CHAPTER 16: DECISION ANALYSIS

16.2-1.

Phillips Petroleum Company developed a decision analysis tool named DISCOVERY to evaluate available investment opportunities and decide on the participation levels. The need for a systematic decision analysis tool arose from the uncertainty associated with various alternatives, the lack of a consistent risk measure across the organization and the scarcity of capital resources. The notion of risk is incorporated in the model with the use of risk-averse exponential utility function. The objective is to maximize expected utility rather than expected return. DISCOVERY provides a decision-tree display of available alternatives at various participation levels. A simple version of the problem is one where Phillips needs to decide first on the participation level and second on whether to drill or not. The exploration of petroleum when drilled is uncertain. The analysis is performed for different levels of risk-aversion and the sensitivity of the decisions to the risk-aversion level is observed. When additional seismic information is available at a cost, the value of information is computed.

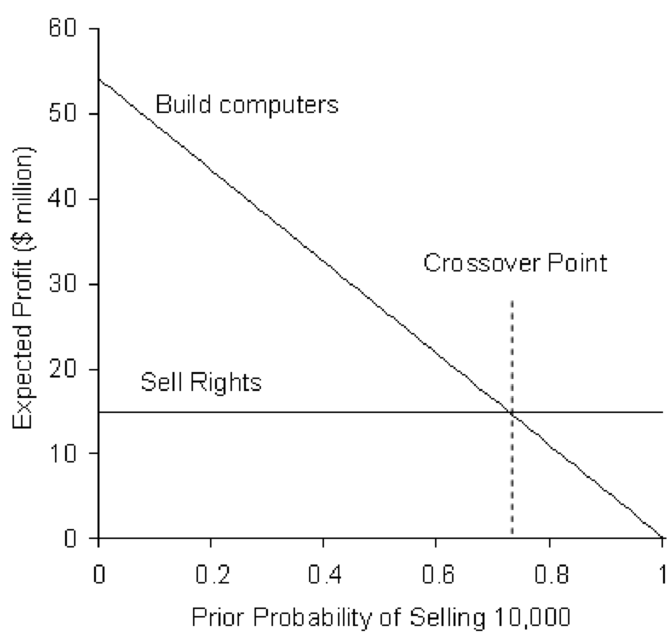
This study "has increased management's awareness of risk and risk tolerance, provided insight into the financial risks associated with its set of investment opportunities, and provided the company a formalized decision model for allocating scarce capital" [p. 55]. The software package developed has been a valuable aid in decision making. It provided a systematic treatment of risk and uncertainty. Other petroleum exploration firms started to use DISCOVERY in analyzing decisions, too.

16.2-2.

(a)

	State of Nature	
Alternative	Sell $10,000$	Sell $100,000$
Build Computers	0	54
Sell Rights	15	15

(b)



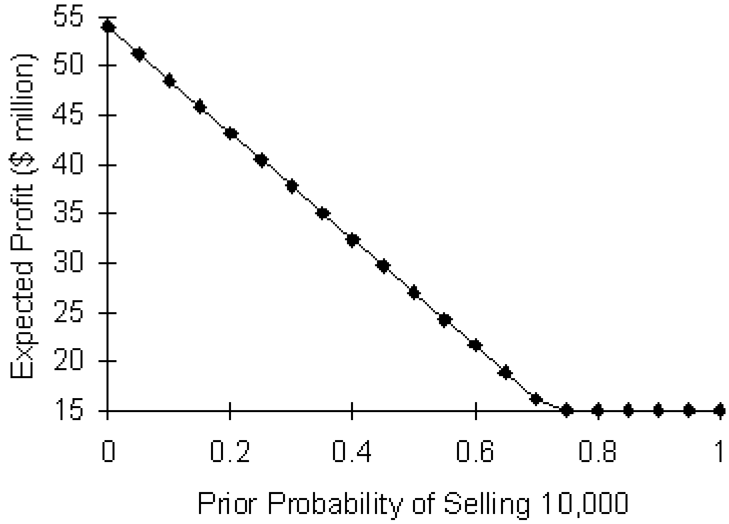
(c) Let p be the prior probability of selling 10,000 computers.

Build: EP $= p(0) + (1-p)(54) = -54p + 54$

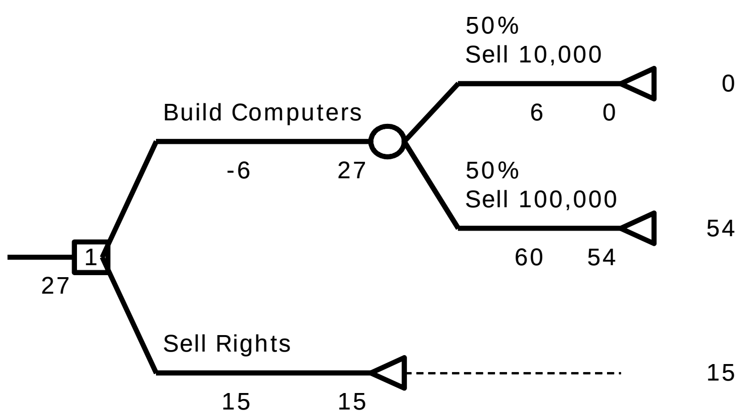
Sell: EP $= p(15) + (1-p)(15) = 15$

The expected profit for Build and Sell is the same when $-54p + 54 = 15 \Rightarrow p = 0.722$. They should build when $p \leq 0.722$ and sell if $p \geq 0.722$.

(d)



(e)



Building computers should be chosen, since it has an expected payoff of $27 million.

16.2-3.

(a)

	State of Nature			
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases
Buy 12 Cases	132	132	132	132
Buy 13 Cases	125	143	143	143
Buy 14 Cases	118	136	154	154
Buy 15 Cases	111	129	147	165
Prior Probability	0.1	0.3	0.4	0.2

(b) According to the maximin payoff criterion, Jean should purchase 12 cases.

	State of Nature				
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Min
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	125
Buy 14 Cases	118	136	154	154	118
Buy 15 Cases	111	129	147	165	111
Prior Probability	0.1	0.3	0.4	0.2	

(c) She will be able to sell 14 cases with highest probability and the maximum possible profit from selling 14 cases is earned when she buys 14 cases. Hence, according to the maximum likelihood criterion, Jean should purchase 14 cases.

(d) According to Bayes' decision rule, Jean should purchase 14 cases.

	State of Nature				Exp.
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Profit
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	141.2
Buy 14 Cases	118	136	154	154	145
Buy 15 Cases	111	129	147	165	141.6
Prior Probability	0.1	0.3	0.4	0.2	

(e) <u>0.2 and 0.5:</u> Jean should purchase 14 cases.

	State of Nature				Exp.
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Profit
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	141.2
Buy 14 Cases	118	136	154	154	146.8
Buy 15 Cases	111	129	147	165	143.4
Prior Probability	0.1	0.2	0.5	0.2	

<u>0.4 and 0.3:</u> Jean should purchase 14 cases.

	State of Nature				Exp.
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Profit
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	141.2
Buy 14 Cases	118	136	154	154	143.2
Buy 15 Cases	111	129	147	165	139.8
Prior Probability	0.1	0.4	0.3	0.2	

<u>0.5 and 0.2:</u> Jean should purchase 14 cases.

	State of Nature				Exp.
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Profit
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	141.2
Buy 14 Cases	118	136	154	154	141.4
Buy 15 Cases	111	129	147	165	138
Prior Probability	0.1	0.5	0.2	0.2	

16.2-4.

(a) The optimal (maximin) actions are conservative and countercyclical investments, both incur a loss of $10 million in the worst case.

(b) The economy is most likely to be stable and the alternative with the highest profit in this state of nature is to make a speculative investment. According to the maximum likelihood criterion, Warren should choose speculative investment.

(c) To maximize his expected payoff, Warren should make a countercyclical investment.

	State of Nature			Exp.
Alternative	Improving	Stable	Worsening	Profit
Conservative	30	5	−10	1.5
Speculative	40	10	−30	−3
Countercyclical	−10	0	15	5
Prior Probability	0.1	0.5	0.4	

16.2-5.

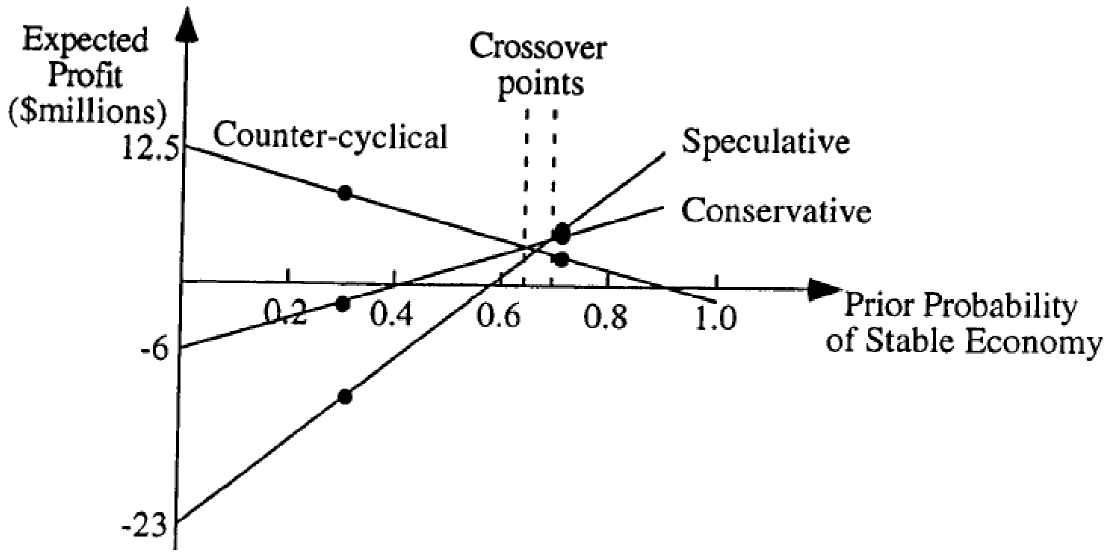
(a) Warren should make a countercyclical investment.

	State of Nature			Exp.
Alternative	Improving	Stable	Worsening	Profit
Conservative	30	5	−10	−1.5
Speculative	40	10	−30	−11
Countercyclical	−10	0	15	8
Prior Probability	0.1	0.3	0.6	

(b) Warren should make a speculative investment.

	State of Nature			Exp.
Alternative	Improving	Stable	Worsening	Profit
Conservative	30	5	−10	4.5
Speculative	40	10	−30	5
Countercyclical	−10	0	15	2
Prior Probability	0.1	0.7	0.2	

(c) The expected profit from countercyclical and conservative investments is the same when $p \approx 0.62$. The expected profit lines for conservative and speculative investments cross at $p \approx 0.68$. Those for countercyclical and speculative investments cross at $p \approx 0.65$; however, this crossover point does not result in a decision shift.



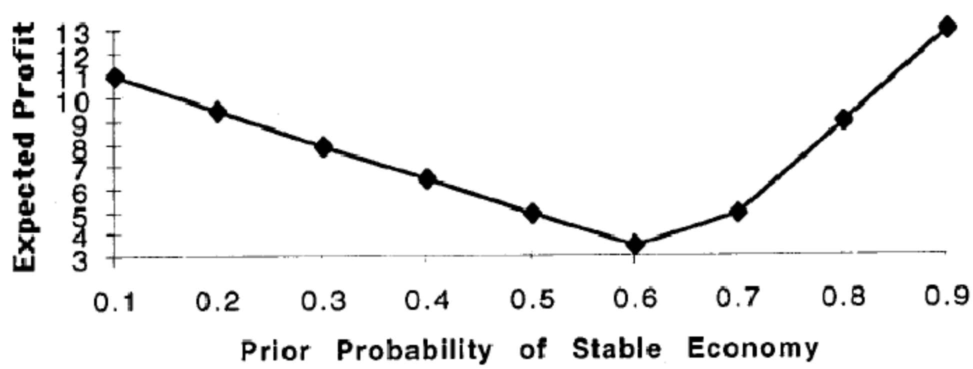
(d) Let p be the prior probability of stable economy.

Conservative:	$\text{EP} = (0.1)(30) + p(5) + (1 - 0.1 - p)(-10) = 15p - 6$
Speculative:	$\text{EP} = (0.1)(40) + p(10) + (1 - 0.1 - p)(-30) = 40p - 23$
Countercyclical:	$\text{EP} = (0.1)(-10) + p(0) + (1 - 0.1 - p)(15) = -15p + 12.5$

Countercyclical and conservative cross when $-15p + 12.5 = 15p - 6 \Rightarrow p = 0.617$.
Conservative and speculative cross when $15p - 6 = 40p - 23 \Rightarrow p = 0.68$.

Accordingly, Warren should choose countercyclical investment when $p < 0.617$, conservative investment when $0.617 \leq p < 0.68$ and speculative investment when $p \geq 0.68$.

(e)



16.2-6.

(a) A_2 should be chosen.

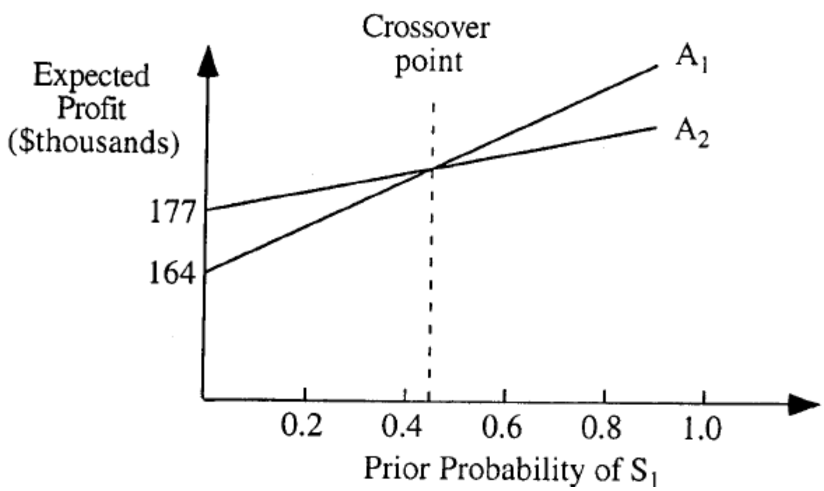
	State of Nature			
Alternative	S_1	S_2	S_3	Min
A_1	220	170	110	110
A_2	200	180	150	150
Prior Probability	0.6	0.3	0.1	

(b) The most likely state of nature is S_1 and the alternative with highest profit in this state is A_1.

(c) A_1 should be chosen.

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A_1	220	170	110	194
A_2	200	180	150	189
Prior Probability	0.6	0.3	0.1	

(d)

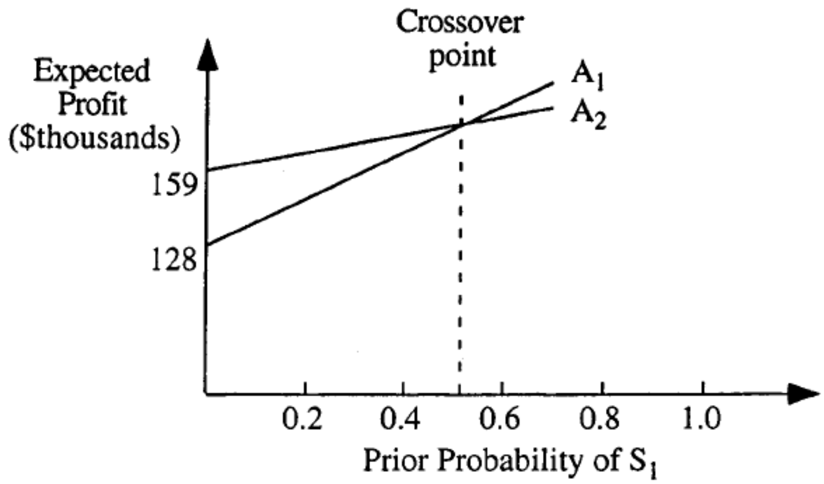


Let p be the prior probability of S_1.

A_1: EP $= p(220) + (1 - 0.1 - p)(170) + (0.1)(110) = 50p + 164$
A_2: EP $= p(200) + (1 - 0.1 - p)(180) + (0.1)(150) = 20p + 177$

A_1 and A_2 cross when $50p + 164 = 20p + 177 \Rightarrow p = 0.433$. They should choose A_2 when $p \leq 0.433$ and A_1 if $p > 0.433$.

(e)

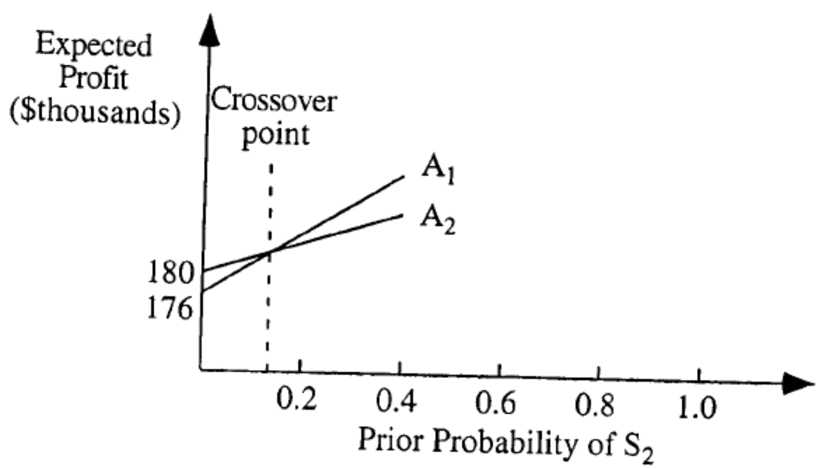


Let p be the prior probability of S_1.

A$_1$: EP $= p(220) + (0.3)(170) + (1 - 0.3 - p)(110) = 110p + 128$
A$_2$: EP $= p(200) + (0.3)(180) + (1 - 0.3 - p)(150) = 50p + 159$

A$_1$ and A$_2$ cross when $110p + 128 = 50p + 159 \Rightarrow p = 0.517$. They should choose A$_2$ when $p \leq 0.517$ and A$_1$ if $p > 0.517$.

(f)



Let p be the prior probability of S_2.

A$_1$: EP $= (0.6)(220) + p(170) + (1 - 0.6 - p)(110) = 60p + 176$
A$_2$: EP $= (0.6)(200) + p(180) + (1 - 0.6 - p)(150) = 30p + 180$

A$_1$ and A$_2$ cross when $60p + 176 = 30p + 180 \Rightarrow p = 0.133$. They should choose A$_2$ when $p \leq 0.133$ and A$_1$ if $p > 0.133$.

(g) A$_1$ should be chosen.

16.2-7.

(a)

	State of Nature		
Alternative	Dry	Moderate	Damp
Crop 1	20	35	40
Crop 2	22.5	30	45
Crop 3	30	25	25
Crop 4	20	20	20
Prior Probability	0.3	0.5	0.2

(b) Grow Crop 1.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	31.5
Crop 2	22.5	30	45	30.75
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.5	0.2	

(c) <u>Prior probability of moderate weather = 0.2:</u> Grow Crop 2.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	33
Crop 2	22.5	30	45	36.25
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.2	0.5	

<u>Prior probability of moderate weather = 0.3:</u> Grow Crop 2.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	32.5
Crop 2	22.5	30	45	33.75
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.3	0.4	

<u>Prior probability of moderate weather = 0.4:</u> Grow Crop 2.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	32
Crop 2	22.5	30	45	32.25
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.4	0.3	

Prior probability of moderate weather $= 0.6$: Grow Crop 1.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	31
Crop 2	22.5	30	45	29.25
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.6	0.1	

16.2-8.

