Exercise 09a

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Proof the idempotence of the 'closing-opening' alternated filter. More formally, proof the theorem:

$$\gamma_B\varphi_B\gamma_B\varphi_B(I) = \gamma_B\varphi_B(I) \quad \forall I$$

Given that:

- Opening (γ) and closing (φ) are idempotent:

$$\gamma_B(I) = \gamma_B\gamma_B(I)$$
$$\varphi_B(I) = \varphi_B\varphi_B(I)$$

- Opening (γ) is anti-extensive and closing (φ) is extensive:

$$\varphi_B(I) \geq \gamma_B(I)$$

We can prove that:

1. The 'opening-closing' is less than or equal to multiple 'opening-closing' operations:

$$\gamma_B\varphi_B(I) = \gamma_B\gamma_B\gamma_B\varphi_B(I)$$
$$(\varphi_B \geq \gamma_B)$$
$$\gamma_B\varphi_B(I) \leq \gamma_B\varphi_B\gamma_B\varphi_B(I)$$

2. The 'opening-closing' is greater than or equal to multiple 'opening-closing' operations:

$$\gamma_B\varphi_B(I) = \gamma_B\varphi_B\varphi_B\varphi_B(I)$$
As opening is anti-extensive and closing is extensive ($\varphi_B \geq \gamma_B$)
$$\gamma_B\varphi_B(I) \geq \gamma_B\varphi_B\gamma_B\varphi_B(I)$$

3. If both expressions are true, then the 'opening-closing' is equal to multiple 'opening-closing' operations, therefore proving that the operation is idempotent:

$$\gamma_B\varphi_B(I) \geq \gamma_B\varphi_B\gamma_B\varphi_B(I) \tag{1}$$
$$\gamma_B\varphi_B(I) \leq \gamma_B\varphi_B\gamma_B\varphi_B(I) \tag{2}$$
$$\gamma_B\varphi_B(I) = \gamma_B\varphi_B\gamma_B\varphi_B(I) \tag{3}$$