CHAPTER 27: FORECASTING

27.4-1.

(a)

$$F_6 = x_5 = 39$$

(b)

$$F_6 = \frac{\sum_{t=1}^{5} x_t}{5} = \frac{5+17+29+41+39}{5} = 26$$

(c)

$$F_6 = \frac{\sum_{t=3}^{5} x_t}{3} = \frac{29+41+39}{3} = 36$$

(d) The demand seems to be rising, so the average forecasting method may be inappropriate, since it uses older, out of data.

27.4-2.

(a)

$$F_6 = x_5 = 13$$

(b)

$$F_6 = \frac{\sum_{t=1}^{5} x_t}{5} = \frac{15+18+12+17+13}{5} = 15$$

(c)

$$F_6 = \frac{\sum_{t=3}^{5} x_t}{3} = \frac{12+17+13}{3} = 14$$

(d) The averaging method seems to be the best, since all five months of data are relevant in determining the forecast of sales for the next month.

27.4-3.

$$F_{t+1} = \frac{1977-1945}{4} + 2083 = 2091$$

27.4-4

$$F_{t+1} = \frac{793-805}{3} + 782 = 778$$

27.4-5.

$$F_{t+1} = \frac{1532-1632}{10} + 1551 = 1541$$

27.4-6.

$$F_{t+1} = \alpha x_t + (1-\alpha)F_t$$

$$F_{t+1}(0.1) = (0.1)(792) + (1-0.1)(782) = 783$$

$$F_{t+1}(0.3) = (0.3)(792) + (1-0.3)(782) = 785$$

$$F_{t+1}(0.5) = (0.5)(792) + (1-0.5)(782) = 787$$

27.4-7.

$$F_{t+1} = \alpha x_t + (1-\alpha)F_t$$
$$F_{t+1}(0.1) = (0.1)(1973) + (1-0.1)(2083) = 2072$$
$$F_{t+1}(0.3) = (0.3)(1973) + (1-0.3)(2083) = 2050$$
$$F_{t+1}(0.5) = (0.5)(1973) + (1-0.5)(2083) = 2028$$

27.4-8.

$$\alpha = 0 \Rightarrow F_{t+1} = F_t = \ldots = F_1$$

The forecast remains equal to the best initial guess for the variable and never changes.

$$\alpha = 1 \Rightarrow F_{t+1} = x_t$$

The forecast always equals the current value of the variable.

27.4-9.

(a) $F_{t+1} = \alpha x_t + (1-\alpha)F_t \Rightarrow x_t = \frac{1}{\alpha}[F_{t+1} - (1-\alpha)F_t] = 2F_{t+1} - F_t$

$\Rightarrow$ Actual demand in April: $2(390) - 380 = 400$

Actual demand in May: $2(380) - 390 = 370$

(b) $F_{\text{Feb}} = 0.5x_{\text{Jan}} + 0.5F_{\text{Jan}}$

$F_{\text{March}} = 0.5x_{\text{Feb}} + 0.5F_{\text{Feb}} = 0.5x_{\text{Feb}} + 0.25x_{\text{Jan}} + 0.25F_{\text{Jan}}$

$x'_{\text{Jan}} = x_{\text{Jan}} + 32$, $x'_{\text{Feb}} = x_{\text{Feb}}$, $F'_{\text{Jan}} = F_{\text{Jan}}$

$\Rightarrow F'_{\text{March}} = F_{\text{March}} + (0.25)(32) = 408$

	Jan	Feb	March	April	May	June
Forecast			408	384	392	381
Actual	400		360	400	370	

27.5-1.

(a)

Quarter	Call Volume	Seasonal Factor
1	6809	$\frac{6809}{7027} = 0.97$
2	6465	$\frac{6465}{7027} = 0.92$
3	6569	$\frac{6569}{7027} = 0.93$
4	8266	$\frac{8266}{7027} = 1.18$

(b)

Quarter	Seasonal Factor	Actual Call Volume	Seasonally Adjusted Call Volume
1	0.97	7257	$\frac{7257}{0.97} = 7481$
2	0.92	7064	$\frac{7064}{0.92} = 7678$
3	0.93	7784	$\frac{7784}{0.93} = 8370$
4	1.18	8724	$\frac{8724}{1.18} = 7393$

(c)

Quarter	Two-year Average	Seasonal Factor
1	7033	$\frac{7033}{7367} = 0.95$
2	6765	$\frac{6765}{7367} = 0.92$
3	7177	$\frac{7177}{7367} = 0.97$
4	8495	$\frac{8495}{7367} = 1.15$

(d)

Quarter	Seasonal Factor	Actual Call Volume	Seasonally Adjusted Call Volume
1	0.95	6992	$\frac{6992}{0.95} = 7360$
2	0.92	6822	$\frac{6822}{0.92} = 7415$
3	0.97	7949	$\frac{7949}{0.97} = 8195$
4	1.15	9650	$\frac{9650}{1.15} = 8391$

27.5-2.

(a)

Quarter	Unemployment Rate	Seasonal Factor
1	0.062	$\frac{0.062}{0.063} = 0.98$
2	0.060	$\frac{0.060}{0.063} = 0.95$
3	0.075	$\frac{0.075}{0.063} = 1.19$
4	0.055	$\frac{0.055}{0.063} = 0.87$

(b)

Quarter	Seasonal Factor	Act. Unemploy. Rate	Seasonally Adj. Unemploy. Rate
1	0.98	0.078	$\frac{0.078}{0.98} = 0.080$
2	0.95	0.074	$\frac{0.074}{0.95} = 0.078$
3	1.19	0.087	$\frac{0.087}{1.19} = 0.073$
4	0.87	0.061	$\frac{0.061}{0.87} = 0.070$

This progression indicates that the state's economy is improving with the unemployment rate decreasing from 8% to 7% (seasonally adjusted) over the four quarters.

27.5-3.

(a)

Quarter	Three-year Average	Seasonal Factor
1	21	$\frac{21}{25} = 0.84$
2	23	$\frac{23}{25} = 0.92$
3	30	$\frac{30}{25} = 1.2$
4	26	$\frac{26}{25} = 1.04$

(b) Seasonally adjusted value: $\frac{28}{1.04} = 27 \Rightarrow$ forecast: $(27)(0.84) = 23$

(c) Quarter 1: seasonally adjusted value: $23/0.84 = 27 \Rightarrow$ forecast:$(27)(0.84) = 23$
Quarter 2: seasonally adjusted value: $25/0.92 = 27 \Rightarrow$ forecast:$(27)(1.20) = 33$
Quarter 3: seasonally adjusted value: $33/1.20 = 27 \Rightarrow$ forecast:$(27)(1.04) = 28$

(d)

Quarter	Seasonal Factor	Avg. House Sales	Seasonally Adjusted Forecast
1	0.84	25	$(25)(0.84) = 21$
2	0.92	25	$(25)(0.92) = 23$
3	1.20	25	$(25)(1.20) = 30$
4	1.04	25	$(25)(1.04) = 26$

27.5-4.

(a) - (b) - (c) - (d) $\alpha = 0.1, \gamma = 0.2$

Year	Quarter	Sales	I	F	S
2010	1	6900	0.965		
	2	6700	0.937		
	3	7900	1.105		
	4	7100	0.993		7150
2011	1	8200	0.997	6900	7285
	2	7000	0.941	6826	7303
	3	7300	1.086	8069	7234
	4	7500	1.001	7183	7266
2012	1	9400	1.049	7245	7482
	2	9200	0.992	7043	7711
	3	9800	1.119	8372	7842
	4	9900	1.047	7849	8047
2013	1	11,400	1.113	8442	8329
	2	10,000	1.029	8260	8505
	3	9400	1.116	9513	8495
	4	8400	1.036	8892	8448
2014	1	8800		9402	8394
	2	7600		8633	8293
	3	7500		9256	8136
	4			8431	

(e) There is a seasonal effect: $1 \underset{\text{down}}{\rightarrow} 2$, and it is incorporated by the parameter I.

(f) There is a substantial error in these estimates, the constant level assumption is not good enough with $\alpha = 0.1$ and $\gamma = 0.2$.

27.6-1.

$F_{t+1} = \alpha x_t + (1-\alpha)F_t + \beta[\alpha(x_t - x_{t-1}) + (1-\alpha)(F_t - F_{t-1})] + (1-\beta)T_{t+1}$

$F_1 = x_0 + T_1 = 3900 + 700 = 4600$

$$F_2 = (0.25)(4600) + (0.75)(4600) + (0.25)[(0.25)(700) + (0.75)(700)] + (0.75)(700)$$
$$= 5300$$

$$F_3 = (0.25)(5300) + (0.75)(5300) + (0.25)[(0.25)(700) + (0.75)(700)] + (0.75)(700)$$
$$= 6000$$

27.6-2.

$$F_{t+1} = \alpha x_t + (1-\alpha)F_t + \beta[\alpha(x_t - x_{t-1}) + (1-\alpha)(F_t - F_{t-1})] + (1-\beta)T_{t+1}$$

$$F_{t+1} = (0.2)(550) + (0.8)(540) + (0.3)[(0.2)(15) + (0.8)(10)] + (0.7)(10) = 552$$

27.6-3.

$$F_{t+1} = \alpha x_t + (1-\alpha)F_t + \beta[\alpha(x_t - x_{t-1}) + (1-\alpha)(F_t - F_{t-1})] + (1-\beta)T_{t+1}$$

$$F_{t+1} = (0.1)(4395) + (0.9)(4975) + (0.2)[(0.1)(280) + (0.9)(255)] + (0.8)(240)$$
$$= 5215$$

27.6-4.

Time Period	True Value	Latest Trend	Estimated Trend	Exponential Smoothing Forecast	Forecasting Error		Smoothing Constants	
1	15		5.00	15	0		α =	0.2
2	21	5.00	5.00	20	1		β =	0.2
3	24	5.20	5.04	25	1			
4	32	4.79	4.99	30	2		**Initial Estimates**	
5	37	5.39	5.07	35	2		Average =	10
6	41	5.38	5.13	41	0		Trend =	5
7	40	5.15	5.14	46	6			
8	47	3.93	4.89	50	3		**Mean Absolute Deviation**	
9	51	4.35	4.79	54	3		MAD =	2.3
10	53	4.19	4.67	58	5			
11		3.66	4.46	62			**Mean Square Error**	
12							MSE =	8.8

Forecast for next production yield: 62%

27.7-1.

Quarter	Forecast	True Value	Error
1	327	345	18
2	332	317	15
3	328	336	8
4	330	311	19

$$\text{MAD} = \frac{\text{sum of forecasting errors}}{\text{number of forecasts}} = \frac{18+15+8+19}{4} = 15$$

$$\text{MSE} = \frac{\text{sum of squares of forecasting errors}}{\text{number of forecasts}} = 243.5$$

27.7-2.

(a) Method 1: $\text{MAD} = \frac{258+499+560+809+609}{5} = 547$

Method 2: $\text{MAD} = \frac{374+471+293+906+396}{5} = 488$

(b) Method 1: $\text{MSE} = 330,905$

Method 2: $\text{MSE} = 285,044$

(c) She can use the older data to calculate more forecasting errors and compare MSE and MAD for a longer time span. This may make her feel more comfortable with her decision.

27.7-3.

(a) $F_{t+1} = \alpha x_t + (1 - \alpha)F_t$

$$F_1 = x_0 = 5000$$

$$F_2 = (0.25)(4600) + (1 - 0.25)(5000) = 4900$$

$$F_3 = (0.25)(5300) + (1 - 0.25)(4900) = 5000$$

(b) MAD $= \frac{400+400+1000}{3} = 600$

(c) MSE $= \frac{400^2+400^2+1000^2}{3} = 440,000$

(d) $F_{t+1} = \; = (0.25)(6000) + (1 - 0.25)(5000) = 5250$

27.7-4.

(a) Since sales are relatively stable, the averaging method would be appropriate for forecasting future sales. This method uses a larger sample size than the last-value method, which should make it more accurate and since the older data is still relevant, it should not be excluded, as would be the case in the moving-average method.

