SUPPLEMENT TO CHAPTER 8

LINEAR GOAL PROGRAMMING AND ITS SOLUTION PROCEDURES

8S-1.

(a) $3x_1 + 4x_2 + 2x_3 - y^+ + y^- = 60$

(b) Let c^+ be the coefficient of y^+ and c^- be the one for y^-, so $c^+ = 2c^-$.

8S-2.

(a)

minimize sum of amounts under market share for product 1 and 2

subject to
$$x_1 + x_2 + x_3 \leq 55$$
$$x_3 \geq 10$$
$$x_1, x_2 \geq 0$$

(b) $y_1 = 0.5x_1 + 0.2x_3 - 15, y_1 = y_1^+ - y_1^-, y_2 = 0.3x_2 + 0.2x_3 - 10, y_2 = y_2^+ - y_2^-$

minimize $y_1^- + y_2^-$

subject to
$$0.5x_1 + 0.2x_3 - y_1^+ + y_1^- = 15$$
$$0.3x_2 + 0.2x_3 - y_2^+ + y_2^- = 10$$
$$x_1 + x_2 + x_3 \leq 55$$
$$x_3 \geq 10$$
$$x_1, x_2, y_1^+, y_1^-, y_2^+, y_2^- \geq 0$$

(c)

	Market Share per $million			Goals				Deviations			Constraints		
	Ad	Ad	Ad	Level				Amount	Amount		Balance		
	Camp. 1	Camp. 2	Camp. 3	Achieved		Goal		Over	Under		(Level-Over+Under)		Goal
Goal 1 (M. Share of Prod. 1)	0.5%		0.2%	15.0%	>=	15%		0.0%	0.0%		15%	=	15%
Goal 2 (M. Share of Prod. 2)		0.3%	0.2%	8.33%	>=	10%		0.0%	1.67%		10%	=	10%
	Ad	Ad	Ad										
	Camp. 1	Camp. 2	Camp. 3	Total		Penalty		Over	Under		Weighted Sum		
Millions of Dollars Spent	13.33	0	41.67	55		Weights		Goal	Goal		of Deviations		
			>=	<=		Goal 1			1		1.67%		
			10	55		Goal 2			1				

8S-3.

(a)
$$6x_1 + 4x_2 + 5x_3 - y_1^+ + y_1^- = 50$$
$$8x_1 + 7x_2 + 5x_3 - y_2^+ + y_2^- = 75$$
$$P = 20x_1 + 15x_2 + 25x_3$$

(b) $Z = 20x_1 + 15x_2 + 25x_3 - 6y_1^+ - 6y_1^- - 3y_2^-$

(c)

maximize $20x_1 + 15x_2 + 25x_3 - 6y_1^+ - 6y_1^- - 3y_2^-$

subject to
$$6x_1 + 4x_2 + 5x_3 - y_1^+ + y_1^- = 50$$
$$8x_1 + 7x_2 + 5x_3 - y_2^+ + y_2^- = 75$$
$$x_1, x_2, x_3, y_1^+, y_1^-, y_2^+, y_2^- \geq 0$$

(d)

	Unit Contribution of Product			Goals			Deviations		Constraints		
	Product	Product	Product	Level			Amount	Amount	Balance		
	1	2	3	Achieved		Goal	Over	Under	(Level-Over+Under)		Goal
Goal 1 (Total Profit)	20	15	25	375		Max					
Goal 2 (Employment Level)	6	4	5	75	=	50	25	0	50.000	=	50
Goal 3 (Earnings Next Year)	8	7	5	75	>=	75	0	0	75.000	=	75
	Product	Product	Product								
	1	2	3				Over	Under	Measure of		
Production Rate	0	0	15			**Benefit**	Goal	Goal	Performance		
						Goal 1	Level Achieved		225		
						Goal 2	-6	-6			
						Goal 3		-3			

8S-4.

(a) No, we would not expect the optimal solution to change. Goal 1 is already met, so increasing the weight on that goal would not change anything. Goal 2 is already exceeded, so decreasing the penalty weight for this goal would only decrease our desire to avoid exceeding this goal.

				Goals			**Deviations**		**Constraints**		
	Contribution per Unit Produced			Level			Amount	Amount	Balance		
	Product 1	Product 2	Product 3	Achieved		Goal	Over	Under	(Level - Over + Under)		Goal
Goal 1 (Profit)	12	9	15	125	>=	125	0	0	125	=	125
Goal 2 (Employment)	5	3	4	48.333333	=	40	8.33333	0	40	=	40
Goal 3 (Investment)	5	7	8	55	<=	55	0	0	55	=	55
	Product 1	Product 2	Product 3			**Penalty**	Over	Under	Weighted Sum		
Units Produced	8.333333333	0	1.666666667			**Weights**	Goal	Goal	of Deviations		
						Profit		7	8.333333333		
						Employment	1	4			
						Investment	3				

(b)

