EM600 - Engineering Economics and Cost Analysis

Lecture 04: *Understanding Rates of Return*

- References:
  - Park, Chan S. Contemporary Engineering Economics. New Jersey: Pearson Prentice Hall, 2006 (Chapter 7)
  - Lang, Hans J. and Merino, Donald M. The Selection Process for Capital Projects. New York: John Wiley & Sons, Inc. 2002 (Chapter 4)
  - Ganguly, A. Engineering Economics Using Excel. New Jersey: SSE, 2008

After completing this module you should understand the following:

- Return on Investment (ROI)
- IRR: Internal Rate of Return
- Incremental IRR
- Mutually Exclusive Alternatives

- Key Definitions: (Lang & Merino)
  - Return on Total Investment: (ROI)

$$\text{ROI} = \frac{\text{Average Net Income per Year}}{\text{Original Book Value or First Cost}}$$

- Limitations:
  - Time value of money is not taken into account.
    - » Cash flows are not discounted.
    - » The contribution of earlier years is understated.
    - » The contribution of later years is overstated.
  - Noncash flow streams e.g depreciation, are involved in calculating the ROI.

- Key Definitions: (Chan S. Park)
  - Internal Rate of Return (IRR):
    - *The internal rate of return is the interest rate charges on the unrecovered balance of the investment such that, when the project terminates, the unrecovered project balance will be zero.*
      - *The IRR equates the present worth, future worth and annual equivalence worth of the entire series of cash flows to zero.*

$$\boxed{\sum PW = 0; \qquad \sum AE = 0; \qquad \sum FW = 0}$$

    - *This internal rate of return is the return that a company would earn if it invested in itself rather than investing the money elsewhere.*

- Key Differences:
  - Return on Investment:
    - Does not account for the time value of money.
    - Undiscounted figure of merit.
  - Internal Rate of Return (IRR):
    - Accounts for the time value of money.
    - Discounted figure of merit.

- Rate of Return Computational Methods:
  - Direct solution method
    - Two-flow transaction project (an investment followed by a single payment).

      OR
    - Project with a service life of two years of return.
    - Set up PW or FW equation using cash flows given and set equation equal to zero.
  - Trial-and-error method
    - Complicated cash flows.
    - Linear Interpolation.
  - Computer solution method
    - Complicated cash flows.
    - Solve graphically.
    - Use Excel IRR function.

- Direct solution method:
  - Example 1: (Chan S. Park, example 7.3)
    - Consider two investment projects with the following cash flow transactions:

n	Project 1	Project 2
0	-$2,000	-$2,000
1	0	$1,300
2	0	$1,500
3	0	-
4	$3,500	-

    - Calculate the IRR, i*, for each project.

- Direct solution method:
  - Example 1: (Chan S. Park, example 7.3)
    - Project 1:
      - **Present Worth (PW) Method:**

$$PW(i^*) = \sum_{n=1}^{N} F(1+i^*)^{-N} + P = 0$$

$$PW(i^*) = \$3{,}500(1+i^*)^{-4} - \$2{,}000 = 0$$

$$\$2{,}000 = \$3{,}500(1+i^*)^{-4} \Rightarrow 0.5714 = \frac{1}{(1+i^*)^4} \Rightarrow (1+i^*)^4 = \frac{1}{0.5714} \Rightarrow (1+i^*)^4 = 1.75$$

$$i^* = (1.75)^{-1/4} - 1$$

$$i^* = 0.1502 = 15.02\%$$

      - **Future Worth (FW) Method:**

$$FW(i^*) = \sum_{n=1}^{N} P(1+i^*)^{N} + F = 0$$

$$FW(i^*) = -\$2{,}000(1+i^*)^{4} + \$3{,}500 = 0$$

$$\$3{,}500 = \$2{,}000(1+i^*)^{4}$$

$$1.75 = (1+i^*)^{4}$$

$$i^* = (1.75)^{1/4} - 1$$

$$i^* = 0.1502 = 15.02\%$$

- Direct solution method:
  - Example 1: (Chan S. Park, example 7.3)
    - Project 2:
      - **Present Worth (PW) Method:**

$$PW(i^*) = \sum_{n=1}^{N} F(1+i^*)^{-N} + P = 0$$

$$PW(i^*) = \$1,300(1+i^*)^{-1} + \$1,500(1+i^*)^{-2} - \$2,000 = 0$$

Transform into a quadratic equation where, $x = (1+i^*)^{-1}$

$$\Rightarrow PW(X) = \$1,500x^2 + \$1,300x - \$2,000 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1,300 \pm \sqrt{1,300^2 - 4(1,500)(2,000)}}{2(1,500)}$$

$$\Rightarrow x = 0.8 \;\text{ OR }\; x = -1.667$$

Solve for i* using the values for x

$$\Rightarrow x = 0.8 \;\therefore\; i^* = \frac{1}{0.8} - 1 = 25\%$$

$$\Rightarrow x = -1.667 \;\therefore\; i^* = \frac{1}{-1.667} - 1 = -160\% \text{ . . . no economic significance}$$

      - **Future Worth (FW) Method yields the same answer.**

- Trial-and-Error Method:
  - **Overview:** Linear Interpolation
    - Definition:
      - *Interpolation is a method of constructing new data points within the range of a discrete set of known data points.*[1]
    - Key Steps:
      - Estimate an IRR value for which PW > 0
      - Estimate an IRR value for which PW < 0
      - Interpolate to find where PW = 0

