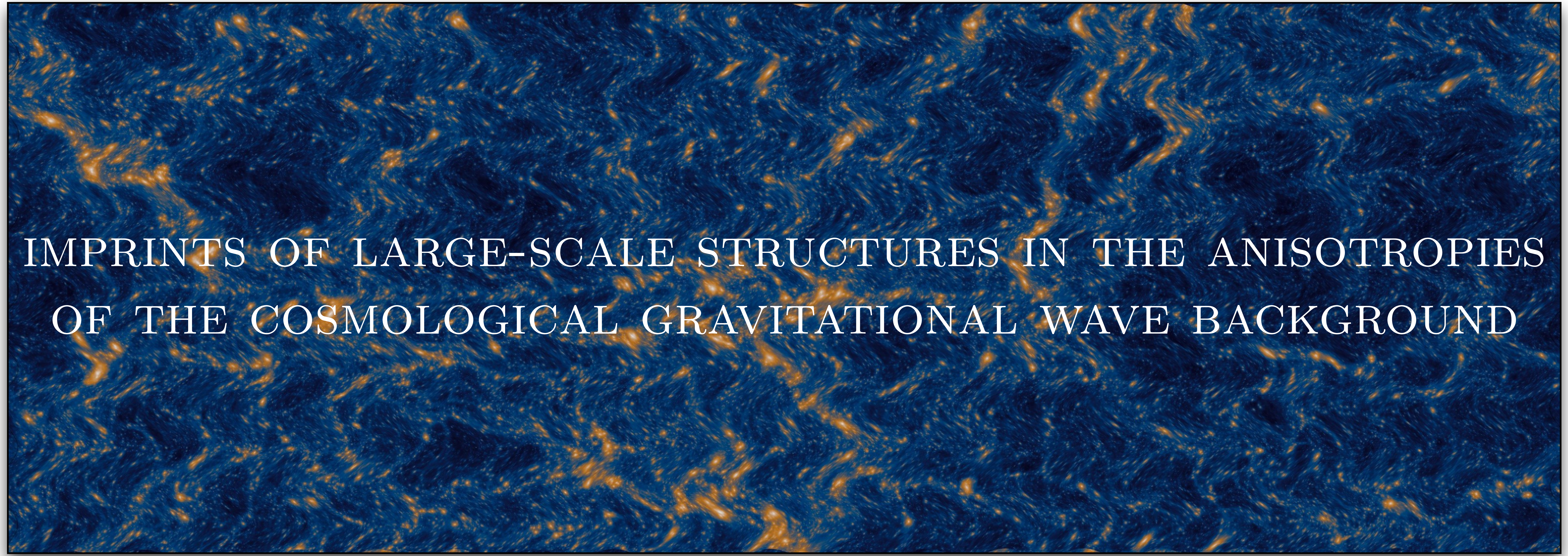

IMPRINTS OF LARGE-SCALE STRUCTURES IN THE ANISOTROPIES OF THE COSMOLOGICAL GRAVITATIONAL WAVE BACKGROUND

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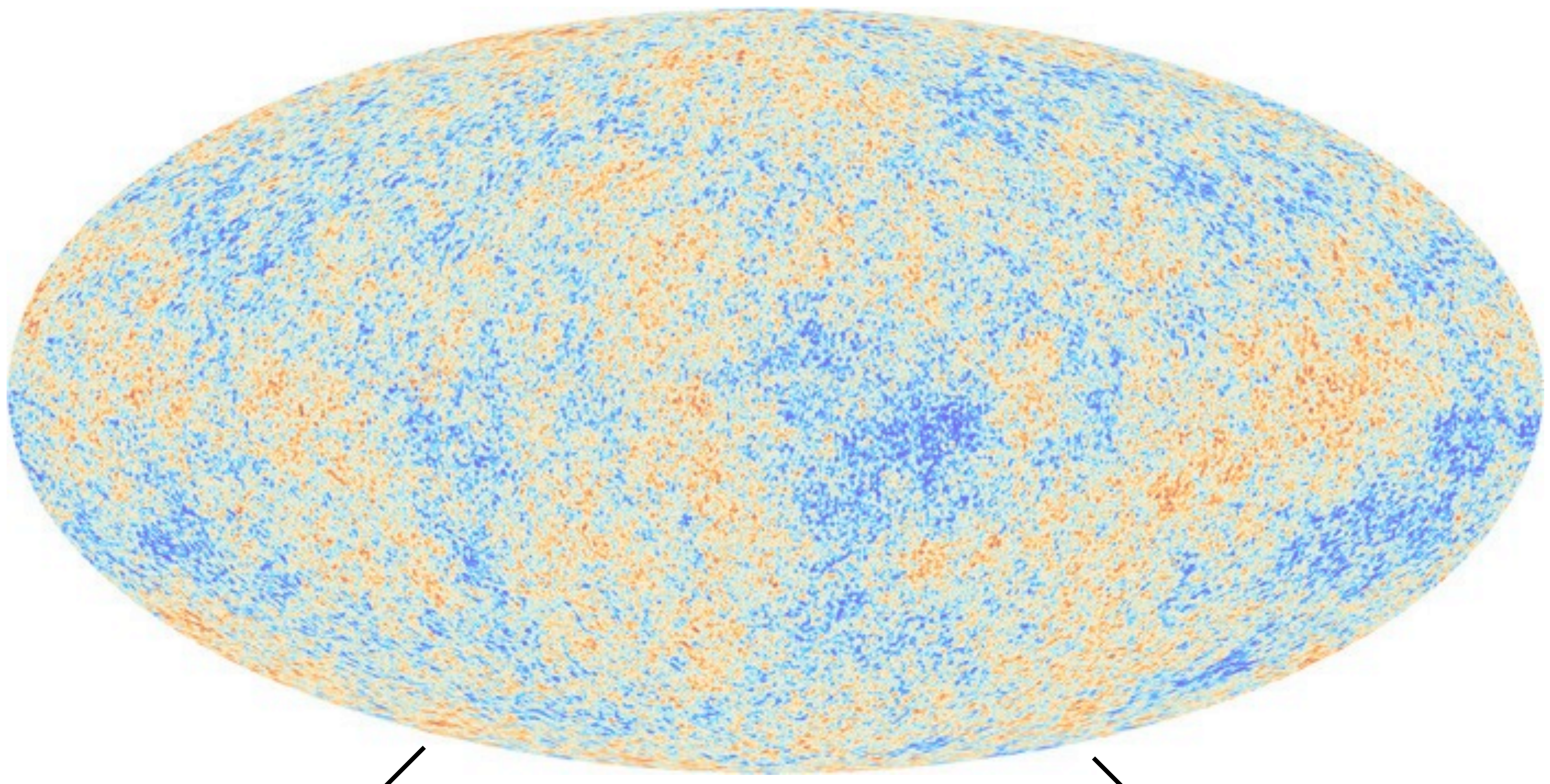