				Goals			**Deviations**		**Constraints**		
	Contribution per Unit Produced			Level			Amount	Amount	Balance		
	Product 1	Product 2	Product 3	Achieved		Goal	Over	Under	(Level - Over + Under)		Goal
Goal 1 (Profit)	12	9	15	140	>=	140	0	0	140	=	140
Goal 2 (Employment)	5	3	4	58.333	=	40	18.333	0	40	=	40
Goal 3 (Investment)	5	7	8	58.333	<=	55	3.333	0	55	=	55
	Product 1	Product 2	Product 3			**Penalty**	Over	Under	Weighted Sum		
Units Produced	11.667	0	0			**Weights**	Goal	Goal	of Deviations		
						Profit		5	46.667		
						Employment	2	4			
						Investment	3				

(c)

				Goals			**Deviations**		**Constraints**		
	Contribution per Unit Produced			Level			Amount	Amount	Balance		
	Product 1	Product 2	Product 3	Achieved		Goal	Over	Under	(Level - Over + Under)		Goal
Goal 1 (Profit)	12	9	15	140	>=	140	0	0	140	=	140
Goal 2 (Employment)	5	3	4	58.333	=	40	18.333	0	40	=	40
Goal 3 (Investment)	5	7	8	58.333	<=	55	3.333	0	55	=	55
	Product 1	Product 2	Product 3			**Penalty**	Over	Under	Weighted Sum		
Units Produced	11.667	0	0			**Weights**	Goal	Goal	of Deviations		
						Profit		7	28.333		
						Employment	1	4			
						Investment	3				

8S-5.

(a)

minimize $\quad 0.01(\text{amount under foreign capital goal})$
$\quad + (\text{amount under citizens fed goal})$
$\quad + (\text{amount under goal for citizens employed})$
$\quad + (\text{amount over goal for citizens employed})$

(b)

minimize $\quad 0.01y_1^- + y_2^- + y_3^+ + y_3^-$

subject to

$$1000x_1 + 1000x_2 + 1000x_3 + x_4 = 15\text{M}$$
$$3000x_1 + 5000x_2 + 4000x_3 - y_1^+ + y_1^- = 70\text{M}$$
$$150x_1 + 75x_2 + 100x_3 - y_2^+ + y_2^- = 1.75\text{M}$$
$$10x_1 + 15x_2 + 12x_3 - y_3^+ + y_3^- = 0.2\text{M}$$
$$x_1, x_2, x_3, x_4, y_1^+, y_1^-, y_2^+, y_2^-, y_3^+, y_3^- \geq 0$$

(c)

				Goals				Deviations			Constraints		
	Contribution per 1000 Acres			Level				Amount	Amount		Balance		
	Crop 1	Crop 2	Crop 3	Achieved		Goal		Over	Under		(Level-Over+Under)		Goal
Goal 1 (Foreign Capital)	$3,000	$5,000	$4,000	$58,333,333	>=	$70,000,000		$0	$11,666,667		$70,000,000	=	$70,000,000
Goal 2 (Citizens Fed)	150	75	100	1,750,000	>=	1,750,000		0	0		1,750,000	=	1,750,000
Goal 3 (Citizens Employed)	10	15	12	183,333	=	200,000		0	16,667		200,000	=	200,000

	Crop 1	Crop 2	Crop 3	Total		**Penalty**	Over	Under		Weighted Sum
Thousands of Acres Planted	8,333	6,667	0	15,000		**Weights**	Goal	Goal		of Deviations
				<=		Goal 1		0.01		133,333
				15,000		Goal 2		1		
						Goal 3	1	1		

(d) minimize $\quad M_2y_1^- + M_1y_2^- + y_3^+ + y_3^-$

subject to

$$1000x_1 + 1000x_2 + 1000x_3 + x_4 = 15\text{M}$$
$$3000x_1 + 5000x_2 + 4000x_3 - y_1^+ + y_1^- = 70\text{M}$$
$$150x_1 + 75x_2 + 100x_3 - y_2^+ + y_2^- = 1.75\text{M}$$
$$10x_1 + 15x_2 + 12x_3 - y_3^+ + y_3^- = 0.2\text{M}$$
$$x_1, x_2, x_3, x_4, y_1^+, y_1^-, y_2^+, y_2^-, y_3^+, y_3^- \geq 0$$

