Melissa Auclaire

Algorithm Design Manual
Skiena Chapter 1
Exercises

Finding Counterexamples

1-1. [3] Show that $a+b$ can be less than $\min(a,b)$.

$a+b$ when

$a=0, b=0$	$a+b=0$	$\min(0,0) \Rightarrow 0$
$a=1, b=1$	$a+b=2$	$\min(1,1) \Rightarrow 1$
$a=9, b=9$	$a+b=18$	$\min(9,9) \Rightarrow 9$
$a=-9, b=9$	$a+b=0$	$\min(-9,9) \Rightarrow -9$
$a=-9, b=-9$	$a+b=-18$	$\min(-9,-9) \Rightarrow -9$

$\therefore$ when $a=-9$ and $b=-9$ $a+b > \min(a,b)$. ∎

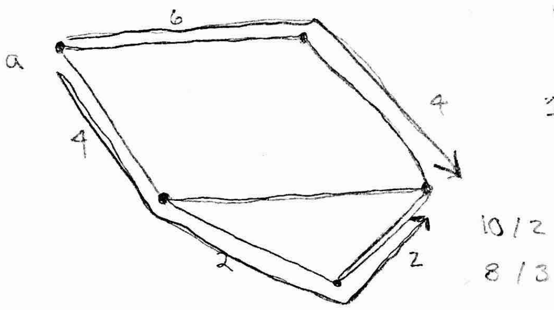
1-2 [3] Show that $a \times b$ can be less than $\min(a,b)$.

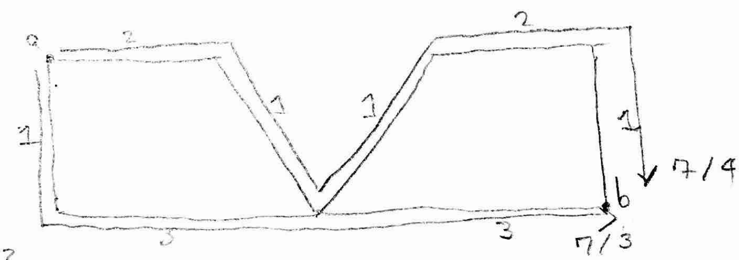
$a \times b$ when

$a=0, b=0$	$a \times b = 0$	$\min(0,0) \Rightarrow 0$
$a=0, b=1$	$a \times b = 0$	$\min(0,1) \Rightarrow 0$
$a=-1, b=1$	$a \times b = -1$	$\min(-1,1) \Rightarrow -1$
$a=-2, b=-1$	$a \times b = 2$	$\min(-2,-1) \Rightarrow -2$

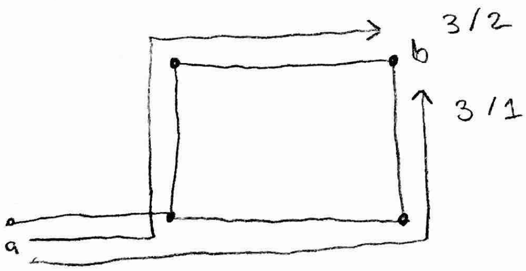
$\therefore$ when $a=-2$ and $b=-1$ $a \times b > \min(a,b)$. ∎

1-3 [5] Design / draw a road network with two points a and b such that the fastest route between a and b is not the shortest route.





1-4 [5] Design / draw a road network with two points a and b such that the shortest route between a and b is not the route with the fewest turns.



1-5 [4] The knapsack problem is as follows: given a set of integers $S = \{s_1, s_2, \ldots, s_n\}$, and a target number T, find a subset of S which adds up exactly to T. For example, there exists a subset within $S = \{1, 2, 5, 9, 10\}$ that adds up to $T = 22$ but not $T = 23$.

Find counterexamples to each of the following algorithms for the knapsack problem. That is, giving an S and T such that the subset is selected using the algorithm does not leave the knapsack completely full, even though such a solution exists.

