EM600 - Engineering Economics and Cost Analysis

Lecture 12: *Decision and Risk Analysis*

- References:
  - Park, Chan S. Contemporary Engineering Economics. New Jersey: Pearson Prentice Hall, 2006 (Chapter 12)
  - Blank, L. and Tarquin, A. Engineering Economy. New York: McGraw-Hill, 2005 (Chapter 18 & 19)
  - Ganguly, A. Engineering Economics Using Excel. New Jersey: SSE, 2008

After completing this module you should understand the following:

- Overview of project risk
- Introduction to probability concepts for investment decisions.
- Probability distribution for NPW decision
- Comparing mutually exclusive risky alternatives
- Overview of risk simulation
- Overview of decision tree analysis in investment decisions

- Risk and Project Risk
  - **Risk:**
    - Definition: (Chan S. Park)
      - *The chance that an investment's actual return will be different than expected.*
    - Risk describes an investment project whose cash flow is not known in advance with absolute certainty.
    - An array of alternative outcomes and their probabilities are known for the investment project in question.

- Risk and Project Risk
  - **Project Risk:**
    - Refers to variability in a project's net present worth, NPW.
    - A greater project risk typically means:
      - A greater variability in a project's NPW
      - The risk is the potential for loss.
  - **Risk Analysis:**
    - Definition: (Chan S. Park)
      - *A technique to identify and assess factors that may jeopardize the success of a project.*

- Recall from previous lecture:
  - Methods of Describing Project Risk
    - **Sensitivity Analysis:** a procedure of identifying the project variables which, when varied, have the greatest effect on project acceptability.
    - **Break-Even Analysis:** a procedure of identifying the value of a particular project variable that causes the project to exactly break even.
    - **Scenario Analysis:** a procedure of comparing a "base case" to one or more additional scenarios, such as best and worst cases, to identify the extreme and most likely project outcomes.

(Chan S. Park, Lecture No46.ppt)

- Probability Concepts for Investment Decisions:
  - Topics for discussion in the proceeding slides:
    - Assessment of probabilities
    - Summary of probabilistic information
    - Joint and conditional probabilities
    - Covariance and coefficient of correlation

- Assessment of probabilities
  - Key terms and their definitions (Chan S. Park):
    - Random Variables
      - *Parameter or variable that can have more than one possible value (not simultaneously).*
      - Discrete random variables
        - » Several isolated / specific values.
      - Continuous random variables
        - » Can have any value within a certain interval / range of values.

- Assessment of probabilities
  - Key terms and their definitions:
    - Probability Distribution
      - Definition: (Blank & Tarquin)
        - » *A probability distribution describes how probability is distributed over the different values of a variable.*
        - » *$P(x)$ = probability that X equals x*
      - The range of probabilities for a given random event made up of either discrete random variables or continuous random variables.
      - All possible outcomes of the random event should be covered by the probability distribution.
      - The total probabilities must sum to 1 (or 100%).

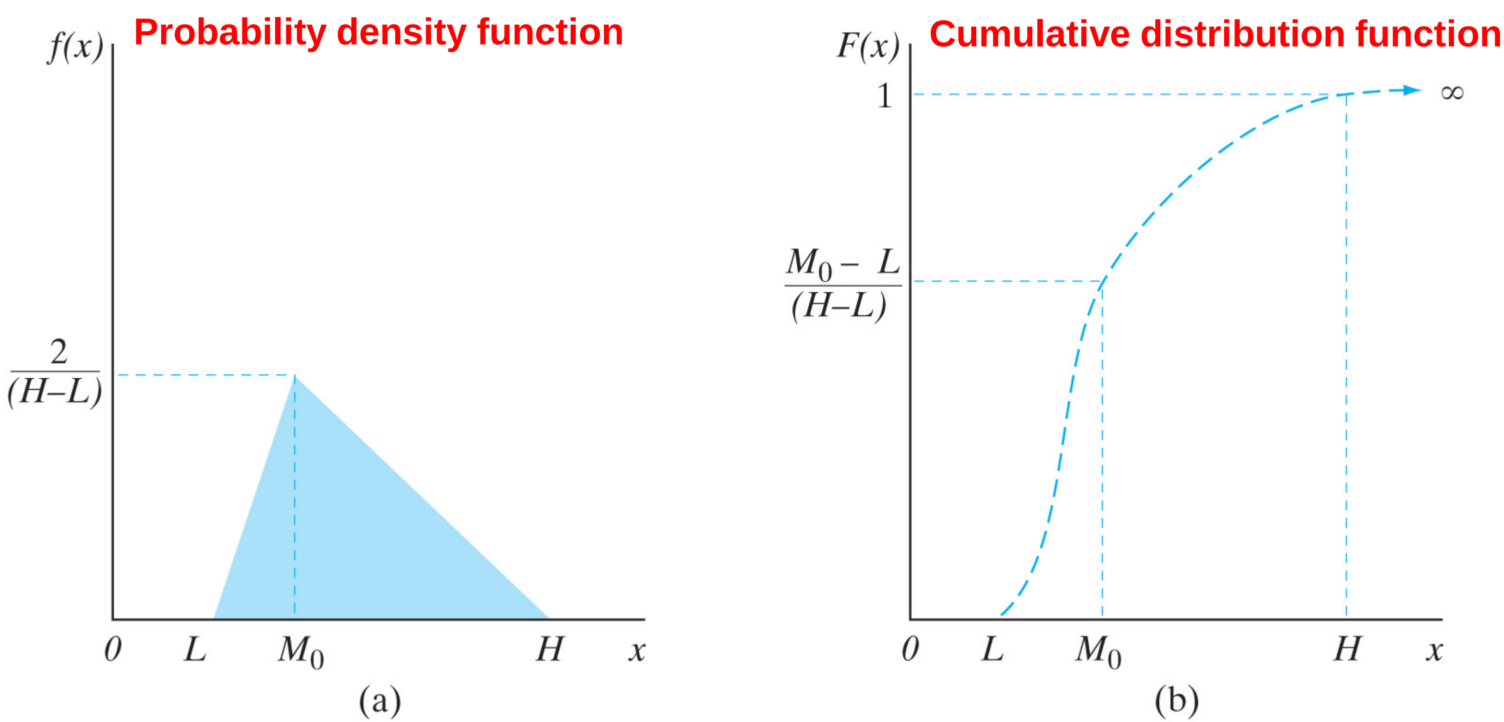
- Assessment of probabilities
  - Types of probability distribution:
    - Continuous probability distribution, for example:
      - Triangular distribution
      - Uniform distribution
      - Normal distribution
    - Discrete probability distribution, for example:
      - Binomial distribution

