CHAPTER 20: SIMULATION

20.1-1.

(a) 0.0000 to 0.4999 correspond to tails.
0.5000 to 0.9999 correspond to heads.

Random observations: 0.6961 = heads, 0.2086 = tails, 0.1457 = tails, 0.3098 = tails, 0.6996 = heads, 0.9617 = heads

(b) 0.0000 to 0.5999 correspond to strikes.
0.6000 to 0.9999 correspond to balls.

Random observations: 0.6961 = ball, 0.2086 = strike, 0.1457 = strike, 0.3098 = strike, 0.6996 = ball, 0.9617 = ball

(c) 0.0000 to 0.3999 correspond to green lights.
0.4000 to 0.4999 correspond to yellow lights.
0.5000 to 0.9999 correspond to red lights.

Random observations: 0.6961 = red, 0.2086 = green, 0.1457 = green, 0.3098 = green, 0.6996 = red, 0.9617 = red

20.1-2.

(a) If it is raining: 0.0000 to 0.5999 correspond to rain next day,
0.6000 to 0.9999 correspond to clear next day.

If it is clear: 0.0000 to 0.7999 correspond to clear next day,
0.8000 to 0.9999 correspond to rain next day.

Day	Random Number	Weather
1	0.6996	Clear
2	0.9617	Rain
3	0.6117	Clear
4	0.3948	Clear
5	0.7769	Clear
6	0.5750	Clear
7	0.6271	Clear
8	0.2017	Clear
9	0.7760	Clear
10	0.9918	Rain

(b)

If Clear, Prob(Stays Clear) =		0.8
If Rain, Prob(Stays Rain) =		0.6
	Random	
Day	Number	Weather
		Clear
1	0.8815	Rain
2	0.0252	Rain
3	0.8081	Clear
4	0.5692	Clear
5	0.0277	Clear
6	0.9160	Rain
7	0.2733	Rain
8	0.0558	Rain
9	0.4683	Rain
10	0.8070	Clear

20.1-3.

(a)

$$P(2) = \frac{4}{25}, P(3) = \frac{7}{25}, P(4) = \frac{8}{25}, P(5) = \frac{5}{25}, P(6) = \frac{1}{25}$$

(b)

Mean: $(2)\frac{4}{25} + (3)\frac{7}{25} + (4)\frac{8}{25} + (5)\frac{5}{25} + (6)\frac{1}{25} = 3.68$ stoves

(c) 0.0000 to 0.1599 correspond to 2 stoves being sold.
0.1600 to 0.4399 correspond to 3 stoves being sold.
0.4400 to 0.7599 correspond to 4 stoves being sold.
0.7600 to 0.9599 correspond to 5 stoves being sold.
0.9600 to 0.9999 correspond to 6 stoves being sold.

(d) $0.4476 \Rightarrow 4$ stoves, $0.9713 \Rightarrow 6$ stoves, $0.0629 \Rightarrow 2$ stoves

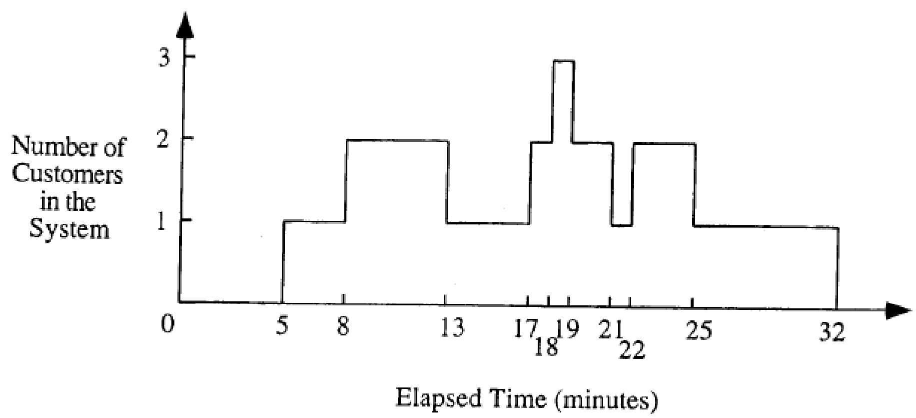
The average of these is $(4 + 6 + 2)/3 = 4$, which exceeds the mean in (b) by 0.32.

(e) Answers will vary. The following 300-day simulation yielded an average demand of 3.723.

Day	Random Number	Demand		Distribution of Demand		
				Probability	Cumulative	Demand
1	0.7167	4		0.16	0	2
2	0.3367	3		0.28	0.16	3
3	0.1763	3		0.32	0.44	4
4	0.9230	5		0.20	0.76	5
5	0.6635	4		0.04	0.96	6
6	0.4588	4				
7	0.2529	3				
297	0.8098	5				
298	0.4217	3				
299	0.4709	4				
300	0.0008	2				
	Average =	3.723				

20.1-4.

(a)



(b)

$$\text{Est}\{P_0\} = \frac{5}{32} = 0.156 \qquad \text{Est}\{P_1\} = \frac{3+4+1+7}{32} = 0.469$$

$$\text{Est}\{P_2\} = \frac{5+1+2+3}{32} = 0.344 \qquad \text{Est}\{P_3\} = \frac{1}{32} = 0.031$$

$$\text{Est}\{L\} = \sum_{n=0}^{3} nP_n = 0 \cdot 0.156 + 1 \cdot 0.469 + 2 \cdot 0.344 + 3 \cdot 0.031 = 1.25 \text{ customers}$$

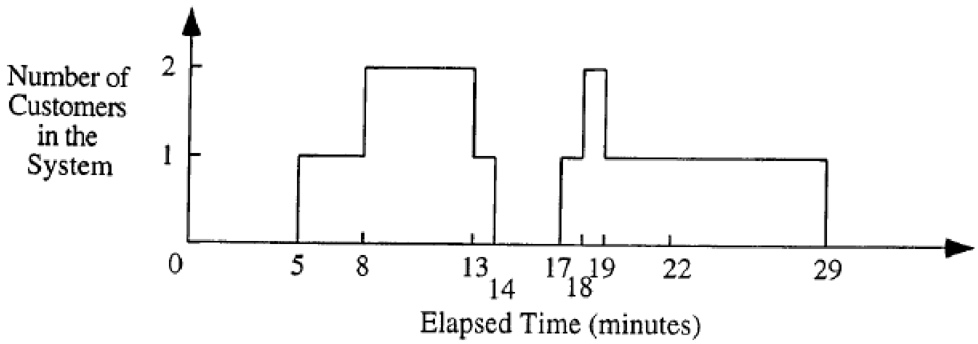
$$\text{Est}\{L_q\} = \sum_{n=1}^{3} (n-1)P_n = 0 \cdot 0.469 + 1 \cdot 0.344 + 2 \cdot 0.031 = 0.406 \text{ customers}$$

Customers	Arrival Time	Service Time	Departure Time	System Time	Wait Time
1	5	8	13	8	0
2	8	6	19	11	5
3	17	2	21	4	2
4	18	4	25	7	3
5	22	7	32	10	3

$$\text{Est}\{W\} = \frac{\text{sum of observed system times}}{\text{number of observed system times}} = \frac{40}{5} = 8 \text{ minutes}$$

$$\text{Est}\{W_q\} = \frac{\text{sum of observed waiting times}}{\text{number of observed waiting times}} = \frac{32}{5} = 2.6 \text{ minutes}$$

(c)



(d)

$\text{Est}\{P_0\} = \frac{5+3}{29} = 0.276 \qquad \text{Est}\{P_1\} = \frac{3+1+1+3+7}{29} = 0.517$

$\text{Est}\{P_2\} = \frac{5+1}{29} = 0.207$

$$\text{Est}\{L\} = \sum_{n=0}^{2} nP_n = 0 \cdot 0.276 + 1 \cdot 0.517 + 2 \cdot 0.207 = 0.931 \text{ customers}$$

$$\text{Est}\{L_q\} = \sum_{n=1}^{2} (n-1)P_n = 0 \cdot 0.517 + 1 \cdot 0.207 = 0.207 \text{ customers}$$

Customers	Arrival Time	Service Time	Departure Time	System Time	Wait Time
1	5	8	13	8	0
2	8	6	14	6	0
3	17	2	19	2	0
4	18	4	22	4	0
5	22	7	29	7	0

$$\text{Est}\{W\} = \frac{\text{sum of observed system times}}{\text{number of observed system times}} = \frac{27}{5} = 5.4 \text{ minutes}$$

$$\text{Est}\{W_q\} = \frac{\text{sum of observed waiting times}}{\text{number of observed waiting times}} = \frac{0}{5} = 0 \text{ minutes}$$

20.1-5.

(a) Interarrival Time ~Exp($\frac{1}{12}$ per minute), Service Time ~Exp($\frac{1}{6}$ per minute)

Next interarrival time: $-12\ln(1 - r_A)$

Next service time: $-6\ln(1 - r_D)$

Let t and $N(t)$ denote the time in minutes and the number of customers in the system at time t respectively. In the table below, N.I.T. stands for Next Interarrival Time and N.S.T. for Next Service Time.

t	$N(t)$	r_A	N.I.T.	r_D	N.S.T.	Next Arriv.	Next Dep.	Next Event
0	0	0.096	1.211	–	–	1.211	–	Arrival
1.211	1	0.596	10.100	0.665	6.562	11.311	7.773	Departure
7.773	0	–	–	–	–	11.311	–	Arrival
11.311	1	0.764	17.327	0.842	11.071	28.638	22.382	Departure
22.382	0	–	–	–	–	28.638	–	Arrival

(b)

$$P\{\text{arrival in two-minute period}\} = 1 - e^{-\frac{5}{10}} = 0.393$$

$$P\{\text{departure in two-minute period}\} = 1 - e^{-1} = 0.632$$

$r_A < 0.392 \Rightarrow$ arrival occurred, $r_A \geq 0.392 \Rightarrow$ arrival did not occur.

$r_D < 0.631 \Rightarrow$ departure occurred, $r_D \geq 0.631 \Rightarrow$ departure did not occur.

Let t and $N(t)$ denote the time in minutes and the number of customers in the system at time t respectively.

t	$N(t)$	r_A	Arrival?	r_D	Departure?
0	0	0.096	Yes	–	–
6	1	0.569	No	0.665	No
12	1	0.764	No	0.842	No
18	1	0.492	No	0.224	Yes
24	0	0.950	No	–	–
30	0	0.610	No	–	–
36	0	0.145	Yes	–	–
42	1	0.484	No	0.552	Yes
48	0	0.350	Yes	–	–

(c) Interarrival Time ~Exp($\frac{1}{5}$), Service Time ~Exp($\frac{1}{10}$)

Current Time	Number of Customers in Queue	Customer Being Served	Next Arrival	Next Service Completion
0	0	Yes	0.4674	0.01815
0.01815	0	No	0.4674	---
0.4674	0	Yes	1.35969	0.76469
0.76469	0	No	1.35969	---
1.35969	0	Yes	1.37764	1.38405
1.37764	1	Yes	1.59748	1.38405
1.38405	0	Yes	1.59748	1.40728
1.40728	0	No	1.59748	---
1.59748	0	Yes	1.81374	1.9353
1.81374	1	Yes	2.07844	1.9353
1.9353	0	Yes	2.07844	2.09178
2.07844	1	Yes	2.09584	2.09178
2.09178	0	Yes	2.09584	2.09842
2.09584	1	Yes	2.3323	2.09842
2.09842	0	Yes	2.3323	2.26042
2.26042	0	No	2.3323	---

2.3323	0	Yes	2.33589	2.33749
2.33589	1	Yes	2.35022	2.33749
2.33749	0	Yes	2.35022	2.34943
2.34943	0	No	2.35022	---
2.35022	0	Yes	2.42298	2.40413
2.40413	0	No	2.42298	---
2.42298	0	Yes	2.42362	2.54099
2.42362	1	Yes	2.55037	2.54099
2.54099	0	Yes	2.55037	2.61055
2.55037	1	Yes	2.74492	2.61055
2.61055	0	Yes	2.74492	2.64271
2.64271	0	No	2.74492	---
2.74492	0	Yes	3.31753	2.80169
2.80169	0	No	3.31753	---
3.31753	0	Yes	3.32939	3.42686
3.32939	1	Yes	3.42362	3.42686
3.42362	2	Yes	3.89049	3.42686
3.42686	1	Yes	3.89049	3.65956
3.65956	0	Yes	3.89049	3.74341
3.74341	0	No	3.89049	---

Average number waiting to begin service: 0.237795
Average number waiting for or in service: 0.753969
Average waiting time excluding service: 0.04015
Average waiting time including service: 0.15169

(d)

	Data				Results	
Number of Servers =	1			Point	95% Confidence Interval	
				Estimate	Low	High
Interarrival Times			L =	1.0176178	0.924695993	1.110539606
Distribution =	Exponential		L_q =	0.51899932	0.435985367	0.602013275
Mean =	0.2		W =	0.20468411	0.187680035	0.221688189
			W_q =	0.10439176	0.088497477	0.120286046
Service Times			P_0 =	0.50138152	0.487114273	0.51564877
Distribution =	Exponential		P_1 =	0.25139558	0.244202927	0.258588225
Mean =	0.1		P_2 =	0.12327761	0.117377792	0.129177429
			P_3 =	0.06076612	0.055624158	0.065908089
			P_4 =	0.02844012	0.02469865	0.032181586
Length of Simulation Run			P_5 =	0.01636786	0.012907523	0.019828193
Number of Arrivals =	10,000		P_6 =	0.00918318	0.006044617	0.012321737
			P_7 =	0.00363525	0.002194291	0.005076203
			P_8 =	0.00176598	0.000781557	0.002750408
Run Simulation			P_9 =	0.00085645	0.000299473	0.001413424
			P_{10} =	0.00086804	-1.54083E-05	0.001751489

(e)

	Data	
$\lambda =$	5	(mean arrival rate)
$\mu =$	10	(mean service rate)
s =	1	(# servers)

	Results
L =	1
L_q =	0.5
W =	0.2
W_q =	0.1
ρ =	0.5

Every measure is inside the 95% confidence level.

20.1-6.

(a) The system is a single-server queueing system with the crew being servers and the machines being customers. The service time has a uniform distribution between 0 and twice the mean. The interarrival time is exponentially distributed with mean being 5 hours. A simulation clock records the amount of simulated time that elapses. The state $N(t)$ of the system at time t is the number of machines that need repair at time t. The breakdowns and repairs that occur over time are randomly generated by generating random observations from the distributions of interarrival and service times. The state of the system needs to be adjusted when a breakdown or repair occurs:

$$\text{Reset } N(t) = \begin{cases} N(t)+1 & \text{if a breakdown occurs at time } t, \\ N(t)-1 & \text{if a repair occurs at time } t. \end{cases}$$

The time on the simulation clock is adjusted by using the next-event time advance procedure. The time t is in hours.

(b) The random numbers r_A and r_D are obtained from Table 20.3 starting from the front of the first row. N.I.T. stands for Next Interarrival Time and N.S.T. for Next Service Time. Interarrival times are computed as $-5 \ln r_A$ and service times correspond to $8\, r_D$. Initially there is one broken machine in the system.

t	$N(t)$	r_A	N.I.T.	r_D	N.S.T.	Next Arriv.	Next Dep.	Next Event
0	1	0.096	11.717	0.569	4.552	11.717	4.552	Departure
4.552	0	–	–	–	–	11.717	–	Arrival
11.717	1	0.665	2.040	0.764	6.112	13.757	17.829	Arrival
13.757	2	0.842	0.860	–	–	14.617	17.829	Arrival
14.617	3	0.492	3.546	–	–	18.163	17.829	Departure
17.829	2	–	–	0.224	1.792	18.163	19.621	Arrival
18.163	3	0.950	0.256	–	–	18.420	19.621	Arrival
18.420	4	0.610	2.471	–	–	20.891	19.621	Departure
19.621	3	–	–	0.145	1.160	20.891	20.781	Departure

(c)

$$P\{\text{arrival in one-hour period}\} = 1 - e^{-1/5} = 0.181$$

$$P\{\text{departure in one-hour period}\} = 1/8 = 0.125$$

$r_A < 0.181 \Rightarrow$ arrival occurred, $r_A \geq 0.181 \Rightarrow$ arrival did not occur.

$r_D < 0.125 \Rightarrow$ departure occurred, $r_D \geq 0.125 \Rightarrow$ departure did not occur.

Let t and $N(t)$ denote the time in hours and the number of broken machines in the system at time t respectively. r_A and r_D are obtained from Table 20.3 starting from the front of the first row.

t	$N(t)$	r_A	Arrival?	r_D	Departure?
0	1				
0	2	0.096	Yes	0.569	No
1	2	0.665	No	0.764	No
2	2	0.842	No	0.492	No
3	2	0.224	No	0.950	No
4	2	0.610	No	0.145	No
5	2	0.484	No	0.552	No
6	2	0.350	No	0.590	No
7	1	0.430	No	0.041	Yes
8	1	0.802	No	0.471	No
9	1	0.255	No	0.799	No
10	1	0.608	No	0.577	No
11	1	0.347	No	0.933	No
12	1	0.581	No	0.173	No
13	0	0.603	No	0.040	Yes
14	0	0.605	No	–	–
15	0	0.842	No	–	–
16	0	0.720	No	–	–
17	0	0.449	No	–	–
18	1	0.076	Yes	–	–
19	1	0.407	No	0.202	No
20	1	0.963	No	0.412	No

(d) Crew size = 2

Current Time	Number of Customers in Queue	Customer Being Served?	Next Arrival	Next Service Completion
0.00000	0	Yes	0.45442	5.06774
0.45442	1	Yes	23.52844	5.06774
5.06774	0	Yes	23.52844	12.56525
12.56525	0	No	23.52844	-
23.52844	0	Yes	24.13347	29.98968
24.13347	1	Yes	35.10738	29.98968
29.98968	0	Yes	35.10738	32.23639
32.23639	0	No	35.10738	-
35.10738	0	Yes	41.89761	39.87832
39.87832	0	No	41.89761	-
41.89761	0	Yes	45.97317	44.93853
44.93853	0	No	45.97317	-
45.97317	0	Yes	48.46326	50.40101
48.46326	1	Yes	51.81284	50.40101
50.40101	0	Yes	51.81284	55.84630
51.81284	1	Yes	52.94219	55.84630
52.94219	2	Yes	89.09479	55.84630
55.84630	1	Yes	89.09479	61.63057
61.63057	0	Yes	89.09479	63.08379
63.08379	0	No	89.09479	-
89.09479	0	Yes	99.09964	94.10255

Average waiting time excluding service: 3.141 hours
Average waiting time including service: 7.982 hours
Average number waiting to begin service: 0.282
Average number waiting or in service: 0.717

Crew size = 3

Current Time	Number of Customers in Queue	Customer Being Served	Next Arrival	Next Service Completion
0.00000	0	Yes	3.23986	5.22623
3.23986	1	Yes	7.47514	5.22623
5.22623	0	Yes	7.47514	10.29107
7.47514	1	Yes	15.15030	10.29107
10.29107	0	Yes	15.15030	15.57362
15.15030	1	Yes	27.53296	15.57362
15.57362	0	Yes	27.53296	16.06349
16.06349	0	No	27.53296	-
27.53296	0	Yes	42.72952	29.37910
29.37910	0	No	42.72952	-
42.72952	0	Yes	46.23502	46.66759
46.23502	1	Yes	48.75186	46.66759
46.66759	0	Yes	48.75186	48.69142
48.69142	0	No	48.75186	-
48.75186	0	Yes	50.75080	54.60197
50.75080	1	Yes	50.88372	54.60197
50.88372	2	Yes	55.78357	54.60197
54.60197	1	Yes	55.78357	59.86150
55.78357	2	Yes	56.25391	59.86150

