# EM600 - Engineering Economics and Cost Analysis

***Lecture 10:*** *Cost Concepts and Capital Budgeting*

- References:
  - Park, Chan S. Contemporary Engineering Economics. New Jersey: Pearson Prentice Hall, 2006 (Chapter 8: 8.1 – 8.4; Chapter 15)
  - Ganguly, A. Engineering Economics Using Excel. New Jersey: SSE, 2008

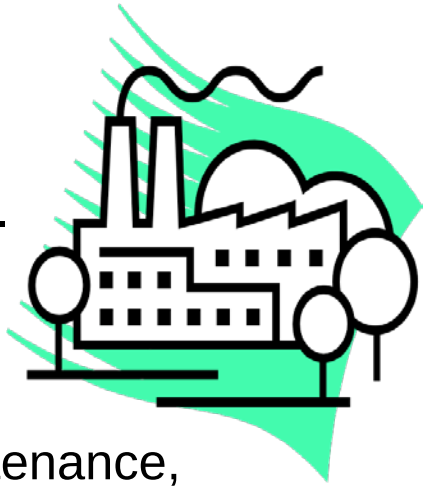
After completing this module you should understand the following:

- General cost concepts including the classification of costs / types of costs
- Introduction to capital budgeting
- Cost of capital
- Choice of MARR
- Capital budgeting decisions.

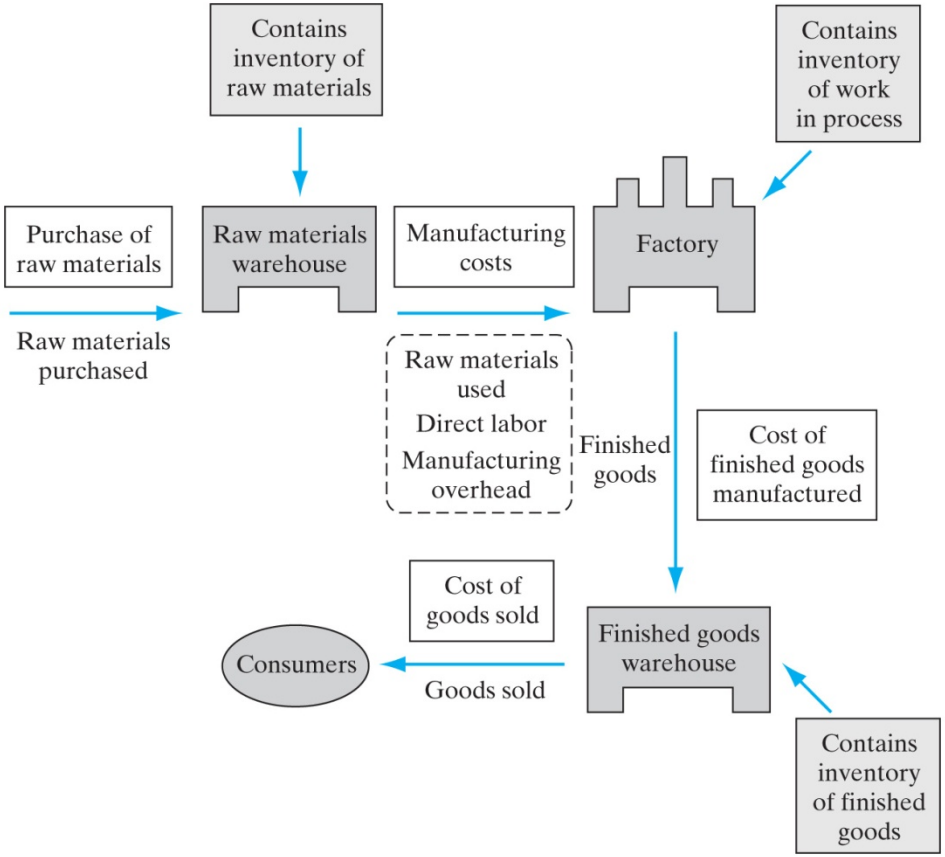
# PART 1

*General Cost Concepts*

- General Cost Terms:
  - Manufacturing Costs:
    - Direct Raw Materials
      - Any material used in the manufacture of a product that can be traced to it.
      - e.g. the active ingredient in a headache tablet is a direct raw material.
    - Direct Labor
      - Cost associated with any direct labor that goes into the manufacture of a final product.
      - e.g. the operators in the manufacture of headache tablets.
    - Manufacturing Overhead
      - All other costs, e.g. indirect materials, maintenance, energy, . . . etc
      - Excludes any direct raw material and labor costs.