- Assessment of probabilities
  - Types of probability distribution:
    - Continuous probability distribution
      - Triangular distribution

$L = \text{minimum}$

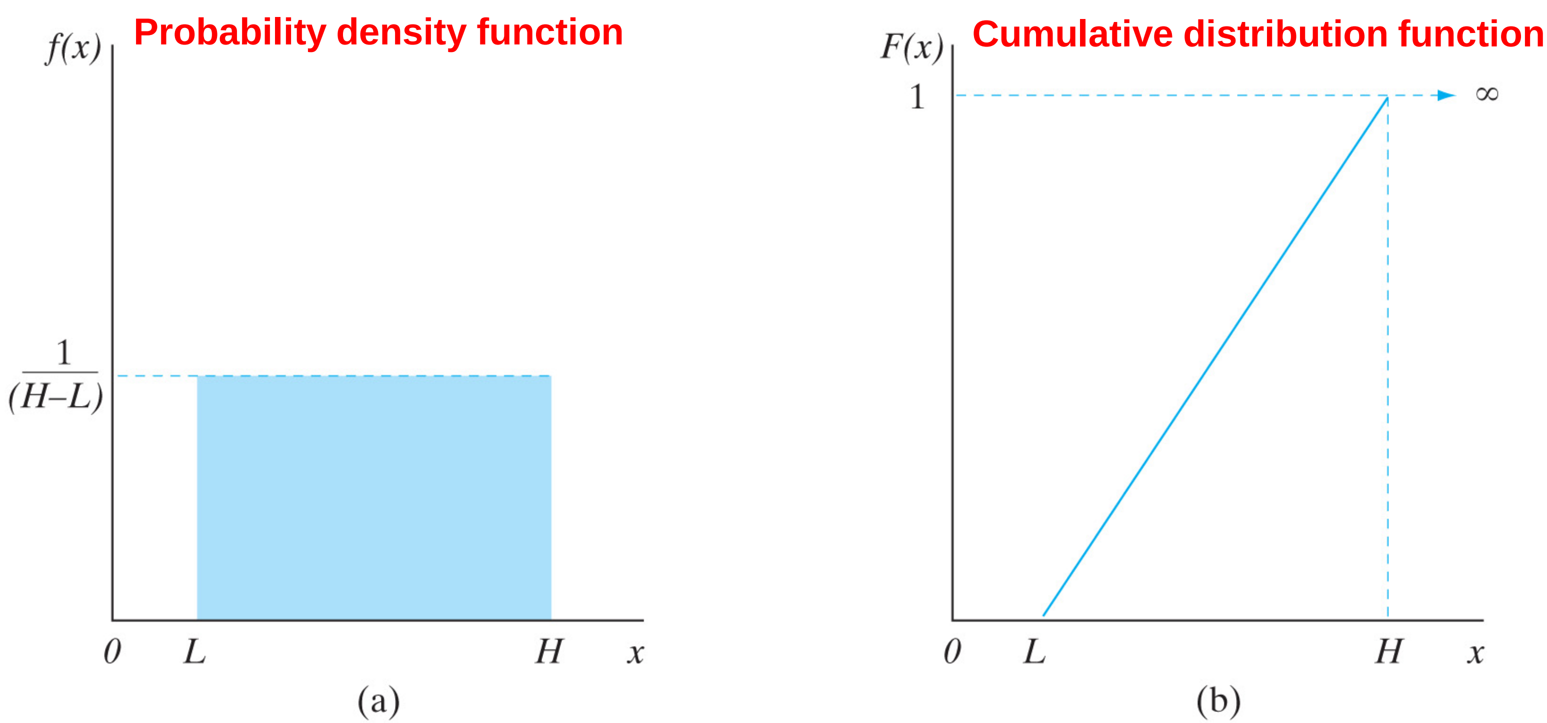
$H = \text{maximum}$

$M_0 = \text{mode}$



Triangular probability distribution (Chan S. Park, Figure 12.3)

