

EM 605

Elements of Operations Research

Solving Linear Programming Problems – Computer Solutions and Sensitivity Analysis

Computer Solution

- Early linear programming used lengthy *manual* mathematical solution procedure called the Simplex Method
- Steps of the simplex method have been programmed in software packages designed for linear programming problems
- Many such packages available currently
- Used extensively in business and government
- This class focuses on Excel Spreadsheets and QM for Windows



Product Mix - Beaver Creek Pottery

- Beaver Creek Pottery is a small crafts operation run by a Native American tribal council. The company employs skilled artisans to produce clay bowls and mugs with authentic Native American designs and colors.
- The two primary resources used by the company are special pottery clay and skilled labor. Given these limited resources, the company wants to know how many bowls and mugs to produce each day in order to maximize profit.
- This is typically called a “product mix” problem

Linear Programming Problem: Standard Form

- *Standard form* requires all variables in the constraint equations to appear on the left of the inequality (or equality) and all numeric values to be on the right-hand side
- Examples:
 - $x_3 \geq x_1 + x_2$ must be converted to $x_3 - x_1 - x_2 \geq 0$
 - $x_1 / (x_2 + x_3) \geq 2$ becomes $x_1 \geq 2(x_2 + x_3)$
and then $x_1 - 2x_2 - 2x_3 \geq 0$

Beaver Creek Pottery

The two products have the following resource requirements for production, and profit per item produced

Bowl Labor: 1 hr/unit

Clay: 4 lb/unit

Profit: 40\$/unit

Mug Labor: 2 hr/unit

Clay: 3 lb/unit

Profit: 50\$/unit

There are 40 hours of labor available each day, and 120 pounds of clay each day, for production

**Please set up the
problem in Excel...and solve it**

Beaver Creek Pottery

X_1 = # of bowls to produce per day

X_2 = # of mugs to produce per day

Maximize $Z = \$40x_1 + \$50x_2$

subject to: $x_1 + 2x_2 \leq 40$

$4x_1 + 3x_2 \leq 120$

$x_1, x_2 \geq 0$

Beaver Creek Pottery

Excel Spreadsheet – Data Screen

[illegible]

Beaver Creek Pottery - Solver Parameter Screen

Solver Parameters

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$J\$21 <= \$H\$5

\$J\$22 <= \$H\$6

Add

Change

Delete

Reset All

Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Beaver Creek Pottery - Solution Screen

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1		Beaver Creek Pottery									Decision Variable Definition									
2				# Bowls		# Mugs					Xb = # of bowls to make									
3		Profit		40.00		50.00					Xm = # of mugs to make									
4								Max	Min											
5		Labor Needed		1		2		40												
6		Clay Needed		4		3		120												
7																				
8		MODEL IN CANONICAL FORM																		
9		Objective Function																		
10		MAXIMIZE $Z = 40X_b + 50X_m$																		
11																				
12		Constraints																		
13		$1X_b + 2X_m \leq 40$																		
14		$4X_b + 3X_m \leq 120$																		
15		$X_b, X_m \geq 0$																		
16																				
17				24		8					Decision Variables									
18																				
19				960.00		400.00				1360.00	Objective Function									
20																				
21				24		16				40	Labor Constraint									
22				96		24				120	Material (clay) Constraint									

Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

☒ Keep Solver Solution

☐ Restore Original Values

☐ Return to Solver Parameters Dialog

☐ Outline Reports

OK

Cancel

Save Scenario...

Reports

Answer
Sensitivity
Limits

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution.
When Simplex LP is used, this means Solver has found a global optimal solution.

Solver Results

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OK

Cancel

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Reports

Answer

Sensitivity

Limits

Solver found a solution. All Constraints and optimality conditions are satisfied.

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 When Simplex LP is used, this means Solver has found a global optimal solution.

Select "Answer" in the Reports section

Beaver Creek Pottery - Answer Report

Microsoft Excel 12.0 Answer Report

Worksheet: [Lecture4a.xlsx]BeaverCreekPottery

Report Created: 9/7/2010 9:01:34 AM

Target Cell (Max)

Cell	Name	Original Value	Final Value	
\$J\$19		0.00	1360.00	\$1,360 is the anticipated profit

Adjustable Cells

Cell	Name	Original Value	Final Value	
\$D\$17	bowls	0	24	# of bowls to make
\$F\$17	cups	0	8	# of mugs to make

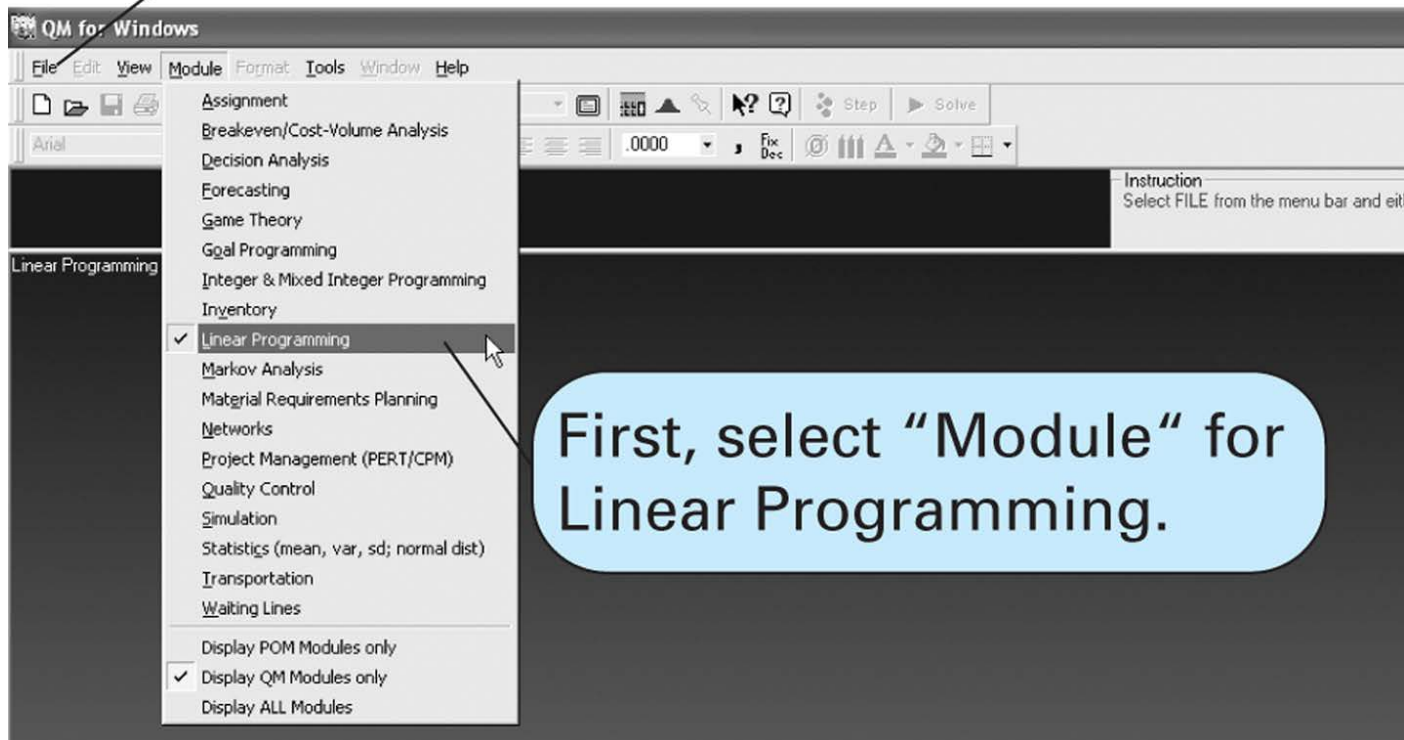
Constraints

Cell	Name	Cell Value	Formula	Status	Slack	
\$J\$21		40	\$J\$21<=40	Binding	0	No unused labor
\$J\$22		120	\$J\$22<=120	Binding	0	No unused material (clay)

Beaver Creek Pottery Example

QM for Windows

Click on "File" and then "New" to enter a new problem.



Beaver Creek Pottery Example

QM for Windows – Data Set Creation

Create data set for Linear Programming

Title: Modify default title

Number of Constraints:

Number of Variables:

Objective

☒ Maximize

☐ Minimize

Row names Column names Overview

☒ Constraint 1, Constraint 2, Constraint 3, ...

☐ a, b, c, d, e, ...

☐ A, B, C, D, E, ...

☐ 1, 2, 3, 4, 5, ...

☐ January, February, March, April, ...

☐ Other

Cancel Help OK

Set number
of constraints
and decision
variables.

Click here when finished.

