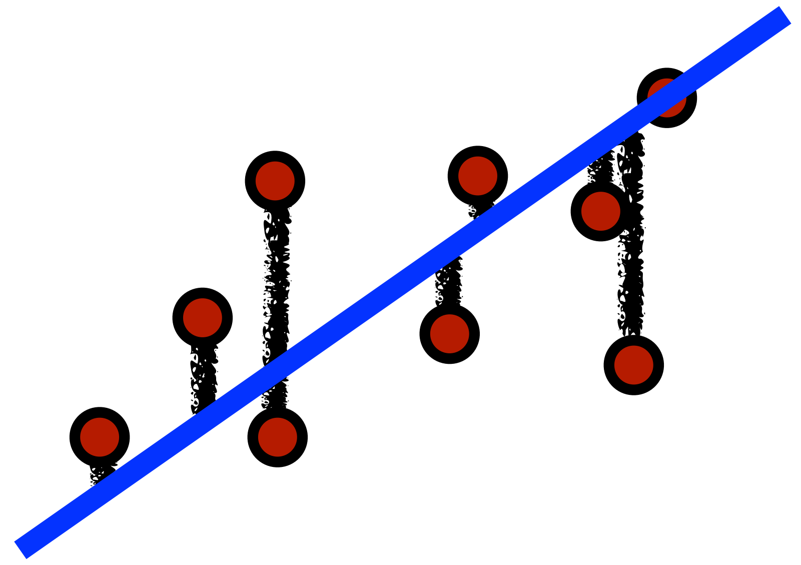
Linear Regression

UCLA Cal databricks™

Regression

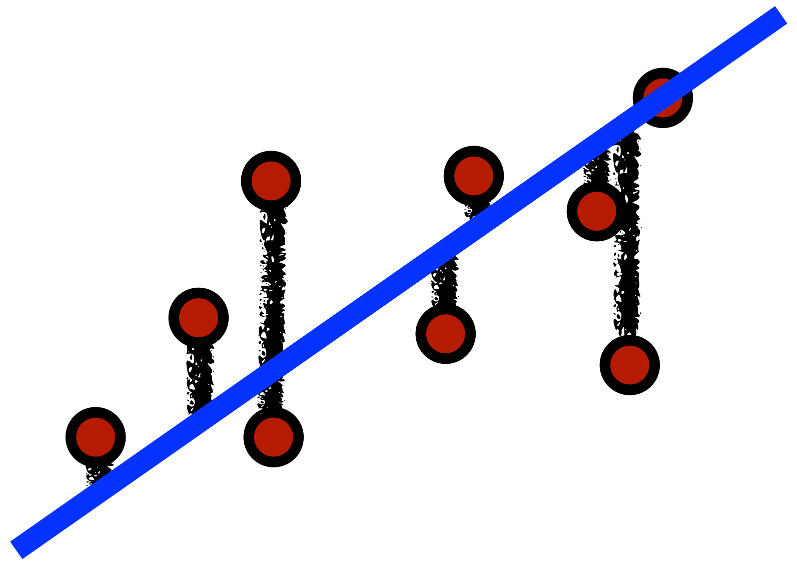


Goal: Learn a mapping from observations (features) to continuous labels given a training set (supervised learning)

Example: Height, Gender, Weight → Shoe Size

- Audio features → Song year
- Processes, memory → Power consumption
- Historical financials → Future stock price
- Many more

Linear Least Squares Regression



Example: Predicting shoe size from height, gender, and weight

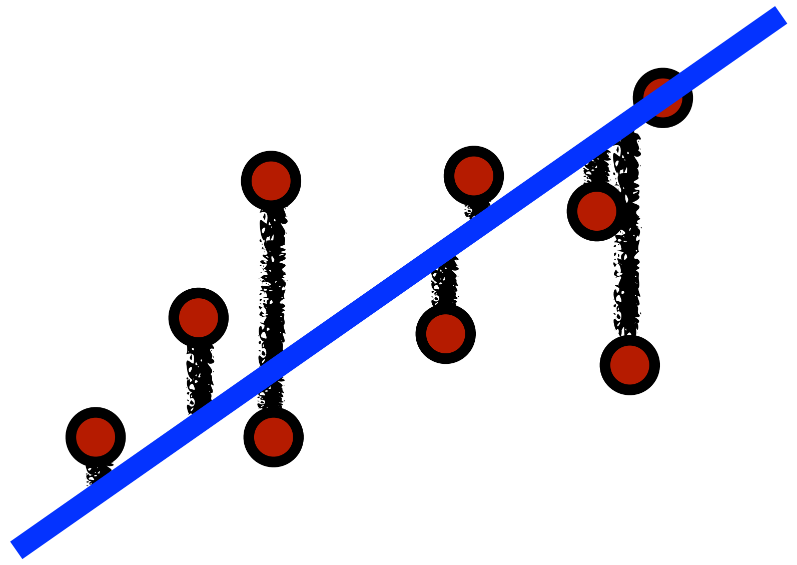
For each observation we have a feature vector, $\mathbf{x}$, and label, y

$$\mathbf{x}^\top = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

We assume a *linear* mapping between features and label:

$$y \approx w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3$$

Linear Least Squares Regression



Example: Predicting shoe size from height, gender, and weight

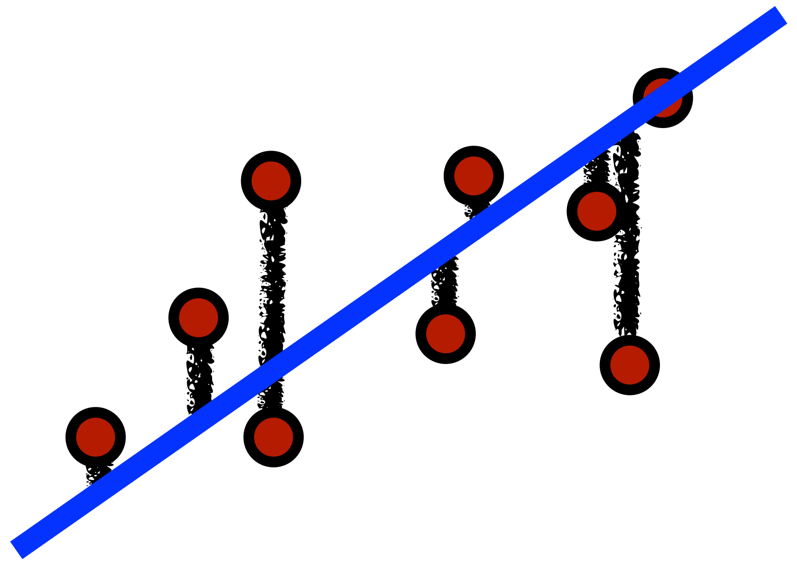
We can augment the feature vector to incorporate offset:

$$\mathbf{x}^\top = \begin{bmatrix} 1 & x_1 & x_2 & x_3 \end{bmatrix}$$

We can then rewrite this linear mapping as scalar product:

$$y \approx \hat{y} = \sum_{i=0}^{3} w_i x_i = \mathbf{w}^\top \mathbf{x}$$

Why a Linear Mapping?



Simple

Often works well in practice

Can introduce complexity via feature extraction

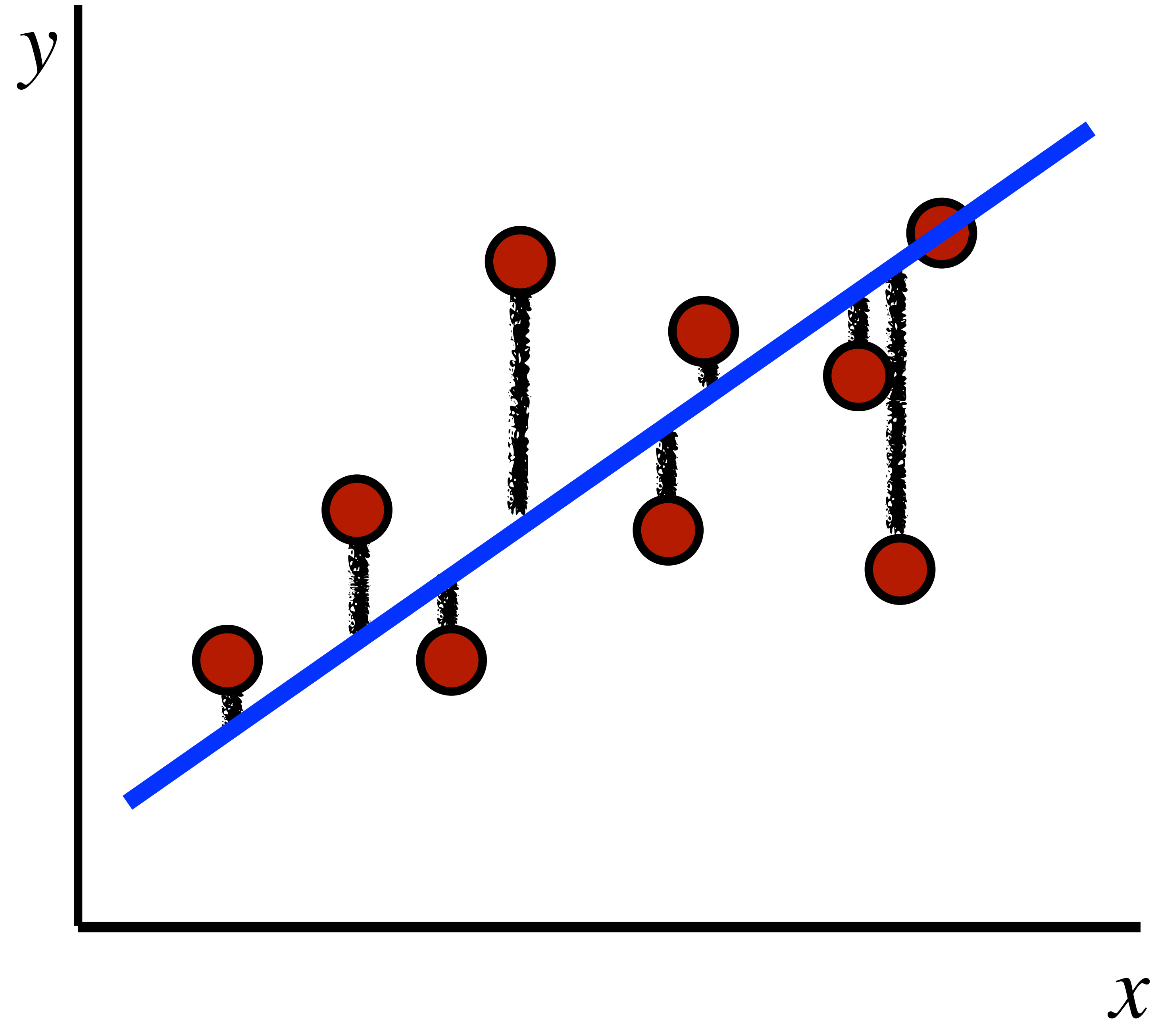
1D Example

Goal: find the line of best fit

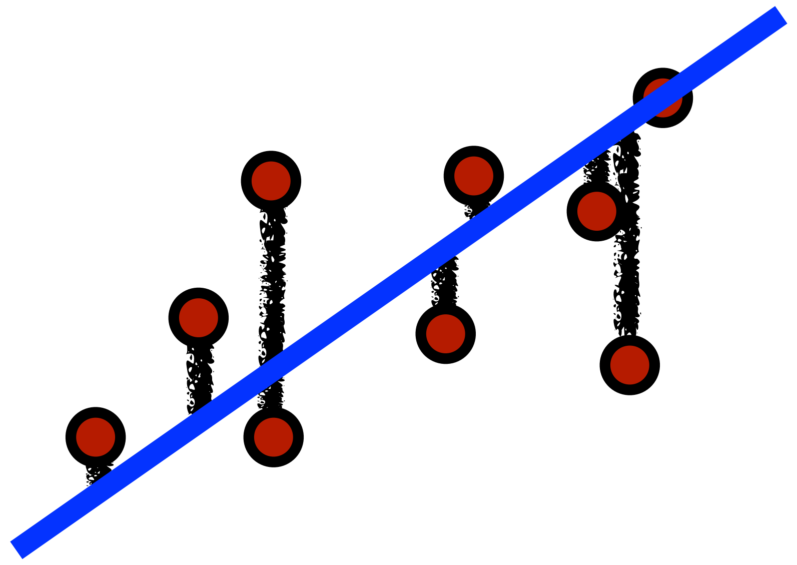
x coordinate: features

y coordinate: labels

$$y \approx \hat{y} = \underbrace{w_0}_{\text{Intercept / Offset}} + \underbrace{w_1}_{\text{Slope}} x$$



Evaluating Predictions



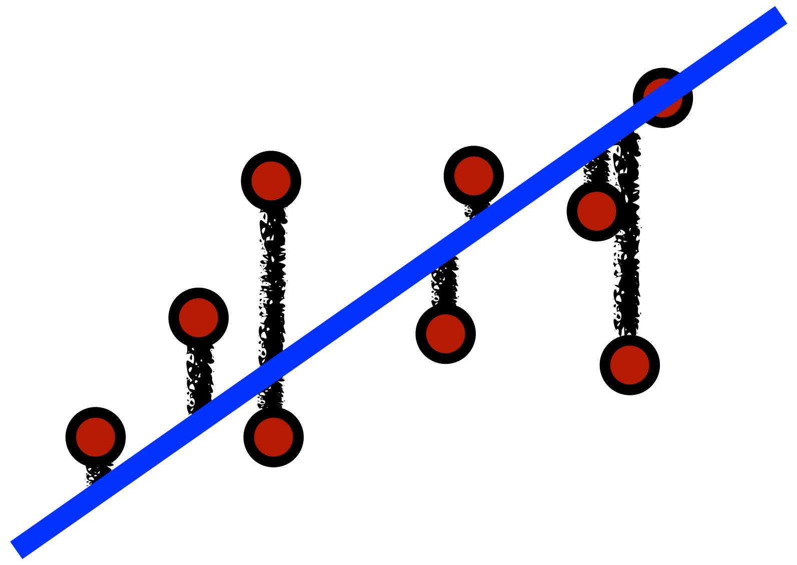
Can measure ‘closeness’ between label and prediction

- Shoe size: better to be off by one size than 5 sizes
- Song year prediction: better to be off by a year than by 20 years

What is an appropriate evaluation metric or ‘loss’ function?

- Absolute loss: $|y - \hat{y}|$
- Squared loss: $(y - \hat{y})^2$ ← Has nice mathematical properties

How Can We Learn Model ($\mathbf{w}$)?



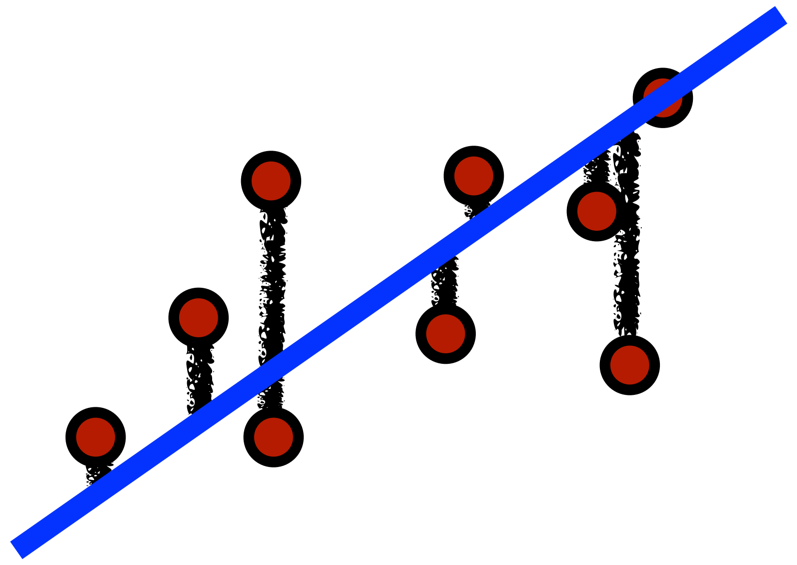
Assume we have n training points, where $\mathbf{x}^{(i)}$ denotes the ith point

Recall two earlier points:

- *Linear* assumption: $\hat{y} = \mathbf{w}^\top \mathbf{x}$
- We use *squared loss*: $(y - \hat{y})^2$

Idea: Find $\mathbf{w}$ that minimizes squared loss over training points:

$$\min_{\mathbf{w}} \sum_{i=1}^{n} (\underbrace{\mathbf{w}^\top \mathbf{x}^{(i)}}_{\hat{y}^{(i)}} - y^{(i)})^2$$



Given n training points with d features, we define:

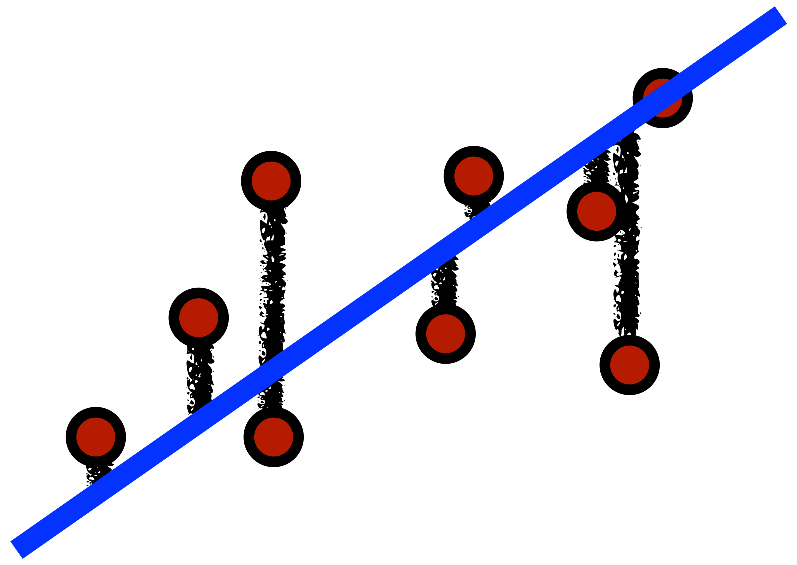
- $\mathbf{X} \in \mathbb{R}^{n \times d}$: matrix storing points
- $\mathbf{y} \in \mathbb{R}^{n}$: real-valued labels
- $\hat{\mathbf{y}} \in \mathbb{R}^{n}$: predicted labels, where $\hat{\mathbf{y}} = \mathbf{X}\mathbf{w}$
- $\mathbf{w} \in \mathbb{R}^{d}$: regression parameters / model to learn

Least Squares Regression: Learn mapping ($\mathbf{w}$) from features to labels that minimizes residual sum of squares:

$$\min_{\mathbf{w}} ||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2$$

Equivalent $\min_{\mathbf{w}} \sum_{i=1}^{n} (\mathbf{w}^\top \mathbf{x}^{(i)} - y^{(i)})^2$ by definition of Euclidean norm

Find solution by setting derivative to zero



1D: $f(w) = ||w\mathbf{x} - \mathbf{y}||_2^2 = \sum_{i=1}^{n} (wx^{(i)} - y^{(i)})^2$

$$\frac{df}{dw}(w) = 2 \underbrace{\sum_{i=1}^{n} x^{(i)} (wx^{(i)} - y^{(i)})}_{w\mathbf{x}^\top \mathbf{x} - \mathbf{x}^\top \mathbf{y}} = 0 \iff w\mathbf{x}^\top \mathbf{x} - \mathbf{x}^\top \mathbf{y} = 0$$

$$\iff w = (\mathbf{x}^\top \mathbf{x})^{-1} \mathbf{x}^\top \mathbf{y}$$

Least Squares Regression: Learn mapping ($\mathbf{w}$) from features to labels that minimizes residual sum of squares:

$$\min_{\mathbf{w}} ||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2$$

Closed form solution: $\mathbf{w} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$ (if inverse exists)

Overfitting and Generalization

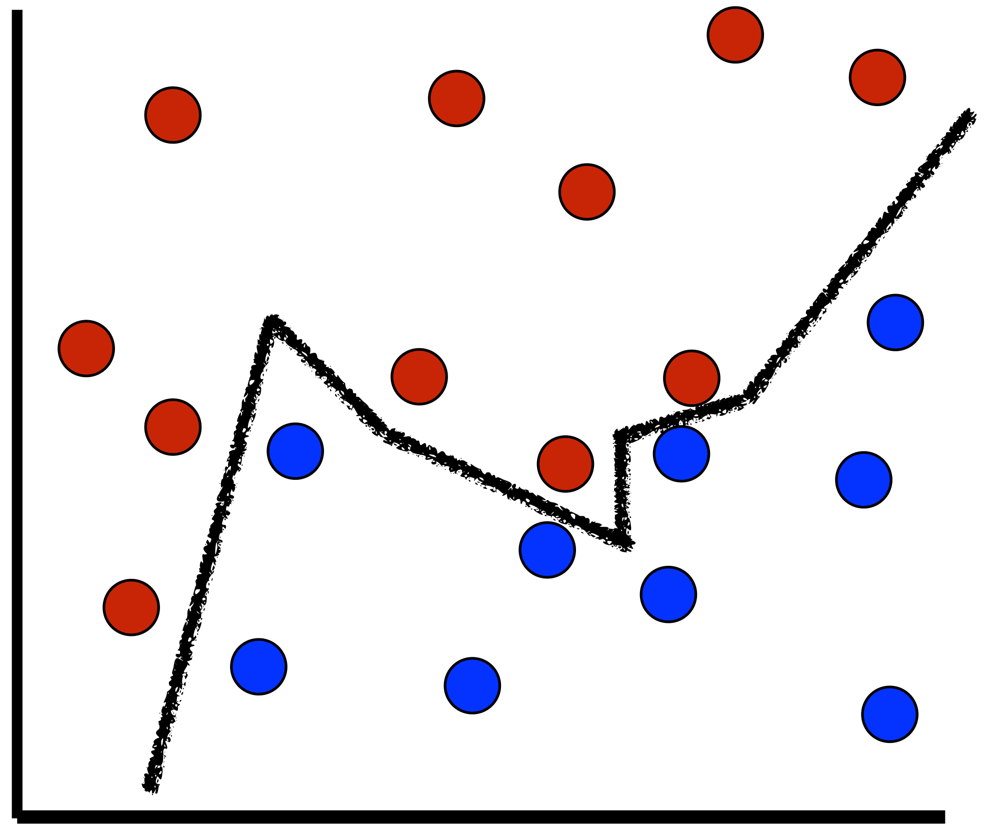
We want good predictions on new data, i.e., ’generalization’

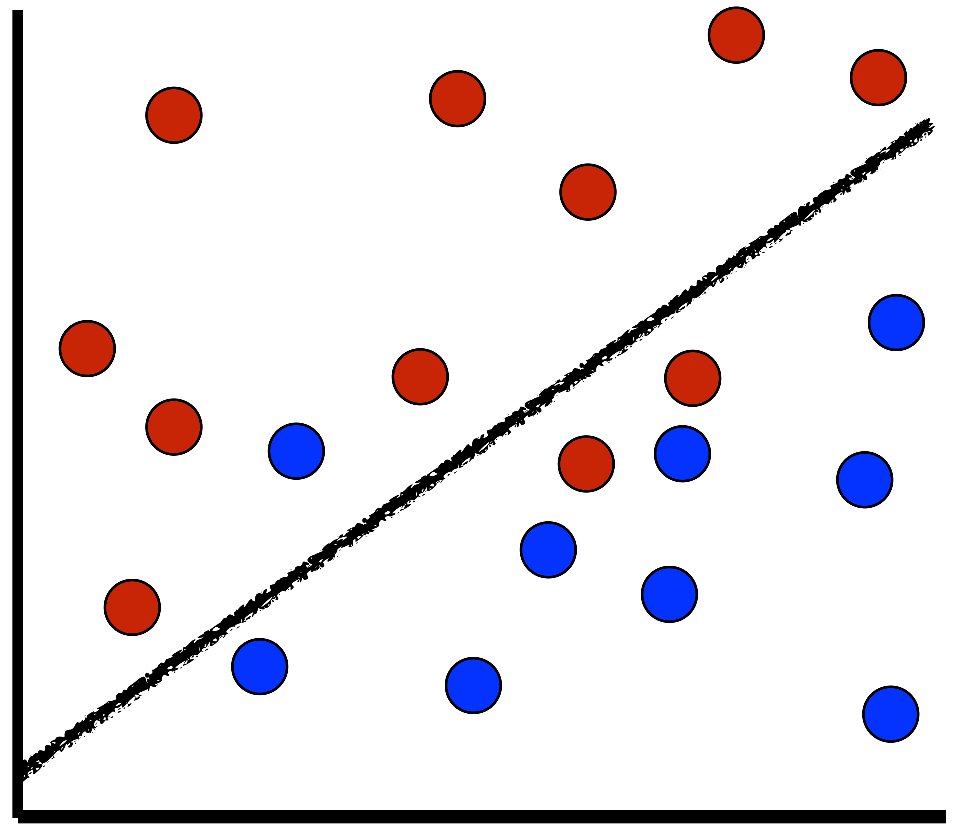
Least squares regression minimizes training error, and could overfit

- Simpler models are more likely to generalize (Occam’s razor)

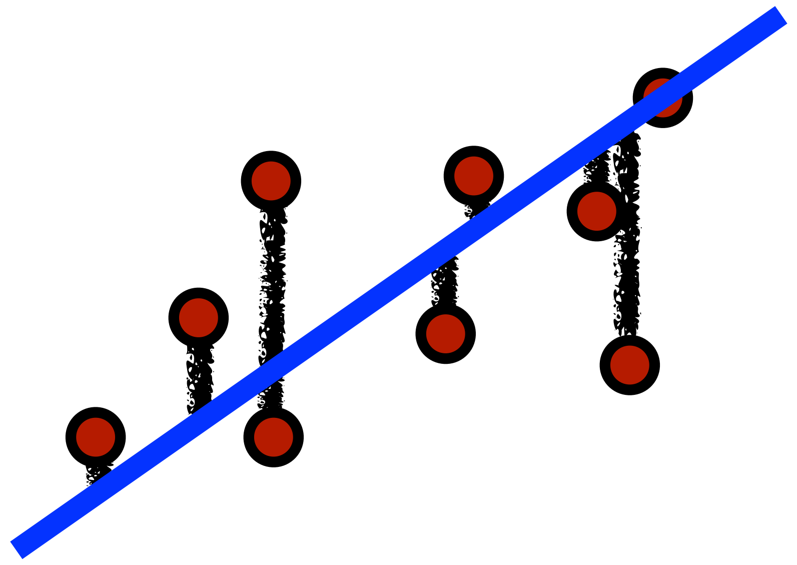
Can we change the problem to penalize for model complexity?