1. "interpolation" Wikipedia.org. August 19 2008. <http://en.wikipedia.org/wiki/Interpolation>

- Trial-and-Error Method:
  - **Overview:** Linear Interpolation
    - Linear interpolation equation:

$$y = y_1 + \frac{(x - x_1)(y_2 - y_1)}{(x_2 - x_1)}$$

where,

y_1 & y_2 = estimated values for the IRR

x_1 & x_2 = corresponding PW values for y_1 and y_2

y = actual IRR at PW = 0

x = PW associated with actual IRR = 0

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- Trial-and-Error Method:
  - **Overview:** Linear Interpolation
    - Key points to remember:
      - The relationship is not truly linear, therefore the closer the PW values (calculated using the estimated IRR values) are to zero, the more accurate the answer will be.
        - » Multiple answers can therefore exist.
      - The MARR is not needed to calculate the IRR.
      - The IRR should lie between the two estimated values.
    - Reference Chapter 2 of Engineering Economics Using Excel (Ganguly) for further detail on Linear Interpolation.

- Trial-and-error method:
  - Example 2: (Chan S. Park, example 7.4)
    - You need to consider a new safety review project for the production plan of a new drug for your company:

n	Costs	Savings	Net Cash Flow
0	-$13,000	$0	-$13,000
1	-$2,300	$6,000	$3,700
2	-$2,300	$7,000	$4,700
3	-$2,300	$9,000	$6,700
4	-$2,300	$9,000	$6,700
5	-$2,300	$9,000	$6,700
6	-$2,300	$9,000	$6,700

    - Calculate i* for this project.

- Trial-and-error method:
  - Example 2: (Chan S. Park, example 7.4)
    - $PW = -P + \sum_{n=1}^{N} A_n (P/F, i, n) + F(P/F, i, N)$

Guess 1: IRR = 30%

$$PW = -\$13{,}000 + \$3{,}700(P/F,30\%,1) + \$4{,}700(P/F,30\%,2) + \$6{,}700\begin{bmatrix}(P/F,30\%,3)\\ +(P/F,30\%,4)\\ +(P/F,30\%,5)\\ +(P/F,30\%,6)\end{bmatrix}$$

$$PW = -\$13{,}000 + \$3{,}700(0.7692) + \$4{,}700(0.5917) + \$6{,}700\begin{bmatrix}(0.4552)\\ +(0.3501)\\ +(0.2693)\\ +(0.2072)\end{bmatrix}$$

$$PW = \$1{,}215$$

- Trial-and-error method:
  - Example 2: (Chan S. Park, example 7.4)
    - $PW = -P + \sum_{n=1}^{N} A_n (P/F, i, N) + F(P/F, i, N)$

      Guess 2: IRR = 35%

$$PW = -\$13{,}000 + \$3{,}700(P/F, 35\%, 1) + \$4{,}700(P/F, 35\%, 2) + \$6{,}700 \begin{bmatrix} (P/F, 35\%, 3) \\ +(P/F, 35\%, 4) \\ +(P/F, 35\%, 5) \\ +(P/F, 35\%, 6) \end{bmatrix}$$

$$PW = -\$13{,}000 + \$3{,}700(0.7407) + \$4{,}700(0.5487) + \$6{,}700 \begin{bmatrix} (0.4604) \\ +(0.3011) \\ +(0.2230) \\ +(0.1652) \end{bmatrix}$$

$$PW = -\$339$$

- Trial-and-error method:
  - Example 2: (Chan S. Park, example 7.4)
    - Use linear interpolation to find where PW = 0

$$y = y_1 + \frac{(x - x_1)(y_2 - y_1)}{(x_2 - x_1)}$$

$$(x, y) = (0, ?); \quad (x_1, y_1) = (\$1215, 30\%); \quad (x_2, y_2) = (-\$339, 35\%)$$