Beaver Creek Pottery Example

QM for Windows: Data Table

QM for Windows - [Data Table]

File Edit View Module Format Tools Window Help

Icons: [New] [Open] [Save] [Print] [Clipboard] [Grid] [Zoom] [100%] [Fit] [Find] [Help] [Step] [Solve]

Font: Arial 8.25 B I U [Align] [Dec] [Fix] [Step] [Solve]

Objective: ☒ Maximize ☐ Minimize

Instruction: Enter the value for clay (lbs) for rhs. Any non-negative value is permissible.

Beaver Creek Pottery Company

	X1	X2		RHS	Equation form
Maximize	40	50			Max $40X_1 + 50X_2$
Labor (hrs)	1	2	\leq	40	$X_1 + 2X_2 \leq 40$
Clay (lbs)	4	3	\leq	120	$4X_1 + 3X_2 \leq 120$

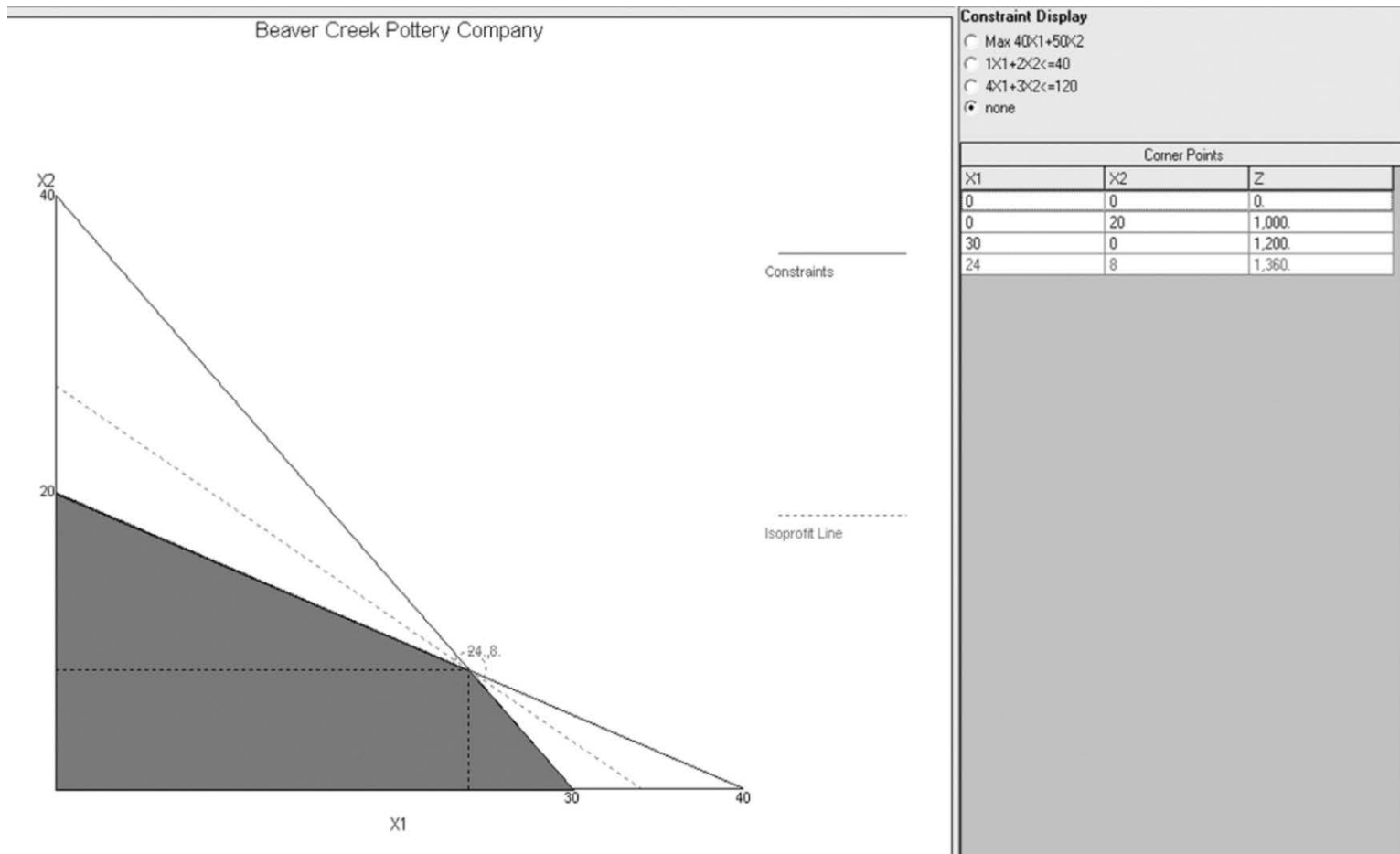
Beaver Creek Pottery Example

QM for Windows: Model Solution

Beaver Creek Pottery Company Solution						
	X1	X2		RHS	Dual	
Maximize	40	50			Max 40X1 +	
Labor (hrs)	1	2	\leq	40	16	
Clay (lbs)	4	3	\leq	120	6	
Solution->	24	8	Optimal Z->	1360	4X1 + 3X2 \leq	

Beaver Creek Pottery Example

QM for Windows: Graphical Display



Beaver Creek Pottery

Sensitivity Analysis

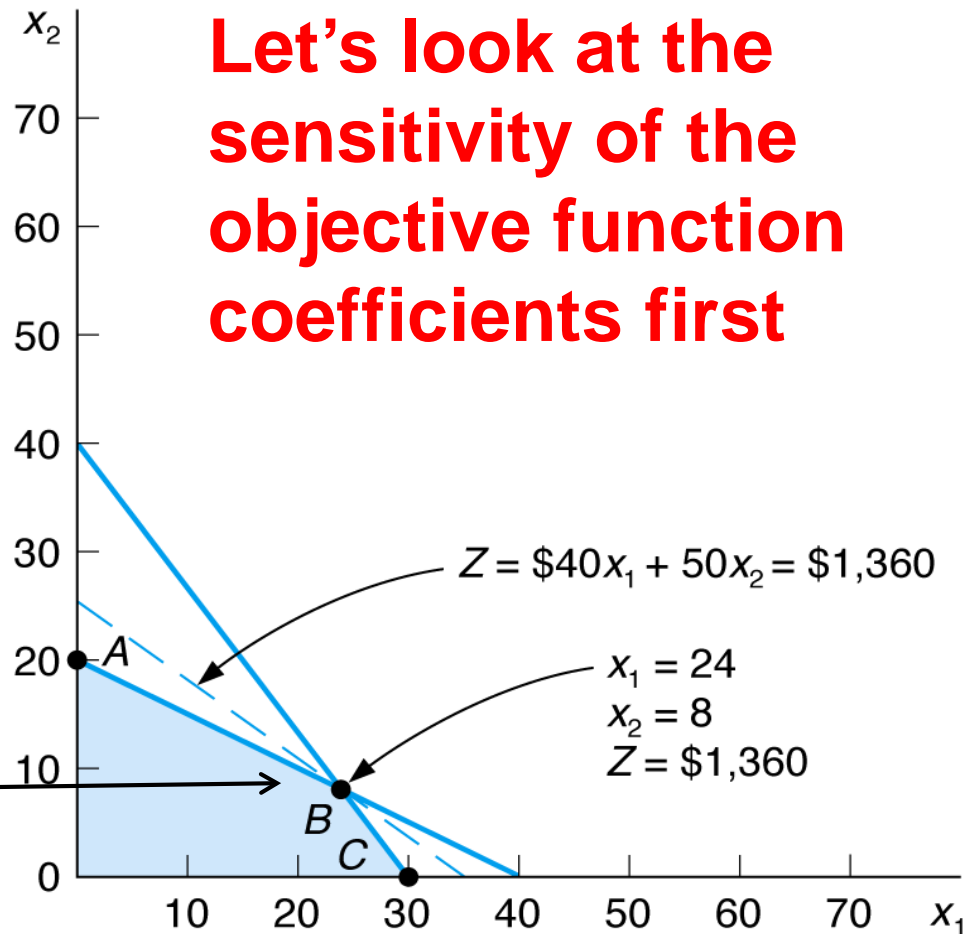
- *Sensitivity analysis* determines the effect on the optimal solution of *changes in parameter values* of the objective function and constraint equations
- Changes may be reactions to anticipated uncertainties in the parameters or to new or changed information concerning the model

Beaver Creek Pottery Sensitivity Analysis

Maximize $Z = \$40x_1 + \$50x_2$
 subject to: $x_1 + 2x_2 \leq 40$
 $4x_1 + 3x_2 \leq 120$
 $x_1, x_2 \geq 0$