(e) Optimal Solution: $(x_1, x_2, x_3) = (50000/6, 20000/6, 0)$ thousand acres

$$Z = (35 \cdot 10^6/3)M_2 + 50000/3.$$

BV	Eq#	Z	x_1	x_2	x_3	x_4	y_1^+	y_1^-	y_2^+	y_2^-	y_3^+	y_3^-	RHS
Z	0	-1	$-150M_1$ $-3000M_2$ -10	$-75M_1$ $-5000M_2$ -15	$-100M_1$ $-4000M_2$ -12	0	M_2	0	M_1	0	2	0	$-175M_1$ $-7000M_2$ -20
x_4	1	0	1000	1000	1000	1	0	0	0	0	0	0	1500
y_1^-	2	0	2000	5000	4000	0	-1	1		0	0	0	7000
y_2^-	3	0	150*	75	100	0	0	0	-1	1	0	0	175
y_3^-	4	0	10	15	12	0	0	0	0	0	-1	1	20
Z	0	-1	0	$-3500M_2$ -10	$-2000M_2$ -12	0	M_2	0	$-20M_2$ $-1/15$	M_1+20M_2 $1/15$	2	0	$3500M_2$ $-25/3$
x_4	1	0	0	500*	1000/3	1	0	0	20/3	$-20/3$	0	0	1000/3
y_1^-	2	0	0	3500	2000	0	-1	1	20	-20	0	0	3500
x_1	3	0	1	1/2	2/3	0	0	0	$-1/150$	1/150	0	0	7/6
y_3^-	4	0	0	10	12	0	0	0	1/15	$-1/15$	-1	1	25/3
Z	0	-1	0	0	$1000/3M_2+4/3$	$7M_2+1/100$	M_2	0	$80/3M_2+1/15$	$M_1-80/3M_2-1/15$	2	0	$-35000/3M_2$ $-50/3$
x_2	1	0	0	1	2/3	1/500	0	0	1/75	$-1/75$	0	0	2/3
y_1^-	2	0	0	0	$-1000/3$	-7	-1	1	$-80/3$	80/3	0	0	35000/3
x_1	3	0	1	0	1/3	$-1/1000$	0	0	0	0	0	0	5/6
y_3^-	4	0	0	0	$-4/3$	$-1/50$	0	0	$-1/15$	1/15	-1	1	6/3

(f) With only $M_1 y_2^-$ in the objective function, we get $y_2^- = Z = 0$, so fix $y_2^- = 0$ and bring $M_2 y_1^-$ into the objective function. Now $y_1^- = 11,666,666\frac{2}{3}$. Fix y_1^- at this value (remembering subtract from RHS) and optimize for the third priority. Then the solution in part (c) is obtained: $(x_1, x_2, y_1^-, y_3^-) = \left(8333\frac{1}{3}, 6666\frac{2}{3}, 11666666\frac{2}{3}, 16666\frac{2}{3}\right)$.

8S-6.

(a) minimize $M_1 y_1^+ + M_2 y_2^+ + M_2 y_2^- + y_3^-$

subject to

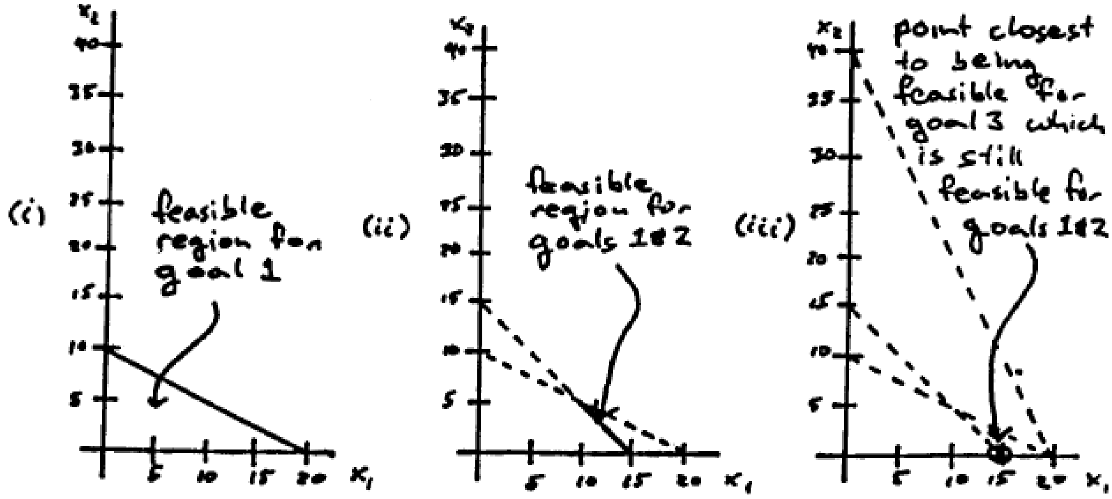
$$x_1 + 2x_2 - y_1^+ + y_1^- = 20$$
$$x_1 + x_2 - y_2^+ + y_2^- = 15$$
$$2x_1 + x_2 - y_3^+ + y_3^- = 40$$
$$x_1, x_2, y_1^+, y_1^-, y_2^+, y_2^-, y_3^+, y_3^- \geq 0$$

(b) - (c)

Optimal Solution: $(x_1, x_2) = (15, 0)$, $Z = 10$

	BV	E	Z	x_1	x_2	y_1^+	y_1^-	y_2^+	y_2^-	y_3^+	y_3^-	RHS
0	Z	0	-1	$-M_2-2$	$-M_2-1$	M_1	0	$2M_2$	0	1	0	$-15M_2-40$
	y_1^-	1	0	1	2	-1	1	0	0	0	0	20
	y_2^-	2	0	1	1	0	0	-1	1	0	0	15
	y_3^-	3	0	2	1	0	0	0	0	-1	1	40
1	Z	0	-1	0	1	M_1	0	M_2-2	M_2+2	1	0	-10
	y_1^-	1	0	0	1	-1	1	1	-1	0	0	5
	x_1	2	0	1	1	0	0	-1	1	0	0	15
	y_3^-	3	0	0	-1	0	0	2	-2	-1	1	10