- General Cost Terms:
  - Manufacturing Costs:



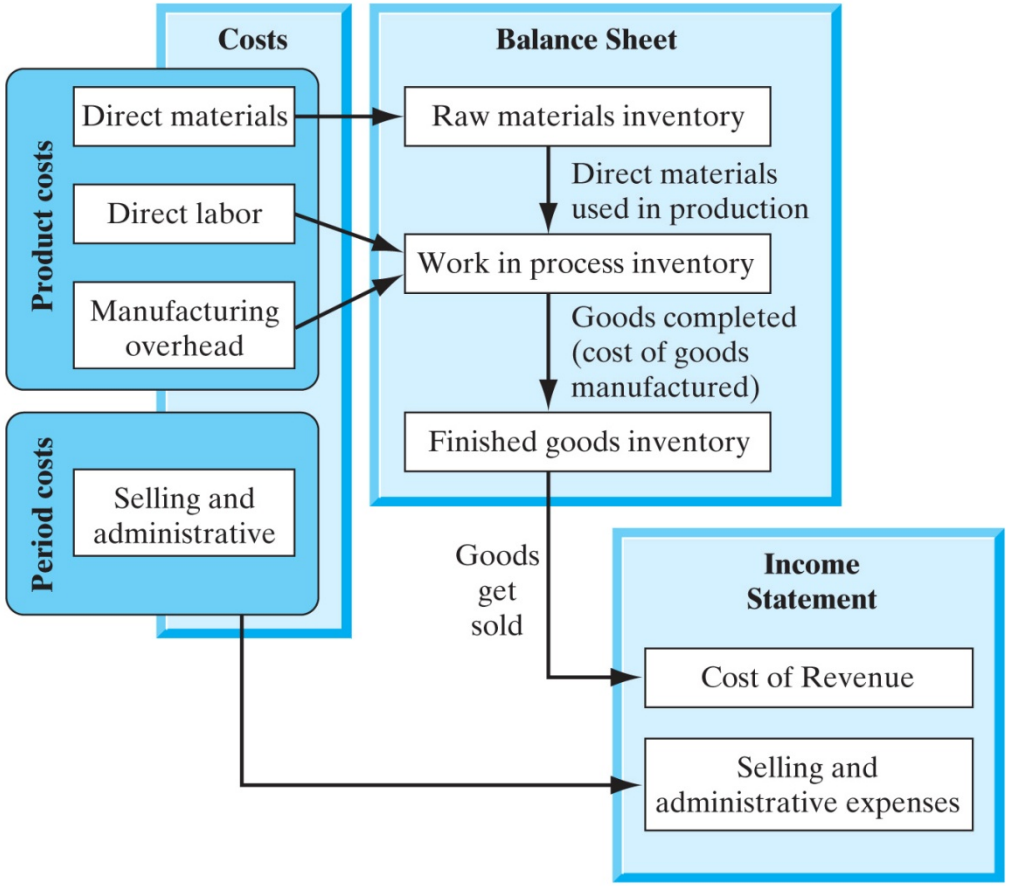

***Various types of manufacturing costs incurred by a manufacturer (Chan S. Park, Figure 8.1)***

- General Cost Terms:
  - Non-manufacturing Costs:
    - Overhead
      - Associated with administrative functions.
    - Marketing
      - All executive, organizational and clerical costs associated with sales activities.
      - e.g. travel, advertising, . . . etc
    - Administrative functions
      - Costs associated with the general management of a company.
      - e.g. public relations, secretarial support, . . .etc

- Classifying Costs for Financial Statements:
  - **Matching Concept:** The costs incurred to generate particular revenue should be recognized as expenses in the same period that the revenue is recognized.
  - **Period Costs:** Those costs that are matched against revenues on a time period basis.
  - **Product Costs:** Those costs that are matched against revenues on a product basis i.e. the costs involved in the purchase or manufacture of goods.

- Classifying Costs for Financial Statements:
  - Examples:
    - Period Costs:
      - General and administrative expenses
      - Marketing expenses
      - Insurance premiums
      - Income taxes
      - Nonmanufacturing costs
    - Product Costs:
      - Direct material costs
      - Direct labor costs
      - Manufacturing overhead

- Classifying Costs for Financial Statements:




***Cost flows and classifications in a manufacturing company (Chan S. Park, Figure 8.3)***

- Cost Classification for Predicting Cost Behavior:
  - Volume index
    - e.g. direct labor hours used, number of kW hours generated, . . . Etc
    - How do costs change in response to changes in the volume index?

- Cost Classification for Predicting Cost Behavior:
  - Cost behaviors
    - Fixed costs
      - A cost that remains constant regardless of any change in a company's activity.
    - Variable costs
      - A cost that changes proportionally to changes in a company's activity, e.g. changing levels of production or sales.
    - Mixed costs
      - Combination of fixed and variable e.g. depreciation
      - Costs are fixed to a set level and vary once this level is exceeded.
    - Average unit cost
      - Activity cost per unit basis
      - Fixed, variable or mixed costs

- Cost Classification for Predicting Cost Behavior:
  - Average unit cost:
    - You have 5000 units to produce.
    - Total labor cost = $40,000
    - Total material cost = $45,000
    - Total overhead cost = $25,000
    - Total fixed cost = $40,000
    - What is the average cost per unit?

$$\text{Total Cost} = \$40{,}000 + \$45{,}000 + \$25{,}000 + \$40{,}000$$

$$\text{Total Cost} = \$150{,}000$$

$$\text{Average Unit Cost} = \frac{\text{Total Cost}}{\#\ \text{Units}} = \frac{\$150{,}000}{5{,}000} = \$30$$