Optimal Solution Point

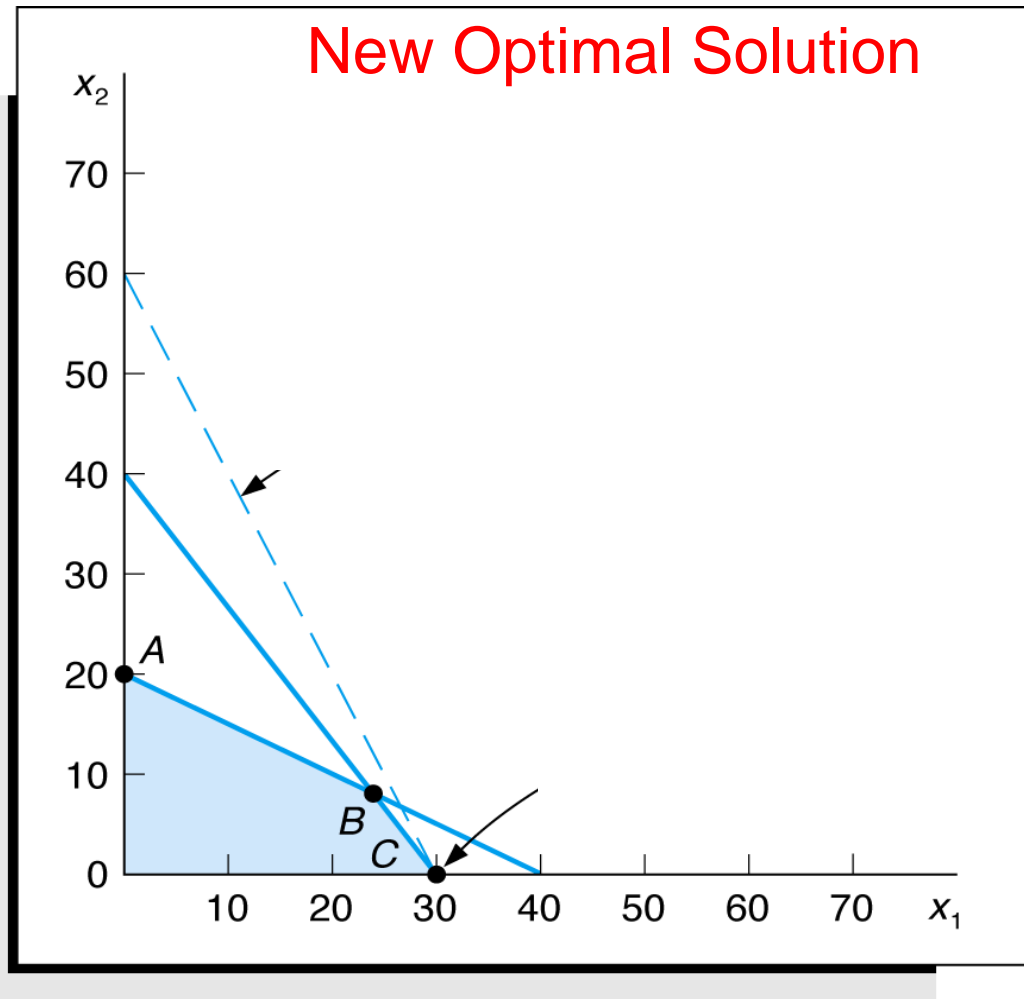
**Let's look at the
sensitivity of the
objective function
coefficients first**



Beaver Creek Pottery

Change x_1 Objective Function Coefficient

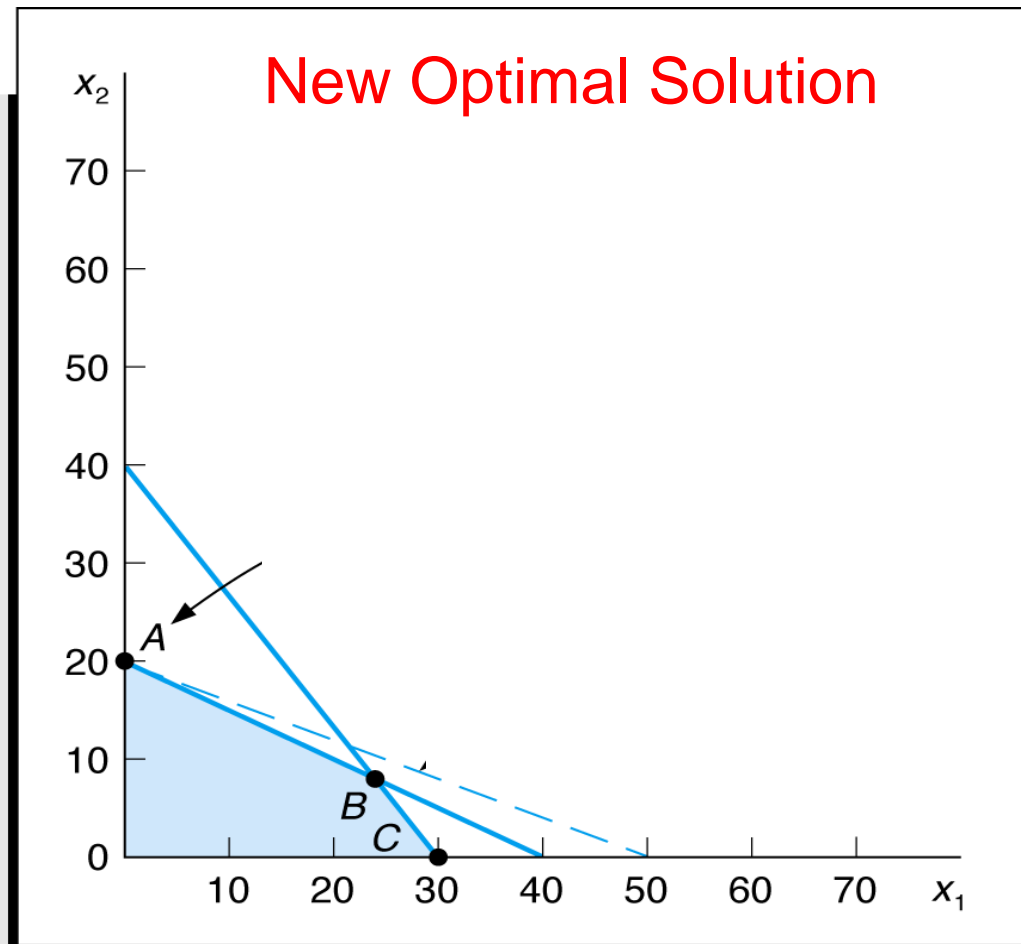
Maximize $Z = \$100x_1 + \$50x_2$
 subject to: $x_1 + 2x_2 \leq 40$
 $4x_1 + 3x_2 \leq 120$
 $x_1, x_2 \geq 0$



Beaver Creek Pottery

Change x_2 Objective Function Coefficient

Maximize $Z = \$40x_1 + \$100x_2$
 subject to: $x_1 + 2x_2 \leq 40$
 $4x_1 + 3x_2 \leq 120$
 $x_1, x_2 \geq 0$



Objective Function Coefficient - Sensitivity Range

- The *sensitivity range* for an objective function coefficient is the *range of values* over which the current optimal solution point will *remain optimal – that is, the value of the decision variables remains the same*
- The sensitivity range for the x_i coefficient is designated as c_i

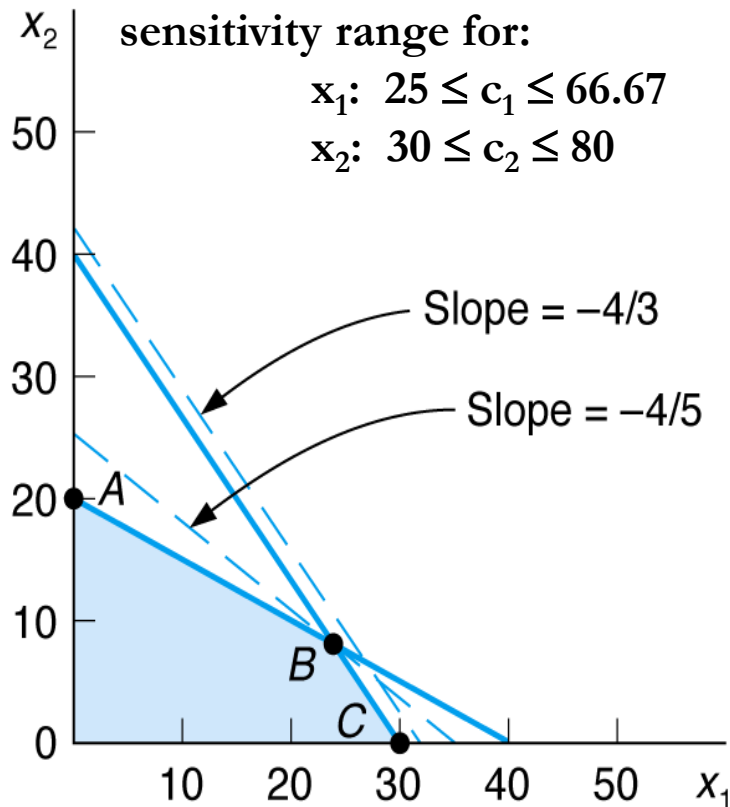
Objective Function Coefficient Sensitivity Range for c_1 and c_2

