$$3M_{\rm Pl}^2 H^2 - \frac{1}{2} e^{2\mathcal{Y}/R_0} \dot{\mathcal{X}}^2 - \frac{1}{2}\dot{\mathcal{Y}}^2 - V = 0,$$

$$\ddot{\mathcal{X}} + 3H\dot{\mathcal{X}} + \frac{2}{R_0}\dot{\mathcal{Y}}\dot{\mathcal{X}} + e^{-2\mathcal{Y}/R_0} V_{\mathcal{X}} = 0,$$

$$\ddot{\mathcal{Y}} + 3H\dot{\mathcal{Y}} - \frac{1}{R_0} e^{2\mathcal{Y}/R_0} \dot{\mathcal{X}}^2 + V_{\mathcal{Y}} = 0.$$

Boundary conditions:

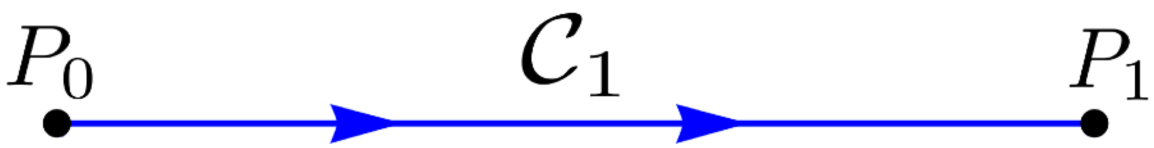
$$\mathcal{Y}(0) = \mathcal{Y}(T) = \mathcal{Y}_0, \quad \mathcal{X}(0) = \mathcal{X}_0, \quad \mathcal{X}(T) = \mathcal{X}_0 + \Delta\mathcal{X}$$

Two-field inflation in hyperbolic spaces

Solutions of the first order system

$$\mathcal{Y}(t) = \mathcal{Y}_0$$

$$\mathcal{X}(t) = R_0 e^{-\mathcal{Y}_0/R_0}\Omega t + \mathcal{X}_0 \qquad \Omega = \frac{\Delta X}{R_0 T} e^{\mathcal{Y}_0/R_0} \qquad \Delta N = HT$$



$$\Delta\phi_{\rm NG} = \int_{\mathcal{C}_1} dt\,\sqrt{\gamma_{ab}\dot{\phi}_0^a\dot{\phi}_0^b}$$

$$\boxed{\Delta\phi_{\rm NG} = e^{\mathcal{Y}_0/R_0}\Delta\mathcal{X}}$$

Geodesic distances in two-field models

The geodesic equations for this realization of the hyperbolic space are

$$\ddot{\mathcal{X}} + \frac{2}{R_0}\dot{\mathcal{X}}\dot{\mathcal{Y}} = 0$$

$$\ddot{\mathcal{Y}} - \frac{1}{R_0}\,e^{2\mathcal{Y}/R_0}\,\dot{\mathcal{X}}^2 = 0$$

Again, we want to solve it attached to the same boundary conditions

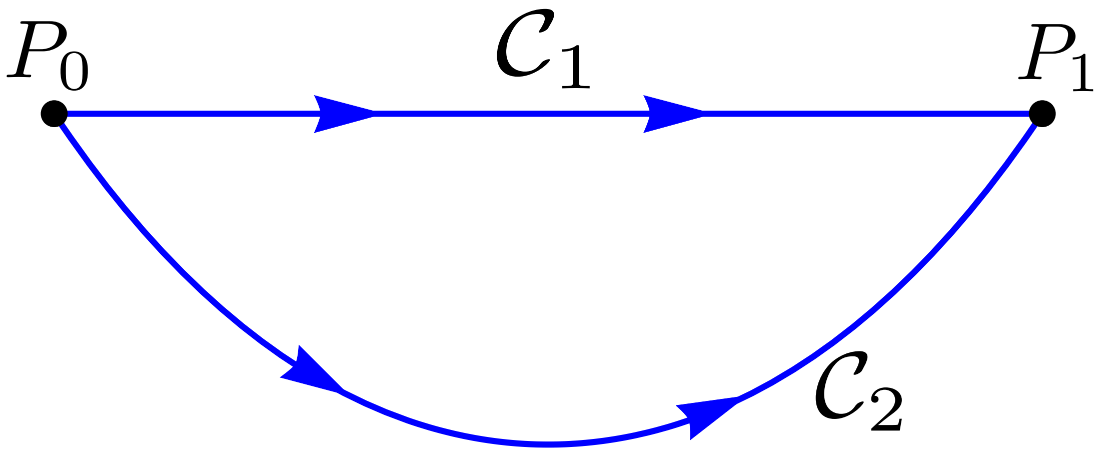
$$\mathcal{Y}(0) = \mathcal{Y}(T) = \mathcal{Y}_0, \quad \mathcal{X}(0) = \mathcal{X}_0, \quad \mathcal{X}(T) = \mathcal{X}_0 + \Delta\mathcal{X}$$

