CHAPTER 12: INTEGER PROGRAMMING

12.1-1.

(a) $x_j = \begin{cases} 1 & \text{if the decision is to build a factory in city } j, \\ 0 & \text{otherwise} \end{cases}$

$y_j = \begin{cases} 1 & \text{if the decision is to build a factory in city } j, \\ 0 & \text{otherwise} \end{cases}$

for $j =$ LA, SF, SD.

maximize $\quad \text{NPV} = 9x_{\text{LA}} + 5x_{\text{SF}} + 7x_{\text{SD}} + 6y_{\text{LA}} + 4y_{\text{SF}} + 5y_{\text{SD}}$

subject to

$$6x_{\text{LA}} + 3x_{\text{SF}} + 4x_{\text{SD}} + 5y_{\text{LA}} + 2y_{\text{SF}} + 3y_{\text{SD}} \leq 10$$
$$y_{\text{LA}} + y_{\text{SF}} + y_{\text{SD}} \leq 1$$
$$-x_{\text{LA}} + y_{\text{LA}} \leq 0$$
$$-x_{\text{SF}} + y_{\text{SF}} \leq 0$$
$$-x_{\text{SD}} + y_{\text{SD}} \leq 0$$
$$x_{\text{LA}}, x_{\text{SF}}, x_{\text{SD}}, y_{\text{LA}}, y_{\text{SF}}, y_{\text{SD}} \text{ binary}$$

(b) - (c)

California Manufacturing Co. Facility Location Problem

NPV ($millions)	Los Angeles	San Francisco	San Diego			
Warehouse	6	4	5			
Factory	8	5	7			
Capital Required ($millions)	Los Angeles	San Francisco	San Diego			
Warehouse	5	2	3	Capital Spent		Capital Available
Factory	6	3	4	10	<=	10
				Total		Maximum
Build?	Los Angeles	San Francisco	San Diego	Warehouses		Warehouses
Warehouse	0	0	1	1	<=	1
	<=	<=	<=			
Factory	0	1	1			
		Total NPV ($millions)	17			

12.1-2.

(a) $M_j = \begin{cases} 1 & \text{if } j \text{ does marketing,} \\ 0 & \text{otherwise} \end{cases}$ $C_j = \begin{cases} 1 & \text{if } j \text{ does cooking,} \\ 0 & \text{otherwise} \end{cases}$

$D_j = \begin{cases} 1 & \text{if } j \text{ does dishwashing,} \\ 0 & \text{otherwise} \end{cases}$ $L_j = \begin{cases} 1 & \text{if } j \text{ does laundry,} \\ 0 & \text{otherwise} \end{cases}$

for $j = \text{E (Eve), S (Steven)}$.

min $\quad \text{T} = 4.5M_\text{E} + 7.8C_\text{E} + 3.6D_\text{E} + 2.9L_\text{E} + 4.9M_\text{S} + 7.2C_\text{S} + 4.3D_\text{S} + 3.1L_\text{S}$

st
$$\begin{aligned}
&M_\text{E} + C_\text{E} + D_\text{E} + L_\text{E} = 2 \\
&M_\text{S} + C_\text{S} + D_\text{S} + L_\text{S} = 2 \\
&M_\text{E} + M_\text{S} = 1 \\
&C_\text{E} + C_\text{S} = 1 \\
&D_\text{E} + D_\text{S} = 1 \\
&L_\text{E} + L_\text{S} = 1 \\
&M_\text{E}, M_\text{S}, C_\text{E}, C_\text{S}, D_\text{E}, D_\text{S}, L_\text{E}, L_\text{S} \text{ binary}
\end{aligned}$$

(b) - (c)

Time Needed (hours)							
	Marketing	Cooking	Dishwashing	Laundry			
Eve	4.5	7.8	3.6	2.9			
Steven	4.9	7.2	4.3	3.1			
Does Task?					Tasks		
	Marketing	Cooking	Dishwashing	Laundry	Performed		
Eve	1	0	1	0	2	=	2
Steven	0	1	0	1	2	=	2
Total	1	1	1	1			
	=	=	=	=			Total Time (hours)
	1	1	1	1			18.4

12.1-3.

(a) $x_j = \begin{cases} 1 & \text{if the decision is to invest in project } j, \\ 0 & \text{otherwise} \end{cases}$

for $j = 1, 2, 3, 4, 5$.

maximize $\quad \text{NPV} = x_1 + 1.8x_2 + 1.6x_3 + 0.8x_4 + 1.4x_5$

subject to $\quad 6x_1 + 12x_2 + 10x_3 + 4x_4 + 8x_5 \leq 20$

$\quad x_1, x_2, x_3, x_4, x_5 \text{ binary}$

(b) - (c)

	Project 1	Project 2	Project 3	Project 4	Project 5			
Estimated Profit	1	1.8	1.6	0.8	1.4			
($million)						Capital		Capital
						Spent		Available
	Capital Required for Project ($million)					($million)		($million)
Capital	6	12	10	4	8	20	<=	20
								Total Profit
	Project 1	Project 2	Project 3	Project 4	Project 5			($million)
Undertake?	1	0	1	1	0			3.4

12.1-4.

(a) $x_j = \begin{cases} 1 & \text{if the decision is to invest in opportunity } j, \\ 0 & \text{otherwise} \end{cases}$

for $j = 1, 2, 3, 4, 5, 6$.

Let p_j denote the estimated profit of opportunity j and c_j the capital required for opportunity j in millions of dollars.

$$\text{maximize} \quad \sum_{j=1}^{6} x_j p_j$$

$$\begin{aligned} \text{subject to} \quad & \sum_{j=1}^{6} x_j c_j \leq 100 \\ & x_1 + x_2 \leq 1 \\ & x_3 + x_4 \leq 1 \\ & x_3 \leq x_1 + x_2 \\ & x_4 \leq x_1 + x_2 \\ & x_j \text{ binary, for } j = 1, \ldots, 6 \end{aligned}$$

(b) Solution: Invest in opportunities 1, 3 and 5.

	Investment Opportunity					
	1	2	3	4	5	6
Estimated Profit	15	12	16	18	9	11
Capital Required	38	33	39	45	23	27
Invest or Not	1	0	1	0	1	0
Total Profit	40					
Total Capital Req.	100	100				
x1 +x2≤1	1	1				
x3 +x4≤1	1	1				
x3≤x1 +x2	1	1				
x4≤x1 +x2	0	1				

12.1-5.

Best Time	Stroke						
	Backstroke	Breaststroke	Butterfly	Freestyle			
Carl	37.7	43.4	33.3	29.2			
Chris	32.9	33.1	28.5	26.4			
David	33.8	42.2	38.9	29.6			
Tony	37.0	34.7	30.4	28.5			
Ken	35.4	41.8	33.6	31.1			
Assignments	Stroke				Total		
	Backstroke	Breaststroke	Butterfly	Freestyle	Assignments		Supply
Carl	0	0	0	1	1	<=	1
Chris	0	0	1	0	1	<=	1
David	1	0	0	0	1	<=	1
Tony	0	1	0	0	1	<=	1
Ken	0	0	0	0	0	<=	1
Total Assigned	1	1	1	1			
	=	=	=	=			Total Time
Demand	1	1	1	1			126.2

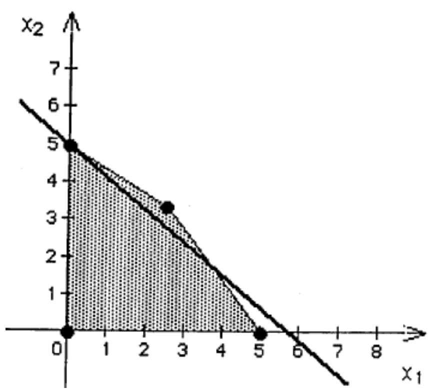
Each swimmer can swim only one stroke and each stroke can be assigned to only one swimmer.

12.1-6.

(a) Let T be the number of tow bars produced and S be the number of stabilizer bars produced.

$$\begin{aligned} \text{maximize} \quad & P = 130T + 150S \\ \text{subject to} \quad & 3.2T + 2.4S \leq 16 \\ & 2T + 3S \leq 15 \\ & T, S \geq 0 \text{ integers} \end{aligned}$$

(b) Optimal Solution: $(T, S) = (0, 5), P = \$750$



(c)

	Tow Bars	Stabilizer Bars			
Unit Profit	$130	$150			
	Resource Usage		Resource		Resource
	per Unit of Activity		Used		Available
Machine 1	3.2	2.4	12	<=	16
Machine 2	2	3	15	<=	15
	Activity 1	Activity 2			Total Profit
Level of Activity	0	5			$750.00

12.1-7.

(a) Let x_{ij} be the number of trucks hauling from pit i to site j and y_{ij} be the number of tons of gravel hauled from pit i to site j, for $i = N, S$ and $j = 1, 2, 3$.

$$\text{minimize } C = 400y_{N1} + 490y_{N2} + 460y_{N3} + 600y_{S1} + 530y_{S2} + 560y_{S3} + 150(x_{N1} + x_{N2} + x_{N3} + x_{S1} + x_{S2} + x_{S3})$$

subject to

$$y_{N1} + y_{N2} + y_{N3} \leq 18$$
$$y_{S1} + y_{S2} + y_{S3} \leq 14$$
$$y_{ij} \leq 5x_{ij}, \text{ for } i = N, S \text{ and } j = 1, 2, 3$$
$$y_{N1} + y_{S1} \geq 10$$
$$y_{N2} + y_{S2} \geq 5$$
$$y_{N3} + y_{S3} \geq 10$$
$$y_{ij} \geq 0,\ x_{ij} \geq 0 \text{ integers, for } i = N, S \text{ and } j = 1, 2, 3$$

(b)

Hauling Cost										
per Ton	Site 1	Site 2	Site 3							
North	$100	$190	$160		Cost per Truck	$50				
South	$180	$110	$140		Capacity per Truck (tons)	5				
Tons of Gravel	Site 1	Site 2	Site 3	Total						
North	10	0	1	11	<=	18				
South	0	5	9	14	<=	14				
Total	10	5	10							
	>=	>=	>=							
	10	5	10		**Tons of Gravel**	<=	**Truck Capacity**	Site 1	Site 2	Site 3
							North	10	0	5
Trucks	Site 1	Site 2	Site 3				South	0	5	10
North	2	0	1		Total Cost					
South	0	1	2		$3,270					

12.2-1.

Answers will vary.

12.2-2.

Answers will vary.

12.2-3.

Answers will vary.

12.2-4.

Answers will vary.

12.3-1.

(a) Let M be a very large number, say 100 million.

max $\quad 70x_1 - 50,000y_1 + 60x_2 - 40,000y_2 + 90x_3 - 70,000y_3 + 80x_4 - 60,000y_4$

st
$$y_1 + y_2 + y_3 + y_4 \leq 2$$
$$y_3 \leq y_1 + y_2$$
$$y_4 \leq y_1 + y_2$$
$$5x_1 + 3x_2 + 6x_3 + 4x_4 \leq 6000 + My_5$$
$$4x_1 + 6x_2 + 3x_3 + 5x_4 \leq 6000 + M(1 - y_5)$$
$$0 \leq xi \leq My_i, \text{ for } i = 1, 2, 3, 4$$
$$y_i \text{ binary, for } i = 1, 2, 3, 4$$

(b)

	Product 1	Product 2	Product 3	Product 4				
Start-up Cost	$50,000	$40,000	$70,000	$60,000				
Marginal Revenue	$70	$60	$90	$80				
							Modified	
					Resource		Resource	Resource
	Resource Used per Unit Produced				Used		Available	Available
Constraint 1	5	3	6	4	6,000	<=	6,000	6,000
Constraint 2	4	6	3	5	12,000	<=	15,999	6,000
	Product 1	Product 2	Product 3	Product 4				
Units Produced	0	2,000	0	0				
	<=	<=	<=	<=				
Only if Setup	0	9,999	0	0	Total		Max Products	
Setup?	0	1	0	0	1	<=	2	
			<=	<=				
		only if (1 or 2)	1	1			Revenue	$120,000
							Setup Cost	$40,000
Which Constraint (0 = Constraint 1, 1 = Constraint 2):				0			Total Profit	$80,000

12.3-2.

$x_1 - x_2 = 0y_1 + 3y_2 - 3y_3 + 6y_4 - 6y_5$, $y_i \in \{0, 1\}$, for $i = 1, \ldots, 5$.

12.3-3.

1. $\quad 3x_1 - x_2 - x_3 + x_4 \leq 12 + My_1$
$$x_1 + x_2 + x_3 + x_4 \leq 15 + M(1 - y_1)$$
$$y_1 \text{ binary}$$

2. $\quad 2x_1 + 5x_2 - x_3 + x_4 \leq 30 + My_2$
$$-x_1 + 3x_2 + 5x_3 + x_4 \leq 40 + My_3$$
$$3x_1 - x_2 + 3x_3 - x_4 \leq 60 + My_4$$
$$y_2 + y_3 + y_4 \leq 1$$
$$y_i \text{ binary, for } i = 2, 3, 4$$

12.3-4.

(a) Let y_1 and y_2 be binary variables that indicate whether or not toys 1 and 2 are produced. Let x_1 and x_2 be the number of toys 1 and 2 that are produced. Also, let z be 0 if factory 1 is used and 1 if factory 2 is used.

$$\begin{aligned}
\text{maximize} \quad & 10x_1 + 15x_2 - 50,000y_1 - 80,000y_2 \\
\text{subject to} \quad & x_1 \leq My_1 \\
& x_2 \leq My_2 \\
& \tfrac{1}{50}x_1 + \tfrac{1}{40}x_2 \leq 500 + Mz \\
& \tfrac{1}{40}x_1 + \tfrac{1}{25}x_2 \leq 700 + M(1-z) \\
& x_1, x_2 \geq 0 \text{ integers} \\
& y_1, y_2, z \text{ binary}
\end{aligned}$$

(b)

	Toy 1	Toy 2				
Start-up Cost	\$50,000	\$80,000				
Unit Profit	\$10	\$15				
					Modified	
	Hours Used per		Resource		Hours	Hours
	Unit Produced		Used		Available	Available
Factory 1	0.02	0.025	560	<=	10,499	500
Factory 2	0.025	0.04	700	<=	700	700
	Toy 1	Toy 2				
Units Produced	28,000	0				
	<=	<=			Gross Profit	\$280,000
Only if Setup	99,999	0			Setup Cost	\$50,000
Setup?	1	0			Net Profit	\$230,000
Which Factory (0 = Factory 1, 1 = Factory 2)?					1	

12.3-5.

(a) Let L, M, and S be the number of long-, medium-, and short-range jets to buy respectively.

$$\begin{aligned}
\text{maximize} \quad & P = 4.2L + 3M + 2.3S \\
\text{subject to} \quad & 67L + 50M + 35S \leq 1500 \\
& L + M + S \leq 30 \\
& \tfrac{5}{3}L + \tfrac{4}{3}M + S \leq 40 \\
& L, M, S \geq 0 \text{ integers}
\end{aligned}$$

(b)

	Long-Range	Medium-Range	Short-Range			
Unit Profit ($million)	4.2	3	2.3			
		Resource Usage per Unit of Activity		Resource Used		Resource Available
Money	67	50	35	1498	<=	1500
Pilots	1	1	1	30	<=	30
Maintenance	1.667	1.333	1	39.33333	<=	40
	Long-Range	Medium-Range	Short-Range			Total Profit
Level of Activity	14	0	16			95.6

(c)
$$L \leq \min\left\{\frac{1500}{67}, \frac{30}{1}, \frac{40}{5/3}\right\} = 24$$

$$M \leq \min\left\{\frac{1500}{50}, \frac{30}{1}, \frac{40}{4/3}\right\} = 30$$

$$S \leq \min\left\{\frac{1500}{35}, \frac{30}{1}, \frac{40}{1}\right\} = 30$$

$$L = 2^0 l_0 + 2^1 l_1 + 2^2 l_2 + 2^3 l_3 + 2^4 l_4$$

$$M = 2^0 m_0 + 2^1 m_1 + 2^2 m_2 + 2^3 m_3 + 2^4 m_4$$

$$S = 2^0 s_0 + 2^1 s_1 + 2^2 s_2 + 2^3 s_3 + 2^4 s_4$$

maximize $\quad P = 4.2\sum_{i=0}^{4} 2^i l_i + 3\sum_{i=0}^{4} 2^i m_i + 2.3\sum_{i=0}^{4} 2^i s_i$

subject to $\quad 67\sum_{i=0}^{4} 2^i l_i + 50\sum_{i=0}^{4} 2^i m_i + 35\sum_{i=0}^{4} 2^i s_i \leq 1500$

$$\sum_{i=0}^{4} 2^i l_i + \sum_{i=0}^{4} 2^i m_i + \sum_{i=0}^{4} 2^i s_i \leq 30$$

$$\frac{5}{3}\sum_{i=0}^{4} 2^i l_i + \frac{4}{3}\sum_{i=0}^{4} 2^i m_i + \sum_{i=0}^{4} 2^i s_i \leq 40$$

l_i, m_i, s_i binary, for $i = 0, 1, 2, 3, 4$

(d) Solution: $l_0 = l_4 = 0, l_1 = l_2 = l_3 = 1, \sum_{i=0}^{4} 2^i l_i = 14$

$m_0 = m_1 = m_2 = m_3 = m_4 = 0, \sum_{i=0}^{4} 2^i m_i = 0$

$s_0 = s_1 = s_2 = s_3 = 0, s_4 = 1, \sum_{i=0}^{4} 2^i s_i = 16$

$P = \$95.6$ (same as in (b))

12.3-6.

(a) $x_1 = y_{11} + 2y_{12}$, $x_2 = y_{21} + 2y_{22}$

$$\begin{array}{ll} \text{maximize} & Z = y_{11} + 2y_{12} + 5y_{21} + 10y_{22} \\ \text{subject to} & y_{11} + 2y_{12} + 10y_{21} + 20y_{22} \leq 20 \\ & y_{11} + 2y_{12} \leq 2 \\ & y_{ij} \text{ binary, for } i, j = 1, 2 \end{array}$$

(b) Solution: $y_{11} = y_{12} = 0 \Rightarrow x_1 = 0$, $y_{21} = 0$, $y_{22} = 1 \Rightarrow x_2 = 2$, $Z = 10$

12.3-7.

