Exercise 02c

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In this exercise we are going to compare the number of operations in two alternatives for computing a morphological dilation with structuring element.

- Let B be the MxM square structuring element.
- Let C be the 1xM 1D horizontal structuring element:

$$\begin{pmatrix} x_0 & \dots & X & \dots & x_{m-1} \end{pmatrix}$$

- Let D be the Mx1 1-D vertical structuring element.

$$\begin{pmatrix} x_0 \\ \vdots \\ X \\ \vdots \\ x_{m-1} \end{pmatrix}$$

- It can be observed that the following property holds:

$$B = dilate_C(D) = dilate_D(C)$$

Estimate the number or 'max' operations that must be computed in order to process a NxN square input image using the following alternatives:

- $dilate_B(I)$
- $dilate_C(dilate_D(I))$

Border effects should not be considered for simplicity, i.e., all image pixels should be treated in the same manner.

The maximum of N numbers can be computed using N-1 elementary maximum operations. Considering this, we can calculate the number of operations required for each case:

- $dilate_B(I)$: For each pixel in I we will compute the maximum value of every pixel in the square structuring element. This means we are going to compute $N \times N$ (each pixel in the image) times $M \times M - 1$ (each pixel in the structuring element minus one):

$$N^{o} of max() = N^2 \times (M^2 - 1)$$

- $dilate_C(dilate_D(I))$: In the first function, we will compute for each pixel in I the maximum value of every pixel in the vertical structuring element. In the second one, we will compute for each pixel in I the maximum value of every pixel in the horizontal structuring element. This means we are going to compute $N \times N$ (each pixel in the first image) times $M \times 1 - 1$ (each pixel in the structuring element minus one) first, and $N \times N$ (each pixel in the second image) times $1 \times M - 1$ (each pixel in the structuring element minus one):

$$N^{o} of max() = N \times N \times (M \times 1 - 1) + N \times N \times (1 \times M - 1)$$
$$N^{o} of max() = 2 \times N^2 \times (M - 1)$$

Comparing both expressions:

$$2 \times N^2 \times (M - 1) = N^2 \times (M^2 - 1)$$
$$2M - 2 = M^2 - 1$$
$$M^2 - 2M + 1 = 0$$
$$M = 1$$

We conclude that assuming $N \in N > 0$, computing $dilate_C(dilate_D(I))$ is faster than $dilate_B(I)$ in terms of number of elementary maximum operations given that $M > 1$, which will always be the case for a valid structuring element.