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Correlated Weights in Infinite Limits of Deep Convolutional Neural Networks

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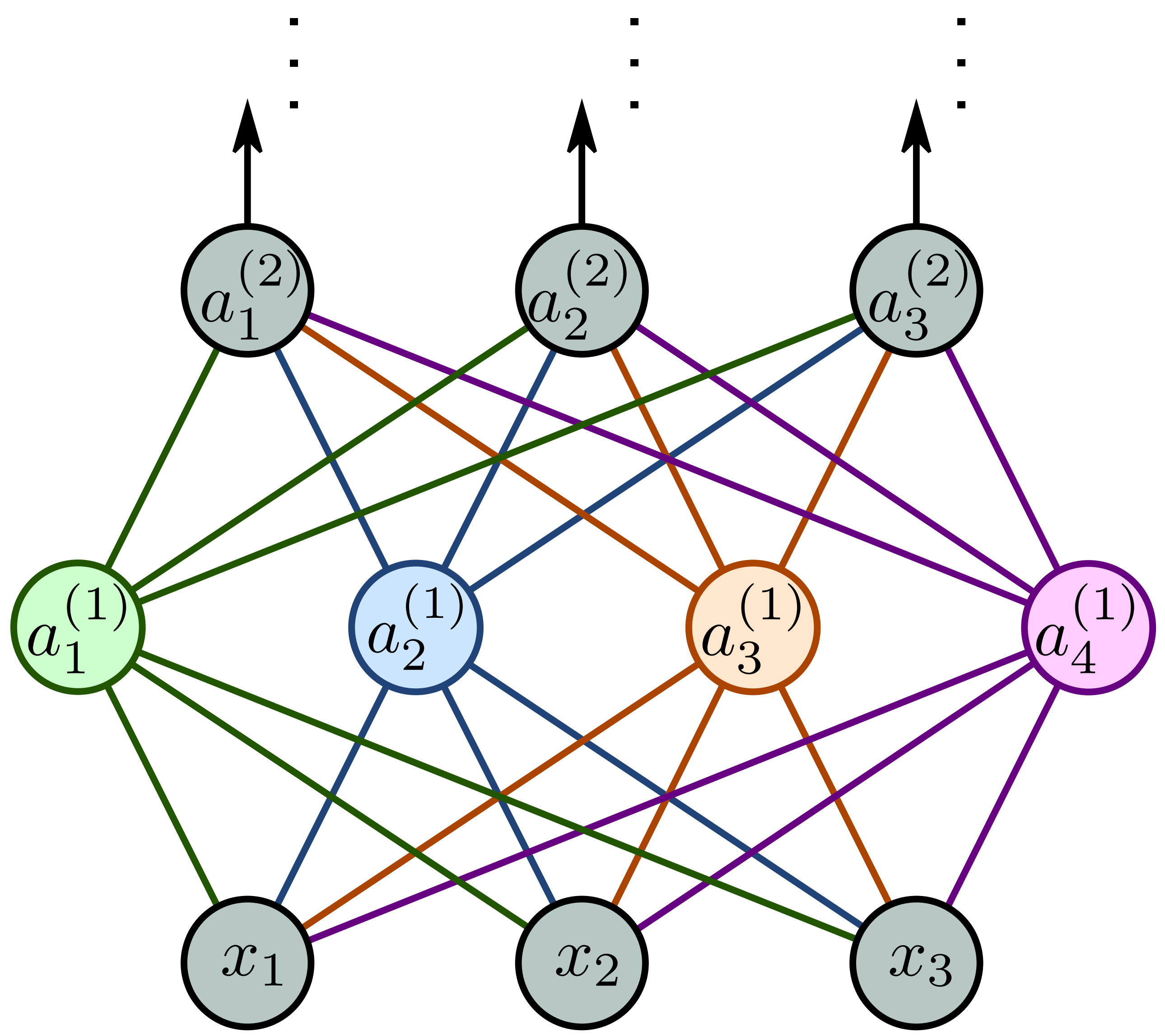
Citations in the description below!