(a) Put the elements of S in the knapsack in left to right order if they fit, i.e. the first-fit algorithm.

(b) Put the elements of S in the knapsack from smallest to largest, i.e. the best fit algorithm.

(c) Put the elements of S in the knapsack from largest to smallest.

a) $S = \{1, 2, 3, 4, 5\}$ where $T = 9$
b) $S = \{1, 2, 3, 4, 5\}$ where $T = 12$
c) $S = \{5, 4, 3, 2, 1\}$ where $T = 9$

★ Ask about it!
Wiki Solutions
a) $S = \{1, 2, 3\}$ $T = 5$
b) $S = \{1, 2, 3\}$ $T = 5$
c) $S = \{4, 3, 2\}$ $T = 5$

1-6 [5] The set cover problem is as follows:
Given a set of subsets $S_1, \ldots, S_m$ of the universal set $U = \{1, \ldots, n\}$, find the smallest subset of subsets $T \subset S$ such that $\cup_{t_i \in T} t_i = U$.
For example, there are the following subsets, $S_1 = \{1, 3, 5\}$, $S_2 = \{2, 4\}$, $S_3 = \{1, 4\}$, and $S_4 = \{2, 5\}$ The set cover would then be S_1 and S_2.
Find a counterexample for the following algorithm: Select the largest subset for the cover, and then delete all its elements from the universal set. Repeat by adding the subset containing the largest number of uncovered elements until all are covered.

$S_1 = \{1, 3, 5\}$ $\quad U = \{1, 2, 3, 4, 5\}$
$S_2 = \{2, 4\}$ $\quad U - S_1 = \{2, 4\}$
$S_3 = \{1, 4\}$ $\quad U + S_2 = \{\}$
$S_4 = \{2, 5\}$

$S_1 = \{1, 2, 6, 8, 10, 14\}$ $\quad U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$
$S_2 = \{11, 12, 13, 14, 15, 16, 9\}$ $\quad U - S_2 = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$
$S_3 = \{3, 9, 11, 12, 13, 15\}$ $\quad U - S_1 = \{3, 4, 5, 7\}$
$S_4 = \{4, 5, 7, 16\}$ $\quad U - S_4 = \{3\}$
$S_5 = \{4, 5, 6\}$ $\quad U - S_6 = \{\}$
$S_6 = \{1, 2, 3\}$
$S_7 = \{7, 8, 10\}$

However:
$U - S_1 - S_3 = \{4, 5, 7, 16\} - S_4 = \{\}$

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Proofs of Correctness

1-7 [3] Prove the correctness of the following recursive algorithm to multiply two natural numbers, for all integer constants $c \geq 2$.

function multiply(y, z)
Comment return the product yz.
1. if z = 0 then return(0) else
2. return (multiply (cy, ⌊z/c⌋) + y · (z mod c))

Base case:

(Floor)

$multiply(0,0) \Rightarrow 0$

$multiply(0,1) \Rightarrow multiply(2(0), \lfloor \frac{1}{2} \rfloor) + 0 \cdot (1 \bmod 2))$
$multiply(0,0) \Rightarrow 0$

$A = multiply(cy, \lfloor [n+1]/c \rfloor)$

$B = y([n+1] \bmod c)$

$multiply(y,z) = A + B$

$c \geq 2 \therefore \lfloor (n+1)/c \rfloor < (n+1)$

$A = multiply(cy, \lfloor [n+1]/c \rfloor) = cy * \lfloor [n+1]/c \rfloor$

$A = cy * \lfloor [n+1]/c \rfloor = y * \lfloor z/c \rfloor c$

$\lfloor z/c \rfloor c + (z \bmod c) = z$

$\lfloor z/c \rfloor c = z - (z \bmod c)$

$A = y * \lfloor z/c \rfloor c = y(z - z \bmod c) = yz - y(z \bmod c)$

$A + B = yz - y(z \bmod c) + y(z \bmod c) = yz$ ∎

Subproof Show that $\lfloor z/c \rfloor c + z \bmod c = z$ where $c >= 2$

Assume $z \bmod c = a$

$\therefore (z-a)/c = \lfloor z/c \rfloor$

$\therefore (z-a)/c * c + a = (z-a) + a = z$

1-8 [3] Prove the correctness of the following algorithm for evaluating a polynomial