(a) Let x_i be the number of units to produce of product $i = 1, 2, 3$.

$$y_i = \begin{cases} 1 & \text{if product } i \text{ is produced,} \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{array}{ll} \text{maximize} & 2x_1 + 3x_2 + 0.8x_3 - 3y_1 - 2y_2 \\ \text{subject to} & 0.2x_1 + 0.4x_2 + 0.2x_3 \leq 1 \\ & x_1 \leq My_1 \\ & x_2 \leq My_2 \\ & 0 \leq x_1 \leq 3 \text{ integer} \\ & 0 \leq x_2 \leq 2 \text{ integer} \\ & 0 \leq x_3 \leq 5 \text{ integer} \\ & y_1, y_2 \text{ binary} \end{array}$$

(b)

	Customer 1	Customer 2	Customer 3			
Startup Cost ($million)	3	2	0	Capacity		Capacity
Marginal net Revenue ($million)	2	3	0.8	Used		Available
Capacity Used per Plane	20%	40%	20%	100%	<=	100%
Maximum Order	3	2	5			
Start Up?	0	1	1	Total Startup Cost		2
					Total Revenue	6.8
Planes to Produce	0	2	1		Total Profit	4.8
	<=	<=	<=			($million)
Maximum Order (if start up)	0	2	5			

12.4-1.

(a) $$y_{ij} = \begin{cases} 1 & \text{if } x_i = j \text{ (i.e., produce } j \text{ units of } i\text{),} \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2, 3$ and $j = 1, 2, 3, 4, 5$.

$$\begin{array}{ll} \text{max} & -y_{11} + 2y_{12} + 4y_{13} + y_{21} + 5y_{22} + y_{31} + 3y_{32} + 5y_{33} + 6y_{34} + 7y_{35} \\ \text{st} & y_{11} + y_{12} + y_{13} \leq 1 \\ & y_{21} + y_{22} \leq 1 \\ & y_{31} + y_{32} + y_{33} + y_{34} + y_{35} \leq 1 \\ & y_{11} + 2y_{12} + 3y_{13} + 2y_{21} + 4y_{22} + y_{31} + 2y_{32} + 3y_{33} + 4y_{34} + 5y_{35} \leq 5 \\ & y_{ij} \text{ binary} \end{array}$$

(b) Solution: $y_{ij} = 0$ except for $(i, j) = (3, 5)$, $y_{35} = 1 \Rightarrow x_3 = 5$, $Z = 7$

(c) $$y_{ij} = \begin{cases} 1 & \text{if } x_i \geq j, \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2, 3$ and $j = 1, 2, 3, 4, 5$.

$$\begin{array}{ll} \max & -y_{11} + 3y_{12} + 2y_{13} + y_{21} + 4y_{22} + y_{31} + 2y_{32} + 2y_{33} + y_{34} + y_{35} \\ \text{st} & y_{13} \leq y_{12} \leq y_{11} \\ & y_{22} \leq y_{21} \\ & y_{35} \leq y_{34} \leq y_{33} \leq y_{32} \leq y_{31} \\ & y_{11} + y_{12} + y_{13} + 2y_{21} + 2y_{22} + y_{31} + y_{32} + y_{33} + y_{34} + y_{35} \leq 5 \\ & y_{ij} \text{ binary} \end{array}$$

(d) Solution: $y_{ij} = 0$ for $i = 1, 2$, $y_{3j} = 1$ for $j = 1, \ldots, 5 \Rightarrow x_3 = 5$, $Z = 7$

12.4-2.

Introduce the binary variables y_1 and y_2 and add constraints $x_1 \leq My_1$, $x_2 \leq My_2$, $y_1 + y_2 = 1$.

12.4-3.

(a) Introduce the binary variables y_1, y_2, and y_3 to represent positive (nonzero) production levels.

$$\begin{array}{ll} \text{maximize} & Z = 50x_1 + 20x_2 + 25x_3 \\ \text{subject to} & 9x_1 + 3x_2 + 5x_3 \leq 500 \\ & 5x_1 + 4x_2 \leq 350 \\ & 3x_1 + 2x_3 \leq 150 \\ & x_3 \leq 20 \\ & x_1 \leq My_1,\ x_2 \leq My_2,\ x_3 \leq My_3 \\ & y_1 + y_2 + y_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \\ & y_1, y_2, y_3 \text{ binary} \end{array}$$

(b)

	Product 1	Product 2	Product 3			
Unit Profit	$50	$20	$25			
				Hours		Hours
	Machine Hours Used per Unit Produced			Used		Available
Milling machine	9	3	5	500	<=	500
Lathe	5	4	0	350	<=	350
Grinder	3	0	2	135.714	<=	150
	Product 1	Product 2	Product 3			Total Profit
Production Rate	45.238	30.952	0			$2,881
	<=	<=	<=			
Only if Produce	999	999	0	Total		
Produce?	1	1	0	2	<=	2
Produce 3 Production Rate		0	<=	20	Sales Potential	

12.4-4.

(a) $$y_{ij} = \begin{cases} 1 & \text{if } x_i = j, \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2$ and $j = 1, 2, 3$.

Work out by hand the objective function contribution for $x_1, x_2 = 0, 1, 2, 3$.

$$\begin{array}{ll} \text{maximize} & 3y_{11} + 8y_{12} + 9y_{13} + 9y_{21} + 24y_{22} + 9y_{23} \\ \text{subject to} & y_{11} + y_{12} + y_{13} \leq 1 \\ & y_{21} + y_{22} + y_{23} \leq 1 \\ & y_{11} + y_{23} \leq 1 \\ & y_{13} + y_{23} \leq 1 \\ & y_{12} + y_{23} \leq 1 \\ & y_{12} + y_{22} \leq 1 \\ & y_{13} + y_{22} \leq 1 \\ & y_{13} + y_{21} \leq 1 \\ & y_{ij} \text{ binary} \end{array}$$

(b) Solution: $y_{ij} = 0$ except $y_{11} = y_{22} = 1 \Rightarrow x_1 = 1$, $x_2 = 2$, $Z = 27$

(c) $$y_{ij} = \begin{cases} 1 & \text{if } x_i \geq j, \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2$ and $j = 1, 2, 3$.

Work out by hand the objective function contribution for $x_1, x_2 = 0, 1, 2, 3$.

$$\begin{array}{ll} \text{maximize} & 3y_{11} + 5y_{12} + y_{13} + 9y_{21} + 15y_{22} - 15y_{23} \\ \text{subject to} & y_{13} \leq y_{12} \leq y_{11} \\ & y_{23} \leq y_{22} \leq y_{21} \\ & y_{11} + y_{23} \leq 1 \\ & y_{12} + y_{22} \leq 1 \\ & y_{13} + y_{21} \leq 1 \\ & y_{ij} \text{ binary} \end{array}$$

(d) Solution: $y_{ij} = 0$ except $y_{11} = y_{21} = y_{22} = 1 \Rightarrow x_1 = 1$, $x_2 = 2$, $Z = 27$

12.4-5.

(a) $x_{ij} = \begin{cases} 1 & \text{if arc from node } i \text{ to node } j \text{ is in the shortest path,} \\ 0 & \text{otherwise} \end{cases}$

$$\begin{array}{lll} \text{min} & 3x_{12} + 6x_{13} + 6x_{24} + 5x_{25} + 4x_{34} + 3x_{35} + 3x_{46} + 2x_{56} & \\ \text{st} & x_{12} + x_{13} = 1 & (1) \\ & x_{24} + x_{25} + x_{34} + x_{35} = 1 & (2) \\ & x_{46} + x_{56} = 1 & (3) \\ & x_{24} + x_{25} \leq x_{12} & (4) \\ & x_{34} + x_{35} \leq x_{13} & (5) \\ & x_{46} \leq x_{24} + x_{34} & (6) \\ & x_{56} \leq x_{25} + x_{35} & (7) \\ & x_{ij} \text{ binary} & \end{array}$$

(1), (2), (3) ensure that exactly one arc is used at each stage and they represent mutually exclusive alternatives. (4), (5), (6) ensure that node i is left only if it is entered and they represent contingent decisions.

(b) Solution: $x_{ij} = 0$ except $x_{12} = x_{25} = x_{56} = 1$, $Z = 10$

Shortest path: $1 \to 2 \to 5 \to 6$

12.4-6.

(a) $y_j = \begin{cases} 1 & \text{if route } j \text{ is chosen,} \\ 0 & \text{otherwise} \end{cases}$

Let x_{ij} be the ijth element of the location/route matrix, for $i = A, \ldots, I$ and $j = 1, \ldots, 10$. Let c_j denote the cost of route j, for $j = 1, \ldots, 10$.

$$\begin{array}{ll} \text{minimize} & \sum_{j=1}^{10} c_j y_j \\ \text{subject to} & \sum_{j=1}^{10} x_{ij} y_j \geq 1, \text{ for } i = A, \ldots, I \\ & \sum_{j=1}^{10} y_j = 3 \\ & y_j \text{ binary, for } j = 1, \ldots, 10 \end{array}$$

(b)

	Route												
	1	2	3	4	5	6	7	8	9	10			
Time (hours)	6	4	7	5	4	6	5	3	7	6			
											Total on		
Location	Delivery Location on Route?										Route		
A	1				1				1		1	>=	1
B		1		1		1			1	1	1	>=	1
C			1	1			1		1		1	>=	1
D	1					1		1			1	>=	1
E			1	1		1					1	>=	1
F		1			1						1	>=	1
G	1						1	1		1	1	>=	1
H			1		1					1	1	>=	1
I		1		1			1				1	>=	1
	Route												
	1	2	3	4	5	6	7	8	9	10	Total		
Do Route?	0	0	0	1	1	0	0	1	0	0	3	<=	3
										Total Time (hours)			12

12.4-7.

$$x_{ij} = \begin{cases} 1 & \text{if tract } j \text{ is assigned to station located in tract } i, \\ 0 & \text{otherwise} \end{cases}$$

Let a_{ij} be the response time to a fire in tract j if that tract is served by a station located in tract i.

min $\quad 2\sum_{i=1}^{5} a_{i1}x_{i1} + \sum_{i=1}^{5} a_{i2}x_{i2} + 3\sum_{i=1}^{5} a_{i3}x_{i3} + \sum_{i=1}^{5} a_{i4}x_{i4} + 3\sum_{i=1}^{5} a_{i5}x_{i5}$

st $\quad \sum_{i=1}^{5} x_{ii} = 2$ (1) Two fire stations have to be located.

$\sum_{i=1}^{5} x_{ij} = 1$, for $j = 1, \ldots, 5$ (2) Each tract needs to be assigned to a station.

$x_{ij} \leq x_{ii}$, for $i = 1, \ldots, 5$ and $j = 1, \ldots, 5$ (3) Tract j can be assigned to the station tract i only if there is a station located in tract i.

x_{ij} binary

(1) and (2) correspond to mutually exclusive alternatives and (3) represent contingent decisions.

12.4-8.

(a) $$x_i = \begin{cases} 1 & \text{if a station is located in tract } i, \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{array}{ll} \text{minimize} & 200x_1 + 250x_2 + 400x_3 + 300x_4 + 500x_5 \\ \text{subject to} & x_1 + x_3 + x_5 \geq 1 \\ & x_1 + x_2 + x_4 \geq 1 \\ & x_2 + x_3 + x_5 \geq 1 \\ & x_2 + x_3 + x_4 + x_5 \geq 1 \\ & x_1 + x_3 + x_4 + x_5 \geq 1 \\ & x_i \text{ binary} \end{array}$$

(b) Yes, this is a set covering problem. The activities are locating stations and the characteristics are the fires. S_i is the set of all locations that could cover a fire in tract i, e.g., $S_1 = \{1, 5\}$. There has to be at least one station, so $\sum_{j \in S_i} x_j \geq 1$ for all i.

(c) Solution: $x_1 = x_2 = 1$, $Z =$ \$450 thousand

12.4-9.

$$x_j = \begin{cases} 1 & \text{if district } j \text{ is chosen,} \\ 0 & \text{otherwise} \end{cases}$$

Let y_j be auxiliary variables that are zero for all j, except for the index of the district with largest c_j that is chosen, y_j is 1.

$$\begin{array}{ll} \text{minimize} & \sum_{j=1}^{N} c_j y_j \\ \text{subject to} & \sum_{j=1}^{N} y_j = 1 \\ & \sum_{j=1}^{N} c_j y_j \geq c_i x_i, \text{ for } i = 1, \ldots, N \\ & \sum_{j=1}^{N} x_j = R \\ & \sum_{j=1}^{N} a_{ij} x_j = 1, \text{ for } i = 1, \ldots, D \\ & x_j, y_j \text{ binary} \end{array}$$

This is a set partitioning problem with additional constraints.

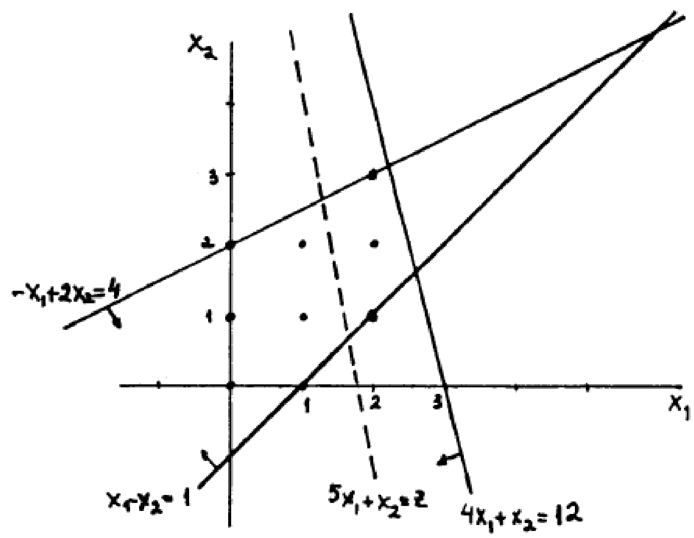
12.5-1.

This study uses integer programming to model employee scheduling problem of Taco Bell restaurants. In this integer program, the decision variables correspond to the number of employees scheduled to start working at time t and to work for s time units. The objective is to minimize the total payroll for the scheduling horizon. At any point in time, the labor requirements in each store have to be met. The total number of employees is bounded above. Without the upper bound, the problem could be solved efficiently as a network flow problem using out-of-kilter algorithms, so the upper bound is eliminated from the constraint set by using generalized Lagrange multipliers.

The new scheduling approach increased labor cost savings significantly. Additional benefits include enhanced flexibility, elimination of variability among stores, improved customer service and quality. Mathematical modeling served as a rational basis for the evaluation of new ideas, buildings, equipment and menu items. It also allowed Taco Bell to eliminate redundant tasks and to schedule balanced workloads. Consequently, productivity is improved and Taco Bell saved $13 million each year in labor costs.

12.5-2.

(a) The dots represent the feasible solutions in the graph below.



Optimal Solution: $(x_1, x_2) = (2, 3), Z = 5x_1 + x_2 = 13$

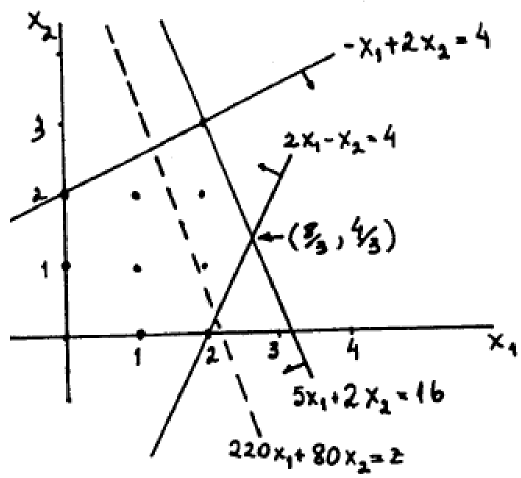
(b) The optimal solution of the LP relaxation is $(x_1, x_2) = (2.6, 1.6), Z = 14.6$. The nearest integer point is $(x_1, x_2) = (3, 2)$, which is not feasible, since $4 \cdot 3 + 2 > 12$.

Rounded Solutions	Violated Constraints	Z
$(3, 2)$	3rd	–
$(3, 1)$	2nd and 3rd	–
$(2, 2)$	none	12
$(2, 1)$	none	11

Hence, none of the feasible rounded solutions is optimal for the IP problem.

12.5-3.

(a) The dots represent the feasible solutions in the graph below.



Optimal Solution: $(x_1, x_2) = (2, 3), Z = 220x_1 + 80x_2 = 680$

(b) The optimal solution of the LP relaxation is $(x_1, x_2) = (8/3, 4/3), Z = 2080/3$. The nearest integer point is $(x_1, x_2) = (3, 1)$, which is not feasible, since $5 \cdot 3 + 2 \cdot 1 > 16$.

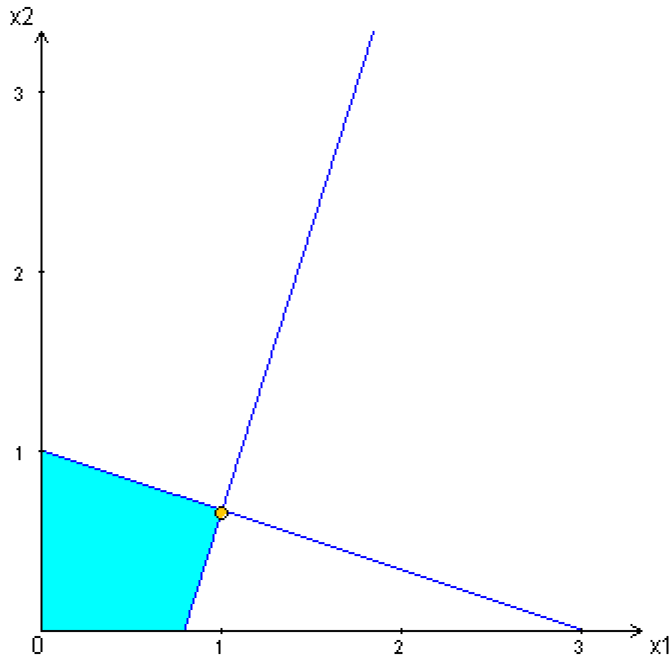
Rounded Solutions	Violated Constraints	Z
$(3, 2)$	2nd	–
$(3, 1)$	2nd and 3rd	–
$(2, 2)$	none	600
$(2, 1)$	none	520

Hence, none of the feasible rounded solutions is optimal for the IP problem.

12.5-4.