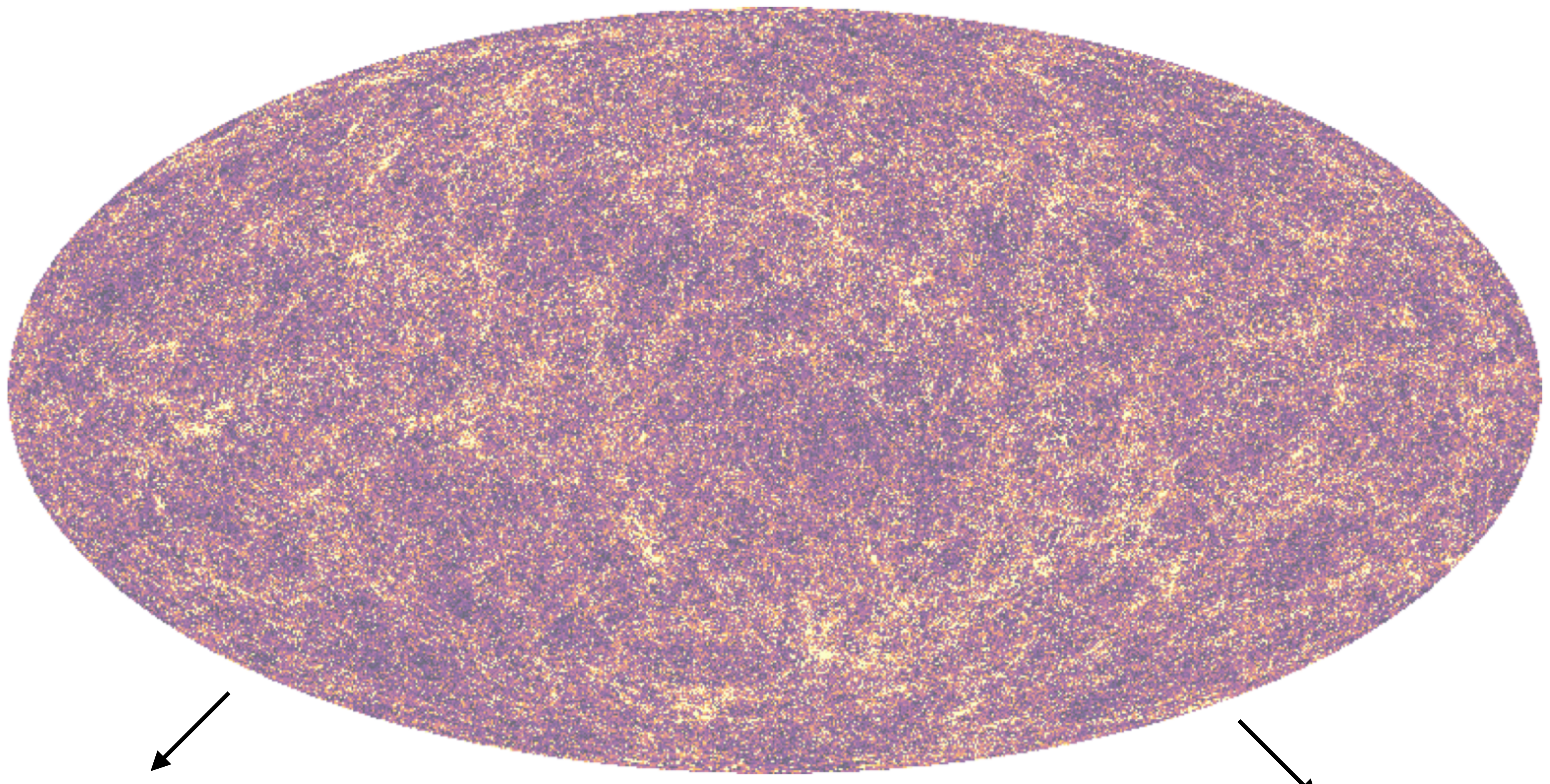
Motivation

“History doesn’t repeat itself, but it often rhymes” – Mark Twain



Background (1965) Anisotropies (1992)



Background (202X ?) Anisotropies (?)

