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Correlated Weights in Infinite Limits of Deep Convolutional Neural Networks

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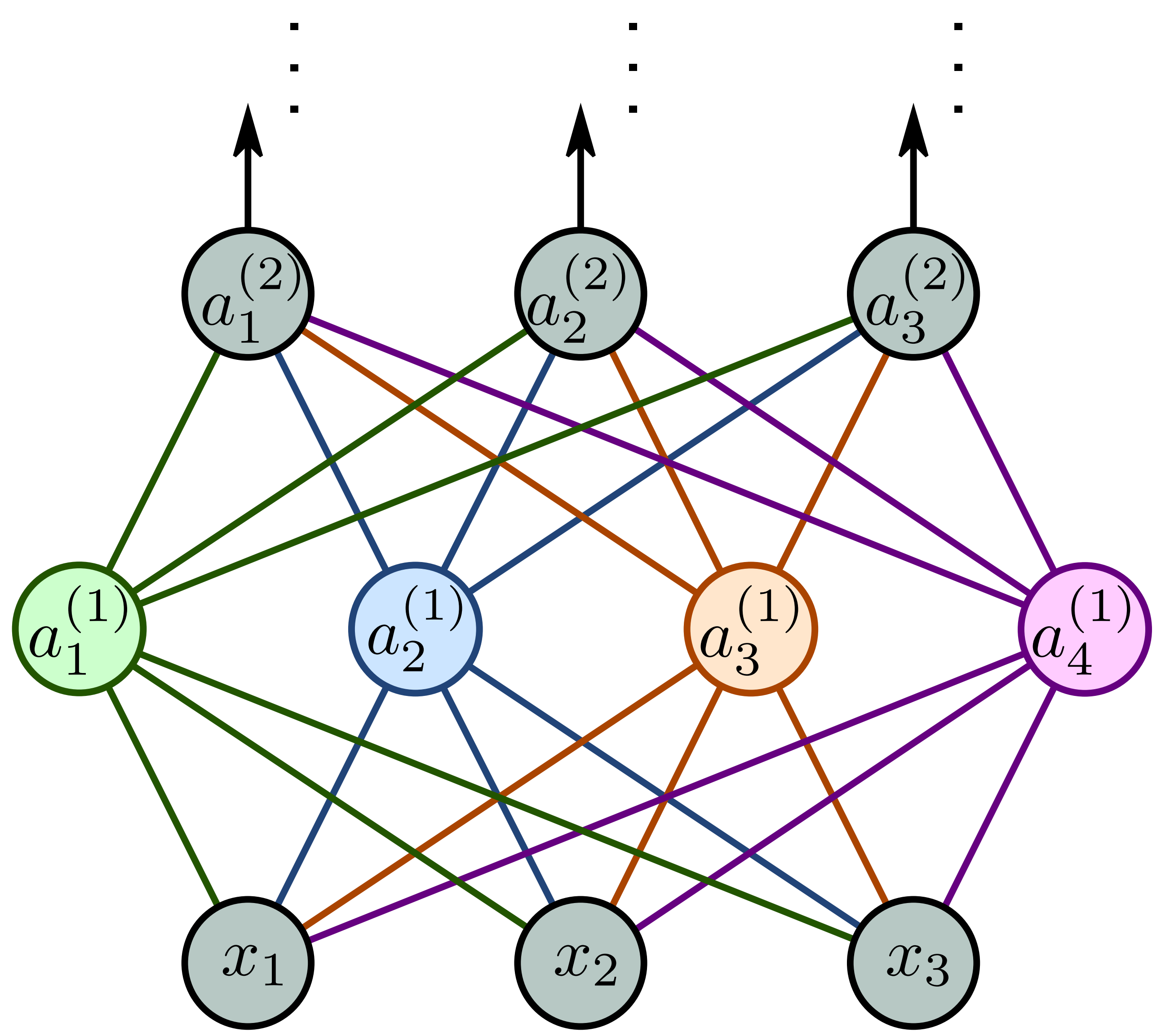
Bayesian neural network