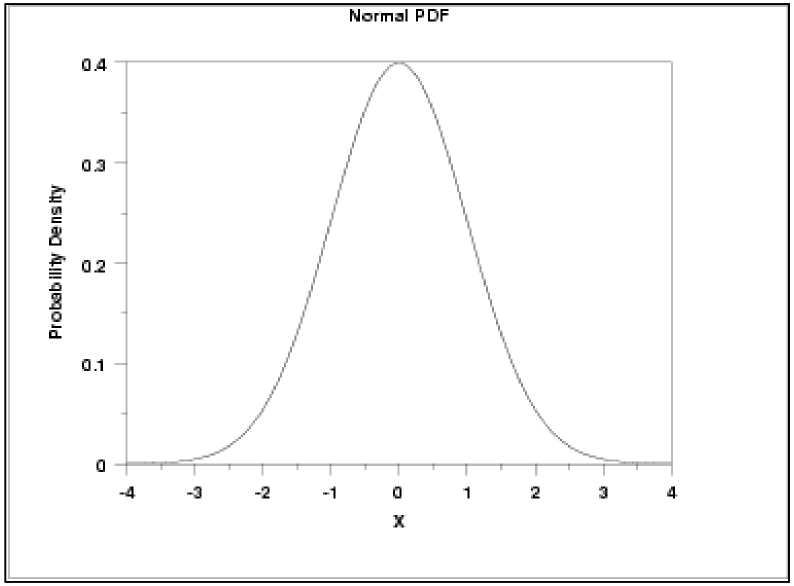
- Assessment of probabilities
  - Types of probability distribution:
    - Continuous probability distribution
      - Uniform distribution



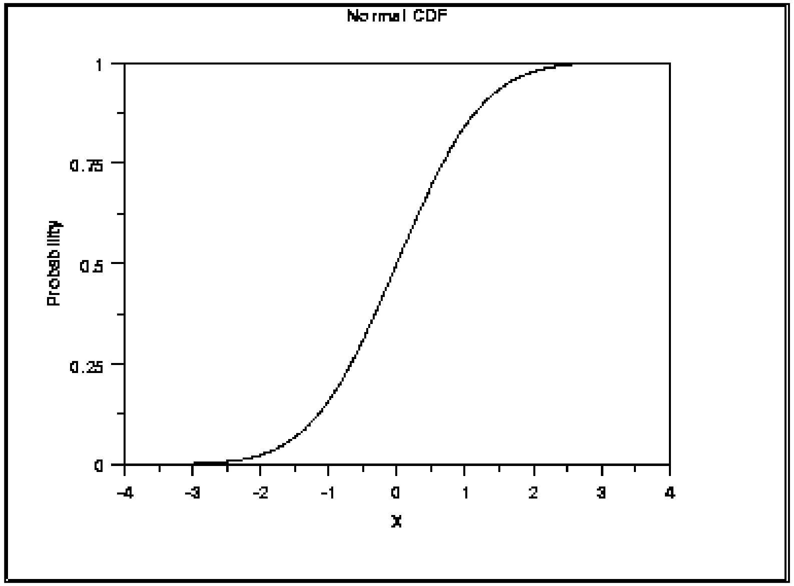
Uniform probability distribution (Chan S. Park, Figure 12.4)

- Assessment of probabilities
  - Types of probability distribution:
    - Continuous probability distribution
      - Normal distribution[1]

Probability density function



Cumulative distribution function



1. Engineering Statistics Handbook. www.itl.nist.gov. August 22, 2008 <http://www.itl.nist.gov/div898/handbook/eda/section3/eda362.htm>

- Assessment of probabilities
  - Key terms and their definitions (Chan S. Park):
    - Cumulative Distribution
      - *The cumulative probability distribution function gives the probability that the random variable X will attain a value smaller than or equal to some value x.*
  - In the case of a continuous random variable, the cumulative distribution rises in a smooth (not stairwise) way.

- Assessment of probabilities
  - The cumulative probability distribution function is given by:

$$F(x)=P(X \le x)=\begin{cases} \sum_{j=1}^{j} p_j & \text{(for a discrete random variable)} \\ \int_L^x f(x)dx & \text{(for a continuous random variable)} \end{cases}$$

where,

p_j = probability of occurance of the $\mathrm{x_j}$ - th value of the discrete random variable

$f(x)$ = a probability function for a continuous random variable

- Summary of probabilistic information
  - Key terms and their definitions:
    - Expected Value:
      - *Is the long-run expected average if the variable is sampled many times.* (Chan S. Park)
      - *Is a weighted average value of the random variable, where the weighting factors are the probabilities of occurrence.* (Blank & Tarquin)
      - Also known as: **mean**.

