EM600 - Engineering Economics and Cost Analysis

Lecture 02: *Understanding Cash Flow Diagrams, Interest Rates and Time Value of Money*

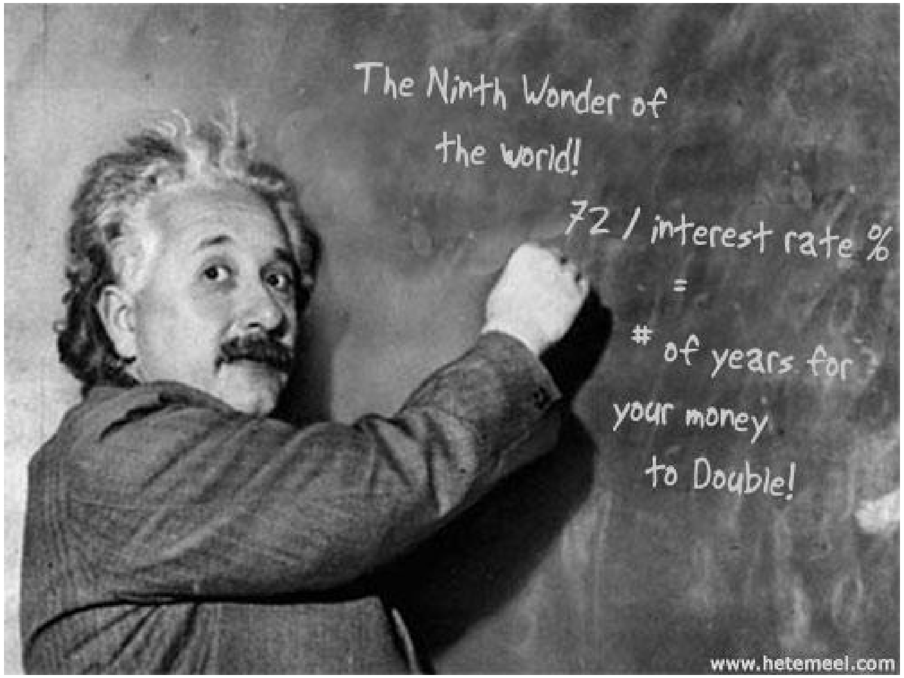
- References:
  - Park, Chan S. Contemporary Engineering Economics. New Jersey: Pearson Prentice Hall, 2006 (Chapter 3 & 4)
  - Ganguly, A. Engineering Economics Using Excel. New Jersey: SSE, 2008

After completing this module you should understand the following:

- Time value of money
- Cash flow diagram: basis, ‘how to’ and types (arithmetic, geometric gradient)
- Overview of simple and compound interests – calculation methods including continuous compounding
- Nominal, periodic and effective interest rates
- Equivalence calculations with nominal and effective interest rates
- Debt Management

Albert Einstein once said:

"The most powerful force in the universe is **compound interest**."[1]



1. "Albert Einstein quotes" ThinkExist.com. Aug 13, 2008 <http://thinkexist.com/quotation/the_most_powerful_force_in_the_universe_is/158830.html>.

- Key Definitions
  - Market Interest Rate
    - *Rates of interest paid on deposits and other investments, determined by the interaction of the supply of and demand for funds in the money market.*[1]
  - Time Value of Money
    - *The idea that a dollar now is worth more than a dollar in the future, even after adjusting for inflation, because a dollar now can earn interest or other appreciation until the time the dollar in the future would be received.*[2]

1. “market interest rate” Bank-Street.co.uk. May 20, 2008 <http://bank-street.co.uk/glossary.html>.
2. "time value of money" InvestorWords.com. WebFinance, Inc. May 20, 2008 <http://www.investorwords.com/4988/time_value_of_money.html>.

- Key Definitions
  - Purchasing Power
    - *The value of money, as measured by the quantity and quality of products and services it can buy.*[1]
  - Actual Dollars
    - *The cash flow measured in terms of dollars at the time of the transaction.*[2]

1. "purchasing power" InvestorWords.com. WebFinance, Inc. May 20, 2008 <http://www.investorwords.com/3959/purchasing_power.html>.
2. Park, Chan S. Contemporary Engineering Economics. New Jersey: Pearson Prentice Hall, 2006 (Chapter 3)

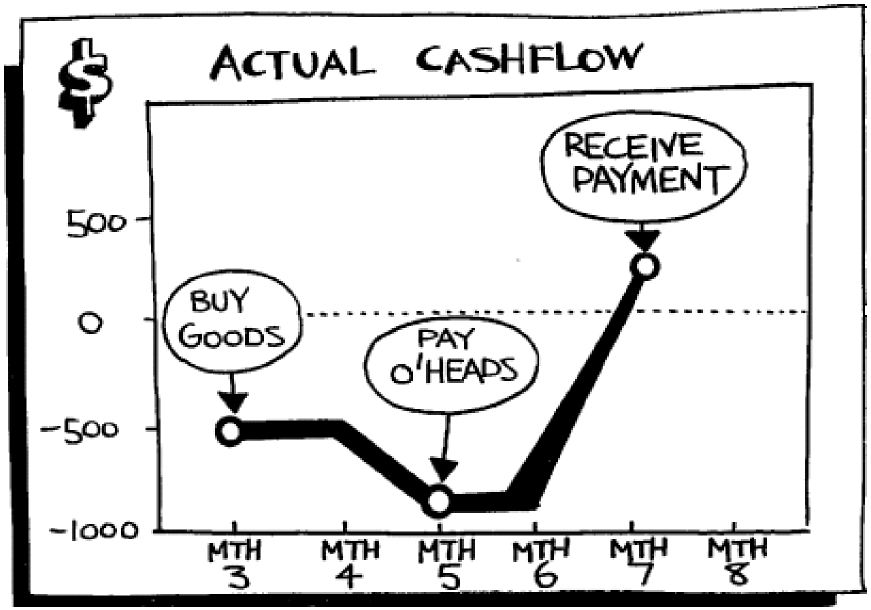
- Useful Terms:

i	=	Interest Rate
N	=	Number of Years (n = 0, 1, . . . , N)
P	=	Present Value / Present Worth (n=0)
F	=	Future Worth (at some time n)
A	=	Annual Worth / Annual Equivalence
S	=	Savage Value (n = N)