The solutions are

$$\mathcal{X}(t) = \mathcal{X}_0 + \frac{\Delta\mathcal{X}}{2} + \frac{1}{2}\sqrt{(\Delta\mathcal{X})^2 + 4R_0^2 e^{-2\mathcal{Y}_0/R_0}}\,\tanh\left[\frac{2}{T}\,\mathrm{arcsinh}\left(\frac{e^{\mathcal{Y}_0/R_0}}{2R_0}\right)\left(t - \frac{T}{2}\right)\right]$$

$$\mathcal{Y}(t) = R_0 \ln\left[\frac{2R_0\cosh\left(\frac{2}{T}\,\mathrm{arcsinh}\left(\frac{e^{\mathcal{Y}_0/R_0}}{2R_0}\right)\left(t - \frac{T}{2}\right)\right)}{\sqrt{(\Delta\mathcal{X})^2 + 4R_0^2 e^{-2\mathcal{Y}_0/R_0}}}\right]$$

Geodesic distances in two-field models



$$\Delta\phi_{\rm G} = \int_{\mathcal{C}_2} dt\,\sqrt{\gamma_{ab}\dot\phi_0^a\dot\phi_0^b}$$

$$\boxed{\Delta\phi_{\rm G} = 2R_0\,\mathrm{arcsinh}\left(e^{\mathcal{Y}_0/R_0}\frac{\Delta\mathcal{X}}{2R_0}\right)}$$

Mixing geodesic and non-geodesics

$$\boxed{\Delta\phi_{\mathrm{G}} = 2R_0 \,\mathrm{arcsinh}\left(\frac{\Delta\phi_{\mathrm{NG}}}{2R_0}\right)}$$

To illustrate

$$R_0 = 0.1 M_{\mathrm{Pl}}$$
$$\Delta\phi_{\mathrm{NG}} = 15 M_{\mathrm{Pl}} \longrightarrow \Delta\phi_{\mathrm{G}} \approx M_{\mathrm{Pl}}$$

At background level we can overcome the SDC

$$\Delta\phi_{\mathrm{G}} < M_{\mathrm{Pl}} < \Delta\phi_{\mathrm{NG}}$$

Does the SDC affects the theory of perturbations?

SDC, the Lyth bound and non-geodesic motion

SDC:

$$\Delta\phi_{\mathrm{G}} < \vartheta\, M_{\mathrm{Pl}}, \qquad \vartheta = \mathcal{O}(1)$$

The modified Lyth bound:

$$\Delta\phi_{\mathrm{NG}} = M_{\mathrm{Pl}}\Delta N\sqrt{\frac{r}{8\beta}}$$

G & N-G distances relation:

$$\Delta\phi_{\mathrm{G}} = 2\sqrt{\frac{2}{|\mathbb{R}|}}\,\mathrm{arcsinh}\left(\frac{1}{2}\sqrt{\frac{|\mathbb{R}|}{2}}\Delta\phi_{\mathrm{NG}}\right)$$

