An Acoustical Domain for Simscape

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1 Why do we need an Acoustical Library

Acoustical transducer devices include audio microphones and loudspeakers, sonar sound sources and receivers, medical ultrasound devices and many similar acoustical devices and systems. Such systems have long been modeled using analog circuit analysis methods. These models use electrical circuits that are analogous to mechanical, acoustical, magnetic and other physical domains. Simscape can model most of these domains. However, the acoustical domain in legacy analog circuit models uses acoustical pressure as the potential (across) variable and acoustical volume velocity as the flow (through) variable. The existing Simscape domains use different variables. While some others are known to have use the existing domains with great success, the author thought it worthwhile, if only for pedagogical reasons, to use the same variables and analysis methods that have been used in nearly a century of published acoustical literature.

2 The Basic Acoustical Library

This library provides an Acoustical Domain for use with Simscape. This domain follows the conventions that have been used for decades with analog circuit models of acoustical systems. An early example of these methods is the book by Olsen[1] (first edition published in 1943). A relevant current reference is Beranek and Mellow[2]. These methods were initially used to model lumped parameter systems, systems that are analogous to simple RLC electrical circuits. Those lumped parameter components are included in the acoustical domain library. An example of the use of these simple components is a simple loudspeaker model in Section 4. The current version of the acoustical library also includes components beyond the simple lumped elements. These include several kinds of acoustical transmission lines, piezoelectric pieces, and assemblies of simpler components such as approximate models radiation impedance and several models of moving coil speakers.

The Acoustical Domain uses the linear approximation for acoustic disturbances in the medium. It cannot deal with any phenomena that produce nonlinear behavior in the medium. For example, it cannot include effects such as

Acoustical Component	Equation	Electrical Component	Equation
Compliance C_a	$U = j\omega C_a\, P$	Capacitance C_e	$i = j\omega C_e\, e$
Inertance M_a	$P = j\omega M_a\, U$	Inductance L_e	$e = j\omega L_e\, i$
Resistance R_a	$P = R_a\, U$	Resistance R_e	$e = R_e\, i$

Table 1: The acoustical lumped elements are analogous to ideal electrical passive components.

- turbulence,
- cavitation in liquid media,
- nonlinear propagation at high pressure amplitudes in tubes.

The user is currently responsible for recognizing when these effects may be present, and to refrain from linear modeling in those cases.

The default parameters for the acoustical medium are those for air at room temperature. To view or change these parameters, the library includes a parameters block. If this block is added to a model and connected to an acoustical network in the model, then the acoustical medium parameters for that network are those in the block.

2.1 Fundamental Acoustical Components

For its simplest lumped elements, this domain is entirely analogous to the electrical domain in the Simscape Foundation Library. The across variable in the acoustical domain is acoustic pressure P, and the through variable is volume velocity U, analogous to electrical voltage and current. Table 1 shows the simple acoustical elements and their analogs from the electrical domain. The final elementary component is the acoustical reference block. Every loop in an acoustical network must include a reference node. The Acoustical Library includes the same basic sources and sensors as the in electrical library. Combinations of these simple elements allow most of the analog circuit models in legacy acoustic literature to be modeled in natural acoustical units.

2.2 Acoustical Medium Parameters

This block can be added to any acoustical network to examine or set the parameters of the medium. The parameters that are available to set are

Density [kg/m^3]
Sound speed [m/s]
Ambient Pressure [Pa]
Specific heat at constant pressure [J/kg/K]
Ratio of specific heats
Viscosity [m^2/s]
Thermal conductivity [W/(m K]

The medium parameters define a linear medium that will generally be a gas. The domain will work well for simple liquid media at fixed temperature and pressure, but it does not attempt to model liquids with dissolved gas, and it does not allow the possibility of cavitation.

2.3 Acoustical Volume or Enclosure

An acoustical enclosure or volume has the same physical characteristics as the fundamental acoustical compliance, but it is characterized by its volume rather than its compliance. The compliance is calculated as $C = {}^{V}\!/_{\gamma P_a}$. where V is the volume, γ is the ratio of specific heats for the medium and P_a is the ambient barometric pressure in the medium.

2.4 Acoustical Transformer

For acoustical waves traveling in a tube of cross sectional area A,the characteristic impedance is ${}^{\rho c}\!/_{A}$.When tubes of different cross sectional area are joined, an acoustical transformer is needed to mediate the change in area. The transformer takes a single parameter that is the ratio of the pressure at port 2 to that at port 1. in the case ot tubes of different area, the pressure ratio would be ${}^{A_1}\!/_{A_2}$,and the ratio of volume velocities is the inverse.

