1) Stage B: Histogram Equalization (mapping of three Intensity level)

Given:

Histogram equalization mapping is:

$$s = T(r) = [(L-1) \cdot CDF(r)]$$

where

- $L = 256$ gray level
- r = Original Intensity
- s = new mapping Intensity
- $CDF(r) = \sum_{k=0}^{r} P(k)$

Assume a Small 8-level

$\therefore L = 8$ Instead of 256

Let the Image have $N = 16$ pixels and histogram:

Intensity r_k	0	1	2	3	4	5	6	7
Count n_k	0	1	2	3	4	3	2	1

~~$N = 0+1+2+3+4$~~

$N = 0+1+2+3+4+3+2+1 = 6$

Step 1: Compute PDF

$$P(r_k) = \frac{n_k}{N}$$

So $-\ P(1) = \frac{1}{16} = 0.0625$

- $P(2) = 2/16 = 0.125$
- $P(3) = 3/16 = 0.1875$
- $P(4) = 4/16 = 0.25$
- $P(5) = 3/16 = 0.1875$
- $P(6) = 2/16 = 0.125$
- $P(7) = 1/16 = 0.0625$

Step2: Compute CDF

$$CDF(r) = \sum_{k=0}^{r} P(k)$$

Compute:

- $CDF(1) = 0.0625$
- $CDF(2) = 0.0625 + 0.125 = 0.1875$
- $CDF(3) = 0.1875 + 0.187 = 0.375$
- $CDF(4) = 0.375 + 0.25 = 0.625$
- $CDF(5) = 0.625 + 0.187 = 0.8125$
- $CDF(6) = 0.8125 + 0.125 = 0.9375$
- $CDF(7) = 0.9375 + 0.0625 = 1.0$

Step3: Mapping function

Since $L = 8$

$$s = [(8-1) \cdot CDF(r)] = [7 \cdot CDF(r)]$$

∴ Mapping for 3 Intensity Level

lets pick 3 levels: $r = 2, 4, 6$

a) for $r = 2$

$$CDF(2) = 0.1875$$

$$S = (7 \times 0.1875) = (1.3125) = 1$$

So: $T(2) = 1$

b) for $r = 4$

$$CDF(4) = 0.625$$

$$S = [7 \times 0.625] = [4.375] = 4$$

So: $T(4) = 4$

c) for $r = 6$

$$CDF(6) = 0.9375$$

$$S = [7 \times 0.9375] = [6.5625] = 6$$

So: $T(6) = 6$

Therefore: $T(2) = 1$, $T(4) = 4$, $T(6) = 6$

Stage C: Manual Median Filter Computation (5x5)

Median filter defination

for a 5x5 window:

$$g(x,y) = \text{median}\{f(x+i,\, y+j)\}$$

wher $i, j \in [-2, 2]$

given 5x5 neighborhood

Assume the following 5x5 pixels values around a centra
pixel

$$\begin{bmatrix} 12 & 15 & 14 & 16 & 18 \\ 11 & 13 & 15 & 17 & 200 \\ 10 & 12 & 14 & 16 & 19 \\ 9 & 11 & 13 & 15 & 18 \\ 8 & 10 & 12 & 14 & 16 \end{bmatrix}$$

Step 1: List all 25 Values

{12, 15, 14, 16, 18, 11, 13, 15, 17, 200, 10, 12, 14, 16, 19, 9, 11, 13,
15, 18, 8, 10, 12, 14, 16}

Step 2: Sort Values in ascending order

{8, 9, 10, 10, 11, 11, 12, 12, 12, 13, 13, 14, 14, 14, 15, 15, 15,
16, 16, 16, 17, 18, 18, 19, 200}

Step 3: Find the median

for 25 Values, median is the 13th value

$\therefore$ Median Index $= \frac{25+1}{2} = 13$ $\therefore$ 13th Value $= 14$

$\therefore \; g(x, y) = 14$

This shows the median filter removes Impulse noise (200) without blurring edges.

Stage D: Manual DFT Coefficient before and after filtering.

Given 4×4 image block $f(x,y)$

$$f(x,y) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

DFT formula

for $M = N = 4$

$$F(u,v) = \sum_{x=0}^{3} \sum_{y=0}^{3} f(x,y) e^{-j2\pi\left(\frac{ux}{4} + \frac{vy}{4}\right)}$$

Compute one Coefficient: $F(0,0)$

where $u = 0, \; v = 0,$

$$e^{-j2\pi(0)} = 1$$

So: $$F(0,0) = \sum_{x=0}^{3} \sum_{y=0}^{3} f(x,y)$$

Sum all Values

Row Sums:

- Row1: $1+2+3+4=10$
- Row2: $5+6+7+8=26$
- Row3: $9+10+11+12=42$
- Row4: $13+14+15+16=58$

Total: $F(0,0) = 10 + 26 + 42 + 58 = 136$

⇒ Now apply notch filter

notch filtering in Sequence domain is:

$$F'(u,v) = H(u,v) \cdot F(u,v)$$

Assume the notch filter attenuates DC Slightly with:

$$H(0,0) = 0.8$$

then: $F'(0,0) = 0.8 \times 136 = 108.8$

∴ before filtering

$$F(0,0) = 136$$

After filtering

$$F'(0,0) = 108.8$$