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

function horner (A, x)
p = A_n
for i from n-1 to 0
p = p * x + A_i
return p

Base case:

for n = 1 horner ([a0], x) = a0

Inductive step:

horner ([a0, ..., a(n+1)], x) =
x * horner ([b0, ..., bn], x) + a0
where bn = a(n+1) ∎

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Induction

1-10 [3] Prove that $\sum_{i=1}^{n} i = n(n+1)/2$ for $n \geq 0$, by induction

Base case: $n = 1$

$$\sum_{i=1}^{1} i = 1 \qquad \frac{1(1+1)}{2} = 1$$

Induction: $n = n+1$

$$\sum_{i=1}^{n+1} i = (n+1) + \sum_{i=1}^{n}$$

$$\sum_{i=1}^{n+1} i = (n+1) + \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n+1} i = \frac{2(n+1)}{2} + \frac{n(n+1)}{2}$$

$$\sum_{i=2}^{n+1} i = \frac{2n+2}{2} + \frac{n^2+n}{2}$$

$$\sum_{i=1}^{n+1} i = \frac{n^2+3n+2}{2} \quad \blacksquare$$

Substitute $n+1$

$$\frac{(n+1)((n+1)+1)}{2}$$

$$\frac{(n^2+2n+1)+(n+1)}{2}$$

$$\frac{n^2+3n+2}{2}$$

1-11 [3] Prove that $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$ for $n \geq 0$, by induction

Base case:

$$\sum_{i=1}^{0} i^2 = 0^2 = 0$$

$$\frac{0(0+1)(2(0)+1)}{6} = 0$$

Inductive step:

$$\sum^{n+1} i^2 = (n+1)^2 + \sum_{i=1}^{n} i^2 =$$

$$(n^2+2n+1) + \frac{n(n+1)(2n+1)}{6} =$$

$$\frac{6n^2+12n+6}{6} + \frac{2n^3+3n^2+n}{6} =$$

$$\frac{2n^3+9n^2+13n+6}{6} \quad \blacksquare$$

$\frac{n(n+1)(2n+1)}{6}$ when $n = n+1$

$$\frac{(n+1)(n+1+1)(2(n+1)+1)}{6} =$$

$$\frac{(n+1)(n+2)(2n+2+1)}{6} =$$

$$\frac{(n^2+3n+2)(2n+3)}{6} =$$

$$\frac{2n^3+9n^2+13n+6}{6}$$

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Induction

1-12 [3] Prove that $\sum_{i=1}^{n} i^3 = n^2(n+1)^2/4$ for $n \geq 0$, by induction.

Base case:

$\sum_{i=1}^{0} i^3 = 0^3 = 0$

$\frac{0^2(0+1)^2}{4} = 0$

Inductive step:

Substitute
$n^2(n+1)^2/4$
$n+1^2((n+1)+1)^2/4$

$\sum_{i=1}^{n+1} i^3 = (n+1)^2((n+1)+1)^2/4 + \sum_{i=1}^{n} i^3$

$\sum_{i=1}^{n+1} i^3 = (n+1)^2((n+1)+1)^2/4 + n^2(n+1)^2/4$

$= ((n+1)(n+1))(n+1+1)((n+1)+1)/4$

$(n+1)^2((n+1)+1)^2/4$ ∎

$(n^2+2n+1)(n^2+2n+1+n+1+n+1+2)/4$

$n^2+2n+1 * \frac{n^2+4n+5}{4}$

$n^2+2n+1 * \frac{1}{4}(n^2+4n+5)$

$(n+1)^2((n+1)+1)^2$

1-13 [3] Prove that

$\sum_{i=1}^{n} i(i+1)(i+2) = n(n+1)(n+2)(n+3)/4$

Substitute
$(n+1)((n+1)+1)((n+1)+2)((n+1)+3)/4$
$= \frac{(n+1)(n+2)(n+3)(n+4)}{4}$