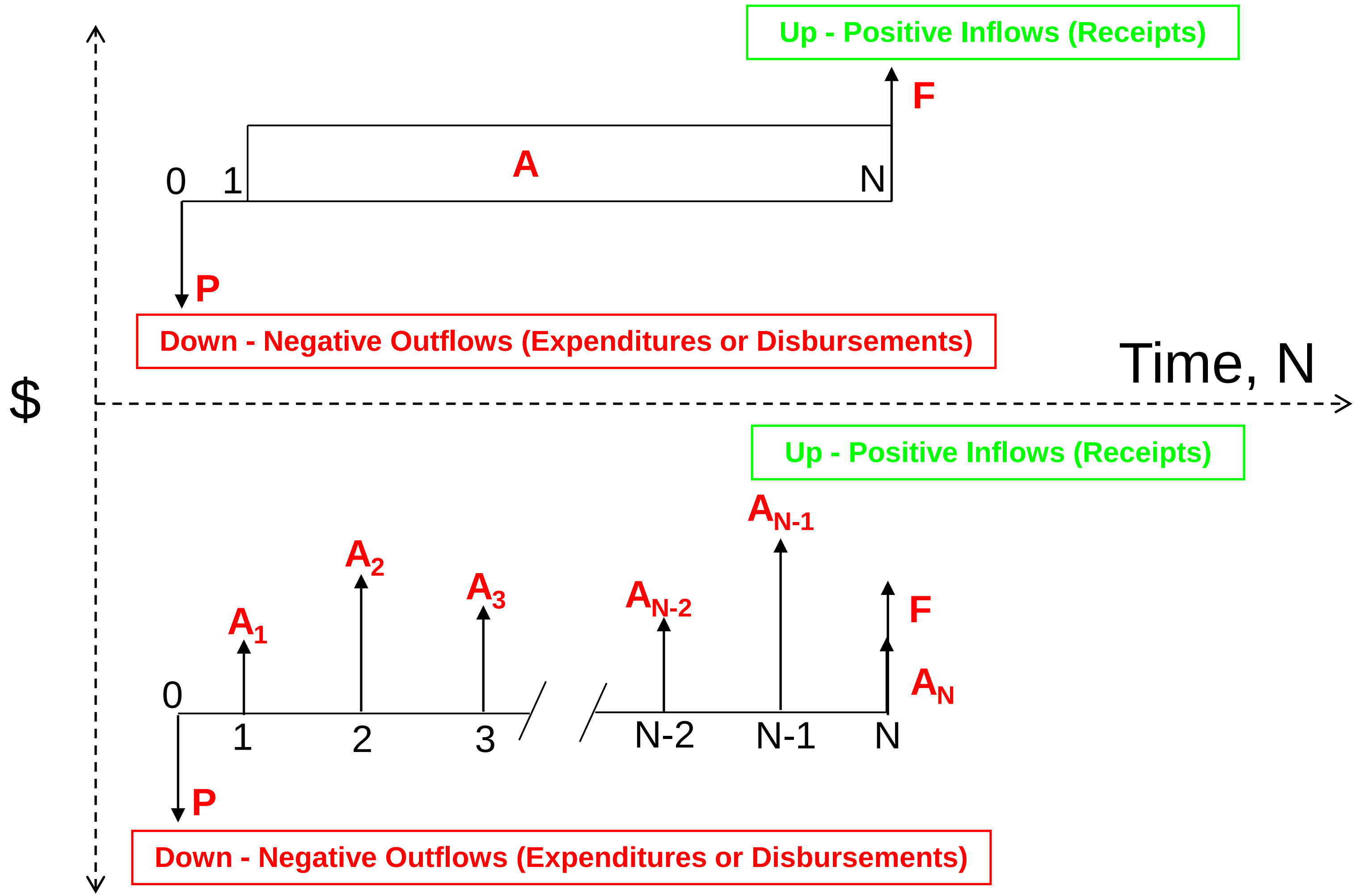
Note:

- P = PV = NPV = PW = NPW
- F = FW
- A = AW = AE (similar to annual cost, AC = EUAC, Equivalent Uniform Annualized Cost)

- Steps to solving a typical cash flow problem:
  - Read the problem & identify key elements (i, N, P,etc)
  - Draw a picture
  - Identify “knowns” and “unknowns”
  - Convert all “knowns” to the same units of time
  - Solve the problem using engineering economic techniques

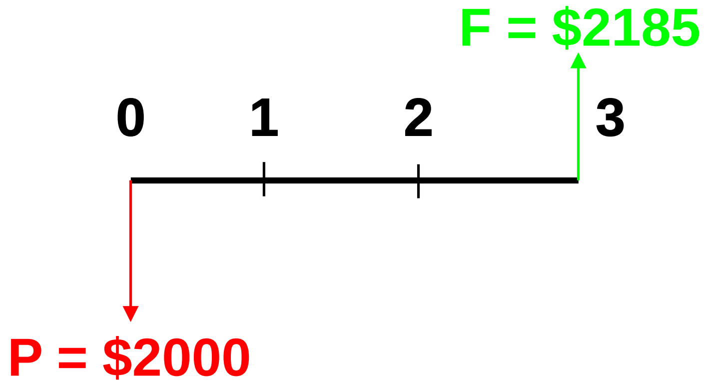


- Typical Cash Flow Diagrams:



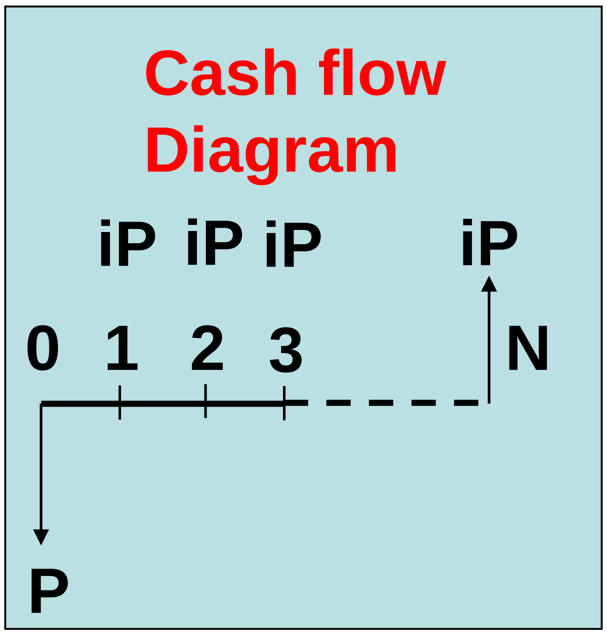
- Example 1:
  - Consider an initial investment of $2000
  - Investment Period is 3 years
  - Interest Rate is 3%
  - Future Worth is $2185
  - Draw the cash flow diagram

i = 0.03 or i = 3%
P = $2000
A = $0
F = $2185
N = 3 years



- Two methods:
  - – Simple Interest
    - Interest is earned only on principal amount during each period.
    - Interest earned during each interest period does not earn additional interest in the remaining periods.
  - – Compounded Interest
    - Interest is earned during each period based on the TOTAL amount at the end of the previous period.
    - TOTAL = original principal + accumulated interest

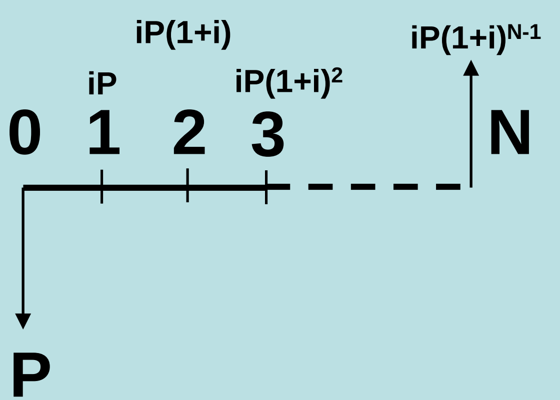
- Equation
  - **Simple interest: $F = P(1 + iN)$**
- Derivation



EOY	Bank Balance at End of Year (EOY)
0	P
1	$P + iP = P(1 + i)$
2	$P + iP + iP = P(1 + 2i)$
3	$P + iP + iP + iP = P(1 + 3i)$
.	.
.	.
.	.
N	$P + (iP)N = P(1 + iN)$

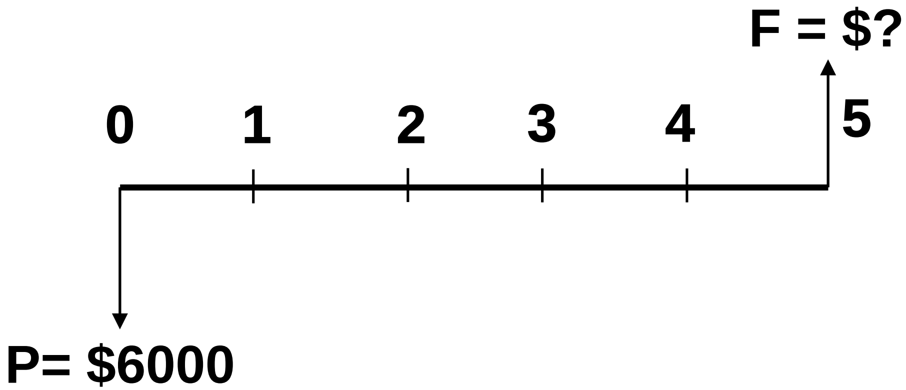
- Equation
  - **Compound interest: $F = P(1 + i)^N$**
- Derivation