PLANCK (2015)

Jenkins & Sakellariadou (2018)

Intro: Stochastic Backgrounds of Gravitational Waves (SGWBs)

$$h_{ij}^{TT}(t,\mathbf{x}) = \sum_{\lambda=+,\times} \int_{-\infty}^{\infty} df \int_{S^2} d^2\hat{\mathbf{n}}\, e^{-2\pi i f(t-\hat{\mathbf{n}}\cdot\mathbf{x}/c)} \epsilon_{ij}^{\lambda}(\hat{\mathbf{n}}) h_\lambda(f,\hat{\mathbf{n}})$$

A SGWB is composed by the superposition of GWs with all possible propagation directions in $\hat{\mathbf{n}}$ which

- $h_\lambda(f,\hat{\mathbf{n}}) \to$ random variables
- The background is Gaussian, stationary, isotropic and unpolarized

$$\langle h_\lambda(f,\hat{\mathbf{n}}) h^*_{\lambda'}(f',\hat{\mathbf{n}}')\rangle = \delta(f-f')\frac{\delta^{(2)}(\hat{\mathbf{n}}-\hat{\mathbf{n}}')}{4\pi}\delta_{\lambda\lambda'}\frac{1}{2}S_h(f)$$

it is described by its energy spectrum

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f}$$

PTA collaborations have reported evidence in favor of a SGWB based on inter-pulsar correlations following the HD curve. [IPTA (2023) = {NANOGrav, EPTA, InPTA, PPTA}, CPTA(2023), MeerKAT (2024), ...]

$$\text{SGWB} = \text{AGWB} + \text{CGWB}$$

Is it possible to untangle them?

SGWB Anisotropies

Variation in the GW enegy flux $\iff$ anisotropies

- Observer-induced: kinematical and detector-related ones.
- *Intrinsic*: source and propagation. ✔

I will focus on the directional $\hat{\mathbf{n}}$ imprints produced by *intrinsic* effects, which unfold as small **anisotropies** in the SGWB energy spectrum

$$\delta\Omega_{\rm GW}(q,\hat{n})$$

It carries information about the GW nature (cosmological or astrophysical)

How do we compute them ?

Statistical (line of sight) approach [Contaldi (2016), Bartolo *et al.* (2019)]

- Each GW mode (graviton) $\rightarrow$ x^μ and p^μ. Statistically described through a distribution function $f(x^\mu, p^\mu)$
- Astrophysical and Cosmological sources set different conditions on the Boltzmann equation
- Once the full distribution function is known, we compute the GW energy-density $\rho_{\rm GW}$ and spectrum $\Omega_{\rm GW}(\mathbf{x},\mathbf{p})$.

Cosmological Gravitational Wave Background (CGWB)

We are interested in solving the Boltzmann equation

$$\frac{df}{d\lambda} = \mathcal{C}[f(\lambda)] + \mathcal{J}[f(\lambda)]$$

Gravitons are massless and *collisionless* modes

The emission term is an extended function of time for Astrophysical sources while constant for Cosmological ones. Such a constant is absorbed as an initial condition

Gravitons propagate along the null geodesics of the perturbed FLRW spacetime

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 + (1-2\Phi)\,\delta_{ij}dx^i dx^j\right]$$

The homogeneous Boltzmann equation, at linear order in the scalar perturbation $\mathbf{\Phi}$ reads

$$\frac{\partial f}{\partial \eta} + n^i \frac{\partial f}{\partial x^i} + \left[\frac{\partial \Phi}{\partial \eta} - n^i \frac{\partial \Phi}{\partial x^i}\right] q \frac{\partial f}{\partial q} = 0, \qquad \hat{\mathbf{n}} \equiv \hat{\mathbf{p}}, \quad q \equiv a(\eta) p$$

CGWB Anisotropies

By expanding the distribution function as $f(\eta, \mathbf{x}, \mathbf{q}) = \bar{f}(\eta, q) + \delta f(\eta, \mathbf{x}, \mathbf{q})$

Zeroth order → $\boxed{\dfrac{\partial \bar{f}}{\partial \eta} = 0 \implies \bar{f} = \bar{f}(q)}$

The background solution is stationary and isotropic, consistent with the properties of the SGWB

It is convenient to write the perturbed part of the distribution as

$$\delta f = -q \frac{\partial \bar{f}}{\partial q} \Gamma(\eta, \mathbf{x}, \mathbf{q})$$