- Intuitively, models with smaller weights are simpler





Given n training points with d features, we define:



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Ridge Regression: Learn mapping ($\mathbf{w}$) that minimizes residual sum of squares along with a regularization term:

$$\min_{\mathbf{w}} \overset{\text{Training Error}}{\overline{||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2}} + \overset{\text{Model Complexity}}{\overline{\lambda||\mathbf{w}||_2^2}}$$

free parameter trades off between training error and model complexity

Closed-form solution: $\mathbf{w} = (\mathbf{X}^\top \mathbf{X} + \lambda \mathbf{I}_d)^{-1} \mathbf{X}^\top \mathbf{y}$

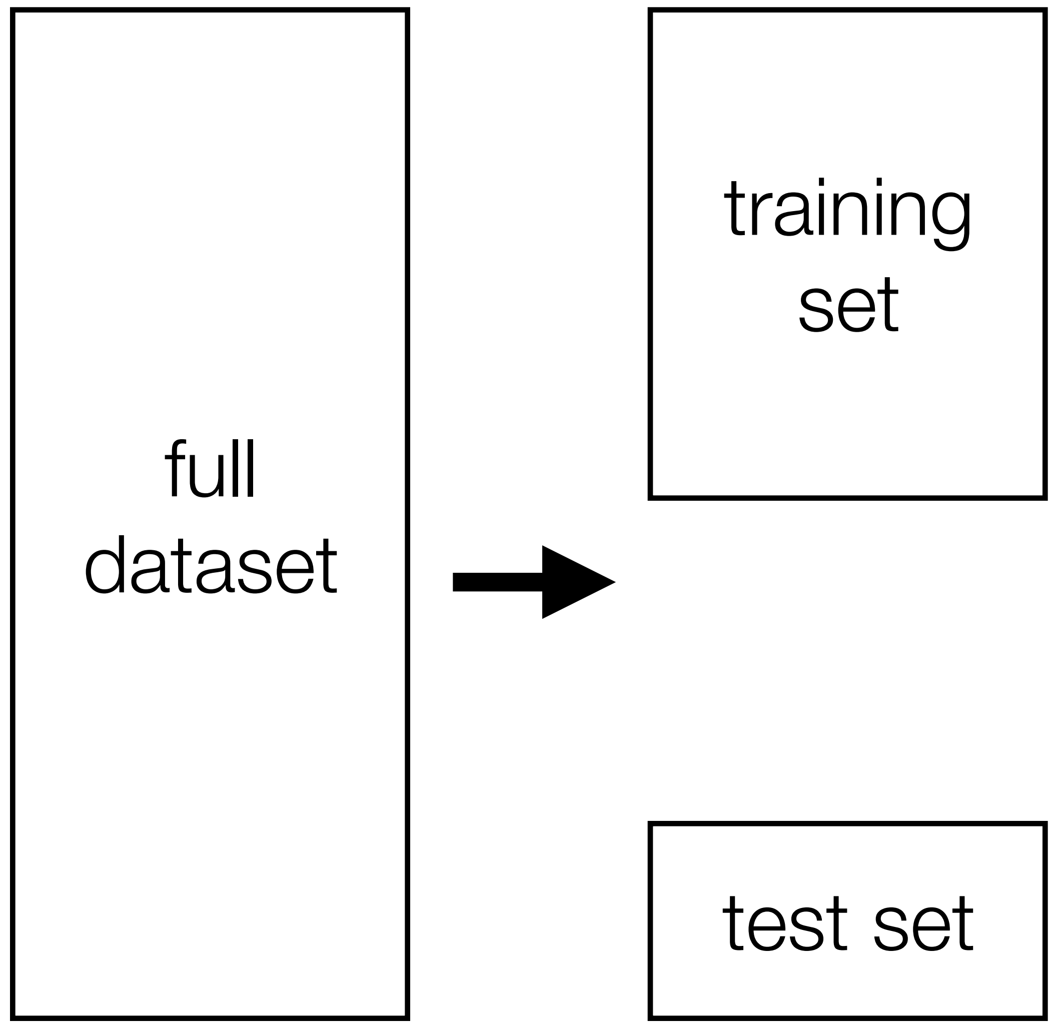
Millionsong
Regression Pipeline

UCLA Cal databricks™

Supervised Learning Pipeline

full dataset

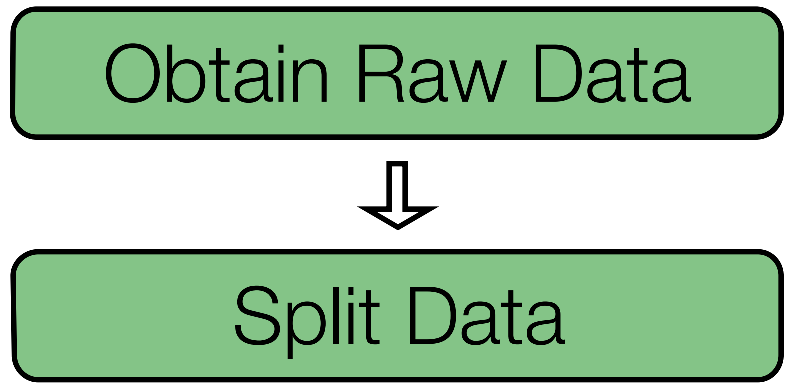
Obtain Raw Data



full dataset

training set

test set



Obtain Raw Data

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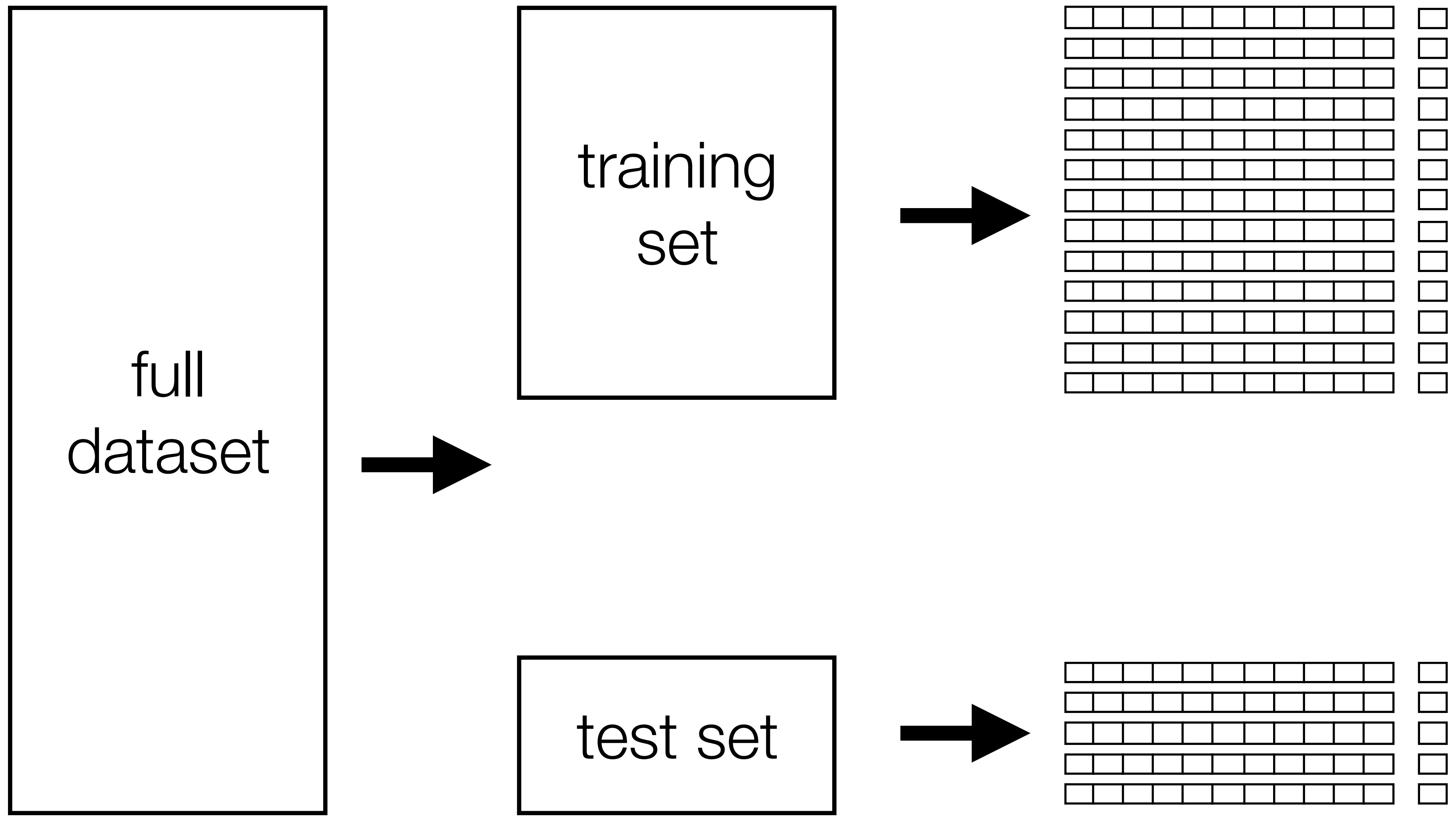
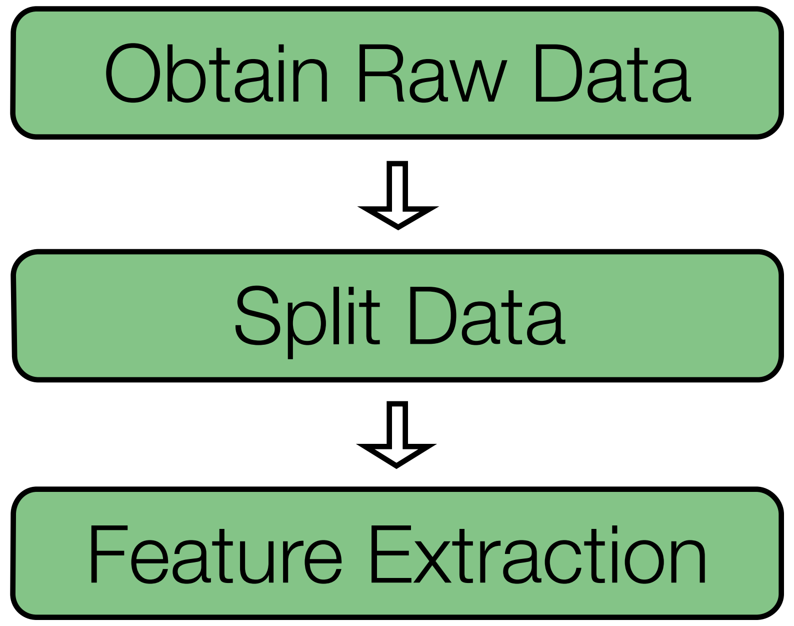
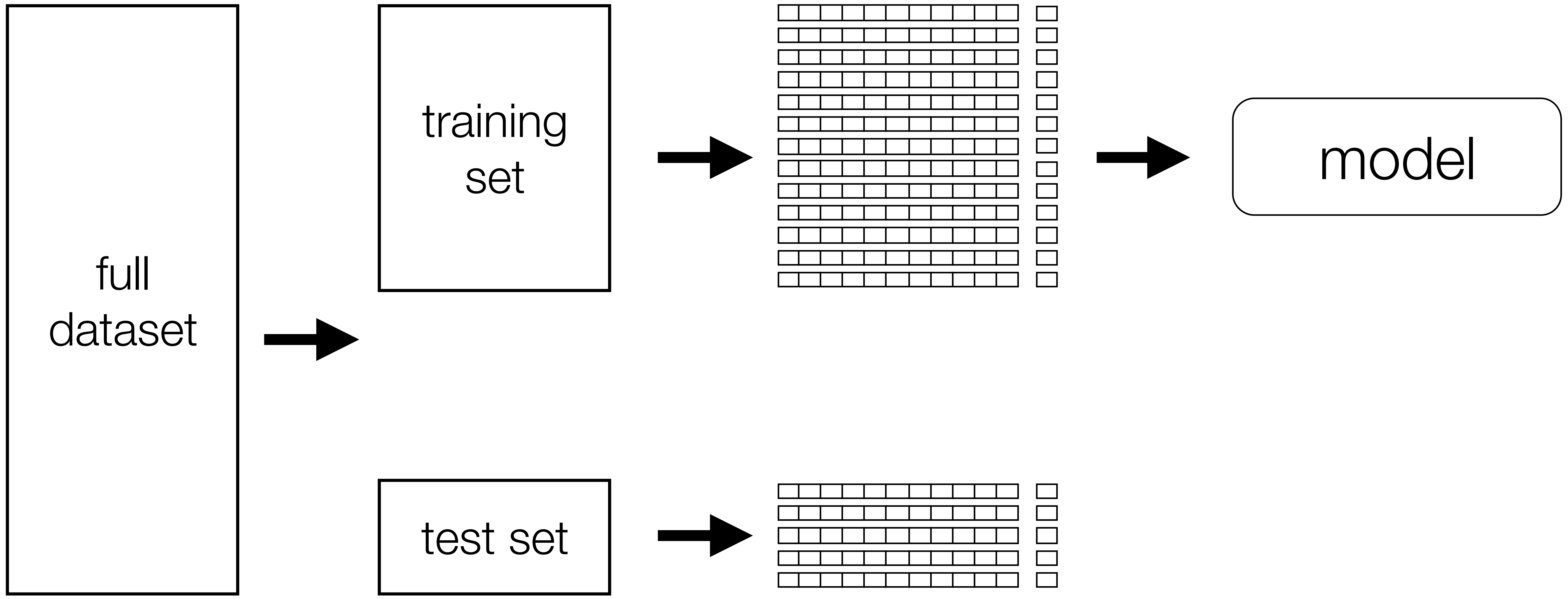
Split Data