Gaussian process (GP)

of *any* architecture

with independent, zero-mean
weight prior,
Gaussian bias prior

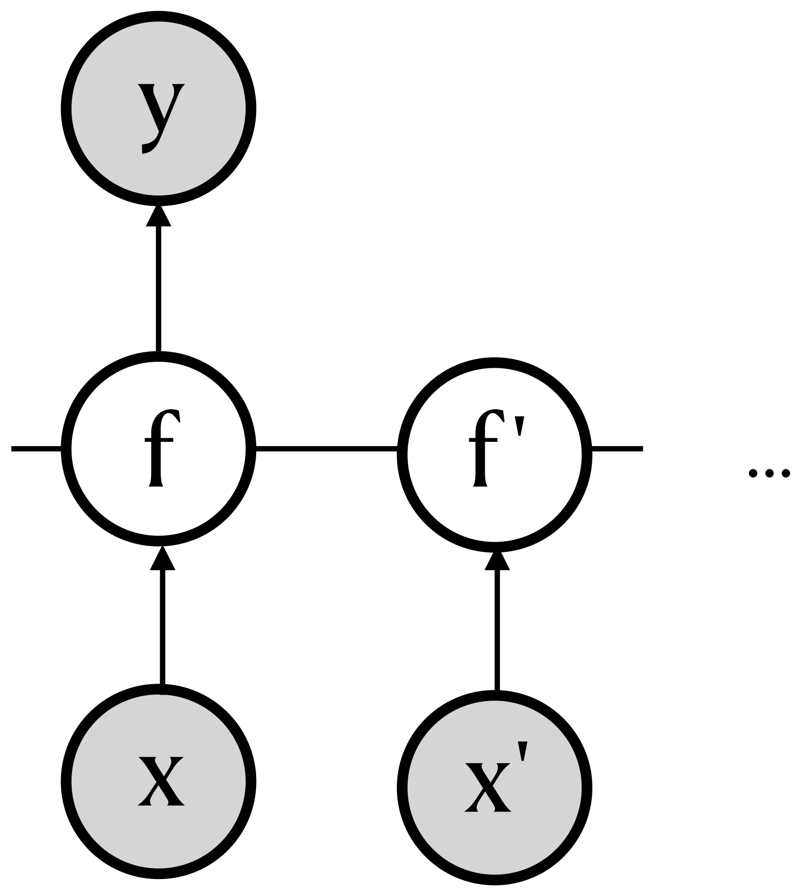


$\lim \text{width} \to \infty$

see e.g. Yang, 2019

First noted by Neal (1996)

Williams & Rasmussen (1995)



Bayesian neural network	Gaussian process
- Hard to infer posterior	+ Easy to infer posterior
+ Successful at modelling functions	+ Successful at modelling functions
+ Learns feature functions from data	- Feature functions fixed (by the kernel function of the GP)

Can GPs really replace NNs?

"Have we thrown the baby out with the bathwater?"

David J. C. MacKay, 1998

This paper: convolutions

Bayesian convolutional NN

\+ Applies the same (random) function to each image patch

↓

Spatially **correlated** activations

Corresponding GP

\- Applies a different random function to each image patch

(Locally connected network, LeCun, 1989. Noted by Novak et al. (2019)

↓

Spatially **uncorrelated** activations

Can we avoid this?