Cash flow Diagram



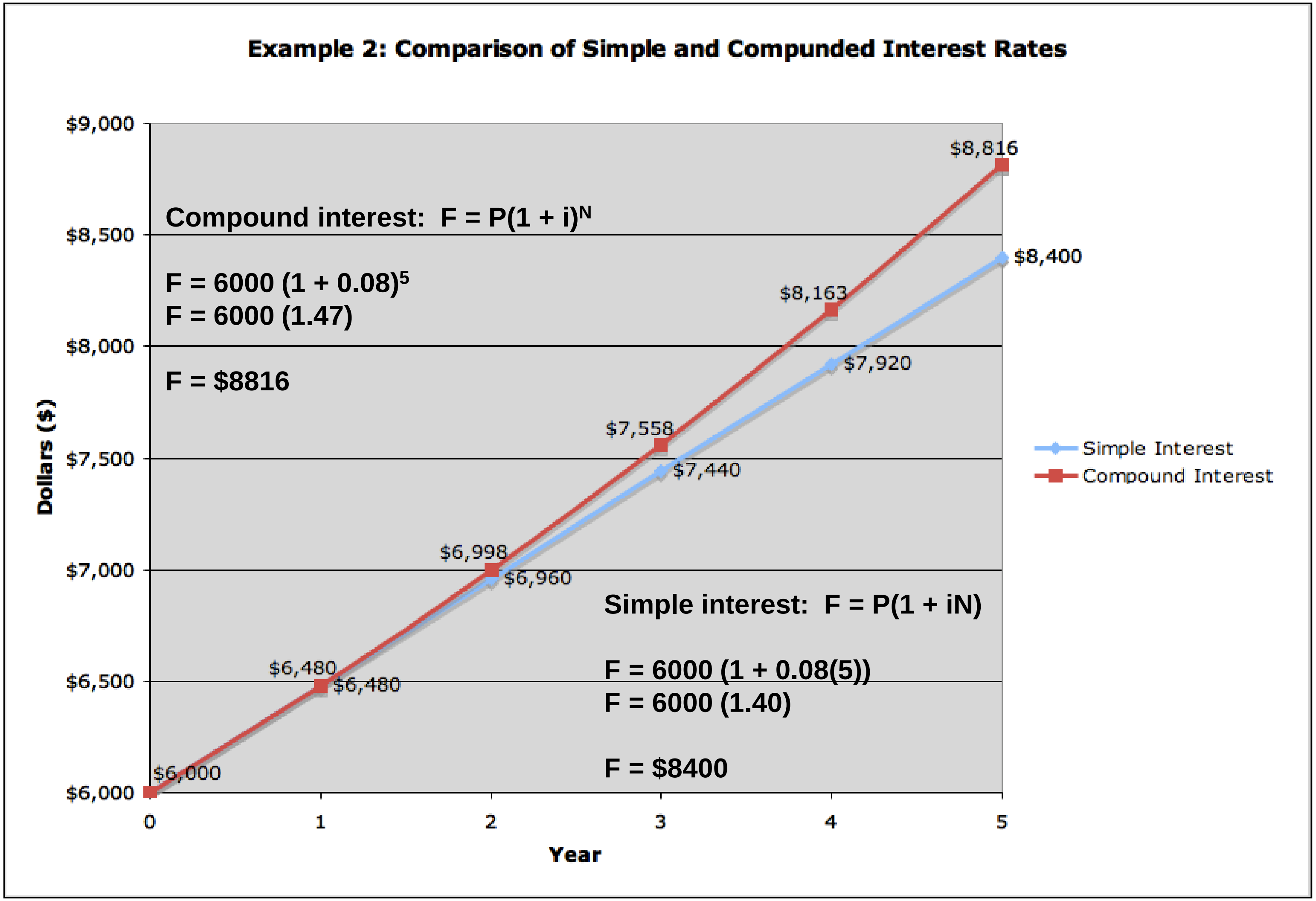
EOY	Bank Balance at End of Year (EOY)
0	P
1	$P + iP = P(1 + i)$
2	$P(1+i) + i[P(1+i)] = P(1 + i)^2$
3	$P(1+i)^2 + i[P(1+i)^2] = P(1 + i)^3$
.	.
.	.
.	.
N	$P(1+i)^{N-1} + i[P(1+i)^{N-1}] = P(1 + i)^N$

- Example 2:
  - $6000 is deposited in your bank account. What is its future value after 5 years assuming:
    - a. 8% simple interest earned annually
    - b. 8% interest compounded annually
  - Cash Flow Diagram

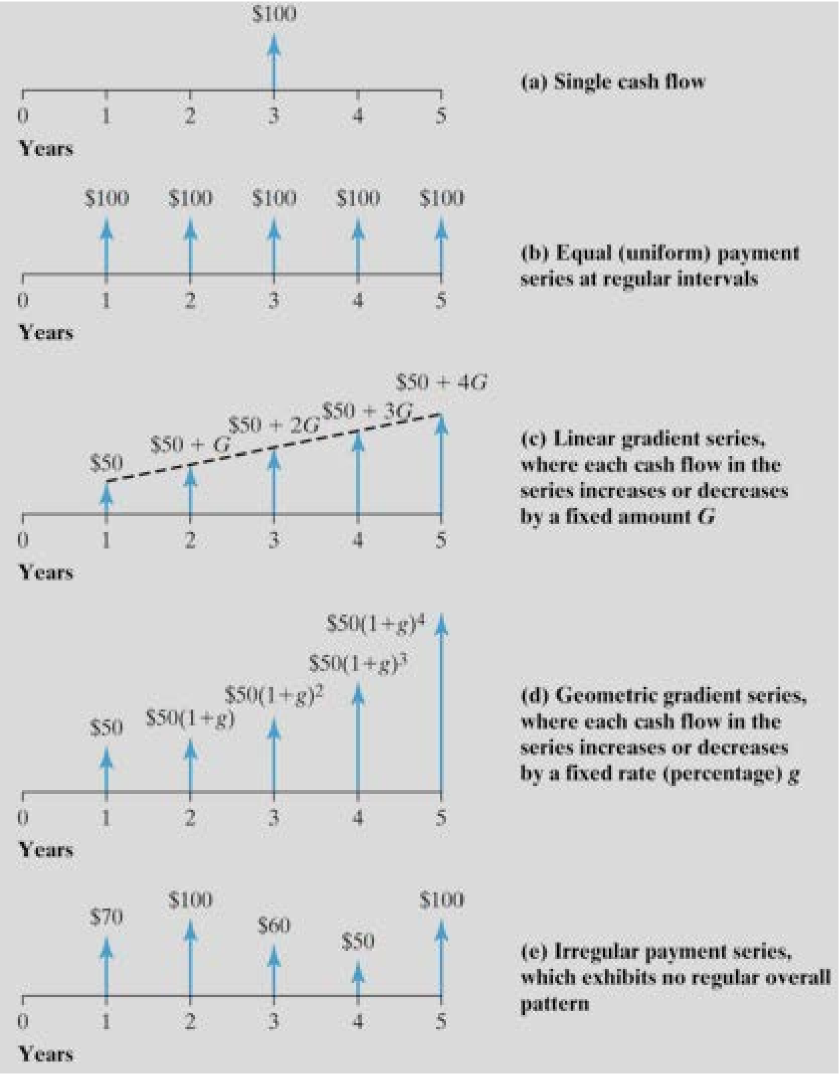


i = 0.08 or i = 8%
P = $6000
F = $?
N = 5 years

- Example 2 contd.