- Future Costs for Business Decisions:
  - Differential (Incremental) Cost and Revenue
    - Method Changes
    - Operations Planning
    - Make-or-Buy Decision
  - Opportunity Cost
  - Sunk Costs
  - Marginal Cost

- Future Costs for Business Decisions:
  - Differential (Incremental) Cost and Revenue
    - Definition: (Chan S. Park)
      - *Incremental Cost: Costs that represent the differences in total costs, which results from selecting one alternative instead of another.*
    - Cost behavior: (Chan S. Park)
      - *Incremental Cost: Increase or decrease with the overall change that a company experiences by producing one additional unit of good.*

- Future Costs for Business Decisions:
  - Differential (Incremental) Cost and Revenue
    - Method Changes
      - The new alternative may be a new method for completing an activity.
      - A cost comparison is required between the existing and the proposed methods.
      - Method changes typically involve an improvement in efficiency e.g. a quicker method, a method that improves the product yield, a reduction in energy, . . . etc

- Future Costs for Business Decisions:
  - Differential (Incremental) Cost and Revenue
    - Operations Planning
      - Demands can vary from high to low over the course of a year.
      - Variation in demand means the shift operating schedule may require adjustments
        - » High demand may require overtime or the introduction of an additional shift for example.
        - » Low demand may require continued operation at low demand levels (inefficient use of resources available) or temporary shutdown for example.

- Future Costs for Business Decisions:
  - Differential (Incremental) Cost and Revenue
    - Make-or-Buy Decision
      - Make: Firm uses internal resources to perform some activities needed to provide products and services.
      - Buy: Firm uses external resources to perform other activities needed to provide products and services.
      - **Question:** Should a function be outsourced or should the company perform the function in-house?
      - Outsourcing examples include customer service activities, waste and energy management activities, . . . etc
      - Opportunity costs are typically considered with make-or-buy decisions
        - » Decision involves more than one alternative.
        - » An opportunity cost is the potential benefit that is given up as you seek an alternative course of action.

- Future Costs for Business Decisions:
  - Opportunity Cost
    - Definition: (Chan S. Park)
      - *The potential benefit that is given up as you seek an alternative course of action.*
    - Important consideration in the decision making process.
    - Is not reported on a company's financial statements.
    - Example:
      - A person who invests $10,000 in a stock denies themselves the interest they could have earned by leaving the $10,000 dollars in a bank account instead. The opportunity cost of the decision to invest in stock is the value of the interest.[1]

1. "opportunity cost examples" Wikipedia.org. Wikipedia. July 31, 2008 <http://en.wikipedia.org/wiki/Opportunity_cost#Examples>.

- Future Costs for Business Decisions:
  - Sunk Costs
    - Definition: (Chan S. Park)
      - *Cost that has already been incurred by past actions.*
      - *Cost that has been incurred and cannot be reversed.*
    - Economic Implications:
      - Not relevant to decisions now or in the future.
      - Only variable costs should be considered.
    - Used by accountants, but not by economists.

- Future Costs for Business Decisions:
  - Marginal Cost
    - Definition (Chan S. Park)
      - *The cost associated with one additional unit of production.*
  - Marginal Revenue
    - Definition: (Chan S. Park)
      - *The revenue that can be obtained from selling one more unit of product.*
  - Marginal Contribution
    - Definition: (Chan S. Park)
      - *The difference between the unit sales price and the unit variable cost.*
    - Also know as: marginal income, unit contribution.

- Future Costs for Business Decisions:
  - Refer to Chan S. Park examples 8.4 and 8.7.

$P$ = Profit  $F$ = Fixed costs

$R$ = Revenue  $V$ = Variable cost per unit

$Q$ = Price per unit  $N$ = Number of units of volume

$$\text{Cost} = \text{Revenue}$$

$$F + NV = NQ$$

$$N_q = \frac{F}{Q - V} \quad \text{at break-even point}$$

$$NQ = \frac{FQ}{Q - V}$$

$$R = \frac{F}{1 - \frac{V}{Q}}$$

$Q - V$ = Marginal contribution per unit ($)

$1 - \frac{V}{Q}$ = Marginal contribution rate (MCR, %)

***(Chan S. Park, Lecture No30.ppt)***

# Product Mix Concepts - Example

| | Product A | Product B |
|---|---|---|
| | | |
| **Selling Price** | **$10** | **$12** |
| **Variable Costs** | **$5** | **$10** |
| **Fixed Costs** | **$2,000** | **$600** |

1. If these products are sold in the ratio of 3A: 4B, what is the break-even point?
2. If the product mix is changed to 5A: 5B, what would happen to the break-even point?
3. In order to maximize the profit, which product mix should be pushed?

