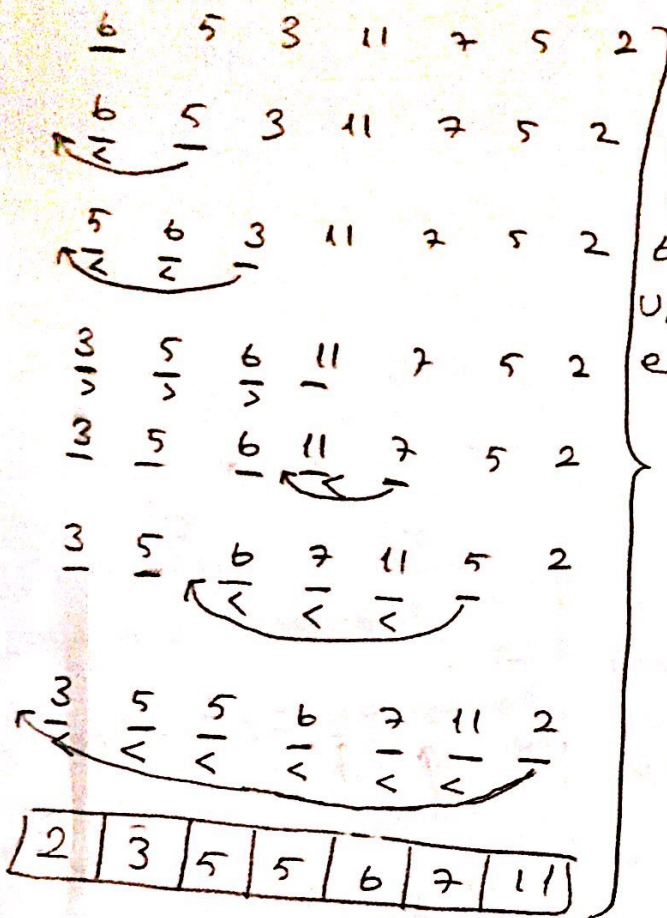


① 6, 5, 3, 11, 7, 5, 2



If the key element is smaller than its predecessor, compare it to the elements before. Move the greater elements one position up to make space for the swapped element.

2.9

```
function (int n) {
    if (n == 1)
        return;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            printf("#");
            break;
        }
    }
}
```

Best case complexity

When the  $n$  is 1, function don't run loop just return. "return" is one process, so the function is  $O(1)$

Worst case Complexity

The second loop will run only once because there is "break" in second loop code scope. The first loop will run  $n$  times. Because they are nested,  $O(1) \times O(n) = O(n)$

Best case  $O(1)$   
 $\Rightarrow$  worst case  $O(n)$



2.5

```
void function (int n) {
```

```
    int count = 0;
```

```
    for (int i = n/3; i <= n; ++i)
```

```
        for (int j = 1; j + n/3 <= n; j++)
```

```
            for (int k = 1; k <= n; k = k * 3)
```

```
                ++count;
```

```
}
```

First loop

The loop runs

$(n - \frac{n}{3})$  times

Second loop

The loop runs

$x$  times.  $\frac{n}{3}$  is constant number because the loop takes this number as top loop.

$$(n - \frac{n}{3}) = x$$

Third Loop

The loop runs  $\log_3 n$  times.

Proof

Suppose that third loop runs  $x$  times.

1, 3, 9, 27, ... n  
 $3^0, 3^1, 3^2, 3^3, \dots, 3^x$

$$3^x = n \Rightarrow \boxed{x = \log_3 n}$$

As these are all loops nested

$$(n - \frac{n}{3}) \times (n - \frac{n}{3}) \times (\log_3 n) \Rightarrow \text{function complexity}$$

$$(\frac{4n^2}{9} \times \log_3 n) = \frac{4n^2 \log_3 n}{9} \Rightarrow \boxed{O(n^2 \log n)}$$

\* Function runs  $O(n^2 \log n)$  stably. Cause of this situation function time complexity is  $\boxed{\Theta(n^2 \log n)}$