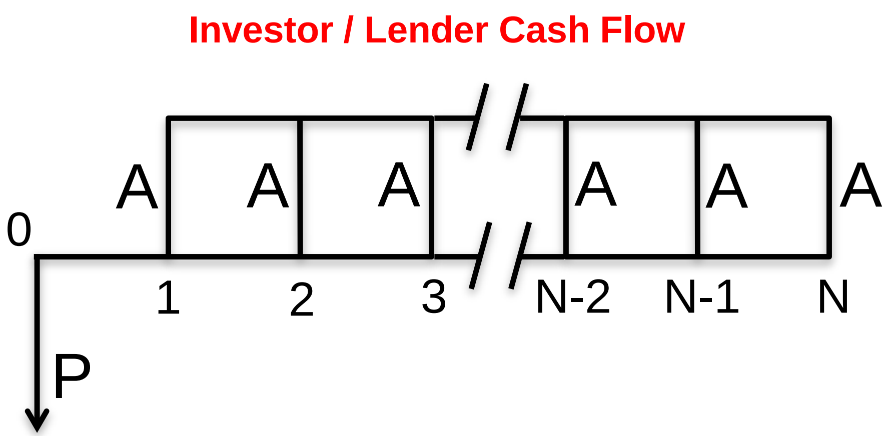


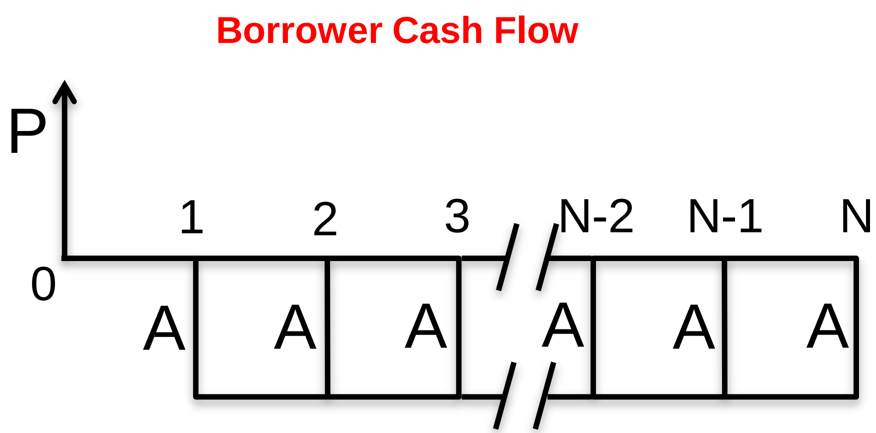
- Single Cash Flow
  - Equivalence relations: P and F
  - Single present or future cash flow
- Equal (Uniform) Series
  - Equivalence relations: P, F and A
  - Series of cash flows of equal amounts at regular intervals
- Linear (Arithmetic) Gradient Series
  - Equivalence relations: P, F and A
  - Fixed amount (G) increase or decrease at regular intervals
- Geometric Gradient Series
  - Equivalence relations: P, F and A
  - Fixed % rate (g) increase or decrease at regular intervals
- Irregular (Mixed) Series
  - Equivalence relations: P, F and A
  - No regular overall pattern (patterns may exist in portions)



Five Types of Cash Flows (Chan S. Park, Figure 3.10)

- Equal (Uniform) Series Example:





Car Repayment Cash Flows (differing viewpoints between the lender and the borrower)

- Compound Amount Factor
  - Used to find the future worth (FW) of a present value
  - Find F given P, i and N
  - $F = P(1 + i)^N = P(F/P, i, N)$

Same as Compound Interest

Single Payment Compound Amount Factor (TABLE FACTOR)

  - Excel Formula = FV(i,N,A,P,Type)
    - Type = 0 (payments at end of period)
    - Type = 1 (payments at start of period)
    - For this class, assume Type = 0 unless otherwise specified

- # Example 3:
  - Apply figures from example 2 to the three different equations presented:

1. STANDARD

- $F = P(1 + i)^N$

  $F = 6000(1+0.08)^5 = 6000(1.47) = \$8816$

2. INTEREST TABLES

- $F = P(F/P, i, N)$

  $F = P(1.4693) = \$8816$

			Interest Table :		8.00	%
	Single Payment			Equal Payment Series		
	Compound Amount Factor	Present Worth Factor	Compound Amount Factor	Sinking Fund Factor	Present Worth Factor	Capit Recove Fact
N	(F/P, i, N)	(P/F, i, N)	(F/A, i, N)	(A/F, i, N)	(P/A, i, N)	(A/P, i
1	1.0800	0.9259	1.0000	1.0000	0.9259	1.080
2	1.1664	0.8573	2.0800	0.4808	1.7833	0.560
3	1.2597	0.7938	3.2464	0.3080	2.5771	0.388
4	1.3605	0.7350	4.5061	0.2219	3.3121	0.301
5	1.4693	0.6806	5.8666	0.1705	3.9927	0.250
6	1.5869	0.6302	7.3359	0.1363	4.6229	0.216

3. EXCEL

- $F = FV(i,N,A,P,Type)$

  $F = FV(8\%,5,0,-6000,0) = \$8816$

- Present Worth Factor
  - Used to find the present worth (PW) of a future value.
  - Find P given F, i and N.
  - Opposite of compounding
  - Known as "discounting".
  - $P = F(1+i)^{-N} = F(P/F, i, N)$

From Compound Interest Formula

Single Payment Discount Amount Factor (TABLE FACTOR)

  - Excel Formula = PV(i,N,A,F,Type)
    - Type assumptions as before

- Present-Worth Factor
  - Problem 1:
    - For a value of $10,000 received in 8 years, at an annual rate of 7%, what is the present worth?
    - How would you solve for i given P, F and N?
    - How would you solve for N given P, F and i?
  - Deliverables:
    - Solve each part using 3 methods:
      - Method 1: Basic equation.
      - Method 2: Equation incorporating the economic tables at the back of Chan S. Park.
      - Method 3: Use Excel

- Compound-Amount Factor
  - Used to find the future worth (FW) of an annuity
  - Find F given A, i and N
  - $F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i, N)$

Equal (Uniform) Payment Series Compound Amount Factor

(TABLE FACTOR)

  - Excel Formula = FV(i,N,A,P,Type)
    - Type = 0 (payments at end of period)
    - Type = 1 (payments at start of period)
    - For this class, assume Type = 0 unless otherwise specified

- Sinking-Fund Factor
  - Used to find the annual worth (AW) of a future value.
  - Find A given F, i and N
  - Sinking Fund:
    - *Interest bearing account into which a fixed sum is deposited each interest period.* (Chan S. Park)
    - Uses
      - Replacing fixed assets
      - Retiring corporate bonds

Equal (Uniform) Payment Series Sinking-Fund Factor

(TABLE FACTOR)

  - $A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$
  - Excel Formula: A = PMT(i,N,P,F,Type)
    - Type = 0 (payments at end of period)
    - Type = 1 (payments at start of period)
    - For this class, assume Type = 0 unless otherwise specified

Capital Recovery (Annuity) Factor

- Used to find the annual worth (AW) of a present value.
- Find A given P, i and N.
- Capital Recovery Factor:
  - *Used to determine the revenue requirements needed to address the upfront capital capital costs for projects.* (Chan S. Park)
- Annuity
  - *A level stream of cash flows for a fixed period of time.* (Chan S. Park)
- $$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P,i,N)$$

Equal (Uniform) Payment Series Capital Recovery Factor

(TABLE FACTOR)

- Excel Formula: A = PMT(i,N,P,F,Type)
  - Type = 0 (payments at end of period)
  - Type = 1 (payments at start of period)
  - For this class, assume Type = 0 unless otherwise specified

- # Present-Worth Factor
  - Used to find the present worth (PW) of an annuity.
  - Find A given P, i and N.
  - Uses:
    - *Used to determine the what should be invested now in order to withdraw A dollars at the end of each of the next N periods.* (Chan S. Park)
  - $P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A, i, N)$

Equal (Uniform) Payment Series Present Worth Factor

(TABLE FACTOR)

  - Excel Formula: A = PMT(i,N,P,F,Type)
    - Type = 0 (payments at end of period)
    - Type = 1 (payments at start of period)
    - For this class, assume Type = 0 unless otherwise specified

- Present-Worth Factor: Linear Gradient
  - Used to find the present worth (PW) for a specified gradient amount.
  - Find P given G, i and N.
  - Uses:
    - *Used to determine the what should be invested now in order to withdraw (N-1)G dollars at the end of each of the next N periods.* (Chan S. Park)
  - $$P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right] = G(P/G, i, N)$$

Gradient Series Gradient Present Worth Factor

(TABLE FACTOR)