(b) Last-Value Method

Time Period	True Value	Last-Value Forecast	Forecasting Error
1	23		
2	24	23	1
3	22	24	2
4	28	22	6
5	22	28	6
6	27	22	5
7	20	27	7
8	26	20	6
9	21	26	5
10	29	21	8
11	23	29	6
12	28	23	5
13		28	

Mean Absolute Deviation	
MAD =	5.2
Mean Square Error	
MSE =	30.6

(c) Averaging Method

Time Period	True Value	Averaging Forecast	Forecasting Error
1	23		
2	24	23	1
3	22	24	2
4	28	23	5
5	22	24	2
6	27	24	3
7	20	24	4
8	26	24	2
9	21	24	3
10	29	24	5
11	23	24	1
12	28	24	4
13		24	

Mean Absolute Deviation	
MAD =	3.0
Mean Square Error	
MSE =	11.1

(d) Moving-Average Method $(n = 3)$

Time Period	True Value	Moving Average Forecast	Forecasting Error
1	23		
2	24		
3	22		
4	28	23	5
5	22	25	3
6	27	24	3
7	20	26	6
8	26	23	3
9	21	24	3
10	29	22	7
11	23	25	2
12	28	24	4
13		27	

Number of previous periods to consider	
n=	3
Mean Absolute Deviation	
MAD =	3.9
Mean Square Error	
MSE =	17.4

(e) Considering the MAD values $(5.2, 3.0, 3.9)$, the averaging method is the best.

(f) Considering the MSE values $(30.6, 11.1, 17.4)$, the averaging method is the best.

(g) Unless there is a reason to believe that sales will not continue to be relatively stable, the averaging method should be the most accurate in the future as well.

27.7-5.

Ben Swanson should choose 0.1 for the smoothing constant.

Smoothing Constant	MAD	MSE
0.1	2.70	9.44
0.2	2.82	10.24
0.3	2.97	11.20
0.4	3.13	12.35
0.5	3.32	13.75

27.7-6.

(a) Answers will vary. The averaging or the moving-average methods seem to do a better job than the last-value method.

(b) For the last-value method, a change in April affects only the forecast of May. For the averaging method, it affects all forecasts after April and for the moving-average method, it affects the forecasts for May, June and July.

(c) Answers will vary. The averaging and the moving-average methods seem to do slightly better than the last-value method.

(d) Answers will vary. The averaging and the moving-average methods seem to do slightly better than the last-value method.

27.7-7.

(a) Since the sales level is shifting significantly from month to month and there is no consistent trend, the last-value method seems to be appropriate. The averaging method will not do as well because it places too much weight on the old data. The moving-average method will be better than the averaging method, but it will lag any short-term trends. The exponential smoothing method will also lag trends by placing too much weight on the old data. Exponential smoothing with trend will likely not to do well because the trend is not consistent.

(b) Last-Value Method

Time Period	True Value	Last-Value Forecast	Forecasting Error
1	126		
2	137	126	11
3	142	137	5
4	150	142	8
5	153	150	3
6	154	153	1
7	148	154	6
8	145	148	3
9	147	145	2
10	151	147	4
11	159	151	8
12	166	159	7
13		166	

Mean Absolute Deviation	
MAD =	5.3
Mean Square Error	
MSE =	36.2

Averaging Method

Time Period	True Value	Averaging Forecast	Forecasting Error
1	126		
2	137	126	11
3	142	132	11
4	150	135	15
5	153	139	14
6	154	142	12
7	148	144	4
8	145	144	1
9	147	144	3
10	151	145	6
11	159	145	14
12	166	147	19
13		148	

Mean Absolute Deviation	
MAD =	10.0
Mean Square Error	
MSE =	131.4

Moving-Average Method

Time Period	True Value	Moving Average Forecast	Forecasting Error
1	126		
2	137		
3	142		
4	150	135	15
5	153	143	10
6	154	148	6
7	148	152	4
8	145	152	7
9	147	149	2
10	151	147	4
11	159	148	11
12	166	152	14
13		159	

Number of previous periods to consider	
n=	3
Mean Absolute Deviation	
MAD =	8.1
Mean Square Error	
MSE =	84.3

Comparing MAD values $(5.3, 10.0, 8.1)$ and MSE values $(36.2, 131.4, 84.3)$, the last-value method is the best.

(c) Using the template for exponential smoothing with an initial estimate of 120, the following forecast errors are obtained for various values of the smoothing constant α.

α	MAD	MSE
0.1	18.5	382.7
0.2	13.0	210.2
0.3	10.1	139.7
0.4	8.7	104.2
0.5	8.0	82.9

Considering both MAD and MSE, a high value of the smoothing constant seems to be appropriate.

(d) Using the template for exponential smoothing with trend using an initial estimate of 120 for the average value and 10 for the trend, the following forecast errors are obtained for various values of the smoothing constants α and β.

α	β	MAD	MSE
0.1	0.1	25.4	919.6
0.1	0.3	21.2	634.1
0.1	0.5	17.7	450.6
0.3	0.1	13.5	261.9
0.3	0.3	9.8	144.1
0.3	0.5	8.8	111.5
0.5	0.1	8.4	116.1
0.5	0.3	7.0	72.2
0.5	0.5	6.5	61.1

Considering both MAD and MSE, high values of the smoothing constants seem to be appropriate.

(e) The management should use the last-value method to forecast sales. Using this method, the forecast for January of the new year is 166. Exponential smoothing with trend using high smoothing constants, e.g., $\alpha = \beta = 0.5$, also works well. With this method, the forecast for January of the new year is 165.

27.7-8.

(a) Answers will vary. The last-value method seems to be the best. Exponential smoothing with trend is a close second.

(b) For the last-value method, a change in April affects only the forecast for May. For the averaging method, exponential smoothing with or without trend, it affects all forecasts after April. For the moving-average method, it affects the forecasts for May, June, and July.

(c) Answers will vary. The last-value method and exponential smoothing seem to do better than the others.

(d) Answers will vary. The last-value method and exponential smoothing seem to do better than the others.

27.7-9.

(a)

α	MAD
0.1	1.51
0.2	1.62
0.3	1.73
0.4	1.84
0.5	1.95

Choose $\alpha = 0.1$.

(b)

α	MAD
0.1	1.84
0.2	1.88
0.3	1.92
0.4	2.00
0.5	2.10

Choose $\alpha = 0.1$.

(c)

α	MAD
0.1	2.82
0.2	2.54
0.3	2.26
0.4	2.06
0.5	1.90

Choose $\alpha = 0.5$.

27.7-10.

(a)

β	MAD
0.1	0.740
0.2	0.749
0.3	0.759
0.4	0.770
0.5	0.782

Choose $\beta = 0.1$.

(b)

β	MAD
0.1	2.61
0.2	2.76
0.3	2.87
0.4	2.99
0.5	3.05

Choose $\beta = 0.1$.

(c)

β	MAD
0.1	5.66
0.2	6.02
0.3	6.23
0.4	6.36
0.5	6.54

Choose $\beta = 0.1$.

27.7-11.

(a) The time series is not stable enough for the moving-average method.

(b)

Time Period	True Value	Moving Average Forecast	Forecasting Error
1	382		
2	405		
3	398		
4	421	395	26
5	426	408	18
6	415	415	0
7	443	421	22
8	451	428	23
9	446	436	10
10	464	447	17
11		454	

Number of previous periods to consider	
n=	3
Mean Absolute Deviation	
MAD =	16.6
Mean Square Error	
MSE =	346.0

(c)

Time Period	True Value	Exponential Smoothing Forecast	Forecasting Error
1	382	380	2
2	405	381	24
3	398	393	5
4	421	396	26
5	426	408	18
6	415	417	2
7	443	416	27
8	451	430	21
9	446	440	6
10	464	443	21
11		454	

Smoothing Constant	
α =	0.5
Initial Estimate	
Average =	380
Mean Absolute Deviation	
MAD =	15
Mean Square Error	
MSE =	323

(d)

Time Period	True Value	Latest Trend	Estimated Trend	Exponential Smoothing Forecast	Forecasting Error
1	382		10.00	380	2
2	405	10.50	10.13	391	14
3	398	13.72	11.02	405	7
4	421	9.21	10.57	414	7
5	426	12.32	11.01	427	1
6	415	10.82	10.96	438	23
7	443	5.33	9.55	441	2
8	451	9.94	9.65	451	0
9	446	9.53	9.62	461	15
10	464	5.87	8.68	466	2
11		8.20	8.56	474	

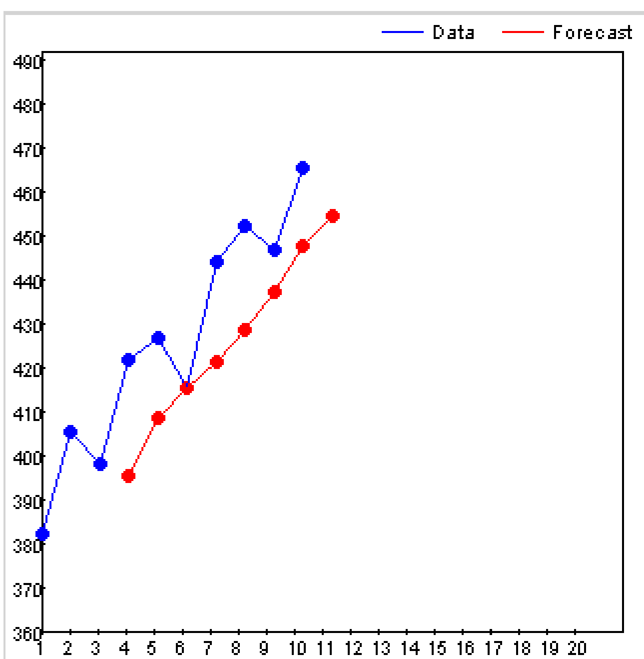
Smoothing Constants	
α =	0.25
β =	0.25
Initial Estimates	
Average =	370
Trend =	10
Mean Absolute Deviation	
MAD =	7.3
Mean Square Error	
MSE =	105.1

(e) Exponential smoothing with a trend is recommended, since it offers the smallest MAD.

27.7-12.

```
Forecasting -- Moving-Average Method
MAD = 16.62          MSE = 345.95
Fetch average of last 3 values
```

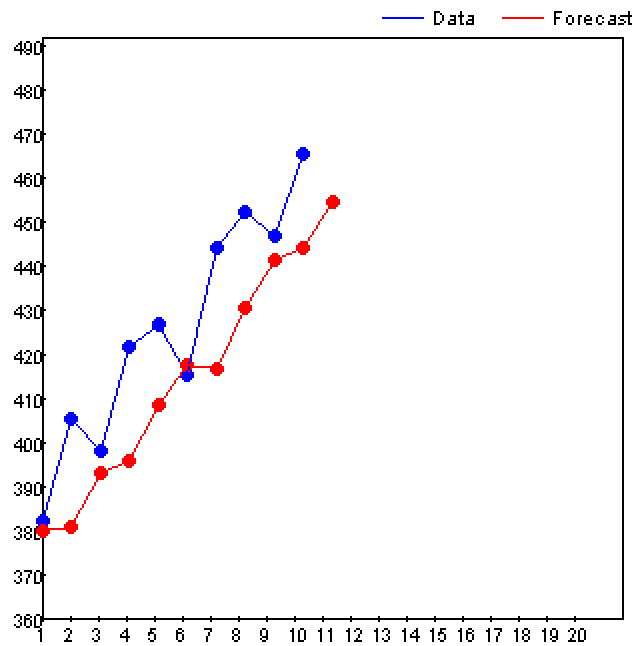
Period	Data	Forecast	Error
1	382		
2	405		
3	398		
4	421	395	26
5	426	408	18
6	415	415	0
7	443	420.67	22.33
8	451	428	23
9	446	436.33	9.67
10	464	446.67	17.33
11	0	453.67	



Moving-Average Method: The forecasts typically lie below the demands.

```
Forecasting -- Exponential Smoothing Method
MAD = 15.14         MSE = 322.97
alpha = 0.5
```

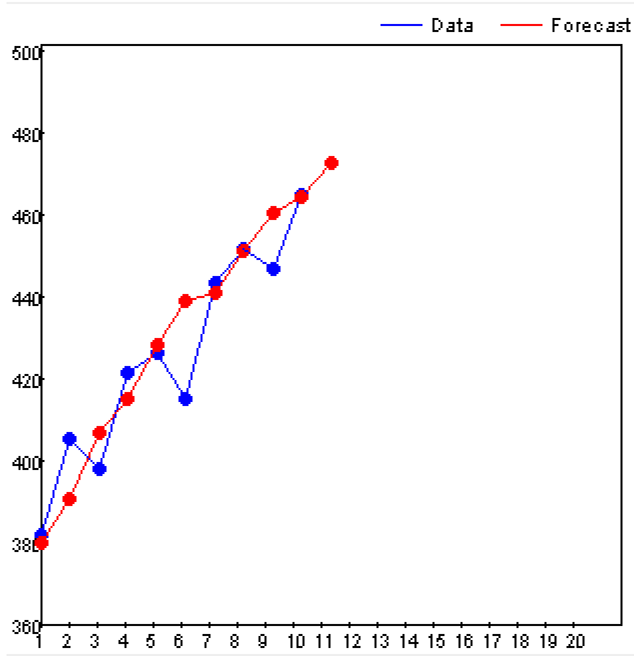
Period	Data	Forecast	Error
1	382	380	2
2	405	381	24
3	398	393	5
4	421	395.5	25.5
5	426	408.25	17.75
6	415	417.12	2.12
7	443	416.06	26.94
8	451	429.53	21.47
9	446	440.27	5.73
10	464	443.13	20.87
11	0	453.57	



Exponential Smoothing: The forecasts typically lie below the demands.

```
Forecasting -- Exponential Smoothing with Trend
MAD = 7.36          MSE = 106.82
alpha = 0.3         beta = 0.3
```

Period	Data	Forecast	Error
1	382	380	2
2	405	390.78	14.22
3	398	406.51	8.51
4	421	414.65	6.35
5	426	427.82	1.82
6	415	438.38	23.38
7	443	440.36	2.64
8	451	450.39	0.61
9	446	459.86	13.86
10	464	463.75	0.25
11	0	471.89	



Exponential Smoothing with Trend: The forecasts are at about the same level as demands (perhaps slightly above). This indicates that exponential smoothing with trend is the best method to use hereafter.

