EM 605 **Spring 2019 Second Exam** **5/8/2019**

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1. Acme's Research Institute, discovers that a computer company wants a new tape drive for a proposed new computer system. Since the company does not have research people available to develop the new drive, it will subcontract the development to an independent research firm. The computer company has offered a fee of $250,000 for the best proposal for developing the new tape drive. The contract will go to the firm with the best technical plan and the highest reputation for technical competence. Acme wants to enter the competition. Management estimates a cost of $50,000 to prepare a proposal with a fifty-fifty chance of winning the contract. However, Acme's engineers are uncertain about how they will develop the tape drive if they are awarded the contract.

Three alternative approaches can be tried. The first approach is a mechanical method with a cost of $120,000, and the engineers are certain they can develop a successful model with this approach. A second approach involves electronic components. The engineers estimate that the electronic approach will cost only $50,000 to develop a model of the tape drive, but with only a 50 percent chance of satisfactory results. A third approach uses magnetic components; this costs $80,000, with a 70 percent chance of success.

Acme engineers are restricted to working on only one approach at a time and they have time to try only two approaches. If it tries either the magnetic or electronic method and the attempt fails, the second choice must be the mechanical method to guarantee a successful model. The management of Acme needs help in incorporating this information into a decision to proceed or not.

2. Joe's Service Station operates with a single gas pump. Cars arrive according to a Poisson distribution at an average rate of 15 cars per hour. On average, Joe takes 3 minutes to service a car, service times follow an exponential distribution.

a. What fraction of the time is Joe busy servicing cars?
b. How many cars can Joe expect to find, on average, in his station?
c. What's the probability that there are at least 2 cars in the station?
d. How long can a driver expect to wait before his car can get gas?
e. Joe is planning to set up an area near the garage, where cars can wait to be serviced. If each car requires 200 sq. ft., on average, how much space would be required?

3. Suppose we want to fill a backpack by selecting some objects among various items. There are *n* different items available and each item *j* has a weight of w_j and a profit of p_j. The backpack can hold a weight of at most *W*. Using a genetic algorithm, describe an appropriate representation and crossover strategy to find an optimal subset of items to maximize the total profits subject to the backpack's weight capacity. (Assume that the profits, weights, and capacity are positive integers.)

4. Biggie's Investments, Inc., is a brokerage firm that manages stock portfolios for a number of clients. One particular portfolio consists of U shares of U.S. Oil and H shares of Huber Steel. The annual return for U.S. Oil is \$3 per share and the annual return for Huber Steel is \$5 per share. U.S. Oil sells for \$25 per share and Huber Steel sells for \$50 per share. The portfolio has \$80,000 to be invested. The portfolio risk index (0.5 per share of U.S. Oil and 0.25 per share for Huber Steel) has a maximum of 700. In addition, the portfolio is limited to a maximum of 1000 shares of U.S. Oil. The linear programming formation that will maximize the total annual return of the portfolio is as follows:

Maximize $3U + 5H$

subject to:

$25U + 50H \le 80{,}000$

$0.5U + 0.25H \le 700$

$1U \le 1000$

$U, H \ge 0$

The computer solution of this problem is shown below:

Original Problem w/answers

(untitled) Solution

	U	H		RHS	Dual
Maximize	3	5			
Available Budget	25	50	<=	80000	.0933
Portfolio Risk Index	.5	.25	<=	700	1.3333
Limit to # of U.S. Oil	1	0	<=	1000	0
Solution->	800	1200	Optimal Z->	8400	

Ranging

(untitled) Solution

Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
U	800	0	3	2.5	10
H	1200	0	5	1.5	6
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Available Budget	.0933	0	80000	65000	140000
Portfolio Risk Index	1.3333	0	700	400	775
Limit to # of U.S. Oil	0	200	1000	800	Infinity

Dual

(untitled) Solution

Original Problem					
Maximize	U	H			
Available Budget	25	50	<=	80000	
Portfolio Risk Index	0.5	0.25	<=	700	
Limit to # of U.S. Oil	1	0	<=	1000	
Dual Problem					
	Available	Portfolio Risk	Limit to # of		
Minimize	80000	700	1000		
U	25	0.5	1	>=	3
H	50	0.25	0	>=	5

a) What is the optimal solution, and what is the value of the total annual return?

b) Which constraints are binding? What is your interpretation of these constraints in terms of the problem?

c) What are the shadow prices for the constraints? Interpret each shadow price and state clearly what it means.

d) Would it be beneficial to increase the maximum amount invested in U.S. Oil? If yes, what is the limit of the increase? If not, why not?