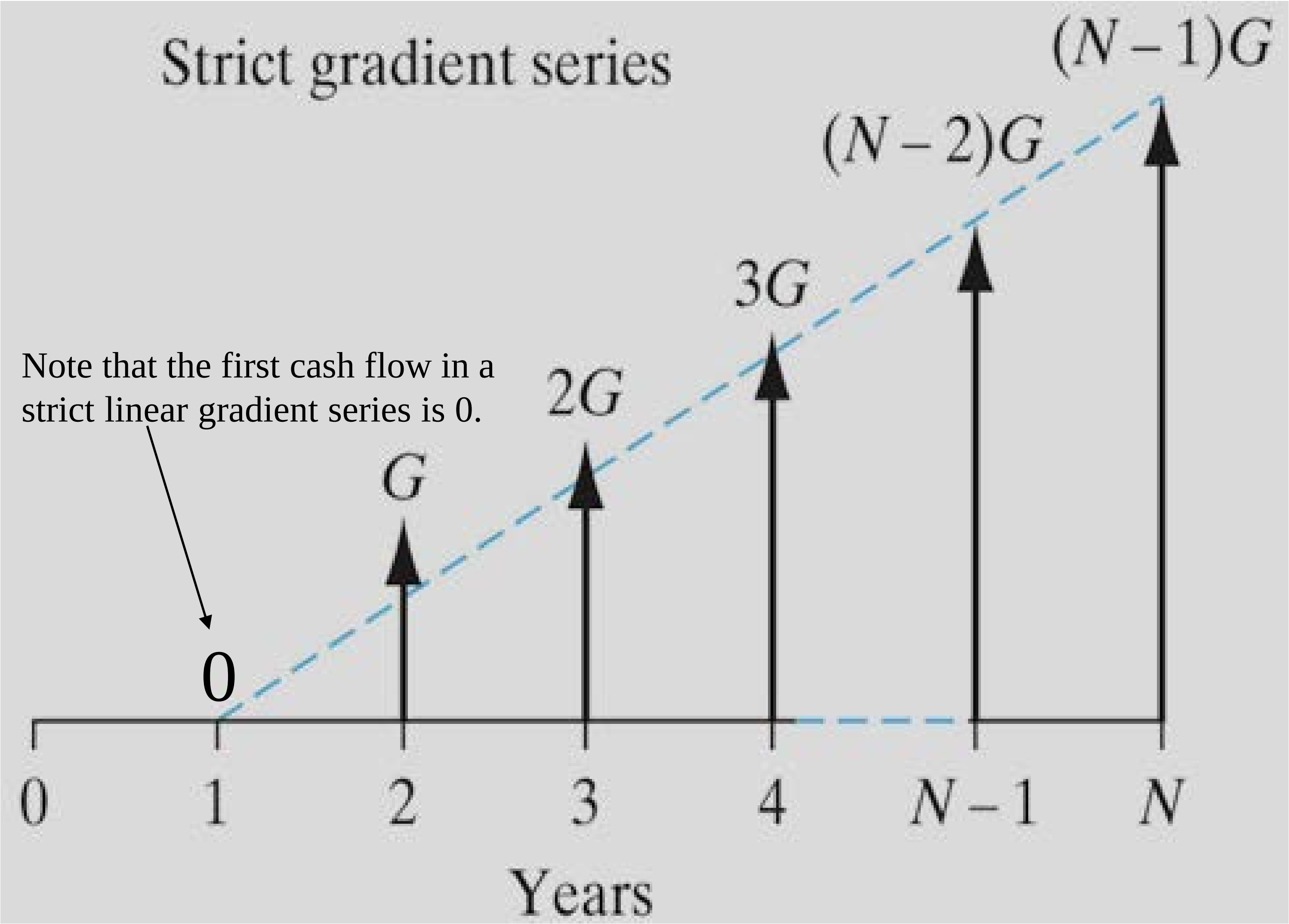
- Gradient-to-Equal-Payment Series Conversion Factor
  - Used to find the annual worth (AW) for a specified gradient amount.
  - Find A given G, i and N.
  - Uses:
    - *Used to determine an equal payment series equivalent to the gradient series.* (Chan S. Park)
  - $A = G\left[\frac{(1+i)^N - iN - 1}{i\left[(1+i)^N - 1\right]}\right] = G(A/G, i, N)$

Gradient Series Gradient Uniform Series Factor

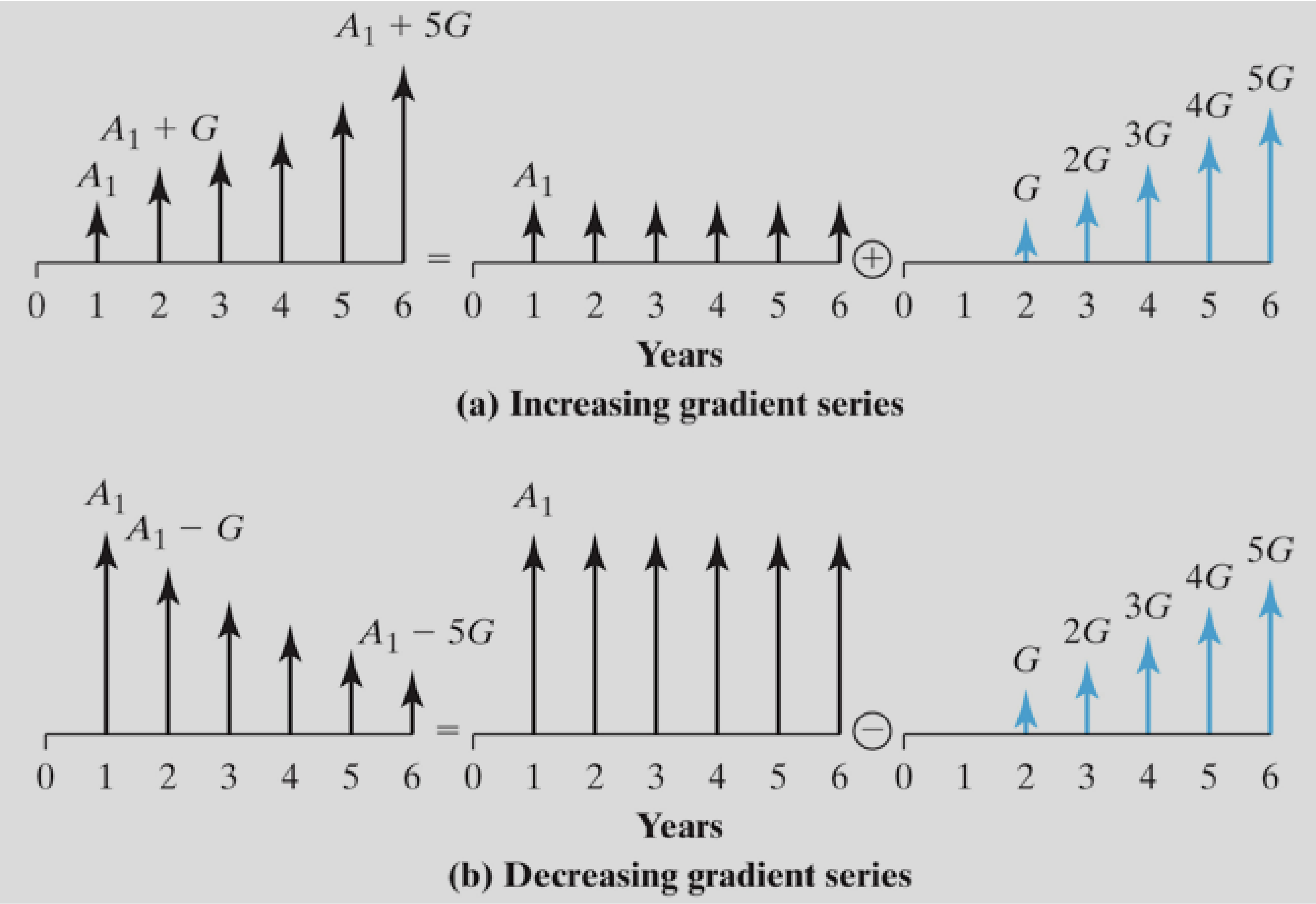
(TABLE FACTOR)

- Types of linear gradient series:
  - Strict gradient series
    - Does not correspond to the form that most engineering economic problems take (Chan S. Park)
  - Composite series
    - Splits the problem into two sets of cash flows -

      1. Equal (uniform) series
      2. Strict gradient series



Strict Gradient Series (Chan S. Park, Figure 3.27)



Composite Series (Chan S. Park, Figure 3.28)

- Present-Worth Factor: Geometric Gradient
  - Find P given A, g, i and N.
  - Geometric Growth:
    - *The year-over-year growth rate of an investment over a specified period of time.* (Chan S. Park)
  - Compound Growth:
    - *Cash flows that increase over time by a constant percentage (g, Geometric Gradient) NOT by a constant amount.* (Chan S. Park)
  - e.g. price changes caused by inflation

$$P = \begin{cases} A_1 \boxed{\left[\dfrac{1-(1+g)^N (1+i)^{-N}}{i-g} \right]} & i \neq g \\ A_1 \boxed{\left[\dfrac{N}{1+i} \right]} & i = g \end{cases}$$

(P/A$_1$, g, i, N)

  - **P = A$_1$(P/A$_1$, g, i, N)**
  - There are NO tables for the geometric gradient.

