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**A transplantation Problem**

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Introduction to Kidney Transplantation

Over the past four years in the United States, the demand for kidney transplants has steadily increased, resulting in a growing number of individuals being waitlisted. While the number of kidney transplants performed has also risen, the rate at which people are joining the waitlist far exceeds the rate at which transplants are completed. This mismatch between supply and demand underscores the critical need to maximize the number of compatible kidney transplants.

**Figure 1: U.S Kidney Transplant Gap** *Data Courtesy United Network for Organ Sharing*

To address this challenge and enhance transplant success rates, it is crucial to examine the factors influencing transplant feasibility. These factors include the medical conditions of both the recipient and the donor, as underlying health issues can make the surgery too risky. Additional considerations include donor availability, the recipient’s age, immunological factors, psychosocial factors, lifestyle habits, and financial concerns. Among these, the most decisive factor is the compatibility between the donor and recipient, which primarily depends on two key variables: the presence of antibodies that could react to donor tissue and blood type compatibility.

This study focuses on blood type compatibility as a critical decision variable to optimize kidney transplants across the United States. By leveraging compatibility criteria, we aim to increase the number of successful transplantations and address the growing demand.

Introduction to Project Criteria

Kidney transplant recipients can only receive a transplant from a donor with a compatible blood type. This requirement creates challenges for recipients with incompatible donors. To address this issue, the Organ Procurement and Transplantation Network (OPTN) facilitates paired donation programs, enabling recipients with incompatible donors to exchange donors with other recipient-donor pairs to achieve compatibility.

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**Figure 2: Sample Kidney Transplant**

For instance, by reference to the example above where Jane needs a kidney transplant and her donor, John, has an incompatible blood type, OPTN could pair them with Kelly and Keith, another recipient-donor pair. If John is compatible with Kelly and Keith is compatible with Jane, both transplants can proceed through this donor swap.

This is a simplified example of a paired donation. In practice, transplant feasibility becomes more complex, requiring a scientific approach to optimize the number of successful transplants.

Criteria for Maximizing Transplants

To maximize the number of kidney transplants:

1. Recipient-Donor Matching**:** Recipients with compatible donors can proceed directly with transplantation, and such pairs are excluded from the pairing process.
2. Pair Swapping**:** Recipients with incompatible donors are paired with other recipient-donor pairs to form compatibility chains.
3. Multi-Pair Networks**:** Beyond two-pair swaps, the study explores the inclusion of three-pair compatibility networks. For example, Pair A, Pair B, and Pair C could form a chain where the donor in Pair C is compatible with the recipient in Pair A, the donor in Pair A is compatible with the recipient in Pair B, and the donor in Pair B is compatible with the recipient in Pair C.
4. Node 1 Cases: When a recipient has a compatible donor, the pair is excluded from the broader pool to prioritize their guaranteed transplant.
5. Non-Obligatory Participation**:** If a recipient is not matched with another donor, their accompanying donor is not required to participate in further transplant cases.

This framework aims to maximize transplant opportunities while respecting ethical considerations and ensuring equitable outcomes for all participants.

Problem Statement

The demand for kidney transplants in the United States has significantly outpaced the availability of compatible donors, resulting in a growing waitlist and unmet medical needs. While paired donation programs offer a solution for recipients with incompatible donors, the current approaches are limited in their ability to optimize the number of transplants due to complex compatibility requirements, ethical constraints, and logistical challenges.

Blood type compatibility and the presence of antibodies remain the most critical factors influencing the feasibility of kidney transplantation. However, many potential transplants are missed due to the lack of efficient pairing mechanisms, particularly for multi-pair compatibility chains involving three or more pairs. Additionally, ethical considerations, such as protecting the rights of donor-recipient pairs with existing compatibility and ensuring voluntary participation, further complicate the process.

This project aims to develop a systematic and scalable method to maximize the number of successful kidney transplants by leveraging compatibility networks, optimizing pairing mechanisms, and adhering to ethical constraints. The proposed solution will explore innovative strategies for pairing incompatible recipient-donor pairs, including multi-pair compatibility chains, while ensuring equitable access and adherence to medical and ethical guidelines.

OR Model in Words and in Math

**Definitions:**

* Dji : Donor j in node i
* Ri: Recipient in node i
* P={(Di,Ri)} : Paired Donor Recipient Pair
* Set of Nodes A: The collection of transplant nodes.
* Allowable Pairs C**:** The set of donor-recipient pairs allowed by blood type or other compatibility constraints.