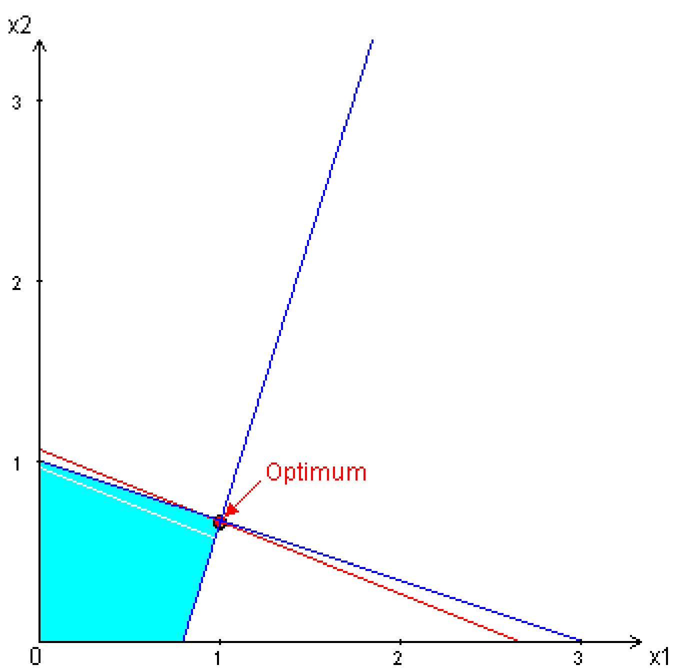
(a)

Solution	Feasible?	$P = 2x_1 + 5x_2$	Optimal?
$(0, 0)$	Yes	0	No
$(1, 0)$	No		
$(0, 1)$	Yes	5	Yes
$(1, 1)$	No		



Optimal Solution: $(x_1, x_2) = (0, 1)$

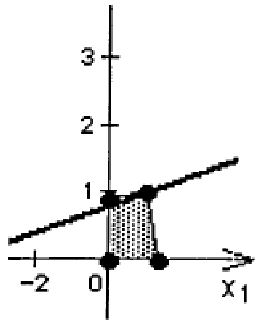
(b) The optimal solution of the LP relaxation is $(x_1, x_2) = (1, 0.667)$. The nearest integer point is $(x_1, x_2) = (1, 1)$, which is not feasible. The other rounded solution is $(1, 0)$, which is not feasible either.



12.5-5.

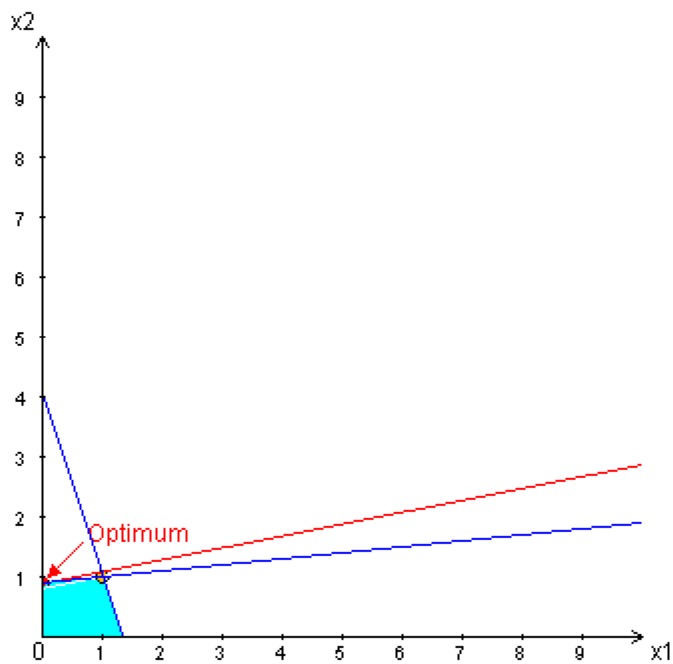
(a)

Solution	Feasible?	$P = -5x_1 + 25x_2$	Optimal?
$(0,0)$	Yes	0	No
$(1,0)$	Yes	-5	No
$(0,1)$	No		
$(1,1)$	Yes	20	Yes



Optimal Solution: $(x_1, x_2) = (1,1)$

(b) The optimal solution of the LP relaxation is $(x_1, x_2) = (0, 0.9)$. The nearest integer point is $(x_1, x_2) = (0,1)$, which is not feasible. The other rounded solution is $(0,0)$, which is feasible, but not optimal.



12.5-6.

(a) TRUE, Sec. 12.5.

(b) TRUE, Sec. 12.5.

(c) FALSE, the result need not be feasible, see Fig. 11.2 for a counterexample. Sec. 12.5, 11th paragraph explains this pitfall.

12.6-1.

```
                                        ,-- X4=1,X=(1,1,1,1,0.75),Z=6,F
                                  --- X3=1,X=(1,1,1,1,0.75),Z=6
                                 /        `-- X4=0,Inf.,F
                         --- X2=1,X=(1,1,0.86,1,1),Z=6.2857
                        /        \
                       /          --- X3=0,X=(1,1,0,0.5,1),Z=3.5,F
                      /
               --- X1=1,X=(1,1,0.86,1,1),Z=6.2857
              /       \
             /         \
            /           \
           /             --- X2=0,X=(1,0,0.67,1,0.83),Z=5.6667,F
          /
         /
        /
  X=(0.67,1,1,1,1),Z=6.3333
        \
         \
          \
           \
            \
             \
              \
               --- X1=0,X=(0,0,1,1,1),Z=6
```

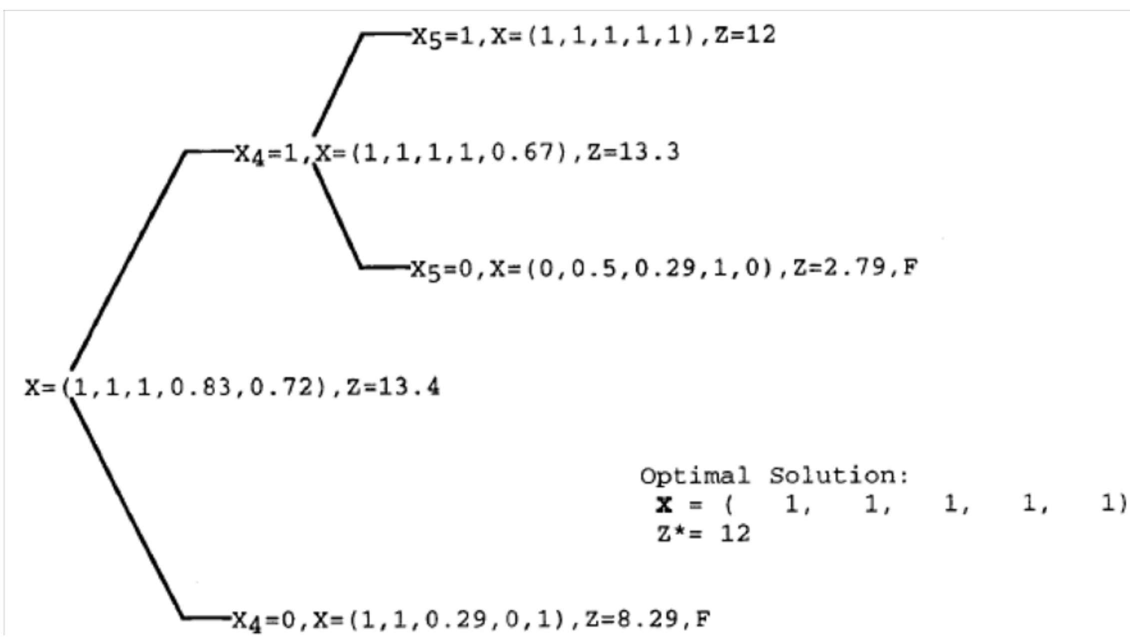
Optimal Solution: $(0, 0, 1, 1, 1)$, $Z = 6$

12.6-2.

```
                          --- X2=1,X=(1,1,1,0,0),Z=18,F
                         /
                        /
                       /
              --- X1=1,X=(1,0,0.67,0,0),Z=9.6667
             /         \
            /           \          --- X3=1,X=(1,0,1,0,0),Z=12
           /             \        /
          /               --- X2=0,X=(1,0,0.67,0,0),Z=9.6667
         /                        \
        /                          --- X3=0,X=(1,0,0,1,1),Z=22,F
       /
X=(0.5,0,0.5,0,0),Z=6
       \
        \
         \
          \
           \
            \
             \
              --- X1=0,Inf.,F
```

Optimal Solution: $(1, 0, 1, 0, 0)$, $Z = 12$

12.6-3.



Optimal Solution: $(1, 1, 1, 1, 1)$, $Z = 12$

12.6-4.

```
                      --- X2=1,Inf.,F
                     /
                    /
                   /
           --- X1=1,X=(1,0.5,0,0.9),Z=11
          /        \
         /          \
        /            \
       /              --- X2=0,X=(1,0,0,0.95),Z=10.5,F
      /
     /
    /
X=(0,1,0,0.9),Z=11
   \                                       ,-- X4=1,Inf.,F
    \                                 --- X3=1,X=(0,1,1,0.4),Z=11
     \                               /        `-- X4=0,X=(0,1,1,0),Z=7,F
      \                       --- X2=1,X=(0,1,0,0.9),Z=11
       \                     /       \        ,-- X4=1,Inf.,F
        \                   /         --- X3=0,X=(0,1,0,0.9),Z=11
         \                 /                  `-- X4=0,X=(0,1,0,0),Z=2,F
           --- X1=0,X=(0,1,0,0.9),Z=11
                   \
                    \
                     \
                      --- X2=0,X=(0,0,0,1),Z=10
```

Optimal Solution: $(0,0,0,1)$, $Z = 10$

12.6-5.

```
Optimal Solution:
  (X1, X2, X3, X4, X5) = (1, 1, 1, 0, 0)
  Z = 1250
```

12.6-6.

(a) FALSE. The feasible region for the IP problem is a subset of the feasible region for the LP relaxation. It is called a relaxation because it relaxes the feasible region.

(b) TRUE. If the optimal solution for the LP relaxation is integer, then it is feasible for the IP problem and since the solution for the latter cannot be better than the solution for the former, it has to be optimal.

(c) FALSE. Figure 12.2 is a counterexample for this statement.

12.6-7.

(a) Initialization: Set $Z^* = +\infty$. Apply the bounding and fathoming steps and the optimality test as described below for the whole problem. If the whole problem is not fathomed, then it becomes the initial subproblem for the first iteration below.

Iteration:

1. Branching: Choose the most recently created unfathomed subproblem (in case of a tie, select the one with the smallest bound). Among the assignees not yet assigned for the current subproblem, choose the first one in the natural ordering to be the branching variable. Subproblems correspond to each of the possible remaining assignments for the branching assignee. Form a subproblem for each remaining assignment by deleting the constraint that each of the unassigned assignees must perform exactly one assignment.

2. Bounding: For each new subproblem, obtain its bound by choosing the cheapest assignee for each remaining assignment and totaling the costs.

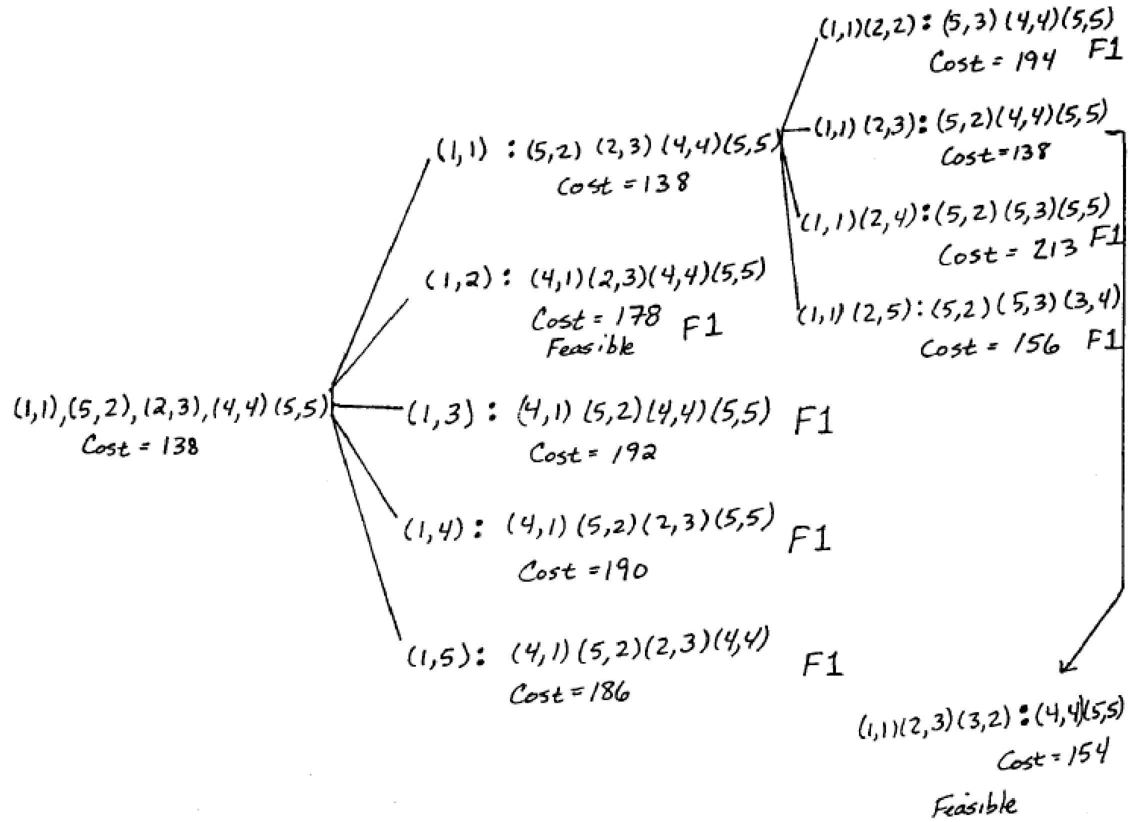
3. Fathoming: For each new subproblem, apply the two fathoming tests:

Test 1. bound $\geq Z^*$

Test 2. The optimal solution for its relaxation is a feasible assignment (If this solution is better than the incumbent, it becomes the new incumbent and Test 1 is reapplied to all unfathomed subproblems with the new smaller Z^*).

Optimality Test: Identical to the one given in the text.

(b) Matchings are indicated with the notation (assignee, assignment).



Optimal matching: $(1,1), (2,3), (3,2), (4,4), (5,5)$, with total cost 154.

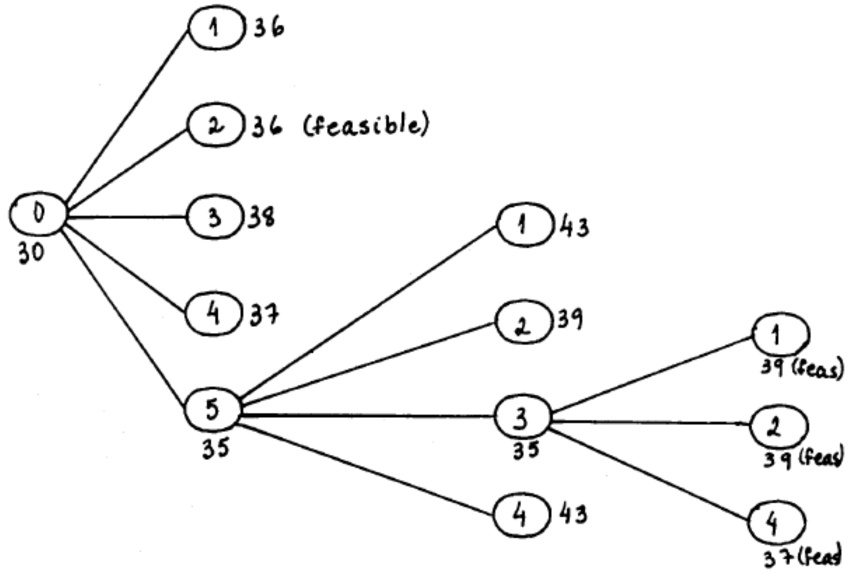
12.6-8.

(a) Branch Step: Use the best bound rule.

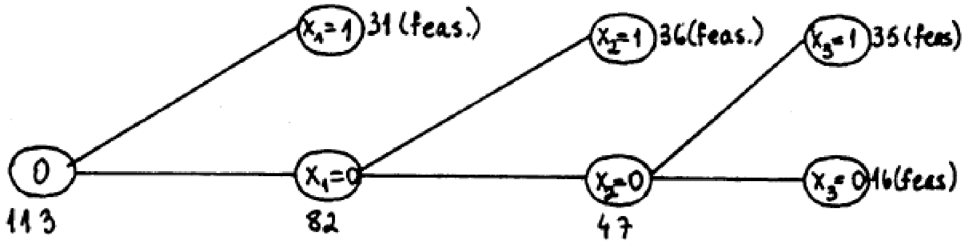
Bound Step: Given a partial sequencing $J_1, \ldots, J_k$ of the first k jobs, a lower bound on the time for the setup of the remaining $5-k$ jobs is found by adding the minimum elements of the columns corresponding to the remaining jobs, excluding those elements in rows "None", $J_1, J_2, \ldots, J_{k-1}$.

Fathoming Step: see the summary of the Branch-and-Bound technique in Sec. 12.6.

(b) The optimal sequence is $2-1-4-5-3$, with a total setup time of 36.



12.6-9.



Optimal Solution: $(x_1, x_2, x_3, x_4) = (0, 1, 1, 0)$, $Z^* = 36$

12.6-10.

(a) The only constraints of the Lagrangian relaxation are nonnegativity and integrality. Since **x** is feasible for an MIP problem, it already satisfies these constraints, so it is feasible for the corresponding Lagrangian relaxation.

(b) **x*** is feasible for an MIP problem, so from (a), it has to be feasible for its Lagrangian relaxation. Also, $A\mathbf{x}^* \leq b$ and $\lambda \geq 0$, so $Z_R^* \geq c\mathbf{x}^* - \lambda(A\mathbf{x}^* - b) \geq c\mathbf{x}^* = Z$.

12.7-1.