The prior distribution is P$\{\theta = \theta_1\} = 2/3$, P$\{\theta = \theta_2\} = 1/3$.

Order 15: $-\text{EP} = 2/3(1.155 \cdot 10^7) + 1/3(1.414 \cdot 10^7) = 1.241 \cdot 10^7$
Order 20: $-\text{EP} = 2/3(1.012 \cdot 10^7) + 1/3(1.207 \cdot 10^7) = 1.077 \cdot 10^7$
Order 25: $-\text{EP} = 2/3(1.047 \cdot 10^7) + 1/3(1.135 \cdot 10^7) = 1.076 \cdot 10^7$

The maximum expected profit, or equivalently the minimum expected cost is that of ordering 25, so the optimal decision under Bayes' decision rule is to order 25.

16.3-1.

This article describes the use of decision analysis at the Workers' Compensation Board of British Columbia (WCB), which is "responsible for the occupational health and safety, rehabilitation, and compensation interests of British Columbia's workers and employers" [p. 15]. The focus of the study is on the short-term disability claims that can later turn into long-term disability claims and can be very costly for the WCB. First, logistic regression is employed to estimate the probability of conversion for each claim. Then using decision analysis, a threshold is determined to classify the claims as high- and low-risk claims. For any fixed conversion probability, the problem consists of a simple decision tree. First the WCB chooses between classifying the claim as high risk or low risk and then whether the claim converts or not determines the actual cost. If the claim is identified as a high-risk claim, the WCB intervenes. The early intervention lowers the costs and ensures faster rehabilitation. The expected total cost is computed for various cutoff points and the point with minimum expected cost is identified as the optimal threshold.

The new policy offers accurate predictions of high-risk claims. As a result, future costs are reduced and injured workers start working sooner. This study is expected to save the WCB $4.7 per year. The scorecard system developed to implement the new policy improved the efficiency of claim management and the productivity of staff. Overall, the benefits accrued from this study paved the way for the WCB's adoption of operations research in other aspects of the organization.

16.3-2.

(a)

	State of Nature	
Alternative	Sell $10,000$	Sell $100,000$
Build Computers	0	54
Sell Rights	15	15
Prior Probability	0.5	0.5
Maximum Payoff	15	54

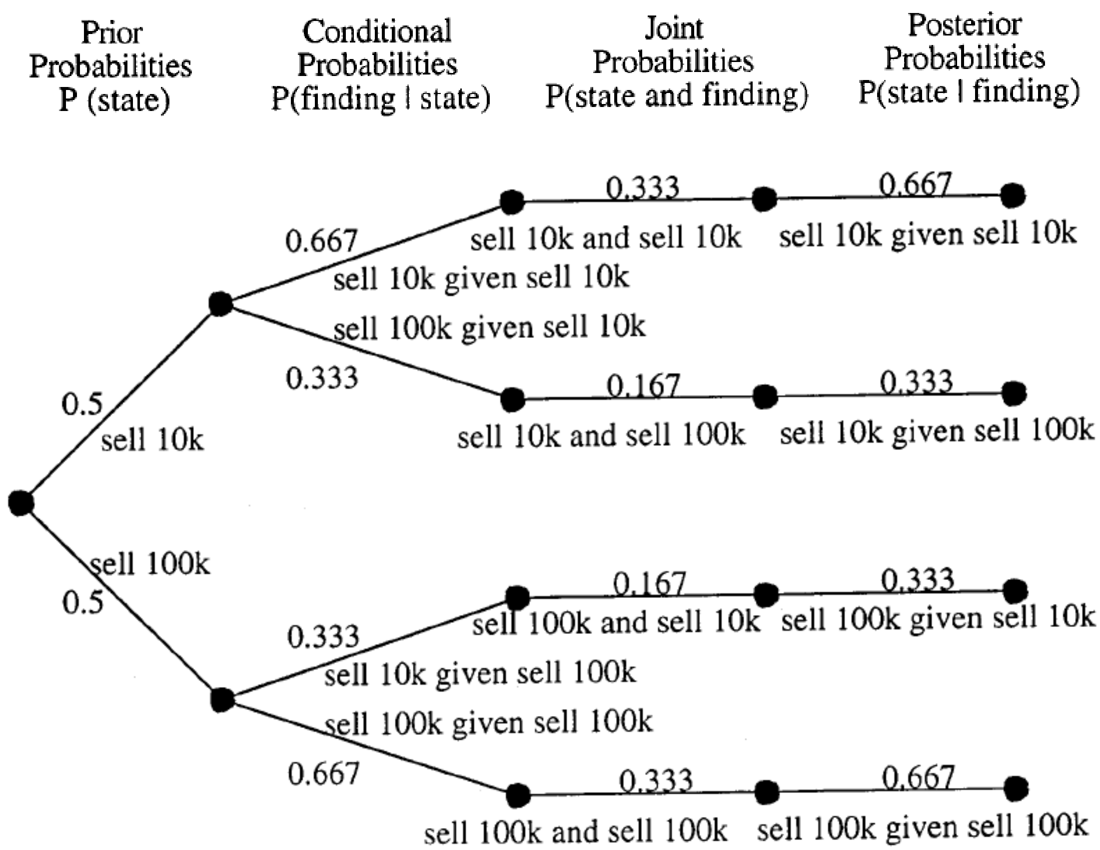
Expected Payoff with Perfect Information: $0.5(15) + 0.5(54) = 34.5$

Expected Payoff without Information: $0.5(0) + 0.5(54) = 27$

EVPI $= 34.5 - 27 =$ \$$7.5$ million

(b) Since the market research will cost \$1 million, it might be worthwhile to perform it.

(c)



(d)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Predict Sell 10,000	Predict Sell 100,000
Sell 10,000	0.5	0.667	0.333
Sell 100,000	0.5	0.333	0.667

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Sell 10,000	Sell 100,000
Predict Sell 10,000	0.5	0.667	0.333
Predict Sell 100,000	0.5	0.333	0.667

(e) EVE $= [0.5(1800) + 0.5(3600)] - 2700 = 0$, so performing the market research is not worthwhile.

16.3-3.

(a) Choose A_2 with expected payoff \$1000.

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A_1	4	0	0	0.8
A_2	0	2	0	1.0
A_3	3	0	1	0.9
Prior Probability	0.2	0.5	0.3	

(b)

	State of Nature		
Alternative	S_1	S_2	S_3
A_1	4	0	0
A_2	0	2	0
A_3	3	0	1
Prior Probability	0.2	0.5	0.3
Maximum Payoff	4	2	1

Expected Payoff with Perfect Information: $0.2(4) + 0.5(2) + 0.3(1) = 2.1$
Expected Payoff without Information: 1.0
EVPI $= 2.1 - 1.0 =$ \$1.1 thousand.

(c) Since the information will cost \$1000 and the value is \$1100, it might be worthwhile to spend the money.

16.3-4.

(a) Choose A_1 with expected payoff \$35.

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A_1	50	100	-100	35
A_2	0	10	-10	1
A_3	20	40	-40	14
Prior Probability	0.5	0.3	0.2	

(b)

	State of Nature		
Alternative	S_1	S_2	S_3
A$_1$	50	100	-100
A$_2$	0	10	-10
A$_3$	20	40	-40
Prior Probability	0.5	0.3	0.2
Maximum Payoff	50	100	-10

Expected Payoff with Perfect Information: $0.5(50) + 0.3(100) + 0.2(-10) = 53$
Expected Payoff without Information: 35
EVPI $= 53 - 35 =$ \$18

(c) Betsy should consider spending up to \$18 to obtain more information.

16.3-5.

(a) Choose A$_3$ with expected payoff \$$35,000$.

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A$_1$	-100	10	100	33
A$_2$	-10	20	50	29
A$_3$	10	10	60	35
Prior Probability	0.2	0.3	0.5	

(b) If S$_1$ occurs for certain, then choose A$_3$ with expected payoff \$$10,000$. If S$_1$ does not occur for certain, then the probability that S$_2$ will occur is $\frac{3}{8}$ and the probability that S$_3$ will occur is $\frac{5}{8}$.

$$\begin{aligned}
&\text{A}_1: && (\tfrac{3}{8})(10) + (\tfrac{5}{8})(100) = 66.25 \\
&\text{A}_2: && (\tfrac{3}{8})(20) + (\tfrac{5}{8})(50) = \ \ 38.75 \\
&\text{A}_3: && (\tfrac{3}{8})(10) + (\tfrac{5}{8})(60) = \ \ 41.25
\end{aligned}$$

Hence, choose A$_1$ which offers an expected payoff of \$$66,250$.

Expected Payoff with Information: $0.2(10) + 0.8(66.25) = 55$
Expected Payoff without Information: 35
EVI $= 55 - 35 =$ \$20 thousand

The maximum amount that should be paid for the information is \$$20,000$. The decision with this information will be to choose A$_3$ if the state of nature is S_1 and A_1 otherwise.

(c) If S$_2$ occurs for certain, then choose A$_2$ with expected payoff \$$20,000$. If S$_2$ does not occur for certain, then the probability that S$_1$ will occur is $\frac{2}{7}$ and the probability that S$_3$ will occur is $\frac{5}{7}$.

$$\begin{aligned}
&\text{A}_1: && (\tfrac{2}{7})(-100) + (\tfrac{5}{7})(100) = 42.857 \\
&\text{A}_2: && (\tfrac{2}{7})(-10) + (\tfrac{5}{7})(50) \quad = 32.857 \\
&\text{A}_3: && (\tfrac{2}{7})(10) + (\tfrac{5}{7})(60) \qquad = 45.714
\end{aligned}$$

Hence, choose A$_3$ which offers an expected payoff of \$$45,714$.

Expected Payoff with Information: $0.3(20) + 0.7(45.714) = 38$
Expected Payoff without Information: 35
EVI $= 38 - 35 =$ \$3 thousand

The maximum amount that should be paid for the information is \$$3000$. The decision with this information will be to choose A_2 if the state of nature is S_2 and A_3 otherwise.

(d) If S_3 occurs for certain, then choose A_1 with expected payoff \$$100,000$. If S_3 does not occur for certain, then the probability that S_1 will occur is $\frac{2}{5}$ and the probability that S_2 will occur is $\frac{3}{5}$.

$$\begin{array}{ll} A_1: & (\frac{2}{5})(-100) + (\frac{3}{5})(100) = -34 \\ A_2: & (\frac{2}{5})(-10) + (\frac{3}{5})(20) \quad = \quad 8 \\ A_3: & (\frac{2}{5})(10) + (\frac{3}{5})(10) \quad\;\; = \quad 10 \end{array}$$

Hence, choose A_3 which offers an expected payoff of \$$10,000$.

Expected Payoff with Information: $0.5(100) + 0.5(10) = 55$
Expected Payoff without Information: 35
EVI $= 55 - 35 =$ \$20 thousand

The maximum amount that should be paid for the information is \$$4,000$. The decision with this information will be to choose A_1 if the state of nature is S_3 and A_3 otherwise.

(e) Expected Payoff with Perfect Information: $0.2(10) + 0.3(20) + 0.5(100) = 58$

Expected Payoff without Information: 35

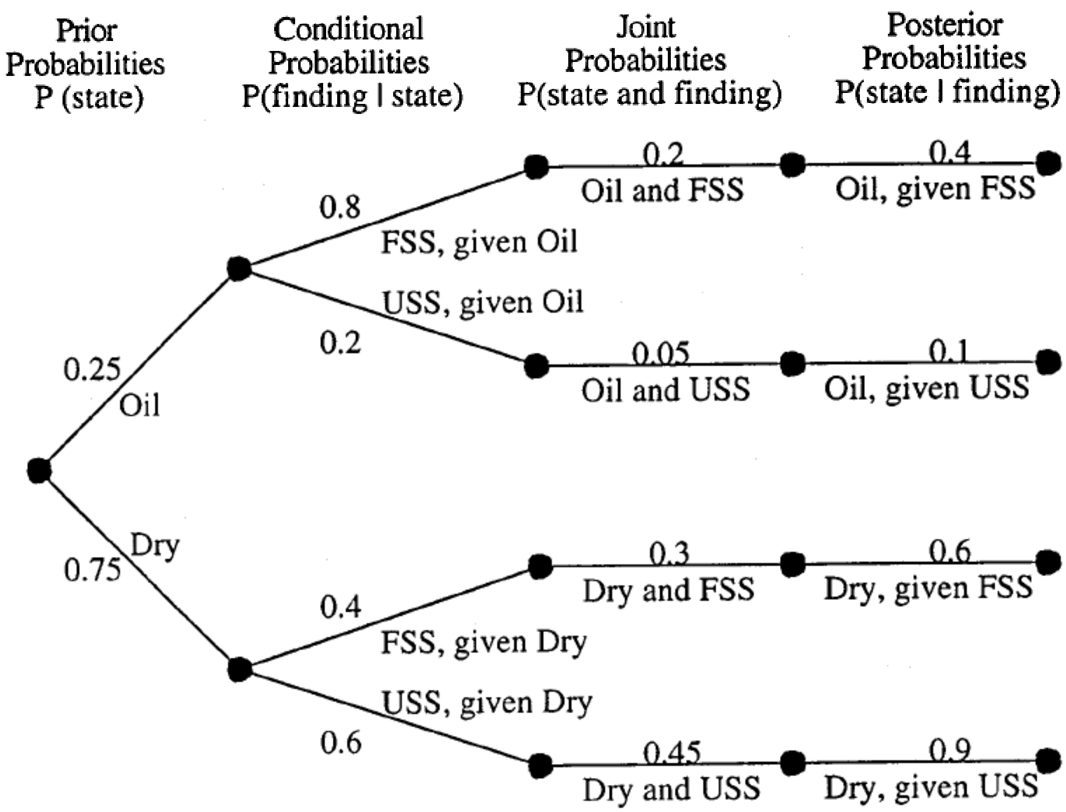
EVPI $= 58 - 35 =$ \$23 thousand

The maximum amount that should be paid for the information is \$$23,000$. The decision with this information will be to choose A_3 if the state of nature is S_1, A_2 if the state of nature is S_2 and A_1 otherwise.

(f) The maximum amount that should be paid for testing is \$$23,000$, since any additional information cannot add more value than perfect information.

16.3-6.

(a)



(b)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	FSS	USS
Oil	0.25	0.8	0.2
Dry	0.75	0.4	0.6

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Oil	Dry
FSS	0.5	0.4	0.6
USS	0.5	0.1	0.9

(c) The optimal policy is to do a seismic survey, to drill if favorable seismic surroundings are obtained, and to sell if unfavorable surroundings are obtained.

16.3-7.

(a) Choose A_1 with expected payoff $100.

	State of Nature		Exp.
Alternative	S_1	S_2	Payoff
A_1	400	−100	100
A_2	0	100	60
Prior Probability	0.4	0.6	

(b)

	State of Nature	
Alternative	S_1	S_2
A_1	400	-100
A_2	0	100
Prior Probability	0.4	0.6
Maximum Payoff	400	100

Expected Payoff with Perfect Information: $0.4(400) + 0.6(100) = 220$

Expected Payoff without Information: 100

EVPI $= 220 - 100 =$ \$120, so it might be worthwhile to do the research.

(c) Let X denote the state of nature and Y denote the prediction. From Bayes' Rule,

$$\text{P}(X = x \text{ and } Y = y) = \text{P}(X = x)\text{P}(Y = y|X = x).$$

(i) $\text{P}(X = \text{S}_1 \text{ and } Y = \text{S}_1) = (0.4)(0.6) = 0.24$
(ii) $\text{P}(X = \text{S}_1 \text{ and } Y = \text{S}_2) = (0.4)(0.4) = 0.16$
(iii) $\text{P}(X = \text{S}_2 \text{ and } Y = \text{S}_1) = (0.6)(0.2) = 0.12$
(iv) $\text{P}(X = \text{S}_2 \text{ and } Y = \text{S}_2) = (0.6)(0.8) = 0.48$

(d) $\text{P}(\text{S}_1) = 0.24 + 0.12 = 0.36$, $\text{P}(\text{S}_2) = 0.16 + 0.48 = 0.64$

(e) Bayes' Rule: $\text{P}(X = x|Y = y) = \frac{\text{P}(X=x \text{ and } Y=y)}{\text{P}(X=x)}$

$\text{P}(\text{S}_1|\text{S}_1) = 0.24/0.36 = 0.667$
$\text{P}(\text{S}_1|\text{S}_2) = 0.16/0.64 = 0.25$
$\text{P}(\text{S}_2|\text{S}_1) = 0.12/0.36 = 0.333$
$\text{P}(\text{S}_2|\text{S}_2) = 0.48/0.64 = 0.75$

(f)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Predict S1	Predict S2
Actual S1	0.4	0.6	0.4
Actual S2	0.6	0.2	0.8

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Actual S1	Actual S2
Predict S1	0.36	0.667	0.333
Predict S2	0.64	0.250	0.750

(g) If S_1 is predicted, then choose A_1 with expected payoff \$233.33.

	State of Nature		Exp.
Alternative	S_1	S_2	Payoff
A_1	400	-100	233.5
A_2	0	100	33.3
Prior Probability	0.667	0.333	

(h) If S_2 is predicted, then choose A_2 with expected payoff $75.

	State of Nature		Exp.
Alternative	S_1	S_2	Payoff
A_1	400	-100	25
A_2	0	100	75
Prior Probability	0.25	0.75	

(i) Given that the research is done, the expected payoff is

$$(0.36)(233.33) + (0.64)(75) - 100 = \$32.$$

(j) The optimal policy is to not do research and to choose A_1.

16.3-8.

(a) EVPI $= [(2/3)(-1.012 \cdot 10^7) + (1/3)(-1.135 \cdot 10^7)] - (-1.076 \cdot 10^7)$

$$= \$230,000.$$

(b)

$$P(\theta = 21 | \text{ 30 spares required}) = \frac{P(\text{30 spares required } |\theta=21)P(\theta=21)}{P(\text{30 spares required } |\theta=21)P(\theta=21)+P(\text{30 spares required } |\theta=24)P(\theta=24)}$$

$$= \frac{(0.013)(2/3)}{(0.013)(2/3)+(0.036)(1/3)} = 0.419$$

$P(\theta = 24 |$ 30 spares required$) = 1 - 0.419 = 0.581$

Order 15: EP $= 0.419(-1.155 \cdot 10^7) + 0.581(-1.414 \cdot 10^7) = -1.305 \cdot 10^7$
Order 20: EP $= 0.419(-1.012 \cdot 10^7) + 0.581(-1.207 \cdot 10^7) = -1.125 \cdot 10^7$
Order 25: EP $= 0.419(-1.047 \cdot 10^7) + 0.581(-1.135 \cdot 10^7) = -1.098 \cdot 10^7$

The optimal alternative is to order 25.

16.3-9.