The linear-order equation for Γ, in Fourier space, reads

$$\frac{\partial \Gamma}{\partial \eta} + ik\mu\Gamma = \frac{\partial \Phi}{\partial \eta} - ik\mu\Phi, \qquad \mu \equiv \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}$$

solved by:

$$\Gamma(\eta, \mathbf{k}, \mathbf{q}) = e^{ik\mu(\eta_{\rm in} - \eta)} \Big[\overbrace{\underbrace{\Gamma(\eta_{\rm in}, \mathbf{k}, q)}_{\text{GW source}}}^{\text{Initial condition}} + \overbrace{\Phi(\eta_{\rm in}, \mathbf{k})}^{\text{Sachs-Wolfe (SW) effect}} \Big] + \overbrace{2 \int_{\eta_{\rm in}}^{\eta} d\eta' e^{ik\mu(\eta' - \eta)} \frac{\partial \Phi(\eta', \mathbf{k})}{\partial \eta'}}^{\text{Integrated SW (ISW) effect}},$$

GW source

Propagation effects

CGWB Energy Spectrum

With the full distribution at hand, we compute the GW energy density as

$$\rho_{\rm GW} = T^{00}_{\rm GW} = \int \sqrt{-g}\, d^3\mathbf{p}\, \frac{p^0 p^0}{p_0} f(x^\mu, p^\mu) = \frac{1}{a^4} \int d^3\mathbf{q}\, q\, f(\eta, \mathbf{x}, \mathbf{q})$$

which is related to the **full** GW spectrum as

$$\rho_{\rm GW} = \rho_c \int d\ln q \int d^2\hat{\mathbf{n}}\, \Omega_{\rm GW}(\eta, \mathbf{x}, \mathbf{q})$$

By splitting the energy spectrum as $\Omega_{\rm GW}(\eta, \mathbf{x}, \mathbf{q}) = \bar{\Omega}_{\rm GW}(\eta, q) + \delta\Omega_{\rm GW}(\eta, \mathbf{x}, \mathbf{q})$, we end up with

Background (monopole) → $\boxed{\bar{\Omega}_{\rm GW}(\eta_0, q)} = \dfrac{4\pi}{\rho_c} \left(\dfrac{q}{a(\eta_0)} \right)^4 \bar{f}(q)$ Model?

Anisotropies (multipoles) → $\delta_{\rm GW}(\eta_0, \mathbf{x}, \mathbf{q}) = \dfrac{\delta\Omega_{\rm GW}(\eta_0, \mathbf{x}, \mathbf{q})}{\Omega_{\rm GW}(\eta_0, q)} = \left[4 - \dfrac{\partial \ln \bar{\Omega}_{\rm GW}(q)}{\partial \ln q} \right] \boxed{\Gamma(\eta_0, \mathbf{x}, \mathbf{q})}$

GWs Sourced by Primordial Scalar Perturbations

CGWB sourced by scalar-induced gravitational waves (SIGWs) [*see* Domenéch (2021) *for a review*]

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 + \left((1-2\Phi)\,\delta_{ij} + \frac{h_{ij}^{(2)}}{2}\right)dx^i dx^j\right]$$

The Einstein equation for the second-order tensor perturbation in Fourier space reads

$$h''_\lambda(\eta,\mathbf{q}) + 2\mathcal{H}h'_\lambda(\eta,\mathbf{q}) + q^2 h_\lambda(\eta,\mathbf{q}) = \mathcal{S}_\lambda(\eta,\mathbf{q})$$

the source term is given by quadratic combinations of the scalar perturbation

$$\mathcal{S}_\lambda(\eta,\mathbf{q}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\epsilon_{ij}^{\lambda}(\mathbf{q})p^i p^j \left[2\Phi_{\mathbf{q}-\mathbf{p}}\Phi_{\mathbf{p}} + \frac{4}{3(1+\omega)\mathcal{H}^2}\left(\Phi'_{\mathbf{q}-\mathbf{p}} + \mathcal{H}\Phi_{\mathbf{q}-\mathbf{p}}\right)\left(\Phi'_{\mathbf{p}} + \mathcal{H}\Phi_{\mathbf{p}}\right)\right]$$

We consider large primordial curvature perturbations ζ, re-entering the horizon during the radiation-dominated (RD) era, where

$$\Phi(\eta,\mathbf{q}) = \frac{2\sqrt{3}}{q\eta}j_1\left(\frac{q\eta}{\sqrt{3}}\right)\zeta(\mathbf{q})$$

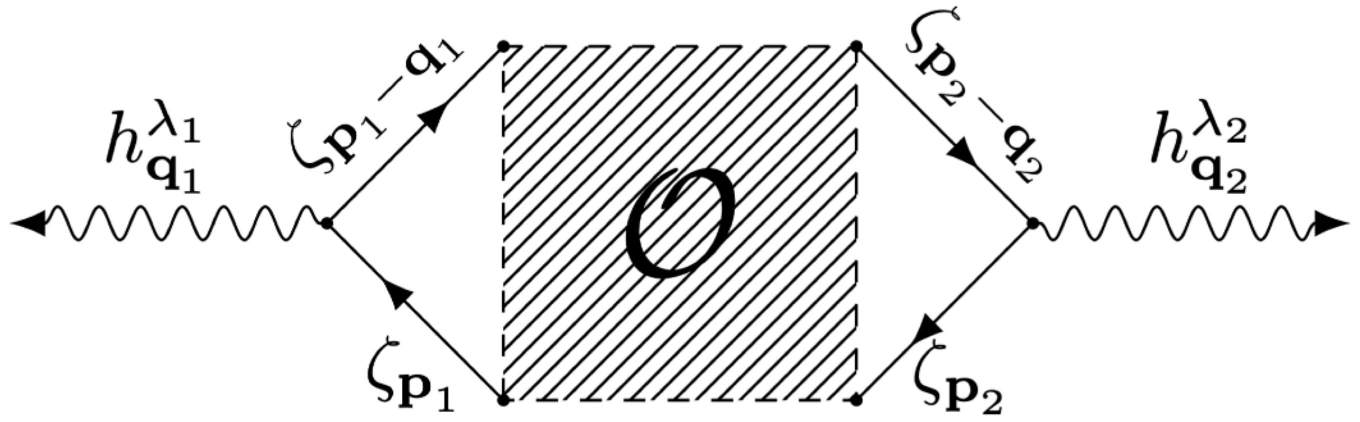
SIGWs - Background Energy Spectrum

By using the Green function method, it is possible to find the solution for the tensor amplitude