$$y = 30\% - \frac{(0 - 1215)(35\% - 30\%)}{(-339 - 1215)}$$

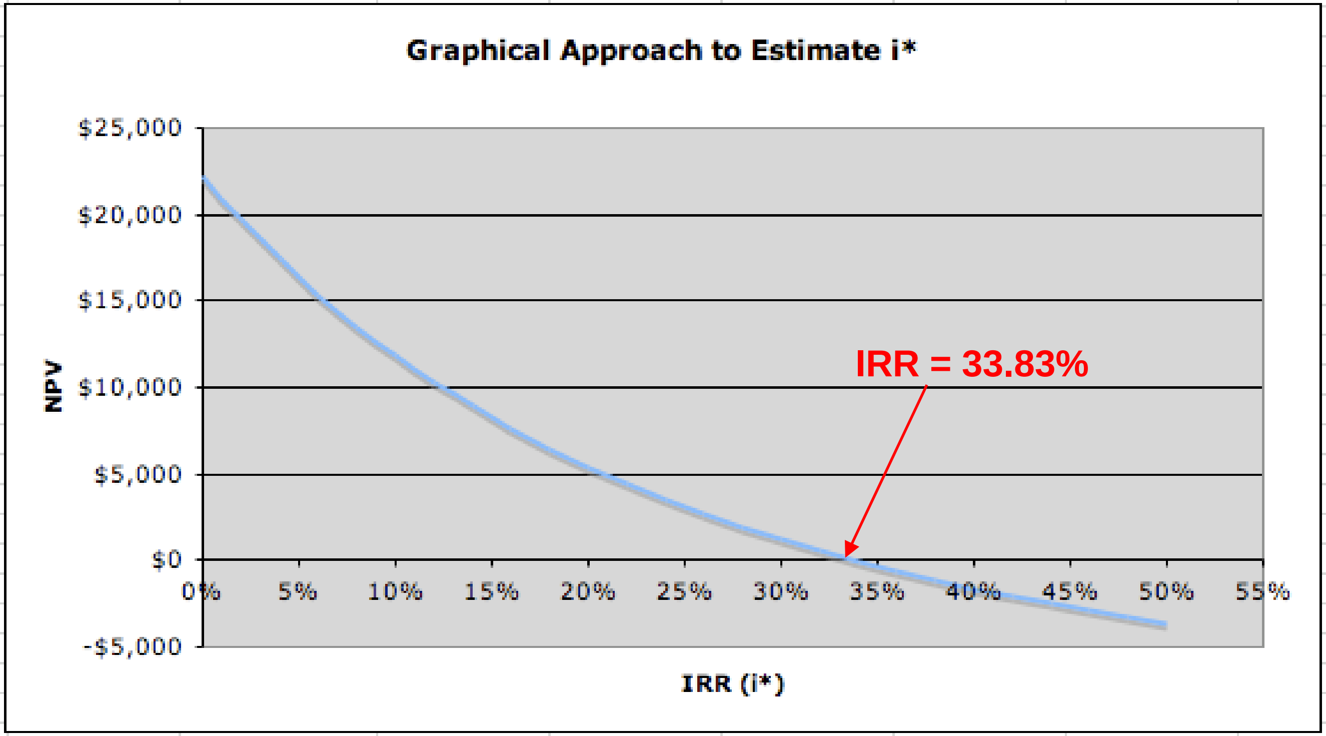
$$y = 33.91\%$$

- Trial-and-error method:
  - Example 2: (Chan S. Park, example 7.4)
    - Remember:
      - The closer the PW values are to 0, the more accurate the linear interpolation will be.
        - » The relationship is not truly linear.
      - For example,
        - » If guesses of 33% and 34% are used instead of 30% and 35%,
        - » PW values of $249 and -$51 and an IRR of 33.83% results.
      - Multiple solutions therefore exist.

- Computer solution method:
  - Example 3: (Chan S. Park, example 7.4)
    - Consider the investment project with the cash flow transactions as detailed in example 2.
    - Calculate i* for this project using the following methods:
      - Graphically
      - Excel IRR function

- Computer solution method:
  - Example 3: (Chan S. Park, example 7.4)
    - Graphical Solution

IRR	NPV
0%	$22,200
2%	$19,666
4%	$17,389
6%	$15,336
8%	$13,481
10%	$11,800
12%	$10,273
14%	$8,884
16%	$7,615
18%	$6,455
20%	$5,392
22%	$4,416
24%	$3,517
26%	$2,689
28%	$1,923
30%	$1,215
32%	$559
34%	-$51
36%	-$617
38%	-$1,145
40%	-$1,638
42%	-$2,098
44%	-$2,528
46%	-$2,932
48%	-$3,310
50%	-$3,665



- Computer solution method:
  - Example 3: (Chan S. Park, example 7.4)
    - Graphical Solution: How to in Excel?
      - Arrange the data so that the x and y values are in adjacent columns.
      - Select the range of x and y values that has to be plotted.
      - Click on the Insert tab from the top of your Excel 2013 spreadsheet.
      - Select Scatter from the list of available charts (should be the last chart listed)
      - This will open up a window with various chart sub-types. Simply click the one you would like to use from this list.
      - Format the graph according to the requirements and convenience.
      - Reference Chapter 1 of Engineering Economics Using Excel (Ganguly) if using earlier versions of MS Excel

- Computer solution method:
  - Example 3: (Chan S. Park, example 7.4)
    - Excel IRR function

	A	B	C	D
1	n	Costs	Savings	Net Cash Flow
2	0	-$13,000	$0	-$13,000
3	1	-$2,300	$6,000	$3,700
4	2	-$2,300	$7,000	$4,700
5	3	-$2,300	$9,000	$6,700
6	4	-$2,300	$9,000	$6,700
7	5	-$2,300	$9,000	$6,700
8	6	-$2,300	$9,000	$6,700
9	IRR			=IRR(D2:D8,)