Average waiting time excluding service: 1.057 hours
Average waiting time including service: 4.943 hours
Average number waiting to begin service: 0.258
Average number waiting or in service: 0.812

Crew size = 4

Current Time	Number of Customers in Queue	Customer Being Served	Next Arrival	Next Service Completion
0.00000	0	Yes	28.44578	3.45477
3.45477	0	No	28.44578	-
28.44578	0	Yes	29.78728	29.41541
29.41541	0	No	29.78728	-
29.78728	0	Yes	32.76097	32.96767
32.76097	1	Yes	38.62356	32.96767
32.96767	0	Yes	38.62356	36.41909
36.41909	0	No	38.62356	-
38.62356	0	Yes	48.10272	40.47197
40.47197	0	No	48.10272	-
48.10272	0	Yes	54.56103	51.69710
51.69710	0	No	54.56103	-
54.56103	0	Yes	55.07481	57.13491
55.07481	1	Yes	57.38123	57.13491
57.13491	0	Yes	57.38123	57.30586
57.30586	0	No	57.38123	-
57.38123	0	Yes	58.73878	58.40348
58.40348	0	No	58.73878	-
58.73878	0	Yes	62.16265	59.34633
59.34633	0	No	62.16265	-
62.16265	0	Yes	65.06976	64.07583

Average waiting time excluding service: 0.227 hours
Average waiting time including service: 2.314 hours
Average number waiting to begin service: 0.036
Average number waiting or in service: 0.372

(e) Crew size = 2

	Data	
Number of Servers =	1	
Interarrival Times		
Distribution =	Exponential	
Mean =	5	
Service Times		
Distribution =	Uniform	
Minimum Value =	0	
Maximum Value =	8	
Length of Simulation Run		
Number of Arrivals =	10,000	
Run Simulation		

	Results		
	Point	95% Confidence Interval	
	Estimate	Low	High
L =	2.79160748	2.487589825	3.095625137
L_q =	1.99914767	1.708379282	2.289916051
W =	14.1265258	12.76418957	15.48886212
W_q =	10.1163976	8.769419342	11.46337583
P_0 =	0.20754019	0.189943343	0.225137028
P_1 =	0.20310594	0.18874546	0.217466416
P_2 =	0.16447728	0.153931932	0.175022627
P_3 =	0.1219194	0.113228192	0.130610613
P_4 =	0.08985019	0.081414391	0.098285997
P_5 =	0.06348721	0.055611754	0.071362667
P_6 =	0.0471849	0.038913859	0.055455941
P_7 =	0.03473004	0.027372469	0.042087618
P_8 =	0.02360601	0.017520132	0.029691895
P_9 =	0.01630948	0.010667279	0.02195168
P_{10} =	0.00959473	0.005360122	0.013829346

Crew size = 3

	Data	
Number of Servers =	1	
Interarrival Times		
Distribution =	Exponential	
Mean =	5	
Service Times		
Distribution =	Uniform	
Minimum Value =	0	
Maximum Value =	6	
Length of Simulation Run		
Number of Arrivals =	10,000	
Run Simulation		

	Results		
	Point	95% Confidence Interval	
	Estimate	Low	High
L =	1.24431993	1.142543237	1.346096621
L_q =	0.64592258	0.554073233	0.737771925
W =	6.29742625	5.859692708	6.735159796
W_q =	3.26897425	2.842349379	3.695599122
P_0 =	0.40160265	0.387532006	0.415673294
P_1 =	0.28107892	0.272769162	0.289388675
P_2 =	0.15597768	0.14921628	0.162739075
P_3 =	0.0793332	0.073170906	0.085495493
P_4 =	0.04275587	0.037208336	0.048303406
P_5 =	0.01981074	0.016184441	0.023437041
P_6 =	0.00908873	0.006525147	0.011652319
P_7 =	0.00422182	0.002175617	0.006268018
P_8 =	0.00178977	0.000376454	0.003203086
P_9 =	0.00161276	-0.000465062	0.003690573
P_{10} =	0.00097323	-0.00055135	0.002497807

Crew size $= 4$

	Data
Number of Servers =	1
Interarrival Times	
Distribution =	Exponential
Mean =	5
Service Times	
Distribution =	Uniform
Minimum Value =	0
Maximum Value =	4
Length of Simulation Run	
Number of Arrivals =	10,000

Run Simulation

Results			
	Point	95% Confidence Interval	
	Estimate	Low	High
L =	0.57461519	0.554018788	0.595211587
L_q =	0.17109501	0.157890875	0.184299136
W =	2.84672424	2.779360906	2.914087583
W_q =	0.84762866	0.79143139	0.903825926
P_0 =	0.59647982	0.587464562	0.605495074
P_1 =	0.27673768	0.27132101	0.282154358
P_2 =	0.09234343	0.087802456	0.096884413
P_3 =	0.02626569	0.0235337	0.028997676
P_4 =	0.00677101	0.005226946	0.008315078
P_5 =	0.00118034	0.000591788	0.001768898
P_6 =	0.00017176	6.54659E-06	0.000336978
P_7 =	2.4834E-05	-1.98852E-05	6.95524E-05
P_8 =	2.5425E-05	-2.43593E-05	7.52092E-05
P_9 =	0	0	0
P_{10} =	0	0	0

According to these simulation runs, a crew size of 4 is enough to get the average waiting time before repair below 3 hours.

(f) λ, $1/\mu$, σ^2, and s denote the mean breakdown rate, the expected repair time, the variance of the repair time, and the number of servers respectively. The variance of a random variable uniformly distributed between a and b is $(b-a)^2/12$.

<u>Crew size $= 2$:</u>

$$\lambda = 0.2, \frac{1}{\mu} = 4, a = 0, b = 8, \sigma^2 = 5.333$$

$$\rho = \frac{\lambda}{\mu} = 0.8$$

$$L_q = \frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)} = 2.133,\ L = \rho + L_q = 2.933$$

$$W_q = \frac{L_q}{\lambda} = 10.667,\ W = W_q + \frac{1}{\mu} = 14.667$$

<u>Crew size $= 3$:</u>

$$\lambda = 0.2, \frac{1}{\mu} = 3, a = 0, b = 6, \sigma^2 = 3$$

$$\rho = \frac{\lambda}{\mu} = 0.6$$

$$L_q = \frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)} = 0.6, \quad L = \rho + L_q = 1.2$$

$$W_q = \frac{L_q}{\lambda} = 3,\ W = W_q + \frac{1}{\mu} = 6$$

<u>Crew size $= 4$:</u>

$$\lambda = 0.2, \frac{1}{\mu} = 2, a = 0, b = 4, \sigma^2 = 1.333$$

$$\rho = \frac{\lambda}{\mu} = 0.4$$

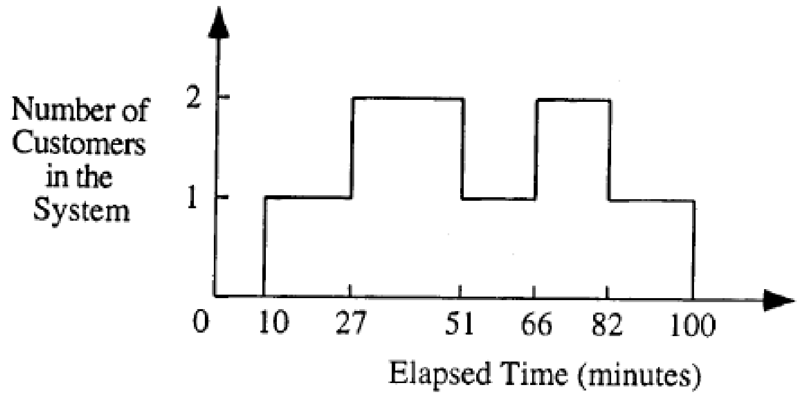
$$L_q = \frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)} = 0.178,\ L = \rho + L_q = 0.578$$

$$W_q = \frac{L_q}{\lambda} = 0.889,\ W = W_q + \frac{1}{\mu} = 2.889$$

A crew size of 3 is enough to have the average waiting time before repair begins no more than 3 hours.

20.1-7.

(a)



(b)

$$\text{Est}\{P_0\} = \tfrac{10}{100} = 0.1 \qquad \text{Est}\{P_1\} = \tfrac{17+15+18}{100} = 0.4$$

$$\text{Est}\{P_2\} = \tfrac{24+16}{100} = 0.4 \qquad \text{Est}\{P_3\} = \tfrac{0}{100} = 0$$

(c)

$$\text{Est}\{L\} = \sum_{n=0}^{3} nP_n = 0 \cdot 0.1 + 1 \cdot 0.4 + 2 \cdot 0.4 + 3 \cdot 0 = 1.2 \text{ customers}$$

$$\text{Est}\{L_q\} = \sum_{n=1}^{3} (n-1)P_n = 0 \cdot 0.4 + 1 \cdot 0.4 + 2 \cdot 0 = 0.4 \text{ customers}$$

(d)

$$\text{Est}\{W\} = \tfrac{\text{sum of observed system times}}{\text{number of observed system times}} = \tfrac{41+55+34}{3} = 43.33 \text{ minutes}$$

$$\text{Est}\{W_q\} = \tfrac{\text{sum of observed waiting times}}{\text{number of observed waiting times}} = \tfrac{0+24+16}{3} = 13.33 \text{ minutes}$$

20.1-8.

(a)

```
Distr. of interarrival times:   Translated Exp. Min  = 0.5    Mean = 1
Distr. of service times:        Erlang          Mean = 1.5    k    = 4
```

Current Time	Number of Customers in Queue	Customer Being Served Server 1	Customer Being Served Server 2	Next Arrival	Next Service Completion Server 1	Next Service Completion Server 2
0	0	Yes	No	1.6685	3.00911	---
1.6685	0	Yes	Yes	2.2903	3.00911	4.06113
2.2903	1	Yes	Yes	3.45204	3.00911	4.06113
3.00911	0	Yes	Yes	3.45204	4.31305	4.06113
3.45204	1	Yes	Yes	3.99204	4.31305	4.06113
3.99204	2	Yes	Yes	5.08213	4.31305	4.06113
4.06113	1	Yes	Yes	5.08213	4.31305	4.82208
4.31305	0	Yes	Yes	5.08213	7.01408	4.82208
4.82208	0	Yes	No	5.08213	7.01408	---
5.08213	0	Yes	Yes	5.80875	7.01408	6.56763
5.80875	1	Yes	Yes	6.42612	7.01408	6.56763
6.42612	2	Yes	Yes	8.45996	7.01408	6.56763
6.56763	1	Yes	Yes	8.45996	7.01408	7.793
7.01408	0	Yes	Yes	8.45996	9.25094	7.793
7.793	0	Yes	No	8.45996	9.25094	---
8.45996	0	Yes	Yes	8.45996	9.25094	8.95073
8.45996	1	Yes	Yes	9.01185	9.25094	8.95073
8.95073	0	Yes	Yes	9.01185	9.25094	10.3732
9.01185	1	Yes	Yes	10.7538	9.25094	10.3732
9.25094	0	Yes	Yes	10.7538	11.0051	10.3732
10.3732	0	Yes	No	10.7538	11.0051	---
10.7538	0	Yes	Yes	11.8319	11.0051	11.9901
11.0051	0	No	Yes	11.8319	---	11.9901
11.8319	0	Yes	Yes	12.5131	13.3238	11.9901
11.9901	0	Yes	No	12.5131	13.3238	---
12.5131	0	Yes	Yes	15.8697	13.3238	14.986
13.3238	0	No	Yes	15.8697	---	14.986
14.986	0	No	No	15.8697	---	---
15.8697	0	Yes	No	18.1124	16.8485	---
16.8485	0	No	No	18.1124	---	---
18.1124	0	Yes	No	19.0569	18.8949	---
18.1124	0	Yes	No	19.0569	18.8949	---
18.8949	0	No	No	19.0569	---	---
19.0569	0	Yes	No	19.8234	21.8863	---
19.8234	0	Yes	Yes	20.7688	21.8863	21.3164
20.7688	0	Yes	Yes	---	21.8863	21.3164

Average number waiting to begin service: 0.186891
Average number waiting for or in service: 1.522408
Average waiting time excluding service: 0.18669
Average waiting time including service: 1.87597

(b) Two Tellers

	Data				Results	
Number of Servers =	2			Point	95% Confidence Interval	
				Estimate	Low	High
Interarrival Times			$L =$	1.83129052	1.751958482	1.910622557
Distribution =	Translated Exponential		$L_q =$	0.3247316	0.267225477	0.382237728
Minimum Value =	0.5		$W =$	1.82063713	1.75482672	1.88644754
Mean =	1		$W_q =$	0.3228425	0.267716503	0.377968501
Service Times			$P_0 =$	0.07904235	0.070993855	0.08709085
Distribution =	Erlang		$P_1 =$	0.33535638	0.317913662	0.352799095
Mean =	1.5		$P_2 =$	0.35727575	0.344441715	0.370109776
k =	4		$P_3 =$	0.15966204	0.146452148	0.17287193
			$P_4 =$	0.04853863	0.03897433	0.058102924
Length of Simulation Run			$P_5 =$	0.01520428	0.008218196	0.022190355
Number of Arrivals =	5,000		$P_6 =$	0.0029803	0.000526793	0.005433813
			$P_7 =$	0.00124712	-0.000891002	0.003385241
			$P_8 =$	0.00062945	-0.000595242	0.001854135
Run Simulation			$P_9 =$	6.3713E-05	-6.02511E-05	0.000187678
			$P_{10} =$	0	0	0

(c) Three Tellers

	Data				Results	
Number of Servers =	3			Point	95% Confidence Interval	
				Estimate	Low	High
Interarrival Times			$L =$	1.50182365	1.471908962	1.531738345
Distribution =	Translated Exponential		$L_q =$	0.01101285	0.008142707	0.013882997
Minimum Value =	0.5		$W =$	1.50567682	1.484221201	1.527132443
Mean =	1		$W_q =$	0.01104111	0.008201326	0.013880889
Service Times			$P_0 =$	0.11036737	0.102191363	0.118543369
Distribution =	Erlang		$P_1 =$	0.4043504	0.393136309	0.415564497
Mean =	1.5		$P_2 =$	0.3693863	0.35881755	0.379955043
k =	4		$P_3 =$	0.10544811	0.097351282	0.113544943
			$P_4 =$	0.00991044	0.007638312	0.012182566
Length of Simulation Run			$P_5 =$	0.00050974	5.55269E-05	0.000963948
Number of Arrivals =	5,000		$P_6 =$	2.7646E-05	-2.6457E-05	8.17495E-05
			$P_7 =$	0	0	0
			$P_8 =$	0	0	0
Run Simulation			$P_9 =$	0	0	0
			$P_{10} =$	0	0	0

(d) Two Tellers

	Data				Results	
Number of Servers =	2			Point	95% Confidence Interval	
				Estimate	Low	High
Interarrival Times			L =	2.59862704	2.245999547	2.951254527
Distribution =	Translated Exponential		L_q =	0.908704	0.583387757	1.234020251
Minimum Value =	0.5		W =	2.335667	2.026140761	2.645193244
Mean =	0.9		W_q =	0.81675051	0.526819275	1.106681746
Service Times			P_0 =	0.04013233	0.033488277	0.046776385
Distribution =	Erlang		P_1 =	0.22981231	0.204481462	0.255143148
Mean =	1.5		P_2 =	0.31263989	0.285380805	0.339898978
k =	4		P_3 =	0.20077625	0.183373045	0.218179462
			P_4 =	0.10913211	0.092724399	0.125539826
Length of Simulation Run			P_5 =	0.0496635	0.036931841	0.062395156
Number of Arrivals =	5,000		P_6 =	0.02257618	0.0131468	0.032005558
			P_7 =	0.00911677	0.002426257	0.015807287
			P_8 =	0.00704568	-0.000481236	0.014572602
Run Simulation			P_9 =	0.00471191	-0.0012258	0.01064961
			P_{10} =	0.00538407	-0.002296853	0.013064986

Three Tellers

	Data				Results	
Number of Servers =	3			Point	95% Confidence Interval	
				Estimate	Low	High
Interarrival Times			L =	1.70776933	1.672660236	1.742878424
Distribution =	Translated Exponential		L_q =	0.02622271	0.020528952	0.031916459
Minimum Value =	0.5		W =	1.54486851	1.521784195	1.567952832
Mean =	0.9		W_q =	0.02372137	0.018683478	0.028759266
Service Times			P_0 =	0.07102483	0.064654631	0.077395038
Distribution =	Erlang		P_1 =	0.34936892	0.337158282	0.361579559
Mean =	1.5		P_2 =	0.40664103	0.396507503	0.416774558
k =	4		P_3 =	0.14889467	0.139047346	0.158742002
			P_4 =	0.02199304	0.017683072	0.026303015
Length of Simulation Run			P_5 =	0.00200283	0.000918837	0.003086824
Number of Arrivals =	5,000		P_6 =	7.4667E-05	-5.7164E-05	0.000206498
			P_7 =	0	0	0
			P_8 =	0	0	0
Run Simulation			P_9 =	0	0	0
			P_{10} =	0	0	0

(e) Let λ denote the average time between customer arrivals. Some performance measures are given for two-teller and three-teller systems in the following tables.

	Two Tellers	Three Tellers
L	1.831	1.502
L_q	0.325	0.011
W	1.821	1.506
W_q	0.323	0.011
Idle	0.414	0.884

$\lambda = 1$

	Two Tellers	Three Tellers
L	2.599	1.708
L_q	0.909	0.026
W	2.336	1.545
W_q	0.817	0.024
Idle	0.270	0.827

$\lambda = 0.9$

The last row corresponds to the probability that at least one of the tellers is idle. For the two-teller system it is $P_0 + P_1$ and for the three-teller system it is $P_0 + P_1 + P_2$. There is a big difference between the idle-time ratios of the two-teller and three-teller systems for both λ values. For this reason, it may be better to hire two tellers. Two tellers also provide reasonable wait times, $W_q = 0.323$ minutes for $\lambda = 1$ and $W_q = 0.817$ minutes for $\lambda = 0.9$. A thorough analysis would also incorporate the cost of hiring and the profit from the completion of each job.