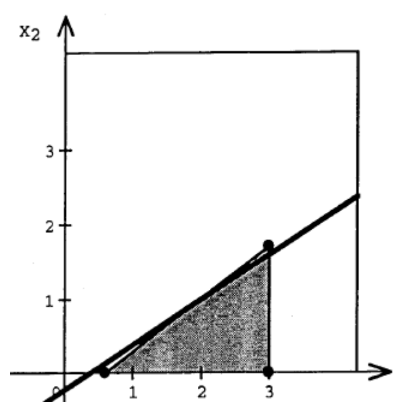
Prior to this study, Waste Management, Inc. (WM) encountered several operational inefficiencies concerning the routing of its trucks. The routes served by different trucks had overlaps and route planners or drivers determined in what order they were going to visit the stops. The result was inefficient sequences and communication gaps between customers and customer-service personnel. The problem is formulated as a mixed integer program, or more specifically as a vehicle routing problem with time windows. The goal is to obtain routes with minimum number of vehicles and travel time, maximum visual attractiveness and a balanced workload. First, a network with nodes that represent actual stops, landfills, lunch break and the depot is constructed. The binary variables x_{ijk} refer to whether arc (i, j) is included in the route of vehicle k or not. The integer variables N_k denote the number of disposal trips and the continuous variables w_{ik} correspond to the beginning time of service for node i by vehicle k. The objective function to be minimized is the total travel time. The constraints make sure that each stop is served by exactly one truck, each truck starts at the depot, the amount of garbage at the stops does not exceed the vehicle capacity and each route includes a lunch break. An iterative two-phase algorithm enhanced with metaheuristics is employed to solve the problem.

Financial benefits of this study include savings of approximately \$18 million in 2003 and estimated savings of \$44 million in 2004. WM expects to save more and to increase its cash flow by \$648 million over a five-year interval. The savings in operational costs over five years is expected to be \$498 million. By using mathematical modeling, WM now generates more efficient routes with minimal overlaps, a reduced number of vehicles and cost-effective sequences. All these contribute to the decrease in operational costs. At the same time, centralized routing made communication in the organization and with the customers easier. Customer-service personnel can now address customer problems more quickly, since they know the routes of the vehicles. As a result, WM provides a more reliable customer service. Operational efficiency also affected the environment and the employees positively. Emissions and noise are reduced. Finally, the benefits from this study led WM to exploit operations research techniques in other operational areas, too.

12.7-2.

(a)

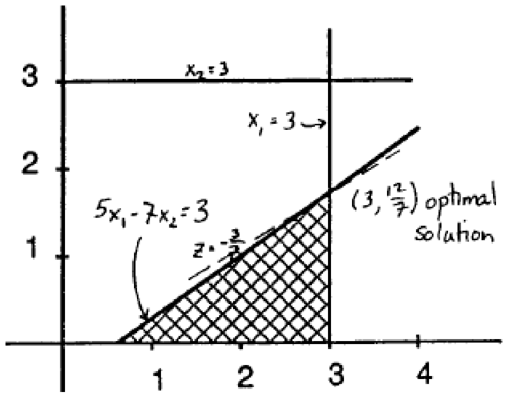
Corner Points	Z
$(3, 1.7143)$	-0.429
$(0.6, 0)$	-1.8
$(3, 0)$	-9



Optimal solution for the LP relaxation: $(3, 1.7143)$ with $Z^* = -0.429$

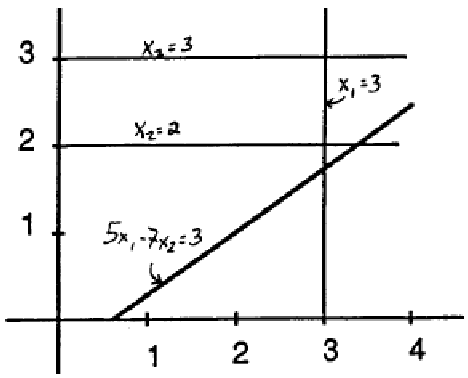
Optimal integer solution: $(2, 1)$ with $Z^* = -1$

(b) LP relaxation of the entire problem:



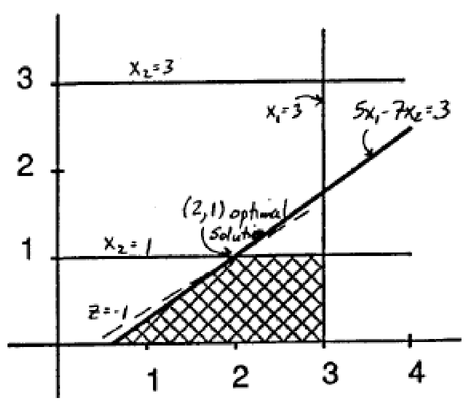
Optimal Solution: $(x_1, x_2) = (3, 12/7)$, $Z = -3/7$

<u>Branch $x_2 \geq 2$:</u>



This subproblem is infeasible, so the branch is fathomed.

<u>Branch $x_2 \leq 1$:</u>



Optimal Solution: $(x_1, x_2) = (2, 1)$, $Z = -1$, feasible for the original problem

Hence, the optimal solution for the original problem is $(x_1, x_2) = (2, 1)$ with $Z = -1$.

(c) Let $x_1 = y_{11} + 2y_{12}$ and $x_2 = y_{21} + 2y_{22}$.

$$\begin{array}{ll} \text{maximize} & Z = -3y_{11} - 6y_{12} + 5y_{21} + 10y_{22} \\ \text{subject to} & 5y_{11} + 10y_{12} - 7y_{21} - 14y_{22} \geq 3 \\ & y_{11}, y_{12}, y_{21}, y_{22} \text{ binary} \end{array}$$

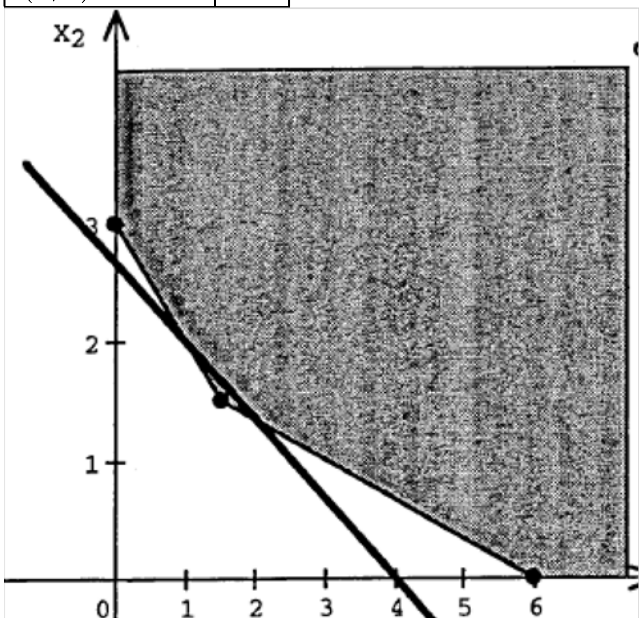
(d) Optimal Solution: $(y_{11}, y_{12}, y_{21}, y_{22}) = (0, 1, 1, 0)$, $Z = -1$, so $x_1 = 2$ and $x_2 = 1$ as in (a).

```
                                        ,-- X4=1,Inf.,F
                                  --- X3=1,X=(1,1,1,0.36),Z=-0.429
                                 /      `-- X4=0,X=(1,1,1,0),Z=-4,F
                        --- X2=1,X=(1,1,0,0.86),Z=-0.429
                       /         \      ,-- X4=1,Inf.,F
                      /           --- X3=0,X=(1,1,0,0.86),Z=-0.429
                     /                  `-- X4=0,X=(1,1,0,0),Z=-9,F
           --- X1=1,X=(1,1,0,0.86),Z=-0.429
          /          \
         /            \
        /              \
       /                --- X2=0,X=(1,0,0,0.14),Z=-1.571,F
      /
     /
    /
X=(1,1,0,0.86),Z=-0.429
    \
     \                            --- X3=1,X=(0,1,1,0),Z=-1
      \                          /
       \                --- X2=1,X=(0,1,0,0.5),Z=-1
        \              /         \
         \            /           --- X3=0,X=(0,1,0,0.5),Z=-1,F
          \          /
           --- X1=0,X=(0,1,0,0.5),Z=-1
                     \
                      \
                       \
                        --- X2=0,Inf.,F
```

12.7-3.

(a)

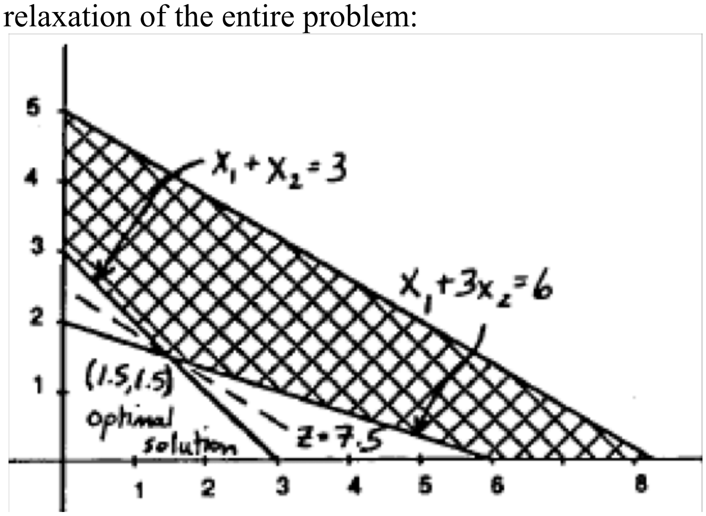
Corner Points	Z
$(1.5, 1.5)$	7.5
$(0, 3)$	9
$(6, 0)$	12



Optimal solution for the LP relaxation: $(1.5, 1.5)$ with $Z^* = 7.5$

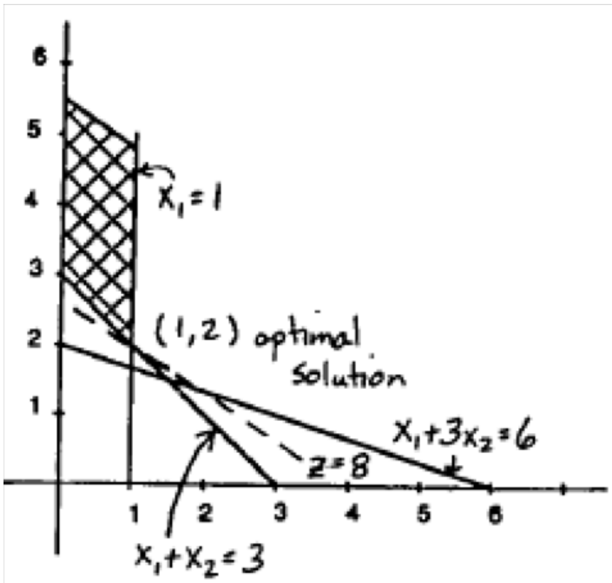
Optimal integer solution: $(1, 2)$ with $Z^* = 8$

(b) LP relaxation of the entire problem:



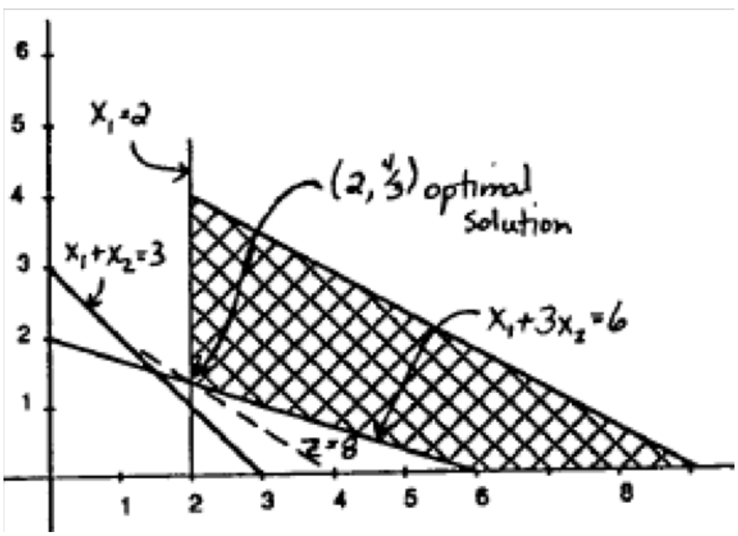
Optimal Solution: $(x_1, x_2) = (1.5, 1.5)$, $Z = 7.5$

Branch $x_1 \leq 1$:



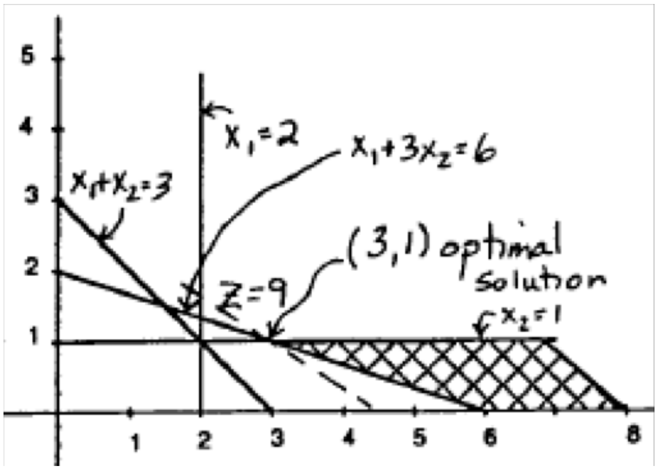
Optimal Solution: $(x_1, x_2) = (1, 2)$, $Z = 8$, feasible for the original problem

Branch $x_1 \geq 2$:



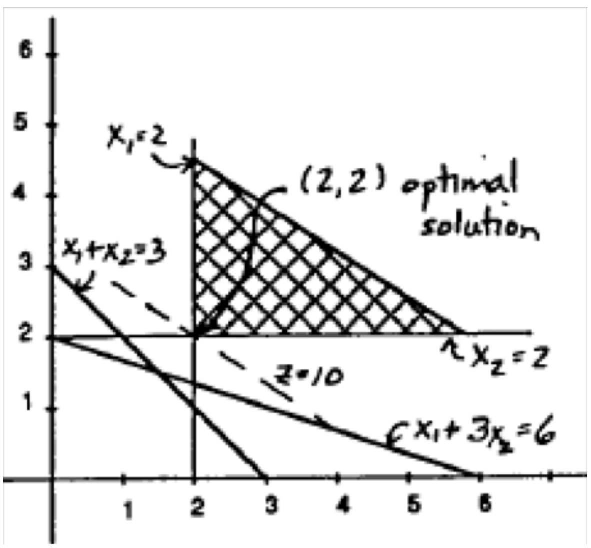
Optimal Solution: $(x_1, x_2) = (2, \frac{4}{3})$, $Z = 8$

Branch $x_2 \leq 1$:



Optimal Solution: $(x_1, x_2) = (3, 1)$, $Z = 9$

Branch $x2 \geq 2$:



Optimal Solution: $(x_1, x_2) = (2, 2)$, $Z = 10$

Hence, the optimal solution for the original problem is $(x_1, x_2) = (1, 2)$ with $Z = 8$.

(c) Let $x_1 = y_{11} + 2y_{12}$ and $x_2 = y_{21} + 2y_{22}$.

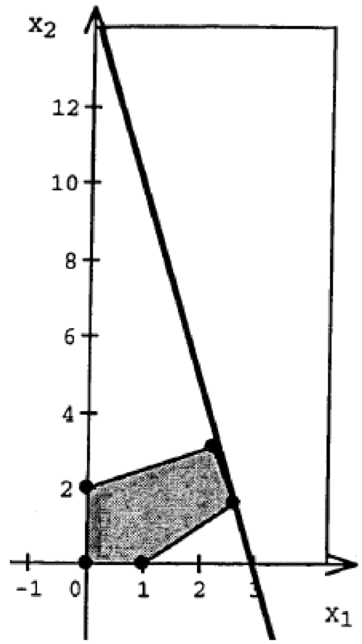
minimize $\quad Z = 2y_{11} + 4y_{12} + 3y_{21} + 6y_{22}$

subject to
$$y_{11} + 2y_{12} + y_{21} + 2y_{22} \geq 3$$
$$y_{11} + 2y_{12} + 3y_{21} + 6y_{22} \geq 6$$
$y_{11}, y_{12}, y_{21}, y_{22}$ binary

(d) Optimal Solution: $(y_{11}, y_{12}, y_{21}, y_{22}) = (1, 0, 0, 1)$, $Z = 8$, so $x_1 = 1$ and $x_2 = 2$ as in part (a).

12.7-4.

(a)

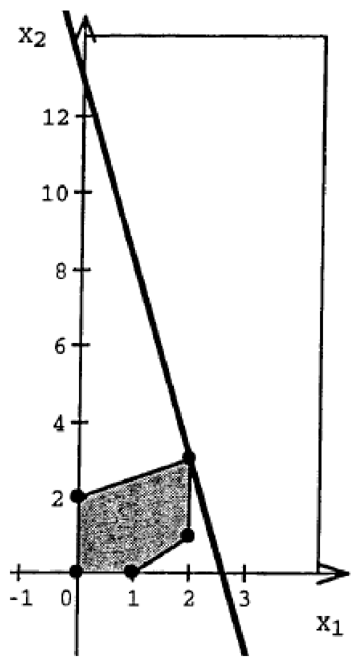


Corner Points	Z
(2.2222, 3.1111)	14.222
(0, 2)	2
(2.6, 1.6)	14.6*
(1, 0)	5
(0, 0)	0

Optimal Solution: $(x_1, x_2) = (2.6, 1.6)$, $Z = 14.6$

<u>Branch $x_1 \geq 3$:</u> Infeasible

<u>Branch $x_1 \leq 2$:</u>

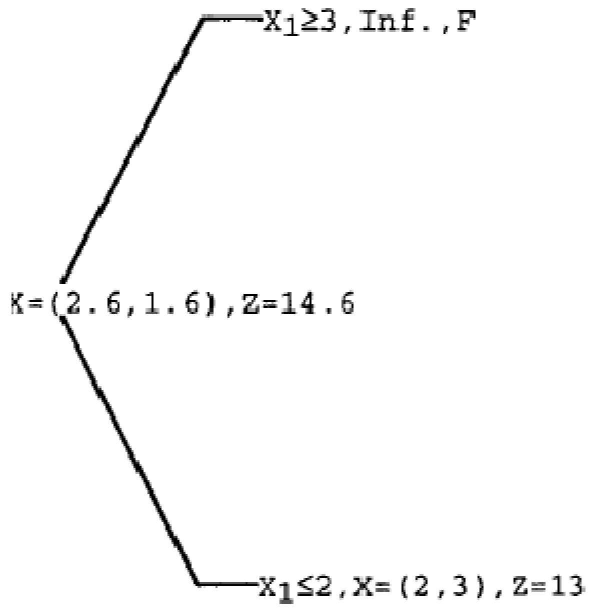


Corner Points	Z
(2, 3)	13*
(0, 2)	2
(2, 1)	11
(1, 0)	5
(0, 0)	0

Optimal Solution: $(x_1, x_2) = (2, 3)$, $Z = 13$, feasible for the original problem

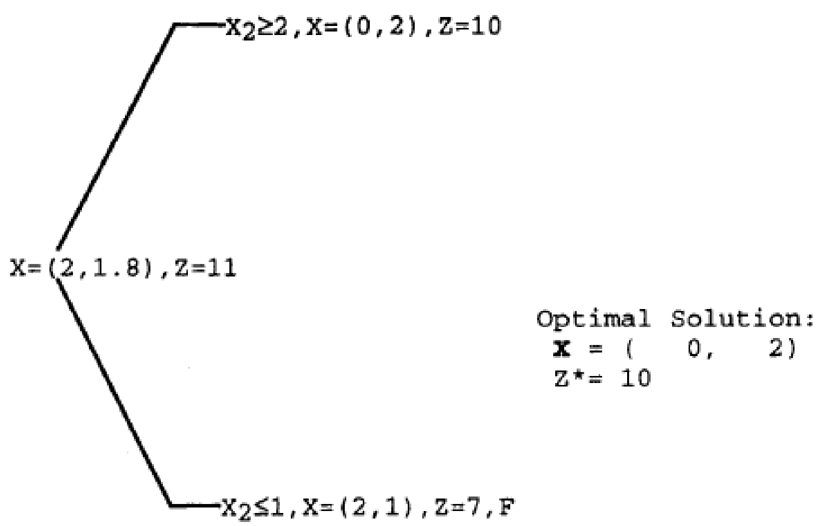
Hence, the optimal solution for the original problem is $(x_1, x_2) = (2, 3)$ with $Z = 13$.