**Decision Variables:**

Xji: Binary indicating if Donor j donates to recipient i

Zk: Node participates in transplant chain (1 if participates, 0 otherwise)

**Objective Function** Maximize the total number of successful kidney transplants across the network. This involves selecting the maximum number of compatible transplants while adhering to logistical and medical constraints.

Maximize: A black and white symbol

Description automatically generated where A is the set of transplant nodes

**Constraints:**

Each donor can donate to at most one recipient:

A black text with a white background

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Each recipient can receive at most one donation:

A number and a symbol

Description automatically generated with medium confidence

If the recipient is incompatible with all donors, then donors in his / her node cannot donate:

A math equation with numbers and symbols

Description automatically generated with medium confidence

Max number of chain lengths is 3. (Number of nodes must be less than 3)

A black and white math symbols

Description automatically generated with medium confidence

Compatibility Constraint:  where C includes pairs allowed by blood type compatibility constraints.

Compatibility X is binary variable. Node Participation Z is binary variable.



C: Allowable Pairs



Python / Gurobi Code

The following Github repository provides the python code as well as pair data.

[GitHub - rgutie171/TTU-Systems-Gutierrez](https://github.com/rgutie171/TTU-Systems-Gutierrez)

Experiment Discussion

The optimization model was written, combined, and solved using the Gurobi solver 11.0.3 in PyCharm 2024.2.4, and the coding language used was Python. The model runs on a Samsung Galaxy Book 4 Laptop with 16 GB of RAM and an Intel(R) Core (TM) i7 – 150U with 10 physical cores, 12 logical processors and using 12 threads. The LIP formulation is solved to optimality with the objective value of 470 Transplants in 52 seconds. Additionally, experiments were conducted to relax and restrict the chain length requirement and was determined to leave it relaxed at a max chain length of 3 to allow for maximum number of transplants.

An Optimal Transplant Plan

To create a comprehensive and effective transplant plan, a Python-based optimization model was developed. This model integrates checks and balances to address the complexities of the maximization transplant problem. These checks include monitoring the number of nodes (donor/recipient pairs) participating in the study and analyzing key statistics such as recipient blood type distributions.

An analysis of the dataset "GutierrezATX.JSON" revealed the following:

* A total of **710 recipients** are awaiting transplants.
* **239 recipients** have blood type “A.”
* **240 recipients** have blood type “O.”
* **231 recipients** have blood type “B.”

Given that recipients with blood type “O” can only receive transplants from donors with the same blood type and noting that no donors in the dataset have blood type “O,” the maximum number of feasible transplants was determined to be **470**.

The optimization model was executed, and the proposed plan successfully achieved the upper bound of **470 transplants**. The output, represented in **Figure 3: Transplant Plan**, maps each recipient node to its recommended donor node. This plan provides a clear, actionable guide for maximizing the number of transplants while adhering to compatibility rules. The sample below shows proposed recipients from node 1 will receive donation from node 203.

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Evaluation of Plans

The proposed transplant plan adheres to all predefined criteria:

1. Required Criteria:
   * Blood type compatibility rules are strictly enforced. For example:
     + Blood type “A” recipients only receive transplants from donors with blood type “A” or “O.”
   * Each donor is assigned to at most one recipient, and each recipient receives from at most one donor.
   * No prohibited matches (e.g., incompatible blood types) are included.
2. Optimality:
   * The model confirms that the plan achieves the maximum feasible number of transplants (470) based on the dataset's constraints.
3. Transparency:
   * The output provides a detailed mapping of donor-recipient pairs.

Despite meeting the defined criteria, the plan has notable limitations:

1. Blood Type “O” Constraints:
   * The lack of blood type “O” donors severely restricts the potential transplants for blood type “O” recipients, leaving **240 recipients** unmatched.
   * This highlights a systemic limitation in the dataset rather than a shortcoming of the optimization process.
2. Static Nature:
   * The plan assumes a static dataset and does not account for potential real-world dynamics, such as new donors entering the system, recipients dropping out, or medical urgency prioritizations.
3. Prohibited Pairing Enforcement:
   * While the plan enforces compatibility, it does not explicitly explore the implications of additional constraints, such as geographic proximity or donor-recipient relationship preferences.

Conclusions

The findings and recommendations from this analysis are summarized below:

1. The proposed transplant plan achieves **470 transplants**, which is the theoretical maximum given the dataset constraints.
2. The plan strictly adheres to required criteria, such as blood type compatibility and one-to-one donor-recipient relationships.
3. The absence of blood type “O” donors is a significant limiting factor, leaving a substantial number of recipients unmatched.