20.1-9.

```
Priority Class 1 (higher priority) customers
    Distr. of interarrival times:   Uniform          Min  = 1     Max  = 3
    Distr. of service times:        Erlang           Mean = 1.5   k    = 4

Priority Class 2 (lower priority) customers
    Distr. of interarrival times:   Translated Exp.  Min  = 0.5   Mean = 1
    Distr. of service times:        Erlang           Mean = 1.5   k    = 4
```

Current Time	# of Customers in Line Class 1	Class 2	Class of Customer Being Served Server 1	Server 2	Next Arrival Class 1	Class 2	Next Service Completion Server 1	Server 2
0	0	0	1	idle	1.19323	0.59076	4.05587	---
0.59076	0	0	1	2	1.19323	1.98438	4.05587	1.75598
1.19323	1	0	1	2	4.03947	1.98438	4.05587	1.75598
1.75598	0	0	1	1	4.03947	2.57211	4.05587	2.6547
2.57211	0	1	1	1	4.03947	4.35852	4.05587	2.6547
2.6547	0	0	1	2	4.03947	4.35852	4.05587	5.02524
4.03947	1	0	1	2	6.60605	4.35852	4.05587	5.02524
4.05587	0	0	1	2	6.60605	4.35852	6.31076	5.02524
4.35852	0	1	1	2	6.60605	5.60471	6.31076	5.02524
5.02524	0	0	1	2	6.60605	5.60471	6.31076	6.5263
5.60471	0	1	1	2	6.60605	6.32351	6.31076	6.5263
6.31076	0	0	2	2	6.60605	6.32351	7.80267	6.5263
6.32351	0	1	2	2	6.60605	7.67972	7.80267	6.5263

6.5263	0	0	2	2	6.60605	7.67972	7.80267	7.41307
6.60605	1	0	2	2	7.66733	7.67972	7.80267	7.41307
7.66733	2	0	2	2	9.48954	7.67972	7.80267	7.41307
7.41307	1	0	2	1	9.48954	7.67972	7.80267	8.0084
7.67972	1	1	2	1	9.48954	8.7606	7.80267	8.0084
7.80267	0	1	1	1	9.48954	8.7606	9.39632	8.0084
8.0084	0	0	1	2	9.48954	8.7606	9.39632	10.085
8.7606	0	1	1	2	9.48954	9.54627	9.39632	10.085
9.39632	0	0	2	2	9.48954	9.54627	11.8025	10.085
9.48954	1	0	2	2	11.0103	9.54627	11.8025	10.085
9.54627	1	1	2	2	11.0103	10.484	11.8025	10.085
10.085	0	1	2	1	11.0103	10.484	11.8025	10.6113
10.484	0	2	2	1	11.0103	11.2066	11.8025	10.6113
10.6113	0	1	2	2	11.0103	11.2066	11.8025	11.1199
11.0103	1	1	2	2	13.2226	11.2066	11.8025	11.1199
11.1199	0	1	2	1	13.2226	11.2066	11.8025	12.4916
11.2066	0	2	2	1	13.2226	11.804	11.8025	12.4916
11.804	0	3	2	1	13.2226	12.6438	11.8025	12.4916
11.8025	0	2	2	1	13.2226	12.6438	13.5255	12.4916
12.4916	0	1	2	2	13.2226	12.6438	13.5255	13.1055
12.6438	0	2	2	2	13.2226	13.1919	13.5255	13.1055
13.1055	0	1	2	2	13.2226	13.1919	13.5255	14.1874
13.1919	0	2	2	2	13.2226	14.1007	13.5255	14.1874
13.2226	1	2	2	2	15.2685	14.1007	13.5255	14.1874
13.5255	0	2	1	2	15.2685	14.1007	15.9147	14.1874

Class 1 Customers: Average number waiting to begin service: 0.209215
Average number waiting for or in service: 1.007613
Average waiting time excluding service: 1.07381
Average waiting time including service: 2.59826

Class 2 Customers: Average number waiting to begin service: 1.406468
Average number waiting for or in service: 2.575706
Average waiting time excluding service: 0.38188
Average waiting time including service: 1.91684

20.1-10.

(a) For parts (a) through (f), each type of car corresponds to an M/M/1 system and they are independent of each other. For parts (g) through (i), the system is an M/M/2 system. Both interarrival and service times are exponentially distributed. A simulation clock records the amount of simulated time that elapses. The state of the system at time t consists of the number $N_J(t)$ of Japanese cars that need to be repaired at time t and the number $N_G(t)$ of German cars that need to be repaired at time t. The breakdowns and repairs that occur over time are generated by random observations with exponential distributions. The state of the system follows the dynamics:

$$N_J(t) = \begin{cases} N_J(t)+1 & \text{if a Japanese car arrives to the shop,} \\ N_J(t)-1 & \text{if a Japanese car is repaired,} \end{cases}$$

$$N_G(t) = \begin{cases} N_G(t)+1 & \text{if a German car arrives to the shop,} \\ N_G(t)-1 & \text{if a German car is repaired.} \end{cases}$$

The time is advanced using the next-event time advance procedure.

(b)

Current Time	Number of Customers in Queue	Customer Being Served	Next Arrival	Next Service Completion
0	0	Yes	0.03044	0.04731
0.03044	1	Yes	0.16674	0.04731
0.04731	0	Yes	0.16674	0.0818
0.0818	0	No	0.16674	——
0.16674	0	Yes	0.32435	0.33876
0.32435	1	Yes	——	0.33876
0.32435	2	Yes	——	0.33876
0.32435	2	Yes	1.47007	0.33954
1.47007	2	Yes	1.73755	1.71047
1.73755	2	Yes	2.05826	1.92858
2.05826	2	Yes	2.15076	2.17713
2.15076	2	Yes	2.5451	2.16401
2.5451	2	Yes	2.64143	2.84944
2.64143	2	Yes	2.67262	2.69043

(c) German Cars

	Data
Number of Servers =	1
Interarrival Times	
Distribution =	Exponential
Mean =	0.25
Service Times	
Distribution =	Exponential
Mean =	0.2
Length of Simulation Run	
Number of Arrivals =	10,000
Run Simulation	

Results	Point	95% Confidence Interval	
	Estimate	Low	High
L =	4.40173207	3.234015207	5.569448935
L_q =	3.59384164	2.443475896	4.74420739
W =	1.09724482	0.823761633	1.370728015
W_q =	0.89585738	0.623425709	1.168289042
P_0 =	0.19210957	0.168354591	0.215864553
P_1 =	0.14288828	0.126749608	0.159026958
P_2 =	0.11977427	0.106778075	0.132770458
P_3 =	0.10506269	0.093858332	0.116267048
P_4 =	0.08706215	0.078233744	0.095890555
P_5 =	0.07384061	0.064676221	0.083005004
P_6 =	0.05661734	0.049562873	0.063671813
P_7 =	0.04646493	0.039897523	0.053032328
P_8 =	0.03867518	0.03211558	0.045234783
P_9 =	0.02872709	0.022373996	0.03508018
P_{10} =	0.02231773	0.016062246	0.028573221

(d) Japanese Cars

	Data			Results		
Number of Servers =	1			Point	95% Confidence Interval	
				Estimate	Low	High
Interarrival Times			$L =$	0.67784073	0.641699572	0.713981894
Distribution =	Exponential		$L_q =$	0.27524319	0.247874873	0.302611514
Mean =	0.5		$W =$	0.33354389	0.318934855	0.348152933
			$W_q =$	0.13543843	0.123198616	0.14767825
Service Times			$P_0 =$	0.59740246	0.586266118	0.608538804
Distribution =	Exponential		$P_1 =$	0.2405621	0.234555839	0.246568367
Mean =	0.2		$P_2 =$	0.09556771	0.090486659	0.100648771
			$P_3 =$	0.03852888	0.034728947	0.04232882
			$P_4 =$	0.01624329	0.013664229	0.018822344
Length of Simulation Run			$P_5 =$	0.00723714	0.005338826	0.009135444
Number of Arrivals =	10,000		$P_6 =$	0.00266085	0.001585541	0.003736161
			$P_7 =$	0.00113441	0.00035521	0.001913604
			$P_8 =$	0.00048975	-2.19148E-05	0.001001412
			$P_9 =$	0.00016031	-7.0534E-05	0.000391154
Run Simulation			$P_{10} =$	1.3099E-05	-1.25453E-05	3.87434E-05

(e)

Current Time	Number of Customers in Queue	Customer Being Served Server 1	Server 2	Next Arrival	Next Service Completion Server 1	Server
0	0	Yes	No	0.03044	0.04731	—
0.03044	0	Yes	Yes	0.76195	0.04731	0.06493
0.04731	0	No	Yes	0.76195	—	0.06493
0.06493	0	No	No	0.76195	—	—
0.76195	0	Yes	No	0.83716	1.01054	—
0.83716	1	Yes	No	0.85615	1.05115	1.07757
0.85615	1	Yes	No	1.17686	1.04719	0.93015
1.17686	1	Yes	No	1.32545	1.49234	1.19012
1.32545	1	Yes	No	1.42178	1.62979	1.3504
1.42178	1	Yes	No	1.48302	1.42802	1.53588
1.48302	1	Yes	No	1.81541	1.57642	1.68985
1.81541	1	Yes	No	1.8777	2.5102	2.03288

(f) German Cars

Data			Results		
Number of Servers =	2		Point Estimate	95% Confidence Interval Low	High
Interarrival Times		L =	0.96734048	0.929376886	1.005304084
Distribution =	Exponential	L_q =	0.15509549	0.135159564	0.175031414
Mean =	0.25	W =	0.23650186	0.229378389	0.243625338
		W_q =	0.03791878	0.033338445	0.042499117
Service Times		P_0 =	0.42259073	0.410429578	0.43475189
Distribution =	Exponential	P_1 =	0.34257354	0.334821362	0.35032571
Mean =	0.2	P_2 =	0.13990835	0.134017407	0.145799301
		P_3 =	0.05765995	0.053234966	0.062084937
		P_4 =	0.02285889	0.019882112	0.025835667
Length of Simulation Run		P_5 =	0.00909262	0.00719708	0.010988165
Number of Arrivals =	10,000	P_6 =	0.00355787	0.002290188	0.004825549
		P_7 =	0.00084521	0.000306354	0.001384058
		P_8 =	0.00055155	3.71251E-05	0.001065979
Run Simulation		P_9 =	0.00026383	-0.000120885	0.000648547
		P_{10} =	5.0835E-05	-4.87318E-05	0.000150401

(g) This option significantly decreases the waiting time for German cars without the added cost of an additional mechanic.

Data			Results		
Number of Servers =	2		Point Estimate	95% Confidence Interval Low	High
Interarrival Times		L =	2.38954746	2.228936755	2.550158169
Distribution =	Exponential	L_q =	1.06377125	0.9248612	1.202681301
Mean =	0.166666667	W =	0.3977351	0.373871382	0.421598812
		W_q =	0.17706246	0.155175541	0.198949387
Service Times		P_0 =	0.20555772	0.195123634	0.215991801
Distribution =	Exponential	P_1 =	0.26310835	0.253142531	0.273074175
Mean =	0.22	P_2 =	0.17488354	0.168697104	0.181069974
		P_3 =	0.12221465	0.116798427	0.12763088
		P_4 =	0.07857001	0.073827865	0.083312156
Length of Simulation Run		P_5 =	0.05118593	0.046911158	0.055460707
Number of Arrivals =	20,000	P_6 =	0.0346834	0.030632654	0.038734137
		P_7 =	0.02396612	0.020211346	0.027720893
		P_8 =	0.01535973	0.012326221	0.018393236
Run Simulation		P_9 =	0.00995903	0.007331075	0.012586987
		P_{10} =	0.00600944	0.004038714	0.00798016

(h)

Part	Est$\{W\}$	W
(c)	1.097	1.000
(d)	0.334	0.333
(f)	0.237	0.238
(g)	0.398	0.390

The results of the simulation were quite accurate.

(i) Answers will vary. The option of training the two current mechanics significantly decreases the waiting time for German cars, without a significant impact on the wait for German cars, and does so without the added cost of a third mechanic. Adding a third mechanic reduces the average wait for German cars even more, but comes with the added cost of a third mechanic.

20.1-11.

(a) There are two independent G/M/1 systems: printers and monitors. For printers, the arrival stream is deterministic; for monitors, the arrival process is uniformly distributed between 10 and 20. The inspection time is exponentially distributed with a mean of 10 minutes. A simulation clock records the amount of simulated time that elapses. The state of the system at time t consists of the number $N_M(t)$ of monitors in the inspection station at time t and the number $N_P(t)$ of printers in the inspection station at time t. The arrivals to the stations and the inspection times are generated by sampling distributions according to interarrival and service time distributions. The system evolves according to the law:

$$N_M(t) = \begin{cases} N_M(t)+1 & \text{if a monitor arrives to the inspection station,} \\ N_M(t)-1 & \text{if a monitor is repaired,} \end{cases}$$

$$N_P(t) = \begin{cases} N_P(t)+1 & \text{if a printer arrives to the inspection station,} \\ N_P(t)-1 & \text{if a printer is repaired.} \end{cases}$$

The time is advanced using the next-event time advance procedure.

(b)

Current Time	Number of Customers in Queue	Customer Being Served	Next Arrival	Next Service Completion
0	0	Yes	18.8535	2.3654
2.3654	0	No	18.8535	——
18.8535	0	Yes	30.1903	20.578
20.578	0	No	30.1903	——
30.1903	0	Yes	45.5138	38.7912
38.7912	0	No	45.5138	40.7702
45.5138	0	Yes	62.9157	57.9432
62.9157	0	Yes	73.018	63.6754
73.018	0	Yes	86.4483	85.0383
86.4483	0	Yes	99.2208	96.0002
99.2208	0	Yes	116.128	105.164
116.128	0	Yes	128.193	116.791
128.193	0	Yes	144.996	143.41
144.996	0	Yes	163.823	147.445

(c)

Current Time	Number of Customers in Queue	Customer Being Served	Next Arrival	Next Service Completion
0	0	Yes	15	1.21772
1.21772	0	No	15	—
15	0	Yes	30	20.4518
20.4518	0	No	30	—
30	0	Yes	45	50.1234
45	0	Yes	60	46.7245
60	0	Yes	75	60.6191
75	0	Yes	90	80.4591
90	0	Yes	105	96.3044
105	0	Yes	120	113.601
120	0	Yes	135	127.452
135	0	Yes	150	136.979

(d) Monitors

	Data				Results	
Number of Servers =	1			Point	95% Confidence Interval	
				Estimate	Low	High
Interarrival Times			L =	1.12531104	1.060901038	1.189721042
Distribution =	Uniform		L_q =	0.46545375	0.411484076	0.519423417
Minimum Value =	10		W =	16.9031435	15.94273751	17.8635494
Maximum Value =	20		W_q =	6.99151716	6.183798912	7.799235411
Service Times			P_0 =	0.34014271	0.327224738	0.353060676
Distribution =	Exponential		P_1 =	0.38164142	0.372075535	0.391207303
Mean =	10		P_2 =	0.16352709	0.155778216	0.171275956
			P_3 =	0.06811831	0.060998944	0.075237683
			P_4 =	0.02888903	0.023293304	0.034484764
Length of Simulation Run			P_5 =	0.01168415	0.007821024	0.015547284
Number of Arrivals =	10,000		P_6 =	0.00424548	0.002158582	0.006332375
			P_7 =	0.00128359	0.000174338	0.002392845
			P_8 =	0.00038837	-0.000179162	0.000955894
Run Simulation			P_9 =	7.9852E-05	-6.35832E-05	0.000223288
			P_{10} =	0	0	0

Printers

	Data
Number of Servers =	1
Interarrival Times	
Distribution =	Constant
Value =	15
Service Times	
Distribution =	Exponential
Mean =	10
Length of Simulation Run	
Number of Arrivals =	10,000

Run Simulation

Results	Point Estimate	95% Confidence Interval Low	High
L =	1.13593344	1.077190283	1.19467659
L_q =	0.46450587	0.415782342	0.513229408
W =	17.0390016	16.15785425	17.92014885
W_q =	6.96758812	6.236735131	7.698441118
P_0 =	0.32857244	0.316172923	0.340971954
P_1 =	0.39246319	0.382956315	0.401970063
P_2 =	0.16410908	0.156167074	0.172051091
P_3 =	0.06829616	0.061331286	0.075261033
P_4 =	0.02874872	0.023039412	0.034458025
P_5 =	0.01227622	0.008565564	0.01598688
P_6 =	0.00479104	0.002763062	0.006819019
P_7 =	0.00070382	0.000113351	0.001294289
P_8 =	3.933E-05	-3.76108E-05	0.00011627
P_9 =	0	0	0
P_{10} =	0	0	0

(e) Monitors

	Data
Number of Servers =	1
Interarrival Times	
Distribution =	Uniform
Minimum Value =	10
Maximum Value =	20
Service Times	
Distribution =	Erlang
Mean =	10
k =	4
Length of Simulation Run	
Number of Arrivals =	10,000

Run Simulation

Results	Point Estimate	95% Confidence Interval Low	High
L =	0.75155745	0.736774593	0.766340302
L_q =	0.08784252	0.078522615	0.097162425
W =	11.3286273	11.11532534	11.54192936
W_q =	1.32409728	1.184672596	1.463521969
P_0 =	0.33628507	0.329400438	0.343169707
P_1 =	0.58164576	0.576199748	0.587091777
P_2 =	0.07650568	0.07048566	0.082525703
P_3 =	0.00535361	0.003131199	0.007576026
P_4 =	0.00020987	-2.40367E-05	0.000443779
P_5 =	0	0	0
P_6 =	0	0	0
P_7 =	0	0	0
P_8 =	0	0	0
P_9 =	0	0	0
P_{10} =	0	0	0

Printers

	Data	
Number of Servers =	1	
Interarrival Times		
Distribution =	Constant	
Value =	15	
Service Times		
Distribution =	Erlang	
Mean =	10	
k =	4	
Length of Simulation Run		
Number of Arrivals =	10,000	
Run Simulation		

	Results		
	Point	95% Confidence Interval	
	Estimate	Low	High
L =	0.73633179	0.722583868	0.750079721
L_q =	0.06997678	0.06156358	0.078389982
W =	11.0449769	10.83875803	11.25119582
W_q =	1.04965172	0.923453699	1.175849737
P_0 =	0.33364499	0.326911198	0.340378775
P_1 =	0.6005944	0.595376773	0.605812026
P_2 =	0.06184121	0.05625764	0.067424777
P_3 =	0.00364284	0.001938538	0.00534714
P_4 =	0.00025637	-0.000124119	0.000636866
P_5 =	2.0194E-05	-1.93445E-05	5.97317E-05
P_6 =	0	0	0
P_7 =	0	0	0
P_8 =	0	0	0
P_9 =	0	0	0
P_{10} =	0	0	0

The new inspection equipment would drastically reduce the average waiting time for both monitors (from 7 minutes to 1.3 minutes) and printers (from 7 minutes to 1 minute).

20.2-1.

Merrill Lynch launched the Management Science Group to deal with the issues raised by the rise of electronic trading in the late 1990s. The group studied various product structure and pricing alternatives. They focused on two main pricing options, viz., an asset-based pricing option and a direct online pricing option. Monte Carlo simulation is applied to simulate the behavior of the clients who choose between the two product and pricing options in the light of economic and qualitative factors. In the simulation model, "the observed system data consist of every revenue-generating component of every account of every client at Merrill Lynch. The output measures are the resulting revenue at the firm level, the compensation impact on each FA, and the percentage of clients considered adverse selectors" [p. 13]. Sensitivity analysis is performed to evaluate various scenarios.

"The benefits were significant and fell into four areas: seizing the marketplace initiative, finding the pricing sweet spot, improving financial performance, and adopting the approach in other strategic initiatives in other strategic initiatives" [p. 15]. As a result of this study, Merrill Lynch also acquired new clients.