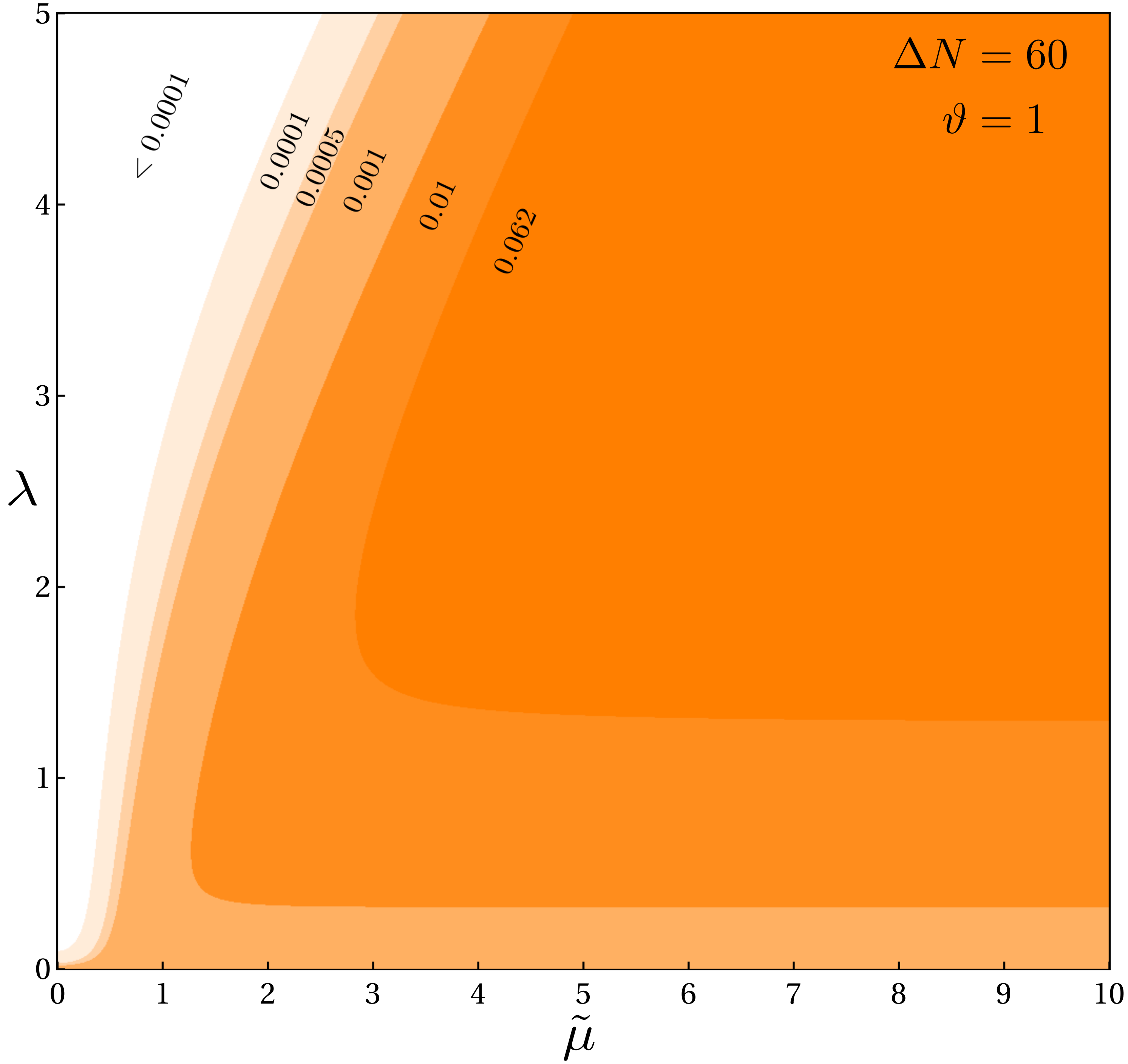
From the equations of motion

$$|\mathbb{R}| = \frac{4\lambda^2\beta}{M_{\mathrm{Pl}}^2\, r}$$

Main Results

The SDC and the perturbations

$$r < \frac{\vartheta^2 \lambda^2}{2} \operatorname{arcsinh}^{-2}\left(\frac{\Delta N |\lambda|}{4}\right) \beta(\lambda, \tilde{\mu})$$