By addressing these limitations and maintaining the rigor of the current optimization approach, the system can improve transplant outcomes and better serve recipients in need.

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pancreas/incompatible-blood-type.html

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**Figure 3: Transplant Plan**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Recipient** | **Donor** |  | **Recipient** | **Donor** |  | **Recipient** | **Donor** |  | **Recipient** | **Donor** |
| 1 | 203 |  | 82 | 9 |  | 151 | 485 |  | 213 | 480 |
| 2 | 376 |  | 83 | 592 |  | 152 | 535 |  | 214 | 583 |
| 3 | 586 |  | 84 | 265 |  | 155 | 651 |  | 218 | 515 |
| 5 | 82 |  | 85 | 83 |  | 156 | 252 |  | 220 | 589 |
| 6 | 652 |  | 87 | 383 |  | 157 | 378 |  | 221 | 429 |
| 8 | 465 |  | 88 | 246 |  | 158 | 482 |  | 223 | 453 |
| 9 | 547 |  | 89 | 666 |  | 159 | 361 |  | 224 | 604 |
| 11 | 363 |  | 93 | 172 |  | 160 | 595 |  | 225 | 434 |
| 14 | 553 |  | 94 | 419 |  | 162 | 165 |  | 226 | 287 |
| 15 | 294 |  | 95 | 75 |  | 164 | 74 |  | 228 | 2 |
| 18 | 299 |  | 96 | 364 |  | 166 | 466 |  | 229 | 664 |
| 19 | 684 |  | 99 | 162 |  | 167 | 52 |  | 230 | 423 |
| 21 | 87 |  | 100 | 631 |  | 169 | 223 |  | 232 | 190 |
| 22 | 580 |  | 103 | 216 |  | 171 | 310 |  | 233 | 642 |
| 25 | 529 |  | 104 | 341 |  | 172 | 183 |  | 234 | 184 |
| 26 | 549 |  | 106 | 334 |  | 173 | 28 |  | 237 | 205 |
| 30 | 343 |  | 107 | 226 |  | 174 | 178 |  | 239 | 319 |
| 32 | 331 |  | 108 | 167 |  | 175 | 196 |  | 240 | 192 |
| 33 | 669 |  | 109 | 254 |  | 176 | 280 |  | 241 | 577 |
| 36 | 58 |  | 111 | 30 |  | 177 | 231 |  | 242 | 256 |
| 37 | 703 |  | 113 | 350 |  | 178 | 179 |  | 243 | 372 |
| 39 | 298 |  | 118 | 185 |  | 179 | 186 |  | 244 | 392 |
| 40 | 497 |  | 119 | 513 |  | 181 | 503 |  | 245 | 673 |
| 41 | 573 |  | 120 | 285 |  | 182 | 312 |  | 246 | 646 |
| 43 | 235 |  | 122 | 249 |  | 183 | 206 |  | 249 | 454 |
| 46 | 527 |  | 124 | 367 |  | 186 | 559 |  | 250 | 37 |
| 47 | 478 |  | 126 | 258 |  | 187 | 475 |  | 251 | 339 |
| 49 | 447 |  | 127 | 54 |  | 189 | 152 |  | 252 | 484 |
| 51 | 455 |  | 129 | 36 |  | 190 | 70 |  | 255 | 414 |
| 52 | 615 |  | 130 | 438 |  | 191 | 259 |  | 256 | 575 |
| 57 | 463 |  | 131 | 630 |  | 192 | 295 |  | 257 | 320 |
| 58 | 504 |  | 132 | 305 |  | 194 | 13 |  | 258 | 215 |
| 60 | 375 |  | 133 | 221 |  | 195 | 253 |  | 259 | 622 |
| 63 | 543 |  | 134 | 228 |  | 196 | 688 |  | 260 | 483 |
| 64 | 302 |  | 135 | 558 |  | 199 | 296 |  | 261 | 61 |
| 65 | 624 |  | 136 | 648 |  | 200 | 50 |  | 262 | 629 |
| 66 | 47 |  | 137 | 393 |  | 201 | 380 |  | 265 | 634 |
| 67 | 693 |  | 139 | 16 |  | 202 | 628 |  | 267 | 67 |