20.2-2.

Answers will vary.

20.3-1.

(a)

n	x_n	x_n+3	$\frac{x_n+3}{10}$	x_{n+1}
0	2	5	$\frac{5}{10}$	5
1	5	8	$\frac{8}{10}$	8
2	8	11	$1\frac{1}{10}$	1
3	1	4	$\frac{4}{10}$	4
4	4	7	$\frac{7}{10}$	7
5	7	10	$1\frac{0}{10}$	0
6	0	3	$\frac{3}{10}$	3
7	3	6	$\frac{6}{10}$	6
8	6	9	$\frac{9}{10}$	9
9	9	12	$1\frac{2}{10}$	2

(b)

n	x_n	$5x_n+1$	$\frac{5x_n+1}{8}$	x_{n+1}
0	1	6	$\frac{6}{8}$	6
1	6	31	$3\frac{7}{8}$	7
2	7	36	$4\frac{4}{8}$	4
3	4	21	$2\frac{5}{8}$	5
4	5	26	$3\frac{2}{8}$	2
5	2	11	$1\frac{3}{8}$	3
6	3	16	$1\frac{0}{8}$	0
7	0	1	$\frac{1}{8}$	1

(c)

n	x_n	$61x_n+27$	$\frac{61x_n+27}{100}$	x_{n+1}
0	10	637	$6\frac{37}{100}$	37
1	37	2284	$22\frac{84}{100}$	84
2	84	5151	$51\frac{51}{100}$	51
3	51	3138	$31\frac{38}{100}$	38
4	38	2345	$23\frac{45}{100}$	45

20.3-2.

(a)

$$U_{n+1} = \frac{x_{n+1}+\frac{1}{2}}{10}, n = 0, 1, \ldots, 9$$

(b)

$$U_{n+1} = \frac{x_{n+1}+\frac{1}{2}}{8}, n = 0, 1, \ldots, 7$$

(c)

$$U_{n+1} = \frac{x_{n+1}+\frac{1}{2}}{100}, n = 0, 1, \ldots, 99$$

20.3-3.

n	x_n	$41x_n+33$	$\frac{41x_n+33}{100}$	x_{n+1}
0	48	2001	$20\frac{1}{100}$	01
1	01	74	$\frac{74}{100}$	74
2	74	3067	$30\frac{67}{100}$	67
3	67	2780	$27\frac{80}{100}$	80
4	80	3313	$33\frac{13}{100}$	13

20.3-4.

n	x_n	$201x_n + 503$	$\frac{201x_n+503}{1000}$	x_{n+1}
0	485	97988	$97\frac{988}{1000}$	988
1	988	199091	$199\frac{91}{1000}$	91
2	91	18794	$18\frac{794}{1000}$	794

20.3-5.

(a)

n	x_n	$13x_n + 15$	$\frac{13x_n+15}{32}$	x_{n+1}
0	14	197	$6\frac{5}{32}$	5
1	5	80	$2\frac{16}{32}$	16
2	16	223	$6\frac{31}{32}$	31
3	31	418	$13\frac{2}{32}$	2
4	2	41	$1\frac{9}{32}$	9

(b)

$$U_{n+1} = \frac{x_{n+1}+\frac{1}{2}}{32}, n = 0, 1, \ldots, 4 \Rightarrow (0.1719, 0.5156, 0.9844, 0.0781, 0.2696)$$

20.3-6.

(a) $x_1 = 7, x_2 = 10, x_3 = 5, x_4 = 9, x_5 = 11, x_6 = 12,$

$x_7 = 6, x_8 = 3, x_9 = 8, x_{10} = 4, x_{11} = 2, x_{12} = 1$

(b) Each integer appears only once in part (a).

(c) $x_{13}, x_{14}, \ldots$ will repeat the cycle $x_1, \ldots, x_{12}$ with length 12.

20.4-1.

(a) Answers will vary.

(b) The formula in cell D10 is $= \text{VLOOKUP}(\text{C10}, \$\text{J}\$8{:}\$\text{K}\$9, 2)$.

Required Difference		3		Distribution of Coin Flips		
Cash At End of Game		$8		Probability	Cumulative	Result
				0.5	0	Heads
	Summary of Game			0.5	0.5	Tails
	Number of Flips	7				
	Winnings	$1				

Flip	Random Number	Result	Total Heads	Total Tails	Stop?
1	0.9683	Tails	0	1	
2	0.8270	Tails	0	2	
3	0.2837	Heads	1	2	
4	0.1236	Heads	2	2	
5	0.8999	Tails	2	3	
6	0.7532	Tails	2	4	
7	0.5228	Tails	2	5	Stop
8	0.3227	Heads	3	5	NA
9	0.4547	Heads	4	5	NA
10	0.2282	Heads	5	5	NA
11	0.0403	Heads	6	5	NA
12	0.6744	Tails	6	6	NA
13	0.0852	Heads	7	6	NA
14	0.9229	Tails	7	7	NA
15	0.9497	Tails	7	8	NA
16	0.4296	Heads	8	8	NA

(c) A simulation with 14 replications:

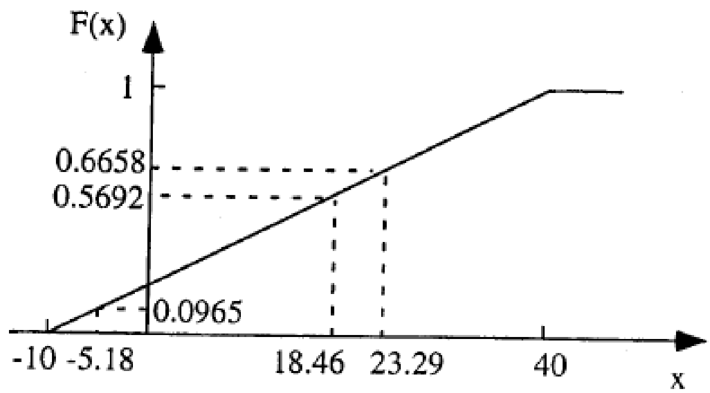
Play	Number of Flips	Winnings
	7	1
1	31	-23
2	3	5
3	7	1
4	29	-21
5	3	5
6	17	-9
7	5	3
8	21	-13
9	3	5
10	9	-1
11	13	-5
12	7	1
13	3	5
14	7	1
Average:	11.286	-3.286

(d) A simulation with 1000 replications:

Play	Number of Flips	Winnings
	7	1
1	13	-5
2	11	-3
3	17	-9
4	13	-5
5	17	-9
6	3	5
7	5	3
8	7	1
9	5	3
10	3	5
11	3	5
12	11	-3
13	11	-3
14	15	-7
15	9	-1
16	9	-1
17	5	3
997	3	5
998	11	-3
999	3	5
1000	5	3
Average:	8.972	-0.972

20.4-2.

(a)



(b) $F(x) = \frac{x+10}{50} \Rightarrow F(-5.18) = 0.0965, F(18.46) = 0.5692, F(23.29) = 0.6658$

(c) If cell A1 contains the uniform random number, then the Excel function is " $= 50*\text{A1} - 10$."

20.4-3.

(a) $r = P\{X \leq x\} = \int_{25}^{x} \frac{dt}{50} = \frac{x-25}{50} \Rightarrow x = 50r + 25$

r	X
0.096	29.80
0.569	53.45
0.665	58.25

(b) $r = P\{X \leq x\} = \int_{-1}^{x} \frac{(t+1)^3}{4} dt = \frac{(x+1)^4}{16} \Rightarrow x = 2r^{1/4} - 1$

r	X
0.096	0.113
0.569	0.737
0.665	0.806

(c) $r = P\{X \leq x\} = \int_{40}^{x} \frac{(t-40)}{200} dt = \frac{(x-40)^2}{400} \Rightarrow x = 20(2 + \sqrt{r})$

r	X
0.096	46.197
0.569	55.086
0.665	56.310

20.4-4.

(a) To determine whether $X = 0$ or X is distributed uniformly between -5 and 15, look at a three-digit random number from Table 20.3.

$000 \leq r \leq 499 \Rightarrow X = 0$.
$500 \leq r \leq 999 \Rightarrow X$ is uniformly distributed.

If $X = 0$, nothing else need to be done. Otherwise, use the next three-digit random number as a decimal to generate X.

$r = P\{X \leq x\} = \int_{-5}^{x} \frac{dt}{20} = \frac{x+5}{20} \Rightarrow x = 20r - 5$

r	
0.096	$X_1 = 0$
0.569	$X_2 \sim U(-5, 15)$
0.665	$X_2 = 20(0.665) - 5 = 8.3$
0.764	$X_3 \sim U(-5, 15)$
0.842	$X_3 = 20(0.842) - 5 = 11.84$

Hence, the sequence is $(0, 8.3, 11.84)$.

(b)

$$P\{1 \leq X \leq 2\} = \int_1^2 (t-1)dt = \frac{1}{2}, P\{2 \leq X \leq 3\} = \int_2^3 (3-t)dt = \frac{1}{2}$$

For $0 \leq r \leq ½$, $r = \int_1^x (t-1)dt = \frac{(x-1)^2}{2} \Rightarrow x = \sqrt{2r} + 1$.

For $½ \leq r \leq 1$, $r = \frac{1}{2} + \int_2^x (3-t)dt = \frac{1}{2} - \frac{(3-x)^2}{2} \Rightarrow x = 3 - \sqrt{2-2r}$.

r	X
0.096	1.438
0.569	2.072
0.665	2.181

(c) Let Z be a Bernoulli random variable with $p = 1/3$, i.e., $P\{Z = 1\} = 1/3$ and $P\{Z = 0\} = 2/3$. Then, X is a random variable denoting the number of trials until the Bernoulli random variable takes the value 1.

$000 \leq r \leq 332 \Rightarrow Z = 1.$
$333 \leq r \leq 999 \Rightarrow Z = 0.$

r	Z	X
096	1	1
569	0	
665	0	
764	0	
842	0	
492	0	
224	1	6
950	0	
610	0	
145	1	3

Hence, the sequence is $(1, 6, 3)$.

20.4-5.

(a) Answers will vary.

(b) 0.0000 to 0.4999 correspond to heads.
0.5000 to 0.9999 correspond to tails.

Group 1: HHH, Group 2: THH, Group 3: HTT, Group 4: THT,
Group 5: THH, Group 6: HHT, Group 7: THT, Group 8: TTH
Number of groups with 0 heads: 0
Number of groups with 1 heads: 4
Number of groups with 2 heads: 3
Number of groups with 3 heads: 1

(c)

Flip	Random Number	Result
1	0.6459	Tails
2	0.3080	Heads
3	0.0353	Heads
Total Number of Heads =		2

(d) Answers will vary. The following eight replications have no replications with no heads, two replications with one head ($\frac{1}{4}$), six replication with two heads ($\frac{3}{4}$), and no replication with three heads. This is not very close to the expected probability distribution.

Replication	Number of Heads
	2
1	1
2	2
3	2
4	1
5	2
6	2
7	2
8	2

(e) Answers will vary. Among the following 800 replications, 96 have no heads ($\frac{96}{800} = 0.12$), 302 have one head ($\frac{302}{800} = 0.378$), 286 have two heads ($\frac{286}{800} = 0.358$), and 116 have three heads ($\frac{116}{800} = 0.145$). This is quite close to the expected probability distribution.

Replication	Number of Heads
	2
1	0
2	1
3	3
4	1
5	2
6	1
7	1
8	3
9	1
10	0
798	1
799	2
800	1
Number with 0 heads =	96
Number with 1 head =	302
Number with 2 heads =	286
Number with 3 heads =	116

20.4-6.

(a)

Summary of Results:

Win? (1=Yes, 0=No)	0
Number of Tosses =	3

Simulated Tosses

Toss	Die 1	Die 2	Sum
1	4	2	6
2	3	2	5
3	6	1	7
4	5	2	7
5	4	4	8
6	1	4	5
7	2	6	8

Results

Win?	Lose?	Continue?
0	0	Yes
0	0	Yes
0	1	No
NA	NA	No
NA	NA	No
NA	NA	No
NA	NA	No

(b) Answers will vary. Below is the results from a 25-replication simulation.

Game	1	2	3	4	5	6	7	8	$9,\ldots,15$	16	17	18	19	$20,\ldots,24$	25
Win?	0	0	1	0	0	0	0	1	0	1	0	1	0	1	0

(c) 9 wins and 16 loses $\Rightarrow P\{\text{win}\} = 9/25$ and $P\{\text{lose}\} = 16/25$

(d)

$$\frac{\overline{X}-0.493}{0.5/\sqrt{n}} \sim N(0,1) \Rightarrow P\left\{\frac{\overline{X}-0.493}{0.5/\sqrt{n}} \leq 1.64\right\} = 0.95$$

$$\Rightarrow P\left\{\overline{X} \leq \frac{0.82}{\sqrt{n}} + 0.493\right\} = 0.95$$

$$\frac{0.82}{\sqrt{n}} + 0.493 = 0.5 \Rightarrow n = 13.689$$

20.4-7.

$$r = P\{X \leq x\} = P\left\{\frac{X-1}{2} \leq \frac{x-1}{2}\right\} = 1 - \Phi\left(\frac{x-1}{2}\right) \Rightarrow x = 2\Phi^{-1}(1-r) + 1$$

We can use r directly instead of $1-r$, since both have uniform distribution. The following values $\Phi^{-1}(r)$ are obtained in Excel using the function NORMINV$(r, 0, 1)$.

r	$\Phi^{-1}(r)$	x
0.096	−1.305	−1.609
0.569	0.174	1.348
0.665	0.426	1.852
0.764	0.719	2.438
0.842	1.003	3.005
0.492	−0.020	0.960
0.224	−0.759	−0.518
0.950	1.645	4.290
0.610	0.279	1.559
0.145	−1.058	−1.116

Average: 1.221

20.4-8.

(a)

r_i^1	r_i^2	r_i^3
0.096	0.764	0.224
0.569	0.842	0.950
0.665	0.492	0.610

$\sum_{i=1}^{3} r_i^k$	$x_k = 20\left(\sum_{i=1}^{3} r_i^k\right) - 25$
1.330	1.6
2.098	17.0
1.784	10.7

(b) $x = 5\Phi^{-1}(r) + 10$

r	$\Phi^{-1}(r)$	x
0.096	-1.305	3.475
0.569	0.174	10.870
0.665	0.426	12.130

20.4-9.

(a)

r_i^1	r_i^2	r_i^3	r_i^4
0.096	0.764	0.224	0.145
0.569	0.842	0.950	0.484
0.665	0.492	0.610	0.552

$\sum_{i=1}^{3} r_i^k$	$x_k = 2\left(\sum_{i=1}^{3} r_i^k\right) - 3$
1.330	-0.340
2.098	1.196
1.784	0.568
1.181	-0.638

Let z_i denote the chi-square observations, for $i = 1, 2$. Then

$$z_1 = x_1^2 + x_2^2 = 1.546 \text{ and } z_2 = x_3^2 + x_4^2 = 0.730.$$

(b)

r	$\Phi^{-1}(r)$
0.096	-1.305
0.569	0.174
0.665	0.426
0.764	0.719

(c) $Y = X_1^2 + X_2^2$

From (a), $Y_1 = 1.546$ and $Y_2 = 0.730$.
From (b), $Y_1 = 1.733$ and $Y_2 = 0.698$.

20.4-10.

(a)

r	$x = -10\ln(r)$
0.096	23.434
0.569	5.639

(b)

r_1	r_2	$x = -5\ln(r_1 r_2)]$
0.096	0.569	14.536
0.665	0.764	3.386

(c)

r_i^1	r_i^2
0.096	0.224
0.569	0.950
0.665	0.610
0.764	0.145
0.842	0.484
0.492	0.552

$\sum_{i=1}^{6} r_i^k$	$x_k = 4\left(\sum_{i=1}^{6} r_i^k\right) - 2$
3.428	11.71
2.965	9.86

20.4-11.

(a)

Uniform Random Number	Random Observation
0.2655	9.22
0.3472	9.49
0.0248	7.25
0.9205	12.21
0.6130	10.38

(b) If cell C4 contains the uniform random number, then the Excel function would be:
$=$ IF(C4<0.2, 7+(2/0.2)*C4, IF(C4<0.8, 9+(2/0.6)*(C4-0.2), 11+(2/0.2)*(C4-0.8))).

20.4-12.

r	$x = -11\ln(1-r)$
0.096	0.101
0.569	0.842
0.665	1.094
0.764	1.444

Hence, the Erlang observation is $\sum_{i=1}^{4} x_k = 3.481$.

20.4-13.

(a) TRUE. Both r_i and $1 - r_i$ are uniformly distributed.

(b) FALSE. Numerically, $\prod r_i \neq \prod(1 - r_i) \Rightarrow \sum x_i \neq \sum y_i$.

(c) TRUE. The sum of independent exponential random variables each with the same mean has Erlang distribution.

20.4-14.

(a) It is not valid, since $P\{x_i = 9\} = P\{\frac{9}{9} \leq r_i < \frac{10}{9}\} = 0$ and r_i wouldn't reach 9. Modify it as $\frac{n-1}{9} \leq r_i < \frac{n}{9}$.

(b) It is valid. When $\frac{n-1}{9} \leq r_i < \frac{n}{9}, n \leq 1 + 9r_i < n + 1$.

(c) It is not valid, since $x'_0 = 4$, $x'_1 = 5$, $x'_2 = 0$, $x'_3 = 7$, $x'_4 = 8$, $x'_5 = 3$, $x'_6 = 2$, $x'_7 = 1$, $x'_8 = 6$, and $x'_8 = 6$, so this method does not cover the number 9. Instead, let $x_i = x'_i + 1$, then it is a valid method.

20.4-15.

r_1	x	r_2	$f(x)$	Accept?
0.096	0.192	0.569	0.192	No
0.665	1.330	0.764	0.670	No
0.842	1.684	0.492	0.316	No
0.224	0.448	0.950	0.448	No
0.610	1.220	0.145	0.780	Yes
0.484	0.968	0.552	0.968	Yes
0.350	0.700	0.590	0.700	Yes

The three samples from the triangular distribution are $1.220, 0.968$, and 0.700.

20.4-16. Let $x = 10r_1 + 10$.

r_1	x	r_2	$f(x)$	Accept?
0.096	10.96	0.569	0.0192	No
0.665	16.65	0.764	0.1350	No
0.842	18.42	0.492	0.1684	No
0.224	12.24	0.950	0.0448	No
0.610	16.10	0.145	0.1220	No
0.484	14.84	0.552	0.0968	No
0.350	13.50	0.590	0.0700	No
0.430	14.30	0.041	0.0860	Yes
0.802	18.02	0.471	0.1604	No
0.255	12.55	0.799	0.0510	No
0.608	16.08	0.577	0.1216	No
0.347	13.47	0.933	0.0694	No
0.581	15.81	0.173	0.1162	No
0.603	16.03	0.040	0.1206	Yes
0.605	16.05	0.842	0.1210	No
0.720	17.20	0.449	0.1440	No
0.076	10.76	0.407	0.0152	No
0.202	12.02	0.963	0.0404	No
0.412	14.12	0.369	0.0824	No
0.976	19.76	0.171	0.1952	Yes

The three samples from the given distribution are $14.30, 16.03$, and 19.76.