- Summary of probabilistic information
  - The expected value function is given by:

$$E[X] = \mu_x = \begin{cases} \sum_{j=1}^{J} (p_j) x_j & \text{(discrete case)} \\ \int_L^H x f(x) dx & \text{(continuous case)} \end{cases}$$

where,

$J =$ number of discrete events

$L =$ lower limit of continuous probability distribution

$H =$ upper limit of continuous probability distribution

$p_j =$ probability of occurance of the x_j - th value of the discrete random variable

$f(x) =$ a probability function for a continuous random variable

- Summary of probabilistic information
  - Key terms and their definitions: (Chan S. Park)
    - Variance:
      - *The variance of a random variable is a measure of its statistical dispersion, indicating how far from the expected value its values typically are.*
      - *The variance tells us the degree of spread, or dispersion, of the distribution on either side of the expected value or the mean value.*

- Summary of probabilistic information
  - The variance function, Var[X], is given by,

$$Var[X] = \sigma_x^2 = \sum_{j=1}^{J} (x_j - \mu)^2 (p_j) \qquad \text{(discrete case)}$$

where,

$J =$ number of discrete events

$p_j =$ probability of occurance of the x_j - th value of the discrete random variable

$\mu = E[X] =$ expected value

- Summary of probabilistic information
  - Key terms and their definitions:
    - Standard Deviation:
      - *The dispersion or spread of values about the expected value E[X] or sample average.* (Blank & Tarquin)
  - The standard deviation function, is given by,

$$\sigma_x = \sqrt{Var[X]}$$

where,

$$Var[X] = \sigma_x^2 = \text{variance of the random variable}$$

- # Joint and conditional probabilities

Dependent Random Variables, X and Y :

$$P(x,y) = P(X = x \mid Y = y) P(Y = y)$$

where,

$P(X = x \mid Y = y) =$ conditional probability of observing $X = x$, given $Y = y$

$P(Y = y) =$ marginal probability of observing $Y = y$

Independent Random Variables, X and Y :

$$P(x,y) = P(x) P(y)$$

where,

$P(x) =$ probability of $X = x$

$P(y) =$ probability of $Y = y$

- Covariance and coefficient of correlation
  - Used to measure dependence for random variables when they are not independent of one another.
  - The covariance function, is given by,

$$Cov(X,Y) = \sigma_{xy}$$

$$Cov(X,Y) = E\{(X - E[X])(Y - E[Y])\}$$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$Cov(X,Y) = \rho_{xy}\sigma_x\sigma_y$$

where,

ρ_{xy} = coefficient of correlation between X and Y

- Covariance and coefficient of correlation
  - Notes:
    - If X tends to fall below its mean, whenever Y exceeds its mean, Cov(X,Y) is positive.
      - The sign of Cov(X,Y) indicates whether X and Y vary directly or inversely with one another.
    - The value of ρ_{xy} can vary within the range of -1 to 1, with ρ_{xy} = 0 indicating no correlation between the two random variables.

- Probability Distribution of NPW
  - Topics for discussion:
    - Procedure for developing an NPW distribution.
    - Aggregating risk over time.
    - Decision rules for comparing mutually exclusive risky alternatives.

- Procedure for developing an NPW distribution.
  - Assumption:
    - All random variables are independent.
  - Steps to develop NPW distribution:
    - Express the NPW as functions of unknown random variables.
    - Determine the probability distribution for each random variable.
    - Determine the joint events and their probabilities.
    - Evaluate the NPW equation at these joint events.
    - Rank the NPW values in increasing order of NPW.

- Procedure for developing an NPW distribution.
  - Example 1: (Chan S. Park, example 12.6)
    - Review the unit demand (X) and unit price (Y) data presented in table 1.
    - Review the after-tax cash flow data (as a function of X and Y) presented in table 2.
    - Deliverables:
      - Develop the NPW distribution for the project.
      - Calculate the mean of the NPW distribution.
      - Calculate the variance of the NPW distribution.

- Procedure for developing an NPW distribution.
  - Example 1: (Chan S. Park, example 12.6)
    - Table 1:

Product Demand (X)		Unit Sale Price (Y)	
Units (x)	P(X = x)	Unit Price (y)	P(Y = y)
1600	0.20	$48	0.30
2000	0.60	$50	0.50
2400	0.20	$53	0.20

- Procedure for developing an NPW distribution.
  - Example 1: (Chan S. Park, example 12.6)
    - Table 2:

Item	**0**	**1**	**2**	**3**	**4**	**5**
Cash inflow:						
Net salvage						$37,389
X(1-0.4)Y		0.6*XY*	0.6*XY*	0.6*XY*	0.6*XY*	0.6*XY*
0.4 (dep)		$7,145	$12,245	$8,745	$6,245	$2,230
Cash outflow:						
Investment	-$125,000					
-*X*(1-0.4)($15)		-9*X*	-9*X*	-9*X*	-9*X*	-9*X*
-(1-0.4)($10,000)		-$6,000	-$6,000	-$6,000	-$6,000	-$6,000
Net Cash Flow	-$125,000	0.6*X*(*Y*-15) + $1145	0.6*X*(*Y*-15) + $6245	0.6*X*(*Y*-15) + $2745	0.6*X*(*Y*-15) + $245	0.6*X*(*Y*-15) +33,619

