



EM600 - Engineering Economics and Cost Analysis

***Lecture 05: Benefit-Cost Analysis and its Implication
for Public Sector Projects***

- References:
 - Park, Chan S. Contemporary Engineering Economics. New Jersey: Pearson Prentice Hall, 2006 (Chapter 16: 16.3, 16.4)
 - Ganguly, A. Engineering Economics Using Excel. New Jersey: SSE, 2008

After completing this module you should understand the following:

- Why Benefit-Cost Analysis (BCA) is used in Public Sector Projects
- Valuation of Benefits and Costs
- Definition of Benefit-Cost Ratio (BCR)
- Calculation of BCR (including Incremental BCR)

- Benefit-Cost Analysis (BCA):
 - Definitions: (Chan S. Park)
 - *A technique designed to determine the feasibility of a project or plan by quantifying its costs or benefits.*
 - *A decision-making tool that is used to systematically develop useful information about the desirable and undesirable effects of public projects.*

- Why use benefit-cost analysis (BCA)?¹
 - Mainly, but not exclusively, used to assess the “value for money” of very large private and public sector projects.
 - Public sector projects tend to include costs and benefits that are less amenable to being expressed in financial or monetary terms (e.g. environmental damage), as well as those that can be expressed in monetary terms.
 - Private sector organizations tend to make much more use of other project appraisal techniques, such as rate of return, where feasible.

1. “cost-benefit analysis” en.wikipedia.org. June 24, 2008 < http://en.wikipedia.org/wiki/Cost-benefit_analysis >.

- **Benefit-Cost Analysis (BCA):**
 - Uses:
 - BCA is commonly used to evaluate public projects.
 - BCA is used to choose among alternatives such as allocation of funds for a mass transit system, a dam with irrigation or an air-traffic control system.
 - Basis for choice:
 - Choose the project for which the benefits exceed the costs by the greatest amount.
 - Mutually Exclusive Alternatives:
 - An incremental benefit-cost ratio must be used.
 - History's role in BCA



- Benefit-Cost Analysis (BCA):
 - Examples:
 - Public transportation systems
 - Environmental regulations on noise and pollution
 - Public-safety programs
 - Public-health programs
 - Flood control
 - Water resource development
 - National defense programs
 - . . . etc



- Benefit-Cost Analysis (BCA):
 - Aims of BCA:
 - Maximize the benefits for any given sets of costs (or budgets).
 - Maximize the net benefits when both benefits and costs vary.
 - Minimize costs to achieve any given level of benefits.



- Framework of benefit-cost analysis:
 - Identify all the users (public) and sponsors (government) of the project.
 - Identify all the users' benefits and disbenefits of the project.
 - Identify all the sponsors' costs.
 - Quantify all benefits, disbenefits and costs in dollars or some other unit of measure.
 - Select an appropriate interest rate at which to discount benefits, disbenefits and costs in the future to a present value.
 - Acceptance Criteria:
 - equivalent users' benefits >> equivalent sponsors' costs

- Valuation of Benefits and Costs:
 - Users' benefits
 - Benefits, disbenefits, secondary effects
 - Primary and secondary benefits
 - Users' benefits, $B = \text{benefits} - \text{disbenefits}$
 - Sponsors' costs
 - Classify expenditure (capital cost + O&M cost) required and savings or revenues realized
 - Sponsors' cost = expenditure - revenue
 - Social discount rate
 - Projects without private counterparts
 - Social discount rate = Government borrowing rate
 - Projects with private counterparts
 - Social discount rate = rate that could have been earned had the funds not been taken from the private sector

- Benefit-Cost Ratio (BCR):
 - The benefit-cost ratio is defined as:
 - $BCR(i) = \frac{B}{C} = \frac{B}{I + C'}$
where,
 $I + C' > 0$
B = users' benefits
I = initial cost of investment
C = sponsors' costs
C' = equivalent annual operating costs
 - ***Note: B, C', and I must be expressed in either present worth, annual worth or future worth equivalents.***
 - Acceptance criteria:
 - If $BCR > 1 \rightarrow$ ACCEPT
 - If $BCR < 1 \rightarrow$ REJECT