20.4-17.

$$\text{size of risk} = \begin{cases} 0 & \text{if } 0 \le U < 0.7 \\ 1 & \text{if } 0.7 \le U < 0.9 \\ 2 & \text{if } 0.9 \le U < 1 \end{cases}$$

$$\text{size of loss } x = \begin{cases} (20U)^2 & \text{if } 0 \le U < ½ \\ 200U & \text{if } U \ge ½ \end{cases}$$

Run 1		Run 2	
U	size	U	size
0.096	0	0.492	0
0.569	0	0.224	0
0.665	0	0.950	2
0.764	1	0.610	0

U	x
0.842	164.4

U	x
0.145	8.41
0.484	91.09

Total loss: $\sum_{i=1}^{4} I_{(\text{size}>0)} \sum_{j=1}^{\text{size}} x_{ij}$

Two simulation runs give 164.4 and 99.5. Actually, 100 runs give 145.

20.4-18.

Since the number N of employees incurring medical expenses has a binomial distribution with $p = 0.9$ and $n = 3$:

$$P\{N = 0\} = C_3^0 \cdot 0.9^0 \cdot 0.1^3 = 0.001,$$

$$P\{N = 1\} = C_3^1 \cdot 0.9^1 \cdot 0.1^2 = 0.027,$$

$$P\{N = 2\} = C_3^2 \cdot 0.9^2 \cdot 0.1^1 = 0.243,$$

$$P\{N = 3\} = C_3^3 \cdot 0.9^3 \cdot 0.1^0 = 0.729.$$

Let $p_0 = 0, p_1 = 0.001, p_2 = 0.028, p_3 = 0.271, p_4 = 1.$

$$N = i \text{ if } p_i \le U < p_{i+1}$$

$$0.01 \Rightarrow N = 1, 0.20 \Rightarrow N = 2$$

$$\text{Total amount} = \begin{cases} 100 & \text{if } 0 \le U < 0.9 \\ 10,000 & \text{if } 0.9 \le U < 1 \end{cases}$$

Only 0.95 causes an actual payment from the insurance company and the total payment is $\$5,000$.

20.5-1.

Answers will vary.

20.5-2.

Answers will vary.

20.6-1.

(a) Answers will vary. A typical set of 5 runs: $(45.72, 44.24, 46.68, 46.24, 47.90)$

(b) Answers will vary. A typical set of 5 runs: $(46.60, 47.06, 46.67, 46.76, 46.84)$

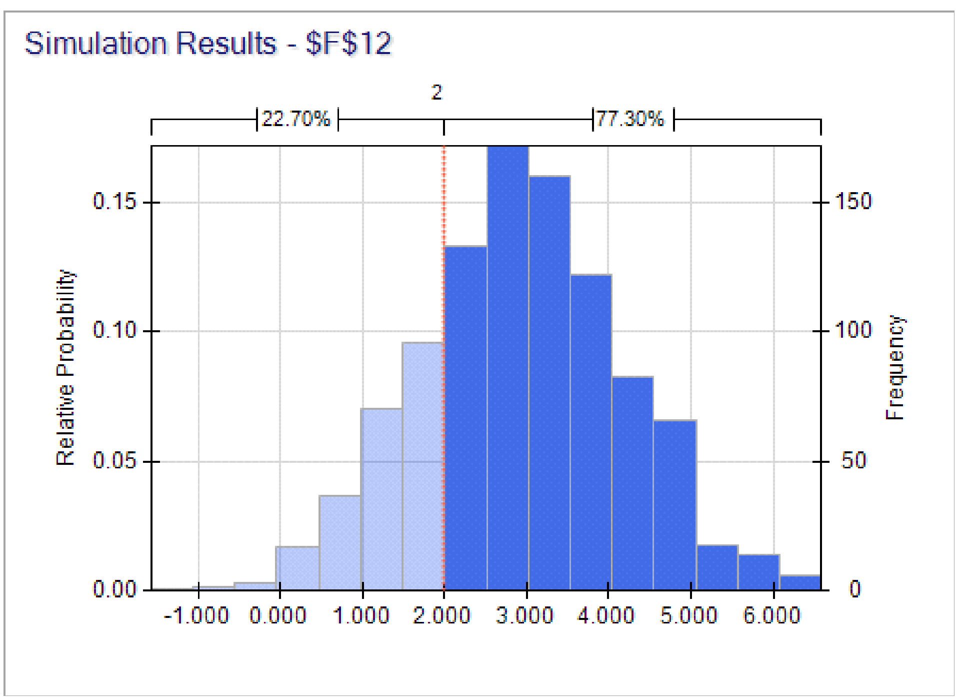
(c) The mean profits in part (b) seem to be more consistent.

20.6-2.

					Now	Year 1	Year 2	Year 3	Year 4	Year 5
Land Purchase	Fixed				-1					
Construction Cost	Triangular(min,likely,max)	-2.4	-2	-1.6		-1.867				
Operating Profit	Normal(mean,s.dev.)	0.7	0.7				1.353	1.440	0.992	0.225
Selling Price	Uniform(min,max)	4	8							5.199
		Total Cash Flow			-1	-1.867	1.35327	1.44046	0.99231	5.42381
		Discount Factor			10%					
						Mean				
	Net Present Value ($million)				3.549	2.925				
Minimum Annual Operating Profit ($million in y2-y5)					0.225	-0.007				

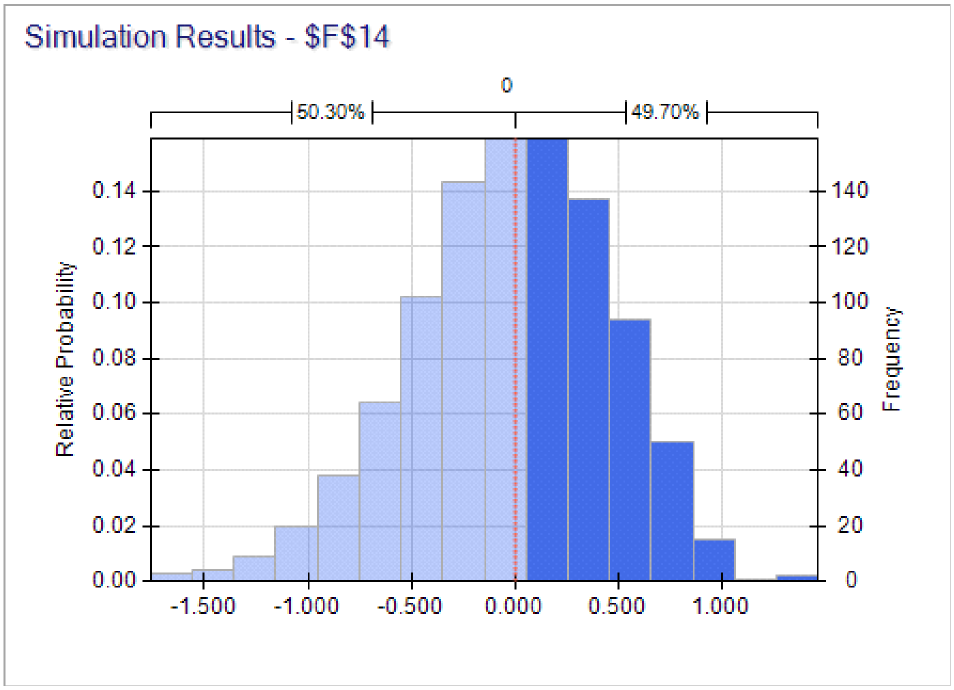
(a) The mean NPV is approximately \$2.9 million.

(b) The probability that the NPV will be at least \$2 million is approximately 77.3%.



(c) The mean value of the minimum annual operating profit is approximately zero.

(d) The probability that the minimum annual operating profit will be at least zero in all four years of operation is approximately 49.7%.



20.6-3.

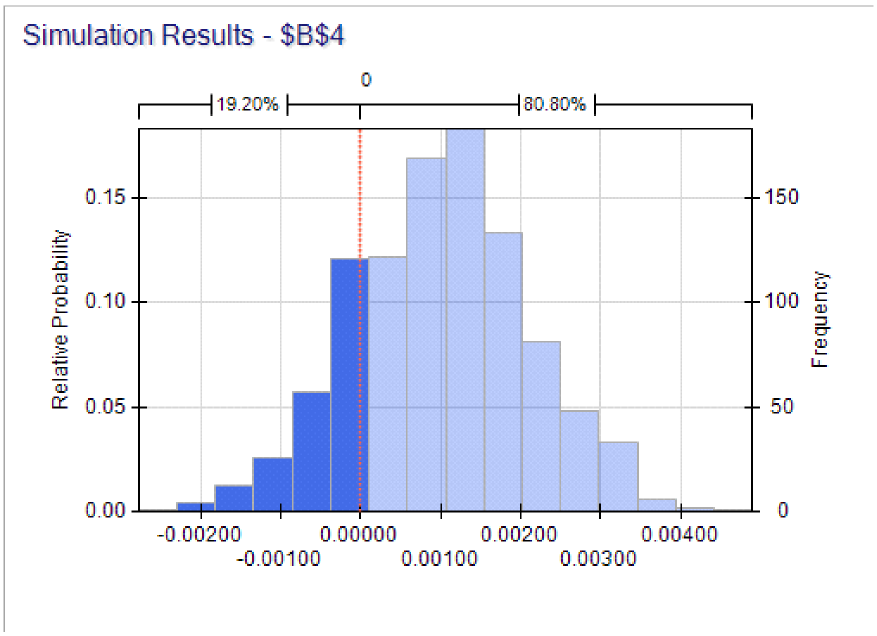
The expected cost with the proposed system of replacing all relays whenever any one of them fails is approximately $2.37 per hour. This is cheaper than the current system of replacing each relay as it fails. Therefore, they should replace all four relays with the first failure.

	Time to Failure (hours)		Min	Max
Relay 1	1,759	Uniform	1,000	2,000
Relay 2	1,354	Uniform	1,000	2,000
Relay 3	1,597	Uniform	1,000	2,000
Relay 4	1,605	Uniform	1,000	2,000
Time to First Failure	1,354			
Time to End of Shutdown	1,356			
Total Cost	$2,800			
Cost per Hour	$2.07			
Mean Cost per Hour	$2.37			

20.6-4.

The chance of negative clearance is approximately 19.2%.

Shaft Radius	1.00122	Triangular(min,likely,max)	1.000	1.001	1.002
Bushing Radius	1.00317	Normal(mean,st.dev.)	1.002	0.001	
Clearance	0.00196				
Mean Clearance	0.00100				



20.6-5.

Toss	Die 1	Die 2	Sum	Win?	Lose?	Continue?		Win
1	6	4	10	No	No	Yes		Game?
2	5	3	8	No	No	Yes		(1=yes,0=no)
3	1	2	3	No	No	Yes		1
4	6	6	12	No	No	Yes		
5	3	3	6	No	No	Yes		Mean (Win Game?)
6	3	2	5	No	No	Yes		0.500
7	4	1	5	No	No	Yes		
8	2	2	4	No	No	Yes		
9	5	5	10	Yes	No	No		
10	4	3	7	#N/A	#N/A	#N/A		
11	6	6	12	#N/A	#N/A	#N/A		
12	3	3	6	#N/A	#N/A	#N/A		
13	1	1	2	#N/A	#N/A	#N/A		
14	4	6	10	#N/A	#N/A	#N/A		
15	3	5	8	#N/A	#N/A	#N/A		
16	2	5	7	#N/A	#N/A	#N/A		
17	3	6	9	#N/A	#N/A	#N/A		
18	2	1	3	#N/A	#N/A	#N/A		
19	3	2	5	#N/A	#N/A	#N/A		
20	6	5	11	#N/A	#N/A	#N/A		
21	6	4	10	#N/A	#N/A	#N/A		
22	4	1	5	#N/A	#N/A	#N/A		
23	3	3	6	#N/A	#N/A	#N/A		
24	6	5	11	#N/A	#N/A	#N/A		
25	6	4	10	#N/A	#N/A	#N/A		
26	5	2	7	#N/A	#N/A	#N/A		
27	2	6	8	#N/A	#N/A	#N/A		
28	1	5	6	#N/A	#N/A	#N/A		
29	6	4	10	#N/A	#N/A	#N/A		
30	2	6	8	#N/A	#N/A	#N/A		

(a) Answers will vary. The standard error is approximately 0.05, so the typical values should be between 0.450 and 0.550.

(b) Answers will vary. The standard error is approximately 0.016, so the typical values should be between 0.484 and 0.516.

(c) Answers will vary. The standard error is approximately 0.005, so the typical values should be between 0.495 and 0.505.

(d) Answers will vary. There is a fair amount of variability in the number of wins, so a large number of iterations, say $10,000$, is necessary to predict the true probability. With $10,000$ iterations, the standard error is 0.005.

20.6-6.

The order quantity that maximizes the mean profit is approximately 55.

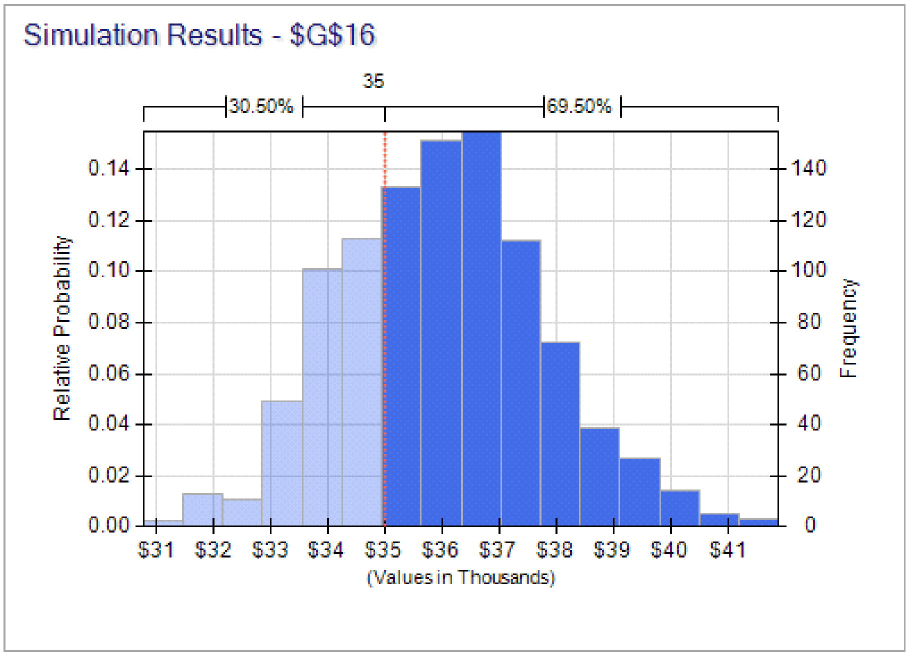
Order Quantity	Mean Profit
50	$46.45
51	$46.74
52	$46.97
53	$47.13
54	$47.22
55	$47.26
56	$47.22
57	$47.13
58	$46.97
59	$46.74
60	$46.45

20.6-7.

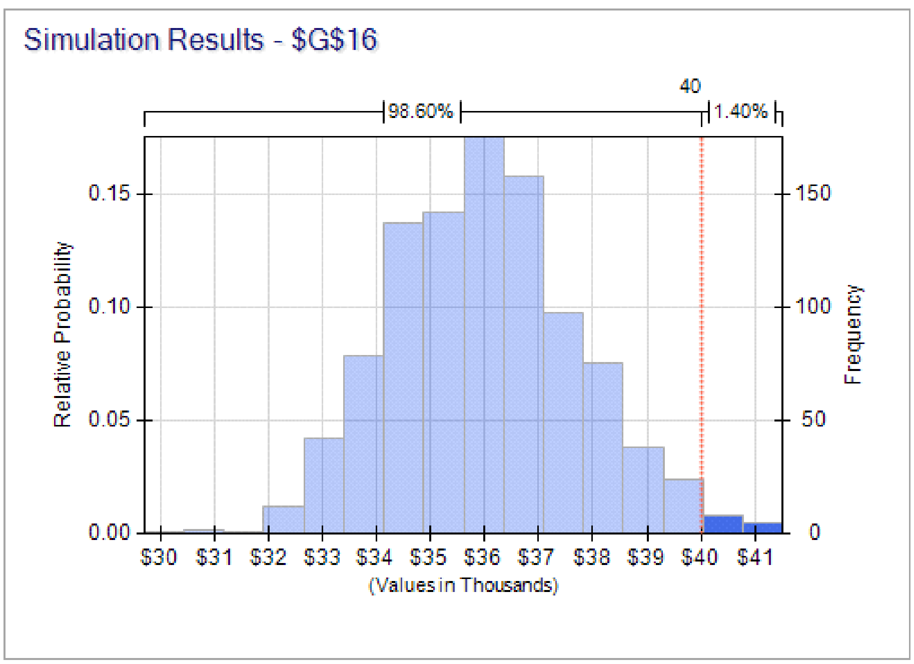
(a) and (b) The expected value of the college fund at year 5 is approximately $36 thousand. The standard deviation of the college fund at year 5 is just over $1700.

	Initial	Annual							
Stock Fund	$3,000	$2,000							
Bond Fund	$3,000	$2,000							
	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5			
Stock Fund Investment	$5,000	$2,000	$2,000	$2,000	$2,000	$2,000			
Stock Fund Start	$5,000	$7,519	$9,962	$12,804	$15,794	$19,738		Mean	St. Dev.
Stock Fund Return (%)	10%	6%	8%	8%	12%		*Normal*	8%	6%
Stock Fund End	$5,519	$7,962	$10,804	$13,794	$17,738				
Bond Fund Investment	$5,000	$2,000	$2,000	$2,000	$2,000	$2,000			
Bond Fund Start	$5,000	$7,298	$9,649	$11,516	$13,722	$16,862			
Bond Fund Return (%)	6%	5%	-1%	2%	8%		*Normal*	4%	3%
Bond Fund End	$5,298	$7,649	$9,516	$11,722	$14,862				
					Total	$36,600			
					Mean	$35,993			
					St. Deviation	$1,729			

(c) The probability that the college fund at year 5 will be at least $35,000 is approximately 69.5%.



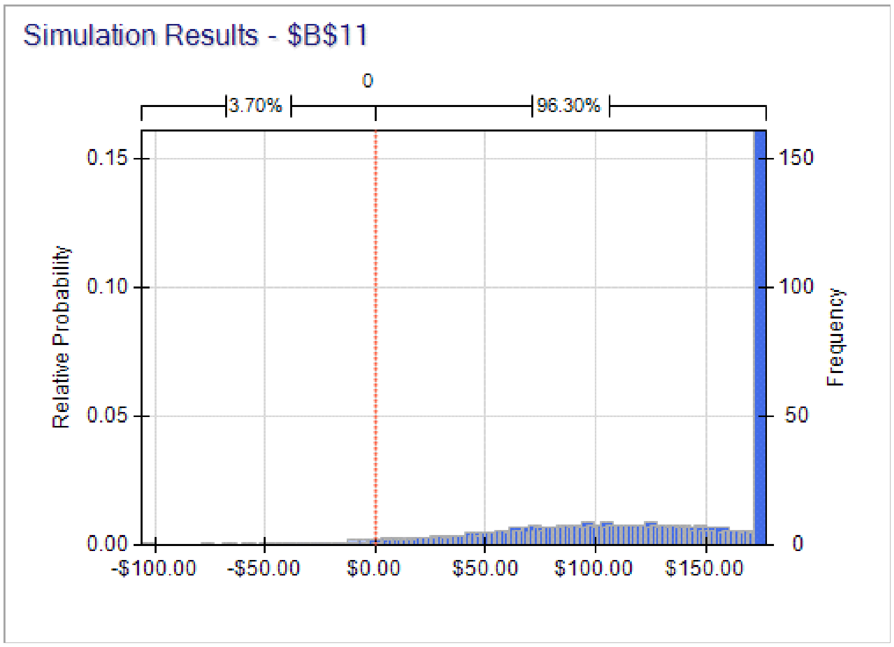
(d) The probability that the college fund at year 5 will be at least $40,000 is approximately 1.4%.