$$h_\lambda(\eta, \mathbf{q}) = 4 \int \frac{d^3\mathbf{p}}{(2\pi)^3} Q_\lambda(\mathbf{q}, \mathbf{p}) \mathcal{I}(|\mathbf{q} - \mathbf{p}|, q, \eta) \zeta(\mathbf{p}) \zeta(\mathbf{q} - \mathbf{p})$$

From which we can compute the *background* energy spectrum as

$$\bar{\Omega}_{\mathrm{GW}}(q) = \frac{q^5}{48\pi^2 a^2 H^2} \sum_{+,\times} \overline{\langle h_\lambda(\eta, \mathbf{q}) h_{\lambda'}(\eta, \mathbf{q}') \rangle}$$

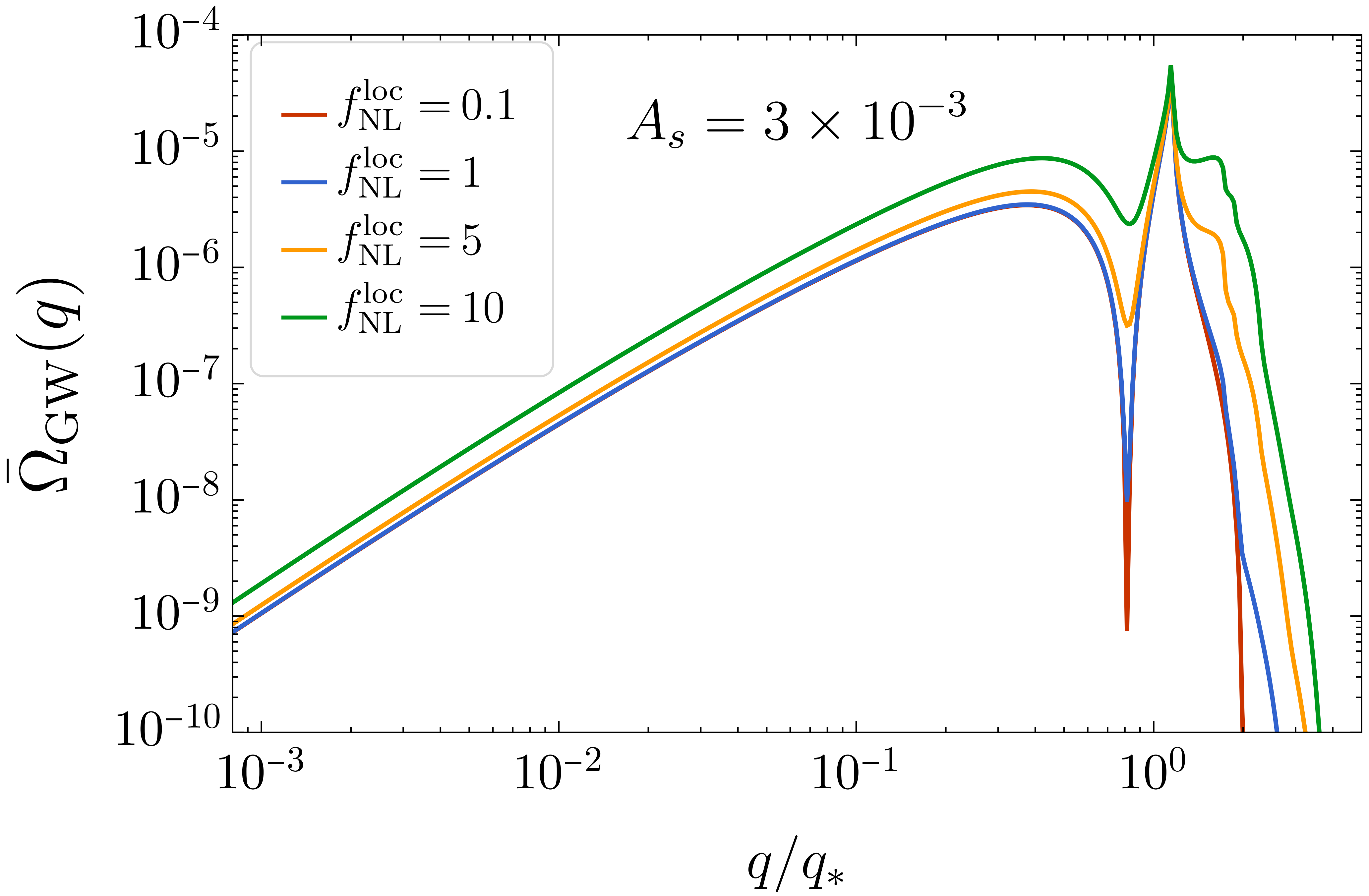


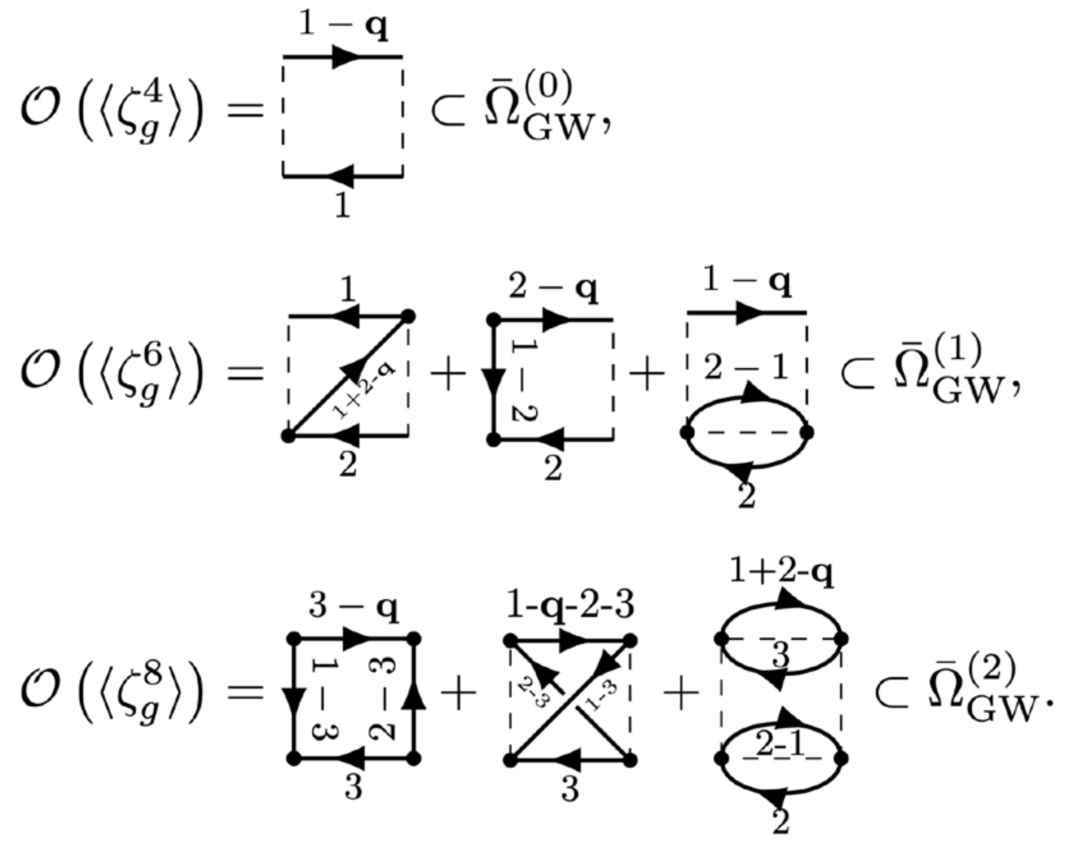
For a non-Gaussian perturbation

$$\zeta(\mathbf{q}) = \zeta_g(\mathbf{q}) + \frac{3}{5} f^{\mathrm{loc}}_{\mathrm{NL}} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \zeta_g(\mathbf{q}) \zeta_g(\mathbf{q} - \mathbf{p})$$

with a monochromatic distribution

$$\Delta^2_{\zeta_g}(q) = A_s q_* \, \delta(q - q_*)$$





[Cai & Sasaki (2018)]
[Unal (2018)]
[Bartolo *et al.* (2019)]
[Adshead, Lozanov & Weiner (2021)]

SIGWs - Anisotropies