objective function $Z = \$40x_1 + \$50x_2$

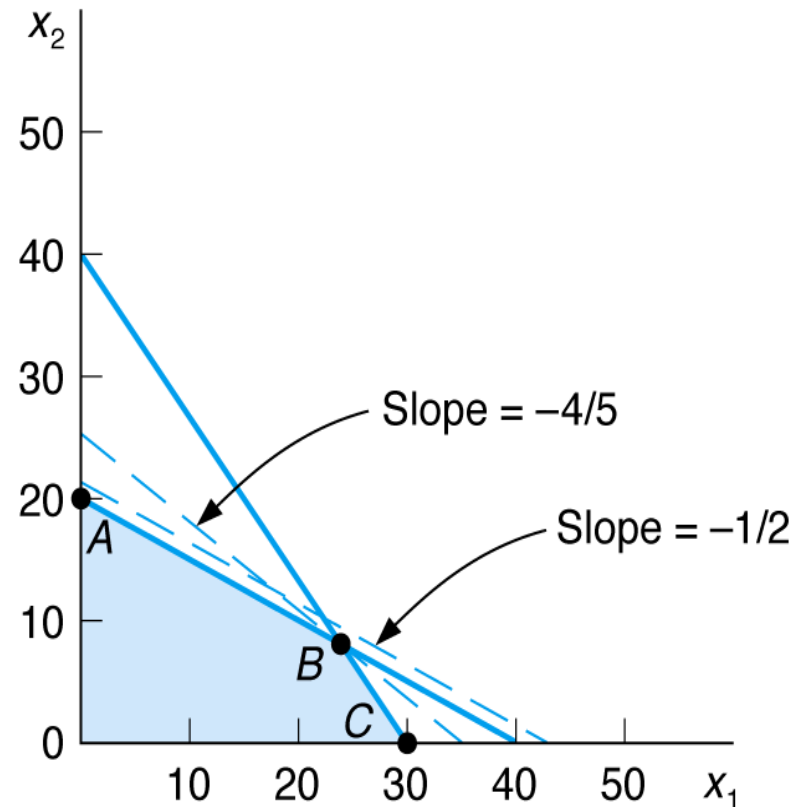
sensitivity range for:

$$x_1: 25 \leq c_1 \leq 66.67$$

$$x_2: 30 \leq c_2 \leq 80$$



(a)



(b)

Objective Function Coefficient Minimization Example

Minimize $Z = \$6x_1 + \$3x_2$
subject to:

$$2x_1 + 4x_2 \geq 16$$

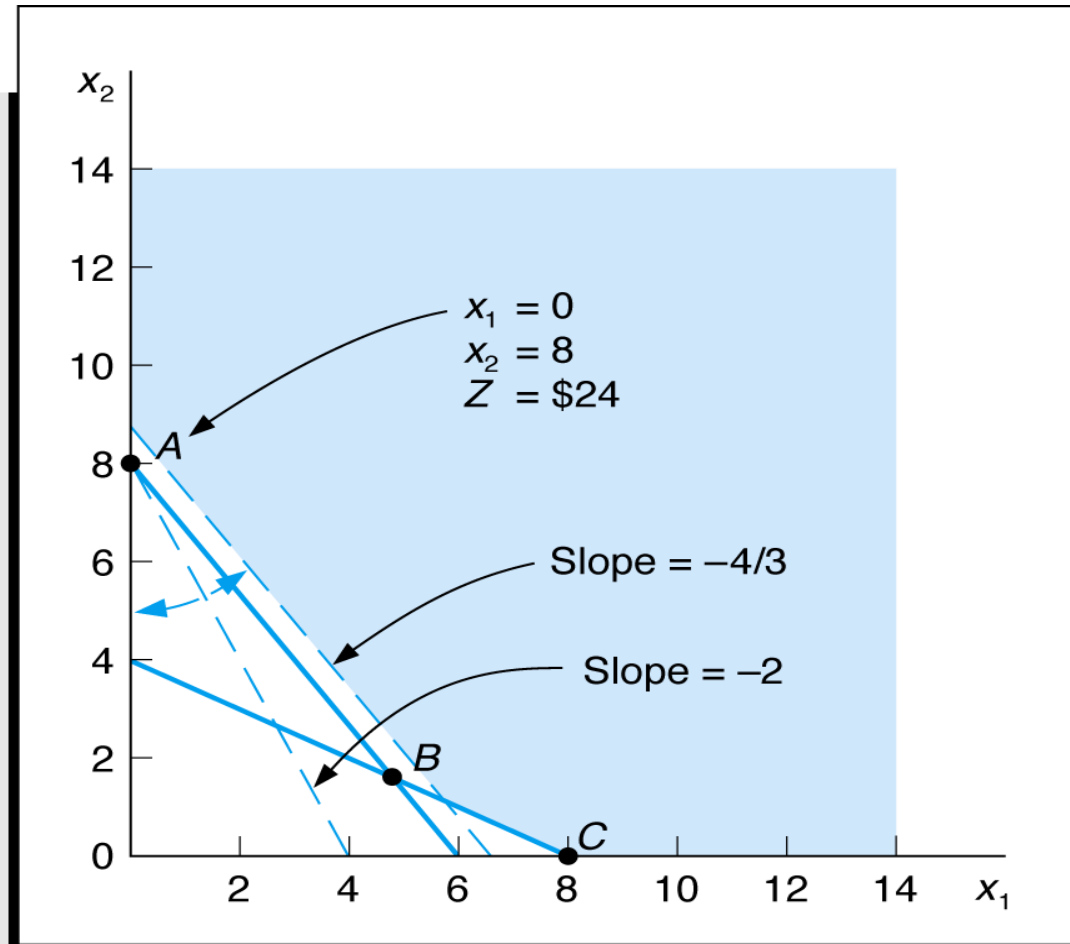
$$4x_1 + 3x_2 \geq 24$$

$$x_1, x_2 \geq 0$$

sensitivity ranges:

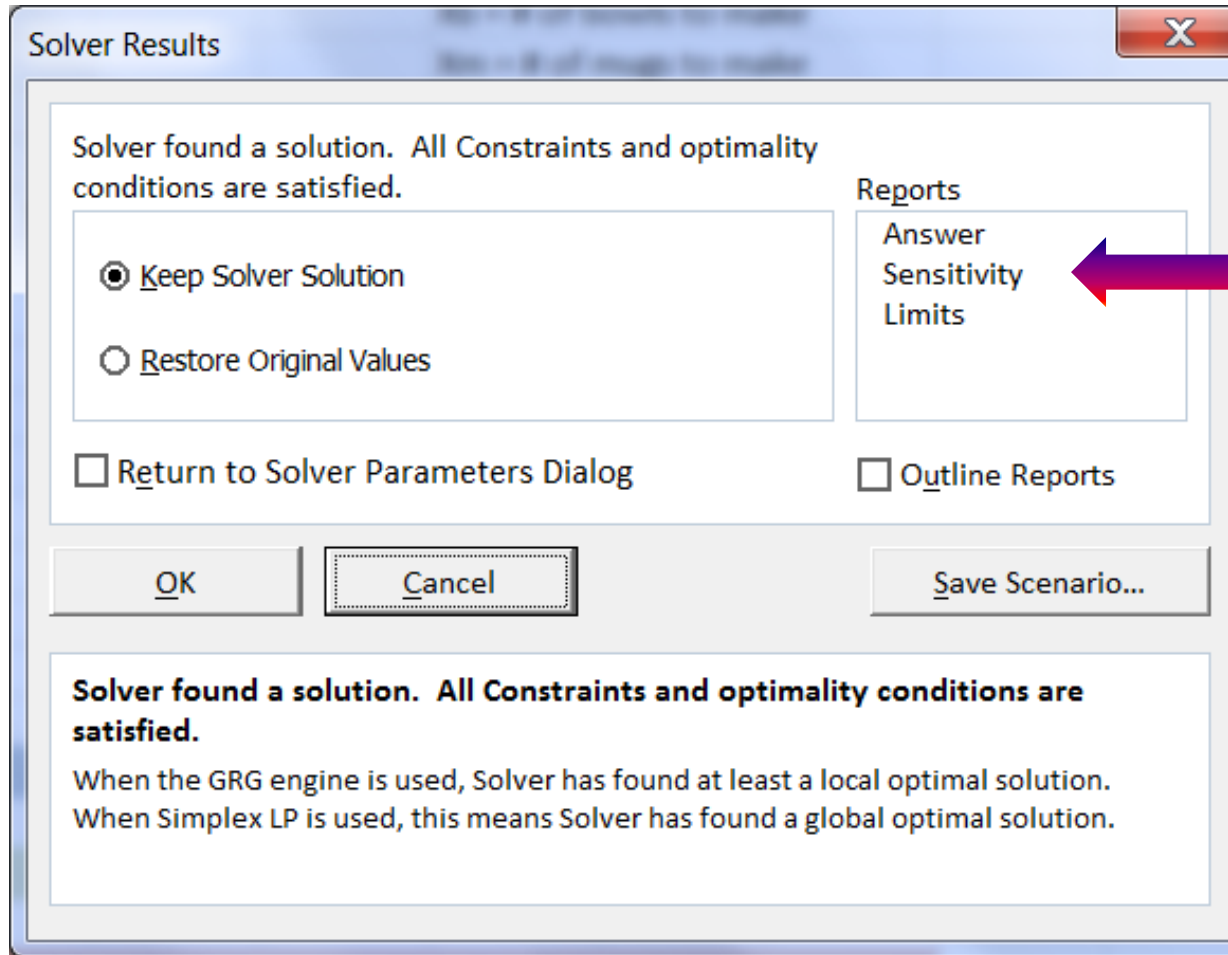
$$4 \leq c_1 \leq \infty$$

$$0 \leq c_2 \leq 4.5$$



The Sensitivity Report

Excel Solver Results



The image shows the 'Solver Results' dialog box in Microsoft Excel. The title bar reads 'Solver Results'. The main text area states: 'Solver found a solution. All Constraints and optimality conditions are satisfied.' Below this, there are two radio buttons: 'Keep Solver Solution' (which is selected) and 'Restore Original Values'. There are also two checkboxes: 'Return to Solver Parameters Dialog' and 'Outline Reports', both of which are currently unchecked. On the right side, there is a 'Reports' section with a list box containing 'Answer', 'Sensitivity', and 'Limits'. A purple arrow points from the text 'You can select more than 1 report at a time' to the 'Sensitivity' option in this list. At the bottom of the dialog, there are three buttons: 'OK', 'Cancel', and 'Save Scenario...'. A status bar at the very bottom of the dialog contains the text: 'Solver found a solution. All Constraints and optimality conditions are satisfied. When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.'

Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

☒ Keep Solver Solution
☐ Restore Original Values

☐ Return to Solver Parameters Dialog

☐ Outline Reports

Reports
Answer
Sensitivity
Limits

OK Cancel Save Scenario...

Solver found a solution. All Constraints and optimality conditions are satisfied.
When the GRG engine is used, Solver has found at least a local optimal solution.
When Simplex LP is used, this means Solver has found a global optimal solution.

You can
select
more than
1 report at
a time



Constraint Coefficient Ranges

Beaver Creek Example Sensitivity Report

Microsoft Excel - Exhibit3.1.xls

File Edit View Insert Format Tools Data Window Help Adobe PDF

100%

Arial 10 B I U

Reply with Changes... End Review...

F39

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2															
3															
4															
5															
6			Adjustable Cells												
7				Final	Reduced	Objective	Allowable	Allowable							
8		Cell	Name	Value	Cost	Coefficient	Increase	Decrease							
9		\$B\$10	Bowls =	24	0	40	26.67	15							
10		\$B\$11	Mugs =	8	0	50	30	20							
11															
12			Constraints												
13				Final	Shadow	Constraint	Allowable	Allowable							
14		Cell	Name	Value	Price	R.H. Side	Increase	Decrease							
15		\$E\$6	labor (hr/unit) Usage	40	16	40	40	10							
16		\$E\$7	clay (lb/unit) Usage	120	6	120	40	60							
17															

Sensitivity ranges for constraint quantity values

This is what Excel says...now let's dissect this picture



Microsoft Excel 12.0 Sensitivity Report

Worksheet: [Lecture4a.xlsx]BeaverCreekPottery

Report Created: 9/7/2010 9:01:34 AM

Note: When we say "change a coefficient in the objective function," we are assuming that the other coefficient(s) in the objective function are held constant. And remember, what we're looking at is the optimal point of 24 bowls and 8 mugs being made. Of course, the PROFIT will change when we change the objective function coefficients, but the NUMBER of bowls and mugs being made optimally WILL NOT change, if we stay within the green limits. Let's prove this! Look at the next tab, "Beaver Creek New Obj. Coef" and change the coefficients within the indicated ranges.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$D\$17	bowls	24	0	40	26.66666667	15
\$F\$17	cups	8	0	50	30	20

OBJECTIVE FUNCTION COEFFICIENTS

In order to keep the same optimal point, the bowl coefficient can be increased by \$26.66 and can be decreased by \$15. The mug coefficient can be increased by \$30 or decreased by \$20

The bowl obj. func. coefficient limits are: $\$25 < C_b < \66.666

The mug obj. func. coefficient limits are: $\$30 < C_c < \80

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$J\$21	Labor	40	16	40	40	10
\$J\$22	Clay	120	6	120	40	60

Objective Function Coefficient Sensitivity Range

QM for Windows

Sensitivity ranges
for objective
function coefficients

Ranging					
Beaver Creek Pottery Company Solution					
Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
X1	24.	0.	40.	25.	66.67
X2	8.	0.	50.	30.	80.
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Labor (hrs)	16.	0.	40.	30.	80.
Clay (lbs)	6.	0.	120.	60.	160.



Changes in Constraint Quantity Value Sensitivity Range

- The *sensitivity range for a right-hand-side* value is the range of values over which the quantity's value can change *without changing the solution variable mix, including the slack variables*
- This means the elements of the solution will remain constant – their numbers may change, but they will remain in the solution set