- Example 1: (BCR)

- The following public project is being considered by the government:

Initial Cost of Expansion, I	\$2,250,000
Annual Costs for O&M, C'	\$70,000
Annual Savings and Benefits, B	\$350,000
Scrap Value after useful life, S	\$325,000
Useful life of investment, N	25
Interest rate, i	7%

- Compute B, C, I, C' and BCR(i) using 3 methods:
 - 1. Present worth equivalence
 - 2. Annual equivalence
 - 3. Future worth equivalence



**EXAMPLE 1: (BCR)
Cash Flow Diagram**

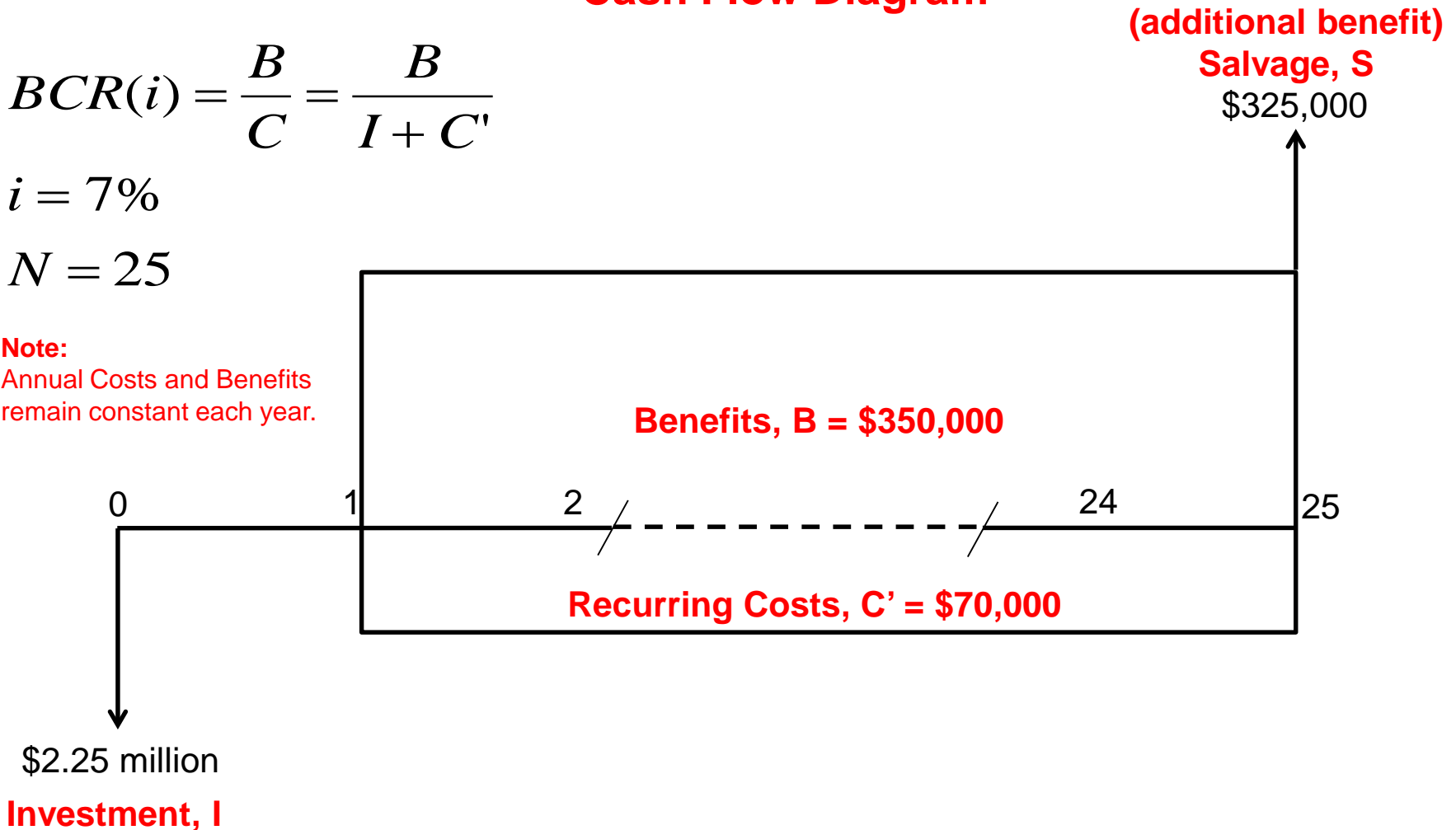
$$BCR(i) = \frac{B}{C} = \frac{B}{I + C'}$$

$$i = 7\%$$

$$N = 25$$

Note:

Annual Costs and Benefits remain constant each year.



• Example 1: (BCR)

– Present Worth:

• Calculate B:

$$PW_B = \$350,000(P/A, 7\%, 25) + \$325,000(P/F, 7\%, 25)$$

$$PW_B = \$350,000(11.6536) + \$325,000(0.1842)$$

$$PW_B = \$4,138,625$$

• Calculate C':

$$PW_{C'} = \$70,000(P/A, 7\%, 25)$$

$$PW_{C'} = \$70,000(11.6536)$$

$$PW_{C'} = \$815,752$$

• Calculate I:

$$PW_I = \$2,250,000$$

EXCEL:

	A	B	C	D
1	I	\$2,250,000		P/A
2	C'	\$70,000		P/F
3	B	\$350,000		A/F
4	S	\$325,000		A/P
5	N	25		F/A
6	i	7%		F/P
7				
8		B	I	C'
9	PW	\$4,138,635	\$2,250,000	\$815,751
10	AW	\$355,138	\$193,074	\$70,000
11	FW	\$22,462,163	\$12,211,723	\$4,427,433

Calculate B: 9 =PV(B6,B5,-B3,,)+PV(B6,B5,-B4,,)

Calculate C': 9 PW \$4,138,635 \$ =PV(B6,B5,-B2,,)

Calculate I: 9 PW \$4,138,635 =B1

- Example 1: (BCR)

- Present Worth:

- Calculate C:

$$PW_C = I + C'$$

$$PW_C = \$2,250,000 + \$815,752$$

$$PW_C = \$3,065,752$$

- Calculate BCR(i):

$$PW_{BCR(i)} = \frac{B}{C} = \frac{B}{I + C'}$$

$$PW_{BCR(10\%)} = \frac{\$4,138,625}{\$2,250,000 + \$815,752} = 1.35$$

- **BCR(10%) = 1.35 > 1**

- **Conclusion: GOOD investment, ACCEPT**



- Example 1: (BCR)
 - Annual Equivalence:

- Calculate B:

$$AE_B = \$350,000 + \$325,000(A/F, 7\%, 25)$$

$$AE_B = \$350,000 + \$325,000(0.0158)$$

$$AE_B = \$355,135$$

- Calculate C':

$$AE_{C'} = \$70,000$$

- Calculate I:

$$AE_I = \$2,250,000(A/P, 7\%, 25)$$

$$AE_I = \$2,250,000(0.0858)$$

$$AE_I = \$193,050$$

EXCEL:

	A	B	C	D
1	I	\$2,250,000		P/A
2	C'	\$70,000		P/F
3	B	\$350,000		A/F
4	S	\$325,000		A/P
5	N	25		F/A
6	i	7%		F/P
7				
8		B	I	C'
9	PW	\$4,138,635	\$2,250,000	\$815,751
10	AW	\$355,138	\$193,074	\$70,000
11	FW	\$22,462,163	\$12,211,723	\$4,427,433

Calculate B:

10 =B3+PMT(B6,B5,,B4,)

Calculate C':

10 AW \$355,138 \$193,074 =B2

Calculate I:

10 AW =PMT(B6,B5,-B1,,)

- Example 1: (BCR)
 - Annual Equivalence:

- Calculate C:

$$AE_C = I + C'$$

$$AE_C = \$193,050 + \$70,000$$

$$AE_C = \$263,050$$

- Calculate BCR(i):

$$AE_{BCR(i)} = \frac{B}{C} = \frac{B}{I + C'}$$

$$AE_{BCR(10\%)} = \frac{\$355,135}{\$193,050 + \$70,000} = 1.35$$

- **BCR(10%) = 1.35 > 1**

- **Conclusion: GOOD investment, ACCEPT**



• Example 1: (BCR)