27.7-13.

(a)

Year	Quarter	True Value		Type of Seasonality	
1	1	25		Quarterly	
1	2	47			
1	3	68			
1	4	42			Estimate for
2	1	27		Quarter	Seasonal Factor
2	2	46		1	0.5497
2	3	72		2	1.0271
2	4	39		3	1.5190
3	1	24		4	0.9042
3	2	49			
3	3	70			
3	4	44			

(b) Forecast: 27 acre-feet

Year	Quarter	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error		
								Type of Seasonality
1	1	25	45					Quarterly
1	2	47	46	45	47	0		
1	3	68	45	46	70	2	Quarter	Seasonal Factor
1	4	42	46	45	40	2	1	0.550
2	1	27	49	46	26	1	2	1.027
2	2	46	45	49	50	4	3	1.519
2	3	72	47	45	68	4	4	0.904
2	4	39	43	47	43	4		
3	1	24	44	43	24	0		
3	2	49	48	44	45	4		
3	3	70	46	48	72	2		
3	4	44	49	46	42	2		
4	1			49	27			
4	2							
4	3							
4	4							
5	1						Mean Absolute Deviation	
5	2						MAD =	2.4
5	3							
5	4						Mean Square Error	
6	1						MSE =	8

(c) Winter: $(49)(0.55) = 27$, Spring: $(49)(1.03) = 50$,

Summer: $(49)(1.52) = 74$, Fall: $(49)(0.9) = 44$

(d) Forecast: 25 acre-feet

Year	Quarter	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error		
								Type of Seasonality
1	1	25	45					Quarterly
1	2	47	46	45	47	0		
1	3	68	45	46	69	1	Quarter	Seasonal Factor
1	4	42	46	45	41	1	1	0.550
2	1	27	49	46	25	2	2	1.027
2	2	46	45	46	48	2	3	1.519
2	3	72	47	46	70	2	4	0.904
2	4	39	43	46	42	3		
3	1	24	44	46	25	1		
3	2	49	48	46	47	2		
3	3	70	46	46	70	0		
3	4	44	49	46	41	3		
4	1			46	25			
4	2							
4	3							
4	4							
5	1						Mean Absolute Deviation	
5	2						MAD =	1.57
5	3							
5	4						Mean Square Error	
6	1						MSE =	3.07

(e) Forecast: 26 acre-feet

Year	Quarter	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	1	25	45			
1	2	47	46			
1	3	68	45			
1	4	42	46			
2	1	27	49	46	25	2
2	2	46	45	47	48	2
2	3	72	47	46	70	2
2	4	39	43	47	42	3
3	1	24	44	46	25	1
3	2	49	48	45	46	3
3	3	70	46	45	69	1
3	4	44	49	45	41	3
4	1			47	26	
4	2					
4	3					
4	4					
5	1					
5	2					
5	3					
5	4					
6	1					
6	2					
6	3					
6	4					

Number of previous periods to consider

n = 4

Type of Seasonality

Quarterly

Quarter	Seasonal Factor
1	0.550
2	1.027
3	1.519
4	0.904

Mean Absolute Deviation

MAD = 2.2

Mean Square Error

MSE = 5.5

(f) Forecast: 25 acre-feet

Year	Quarter	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	1	25	45	46	25	0
1	2	47	46	46	47	0
1	3	68	45	46	70	2
1	4	42	46	46	41	1
2	1	27	49	46	25	2
2	2	46	45	46	47	1
2	3	72	47	46	70	2
2	4	39	43	46	42	3
3	1	24	44	46	25	1
3	2	49	48	46	47	2
3	3	70	46	46	70	0
3	4	44	49	46	41	3
4	1			46	25	
4	2					
4	3					
4	4					
5	1					
5	2					
5	3					
5	4					
6	1					
6	2					
6	3					
6	4					
7	1					
7	2					

Smoothing Constant

$\alpha =$ 0.1

Initial Estimate

Average = 46

Type of Seasonality

Quarterly

Quarter	Seasonal Factor
1	0.550
2	1.027
3	1.519
4	0.904

Mean Absolute Deviation

MAD = 1.4

Mean Square Error

MSE = 2.7

(g) Exponential smoothing results in the lowest MAD value, 1.4.

(h) Exponential smoothing gives the lowest MSE value, 2.7.

27.7-14.

(a)

Year	Quarter	True Value			
				Type of Seasonality	
1	1	23		Quarterly	
1	2	22			
1	3	31			
1	4	26			Estimate for
2	1	19		Quarter	Seasonal Factor
2	2	21		1	0.8400
2	3	27		2	0.9200
2	4	24		3	1.2000
3	1	21		4	1.0400
3	2	26			
3	3	32			
3	4	28			

(b)

Year	Quarter	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error			Type of Seasonality
1	1	23	27						Quarterly
1	2	22	24	27	25	3			
1	3	31	26	24	29	2		Quarter	Seasonal Factor
1	4	26	25	26	27	1		1	0.840
2	1	19	23	25	21	2		2	0.920
2	2	21	23	23	21	0		3	1.200
2	3	27	23	23	27	0		4	1.040
2	4	24	23	23	23	1			
3	1	21	25	23	19	2			
3	2	26	28	25	23	3			
3	3	32	27	28	34	2			
3	4	28	27	27	28	0			
4	1			27	23				
4	2								
4	3								
4	4								
5	1							Mean Absolute Deviation	
5	2							MAD =	1.5
5	3								
5	4							Mean Square Error	
6	1							MSE =	3

(c)

Year	Quarter	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	1	23	27			
1	2	22	24	27	25	3
1	3	31	26	26	31	0
1	4	26	25	26	27	1
2	1	19	23	26	21	2
2	2	21	23	25	23	2
2	3	27	23	25	30	3
2	4	24	23	24	25	1
3	1	21	25	24	20	1
3	2	26	28	24	22	4
3	3	32	27	25	30	2
3	4	28	27	25	26	2
4	1			25	21	
4	2					
4	3					
4	4					
5	1					
5	2					
5	3					
5	4					
6	1					

Type of Seasonality: Quarterly

Quarter	Seasonal Factor
1	0.840
2	0.920
3	1.200
4	1.040

Mean Absolute Deviation
MAD = 1.94

Mean Square Error
MSE = 4.85

(d)

Year	Quarter	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	1	23	27			
1	2	22	24			
1	3	31	26			
1	4	26	25			
2	1	19	23	26	21	2
2	2	21	23	24	22	1
2	3	27	23	24	29	2
2	4	24	23	23	24	0
3	1	21	25	23	19	2
3	2	26	28	23	21	5
3	3	32	27	25	30	2
3	4	28	27	26	27	1
4	1			27	22	
4	2					
4	3					
4	4					
5	1					
5	2					
5	3					
5	4					
6	1					
6	2					
6	3					
6	4					

Number of previous periods to consider
n = 4

Type of Seasonality: Quarterly

Quarter	Seasonal Factor
1	0.840
2	0.920
3	1.200
4	1.040

Mean Absolute Deviation
MAD = 2.0

Mean Square Error
MSE = 5.3

(e)

Year	Quarter	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	1	23	27	25	21	2
1	2	22	24	26	24	2
1	3	31	26	25	30	1
1	4	26	25	25	26	0
2	1	19	23	25	21	2
2	2	21	23	25	23	2
2	3	27	23	24	29	2
2	4	24	23	24	25	1
3	1	21	25	24	20	1
3	2	26	28	24	22	4
3	3	32	27	25	30	2
3	4	28	27	25	26	2
4	1			26	22	
4	2					
4	3					
4	4					
5	1					
5	2					
5	3					
5	4					
6	1					
6	2					
6	3					
6	4					
7	1					
7	2					

Smoothing Constant

α = 0.25

Initial Estimate

Average = 25

Type of Seasonality

Quarterly

Quarter	Seasonal Factor
1	0.840
2	0.920
3	1.200
4	1.040

Mean Absolute Deviation

MAD = 1.7

Mean Square Error

MSE = 3.6

(f)

Year	Quarter	True Value	Seasonally Adjusted Value	Latest Trend	Estimated Trend	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	1	23	27		0	25	21	2
1	2	22	24	1	0	26	24	2
1	3	31	26	0	0	25	30	1
1	4	26	25	0	0	26	27	1
2	1	19	23	0	0	25	21	2
2	2	21	23	-1	0	25	23	2
2	3	27	23	-1	0	24	29	2
2	4	24	23	-1	0	23	24	0
3	1	21	25	0	0	23	19	2
3	2	26	28	0	0	23	21	5
3	3	32	27	1	0	25	29	3
3	4	28	27	1	0	25	26	2
4	1			1	0	26	22	
4	2							
4	3							
4	4							
5	1							
5	2							
5	3							
5	4							
6	1							
6	2							
6	3							
6	4							
7	1							
7	2							
7	3							
7	4							

Smoothing Constant

α = 0.25

β = 0.25

Initial Estimate

Average = 25

Trend = 0

Type of Seasonality

Quarterly

Quarter	Seasonal Factor
1	0.840
2	0.920
3	1.200
4	1.040

Mean Absolute Deviation

MAD = 2

Mean Square Error

MSE = 4

(g) Using the last-value method with seasonality ($\text{MAD} = 1.5$), the forecast for first quarter is 23 houses.

(h) Quarter 2: $(27)(0.92) = 25$, Quarter 3: $(27)(1.2) = 32$, Quarter 4: $(27)(1.04) = 28$

27.7-15.