Initial condition anisotropy → Comes from the coupling of short- and long-wavelength fluctuations $\zeta_g = \zeta_S(\mathbf{q}) + \zeta_L(\mathbf{k})$ through $f_{\rm NL}^{\rm loc}$

$$\delta^0_{\rm GW}(\eta_{\rm in}, \mathbf{k}, q) = \frac{3}{5} f_{\rm NL}^{\rm loc} \frac{\Omega_{\rm NG}(\eta_{\rm in}, q)}{\bar{\Omega}_{\rm GW}(\eta_{\rm in}, q)} \zeta_L(\mathbf{k})$$

[Bartolo *et al.* (2019)]
[Wang, Zhao, Li & Zhu (2018)]

Propagation effect anisotropies → Produced when the long-wavelength fluctuation ζ_L re-enters the horizon in the matter-dominated (MD) universe $\Phi(\eta, \mathbf{k}) = \frac{3}{5} T(k) g(\eta) \zeta_L(\mathbf{k})$

SW effect: $\Phi(\eta_{\rm in}, \mathbf{k}) = \frac{3}{5} \zeta_L(\mathbf{k})$

ISW effect: the growth rate must be taken into account up to the present epoch. The relevant quantity that enters when computing *angular correlations* is

$$\frac{I_\ell^{\rm ISW}(\eta_{\rm in}, k)}{T(k)(4 - n_{\rm GW})} = \frac{6}{5} \int_{\eta_{\rm in}}^{\eta_0} d\eta \, g'(\eta) j_\ell(k(\eta_0 - \eta))$$