③ define function merge(arr, l, m, r):

set num1 to (m-l+1)

set num2 to (r-m)

set l-arr to [0] \* (num1)

set r-arr to [0] \* (num2)

for i in range(0, num1):

set l-arr[i] to arr[l+i]

for i in range(0, num2):

set r-arr[i] to arr[m+1+i]

set i to 0

set j to 0

set k to l

while i < num1 and j < num2:

if l-arr[i] <= r-arr[j]:

set arr[k] to l-arr[i]

i += 1

else:

set arr[k] to r-arr[j]

j += 1

k += 1

while i < num1:

set arr[k] to l-arr[i]

i += 1

k += 1



define function mergeSort(arr, l, r):

if  $l < r$

set  $m$  to  $(l + (r - 1)) // 2$

mergeSort(arr, l, m)

mergeSort(arr, m+1, r)

merge(arr, l, m, r)

define function productPairs\_helper(arr, target, pairs):

mergeSort(arr, 0, len(arr)-1)

set beg to 0

set end to len(arr)-1

while beg < end:

set temp to  $arr[beg] * arr[end]$

if temp equals target:

pairs.append(arr[beg], arr[end])

if temp < target:

beg += 1

else:

end -= 1



Define function productPairs (arr, target, pairs):

productPairs-helper(arr, target, pairs)

if len(pairs) equals 0:

Output("No pairs...")

else:

Output("Pairs: " + str(pairs))

set arr to [1, 2, 3, 6, 5, 4]

set target to 6

set pairs to []

productPairs(arr, target, pairs)

\* First we need to sort the array from smallest to largest. I used merge sort algorithm in this part because an  $n \log n$  solution was requested. After sorting the array, we can find pairs in linear time by performing a binary search.

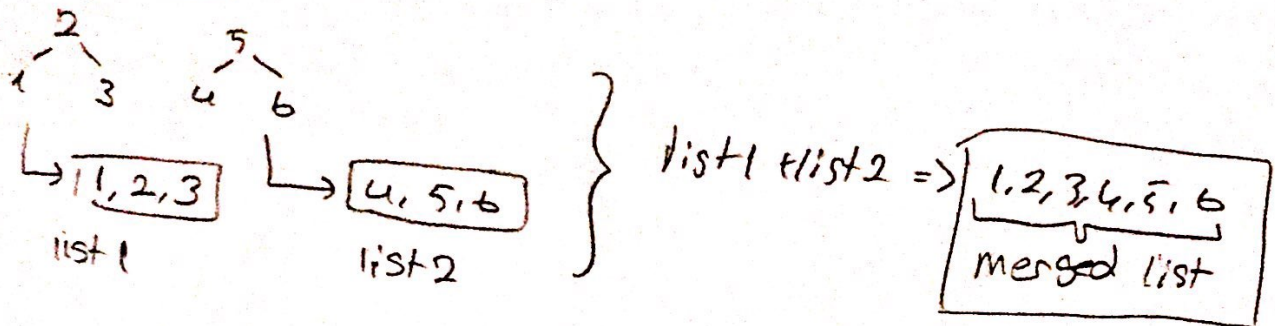
Time Complexity =  $\Theta(n \log n)$



④ Do inorder traversal of first and second tree and store on list. The complexity is the same for two operations on two trees, because trees have  $n$ -nodes.

And then merge two list to one list.

For example



Construct a new balanced tree from the merged list.  
This step takes  $O(2n) = O(n)$  times

⑤ define function isSubarray (arr1, arr2):

initialize hashset

for  $i$  in range(0, length of arr1):  
add hashset element of arr1[i]

for  $i$  in range(0, length of arr2):

if arr2[i] in hashset:

continue

else

return False

return True

\* The algorithm runs

$\Rightarrow$  linear time because adding hashset  $O(1)$  time but if "collision" exist then it will run  $O(n)$  time in worst case

The function generally runs

$O(n)$  linear time