Base case:

$\sum_{i=1}^{0} i(i+1)(i+2) = 0(0+1)(0+2) = 0$

$0(0+1)(0+2)(0+3)/4 = 0$

Inductive Step

$\sum_{i=1}^{n+1} i(i+1)(i+2) = (n+1)((n+1)+1)((n+1)+2) + \sum_{i=1}^{n} i(i+1)(i+2)$

$\sum_{i=1}^{n+1} i(i+1)(i+2) = (n+1)((n+1)+1)((n+1)+2) + n(n+1)(n+2)(n+3)/4$

$= (n+1)(n+2)(n+3) + n(n+1)(n+2)(n+3)/4$

$= \frac{4(n+1)(n+2)(n+3)}{4} + \frac{n(n+1)(n+2)(n+3)}{4}$

$= \frac{(n+1)(n+2)(n+3)(n+4)}{4}$ ∎

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1-14 [5] Prove by induction on $n \geq 1$ that for every $a \neq 1$,

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$$

Base Case:

$$\sum_{i=0}^{0} a^0 = 1 \qquad \sum_{i=0}^{1} a^1 = a^1 + 1$$

$$\frac{a^{0+1} - 1}{a - 1} \quad 1 \quad \frac{a^1 - 1}{a - 1} \qquad \frac{a^{1+1} - 1}{a^1 - 1} = \frac{a^2 - 1}{a - 1} = a + 1$$

$$= \frac{a - 1}{a - 1} = 1$$

Inductive step: ☆ Ask about this

1-15 [3] Prove by induction that for $n \geq 1$,

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

Base case: $n = 1$

$$\frac{1}{i(i+1)} = \frac{n}{n+1}$$

$$\frac{1}{1(1+1)} = \frac{1}{1+1} = \frac{1}{2}$$

Inductive step:

$$\sum_{i=1}^{n+1} = \frac{n+1}{n+1+1} = \frac{n}{n+1} + \frac{1}{(n+1)(n+1+1)}$$

$$\frac{n+1}{n+2} = \frac{n(n+2)}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)}$$

$$\frac{n+1}{n+2} = \frac{n(n+2)+1}{(n+1)(n+2)}$$

$$\frac{n+1}{n+2} = \frac{n^2+2n+1}{(n+1)(n+2)}$$

$$\frac{n+1}{n+2} = \frac{(n+1)(n+1)}{(n+1)(n+2)}$$

$$\frac{n+1}{n+2} = \frac{(n+1)}{(n+2)} = \frac{n+1}{n+1+1} \blacksquare$$

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Induction contd.

1-16 [3] Prove by induction that $n^3 + 2n$ is divisible by 3 for all $n \geq 0$.

Base case: $n = 0$

$0^3 + 2(0) = 0/3 = 0$

Inductive step:

$$\frac{(n+1)^3 + 2(n+1)}{3} = \frac{n^3 + 3n^2 + 3n + 1 + 2n + 2}{3}$$

$$\frac{n^3 + 2n}{3} + \frac{3n^2 + 3n + 3}{3} \quad \blacksquare$$

1-17 [3] Prove by induction that a tree with n vertices has exactly n-1 edges.

Ex.