(a)

	State of Nature		
Alternative	Poor Risk	Average Risk	Good Risk
Extend Credit	$-15,000$	$10,000$	$20,000$
Not Extend Credit	0	0	0
Prior Probability	0.2	0.5	0.3

(b) Choose to extend credit with expected payoff $8,000.

	State of Nature			Exp.
Alternative	Poor Risk	Average Risk	Good Risk	Payoff
Extend Credit	$-15,000$	$10,000$	$20,000$	$8,000$
Not Extend Credit	0	0	0	0
Prior Probability	0.2	0.5	0.3	

(c)

	State of Nature		
Alternative	Poor Risk	Average Risk	Good Risk
Extend Credit	$-15,000$	$10,000$	$20,000$
Not Extend Credit	0	0	0
Prior Probability	0.2	0.5	0.3
Maximum Payoff	0	$10,000$	$20,000$

Expected Payoff with Perfect Information:

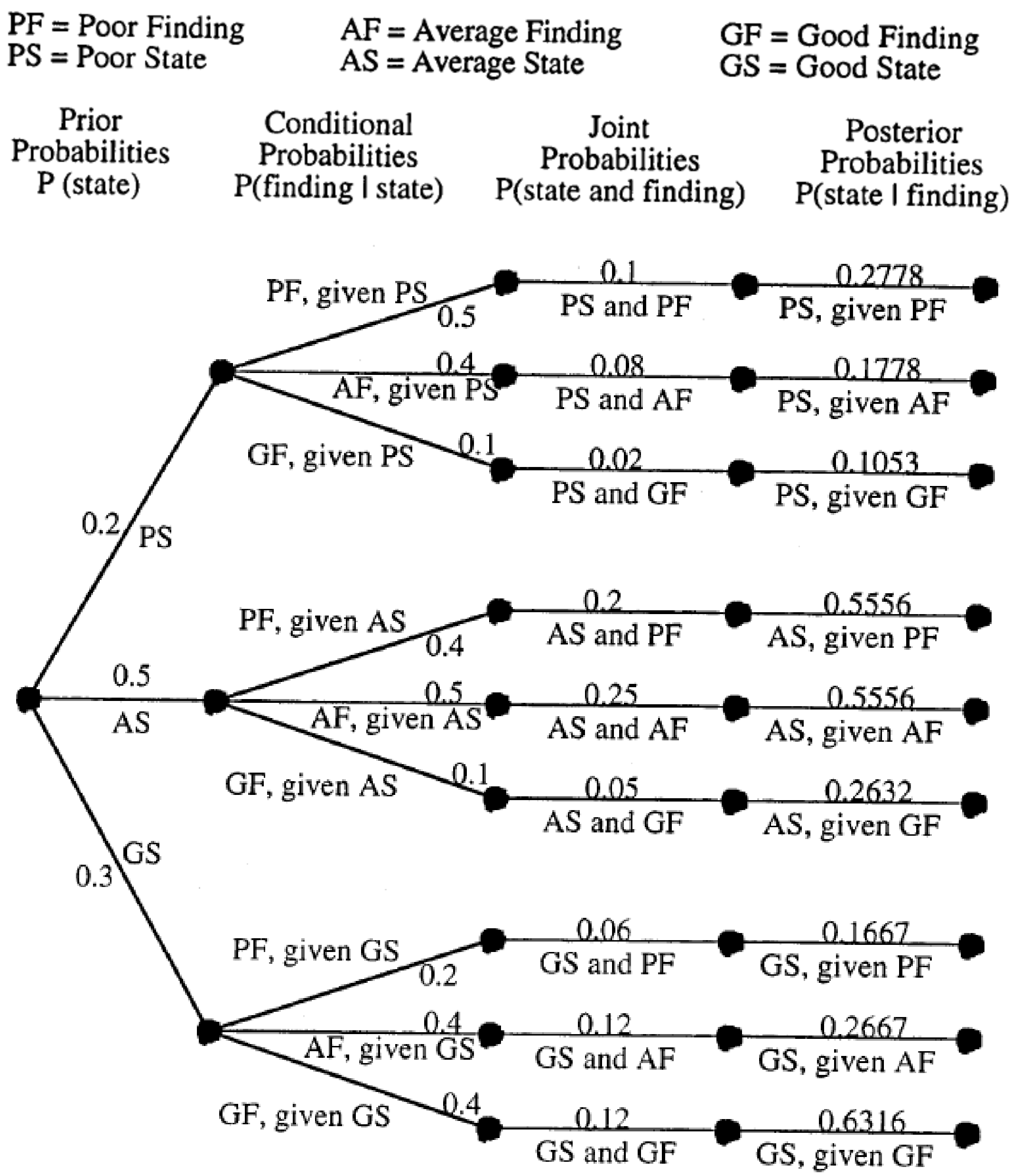
$$0.2(0) + 0.3(10,000) + 0.4(20,000) = 11,000$$

Expected Payoff without Information: $8,000$

EVPI $= 11,000 - 8,000 = \$3,000$

Hence, the credit-rating organization should not be used.

(d)

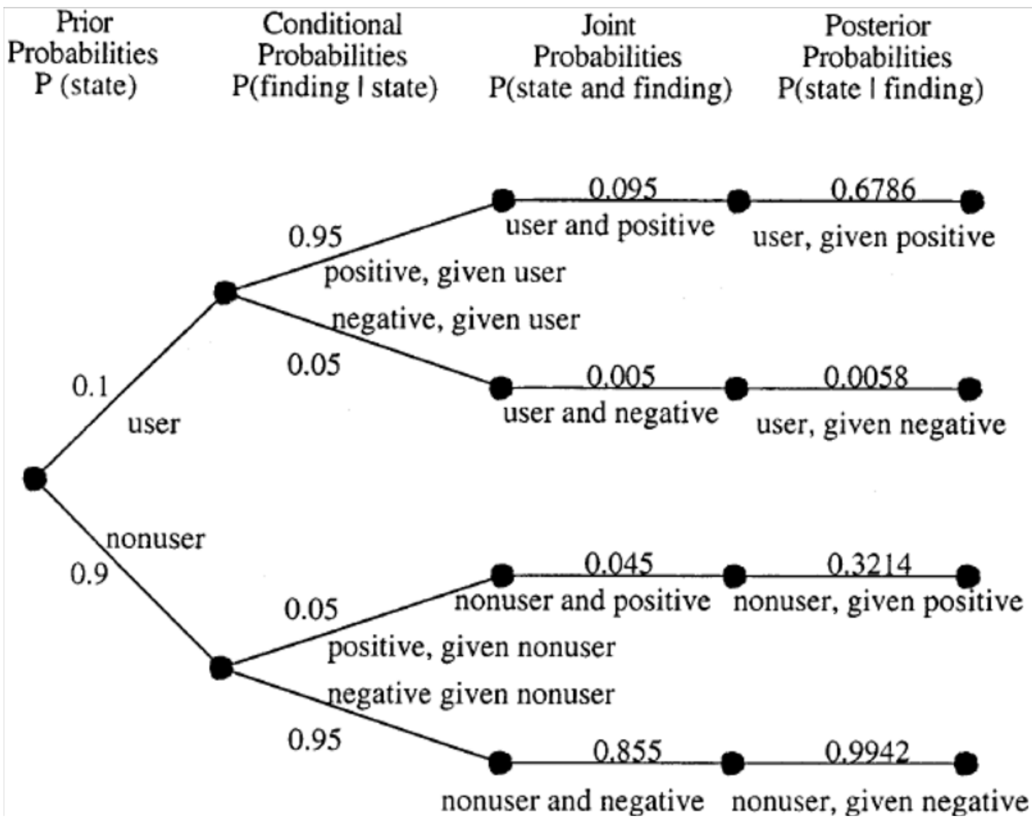


(e)

Data:		P(Finding | State)		
State of	Prior	Finding		
Nature	Probability	Poor	Average	Good
Poor	0.2	0.5	0.4	0.1
Average	0.5	0.4	0.5	0.1
Good	0.3	0.2	0.4	0.4
Posterior		P(State | Finding)		
Probabilities:		State of Nature		
Finding	P(Finding)	Poor	Average	Good
Poor	0.360	0.278	0.556	0.167
Average	0.450	0.178	0.556	0.267
Good	0.190	0.105	0.263	0.632

(f) Vincent should not get the credit rating and extend credit.

16.3-10.



(a) Given that the test is positive, the athlete is a drug user with probability 0.6786.

(b) Given that the test is positive, the athlete is not a drug user with probability 0.3214.

(c) Given that the test is negative, the athlete is a drug user with probability 0.0058.

(d) Given that the test is negative, the athlete is not a drug user with probability 0.9942.

(e) The answers in Excel agree with those found in parts (a), (b), (c), and (d).

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Positive	Negative
User	0.1	0.95	0.05
Nonuser	0.9	0.05	0.95

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	User	Nonuser
Positive	0.14	0.6786	0.3214
Negative	0.86	0.0058	0.9942

16.3-11.

(a)

	State of Nature	
Alternative	Successful	Unsuccessful
Develop New Product	$1,500,000$	$-1,800,000$
Not Develop New Product	0	0
Prior Probability	0.667	0.333

(b) Choose to develop new product with expected payoff $\$400,000$.

	State of Nature		Exp.
Alternative	Successful	Unsuccessful	Payoff
Develop New Product	$1,500,000$	$-1,800,000$	$400,000$
Not Develop New Product	0	0	0
Prior Probability	0.667	0.333	

(c)

	State of Nature	
Alternative	Successful	Unsuccessful
Develop New Product	$1,500,000$	$-1,800,000$
Not Develop New Product	0	0
Prior Probability	0.667	0.333
Maximum Payoff	$1,500,000$	0

Expected Payoff with Perfect Information: $0.667(1,500,000) + 0.333(0) = 1,000,000$

Expected Payoff without Information: $400,000$

EVPI $= 1,000,000 - 400,000 = \$600,000$

This indicates that consideration should be given to conducting the market survey.

(d)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Predict Successful	Predict Unsuccessful
Successful	0.667	0.8	0.2
Unsuccessful	0.333	0.3	0.7

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Successful	Unsuccessful
Predict Successful	0.633	0.8421	0.1579
Predict Unsuccessful	0.367	0.3636	0.6364

(e)

Action	Prediction	Expected Payoff
Develop product	Successful	$[0.8421(1.5) + 0.1579(-1.8)] \cdot 10^6 = \$979,000$
Not develop product	Successful	0
Develop product	Unsuccessful	$[0.3636(1.5) + 0.6364(-1.8)] \cdot 10^6 = -\$600,000$
Not develop product	Unsuccessful	0

It is optimal to develop the product if it is predicted to be successful and to not develop otherwise. Let S be the event that the product is predicted to be successful. Then,

$$\text{P(S)} = \text{P(S}|\theta_1)\text{P}(\theta_1) + \text{P(S}|\theta_2)\text{P}(\theta_2) = 0.8(2/3) + 0.2(1/3) = 0.6.$$

The expected payoff given the information is $0.6(979,000) + 0.4(0) = \$587,000$, so

$$\text{EVE} = 587,000 - 400,000 = \$187,000 < \$300,000 = \text{Cost of survey}.$$

Hence, the optimal strategy is to not conduct the market survey, and to market the product.

16.3-12.

(a)

	State of Nature	
Alternative	$p = 0.05$	$p = 0.25$
Screen	$-1,500$	$-1,500$
Not Screen	-750	$-3,750$
Prior Probability	0.8	0.2

(b) Choose to not screen with expected loss $\$1,350$.

	State of Nature		Exp.
Alternative	$p = 0.05$	$p = 0.25$	Payoff
Screen	$-1,500$	$-1,500$	$-1,500$
Not Screen	-750	$-3,750$	$-1,350$
Prior Probability	0.8	0.2	

(c)

	State of Nature	
Alternative	$p = 0.05$	$p = 0.25$
Screen	$-1,500$	$-1,500$
Not Screen	-750	$-3,750$
Prior Probability	0.8	0.2
Maximum Payoff	-750	$-1,500$

Expected Payoff with Perfect Information: $0.8(-750) + 0.2(-1,500) = -900$

Expected Payoff without Information: $-1,350$

EVPI $= -900 - (-1,350) =$ \$450

This indicates that consideration should be given to inspecting the single item.

(d)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Defective	Nondefective
p = 0.05	0.8	0.05	0.95
p = 0.25	0.2	0.25	0.75

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	p = 0.05	p = 0.25
Defective	0.09	0.4444	0.5556
Nondefective	0.91	0.8352	0.1648

(e) P(defective) $= (0.05)(0.8) + (0.25)(0;2) = 0.09$ and P(nondefective) $= 0.91$

EVE $= [(0.09)(-1500) + (0.91)(-1245)] - (-1350) = 82.05$

Since the cost of the inspection is \$125 $>$ \$82.05, inspecting the single item is not worthwhile.

(f) If defective:

EP(screen, θ|defective) $= 0.444(-1500) + 0.556(-1500) = -1500$
EP(no screen, θ|defective) $= 0.444(-750) + 0.556(-3750) = -2418$

If nondefective:

EP(screen, θ|defective) $= -1500$
EP(no screen, θ|defective) $= 0.835(-750) + 0.165(-3750) = -1245$

Hence, the optimal policy with experimentation is to screen if defective is found and not screen if nondefective is found. On the other hand, from part (e), inspecting a single item, in other words experimenting is not worthwhile. Using part (b), the overall optimal policy is to not inspect the single items, to not screen each item in the lot, instead, rework each item that is ultimately found to be defective.

16.3-13.

(a) Say coin 1 tossed: EP $= 0.6(0) + 0.4(-1) = -0.4$
Say coin 2 tossed: EP $= 0.6(-1) + 0.4(0) = -0.6$

The optimal alternative is to say coin 1 is tossed.

(b) If the outcome is heads (H):

$$\text{P(coin 1|H)} = \frac{\text{P(H|coin 1)P(coin 1)}}{\text{P(H|coin 1)P(coin 1)+P(H|coin 2)P(coin 2)}} = \frac{0.3(0.6)}{0.3(0.6)+0.6(0.4)} = \frac{3}{7}$$

$$\text{P(coin 2|H)} = \frac{4}{7}$$

Say coin 1: EP $= \frac{3}{7}(0) + \frac{4}{7}(-1) = -\frac{4}{7}$

Say coin 2: EP $= \frac{3}{7}(-1) + \frac{4}{7}(0) = -\frac{3}{7}$

The optimal alternative is to say coin 2.

If the outcome is tails (T):

$$\text{P(coin 1|T)} = \frac{\text{P(T|coin 1)P(coin 1)}}{\text{P(T|coin 1)P(coin 1)+P(T|coin 2)P(coin 2)}} = \frac{0.7(0.6)}{0.7(0.6)+0.4(0.4)} = 0.7241$$

$$\text{P(coin 2|T)} = 0.2759$$

Say coin 1: EP $= 0.7241(0) + 0.2759(-1) = -0.2759$

Say coin 2: EP $= 0.7241(-1) + 0.2759(0) = -0.7241$

The optimal alternative is to say coin 1.

16.3-14.

(a)

	State of Nature	
Alternative	Coin 1	Coin 2
Predict 0 H	4	36
Predict 1 H	32	48
Predict 2 H	64	16
Prior probabilities	0.5	0.5

Predict 0 H: EP $= 0.5(4) + 0.5(36) = 20$
Predict 1 H: EP $= 0.5(32) + 0.5(48) = 40$
Predict 2 H: EP $= 0.5(64) + 0.5(16) = 40$

The optimal alternative is to predict one or two heads with expected payoff of $40.

(b)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Heads	Tails
Coin 1	0.5	0.8	0.2
Coin 2	0.5	0.4	0.6

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Coin 1	Coin 2
Heads	0.6	0.6667	0.3333
Tails	0.4	0.2500	0.7500

(c) If the outcome is heads (H):

Predict 0 H: EP $= 0.667(4) + 0.333(36) = 14.67$
Predict 1 H: EP $= 0.667(32) + 0.333(48) = 37.33$
Predict 2 H: EP $= 0.667(64) + 0.333(16) = 48$

The optimal alternative is to predict two heads.

If the outcome is tails (T):

Predict 0 H: EP $= 0.25(4) + 0.75(36) = 28$
Predict 1 H: EP $= 0.25(32) + 0.75(48) = 44$
Predict 2 H: EP $= 0.25(64) + 0.75(16) = 28$

The optimal alternative is to predict one heads.

The expected payoff $= 0.6(\$48) + 0.4(\$44) = \$46.40$.

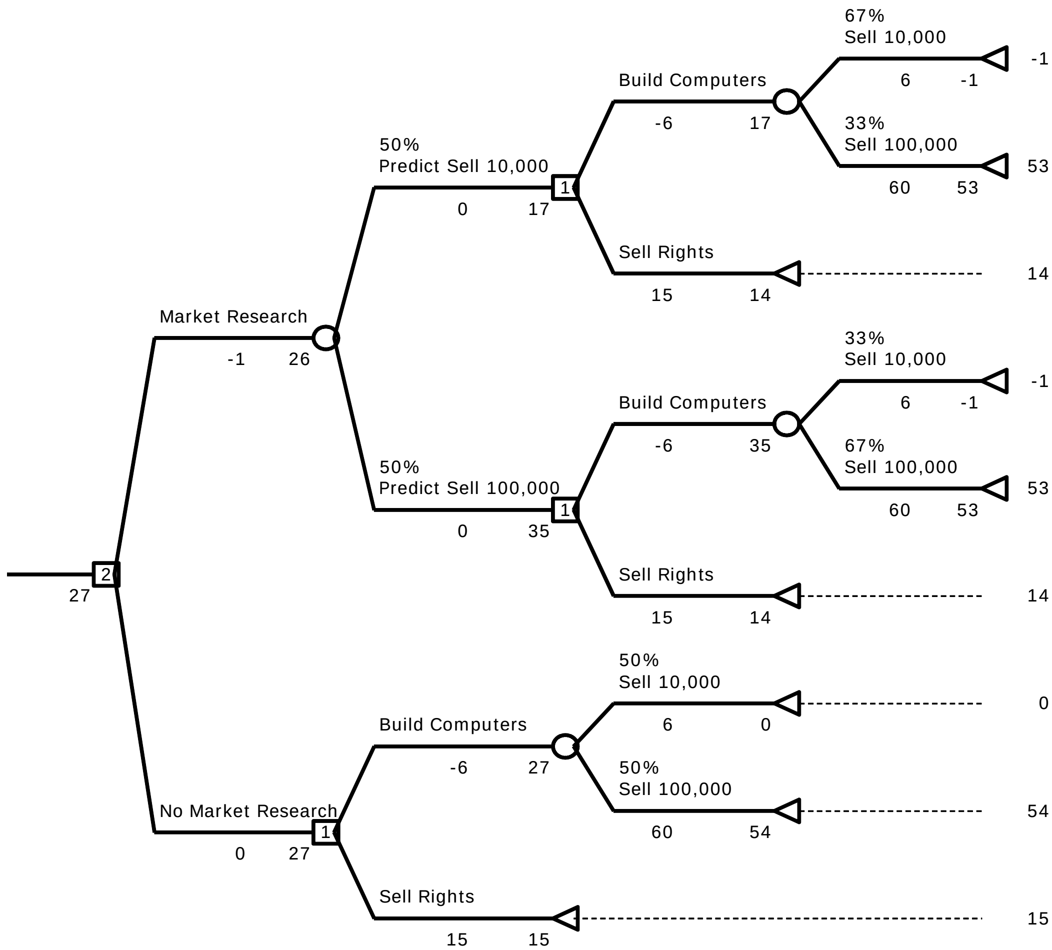
(d) EVE $= \$46.40 - \$40 = \$6.40 < \$30$, so it is better to not pay for the experiment and choose to predict either one or two heads.