2.5 Mechanical to Acoustical Converter (Mechanical to Acoustical Transformer)

The library also includes a mechanical-to-acoustic converter component, that connects a vibrating mechanical piston to the adjacent acoustical domain. The converter implements the transformation

$$F = S_d P$$
$$U = S_d v$$

from force F and velocity v in the mechanical domain to pressure P and volume velocity U in the acoustical domain, where S_d is the area of the moving surface. This would be the correct interface component to connect the motion of a piston of area S_d to an acoustical system that models radiation from the piston.

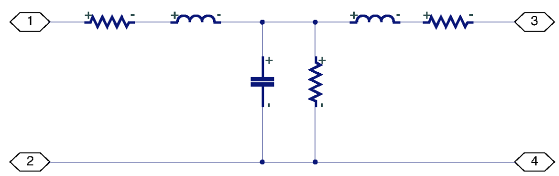

Figure 1: This circuit models a short segment of an acoustical line. A cascade of many such segments can model transmission in a thin tube.

2.6 Acoustical Tubes and Horns

The current version of the acoustical library includes three models for acoustical propagation in tubes. The first is a cylindrical tube constructed as a cascade of segments shown in Figure 1. This is a lossy tube in which the loss is independent of frequency. The reactive components in the Figure 1 are calculated from the length and radius of the tube, and the damping values provided as parameters.

The second kind of tube is a cylindrical tube with thermoviscous losses at the walls. This tube follows the model provided in Thompson, et. al.[4]. The third kind of tube is a conical tube, as described by Benade[5].

2.7 Radiation Impedance for a Circular Piston in a Rigid Baffle

A simple acoustical circuit whose impedance is a reasonable approximation to the radiation impedance of a circular piston in a rigid baffle is described in Section 3. An implementation of this circuit is included in the library. This component requires only the piston area as a parameter for the calculation. It uses the density and sound speed of the medium defined in the acoustical domain.

2.8 Far Field Radiated Pressure Calculation

The acoustical library includes a block that uses the volume velocity through the block to calculate the far field pressure magnitude that would be radiated by a simple acoustical source. The calculation assumes the source radiates into a half space, and provides the pressure magnitude referred to 1 m from the source. The calculation is

$$P_{ff} = \frac{\omega\rho}{2\pi}U.$$

With this definition, the block is actually intended for use in the frequency domain, after the circuit is linearized. However it also calculates the result of

$$P_{ff} = \frac{\rho}{2\pi}\frac{dU}{dt},$$

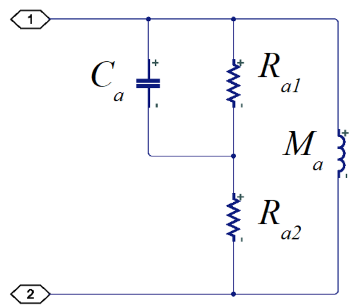

Figure 2: Beranek [3, 2] has shown that this acoustical circuit is a reasonably good approximation to the radiation impedance of a circular piston in an infinite rigid baffle. Values for the circuit components are given in the text.

which is a proper calculation of the time domain waveform radiated to the far field, referenced to its value at 1 m from the source..

3 Simple Example of the Use of the Library

An assembly of the lumped elements can provide a reasonable approximation to the radiation impedance of transducers in simple geometries. This model was first included in Beranek's *Acoustics* text in 1954[3], but is currently more easily found in Beranek and Mellow[2]. Beranek showed that the acoustical model in Figure 2 is a reasonable approximation to the radiation impedance of a circular piston of radius a in an infinite rigid baffle if the circuit components are

$$\begin{aligned} M_a &= \frac{8\rho}{3\pi^2 a}\left[\frac{\mathrm{kg}}{\mathrm{m}^4}\right] \\ R_{a1} &= 0.1404\frac{\rho c}{a^2}\left[\frac{\mathrm{N\,s}}{\mathrm{m}^5}\right] \\ R_{a2} &= \frac{\rho c}{\pi a^2}\left[\frac{\mathrm{N\,s}}{\mathrm{m}^5}\right] \\ C_a &= 5.94\frac{a^3}{\rho c^2}\left[\frac{\mathrm{m}^5}{\mathrm{N}}\right] \end{aligned}$$

where ρ and c are the density and sound speed in the radiating medium. This compound component might be implemented in Simscape as a subsystem, or it could be written into .ssc code. The acoustical library actually includes an acoustical component that implements this Beranek radiation impedance approximation.