(a) Last-Value Method with Seasonality

Year	Month	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan	68	76			
1	Feb	71	81	76	66	5
1	Mar	66	73	81	73	7
1	Apr	72	77	73	67	5
1	May	77	80	77	74	3
1	June	85	78	80	87	2
1	July	94	80	78	91	3
1	Aug	96	83	80	92	4
1	Sep	80	82	83	81	1
1	Oct	73	80	82	75	2
1	Nov	84	80	80	84	0
1	Dec	89	82	80	86	3
2	Jan			82	74	
2	Feb					
2	Mar					
2	Apr					
2	May					
2	June					
2	July					
2	Aug					
2	Sep					

Type of Seasonality: Monthly

Month	Seasonal Factor
Jan	0.900
Feb	0.880
Mar	0.910
Apr	0.930
May	0.960
June	1.090
July	1.170
Aug	1.150
Sep	0.970
Oct	0.910
Nov	1.050
Dec	1.080

Mean Absolute Deviation

MAD = 3.07

Mean Square Error

MSE = 12.89

Averaging Method with Seasonality:

Year	Month	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan	68	76			
1	Feb	71	81	76	66	5
1	Mar	66	73	78	71	5
1	Apr	72	77	76	71	1
1	May	77	80	77	73	4
1	June	85	78	77	84	1
1	July	94	80	77	91	3
1	Aug	96	83	78	89	7
1	Sep	80	82	79	76	4
1	Oct	73	80	79	72	1
1	Nov	84	80	79	83	1
1	Dec	89	82	79	86	3
2	Jan			79	71	
2	Feb					
2	Mar					
2	Apr					
2	May					
2	June					
2	July					
2	Aug					
2	Sep					

Type of Seasonality: Monthly

Month	Seasonal Factor
Jan	0.900
Feb	0.880
Mar	0.910
Apr	0.930
May	0.960
June	1.090
July	1.170
Aug	1.150
Sep	0.970
Oct	0.910
Nov	1.050
Dec	1.080

Mean Absolute Deviation

MAD = 3.12

Mean Square Error

MSE = 13.07

Moving-Average Method with Seasonality

Year	Month	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan	68	76			
1	Feb	71	81			
1	Mar	66	73			
1	Apr	72	77	76	71	1
1	May	77	80	77	74	3
1	June	85	78	77	84	1
1	July	94	80	79	92	2
1	Aug	96	83	80	91	5
1	Sep	80	82	81	78	2
1	Oct	73	80	82	75	2
1	Nov	84	80	82	86	2
1	Dec	89	82	81	87	2
2	Jan			81	73	
2	Feb					
2	Mar					
2	Apr					
2	May					
2	June					
2	July					
2	Aug					
2	Sep					
2	Oct					
2	Nov					
2	Dec					

Number of previous periods to consider

n = 3

Type of Seasonality

Monthly

Month	Seasonal Factor
Jan	0.900
Feb	0.880
Mar	0.910
Apr	0.930
May	0.960
June	1.090
July	1.170
Aug	1.150
Sep	0.970
Oct	0.910
Nov	1.050
Dec	1.080

Mean Absolute Deviation

MAD = 2.18

Mean Square Error

MSE = 5.79

Exponential Smoothing Method with Seasonality

Year	Month	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan	68	76	80	72	4
1	Feb	71	81	79	70	1
1	Mar	66	73	79	72	6
1	Apr	72	77	78	73	1
1	May	77	80	78	75	2
1	June	85	78	78	85	0
1	July	94	80	78	92	2
1	Aug	96	83	79	91	5
1	Sep	80	82	80	77	3
1	Oct	73	80	80	73	0
1	Nov	84	80	80	84	0
1	Dec	89	82	80	87	2
2	Jan			81	73	
2	Feb					
2	Mar					
2	Apr					
2	May					
2	June					
2	July					
2	Aug					
2	Sep					
2	Oct					
2	Nov					
2	Dec					
3	Jan					
3	Feb					

Smoothing Constant

$\alpha =$ 0.2

Initial Estimate

Average = 80

Type of Seasonality

Monthly

Month	Seasonal Factor
Jan	0.900
Feb	0.880
Mar	0.910
Apr	0.930
May	0.960
June	1.090
July	1.170
Aug	1.150
Sep	0.970
Oct	0.910
Nov	1.050
Dec	1.080

Mean Absolute Deviation

MAD = 2.34

Mean Square Error

MSE = 9.31

Method	MAD	MSE
Last-Value	3.07	12.89
Averaging	3.12	13.07
Moving-Average	2.18	5.79
Exponential Smoothing	2.34	9.31

(b) The moving-average method with seasonality has the lowest MAD value. With this method, the forecast for January is 73 passengers.

27.7-16.

(a)

Method	MAD	MSE
Last-Value	2.46	8.34
Averaging	7.06	74.73
Moving-Average	2.79	9.68
Exp. Smoothing	4.28	25.87

(b) Forecast: 94

Year	Month	True Value	Seasonally Adjusted Value	Latest Trend	Estimated Trend	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan	75	83		2	82	74	1
1	Feb	76	86	2	2	84	74	2
1	Mar	81	89	2	2	87	79	2
1	Apr	84	90	3	2	90	83	1
1	May	85	89	2	2	92	88	3
1	June	99	91	2	2	93	102	3
1	July	107	91	2	2	95	111	4
1	Aug	108	94	1	2	96	110	2
1	Sep	94	97	1	2	97	95	1
1	Oct	90	99	2	2	99	90	0
1	Nov	106	101	2	2	101	106	0
1	Dec	110	102	2	2	103	111	1
2	Jan			2	2	104	94	
2	Feb							
2	Mar							
2	Apr							
2	May							
2	June							
2	July							
2	Aug							
2	Sep							
2	Oct							
2	Nov							
2	Dec							
3	Jan							
3	Feb							
3	Mar							
3	Apr							

Smoothing Constant	
α =	0.2
β =	0.2
Initial Estimate	
Average =	80
Trend =	2
	Type of Seasonality
	Monthly
Month	**Seasonal Factor**
Jan	0.900
Feb	0.880
Mar	0.910
Apr	0.930
May	0.960
June	1.090
July	1.170
Aug	1.150
Sep	0.970
Oct	0.910
Nov	1.050
Dec	1.080
Mean Absolute Deviation	
MAD =	1.66
Mean Square Error	
MSE =	4.21

MAD and MSE values are lower than those in (a).

(c) MAD and MSE values obtained are higher than the ones in (b).

Year	Month	True Value	Seasonally Adjusted Value	Latest Trend	Estimated Trend	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan	68	76		0	80	72	4
1	Feb	71	81	-1	0	79	69	2
1	Mar	66	73	0	0	79	72	6
1	Apr	72	77	-1	0	77	72	0
1	May	77	80	0	0	77	74	3
1	June	85	78	0	0	77	84	1
1	July	94	80	0	0	77	90	4
1	Aug	96	83	0	0	78	89	7
1	Sep	80	82	1	0	79	77	3
1	Oct	73	80	1	0	80	73	0
1	Nov	84	80	0	0	80	84	0
1	Dec	89	82	0	0	80	87	2
2	Jan	75	83	1	0	81	73	2
2	Feb	76	86	1	0	82	72	4
2	Mar	81	89	1	1	83	76	5
2	Apr	84	90	2	1	85	79	5
2	May	85	89	2	1	87	84	1
2	June	99	91	1	1	89	97	2
2	July	107	91	1	1	90	106	1
2	Aug	108	94	1	1	92	105	3
2	Sep	94	97	2	1	93	91	3
2	Oct	90	99	2	1	96	87	3
2	Nov	106	101	2	2	98	103	3
2	Dec	110	102	2	2	100	108	2
3	Jan			2	2	102	92	
3	Feb							
3	Mar							
3	Apr							

Smoothing Constant	
$\alpha =$	0.2
$\beta =$	0.2
Initial Estimate	
Average =	80
Trend =	0
	Type of Seasonality
	Monthly
Month	**Seasonal Factor**
Jan	0.900
Feb	0.880
Mar	0.910
Apr	0.930
May	0.960
June	1.090
July	1.170
Aug	1.150
Sep	0.970
Oct	0.910
Nov	1.050
Dec	1.080
Mean Absolute Deviation	
MAD =	2.74
Mean Square Error	
MSE =	10.44

(d) Exponential smoothing with seasonality and trend (with parameters as in (b) should be used.

27.7-17.

(a) Based on past sales:

		True			
Year	Month	Value		Type of Seasonality	
1	Jan	352		Monthly	
1	Feb	329			
1	Mar	365			
1	Apr	358			Estimate for
1	May	412		Month	Seasonal Factor
1	June	446		Jan	0.8082
1	July	420		Feb	0.8074
1	Aug	471		Mar	0.8764
1	Sep	355		Apr	0.9213
1	Oct	312		May	1.016
1	Nov	567		June	1.105
1	Dec	533		July	1.018
2	Jan	317		Aug	1.189
2	Feb	331		Sep	0.806
2	Mar	344		Oct	0.761
2	Apr	386		Nov	1.437
2	May	423		Dec	1.256
2	June	472			
2	July	415			
2	Aug	492			
2	Sep	340			
2	Oct	301			
2	Nov	629			
2	Dec	505			
3	Jan	338			
3	Feb	346			
3	Mar	383			
3	Apr	404			
3	May	431			
3	June	459			
3	July	433			
3	Aug	518			
3	Sep	309			
3	Oct	335			
3	Nov	594			
3	Dec	527			

(b) Moving Average with Seasonality

Year	Month	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan					
1	Feb					
1	Mar					
1	Apr					
1	May					
1	June					
1	July					
1	Aug					
1	Sep					
1	Oct	335	440			
1	Nov	594	413			
1	Dec	527	420			
2	Jan	364	450	424	343	21
2	Feb	343	425	428	345	2
2	Mar	391	446	432	378	13
2	Apr	437	474	440	406	31
2	May	458	451	448	456	2
2	June	494	447	457	505	11
2	July	468	460	457	465	3
2	Aug	555	467	453	538	17
2	Sep	387	480	458	369	18
2	Oct	364	478	469	357	7
2	Nov	662	461	475	683	21
2	Dec	581	463	473	594	13
3	Jan			467	378	

Number of previous periods to consider

n = 3

Type of Seasonality

Monthly

Month	Seasonal Factor
Jan	0.808
Feb	0.807
Mar	0.876
Apr	0.921
May	1.016
June	1.105
July	1.018
Aug	1.189
Sep	0.806
Oct	0.761
Nov	1.437
Dec	1.256

Mean Absolute Deviation

MAD = 13.30

Mean Square Error

MSE = 249.09

(c) Exponential Smoothing with Seasonality

Year	Month	True Value	Seasonally Adjusted Value	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan	364	450	420	339	25
1	Feb	343	425	426	344	1
1	Mar	391	446	426	373	18
1	Apr	437	474	430	396	41
1	May	458	451	439	446	12
1	June	494	447	441	488	6
1	July	468	460	442	450	18
1	Aug	555	467	446	530	25
1	Sep	387	480	450	363	24
1	Oct	364	478	456	347	17
1	Nov	662	461	461	662	0
1	Dec	581	463	461	579	2
2	Jan			461	373	
2	Feb					
2	Mar					
2	Apr					
2	May					
2	June					
2	July					
2	Aug					
2	Sep					
2	Oct					
2	Nov					
2	Dec					
3	Jan					
3	Feb					

Smoothing Constant

$\alpha =$ 0.2

Initial Estimate

Average = 420

Type of Seasonality

Monthly

Month	Seasonal Factor
Jan	0.808
Feb	0.807
Mar	0.876
Apr	0.921
May	1.016
June	1.105
July	1.018
Aug	1.189
Sep	0.806
Oct	0.761
Nov	1.437
Dec	1.256

Mean Absolute Deviation

MAD = 15.83

Mean Square Error

MSE = 384.99

(d) Exponential Smoothing with Seasonality and Trend

Year	Month	True Value	Seasonally Adjusted Value	Latest Trend	Estimated Trend	Seasonally Adjusted Forecast	Actual Forecast	Forecasting Error
1	Jan	364	450		0	420	339	25
1	Feb	343	425	6	1	427	345	2
1	Mar	391	446	1	1	428	375	16
1	Apr	437	474	5	2	433	399	38
1	May	458	451	10	3	445	452	6
1	June	494	447	5	4	450	497	3
1	July	468	460	3	4	453	461	7
1	Aug	555	467	5	4	458	545	10
1	Sep	387	480	6	4	464	374	13
1	Oct	364	478	7	5	472	359	5
1	Nov	662	461	6	5	479	688	26
1	Dec	581	463	2	4	479	602	21
2	Jan			1	4	480	388	
2	Feb							
2	Mar							
2	Apr							
2	May							
2	June							
2	July							
2	Aug							
2	Sep							
2	Oct							
2	Nov							
2	Dec							
3	Jan							
3	Feb							
3	Mar							
3	Apr							

Smoothing Constant

$\alpha =$	0.2
$\beta =$	0.2

Initial Estimate

Average =	420
Trend =	0

Type of Seasonality: Monthly

Month	Seasonal Factor
Jan	0.808
Feb	0.807
Mar	0.876
Apr	0.921
May	1.016
June	1.105
July	1.018
Aug	1.189
Sep	0.806
Oct	0.761
Nov	1.437
Dec	1.256

Mean Absolute Deviation

MAD = 14.26

Mean Square Error

MSE = 314.71

(e) The moving-average method results in the best MAD value (13.30) and the best MSE value (249.09).

(f)

Month	Avg. Forecast	Forecasting Error
January	341	23
February	345	2
March	375	16
April	400	37
May	451	7
June	497	3
July	459	9
August	537	18
September	369	18
October	354	10
November	677	15
December	592	12

MAD = 14.17

(g) The moving-average method performed better than the average of all three, so it should be used next year.

27.7-18.