$v = 5$
$e = v - 1$

$T_v(n) = n - 1 = T_e(n) - 1$

Base case: • $v = 1$, $e = 0 = v - 1$ $\quad T_v(1) = 1 - 1 = 0$

Inductive step:

$T_v(n) - T_e(n) = 1$

$T_v(n+1) = (n+1) - 1$
$T_v(n+1) = n$
$T_v(n) + 1 = n$
$n - 1 + 1 = n$
$n = n \quad \blacksquare$

1-18 [3] Prove by mathematical induction that the sum of the cubes of the first n positive integers is equal to the square of the sum of these integers, i.e.

$$\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$$

Base Case:

$$\sum_{i=1}^{1} i^3 = \left(\sum_{i=1}^{1}\right)^2$$

$$\Leftrightarrow 1^3 = 1^2 = 1$$

Inductive Step:

$$\sum_{i=1}^{n+1} i^3 = \left(\sum_{i=1}^{n+1} i\right)^2$$

$$\Leftrightarrow \sum_{i=1}^{n} i^3 + (n+1)^3 = \left(\sum_{i=1}^{n} i + (n+1)\right)^2$$

$$= \left(\sum_{i=1}^{n} i\right)^2 + 2\left(\sum_{i=1}^{n} i\right)(n+1) + (n+1)^2$$

$$= \left(\sum_{i=1}^{n} i\right)^2 + 2\left(\frac{n(n+1)}{2}\right)(n+1) + (n+1)^2$$

1-18 contd.

$$\sum_{i=1}^{n} i^3 + (n+1)^3 = \left(\sum_{i=1}^{n} i\right)^2 + n(n+1)^2 + (n+1)^2$$

$$\sum_{i=1}^{n} i^3 + (n+1)^3 = \left(\sum_{i=1}^{n} i\right)^2 + (n+1)(n+1)^2$$

$$\Leftrightarrow \sum_{i=1}^{n} i^3 + (n+1)^3 = \left(\sum_{i=1}^{n} i\right)^2 + (n+1)^3$$

$$\Leftrightarrow \sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2 \quad \blacksquare$$

Estimation

1-19 [3] Do all the books you own total at least 1 million pages? How many total textbooks are stored in your school library?

I currently own at present (in my possession)

15 textbooks

14 non fiction & novels

6 Picture Books

Assuming

Textbook ≈ 750 [pg]

Non-fiction / noven ≈ 400 [pg]

Picture Book ≈ 100 [pg]

The total pages of the books I currently own at the present moment are

(15) 750 + (14) 400 + (6) 100 = 17450 pages < 1 million pages

(Note I own many books that are not in my current possesion & thus uncoutable !!)

If a 750 [pg] textbook is ≈ 2", and a shelf in my school's library is 8 shelves high and roughly 20 ft wide, 20 per floor & 4 floors then

1 shelf contains 16 × 20 ft = 320 ft which is 3840 inches / shelf.
There are 20 shelves per floor therefore there are 76800 inches of books per flo
There are 4 floors, so there are ≈ 307,200 inches of books in the entire libra

$\therefore \frac{307,200}{2} \cdot 750 \approx 115\,200\,000 \Rightarrow$ over 115 million pages > 1 million pa

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1-20 [3] How many words are there in this textbook?

There are roughly 750 pages in this textbook.
Assuming an average of 150 words per page, there are
$750 * 150 \approx 112,500$ words.

1-21 [3] How many hours are in 1 million seconds? How many days?
Answer these questions by doing all arithmetic in your head.

72,000 seconds in a day
504,000 seconds in a week
1 million seconds is roughly 2 weeks

60 seonds in min
3600 seconds in hour
85,200 seconds in day

1-22 [3] Estimate how many cities and towns there are in the United States.

Assuming there are 3 cities and 25 towns per state:

$(3 \times 50) + (25 \times 50) \approx 150 + 1250 \approx 1400$ cities and towns in the US

1-23 [3] Estimate how many cubic miles of water flow out of the mouth of the Mississippi River each day. Do not look up any supplemental facts. Describe all assumptions that you made before arriving at your answer.