| 69 | 486 |  | 140 | 43 |  | 203 | 384 |  | 268 | 645 |
| 71 | 499 |  | 143 | 247 |  | 206 | 551 |  | 269 | 491 |
| 75 | 72 |  | 145 | 267 |  | 207 | 605 |  | 270 | 532 |
| 77 | 182 |  | 149 | 27 |  | 210 | 460 |  | 271 | 572 |
| 78 | 316 |  | 150 | 517 |  | 212 | 161 |  | 272 | 635 |
|  |  |  |  |  |  |  |  |  |  |  |
| **Recipient** | **Donor** |  | **Recipient** | **Donor** |  | **Recipient** | **Donor** |  | **Recipient** | **Donor** |
| 276 | 557 |  | 336 | 159 |  | 404 | 80 |  | 467 | 656 |
| 277 | 422 |  | 337 | 469 |  | 405 | 424 |  | 468 | 561 |
| 278 | 500 |  | 338 | 270 |  | 406 | 55 |  | 469 | 435 |
| 281 | 578 |  | 339 | 479 |  | 407 | 569 |  | 470 | 33 |
| 282 | 530 |  | 343 | 472 |  | 408 | 458 |  | 471 | 686 |
| 283 | 509 |  | 346 | 149 |  | 409 | 169 |  | 473 | 689 |
| 285 | 398 |  | 347 | 330 |  | 410 | 25 |  | 475 | 585 |
| 286 | 523 |  | 348 | 619 |  | 413 | 155 |  | 477 | 546 |
| 287 | 685 |  | 349 | 574 |  | 416 | 300 |  | 478 | 79 |
| 289 | 644 |  | 350 | 7 |  | 417 | 418 |  | 479 | 44 |
| 290 | 473 |  | 351 | 97 |  | 418 | 34 |  | 480 | 95 |
| 291 | 655 |  | 352 | 640 |  | 419 | 318 |  | 483 | 266 |
| 292 | 541 |  | 353 | 588 |  | 421 | 151 |  | 484 | 53 |
| 294 | 452 |  | 354 | 534 |  | 422 | 519 |  | 485 | 210 |
| 295 | 176 |  | 355 | 440 |  | 423 | 84 |  | 486 | 73 |
| 296 | 230 |  | 359 | 508 |  | 425 | 683 |  | 487 | 582 |
| 297 | 637 |  | 360 | 533 |  | 428 | 338 |  | 489 | 653 |
| 298 | 665 |  | 361 | 659 |  | 429 | 443 |  | 490 | 662 |
| 302 | 229 |  | 366 | 326 |  | 431 | 336 |  | 491 | 284 |
| 304 | 700 |  | 367 | 307 |  | 433 | 59 |  | 493 | 335 |
| 305 | 439 |  | 371 | 432 |  | 434 | 337 |  | 494 | 560 |
| 306 | 493 |  | 372 | 706 |  | 435 | 565 |  | 495 | 24 |
| 307 | 22 |  | 373 | 550 |  | 436 | 391 |  | 497 | 272 |
| 309 | 281 |  | 374 | 98 |  | 437 | 195 |  | 499 | 348 |
| 310 | 35 |  | 375 | 301 |  | 438 | 145 |  | 500 | 244 |
| 311 | 417 |  | 376 | 188 |  | 440 | 606 |  | 502 | 15 |
| 312 | 242 |  | 377 | 328 |  | 441 | 457 |  | 503 | 411 |
| 314 | 462 |  | 379 | 340 |  | 442 | 368 |  | 505 | 395 |
| 315 | 678 |  | 381 | 487 |  | 443 | 436 |  | 508 | 48 |
| 316 | 232 |  | 382 | 467 |  | 445 | 611 |  | 510 | 699 |
| 317 | 352 |  | 383 | 309 |  | 447 | 625 |  | 512 | 345 |
| 318 | 620 |  | 384 | 471 |  | 448 | 66 |  | 513 | 488 |
| 321 | 194 |  | 385 | 600 |  | 449 | 502 |  | 515 | 212 |
| 322 | 65 |  | 386 | 164 |  | 452 | 147 |  | 517 | 174 |
| 323 | 71 |  | 389 | 144 |  | 453 | 357 |  | 521 | 587 |
| 324 | 409 |  | 391 | 405 |  | 454 | 692 |  | 522 | 136 |
| 325 | 579 |  | 392 | 698 |  | 456 | 69 |  | 523 | 654 |
| 328 | 633 |  | 394 | 404 |  | 457 | 227 |  | 527 | 292 |