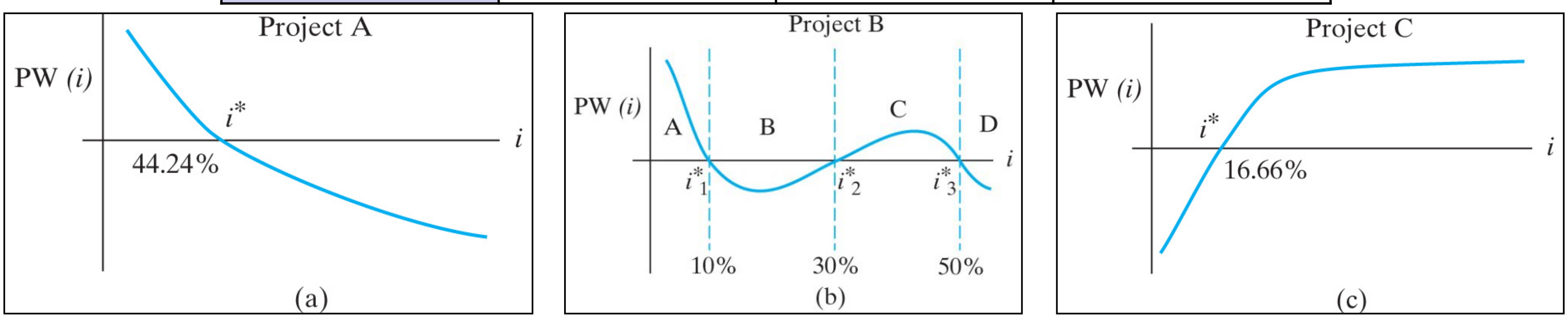
9	IRR			33.83%

- Excel Formula: IRR(values,guess)
  - » Values = cash flow (including initial investment and salvage value if any)
  - » Guess = an IRR value can be estimated. This can be left blank as in this example.
  - » Reference Chapter 2 of Engineering Economics Using Excel (Ganguly) for further detail.

- Decision Rules and IRR:
  - Introduction:
    - Simple Investment:
      - An investment in which the initial cash flows are negative and only one sign change occurs in the remaining cash flow series.
    - Simple Borrowing:
      - An investment in which the initial cash flows are positive and only one sign change occurs in the remaining cash flow series.
    - Nonsimple Investment:
      - An investment in which one or more sign change occurs in the cash flow series.

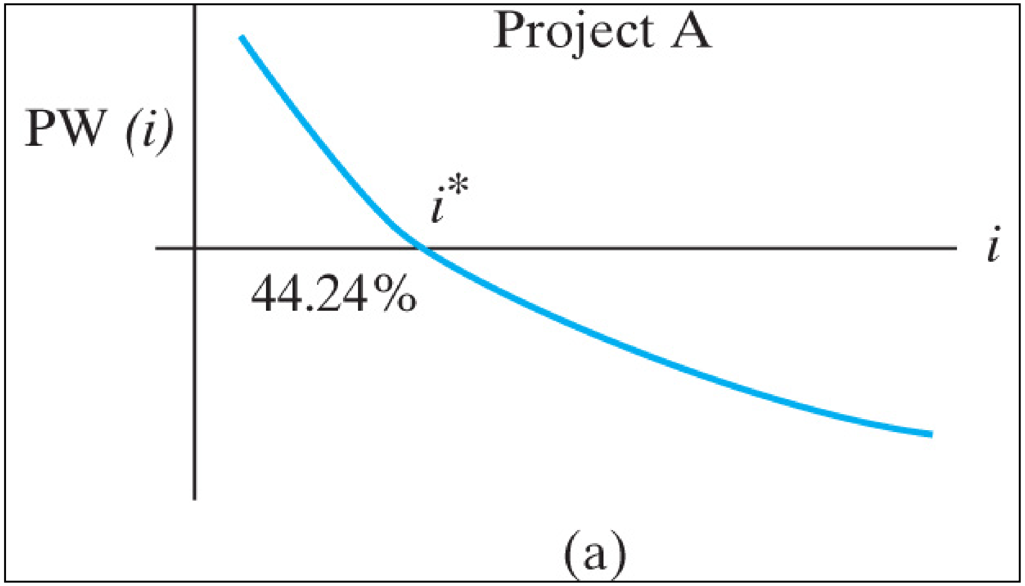
- # Decision Rules and IRR:
  - Introduction: (Chan S. Park, example 7.1)
    - Simple and Nonsimple Investments:

Period n	Net Cash Flow		
	Project A	Project B	Project C
0	-$1,000	-$1,000	$1,000
1	-$500	$3,900	-$450
2	$800	-$5,030	-$450
3	$1,500	$2,145	-$450
4	$2,000		



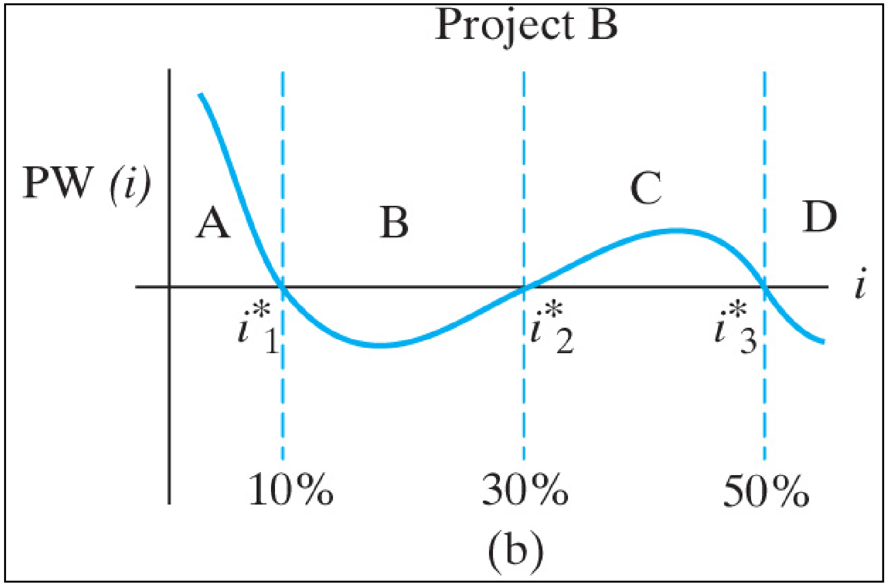
Present-Worth Profiles (Chan S. Park, Figure 7.1)

- Decision Rules and IRR:
  - Introduction: (Chan S. Park, example 7.1)
    - Simple and Nonsimple Investments:
      - Project A represents many common simple investments.
      - The NPW profile (curve) crosses the x-axis only once.