16.4-1.

Driven by "the pressure to reduce costs and deliver high-impact technology quickly while justifying investments" [p. 57], Westinghouse initiated this study to evaluate R and D efforts effectively. At any point in time, the firm chooses between launching, delaying and abandoning an innovation. When the launch is delayed, there is a chance of losing the opportunity. R and D is hence treated as a call option with flexibility. The value of the innovation and the optimal decision rule in subsequent stages are found by using dynamic programming. This value is then used in the analysis of the decision tree constructed to find the present value of the project. In this tree, decisions consist of whether to fund or not at different stages and each decision node is followed by a chance node that represents either a technical milestone or strategic fit. Sensitivity analysis is performed to ensure robustness of the model.

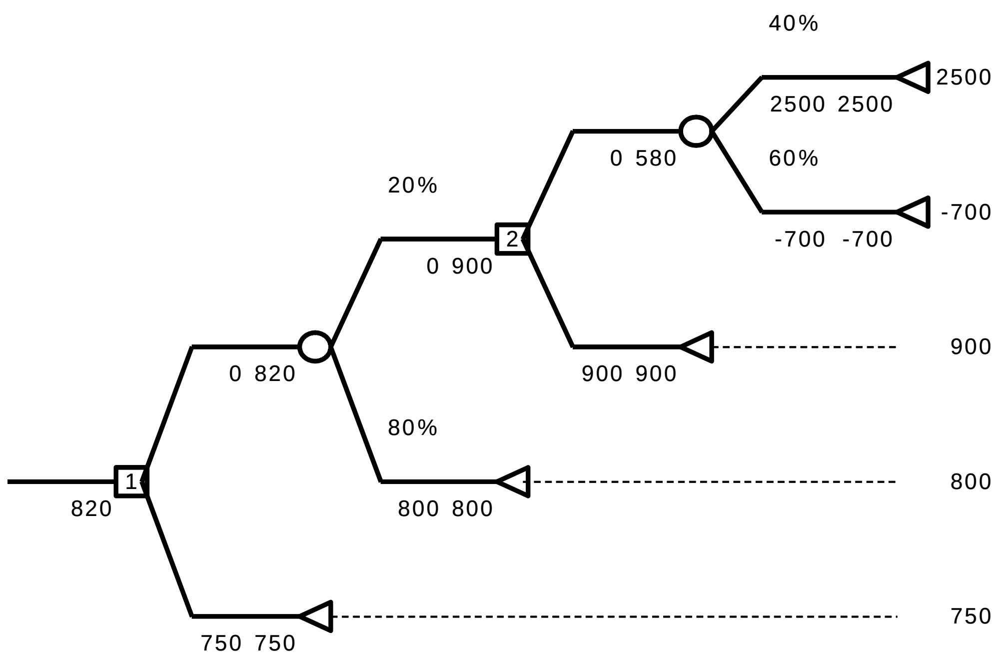
As a result of this study, explicit decision rules for funding R and D projects are obtained. Including flexibility in the model yields a more realistic model. The new system helps identifying cost-effective research portfolios with simplified data acquisition and easy implementation.

16.4-2.



The optimal policy is to build the computers without doing market research.

16.4-3.



16.4-4.

(a)

	State of Nature	
Alternative	W	L
Hold Campaign	3	-2
Not Hold Campaign	0	0
Prior Probability	0.6	0.4

(b) Choose to hold the campaign with expected payoff \$1 million.

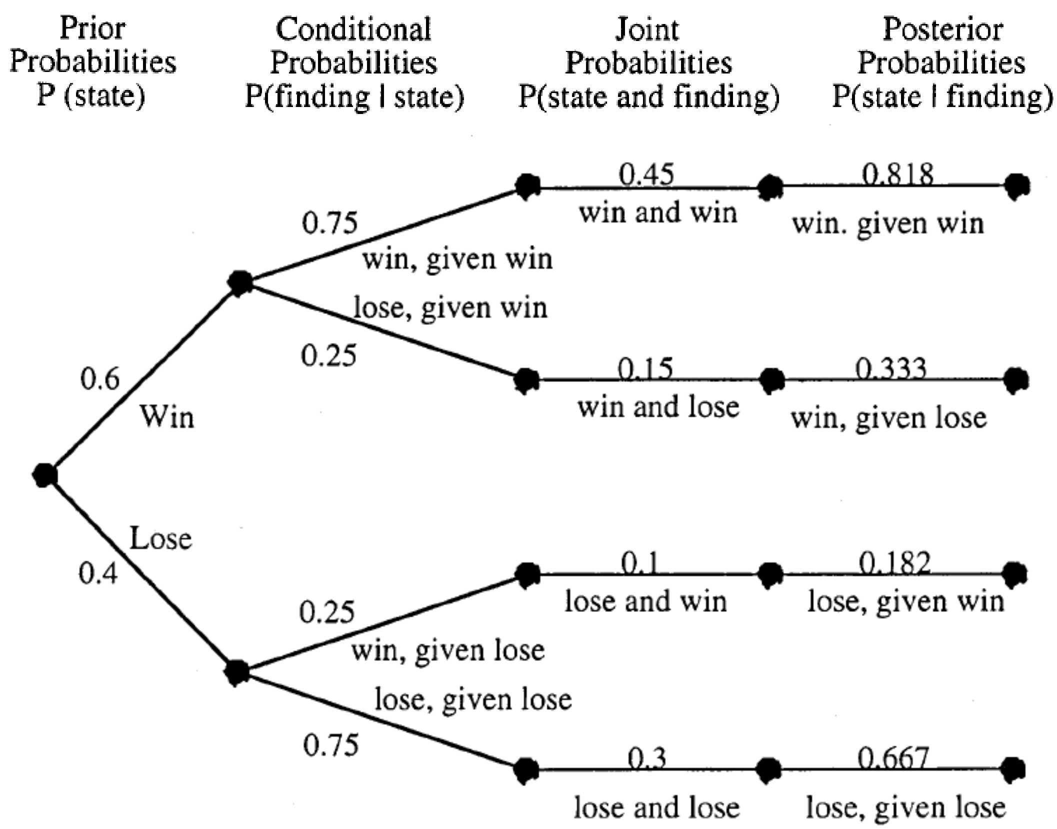
	State of Nature		Exp.
Alternative	W	L	Payoff
Hold Campaign	3	-2	1
Not Hold Campaign	0	0	0
Prior Probability	0.6	0.4	

(c)

	State of Nature	
Alternative	W	L
Hold Campaign	3	-2
Not Hold Campaign	0	0
Prior Probability	0.6	0.4
Maximum Payoff	3	0

Expected Payoff with Perfect Information: $0.6(3) + 0.4(0) = 1.8$
Expected Payoff without Information: 1
EVPI $= 1.8 - 1 =$ \$0.8 million

(d)

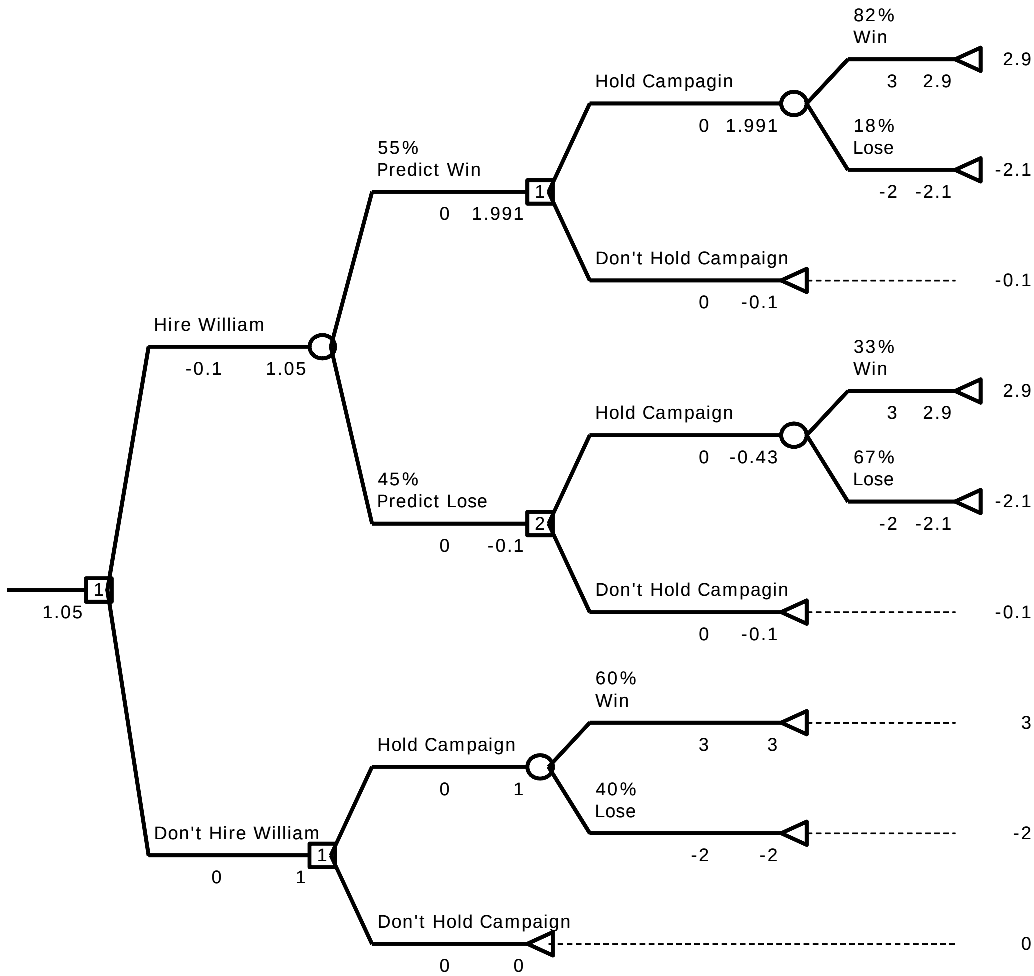


(e)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Predict W	Predict L
Winning Season	0.6	0.75	0.25
Losing Season	0.4	0.25	0.75

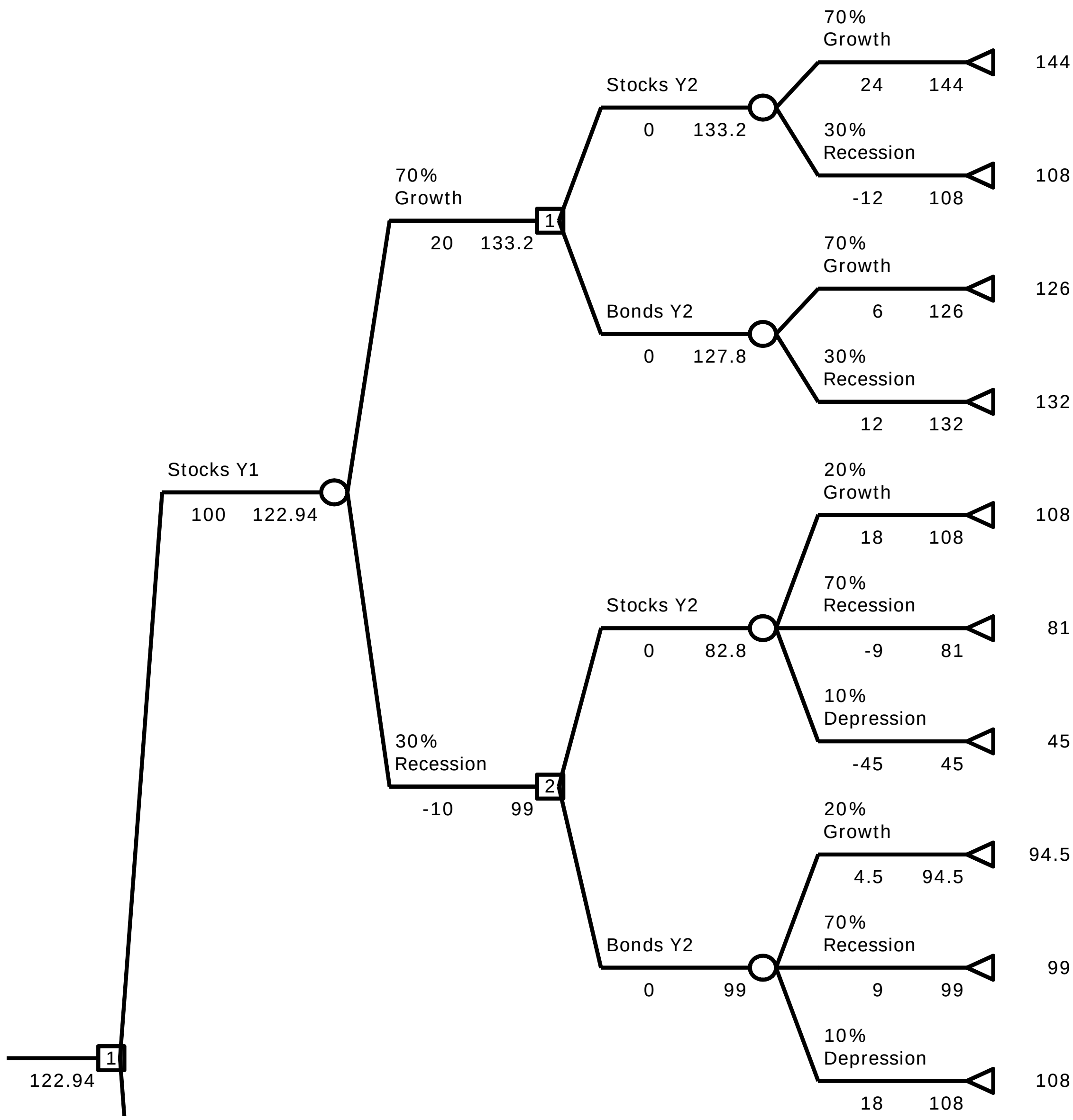
Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Winning Season	Losing Season
Predict W	0.55	0.818	0.182
Predict L	0.45	0.333	0.667

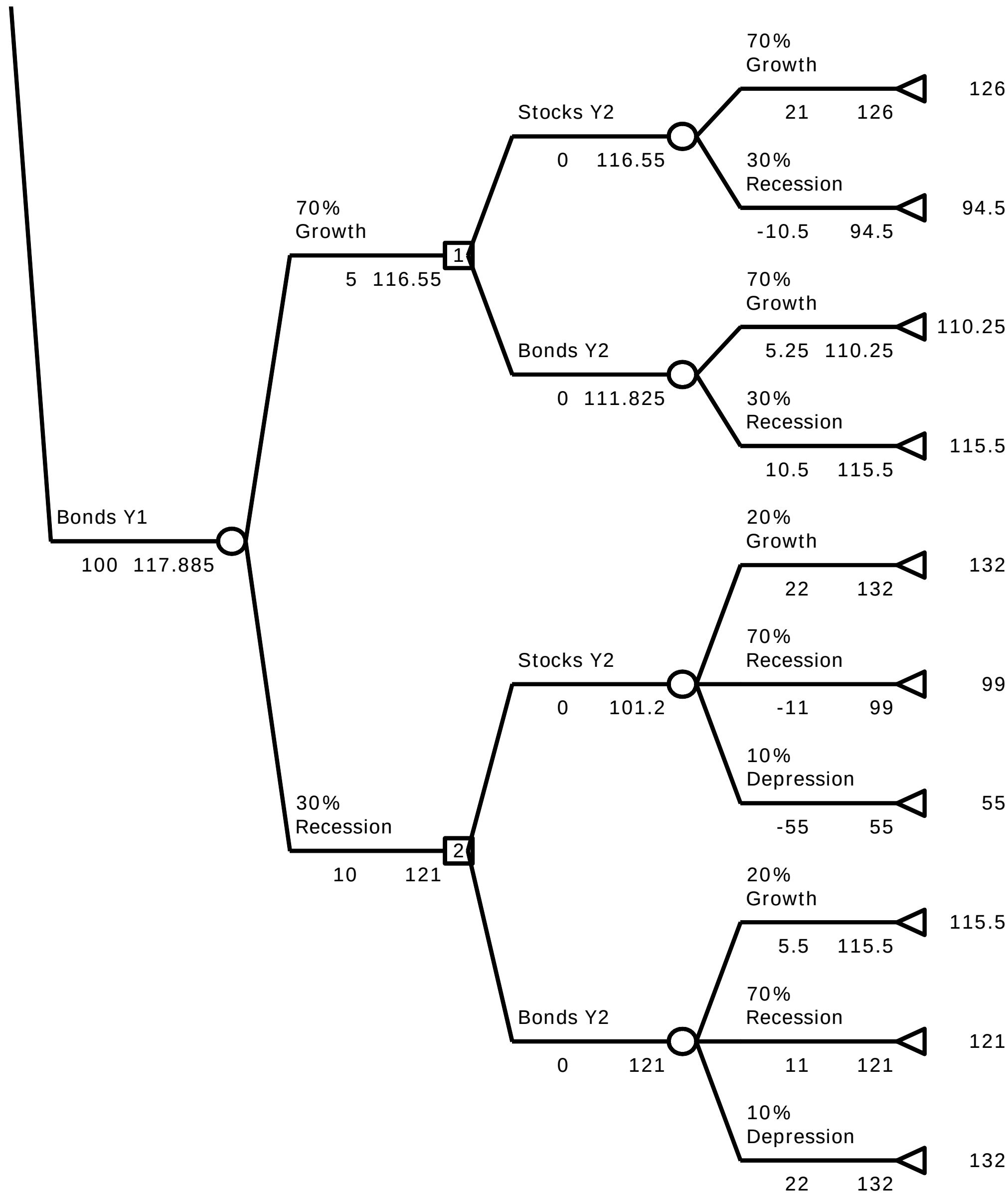
(f) Leland University should hire William. If he predicts a winning season, then they should hold the campaign and if he predicts a losing season, then they should not hold the campaign.



16.4-5.