# Product Mix Example (Solution)

1. If these products are sold in the ratio of 3A: 4B, what is the break-even point?

**Answer:** The product mix given in the problem is 3A : 4B. Therefore, B = 0.75A
Now, at Break-Even level, TOTAL REVENUE = TOTAL COST
Therefore, 10A + 12 * (0.75A) = 5A + 10 * (0.75A) + 2000 + 600
Now, solving for A, we get A = 400 units. Then B = 0.75A = 300 Units

2. If the product mix is changed to 5A: 5B, what would happen to the break-even point?

**Answer:** The new product mix given in the problem is 5A: 5B. Therefore, A = B
Now, at Break-Even level, TOTAL REVENUE = TOTAL COST
Therefore, 10A + 12A = 5A + 10A + 2000 + 600. Now, solving for A, we get A = B = 371.43 units

3. In order to maximize the profit, which product mix should be pushed?

**Answer:** In order to arrive at a decision regarding which product to push, we need to compare the Marginal Contribution per unit
of both products.
*MCA = Unit Selling Price - Variable Cost* = $ 1 0 - $ 5 = $5.00
*MCB = Unit Selling Price - Variable Cost* $ 1 2 - $ 10 = $2.00

***Since the MCR of Product A > MCR of Product B, Product A will have to be pushed*.**

# PART 2

## *Capital Budgeting*

*"the planning process used to determine a firm's long term investments" (Chan S. Park)*

- Methods of Financing:
  - Equity financing
    - Capital from either retained earnings or funds raised from an issuance of stock.
  - Debt Financing
    - Money raised through loans or by an issuance of bonds.
  - Capital Structure
    - Well managed firms establish a target capital structure and strive to maintain the debt ratio.

- Methods of Financing:
  - Equity financing
    - Use of retained earnings otherwise paid to stockholders in the form of dividends.
    - Issuance of stock:
      - Two types of securities:
        - » Common Stock
        - » Preferred Stock
      - Floatation costs:
        - » Costs associated with the issuance of new securities.
        - » e.g. bankers' fees, lawyers' fees, . . . etc

- Methods of Financing:
  - Debt financing
    - Money raised through loans or by an issuance of bonds.
    - Types:
      - Term Loans:
        - » Short term borrowing from financial institutions.
        - » Equal repayments over the term of a loan.
          - Interest repayments decrease.
          - Principal repayments increase.
      - Bond Financing:
        - » Sale of long term bonds.
        - » Principal is paid in a lump sum once the bond matures.
        - » Interest is paid annually.
        - » Flotation costs for bond issuance.

- Methods of Financing:
  - Capital Structure
    - The means by which a firm is financed.
    - Ratio of total debt to total capital.
      - Mixed financing:
        - » Retained earnings
        - » Borrow from a financial institution or issue stock
    - Also known as: debt ratio, or, debt-equity ratio.
    - Firms usually establish a target capital structure or a target debt ratio.
      - Trade-off between risk and return
      - The interest from debt is tax deductible
      - Financial flexibility

- Cost of Capital
  - Three main areas of focus:
    - Cost of equity.
    - Cost of debt.
    - Calculating the cost of capital.


- Cost of Equity
  - Definition: (Chan S. Park)
    - $i_e$ *: the minimum rate of return a firm must offer stockholders to compensate for waiting for their returns, and for bearing some risk.*
  - Involves an opportunity cost
    - If retained earnings are reinvested by the firm, dividends are not paid out to the stockholders.
    - Stockholders could have received the earnings as dividends and invested in other financial assets of their own choice.

- Cost of Equity
  - Based on Financing Sources:
    - Cost of retained earnings:

$$k_r = \frac{D_1}{P_0} + g \qquad \text{where,}$$

$k_r$ = rate of return required by shareholders; $D_1$ = first year dividend;

$P_0$ = market price; $g$ = annual growth rate

    - Flotation costs for common stock:

$$k_e = \frac{D_1}{P_0(1 - f_c)} + g \qquad \text{where,}$$

$k_e$ = cost of common equity; $D_1$ = first year dividend;

$P_0$ = market price; $f_c$ = flotation cost as a percentage of the stock price;

$g$ = annual growth rate

- Cost of Equity
  - Based on Financing Sources:
    - Flotation costs for preferred stock:

$$k_p = \frac{D^*}{P^*(1 - f_c)} \qquad \text{where,}$$

$k_p$ = cost of preferred equity; $D^*$ = fixed annual dividend;

$P^*$ = issuing price; $f_c$ = flotation cost as a percentage of the stock price

- Cost of Equity
  - Based on Financing Sources:
    - Cost of Equity:

$$i_e = \left(\frac{c_r}{c_e}\right)k_r + \left(\frac{c_c}{c_e}\right)k_e + \left(\frac{c_p}{c_e}\right)k_p \qquad \text{where,}$$

$i_e$ = weighted-average cost of equity for a project;

$c_r$ = amount of equity financed from retained earnings;

$c_c$ = amount of equity financed from issuing new stock;

$c_p$ = amount of equity financed from issuing preferred stock;

$k_r$ = rate of return required by shareholders (%);

$k_e$ = cost of common equity (%);

$k_p$ = cost of preferred equity (%);