This time integral receives contributions only at *late-times* afther the MD-ΛM equivalence η_Λ , when the growth of structure slows down and the gravitational potential decay

If Λ dominates the expansion, $g'(\eta) \simeq -\frac{4}{5} (\eta/\eta_0)^3$ and the exact solution of the previous integral is found to be

$$\frac{I_\ell^{\rm ISW}}{4 - n_{\rm GW}} = \frac{12}{25} \frac{\sqrt{\pi}}{2^\ell} \frac{(\eta_0 - \eta_\Lambda)^{\ell+1} k^\ell}{\Gamma\left(\ell + \frac{3}{2}\right)} \sum_{n=0}^{3} \binom{3}{n} \frac{((\eta_0 - \eta_\Lambda)/\eta_0)^n}{\ell + 1 + n} \, {}_1F_2\left[\frac{\ell + 1 + n}{2}; \frac{3}{2} + \ell, \frac{\ell + 3 + n}{2}; -\frac{k^2 (\eta_0 - \eta_\Lambda)}{4}\right]$$

(Auto- and Cross-) Correlations

In spherical harminonics, $\delta_{\rm GW}^{\ell m}$ the angular power-spectrum of CGWB anisotropies $\langle \delta_{\rm GW}^{\ell m} \delta_{\rm GW}^{*\,\ell' m'} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell^{\rm GW}$

$$C_\ell^{\rm GW} = \frac{2\pi \Delta_L^2}{\ell(\ell+1)} J_{\rm in}^2(q) + C_\ell^{\rm ISW}$$

early-time contributions (Initial condition + SW): $\quad J_{\rm in}(q) = \frac{3}{5}\left[f_{\rm NL}^{\rm loc}\Omega_{\rm NG}(q)/\bar{\Omega}_{\rm GW}(q) + (4 - n_{\rm GW})\right]$

late-time contribution (ISW): $\quad C_\ell^{\rm ISW} = \int_0^\infty d\ln k\, \Delta_L^2(k) \left[I_\ell^{\rm ISW}(\eta_\Lambda, k)\right]^2$

What is the status of correlations involving SGWB anisotropies?

- GW $\times$ GW : AGWB [Cusin, Pitrou & Uzan (2018)]. CGWB [Bartolo *et al.* (2019), Wang *et al.* (2023)].
- GW $\times$ CMB : AGWB [Perna *et al.* (2023)], CGWB [Adshead *et al.* (2020), Braglia & Kuroyanagi (2021), Zhao *et al.* (2024)], Both [Ricciardone *et al.* (2021)]
- GW $\times$ LSS : AGWB [Scelfo *et al.* (2018), Bosi, Bellomo & Raccanelli (2023), Semenzato *et al.* (2024), Pedrotti *et al.* (2024), Cusin et *al.* (2025)]. CGWB [RB & Riquelme (2025)]

Galaxy density contrast δ_g as LSS tracer $\rightarrow \quad \delta_{\rm g}^{\ell m} = 4\pi(-i)^\ell \int \frac{d^3\mathbf{k}}{(2\pi)^3} Y_{\ell m}^*(\hat{\mathbf{k}}) I_\ell^{\rm gal}(z,k)\zeta_L(k)$

$$\frac{I_\ell^{\rm gal}(z,k)}{k^2 T(k)} = \frac{2}{5}\int dz\, \frac{b(z,k)D(z)}{\Omega_{\rm m}H_0}\frac{dN(z)}{dz} j_\ell(k\chi(z)) \qquad b(z,k) = b_g(z) + \frac{3 f_{\rm NL}^{\rm loc}\Omega_{\rm m}H_0^2(b_g-1)\delta_c}{k^2 T(k) D(z)}$$