(b) Optimal Solution: $(x_1, x_2) = (2, 3)$, $Z = 13$



(c) Solution: $(x_1, x_2) = (2, 3)$, $Z = 13$

12.7-5.



12.7-6.

```
                --- X1>=15,X=(15,0,14.1),Z=95.529,F
               /
              /
             /
            /
           /
          /
         /
X=(14.1,0,15.9),Z=95.719
         \
          \
           \
            \          --- X2>=1,X=(13.6,1,15.4),Z=95.528,F
             \        /
              \      /
               \    /
                --- X1<=14,X=(14,0.13,15.9),Z=95.693
                    \
                     \
                      \
                       --- X2<=0,X=(14,0,16),Z=95.6
```

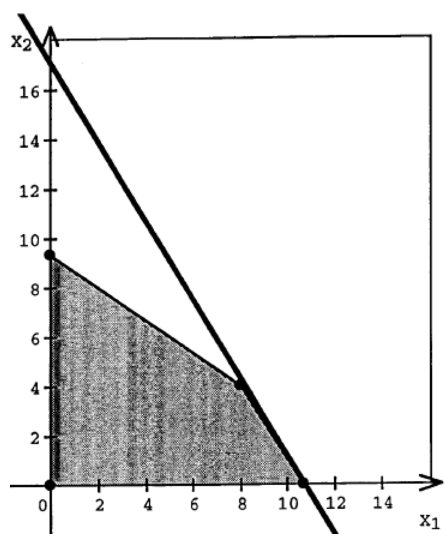
Optimal Solution: $\mathbf{x} = (14, 0, 16)$, $Z = 95.6$.

12.7-7.

(a) Let x_i be the number of ¼ units of product i to be produced, for $i = 1, 2$.

$$\begin{array}{ll} \text{maximize} & 4x_1 + 2.5x_2 \\ \text{subject to} & \frac{3}{4}x_1 + \frac{1}{2}x_2 \leq 8 \\ & \frac{1}{2}x_1 + \frac{3}{4}x_2 \leq 7 \\ & x_1, x_2 \geq 0 \text{ integers} \end{array}$$

(b)

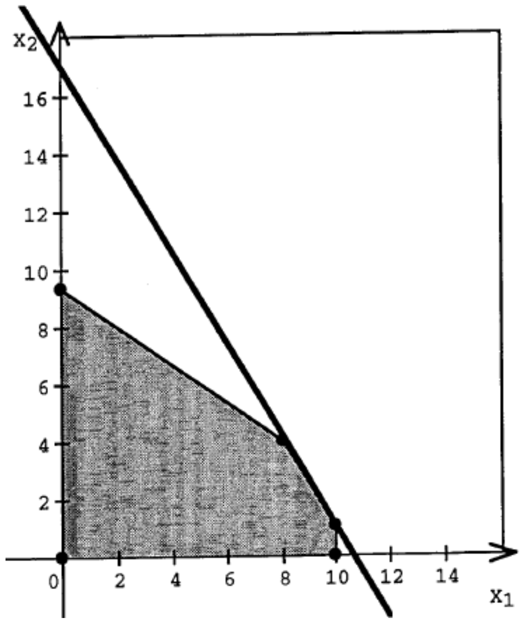


Corner Points	Z
(8, 4)	42
(10.667, 0)	42.667*
(0,9.3333)	23.333
(0, 0)	0

Optimal Solution: $(x_1, x_2) = (10.667, 0)$, $Z = 42.667$

(c) Branch $x_1 \geq 11$: Infeasible

Branch $x_1 \leq 10$:

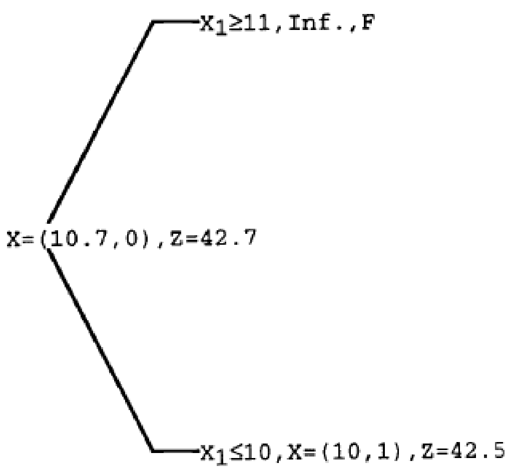


Corner Points	Z
(8, 4)	42
(10, 1)	42.5*
(0, 9.3333)	23.333
(10, 0)	40
(0, 0)	0

Optimal Solution: $(x_1, x_2) = (10, 1)$, $Z = 42.5$, feasible for the original problem

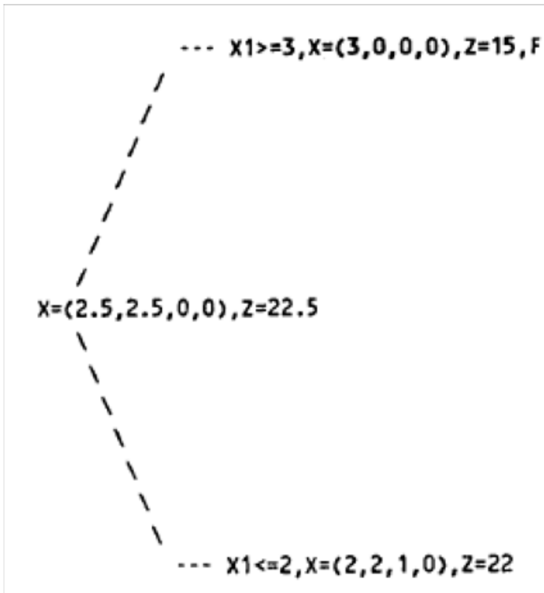
Hence, the optimal solution for the original problem is $(x_1, x_2) = (10, 1)$ with $Z = 42.5$.

(d) Optimal Solution: $(x_1, x_2) = (10, 1)$, $Z = 42.5$



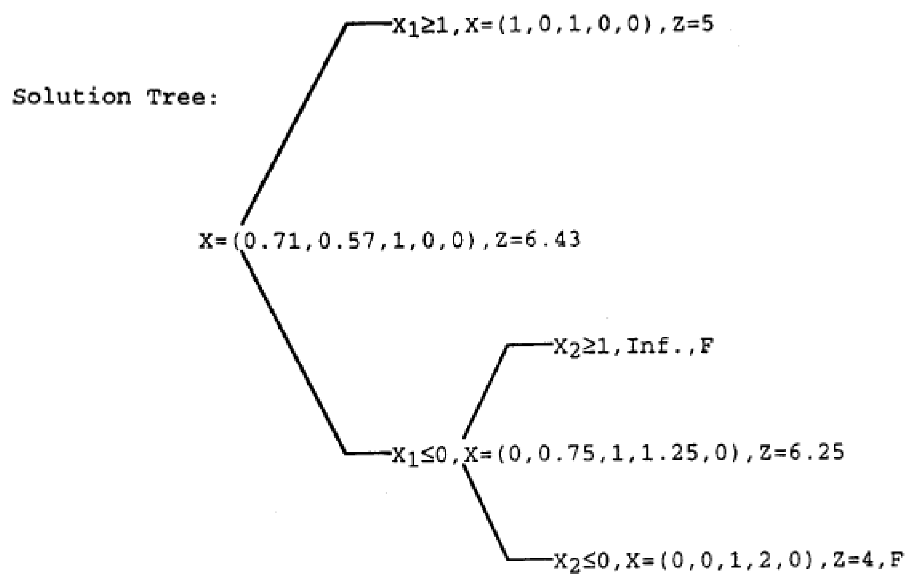
(e) Solution: $(x_1, x_2) = (10, 1)$, $Z = 42.5$

12.7-8.



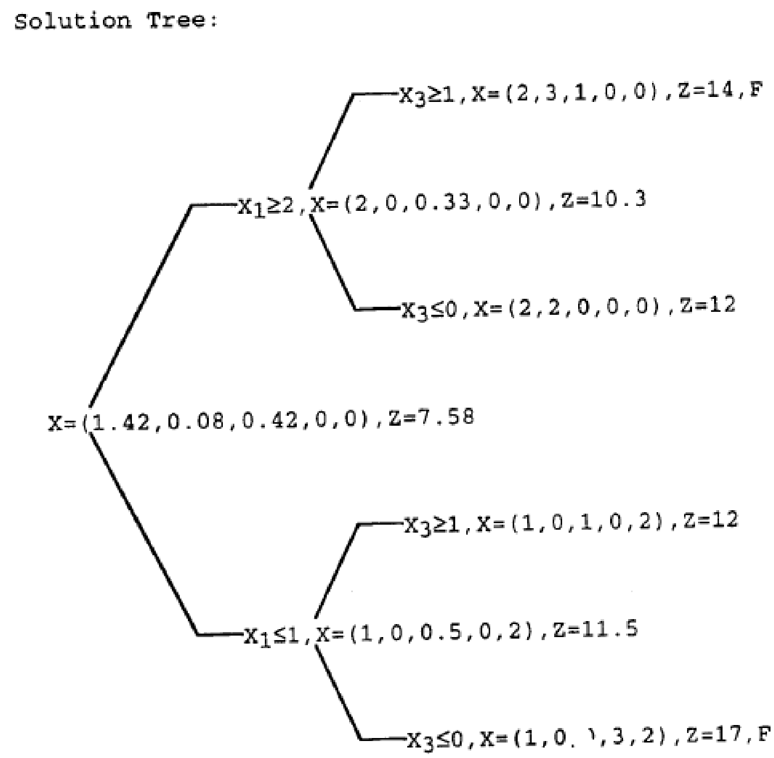
Optimal Solution: $\mathbf{x} = (2, 2, 1, 0)$, $Z = 22$

12.7-9.



Optimal Solution: $\mathbf{x} = (1, 0, 1, 0, 0)$, $Z = 5$

12.7-10.



Optimal Solution: $\mathbf{x} = (1, 0, 1, 0, 2)$ and $\mathbf{x} = (2, 2, 0, 0, 0)$, $Z = 12$

12.8-1.

(a) $x_1 = 0, x_3 = 0$

(b) $x_1 = 0$

(c) $x_1 = 1, x_3 = 1$

12.8-2.

(a) $x_1 = 0$

(b) $x_1 = 1, x_2 = 0$

(c) $x_1 = 0, x_2 = 1$

12.8-3.

From the first equation, , $x_3 = 0$. Then, this equation becomes redundant. From the third equation, $x_5 = 0$ and $x_6 = 1$. Now, this equation is redundant, too. Since $x_6 = 1$, from the second equation, $x_2 = x_4 = 0$ and this equation becomes redundant. Finally, the fourth equation reduces to $x_1 = 0$. Consequently, all equations become redundant. The solution is then fixed to $(0, 0, 0, 0, 0, 1, x_7)$.

12.8-4.

(a) Redundant. Even if all the variables are set to their upper bounds, $x_i = 1$, $2 + 1 + 2 \leq 5$.

(b) Not redundant. For example, $(1, 0, 1)$ violates this constraint.

(c) Not redundant. For example $(0.0.0)$ violates this constraint.

(d) Redundant. The least value of $3x_1 - x_2 - 2x_3$ is attained by $(0, 1, 1)$ and it is -3, so the constraint is still satisfied.

12.8-5.

$$
\begin{aligned}
& \qquad 4x_1 - 3x_2 + x_3 + 2x_4 \leq 5 \\
& b = 5, S = 7, S < b + |a_1| \Rightarrow \overline{a}_1 = S - b = 2, \overline{b} = S - a_1 = 3 \\
& \Rightarrow \qquad 2x_1 - 3x_2 + x_3 + 2x_4 \leq 3 \\
& b = 3, S = 5, S < b + |a_2| \Rightarrow \overline{a}_2 = b - S = -2 \\
& \Rightarrow \qquad 2x_1 - 2x_2 + x_3 + 2x_4 \leq 3 \\
& b = 3, S = 5, S \geq b + |a_j| \text{ for } j = 1, 2, 3, 4
\end{aligned}
$$

12.8-6.

$$
\begin{aligned}
& \qquad 3x_1 - 2x_2 + x_3 \leq 3 \\
& S = 4 < 3 + |a_1| = 6 \\
& \overline{a}_1 = S - b = 1,\ \overline{b} = S - a_1 = 1 \\
& \Rightarrow \qquad x_1 - 2x_2 + x_3 \leq 1 \\
& S = 2 < 1 + |a_2| \Rightarrow \overline{a}_2 = b - S = -1 \\
& \Rightarrow \qquad x_1 - x_2 + x_3 \leq 1
\end{aligned}
$$

12.8-7.

$$
\begin{aligned}
& \qquad x_1 - x_2 + 3x_3 + 4x_4 \geq 1 \\
& \Leftrightarrow \qquad -x_1 + x_2 - 3x_3 - 4x_4 \leq -1 \\
& b = -1, S = 1, S < b + |a_3| \Rightarrow \overline{a}_3 = b - S = -2 \\
& \Rightarrow \qquad -x_1 + x_2 - 2x_3 - 4x_4 \leq -1 \\
& b = -1, S = 1, S < b + |a_4| \Rightarrow \overline{a}_4 = b - S = -2 \\
& \Rightarrow \qquad -x_1 + x_2 - 2x_3 - 2x_4 \leq -1 \\
& b = -1, S = 1, S \geq b + |a_j| \text{ for } j = 1, 2, 3, 4
\end{aligned}
$$

12.8-8.

(a) $\qquad x_1 + 3x_2 - 4x_3 \leq 2$

$$
\begin{aligned}
& b = 2, S = 4, S < b + |a_2| \Rightarrow \overline{a}_2 = S - b = 2, \overline{b} = S - a_2 = 1 \\
& \Rightarrow \qquad x_1 + 2x_2 - 4x_3 \leq 1 \\
& b = 1, S = 3, S < b + |a_3| \Rightarrow \overline{a}_3 = b - S = -2 \\
& \Rightarrow \qquad x_1 + 2x_2 - 2x_3 \leq 1 \\
& b = 1, S = 3, S \geq b + |a_j| \text{ for } j = 1, 2, 3
\end{aligned}
$$

(b) $3x_1 - x_2 + 4x_3 \geq 1$

$\Leftrightarrow \quad -3x_1 + x_2 - 4x_3 \leq -1$

$b = -1, S = 1, S < b + |a_1| \Rightarrow \overline{a}_1 = b - S = -2$

$\Rightarrow \quad -2x_1 + x_2 - 4x_3 \leq -1$

$b = -1, S = 1, S < b + |a_3| \Rightarrow \overline{a}_3 = b - S = -2$

$\Rightarrow \quad -2x_1 + x_2 - 2x_3 \leq -1$

$b = -1, S = 1, S \geq b + |a_j|$ for $j = 1, 2, 3$

12.8-9.

The minimum cover for the constraint $2x_1 + 3x_2 \leq 4$ is $\{x_1, x_2\}$, so the resulting cutting plane is $x_1 + x_2 \leq 1$, which is the same constraint obtained using the tightening procedure.

12.8-10.

$\{x_2, x_4\} \rightarrow x_2 + x_4 \leq 1$

$\{x_3, x_4\} \rightarrow x_3 + x_4 \leq 1$

$\{x_1, x_2, x_3\} \rightarrow x_1 + x_2 + x_3 \leq 2$

12.8-11.

$\{x_1, x_4\} \rightarrow x_1 + x_4 \leq 1$

$\{x_2, x_4\} \rightarrow x_2 + x_4 \leq 1$

$\{x_1, x_2, x_3\} \rightarrow x_1 + x_2 + x_3 \leq 2$

12.8-12.

$\{x_1, x_4\} \rightarrow x_1 + x_4 \leq 1$

$\{x_2, x_4\} \rightarrow x_2 + x_4 \leq 1$

$\{x_3, x_4\} \rightarrow x_3 + x_4 \leq 1$

$\{x_1, x_2, x_3\} \rightarrow x_1 + x_2 + x_3 \leq 2$

12.8-13.

$\{x_1, x_3\} \rightarrow x_1 + x_3 \leq 1$

$\{x_1, x_5\} \rightarrow x_1 + x_5 \leq 1$

$\{x_2, x_3\} \rightarrow x_2 + x_3 \leq 1$

$\{x_3, x_4\} \rightarrow x_3 + x_4 \leq 1$

$\{x_3, x_5\} \rightarrow x_3 + x_5 \leq 1$

$\{x_4, x_5\} \rightarrow x_4 + x_5 \leq 1$

$\{x_1, x_2, x_4\} \rightarrow x_1 + x_2 + x_4 \leq 2$

12.8-14.