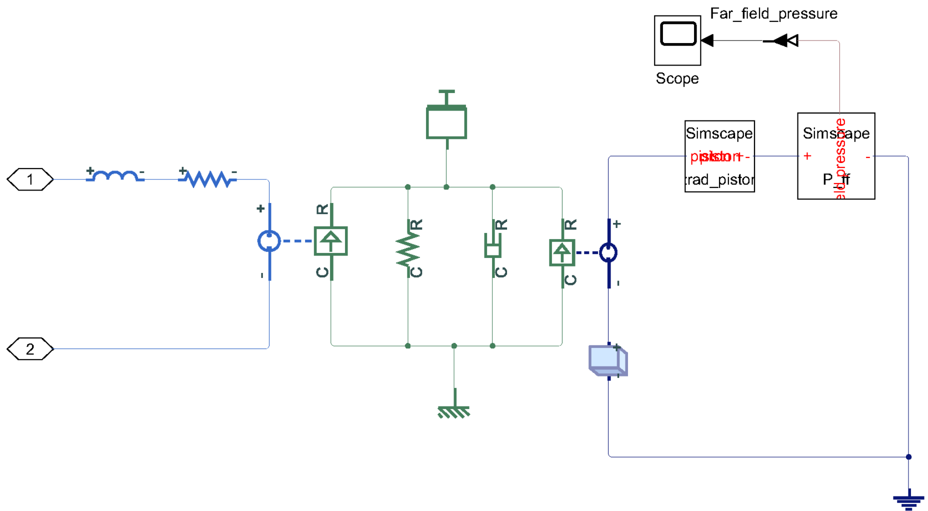

Figure 3: The electroacoustic model of a moving coil loudspeaker in a sealed enclosure uses several components from the acoustical library.

4 Simple Model of a Moving Coil Loudspeaker

A simple electromechanical model of a moving coil .loudspeaker can be made using standard components from the Simscape Simscape Foundation Library, as shown in Figure 3. The electrical domain contains components for the resistance and inductance of the coil. The electrical to mechanical converter implements the standard magnetic force coupling between the electrical and mechanical domains. The components in the mechanical domain are the moving mass of the speaker, the mounting stiffness and the mechanical damping in the speaker cone. This electromechanical model is sometimes used alone with the assumption that the radiation impedance is negligible. Sometimes it is loaded with the mechanical equivalent of the acoustical radiation impedance. The components in the acoustical library allow more comprehensive modeling of speaker systems with the acoustical components modeled in their natural units.

The mechanical to acoustical converter described in Section 2.5 creates the coupling to the acoustical domain. The transformer uses the area and velocity of the speaker cone to create the volume velocity in the acoustical domain. The flow from the + terminal (top in the figure) of this converter is the acoustical volume velocity from the front side of the speaker cone. The front side of the speaker cone connects to the Simscape component that is the radiation impedance described in Section 2.7. The flow on the front side of the cone also passes through the far field pressure calculation block. The acoustical flow from the - terminal of the converter (bottom in the figure) is the flow from the back side of the speaker cone. In this simple example, the back side connects to the speaker enclosure that here is assumed to be a sealed back volume.

The acoustical library includes a component that is the the electrical and mechanical domains and the mechanical to acoustical converter in the model of Figure 3. That allows the model to be simplified to the equivalent model of Figure 4.

A fairly simple modification of the enclosed speaker model can include a port in the enclosure. The acoustical library includes a component that is a cylindrical tube. Figure 4 shows that tube connected at its left end to the pressure in the enclosure and its right end radiating to the far field. The port tube is connected to a radiation impedance with the proper area for the tube. This creates the situation that the volume velocity of the speaker cone radiates to the far field and the volume velocity of the port tube also radiates to the far field. With the assumption that the speaker and the port tube are closely spaced relative to the wavelength. The far field pressure can be calculated by adding the volume velocities from the two sources to calculate the far field pressure. The several components on the right and top of the diagram accomplish this calculation.

The model in Figure 4 can be used to show that the radiation from the speaker cone is much larger than that from the port tube for all frequencies except a small band around the resonance of the port and enclosure. For the lowest part of this band at and below the port resonance, the port radiation dominates. For a well designed system, just above the port resonance frequency, the radiation from the speaker and the port are approximately equal.