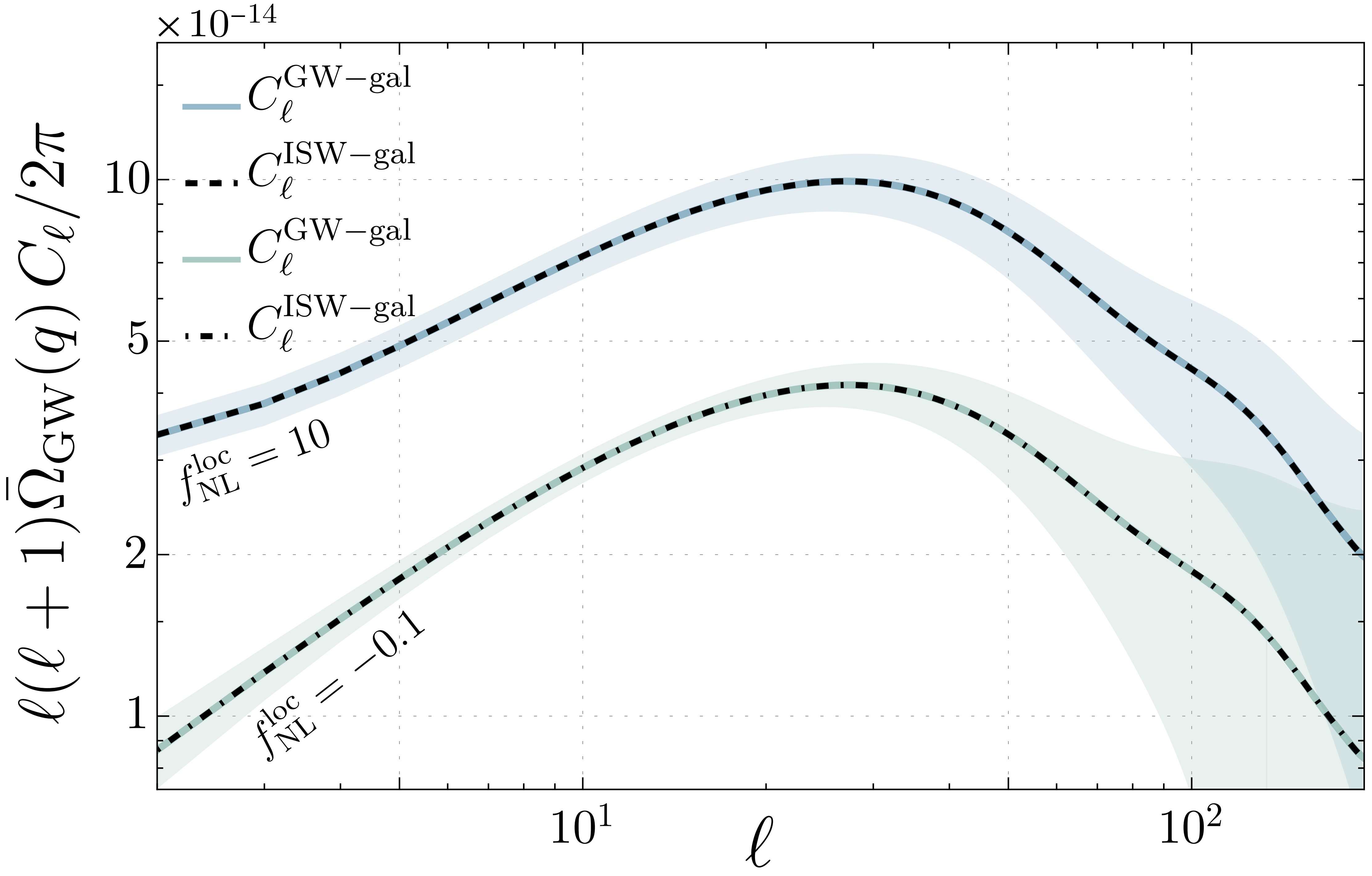
Imprints of LSS in the Anisotropies of the CGWB

The cross-correlation of CGWB anisotropies with the galaxy density contrast $\langle \delta_{\rm GW}^{\ell m} \delta_{\rm g}^{*\,\ell' m'} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell^{\text{GW-gal}}$ is given by

$$C_\ell^{\text{GW-gal}} = \int_0^\infty d\ln k\, \Delta_L^2(k) I_\ell^{\rm GW}(\eta_\Lambda^{\rm in}, k) I_\ell^{\rm gal}(\eta_z, k)$$

with $I_\ell^{\rm GW}(\eta_\Lambda^{\rm in}, k) = J_{\rm in}(q) j_\ell(k(\eta_0 - \eta_{\rm in})) + I_\ell^{\rm ISW}(\eta_\Lambda, k)$

the numerical solution computed with a modified version of `GW_CLASS` [Schulze *et al.* (2023)]



Completely driven by the ISW effect!

The largest contribution from $J_{\rm in}$ is of the form $\frac{1}{\sqrt{\ell}}\left(\frac{\eta_0 - \eta_z}{\eta_0 - \eta_{\rm in}}\right)^\ell$, so that

$$C_\ell^{\text{GW-gal}} \simeq C_\ell^{\text{ISW-gal}} = \int_0^\infty d\ln k\, \Delta_L^2(k) I_\ell^{\rm ISW}(\eta_\Lambda, k) I_\ell^{\rm gal}(\eta_z, k)$$

At large-scales can be estimated as

$$C_{\ell \lesssim 25}^{\text{ISW-gal}} \simeq \frac{12\sqrt{\pi}\Delta_L^2}{25}\frac{\chi(z_{\rm bin})(4 - n_{\rm GW})}{\ell(\ell+1)\sqrt{(\ell+1)}} \times \left(\frac{\ell\, D(z_{\rm bin}) b_g(z_{\rm bin})}{\chi(z_{\rm bin})^2 \Omega_{\rm m} H_0} + \frac{3}{2}(b_g(z_{\rm bin}) - 1)\delta_c H_0^2 f_{\rm NL}^{\rm loc}\right)$$

Different from AGWB × LSS $C_\ell^{\text{AGW-gal}} \propto (\ell + 1/2)^{-1}$

We also found SNR ~ 15 and $\sigma(f_{\rm NL}^{\rm loc}) \sim 10$

To Sum Up

- In 2505.15084 we computed the CGWB $\times$ LSS correlation.
- Entirely driven by the graviton integrated Sachs-Wolfe (ISW) effect.
- CGWB $\times$ LSS $\neq$ AGWB $\times$ LSS $\Longrightarrow$ It can be used to distinguish among sources.
- $\times$-correlation *can* be used to constrain primordial non-Gaussianities $f_{\rm NL}^{\rm loc}$.
- ISW is sensitive to the late-time expansion. $\times$-correlation *might* be used as as tool to test dark energy / modified gravity scenarios.

Thanks for your attention!