- Procedure for developing an NPW distribution.
  - Example 1: (Chan S. Park, example 12.6)
    - Present Worth of Cash Inflows:

$$PW = 0.6XY(P/A,15\%,5) + \$7{,}145(P/F,15\%,1) + \ldots + \$2{,}230(P/F,15\%,5) + \$37{,}389(P/F,15\%,5)$$
$$PW = 0.6XY(3.3522) + \$7{,}145(0.8696) + \ldots + \$2{,}230(0.4972) + \$37{,}389(0.4972)$$
$$PW = 2.01132XY + \$44{,}491$$

    - Present Worth of Cash Outflows:

$$PW = -\$125{,}000 - 9X(P/A,15\%,5) - \$6{,}000(P/A,15\%,5)$$
$$PW = -\$125{,}000 - 9X(3.3522) - \$6{,}000(3.3522)$$
$$PW = -30.1698X - \$145{,}113$$

    - Net Present Worth:

$$NPW = PW_{\text{inflow}} + PW_{\text{outflow}}$$
$$NPW = 2.01132XY + \$44{,}491 + (-30.1698X - \$145{,}113)$$
$$NPW = 2.01132X(Y - 15) - \$100{,}622$$

- Procedure for developing an NPW distribution.
  - Example 1: (Chan S. Park, example 12.6)
    - Combinations for unit demand and unit price, the joint probabilities and the associated NPW distribution:

Unit Demand, X	Unit Price, Y	Joint Prob	NPW
1600	$48	0.06	$5,576
1600	$50	0.10	$12,012
1600	$53	0.04	$21,666
2000	$48	0.18	$32,125
2000	$50	0.30	$40,170
2000	$53	0.12	$52,238
2400	$48	0.06	$58,675
2400	$50	0.10	$68,329
2400	$53	0.04	$82,810

$$\boxed{NPW = 2.01132X(Y-15) - \$100,622}$$

- Procedure for developing an NPW distribution.
  - Example 1: (Chan S. Park, example 12.6)
    - Mean of the NPW distribution:

		pj	xj	
Unit Demand, X	**Unit Price, Y**	**Joint Prob**	**NPW**	$p_j x_j$
1600	$48	0.06	$5,576	$335
1600	$50	0.10	$12,012	$1,201
1600	$53	0.04	$21,666	$867
2000	$48	0.18	$32,125	$5,783
2000	$50	0.30	$40,170	$12,051
2000	$53	0.12	$52,238	$6,269
2400	$48	0.06	$58,675	$3,520
2400	$50	0.10	$68,329	$6,833
2400	$53	0.04	$82,810	$3,312
			E[NPW]	**$40,170**

$$\boxed{E[X] = \mu_x = \sum_{j=1}^{J} (p_j) x_j}$$

- Procedure for developing an NPW distribution.
  - Example 1: (Chan S. Park, example 12.6)
    - Variance of the NPW distribution:

		pj	xj	μ		
Unit Demand, X	**Unit Price, Y**	**Joint Prob**	**NPW**	**E[NPW]**	**$(x_j - \mu)^2$**	**$(x_j - \mu)^2(p_j)$**
1600	$48	0.06	$5,576	$40,170	$1,196,793,545	$71,807,613
1600	$50	0.10	$12,012	$40,170	$792,899,996	$79,290,000
1600	$53	0.04	$21,666	$40,170	$342,403,345	$13,696,134
2000	$48	0.18	$32,125	$40,170	$64,726,530	$11,650,775
2000	$50	0.30	$40,170	$40,170	$0	$0
2000	$53	0.12	$52,238	$40,170	$145,634,693	$17,476,163
2400	$48	0.06	$58,675	$40,170	$342,403,345	$20,544,201
2400	$50	0.10	$68,329	$40,170	$792,899,996	$79,290,000
2400	$53	0.04	$82,810	$40,170	$1,818,168,236	$72,726,729
					Var[NPW]	**$366,481,614**
					Std Dev, s	**$19,144**

$$\boxed{Var[X] = \sigma_x^2 = \sum_{j=1}^{J}(x_j - \mu)^2 (p_j)}$$

- Aggregating risk over time.
  - Involves:
    - Determining the mean and variance of cash flows in each period.
    - Aggregating the risk over the project life in terms of NPW.
  - Two cases:
    - **Case 1:** Independent random variables
    - **Case 2:** Dependent random variables
    - **NOTES:**
      - Case 1 is only being considered for this class.
      - Case 2 is outside of the scope for this class
  - Review Chan S. Park, example 12.7

- Aggregating risk over time.
  - **Case 1:** Independent random variables
    - Assumptions:
      - Cash flows are independent from period to period.

$$E\left[PW(i)\right]=\sum_{n=0}^{N}\frac{E(A_n)}{(1+i)^n}$$

and

$$Var\left[PW(i)\right]=\sum_{n=0}^{N}\frac{Var(A_n)}{(1+i)^{2n}}$$

where,

A_n = cash flow in period n.

$E(A_n)$ = expected cash flow in period n.

$Var(A_n)$ = variance of the cash flows in period n.

- Aggregating risk over time.
  - **Case 2:** Dependent random variables
    - For information only.
    - A statistical independence among cash flows cannot be assumed (must consider the degree of dependence).
    - Expected value calculation is not affected by this and is as per Case 1.