– Future Worth:

• Calculate B:

$$FW_B = \$350,000(F/A, 7\%, 25) + \$325,000$$

$$FW_B = \$350,000(63.2490) + \$325,000$$

$$FW_B = \$22,462,150$$

• Calculate C':

$$FW_{C'} = \$70,000(F/A, 7\%, 25)$$

$$FW_{C'} = \$70,000(63.2490)$$

$$FW_{C'} = \$4,427,430$$

• Calculate I:

$$FW_I = \$2,250,000(F/P, 7\%, 25)$$

$$FW_I = \$2,250,000(5.4274)$$

$$FW_I = \$12,211,650$$

EXCEL:

	A	B	C	D
1	I	\$2,250,000		P/A
2	C'	\$70,000		P/F
3	B	\$350,000		A/F
4	S	\$325,000		A/P
5	N	25		F/A
6	i	7%		F/P
7				
8		B	I	C'
9	PW	\$4,138,635	\$2,250,000	\$815,751
10	AW	\$355,138	\$193,074	\$70,000
11	FW	\$22,462,163	\$12,211,723	\$4,427,433

Calculate B:

11 =FV(B6,B5,-B3,,)+B4

Calculate C':

11 FW \$22,462,163 \$=FV(B6,B5,-B2,,)

Calculate I:

11 FW =FV(B6,B5,-B1,)

- Example 1: (BCR)

- Present Worth:

- Calculate C:

$$FW_C = I + C'$$

$$FW_C = \$12,211,650 + \$4,427,430$$

$$FW_C = \$16,639,080$$

- Calculate BCR(i):

$$FW_{BCR(i)} = \frac{B}{C} = \frac{B}{I + C'}$$

$$FW_{BCR(10\%)} = \frac{\$22,462,150}{\$12,211,650 + \$4,427,430} = 1.35$$

- **BCR(10%) = 1.35 > 1**

- **Conclusion: GOOD investment, ACCEPT**



- Example 1: (BCR)
 - Summary

	A	B	C	D	E	F
1	I	\$2,250,000		P/A	11.6536	
2	C'	\$70,000		P/F	0.1842	
3	B	\$350,000		A/F	0.0158	
4	S	\$325,000		A/P	0.0858	
5	N	25		F/A	63.2490	
6	i	7%		F/P	5.4274	
7						
8		B	I	C'	C	BCR
9	PW	\$4,138,635	\$2,250,000	\$815,751	\$3,065,751	1.35
10	AW	\$355,138	\$193,074	\$70,000	\$263,074	1.35
11	FW	\$22,462,163	\$12,211,723	\$4,427,433	\$16,639,156	1.35
12						
13		B	I	C'	C	BCR
14	PW	\$4,138,625	\$2,250,000	\$815,752	\$3,065,752	1.35
15	AW	\$355,135	\$193,050	\$70,000	\$263,050	1.35
16	FW	\$22,462,150	\$12,211,650	\$4,427,430	\$16,639,080	1.35

FINANCIAL
FACTORS

EXCEL
FINANCIAL
FUNCTIONS

FINANCIAL
FACTORS

- Example 2: (Chan S. Park, example 16.1)
 - A public project being considered by a local government has the following estimated benefit-cost profile:

n	b_n	c_n
0		\$10
1		\$10
2	\$20	\$5
3	\$30	\$5
4	\$30	\$8
5	\$20	\$8

- Assume $i = 10\%$, $N = 5$
- Compute B , C , I , C' and $BCR(i)$



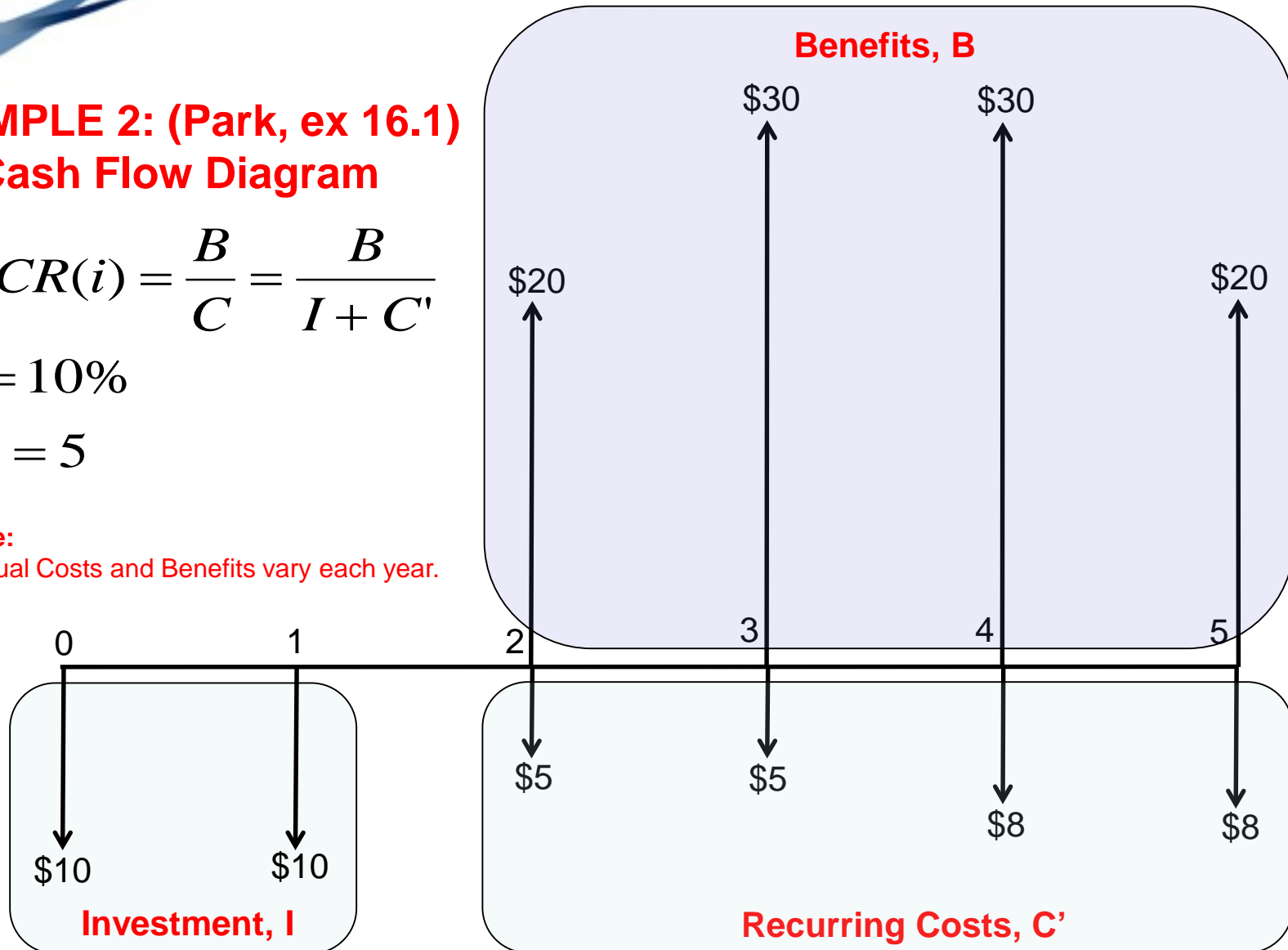
EXAMPLE 2: (Park, ex 16.1)
Cash Flow Diagram

$$BCR(i) = \frac{B}{C} = \frac{B}{I + C'}$$

$$i = 10\%$$

$$N = 5$$

Note:
Annual Costs and Benefits vary each year.