Note:

$$c_r + c_c + c_p = c_e$$

- Cost of Equity
  - ***– Based on Financing Sources***
  - – Example 1: (Adapted fromChan S. Park, example 15.4)
    - Alpha Pharmaceutical, Inc. Corporation needs to raise $10 million for plant modernization.
    - Alpha's target capital structure calls for a debt ratio of 0.4.
    - $6 million needs to be financed from equity from the following sources:

| Sources | Amount | Fraction |
|---|---|---|
| Retained earnings | $1,000,000 | 0.1667 |
| New Common Stock | $4,000,000 | 0.6667 |
| Preferred Stock | $1,000,000 | 0.1667 |

- Cost of Equity
  - ***– Based on Financing Sources***
  - – Example 1: (Adapted from Chan S. Park, example 15.4)
    - Financial Data:

| | Common Stock | Preferred Stock |
|---|---|---|
| Market Price | $40 | $100 |
| Annual Cash Dividend | $5 (EOY 1) | $9 |
| Annual Cash Dividend Growth Rate | 8% | - |
| Issue Price | $40 | $95 |
| Flotation Costs | 12.4% | 6% (market price) |

- Cost of Equity
  - ***– Based on Financing Sources***
  - – Example 1: (Chan S. Park, example 15.4)
    - Deliverables:
      - – Calculate the cost of equity required to finance the plant modernization.
        - » Cost of retained earnings
        - » Cost of new common stock
        - » Cost of preferred stock
        - » Cost of equity

- Cost of Equity
  - ***– Based on Financing Sources***
  - – Example 1: (Chan S. Park, example 15.4)
    - Cost of retained earnings (common stock)

$$k_r = \frac{D_1}{P_0} + g \qquad \text{where,}$$

$k_r$ = rate of return required by shareholders; $D_1$ = first year dividend;

$P_0$ = market price; $g$ = annual growth rate

$$k_r = \frac{D_1}{P_0} + g = \frac{\$5}{\$40} + 8\% = 20.5\%$$

$k_r = 20.5\%$ rate of return required by shareholders where dividends are not paid out.

- Cost of Equity
  - ***– Based on Financing Sources***
  - – Example 1: (Chan S. Park, example 15.4)
    - Cost of new common stock

$$k_e = \frac{D_1}{P_0(1-f_c)} + g \qquad \text{where,}$$

$k_e$ = cost of common equity (%); $D_1$ = first year dividend;

$P_0$ = market price; $f_c$ = flotation cost as a percentage of the stock price;

$g$ = annual growth rate

$$k_e = \frac{D_1}{P_0(1-f_c)} + g = \frac{\$5}{\$40(1-12.4\%)} + 8\%$$

$k_e$ = 22.27% required rate of return associated with the cost of issuing new stock

- Cost of Equity
  - ***– Based on Financing Sources***
  - – Example 1: (Chan S. Park, example 15.4)
    - Cost of preferred stock

$$k_p = \frac{D^*}{P^*(1-f_c)} \qquad \text{where,}$$

$k_p$ = cost of preferred equity (%); $D^*$ = fixed annual dividend;

$P^*$ = issuing price; $f_c$ = flotation cost as a percentage of the stock price

$$k_p = \frac{D^*}{P^*(1-f_c)} = \frac{\$9}{\$95(1-6\%)}$$

$k_p$ = 10.08% required rate of return associated with the cost of issuing preferred stock

- Cost of Equity
  - ***– Based on Financing Sources***
  - – Example 1: (Chan S. Park, example 15.4)
    - Cost of equity

$$i_e = \left(\frac{c_r}{c_e}\right)k_r + \left(\frac{c_c}{c_e}\right)k_e + \left(\frac{c_p}{c_e}\right)k_p \qquad \text{where,}$$

$i_e$ = weighted-average cost of equity for a project;

$c_r$ = amount of equity financed from retained earnings;

$c_c$ = amount of equity financed from issuing new stock;

$c_p$ = amount of equity financed from issuing preferred stock;

$k_r$ = rate of return required by shareholders;

$k_e$ = cost of common equity

$k_p$ = cost of preferred equity;

- Cost of Equity
  - ***– Based on Financing Sources***
  - – Example 1: (Chan S. Park, example 15.4)
    - Cost of equity

$$i_e = \left(\frac{c_r}{c_e}\right)k_r + \left(\frac{c_c}{c_e}\right)k_e + \left(\frac{c_p}{c_e}\right)k_p$$

$$i_e = \left(\frac{\$1{,}000{,}000}{\$6{,}000{,}000}\right)20.5\% + \left(\frac{\$4{,}000{,}000}{\$6{,}000{,}000}\right)22.27\% + \left(\frac{\$1{,}000{,}000}{\$6{,}000{,}000}\right)10.08\%$$

$$i_e = (16.67\%)20.5\% + (66.67\%)22.27\% + (16.67\%)10.08\%$$

$i_e$ = weighted - average cost of equity for a project;

$i_e = 19.94\%$

**If Alpha finances the project entirely from its equity funds, the project must earn at least a 19.94% return on investment.**

- Cost of Equity
  - Based on Financial Markets
    - Key Definitions: (Chan S. Park)
      - Inflation Risk or Inflation Premium:
        - » *The possibility that the value of assets or income will decrease as inflation shrinks the purchasing power of a currency.*
      - Risk Premium:
        - » *The return in excess of the risk-free rate of return that an investment is expected to yield.*
      - Beta: ($\beta$)
        - » *A measure of the volatility or systemic risk, of a security or portfolio in comparison to the market as a whole.*