(d)



(e) minimize $Z_1 = M_1 y_1^+$

subject to
$$\begin{aligned}
&x_1 + 2x_2 - y_1^+ + y_1^- = 20\\
&[x_1 + x_2 - y_2^+ + y_2^- = 15]\\
&[2x_1 + x_2 - y_3^+ + y_3^- = 40]\\
&x_1, x_2 \geq 0
\end{aligned}$$

The feasible region is a shown in figure (i) of part (d). Fix $y_1^+ = 0$.

minimize $Z_2 = M_2 y_2^+ + M_2 y_2^-$

subject to
$$\begin{aligned}
&x_1 + 2x_2 - y_1^+ + y_1^- = 20\\
&x_1 + x_2 - y_2^+ + y_2^- = 15\\
&[2x_1 + x_2 - y_3^+ + y_3^- = 40]\\
&x_1, x_2 \geq 0
\end{aligned}$$

The feasible region is a shown in figure (ii) of part (d). Fix $y_1^+ = y_2^+ = y_2^- = 0$.

minimize $Z_3 = y_3^-$

subject to
$$\begin{aligned}
&x_1 + 2x_2 - y_1^+ + y_1^- = 20\\
&x_1 + x_2 - y_2^+ + y_2^- = 15\\
&2x_1 + x_2 - y_3^+ + y_3^- = 40\\
&x_1, x_2 \geq 0
\end{aligned}$$

The solution is $(15, 0)$ with $Z_3 = 10$.

8S-7.

(a) minimize $M_1 y_1^+ + M_2 y_2^- + y_3^-$

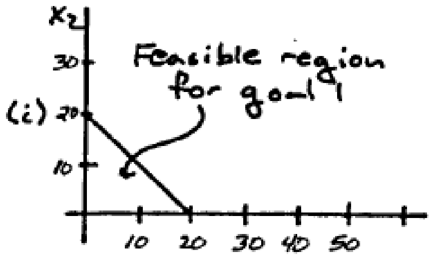
subject to
$$\begin{aligned}
&x_1 + x_2 - y_1^+ + y_1^- = 20\\
&x_1 + x_2 - y_2^+ + y_2^- = 30\\
&x_1 + 2x_2 - y_3^+ + y_3^- = 50\\
&x_1, x_2, y_1^+, y_1^-, y_2^+, y_2^-, y_3^+, y_3^- \geq 0
\end{aligned}$$

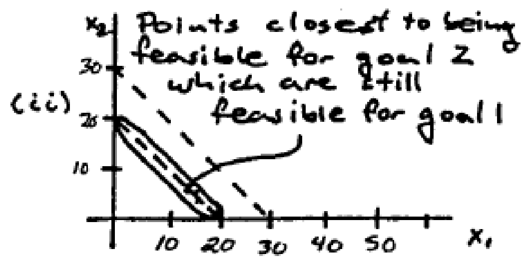
(b) - (c)

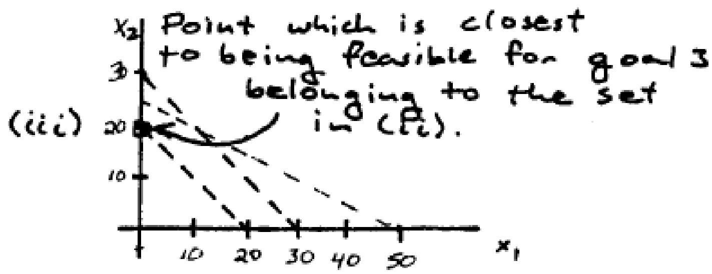
Optimal Solution: $(x_1, x_2) = (0, 20)$, $Z = 10M_2 + 10$

	BV	E	Z	x_1	x_2	y_1^+	y_1^-	y_2^+	y_2^-	y_3^+	y_3^-	RHS
0	Z	0	-1	$-M_2-1$	$-M_2-2$	M_1	0	M_2	0	1	0	$-30M_2-50$
	y_1^-	1	0	1	1	-1	1	0	0	0	0	20
	y_2^-	2	0	1	1	0	0	-1	1	0	0	30
	y_3^-	3	0	1	2	0	0	0	0	-1	1	50
1	Z	0	-1	1	0	M_1-M_2-2	M_2+2	M_2	0	1	0	$-10M_2-10$
	x_2	1	0	1	1	-1	1	0	0	0	0	20
	y_2^-	2	0	0	0	1	-1	-1	1	0	0	10
	y_3^-	3	0	-1	0	2	-2	0	0	-1	1	10

(d)







8S-8.