Supervised Learning Pipeline





Supervised Learning Pipeline



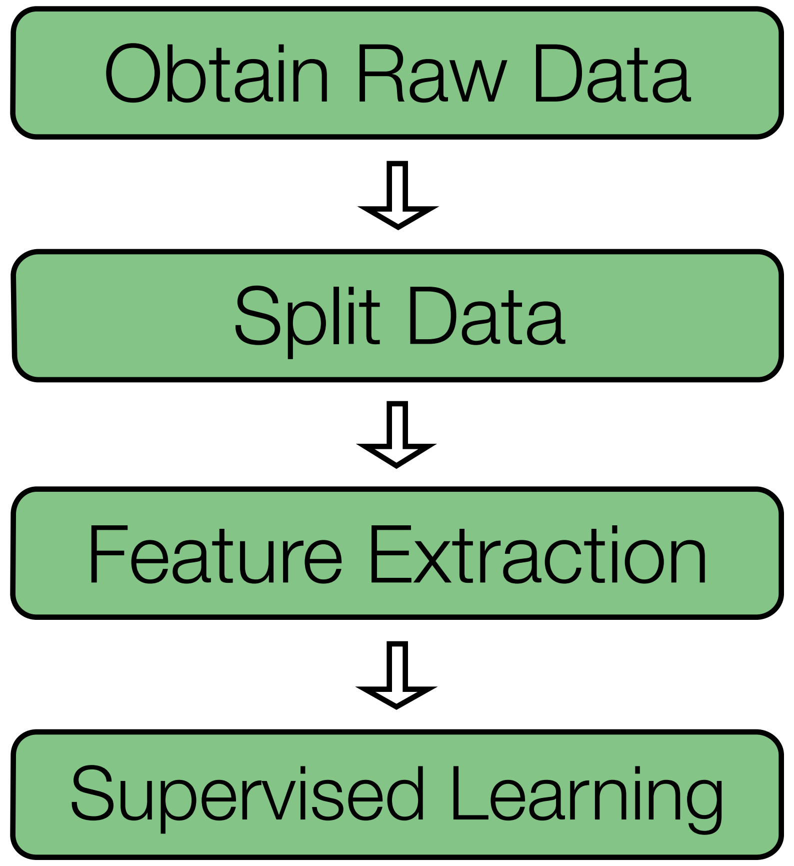
full dataset

training set

test set

model

Supervised Learning Pipeline

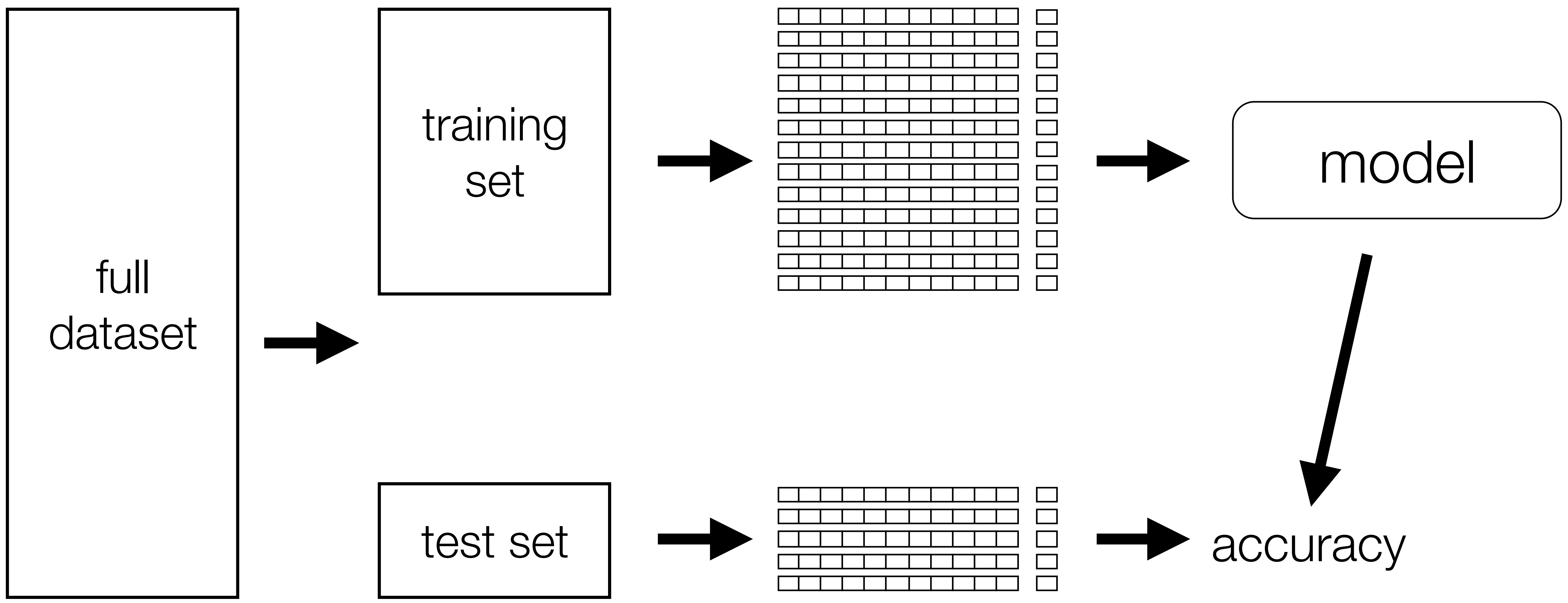


Obtain Raw Data

Split Data

Feature Extraction

Supervised Learning



full dataset

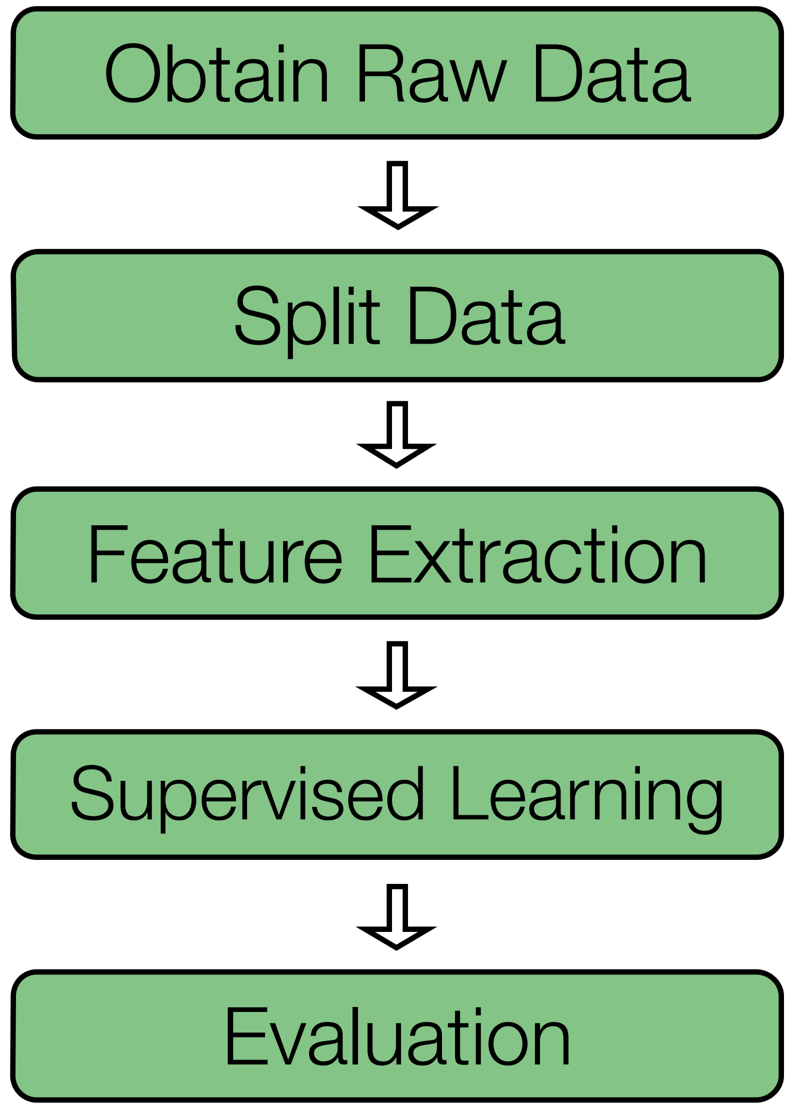
training set

test set

model

accuracy

Supervised Learning Pipeline



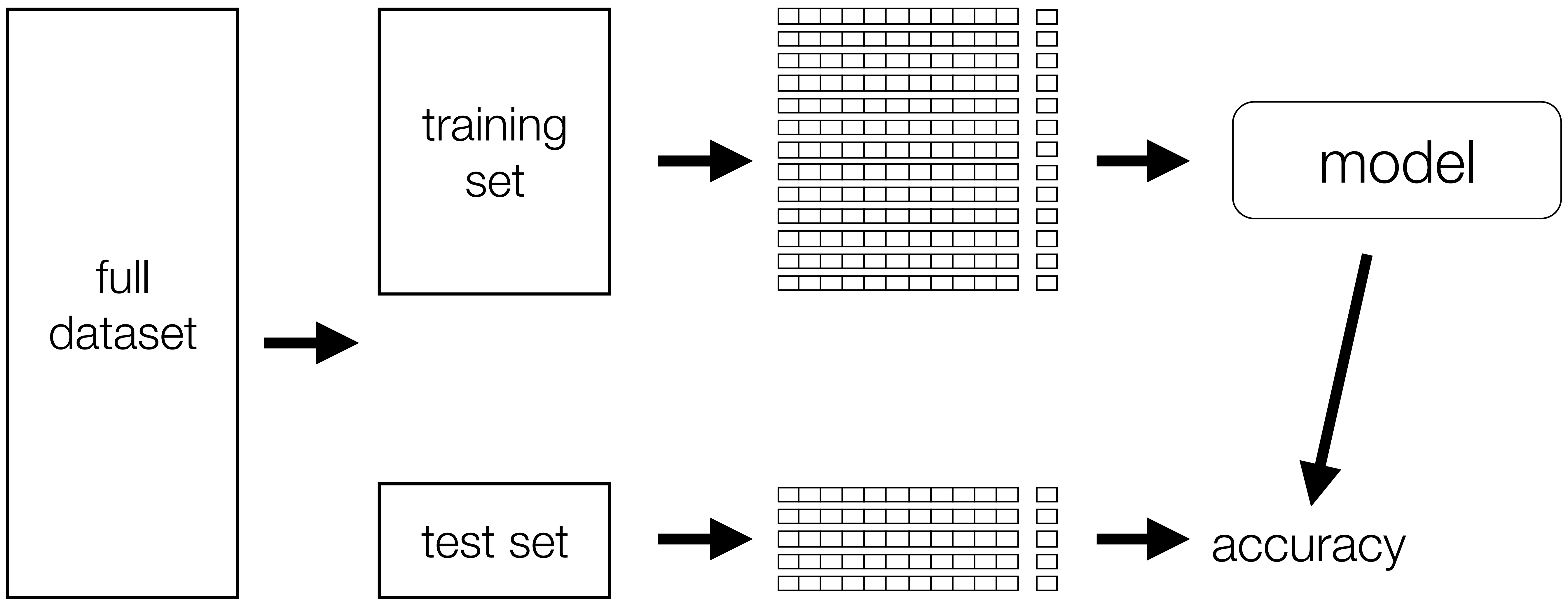
Obtain Raw Data

Split Data

Feature Extraction

Supervised Learning

Evaluation



full dataset

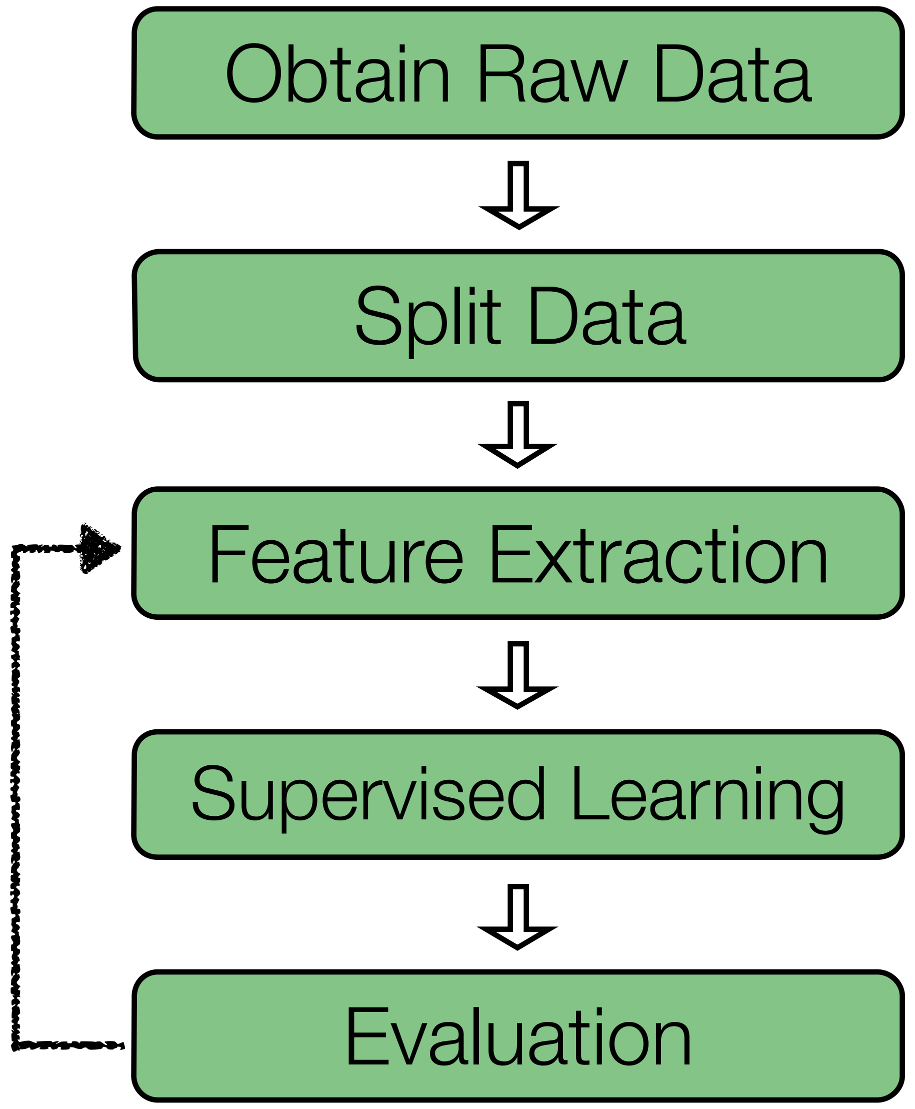
training set

test set

model

accuracy

Supervised Learning Pipeline



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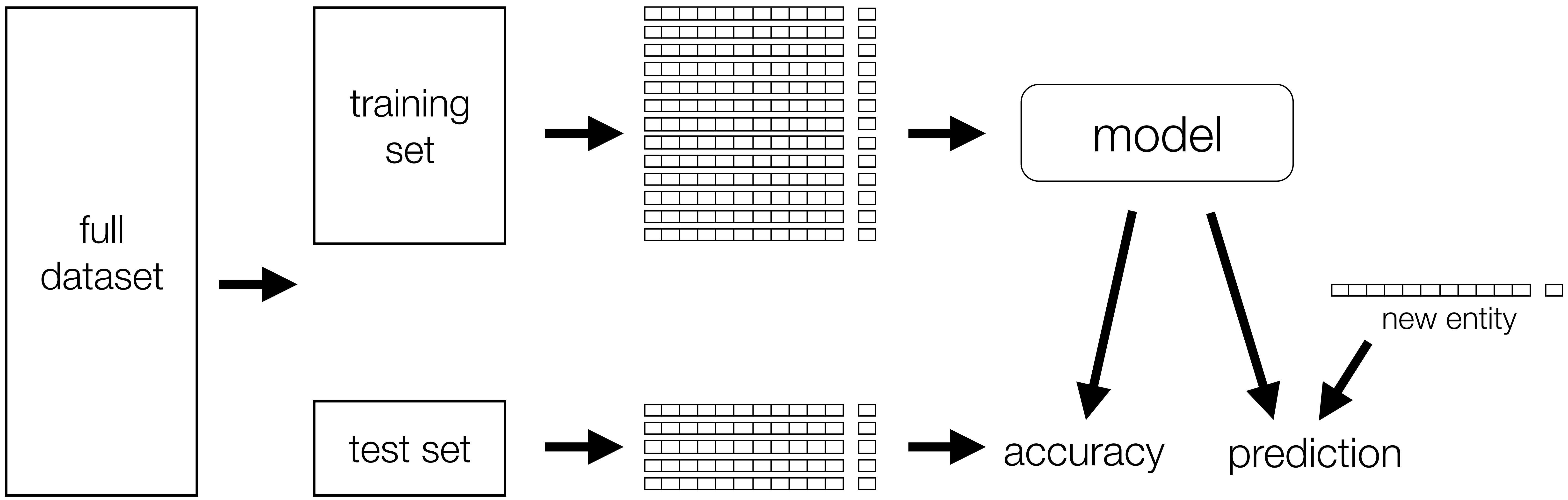


full dataset

training set

test set

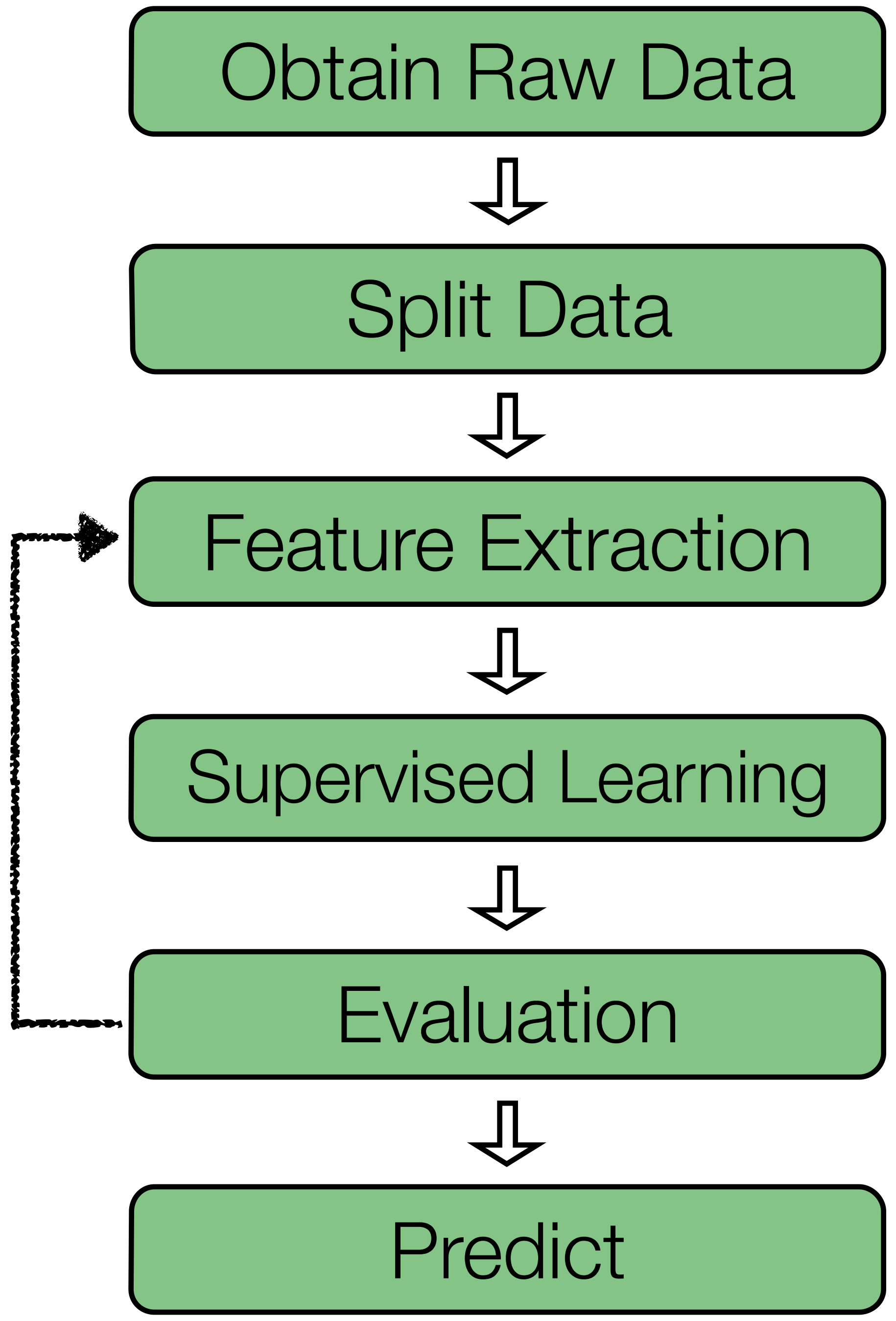
model

new entity

accuracy

prediction

Supervised Learning Pipeline



Obtain Raw Data

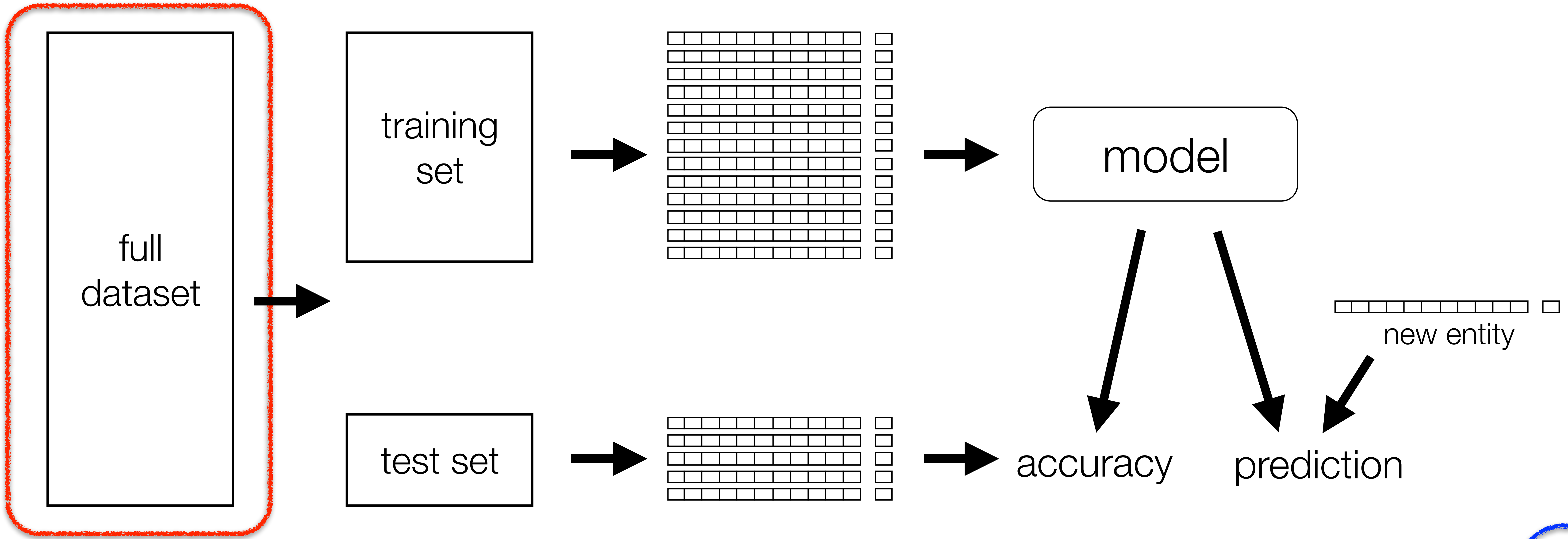
Split Data

Feature Extraction

Supervised Learning

Evaluation

Predict



full dataset

training set

test set

model

accuracy

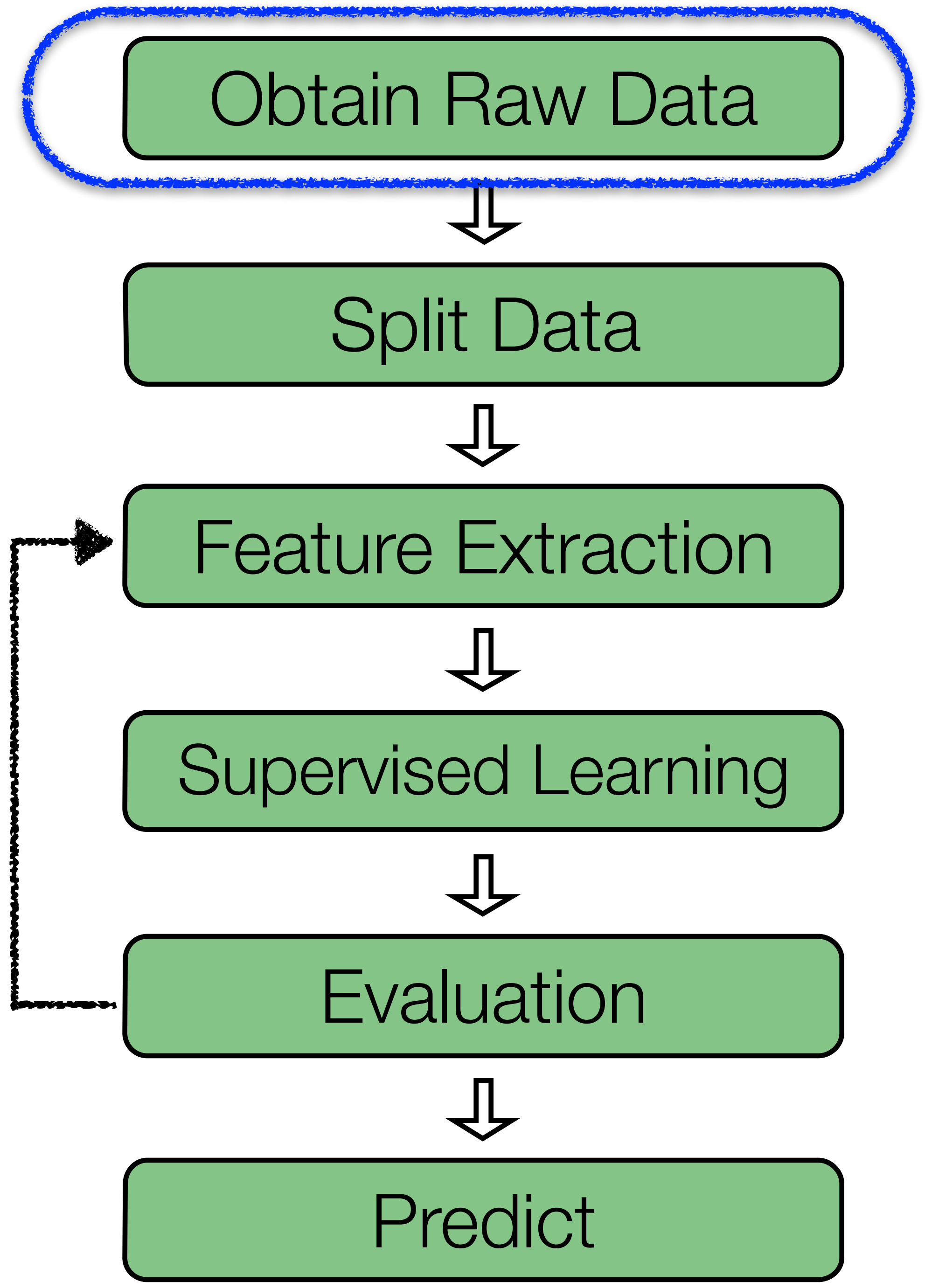
prediction

new entity

Goal: Predict song’s release year from audio features

Raw Data: Millionsong Dataset from UCI ML Repository

- Western, commercial tracks from 1980-2014
- 12 timbre averages (features) and release year (label)



Obtain Raw Data

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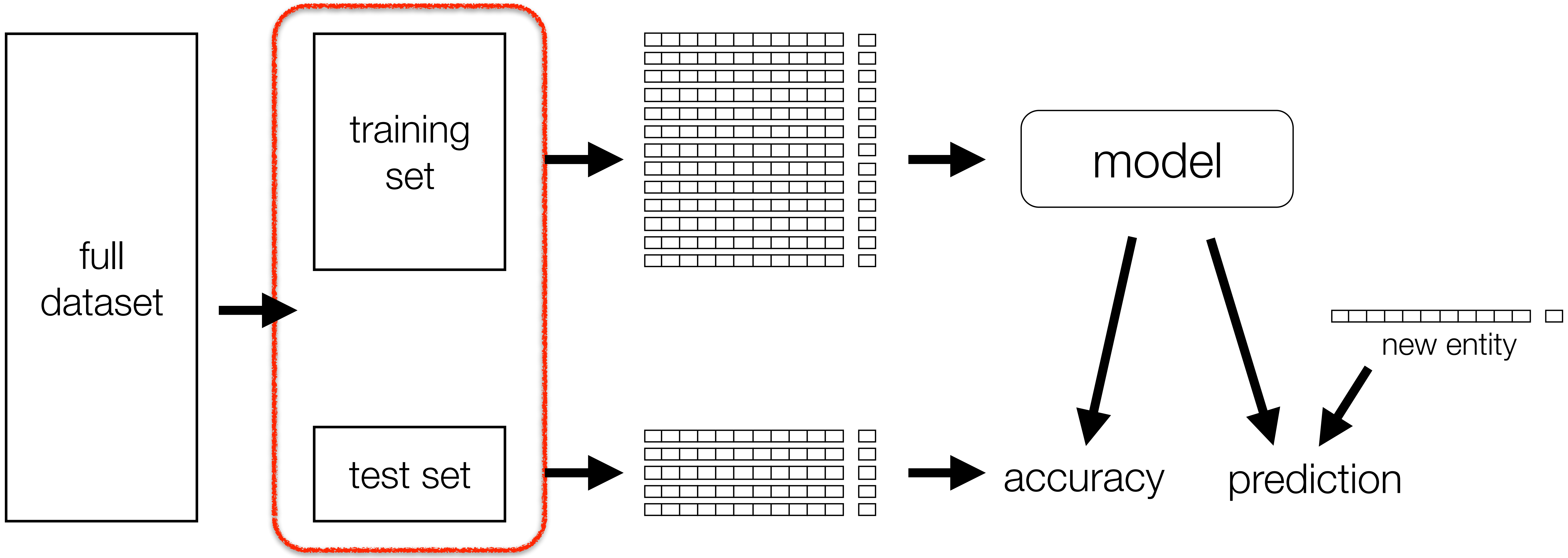
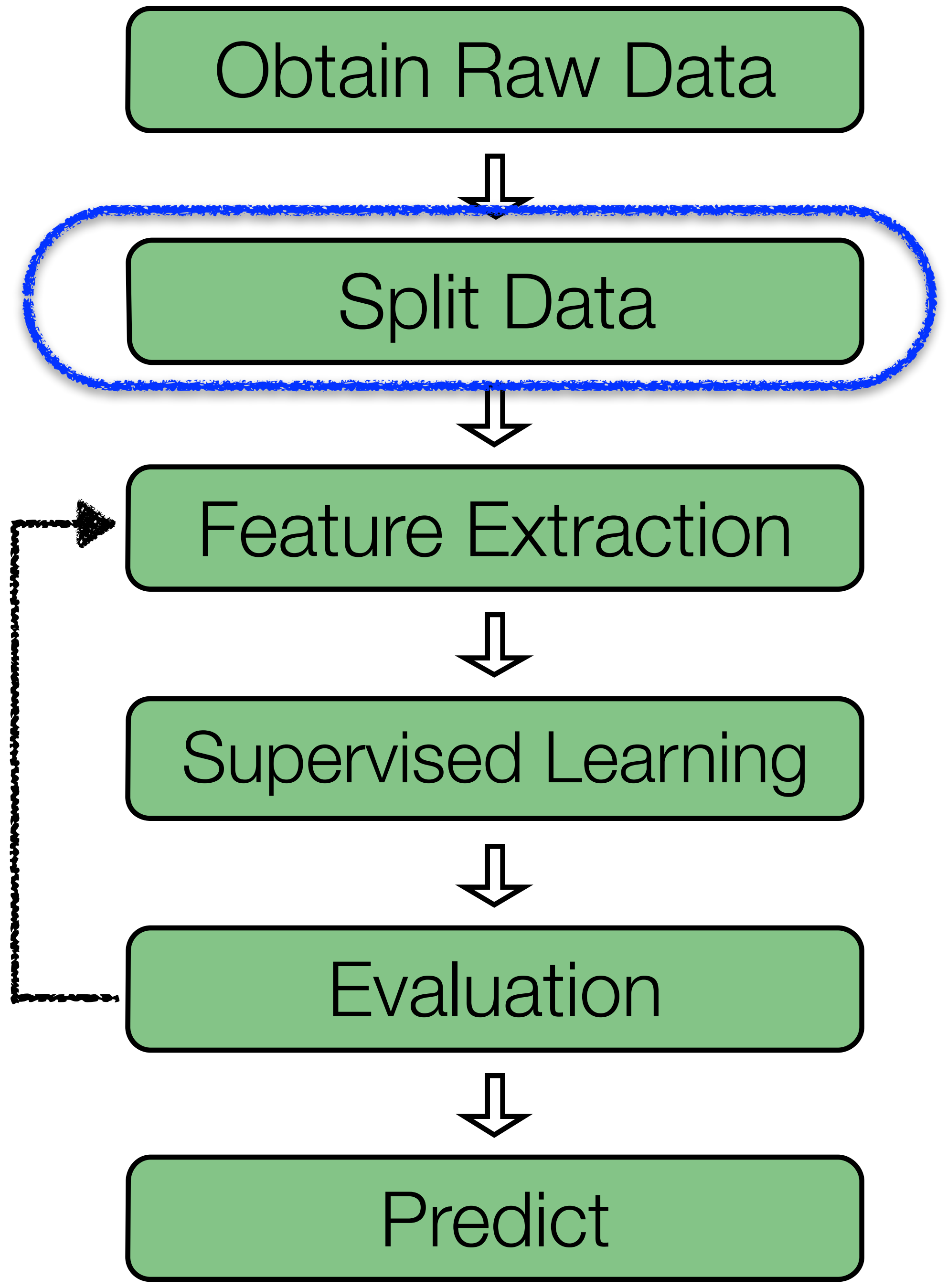
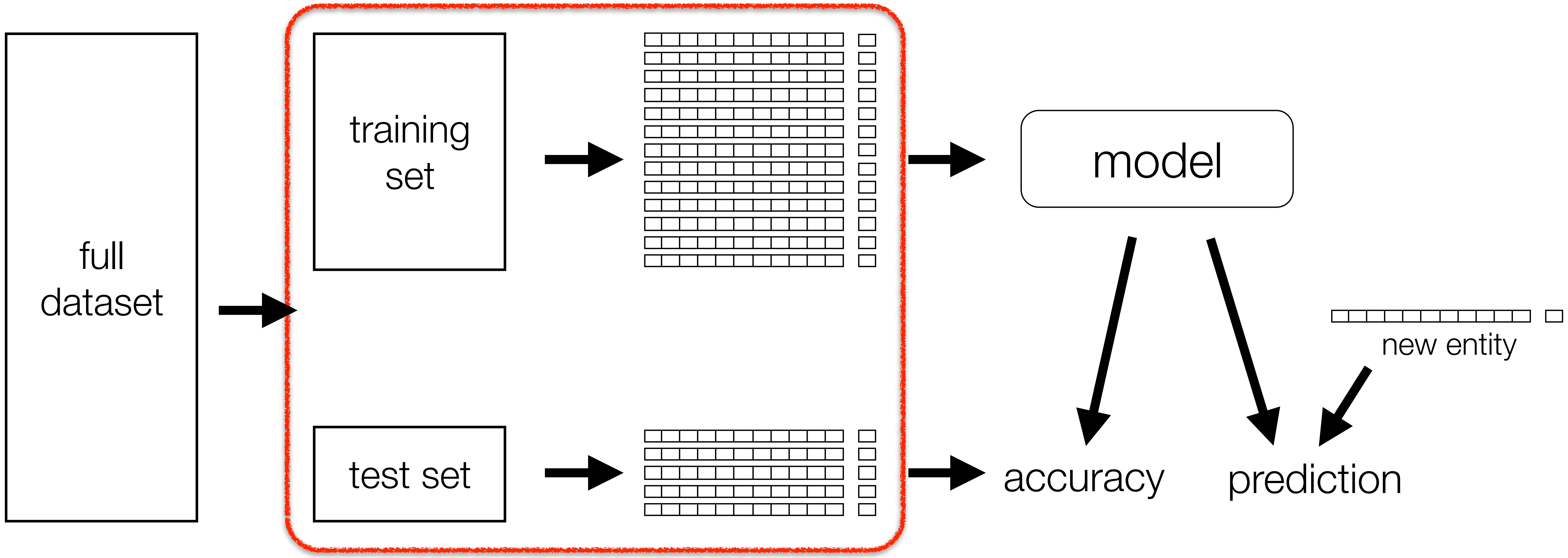
Predict