Changes in Constraint Quantity Values

Increasing the Labor Constraint

Beaver Creek Pottery Example

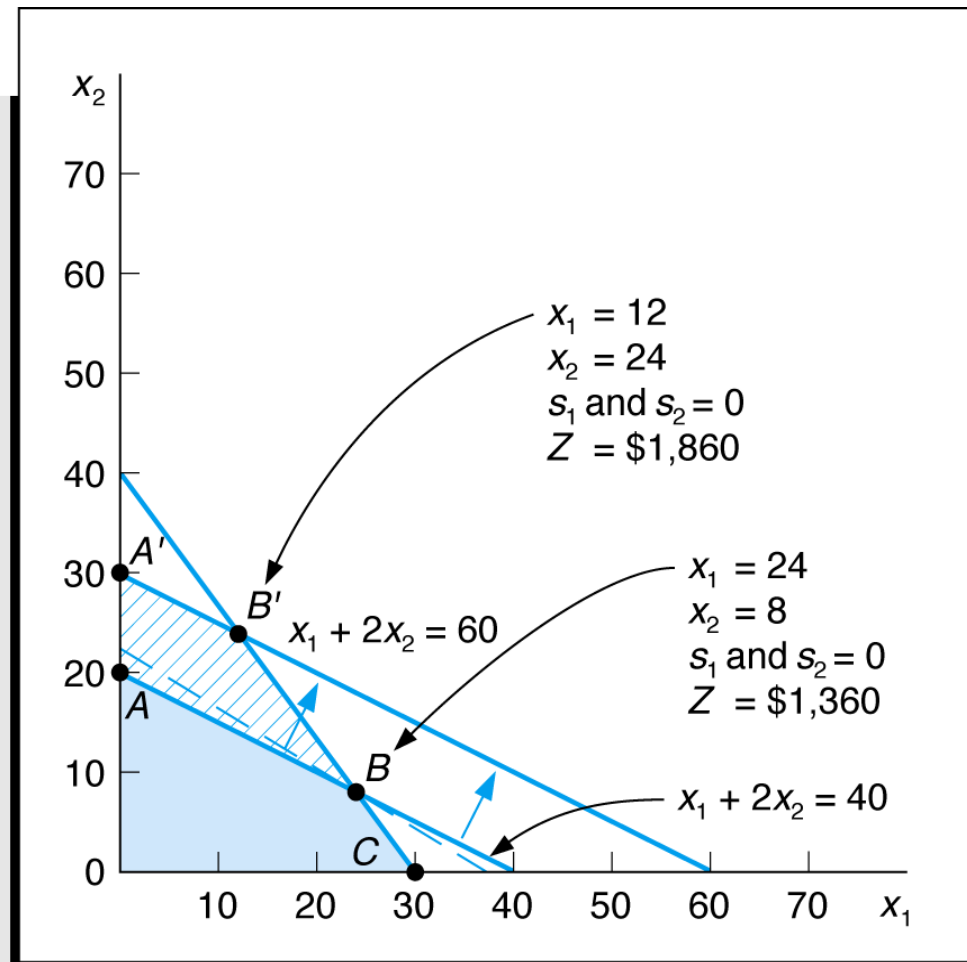
Maximize $Z = \$40x_1 + \$50x_2$

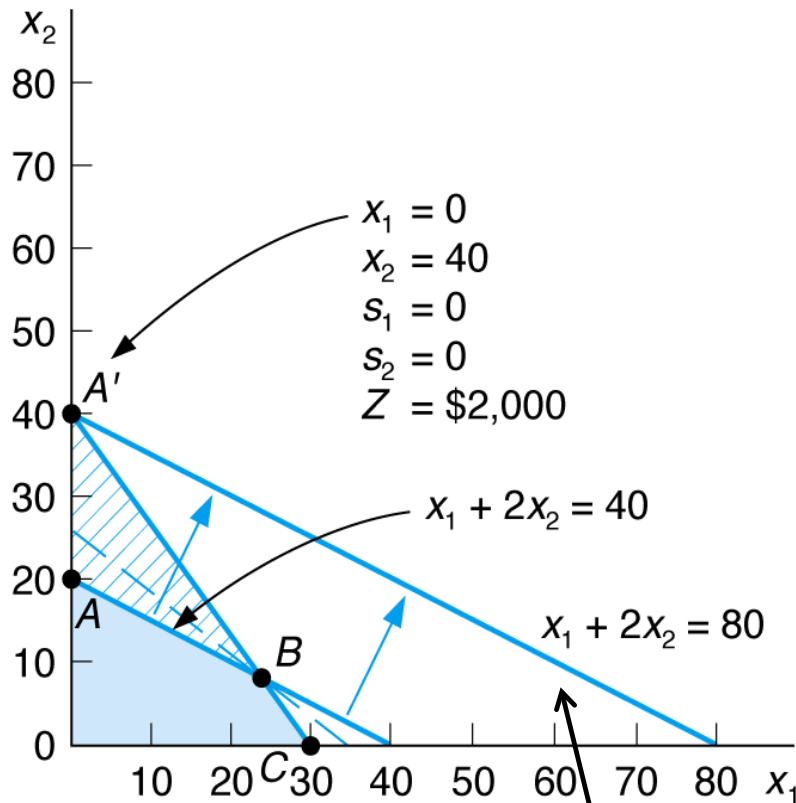
subject to:

$$x_1 + 2x_2 + s_1 = 40$$

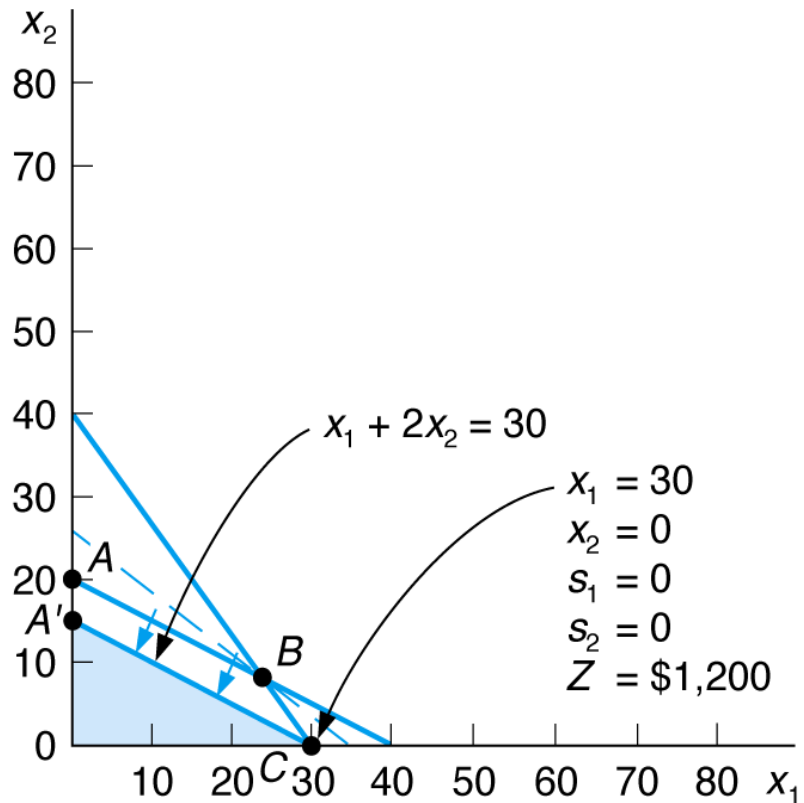
$$4x_1 + 3x_2 + s_2 = 120$$

$$x_1, x_2 \geq 0$$





(a)



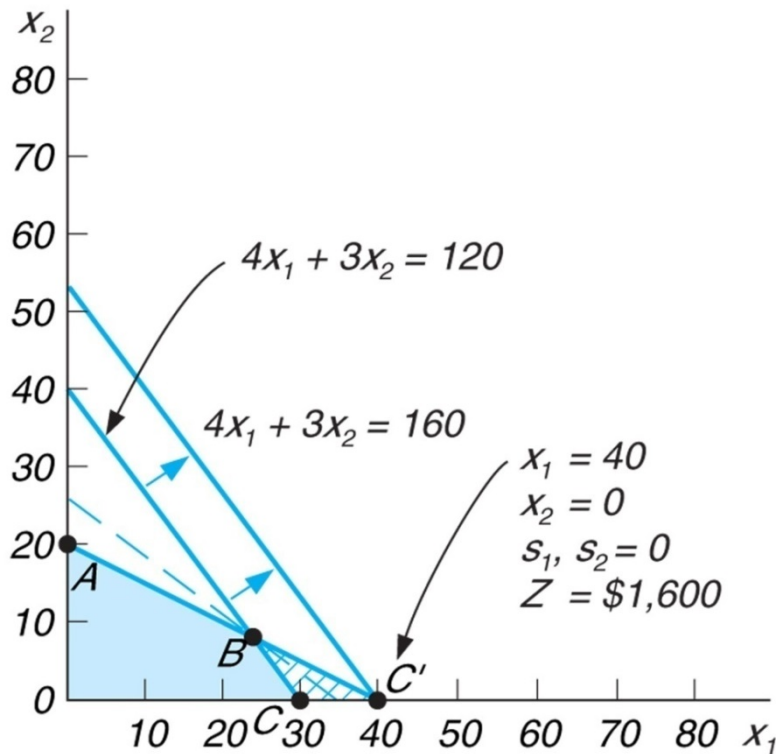
(b)

Increasing labor hrs from 40 to 80

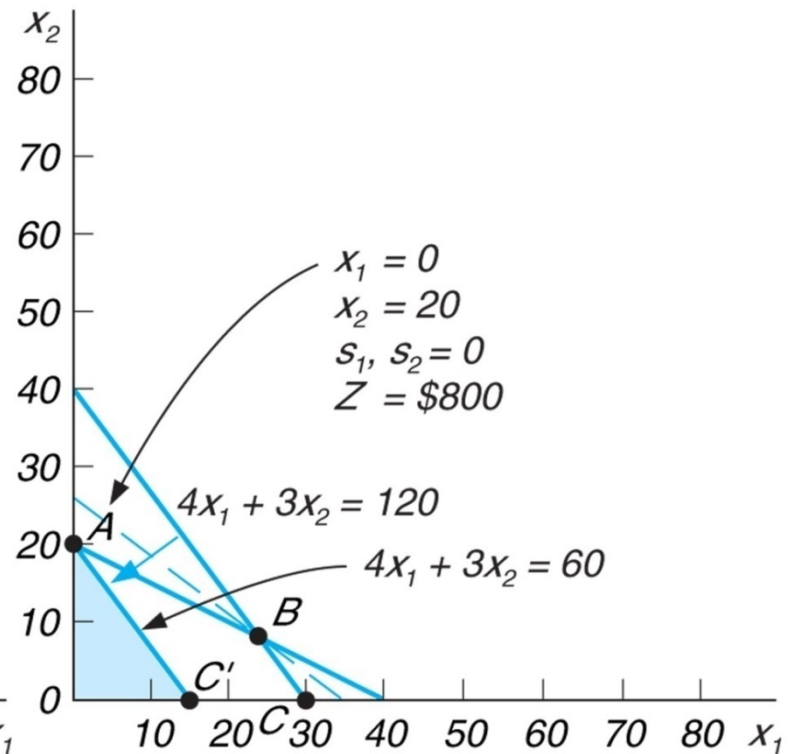
Decreasing labor hrs from 40 to 30

Changes in Constraint Quantity Values

Sensitivity Range for Clay Constraint



(a)