- Present-Worth
  - – For an uneven series, the problem must be decomposed into single payments:
    - Calculate the present value of each individual payment
    - Sum the results
- Future-Worth
  - – Use the present worth and find its equivalent future worth [F = P(F/P,i,N)]
- Annual-Worth
  - – Use the present worth and find its equivalent annual worth [A = P(A/P,i,N)]

- Nominal Interest Rate:
  - Also known as *"annual percentage rate (APR)"*
  - Definition: (Chan S. Park)
    - *Yearly cost of a loan including interest, insurance, and the origination fee, expressed as a percentage.*
  - e.g.
    - 18% APR, compounded monthly = 1.5% per month
    - 2.0% per month = 24% APR
  - Does NOT represent the amount of interest earned in a year.
  - r = nominal interest rate
  - Excel Formula:
    - NOMINAL(effect_rate, npery)
    - effect_rate = effective annual interest rate, i_a
    - npery = # compounding periods per year, M

- Periodic Interest Rate:
  - The interest rate per compounding period is called a periodic interest rate (or periodic rate).
  - Definition: (Chan S. Park)
    - *The interest the lender will charge on the amount you borrow. If the lender also charges fees, the periodic interest rate will not be the true interest rate.*
  - $i_m = \dfrac{r}{M}$ where,

    i_m = periodic interest rate

    r = nominal interest rate

    M = # of compounding (interest) periods per year

- Effective Interest Rate:
  - Also known as *"annual percentage yield (APY)"*
  - Represents the interest earned in a year.
  - Definition: (Chan S. Park)
    - *Rate actually earned or paid in one year, taking into account the affect of compounding.*
    - $i_a = \left(1 + \frac{r}{M}\right)^M - 1$ where,

      i_a = effective annual interest rate

      r = nominal interest rate

      M = # of compounding (interest) periods per year
    - Excel Formula:
      - EFFECT(nominal_rate, npery)
      - nominal_rate = nominal interest rate, r
      - npery = # compounding periods per year, M

- Continuous Compounding
  - Definition:
    - *The process of calculating interest and adding it to existing principal and interest at infinitely short time intervals.* (Chan S. Park)
    - $i_a = e^r - 1$, where

      r = nominal interest rate

	Compounding Period Description and Number of Periods per Year									
	Annually		Semi-annually		Quarterly		Monthly		Daily	
	1		2		4		12		365	
r	i_a	i_m	i_a	i_m	i_a	i_m	i_a	i_m	i_a	i_m
4.00%	4.00%	4.00%	4.04%	2.00%	4.06%	1.00%	4.07%	0.33%	4.08%	0.01%
5.00%	5.00%	5.00%	5.06%	2.50%	5.09%	1.25%	5.12%	0.42%	5.13%	0.01%
6.00%	6.00%	6.00%	6.09%	3.00%	6.14%	1.50%	6.17%	0.50%	6.18%	0.02%
7.00%	7.00%	7.00%	7.12%	3.50%	7.19%	1.75%	7.23%	0.58%	7.25%	0.02%
8.00%	8.00%	8.00%	8.16%	4.00%	8.24%	2.00%	8.30%	0.67%	8.33%	0.02%
9.00%	9.00%	9.00%	9.20%	4.50%	9.31%	2.25%	9.38%	0.75%	9.42%	0.02%
10.00%	10.00%	10.00%	10.25%	5.00%	10.38%	2.50%	10.47%	0.83%	10.52%	0.03%
11.00%	11.00%	11.00%	11.30%	5.50%	11.46%	2.75%	11.57%	0.92%	11.63%	0.03%
12.00%	12.00%	12.00%	12.36%	6.00%	12.55%	3.00%	12.68%	1.00%	12.75%	0.03%

r = nominal interest rate
i_a = effective annual interest rate
i_m = periodic interest rate

- Economic Equivalence:
  - Definition: (Chan S. Park)
    - *The process of comparing two different cash amounts at different points in time.*
  - Can assess:
    - Single Payments
    - Series of Payments

- Economic Equivalence:
  - Guiding Principles (Chan S. Park)
    - Equivalence calculations made to compare alternatives require a common time basis.
    - Equivalence depends on interest rate.
    - Equivalence calculations may require the conversion of multiple payment cash flows to a single cash flow.
    - Equivalence is maintained regardless of point of view.

- Economic Equivalence:
  - Sample problems:
    - You want to deposit $1000 for 3 years. Is it better to have 5% simple interest or 4% compounded annually?
    - Suppose you borrow $5000 for a used car from your parents at 9% interest. When you graduate from Stevens in 2 years how much will you owe them?
    - I want to retire in 15 years and buy a motorcycle. Having studied the price increases for a Harley Davidson motorcycle, I estimate I'll need $25,000. If my mutual funds are paying 13%, how much will I need to deposit now so I can buy my bike?

- Types of unconventional equivalence calculations:
  - – Composite cash flows
    - Refer to examples 3.25 & 3.26 (Chan S. Park)
  - – Determining an interest rate to establish economic equivalence
    - Refer to example 3.27 (Chan S. Park)
    - Manual = Interpolate
    - Excel = Use the Goal Seek Function

- Compounding and Payment Period must be in the same order.
- Possible situations:
  - Payment Period = Compounding Period
  - Payment Period < Compounding Period
  - Payment Period > Compounding Period

- **Payment Period = Compounding Period.**
  - Identify # compounding periods per year, M
    - M = K (payment period)
    - M = CK, therefore, C = 1
  - Calculate effective interest rate per period (periodic interest rate),
    - $i = \dfrac{r}{M}$
  - Determine # of compounding periods,
    - N = M × (number of years)
  - Calculate PW, AW or FW using i and N

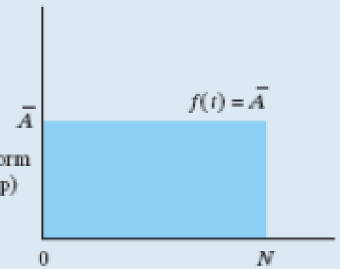
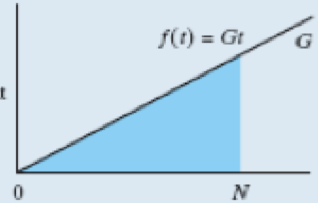
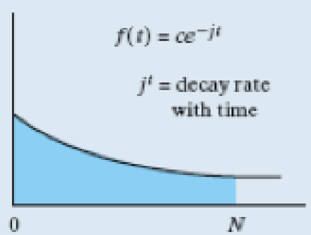
- Note:
  - The effective interest rate can be assessed per payment period (periodic interest rate).
  - $i = \left(1 + \frac{r}{M}\right)^{C} - 1 = \left(1 + \frac{r}{CK}\right)^{C} - 1$ where,

    M = number of interest periods per year

    C = number of interest periods per payment period

    K = Number of payment periods per year

- **Payment Period < 'OR' > Compounding Period.**
  - Identify # compounding periods per year (M), the number of payment periods per year (K), the number of interest periods per payment period (C).
  - Calculate effective interest rate per period (periodic interest rate),
    - Discrete: $$i = \left(1 + \frac{r}{M}\right)^{C} - 1$$
    - Continuous: $$i = e^{r/K} - 1$$
  - Determine # of compounding periods,
    - N = K × (number of years)
  - Calculate PW, AW or FW using i and N