Split Data: Train on training set, evaluate with test set

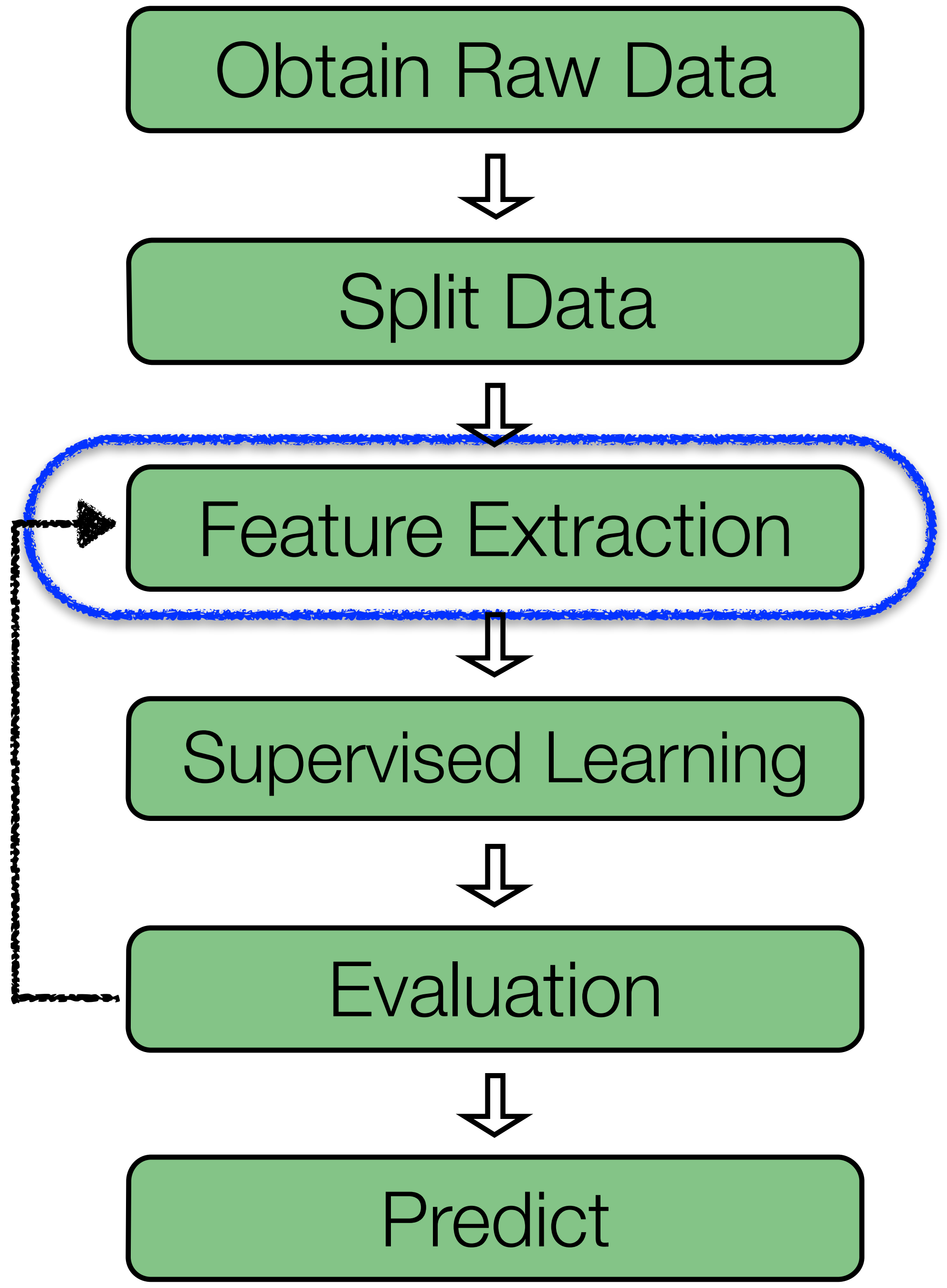
- Test set simulates unobserved data
- Test error tells us whether we've generalized well





Feature Extraction: Quadratic features

- Compute pairwise feature interactions
- Captures covariance of initial timbre features
- Leads to a non-linear model relative to raw features



Given 2 dimensional data, quadratic features are:

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^\top \implies \mathbf{\Phi}(\mathbf{x}) = \begin{bmatrix} x_1^2 & x_1 x_2 & x_2 x_1 & x_2^2 \end{bmatrix}^\top$$

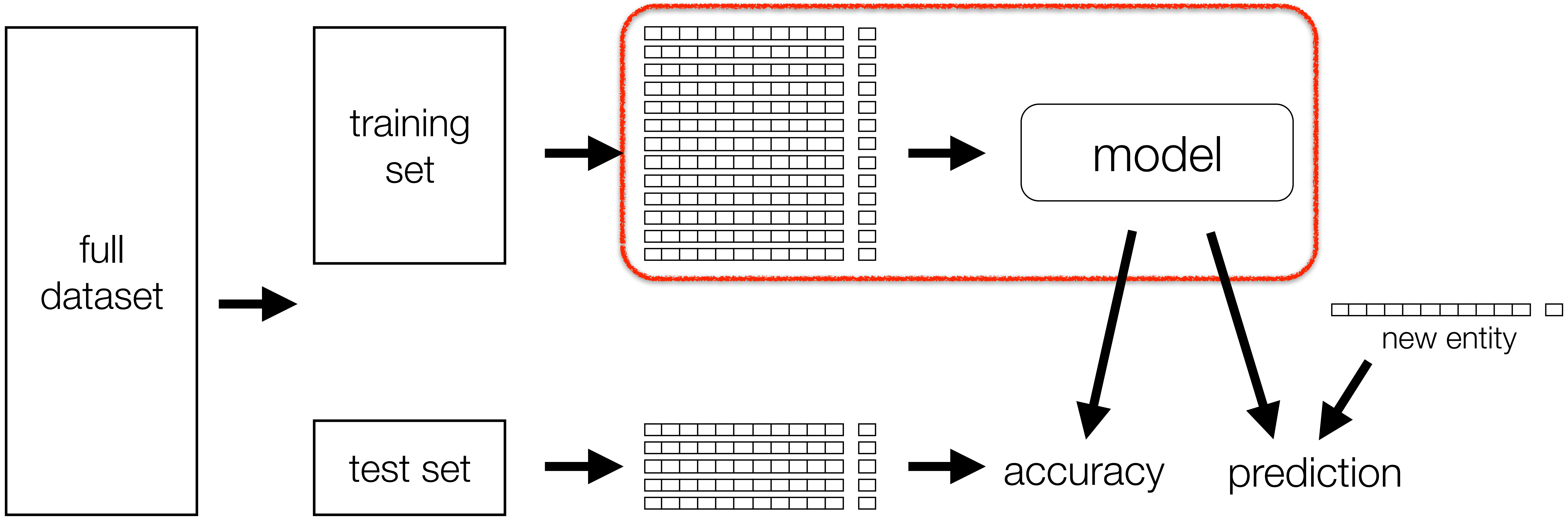
$$\mathbf{z} = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^\top \implies \mathbf{\Phi}(\mathbf{z}) = \begin{bmatrix} z_1^2 & z_1 z_2 & z_2 z_1 & z_2^2 \end{bmatrix}^\top$$

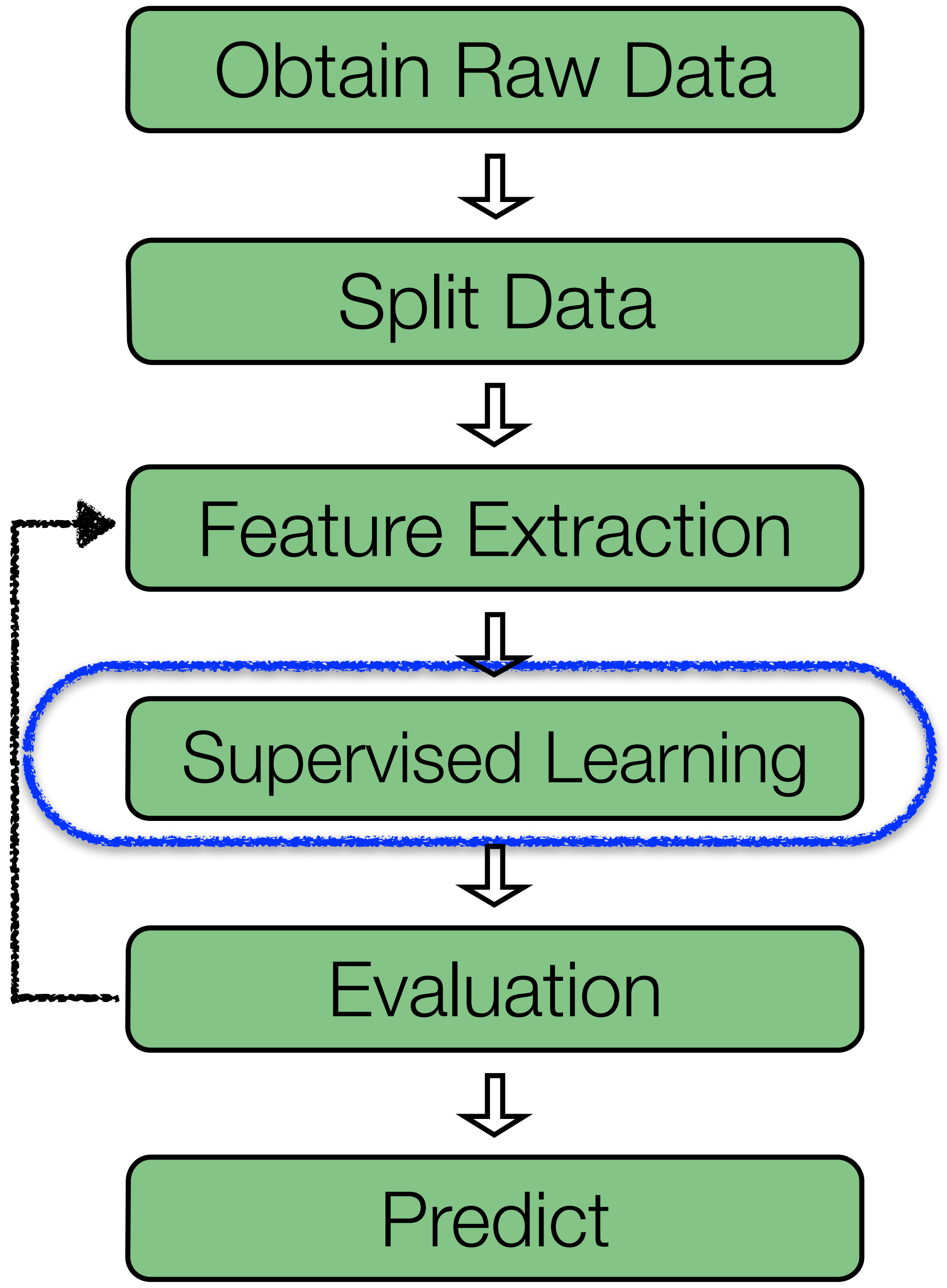
More succinctly:

$$\mathbf{\Phi}'(\mathbf{x}) = \begin{bmatrix} x_1^2 & \sqrt{2} x_1 x_2 & x_2^2 \end{bmatrix}^\top \qquad \mathbf{\Phi}'(\mathbf{z}) = \begin{bmatrix} z_1^2 & \sqrt{2} z_1 z_2 & z_2^2 \end{bmatrix}^\top$$

Equivalent inner products:

$$\mathbf{\Phi}(\mathbf{x})^\top \mathbf{\Phi}(\mathbf{z}) = \sum x_1^2 z_1^2 + 2 x_1 x_2 z_1 z_2 + x_2^2 z_2^2 = \mathbf{\Phi}'(\mathbf{x})^\top \mathbf{\Phi}'(\mathbf{z})$$

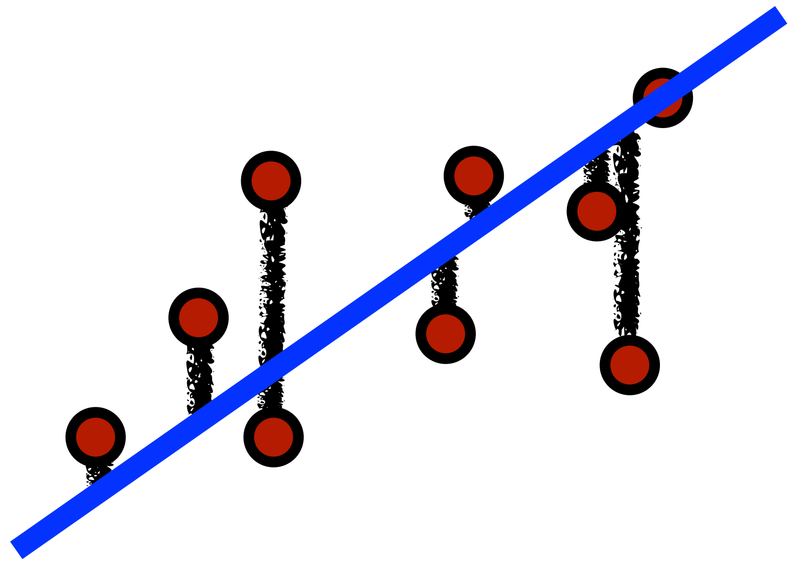




Supervised Learning: Least Squares Regression

- Learn a mapping from entities to continuous labels given a training set
- Audio features → Song year

Given n training points with d features, we define:



- $\mathbf{X} \in \mathbb{R}^{n \times d}$: matrix storing points
- $\mathbf{y} \in \mathbb{R}^{n}$: real-valued labels
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- $\mathbf{w} \in \mathbb{R}^{d}$: regression parameters / model to learn

Ridge Regression: Learn mapping ($\mathbf{w}$) that minimizes residual sum of squares along with a regularization term:

$$\min_{\mathbf{w}} \overset{\text{Training Error}}{\overline{||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2}} + \overset{\text{Model Complexity}}{\overline{\lambda||\mathbf{w}||_2^2}}$$

Closed-form solution: $\mathbf{w} = (\mathbf{X}^\top \mathbf{X} + \lambda \mathbf{I}_d)^{-1} \mathbf{X}^\top \mathbf{y}$

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free parameter trades off between training error and model complexity

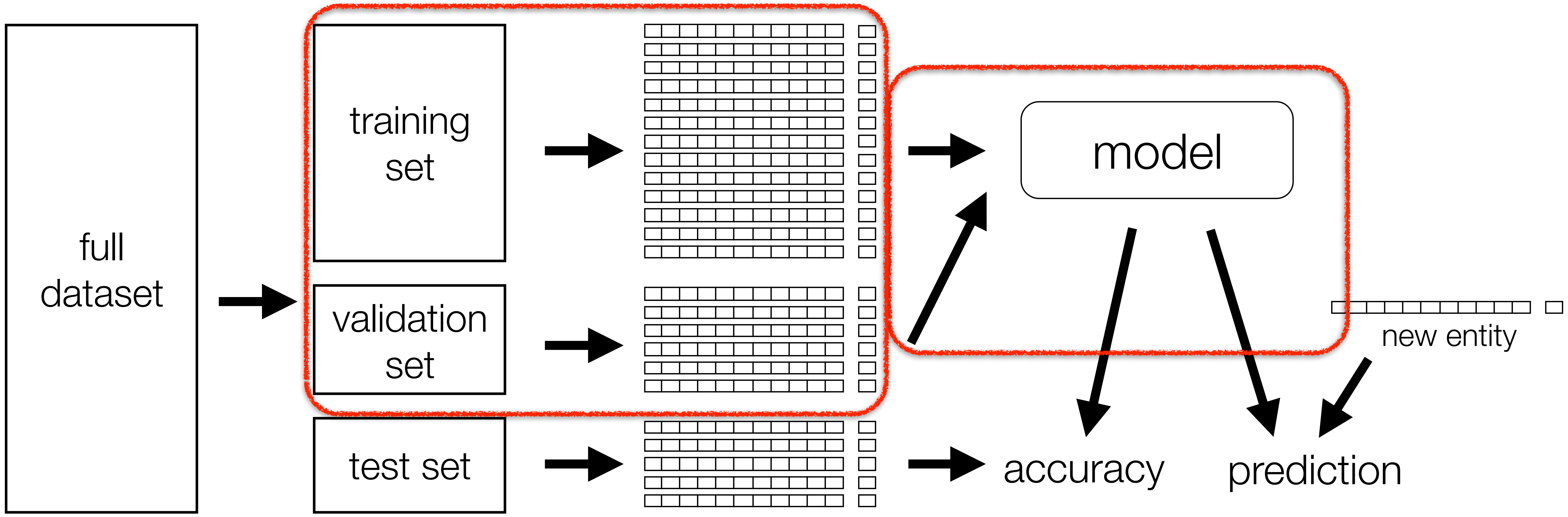
How do we choose a good value for this free parameter?

- Most methods have free parameters / ‘hyperparameters’ to tune

First thought: Search over multiple values, evaluate each on test set

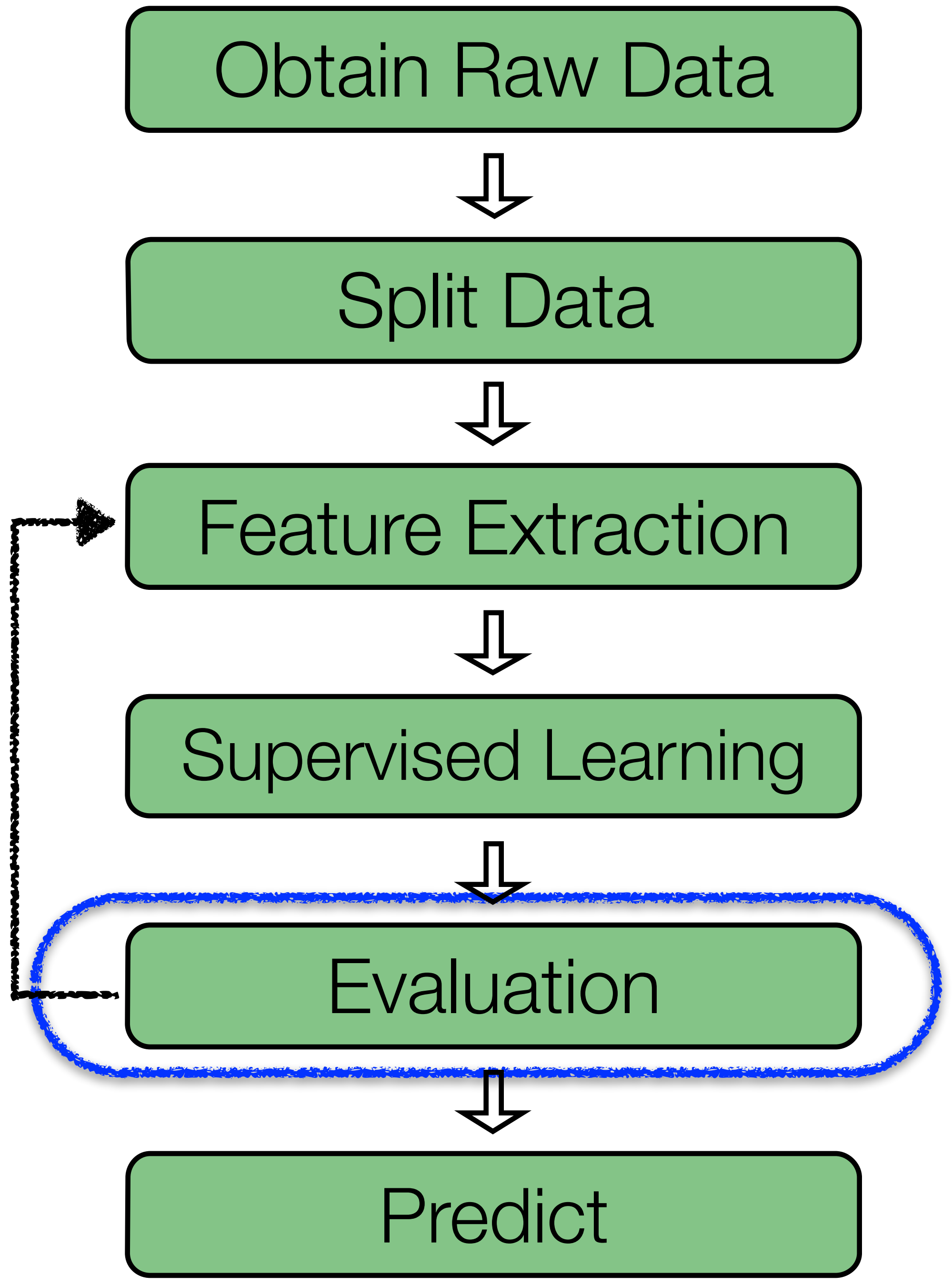
- But, goal of test set is to simulate unobserved data
- We may overfit if we use it to choose hyperparameters

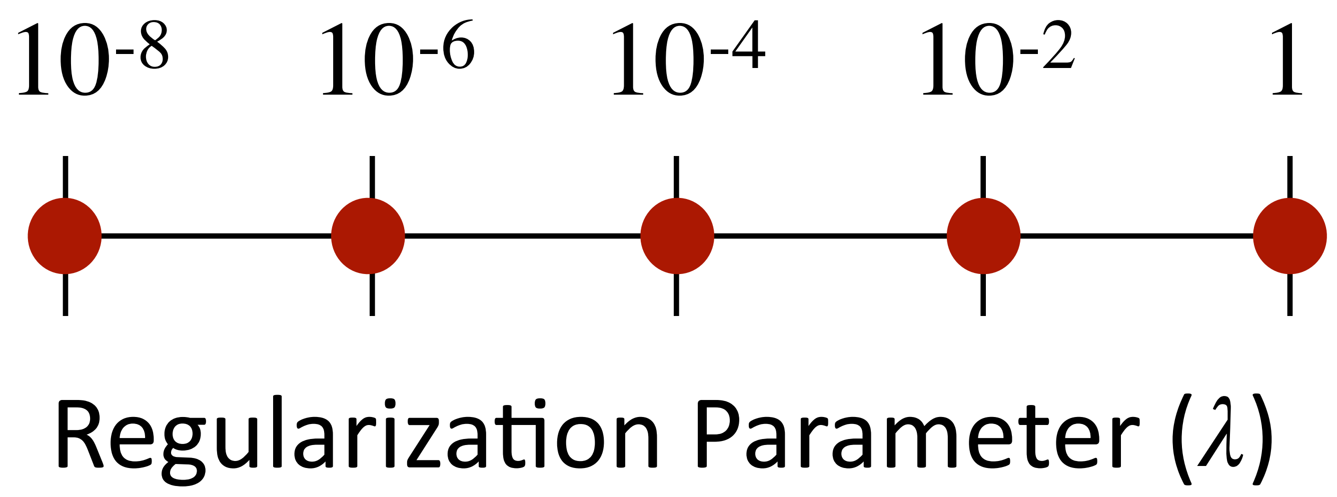
Second thought: **Create another hold out dataset for this search**