(b)

Increasing clay from 120 to 160

Decreasing clay from 120 to 60

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [Lecture4a.xlsx]BeaverCreekPottery
Report Created: 9/7/2010 9:01:34 AM

Note: When we say "change a coefficient in the objective function," we are assuming that the other coefficient(s) in the objective function are held constant. And remember, what we're looking at is the optimal point of 24 bowls and 8 mugs being made. Of course, the PROFIT will change when we change the objective function coefficients, but the NUMBER of bowls and mugs being made optimally WILL NOT change, if we stay within the green limits. Let's prove this! Look at the next tab, "Beaver Creek New Obj. Coef" and change the coefficients within the indicated ranges.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$D\$17	bowls	24	0	40	26.66666667	15
\$F\$17	cups	8	0	50	30	20

OBJECTIVE FUNCTION COEFFICIENTS

In order to keep the same optimal point, the bowl coefficient can be increased by \$26.66 and can be decreased by \$15. The mug coefficient can be increased by \$30 or decreased by \$20

The bowl obj. func. coefficient limits are: $\$25 < C_b < \66.666

The mug obj. func. coefficient limits are: $\$30 < C_c < \80

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$J\$21	Labor	40	16	40	40	10
\$J\$22	Clay	120	6	120	40	60

RHS Values for CONSTRAINTS

If available labor is between 30 (= 40 -10) and 80 (=40+80), shadow price of \$16.00 remains in effect

If available clay is between 60 (= 120 -60) and 160 (=120+40), shadow price of \$6.00 remains in effect

What this means is that, if you're within the limits for the RHS of the constraint equations, you're able to predict the impact of additional resources on the obj. function.



Constraint Quantity Value Ranges QM for Windows

Beaver Creek Pottery Company Solution					
Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
X1	24	0	40	25	66.6667
X2	8	0	50	30	80
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Labor (hrs)	16	0	40	30	80
Clay (lbs)	6	0	120	60	160

Sensitivity ranges
for constraint
quantity values

Shadow Prices (Dual Variable Values)

- Defined as the *marginal value* of one additional unit of resource
- The *sensitivity range* for a constraint quantity value is also the range over which the *shadow price is valid*

Beaver Creek Pottery Shadow Prices

Notice the shadow price for labor is \$16. What happens if we get an additional 4 people on the payroll, would that affect our profit and product mix, or not? If yes, how?

We currently have 5 people working 8 hrs/day = $5 * 8 = 40$ hrs/day of labor
This would make it 9 people or $9 * 8$ hrs = 72 hours/day of labor

Maximize $Z = \$40x_1 + \$50x_2$
subject to:

$$x_1 + 2x_2 \leq 40 \text{ hr of labor}$$

$$4x_1 + 3x_2 \leq 120 \text{ lb of clay}$$

$$x_1, x_2 \geq 0$$

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$10	Bowls =	24		40	26.67	15
\$B\$11	Mugs =	8		50	30	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$6	labor (hr/unit) Usage	40	16	40	40	10
\$E\$7	clay (lb/unit) Usage	120	6	120	40	60

Shadow prices
(dual values)



Microsoft Excel 12.0 Sensitivity Report

Worksheet: [Lecture4a.xlsx]BeaverCreekPottery

Report Created: 9/7/2010 9:01:34 AM

Note: When we say "change a coefficient in the objective function," we are assuming that the other coefficient(s) in the objective function are held constant. And remember, what we're looking at is the optimal point of 24 bowls and 8 mugs being made. Of course, the PROFIT will change when we change the objective function coefficients, but the NUMBER of bowls and mugs being made optimally WILL NOT change, if we stay within the green limits. Let's prove this! Look at the next tab, "Beaver Creek New Obj. Coef" and change the coefficients within the indicated ranges.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$D\$17	bowls	24	0	40	26.66666667	15
\$F\$17	cups	8	0	50	30	20

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The bowl obj. func. coefficient limits are: $\$25 < C_b < \66.666

The mug obj. func. coefficient limits are: $\$30 < C_c < \80

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$J\$21	Labor	40	16	40	40	10
\$J\$22	Clay	120	6	120	40	60

RHS Values for CONSTRAINTS

If available labor is between 30 (= 40 -10) and 80 (=40+80), shadow price of \$16.00 remains in effect

If available clay is between 60 (= 120 -60) and 160 (=120+40), shadow price of \$6.00 remains in effect

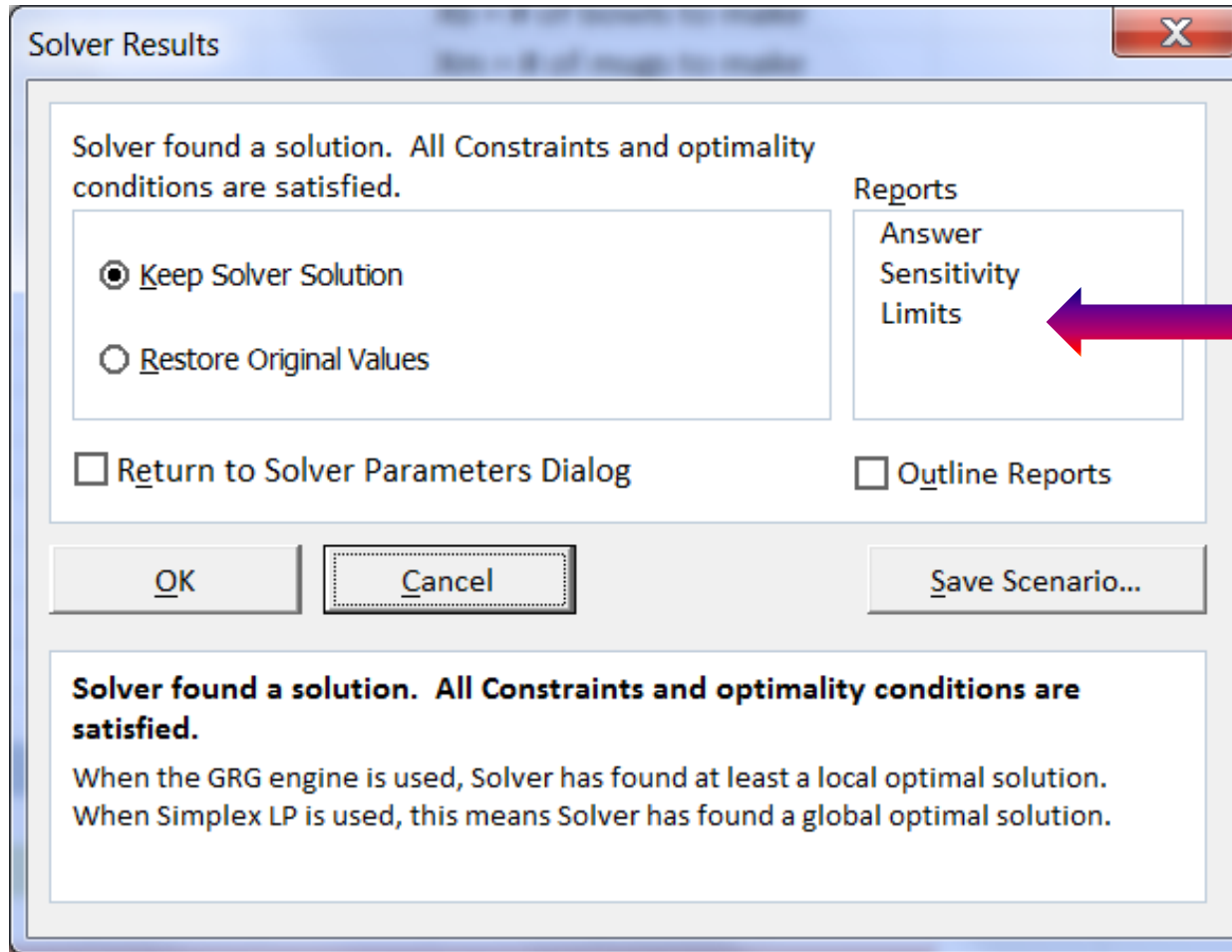
What this means is that, if you're within the limits for the RHS of the constraint equations, you're able to predict the impact of additional resources on the obj. function.

The shadow price for labor is \$16.00, which means that for every additional hour of labor, the profit will increase by \$16.00.

The shadow price for clay is \$6.00, meaning for every additional pound of clay obtained made, the profit will increase by \$6.00.