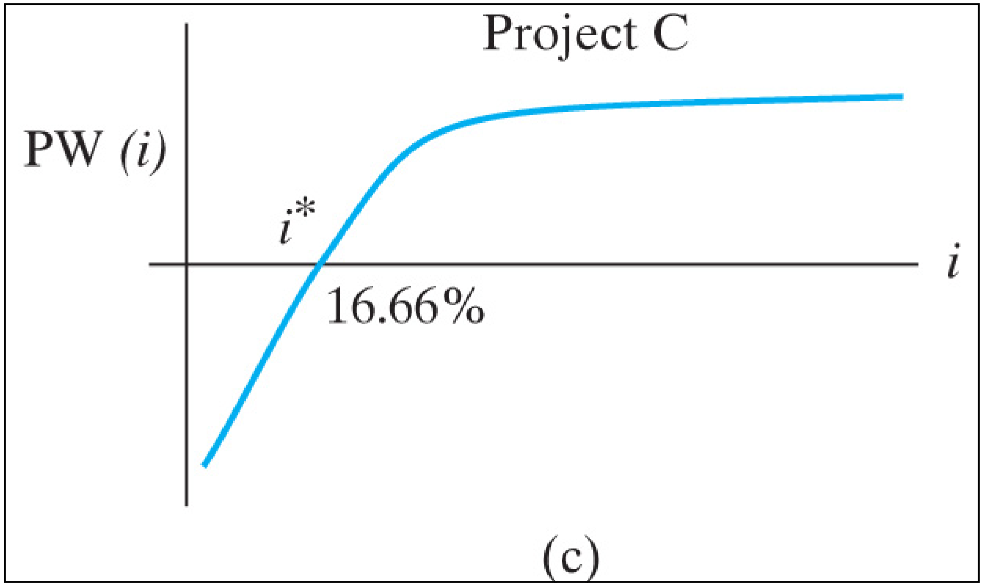
Present-Worth Profiles (Chan S. Park, Figure 7.1)

- Decision Rules and IRR:
  - Introduction: (Chan S. Park, example 7.1)
    - Simple and Nonsimple Investments:
      - Project B represents a nonsimple investment.
      - The NPW profile (curve) crosses the x-axis at multiple points.



Present-Worth Profiles (Chan S. Park, Figure 7.1)

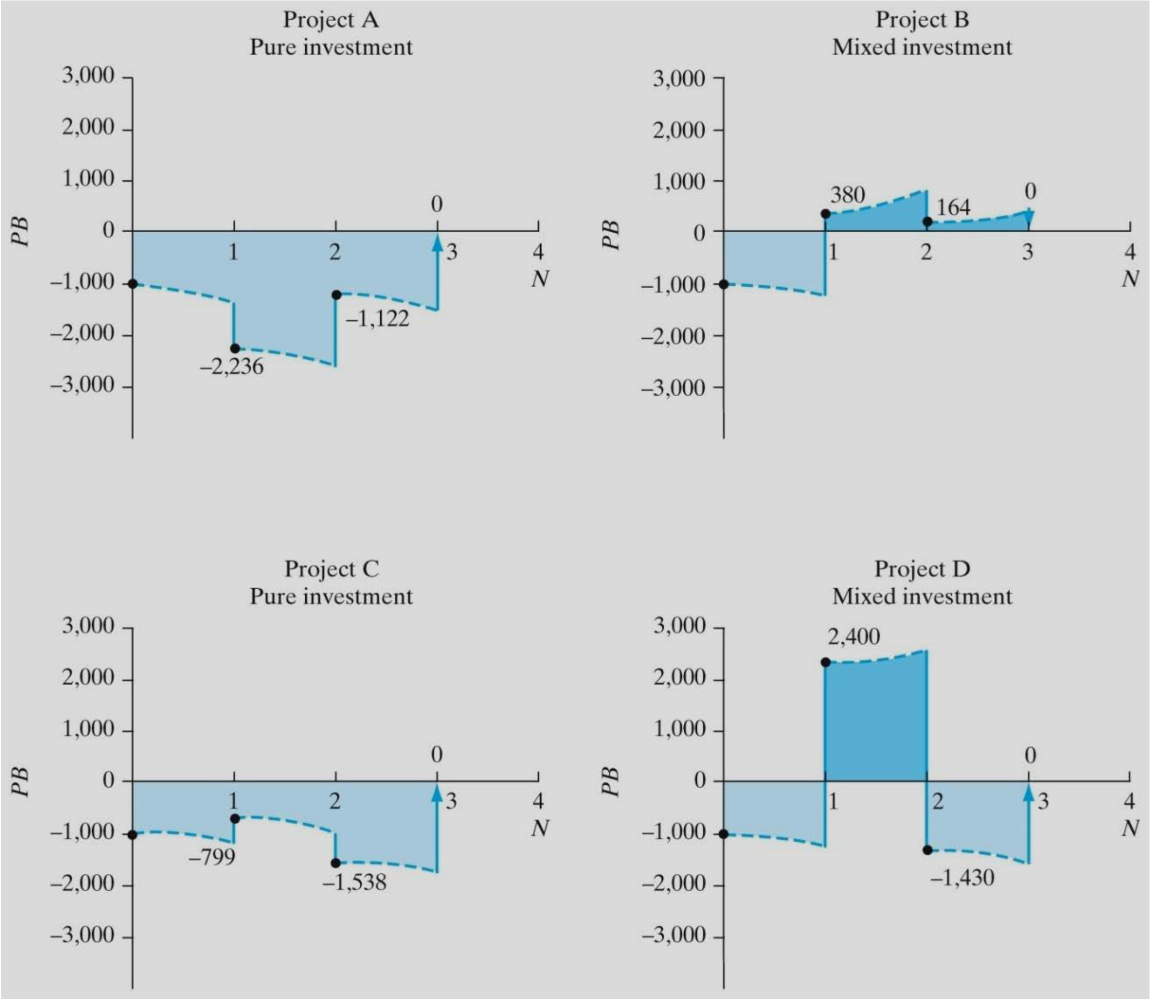
- Decision Rules and IRR:
  - Introduction: (Chan S. Park, example 7.1)
    - Simple and Nonsimple Investments:
      - Project C represents a simple borrowing cashflow.
      - There is only 1 sign change, however, the first cashflow is positive.