Therefore, $Var\left[PW(i)\right]=\sum_{n=0}^{N}\frac{Var(A_n)}{(1+i)^{2n}}+2\sum_{n=0}^{N-1}\sum_{s=n+1}^{N}\frac{\rho_{ns}\sigma_n\sigma_s}{(1+i)^{n+s}}$

where,

ρ_{ns} = correlation coefficient (degree of dependence) between A_n and A_s.

$\rho_{ns}=0 \Rightarrow$ no correlation exists.

$\rho_{ns}>0 \Rightarrow$ positive correlation exists.

$\rho_{ns}<0 \Rightarrow$ negative correlation exists.

$s = n + 1$

- Decision rules for comparing mutually exclusive risky alternatives.
  - If EA > EB and VA ≤ VB ➔ select A.
  - If EA < EB and VA ≥ VB ➔ select B.
  - If EA = EB and VA < VB ➔ select A.
  - If EA = EB and VA > VB ➔ select B.
  - If EA = EB and VA = VB ➔ Indifferent.
  - If EA > EB and VA > VB ➔ Not clear.
  - If EA < EB and VA < VB ➔ Not clear.

- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Adapted from Chan S. Park, example 12.8)
    - Your boss has asked you to view the data presented in Table 1 for four different models of a fermentation machine needed for a new vaccine that your company will be producing.
    - Deliverables:
      - Evaluate the expected return and risk for each model configuration.
      - Make recommendations on which (if any) model should be selected.

- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Adapted from Chan S. Park, example 12.8)
    - Table 1:

	Probabilities			
NPW	**Model 1**	**Model 2**	**Model 3**	**Model 4**
$1,000	0.35	0.10	0.40	0.20
$1,500	0.00	0.45	0.00	0.40
$2,000	0.40	0.00	0.25	0.00
$2,500	0.00	0.35	0.00	0.30
$3,000	0.20	0.00	0.20	0.00
$3,500	0.00	0.00	0.00	0.00
$4,000	0.05	0.00	0.15	0.00
$4,500	0.00	0.10	0.00	0.10

- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Adapted from Chan S. Park, example 12.8)
    - Calculate the mean and variance:

	Probabilities			
NPW	**Model 1**	**Model 2**	**Model 3**	**Model 4**
$1,000	0.35	0.10	0.40	0.20
$1,500	0.00	0.45	0.00	0.40
$2,000	0.40	0.00	0.25	0.00
$2,500	0.00	0.35	0.00	0.30
$3,000	0.20	0.00	0.20	0.00
$3,500	0.00	0.00	0.00	0.00
$4,000	0.05	0.00	0.15	0.00
$4,500	0.00	0.10	0.00	0.10
E[NPW]	$1,950	$2,100	$2,100	$2,000
Var[NPW]	$747,500	$915,000	$1,190,000	$1,000,000

- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Chan S. Park, example 12.8)
    - Calculate the mean and variance:

	Probabilities			
NPW	**Model 1**	**Model 2**	**Model 3**	**Model 4**
$1,000	0.35	0.10	0.40	0.20
$1,500	0.00	0.45	0.00	0.40
$2,000	0.40	0.00	0.25	0.00
$2,500	0.00	0.35	0.00	0.30
$3,000	0.20	0.00	0.20	0.00
$3,500	0.00	0.00	0.00	0.00
$4,000	0.05	0.00	0.15	0.00
$4,500	0.00	0.10	0.00	0.10
E[NPW]	$1,950	$2,100	$2,100	$2,000
Var[NPW]	$747,500	$915,000	$1,190,000	$1,000,000

$$\boxed{E\left[NPW\right]_{\text{model 1}} = (\$1{,}000 \times 0.35) + (\$1{,}500 \times 0) + \ldots + (\$4{,}500 \times 0)}$$

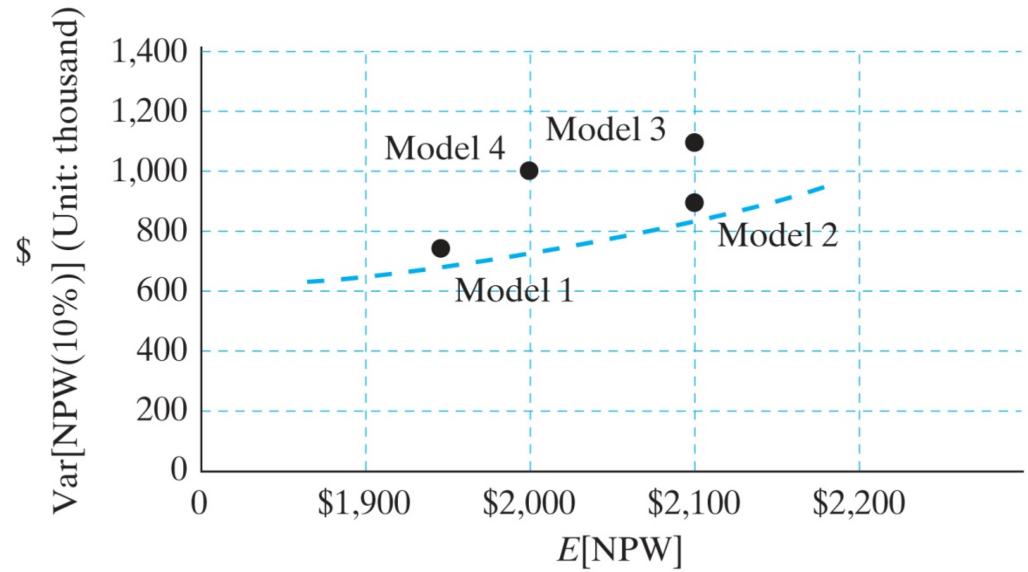
- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Chan S. Park, example 12.8)
    - Calculate the mean and variance:

	Probabilities			
NPW	**Model 1**	**Model 2**	**Model 3**	**Model 4**
$1,000	0.35	0.10	0.40	0.20
$1,500	0.00	0.45	0.00	0.40
$2,000	0.40	0.00	0.25	0.00
$2,500	0.00	0.35	0.00	0.30
$3,000	0.20	0.00	0.20	0.00
$3,500	0.00	0.00	0.00	0.00
$4,000	0.05	0.00	0.15	0.00
$4,500	0.00	0.10	0.00	0.10
E[NPW]	$1,950	$2,100	$2,100	$2,000
Var[NPW]	$747,500	$915,000	$1,190,000	$1,000,000

$$\boxed{Var[NPW]_{\text{model 1}} = (\$1{,}000 - \$1950)^2(0.35) + (\$1{,}500 - \$1950)^2(0) + \ldots + (\$4{,}500 - \$1950)^2(0)}$$

- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Chan S. Park, example 12.8)
    - Which model should be selected?

	Model 1	Model 2	Model 3	Model 4
E[NPW]	$1,950	$2,100	$2,100	$2,000
Var[NPW]	$747,500	$915,000	$1,190,000	$1,000,000



(Chan S. Park, Figure 12.9)

- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Chan S. Park, example 12.8)
    - Which model should be selected?
      - Decision solely based on E[NPW]
        - » Select model 2 or model 3.
      - Decision accounting for both the E[NPW] and the Var[NPW]
        - » Choice not as obvious.

- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Chan S. Park, example 12.8)
    - Which model should be selected?
      - Decision accounting for both the E[NPW] and the Var[NPW]
        - » Model 2 v Model 3:

          $E[NPW]_{\text{Model 2}} = E[NPW]_{\text{Model 3}}$

          $Var[NPW]_{\text{Model 2}} < Var[NPW]_{\text{Model 3}}$

          Select Model 2 for next comparison
        - » Model 2 v Model 4:

          $E[NPW]_{\text{Model 2}} > E[NPW]_{\text{Model 4}}$

          $Var[NPW]_{\text{Model 2}} < Var[NPW]_{\text{Model 4}}$

          Select Model 2 for next comparison

- Decision rules for comparing mutually exclusive risky alternatives.
  - Example 2: (Chan S. Park, example 12.8)
    - Which model should be selected?
      - Decision accounting for both the E[NPW] and the Var[NPW]
        - » Model 2 v Model 1:

          $E[NPW]_{\text{Model 2}} > E[NPW]_{\text{Model 1}}$

          $Var[NPW]_{\text{Model 2}} > Var[NPW]_{\text{Model 1}}$

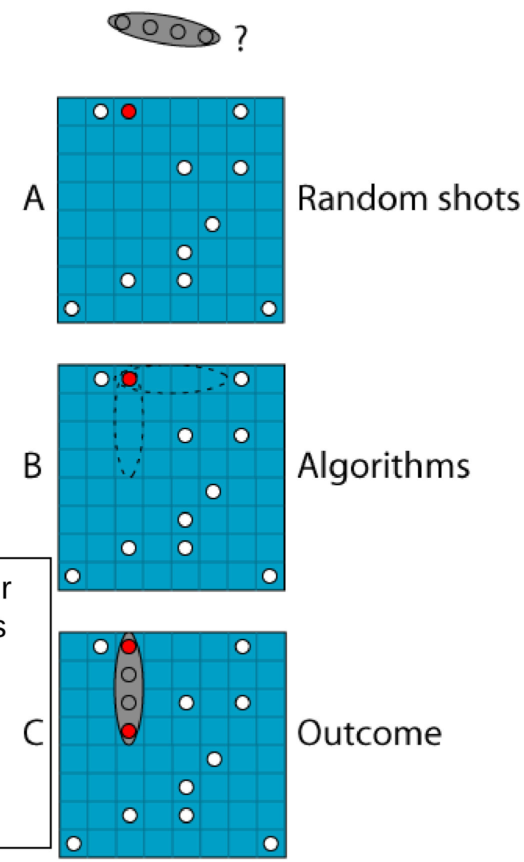
          Can’t decide.
      - Conclusion:
        - » Decision maker has to make a trade-off between the incremental expected return ($150) and the incremental risk ($167,500)

- Risk Simulation
  - Used where standard analytical methods are not suitable especially when multiple random variables are involved.
  - Computer simulation using computational algorithms is the most common way to build, execute and analyze risk models.
  - Excel software has the capability to perform a computer simulation and add-ins such as @RISK[1] can also be used.