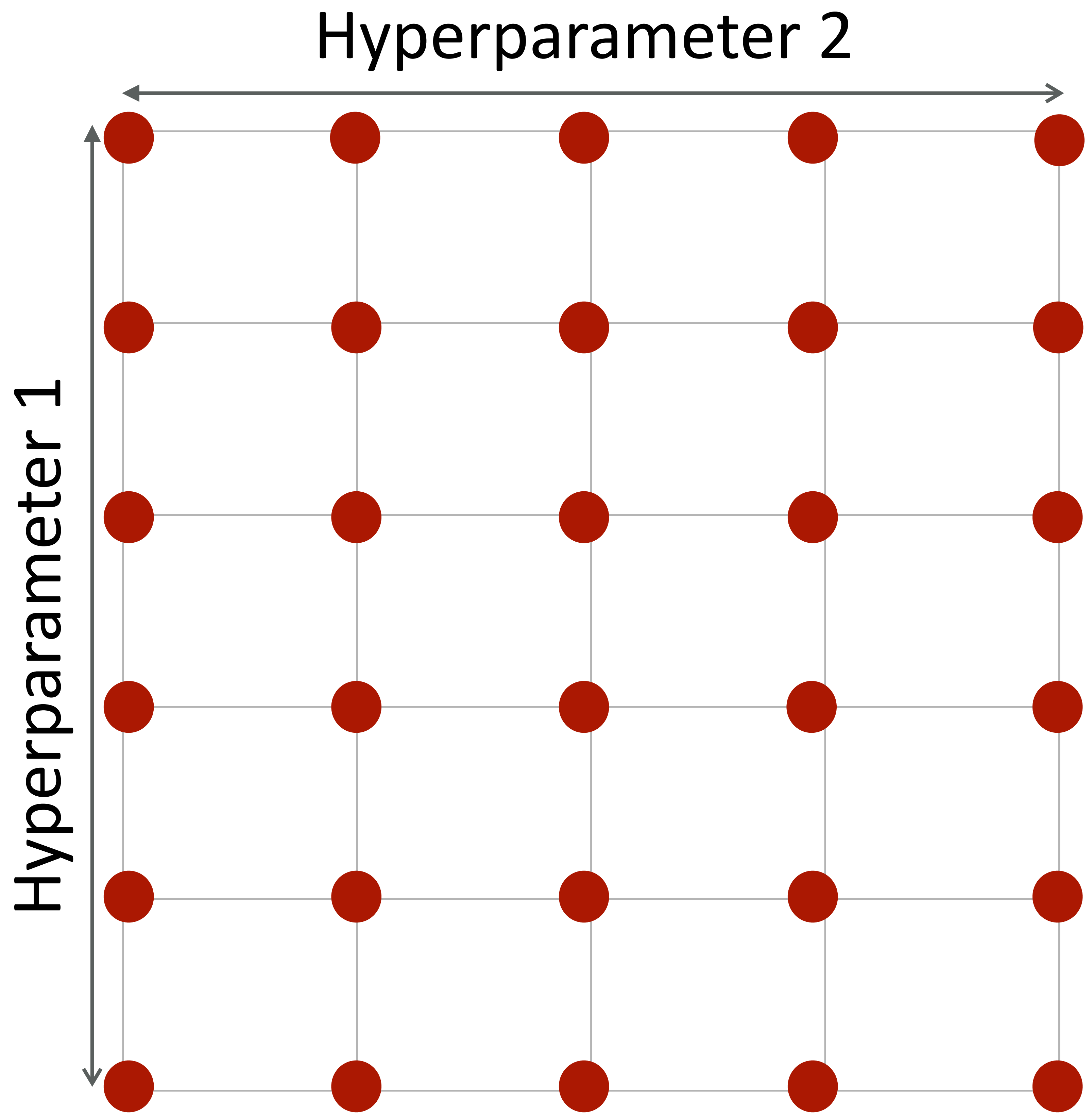


Evaluation (Part 1): Hyperparameter tuning

- *Training*: train various models
- *Validation*: evaluate various models (e.g., Grid Search)
- Test: evaluate final model’s accuracy



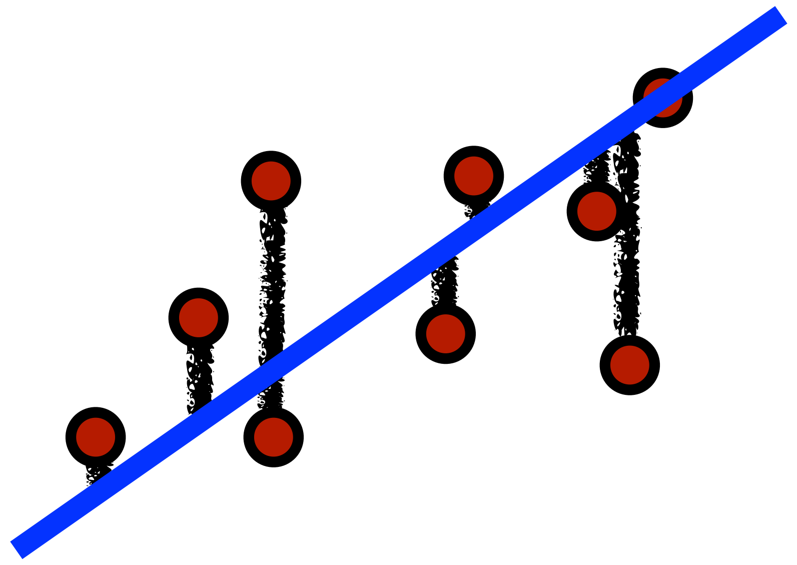




Grid Search: Exhaustively search through hyperparameter space

- Define and discretize search space (linear or log scale)
- Evaluate points via validation error

Evaluating Predictions



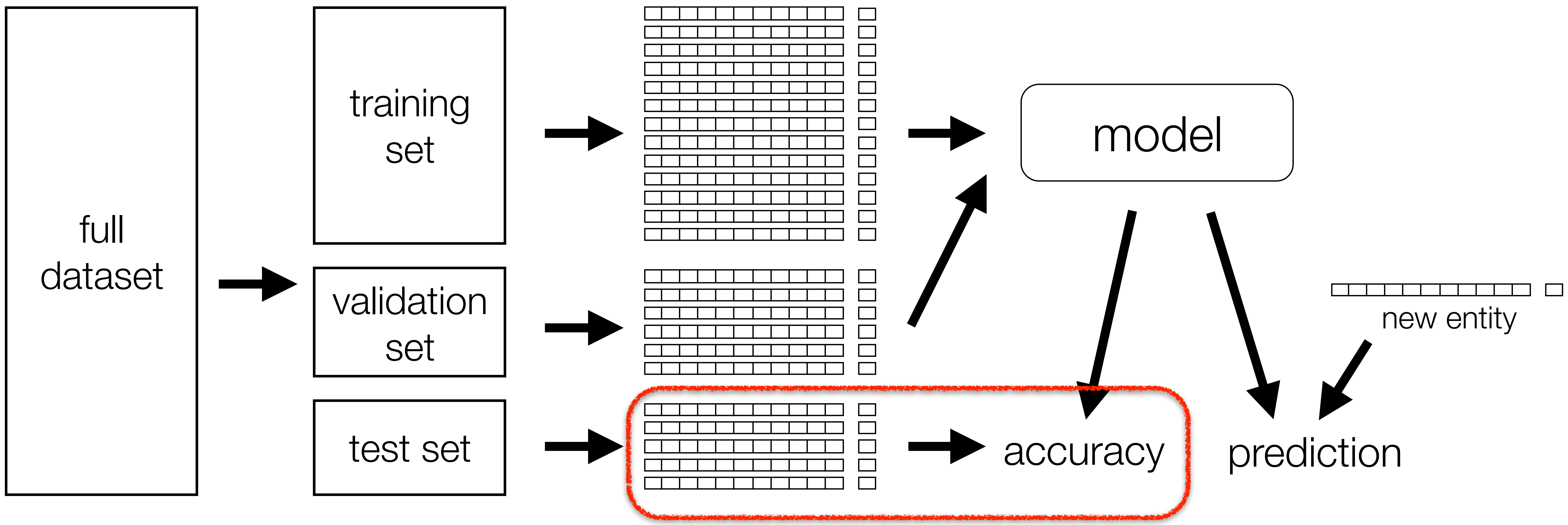
How can we compare labels and predictions for n validation points?

Least squares optimization involves squared loss, $(y - \hat{y})^2$, so it seems reasonable to use mean squared error (**MSE**):

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (\hat{y}^{(i)} - y^{(i)})^2$$

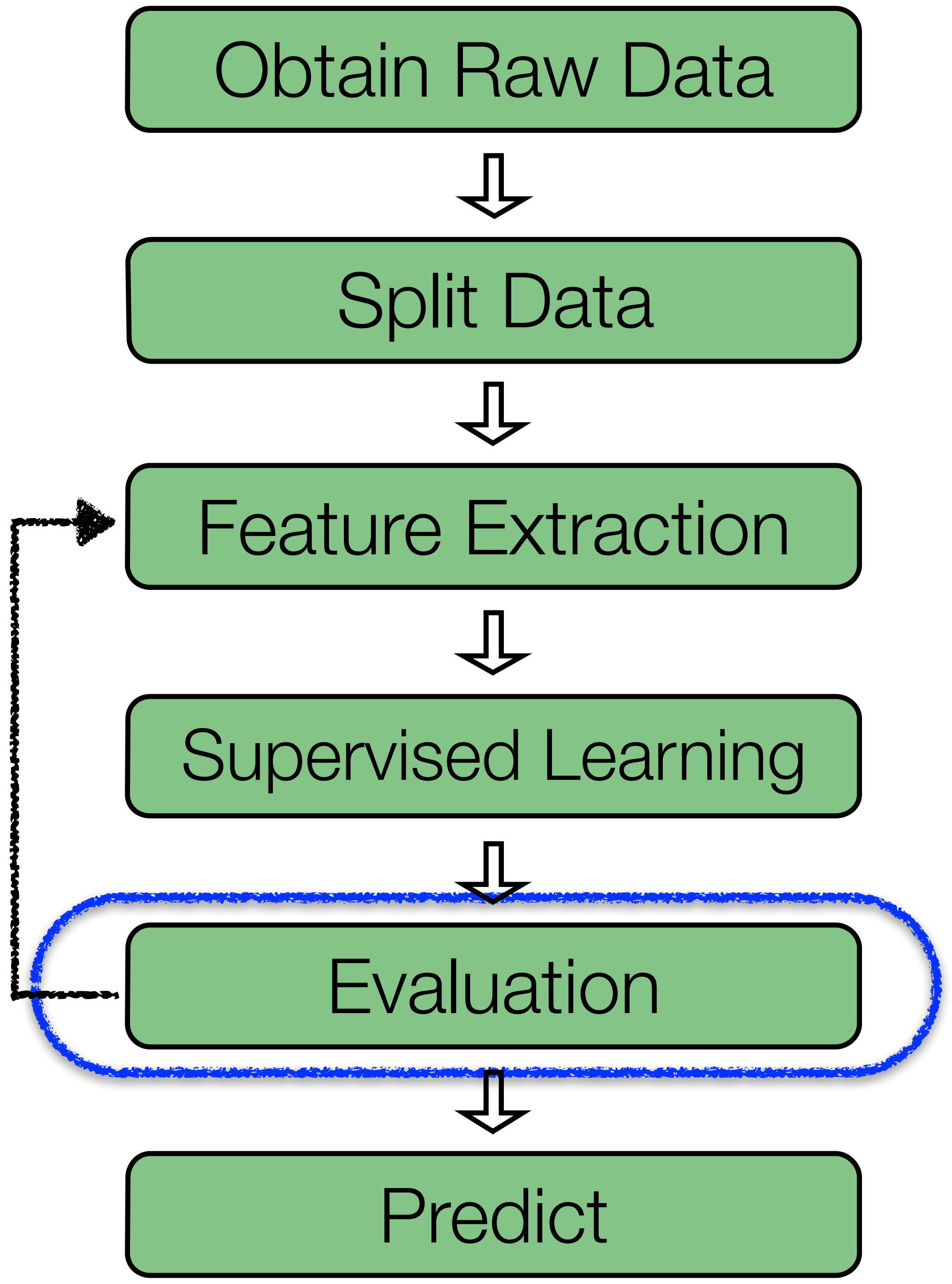
But MSE’s unit of measurement is square of quantity being measured, e.g., “squared years” for song prediction

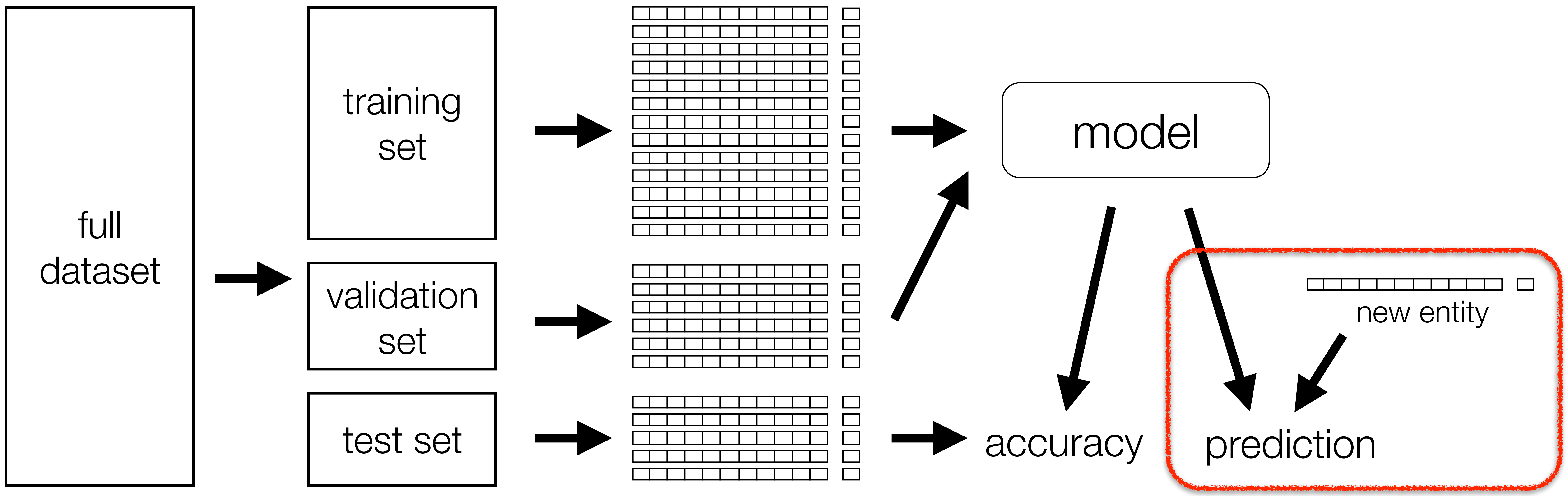
More natural to use root-mean-square error (**RMSE**), i.e., $\sqrt{\text{MSE}}$



Evaluation (Part 2): Evaluate final model

- Training set: train various models
- Validation set: evaluate various models
- *Test set*: evaluate final model's accuracy





full dataset

training set

validation set

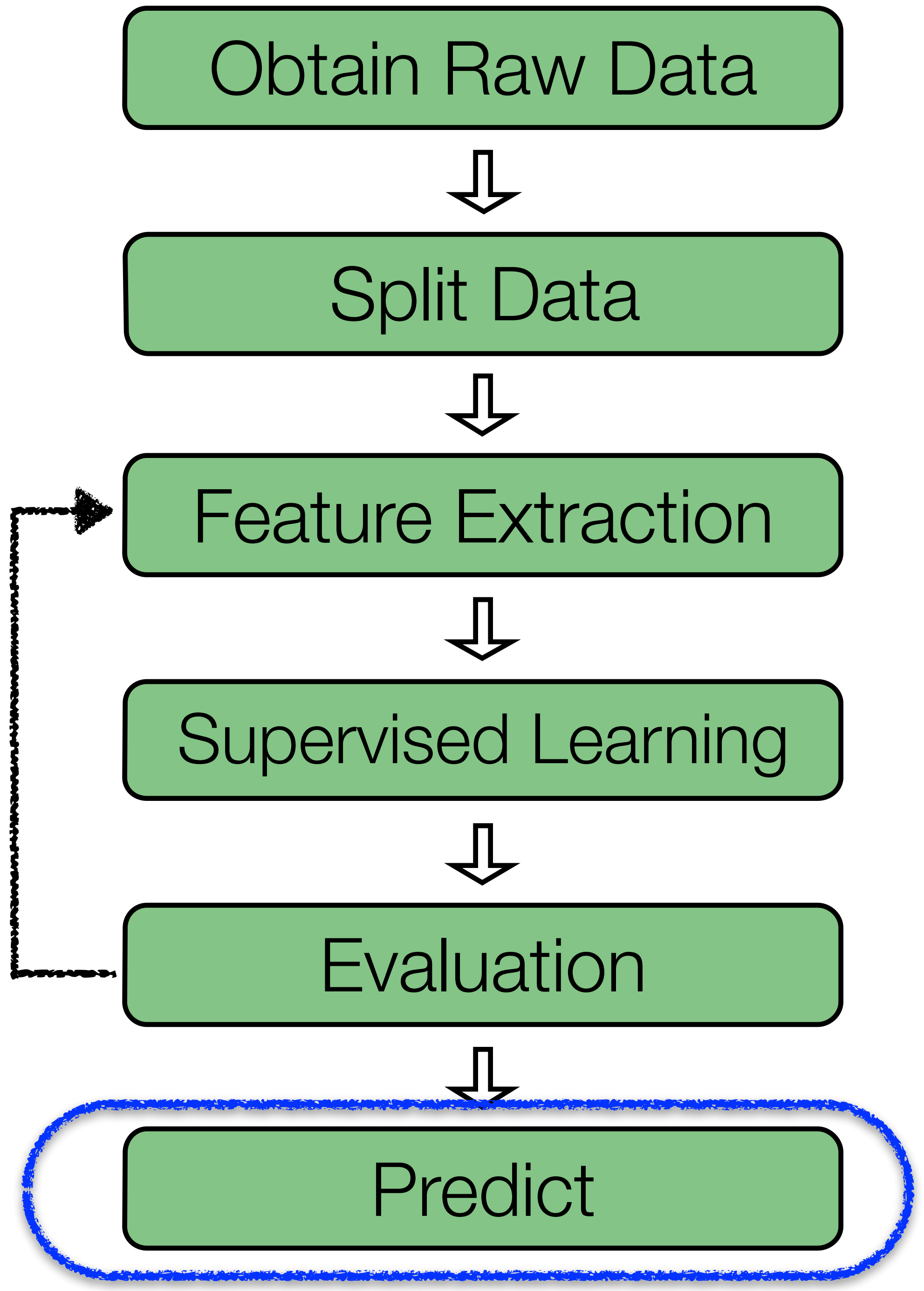
test set

model

accuracy

new entity

prediction



Obtain Raw Data

Split Data

Feature Extraction

Supervised Learning

Evaluation

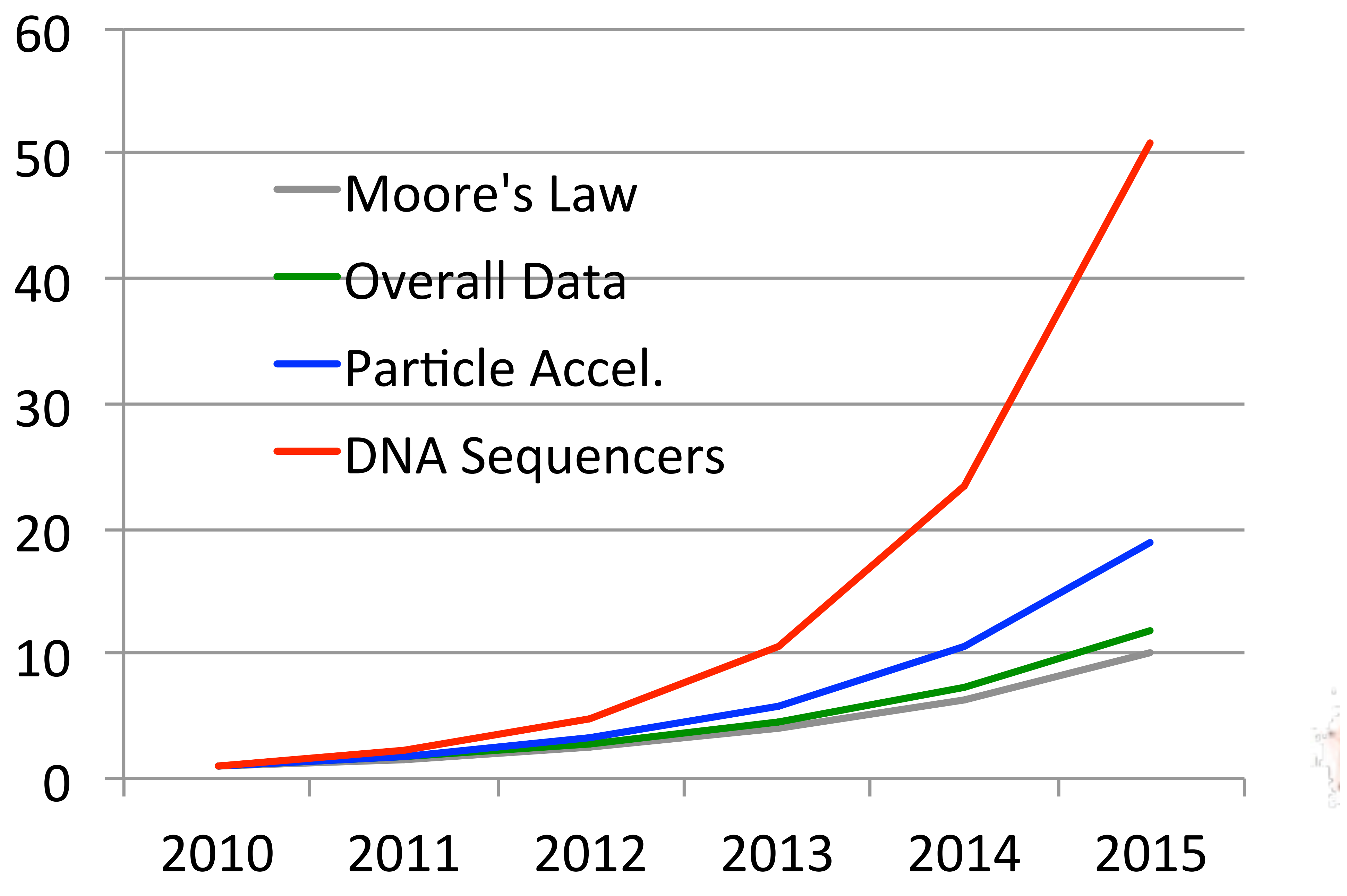
Predict

Predict: Final model can then be used to make predictions on future observations, e.g., new songs

Distributed ML: Computation and Storage

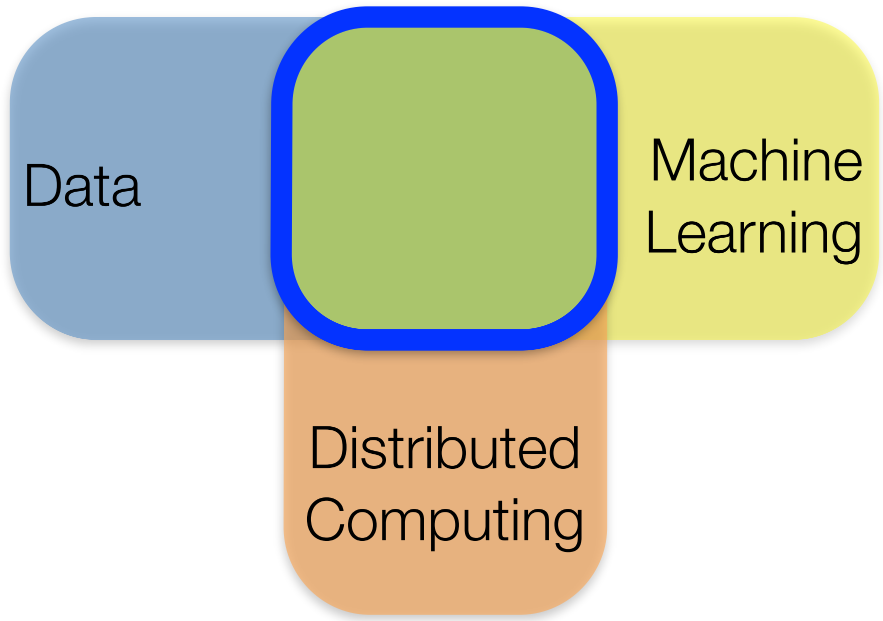
Challenge: Scalability

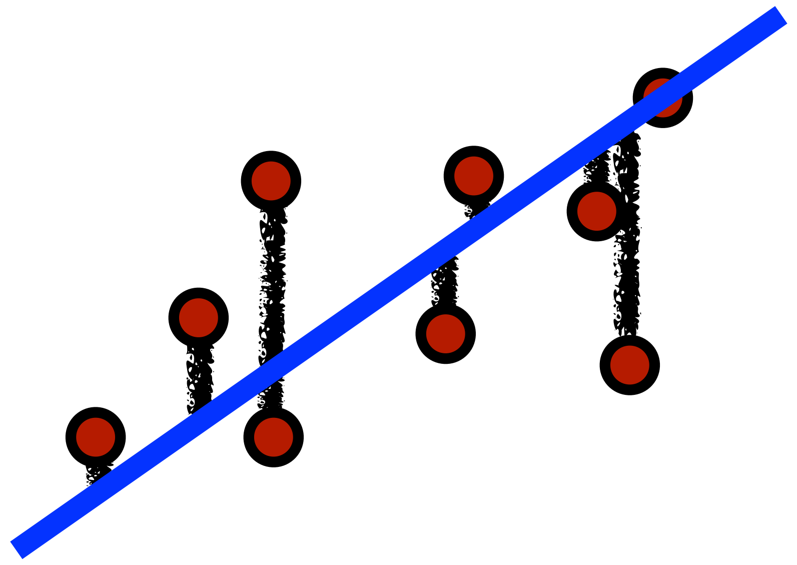
Classic ML techniques are not always suitable for modern datasets



Data Grows Faster than Moore's Law

[IDC report, Kathy Yelick, LBNL]





Least Squares Regression: Learn mapping ($\mathbf{w}$) from features to labels that minimizes residual sum of squares:

$$\min_{\mathbf{w}} ||\mathbf{X}\mathbf{w} - \mathbf{y}||_2^2$$

Closed form solution: $\mathbf{w} = (\mathbf{X}^\top \mathbf{X})^{-1}\mathbf{X}^\top \mathbf{y}$ (if inverse exists)

How do we solve this computationally?