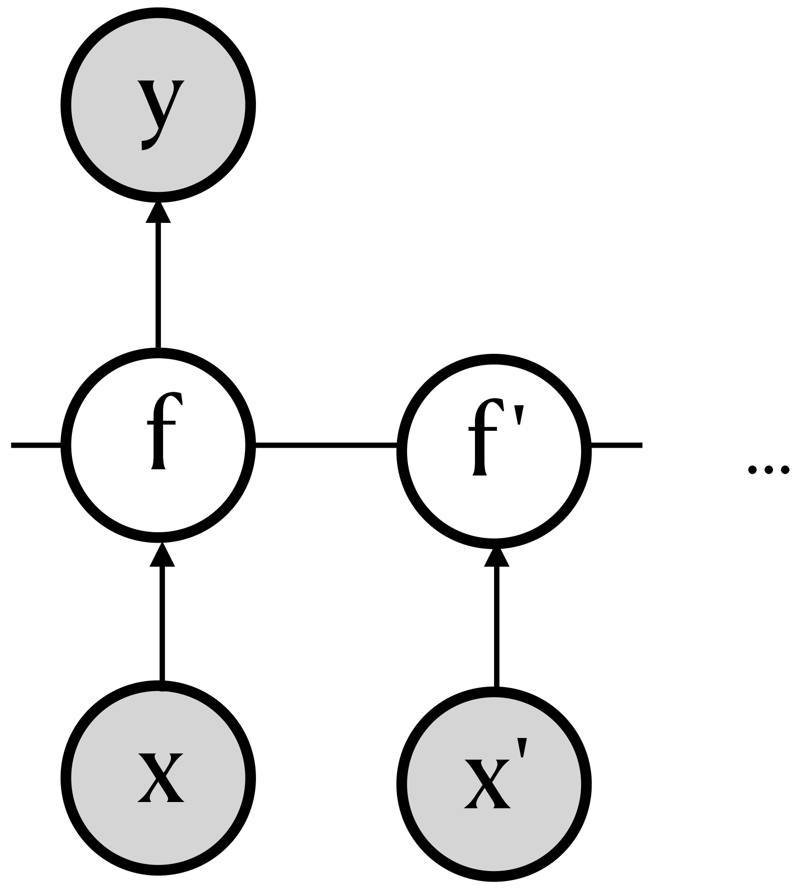
Bayesian neural network

Gaussian process



$\lim \text{width} \to \infty$

see e.g. Yang, 2019



First noted by Neal (1996)

Bayesian neural network

- hard to infer posterior

+ Learns feature functions from data

Gaussian process

+ Easy to infer posterior

- Feature functions fixed (by the kernel function of the GP)

Can GPs really replace NNs?

Have we thrown the baby out with the bathwater?

Bayesian convolutional neural network

- The same function is applied to all patches of the image
  - The output of different patches in different locations is correlated

Gaussian process

- A different (random) function is applied to each patch
- The activation for different patches is uncorrelated and independent

The infinite limit of mean-pooling restores the importance of these correlations,

But for Bayesian CNNs, these correlations matter even without pooling

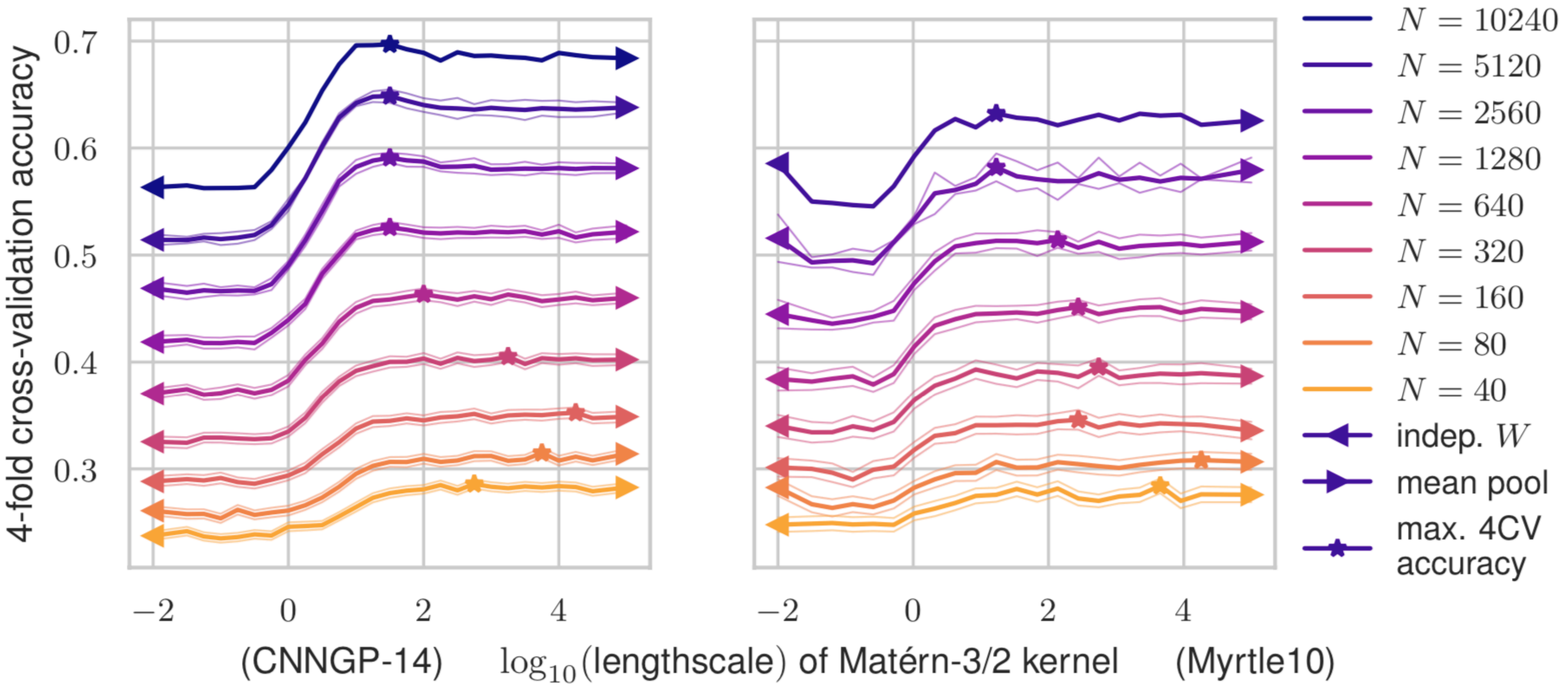
Can we keep them in the infinite limit without changing the architecture?