If $z_i = z_i^+ - z_i^-$, where $z_i^+, z_i^- \geq 0$, then $|z_i| = z_i^+ + z_i^-$.

(a) minimize $\sum_{i=1}^{n}(z_i^+ + z_i^-)$

subject to
$$z_i^+ - z_i^- = y_i - (a + bx_i),\ i = 1, 2, \ldots, n$$
$$z_i^+, z_i^- \geq 0,\ i = 1, 2, \ldots, n$$

(b) minimize z

subject to
$$z_i^+ - z_i^- = y_i - (a + bx_i),\ i = 1, 2, \ldots, n$$
$$0 \leq z_i^+ \leq z,\ i = 1, 2, \ldots, n$$
$$0 \leq z_i^- \leq z,\ i = 1, 2, \ldots, n$$

Case 8S.1 A Cure for Cuba

a) We need to develop a goal programming problem whose solution characterizes Mr. Baker's shipping policy. The decision variables are the number (in 1000's) of basic, advanced, and supreme packages to send, and the number of doctors to send. Note: measuring most variables in 1000's greatly improves the reliability of Solver.

Mr. Baker faces three hard constraints. Because of the size limitation, the total number of package must not exceed 40,000. Second, the total weight can not exceed 6 million pounds. Finally, the total number of Supreme packages cannot exceed 100 times the number of doctors. These constraints are included in the spreadsheet as follows.

TotalPackages (E14) ≤ SizeLimit (E16)
TotalWeight (E10) ≤ WeightRestriction (G10)
SupremePackages (D14) ≤ SafetyRestriction (D16)

In addition, we need to include three constraints for Mr. Baker's goals. We measure the deviations from the goals using changing cells (Deviations in I4:J6), and enforce the correct value of these changing cells with the constraints in columns L through N.

Finally, the penalty weights are entered in I15:J17, and the weight sum of deviations calculated in L15.

The spreadsheet follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1					Goals				Deviations			Constraints			
2					Level				Amount	Amount		Balance			
3		Basic	Advanced	Supreme	Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Goal 1 (Cost)	$300	$350	$720	21,000	<=	20,000		1,000	0		20,000	=	20,000	$thousand
5	Goal 2 (Packages Sent)	1	1	1	40	>=	3		37	0		3	=	3	thousand
6	Goal 3 (Population Reached)	30	35	54	1,488	>=	2,200		0	712		2,200	=	2,200	thousand
7															
8					Total		Weight								
9					Weight		Restriction								
10	Weight	120	180	220	6,000	<=	6,000	thousand pounds							
11															
12					Total										
13		Basic	Advanced	Supreme	Packages		**Penalty**		Over	Under		Weighted Sum			
14	Packages Sent (thousands)	28	0	12	40		**Weights**		Goal	Goal		of Deviations			
15				<=	<=		Goal 1		0.001			50.84			
16	Doctors	120	Safety	12	40		Goal 2			1					
17			Restriction	0.1	Size Limit		Goal 3			0.07					
18				per Doctor											
19	Cost per Doctor ($thousand)	33													

Mr. Baker should send 28,000 basic packages and 12,000 supreme packages along with 120 doctors to Cuba.

b) The penalty weight for being under goal 3 changes. One-half percent of the population is 55,000. Therefore, the new penalty weight is 10 points / 55 (thousand people) = 0.182. The new solution follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1					Goals				Deviations			Constraints			
2					Level				Amount	Amount		Balance			
3		Basic	Advanced	Supreme	Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Goal 1 (Cost)	$300	$350	$720	21,000	<=	20,000		1,000	0		20,000	=	20,000	$thousand
5	Goal 2 (Packages Sent)	1	1	1	40	>=	3		37	0		3	=	3	thousand
6	Goal 3 (Population Reached)	30	35	54	1,488	>=	2,200		0	712		2,200	=	2,200	thousand
7															
8					Total		Weight								
9					Weight		Restriction								
10	Weight	120	180	220	6,000	<=	6,000	thousand pounds							
11															
12					Total										
13		Basic	Advanced	Supreme	Packages		**Penalty**		Over	Under		Weighted Sum			
14	Packages Sent (thousands)	28	0	12	40		**Weights**		Goal	Goal		of Deviations			
15				<=	<=		Goal 1		0.001			130.45			
16	Doctors	120	Safety	12	40		Goal 2			1					
17			Restriction	0.1	Size Limit		Goal 3			0.182					
18				per Doctor											
19	Cost per Doctor ($thousand)	33													

The optimal shipping policy did not change. The plan appears to be insensitive to increases in the penalty weight for violating the goal to reach at least 20% of the Cuban population.