- Computational profile similar for Ridge Regression

Computing Closed Form Solution

$$\mathbf{w} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$$

Computation: O($nd^2 + d^3$) operations

Consider number of arithmetic operations (+, –, ×, /)

Computational bottlenecks:

- Matrix multiply of $\mathbf{X}^\top \mathbf{X}$: O(nd^2) operations
- Matrix inverse: O(d^3) operations

Other methods (Cholesky, QR, SVD) have same complexity

Storage Requirements

$$\mathbf{w} = (\mathbf{X}^\top \mathbf{X})^{-1}\mathbf{X}^\top \mathbf{y}$$

Computation: O($nd^2 + d^3$) operations

Storage: O($nd + d^2$) floats

Consider storing values as floats (8 bytes)

Storage bottlenecks:

- $\mathbf{X}^\top \mathbf{X}$ and its inverse: O(d^2) floats
- $\mathbf{X}$: O(nd) floats

Big n and Small d

$$\mathbf{w} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$$

Computation: O($\underline{nd^2}$ + d^3) operations

Storage: O($\underline{nd}$ + d^2) floats

Assume O(d^3) computation and O(d^2) storage feasible on single machine

Storing $\mathbf{X}$ and computing $\mathbf{X}^\top \mathbf{X}$ are the bottlenecks

Can distribute storage and computation!

- Store data points (rows of $\mathbf{X}$) across machines
- Compute $\mathbf{X}^\top \mathbf{X}$ as a sum of outer products

Matrix Multiplication via Inner Products

Each entry of output matrix is result of inner product of inputs matrices

$$\begin{bmatrix} 9 & 3 & 5 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 28 & \\ & \end{bmatrix}$$

$$9 \times 1 + 3 \times 3 + 5 \times 2 = 28$$

Matrix Multiplication via Inner Products

Each entry of output matrix is result of inner product of inputs matrices

$$\begin{bmatrix} 9 & 3 & 5 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 28 & 18 \\ & \end{bmatrix}$$

Matrix Multiplication via Inner Products

Each entry of output matrix is result of inner product of inputs matrices

$$\begin{bmatrix} \boxed{\begin{matrix} 9 & 3 & 5 \end{matrix}} \\ \begin{matrix} 4 & 1 & 2 \end{matrix} \end{bmatrix} \begin{bmatrix} \begin{matrix} 1 \\ 3 \\ 2 \end{matrix} & \boxed{\begin{matrix} 2 \\ -5 \\ 3 \end{matrix}} \end{bmatrix} = \begin{bmatrix} 28 & 18 \\ 11 & 9 \end{bmatrix}$$

Matrix Multiplication via Outer Products

Output matrix is **sum of outer products** between corresponding rows and columns of input matrices

$$\begin{bmatrix} \boxed{\begin{matrix} 9 \\ 4 \end{matrix}} & \begin{matrix} 3 \\ 1 \end{matrix} & \begin{matrix} 5 \\ 2 \end{matrix} \end{bmatrix} \begin{bmatrix} \boxed{\begin{matrix} 1 & 2 \end{matrix}} \\ \begin{matrix} 3 & -5 \end{matrix} \\ \begin{matrix} 2 & 3 \end{matrix} \end{bmatrix} = \begin{bmatrix} \qquad\qquad & \end{bmatrix}$$

$$\begin{bmatrix} 9 & 18 \\ 4 & 8 \end{bmatrix}$$

Matrix Multiplication via Outer Products

Output matrix is **sum of outer products** between corresponding rows and columns of input matrices

$$\left[\begin{matrix}9\\4\end{matrix}\quad\boxed{\begin{matrix}3\\1\end{matrix}}\quad\begin{matrix}5\\2\end{matrix}\right]\left[\begin{matrix}1 \qquad 2\\ \boxed{3 \qquad -5}\\ 2 \qquad 3\end{matrix}\right]=\begin{bmatrix} & \\ & \end{bmatrix}$$

$$\begin{bmatrix}9 & 18\\ 4 & 8\end{bmatrix}+\begin{bmatrix}9 & -15\\ 3 & -5\end{bmatrix}$$

Matrix Multiplication via Outer Products

Output matrix is **sum of outer products** between corresponding rows and columns of input matrices

$$\begin{bmatrix} 9 & 3 & 5 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

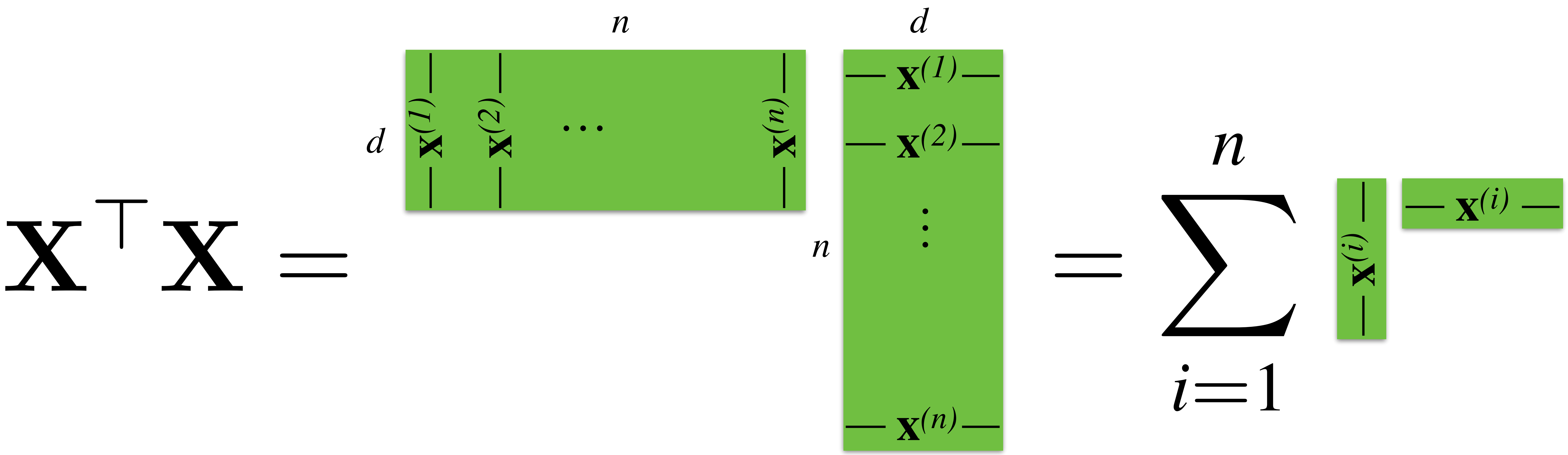
$$\begin{bmatrix} 9 & 18 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 9 & -15 \\ 3 & -5 \end{bmatrix} + \begin{bmatrix} 10 & 15 \\ 4 & 6 \end{bmatrix}$$

Matrix Multiplication via Outer Products

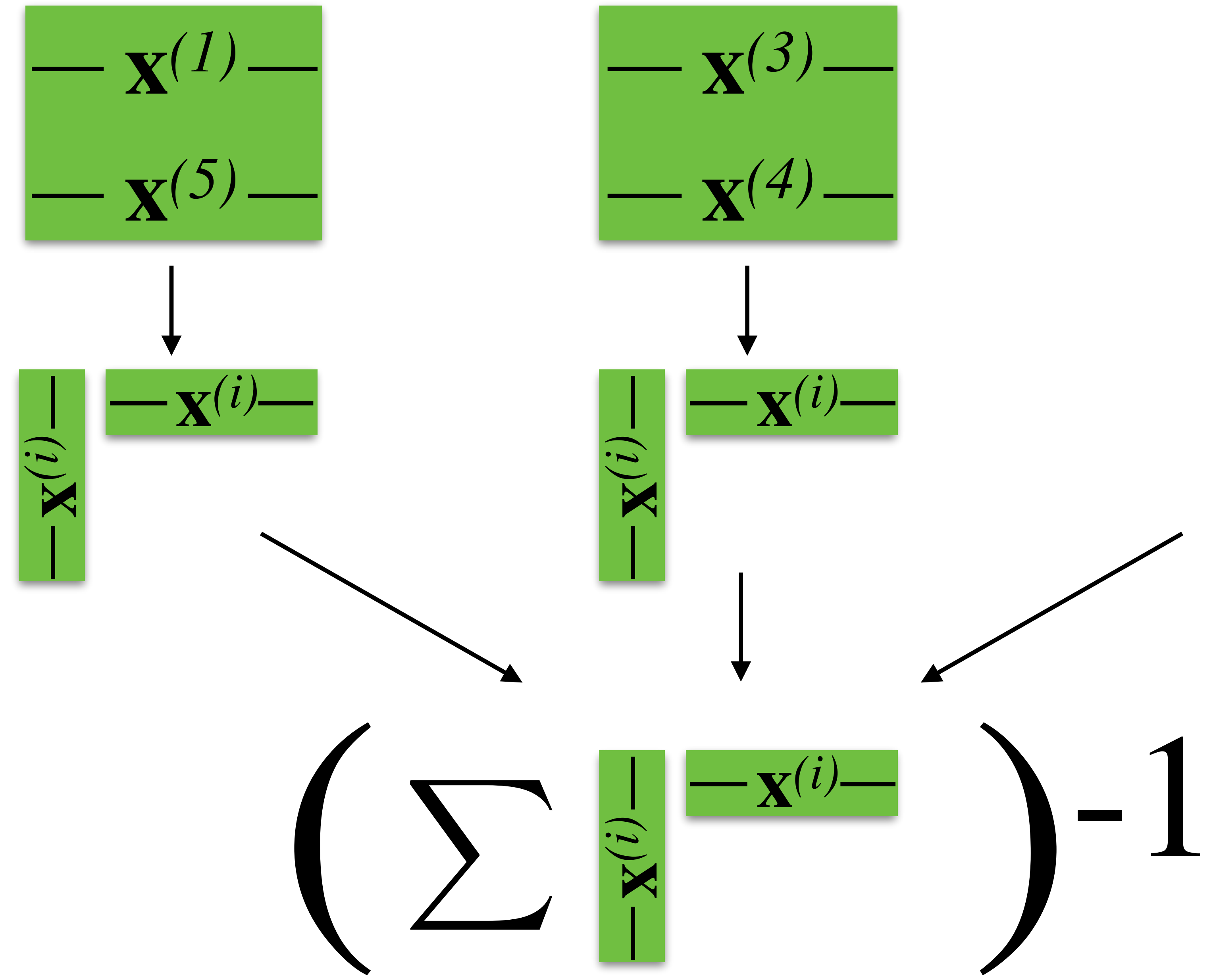
Output matrix is **sum of outer products** between corresponding rows and columns of input matrices

$$\left[\begin{matrix}9 & 3 \\ 4 & 1\end{matrix}\;\boxed{\begin{matrix}5 \\ 2\end{matrix}}\right]\left[\begin{array}{c}\begin{matrix}1 & 2 \\ 3 & -5\end{matrix} \\ \boxed{\begin{matrix}2 & 3\end{matrix}}\end{array}\right]=\begin{bmatrix}28 & 18 \\ 11 & 9\end{bmatrix}$$

$$\begin{bmatrix}9 & 18 \\ 4 & 8\end{bmatrix}+\begin{bmatrix}9 & -15 \\ 3 & -5\end{bmatrix}+\begin{bmatrix}10 & 15 \\ 4 & 6\end{bmatrix}$$



Example: n = 6; 3 workers



workers:

map:

reduce:

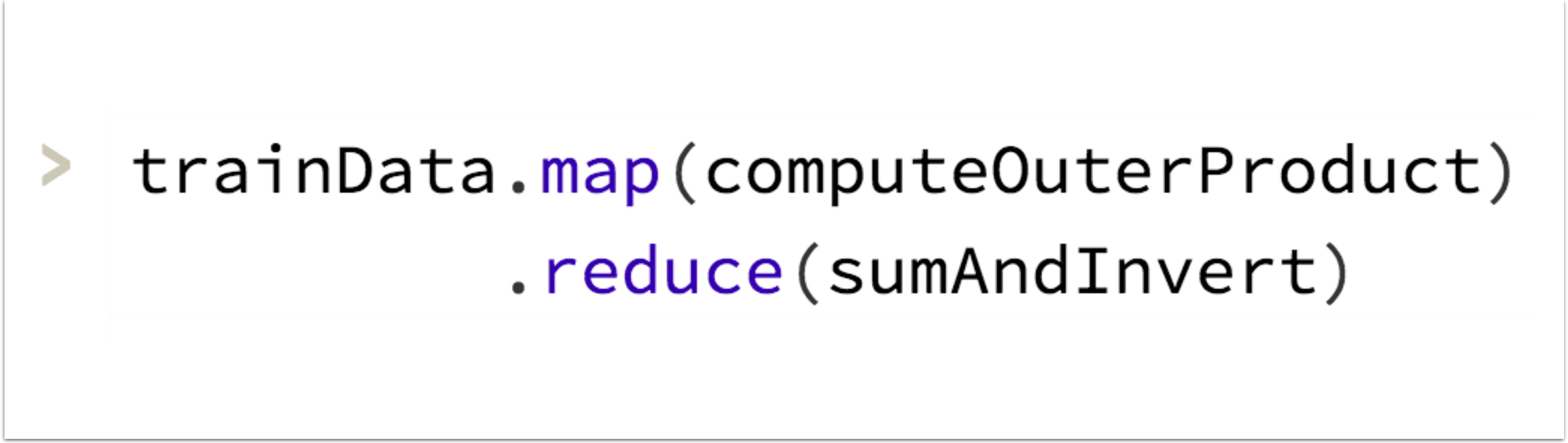
O(nd) Distributed Storage

O(nd^2) Distributed Computation

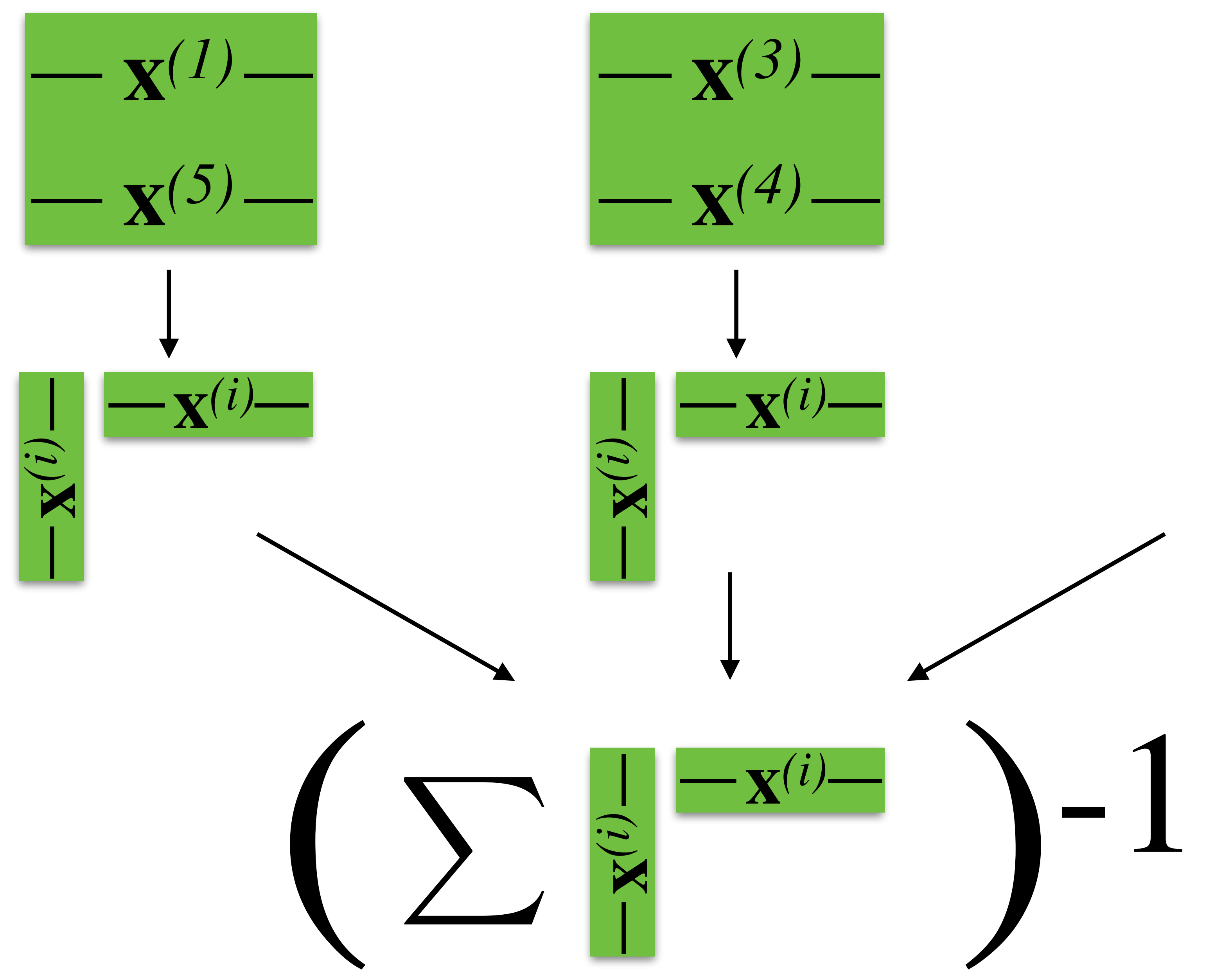
O(d^2) Local Storage

O(d^3) Local Computation

O(d^2) Local Storage



```
> trainData.map(computeOuterProduct)
             .reduce(sumAndInvert)
```



workers:

O(nd) Distributed Storage

map:

O(nd^2) Distributed Computation

O(d^2) Local Storage

reduce:

O(d^3) Local Computation

O(d^2) Local Storage

Distributed ML: Computation and Storage, Part II

UCLA

Cal

databricks™

Big n and Small d

$$\mathbf{w} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$$

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Storage: O($\underline{nd}$ + d^2) floats

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Big n and Small d

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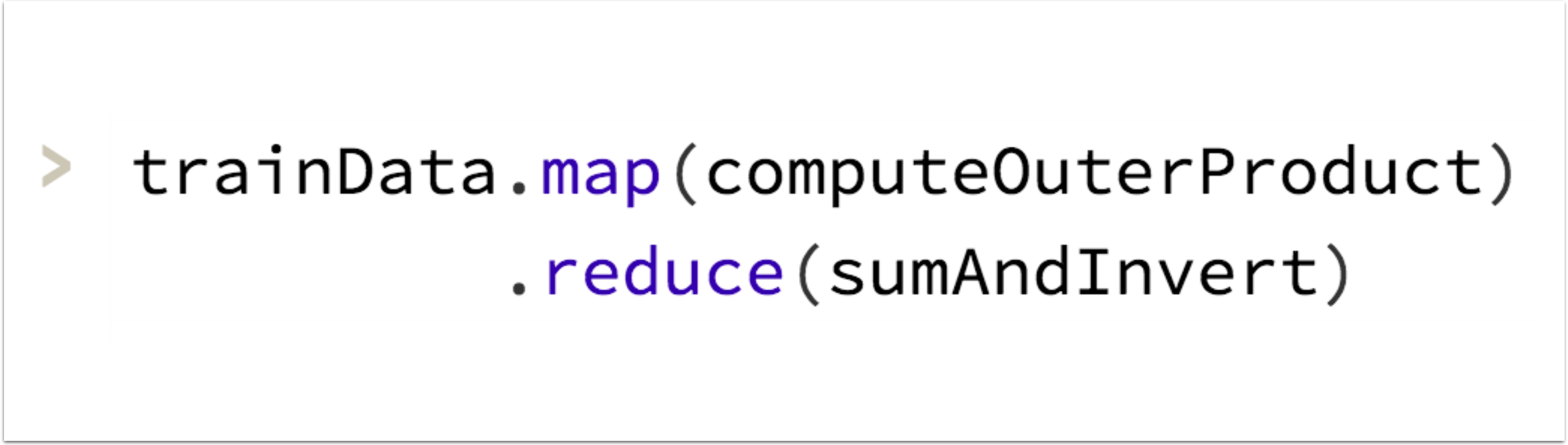
Computation: $\mathrm{O}(\underline{nd^2} + d^3)$ operations

Storage: $\mathrm{O}(\underline{nd} + d^2)$ floats



```
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           .reduce(sumAndInvert)
```

Big n and Big d

$$\mathbf{w} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$$

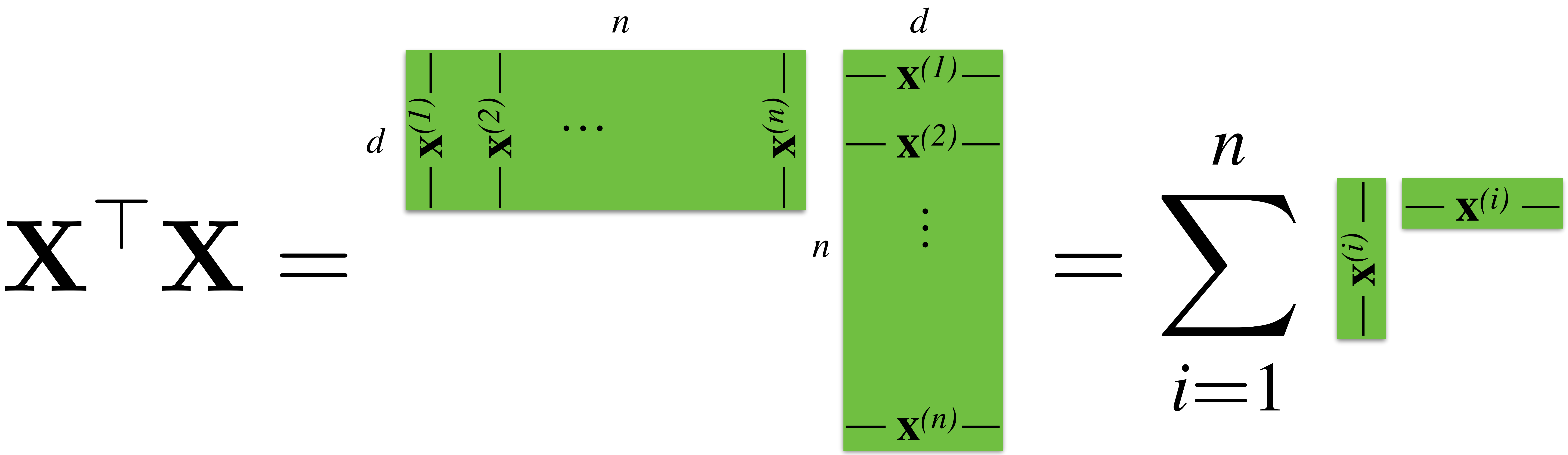
Computation: O($\underline{nd^2}$ + $\underline{d^3}$) operations

Storage: O($\underline{nd + d^2}$) floats

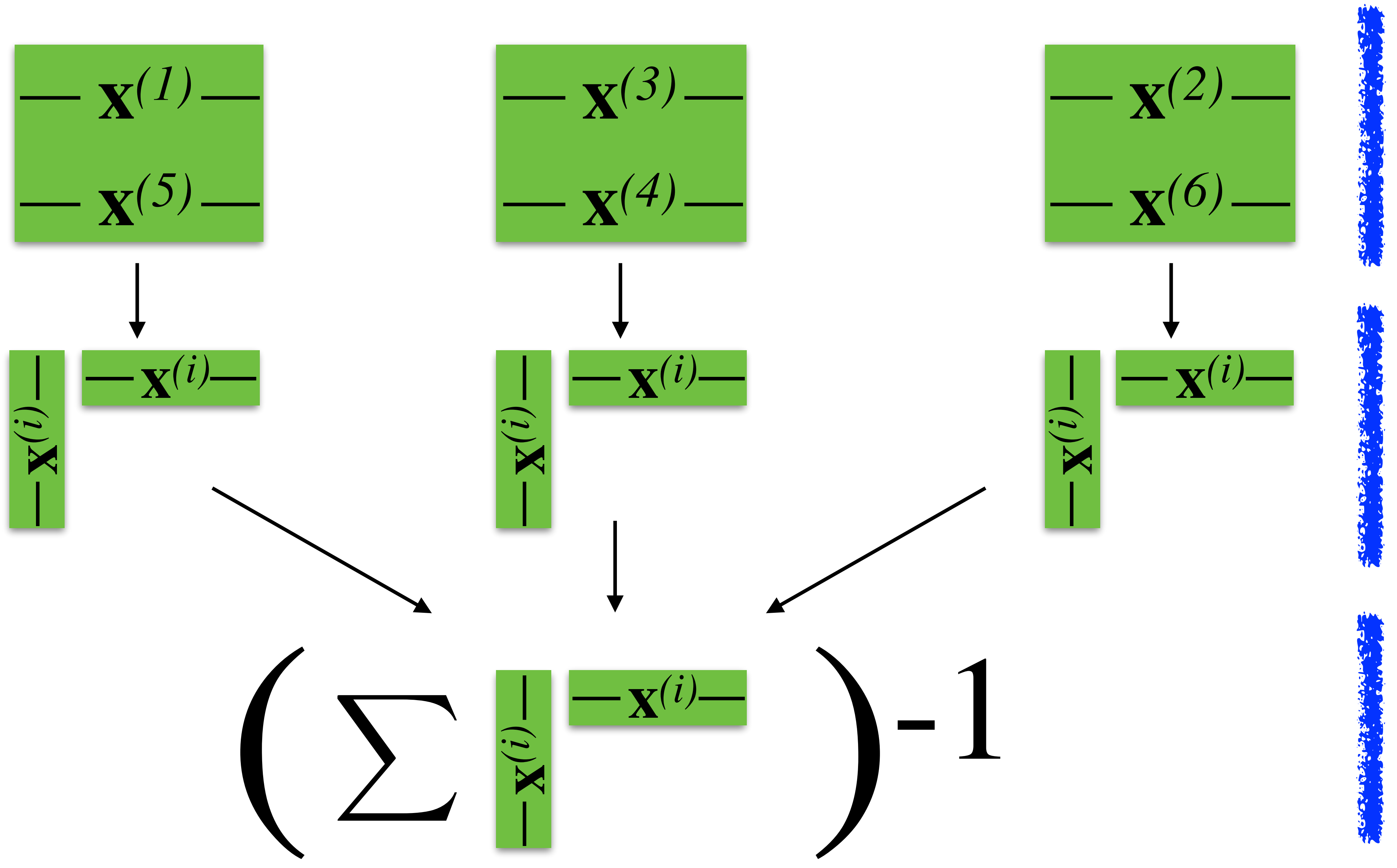
As before, storing $\mathbf{X}$ and computing $\mathbf{X}^\top \mathbf{X}$ are bottlenecks

Now, storing and operating on $\mathbf{X}^\top \mathbf{X}$ is also a bottleneck

- Can't easily distribute!