Yes!

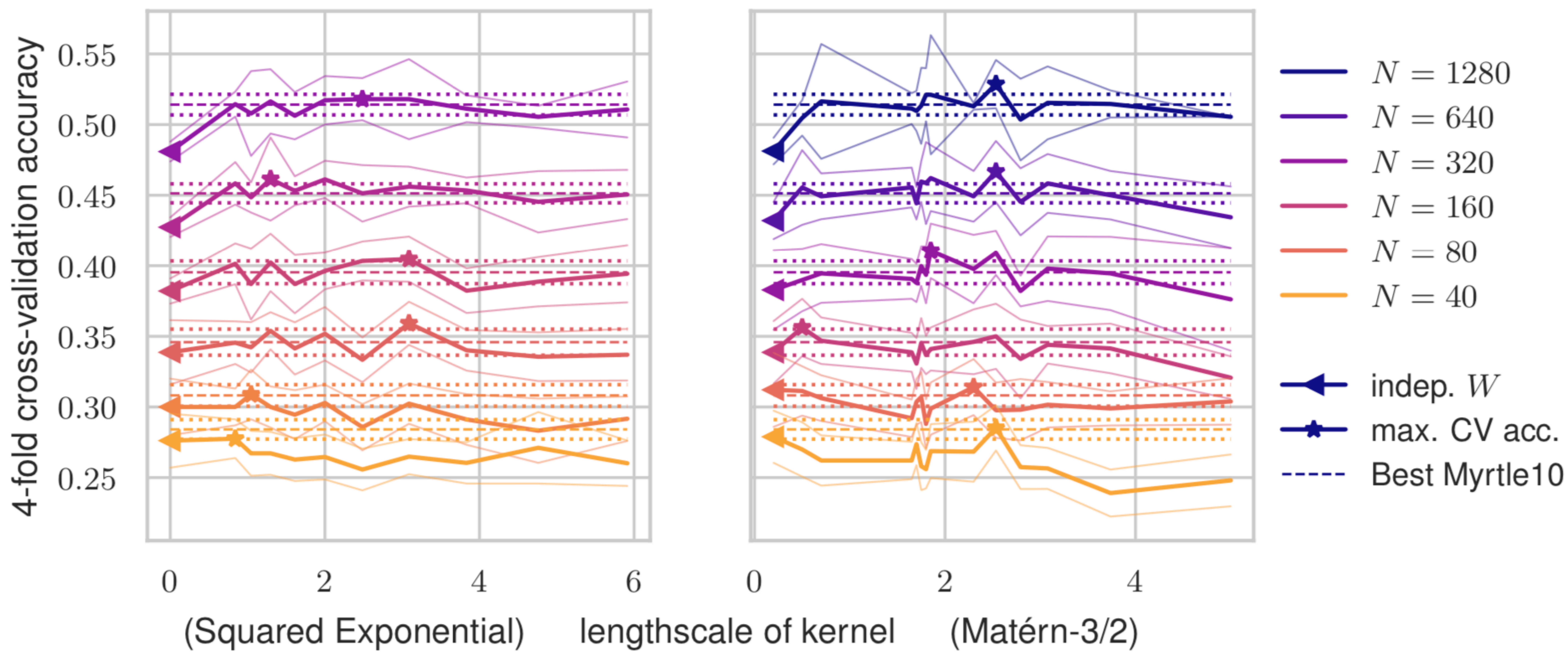
spatial correlation in the weights → spatial correlation between patches in the infinite limit

d-dimensional convolution of weights in the NN → 2d-dimensional convolution of covariance tensors in the kernel

Generalizes mean-pooling (all-ones covariance) and independent weights.



4-fold cross-validation accuracy
0.7
0.6
0.5
0.4
0.3
−2
0
2
4
(CNNGP-14)
log10(lengthscale) of Matérn-3/2 kernel
(Myrtle10)
N = 10240
N = 5120
N = 2560
N = 1280
N = 640
N = 320
N = 160
N = 80
N = 40
indep. W
mean pool
max. 4CV accuracy



4-fold cross-validation accuracy
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0
2
4
6
0
2
4
(Squared Exponential)
lengthscale of kernel
(Matérn-3/2)
$N = 1280$
$N = 640$
$N = 320$
$N = 160$
$N = 80$
$N = 40$
indep. W
max. CV acc.
Best Myrtle10

References

- Yang, Greg, ...
- Neal, Radford, ...
- MacKay, D.J., ...