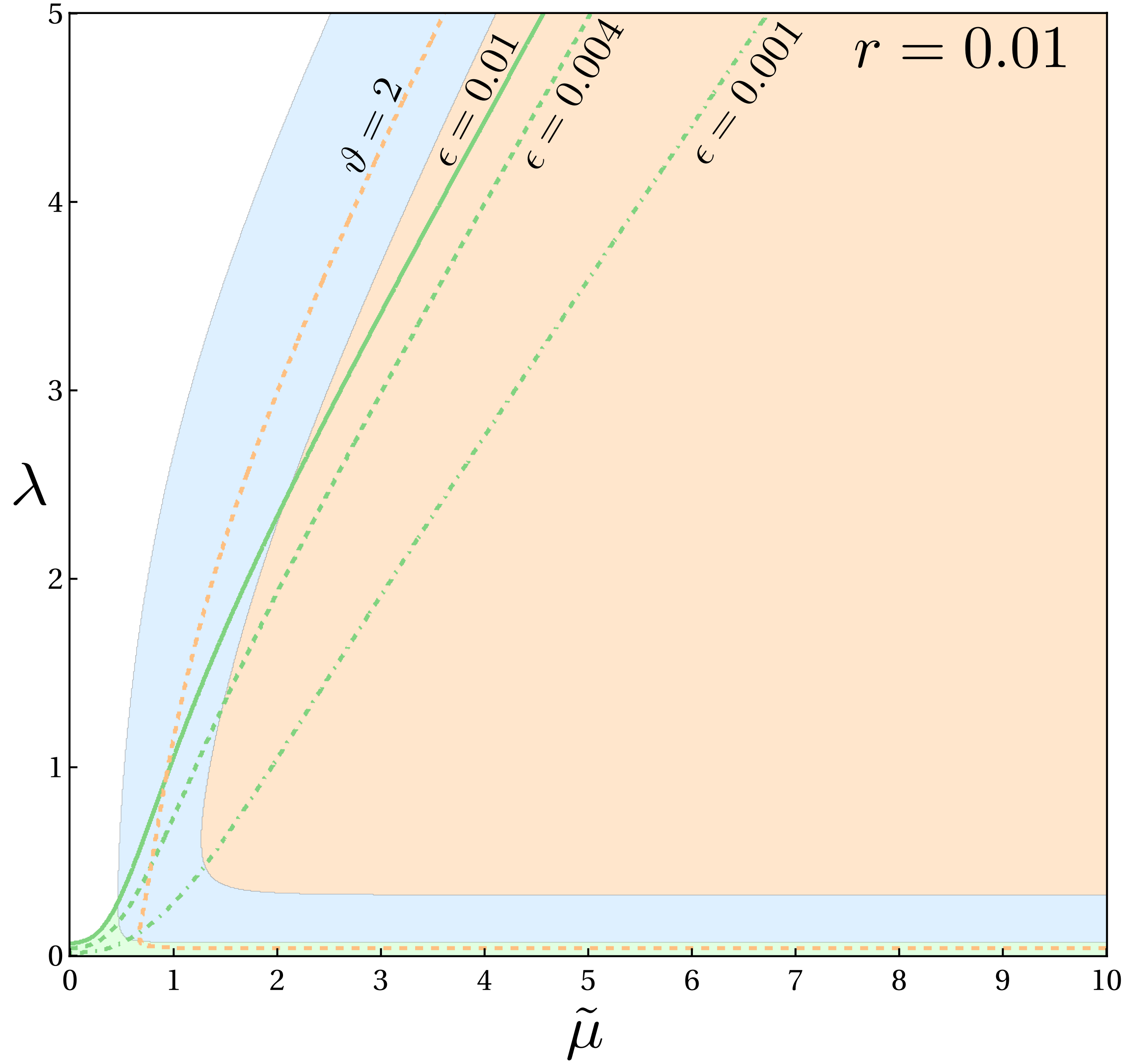


Bounds involving $\beta(\lambda, \tilde{\mu})$

Slow-roll: $$\epsilon \ll 1 \Rightarrow r \ll 16\beta$$

Sub-Planckian: $$R_0 < M_{\rm Pl} \Rightarrow r \ll 2\beta\lambda^2$$

SDC: $$r < \frac{\vartheta^2\lambda^2}{2}\text{arcsinh}^{-2}\left(\frac{\Delta N|\lambda|}{4}\right)\beta(\lambda, \tilde{\mu})$$