1. Risk simulation software developed by Palisade Corporation, Ithaca, NY 14850

- Risk Simulation
  - Logical steps for a computer program that simulates investment scenarios. (Chan S. Park)
    - **Step 1:** Identify all the variables that affect the measure of investment worth (e.g. NPW after taxes)
    - **Step 2:** Identify the relationships among all the variables. The relationships of interest are expressed by the equations or the series of numerical computations by which we compute the NPW of an investment project. These equations make up the model we are trying to analyze.
    - **Step 3:** Classify the variables into 2 groups: the parameters whose values are known with certainty, and the random variables for which exact values cannot be specified at the time of decision making.

- Risk Simulation
  - Logical steps for a computer program that simulates investment scenarios. (Chan S. Park)
    - **Step 4:** Define distributions for all the random variables.
    - **Step 5:** Perform Monte Carlo sampling and describe the resulting NPW distribution.
    - **Step 6:** Compute the distribution parameters and prepare graphic displays of the results of the simulation.

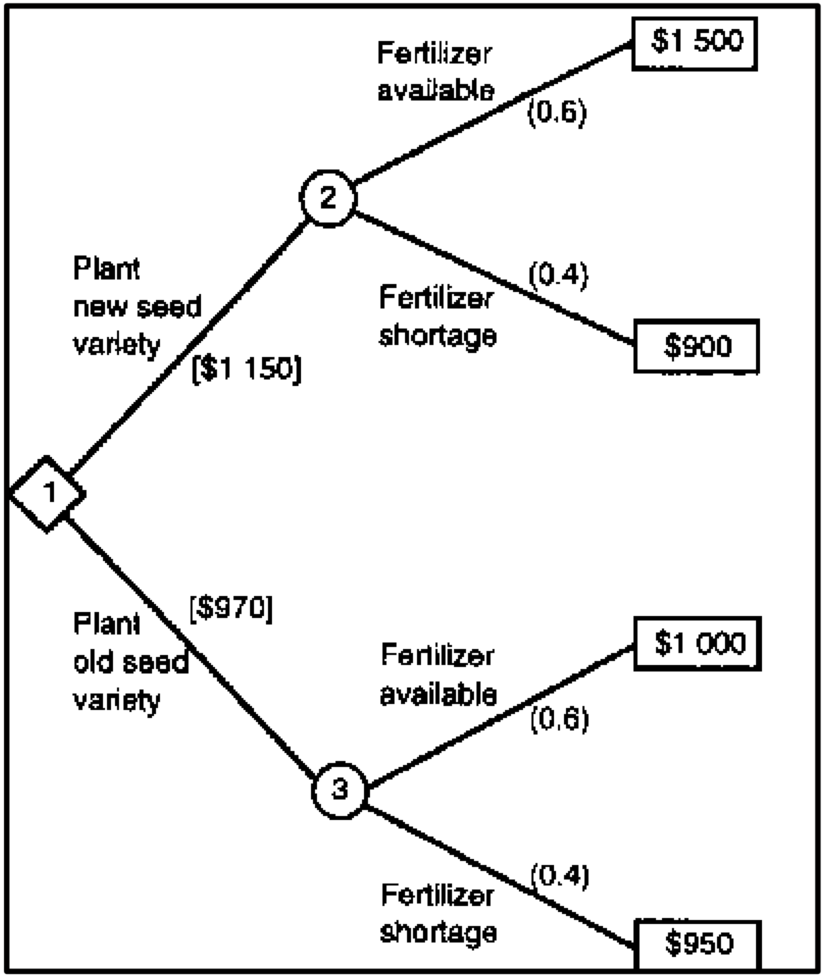


The Monte Carlo method can be illustrated as a game of battleship. First a player makes some random shots. Next the player applies algorithms (ie. a battleship is four dots in the vertical or horizontal direction). Finally based on the outcome of the random sampling and the algorithm the player can determine the likely locations of the other player's ships.
("monte carol method" Wikipedia, August 12, 2008
<http://en.wikipedia.org/wiki/Monte_Carlo_method>)

- Decision Tree Analysis
  - Definition: (Chan S. Park)
    - *Technique that can facilitate investment decision making when uncertainty prevails, especially when the problem involves a sequence of decisions.*
  - Decision criterion / objective must be selected.
    - e.g. maximize profit, minimize cost, . . . etc.
  - Each possible decision and outcome should be calculated.

- Decision Tree Analysis
  - A decision tree includes: (Blank & Tarquin)
    - More than one stage of alternative selection.
    - Selection of an alternative at one stage that leads to another stage.
    - Expected results from a decision at each stage.
    - Probability estimates for each outcome.
    - Estimates of economic value (cost, revenue) for each outcome.
    - Measure of worth as the selection criterion e.g. E(PW).

- Decision Tree Analysis
  - Constructing a decision tree:
    - Constructed from left to right.
    - Includes each possible decision and outcome.
    - **Square =** decision node.
    - **Branch =** possible alternatives indicated on the branches from the decision node.
    - **Circle =** probability node.



Questions?