- Example 2: (Chan S. Park, example 16.1)

- Calculate B:

$$B = 20(P/F, 10\%, 2) + 30(P/F, 10\%, 3) + 30(P/F, 10\%, 4) + 20(P/F, 10\%, 5)$$

$$B = 20(0.8264) + 30(0.7513) + 30(0.6830) + 20(0.6209)$$

$$B = \$71.98$$

- Calculate C':

$$C' = 5(P/F, 10\%, 2) + 5(P/F, 10\%, 3) + 8(P/F, 10\%, 4) + 8(P/F, 10\%, 5)$$

$$C' = 5(0.8264) + 5(0.7513) + 8(0.6830) + 8(0.6209)$$

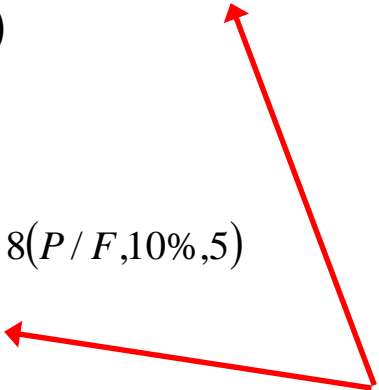
$$C' = \$18.32$$

- Calculate I:

$$I = 10 + 10(P/A, 10\%, 1)$$

$$I = 10 + 10(0.9091)$$

$$I = \$19.09$$



Annual Costs and Benefits are not equal for each year, therefore treat each annual cost or benefit as a future value and discount it to a present value.

- Example 2: (Chan S. Park, example 16.1)

- Calculate C:

$$C = I + C'$$

$$C = \$19.09 + \$18.32$$

$$C = \$37.41$$

- Calculate BCR(i):

$$BCR(i) = \frac{B}{C} = \frac{B}{I + C'}$$

$$BCR(10\%) = \frac{\$71.89}{\$19.09 + \$18.32} = 1.92$$

- $BCR(10\%) = 1.92 > 1$

- Conclusion: GOOD investment, ACCEPT



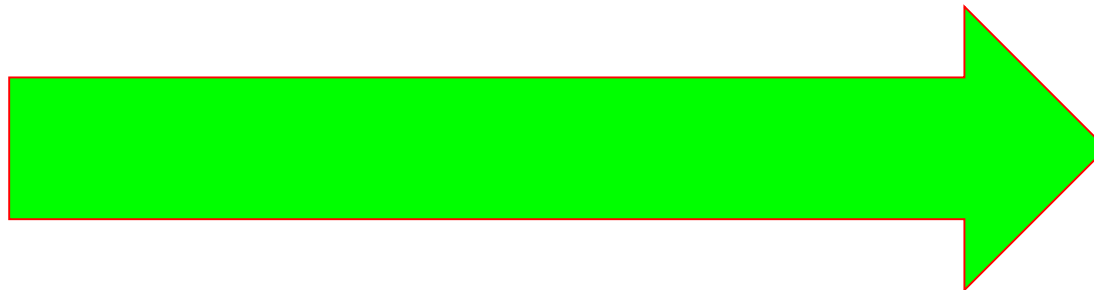
- Benefit-Cost Analysis
 - Mutually Exclusive Alternatives
 - **Method for Incremental Analysis**
 - **Step 1:** Calculate the BCR for each alternative. Eliminate any alternative with a $BCR < 1$.
 - **Step 2:** Calculate $(I + C')$ for each alternative.
 - **Step 3:** Arrange the alternatives in order of increasing value of $(I + C')$.
 - **Step 4:** The smallest value for $(I + C')$ is alternative j ; the next smallest value for $(I + C')$ is alternative k .
 - **Step 5:** Calculate $BCR(i)$ incrementally:

$$BCR(i)_{k-j} = \frac{B_k - B_j}{(I + C')_k - (I + C')_j}$$

where, $(I + C')_k > (I + C')_j$ for alternatives k and j

- Benefit-Cost Analysis
 - Mutually Exclusive Alternatives
 - **Method for Incremental Analysis**
 - **Step 6:** Evaluate the $BCR(i)$ value
 - » $BCR(i)_{k-j} > 1$ choose alternative k
 - » $BCR(i)_{k-j} < 1$ choose alternative j
 - **Step 7:** The alternative chosen in step 6 becomes alternative j for the next comparison. The next alternative (3rd smallest) in the ordered (I + C') list becomes alternative k for the next comparison.
 - **Step 8:** Calculate $BCR(i)$ incrementally for the two new alternatives and evaluate the $BCR(i)$ value.