20.6-8.

(a) The mean profit is approximately $107. There is an approximately 96.3% change of making at least $0 profit.

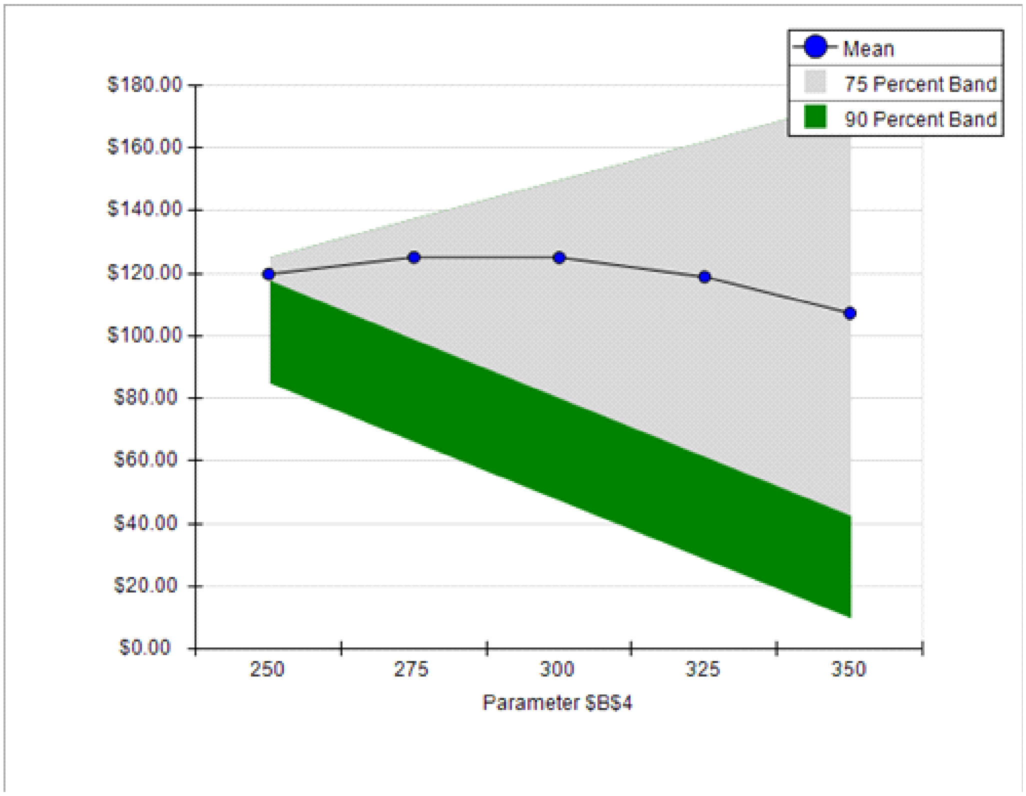
Purchase Price	$0.75			
Selling Price	$1.25			
Order Quantity	350			
			Mean	St. Dev.
Demand	298.7126	*Normal*	300	50
Rounded Demand	299			
Revenue	$373.75			
Purchase Cost	$262.50			
Total Profit	$111.25			
Mean Total Profit	$107.29			



(b) An order quantity of 275 maximizes the mean profit. An order quantity of 300 is also very close to maximizing the mean profit. The order quantity that actually maximizes the mean profit is probably somewhere between these two quantities.

Order Quantity	Mean Profit
250	$119.80
275	$125.14
300	$125.07
325	$118.89
350	$107.30

(c)



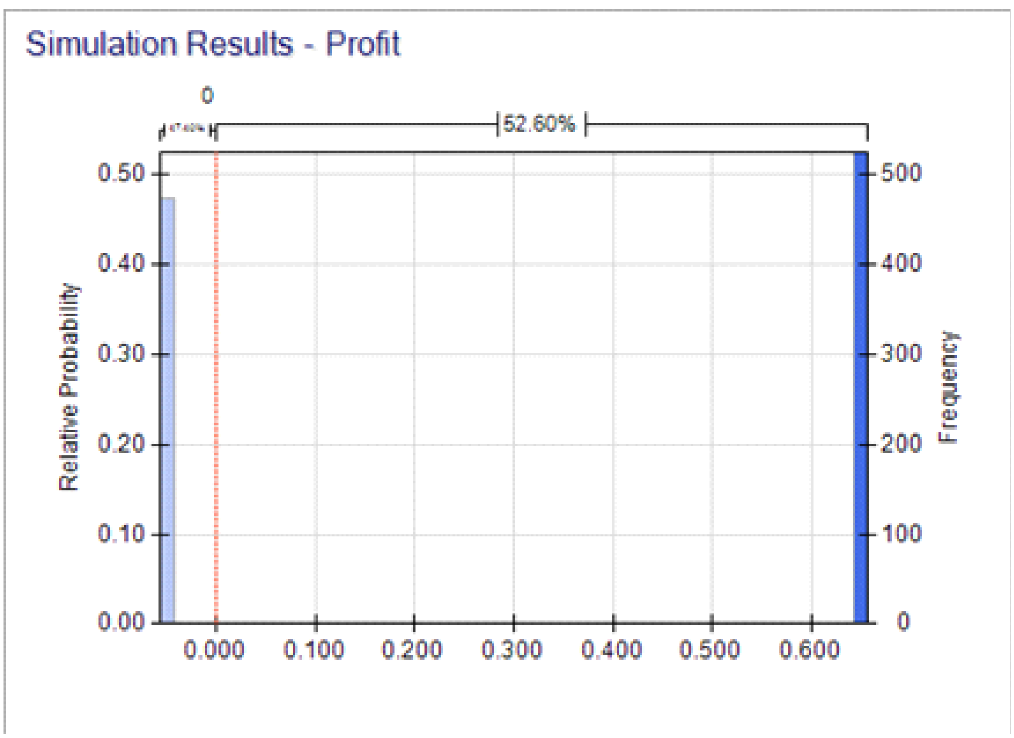
(d) An order quantity of approximately 287 maximizes Michael's mean profit.

Purchase Price	$0.75			
Selling Price	$1.25			
Order Quantity	287			
			Mean	St. Dev.
Demand	348.3337	*Normal*	300	50
Rounded Demand	348			
Revenue	$358.75			
Purchase Cost	$215.25			
Total Profit	$143.50			
Mean Total Profit	$125.85			

20.6-9.

(a) The mean profit is approximately \$0.3 million. The probability of winning the bid is approximately 52.6%.

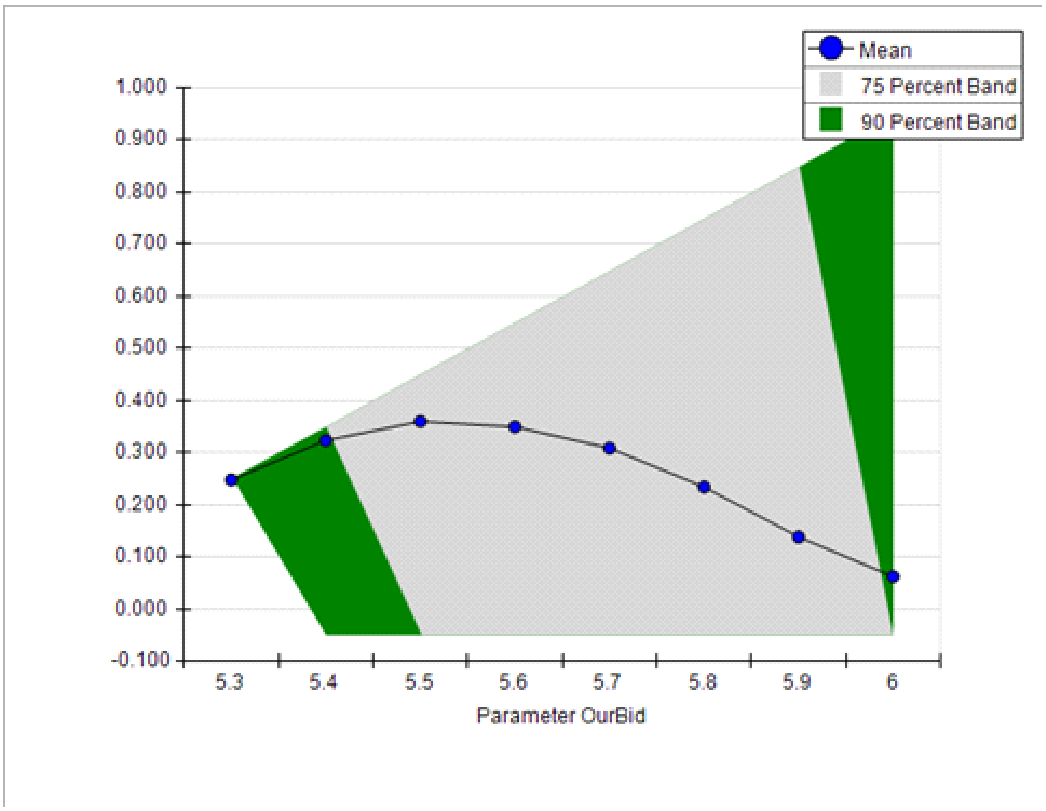
Data				
Our Project Cost ($million)	5.000			
Our Bid Cost ($million)	0.050			
Competitor Bids	Competitor 1	Competitor 2	Competitor 3	Competitor 4
Bid ($million)	5.824	5.959	6.114	6.136
Distribution	*Triangular*	*Triangular*	*Triangular*	*Triangular*
Competitor Distribution Parameters (Proportion of Our Project Cost)				
Minimum	105%	105%	105%	105%
Most Likely	120%	120%	120%	120%
Maximum	140%	140%	140%	140%
Competitor Distribution Parameters ($millions)				
Minimum	5.250	5.250	5.250	5.250
Most Likely	6.000	6.000	6.000	6.000
Maximum	7.000	7.000	7.000	7.000
Minimum Competitor Bid ($million)	5.824			
Our Bid ($million)	5.700			
Win Bid?	1	(1=yes, 0=no)		
Profit ($million)	0.650			
Mean Profit ($million)	0.303			



(b) A bid of approximately $5.5 million maximizes RPI's mean profit.

OurBid	Mean Profit ($million)
5.3	0.248
5.4	0.323
5.5	0.364
5.6	0.356
5.7	0.313
5.8	0.234
5.9	0.140
6.0	0.061

(c)



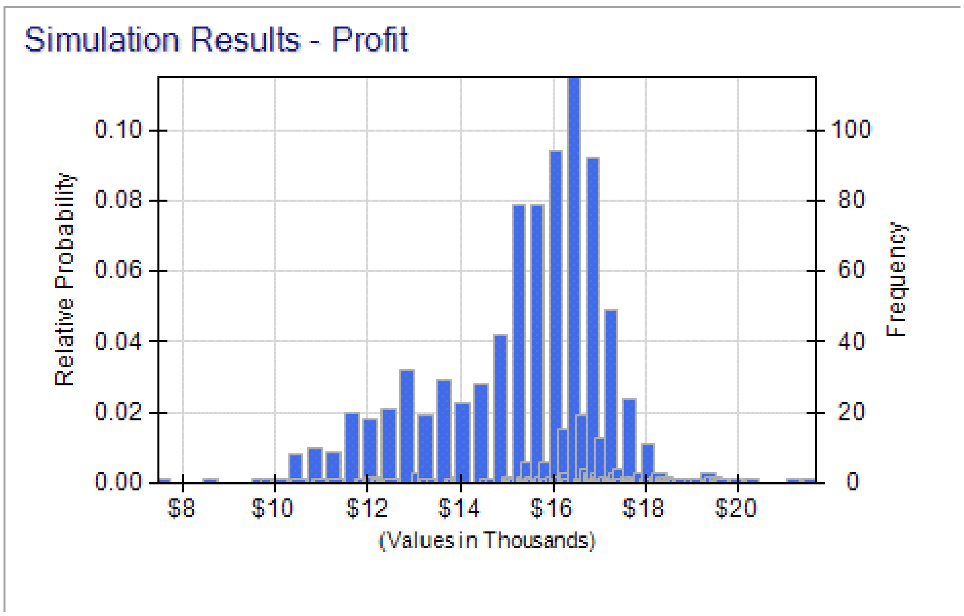
(d) The optimal bid is approximately $5.57 million, as found by Solver.

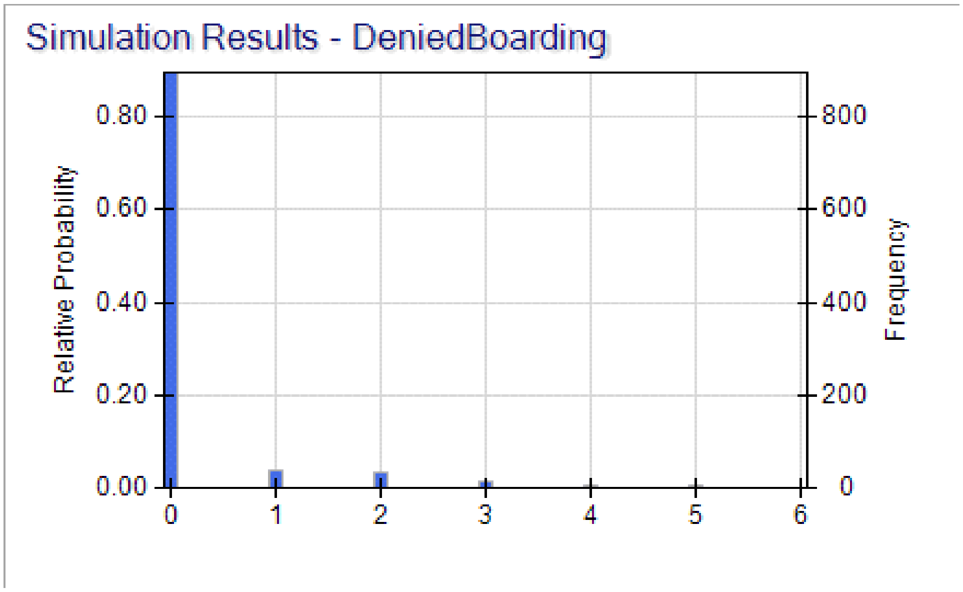
Data				
Our Project Cost ($million)	5.000			
Our Bid Cost ($million)	0.050			
Competitor Bids	Competitor 1	Competitor 2	Competitor 3	Competitor 4
Bid ($million)	6.133	5.864	6.312	6.341
Distribution	*Triangular*	*Triangular*	*Triangular*	*Triangular*
Competitor Distribution Parameters (Proportion of Our Project Cost)				
Minimum	105%	105%	105%	105%
Most Likely	120%	120%	120%	120%
Maximum	140%	140%	140%	140%
Competitor Distribution Parameters ($millions)				
Minimum	5.250	5.250	5.250	5.250
Most Likely	6.000	6.000	6.000	6.000
Maximum	7.000	7.000	7.000	7.000
Minimum Competitor Bid ($million)	5.864			
Our Bid ($million)	5.569			
Win Bid?	1	(1=yes, 0=no)		
Profit ($million)	0.519			
Mean Profit ($million)	0.366			

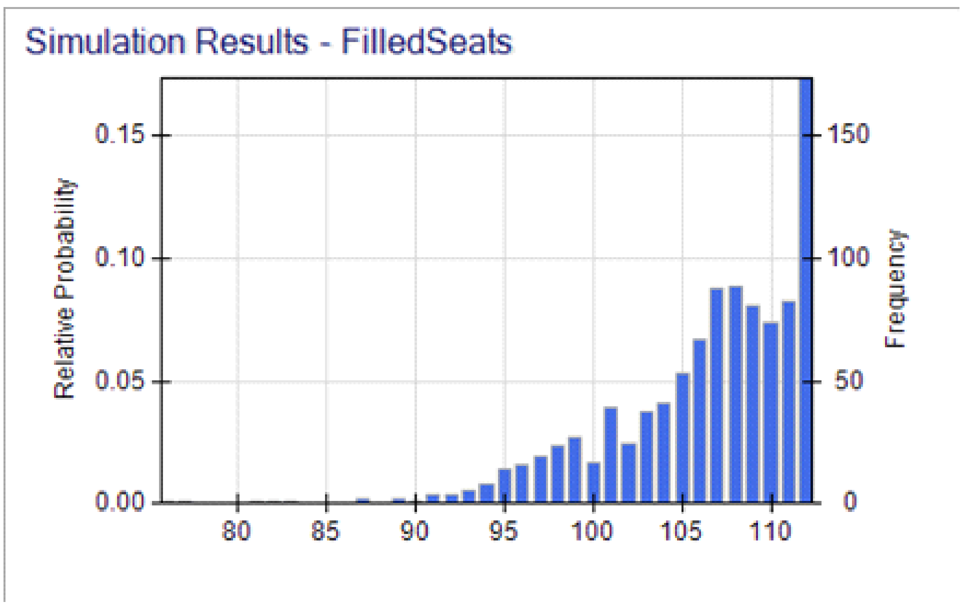
20.6-10.

(a)

Airline Overbooking				
		Discount		
Data		**Reservations**		
Seats Available	112	**to Accept**	50	
Fixed Cost	$10,000			
Discount Fare	$150	**Total**		
Full Coach Fare	$400	**Reservations**		
Cost of Bumping	$600	**to Accept**	112	
Discount Ticket Demand (Triangular)				
Minimum	50	Discount-Fare Demand	77.01	
Most Likely	90	Rounded	77	
Maximum	150	Tickets Purchased	50	
Probability to Show Up	95%	Number that Show	47	
Full-Coach Ticket Demand (Uniform)		Full Coach Demand	52.32	
Minimum	30	Rounded	52	
Maximum	70	Tickets Purchased	52	
Probability to Show Up	85%	Number that Show	47	
				Mean
		Number Denied Boarding	0	0.00
		Number of Filled Seats	94	89.33
		Revenue (Discount Fare)	$7,500	
		Revenue (Full Coach)	$18,800	
		Bumping Cost	$0	
		Fixed Cost	$10,000	Mean
		Profit	$16,300	$14,231







(b)

DiscountReservationsToAccept	TotalReservationsToAccept				
	112	**117**	**122**	**127**	**132**
50	$14,220	$14,453	$14,492	$14,492	$14,492
60	$14,617	$15,271	$15,657	$15,722	$15,713
70	$14,183	$15,232	$15,925	$16,024	$15,860
80	$13,100	$14,479	$15,289	$15,220	$14,879
90	$11,830	$13,347	$14,132	$13,912	$13,406

(c) They should accept approximately 68 discount reservations and up to approximately 125 total in order to maximize mean profit, as found by Solver.