Type of Cash Flow	Cash Flow Function	Parameters Find	Parameters Given	Algebraic Notation	Factor Notation
Uniform (step)		P	$\bar{A}$	$\bar{A}\left[\frac{e^{rN}-1}{re^{rN}}\right]$	$(P/\bar{A}, r, N)$
		$\bar{A}$	P	$P\left[\frac{re^{rN}}{e^{rN}-1}\right]$	$(\bar{A}/P, r, N)$
		F	$\bar{A}$	$\bar{A}\left[\frac{e^{rN}-1}{r}\right]$	$(F/\bar{A}, r, N)$
		$\bar{A}$	F	$F\left[\frac{r}{e^{rN}-1}\right]$	$(\bar{A}/P, r, N)$
Gradient (ramp)		P	G	$\frac{G}{r^2}(1 - e^{-rN}) - \frac{G}{r}(Ne^{-rN})$	
Decay		P	c, j	$\frac{c}{r+j}(1 - e^{-(r+j)N})$	

Summary of Interest Factors for Typical Continuous Cash Flows with Continuous Compounding (Chan S. Park, Table 4.2)

- Example 4:
  - Find the effective interest rate per quarter at a nominal interest rate of 8%,
    - a. compounded weekly.
      - 52 weeks/year
    - b. compounded daily.
      - 365 days/year

- Example 4:
  - a. Weekly Compounding
    - Given:
      - r = 8% per year
      - M = 52 weeks (compounding periods per year)
      - C = 52/4 = 13 periods per quarter
    - Find the effective rate, i

$$i = \left(1 + \frac{r}{M}\right)^{C} - 1 = \left(1 + \frac{8\%}{52}\right)^{13} - 1$$

$$i = 2.02\% \text{ per quarter}$$

- Example 4:
  - b. Daily Compounding
    - Given:
      - r = 8% per year
      - M = 365 days (compounding periods per year)
      - C = 365/4 = 91.25 periods per quarter
    - Find the effective rate, i

$$i = \left(1 + \frac{r}{M}\right)^{C} - 1 = \left(1 + \frac{8\%}{365}\right)^{91.25} - 1$$

$$i = 2.02\% \text{ per quarter}$$

- Example 5: (Chan S. Park, example 4.9)
  - You own a small pill bottle manufacturing company and generate \$200 cash each day. This daily cash flow is deposited into a special business account for 15 months. The account earns an interest rate of 6%. Compare the accumulated cash values at the end of 15 months, assuming
    - a. daily compounding
    - b. continuous compounding

- Example 5: (Chan S. Park, example 4.9)
  - a. Daily Compounding
    - Given:
      - A = \$200 per day
      - r = 6% per year
      - M = 365 (compounding periods per year)
      - N = 15 months = 455 days
    - Find: F

Payment Period =
Compounding Period

- $i = \frac{r}{M} = \frac{6\%}{365} = 0.01644\% \cdot per \cdot day$

$$F = A(F/A, i, N) = \$200(F/A, 0.01644\%, 455) = \$200(472.4095) = \$94,482$$

- Example 5: (Chan S. Park, example 4.9)
  - b. Continuous Compounding
    - Given:
      - $\bar{A}$ = \$200 × 365 = \$73,000 per year for N years
      - r = 6% per year, compounded continuously
      - N = 15 months = 1.25 years
    - Find: F
      - $F = \bar{A}\left[\frac{e^{rN}-1}{r}\right] = \$73{,}000\left[\frac{e^{0.06\times 1.25}-1}{0.06}\right] = \$73{,}000(1.298) = \$94{,}759$

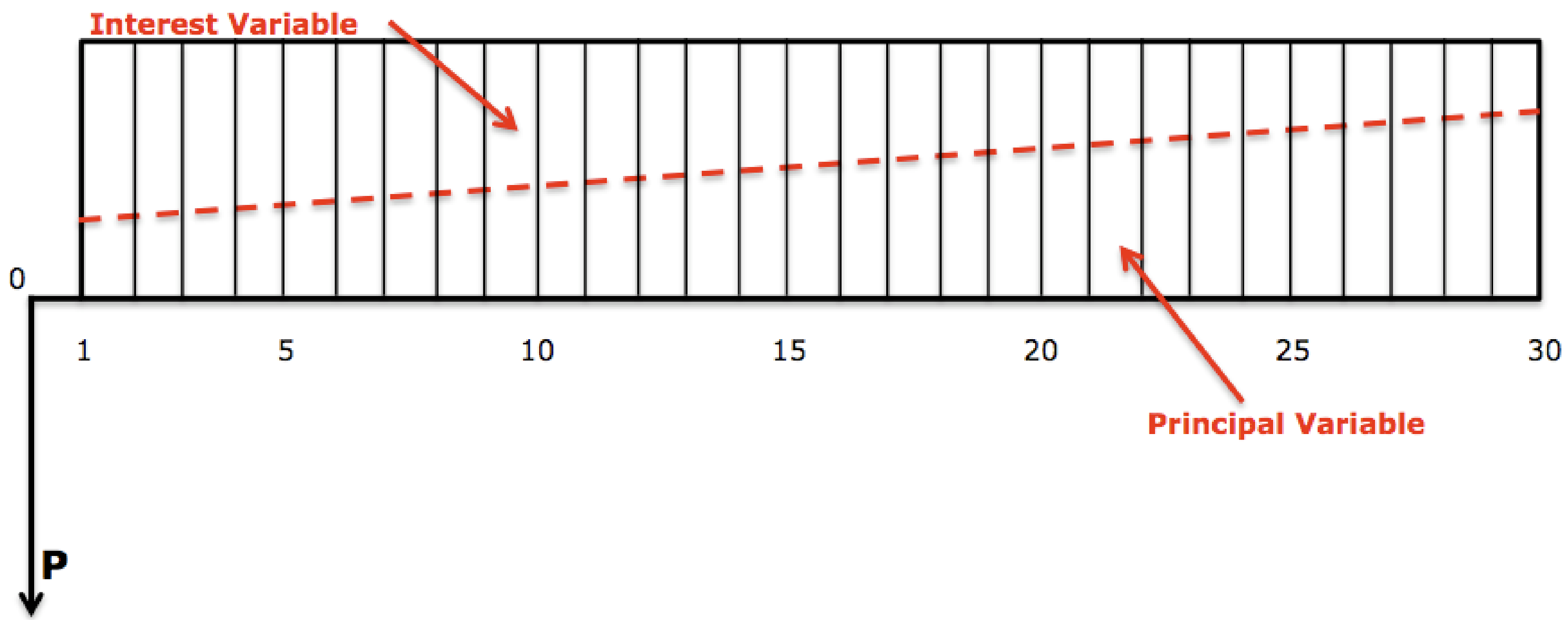
- Commercial Loans
- Loan versus Lease Financing
- Home Mortgage

- Two Types:
  - Amortized Loan
    - *A loan that is repaid in equal periodic amounts.*[1]
    - *A loan with scheduled periodic payments of both principal and interest.*[2]
  - Add-On Interest Loan
    - *A method of computing interest whereby interest charges are made for the entire principal amount for the entire term, regardless of any repayments of principal made.*[1]
    - Outside of the scope of this class.