This paper: convolutions

Bayesian convolutional NN		Corresponding GP
Spatial correlation in weight prior	→	Spatial correlation between activations in the ∞-width limit
D-dimensional weight convolution	→	2D-dimensional covariance tensor convolution

This paper: generalize

Independent weights

$$\mathrm{Cov} = \begin{array}{|c|c|c|c|}\hline 1 & & & \\ \hline & 1 & & \\ \hline & & 1 & \\ \hline & & & 1 \\ \hline \end{array}$$

$\longleftrightarrow$

Interpolate

(stationary kernel
on positions)

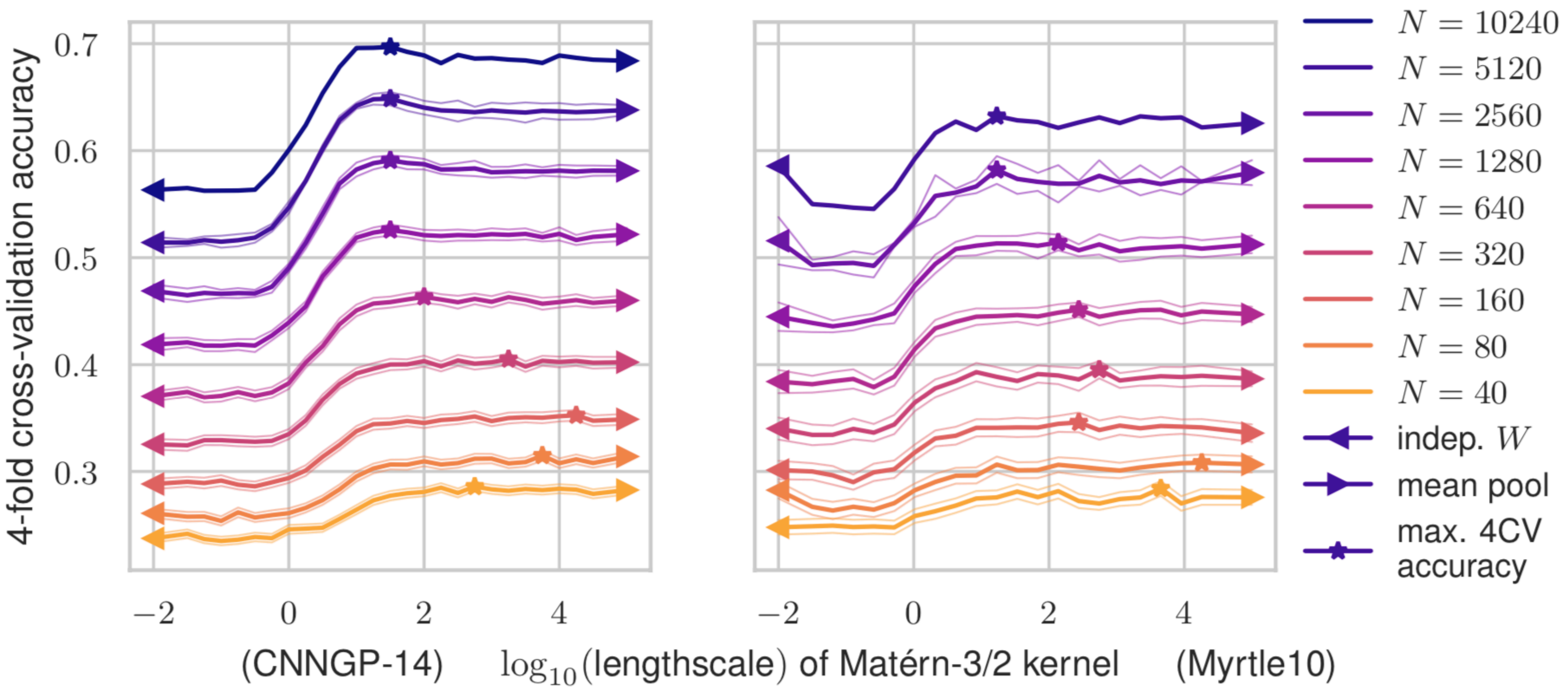
Mean pooling

$$\mathrm{Cov} = \frac{1}{16} \begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ \hline \end{array}$$

Convolutional Gaussian processes
(van der Wilk et al., 2017)