- Benefit-Cost Analysis
 - Mutually Exclusive Alternatives
 - **Method for Incremental Analysis**
 - Continue until you reach the bottom of the list.
 - The alternative selected during the last pairing is the overall best alternative.
 - **Examples 3 and 4 illustrate this process.**



- Example 3: (Chan S. Park, example 16.2)
 - Mutually Exclusive Alternatives (BCR)
 - Consider the following 3 alternatives:

	A1	A2	A3
I	\$5,000	\$20,000	\$14,000
B	\$12,000	\$35,000	\$21,000
C'	\$4,000	\$8,000	\$1,000



- Note:
 - Projects are mutually exclusive.
 - Each project has the same service life.
 - Interest rate, $i = 10\%$
- Using incremental BCR analysis which is the best alternative?

- Example 3: (Chan S. Park, example 16.2)
 - Mutually Exclusive Alternatives (BCR)
 - Calculate BCR for each alternative:

$$BCR(i) = \frac{B}{C} = \frac{B}{I + C'}$$

	A1	A2	A3
I	\$5,000	\$20,000	\$14,000
B	\$12,000	\$35,000	\$21,000
C'	\$4,000	\$8,000	\$1,000
BCR(10%)	1.33	1.25	1.40
Status?	ACCEPT	ACCEPT	ACCEPT

Each $BCR > 1$, therefore “do-nothing” alternative can be dropped.

- Example 3: (Chan S. Park, example 16.2)
 - Mutually Exclusive Alternatives (BCR)
 - Note on calculated BCR for each alternative:

	A1	A2	A3
I	\$5,000	\$20,000	\$14,000
B	\$12,000	\$35,000	\$21,000
C'	\$4,000	\$8,000	\$1,000
BCR(10%)	1.33	1.25	1.40
Status?	ACCEPT	ACCEPT	ACCEPT

Each $BCR > 1$, therefore “do-nothing” alternative can be dropped.

- Cannot compare these BCR values directly with each other.
- The calculated BCR does not represent the magnitude of the investment.
- Incremental BCR analysis must be used.

- Example 3: (Chan S. Park, example 16.2)
 - Mutually Exclusive Alternatives (BCR)
 - Calculate $(I + C')$ for each alternative:

	A1	A2	A3
I	\$5,000	\$20,000	\$14,000
C'	\$4,000	\$8,000	\$1,000
$I + C'$	\$9,000	\$28,000	\$15,000

- Rank lowest to highest $(I + C')$

	A1	A2	A3
$I + C'$	\$9,000	\$28,000	\$15,000
RANK	1	3	2

- Example 3: (Chan S. Park, example 16.2)
 - Mutually Exclusive Alternatives (BCR)

- 1st incremental comparison:

- Alternative j = Project A1
 - Alternative k = Project A3

$$BCR(i)_{k-j} = \frac{B_k - B_j}{(I + C')_k - (I + C')_j}$$

$$BCR(10\%)_{A3-A1} = \frac{B_{A3} - B_{A1}}{(I + C')_{A3} - (I + C')_{A1}}$$

$$BCR(10\%)_{A3-A1} = \frac{\$21,000 - \$12,000}{\$15,000 - \$9,000} = 1.5$$

- $BCR(10\%)_{A3-A1} > 1 \Rightarrow$ select A3 for next comparison.

- Example 3: (Chan S. Park, example 16.2)
 - Mutually Exclusive Alternatives (BCR)

- 2nd incremental comparison:

- Alternative j = Project A3
- Alternative k = Project A2

$$BCR(i)_{k-j} = \frac{B_k - B_j}{(I + C')_k - (I + C')_j}$$

$$BCR(10\%)_{A2-A3} = \frac{B_{A2} - B_{A3}}{(I + C')_{A2} - (I + C')_{A3}}$$

$$BCR(10\%)_{A2-A3} = \frac{\$35,000 - \$21,000}{\$28,000 - \$15,000} = 1.08$$

- $BCR(10\%)_{A2-A3} > 1 \Rightarrow$ select A2.