Quarter	Sales	Forecast a)	Squared Error a)	Forecast b)	Squared Error b)	Forecast c)	Squared Error c)	Forecast d)	Squared Error d)
1	6900								
2	6700								
3	7900			6880	1040400	6840	1123600	6500	1960000
4	7100			6982	13924	7158	3364	6846	64516
5	8200	7150	1102500	6994	1454918	7141	1122328	6871	1766082
6	7000	7475	225625	7114	13092	7458	210149	7338	114384
7	7300	7550	62500	7103	38818	7321	437	7275	637
8	7500	7400	10000	7123	142370	7315	34364	7323	31458
9	9400	7500	3610000	7160	5015754	7370	4119934	7432	3872612
10	9200	7800	1960000	7384	3296509	7979	1490434	8256	891432
11	9800	8350	2102500	7566	4991051	8345	2115813	8857	888430
12	9900	8975	855625	7789	4454884	8782	1250390	9543	127179
13	11400	9575	3330625	8000	11557236	9117	5210929	10086	1727552
14	10000	10075	5625	8340	2754386	9802	39173	11034	1068141
15	9400	10275	765625	8506	798647	9861	212940	11184	3182671
16	8400	10175	3150625	8596	38297	9723	1750377	10949	6496332
17	8800	9800	1000000	8576	50119	9326	276795	10255	2116296
18	7600	9150	2402500	8599	997030	9168	2459499	9758	4656936
19	7500	8550	1102500	8499	997327	8698	1434713	8856	1838858
	MSE		1445750		2214986		1344426		1811971

SMALLEST

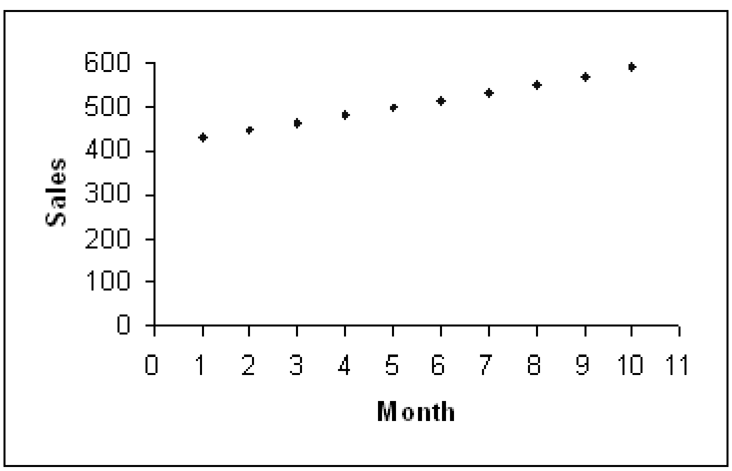
27.7-19.

Quarter	Sales	Forecast a)	Squared Error a)	Forecast b)	Squared Error b)	Forecast c)	Squared Error c)	Forecast d)	Squared Error d)
1	546								
2	528								
3	530			544	202	541	112	510	400
4	508			543	1210	537	866	500	67
5	647	528	14161	539	11599	523	15277	487	25665
6	594	553	1661	550	1930	560	1124	534	3622
7	665	570	9073	554	12218	631	1149	556	11828
8	630	604	702	566	4158	641	127	603	727
9	736	634	10404	572	26907	655	6642	628	11727
10	724	656	4590	588	18396	679	2030	687	1404
11	813	689	15438	602	44549	732	6496	727	7314
	MSE		8004		13463		3758		6973

smallest

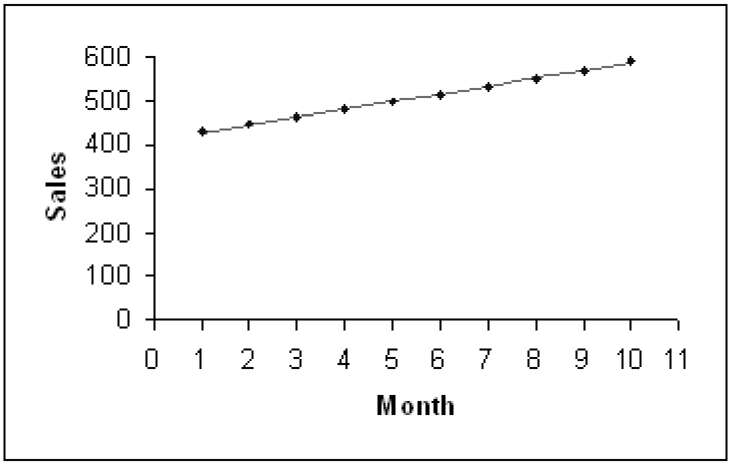
27.9-1.

(a)



(b) $y = 410 + 17.6x$

(c)



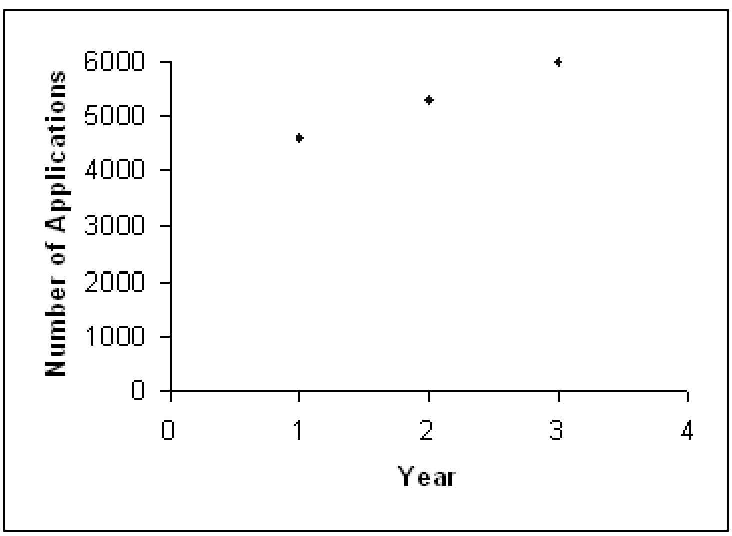
(d) $y = 410 + (17.6)(11) = 604$

(e) $y = 410 + (17.6)(20) = 762$

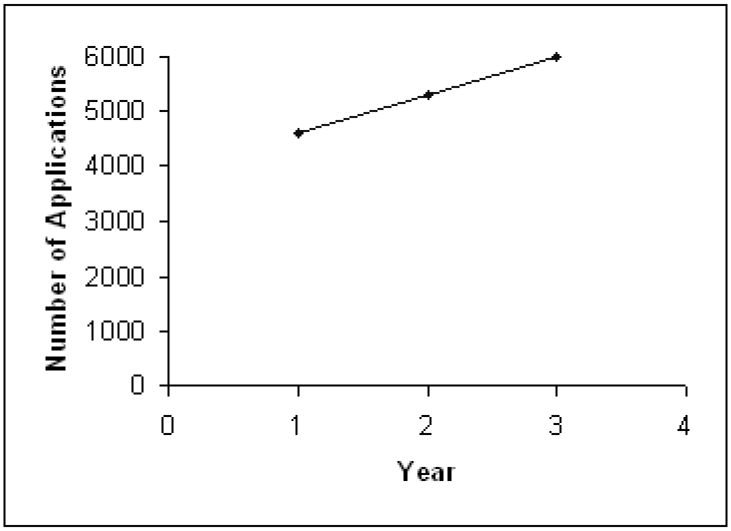
(f) The average growth in monthly sales is 17.6.

27.9-2.

(a)



(b)

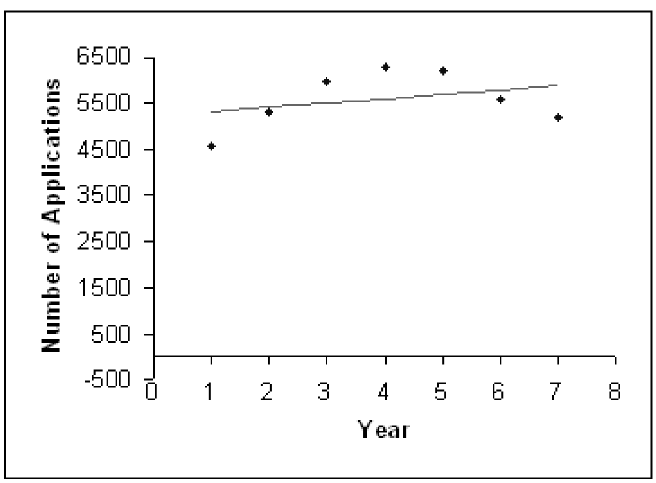


(c) $y = 3900 + 700x$

(d) $y(\text{Year } 4) = 3900 + (700)(4) = 6700$
$y(\text{Year } 5) = 3900 + (700)(5) = 7400$
$y(\text{Year } 6) = 3900 + (700)(6) = 8100$
$y(\text{Year } 7) = 3900 + (700)(7) = 8800$
$y(\text{Year } 8) = 3900 + (700)(8) = 9500$

(e) It does not make sense to use the forecast obtained earlier, 9500. The relationship between the variables has changed and thus the linear regression that was used is no longer appropriate.

(f)



$y = 5228 + 92.9x$
$y = 5228 + (92.9)(8) = 5971$

The linear regression line does not provide a close fit to the data. Consequently, the forecast that it provides for year 8 is not likely to be accurate. It does not make sense to continue to use a linear regression line when changing conditions cause a large shift in the underlying trend in the data.

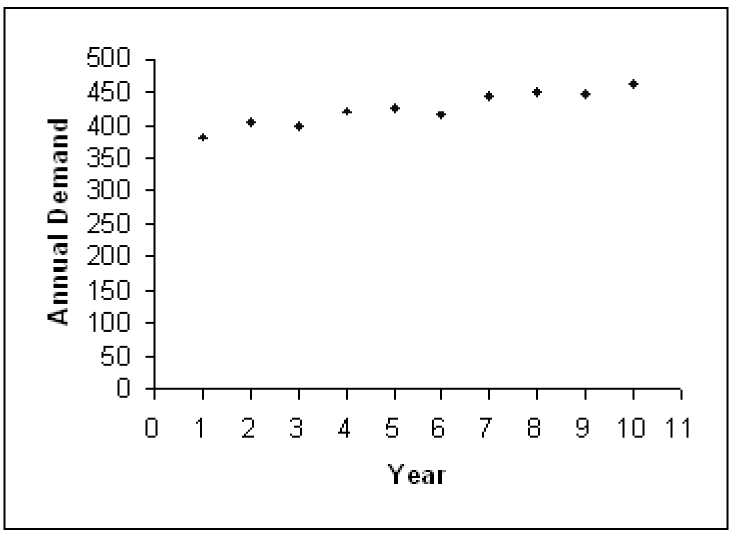
(g)

Time Period	True Value	Latest Trend	Estimated Trend	Exponential Smoothing Forecast	Forecasting Error		Smoothing Constants	
1	4,600		700.00	4,600	0		α =	0.5
2	5,300	700.00	700.00	5,300	0		β =	0.5
3	6,000	700.00	700.00	6,000	0			
4	6,300	700.00	700.00	6,700	400		**Initial Estimates**	
5	6,200	500.00	600.00	7,100	900		Average =	3,900
6	5,600	150.00	375.00	7,025	1,425		Trend =	700
7	5,200	-337.50	18.75	6,331	1,131			
8		-546.88	-264.06	5,502			**Mean Absolute Deviation**	
9							MAD =	550.9
10								
11							**Mean Square Error**	
12							MSE =	611,478.8

Casual forecasting takes all the data into account, even the data from before changing conditions cause a shift. Exponential smoothing with trend adjusts to shifts in the underlying trend.

27.9-3.