5. A farmer in the Midwest has 1000 acres of land on which she intends to plant corn, wheat, and soybeans. Each acre of corn costs $100 for preparation, 7 worker-days of labor, and yields a profit of $30. An acre of wheat costs $120 to prepare, requires 10 worker-days of labor, and yields $40 profit. An acre of soybeans costs $70 to prepare, requires 8 worker-days, and yields $20 profit. The farmer has taken out a loan of $80,000 for crop preparation and has contracted with a union for 6,000 worker days of labor. A midwestern granary has agreed to purchase200 acres of corn, 500 acres of wheat and 300 acres of soybeans. The farmer has established the following goals in the order of their importance:

1) To maintain good relations with the union, the labor contract must be honored; that is, the full 6,000 hours of labor contracted for must be used.

2) Preparation costs should not exceed the loan amount, so that additional loans will not have to be secured.

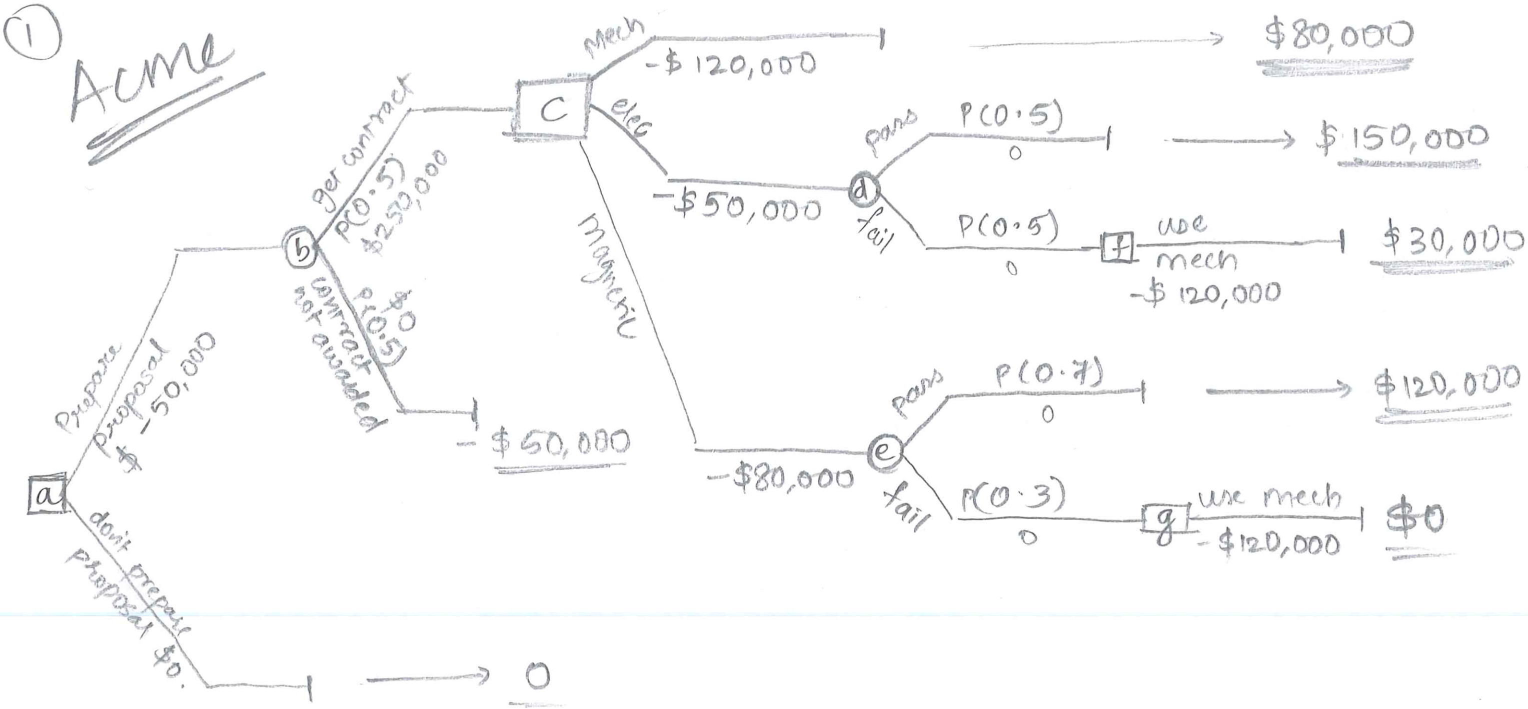
3) The farmer desires a profit of at least $105,000 to remain in good financial condition.