c) The doctors needed per thousand supreme packages changes from 0.1 to 0.075. The new solution follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1					Goals				Deviations			Constraints			
2					Level				Amount	Amount		Balance			
3		Basic	Advanced	Supreme	Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Goal 1 (Cost)	$300	$350	$720	22,320	<=	20,000		2,320	0		20,000	=	20,000	$thousand
5	Goal 2 (Packages Sent)	1	1	1	40	>=	3		37	0		3	=	3	thousand
6	Goal 3 (Population Reached)	30	35	54	1,488	>=	2,200		0	712		2,200	=	2,200	thousand
7															
8					Total		Weight								
9					Weight		Restriction								
10	Weight	120	180	220	6,000	<=	6,000	thousand pounds							
11															
12					Total										
13		Basic	Advanced	Supreme	Packages		**Penalty**		Over	Under		Weighted Sum			
14	Packages Sent (thousands)	28	0	12	40		**Weights**		Goal	Goal		of Deviations			
15				<=	<=		Goal 1		0.001			131.77			
16	Doctors	160	Safety	12	40		Goal 2			1					
17			Restriction	0.075	Size Limit		Goal 3			0.182					
18				per Doctor											
19	Cost per Doctor ($thousand)	33													

While the number of packages Mr. Baker should ship has not changed, the number of doctors is now 160.

d) The budget restriction is now a hard constraint and the penalty variables for the cost goal can be eliminated.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1						Goals			Deviations			Constraints			
2					Level				Amount	Amount		Balance			
3		Basic	Advanced	Supreme	Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Cost (Hard Constraint)	$300	$350	$720	20,000	<=	20,000								$thousand
5	Goal 2 (Packages Sent)	1	1	1	40	>=	3		37	0		3	=	3	thousand
6	Goal 3 (Population Reached)	30	35	54	1,465	>=	2,200		0	735.5		2,200	=	2,200	thousand
7															
8					Total		Weight								
9					Weight		Restriction								
10	Weight	120	180	220	6,000	<=	6,000	thousand pounds							
11															
12					Total										
13		Basic	Advanced	Supreme	Packages										
14	Packages Sent (thousands)	27	2.5	10.5	40			**Penalty**	Over	Under		Weighted Sum			
15				<=	<=			**Weights**	Goal	Goal		of Deviations			
16	Doctors	105	Safety	10.5	40			Goal 2		1		51.49			
17			Restriction	0.1	Size Limit			Goal 3		0.07					
18				per Doctor											
19	Cost per Doctor ($thousand)	33													

Mr. Baker should send 27,000 basic packages, 2,500 advanced packages, and 10,500 supreme packages along with 105 doctors to Cuba.

e) We start by minimizing the amount over goal 1 (total cost ≤ $20 million).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1						Goals			Deviations			Constraints			
2					Level				Amount	Amount		Balance			
3		Basic	Advanced	Supreme	Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Goal 1 (Cost)	$300	$350	$720	20,000	<=	20,000		0	0		20,000	=	20,000	$thousand
5	Goal 2 (Packages Sent)	1	1	1	19.024	>=	3		16.024	0		3	=	3	thousand
6	Goal 3 (Population Reached)	30	35	54	1,027	>=	2,200		0	1,173		2,200	=	2,200	thousand
7															
8					Total		Weight		Minimize Over Goal 1						
9					Weight		Restriction								
10	Weight	120	180	220	4,185	<=	6,000								
11															
12					Total										
13		Basic	Advanced	Supreme	Packages										
14	Packages Sent (thousands)	0	0	19.024	19.02361111										
15				<=	<=										
16	Doctors	191	Safety	19.1	40										
17			Restriction	0.1	Size Limit										
18				per Doctor											
19	Cost per Doctor ($thousand)	33													

Then, since goal 2 is already met, we move on to goal 3. We minimize the amount under goal 3 (population reached ≥ 20%), while constraining (amount over goal 1 = 0) and (amount under goal 2 = 0).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1						Goals			Deviations			Constraints			
2					Level				Amount	Amount		Balance			
3		Basic	Advanced	Supreme	Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Goal 1 (Cost)	$300	$350	$720	20,000	<=	20,000		0	0		20,000	=	20,000	$thousand
5	Goal 2 (Packages Sent)	1	1	1	40	>=	3		37	0		3	=	3	thousand
6	Goal 3 (Population Reached)	30	35	54	1,464	>=	2,200		0	735		2,200	=	2,200	thousand
7															
8					Total		Weight		Minimize Under Goal 3						
9					Weight		Restriction		(Over Goal 1 = 0)						
10	Weight	120	180	220	6,000	<=	6,000		(Under Goal 2 = 0)						
11															
12					Total										
13		Basic	Advanced	Supreme	Packages										
14	Packages Sent (thousands)	27	2.5	10.5	40										
15				<=	<=										
16	Doctors	105	Safety	10.5	40										
17			Restriction	0.1	Size Limit										
18				per Doctor											
19	Cost per Doctor ($thousand)	33													

Mr. Baker should send 27 thousand basic packages, 2,500 advanced packages, and 10,500 supreme packages, along with 105 doctors.

Case 8S.2 Airport Security

a) The two decisions to be made are how much to spend on the two security systems. Hence, we define the following two variables.

Let PS = thousands of dollars spent per portal system
SS = thousands of dollars spent per screening system.

b) Preemptive goal programming is appropriate because there is a clear order of priorities.