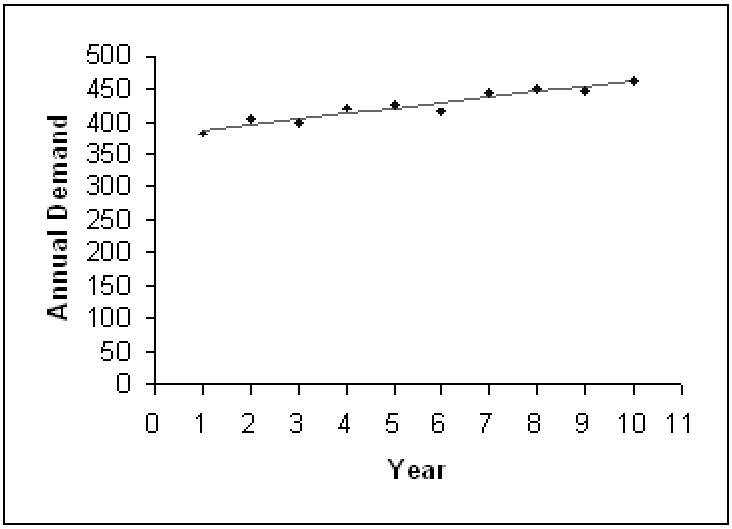
(a)



(b) $y = 380.27 + 8.15x$

Time Period	Independent Variable	Dependent Variable	Estimate	Estimation Error	Square of Error		Linear Regression Line	
							y = a + bx	
1	1	382	388	6.42	41		a =	380.27
2	2	405	397	8.43	71		b =	8.15
3	3	398	405	6.72	45			
4	4	421	413	8.13	66			
5	5	426	421	4.98	25		**Estimator**	
6	6	415	429	14.18	201		If x =	5,000
7	7	443	437	5.67	32			
8	8	451	445	5.52	30		then y=	41,137.84
9	9	446	454	7.63	58			
10	10	464	462	2.22	5			

(c)



(d) $y = 380 + (8.15)(11) = 470$

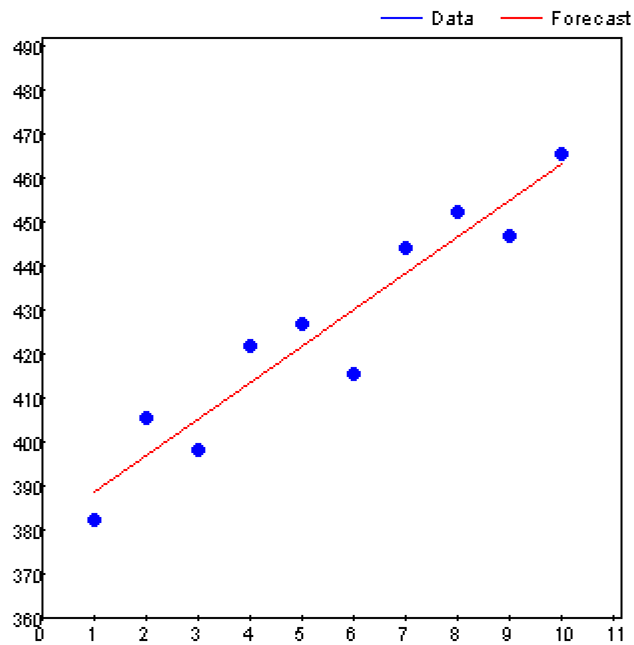
(e) $y = 380 + (8.15)(15) = 503$

(f) The average growth per year is 8.15 tons.

(g)

```
Forecasting -- Linear Regression Method
MAD = 6.99          MSE = 57.5
a = 380.27      b = 8.15           y = 380.27 + 8.15x
```

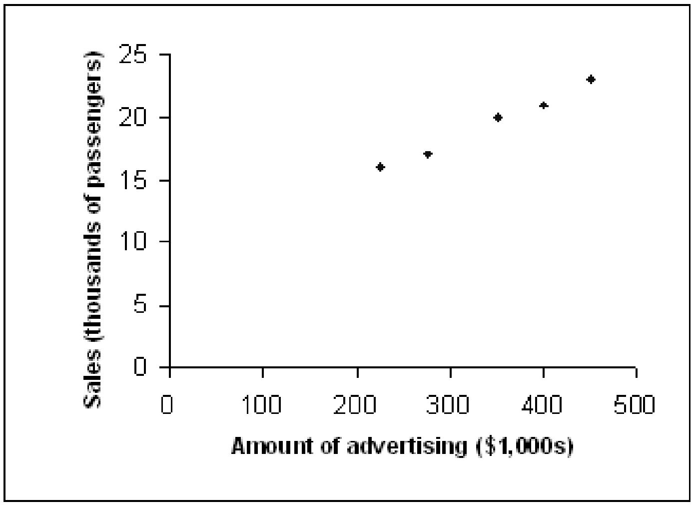
x	y	Forecast	Error
1	382	388.42	6.42
2	405	396.57	8.43
3	398	404.72	6.72
4	421	412.87	8.13
5	426	421.02	4.98
6	415	429.18	14.18
7	443	437.33	5.67
8	451	445.48	5.52
9	446	453.63	7.63
10	464	461.78	2.22



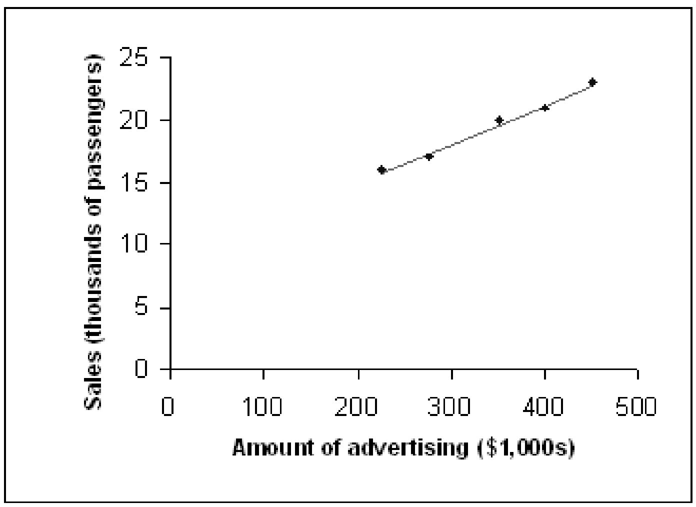
27.9-4.

(a) The amount of advertising is the independent variable and sales is the dependent variable.

(b)



(c) $y = 8.71 + 0.031x$



(d) $y = 8.71 + (0.031)(300) = 18,000$ passengers

(e) $22 = 8.71 + (0.031)(x) \Rightarrow x = \$429,000$

(f) An increase of 31 passengers can be attained.

27.9-5.

(a) If the sales increase from 16 to 19 when the amount of advertising is 225, then the linear regression line shifts below this point. The line actually shifts up, but not as much as the data point has shifted up.

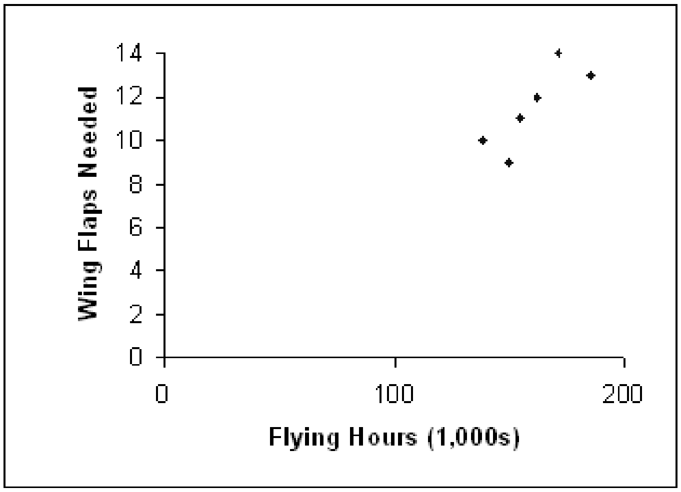
(b) If the sales increase from 23 to 26 when the amount of advertising is 450, then the linear regression line shifts below this point. The line actually shifts up, but not as much as the data point has shifted up.

(c) If the sales increase from 20 to 23 when the amount of advertising is 350, then the linear regression line shifts below this point. The line actually shifts up, but not as much as the data point has shifted up.

27.9-6.

(a) The number of flying hours is the independent variable and the number of wing flaps needed is the dependent variable.

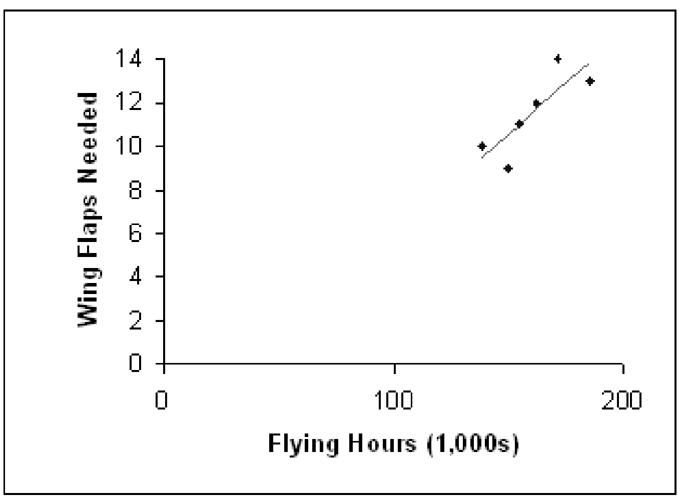
(b)



(c) $y = -3.382 + 0.093x$

Time Period	Independent Variable	Dependent Variable	Estimate	Estimation Error	Square of Error		Linear Regression Line y = a + bx	
1	162	12	12	0.30	0		a =	-3.382
2	149	9	10	1.49	2		b =	0.093
3	185	13	14	0.84	1			
4	171	14	13	1.46	2			
5	138	10	9	0.53	0		Estimator	
6	154	11	11	0.04	0		If x =	150
7								
8							then y=	10.584

(d)



(e) $y = -3.382 + (0.093)(150) = 11$

(f) $y = -3.382 + (0.093)(200) = 15$

27.9-7. Joe should use the linear regression line $y = -9.95 + 0.097x$ to develop a forecast for jobs in the future.

Time Period	Independent Variable	Dependent Variable	Estimate	Estimation Error	Square of Error
1	323	24	22	2.48	6
2	359	23	25	2.02	4
3	396	28	29	0.63	0
4	421	32	31	0.93	1
5	457	34	35	0.57	0
6	472	37	36	0.97	1
7	446	33	34	0.50	0
8	407	30	30	0.30	0
9	374	27	26	0.51	0
10	343	22	23	1.47	2

Linear Regression Line	
y = a + bx	
a =	-9.954
b =	0.097

27.9-8.

(a) $\widehat{y}(x) = 121.04 - 1.0346x \Rightarrow \widehat{y}(55) = 64.137$

(b) $$t_{0.025:5} = 2.571,\, s_{y|x} = 6.34$$

$$\sqrt{1 + \tfrac{1}{7} + \tfrac{(x_t - \overline{x})^2}{\sum (x_i - \overline{x})^2}} = 1.0735$$

The 95% prediction interval is $[46.64, 81.64]$.

(c) By interpolation:

$$t_{0.0125:5} = 3.365 - \tfrac{0.0025}{0.015}(3.365 - 2.571) = 3.233$$

The simultaneous 95% prediction interval is $[42.13, 86.14]$.

(d) By interpolation:

$$c^{**} = 10.722 + \tfrac{1}{2}(11.150 - 10.722) = 10.936$$

The simultaneous tolerance interval is $[37.1, 91.2]$.

27.9-9.

(a) $$\sum_{i=1}^{5} x = 20,\, \sum_{i=1}^{5} y = 40,\, \sum_{i=1}^{5} xy = 242,\, \sum_{i=1}^{5} x^2 = 120$$

$$\Rightarrow \widehat{y}(x) = -0.2 + 2.05x \Rightarrow \widehat{y}(10) = 20.3$$

(b) $$s_{y|x}^2 = 0.6333,\, t_{0.025:3} = 3.182$$

$$\sqrt{\tfrac{1}{5} + \tfrac{(x_t - \overline{x})^2}{\sum (x_i - \overline{x})^2}} = \sqrt{1.1}$$

The 95% prediction interval is $[17.64, 22.9]$.

(c)

$$\sqrt{1 + \tfrac{1}{5} + \tfrac{(x_t - \overline{x})^2}{\sum (x_i - \overline{x})^2}} = \sqrt{2.1}$$

The 95% prediction interval is $[16.630, 23.970]$.

(d) By interpolation:

$$c^{**} = 11.150 + \tfrac{1}{2}(14.953 - 11.150) = 13.0515$$

The simultaneous tolerance interval is $[9.406, 31.194]$.

27.9-10.

(a)

$$k = \frac{\sum_{i=1}^{5} x_i y_i - \left(\sum_{i=1}^{5} x_i \sum_{i=1}^{5} y_i\right)/5}{\sum_{i=1}^{5} x_i^2 - \left(\sum_{i=1}^{5} x_i\right)^2/5} = \frac{19.96-0}{10-0} = 1.996$$

$$\log g = \frac{\sum_{i=1}^{5}(y_i - kx_i)}{5} = \frac{0.08}{5} = 0.016$$

$$\Rightarrow \log r = 0.016 + 1.996 \log t$$

$$\log t = 3 \Rightarrow \log r = 0.016 + 1.996 \times 3 = 6.004$$

The forecast for the distance traveled when $\log\ t = 3$ is then $10^{6.004}$, which is approximately one million.