Example: n = 6; 3 workers



workers:

O(nd) Distributed Storage

map:

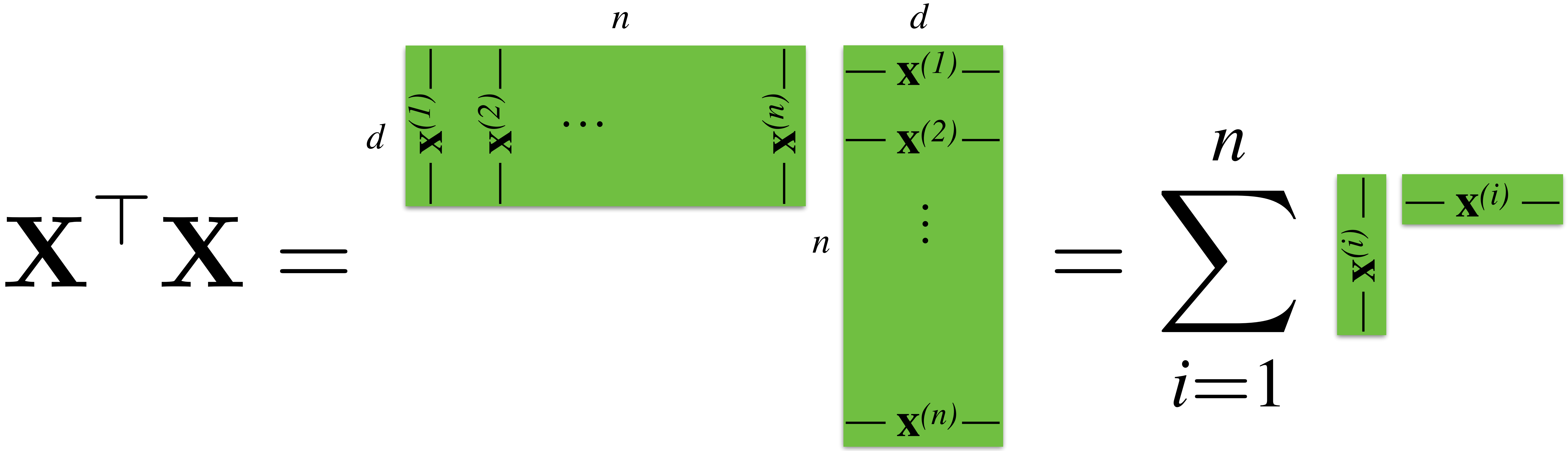
O(nd^2) Distributed Computation

O(d^2) Local Storage

reduce:

O(d^3) Local Computation

O(d^2) Local Storage

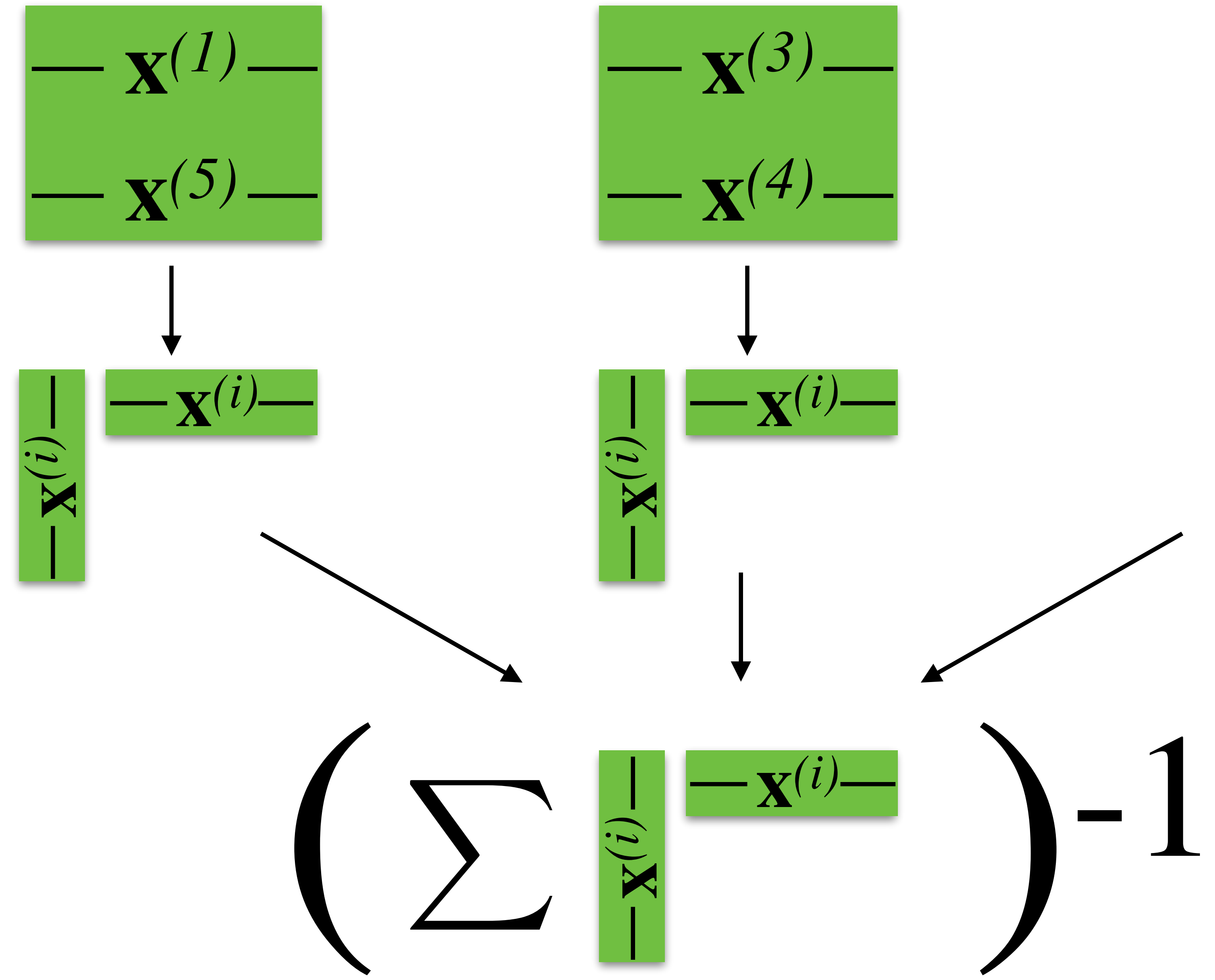


Example: n = 6; 3 workers

workers:

map:

reduce:



O(nd) Distributed Storage

O(nd^2) Distributed Computation

O(d^2) Local Storage

O(d^3) Local Computation

O(d^2) Local Storage

Big n and Big d

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Storage: O($\underline{nd + d^2}$) floats

As before, storing $\mathbf{X}$ and computing $\mathbf{X}^\top \mathbf{X}$ are bottlenecks

Now, storing and operating on $\mathbf{X}^\top \mathbf{X}$ is also a bottleneck

- Can't easily distribute!

1st Rule of thumb

Computation and storage should be linear (in n, d)

Big n and Big d

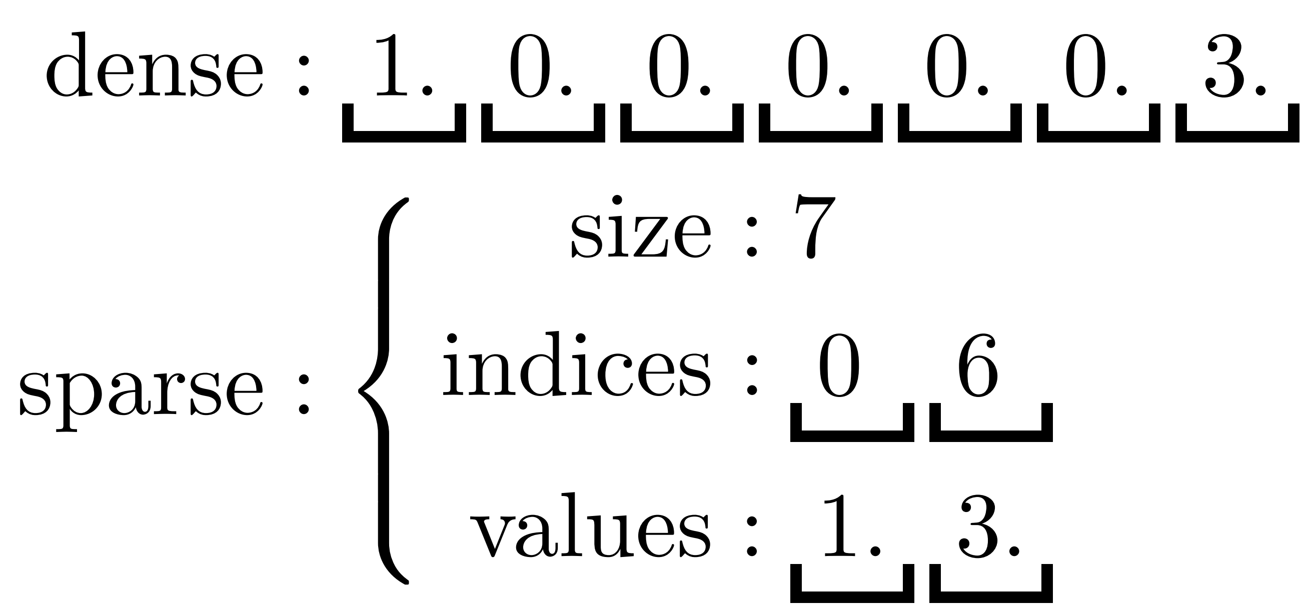
We need methods that are linear in time and space

One idea: **Exploit sparsity**

- Explicit sparsity can provide orders of magnitude storage and computational gains

Sparse data is prevalent

- Text processing: bag-of-words, n-grams
- Collaborative filtering: ratings matrix
- Graphs: adjacency matrix
- Categorical features: one-hot-encoding
- Genomics: SNPs, variant calling



dense : 1. 0. 0. 0. 0. 0. 3.

sparse : size : 7

indices : 0 6

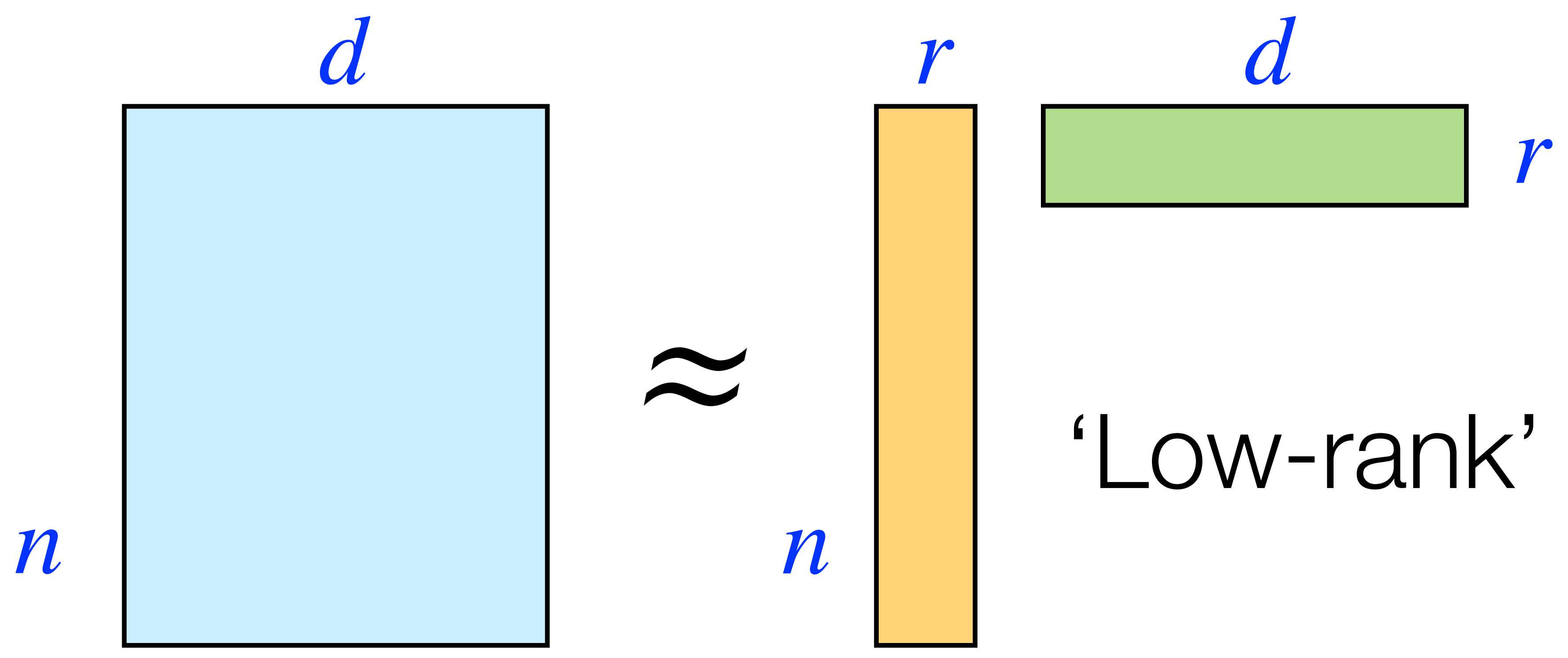
values : 1. 3.

Big n and Big d

We need methods that are linear in time and space

One idea: **Exploit sparsity**

- Explicit sparsity can provide orders of magnitude storage and computational gains
- Latent sparsity assumption can be used to reduce dimension, e.g., PCA, low-rank approximation (unsupervised learning)



Big n and Big d

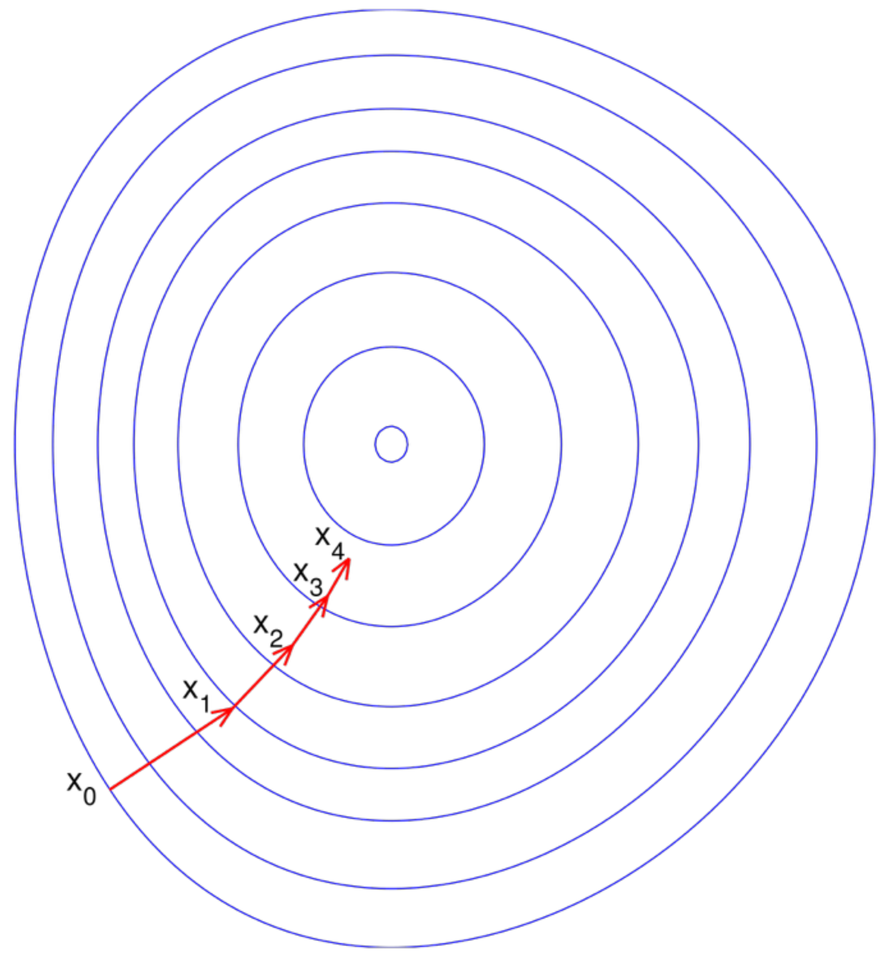
We need methods that are linear in time and space

One idea: **Exploit sparsity**

- Explicit sparsity can provide orders of magnitude storage and computational gains
- Latent sparsity assumption can be used to reduce dimension, e.g., PCA, low-rank approximation (unsupervised learning)

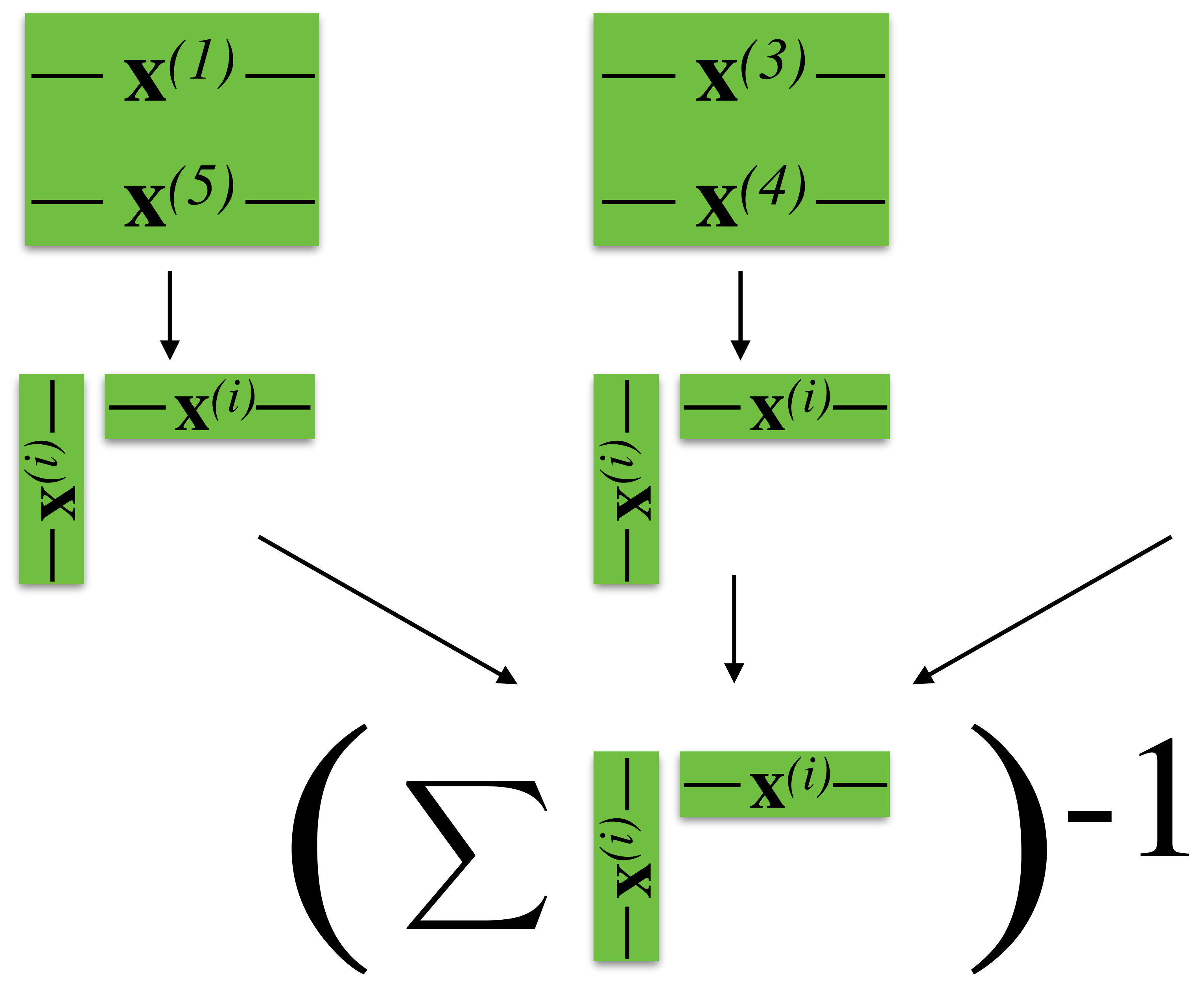
Another idea: **Use different algorithms**

- Gradient descent is an iterative algorithm that requires O(nd) computation and O(d) local storage per iteration



Closed Form Solution for Big n and Big d

Example: n = 6; 3 workers



workers:

O(nd) Distributed Storage

map:

O(nd^2) Distributed Computation

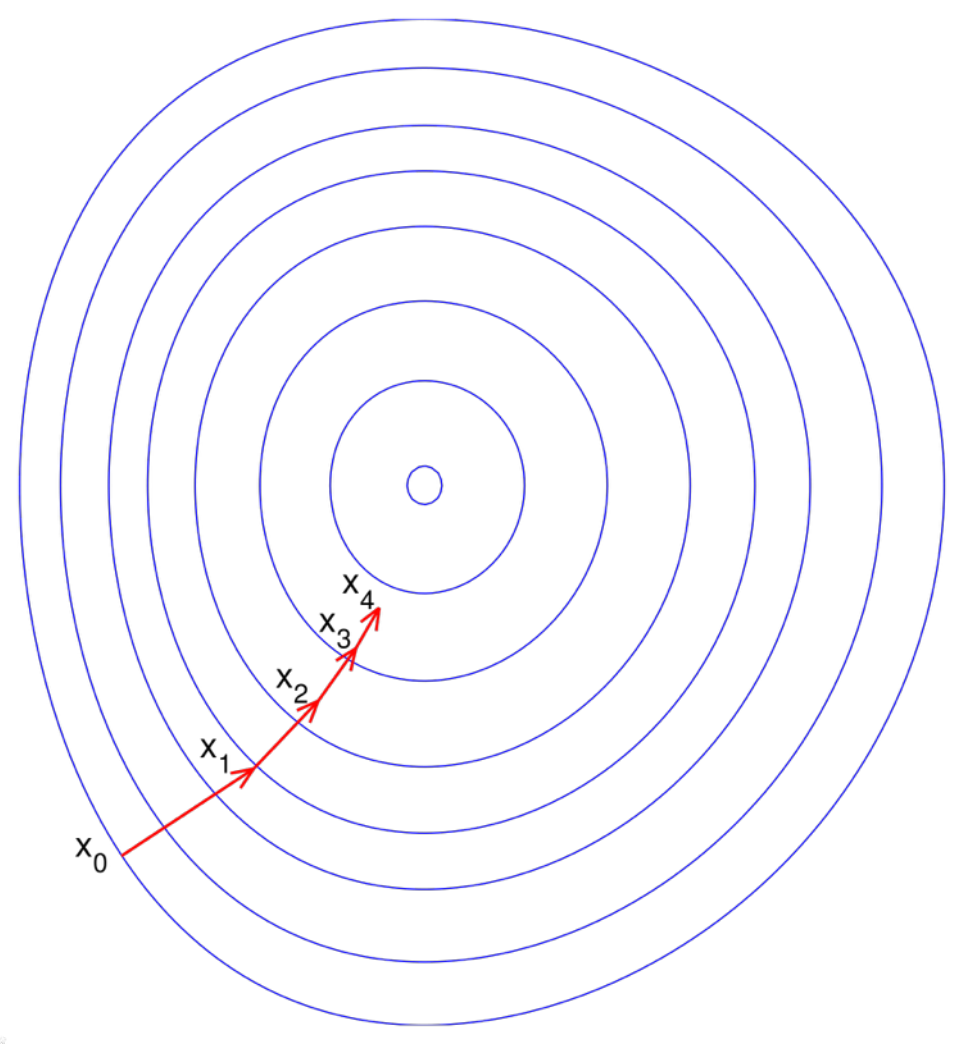
O(d^2) Local Storage

reduce:

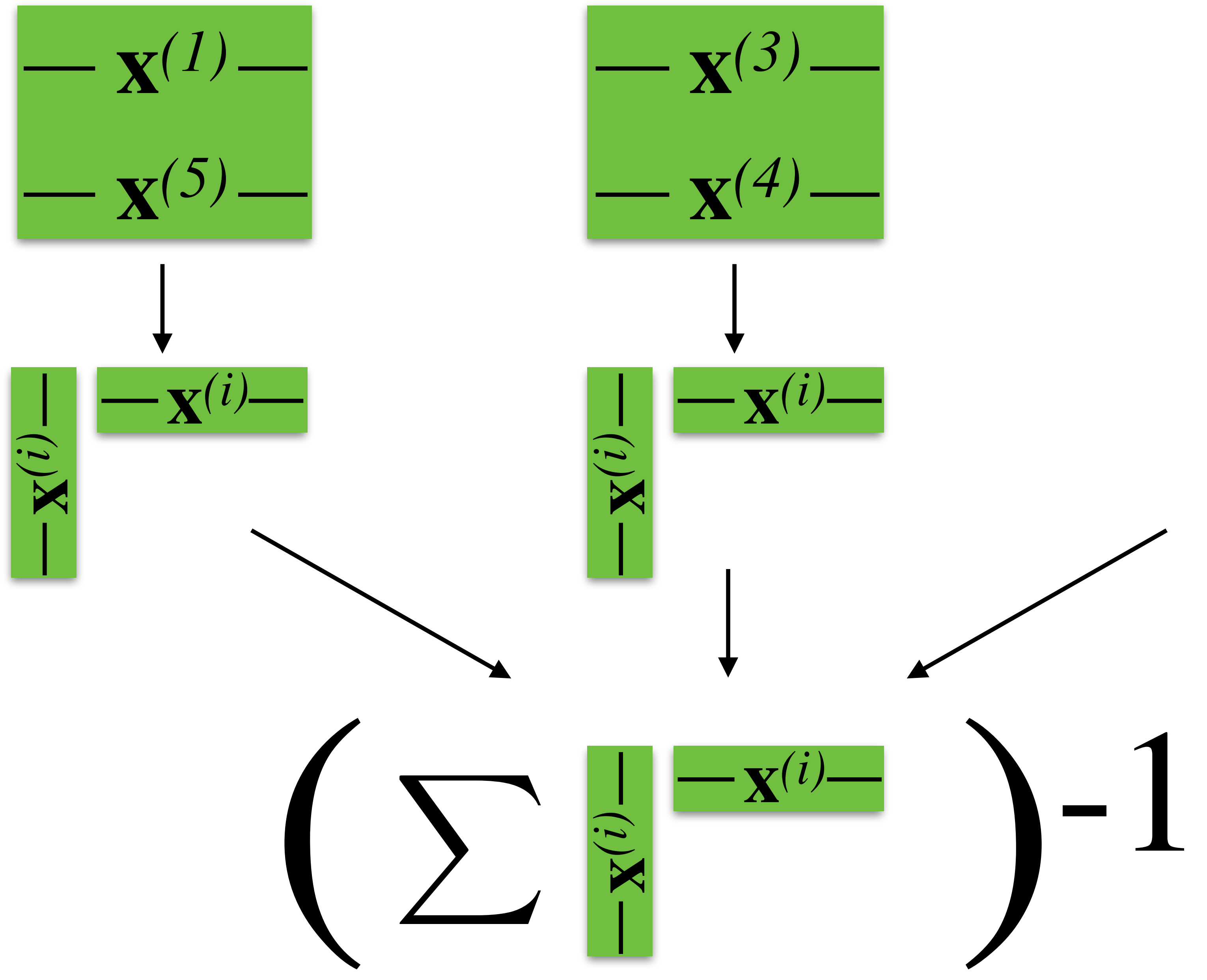
O(d^3) Local Computation

O(d^2) Local Storage

Gradient Descent for Big n and Big d



Example: n = 6; 3 workers



workers:

— $\mathbf{x}^{(1)}$ —
— $\mathbf{x}^{(5)}$ —

— $\mathbf{x}^{(3)}$ —
— $\mathbf{x}^{(4)}$ —

— $\mathbf{x}^{(2)}$ —
— $\mathbf{x}^{(6)}$ —

O(nd) Distributed Storage

map:

| — $\mathbf{x}^{(i)}$ — | — $\mathbf{x}^{(i)}$ — |

| — $\mathbf{x}^{(i)}$ — | — $\mathbf{x}^{(i)}$ — |

| — $\mathbf{x}^{(i)}$ — | — $\mathbf{x}^{(i)}$ — |

O(nd^2) Distributed Computation

O(d^2) Local Storage

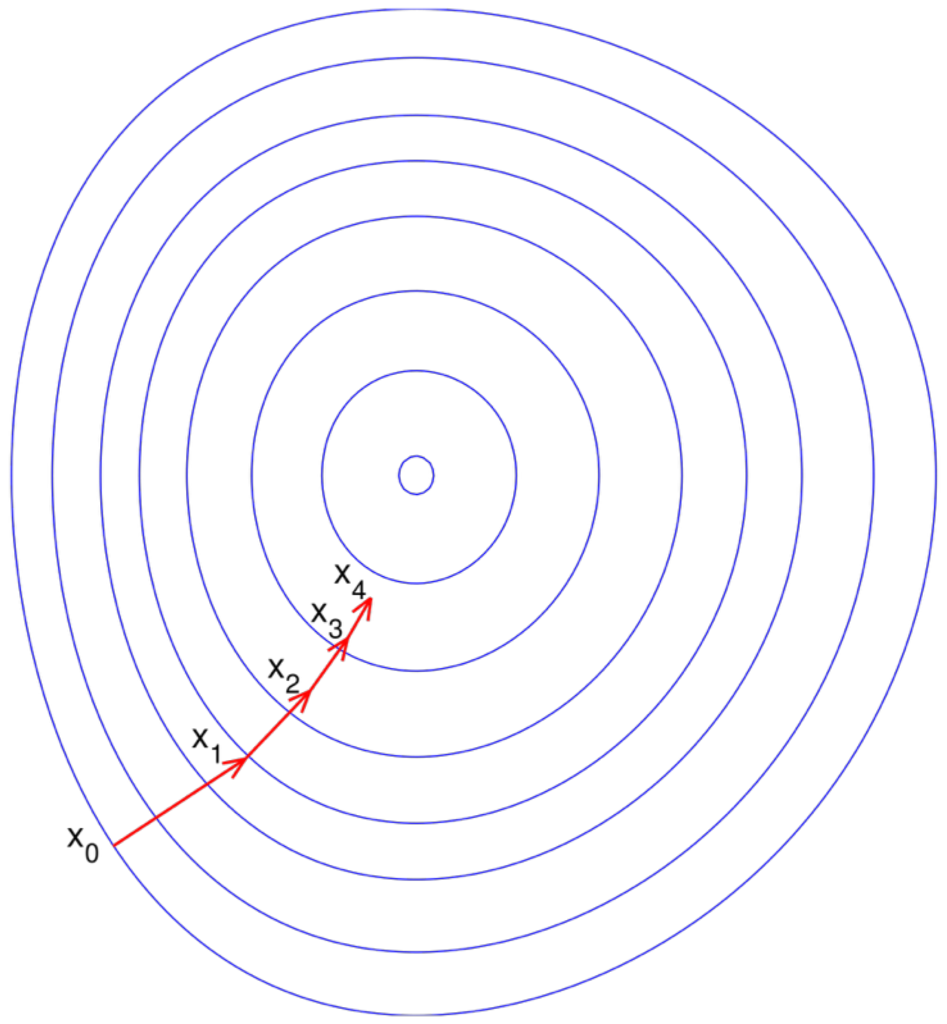
reduce:

$$\Big(\sum \,|\,\mathbf{x}^{(i)}\,|\ \text{—}\,\mathbf{x}^{(i)}\,\text{—}\Big)^{-1}$$

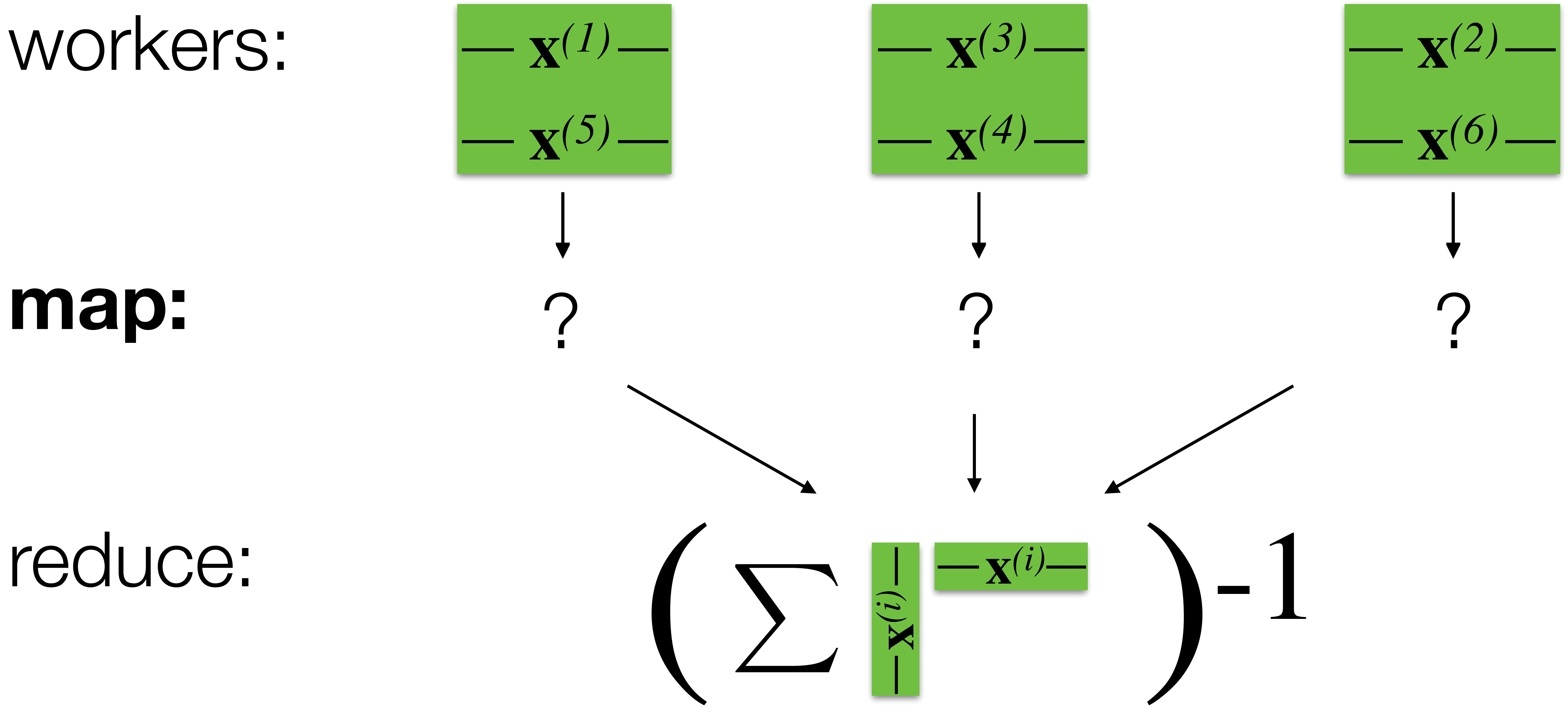
O(d^3) Local Computation

O(d^2) Local Storage

Gradient Descent for Big n and Big d



Example: $n = 6$; 3 workers



workers: $—\mathbf{x}^{(1)}—$ $—\mathbf{x}^{(5)}—$ | $—\mathbf{x}^{(3)}—$ $—\mathbf{x}^{(4)}—$ | $—\mathbf{x}^{(2)}—$ $—\mathbf{x}^{(6)}—$

map: ? ? ?

reduce: $\left(\sum \mathbf{x}^{(i)} \mathbf{x}^{(i)}\right)^{-1}$

$O(nd)$ Distributed Storage

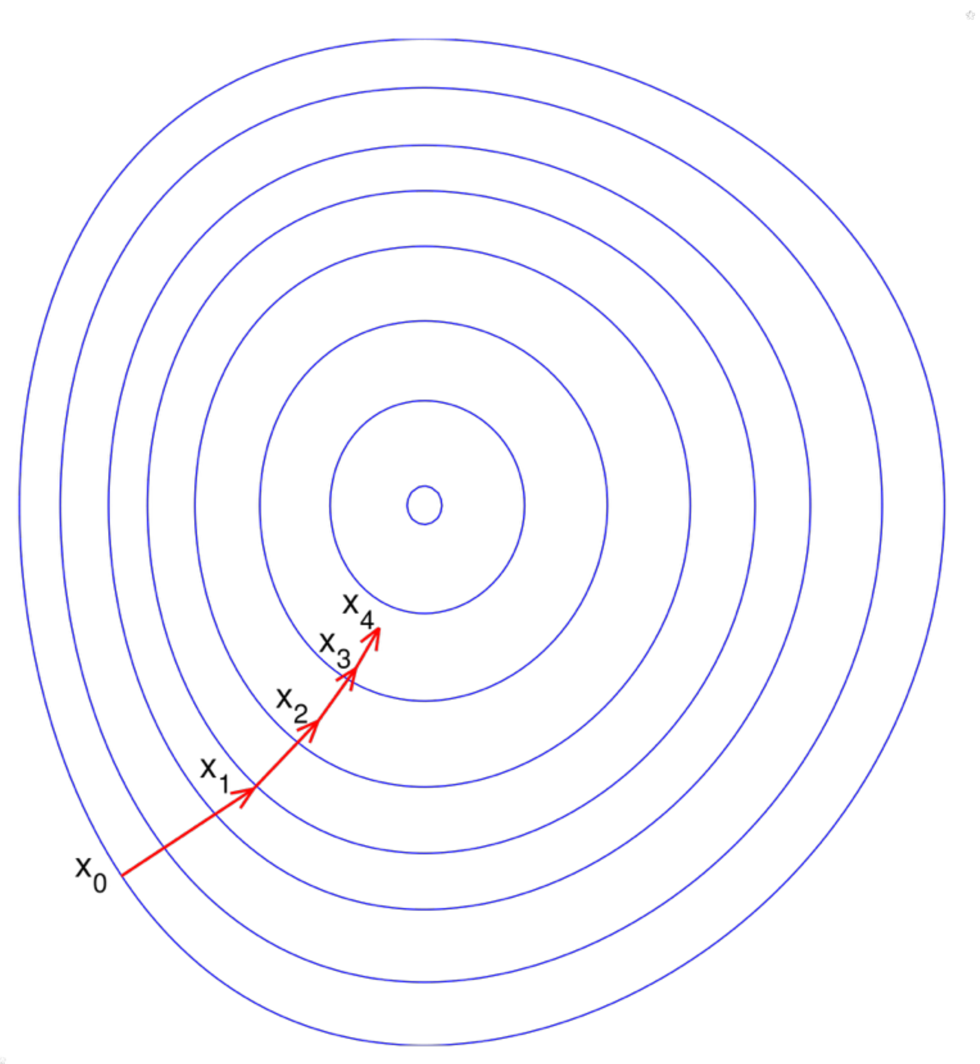
$O(\boldsymbol{nd})$ ~~$O(nd^2)$~~ Distributed Computation

$O(\boldsymbol{d})$ ~~$O(d^2)$~~ Local Storage

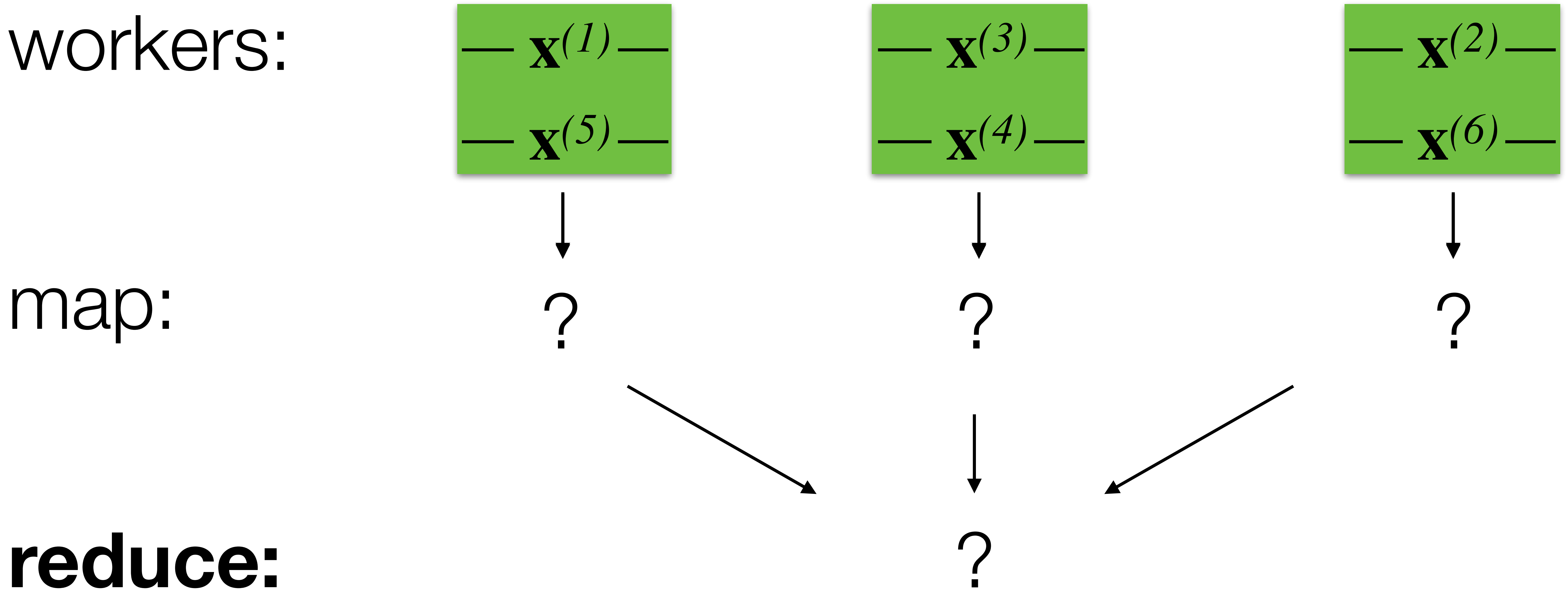
$O(d^3)$ Local Computation

$O(d^2)$ Local Storage

Gradient Descent for Big n and Big d



Example: n = 6; 3 workers



workers: — $\mathbf{x}^{(1)}$ — — $\mathbf{x}^{(5)}$ — | — $\mathbf{x}^{(3)}$ — — $\mathbf{x}^{(4)}$ — | — $\mathbf{x}^{(2)}$ — — $\mathbf{x}^{(6)}$ —

map: ? ? ?

reduce: ?

O(nd) Distributed Storage

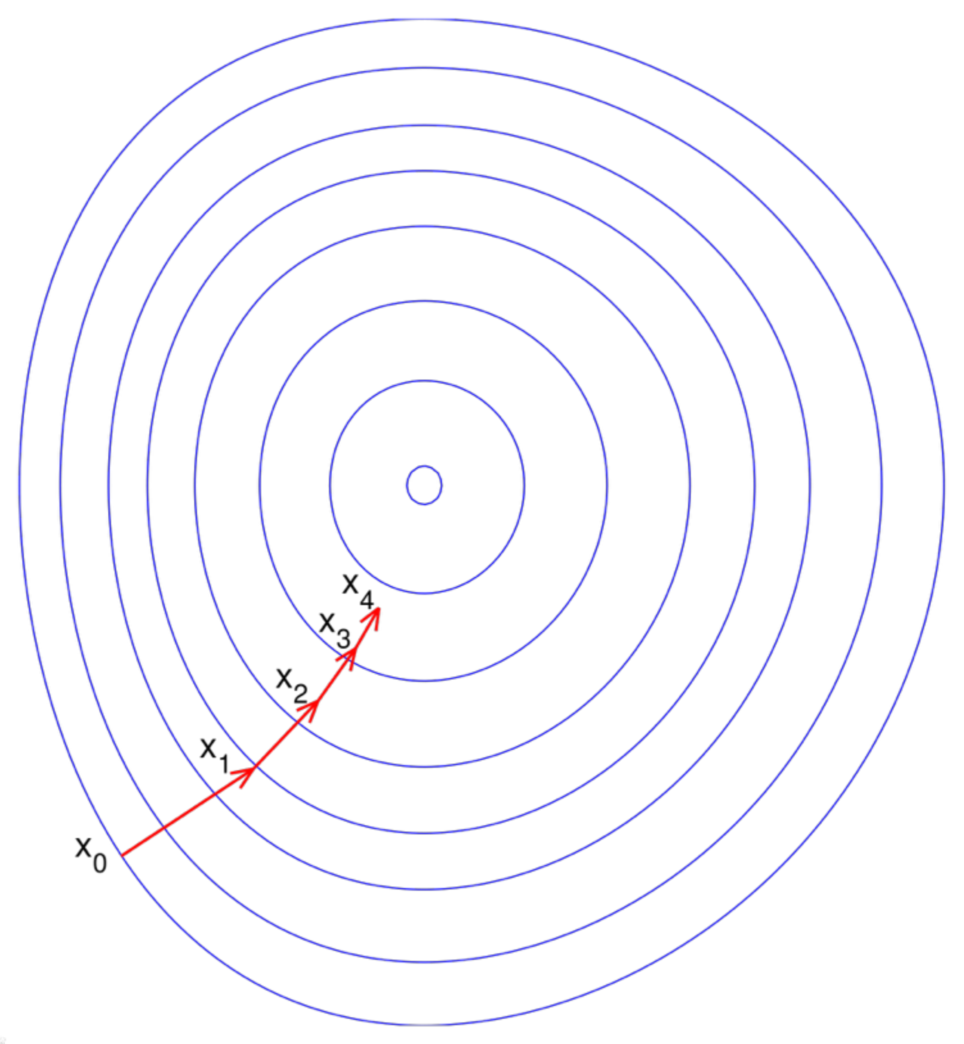
O($\boldsymbol{nd}$) ~~O(nd^2)~~ Distributed Computation

O($\boldsymbol{d}$) ~~O(d^2)~~ Local Storage

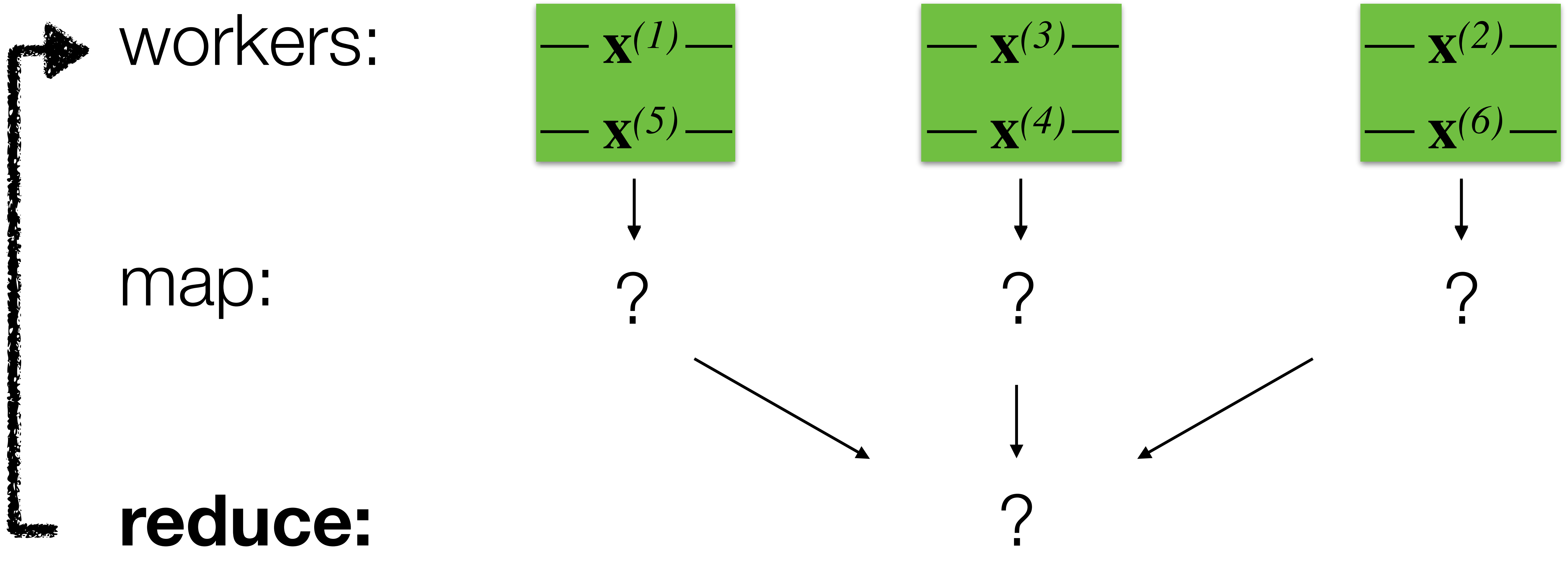
O($\boldsymbol{d}$) ~~O(d^3)~~ Local Computation

O($\boldsymbol{d}$) ~~O(d^2)~~ Local Storage

Gradient Descent for Big n and Big d



Example: $n = 6$; 3 workers



workers:

— $\mathbf{x}^{(1)}$ —
— $\mathbf{x}^{(5)}$ —

— $\mathbf{x}^{(3)}$ —
— $\mathbf{x}^{(4)}$ —

— $\mathbf{x}^{(2)}$ —
— $\mathbf{x}^{(6)}$ —

map:

? ? ?

reduce:

?

$O(nd)$ Distributed Storage

$O(\boldsymbol{nd})$ ~~$O(nd^2)$~~ Distributed Computation

$O(\boldsymbol{d})$ ~~$O(d^2)$~~ Local Storage

$O(\boldsymbol{d})$ ~~$O(d^3)$~~ Local Computation

$O(\boldsymbol{d})$ ~~$O(d^2)$~~ Local Storage