The Limits Report

Excel Solver Results



The image shows the 'Solver Results' dialog box in Microsoft Excel. The title bar reads 'Solver Results'. The main text area states: 'Solver found a solution. All Constraints and optimality conditions are satisfied.' Below this, there are two radio buttons: 'Keep Solver Solution' (which is selected) and 'Restore Original Values'. To the right, under the heading 'Reports', there is a list box containing 'Answer', 'Sensitivity', and 'Limits'. A purple arrow points to the 'Limits' option in this list. Below the list box is a checkbox labeled 'Outline Reports'. At the bottom of the dialog, there are three buttons: 'OK', 'Cancel', and 'Save Scenario...'. At the very bottom, there is a text box containing the following text: 'Solver found a solution. All Constraints and optimality conditions are satisfied. When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.'

Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

☒ Keep Solver Solution
☐ Restore Original Values

☐ Return to Solver Parameters Dialog

Reports

Answer
Sensitivity
Limits

☐ Outline Reports

OK Cancel Save Scenario...

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution.
When Simplex LP is used, this means Solver has found a global optimal solution.

You can
select
more than
1 report at
a time

The Limits Report

Microsoft Excel 12.0 Limits Report

Worksheet: [Lecture4a.xlsx]Limits Report 1

Report Created: 9/7/2010 3:17:06 PM

Target		
Cell	Name	Value
\$J\$19		1360.00

\$1,360 is the anticipated profit

Adjustable		
Cell	Name	Value
\$D\$17	bowls	24
\$F\$17	mugs	8

These are the values of the decision variables at the optimal point: 24 bowls, 8 mugs.

Lower Target		Upper Target		
Limit	Result	Limit	Result	
0	400	24	1360	These values represent the upper limit of the feasible space, which (not surprisingly) is the optimal solution.
0	960	8	1360	Both decision variables are held at their optimal point.

This represents what happens when the values of the decision variables are alternately set to lowest possible value

- If the $X_b = 0$ and $X_m = 8$, the profit is \$400

- If the $X_b = 24$ and $X_m = 0$, the profit is \$960



Other Forms of Sensitivity Analysis

These alterations require that you reformulate and resolve the problem:

- Changing individual constraint parameter coefficients

- Adding new constraints

- Adding new variables

Changing a Constraint Parameter

Original problem:

Maximize $Z = \$40x_1 + \$50x_2$

subject to: $x_1 + 2x_2 \leq 40$

$4x_1 + 3x_2 \leq 120$

$x_1, x_2 \geq 0$

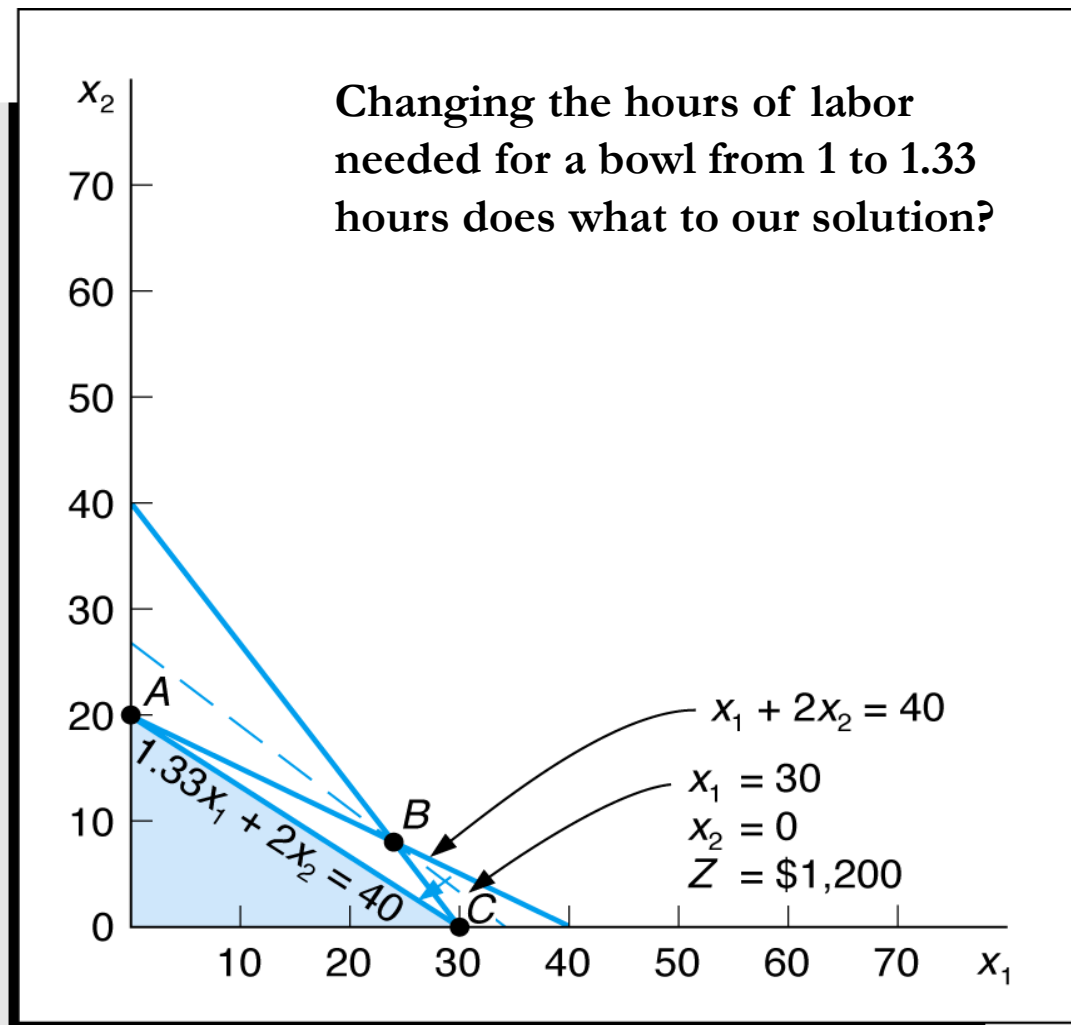
NEW problem:

Maximize $Z = \$40x_1 + \$50x_2$

subject to: **1.33** $x_1 + 2x_2 \leq 40$

$4x_1 + 3x_2 \leq 120$

$x_1, x_2 \geq 0$



Adding a New Constraint

Adding a new constraint to Beaver Creek Model:

$$0.20x_1 + 0.10x_2 \leq 5 \text{ hours for packaging}$$

Original solution: 24 bowls, 8 mugs, \$1,360 profit

Exhibit3.17.xlsx [Compatibility Mode] - Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	The Beaver Creek Pottery Company												
2													
3	Products:		Bowl	Mug									
4	Profit per unit:		40	50									
5	Resources:				Usage	Constraints	Available	Left over					
6	labor (hr/unit)		1	2	40	<=	40	0					
7	clay (lb/unit)		4	3	110	<=	110	0					
8	packaging(hr/unit)		0.2	0.1	5	<=	5	0					
9													
10	Production:												
11	Bowls =	20											
12	Mugs =	10											
13	Profit =	1300											
14													

Added constraint for packaging

Constraint E8 ≤ G8 added to Solver

Adding a New Variable

Adding a new variable to the Beaver Creek model, x_3 , for a third product, cups

Maximize $Z = \$40x_1 + 50x_2 + 30x_3$

subject to:

$$x_1 + 2x_2 + 1.2x_3 \leq 40 \text{ hr of labor}$$

$$4x_1 + 3x_2 + 2x_3 \leq 120 \text{ lb of clay}$$

$$x_1, x_2, x_3 \geq 0$$

Solving model shows that this particular change has no effect on the original solution (i.e., the model is not sensitive to this change)



There are 4 Categories of Solutions

- Unique - single solution that optimizes the problem
- Alternate - more than one solution optimizes the problem (great from a managerial perspective)
- Unbounded - a better solution can always be found
- Infeasible - the problem cannot be solved based on current model, so back to the drawing board

- Two airplane parts: no.1 (profit \$650), no. 2 (profit \$910)
- Three manufacturing stages (hrs):

	<u>stamping</u>	<u>drilling</u>	<u>finishing</u>
▶ No.1	4	6.2	9.1
▶ <u>No. 2</u>	<u>7.5</u>	<u>4.9</u>	<u>4.1</u>
▶ Avail hrs	105	90	110

**Set up and
solve this
problem**

- Decision variables:
 - ▶ x_1 (number of part no. 1 to produce)
 - ▶ x_2 (number of part no. 2 to produce)

**You're CEO of a huge
manufacturing company,
Airplane Parts 1 & 2, Inc.**

- Model: Maximize $Z = \$650x_1 + \$910x_2$

- subject to:

$$4x_1 + 7.5x_2 \leq 105 \quad (\text{stamping, hr})$$

$$6.2x_1 + 4.9x_2 \leq 90 \quad (\text{drilling, hr})$$

$$9.1x_1 + 4.1x_2 \leq 110 \quad (\text{finishing, hr})$$

$$x_1, x_2 \geq 0$$



Check out the Lecture Problems!