Present-Worth Profiles (Chan S. Park, Figure 7.1)

- Decision Rules and IRR:
  - Introduction:
    - Pure Investment:
      - An investment in which the firm never borrows money from the project.
      - Project balances (PB) are ≤ zero throughout the life of the investment with the first cashflow being negative.
      - Simple investments will always be pure investments.
    - Mixed investment:
      - An investment in which the firm borrows money from the project during the investment period.
      - PB > zero at some point during the life of the investment. Here the firms acts as a borrower, not an investor.

Decision Rules: Internal Rate of Return



Examples of Pure and Mixed investment projects (Chan S. Park, Figure 7.4)

- Recall the following important definitions:
  - MARR:
    - **M**inimum **A**ttractive **R**ate of **R**eturn
    - The minimum interest rate that the firm wants to earn on its investment.
  - IRR:
    - **I**nternal **R**ate of **R**eturn
    - The actual interest rate that the firm earns on its investment.
    - The IRR equates the present worth, future worth and annual equivalence worth of the entire series of cash flows to zero.

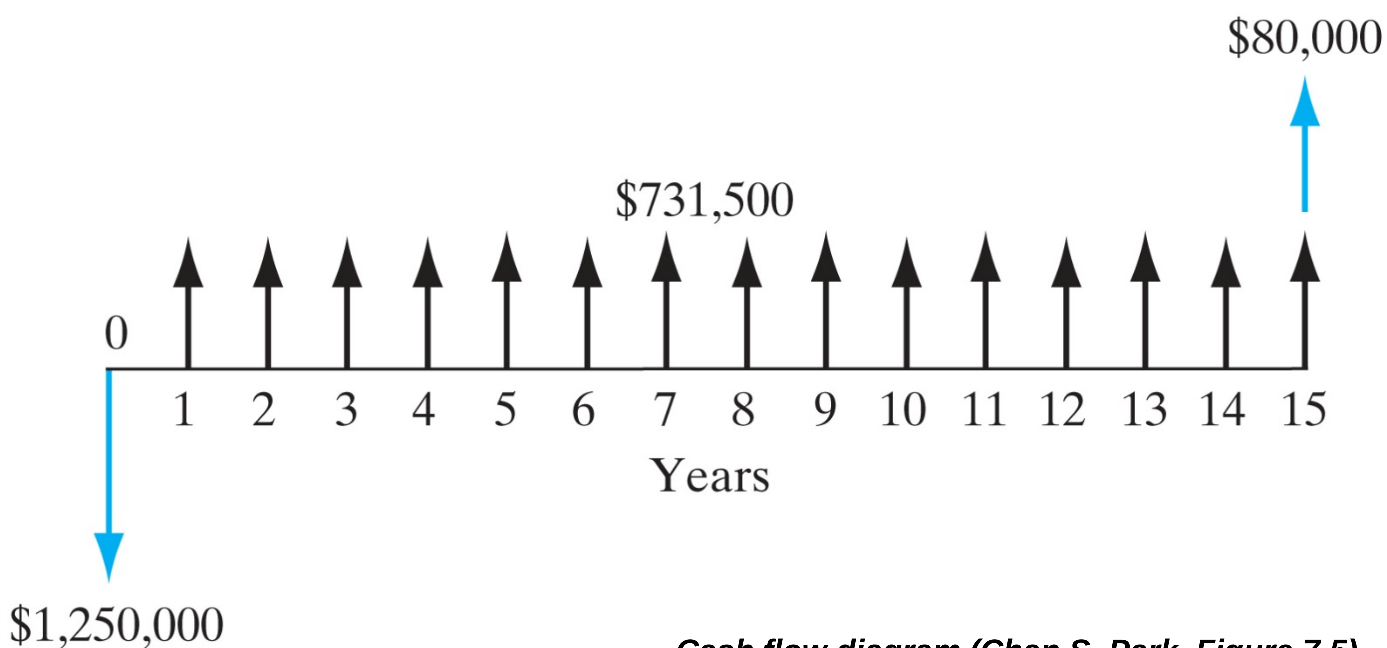
- Decision rules* for ***pure investment projects***:
  - If IRR > MARR ➔ ACCEPT
  - If IRR = MARR ➔ INDIFFERENT
  - If IRR < MARR ➔ REJECT

* **Note:**

Only applicable for single project evaluation.