Priority 1 is met by all possible systems.

Priority 2 (hereafter referred to as goal 1) is that the false alarm rate should not exceed 10%. The false alarm rate of the two systems is as follows:

Portal System: $10\% - (1\%)(PS - 90) / 15$

Screening System: $6\% - (1\%)(SS - 60) / 30$

Goal 1 is thus

$$[10\% - (1\%)(PS - 90) / 15] + [6\% - (1\%)(SS - 60) / 30] \leq 10\%$$

Priority 3 (hereafter referred to as goal 2) is that the first budgetary guideline should be met (total expenditures ≤ $250,000). Goal 2 is thus

$$PS + SS \leq 250$$

Priority 4 (hereafter referred to as goal 3) is that the second budgetary guideline should be met (average total maintenance cost $30,000). The maintenance cost of the two systems is as follows:

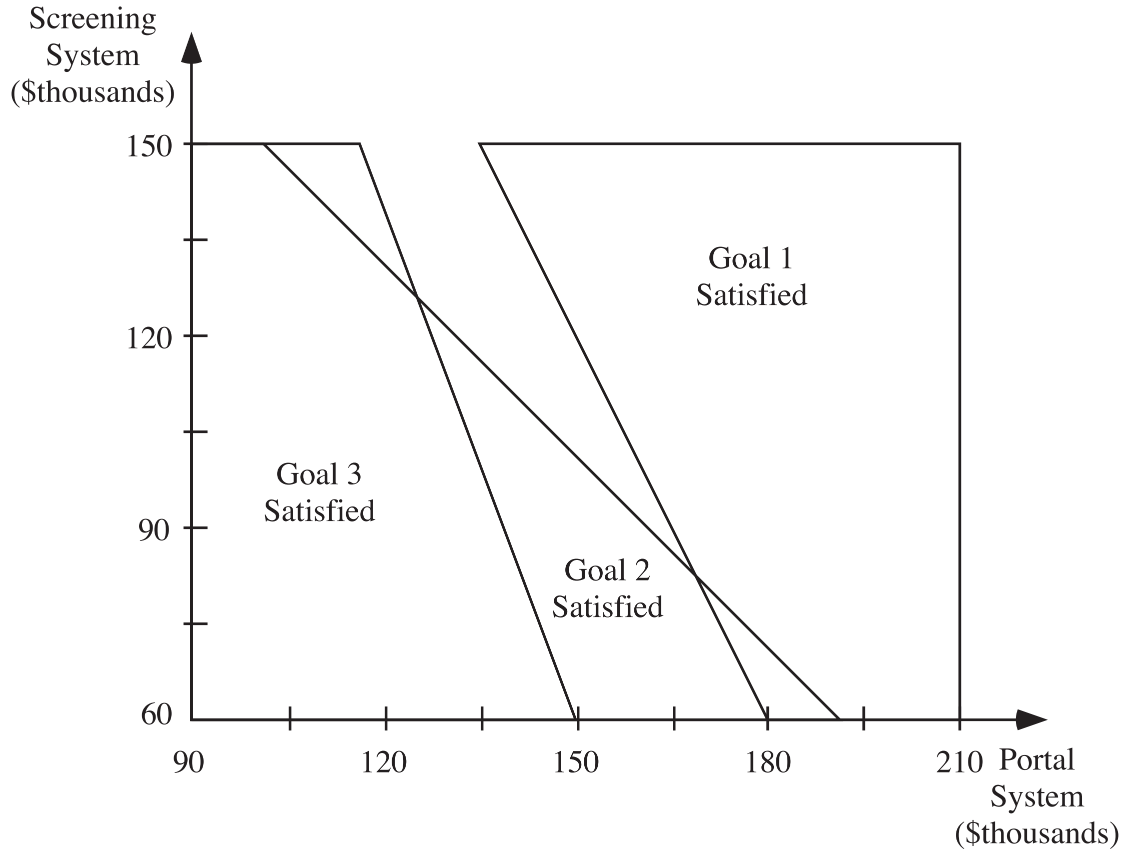
Portal System: $15 + (PS - 90) / 10$

Screening System: $9 + (SS - 60) / 25$

Goal 3 is thus

$$[15 + (PS - 90) / 10] + [9 + (SS - 60) / 25] \leq 30$$

c)



Goal 1 is satisfied inside the rightmost polygon. Goal 2 is satisfied in the polygon in the middle. The small triangle with vertices at (180, 60), (170, 80), (190, 60) is the only area where both goal 1 and goal 2 are satisfied.

Applying preemptive goal programming, the first solution will be somewhere inside the region where goal 1 is satisfied.

The second solution (minimizing the amount over goal 2 while constraining goal 1 to be met) will give a solution inside the small triangle where both goal 1 and goal 2 are met.