(a)

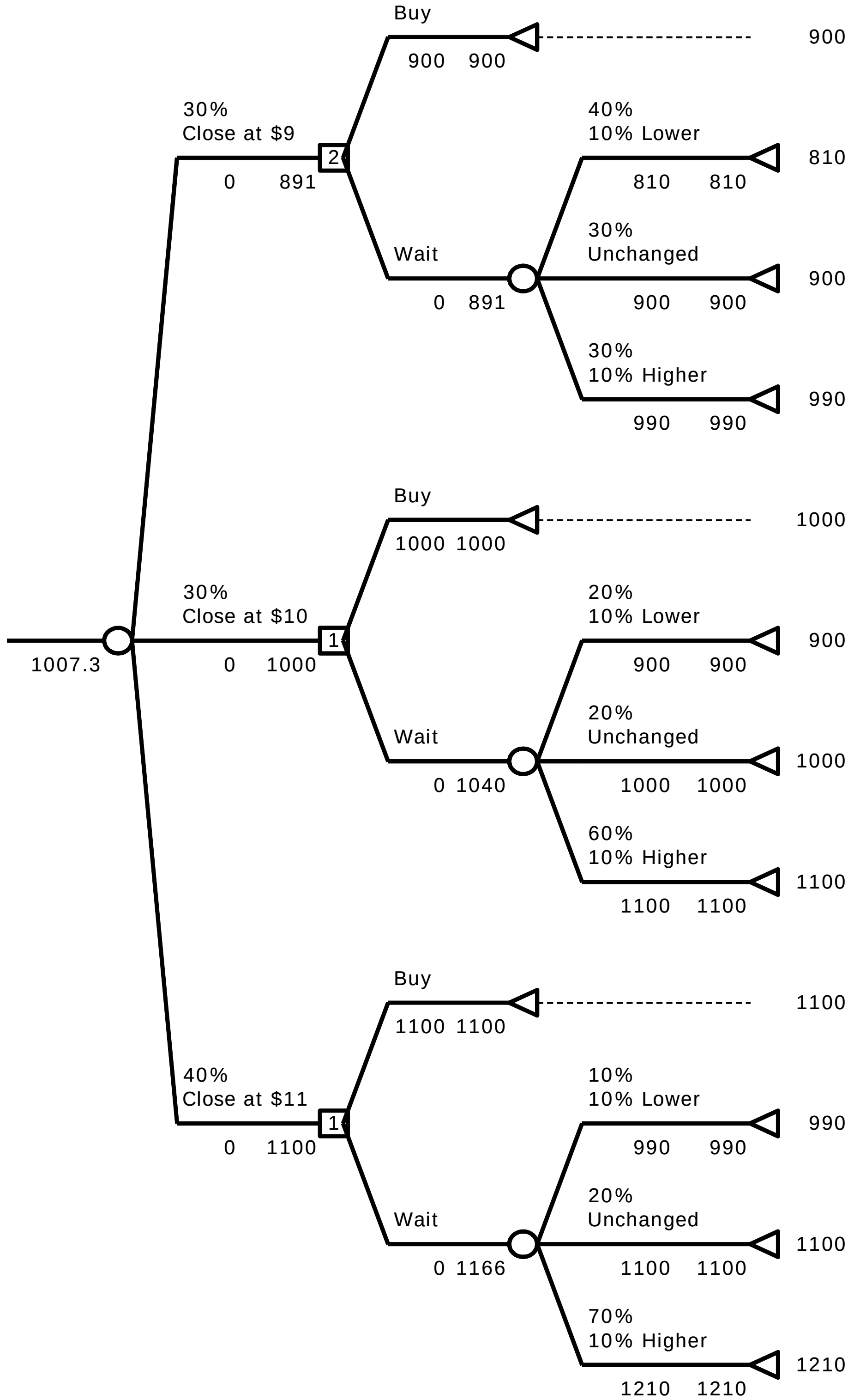

70%
Growth
24
144
144
Stocks Y2
0
133.2
30%
Recession
-12
108
108
70%
Growth
20
133.2
1
70%
Growth
6
126
126
Bonds Y2
0
127.8
30%
Recession
12
132
132
Stocks Y1
100
122.94
20%
Growth
18
108
108
70%
Recession
-9
81
81
Stocks Y2
0
82.8
10%
Depression
-45
45
45
30%
Recession
-10
99
2
20%
Growth
4.5
94.5
94.5
70%
Recession
9
99
99
Bonds Y2
0
99
10%
Depression
18
108
108
1
122.94



(b) The comptroller should invest in stocks the first year. If growth occurs in the first year, then she should invest in stocks again the second year. If recession occurs in the first year, then she should invest in bonds the second year.

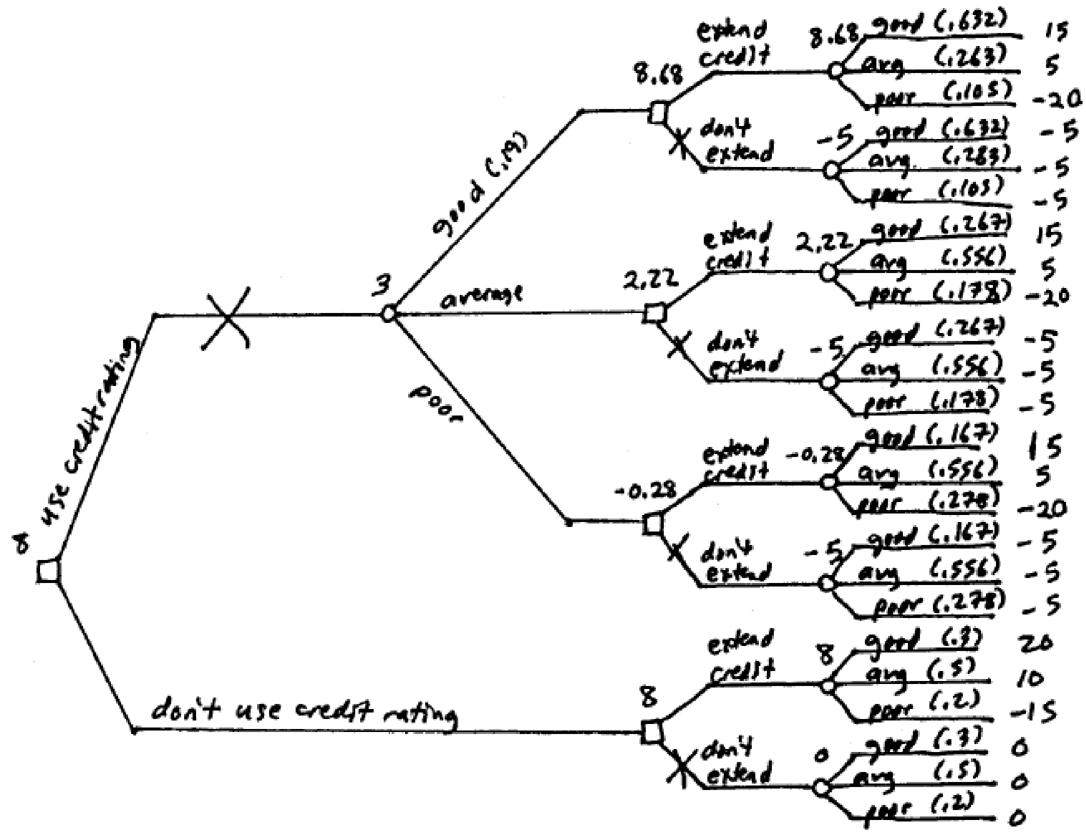
16.4-6.

The optimal policy is to wait until Wednesday to buy if the price is $9 on Tuesday. If the price is $10 or $11 on Tuesday, then buying on Tuesday is optimal.



16.4-7.

(a)



(b) Prior Distribution:

	θ_1	θ_2	θ_3
$\mathrm{P}_\theta(k)$	0.2	0.5	0.3

	$Q_{X\mid\theta=k}(x)$		
x	θ_1	θ_2	θ_3
X_1	0.5	0.4	0.2
X_2	0.4	0.5	0.4
X_3	0.1	0.1	0.4

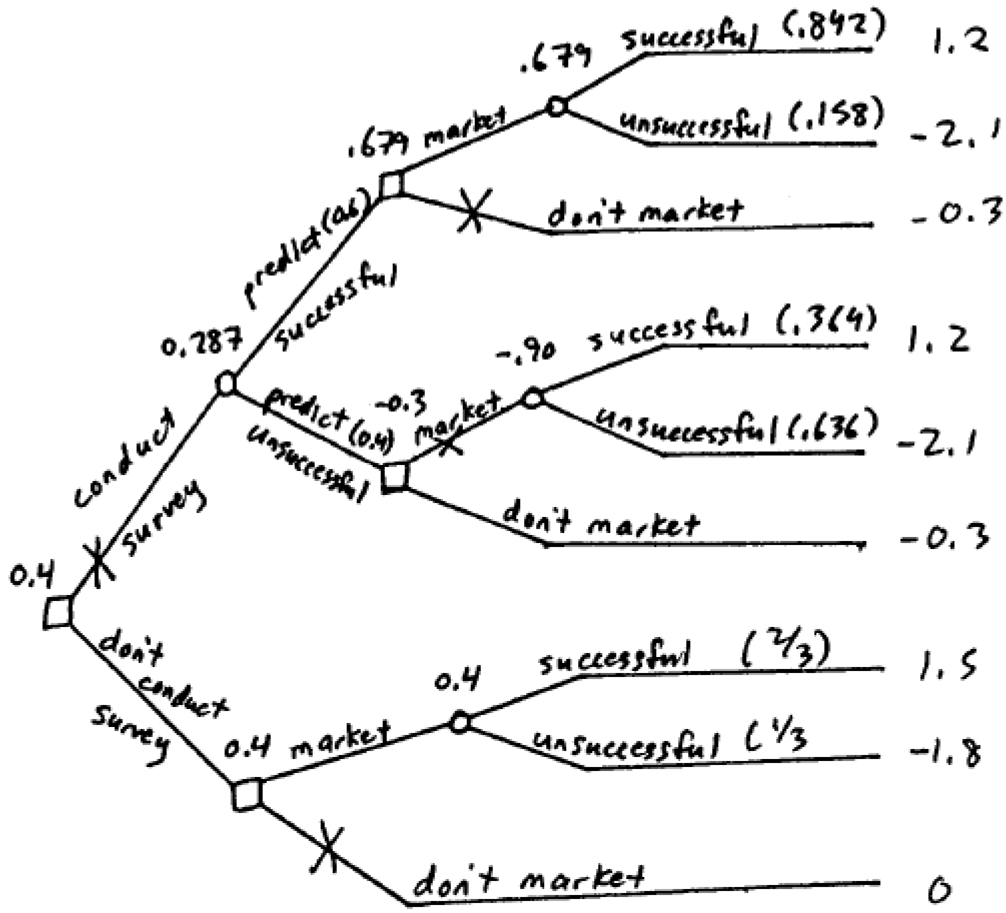
Posterior Distribution:

	$h_{\theta\mid X=x}(k)$		
x	θ_1	θ_2	θ_3
X_1	0.278	0.556	0.167
X_2	0.178	0.556	0.267
X_3	0.105	0.263	0.632

(c) It is optimal to not use credit rating, but to extend credit, see part (a).

16.4-8.

(a)



(b) Prior Distribution:

	θ_1	θ_2
$P_\theta(k)$	0.667	0.333

	$Q_{X\mid\theta=k}(x)$	
x	θ_1	θ_2
X_1	0.8	0.3
X_2	0.2	0.7

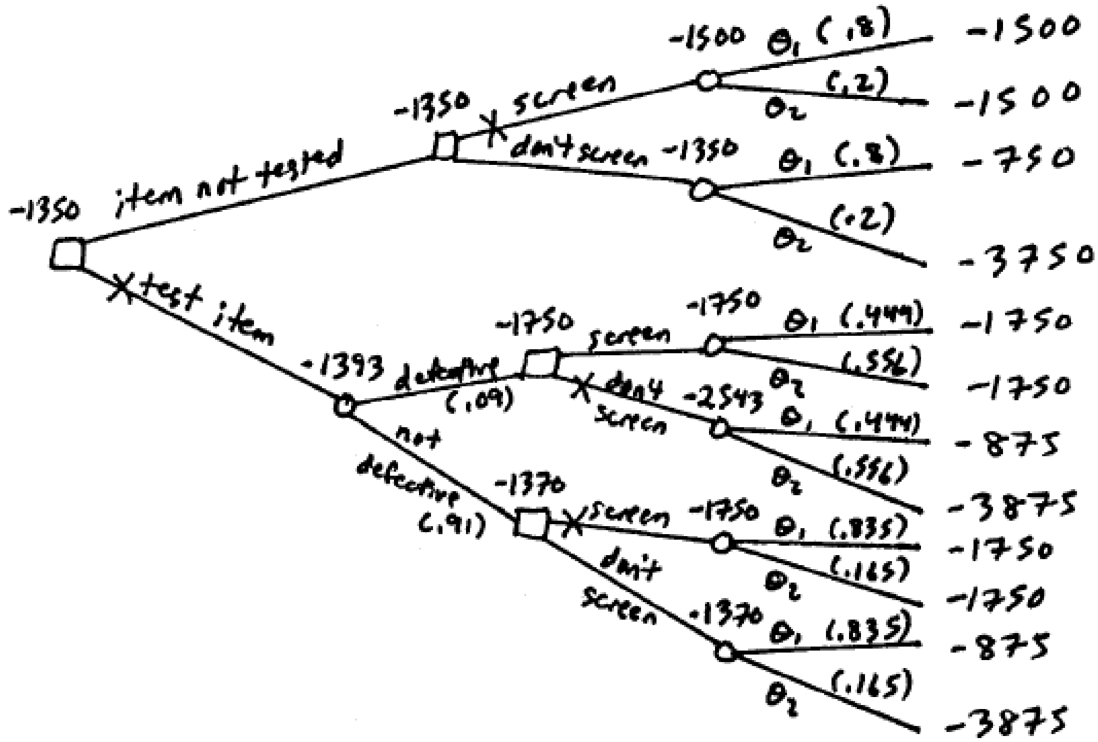
Posterior Distribution:

	$h_{\theta\mid X=x}(k)$	
x	θ_1	θ_2
X_1	0.842	0.158
X_2	0.364	0.636

(c) It is optimal to not conduct a survey, but to market the new product, see part (a).

16.4-9.

(a)



(b) Prior Distribution:

	θ_1	θ_2
$P_\theta(k)$	0.8	0.2

	$Q_{X\|\theta=k}(x)$	
x	θ_1	θ_2
X_1	0.95	0.75
X_2	0.05	0.25

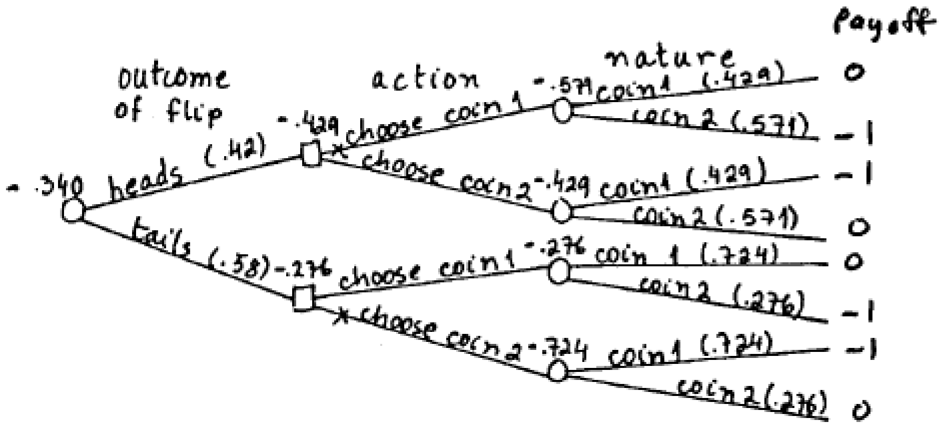
Posterior Distribution:

	$h_{\theta\|X=x}(k)$	
x	θ_1	θ_2
X_1	0.835	0.165
X_2	0.444	0.556

(c) It is optimal to not test and to not screen, see part (a).

16.4-10.

(a)



(b) Prior Distribution:

	θ_1	θ_2
$\mathrm{P}_\theta(k)$	0.6	0.4

	$Q_{X\|\theta=k}(x)$	
x	θ_1	θ_2
X_1	0.3	0.6
X_2	0.7	0.4

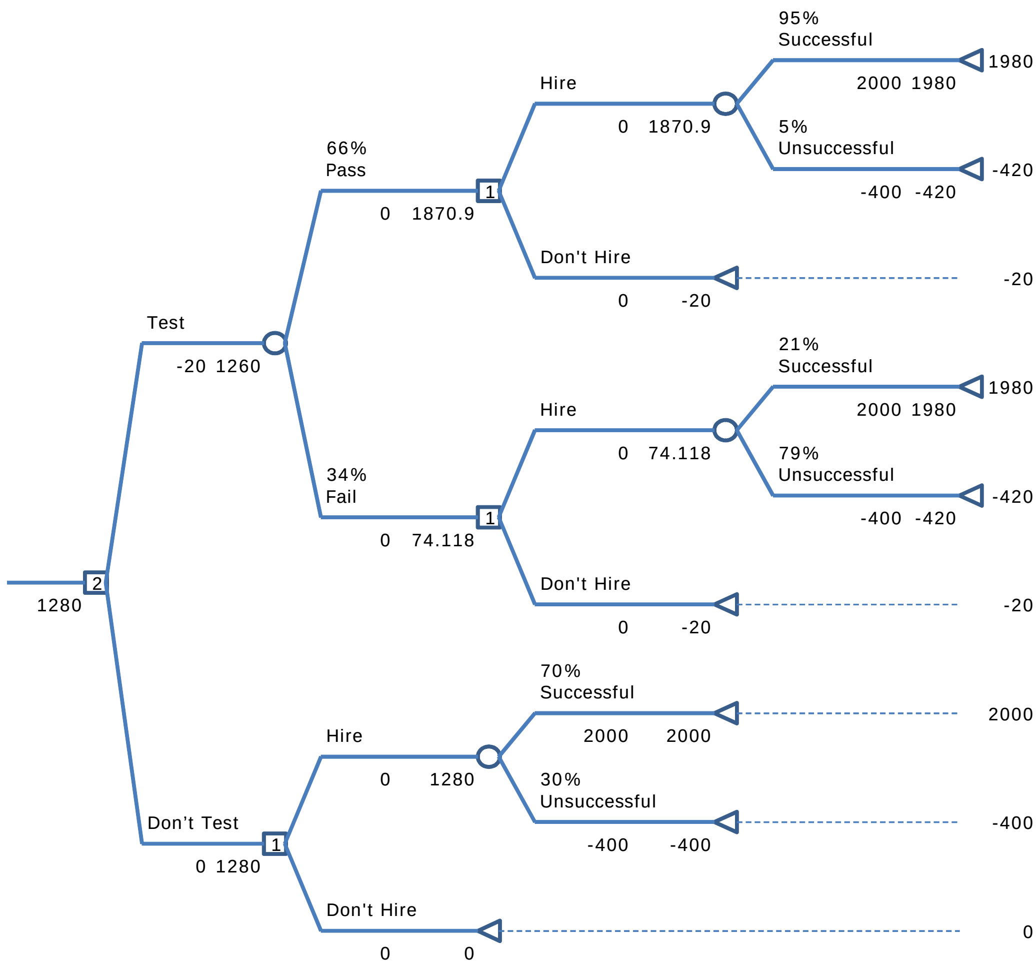
Posterior Distribution:

	$h_{\theta\|X=x}(k)$	
x	θ_1	θ_2
X_1	0.429	0.571
X_2	0.724	0.276

(c) It is optimal to choose coin 1 if the outcome is tails and coin 2 if the outcome is heads, see part (a).

16.4-11.