(b)

$\log t$	$\log r$	$\widehat{E}(\log r)$
-2.0	-3.95	–
-1.0	-2.12	–
0.0	0.08	-3.767
1.0	2.20	-3.382
2.0	3.87	-2.824
3.0	–	-2.155

(c)

$\log t$	$\log r$	$\alpha x + (1-\alpha)F$	Trend	$\widehat{E}(\log r)$
-2.0	-3.95	-3.950	1.996	–
-1.0	-2.12	-1.971	1.994	–
0.0	0.08	0.029	1.995	0.024
1.0	2.20	2.042	1.997	2.024
2.0	3.87	4.022	1.995	4.039
3.0	–	–	–	6.017

27.9-11.

$$Q = \sum_{i=1}^{n}(y_i - bx_i)^2 \Rightarrow \frac{dQ}{db} = \sum_{i=1}^{n} -2x_i(y_i - bx_i) = 0 \Rightarrow B = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$$

Cases

27-1 a) We need to forecast the call volume for each day separately.

1) To obtain the seasonally adjusted call volume for the past 13 weeks, we first have to determine the seasonal factors. Because call volumes follow seasonal patterns within the week, we have to calculate a seasonal factor for Monday, Tuesday, Wednesday, Thursday, and Friday. We use the Template for Seasonal Factors. The 0 values for holidays should not factor into the average. Leaving them blank (rather than 0) accomplishes this. (A blank value does not factor into the AVERAGE function in Excel that is used to calculate the seasonal values.) Using this template (shown on the following page, the seasonal factors for Monday, Tuesday, Wednesday, Thursday, and Friday are 1.238, 1.131, 0.999, 0.850, and 0.762, respectively.

	A	B	C	D	E	F	G
1	**Template for Seasonal Factors**						
2							
3				**True**			
4		**Week**	**Day**	**Value**		**Type of Seasonality**	
5		44	Mon	1,130		Daily	
6		44	Tue	851			
7		44	Wed	859			
8		44	Thur	828			**Estimate for**
9		44	Fri	726		**Day**	**Seasonal Factor**
10		45	Mon	1,085		Mon	1.238
11		45	Tue	1,042		Tue	1.131
12		45	Wed	892		Wed	0.999
13		45	Thur	840		Thur	0.850
14		45	Fri	799		Fri	0.762
15		46	Mon	1,303			
16		46	Tue	1,121			
17		46	Wed	1,003			
18		46	Thur	1,113			
19		46	Fri	1,005			
20		47	Mon	2,652			
21		47	Tue	2,825			
22		47	Wed	1,841			
23		47	Thur				
24		47	Fri				
25		48	Mon	1,949			Average Call Volume
26		48	Tue	1,507			1,025
27		48	Wed	989			
28		48	Thur	990			
29		48	Fri	1,084			
30		49	Mon	1,260			
31		49	Tue	1,134			
32		49	Wed	941			
33		49	Thur	847			
34		49	Fri	714			
35		50	Mon	1,002			
36		50	Tue	847			
37		50	Wed	922			
38		50	Thur	842			
39		50	Fri	784			
40		51	Mon	823			
41		51	Tue				
42		51	Wed				
43		51	Thur	401			
44		51	Fri	429			
45		52/1	Mon	1,209			
46		52/1	Tue	830			
47		52/1	Wed				
48		52/1	Thur	1,082			
49		52/1	Fri	841			
50		2	Mon	1,362			
51		2	Tue	1,174			
52		2	Wed	967			
53		2	Thur	930			
54		2	Fri	853			
55		3	Mon	924			
56		3	Tue	954			
57		3	Wed	1,346			
58		3	Thur	904			
59		3	Fri	758			
60		4	Mon	886			
61		4	Tue	878			
62		4	Wed	802			
63		4	Thur	945			
64		4	Fri	610			
65		5	Mon	910			
66		5	Tue	754			
67		5	Wed	705			
68		5	Thur	729			
69		5	Fri	772			

2) To forecast the call volume for the next week using the last-value forecasting method, we need to use the Last Value with Seasonality template. To forecast the next week, we need only start with the last Friday value since the Last Value method only looks at the previous day.

	A	B	C	D	E	F	G	H	I	J	K
1	**Template for Last-Value Forecasting Method with Seasonality**										
2											
3					**Seasonally**	**Seasonally**					
4				**True**	**Adjusted**	**Adjusted**	**Actual**	**Forecasting**			
5		**Week**	**Day**	**Value**	**Value**	**Forecast**	**Forecast**	**Error**			**Type of Seasonality**
6		5	Mon								Daily
7		5	Tue								
8		5	Wed							**Day**	**Seasonal Factor**
9		5	Thur							Mon	1.238
10		5	Fri	772	1,013					Tue	1.131
11		6	Mon	1,254	1,013	1,013	1,254	0		Wed	0.999
12		6	Tue	1,146	1,013	1,013	1,146	0		Thur	0.850
13		6	Wed	1,012	1,013	1,013	1,012	0		Fri	0.762
14		6	Thur	860	1,012	1,013	860	0			
15		6	Fri	771	1,012	1,012	771	0			

The forecasted call volume for the next week is 5,045 calls: 1,254 calls are received on Monday, 1,148 calls are received on Tuesday, 1,012 calls are received on Wednesday, 860 calls are received on Thursday, and 771 calls are received on Friday.

3) To forecast the call volume for the next week using the averaging forecasting method, we need to use the Averaging with Seasonality template.

	A	B	C	D	E	F	G	H	I	J	K
1	Template for Averaging Forecasting Method with Seasonality										
2											
3					Seasonally	Seasonally					
4				True	Adjusted	Adjusted	Actual	Forecasting			
5		Week	Day	Value	Value	Forecast	Forecast	Error			Type of Seasonality
6		44	Mon	1,130	913						Daily
7		44	Tue	851	752	913	1,033	182			
8		44	Wed	859	860	833	832	27		Day	Seasonal Factor
9		44	Thur	828	975	842	715	113		Mon	1.238
10		44	Fri	726	953	875	667	59		Tue	1.131
11		45	Mon	1,085	877	890	1,102	17		Wed	0.999
12		45	Tue	1,042	921	888	1,005	37		Thur	0.850
13		45	Wed	892	893	893	892	0		Fri	0.762
14		45	Thur	840	989	893	759	81			
15		45	Fri	799	1,048	903	689	110			
16		46	Mon	1,303	1,053	918	1,136	167			
17		46	Tue	1,121	991	930	1,052	69			
18		46	Wed	1,003	1,004	935	934	69			
19		46	Thur	1,113	1,310	941	799	314			
20		46	Fri	1,005	1,319	967	737	268			
21		47	Mon	2,652	2,143	990	1,226	1,426			
22		47	Tue	2,825	2,497	1,062	1,202	1,623		Mean Absolute Deviation	
23		47	Wed	1,841	1,842	1,147	1,146	695		MAD =	267.27
24		47	Thur	0	0	1,185	1,007	1,007			
25		47	Fri	0	0	1,123	856	856		Mean Square Error	
26		48	Mon	1,949	1,575	1,067	1,321	628		MSE =	187,916.17
27		48	Tue	1,507	1,332	1,091	1,234	273			
28		48	Wed	989	990	1,102	1,101	112			
29		48	Thur	990	1,165	1,097	932	58			
30		48	Fri	1,084	1,422	1,100	838	246			
31		49	Mon	1,260	1,018	1,113	1,377	117			
32		49	Tue	1,134	1,002	1,109	1,255	121			
33		49	Wed	941	942	1,105	1,104	163			
34		49	Thur	847	997	1,099	934	87			
35		49	Fri	714	937	1,096	835	121			
36		50	Mon	1,002	810	1,091	1,350	348			
37		50	Tue	847	749	1,081	1,224	377			
38		50	Wed	922	923	1,071	1,070	148			
39		50	Thur	842	991	1,067	906	64			
40		50	Fri	784	1,029	1,064	811	27			
41		51	Mon	823	665	1,063	1,316	493			
42		51	Tue	0	0	1,052	1,191	1,191			
43		51	Wed	0	0	1,024	1,023	1,023			
44		51	Thur	401	472	997	847	446			
45		51	Fri	429	563	983	750	321			
46		52/1	Mon	1,209	977	973	1,204	5			
47		52/1	Tue	830	734	973	1,101	271			
48		52/1	Wed	0	0	967	967	967			
49		52/1	Thur	1,082	1,274	945	803	279			
50		52/1	Fri	841	1,103	952	726	115			
51		2	Mon	1,362	1,100	956	1,183	179			
52		2	Tue	1,174	1,038	959	1,085	89			
53		2	Wed	967	968	960	960	7			
54		2	Thur	930	1,095	961	816	114			
55		2	Fri	853	1,119	963	734	119			
56		3	Mon	924	746	966	1,196	272			
57		3	Tue	954	843	962	1,089	135			
58		3	Wed	1,346	1,347	960	959	387			
59		3	Thur	904	1,064	967	822	82			
60		3	Fri	758	995	969	738	20			
61		4	Mon	886	716	969	1,200	314			
62		4	Tue	878	776	965	1,092	214			
63		4	Wed	802	803	962	961	159			
64		4	Thur	945	1,112	959	815	130			
65		4	Fri	610	800	961	733	123			
66		5	Mon	910	735	959	1,187	277			
67		5	Tue	754	666	955	1,081	327			
68		5	Wed	705	706	950	950	245			
69		5	Thur	729	858	947	804	75			
70		5	Fri	772	1,013	945	720	52			
71		6	Mon	1171	946	946	1,171	0			
72		6	Tue	1071	947	946	1,071	0			
73		6	Wed	945	946	946	945	0			
74		6	Thur	804	946	946	804	0			
75		6	Fri	721	946	946	721	0			

The forecasted call volume for the next week is 4,712 calls: 1,171 calls are received on Monday, 1,071 calls are received on Tuesday, 945 calls are received on Wednesday, 804 calls are received on Thursday, and 721 calls are received on Friday.

4) To forecast the call volume for the next week using the moving-average forecasting method, we need to use the Moving Averaging with Seasonality template. Since only the past 5 days are used in the forecast, we start with Monday of the last week to forecast through Friday of the next week.

	A	B	C	D	E	F	G	H	I	J	K
1	Template for Moving-Average Forecasting Method with Seasonality										
2											
3					Seasonally	Seasonally					
4				True	Adjusted	Adjusted	Actual	Forecasting		Number of previous	
5		Week	Day	Value	Value	Forecast	Forecast	Error		periods to consider	
6		5	Mon	910	735					n =	5
7		5	Tue	754	666						
8		5	Wed	705	706						Type of Seasonality
9		5	Thur	729	858						Daily
10		5	Fri	772	1,013						
11		6	Mon	985	796	796	985	0		Day	Seasonal Factor
12		6	Tue	914	808	808	914	0		Mon	1.238
13		6	Wed	835	836	836	835	0		Tue	1.131
14		6	Thur	732	862	862	732	0		Wed	0.999
15		6	Fri	658	863	863	658	0		Thur	0.850
16		7	Mon			833	1,031			Fri	0.762

The forecasted call volume for the next week is 4,124 calls: 985 calls are received on Monday, 914 calls are received on Tuesday, 835 calls are received on Wednesday, 732 calls are received on Thursday, and 658 calls are received on Friday.

5) To forecast the call volume for the next week using the exponential smoothing forecasting method, we need to use the Exponential with Seasonality template. We start with the initial estimate of 1,125 calls (the average number of calls on non-holidays during the previous 13 weeks).

