Correlated Weights in Infinite Limits
of Deep Convolutional Neural Networks

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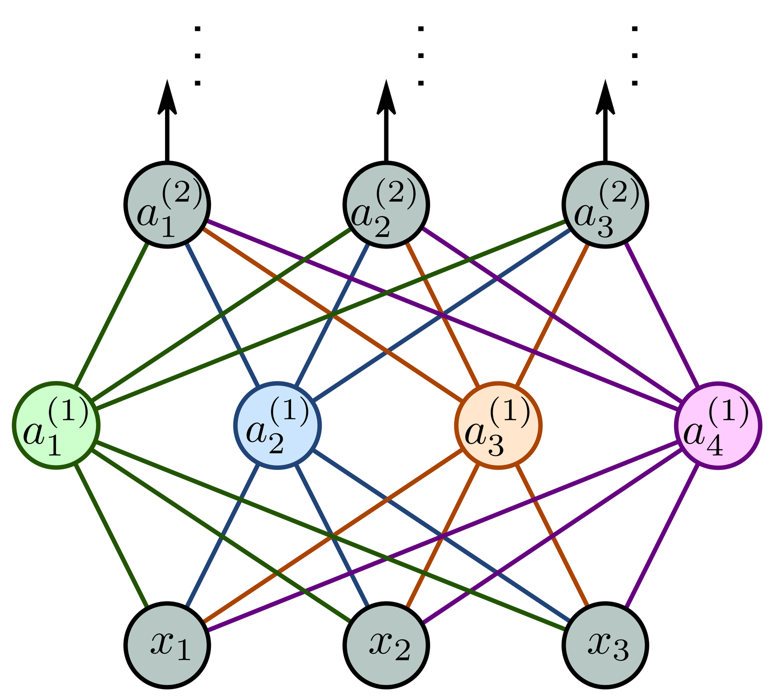
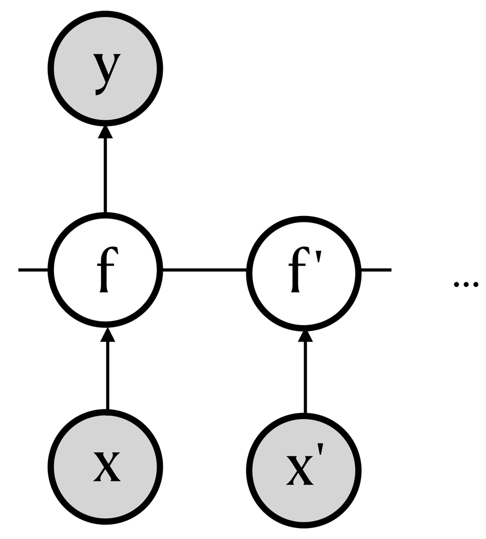


lim width $\to \infty$

see e.g. Yang, 2019

First noted by Neal (1996)



Bayesian convolutional NN

- Hard to infer posterior

+ Learns feature functions from data

Gaussian process (GP)

+ Easy to infer posterior

- Feature functions fixed (by the kernel function of the GP)

With independent, zero-mean weight prior...

+ Applies the same (random) function to each image patch

↓

Spatially **correlated** activations

- Applies a different random function to each image patch

(Locally connected network, LeCun, 1989. Noted by Novak et al. (2019)

↓

Spatially **uncorrelated** activations

The GP loses spatial correlations? How do we solve this?

Spatial correlation in weight prior → Spatial correlation between activations in the ∞-width limit

D-dimensional weight convolution → 2D-dimensional covariance tensor convolution

$$Z_{i,\boldsymbol{q}}^{(\ell)}(\boldsymbol{X}) = \sum_{j=1}^{C^{(\ell-1)}} \sum_{\boldsymbol{p}=\boldsymbol{1}}^{\boldsymbol{P}^{(\ell)}} W_{i,j,\boldsymbol{p}}^{(\ell)} A_{j,\tilde{\boldsymbol{q}}(\boldsymbol{p})}^{(\ell-1)}(\boldsymbol{X})$$

$$K_{\boldsymbol{q},\boldsymbol{q}'}^{(\ell)}(\boldsymbol{X},\boldsymbol{X}') = \sum_{\boldsymbol{p}=\boldsymbol{1}}^{\boldsymbol{P}^{(\ell)}} \sum_{\boldsymbol{p}'=\boldsymbol{1}}^{\boldsymbol{P}^{(\ell)}} \Sigma_{\boldsymbol{p},\boldsymbol{p}'}^{(\ell)} V_{\tilde{\boldsymbol{q}}(\boldsymbol{p}),\tilde{\boldsymbol{q}}'(\boldsymbol{p}')}^{(\ell-1)}(\boldsymbol{X},\boldsymbol{X}')$$

Consequences:

- Previously, only mea-pooling would add spatial correlations to the GP limit
  - Recover and **interpolate** independent weights and mean pooling

- Intermediate-correlation weights have **better performance**, for the GP limit (this paper), and in finite BNNs (Fortuin et al. 2021)

References:

- Fortuin, V., Garriga-Alonso, A., Wenzel, F., Rätsch, G., Turner, R., van der Wilk, M., & Aitchison, L. (2021). Bayesian neural network priors revisited. https://arxiv.org/abs/2102.06571
- Greg Yang. Wide feedforward or recurrent neural networks of any architecture are Gaussian processes. NeurIPS 2019.
- Radford M. Neal. Bayesian learning for neural networks, volume 118. Springer, 1996.