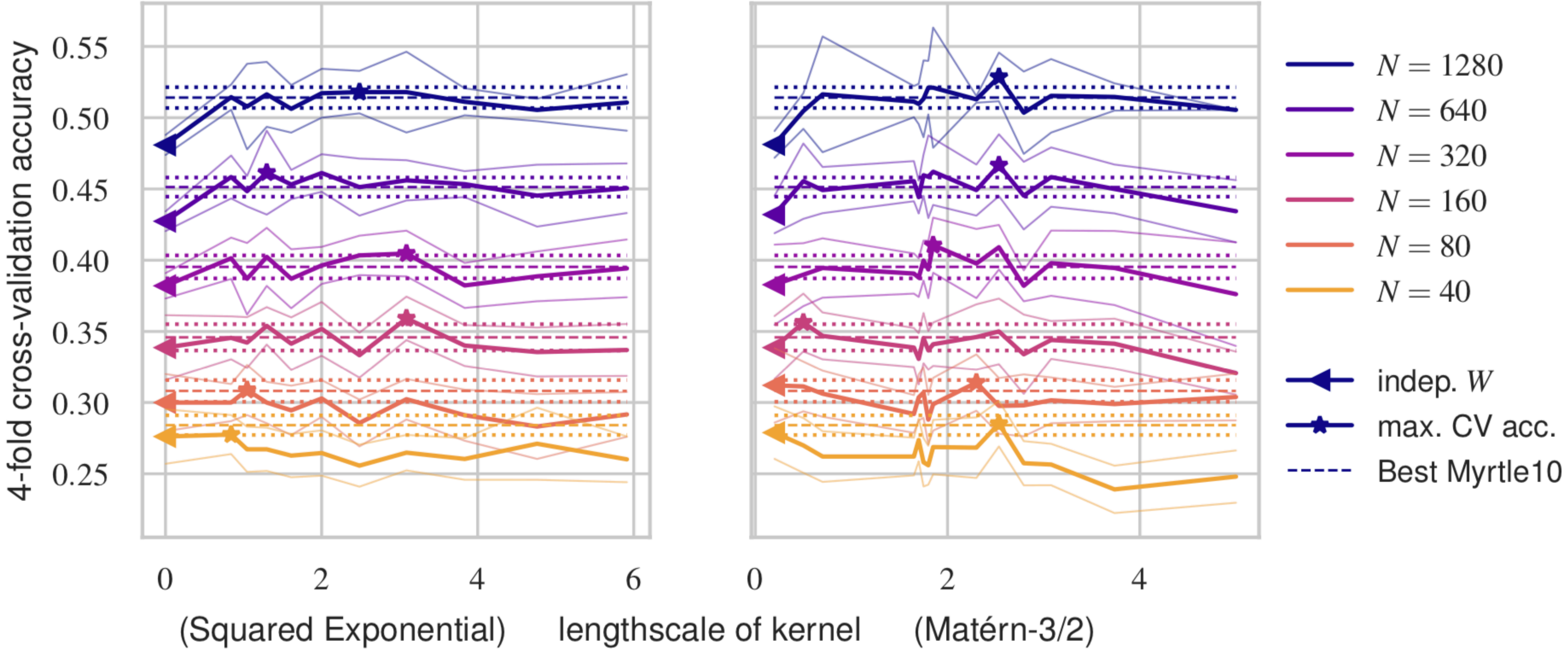
Experiments

Last layer correlation



Dataset: CIFAR-10

Intermediate layers correlation



Dataset: CIFAR-10. Network: Myrtle10 (Shankar et al. 2020).
Replace mean-pooling and convolution by just convolution.

Take-home message

- Infinite limit of independent-weight CNN has no spatial correlations.
  - Recover them with spatial correlations in the weights.
  - Successful prior for finite Bayesian NNs (Fortuin et al., 2021)

- Prior and kernel generalize existing full-independence and mean-pooling

- Competitive performance by tuning continuous
  "spatial correlation" parameters

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