(a)



(b)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Pass Test	Fail Test
Successful	0.7	0.9	0.1
Not Successful	0.3	0.1	0.9

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Successful	Not Successful
Pass Test	0.6600	0.9545	0.0455
Fail Test	0.3400	0.2059	0.7941

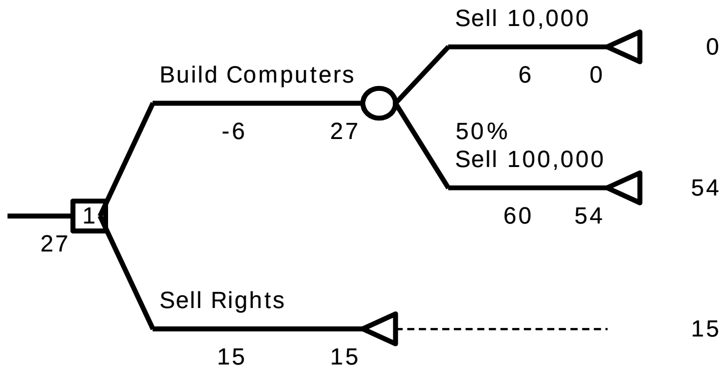
(c) The optimal policy is to not pay for testing and to hire Matthew.

(d) Even if the fee is zero, hiring Matthew without any further investigation is optimal, so Western Bank should not pay anything for the detailed report.

16.5-1.

(a)

	State of Nature	
Alternative	Sell $10,000$	Sell $100,000$
Build Computers	0	54
Sell Rights	15	15



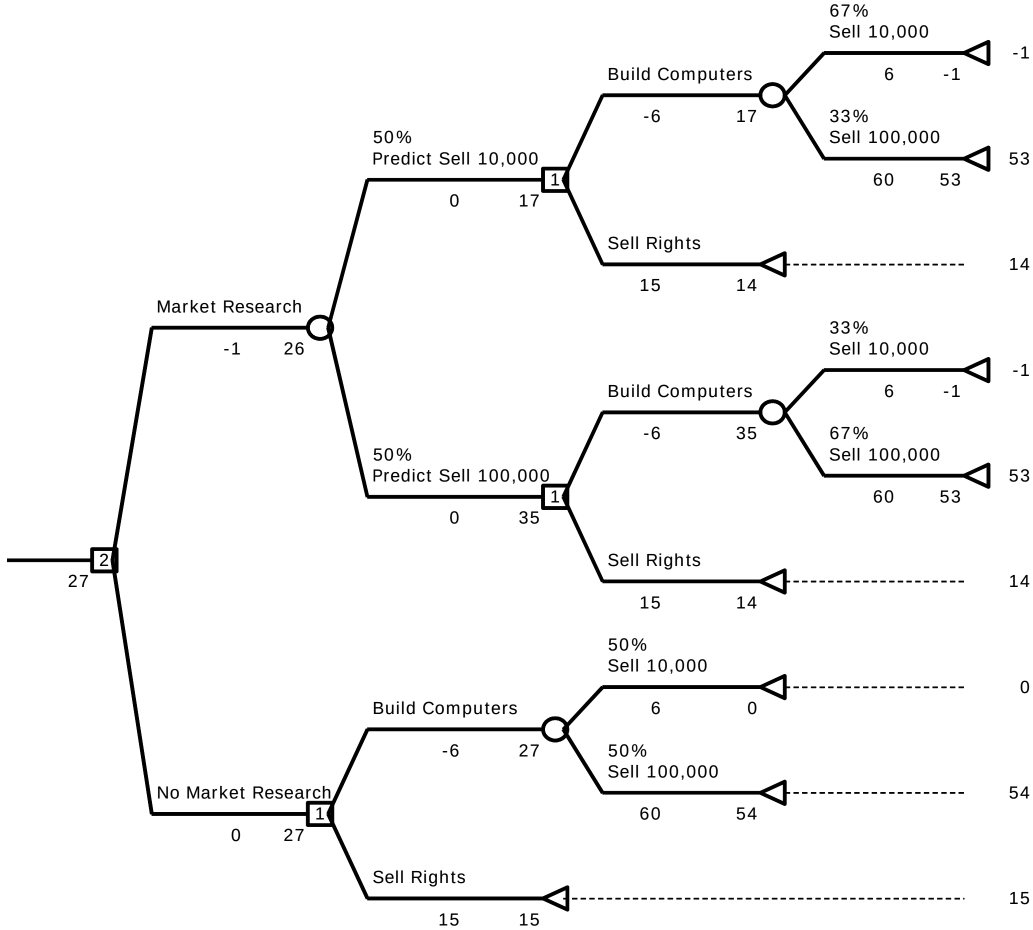
They should build computers with an expected payoff of $\$27$ million.

(b)

Prob(Sell 10,000)	Decision	Expected Payoff ($million)
	Build	27
0	Build	54
0.1	Build	48.6
0.2	Build	43.2
0.3	Build	37.8
0.4	Build	32.4
0.5	Build	27
0.6	Build	21.6
0.7	Build	16.2
0.8	Sell	15
0.9	Sell	15
1	Sell	15

16.5-2.

(a) The optimal policy is to not do market research and build the computers. The expected payoff is $27 million.



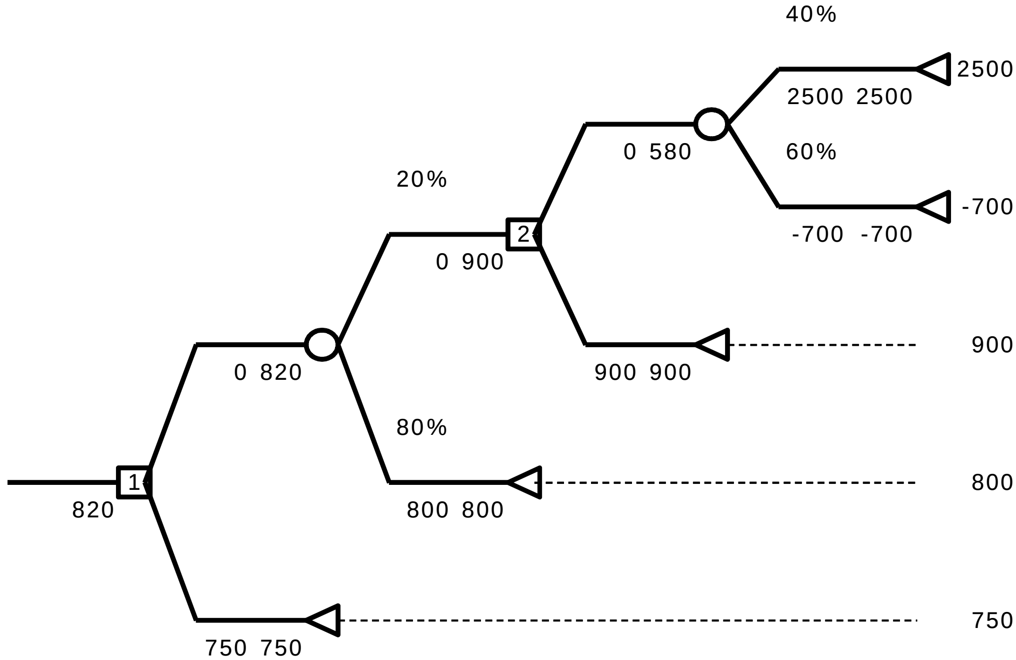
(b)

Prior(Sell 10,000)	Action 1	Action 2	Expected Payoff
	No Market Research	Build	27
0	No Market Research	Build	54
0.1	No Market Research	Build	48.6
0.2	No Market Research	Build	43.2
0.3	No Market Research	Build	37.8
0.4	No Market Research	Build	32.4
0.5	No Market Research	Build	27
0.6	No Market Research	Build	21.6
0.7	Market Research	Build if Predict Sell 100,000	18.3
0.8	Market Research	Build if Predict Sell 100,000	15.2
0.9	No Market Research	Sell	15
1	No Market Research	Sell	15

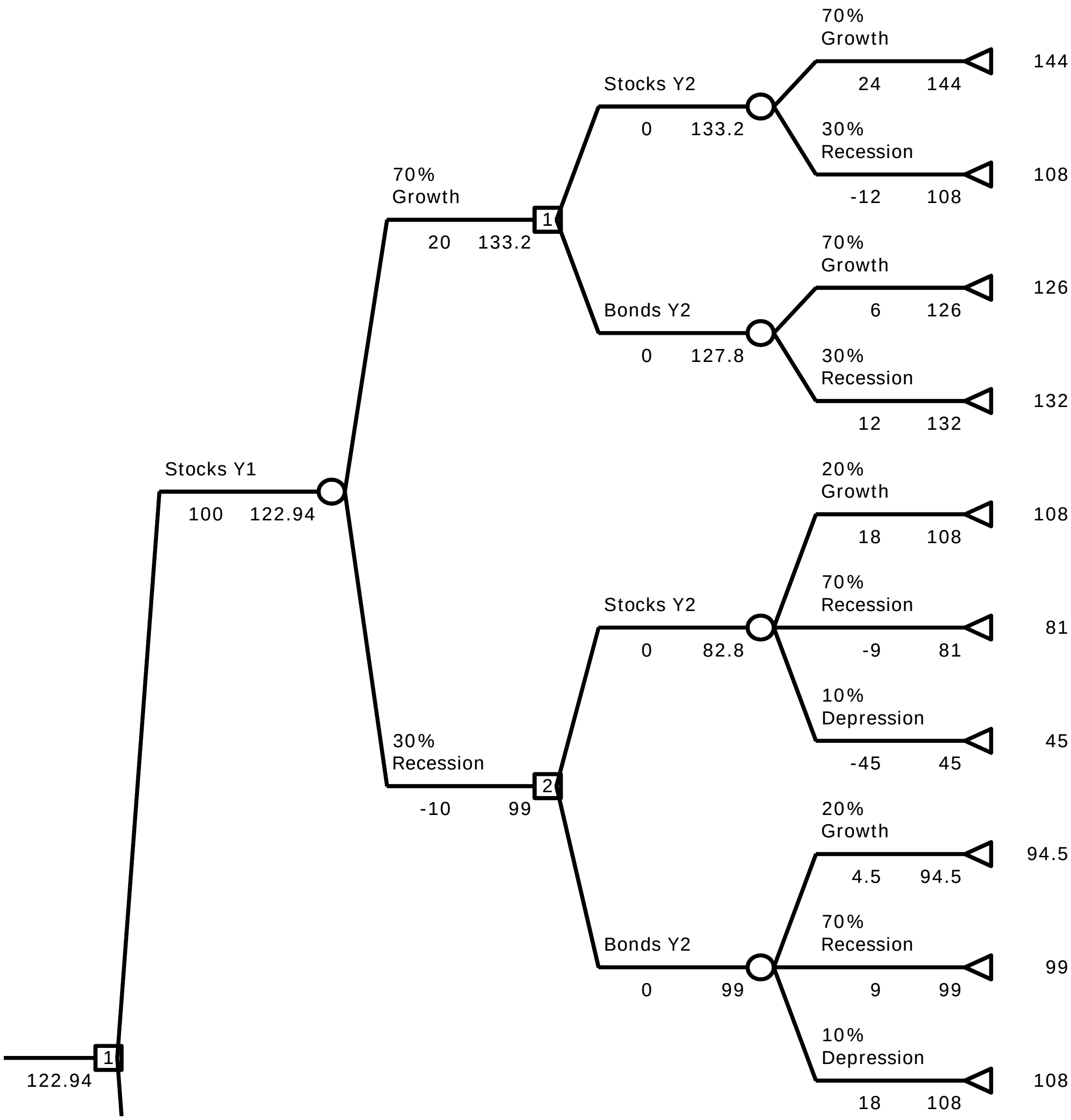
(c) If the rights can be sold for $16.5 or $13.5 million, the optimal policy is still to build the computers with an expected payoff of $27 million. If the cost of setting up the assembly line is $5.4 million or $6.6 million, the optimal policy is still to build the computers with an expected payoff of $27.6 or $26.4 million respectively. If the difference between the selling price and the variable cost of each computer is $540 or $660, the optimal policy is still to build the computers with an expected payoff of $23.7 or $33.3 million respectively. For each combination of financial data, the expected payoff is as shown in the following table. In all cases, the optimal policy is to build the computers without doing market research.

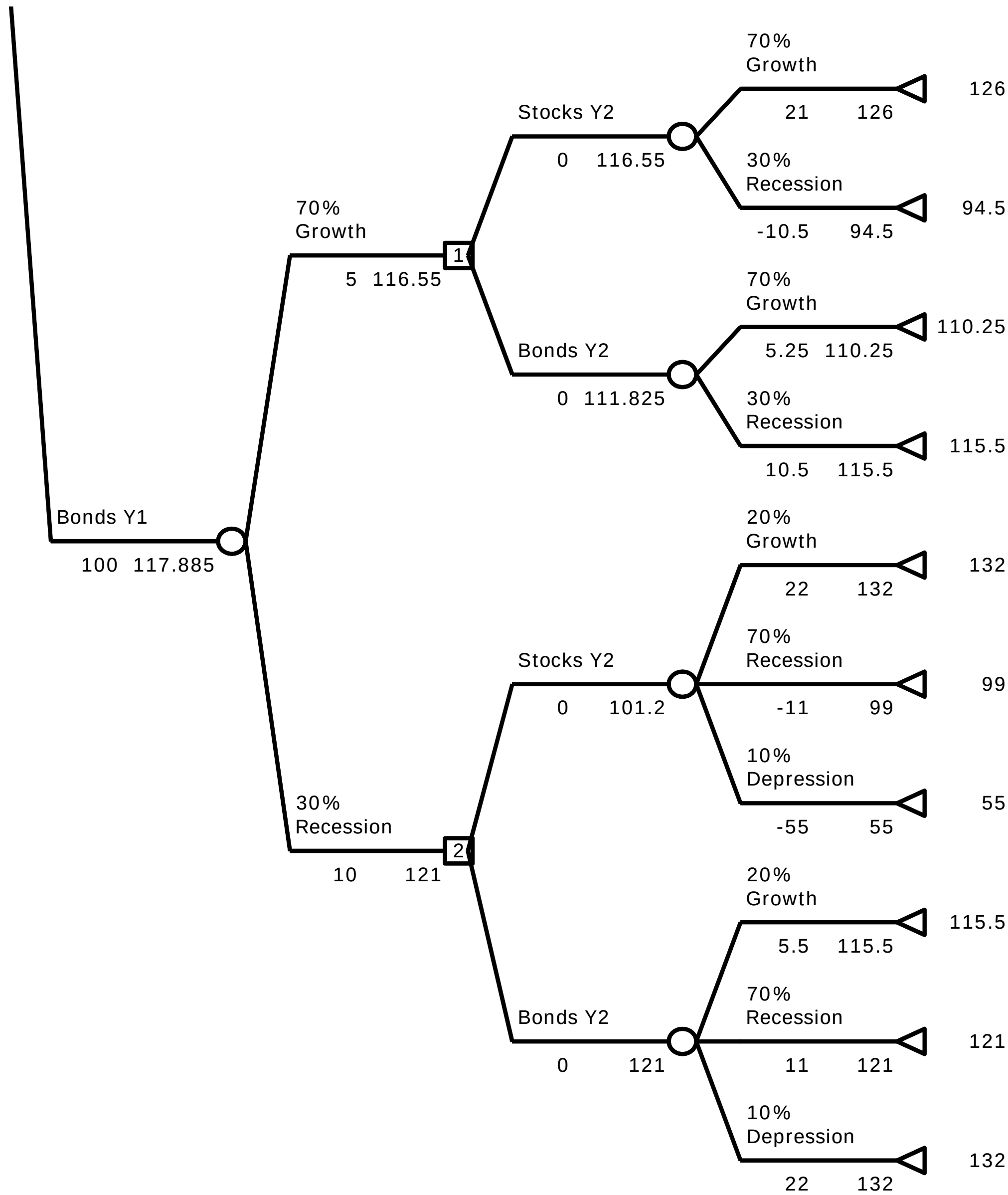
Sell Rights	Cost of Assembly Line	Selling Price – Variable Cost	Expected Payoff
$13.5 million	$5.4 million	$540	$23.4 million
$13.5 million	$5.4 million	$660	$30.9 million
$13.5 million	$6.6 million	$540	$23.1 million
$13.5 million	$6.6 million	$660	$29.7 million
$16.5 million	$5.4 million	$540	$24.3 million
$16.5 million	$5.4 million	$660	$30.9 million
$16.5 million	$6.6 million	$540	$23.1 million
$16.5 million	$6.6 million	$660	$29.7 million

16.5-3.

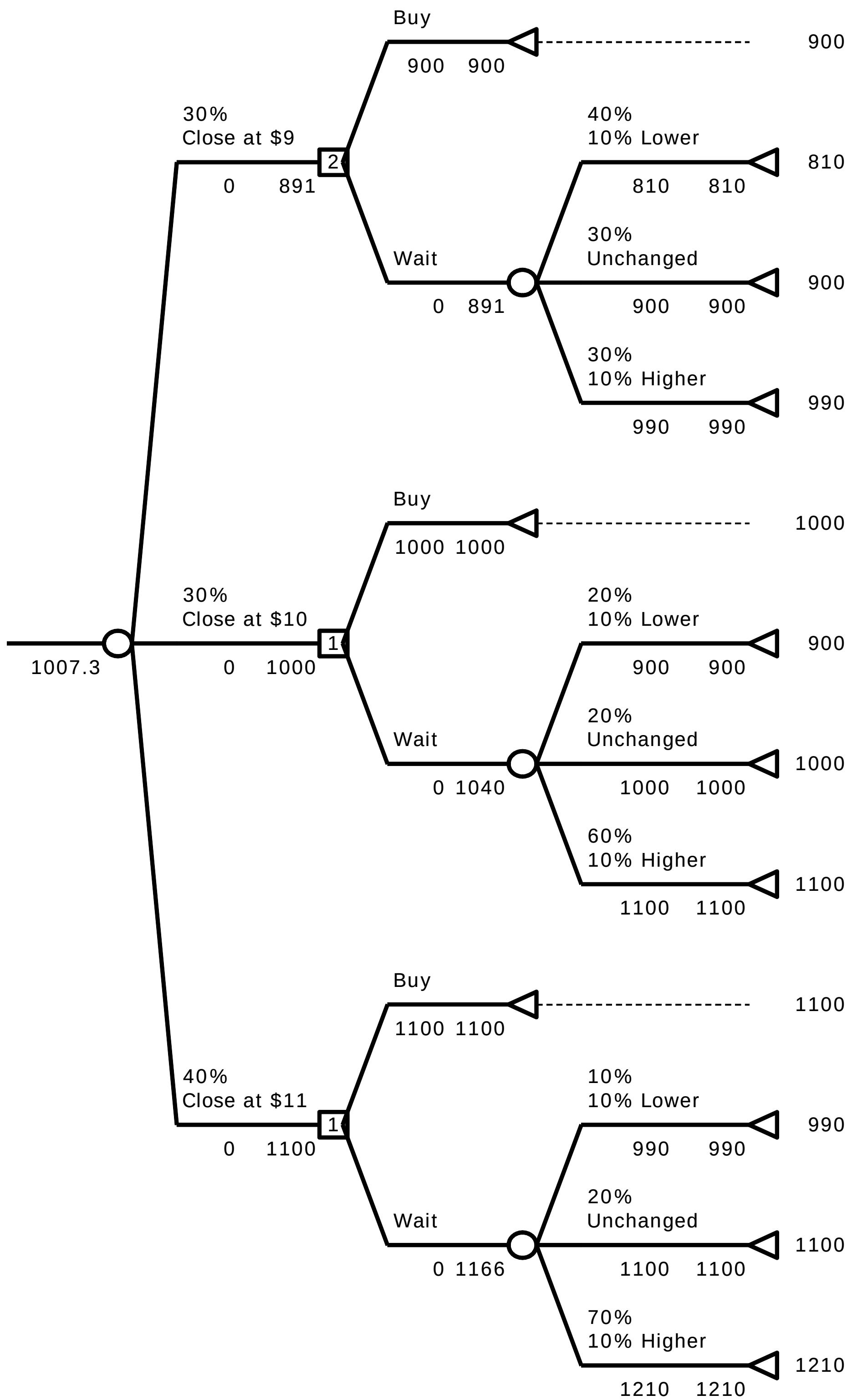


16.5-4.