- Cost of Equity
  - Based on Financial Markets
    - Expected return on a risky asset has 3 sources:
      - Risk-free interest rate = compensation from the opportunity cost incurred in holding the asset
      - Inflation premium = compensation for the declining purchasing power of the investment over time
      - Risk premium = compensation for bearing risk

$$\begin{pmatrix}\text{Expected Return}\\ \text{on Risky Asset}\end{pmatrix} = \begin{pmatrix}\text{Risk - free}\\ \text{interest rate}\end{pmatrix} + \begin{pmatrix}\text{Inflation}\\ \text{premium}\end{pmatrix} + \begin{pmatrix}\text{Risk}\\ \text{premium}\end{pmatrix}$$

$$= \begin{pmatrix}\text{Interest rate on}\\ \text{government bond}\end{pmatrix} + \begin{pmatrix}\text{Risk}\\ \text{premium}\end{pmatrix}$$

- Cost of Equity
  - Based on Financial Markets
    - The cost of equity is the risk-free cost of debt (20-year U.S. Treasury Bills around 7%) plus a premium for taking a risk as to whether a return will be received.
    - The premium is the average return on the market (say 12.5%) less the risk-free cost of debt. This premium is multiplied by Beta, $\beta$, a measure of stock price volatility.
    - Beta quantifies risk and is an approximate measure of stock price volatility. It measures one firm's stock price compared (relative) with the market stock prices as a whole.
    - A number greater than one means that the stock is more volatile than the market on average; a number less than one means that the stock is less volatile than the market on average.

***(Chan S. Park, Lecture No61.ppt)***

- Cost of Equity
  - Based on Financial Markets:

$i_e = r_f + \beta\left[r_M - r_f\right]$ where,

$i_e$ = cost of equity for a project;

$r_f$ = risk free interest rate;

$r_M$ = market rate of return

Note:

$r_f$ is commonly referenced to the U.S. Treasury Bond yield, adjusted for inflation.

$r_M$ is commonly referenced to the average return on S&P 500 stock index funds, adjusted for inflation.

- Cost of Equity
  - ***Based on Financial Markets***
  - Example 2: (Chan S. Park, example 15.5)
    - Alpha Corporation needs to raise $10 million for plant modernization.
    - Alpha's target capital structure calls for a debt ratio of 0.4.
    - $6 million needs to be financed from equity from the financial market.
    - Financial Data:

| | Value | Comment |
|---|---|---|
| Beta, $\beta$ | 1.99 | >1; firm is more risky than market average |
| Risk Free Interest Rate | 6% | Adjusted to reflect inflation in the economy |
| Average Market Return | 13% | |

- Cost of Equity
  - ***Based on Financial Markets***
  - Example 2: (Chan S. Park, example 15.5)
    - Deliverables:
      - Calculate the cost of equity required to finance the plant modernization.

- Cost of Equity
  - ***– Based on Financial Markets***
  - – Example 2: (Chan S. Park, example 15.5)
    - Cost of Equity:

$$i_e = r_f + \beta\left[r_M - r_f\right] \qquad \text{where,}$$

$i_e$ = cost of equity for a project;

$r_f$ = risk free interest rate;

$r_M$ = market rate of return

$$i_e = r_f + \beta\left[r_M - r_f\right] = 6\% + 1.99\left[13\% - 6\%\right] = 19.93\%$$

**If Alpha finances the project entirely from its equity funds, the project must earn at least a 19.93% return on investment.**

- Cost of Debt
  - Definition: (Chan S. Park)
    - *The effective rate a company pays on its current debt.*

$$i_d = \left(\frac{c_s}{c_d}\right) k_s (1 - t_m) + \left(\frac{c_b}{c_d}\right) k_b (1 - t_m) \qquad \text{where,}$$

$i_d$ = after-tax cost of debt for a project;

$c_s$ = amount of short-term loan;

$c_b$ = amount of bond financing;

$k_s$ = before tax interest on the term loan;

$k_b$ = before tax interest on the bond;

$t_m$ = firm's marginal tax rate

Note :

$c_s + c_b = c_d$

- Cost of Debt
  - Example 3: (Chan S. Park, example 15.6)
    - Alpha Corporation needs to raise $10 million for plant modernization.
    - Alpha's target capital structure calls for a debt ratio of 0.4.
    - $4 million needs to be financed from a term loan and issuing 20-year par bonds.
    - Financial Data:

| Source | Amount | Fraction | Interest Rate | Flotation Cost |
|---|---|---|---|---|
| Term Loan | $1,000,000 | 0.25 | 12.00% | |
| Bonds | $3,000,000 | 0.75 | 10.00% | 6.00% |

      - Bond Issue Price = $940 (flotation costs deducted)
      - Marginal Tax Rate = 38%

- Cost of Debt
  - Example 3: (Chan S. Park, example 15.6)
    - Deliverables:
      - Calculate the cost of debt required to finance the plant modernization.