The third solution (minimizing the amount over goal 3 while constraining goal 1 and 2 to be met) will pick the solution inside the small triangle (since goal 1 and 2 must remain to be met) that is closest to meeting goal 3. This occurs at (170, 80). That is, they should spend $170 thousand on the portal system and $80 thousand on the screening system.

d) We start by minimizing the amount over goal 1 (false alarm rate ≤ 10%).

	A	B	C	D	E	F	G	H	I	J	K	L
1			Goals			Deviations			Constraints			
2		Level				Amount	Amount		Balance			
3		Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Goal 1 (False Alarm Rate)	10%	<=	10%		0	0		10%	=	10%	
5	Goal 2 (Total Expenditure)	250	<=	250		0	0		250	=	250	($thousand)
6	Goal 3 (Maintenance Cost)	32.8	<=	30		2.8	0		30	=	30	($thousand)
7												
8		Portal	Screening			Minimize Over Goal 1						
9		System	System									
10	Minimum	90	60									
11		<=	<=									
12	Expenditure ($thousand/system)	170	80									
13		<=	<=									
14	Maximum	210	150									
15												
16	False Alarm Rate	5%	5%									
17	Base Rate	10%	6%									
18	Minus 1% per ($x thousand)	15	30									
19												
20	Maintenance Cost ($thousand)	23	9.8									
21	Base Rate	15	9									
22	Plus $1 per $x	10	25									

Since goal 2 is already met, we move on to minimizing the amount over goal 3 (maintenance cost ≤ $30,000), while constraining (amount over goal 1 = 0) and (amount over goal 2 = 0).

	A	B	C	D	E	F	G	H	I	J	K	L
1			Goals			Deviations			Constraints			
2		Level				Amount	Amount		Balance			
3		Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Goal 1 (False Alarm Rate)	10%	<=	10%		0	0		10%	=	10%	
5	Goal 2 (Total Expenditure)	250	<=	250		0	0		250	=	250	($thousand)
6	Goal 3 (Maintenance Cost)	32.8	<=	30		2.8	0		30	=	30	($thousand)
7												
8		Portal	Screening			Minimize Over Goal 3						
9		System	System			(Over Goal 1 = 0)						
10	Minimum	90	60			(Over Goal 2 = 0)						
11		<=	<=									
12	Expenditure ($thousand/system)	170	80									
13		<=	<=									
14	Maximum	210	150									
15												
16	False Alarm Rate	5%	5%									
17	Base Rate	10%	6%									
18	Minus 1% per ($x thousand)	15	30									
19												
20	Maintenance Cost ($thousand)	23	9.8									
21	Base Rate	15	9									
22	Plus $1 per $x	10	25									

e) The first two goals are now hard constraints, and we minimize the amount over goal 3.

	A	B	C	D	E	F	G	H	I	J	K	L
1		Goals				Deviations			Constraints			
2		Level				Amount	Amount		Balance			
3		Achieved		Goal		Over	Under		(Level-Over+Under)		Goal	
4	Goal 1 (False Alarm Rate)	10%	<=	10%	Hard Constraint							
5	Goal 2 (Total Expenditure)	250	<=	250	Hard Constraint							($thousand)
6	Goal 3 (Maintenance Cost)	32.8	<=	30		2.8	0		30	=	30	($thousand)
7												
8		Portal	Screening			Minimize Over Goal 1						
9		System	System									
10	Minimum	90	60									
11		<=	<=									
12	Expenditure ($thousand/system)	170	80									
13		<=	<=									
14	Maximum	210	150									
15												
16	False Alarm Rate	5%	5%									
17	Base Rate	10%	6%									
18	Minus 1% per ($x thousand)	15	30									
19												
20	Maintenance Cost ($thousand)	23	9.8									
21	Base Rate	15	9									
22	Plus $1 per $x	10	25									

If the linear program had no feasible solution, this would imply that it is not possible to meet all of the higher priority goals that were turned into hard constraints.

f) We no longer use goal programming. The goal is to minimize the total false alarm rate subject to meeting the first budgetary guideline (total expenditure), but ignoring the second budgetary guideline (maintenance cost). The spreadsheet model follows.

	A	B	C	D
1		Level		
2		Achieved		Maximum
3	Total False Alarm Rate	9%		Expenditure
4	Total Expenditure	250	<=	250
5	Maintenance Cost	34		
6				
7		Portal	Screening	
8		System	System	
9	Minimum	90	60	
10		<=	<=	
11	Expenditure ($thousand/system)	190	60	
12		<=	<=	
13	Maximum	210	150	
14				
15	False Alarm Rate	3%	6%	
16	Base Rate	10%	6%	
17	Minus 1% per ($x thousand)	15	30	
18				
19	Maintenance Cost ($thousand)	25	9	
20	Base Rate	15	9	
21	Plus $1 per $x	10	25	

The total false alarm rate can be lowered to 9% by ignoring the second budgetary guideline (maintenance cost).

g) Further what-if analysis might look at how low the false-alarm rate can be lowered by ignoring the first budgetary guideline, but meeting the second. Also, it would be interesting to look at how the minimum false alarm rate changes as both of the budgetary guidelines are varied.