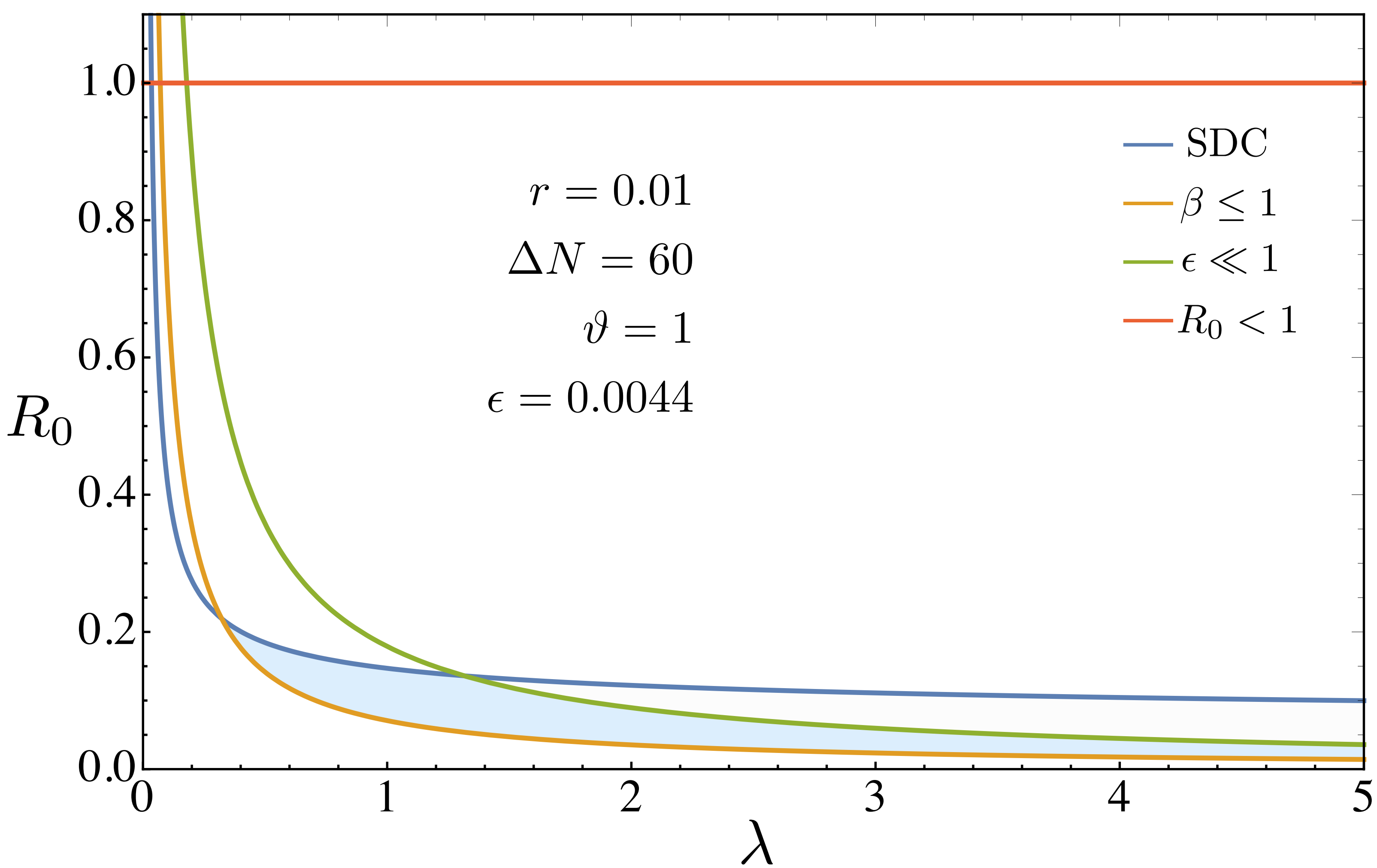
Bounds involving R_0

Slow-roll:
$$R_0 < \frac{2\sqrt{2}}{\lambda}$$

Sub-Planckian:
$$R_0 < M_{\text{Pl}}$$

SDC:
$$R_0 < \frac{\vartheta}{2\,\text{arcsinh}\left(\frac{\Delta N \lambda}{4}\right)}$$

Subluminality:
$$\beta \leq 1 \Rightarrow R_0 \geq \frac{r}{2\lambda}$$



Summary

- Geodesic-non-geodesic relation in two-field hyperbolic inflation

- Sizable primordial gravitational waves without the need of super-Planckian geodesic field displacements

- Novel bounds over the multi-field parameter space

Merci !