70%
Growth
24
144
144
Stocks Y2
0
133.2
30%
Recession
-12
108
108
70%
Growth
20
133.2
70%
Growth
6
126
126
Bonds Y2
0
127.8
30%
Recession
12
132
132
Stocks Y1
100
122.94
20%
Growth
18
108
108
Stocks Y2
0
82.8
70%
Recession
-9
81
81
10%
Depression
-45
45
45
30%
Recession
-10
99
20%
Growth
4.5
94.5
94.5
Bonds Y2
0
99
70%
Recession
9
99
99
10%
Depression
18
108
108
122.94


Bonds Y1
100 117.885
70%
Growth
5 116.55
Stocks Y2
0 116.55
70%
Growth
21 126
126
30%
Recession
-10.5 94.5
94.5
Bonds Y2
0 111.825
70%
Growth
5.25 110.25
110.25
30%
Recession
10.5 115.5
115.5
30%
Recession
10 121
Stocks Y2
0 101.2
20%
Growth
22 132
132
70%
Recession
-11 99
99
10%
Depression
-55 55
55
Bonds Y2
0 121
20%
Growth
5.5 115.5
115.5
70%
Recession
11 121
121
10%
Depression
22 132
132

16.5-5.

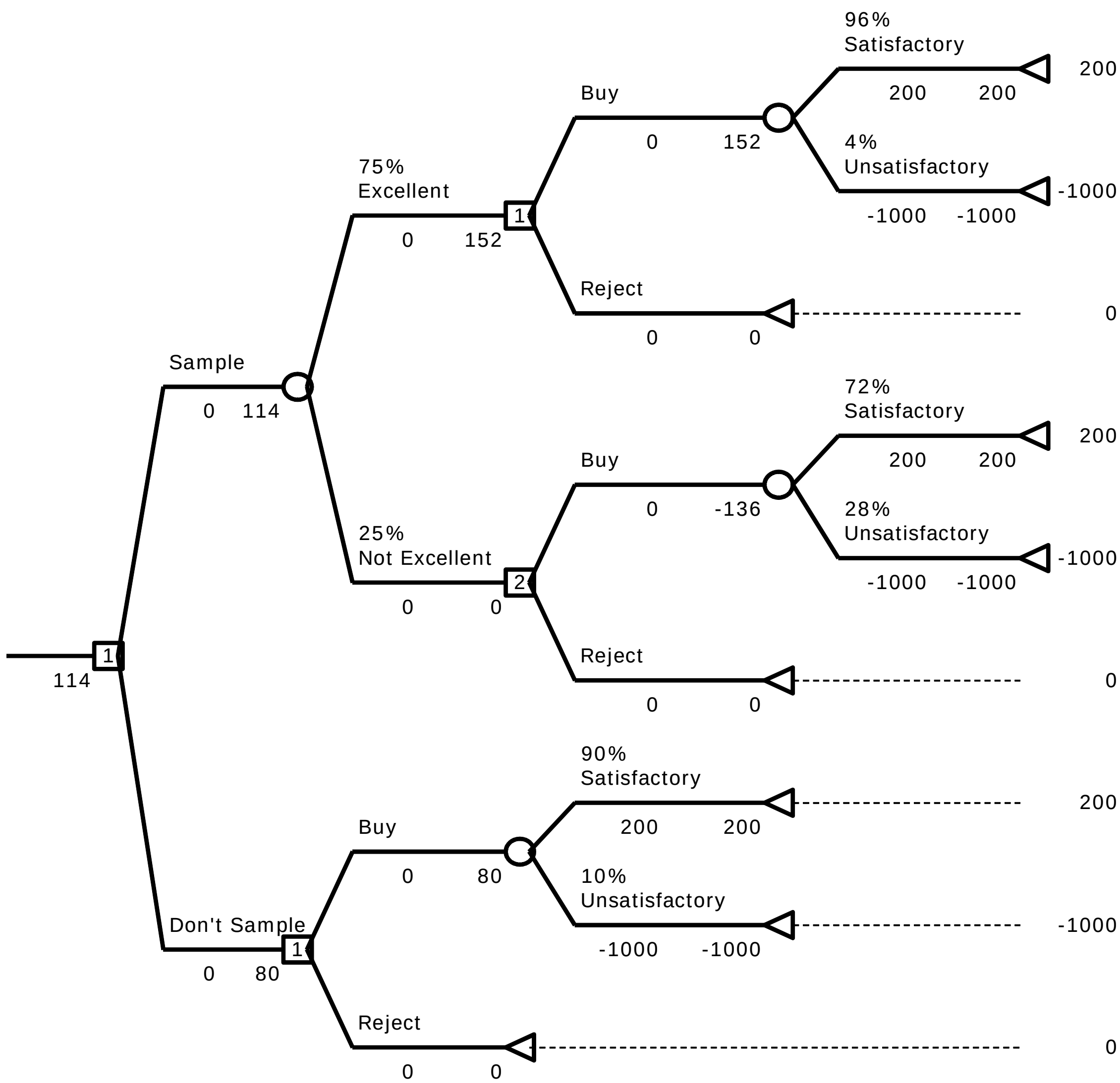


The optimal policy is to wait until Wednesday to buy if the price is $9 on Tuesday. If the price is $10 or $11 on Tuesday, then buy on Tuesday.

16.5-6.

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Excellent	Not Excellent
Satisfactory Box	0.9	0.8	0.2
Unsatisfactory Box	0.1	0.3	0.7

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Satisfactory Box	Unsatisfactory Box
Excellent	0.75	0.960	0.040
Not Excellent	0.25	0.720	0.280



The optimal policy is to sample the fruit and buy if it is excellent and reject if it is unsatisfactory.

16.5-7.

(a) Choose to introduce the new product with expected payoff of $12.5 million.

	State of Nature		Exp.
Alternative	Successful	Unsuccessful	Payoff
Introduce New Product	$40 million	$-$15 million	$12.5 million
Don't Introduce New Product	0	0	0
Prior Probabilities	0.5	0.5	

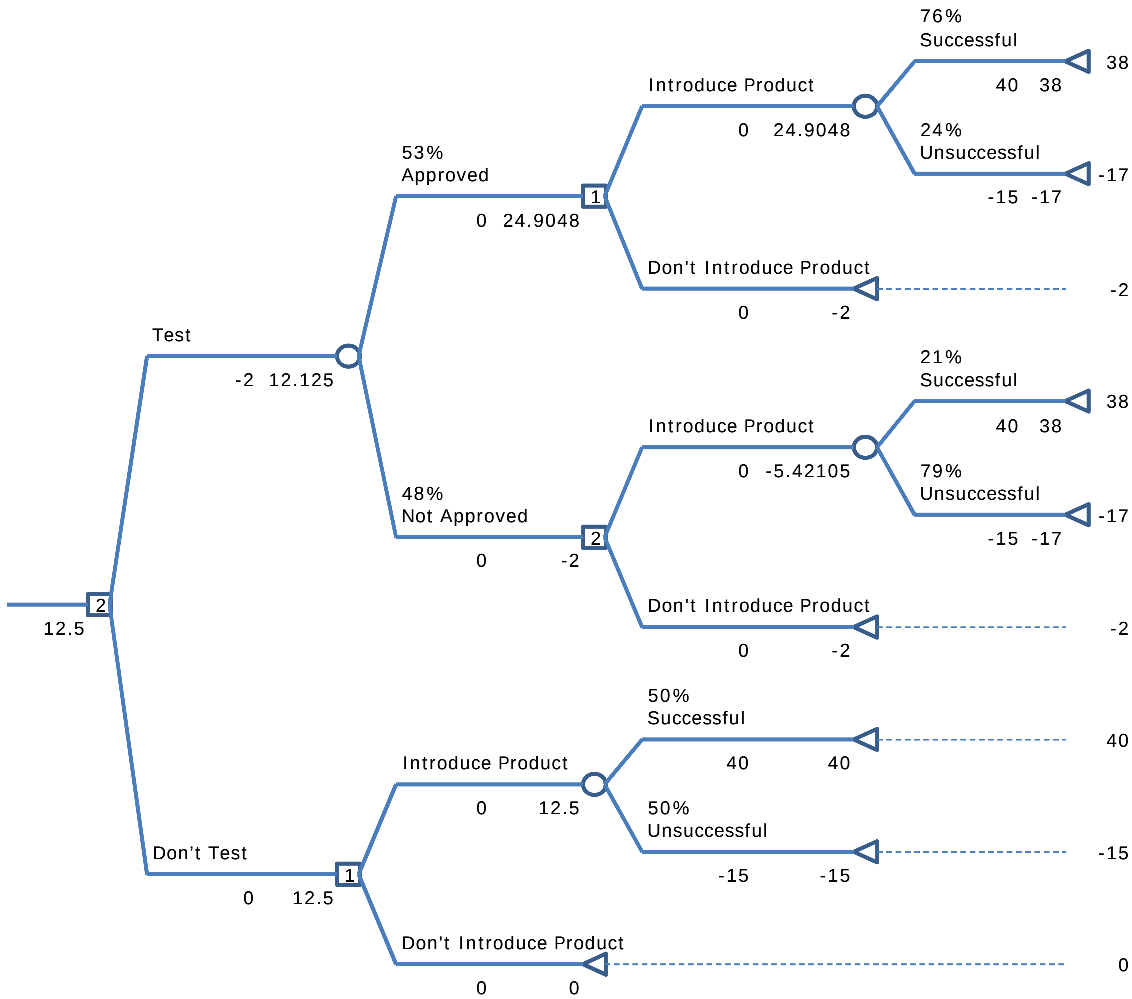
(b) With perfect information, Morton Ward should introduce the product if it will be successful and not introduce it if it will not be successful.

Expected Payoff with Perfect Information: $0.5(40) + 0.5(0) = 20$
Expected Payoff without Information: 12.5
EVPI $= 20 - 12.5 =$ $7.5 million

(c) The optimal policy is to not test but to introduce the new product, with expected payoff $12.5 million.

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Approved	Not Approved
Successful	0.5	0.8	0.2
Unsuccessful	0.5	0.25	0.75

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Successful	Unsuccessful
Approved	0.525	0.762	0.238
Not Approved	0.475	0.211	0.789



(d)

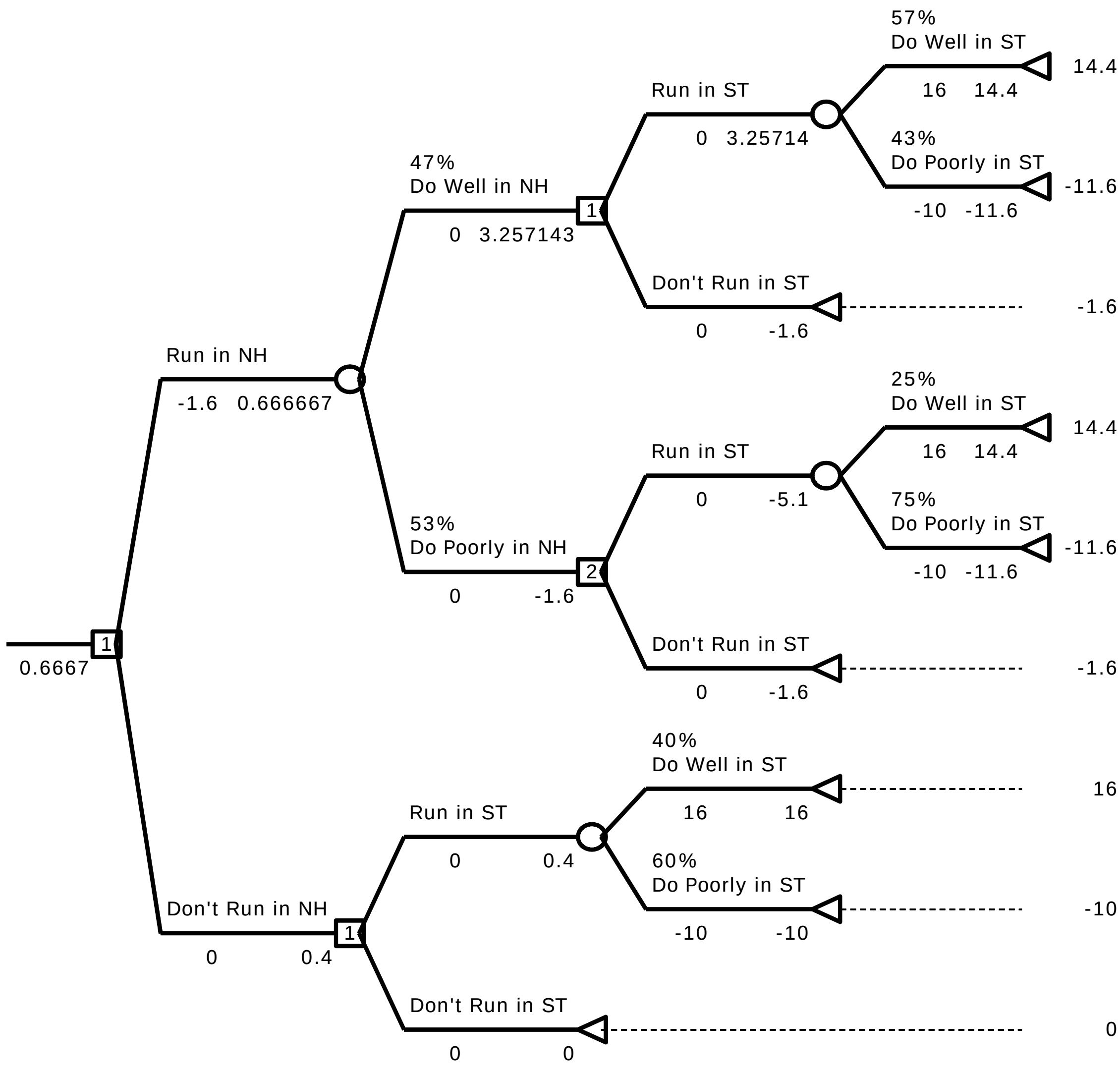
Prior(Successful)	Action 1	Action 2	Expected Payoff
	Don't Test	Introduce	12.5
0	Don't Test	Don't Introduce	0
0.1	Don't Test	Don't Introduce	0
0.2	Test	Introduce if Approved	1.4
0.3	Test	Introduce if Approved	4.975
0.4	Test	Introduce if Approved	8.55
0.5	Don't Test	Introduce	12.5
0.6	Don't Test	Introduce	18
0.7	Don't Test	Introduce	23.5
0.8	Don't Test	Introduce	29
0.9	Don't Test	Introduce	34.5
1	Don't Test	Introduce	40

(e) If the net profit if successful is only $30 million, then the optimal policy is to test and to introduce the product only if the test market approves. The expected payoff is $8.125 million. If the net profit if successful is $50 million, then the optimal policy is to skip the test and to introduce the product, with an expected payoff of $17.5 million. If the net loss if unsuccessful is only $11.25 million, then the optimal policy is to skip the test and to introduce the product, with an expected payoff of $14.375 million. If the net loss if unsuccessful is $18.75 million, then the optimal policy is to conduct the test and to introduce the product only if the test market approves. The expected payoff is $11.656 million. For each combination of financial data, the expected payoff and the optimal policy are as shown below.

Successful	Unsuccessful	Optimal Policy	Expected Profit
$30 million	-$11.25 million	Skip Test, Introduce Product	$9.375 million
$30 million	-$18.75 million	Test, Introduce Product if Approved	$7.656 million
$50 million	-$11.25 million	Skip Test, Introduce Product	$19.375 million
$50 million	-$18.75 million	Test, Introduce if Approved	$15.656 million

16.5-8.

(a) Chelsea should run in the NH primary. If she does well, then she should run in the ST primaries. If she does poorly in the NH primary, then should not run the ST primaries. The expected payoff is $666, 667.