- Cost of Debt
  - Example 3: (Chan S. Park, example 15.6)
    - Cost of Debt:

$$i_d = \left(\frac{c_s}{c_d}\right) k_s (1 - t_m) + \left(\frac{c_b}{c_d}\right) k_b (1 - t_m) \qquad \text{where,}$$

$i_d$ = after-tax cost of debt for a project;

$c_s$ = amount of short-term loan;

$c_b$ = amount of bond financing;

$k_s$ = before tax interest on the term loan;

$k_b$ = before tax interest on the bond;

$t_m$ = firm's marginal tax rate

**NOTE:** The before tax interest rate on the bond ($k_b$) needs to be recalculated taking the 6% flotation cost into account.

- Cost of Debt
  - Example 3: (Chan S. Park, example 15.6)
    - Cost of Debt:
      - Calculate before tax interest rate on the bond ($k_b$):
        - » P = $940
        - » A = $1,000 × 10% = $100
        - » F = $1,000
        - » N = 20
        - » i = $k_b$ = ?
        - » $940 = $100(P/A, $k_b$, 20) + $1,000(P/F, $k_b$, 20)
        - » Using linear interpolation, $k_b$ = 10.74%

- Cost of Debt
  - Example 3: (Chan S. Park, example 15.6)
    - Cost of Debt:

$$i_d = \left(\frac{c_s}{c_d}\right) k_s (1 - t_m) + \left(\frac{c_b}{c_d}\right) k_b (1 - t_m)$$

$$i_d = \left(\frac{\$1{,}000{,}000}{\$4{,}000{,}000}\right)(12.00\%)(1 - 38\%) + \left(\frac{\$3{,}000{,}000}{\$4{,}000{,}000}\right)(10.74\%)(1 - 38\%)$$

$$i_d = 6.85\%$$

- Cost of Capital
  - Definition: (Chan S. Park)
    - *Cost of Capital for a firm is a weighted sum of the cost of equity and the cost of debt.*
  - Weighted-average cost of capital:
    - Represents a composite index reflecting the cost of raising funds from different sources.
  - Marginal cost of capital: (Chan S. Park)
    - *The marginal cost of capital is defined as the cost of obtaining another dollar of new capital. The marginal cost rises as more and more capital is raised during a given period.*

- Calculate: Cost of Capital
  - Weighted-average cost of capital:

$$k = \frac{i_d c_d}{V} + \frac{i_e c_e}{V} \qquad \text{where,}$$

$k$ = Tax-adjusted weighted average cost of capital;

$c_d$ = Total debt capital (e.g. bonds) in dollars;

$c_e$ = Total equity capital in dollars;

$V = c_d + c_e$

$i_e$ = Average equity interest rate per period, taking into account all equity sources;

$i_d$ = After-tax average borrowing interest rate per period, taking into account all debt sources.

- Choice of MARR
  - Definition of MARR: (Chan S. Park)
    - ***M**inimum **A**ttractive **R**ate of **R**eturn*
    - *The required return necessary to make a capital budgeting project – such as building a new factory – worthwhile.*
    - *Project Risk Level:*
      - *High = High MARR*
      - *Low = Low MARR*
    - *Typical MARR values:*
      - *Public / Government project = 5% to 8%*
      - *Private project = 15% and up*

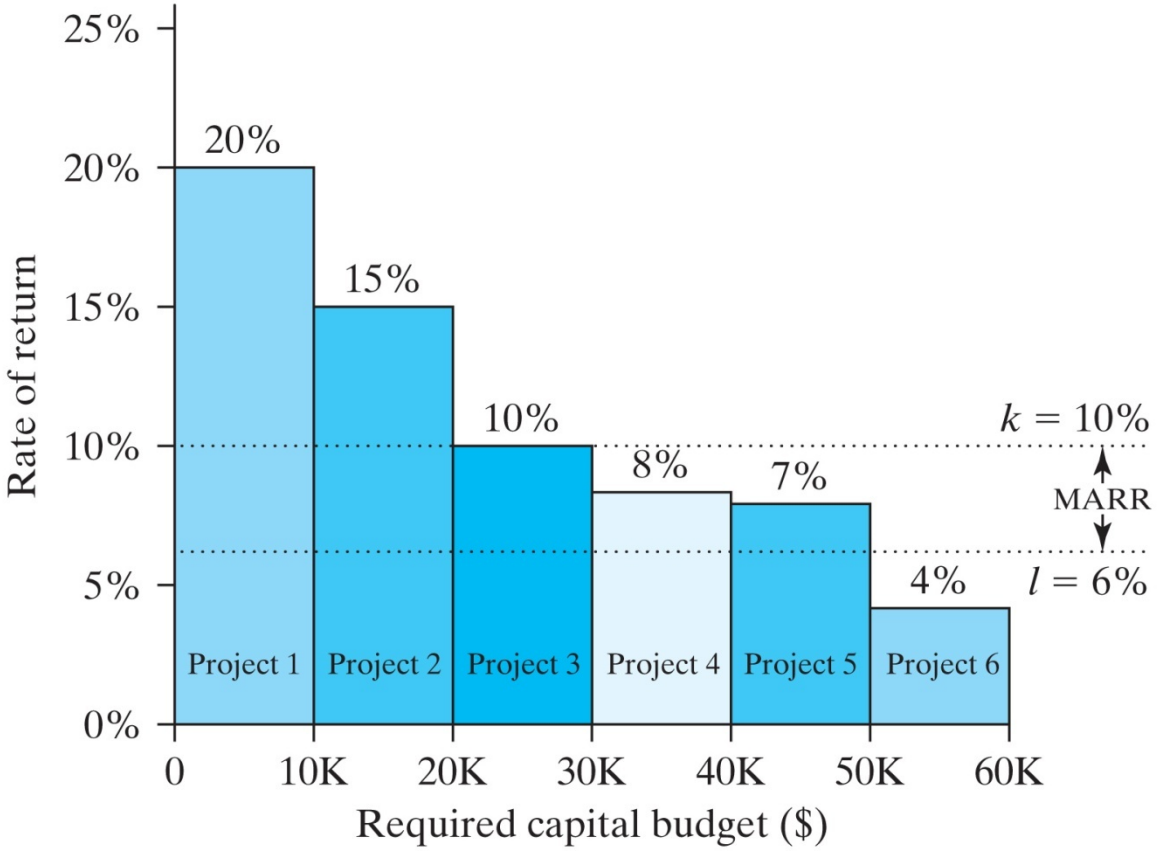
- Choice of MARR
  - Project financing is known:
    - MARR represents the cost of equity.
    - MARR = $i_e$
    - Refer to Chan S. Park, example 15.8.
  - Project financing is unknown:
    - MARR represents the weighted cost of capital.
    - MARR = $k$
    - Refer to Chan S. Park, example 15.9.
  - Capital rationing:
    - Driven by the borrowing and lending opportunities.
      - Borrowing rate = $k$
      - Lending rate = $l$
      - MARR = $i$, where $l < i < k$