	B	C	D	E	F	G	H	J	K
1	**Template for Exponential Smoothing Forecasting Method with Seasonality**								
2									
3				**Seasonally**	**Seasonally**				
4			**True**	**Adjusted**	**Adjusted**	**Actual**	**Forecasting**	**Smoothing Constant**	
5	**Week**	**Day**	**Value**	**Value**	**Forecast**	**Forecast**	**Error**	α =	0.1
6	44	Mon	1,130	913	1,025	1,269	139		
7	44	Tue	851	752	1,014	1,147	296	**Initial Estimate**	
8	44	Wed	859	860	988	987	128	Average =	1,025
9	44	Thur	828	975	975	828	0		
10	44	Fri	726	953	975	743	17		**Type of Seasonality**
11	45	Mon	1,085	877	973	1,204	119		Daily
12	45	Tue	1,042	921	963	1,089	47		
13	45	Wed	892	893	959	958	66	**Day**	**Seasonal Factor**
14	45	Thur	840	989	952	809	31	Mon	1.238
15	45	Fri	799	1,048	956	728	71	Tue	1.131
16	46	Mon	1,303	1,053	965	1,195	108	Wed	0.999
17	46	Tue	1,121	991	974	1,102	19	Thur	0.850
18	46	Wed	1,003	1,004	976	975	28	Fri	0.762
19	46	Thur	1,113	1,310	978	831	282		
20	46	Fri	1,005	1,319	1,012	771	234		
21	47	Mon	2,652	2,143	1,042	1,290	1,362		
22	47	Tue	2,825	2,497	1,152	1,304	1,521		
23	47	Wed	1,841	1,842	1,287	1,286	555		
24	47	Thur	0	0	1,342	1,140	1,140		
25	47	Fri	0	0	1,208	921	921		
26	48	Mon	1,949	1,575	1,087	1,346	603		
27	48	Tue	1,507	1,332	1,136	1,285	222	**Mean Absolute Deviation**	
28	48	Wed	989	990	1,156	1,155	166	MAD =	261.3
29	48	Thur	990	1,165	1,139	968	22		
30	48	Fri	1,084	1,422	1,142	870	214	**Mean Square Error**	
31	49	Mon	1,260	1,018	1,170	1,448	188	MSE =	171,377.0
32	49	Tue	1,134	1,002	1,155	1,306	172		
33	49	Wed	941	942	1,139	1,138	197		
34	49	Thur	847	997	1,120	951	104		
35	49	Fri	714	937	1,107	844	130		
36	50	Mon	1,002	810	1,090	1,350	348		
37	50	Tue	847	749	1,062	1,202	355		
38	50	Wed	922	923	1,031	1,030	108		
39	50	Thur	842	991	1,020	867	25		
40	50	Fri	784	1,029	1,017	775	9		
41	51	Mon	823	665	1,018	1,260	437		
42	51	Tue	0	0	983	1,112	1,112		
43	51	Wed	0	0	885	884	884		
44	51	Thur	401	472	796	676	275		
45	51	Fri	429	563	764	582	153		
46	52/1	Mon	1,209	977	744	921	288		
47	52/1	Tue	830	734	767	868	38		
48	52/1	Wed	0	0	764	763	763		
49	52/1	Thur	1,082	1,274	687	584	498		
50	52/1	Fri	841	1,103	746	568	273		
51	2	Mon	1,362	1,100	782	968	394		
52	2	Tue	1,174	1,038	814	920	254		
53	2	Wed	967	968	836	835	132		
54	2	Thur	930	1,095	849	721	209		
55	2	Fri	853	1,119	874	666	187		
56	3	Mon	924	746	898	1,112	188		
57	3	Tue	954	843	883	999	45		
58	3	Wed	1,346	1,347	879	878	468		
59	3	Thur	904	1,064	926	787	117		
60	3	Fri	758	995	940	716	42		
61	4	Mon	886	716	945	1,170	284		
62	4	Tue	878	776	922	1,043	165		
63	4	Wed	802	803	908	907	105		
64	4	Thur	945	1,112	897	762	183		
65	4	Fri	610	800	919	700	90		
66	5	Mon	910	735	907	1,122	212		
67	5	Tue	754	666	890	1,007	253		
68	5	Wed	705	706	867	867	162		
69	5	Thur	729	858	851	723	6		
70	5	Fri	772	1,013	852	649	123		
71	6	Mon	1074	868	868	1,074	0		
72	6	Tue	982	868	868	982	0		
73	6	Wed	867	868	868	867	0		
74	6	Thur	737	867	868	737	0		
75	6	Fri	661	867	868	661	0		

The forecasted call volume for the next week is 4,322 calls: 1,074 calls are received on Monday, 982 calls are received on Tuesday, 867 calls are received on

Wednesday, 737 calls are received on Thursday, and 661 calls are received on Friday.

b) To obtain the mean absolute deviation for each forecasting method, we simply need to subtract the true call volume from the forecasted call volume for each day in the sixth week. We then need to take the absolute value of the five differences. Finally, we need to take the average of these five absolute values to obtain the mean absolute deviation.

1) The spreadsheet for the calculation of the mean absolute deviation for the last-value forecasting method follows.

	A	B	C	D	E	F	G	H
1	**Last Value**							
2								
3			**True**	**Actual**	**Forecast**			
4	**Week**	**Day**	**Value**	**Forecast**	**Error**			
5	6	Monday	723	1,254	531			
6	6	Tuesday	677	1,146	469			
7	6	Wednesday	521	1,012	491			
8	6	Thursday	571	860	289		**Mean Absolute Deviation**	
9	6	Friday	498	771	273		MAD =	410.6

This method is the least effective of the four methods because this method depends heavily upon the <u>average</u> seasonality factors. If the average seasonality factors are not the true seasonality factors for week 6, a large error will appear because the average seasonality factors are used to transform the Friday call volume in week 5 to forecasts for <u>all</u> call volumes in week 6. We calculated in part (a) that the call volume for Friday is 0.762 times lower than the overall average call volume. In week 6, however, the call volume for Friday is only 0.83 times lower than the average call volume over the week. Also, we calculated that the call volume for Monday is 1.34 times higher than the overall average call volume. In Week 6, however, the call volume for Monday is only 1.21 times higher than the average call volume over the week. These differences introduce a large error.

2) The spreadsheet for the calculation of the mean absolute deviation for the averaging forecasting method appears below.

	A	B	C	D	E	F	G	H
1	**Averaging**							
2								
3			**True**	**Actual**	**Forecast**			
4	**Week**	**Day**	**Value**	**Forecast**	**Error**			
5	6	Monday	723	1,171	448			
6	6	Tuesday	677	1,071	394			
7	6	Wednesday	521	945	424			
8	6	Thursday	571	804	233		**Mean Absolute Deviation**	
9	6	Friday	498	721	223		MAD =	344.4

This method is the second-most effective of the four methods. Again, the reason lies in the average seasonality factors. Applying the average seasonality factors to an average call volume yields a much more accurate result than applying average seasonality factors to only one call volume. This method is not the most effective method, however, because the centralized call center experiences not only daily seasonality, but also weekly seasonality. For example, the call volumes in weeks 45 and 46 are much greater than the call volumes in week 6. Therefore, these larger call volumes inflate the average call volume, which in turn inflates the forecasts for Week 6.

3)The spreadsheet for the calculation of the mean absolute deviation for the moving-average forecasting method appears below.

	A	B	C	D	E	F	G	H
1	**Moving Average**							
2								
3			**True**	**Actual**	**Forecast**			
4	**Week**	**Day**	**Value**	**Forecast**	**Error**			
5	6	Monday	723	985	262			
6	6	Tuesday	677	914	237			
7	6	Wednesday	521	835	314			
8	6	Thursday	571	732	161		**Mean Absolute Deviation**	
9	6	Friday	498	658	160		MAD =	226.8

This method is the most effective of the four methods because this method only uses the average week 5 call volume to forecast the call volumes for week 6. Again, applying the average seasonality factors to an average call volume yields a much more accurate result than applying average seasonality factors to only one call volume. Also, the average call volume used in this method is not overly inflated since it is an average of the week 5 call volumes, which are closer to the week 6 call volumes than any other of the 13 weeks.

4) The spreadsheet for the calculation of the mean absolute deviation for exponential forecasting method follows.

	A	B	C	D	E	F	G	H
1	**Exponential Smoothing**							
2								
3			**True**	**Actual**	**Forecast**			
4	**Week**	**Day**	**Value**	**Forecast**	**Error**			
5	6	Monday	723	1,074	351			
6	6	Tuesday	677	982	305			
7	6	Wednesday	521	867	346			
8	6	Thursday	571	737	166		**Mean Absolute Deviation**	
9	6	Friday	498	661	163		MAD =	266.2

This method is nearly as effective as the moving average. This method is a little more effective than the averaging forecasting method because the smoothing constant causes less weight to be placed on the call volumes in the earlier weeks.

c) This problem is simply a linear regression problem.

1) To find a mathematical relationship, we use the Linear Regression template. The decentralized case volumes are the independent variables, and the centralized case volumes are the dependent variables. Substituting the case volume data, we obtain the following spreadsheet. The relationship is $y = 1576 + 0.756x$, where x is the decentralized case volume, and y is the estimated centralized case volume.

	A	B	C	D	E	F	G	H	I	J
1	**Template for Linear Regression**									
2										
3			**Independent**	**Dependent**		**Estimation**	**Square**		**Linear Regression Line**	
4		**Week**	**Variable**	**Variable**	**Estimate**	**Error**	**of Error**		**y = a + bx**	
5		44	612	2,052	2,038	13.84	192		a =	1,576
6		45	721	2,170	2,121	49.45	2,445		b =	0.76
7		46	693	2,779	2,099	679.61	461,872			
8		47	540	2,334	1,984	350.27	122,690			
9		48	1,386	2,514	2,623	109.26	11,938		**Estimator**	
10		49	577	1,713	2,012	298.70	89,221		If x =	613
11		50	405	1,927	1,882	45.32	2,054			
12		51	441	1,167	1,909	741.89	550,400		then y=	2,038.9
13		52/1	655	1,549	2,071	521.66	272,132			
14		2	572	2,126	2,008	118.08	13,943			
15		3	475	2,337	1,935	402.41	161,932			
16		4	530	1,916	1,976	60.17	3,620			
17		5	595	2,098	2,025	72.69	5,284			

2) To forecast the week 6 call volume for the centralized call center, we simply input the week 6 decentralized case volume for the value of *x* in the Estimator section of the Linear Regression Spreadsheet (as shown in part 1 above). The value of *y* then represents the week 6 centralized case volume. We multiply this value of y by 1.5 to obtain the week 6 centralized call volume. Thus, the forecasted number of calls is 1.5 * 2,038.9 = 3,058.

We then break this weekly call volume into daily call volume. We do this conversion by dividing the weekly call volume by the sum of the seasonal factors calculated in part (a) and then multiplying this weekly call volume by the appropriate seasonal factor to find the call volume for each of the five days of the week. The spreadsheet showing these calculations follows:

	A	B	C
1	Week 6 Call Volume	3058	
2	Daily Call Volume	611.6	
3			
4		Seasonal	Forecasted
5	Day	Factor	Call Volume
6	Monday	1.238	757
7	Tuesday	1.131	692
8	Wednesday	0.999	611
9	Thursday	0.850	520
10	Friday	0.762	466

The forecasted call volume for week 6 is 3,046 calls: 757 calls are received on Monday, 692 calls are received on Tuesday, 611 calls are received on Wednesday, 520 calls are received on Thursday, and 466 calls are received on Friday.

3) To calculate the mean absolute deviation, we need to subtract the true call volume from the forecasted call volume for each day in the sixth week. We then need to take the absolute value of the five differences. Finally, we need to take the average of these five absolute values to obtain the mean absolute deviation.

The spreadsheet for the calculation of the mean absolute deviation follows.

	A	B	C	D	E	F	G	H
1	**Causal Forecasting**							
2								
3			**True**	**Actual**	**Forecast**			
4	**Week**	**Day**	**Value**	**Forecast**	**Error**			
5	6	Monday	723	757	34			
6	6	Tuesday	677	692	15			
7	6	Wednesday	521	611	90			
8	6	Thursday	571	520	51		**Mean Absolute Deviation**	
9	6	Friday	498	466	32		MAD =	44.4

This forecasting method is by far the most effective method. The centralized center performs the same services and serves the same population as the decentralized center. Therefore, the call volume trends are the same. Once we have a factor to scale the decentralized call volumes to the centralized call volumes, we have a very effective forecasting method.

d) We would definitely recommend using the causal forecasting method implemented in part (c) because it yields the lowest error. The causal method shows us that the call volume trends remain relatively the same year after year. We had to convert between case volumes and call volumes in part (c), however, and such a conversion introduces error. For example, what if a case generates a higher or lower number of calls? We therefore recommend that call volume data be meticulously recorded as the centralized center continues its operation. Once one year's worth of call volumes have been collected, the causal forecasting model should be updated. The model should be updated to use the historical centralized call volume data instead of the historical decentralized case volume data.