4) Contracting for excess labor should be avoided.

5) The farmer would like to use as much of the available acreage as possible.

Formulate a model to determine the number of acres of each crop the farmer should plant to satisfy the goals in the best possible way.

① Acme



① At node f "choose mechanical method"
Expected payoff = $30,000
At node g "choose mechanical method"
Expected payoff = $0

working
12000
x 0.7
84000.0

② At node e
Expected payoff
= (0.7 x 120,000) + (0.3 x 0)
= $84,000

At node d
Expected payoff
= (0.5 x 150000) + (0.5 x 30,000)
= 75000 + 15000
= $90000

③ At node C "choose electronic method"
90,000 > 80000 & 90 > 84000
∴ Expected payoff = $90,000

④ At node b
Expected payoff
= (0.5 x 90,000) + (0.5 x -50000)
= 45000 - 25000
= $20,000

⑤ At node A "choose to prepare proposal"
20000 > 0
∴ Expected payoff
= $20,000

Flow

1. prepare proposal
2. if awarded with contract
   (i) go with electronic method
   (ii) if electronic model works
      - get \$150,000 profit
      - End.
   (iii) else if electronic method fails.
      - work with mechanical method
      - get \$ 30,000 profit
      - End.

② **Joe's**

$1 - 3$ $\quad$ $= \frac{60}{3}$ 20

$? - 60$

Arrival Rate (λ) = 15 cars / hour.

Service Rate (μ) = 1 car / 3 min

= 20 cars / hour

a) What fraction of time is Joe busy serving cars? (U)

i.e. Utilization factor (U)

$U = \frac{\lambda}{\mu} = \frac{15}{20} = \frac{3}{4}$ OR $\underline{0.75}$ ← probability of Joe being busy!

b) How many cars can Joe expect to find, on average, in his station?

i.e. Number of customers in system on an average (L)

$L = \frac{\lambda}{\mu - \lambda} = \frac{15}{20-15} = \frac{15}{5} = \underline{3}$ cars.

c) Probability of $\underline{2}$ at least cars in the station?

i.e. $P_n = ?$

$P_n = \left(\frac{\lambda}{\mu}\right)^n \times P_0$

$P_0 = \left(1 - \frac{\lambda}{\mu}\right) = \left(1 - \frac{15}{20}\right) = (1 - 0.75) = 0.25$

$\therefore P_n$, where $n = 2$ becomes

$P_2 = \left(\frac{15}{20}\right)^2 \times 0.25$

$= (0.75)^2 \times 0.25$

$= \underline{0.140625}$ ← Probability of at least 2 cars in the station

$$\begin{array}{r} 75 \\ \times\ 75 \\ \hline 375 \\ 525\times \\ \hline 0.5625 \end{array}$$

$$\begin{array}{r} 5625 \\ \times\ \ 25 \\ \hline 0.140625 \end{array}$$

d) How long can a driver expect to wait before his car can get gas?

i.e. $W_q = ?$

$$0.15 \times 60 = 9.0$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{15}{20(20-15)} = \frac{15}{20(5)} = 0.15 \text{ hours}$$

$$= \underline{9 \text{ mins}}$$

e) Space required?

Assuming Joe is calculating space required by looking at the average no. of customers in queue.

$$\text{i.e. } L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{15 \times 15}{20(20-15)} = \frac{\cancel{15}^{3} \times \cancel{15}^{3}}{\cancel{20}_{4} \times \cancel{5}} = \frac{9}{4}$$

$$= \underline{2.25}$$

cars in queue.

Now,

200 sq. ft per car

$\therefore$ 2.25 cars will take

200 sq. ft $\times$ 2.25 cars

$$2.25 \times 2 = 4.50$$

= 450 sq. ft

(3)

Items 'j' range from 1 to n

↳ $j \in (1, 2, \cdots n)$

Weight (w_j) is an integer

↳ $j \in (1, 2, \cdots n)$

Profit (p_j) is also an integer

↳ $j \in (1, 2, \cdots n)$

Capacity of bag = W

We have to maximize profits

$$\sum_{j=1}^{n} p_i x_j \qquad x \in \{0, 1\} \quad \begin{matrix} 0 = \text{don't take} \\ 1 = \text{take} \end{matrix}$$

where x is the decision variable for including item 'j' in the backpack.