(1) $3x_2 + x_4 + x_5 \geq 3 \Rightarrow x_2 = 1$

(2) $x_1 + x_2 \leq 1$ and $x_2 = 1 \Rightarrow x_1 = 0$

(3) $x_2 + x_4 - x_5 - x_6 \leq -1$ and $x_2 = 1 \Rightarrow x_4 = 0, x_5 = x_6 = 1$

(4) $x_2 + 2x_6 + 3x_7 + x_8 + 2x_9 \geq 4$ and $x_2 = x_6 = 1$

$\Rightarrow 3x_7 + x_8 + 2x_9 \geq 1 \Rightarrow x_7 + x_8 + x_9 \geq 1$

(5) $-x_3 + 2x_5 + x_6 + 2x_7 - 2x_8 + x_9 \leq 5$ and $x_5 = x_6 = 1$

$\Rightarrow -x_3 + 2x_7 - 2x_8 + x_9 \leq 2 \Rightarrow -x_3 + x_7 - x_8 + x_9 \leq 1$

Hence, the problem is reduced to finding binary variables x_3, x_7, x_8, x_9 that

$$\begin{array}{ll} \text{maximize} & x_3 + 2x_7 + x_8 + 3x_9 \\ \text{subject to} & \quad\;\; x_7 + x_8 + x_9 \geq 1 \\ & -x_3 + x_7 - x_8 + x_9 \leq 1. \end{array}$$

The objective is maximized when all variables with positive coefficients are set to their upper bounds, so when $x_3 = x_7 = x_8 = x_9 = 1$. This solution also satisfies the constraints, so it is optimal.
Optimal Solution: $\mathbf{x} = (0,1,1,0,1,1,1,1,1)$, $Z = 15$

12.9-1.

Since the variables x_1, x_2, x_3 take values from the set $\{1,2,3\}$ and all the variables must have different values, $x_4 \in \{4\}$. There are two feasible solutions :

$(x_1, x_2, x_3, x_4) = (1,2,3,4)$ with $Z = 3(1) + 2(2) + 4(3) + 1(4) = 23$,
$(x_1, x_2, x_3, x_4) = (3,1,2,4)$ with $Z = 3(3) + 2(1) + 4(2) + 1(4) = 23$.

Either feasible solution is optimal.

12.9-2.

$\underline{x_1 = 12}$: $x_4 = 6$, $x_2 = 3$, and $x_3 = 9$, but $12 + 9 + 6 > 25$, so this is not feasible.

$\underline{x_1 = 6}$: $x_2 = 3$, $x_4 = 12$, and $x_3 = 9$, $6 + 9 + 12 > 25$, so this is not feasible.

$\underline{x_1 = 3}$: $x_2 = 6$, $x_4 = 12$, and $x_3 = 9$, $3 + 9 + 12 \leq 25$, so this is feasible. There are two feasible solutions, $(3,6,9,12,15)$ with $Z = 138$ and $(3,6,9,12,18)$ with $Z = 99$. Hence, $(3,6,9,12,15)$ is optimal.

12.9-3.

$\underline{x_1 = 25}$: $x_4 = 20$ and $x_3 = 30$, but $25 + 30 > 55$, so this is not feasible.

$\underline{x_1 = 30}$: $x_3 \in \{20, 25\}$, but $30 + 25 > 55$, so $x_3 = 20$ and $x_4 = 25$. There are two feasible solutions, $(30,35,20,25)$ with $Z = 11825$ and $(30,40,20,25)$ with $Z = 11950$, so $(30,40,20,25)$ is optimal.

12.9-4.

Let y_i denote the task to which the assignee i is assigned.

$$\begin{array}{ll} \text{minimize} & z_1 + z_2 + z_3 + z_4 \\ \text{subject to} & \text{element}(y_1, [13, 16, 12, 11], z_1) \\ & \text{element}(y_2, [15, \text{M}, 13, 20], z_2) \\ & \text{element}(y_3, [5, 7, 10, 6], z_3) \\ & \text{element}(y_4, [0, 0, 0, 0], z_4) \\ & \text{all-different}(y_1, y_2, y_3, y_4) \\ & y_i \in \{1, 2, 3, 4\}, \text{ for } i = 1, 2, 3, 4 \end{array}$$

12.9-5.

Relabel Carl, Chris, David, Tony and Ken as assignee $1, 2, 3, 4, 5$ respectively. Relabel Backstroke, Breaststroke, Butterfly, Freestyle and Dummy as tasks $1, 2, 3, 4, 5$ respectively. Let y_i be the task to which assignee i is assigned.

$$\begin{array}{ll} \text{minimize} & z_1 + z_2 + z_3 + z_4 + z_5 \\ \text{subject to} & \text{element}(y_1, [37.7, 43.4, 33.3, 29.2, 0], z_1) \\ & \text{element}(y_2, [32.9, 33.1, 28.5, 26.4, 0], z_2) \\ & \text{element}(y_3, [33.8, 42.2, 38.9, 29.6, 0], z_3) \\ & \text{element}(y_4, [37.0, 34.7, 30.4, 28.5, 0], z_4) \\ & \text{element}(y_5, [35.4, 41.8, 33.6, 31.1, 0], z_5) \\ & \text{all-different}(y_1, y_2, y_3, y_4, y_5) \\ & y_i \in \{1, 2, 3, 4, 5\}, \text{ for } i = 1, 2, 3, 4, 5 \end{array}$$

12.9-6.

Let y_i be the number of study days allocated to course i for $i = 1, 2, 3, 4$.

$$\begin{array}{ll} \text{minimize} & z_1 + z_2 + z_3 + z_4 \\ \text{subject to} & \text{element}(y_1, [1, 3, 6, 8], z_1) \\ & \text{element}(y_2, [5, 6, 8, 8], z_2) \\ & \text{element}(y_3, [4, 6, 7, 9], z_3) \\ & \text{element}(y_4, [4, 4, 5, 8], z_4) \\ & y_1 + y_2 + y_3 + y_4 \leq 7 \\ & y_i \in \{1, 2, 3, 4\}, \text{ for } i = 1, 2, 3, 4 \end{array}$$

12.9-7.

Let y_i be the number of crates allocated to store i for $i = 1, 2, 3$.

$$\begin{array}{ll} \text{minimize} & z_1 + z_2 + z_3 \\ \text{subject to} & \text{element}(y_1 + 1, [0, 5, 9, 14, 17, 21], z_1) \\ & \text{element}(y_2 + 1, [0, 6, 11, 15, 19, 22], z_2) \\ & \text{element}(y_3 + 1, [0, 4, 9, 13, 18, 20], z_3) \\ & y_1 + y_2 + y_3 \leq 5 \\ & y_i \in \{0, 1, 2, 3, 4, 5\}, \text{ for } i = 1, 2, 3 \end{array}$$

12.9-8.

$$\begin{aligned} \text{minimize} \quad & Z = \sum_{j=1}^{n} c_{x_j x_{j+1}} \\ \text{subject to} \quad & x_j \in \{2, 3, \ldots, n\}, \text{ for } j = 2, 3, \ldots, n \\ & x_1 = 1 \\ & x_{n+1} = 1 \\ & \text{all-different}(x_2, \ldots, x_n) \end{aligned}$$

12.10-1.

Answers will vary.

12.10-2.

Answers will vary.

Case 12.1

a) With this approach, we need to formulate an integer program for each month and optimize each month individually.

In the first month, Emily does not buy any servers since none of the departments implement the intranet in the first month.

In the second month she must buy computers to ensure that the Sales Department can start the intranet. Emily can formulate her decision problem as an integer problem (the servers purchased must be integer. Her objective is to minimize the purchase cost. She has to satisfy to constraints. She cannot spend more than $9500 (she still has her entire budget for the first two months since she didn't buy any computers in the first month) and the computer(s) must support at least 60 employees. She solves her integer programming problem using the Excel solver.

	A	B	C	D	E	F	G	H
1		Standard	Enhanced	SGI	Sun			
2		Intel	Intel	Workstation	Workstation			
3	Original Cost	$2,500	$5,000	$10,000	$25,000			
4	Discount	0%	0%	10%	25%			
5	Unit Cost	$2,500	$5,000	$9,000	$18,750			
6						Total		Support
7		Number of Employees Server Supports				Support		Needed
8	Support	30	80	200	2,000	80	>=	60
9								
10						Budget		Budget
11		Budget Spent per Server Purchased				Spent		Available
12	Budget	$2,500	$5,000	$9,000	$18,750	$5,000	<=	$9,500
13								
14		Standard	Enhanced	SGI	Sun			Total
15	Servers	Intel	Intel	Workstation	Workstation			Cost
16	Purchased	0	1	0	0			$5,000

Note, that there is a second optimal solution to this integer programming problem. For the same amount of money Emily could buy two standard PC's that would also support 60 employees. However, since Emily knows that she needs to support more employees in the near future, she decides to buy the enhanced PC since it supports more users.

For the third month Emily needs to support 260 users. Since she has already computing power to support 80 users, she now needs to figure out how to support additional 180 users at minimum cost. She can disregard the constraint that the Manufacturing Department needs one of the three larger servers, since she already bought such a server in the previous month. Her task leads her to the following integer programming problem and solution.

	A	B	C	D	E	F	G	H
1		Standard	Enhanced	SGI	Sun			
2		Intel	Intel	Workstation	Workstation			
3	Original Cost	$2,500	$5,000	$10,000	$25,000			
4	Discount	0%	0%	0%	0%			
5	Unit Cost	$2,500	$5,000	$10,000	$25,000			
6						Total		Support
7		Number of Employees Server Supports				Support		Needed
8	Support	30	80	200	2,000	200	>=	180
9								
10		Standard	Enhanced	SGI	Sun			Total
11	Servers	Intel	Intel	Workstation	Workstation			Cost
12	Purchased	0	0	1	0			$10,000

Emily decides to buy one SGI Workstation in month 3. The network is now able to support 280 users.

In the fourth month Emily needs to support a total of 290 users. Since she has already computing power to support 280 users, she now needs to figure out how to support additional 10 users at minimum cost. This task leads her to the following integer programming problem:

	A	B	C	D	E	F	G	H
1		Standard	Enhanced	SGI	Sun			
2		Intel	Intel	Workstation	Workstation			
3	Original Cost	$2,500	$5,000	$10,000	$25,000			
4	Discount	0%	0%	0%	0%			
5	Unit Cost	$2,500	$5,000	$10,000	$25,000			
6						Total		Support
7		Number of Employees Server Supports				Support		Needed
8	Support	30	80	200	2,000	30	>=	10
9								
10		Standard	Enhanced	SGI	Sun			Total
11	Servers	Intel	Intel	Workstation	Workstation			Cost
12	Purchased	1	0	0	0			$2,500

Emily decides to buy a standard PC in the fourth month. The network is now able to support 310 users.

Finally, in the fifth and last month Emily needs to support the entire company with a total of 365 users. Since she has already computing power to support 310 users, she now needs to figure out how to support additional 55 users at minimum cost. This task leads her to the following integer programming problem and solution.

	A	B	C	D	E	F	G	H
1		Standard	Enhanced	SGI	Sun			
2		Intel	Intel	Workstation	Workstation			
3	Original Cost	$2,500	$5,000	$10,000	$25,000			
4	Discount	0%	0%	0%	0%			
5	Unit Cost	$2,500	$5,000	$10,000	$25,000			
6						Total		Support
7		Number of Employees Server Supports				Support		Needed
8	Support	30	80	200	2,000	80	>=	55
9								
10		Standard	Enhanced	SGI	Sun			Total
11	Servers	Intel	Intel	Workstation	Workstation			Cost
12	Purchased	0	1	0	0			$5,000

Emily decides to buy another enhanced PC in the fifth month. (Note that again she could have also bought two standard PC's, but clearly the enhanced PC provides more room for the workload of the system to grow.) The entire network of CommuniCorp consists now of 1 standard PC, 2 enhanced PC's and 1 SGI workstation and it is able to support 390 users. The total purchase cost for this network is $22,500.

b) Due to the budget restriction and discount in the first two months Emily needs to distinguish between the computers she buys in those early months and in the later months. Therefore, Emily uses two variables for each server type.

Emily essentially faces four constraints. First, she must support the 60 users in the sales department in the second month. She realizes that, since she no longer buys the computers sequentially after the second month, that it suffices to include only the constraint on the network to support the all users in the entire company. This second constraint requires her to support a total of 365 users. The third constraint requires her to buy at least one of the three large servers. Finally, Emily has to make sure that she stays within her budget during the second month.

	A	B	C	D	E	F	G	H
1		Standard	Enhanced	SGI	Sun			
2		Intel	Intel	Workstation	Workstation			
3	Month 3-5 Cost	$2,500	$5,000	$10,000	$25,000			
4	Month 2 Discount	0%	0%	10%	25%			
5	Month 2 Cost	$2,500	$5,000	$9,000	$18,750			
6						Total		Support
7	Support	Number of Employees Server Supports				Support		Needed
8	Month 2	30	80	200	2,000	200	>=	60
9	Month 3-5	30	80	200	2,000	400	>=	365
10								
11						Budget		Budget
12	Budget	Budget Spent per Server Purchased				Spent		Available
13	Month 2	$2,500	$5,000	$9,000	$18,750	$9,000	<=	$9,500
14								
15	Server Purchases	Standard	Enhanced	SGI	Sun			
16		Intel	Intel	Workstation	Workstation			
17	Month 2	0	0	1	0	Month 2 Cost		$9,000
18	Month 3-5	0	0	1	0	Month 3-5 Cost		$10,000
19	Total Purchases	0	0	2	0	Total Cost		$19,000
20								
21								
22	Total Advanced Servers		2	>=	1	Advanced Servers Needed		

Emily should purchase a discounted SGI workstation in the second month, and another regular priced one in the third month. The total purchase cost is $19,000.

c) Emily's second method in part (b) finds the cost for the best overall purchase policy. The method in part (a) only finds the best purchase policy for the given month, ignoring the fact that the decision in a particular month has an impact on later decisions. The method in (a) is very short-sighted and thus yields a worse result that the method in part (b).

d) Installing the intranet will incur a number of other costs. These costs include:

Training cost,
Labor cost for network installation,
Additional hardware cost for cabling, network interface cards, necessary hubs, etc.,
Salary and benefits for a network administrator and web master,
Cost for establishing or outsourcing help desk support.

e) The intranet and the local area network are complete departures from the way business has been done in the past. The departments may therefore be concerned that the new technology will eliminate jobs. For example, in the past the manufacturing department has produced a greater number of pagers than customers have ordered. Fewer employees may be needed when the manufacturing department begins producing only enough pagers to meet orders. The departments may also become territorial about data and procedures, fearing that another department will encroach on their business. Finally, the departments may be concerned about the security of their data when sending it over the network.

Case 12.2

a) We want to maximize the number of pieces displayed in the exhibit. For each piece, we therefore need to decide whether or not we should display the piece. Each piece becomes a binary decision variable. The decision variable is assigned 1 if we want to display the piece and assigned 0 if we do not want to display the piece.

We group our constraints into four categories – the artistic constraints imposed by Ash, the personal constraints imposed by Ash, the constraints imposed by Celeste, and the cost constraint. We now step through each of these constraint categories.

<u>Artistic Constraints Imposed by Ash</u>

Ash imposes the following constraints that depend upon the type of art that is displayed. The constraints are as follows:

1. Ash wants to include only one collage. We have four collages available: "Wasted Resources" by Norm Marson, "Consumerism" by Angie Oldman, "My Namesake" by Ziggy Lite, and "Narcissism" by Ziggy Lite. A constraint forces us to include exactly one of these four pieces (D36=D38 in the spreadsheet model that follows).

2. Ash wants at least one wire-mesh sculpture displayed if a computer-generated drawing is displayed. We have three wire-mesh sculptures available and two computer-generated drawings available. Thus, if we include either one or two computer-generated drawings, we have to include at least one wire-mesh sculpture. Therefore, we constrain the total number of wire-mesh sculptures (total) to be at least (1/2) time the total number of computer-generated drawings (L40 ≥ N40).

3. Ash wants at least one computer-generated drawing displayed if a wire-mesh sculpture is displayed. We have two computer-generated drawings available and three wire-mesh sculptures available. Thus, if we include one, two, or three wire-mesh sculptures, we have to include either one or two computer-generated drawings. Therefore, we constraint the total number of wire-mesh sculptures (total) to be at least (1/3) times the total number of computer-generated drawings (L41 ≥ N41).

4. Ash wants at least one photo-realistic painting displayed. We have three photo-realistic paintings available: "Storefront Window" by David Lyman, "Harley" by David Lyman, and "Rick" by Rick Rawls. At least one of these three paintings has to be displayed (G36 ≥ G38).

5. Ash wants at least one cubist painting displayed. We have three cubist paintings available: "Rick II" by Rick Rawls, "Study of a Violin" by Helen Row, and "Study of a Fruit Bowl" by Helen Row. At least one of these three paintings has to be displayed (H36 ≥ H38).

6. Ash wants at least one expressionist painting displayed. We have only one expressionist painting available: "Rick III" by Rick Rawls. This painting has to be displayed (I36 ≥ I38).

7. Ash wants at least one watercolor painting displayed. We have six watercolor paintings available: "Serenity" by Candy Tate, "Calm Before the Storm" by Candy Tate, "All That Glitters" by Ash Briggs, "The Rock" by Ash Briggs, "Winding Road" by Ash Briggs, and "Dreams Come True" by Ash Briggs. At least one of these six paintings has to be displayed (J36 ≥ J38).

8. Ash wants at least one oil painting displayed. We have five oil paintings available: "Void" by Robert Bayer, "Sun" by Robert Bayer, "Beyond" by Bill Reynolds, "Pioneers" by Bill Reynolds, and "Living Land" by Bear Canton. At least one of these five paintings has to be displayed (K36 ≥ K38).

9. Finally, Ash wants the number of paintings to be no greater than twice the number of other art forms. We have 18 paintings available and 16 other art forms available. We classify the following pieces as paintings: "Serenity," "Calm Before the Storm," "Void," "Sun," "Storefront Window," "Harley," "Rick," "Rick II," "Rick III," "Beyond," "Pioneers," "Living Land," "Study of a Violin," "Study of a Fruit Bowl," "All That Glitters," "The Rock," "Winding Road," and "Dreams Come True." The total number of these paintings that we display has to be less than or equal to twice the total number of other art forms we display (L42 ≤ N42).

<u>Personal Constraints Imposed by Ash</u>

1. Ash wants all of his own paintings included in the exhibit, so we must include "All That Glitters," "The Rock," "Winding Road," and "Dreams Come True." (In the spreadsheet model, we constraint the total number of Ash paintings to equal 4: N36=N38.)

2. Ash wants all of Candy Tate's work included in the exhibit, so we must include "Serenity" and "Calm Before the Storm." (In the spreadsheet model, we constrain the total number of Candy Tate works to equal 2: O36=O38.)

3. Ash wants to include at least one piece from David Lyman, so we have to include one or more of the pieces "Storefront Window" and "Harley" (P36 ≥ P38).