Airline Overbooking				
		Discount		
Data		Reservations		
Seats Available	112	to Accept	68	
Fixed Cost	$10,000			
Discount Fare	$150	Total		
Full Coach Fare	$400	Reservations		
Cost of Bumping	$600	to Accept	125	
Discount Ticket Demand (Triangular)				
Minimum	50	Discount-Fare Demand	73.05	
Most Likely	90	Rounded	73	
Maximum	150	Tickets Purchased	68	
Probability to Show Up	95%	Number that Show	63	
Full-Coach Ticket Demand (Uniform)		Full Coach Demand	31.78	
Minimum	30	Rounded	32	
Maximum	70	Tickets Purchased	32	
Probability to Show Up	85%	Number that Show	30	
				Mean
		Number Denied Boarding	0	0.69
		Number of Filled Seats	93	104.28
		Revenue (Discount Fare)	$10,200	
		Revenue (Full Coach)	$12,000	
		Bumping Cost	$0	
		Fixed Cost	$10,000	Mean
		Profit	$12,200	$16,045

20.7-1.

Answers will vary.

20.7-2.

Answers will vary.

Case 20.1 Reducing In-Process Inventory (Revisited)

a) Status quo at the presses – 7.5 sheets of in-process inventory.

	A	B	C	D	E	F	G	H
1	Template for Queueing Simulation							
2								
3			Data			Results		
4		Number of Servers =	10			Point	95% Confidence Interval	
5						Estimate	Low	High
6		Interarrival Times			L =	7.48596004	7.122474949	7.849445126
7		Distribution =	Exponential		L_q =	0.55020043	0.347368991	0.753031867
8		Mean =	0.142857143		W =	1.0770836	1.036422591	1.117744603
9					W_q =	0.07916311	0.050901621	0.107424593
10								
11		Service Times			P_0 =	0.00110924	0.000312762	0.001905718
12		Distribution =	Exponential		P_1 =	0.00582387	0.003292739	0.008355008
13		Mean =	1		P_2 =	0.02306409	0.018701971	0.027426208
14					P_3 =	0.05166684	0.043052172	0.060281501
15					P_4 =	0.0866959	0.077527167	0.09586463
16		Length of Simulation Run			P_5 =	0.12118604	0.112124348	0.130247735
17		Number of Arrivals =	10,000		P_6 =	0.14062225	0.13442836	0.14681614
18					P_7 =	0.14294653	0.134902634	0.150990419
19					P_8 =	0.12452751	0.11900339	0.130051626
20					P_9 =	0.08806336	0.084082813	0.092043901
21		Run Simulation			P_{10} =	0.06192446	0.055935883	0.067913035

Status quo at the inspection station – 3.6 wing sections of in-process inventory.

	A	B	C	D	E	F	G	H
1	Template for Queueing Simulation							
2								
3			Data			Results		
4		Number of Servers =	1			Point	95% Confidence Interval	
5						Estimate	Low	High
6		Interarrival Times			L =	3.57765981	3.096884037	4.058435589
7		Distribution =	Exponential		L_q =	2.71234549	2.244158962	3.180532014
8		Mean =	0.142857143		W =	0.51681506	0.454627294	0.57900283
9					W_q =	0.39181506	0.329627294	0.45400283
10								
11		Service Times			P_0 =	0.13468567	0.118076335	0.151295015
12		Distribution =	Constant		P_1 =	0.18444199	0.164766618	0.204117359
13		Value =	0.125		P_2 =	0.16054199	0.145653686	0.175430299
14					P_3 =	0.12577666	0.114607169	0.136946159
15					P_4 =	0.09279878	0.083029162	0.102568391
16		Length of Simulation Run			P_5 =	0.07546784	0.065828646	0.085107034
17		Number of Arrivals =	10,000		P_6 =	0.0548405	0.045754492	0.063926513
18					P_7 =	0.04326737	0.033313657	0.053221074
19					P_8 =	0.03643173	0.026094365	0.046769093
20					P_9 =	0.02983638	0.020206033	0.039466733
21		Run Simulation			P_{10} =	0.02245891	0.014710033	0.030207788

Inventory cost = (7.5 + 3.6)($8/hour) = $88.80 / hour
Machine cost = (10)($7/hour) = $70 / hour
Inspector cost = $17 / hour

Total cost = $175.80 / hour

b) Proposal 1 will increase the in-process inventory at the presses to 10.6 sheets since the mean service rate has decreased.

	A	B	C	D	E	F	G	H
1	**Template for Queueing Simulation**							
2								
3			**Data**				**Results**	
4		Number of Servers =	10			Point	95% Confidence Interval	
5						Estimate	Low	High
6		**Interarrival Times**			L =	10.6124208	10.07045277	11.15438883
7		Distribution =	Exponential		L_q =	2.34034351	1.812410733	2.868276277
8		Mean =	0.142857143		W =	1.5192496	1.422904809	1.615594383
9					W_q =	0.33503816	0.255248897	0.41482742
10								
11		**Service Times**			P_0 =	0.00034416	-0.000135983	0.000824295
12		Distribution =	Exponential		P_1 =	0.00330079	0.002146705	0.004454878
13		Mean =	1.2		P_2 =	0.00683624	0.005191338	0.008481139
14					P_3 =	0.0225304	0.017788623	0.027272181
15					P_4 =	0.0437143	0.041059108	0.0463695
16		**Length of Simulation Run**			P_5 =	0.06530488	0.058747044	0.071862716
17		Number of Arrivals =	10,000		P_6 =	0.08305601	0.074729232	0.091382794
18					P_7 =	0.09066307	0.081970997	0.099355138
19					P_8 =	0.09495054	0.09393376	0.095967318
20					P_9 =	0.09944674	0.090813615	0.108079863
21		Run Simulation			P_{10} =	0.08672109	0.077951648	0.095490525

The in-process inventory at the inspection station will not change.

Inventory cost = (10.6 + 3.6)($8/hour) = $113.60 / hour
Machine cost = (10)($6.50) = $65 / hour
Inspector cost = $17 / hour

Total cost = $195.60 / hour

This total cost is higher than for the status quo so should not be adopted. The main reason for the higher cost is that slowing down the machines won't change in-process inventory for the inspection station.

c) Proposal 2 will increase the in-process inventory at the inspection station to 4.2 wing sections since the variability of the service rate has increased.

	A	B	C	D	E	F	G	H
1	Template for Queueing Simulation							
2								
3			Data				Results	
4		Number of Servers =	1			Point	95% Confidence Interval	
5						Estimate	Low	High
6		Interarrival Times			L =	4.15349196	3.51945922	4.787524705
7		Distribution =	Exponential		L_q =	3.31066782	2.691612222	3.929723426
8		Mean =	0.142857143		W =	0.58953022	0.506614288	0.672446148
9					W_q =	0.46990309	0.387653637	0.552152552
10								
11		Service Times			P_0 =	0.15717586	0.13797724	0.176374483
12		Distribution =	Erlang		P_1 =	0.16164362	0.143938659	0.179348578
13		Mean =	0.12		P_2 =	0.1417251	0.127306603	0.156143599
14		k =	2		P_3 =	0.11157869	0.100074725	0.123082653
15					P_4 =	0.08340382	0.074497166	0.092310469
16		Length of Simulation Run			P_5 =	0.0729656	0.064546969	0.081384232
17		Number of Arrivals =	10,000		P_6 =	0.05422094	0.04655526	0.061886616
18					P_7 =	0.04033746	0.033104015	0.047570898
19					P_8 =	0.03068653	0.023437928	0.037935133
20					P_9 =	0.02468793	0.018553583	0.030822285
21		Run Simulation			P_{10} =	0.02288346	0.016278465	0.029488445

The in-process inventory at the presses will not change.

Inventory cost = (7.5 + 4.2)($8/hour) = $93.60 / hour
Machine cost = (10)($7/hour) = $70 / hour
Inspector cost = $17 / hour

Total cost = $180.60 / hour

This total cost is higher than for the status quo so should not be adopted. The main reason for the higher cost is the increase in the service rate variability (Erlang rather than constant) and the resulting increase in the in-process inventory.

d) They should consider *increasing* power to the presses (increasing there cost to $7.50 per hour but reducing their average time to form a wing section to 0.8 hours). This would decrease the in-process inventory at the presses to 5.7.

	A	B	C	D	E	F	G	H
1	**Template for Queueing Simulation**							
2								
3			**Data**				**Results**	
4		Number of Servers =	10			Point	95% Confidence Interval	
5						Estimate	Low	High
6		**Interarrival Times**			L =	5.74237458	5.581608211	5.903140941
7		Distribution =	Exponential		L_q =	0.11624317	0.076181593	0.156304743
8		Mean =	0.142857143		W =	0.81487258	0.801697429	0.828047729
9					W_q =	0.01649551	0.011001805	0.021989206
10								
11		**Service Times**			P_0 =	0.00445475	0.002433487	0.00647602
12		Distribution =	Exponential		P_1 =	0.0241519	0.019051394	0.0292524
13		Mean =	0.8		P_2 =	0.06075455	0.0522877	0.069221409
14					P_3 =	0.10828334	0.096234	0.120332681
15					P_4 =	0.14577459	0.138731319	0.152817867
16		**Length of Simulation Run**			P_5 =	0.1580859	0.148929657	0.167242144
17		Number of Arrivals =	10,000		P_6 =	0.14882682	0.137378613	0.160275035
18					P_7 =	0.12347465	0.116102784	0.13084652
19					P_8 =	0.0909915	0.084900257	0.097082738
20					P_9 =	0.05514285	0.050413495	0.059872212
21		Run Simulation			P_{10} =	0.03360049	0.029185971	0.038015016

Inventory cost = (5.7 + 3.6)($8/hour) = $74.40 / hour
Machine cost = (10)($7.50/hour) = $75 / hour
Inspector cost = $17 / hour

Total cost = $166.40 / hour

This total cost is lower than the status quo and both proposals.

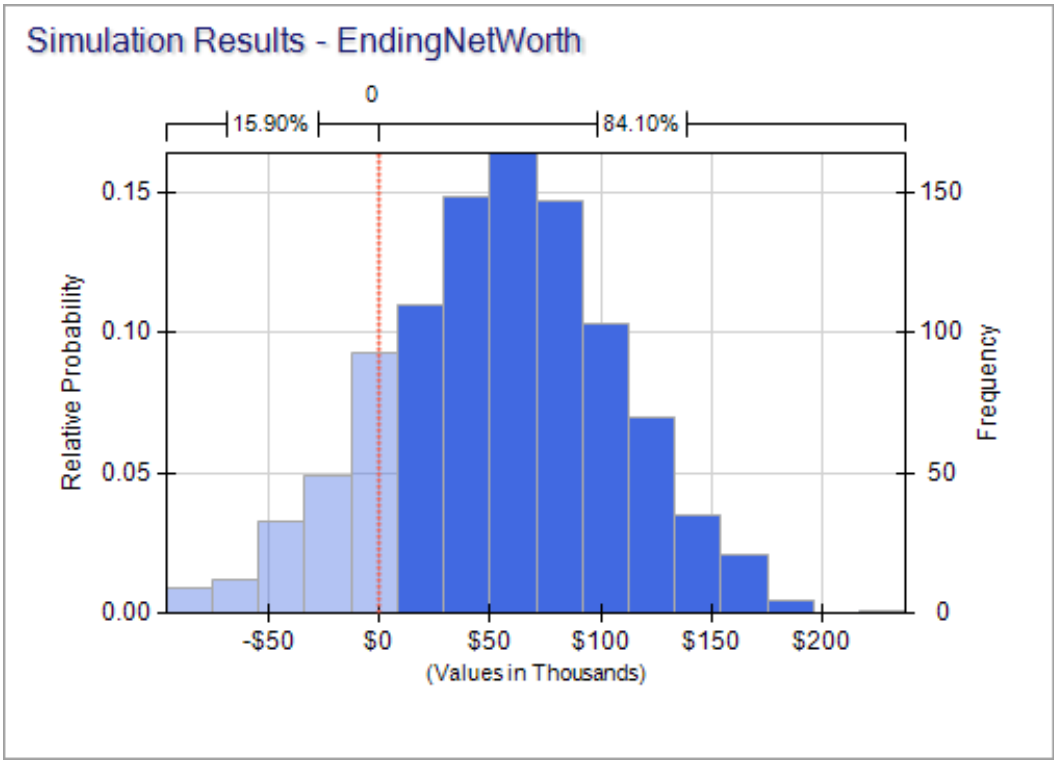
Case 20.2 Action Adventures

a) The spreadsheet model is spread over the next two pages:

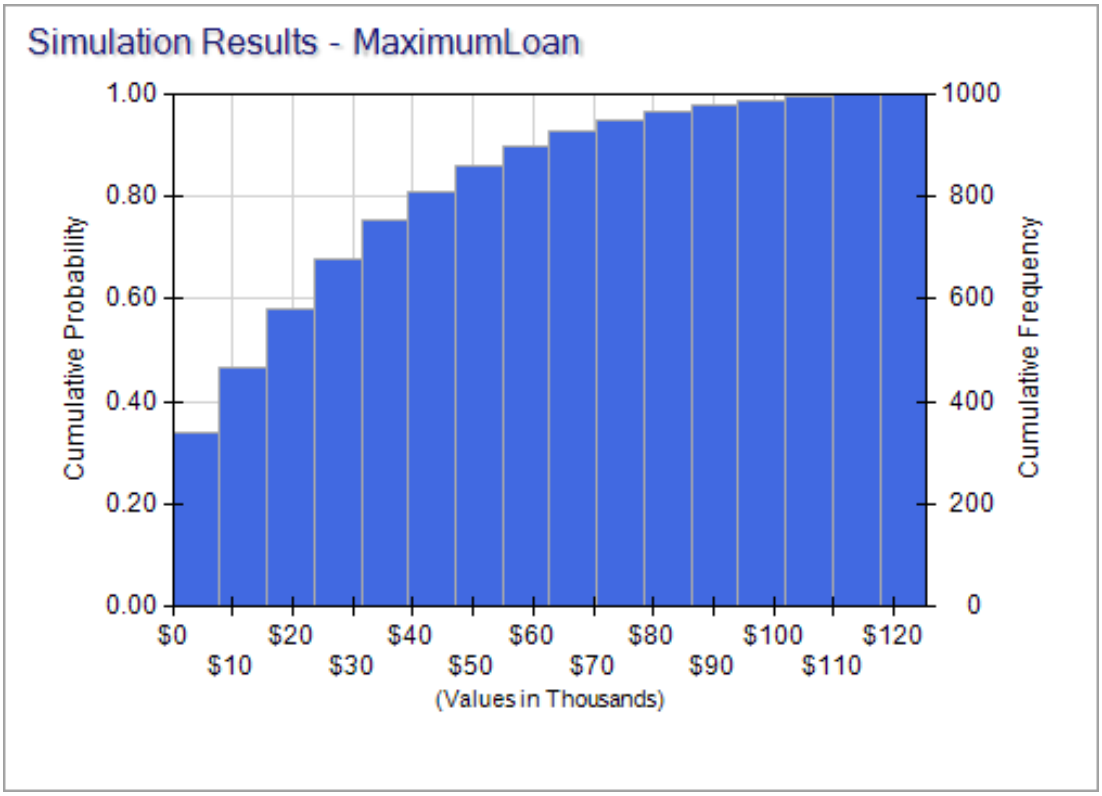
	A	B	C	D	E	F	G	H	I
1		Cost & Revenue Data				Interest Rate Data			
2	Selling Price	$10			Initial Prime Rate		5%		
3	Replacement Part Cost	$5,000			Loan Rate Prime Gap		2%		
4	Monthly Fixed Cost	$15,000			Loan Rate Maximum		9%		
5	Minimum Balance	$20,000			Savings Rate Prime Gap		-2%		
6	Starting Balance	$25,000			Savings Rate Minimum		2%		
7									
8	**Sales**	Dec	Jan	Feb	Mar	Apr	May	June	July
9	Seasonality Index	1.18	0.79	0.88	0.95	1.05	1.09	0.84	0.74
10	Base Sales	6,000	5,250	5,534	4,937	5,562	5,706	5,647	5,137
11	Actual Sales	7,080	4,148	4,870	4,690	5,840	6,219	4,743	3,801
12	Fraction Cash Customers	42%	42%	40%	45%	41%	32%	38%	39%
13									
14	**Interest Rates**								
15	Prime Rate Change		0.00%	-0.25%	0.50%	0.25%	0.00%	0.00%	0.00%
16	Prime Rate	5.00%	5.00%	4.75%	5.25%	5.50%	5.50%	5.50%	5.50%
17	Loan Interest Rate	7.00%	7.00%	6.75%	7.25%	7.50%	7.50%	7.50%	7.50%
18	Savings Interest Rate	3.00%	3.00%	2.75%	3.25%	3.50%	3.50%	3.50%	3.50%
19									
20	**Manufacturing Costs**								
21	Replacement Parts Needed		3	0	0	1	0	0	0
22									
23	Variable Cost		$6.25	$7.96	$6.87	$6.85	$7.24	$6.39	$7.63
24									
25	**Cash Flows**								
26	Beginning Balance		$25,000	$28,415	$20,000	$22,627	$20,000	$20,000	$22,377
27	Cash Receipts		$17,517	$19,344	$21,251	$24,195	$19,934	$18,177	$15,007
28	30-Day Credit Receipts		$41,064	$23,960	$29,351	$25,653	$34,203	$42,257	$29,254
29	Fixed Cost		-$15,000	-$15,000	-$15,000	-$15,000	-$15,000	-$15,000	-$15,000
30	Total Variable Cost		-$25,916	-$38,784	-$32,231	-$39,993	-$45,055	-$30,309	-$28,994
31	Repair Cost		-$15,000	$0	$0	-$5,000	$0	$0	$0
32	Loan Payoff		$0	$0	-$1,212	$0	-$6,783	-$12,509	$0
33	Loan Interest		$0	$0	-$82	$0	-$509	-$938	$0
34	Savings Interest		$750	$852	$550	$735	$700	$700	$783
35	Balance Before Loan		$28,415	$18,788	$22,627	$13,217	$7,491	$22,377	$23,427
36	New Loan		$0	$1,212	$0	$6,783	$12,509	$0	$0
37	Ending Balance	$25,000	$28,415	$20,000	$22,627	$20,000	$20,000	$22,377	$23,427
38			>=	>=	>=	>=	>=	>=	>=
39	Minimum Balance		$20,000	$20,000	$20,000	$20,000	$20,000	$20,000	$20,000
40									
41		Simulated							
42		Value	Mean						
43	Ending Net Worth	$14,935	$54,298						
44									
45	Maximum Loan	$36,775	$25,990						

	A	J	K	L	M	N	O	P	Q	R
1										
2	Selling Price									
3	Replacement Part Cost									
4	Monthly Fixed Cost									
5	Minimum Balance									
6	Starting Balance									
7										
8	**Sales**	August	Sept	October	November	December	January			
9	Seasonality Index	0.98	1.06	1.1	1.16	1.18				
10	Base Sales	5,614	5,652	5,354	5,729	5,549	*Normal*	prev mo.	500	
11	Actual Sales	5,501	5,991	5,889	6,645	6,548				
12	Fraction Cash Customers	37%	33%	38%	43%	39%	*Triangular*	28%	40%	48%
13										
14	**Interest Rates**									
15	Prime Rate Change	0.00%	0.00%	0.00%	0.00%	0.00%	*Custom*	-0.50%	0.05	
16	Prime Rate	4.75%	4.75%	4.75%	4.75%	4.75%	*Discrete*	-0.25%	0.1	
17	Loan Interest Rate	6.75%	6.75%	6.75%	6.75%	6.75%		0%	0.7	
18	Savings Interest Rate	2.75%	2.75%	2.75%	2.75%	2.75%		0.25%	0.1	
19								0.50%	0.05	
20	**Manufacturing Costs**									
21	Replacement Parts Needed	1	1	1	0	1	*Binomial*	10%	8	
22										
23	Variable Cost	$7.18	$7.49	$7.05	$7.15	$6.63	*Uniform*	$6	$8	
24										
25	**Cash Flows**									
26	Beginning Balance	$27,676	$20,000	$20,000	$20,000	$20,000	$20,000			
27	Cash Receipts	$20,491	$19,687	$22,385	$28,906	$25,395				
28	30-Day Credit Receipts	$25,782	$34,523	$40,226	$36,509	$37,548	$40,081			
29	Fixed Cost	-$15,000	-$15,000	-$15,000	-$15,000	-$15,000				
30	Total Variable Cost	-$39,486	-$44,866	-$41,494	-$47,529	-$43,428				
31	Repair Cost	-$5,000	-$5,000	-$5,000	$0	-$5,000				
32	Loan Payoff	$0	-$4,776	-$15,204	-$14,563	-$12,111	-$12,863			
33	Loan Interest	$0	-$322	-$1,026	-$983	-$818	-$868			
34	Savings Interest	$761	$550	$550	$550	$550	$550			
35	Balance Before Loan	$15,224	$4,796	$5,437	$7,889	$7,137	$46,899			
36	New Loan	$4,776	$15,204	$14,563	$12,111	$12,863				
37	Ending Balance	$20,000	$20,000	$20,000	$20,000	$20,000				
38		>=	>=	>=	>=	>=				
39	Minimum Balance	$20,000	$20,000	$20,000	$20,000	$20,000				

b) As seen on the spreadsheet in part a, the mean ending net worth is approximately \$54.4 thousand. The probability that it will be greater than \$0 is approximately 84.1%.



c) The maximum short-term loan is shown in row 45 of the spreadsheet. The maximum short-term loan averages just over $25 thousand. However, to be fairly sure that the credit limit is high enough, it should probably be set quite a bit higher. The cumulative chart shows the probability that any given credit limit will be large enough. For example, a $70 thousand credit limit has about a 95% chance of being sufficient.