Mississipi River Assumptions

$\approx$ 30 mi wide
$\approx$ 1000 mi long
$\approx$ 2 mi deep
Thus $1000 \times 30 \times 2 \approx 60000$ $[mi]^3$ of water. Mouth of river is also 30 [mi] wide & water flows at 5 mph
Therefore the area of displaced water is
$30 [mi] \times 2 [mi] \times 5 [mi] \approx 300 [mi]^3$ per day

1-24 [3] Is disk drive access time normally measured in milliseconds (thousandths of a second) or microseconds (millionths of a second)? Does your RAM memory access a word in more or less than a microsecond? How many instructions can your CPU execute in 1 year if your machine is left running all the time?

I would imagine it is microseconds because the amount of data and how it is stored would influence the number of read operations. Access time would need to include all of this data. RAM would be accessed in less than a microsecond because if it weren't cached in a register or L2 it would travel along a bus based on a signal from software to hardware. The distance of the bus

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1-24 |
is less than the distance it would take for the electrical signal to fetch from RAM, and then for the accumulator to subsequently load the data into a register by far, so I would assume it would be $< 1\mu s$ as long as the OS hasn't left it that way due to legacy. My CPU can most likely complete billions of instructions in parallel per second. I would assume a core i-7 can complete 8 billion, so the amount of instructions it would be able to complete in a year would be:

$$8 \text{ billion} \times 60[s] \times 60[m] \times 24[h] \times 7[d] \times 52[w]$$
$$\approx 2.51 \times 10^{17}.$$

1-25 [4] A sorting algorithm takes 1 second to sort 1,000 items on your local machine. How long will it take to sort 10,000 items ...

(a) If you believe that the algorithm takes time proportional to n^2, and
(b) If you believe that the algorithm takes time roughly proportional to $n \log n$?

(a) If the algorithm were $O(n^2)$ in the worst case / average cases then the difference would be

1000^2 vs. 10000^2

$1{,}000{,}000$ to $100{,}000{,}000$

Thus, 100 times slower to sort 10,000 items $\Rightarrow$ 100 [s]

(b) If the algorithm were $O(n \log n)$ in the worst / average cases then the difference would be

$1000 \log_{10}(1000)$ vs $10000 \log_{10}(10000)$

3000 to 40000

Thus $13.\bar{3}$ times slower to sort 10,000 items $\Rightarrow$ $13.\bar{3}$ [s]

Implementation Projects

1-26 [5] Implement the two TSP heuristics of Section 1.1 (page 5). Which of them gives better-quality solutions in practice? Can you devise a heuristic that works better than both of them?

Nearest neighbor
Closest Pair

Closet Pair performs better in general because it is a greedy algorithm which takes into account all points globally as oppossed to the Nearest Neighbor Heuristic, which only looks at the nearest neighbor's shortest distance.
(Will design in code)

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1-27 [5] Describe how to test whether a given set of tickets establishes sufficient coverage in the Lotto problem of Section 1.6 (page 23). Write a program to find good ticket sets.

Testing: Test edge cases, and number of iterations.

(Program will be written)

Interview Problems

1-28 [5] Write a function to perform integer division without using either the * or / operators. Find a fast way to do it.

```
int division( int dividend, int divisor) {
    int quotient = 1;
    if ( divisor == dividend ) {
        remainder = 0;
        return 1;
    } else if ( dividend < divisor) {
        remainder = dividend;
        return 0;
    }

    do {
        divisor = divisor << 1;
        quotient = quotient << 1;
    } while ( divisor <= dividend );

    quotient = quotient + division ( dividend - divisor , divisor);
    return quotient;
}
```

1-29 [5] There are 25 horses. At most, 5 horses can race together at a time. You must determine the fastest, second fastest, and third fastest horses. Find the minimum number of races in which this can be done.

7 Races

Race 5 times with 5 Pairs of Horses

Race 1 time with 5 winning horses

Race 1 time with 5 second place horses

- The fastest horses are y, j and g

$a_1 b_4 c_5 d_3 e_2$ $p_1 q_4 r_3 s_2 t_5$

$f_4 g_2 h_3 i_5 j_1$ $u_2 v_3 w_4 x_5 y_1$

$k_5 l_1 m_4 n_3 o_2$

y_{1_1} is fastest

j_3 l_4 p_5 q_6 2 races

g_2 s_2 u_2 o_2 e_2 1 race

y_{1_1} j_{1_2} l_{1_3} p_{1_4} a_{1_5}

g_{2_1} s_{2_2} v_{2_3} o_{2_4} e_{2_5}

3 4 5 6 7

1-30 [3] How many piano tuners are there in the entire world?