| 329 | 412 |  | 395 | 91 |  | 458 | 400 |  | 528 | 671 |
| 330 | 6 |  | 398 | 528 |  | 461 | 505 |  | 530 | 94 |
| 331 | 522 |  | 399 | 623 |  | 462 | 177 |  | 531 | 661 |
| 333 | 217 |  | 400 | 325 |  | 463 | 207 |  | 533 | 451 |
| 334 | 347 |  | 402 | 415 |  | 464 | 40 |  | 534 | 394 |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| **Recipient** | **Donor** |  | **Recipient** | **Donor** |  | **Recipient** | **Donor** |  |  |  |
| 535 | 153 |  | 591 | 650 |  | 660 | 239 |  |  |  |
| 536 | 552 |  | 592 | 269 |  | 663 | 288 |  |  |  |
| 538 | 42 |  | 593 | 181 |  | 664 | 567 |  |  |  |
| 539 | 697 |  | 594 | 274 |  | 665 | 691 |  |  |  |
| 540 | 140 |  | 595 | 470 |  | 666 | 674 |  |  |  |
| 541 | 494 |  | 596 | 150 |  | 667 | 137 |  |  |  |
| 542 | 594 |  | 597 | 306 |  | 668 | 356 |  |  |  |
| 543 | 355 |  | 598 | 138 |  | 669 | 627 |  |  |  |
| 546 | 420 |  | 599 | 263 |  | 673 | 297 |  |  |  |
| 547 | 680 |  | 600 | 197 |  | 674 | 160 |  |  |  |
| 548 | 321 |  | 602 | 525 |  | 675 | 598 |  |  |  |
| 549 | 608 |  | 603 | 514 |  | 676 | 539 |  |  |  |
| 551 | 264 |  | 604 | 241 |  | 677 | 29 |  |  |  |
| 552 | 495 |  | 606 | 540 |  | 679 | 426 |  |  |  |
| 553 | 198 |  | 608 | 492 |  | 680 | 89 |  |  |  |
| 554 | 49 |  | 609 | 385 |  | 681 | 19 |  |  |  |
| 555 | 311 |  | 612 | 93 |  | 682 | 511 |  |  |  |
| 556 | 581 |  | 613 | 143 |  | 683 | 283 |  |  |  |
| 557 | 390 |  | 614 | 675 |  | 687 | 524 |  |  |  |
| 558 | 643 |  | 615 | 204 |  | 688 | 323 |  |  |  |
| 559 | 401 |  | 617 | 187 |  | 689 | 81 |  |  |  |
| 560 | 518 |  | 620 | 679 |  | 690 | 262 |  |  |  |
| 561 | 90 |  | 621 | 245 |  | 691 | 238 |  |  |  |
| 562 | 342 |  | 622 | 396 |  | 692 | 571 |  |  |  |
| 563 | 369 |  | 624 | 290 |  | 693 | 617 |  |  |  |
| 564 | 374 |  | 627 | 636 |  | 694 | 687 |  |  |  |
| 565 | 464 |  | 630 | 327 |  | 695 | 649 |  |  |  |
| 567 | 1 |  | 631 | 139 |  | 696 | 596 |  |  |  |
| 568 | 708 |  | 632 | 60 |  | 697 | 626 |  |  |  |
| 569 | 237 |  | 633 | 610 |  | 699 | 92 |  |  |  |
| 570 | 218 |  | 634 | 531 |  | 700 | 658 |  |  |  |
| 572 | 201 |  | 637 | 639 |  | 701 | 702 |  |  |  |
| 573 | 96 |  | 641 | 386 |  | 702 | 4 |  |  |  |
| 574 | 450 |  | 642 | 173 |  | 703 | 332 |  |  |  |
| 576 | 612 |  | 644 | 410 |  | 704 | 146 |  |  |  |
| 579 | 570 |  | 647 | 681 |  | 705 | 222 |  |  |  |
| 580 | 556 |  | 648 | 389 |  | 706 | 17 |  |  |  |
| 581 | 427 |  | 650 | 690 |  | 708 | 68 |  |  |  |
| 582 | 638 |  | 651 | 663 |  | 709 | 8 |  |  |  |
| 586 | 602 |  | 655 | 576 |  | 710 | 421 |  |  |  |
| 588 | 670 |  | 657 | 701 |  |  |  |  |  |  |
| 589 | 171 |  | 658 | 402 |  |  |  |  |  |  |
| 590 | 597 |  | 659 | 647 |  |  |  |  |  |  |