- Choice of MARR
  - Capital rationing:

| Conditions | MARR |
|---|---|
| A firm borrows some capital from lending institutions at the borrowing rate $k$ and some from its investment pool at the lending rate $l$. | $l < \text{MARR} < k$ |
| A firm borrows all capital from lending institutions at the borrowing rate $k$. | $\text{MARR} = k$ |
| A firm borrows all capital from its investment pool at the lending rate $l$. | $\text{MARR} = l$ |

***(Chan S. Park, Chapter 15, summary)***

- Choice of MARR
  - Capital rationing: (Example 15.10, Chan S. Park)




***A range of MARRs as a function of a budget under capital rationing (Chan S. Park, Figure 15.4)***

- Capital Budgeting Decisions:
  - Evaluation of multiple investment alternatives.
  - Formulation of mutually exclusive alternatives.
  - Capital budgeting decisions with limited budgets.

- Evaluation of multiple investment alternatives:
  - Independent projects:
    - A project that may be accepted or rejected without influencing the decision to accept or reject another independent project.
    - Separate evaluation required.
  - Dependent projects:
    - A project that may be accepted or rejected and influences the decision to accept or reject another dependent project.
    - e.g. mutually exclusive projects, contingent projects

- Formulation of mutually exclusive alternatives.
  - Independent projects: (A & B)

| Alternative | Description | $X_a$ | $X_b$ |
|---|---|---|---|
| **1** | Reject A, Reject B | 0 | 0 |
| **2** | Accept A, Reject B | 1 | 0 |
| **3** | Reject A, Accept B | 0 | 1 |
| **4** | Accept A, Accept B | 1 | 1 |

- Formulation of mutually exclusive alternatives.
  - Mutually exclusive projects: (A1, A2; B1, B2)

| Alternative | $(X_{A1}, X_{A2})$ | $(X_{B1}, X_{B2})$ |
|---|---|---|
| **1** | (0,0) | (0,0) |
| **2** | (1,0) | (0,0) |
| **3** | (0,1) | (0,0) |
| **4** | (0,0) | (1,0) |
| **5** | (0,0) | (0,1) |
| **6** | (1,0) | (1,0) |
| **7** | (0,1) | (1,0) |
| **8** | (1,0) | (0,1) |
| **9** | (0,1) | (0,1) |

Selection of A1 or A2 is independent of the selection of either B1 or B2.

A1 and A2 cannot be selected together.

B1 and B2 cannot be selected together.

- Formulation of mutually exclusive alternatives.
  - Contingent projects: (A, B and C)

| Alternative | $X_A$ | $X_B$ | $X_C$ |
|---|---|---|---|
| **1** | 0 | 0 | 0 |
| **2** | 1 | 0 | 0 |
| **3** | 1 | 1 | 0 |
| **4** | 1 | 1 | 1 |

Selection of A is independent.

Selection of B is contingent on the selection of A.

Selection of C is contingent on the selection of A and B.

- Capital budgeting decisions with limited budgets.
  - Enumerate all investment alternatives as before.
  - Eliminate any alternatives that exceed the budget.
  - Select the group of projects that maximizes the total PW of future cash flows over the required investment outlays.
  - Refer to Chan S. Park, example 15.11.

# Questions?