4. Ash wants to include at least one piece from Rick Rawls, so we have to include one or more of the pieces "Rick," "Rick II," and "Rick III" (Q36 ≥ Q38)

5. Ash wants to display as many pieces from David Lyman as from Rick Rawls. Therefore we constrain the total number of David Lyman works to equal the total number of Rick Rawls works (L43 = N43).

6. Finally, Ash wants at most one piece from Ziggy Lite displayed. We can therefore include no more than one of "My Namesake" and "Narcissism" (R36 ≤ R38).

<u>Constraints Imposed by Celeste</u>

1. Celeste wants to include at least one piece from a female artist for every two pieces included from a male artist. We have 11 pieces by female artists available: "Chaos Reigns" by Rita Losky, "Who Has Control?" by Rita Losky, "Domestication" by Rita Losky, "Innocence" by Rita Losky, "Serenity" by Candy Tate, "Calm Before the Storm" by Candy Tate, "Consumerism" by Angie Oldman, "Reflection" by Angie Oldman, "Trojan Victory" by Angie Oldman, "Study of a Violin" by Helen Row, and "Study of a Fruit Bowl" by Helen Row. The total number of these pieces has to be greater-than-or-equal-to (1/2) times the total number of pieces by male artists (L44 ≥ N44).

2. Celeste wants at least one of the pieces "Aging Earth" and "Wasted Resources" displayed in order to advance environmentalism (V36 ≥ V38).

3. Celeste wants to include at least one piece by Bear Canton, so we must include one or more of the pieces "Wisdom," "Superior Powers," and "Living Land" to advance Native American rights (W36 ≥ W38).

4. Celeste wants to include one or more of the pieces "Chaos Reigns," "Who Has Control," "Beyond," and "Pioneers" to advance science (X36 ≥ X38).

5. Celeste knows that the museum only has enough floor space for four sculptures. We have six sculptures available: "Perfection" by Colin Zweibell, "Burden" by Colin Zweibell, "The Great Equalizer" by Colin Zweibell, "Aging Earth" by Norm Marson, "Reflection" by Angie Oldman, and "Trojan Victory" by Angie Oldman. We can only include a maximum of four of these six sculptures (Y36 ≤ Y38).

6. Celeste also knows that the museum only has enough wall space for 20 paintings, collages, and drawings. We have 28 paintings, collages, and drawings available: "Chaos Reigns," "Who Has Control," "Domestication," "Innocence," "Wasted Resources," "Serenity," "Calm Before the Storm," "Void," "Sun," "Storefront Window," "Harley," "Consumerism," "Rick," "Rick II," "Rick III," "Beyond," "Pioneers," "Wisdom," "Superior Powers," "Living Land," "Study of a Violin," "Study of a Fruit Bowl," "My Namesake," "Narcissism," "All That Glitters," "The Rock," "Winding Road," and "Dreams Come True." We can only include a maximum of 20 of these 28 wall pieces (Z36 ≤ Z38).

7. Finally, Celeste wants "Narcissism" displayed if "Reflection" is displayed. So if the decision variable for "Reflection" is 1, the decision variable for "Narcissism" must also be 1. However, the decision variable for "Narcissism" can still be 1 even if the decision variable for "Reflection" is 0 (L45 ≥ N45).

Cost Constraint

The cost of all of the pieces displayed has to be less than or equal to $4 million ($C36 \leq C38$).

The problem formulation in an Excel spreadsheet follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA
1	Artist	Piece	Price ($thousand)	Collage?	Wire-Mesh Sculpture?	Computer-Generated Drawing?	Photo-Realistic Painting?	Cubist Painting?	Expressional Painting?	Water-Color Painting?	Oil Painting?	Painting?	Other Art Form?	Ash Briggs?	Candy Tate?	David Lyman?	Rick Rawls?	Ziggy Lite?	Female Artist?	Male Artist?	Advances Environmentalism?	Advances Native American Rights?	Advances Science?	Sits on Floor?	Hangs on Wall?		Include?
2	Colin Zweibell	Perfection	300		1								1							1				1			0
3	Colin Zweibell	Burden	250		1								1							1				1			0
4	Colin Zweibell	The Great Equalizer	125		1								1							1				1			1
5	Rita Losky	Chaos Reigns	400			1							1						1				1		1		1
6	Rita Losky	Who Has Control?	500			1							1						1				1		1		0
7	Rita Losky	Domestication	400										1						1						1		0
8	Rita Losky	Innocence	550										1						1						1		0
9	Norm Marson	Aging Earth	700										1							1	1			1			0
10	Norm Marson	Wasted Resources	575	1									1							1	1				1		1
11	Candy Tate	Serenity	200							1		1			1				1						1		1
12	Candy Tate	Calm before the Storm	225							1		1			1				1						1		1
13	Robert Bayer	Void	150								1	1								1					1		1
14	Robert Bayer	Sun	150								1	1								1					1		0
15	David Lyman	Storefront Window	850				1					1				1				1					1		0
16	David Lyman	Harley	750				1					1				1				1					1		1
17	Angie Oldman	Consumerism	400	1									1						1						1		0
18	Angie Oldman	Reflection	175										1						1					1			1
19	Angie Oldman	Trojan Victory	450										1						1					1			0
20	Rick Rawls	Rick	500				1					1					1			1					1		0
21	Rick Rawls	Rick II	500					1				1					1			1					1		0
22	Rick Rawls	Rick III	500						1			1					1			1					1		1
23	Bill Reynolds	Beyond	650								1	1								1			1		1		0
24	Bill Reynolds	Pioneers	650								1	1								1			1		1		0
25	Bear Canton	Wisdom	250										1							1		1			1		1
26	Bear Canton	Superior Powers	350										1							1		1			1		0
27	Bear Canton	Living Land	450								1	1								1		1			1		0
28	Helen Row	Study of a Violin	400					1				1							1						1		0
29	Helen Row	Study of a Fruit Bowl	400					1				1							1						1		1
30	Ziggy Lite	My Namesake	300	1									1					1		1					1		0
31	Ziggy Lite	Narcissism	300	1									1					1		1					1		0
32	Ash Briggs	All That Glitters	50							1		1		1						1					1		1
33	Ash Briggs	The Rock	50							1		1		1						1					1		1
34	Ash Briggs	Winding Road	50							1		1		1						1					1		1
35	Ash Briggs	Dreams Come True	50							1		1		1						1					1		1
36		Total	3,950	1	1	1	1	1	1	6	1	10	5	4	2	1	1	0	5	10	1	1	1	2	13		15
37			<=	=			>=	>=	>=	>=	>=			=	=	>=	>=	<=			>=	>=	>=	<=	<=		
38		Budget	4,000	1			1	1	1	1	1			4	2	1	1	1			1	1	1	4	20		
39																											
40											Wire Mesh Sculpture	1	>=	0.5	0.5	times		Computer-Generated Drawing									
41											Computer-Generated Drawing	1	>=	0.33	0.33	times		Wire Mesh Sculpture									
42											Paintings	10	<=	10	2	times		Other Art Forms									
43											David Lyman Pieces	1	=	1	Rick Rawls Pieces												
44											Female Artist Pieces	5	>=	5	0.5	times		Male Artist Pieces									
45											"Reflection"	1	>=	0	"Narcissism"												

	K	L	M	N	O	P	Q	R
40	Wire Mesh Sculpture	=E36	>=	=O40*F36	=1/SUM(F2:F35)	times		Computer-Generated Drawing
41	Computer-Generated Drawing	=F36	>=	=O41*E36	=1/SUM(E2:E35)	times		Wire Mesh Sculpture
42	Paintings	=L36	<=	=O42*M36	2	times		Other Art Forms
43	David Lyman Pieces	=P36	=	=Q36	Rick Rawls Pieces			
44	Female Artist Pieces	=S36	>=	=O44*T36	0.5	times		Male Artist Pieces
45	"Reflection"	=AA18	>=	=AA31	"Narcissism"			

In the optimal solution, 15 pieces are displayed at a cost of $3.95 million. The following pieces are displayed:

1. “The Great Equalizer” by Colin Zweibell
2. “Chaos Reigns” by Rita Losky
3. “Wasted Resources” by Norm Marson
4. “Serenity” by Candy Tate
5. “Calm Before the Storm” by Candy Tate
6. “Void” by Robert Bayer (or “Sun” by Robert Bayer)
7. “Harley” by David Lyman
8. “Reflection” by Angie Oldman
9. “Rick III” by Rick Rawls
10. “Wisdom” by Bear Canton
11. “Study of a Fruit Bowl” by Helen Row (or “Study of a Violin”)
12. “All That Glitters” by Ash Briggs
13. “The Rock” by Ash Briggs
14. “Winding Road” by Ash Briggs
15. “Dreams Come True” by Ash Briggs

b) The formulation of this problem is the same as the formulation in part (a) except that the objective function from part (a) now becomes a constraint and the cost constraint from part (a) now becomes the objective function. Thus, we have the new constraint that we need to select 20 or more pieces to display in the exhibit. We also have the new objective to minimize the cost of the exhibit.

The new formulation of the problem in an Excel follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA
1	Artist	Piece	Price ($thousand)	Collage?	Wire-Mesh Sculpture?	Computer-Generated Drawing?	Photo-Realistic Painting?	Cubist Painting?	Expressional Painting?	Water-Color Painting?	Oil Painting?	Painting?	Other Art Form?	Ash Briggs?	Candy Tate?	David Lyman?	Rick Rawls?	Ziggy Lite?	Female Artist?	Male Artist?	Advances Environmentalism?	Advances Native American Rights?	Advances Science?	Sits on Floor?	Hangs on Wall?		Include?
2	Colin Zweibell	Perfection	300		1								1							1				1			1
3	Colin Zweibell	Burden	250		1								1							1				1			1
4	Colin Zweibell	The Great Equalizer	125		1								1							1				1			1
5	Rita Losky	Chaos Reigns	400			1							1						1				1		1		1
6	Rita Losky	Who Has Control?	500			1							1						1				1		1		0
7	Rita Losky	Domestication	400										1						1						1		1
8	Rita Losky	Innocence	550										1						1						1		0
9	Norm Marson	Aging Earth	700										1							1	1			1			0
10	Norm Marson	Wasted Resources	575	1									1							1	1				1		1
11	Candy Tate	Serenity	200							1		1			1				1						1		1
12	Candy Tate	Calm before the Storm	225							1		1			1				1						1		1
13	Robert Bayer	Void	150								1	1								1					1		1
14	Robert Bayer	Sun	150								1	1								1					1		1
15	David Lyman	Storefront Window	850				1					1				1				1					1		0
16	David Lyman	Harley	750				1					1				1				1					1		1
17	Angie Oldman	Consumerism	400	1									1						1						1		0
18	Angie Oldman	Reflection	175										1						1					1			1
19	Angie Oldman	Trojan Victory	450										1						1					1			0
20	Rick Rawls	Rick	500				1					1					1			1					1		0
21	Rick Rawls	Rick II	500					1				1					1			1					1		0
22	Rick Rawls	Rick III	500						1			1					1			1					1		1
23	Bill Reynolds	Beyond	650								1	1								1			1		1		0
24	Bill Reynolds	Pioneers	650								1	1								1			1		1		0
25	Bear Canton	Wisdom	250										1							1		1			1		1
26	Bear Canton	Superior Powers	350										1							1		1			1		0
27	Bear Canton	Living Land	450								1	1								1		1			1		0
28	Helen Row	Study of a Violin	400					1				1							1						1		1
29	Helen Row	Study of a Fruit Bowl	400					1				1							1						1		1
30	Ziggy Lite	My Namesake	300	1									1					1		1					1		0
31	Ziggy Lite	Narcissism	300	1									1					1		1					1		0
32	Ash Briggs	All That Glitters	50							1		1		1						1					1		1
33	Ash Briggs	The Rock	50							1		1		1						1					1		1
34	Ash Briggs	Winding Road	50							1		1		1						1					1		1
35	Ash Briggs	Dreams Come True	50							1		1		1						1					1		1
36		Total	5,450	1	3	1	1	2	1	6	2	12	8	4	2	1	1	0	7	13	1	1	1	4	16		20
37				=			>=	>=	>=	>=	>=			=	=	>=	>=	<=			>=	>=	>=	<=	<=		>=
38				1			1	1	1	1	1			4	2	1	1	1			1	1	1	4	20		20
39																											
40											Wire Mesh Sculpture	3	>=	0.5	0.5	times		Computer-Generated Drawing									
41											Computer-Generated Drawing	1	>=	1.00	0.33	times		Wire Mesh Sculpture									
42											Paintings	12	<=	16	2	times		Other Art Forms									
43											David Lyman Pieces	1	=	1	Rick Rawls Pieces												
44											Female Artist Pieces	7	>=	6.5	0.5	times		Male Artist Pieces									
45											"Reflection"	1	>=	0	"Narcissism"												

In the optimal solution, exactly 20 pieces are displayed at a cost of $5.45 million – $1.45 million more than Ash decided to allocate in part (a). All pieces from part (a) are displayed in addition to the following five new pieces:

1. “Perfection” by Colin Zweibell
2. “Burden” by Colin Zweibell
3. “Domestication” by Rita Losky
4. “Sun” (or “Void”) by Robert Bayer
5. “Study of a Violin” (or “Study of a Fruit Bowl”) by Helen Row

c) This problem is also a cost minimization problem. The problem formulation is the same as that used in part (b). A new constraint is added, however. The patron wants all of Rita’s pieces displayed. Rita has four pieces: “Chaos Reigns,” “Who Has Control?,” “Domestication,” and “Innocence.” All of these four pieces must be displayed.

   The problem formulation in Excel follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB
1	Artist	Piece	Price ($thousand)	Collage?	Wire-Mesh Sculpture?	Computer-Generated Drawing?	Photo-Realistic Painting?	Cubist Painting?	Expressional Painting?	Water-Color Painting?	Oil Painting?	Painting?	Other Art Form?	Ash Briggs?	Candy Tate?	David Lyman?	Rick Rawls?	Ziggy Lite?	Rita Losky?	Female Artist?	Male Artist?	Advances Environmentalism?	Advances Native American Rights?	Advances Science?	Sits on Floor?	Hangs on Wall?		Include?
2	Colin Zweibell	Perfection	300		1								1								1				1			0
3	Colin Zweibell	Burden	250		1								1								1				1			1
4	Colin Zweibell	The Great Equalizer	125		1								1								1				1			1
5	Rita Losky	Chaos Reigns	400			1							1						1	1				1		1		1
6	Rita Losky	Who Has Control?	500			1							1						1	1				1		1		1
7	Rita Losky	Domestication	400										1						1	1						1		1
8	Rita Losky	Innocence	550										1						1	1						1		1
9	Norm Marson	Aging Earth	700										1								1	1			1			0
10	Norm Marson	Wasted Resources	575	1									1								1	1				1		1
11	Candy Tate	Serenity	200							1		1			1					1						1		1
12	Candy Tate	Calm before the Storm	225							1		1			1					1						1		1
13	Robert Bayer	Void	150								1	1									1					1		1
14	Robert Bayer	Sun	150								1	1									1					1		1
15	David Lyman	Storefront Window	850				1					1				1					1					1		0
16	David Lyman	Harley	750				1					1				1					1					1		1
17	Angie Oldman	Consumerism	400	1									1							1						1		0
18	Angie Oldman	Reflection	175										1							1					1			1
19	Angie Oldman	Trojan Victory	450										1							1					1			0
20	Rick Rawls	Rick	500				1					1					1				1					1		0
21	Rick Rawls	Rick II	500					1				1					1				1					1		0
22	Rick Rawls	Rick III	500						1			1					1				1					1		1
23	Bill Reynolds	Beyond	650								1	1									1			1		1		0
24	Bill Reynolds	Pioneers	650								1	1									1			1		1		0
25	Bear Canton	Wisdom	250										1								1		1			1		1
26	Bear Canton	Superior Powers	350										1								1		1			1		0
27	Bear Canton	Living Land	450								1	1									1		1			1		0
28	Helen Row	Study of a Violin	400					1				1								1						1		0
29	Helen Row	Study of a Fruit Bowl	400					1				1								1						1		1
30	Ziggy Lite	My Namesake	300	1									1					1			1					1		0
31	Ziggy Lite	Narcissism	300	1									1					1			1					1		0
32	Ash Briggs	All That Glitters	50							1		1		1							1					1		1
33	Ash Briggs	The Rock	50							1		1		1							1					1		1
34	Ash Briggs	Winding Road	50							1		1		1							1					1		1
35	Ash Briggs	Dreams Come True	50							1		1		1							1					1		1
36		Total	5,800	1	2	2	1	1	1	6	2	11	9	4	2	1	1	0	4	8	12	1	1	2	3	17		20
37				=			>=	>=	>=	>=	>=			=	=	>=	>=	<=	>=			>=	>=	>=	<=	<=		>=
38				1			1	1	1	1	1			4	2	1	1	1	4			1	1	1	4	20		20
39																												
40											Wire Mesh Sculpture	2	>=	1	0.5	times	Computer-Generated Drawing											
41											Computer-Generated Drawing	2	>=	0.67	0.33	times	Wire Mesh Sculpture											
42											Paintings	11	<=	18	2	times	Other Art Forms											
43											David Lyman Pieces	1	=	1	Rick Rawls Pieces													
44											Female Artist Pieces	8	>=	6	0.5	times	Male Artist Pieces											
45											"Reflection"	1	>=	0	"Narcissism"													

In the optimal solution, exactly 20 pieces are displayed at a total cost of $5.8 million. The patron has to pay $1.8 million. The following pieces are displayed:

1. "Burden" by Colin Zweibell
2. "The Great Equalizer" by Colin Zweibell
3. "Chaos Reigns" by Rita Losky
4. "Who Has Control?" by Rita Losky
5. "Domestication" by Rita Losky
6. "Innocence" by Rita Losky
7. "Wasted Resources" by Norm Marson
8. "Serenity" by Candy Tate
9. "Calm Before the Storm" by Candy Tate
10. "Void" by Robert Bayer
11. "Sun" by Robert Bayer
12. "Harley" by David Lyman
13. "Reflection" by Angie Oldman
14. "Rick III" by Rick Rawls
15. "Wisdom" by Bear Canton
16. "Study of a Fruit Bowl" (or "Study of a Violin") by Helen Row
17. "All That Glitters" by Ash Briggs
18. "The Rock" by Ash Briggs
19. "Winding Road" by Ash Briggs
20. "Dreams Come True" by Ash Briggs

Case 12.3

a) We want to maximize the total number of kitchen sets, so each of the 20 kitchen sets becomes a decision variable. But the kitchen sets are not our only decision variables. Because we assume that any particular item composing a kitchen set is replenished immediately, we only need to stock one of each item. A particular item may compose multiple kitchen sets. For example, tile T1 is part of kitchen sets 3, 7, 10, and 17. So a kitchen set exists when all of the items composing that kitchen set are in stock. Therefore, each of 30 items also becomes a decision variable. These decision variables are binary decision variables. If a kitchen set or item is in stock, the decision variable is 1. If a kitchen set or item is not in stock, the decision variable is 0.