1. Chan S. Park
2. "amortized loan" Investopedia.com. Investopedia ®. May 27, 2008 <www.investopedia.com/terms/a/amortized_loan.asp>

- Amortized Loan
  - Based on Compound Interest method.
  - Common in various types of commercial lending.
  - Calculations:
    - For a given P, i and N, calculate A (sum of principal and interest repayment)
    - Tabular method:
      - For each payment record the principal paid, the interest paid and the loan balance.
    - Remaining-Balance method:
      - B_n = remaining balance after n periods = A(P/A, i, N-n)
      - I_n = interest payment during period n = $B_{n-1} \times i$
        I_n = A(P/A, i, N-n+1) $\times$ i
      - P_n = interest payment during period n = A(P/F, i, N-n+1)

- Amortized Loan
  - Sample Cash Flow Diagram:



- Amortized Loans
  - Example 6: (Chan S. Park, example 4.12)
    - Suppose you secure a home improvement loan in the amount of $5000 from a local bank. The monthly payment is computed as follows:
      - Contract amount: $5000
      - Contract Period: 24 months
      - APR: 12%
      - Monthly Installments, A: $235.37
    - Show how A is calculated.
    - Using the tabular method find P_n, I_n and B_n for each n
    - Using the remaining-balance method find P_6, I_6, B_6

- Amortized Loans
  - Example 6: (Chan S. Park, example 4.12)
    - ***Calculate monthly installments, A:***

      A = P (A/P, i, N)

      Given: P = $5000; N = 24 months; r = 12%

      Need i (effective interest rate per payment period / periodic interest rate) and A

$$i = \frac{r}{M} = \frac{12\%}{12} = 1\% \cdot per \cdot month$$

Payment Period = Compounding Period

By Hand:

$$A = 5{,}000(A/P, 1\%, 24)$$

$$A = 5{,}000(0.0471)$$

$$A = \$235.50$$

Excel:

$$A = PMT(1\%, 24, 5000,, 0)$$

$$A = \$235.37$$

- Amortized Loans
  - Example 6: (Chan S. Park, example 4.12)
    - Using the tabular method find P_n, I_n and B_n for each n
      - Best tool for this exercise ➔ EXCEL

Payment No.	Size of Payment	Principal Payment	Interest Payment	Loan Balance
1	$235.37	$185.37	$50.00	$4,814.63
2	235.37	187.22	48.15	4,627.41

Monthly installments, 'A_n' as calculated, $235.37

$P_n = A_n - I_n$

$I_n = i \times$ balance at end of previous period

$I_1 = 0.01 \times \$5000 = \$50$

$I_2 = 0.01 \times \$4814.63 = \$48.15$

$B_n = B_{n-1} - P_n$

Payment No.	Size of Payment	Principal Payment	Interest Payment	Loan Balance
1	$235.37	$185.37	$50.00	$4,814.63
2	235.37	187.22	48.15	4,627.41
3	235.37	189.09	46.27	4,438.32
4	235.37	190.98	44.38	4,247.33
5	235.37	192.89	42.47	4,054.44
6	235.37	194.83	40.54	3,859.62
7	235.37	196.77	38.60	3,662.85
8	235.37	198.74	36.63	3,464.11
9	235.37	200.73	34.64	3,263.38
10	235.37	202.73	32.63	3,060.65
11	235.37	204.76	30.61	2,855.89
12	235.37	206.81	28.56	2,649.08
13	235.37	208.88	26.49	2,440.20
14	235.37	210.97	24.40	2,229.24
15	235.37	213.08	22.29	2,016.16
16	235.37	215.21	20.16	1,800.96
17	235.37	217.36	18.01	1,583.60
18	235.37	219.53	15.84	1,364.07
19	235.37	221.73	13.64	1,142.34
20	235.37	223.94	11.42	918.40
21	235.37	226.18	9.18	692.21
22	235.37	228.45	6.92	463.77
23	235.37	230.73	4.64	233.04
24	235.37	233.04	2.33	0.00

Creating a loan repayment schedule with excel (Chan S. Park, Table 4.3)

- Amortized Loans
  - Example 6: (Chan S. Park, example 4.12)
    - Remaining-balance method: find P_6, I_6, B_6

$$P_6 = 235.37(P/F,0.01,24-6+1) = 235.37(P/F,0.01,19) = 235.37(0.8277) = \$195.23$$

$$Excel: P_6 = PV(1\%,19,,235.37,0) = \$194.83$$

$$I_6 = 235.37(P/A,0.01,24-6+1)(0.01) = 235.37(17.2260)(0.01) = \$40.54$$

$$B_6 = 235.37(P/A,0.01,24-6) = 235.37(16.3983) = \$3859.67$$

$$Excel: B_6 = PV(1\%,18,235.37,,0) = \$3859.66$$

- Add-On Interest Loan
  - Based on Simple Interest method.
  - Common in financing appliances and furniture.
  - Method:

Simple interest

$$Total \cdot Add - On \cdot Interest = P(i)(N)$$

$$\Pr incipal + Total \cdot Add - On \cdot Interest = P + P(i)(N) = P(1 + iN)$$

$$Monthly \cdot Installments : A = \frac{P(1 + iN)}{12N}$$

- Example 7:
  - **Lexus**

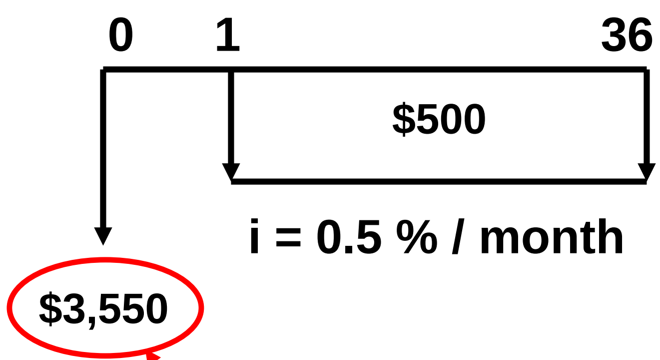
  - Better to lease for 36 months?

    OR

  - Purchase new through loans and keep for 36 months?

- ***DATA:***
  - *MSRP $40,000*
  - *Purchase Price $38,000*
  - *Residual Value at year 3, $20,000*
  - *Lease Term 36 months*
  - *Mileage Allowance 36,000*
  - *Down Payment, $3,000*
  - *Security Deposit $0*
  - *Monthly Leased Payment $500*
  - *Amount Due at Signing $500 plus tax & license of $50*
  - *MARR 0.5% / month*
  - *Interest loan, 1% / month*

- ***Option 1:***
  - ***Lease for 36 months***



Down Payment $3,000

Security Deposit $0

Amount Due at Signing $500 plus tax & license of $50

- Parameters:
  - P = $3,550
  - A = $500
  - i = 0.5%/month
  - N = 36 months
- Annual Worth, AW

$$AW = A - P(A/P,i,N)$$

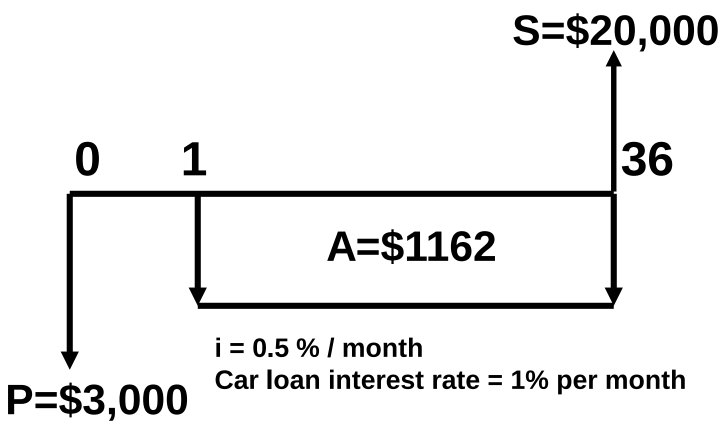
$$AW = -500 - 3,550(A/P,0.5\%,36)$$

$$AW = -500 - 3,550(0.0304)$$

$$AW = (\$607.92)$$

- Annual Cost = $608

- ***Option 2:***
  - ***Purchase new through loans and keep for 36 months***



  - Calculate A (Cost of loan):

$$A = P(A/P,i,N)$$
$$A = (38{,}000 - 3{,}000)(A/P,1\%,36)$$
$$A = 35{,}000(0.0332)$$
$$A = \$1162$$

- Parameters:
  - P = $3,000
  - A = $1162
  - S = F = $20,000
  - i = 0.5%/month
  - N = 36 months
- Annual Worth, AW

$$AW = A - P(A/P,i,N) + F(A/F,i,N)$$
$$AW = -1162 - 3{,}000(A/P,0.5\%,36)$$
$$+20{,}000(A/F,0.5\%,36)$$
$$AW = -1162 - 3{,}000(0.0304) + 20{,}000(0.0254)$$
$$AW = (\$745.20)$$

- Annual Cost = $745

- The right investment is a balance of three things:
  - Liquidity
    - How accessible is your money?
    - How quickly can your investment be converted to cash?
    - Short Term versus Long Term investment
  - Risk
    - How safe is your money?
    - Will you make or lose money?
    - State of the economy.
  - Return
    - How much profit do you expect from your investment?