Mutually exclusive investment projects need the ***incremental analysis approach.***

- Example 4: (Chan S. Park, example 7.7)
  - For the following cash flows:
    - Calculate the IRR for the investment.
    - Should the investment be accepted or rejected? (MARR = 18%)



Cash flow diagram (Chan S. Park, Figure 7.5)

- Example 4: (Chan S. Park, example 7.7)
  - Trial-and-error:

$$PW = -P + A(P/A, i, N) + F(P/F, i, N)$$

Guess 1: IRR = 55%

$$PW = -\$1{,}250{,}000 + \$731{,}500(P/A, 55\%, 15) + \$80{,}000(P/F, 55\%, 15)$$

$$PW = \$78{,}254$$

Guess 2: IRR = 60%

$$PW = -\$1{,}250{,}000 + \$731{,}500(P/A, 60\%, 15) + \$80{,}000(P/F, 60\%, 15)$$

$$PW = -\$31{,}821$$

Using linear interpolation as before:

$$\boxed{y = i^* = 58.55\%}$$

  - Excel IRR function:
    - i* = 58.47%

i* > MARR → ACCEPT

- Mutually Exclusive Alternatives:
  - Two situations:
    - Alternatives with the same economic service life.
    - Alternatives that have unequal service lives.
  - Incremental IRR analysis is required because:
    - IRR is a relative (percentage) measure.
    - IRR ignores the scale of the investment
    - IRR cannot be analyzed in the same way as the 3 worths.

- Incremental IRR:
  - What is Incremental IRR?
    - The internal rate of return is calculated based on the incremental investment.
    - The incremental investment is calculated based on choosing a large project over a smaller project.
  - Incremental IRR considers ***increments of investment,*** therefore:

Cash Flow Difference = higher Investment Cost Project B
- lower investment cost project A

    - This ranking is IMPORTANT
    - Investment cost = P

- Incremental IRR:
  - Decision rules:
    - If $IRR_{B\text{-}A} > MARR$ → select B
    - If $IRR_{B\text{-}A} = MARR$ → select either project
    - If $IRR_{B\text{-}A} < MARR$ → select A

Mutually Exclusive Alternatives: Internal Rate of Return

- Incremental IRR
  - Initial investments are equal?
    - Set up the increment so that the first non zero flow is negative.
    - e.g.

n	A	B	A - B
0	-$10,000	-$10,000	$0
1	$650	$6,740	-$6,090
2	$4,125	$3,350	$775
3	$6,950	$2,200	$4,750
4	$3,880	$1,470	$2,410
IRR	**17%**	**19%**	**12.88%**

- Incremental IRR
  - More than 2 mutually exclusive alternatives?
    - Compare in pairs by successive examination.
      - Consider 3 projects A, B, C
      - Verify IRR_A and IRR_B and IRR_C are each > MARR
        - » Any project whose IRR < MARR can be ruled out
      - Compare each incremental pair:
        - » A and B
        - » A and C
        - » B and C
      - Find the best alternative.

- Incremental IRR
  - More than 2 mutually exclusive alternatives?
    - e.g. MARR = 15%, select best alternative

n	A	B	C	A - B	C - A	C - B
0	($2,500)	($1,500)	($4,000)	($1,000)	($1,500)	($2,500)
1	$2,000	$1,300	$2,500	$700	$500	$1,200
2	$1,500	$1,000	$2,500	$500	$1,000	$1,500
3	$1,300	$1,000	$1,500	$300	$200	$500
IRR	**45.69%**	**56.49%**	**31.63%**	**27.61%**	**7.16%**	**15.17%**

- **Select alternative A.**

Select A **Select A** **Select C**

Note - A already selected over C

- Incremental IRR
  - Mutually exclusive alternatives with equal revenues?
    - Comparison is on a cost only basis.
    - Calculate the IRR based on incremental cash flows.
    - Service projects is an example of this type of problem, e.g.
      - alternative manufacturing systems (batch / continuous, flexible / dedicated)
      - alternative equipment options
      - alternative power supply / heating / chilling options
      - . . . etc

- Incremental IRR
  - Example 5: (Chan S. Park, example 7.13)
    - Based on the following data and a MARR of 15% (N = 6), using IRR, which manufacturing option should be chosen?