A handful of constraints exist in this problem.

1. We cannot indicate that a kitchen set is in stock unless all the items composing that kitchen set are also in stock. Thus, a kitchen set decision variable is 1 only if all the decision variables for the items composing that kitchen set are also 1. For example, for set 1 this constraint equals (Set 1) <= (T2+W2+L4+C2+O4+S2+D2+R2) / 8.
2. Each kitchen set requires 20 square feet of tile. Thus, if a particular tile is in stock, 20 square feet of that tile are in stock. The warehouse can only hold 50 square feet of tile, so only a maximum of two different styles of tile can be in stock.
3. Each kitchen set requires five rolls of wallpaper. Thus, if a particular style of wallpaper is in stock, five rolls of that wallpaper are in stock. The warehouse can only hold 12 rolls of wallpaper, so only a maximum of two different styles of wallpaper can be in stock.
4. A maximum of two different styles of light fixtures can be in stock.
5. A maximum of two different styles of cabinets can be in stock.
6. A maximum of three different styles of countertops can be in stock.
7. A maximum of two different sinks can be in stock.
8. A combination of four different styles of dishwashers and ranges can be held in stock.

The problem formulated in an Excel spreadsheet follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI
1		Floor Tile				Wallpaper				Light Fixtures				Cabinets				Countertops				Dwash		Sinks				Ranges					Set In		Fraction of
2		T1	T2	T3	T4	W1	W2	W3	W4	L1	L2	L3	L4	C1	C2	C3	C4	O1	O2	O3	O4	D1	D2	S1	S2	S3	S4	R1	R2	R3	R4		Stock?		Items in Stock
3	Set 1		1				1						1		1						1		1		1				1				0	<=	0.625
4	Set 2		1			1				1							1				1		1				1		1				0	<=	0.75
5	Set 3	1						1			1			1				1				1				1				1			0	<=	0.625
6	Set 4			1				1				1				1				1		1		1				1					0	<=	0.5
7	Set 5				1				1	1				1					1			1			1			1					0	<=	0.375
8	Set 6		1				1				1						1				1		1			1					1		0	<=	0.625
9	Set 7	1						1					1			1			1			1		1				1					0	<=	0.375
10	Set 8		1			1						1		1				1					1			1					1		1	<=	1
11	Set 9		1			1					1					1			1				1		1				1				0	<=	0.625
12	Set 10	1				1				1				1						1		1					1			1			0	<=	0.5
13	Set 11			1		1						1				1		1				1		1						1			0	<=	0.75
14	Set 12		1				1			1					1				1				1				1		1				0	<=	0.75
15	Set 13				1				1			1				1		1				1			1					1			0	<=	0.375
16	Set 14				1				1				1	1						1				1				1					0	<=	0.286
17	Set 15			1				1		1				1				1								1				1			1	<=	1
18	Set 16			1				1					1	1						1					1			1					0	<=	0.429
19	Set 17	1							1		1					1				1							1			1			0	<=	0.143
20	Set 18		1					1				1			1						1			1					1				1	<=	1
21	Set 19		1						1				1				1				1				1						1		0	<=	0.429
22	Set 20		1					1		1				1					1							1					1		1	<=	1
23																																			
24	Item In Stock?	0	1	1	0	1	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0	1	1	0	1	0	0	1	1	1				
25		(>= 20 sq. ft.)				(>= 5 rolls)																													
26									Floor Tile	Wallpaper	Light Fixtures	Cabinets	Countertops	Sinks	Dwashers & Ranges																		Total Sets in Stock		
27								Total Items	2	2	2	2	3	2	4																		4		
28									<=	<=	<=	<=	<=	<=	<=																				
29								Capacity	2	2	2	2	3	2	4																				

b) We should stock the following items: T2, T3; W1, W3; L1, L3; C1, C2; O1, O2, O4; D2; S1, S3; and R2, R3, and R4. This combination allows four different kitchen sets to be in stock—set 8, 15, 18, and 20.

Note that this is not a unique solution. The value of the objective function is always four complete kitchen sets, but the *specific* items and kitchen sets stocked may be different. Throughout this solution, we will refer to the optimal solution shown above, but because other optimal solutions exist, student answers may differ from the solution somewhat.

c) We model this new problem by changing the capacity constraint for the dishwashers and ranges. Now, instead of being able to stock a combination of only four different styles of dishwashers and ranges, we can stock a maximum of two different styles of dishwashers and a maximum of three different styles of ranges. Because we only have two different styles of dishwashers available, we now effectively do not have a constraint on the number of dishwashers we can carry.

The formulation of the problem in Excel follows:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI
1		Floor Tile				Wallpaper				Light Fixtures				Cabinets				Countertops				Dwash		Sinks				Ranges					Set In		Fraction of
2		T1	T2	T3	T4	W1	W2	W3	W4	L1	L2	L3	L4	C1	C2	C3	C4	O1	O2	O3	O4	D1	D2	S1	S2	S3	S4	R1	R2	R3	R4		Stock?		Items in Stock
3	Set 1		1				1						1		1						1		1		1				1				0	<=	0.25
4	Set 2		1			1				1							1				1		1				1		1				0	<=	0.5
5	Set 3	1						1			1			1				1				1				1				1			0	<=	0.75
6	Set 4			1				1				1				1				1		1		1				1					1	<=	1
7	Set 5				1				1	1				1					1			1			1			1					0	<=	0.625
8	Set 6		1				1				1						1				1		1			1					1		0	<=	0.5
9	Set 7	1						1					1			1			1			1		1				1					0	<=	0.75
10	Set 8		1			1						1		1				1					1			1					1		1	<=	1
11	Set 9		1			1					1					1			1				1		1				1				0	<=	0.625
12	Set 10	1				1				1				1						1		1					1			1			0	<=	0.75
13	Set 11			1		1						1				1		1				1		1						1			1	<=	1
14	Set 12		1				1			1					1				1				1				1		1				0	<=	0.5
15	Set 13				1				1			1				1		1				1			1					1			0	<=	0.625
16	Set 14				1				1				1	1						1				1				1					0	<=	0.571
17	Set 15			1				1		1				1				1								1				1			1	<=	1
18	Set 16			1				1					1	1						1					1			1					0	<=	0.714
19	Set 17	1							1		1					1				1							1			1			0	<=	0.429
20	Set 18		1					1				1			1						1			1					1				0	<=	0.571
21	Set 19		1						1				1				1				1				1						1		0	<=	0.286
22	Set 20		1					1		1				1					1							1					1		1	<=	1
23																																			
24	Item In Stock?	0	1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	1	1	0	1	1	1	0	1	0	1	0	1	1				
25		(>= 20 sq. ft.)				(>= 5 rolls)																													
26									Floor Tile	Wallpaper	Light Fixtures	Cabinets	Countertops	Sinks	Ranges																		Total Sets in Stock		
27								Total Items	2	2	2	2	3	2	3																		5		
28									<=	<=	<=	<=	<=	<=	<=																				
29								Capacity	2	2	2	2	3	2	3																				

With the extra space, the number of kitchen sets we can stock increases from four to five—now sets 4, 8, 11, 15, and 20. To keep these five sets in stock, we stock the following items: T2, T3; W1, W3; L1, L3; C1, C3; O1, O2, O3; D1, D2; S1, S3; R1, R3, and R4. (This optimal solution is not unique.) The new space vacated by the nursery department provides us with the space to stock the new range.

d) With the additional space, our constraints change. We eliminate the constraints limiting the maximum number of different styles of sinks and countertops we can stock. Instead of stocking two of the four styles of light fixtures, we can now stock three of the four styles of light fixtures. Finally, instead of stocking only two of the four cabinet styles, we can now stock three of the four cabinet styles.

The problem formulated in Excel follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI
1		Floor Tile				Wallpaper				Light Fixtures				Cabinets				Countertops				Dwash		Sinks				Ranges					Set In		Fraction of
2		T1	T2	T3	T4	W1	W2	W3	W4	L1	L2	L3	L4	C1	C2	C3	C4	O1	O2	O3	O4	D1	D2	S1	S2	S3	S4	R1	R2	R3	R4		Stock?		Items in Stock
3	Set 1		1				1						1		1						1		1		1				1				0	<=	0.5
4	Set 2		1			1				1							1				1		1				1		1				0	<=	0.5
5	Set 3	1						1			1			1				1				1				1				1			0	<=	0.75
6	Set 4			1				1				1				1				1		1		1				1					1	<=	1
7	Set 5				1				1	1				1					1			1			1			1					0	<=	0.75
8	Set 6		1				1				1						1				1		1			1					1		0	<=	0.5
9	Set 7	1						1					1			1			1			1		1				1					0	<=	0.875
10	Set 8		1			1						1		1				1					1			1					1		1	<=	1
11	Set 9		1			1					1					1			1				1		1				1				0	<=	0.75
12	Set 10	1				1				1				1						1		1					1			1			0	<=	0.75
13	Set 11			1		1						1				1		1				1		1						1			1	<=	1
14	Set 12		1				1			1					1				1				1				1		1				0	<=	0.5
15	Set 13				1				1			1				1		1				1			1					1			0	<=	0.75
16	Set 14				1				1				1	1						1				1				1					0	<=	0.714
17	Set 15			1				1		1				1				1								1				1			1	<=	1
18	Set 16			1				1					1	1						1					1			1					1	<=	1.000
19	Set 17	1							1		1					1				1							1			1			0	<=	0.429
20	Set 18		1					1				1			1						1			1					1				0	<=	0.571
21	Set 19		1						1				1				1				1				1						1		0	<=	0.571
22	Set 20		1					1		1				1					1							1					1		1	<=	1
23																																			
24	Item In Stock?	0	1	1	0	1	0	1	0	1	0	1	1	1	0	1	0	1	1	1	0	1	1	1	1	1	0	1	0	1	1				
25		(>= 20 sq. ft.)				(>= 5 rolls)																													
26									Floor Tile	Wallpaper	Light Fixtures	Cabinets	Countertops	Sinks	Ranges																		Total Sets in Stock		
27								Total Items	2	2	3	2	3	3	3																		6		
28									<=	<=	<=	<=	<=	<=	<=																				
29								Capacity	2	2	3	3	4	4	3																				

With the extra space, we are now able to stock six complete kitchen sets—set 4, 8, 11, 15, 16, and 20. The following items are stocked: T2, T3; W1, W3, L1, L3, L4; C1, C3, O1, O2, O3; D1, D2; S1, S2, S3; R1, R3, R4. (Again, this optimal solution is not unique.)

e) If the items composing a kitchen set could not be replenished immediately, we could not formulate this problem as a binary integer program. We would have to formulate the problem as an integer program since we may have to store more than one kitchen component or kitchen set to ensure that we meet demand.

The assumption of immediate replenishment is justified if the average time to replenish the component is significantly less than the average time between demands for that component.

Case 12.4

a) Let x_{ij} = 1 if students from area *i* are assigned to school *j*; 0 if not.
C_{ij} = bussing cost
S_i = student population of area *i*
K_j = capacity of school *j*
P_{ik} = % of students in area *i* in grade *k*
(for *i* = 1, 2, 3, 4, 5, 6   *j* = 1, 2, 3   and   *k* = 6, 7, 8)

and x_{ij} = are binary variables (for *i* = 1, 2, 3, 4, 5, 6 and *j* = 1, 2, 3).

Note $x_{21} = x_{43} = x_{52} = 0$ due to infeasibility.

b) The models really aren't too different. x_{ij} are binary here, which amounts to forcing their value in the LP of Case 4.3 to be either 0 or S_i. We can leave out the three variables known to be 0, and also 9 redundant constraints. The LP-relaxation of this model, with $0 \le x_{ij} \le 1$ would allow us to interpret x_{ij} as the fraction of students from area *i* to be assigned to school *j*. This obviously would be a more general model, equivalent to that in Case 4.3.

c) Since a residential area cannot be split across multiple schools, the decision becomes which school to send each area's students to. This is formulated with a binary variable for each area/school combination, representing the yes-or-no decision of whether that area should be assigned to that school. Each area can only be sent to a single school, represented by the constraints TotalAssignments (E14:E19) = Supply (G14:G19).

The number of students in each school is then calculated in StudentAssignments (B24:D29) based upon the results in AreaAssignments (B14:D19).

	A	B	C	D	E	F	G	H
1	**Data:**		Percentage	Percentage	Percentage			
2		Number	in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	of Students	Grade	Grade	Grade	School 1	School 2	School 3
4	1	450	32%	38%	30%	$300	$0	$700
5	2	600	37%	28%	35%	–	$400	$500
6	3	550	30%	32%	38%	$600	$300	$200
7	4	350	28%	40%	32%	$200	$500	–
8	5	500	39%	34%	27%	$0	–	$400
9	6	450	34%	28%	38%	$500	$300	$0
10								
11								
12	**Area Assignments**				Total			
13		School 1	School 2	School 3	Assignments		Supply	
14	Area 1	1	0	0	1	=	1	
15	Area 2	0	1	0	1	=	1	
16	Area 3	0	0	1	1	=	1	
17	Area 4	0	1	0	1	=	1	
18	Area 5	0	0	1	1	=	1	
19	Area 6	1	0	0	1	=	1	
20								
21								
22	**Student Assignments**							
23		School 1	School 2	School 3				
24	Area 1	450	0	0				
25	Area 2	0	600	0				
26	Area 3	0	0	550			Total	
27	Area 4	0	350	0			Bussing	
28	Area 5	0	0	500			Cost	
29	Area 6	450	0	0			$1,085,000	
30	Total In School	900	950	1,050				
31		<=	<=	<=				
32	Capacity	900	1,100	1,000				
33								
34								
35	**Grade Constraints:**							
36		270	285	315	30%	of total in school		
37		<=	<=	<=				
38	6th Graders	297	320	360				
39	7th Graders	297	308	346				
40	8th Graders	306	322	344				
41		<=	<=	<=				
42		324	342	378	36%	of total in school		

d) Without prohibiting the splitting of residential areas, the total cost was $555,556. Thus, adding this restriction increases the cost by $1,085,000 - $555,556 = $529,443.

e) As shown in the spreadsheet, the solution remains the same, but the bussing costs are reduced to $975,000.

	A	B	C	D	E	F	G	H
1	**Data:**		Percentage	Percentage	Percentage			
2		Number	in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	of Students	Grade	Grade	Grade	School 1	School 2	School 3
4	1	450	32%	38%	30%	$300	$0	$700
5	2	600	37%	28%	35%	–	$400	$500
6	3	550	30%	32%	38%	$600	$300	$0
7	4	350	28%	40%	32%	$0	$500	–
8	5	500	39%	34%	27%	$0	–	$400
9	6	450	34%	28%	38%	$500	$300	$0
10								
11								
12	**Area Assignments**				Total			
13		School 1	School 2	School 3	Assignments		Supply	
14	Area 1	1	0	0	1	=	1	
15	Area 2	0	1	0	1	=	1	
16	Area 3	0	0	1	1	=	1	
17	Area 4	0	1	0	1	=	1	
18	Area 5	0	0	1	1	=	1	
19	Area 6	1	0	0	1	=	1	
20								
21								
22	**Student Assignments**							
23		School 1	School 2	School 3				
24	Area 1	450	0	0				
25	Area 2	0	600	0				
26	Area 3	0	0	550			Total	
27	Area 4	0	350	0			Bussing	
28	Area 5	0	0	500			Cost	
29	Area 6	450	0	0			$975,000	
30	Total In School	900	950	1,050				
31		<=	<=	<=				
32	Capacity	900	1,100	1,000				
33								
34								
35	**Grade Constraints:**							
36		270	285	315	30%	of total in school		
37		<=	<=	<=				
38	6th Graders	297	320	360				
39	7th Graders	297	308	346				
40	8th Graders	306	322	344				
41		<=	<=	<=				
42		324	342	378	36%	of total in school		

f) Again, the solution remains the same, but the bussing costs are reduced to $840,000.

	A	B	C	D	E	F	G	H
1	**Data:**		Percentage	Percentage	Percentage			
2		Number	in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	of Students	Grade	Grade	Grade	School 1	School 2	School 3
4	1	450	32%	38%	30%	$0	$0	$700
5	2	600	37%	28%	35%	–	$400	$500
6	3	550	30%	32%	38%	$600	$0	$0
7	4	350	28%	40%	32%	$0	$500	–
8	5	500	39%	34%	27%	$0	–	$400
9	6	450	34%	28%	38%	$500	$0	$0
10								
11								
12	**Area Assignments**				Total			
13		School 1	School 2	School 3	Assignments		Supply	
14	Area 1	1	0	0	1	=	1	
15	Area 2	0	1	0	1	=	1	
16	Area 3	0	0	1	1	=	1	
17	Area 4	0	1	0	1	=	1	
18	Area 5	0	0	1	1	=	1	
19	Area 6	1	0	0	1	=	1	
20								
21								
22	**Student Assignments**							
23		School 1	School 2	School 3				
24	Area 1	450	0	0				
25	Area 2	0	600	0				
26	Area 3	0	0	550			Total	
27	Area 4	0	350	0			Bussing	
28	Area 5	0	0	500			Cost	
29	Area 6	450	0	0			$840,000	
30	Total In School	900	950	1,050				
31		<=	<=	<=				
32	Capacity	900	1,100	1,000				
33								
34								
35	**Grade Constraints:**							
36		270	285	315	30%	of total in school		
37		<=	<=	<=				
38	6th Graders	297	320	360				
39	7th Graders	297	308	346				
40	8th Graders	306	322	344				
41		<=	<=	<=				
42		324	342	378	36%	of total in school		

g) For all three options, the assignments of areas to schools are identical. For the current alternative, the bussing costs are $1,085,000. For option 1, the bussing costs are $975,000 (a reduction of $110,000). This savings results from the fact that students from area 3 would no longer be bussed to school 3. For option 2, the bussing costs are $840,000 (a reduction of $135,000 over option 1, and $245,000 over the current alternative). This additional savings results from the fact that students would no longer be bussed from area 1 to school 1.

h) Arguments can be made for all three alternatives. Answers will vary.