The backpack has a capacity W.

constraint

$$\therefore \sum_{j=1}^{n} w_j x_j \leq W \qquad x \in \{0, 1\} \quad \begin{matrix} 0 = \text{don't take} \\ 1 = \text{take} \end{matrix}$$

where w = weight of item j

x = Decision variable for item j &

j = item we have ~~to~~. from 1 to n.

W = capacity of backpack.

Mutation Rate is assumed as 0.1 as backpack are relatively small objects

2 parents have 2 children

Assuming uniform crossover.

if j lies in range $- 0 < x < 31$

$w - (0 - 3)$ pounds (weight of j)

$W - 20$ pounds (capacity of backpack)

$P - \$(10 - 50)$ profit from items

$x = 0 \leftarrow$ decide not to take in backpack

$1 \leftarrow$ decide to take in backpack.

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initial pop.

j	Pj ($)	w_j	Σ	
15	12	2	24	← least fit
4	27	3	81	most fit
8	41	1	41	least fit
23	15	3	45	
10	27	3	81	most fit
9	30	2	60	
5	13	1	13	← least fit
18	50	1	50	
30	39	3	117	most fit
21	38	3	114	

Parents		Children	
30	11110	01110	14
10	01010	01010	10
		mutation	
9	01001	00001	1
23	10111	01101	13
		mut	
21	10101	10100	20
5	00101	00010	2
		mutation	

The decision will be based on the weight of items bag can carry after the stopping condition is met.

The fitness will determine if the item has to be selected or not.

(4) Biggie's Investment, Inc.

a) 800 shares of U.S. Oil + 1200 shares of Huber Steel. is the optimal solution point, keeping the constraints in check. &

Value of annual return is $8400

b) Budget & the portfolio Risk Index are binding but the limit set on the no. of shares for U.S. Oil is not binding.

Budget $25U + 50H = \$80{,}000$ &

Risk Index $0.5U + 0.25H = 700$

All of the budget is used to the maximum.

c) Shadow price for budget is $0.09333 &

Shadow price for Risk Index is $1.3333

which means for each unit change in any of ~~them~~ the constraints, ~~their~~ optimal solution will change proportionally.

~~An addition unit Limit do~~

d) It wouldn't be beneficial to increase the max amount on U.S. Oil as its upper limit, as per the reports, is ∞.

i.e. any increase in the will ~~still~~ not change the current ~~annual returns~~ optimal result.

(Shadow price is $0).

(15) Midwestern Farmer.

x_1 = corn
x_2 = wheat } (acres of)
x_3 = soybeans.

Our objective → Minimize deviations with Priorities in check.

$$\text{Minimize } P_1 d_1^-, \; P_2 d_2^+, \; P_3 d_3^-, \; P_4 d_1^+, \; P_5 d_5^+$$

1. Labor contract

Original $7x_1 + 10x_2 + 8x_3 \leq 6000$

With P1 we want to reach 6000 worker - labor days.

Now,

$$7x_1 + 10x_2 + 8x_3 + d_1^- - d_1^+ = 6000$$

where we want to minimize d_1^-

2. Prep. costs

Original $100x_1 + 120x_2 + 70x_3 \leq 80,000$

With P2 we dont want to exceed the prep costs than the loan costs.

$\therefore$

$$100x_1 + 120x_2 + 70x_3 + d_2^- - d_2^+ = 80,000$$

where we want to minimize d_2^+

3. Profit

Original $30x_1 + 40x_2 + 20x_3 \geq \$105,000$

With P3 we want to minimize losses

$$\therefore \; 30x_1 + 40x_2 + 20x_3 + d_3^- - d_3^+ = 105000$$

where we want to minimize d_3^-

4. Excess Labour

Change in P1 goal

$$d_1^- + d_4^- - d_4^+ = 0$$

where, we Minimize d_4^+

5. Land use

Original $200x_1 + 500x_2 + 300x_3 \leq 1000$.

With P_5 we want to utilize as much land as possible.

$$200x_1 + 500x_2 + 300x_3 + d_5^- - d_5^+ = 1000$$

here we minimize d_5^+

6. Non-negetivity

$x_1, x_2, x_3, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-,$
$d_4^+, d_4^-, d_5^+, d_5^- \geq 0$