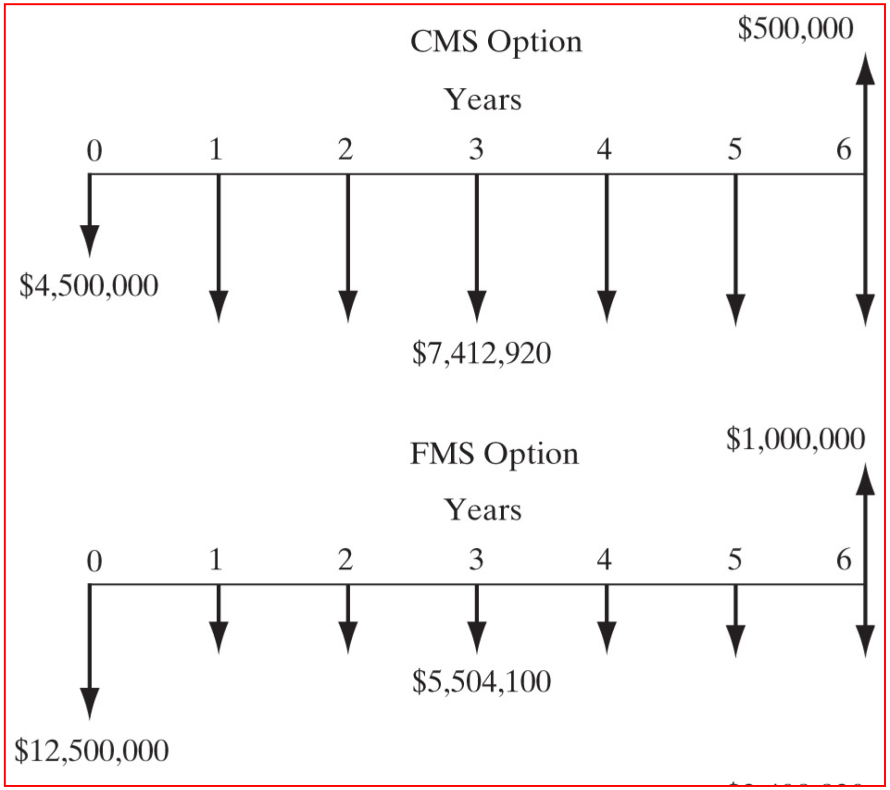
Items	CMS Option	FMS Option
Annual O&M costs:		
Annual labor cost	$1,169,600	$707,200
Annual material cost	$832,320	$598,400
Annual overhead cost	$3,150,000	$1,950,000
Annual tooling cost	$470,000	$300,000
Annual inventory cost	$141,000	$31,500
Annual income taxes	$1,650,000	$1,917,000
Total annual costs	$7,412,920	$5,504,100
Investment	$4,500,000	$12,500,000
Net salvage value	$500,000	$1,000,000

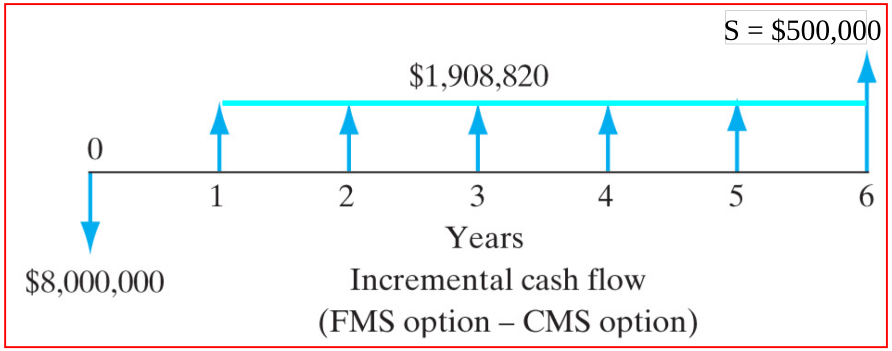
- Incremental IRR
  - Example 5: (Chan S. Park, example 7.13)
    - Assumption:
      - Both manufacturing systems would yield the same level of revenues over the analysis period.
    - Comparison basis:
      - Cost only as the revenues are equal.
    - Approach:
      - Calculate IRR based on the incremental cash flows.
      - $I_{FMS} > I_{CMS}$
      - Therefore the incremental cash flow is FMS – CMS.

- Incremental IRR
  - Example 5: (Chan S. Park, example 7.13)

n	CMS Option	FMS Option	Incremental (FMS-CMS)
0	-$4,500,000	-$12,500,000	-$8,000,000
1	-7,412,920	-5,504,100	1,908,820
2	-7,412,920	-5,504,100	1,908,820
3	-7,412,920	-5,504,100	1,908,820
4	-7,412,920	-5,504,100	1,908,820
5	-7,412,920	-5,504,100	1,908,820
6	-7,412,920	-5,504,100	1,908,820
Salvage	**$500,000**	**$1,000,000**	**$500,000**

- Incremental IRR
  - Example 5: (Chan S. Park, example 7.13)





Comparison of mutually exclusive alternatives on a cost only basis (Chan S. Park, Figure 7.11)

- Incremental IRR
  - Example 5: (Chan S. Park, example 7.13)
    - **To calculate the incremental IRR, calculate *i* where PW = 0**

$$\text{Objective: Calculate } i, \text{ where } PW_{FMS\text{-}CMS} = 0$$

$$PW_{FMS-CMS} = -P + A(P/A,i,N) + S(P/A,i,N)$$
$$PW_{FMS-CMS} = -\$8{,}000{,}000 + \$1{,}908{,}820(P/A,i,6) + \$500{,}000(P/F,i,6)$$

$$\text{Solving for } i \text{ using trial-and-error yields,}$$
$$i = 12.43\%$$

- **Conclusion:**
  - **$IRR_{FMS\text{-}CMS}$ = 12.43%**
  - **MARR = 15%**
  - **$IRR_{FMS\text{-}CMS}$ < MARR**
  - **Therefore, select CMS**

Questions?