Case 20.3 Planning Planers

Current Situation: A simulation run (shown below) indicates that the average number of jobs in the system is 2.0. Of these, half will be platen castings (1) and half will be housing castings (1). The waiting cost is therefore ($200)(1) + ($100)(1) = $300 / hour.

	A	B	C	D	E	F	G	H
1	Template for Queueing Simulation							
2								
3			Data			Results		
4		Number of Servers =	2			Point	95% Confidence Interval	
5						Estimate	Low	High
6		Interarrival Times			$L =$	1.98365641	1.870700578	2.096612244
7		Distribution =	Exponential		$L_q =$	0.66628639	0.575783306	0.756789465
8		Mean =	15		$W =$	30.0811805	28.73655618	31.4258049
9					$W_q =$	10.1039076	8.845870432	11.36194473
10								
11		Service Times			$P_0 =$	0.19988054	0.188799674	0.210961405
12		Distribution =	Translated Exponential		$P_1 =$	0.2828689	0.271597628	0.294140162
13		Minimum Value =	10		$P_2 =$	0.21948306	0.211376682	0.227589435
14		Mean =	20		$P_3 =$	0.13257277	0.125756108	0.13938943
15					$P_4 =$	0.0722497	0.0660523	0.078447105
16		Length of Simulation Run			$P_5 =$	0.04178641	0.036150923	0.047421901
17		Number of Arrivals =	10,000		$P_6 =$	0.02261418	0.018147395	0.027080961
18					$P_7 =$	0.0129863	0.009197547	0.016775062
19					$P_8 =$	0.00771744	0.004659116	0.010775773
20					$P_9 =$	0.003861	0.001884639	0.005837354
21		Run Simulation			$P_{10} =$	0.00185903	0.000591296	0.003126755

Proposal 1: A simulation run (shown below) indicates that the average number of jobs in the system with three planers is approximately 1.4. Of these, half will be platen castings (0.7) and half will be housing castings (0.7). The waiting cost is therefore ($200)(0.7) + ($100)(0.7) = $210 / hour. The savings ($90 / hour) is substantially more than the added cost of the third planer ($30 / hour), so this looks to be worthwhile. The net savings would be $60 / hour.

	A	B	C	D	E	F	G	H
1	Template for Queueing Simulation							
2								
3			Data			Results		
4		Number of Servers =	3			Point	95% Confidence Interval	
5						Estimate	Low	High
6		Interarrival Times			$L =$	1.42409865	1.380256124	1.467941167
7		Distribution =	Exponential		$L_q =$	0.09771456	0.076609893	0.11881923
8		Mean =	15		$W =$	21.4712624	21.07385924	21.86866564
9					$W_q =$	1.47325117	1.168148546	1.778353785
10								
11		Service Times			$P_0 =$	0.25534157	0.245577077	0.265106061
12		Distribution =	Translated Exponential		$P_1 =$	0.33796761	0.329893149	0.346042064
13		Minimum Value =	10		$P_2 =$	0.231656	0.224819899	0.238492092
14		Mean =	20		$P_3 =$	0.11158027	0.106409547	0.116750986
15					$P_4 =$	0.04233244	0.038546771	0.046118111
16		Length of Simulation Run			$P_5 =$	0.01406273	0.011836939	0.016288531
17		Number of Arrivals =	10,000		$P_6 =$	0.00409905	0.002819395	0.005378708
18					$P_7 =$	0.00133435	0.000425396	0.002243309
19					$P_8 =$	0.00080672	-0.000131529	0.001744969
20					$P_9 =$	0.00036429	-0.000147459	0.000876045
21		Run Simulation			$P_{10} =$	0.00025729	-0.000188128	0.000702701

Proposal 2: A simulation run (shown below) indicates that the average number of jobs in the system with constant interarrival times is approximately 1.4. Of these, half will be platen castings (0.7) and half will be housing castings (0.7). The waiting cost is therefore (\$200)(0.7) + (\$100)(0.7) = \$210 / hour. The savings (\$90 / hour) is somewhat more than the added cost of changing the preceding production cost (\$60 / hour). The net savings (\$30) is less than for proposal 1, so this option is less worthwhile.

	A	B	C	D	E	F	G	H
1	**Template for Queueing Simulation**							
2								
3			**Data**				**Results**	
4		Number of Servers =	2			Point	95% Confidence Interval	
5						Estimate	Low	High
6		**Interarrival Times**			$L =$	1.40396168	1.383412164	1.424511205
7		Distribution =	Constant		$L_q =$	0.06455913	0.055139154	0.073979097
8		Value =	15		$W =$	21.0594253	20.75118246	21.36766807
9					$W_q =$	0.96838689	0.82708731	1.109686461
10								
11		**Service Times**			$P_0 =$	0.05142644	0.049243293	0.053609594
12		Distribution =	Translated Exponential		$P_1 =$	0.55774455	0.547877528	0.56761158
13		Minimum Value =	10		$P_2 =$	0.33345643	0.325678768	0.341234095
14		Mean =	20		$P_3 =$	0.05060063	0.045262513	0.055938746
15					$P_4 =$	0.00635733	0.004037203	0.008677449
16		**Length of Simulation Run**			$P_5 =$	0.00041461	2.30036E-06	0.000826928
17		Number of Arrivals =	10,000		$P_6 =$	0	0	0
18					$P_7 =$	0	0	0
19					$P_8 =$	0	0	0
20					$P_9 =$	0	0	0
21		Run Simulation			$P_{10} =$	0	0	0

Proposal 1 and 2: A simulation run (shown below) indicates that the average number of jobs in the system with both three planers and constant interarrival times is approximately 1.33. Of these, half will be platen castings (0.665) and half will be housing castings (0.665). The waiting cost is therefore (\$200)(0.665) + (\$100)(0.665) = \$200 / hour. The savings (\$85 / hour) is less than the combined cost of adding a third planer and changing the preceding production cost (\$90 / hour), so this combined option does not appear to be worthwhile.

	A	B	C	D	E	F	G	H
1	**Template for Queueing Simulation**							
2								
3			**Data**				**Results**	
4		Number of Servers =	3			Point	95% Confidence Interval	
5						Estimate	Low	High
6		**Interarrival Times**			$L =$	1.32985569	1.316690172	1.343021211
7		Distribution =	Constant		$L_q =$	0.00052554	0.000184596	0.00086648
8		Value =	15		$W =$	19.9478354	19.75035259	20.14531816
9					$W_q =$	0.00788307	0.002768944	0.012997201
10								
11		**Service Times**			$P_0 =$	0.05754771	0.055474824	0.059620587
12		Distribution =	Translated Exponential		$P_1 =$	0.58946474	0.581401676	0.5975278
13		Minimum Value =	10		$P_2 =$	0.31909725	0.311531281	0.326663225
14		Mean =	20		$P_3 =$	0.03336476	0.03022801	0.036501519
15					$P_4 =$	0.00052554	0.000184596	0.00086648
16		**Length of Simulation Run**			$P_5 =$	0	0	0
17		Number of Arrivals =	10,000		$P_6 =$	0	0	0
18					$P_7 =$	0	0	0
19					$P_8 =$	0	0	0
20					$P_9 =$	0	0	0
21		Run Simulation			$P_{10} =$	0	0	0

Overall recommendation: Proposal 1 appears to be the most worthwhile, with a net savings of about \$36 / hour over the current situation. Other proposals that may be worth looking into should include giving priority to platen castings, because of the higher waiting cost for that type of job.

Case 20.4 Pricing Under Pressure

a) Before we begin the formal problem, we must first calculate the mean μ and standard deviation σ of the normally distributed random variable N. We are told that the annual interest rate will be used to estimate μ and the historical annual volatility will be used to estimate σ. Because the case is simulating weekly – not yearly – change, we must convert these yearly values to weekly values.

We first convert the annual interest rate r = 8% to a weekly interest rate w with the following formula:

$$\begin{aligned} w &= (1 + r)^{(1/52)} - 1 \\ &= (1 + 0.08)^{(1/52)} - 1 \\ &= (1.08)^{(1/52)} - 1 \\ &= 0.00148 \end{aligned}$$

We next convert the annual volatility $V_a = 0.30$ to a weekly volatility V_w with the following formula:

$$\begin{aligned} V_w &= V_a / \sqrt{52} \\ &= 0.30 / \sqrt{52} \\ &= 0.0416 \end{aligned}$$

Once we have the weekly interest rate and volatility, we can calculate μ and σ.

$$\begin{aligned} \mu &= w - 0.5(V_w)^2 \\ &= 0.00148 - 0.5(0.0416)^2 \\ &= 0.0006 \end{aligned}$$

$$\begin{aligned} \sigma &= V_w \\ &= 0.0416 \end{aligned}$$

1. One component appears in this system: the stock price. The stock price in the previous week is used to calculate the stock price in the next week. The relationship between the stock price in the previous week and the stock price in the next week is given by $s_n = e^N s_c$.

2. State of the system: $P(t)$ = price of the stock at time t.

3. This simulation requires generating a series of random observations from the normal distribution. Each random observation is a normally distributed random variable that determines the increase or decrease of the stock price at the end of next week. The random variable is substituted for N in the following equation:

$$s_n = e^N s_c$$

To generate a series of random variables, we define an uncertain variable cell with normal distribution, where $\mu = 0.0006$ and $\sigma = 0.0416$.

4. The formula $s_n = e^N s_c$ gives us a procedure for changing the price (the state of the system) when an event occurs.

5. In this simulation, the time periods are fixed. We have a twelve-week period, and we need to calculate the change in the stock price each week. We have a formula $s_n = e^N s_c$ that relates the stock price at the end of the next week to the stock price at the end of the previous week. Thus, we do not have to worry about advancing the clock. We simply have to generate N for each of the twelve weeks.

6. We need to build a spreadsheet using RSPE. We start with the current stock price of $42.00. We then use the formula $s_n = e^N s_c$ to calculate the stock price at the end of each of the twelve weeks. We substitute a RSPE uncertain variable cell with normal distribution (with mean $\mu = 0.0006$, and standard deviation $\sigma = 0.0416$) for N.

We then use the stock price at the end of the twelfth week to calculate the value of the option at the end of the twelfth week. If the stock price at the end of the twelfth week is greater than the exercise price of $44.00, the value of the option is the difference between the value of the stock at the end of the twelfth week and the exercise price. If the stock price at the end of the twelfth week is less than or equal to the exercise price of $44.00, the value of the option is $0.

Finally, we need to discount the value of the option at the end of the twelfth week to the value of the option in today's dollars using the following formula:

$$(\text{Value of the option at the end of the twelfth week}) / (1.00148)^{12}$$

The spreadsheet model is shown below. The uncertain variable cells are the N values (B8:B19), the result cell is the price of the option today (C22), and the statistic cell is the mean price of the option today (C23).

	A	B	C	D	E	F
1	**Simulation Model to Estimate Option Value**					
2						
3		Current Stock Price	$42.00		Annual Interest Rate	8%
4		Exercise Price	$44.00		Weekly Interest Rate	0.148%
5						
6			Stock Price at		Annual Volatility	30%
7	Week	N	End of Week		Weekly Volatility	4.160%
8	1	0.002944088	$42.12			
9	2	-0.041470376	$40.41		μ =	0.0006
10	3	0.051283439	$42.54		σ =	0.0416
11	4	0.012944376	$43.09			
12	5	-0.026906539	$41.95			
13	6	-0.079994242	$38.72			
14	7	0.006708864	$38.99			
15	8	0.00707491	$39.26			
16	9	-0.003553399	$39.12			
17	10	0.093240243	$42.95			
18	11	-0.086581855	$39.38			
19	12	-0.017522862	$38.70			
20						
21	Price of Option at end of Week 12		$0.00			
22	Price of Option Today		$0.00			
23	Mean(Price of Option Today)		$1.91			

	A	B	C
3		Current Stock Price	42
4		Exercise Price	44
5			
6			Stock Price at
7	Week	N	End of Week
8	1	=PsiNormal(Mean,StandardDeviation)	=CurrentStockPrice*EXP(B8)
9	2	=PsiNormal(Mean,StandardDeviation)	=EXP(B9)*C8
10	3	=PsiNormal(Mean,StandardDeviation)	=EXP(B10)*C9
11	4	=PsiNormal(Mean,StandardDeviation)	=EXP(B11)*C10
12	5	=PsiNormal(Mean,StandardDeviation)	=EXP(B12)*C11
13	6	=PsiNormal(Mean,StandardDeviation)	=EXP(B13)*C12
14	7	=PsiNormal(Mean,StandardDeviation)	=EXP(B14)*C13
15	8	=PsiNormal(Mean,StandardDeviation)	=EXP(B15)*C14
16	9	=PsiNormal(Mean,StandardDeviation)	=EXP(B16)*C15
17	10	=PsiNormal(Mean,StandardDeviation)	=EXP(B17)*C16
18	11	=PsiNormal(Mean,StandardDeviation)	=EXP(B18)*C17
19	12	=PsiNormal(Mean,StandardDeviation)	=EXP(B19)*C18
20			
21		Price of Option at end of Week 12	=IF(C19>ExercisePrice,C19-ExercisePrice,0)
22		Price of Option Today	=C21/(1+WeeklyInterestRate)^12 + PsiOutput()
23		Mean(Price of Option Today)	=PsiMean(C22)

Range Name	Cells
AnnualInterestRate	F3
AnnualVolatility	F6
CurrentStockPrice	C3
ExercisePrice	C4
Mean	F9
PriceOfOption	C22
StandardDeviation	F10
WeeklyInterestRate	F4
WeeklyVolatility	F7

	E	F
3	Annual Interest Rate	0.08
4	Weekly Interest Rate	=((1+AnnualInterestRate)^(1/52))-1
5		
6	Annual Volatility	0.3
7	Weekly Volatility	=AnnualVolatility/SQRT(52)
8		
9	m=	=WeeklyInterestRate-0.5*(WeeklyVolatility^2)
10	s =	=WeeklyVolatility

The mean of the "Price of Option Today" is the price of the option in today's dollars. The simulation results after 100, 1,000, and 10,000 trials will vary. Typical mean values might be $1.70, $1.91, and $1.87. The variation is significantly reduced with more trials (the mean standard error drops from $0.26 to $0.11 to $0.03 at 100, 1,000, and 10,000 trials, respectively).

b) Using the Black-Scholes Formula, the price of the option is $1.88. The spreadsheet used to calculate the Black-Scholes Formula in Excel follows:

	A	B	C	D	E	F
1	**Black-Scholes Calculation of Option Value**					
2						
3		Current Stock Price	$42.00		**Black-Scholes**	
4					d1 =	-0.127503153
5		Weeks to exercise date	12		d2 =	-0.271618491
6		Exercise Price	$44.00			
7		Exercise Price Present Value	$43.23		N[d1] =	0.449271051
8					N[d2] =	0.39295775
9		Annual Interest Rate	8%			
10		Weekly Interest Rate	0.148%		Value =	$1.88
11						
12		Annual Volatility	30%			
13		Weekly Volatility	4.160%			
14						
15		μ =	0.0006			
16		σ =	0.0416			

	E	F
3	**Black-Scholes**	
4	d1 =	=LN(CurrentStockPrice/ExercisePricePV)/(StandardDeviation*SQRT(WeeksToExerciseDate))+StandardD
5	d2 =	=d_1-StandardDeviation*SQRT(WeeksToExerciseDate)
6		
7	N[d1] =	=NORMSDIST(d_1)
8	N[d2] =	=NORMSDIST(d_2)
9		
10	Value =	=Nd1*CurrentStockPrice-Nd2*ExercisePricePV

Range Name	Cells
AnnualInterestRate	C9
AnnualVolatility	C12
CurrentStockPrice	C3
d_1	F4
d_2	F5
ExercisePrice	C6
ExercisePricePV	C7
Mean	C15
Nd1	F7
Nd2	F8
StandardDeviation	C16
Value	F10
WeeklyInterestRate	C10
WeeklyVolatility	C13
WeeksToExerciseDate	C5

The price of the option obtained by simulation and the price of the option obtained by the Black-Scholes formula are fairly close. The 1,000-iteration simulation price is off by just thirteen cents.

c) No, a random walk does not completely describe the price movement of the stock because the random walk assumes a consistent lognormal increase or decrease in the price of the stock. The price of the stock could change according to a different distribution, however, especially if an event occurs to trigger a dramatic increase or decrease in the stock. In this case, the European Space Agency may award Fellare the International Space Station contract. The award notice would most likely trigger a dramatic movement in the stock. The random walk does not take into account this dramatic event.