Assume: 1 in 100 people play Piano
that for every 100 piano players there is 1 piano tuner

If World Population is 7,800,000,000

7,800,000,000 ÷ 100 = 78,000,000 ÷ 100 ≈ 780,000 piano tuners

1-31 [3] How many gas stations are there in the United States.

Assume: U.S. population is 320,000,000
Number of cars is 1/3 of population 105,600,000
The continental US is 3 million sq. miles
3/4's is uninhabited thus 750,000 sq miles habited

If there is 1 gas station per inhabited sq. mile

750,000 gas stations

Check: Each station would service 140.8 cars on a regular basis. If vehicles need service at 4x / month, and the average service is \$30.00:

140.8 × 30 × 4 = \$16,896 per month income.

Assuming \$10,000 per month in bulk fuel costs, \$2000 a month rent and \$2000 upkeep, this would allow the gas stations to remain profitable, before food and miscellaneous items are added in, fitting with our model.

∴ ≈ 750,000 gas stations

1-32 [3] How much does the ice in a hockey rink weigh?

1 liter is defined as

1000 [g]^3. If a hockey rink were 100 [m] × 100 [m], → 10000 [m]^2 of surface area

If the sheet of ice is 0.25 [m] thick:

0.25 [m] = 250 [cm]

1.0000 [m]^2 = 1000000 [cm]^2

The total volume of the ice would be ≈ 250,000,000 [cm]^3

If 1 [l] = 1000 [cm]^3 then $\frac{250{,}000{,}000\ [cm]^3}{1000\ [cm]^3}$ = 250,000 [l]

∴ Thus 250,000 [kg] would be the weight of the ice sheet.

1-33 [3] How many miles of road are there in the United States?

The United States is roughly 2000 miles across, and 1500 miles from North to South. Thus, it is roughly 3 million sq. miles in area.

There are roads in most towns and municipalities as well as the interstate system.

Assuming this is true, let's assume 600 interstates of 100 [mi] each

600 × 100 [mi] = 60,000 [mi]. Earlier we estimated 1400 cities and towns at 30 [mi]^2 of road each on average.

1400 × 30 [mi^2] = 42,000 [mi]^2

If the width of a road is 100 [ft] and 5280 [ft] in one mile

42,000 [mi]^2 = 700 [mi] × 60 [mi]

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Exercises

1-33 [3] contd.

$$60\,[mi] \times 5280\,[ft] = \frac{316{,}800\,[ft]}{100\,[ft]} = 3168 \text{ roads of } 700\,[mi] \text{ length}$$

$$= \underbrace{2{,}217{,}600\,[mi]}_{\text{other roads}} + \underbrace{60{,}000\,[mi]}_{\text{interstate}} = 2{,}277{,}600\,[mi] \text{ of road in the US.}$$

1-34 [3] On average, how many times would you have to flip open the Manhattan phone book at random in order to find a specific name.

Assuming 500,000 Residents in Manhattan.

Each page contains 200 names

$$\frac{500{,}000}{200} = 2500 \text{ pages}$$

An average find is reached at 1250 flips, assuming no duplicates.

1-9 [3] Prove the correctness of the following sorting algorithm.

```
function bubblesort (A: list[1,...,n])
    var int i, j
    for i from n to 1
        for j from 1 to i-1
            if (A[j] > A[j+1])
                swap the values of A[j] and A[j+1]
```

```
for i from n+1 to 1
    for j from 1 to i-1
        if (A[j] > A[j + 1])
            swap the values of A[j] and A[j+1]
```