References

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[5] A. H. Benade, "Equivalent circuits for conical waveguides," *J. Acoust. Soc. Am.* **83**, 1764 (1988); https://doi.org/10.1121/1.396510.

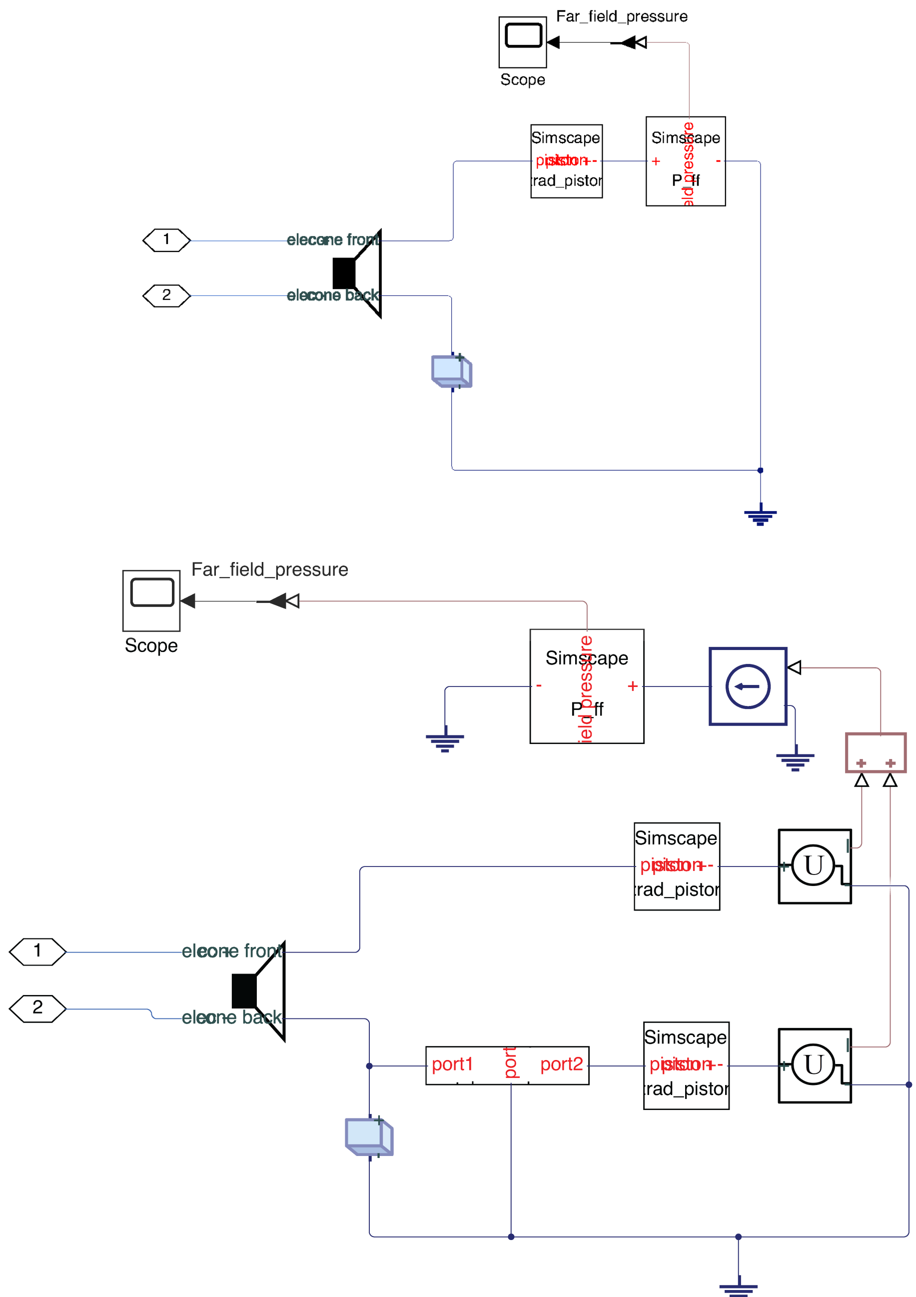

Figure 4: (top)The drawing of the model of Figure 3 is simplified using the Simple Speaker component in the library. (bottom) The addition of a port tube and a second radiation impedance models the ported speaker.