171044033 HW-3

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a) $T(n) = \underbrace{27}_{a}\, T(n/\underbrace{3}_{b}) + n^{\overset{c}{2}}$

$= \boxed{\log_3 27 = 3}$ $\boxed{3 > \overset{c}{2}}$ $\longrightarrow \boxed{T(n) = \Theta(n^3)}$

b) $T(n) = \underbrace{9}_{a}\, T(n/\underbrace{4}_{b}) + n^{\overset{c}{1}}$

$\boxed{\log_4 9 \cong 1,5}$ $\boxed{1,5 > \overset{c}{1}}$ $\longrightarrow \boxed{T(n) = \Theta(n^{1,5})}$

c) $T(n) = 2T(n/4) + \sqrt{n} = \underbrace{2}_{a}\, T(n/\underbrace{4}_{b}) + n^{\overset{c}{\frac{1}{2}}}$

$\boxed{\log_4 2 = \frac{1}{2}}$ $\boxed{\frac{1}{2} = \overset{c}{\frac{1}{2}}}$ $\longrightarrow \boxed{T(n) = \Theta(n^{\frac{1}{2}} \log n)}$

d) $T(n) = 2T(\sqrt{n}) + 1$ $\boxed{n = 2^x}$

$T(2^x) = 2T(2^{x/2}) + 1$ $\quad T(2^x) = f(x)$

$T(2^x) = f(x) = 2f(\frac{x}{2}) + 1$

$\log_2 2 > 0 \longrightarrow \Theta(x) = T(2^x) \boxed{= T(n)}$

e) $T(n) = 2T(n-2), \quad T(0)=1, \ T(1)=1$

$T(n) = r^n$

$\Rightarrow r^n = 2r^{n-2}$

$r^2 = 2$ $\quad \boxed{r_1 = \sqrt{2} \qquad r_2 = -\sqrt{2}}$

$T(n) = c_1(r_1) + c_2(-r_2)$

$1 = c_1 + c_2$

$1 = \sqrt{2}c_1 - \sqrt{2}c_2$

$c_1 = \frac{2+\sqrt{2}}{4} \qquad c_2 = \frac{2-\sqrt{2}}{4}$ $\quad \boxed{T(n) = \Theta\left(\frac{2+\sqrt{2}}{4}\sqrt{2} + \frac{2-\sqrt{2}}{4} \cdot (-\sqrt{2})^n\right)}$

f) $T(n) = \underset{a}{4}T(n/\underset{b}{2}) + n^{\overset{c}{1}} \quad T(1)=1$

$\boxed{\log_2 4 = 2}$ $\quad \boxed{2 > 1}$ $\longrightarrow \boxed{T(n) = \Theta(n^{\log_2 4}) = \Theta(n^2)}$

g) $T(n) = 2T(\sqrt[3]{n}) + 1 \qquad T(3) = 1$

$n = 3^x$

$T(3^x) = 2T(3^{\frac{x}{3}}) + 1$

$f(x) = T(3^x)$

$f(x) = \underset{a}{2} f(\frac{x}{\underset{b}{3}}) + 1 \quad c=0 \longrightarrow \log_3 2 > 0 \quad \Theta(x^{\log_3 2})$

$\longrightarrow x = \log_3 n \rightarrow \boxed{T(n) = \Theta(\log_3 n^{\log_3 2})}$

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(2) function f(n)

```
if n <= 1 :
    Print_line ("*A")
else :
    for i=1 to n
        f(n/2)
    end for
```

} Sub problems of algorithm

$\boxed{\text{Sub problem size} = n/2}$ $\boxed{\text{How may print?} = 1}$

$$\Rightarrow T(n) = \underbrace{n}_{a} . T(n/\underbrace{2}_{b}) + 1 . n^{\underbrace{0}_{c}}$$

$\boxed{\log_2^n > 0}$ $\boxed{T(n) = \theta(n^{\log_2^n})}$

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(3) Algorithm Function_f (A [0..n-1])

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if n=2 and A[0]>A[1]
    then swap (A[0],A[1])
if n>2 then {
    Function_f (A[0..ceil (2n/3)])
    Function_f (A[floor(n/3)..n])
    Function_f (A[0...ceil(2n/3)])
}
```

constant time $\Theta(1)$

$$T(n) = 3T\left(\frac{2}{3}n\right) + 1$$

$$T(n) = 3T(n/(3/2)) + 1$$

Master Theorem

$$\log_{3/2}^{3} > 0 \rightarrow T(n) = \Theta(n) = (n^{\log_{3/2}^{3}}) = n^{2.709}$$

$$\boxed{\cong > \theta(n^{2.709})}$$

(4)

Average case of quick sort is $\Theta(n\log n)$. Average case of insertion sort, the algorithm iterates for n index of array and each item. The algorithm compares with this items which are on $i-1$ to 0. item is greather than any item on index t where $0 \leq t < i-1$.

The best case $O(n)$. Since algorithm iterates n times independent from the input. The number of swaps for this algorithm is strongly connected with reversed item pairs because the algorithm must be swap each item if they are not ordered. For average case we use compute the number of swaps of permutaties of input. It is impossible especially if the numbers of swaps to calculate average case. Expected value is equal to sum of probablity of occurence for each sub-situation. In this case the sub situation can be item k > item and k < item. Therefore in average case we need Σ swap operation's. This sum equal to $\Sigma\, 1/2$ which is also equal to $n/2$ so the average case becomes $\Theta(n * n/2) = \Theta(n^2)$

In my example unsorted array is [20, 17, 8, 15, 0, 6]

In insertion sort swap	13
In quick sort swap	7

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a) Dividing 5 sub problems where the size of each sub problems is one of third

$T(n) = \underbrace{5}_{a} T(\frac{n}{\underbrace{3}_{b}}) + n^{\overbrace{2}^{c}}$

Master theorem $\rightarrow$ $\log_3 5 \cong 1,4$ $c=2$

$\boxed{= T(n) = \Theta(n^2)}$

b) Dividing 2 subproblems, half of size

$T(n) = \underbrace{2}_{a} T(\frac{n}{\underbrace{2}_{b}}) + n^{\overbrace{2}^{c}}$

$\boxed{\log_2 2 = 1 < \underbrace{2}_{c}}$ $\rightarrow$ $\boxed{T(n) = \Theta(n^2)}$

c) $T(n) = T(n-1) + n$

$T(n) = T(n-1) + n$

$T(n) = [T(n-2) + n - 1] + n$

$T(n) = [T(n-2) + n - 1 + n]$

$T(n) = [T(n-3) + n - 2] + n - 1 + n$

$T(n) = T(n-x) + n - (x-1) + \ldots + x - 1 + x$

Assume that $n - x = 0$

$T(n) = T(0) + 1 + 2 + 3 + 4 + 5 \ldots + n$

$= 1 + \frac{n^2+1}{2} \Rightarrow \boxed{\Theta(n^2)}$

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