- Example 4:

- Mutually Exclusive Alternatives (BCR)

- Consider the following 5 alternatives for a machine to produce a new smallpox vaccine:

	A1	A2	A3	A4	A5
I	\$37,500	\$20,000	\$28,000	\$10,000	\$35,000
B	\$67,500	\$31,150	\$42,000	\$35,000	\$47,500
C'	\$12,500	\$12,500	\$5,750	\$9,000	\$11,500



- Note:
 - Projects are mutually exclusive.
 - Each project has the same service life.
 - Interest rate, $i = 8\%$
 - Using incremental BCR analysis which is the best alternative?

- Example 4:
 - Mutually Exclusive Alternatives (BCR)
 - Calculate BCR for each alternative:

$$BCR(i) = \frac{B}{C} = \frac{B}{I + C'}$$

	A1	A2	A3	A4	A5
I	\$37,500	\$20,000	\$28,000	\$10,000	\$35,000
B	\$67,500	\$31,150	\$42,000	\$35,000	\$47,500
C'	\$12,500	\$12,500	\$5,750	\$9,000	\$11,500
BCR(8%)	1.35	0.96	1.24	1.84	1.02
Status?	ACCEPT	REJECT	ACCEPT	ACCEPT	ACCEPT

- Example 4:
 - Mutually Exclusive Alternatives (BCR)
 - Calculate $(I + C')$ for each alternative:

	A1	A2	A3	A4	A5
I	\$37,500	\$20,000	\$28,000	\$10,000	\$35,000
C'	\$12,500	\$12,500	\$5,750	\$9,000	\$11,500
I + C'	\$50,000	N/A	\$33,750	\$19,000	\$46,500

- Rank lowest to highest $(I + C')$

	A1	A2	A3	A4	A5
I + C'	\$50,000	N/A	\$33,750	\$19,000	\$46,500
RANK	4	N/A	2	1	3

- Example 4:
 - Mutually Exclusive Alternatives (BCR)
 - 1st incremental comparison:
 - Alternative j = Project A4
 - Alternative k = Project A3

$$BCR(i)_{k-j} = \frac{B_k - B_j}{(I + C')_k - (I + C')_j}$$

$$BCR(8\%)_{A3-A4} = \frac{B_{A3} - B_{A4}}{(I + C')_{A3} - (I + C')_{A4}}$$

$$BCR(8\%)_{A3-A4} = \frac{\$42,000 - \$35,000}{\$33,750 - \$19,000} = 0.47$$

- $BCR(8\%)_{A3-A4} < 1 \Rightarrow$ select A4 for next comparison.

- Example 4:
 - Mutually Exclusive Alternatives (BCR)
 - 2nd incremental comparison:
 - Alternative j = Project A4
 - Alternative k = Project A5

$$BCR(i)_{k-j} = \frac{B_k - B_j}{(I + C')_k - (I + C')_j}$$

$$BCR(8\%)_{A5-A4} = \frac{B_{A5} - B_{A4}}{(I + C')_{A5} - (I + C')_{A4}}$$

$$BCR(8\%)_{A5-A4} = \frac{\$47,500 - \$35,000}{\$46,500 - \$19,000} = 0.45$$

- $BCR(8\%)_{A5-A4} < 1 \Rightarrow$ select A4 for next comparison.

- Example 4:
 - Mutually Exclusive Alternatives (BCR)
 - 3rd incremental comparison:
 - Alternative j = Project A4
 - Alternative k = Project A1

$$BCR(i)_{k-j} = \frac{B_k - B_j}{(I + C')_k - (I + C')_j}$$

$$BCR(8\%)_{A1-A4} = \frac{B_{A1} - B_{A4}}{(I + C')_{A1} - (I + C')_{A4}}$$

$$BCR(8\%)_{A1-A4} = \frac{\$67,500 - \$35,000}{\$50,000 - \$19,000} = 1.05$$

– $BCR(8\%)_{A1-A4} > 1 \Rightarrow$ select A1.