(b)

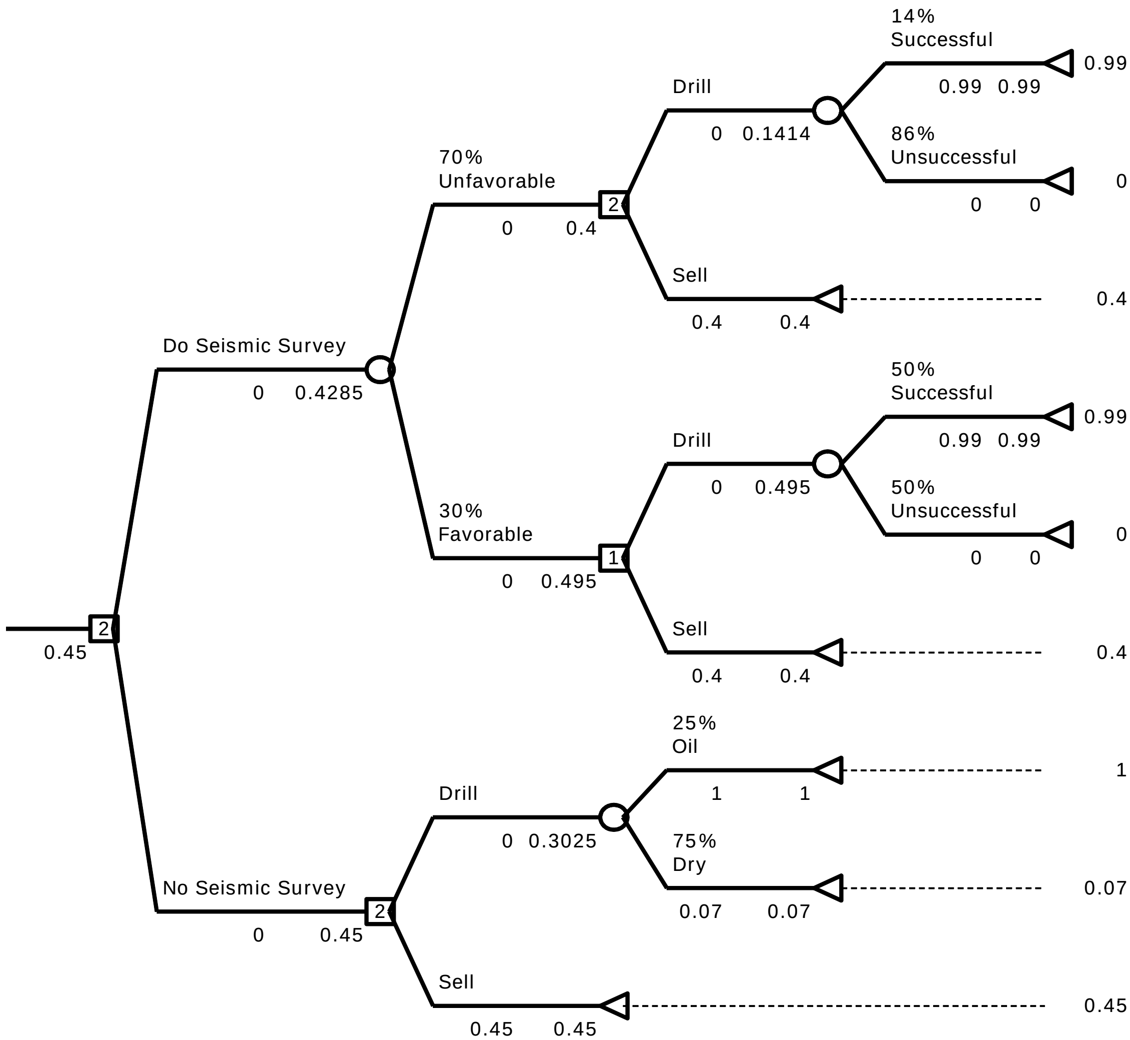
Prior(Well in NH)	Action 1	Action 2	Expected Payoff
	Run in NH	Run in ST if do well in NH	0.6667
0.0000	Don't Run in NH	Run in ST	0.4000
0.0667	Don't Run in NH	Run in ST	0.4000
0.1333	Don't Run in NH	Run in ST	0.4000
0.2000	Don't Run in NH	Run in ST	0.4000
0.2667	Don't Run in NH	Run in ST	0.4000
0.3333	Don't Run in NH	Run in ST	0.4000
0.4000	Don't Run in NH	Run in ST	0.4000
0.4667	Run in NH	Run in ST if do well in NH	0.6667
0.5333	Run in NH	Run in ST if do well in NH	0.9905
0.6000	Run in NH	Run in ST if do well in NH	1.3143
0.6667	Run in NH	Run in ST if do well in NH	1.6381
0.7333	Run in NH	Run in ST if do well in NH	1.9619
0.8000	Run in NH	Run in ST if do well in NH	2.2857
0.8667	Run in NH	Run in ST if do well in NH	2.6095
0.9333	Run in NH	Run in ST if do well in NH	2.9333
1.0000	Run in NH	Run in ST if do well in NH	3.2571

(c) If the payoff for doing well in ST is only $\$12$ million, Chelsea should not run in either NH or ST, with expected payoff of $\$0$. If the payoff for doing well in ST is $\$20$ million, Chelsea should not run in NH, but run in ST, with expected payoff of $\$2$ million. If the loss for doing poorly in ST is $\$7.5$ million, Chelsea should not run in NH, but run in ST, with expected payoff of $\$1.9$ million. If the loss for doing poorly in ST is only $\$12.5$ million, Chelsea should run in NH and run in ST if she does well in NH, with expected payoff of $\$166,667$. For each combination of financial data, the expected payoff and the optimal policy is as shown below.

Well in ST	Poorly in ST	Optimal Policy	Expected Funds
$12 million	-$7.5 million	Run in ST Only	$300,000
$12 million	-$12.5 million	Don't Run in Either	$0
$20 million	-$7.5 million	Run in ST Only	$3.5 million
$20 million	-$12.5 million	Run in NH, Run in ST if Well	$1.233 million

16.6-1.

(a) - (b) The optimal policy is to not conduct a survey and to sell the land.



16.6-2.

(a) Choose to not buy insurance with expected payoff $\$249,840$.

	State of Nature		Exp.
Alternative	Earthquake	No Earthquake	Payoff
Buy Insurance	$249,820$	$249,820$	$249,820$
Not Buy Insurance	$90,000$	$250,000$	$249,840$
Prior Probability	0.001	0.999	

(b) $U(\text{insurance}) = U(250,000 - 180) = \sqrt{249,820} = 499.82$

$U(\text{no insurance}) = 0.999U(250,000) + 0.001U(90,000) = 499.8$

The optimal policy is to buy insurance.

16.6-3.

Expected utility of $19,000: $U(19) = \sqrt{25} = 5$

Expected utility of investment: $0.3U(10) + 0.7U(30) = 0.3\sqrt{16} + 0.7\sqrt{36} = 5.4$

Choose the investment to maximize expected utility.

16.6-4.

Expected utility of $\mathrm{A}_1 =$ Expected utility of A_2

$$pU(10) + (1-p)U(30) = U(19)$$

$$0.3U(10) + 0.7(20) = 16.7 \Rightarrow U(10) = 9$$

16.6-5.

(a) Expected utility of $\mathrm{A}_1 =$ Expected utility of A_2

$$pU(10) + (1-p)U(0) = U(1)$$

$$0.125U(10) + 0.875(0) = 1 \Rightarrow U(10) = 8$$

(b) Expected utility of $\mathrm{A}_1 =$ Expected utility of A_2

$$pU(10) + (1-p)U(0) = U(5)$$

$$0.5625(8) + 0.4375(0) = U(5) \Rightarrow U(5) = 4.5$$

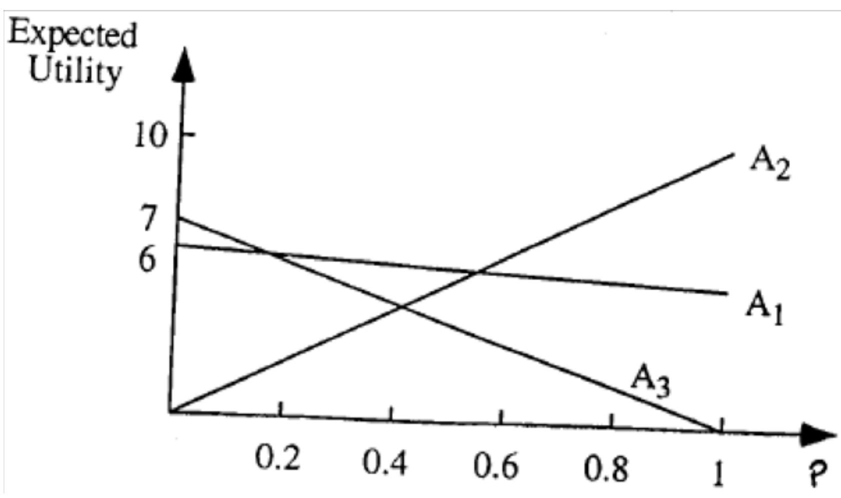
(c) Answers will vary.

16.6-6.

(a) Expected utility of $\mathrm{A}_1 = pU(25) + (1-p)U(36) = 5p + 6(1-p) = 6 - p$

Expected utility of $\mathrm{A}_2 = pU(100) + (1-p)U(0) = 10p + 0 = 10p$

Expected utility of $\mathrm{A}_3 = pU(0) + (1-p)U(49) = 0 + 7(1-p) = 7 - 7p$



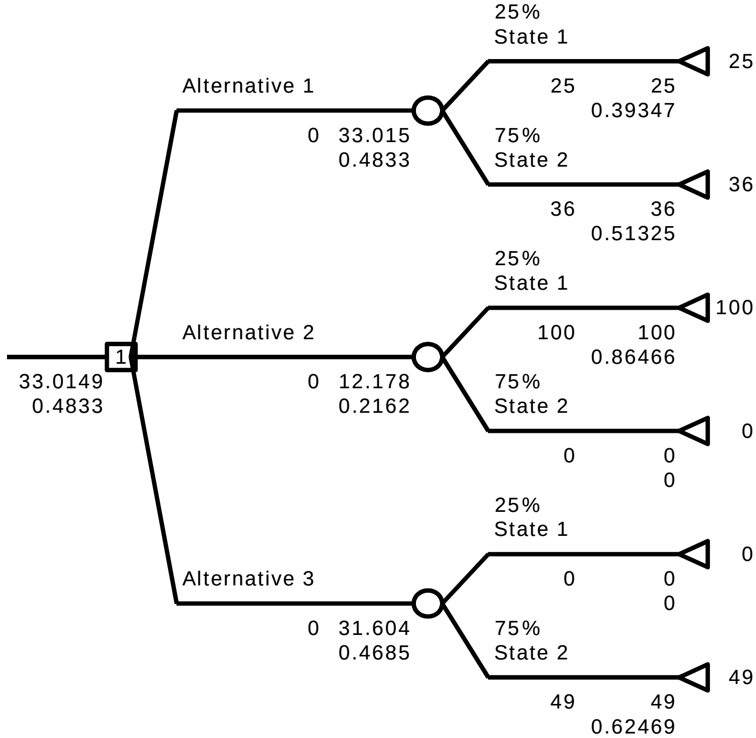
A_1 and A_3 cross when $6 - p = 7 - 7p \Rightarrow p = \frac{1}{6}$.

A_1 and A_2 cross when $6 - p = 10p \Rightarrow p = \frac{2}{3}$.

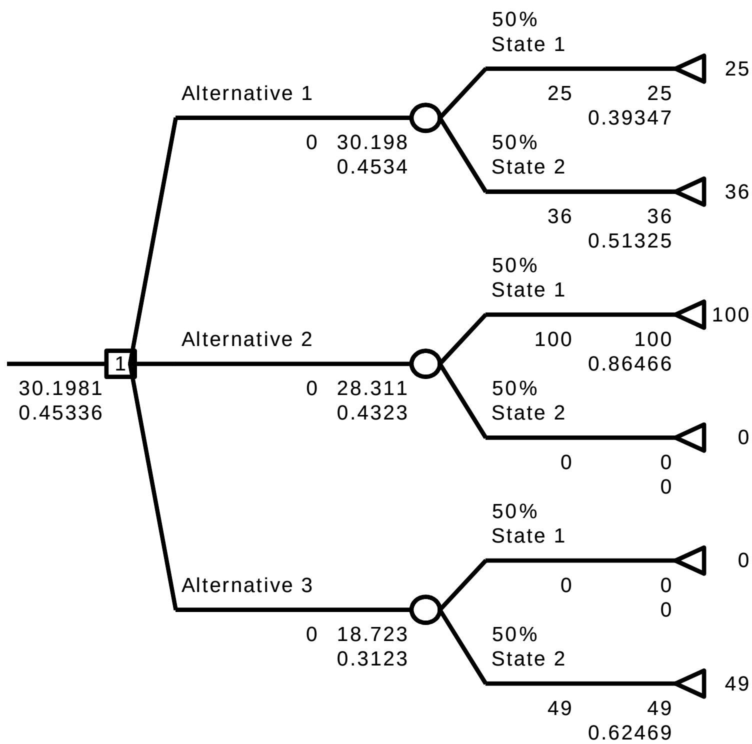
Thus, A_3 is best when $p \leq \frac{1}{6}$, A_1 is best when $\frac{1}{6} \leq p \leq \frac{2}{3}$, and A_2 is best when $p \geq \frac{2}{3}$.

(b) $U(M) = 50(1 - e^{-M/50})$

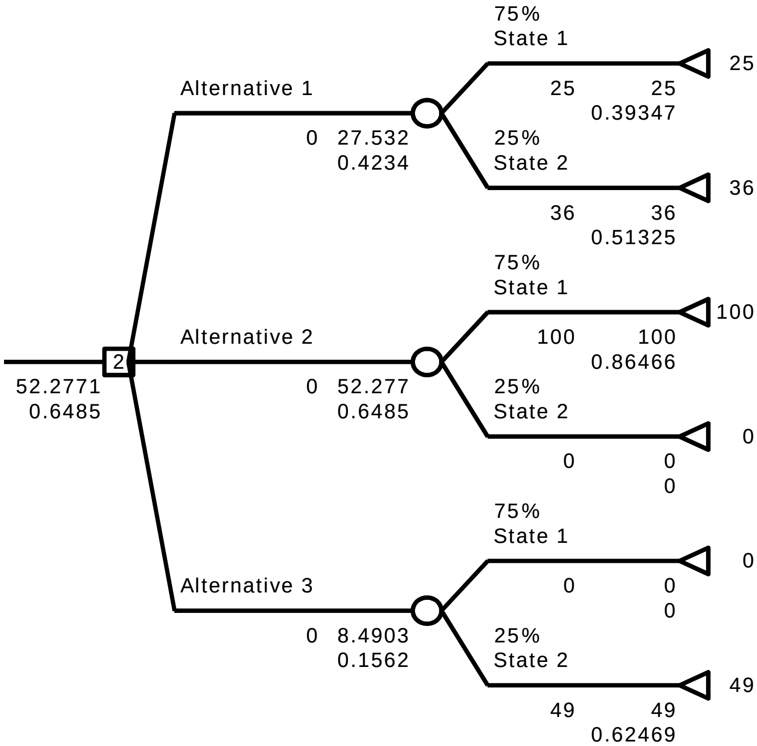
$p = 25\%$, alternative 1 is optimal



$p = 50\%$, alternative 1 is optimal

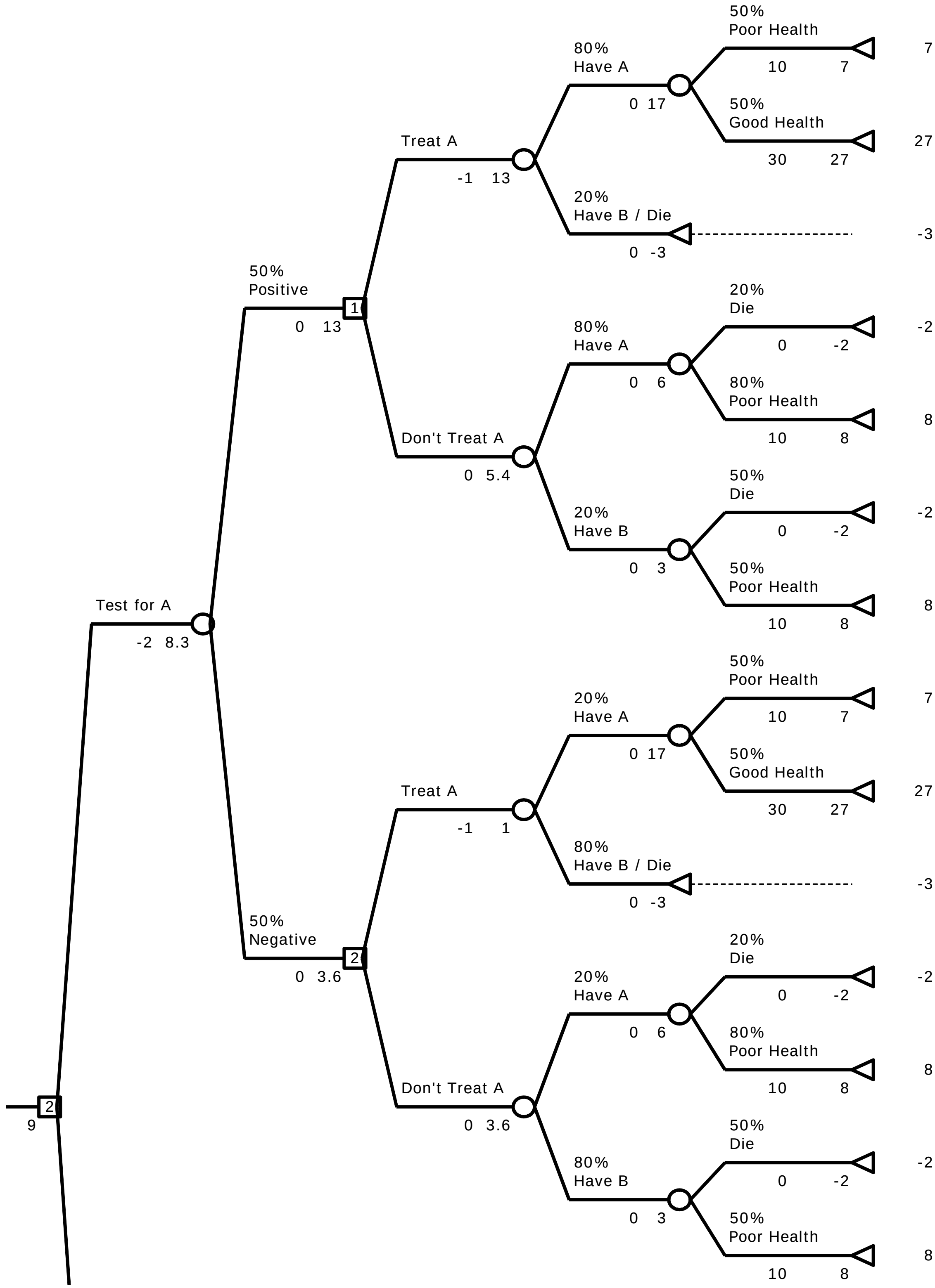


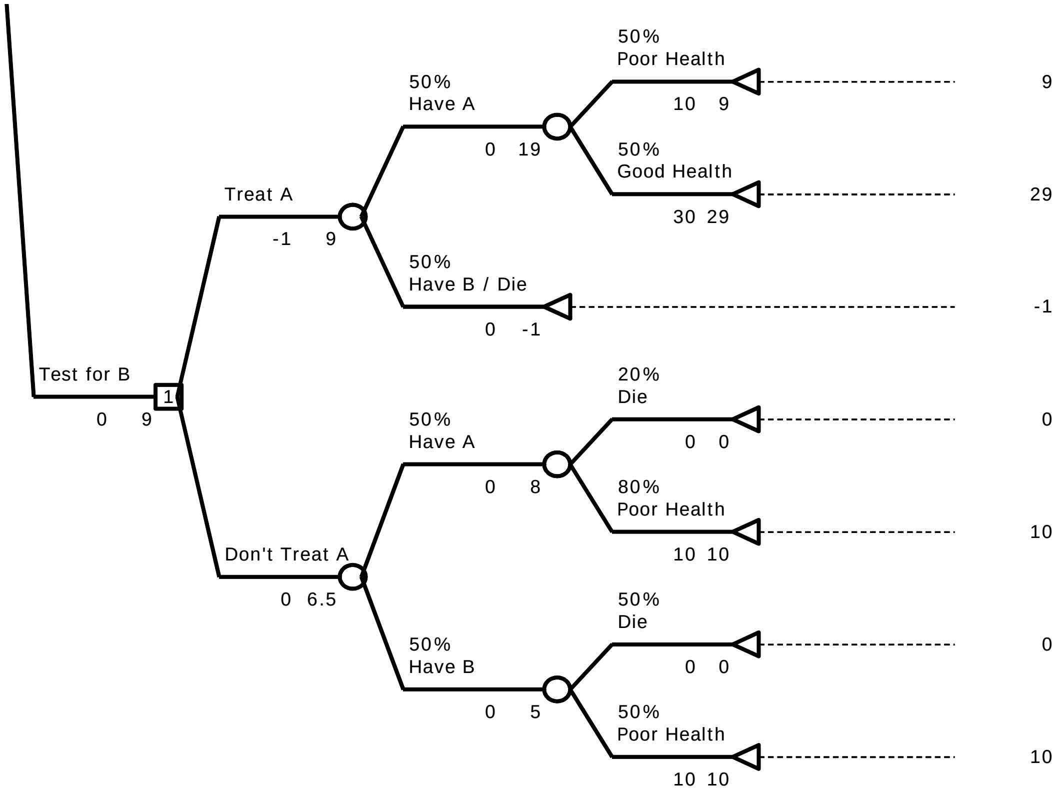
$p = 75\%$, alternative 2 is optimal



16.6-7.

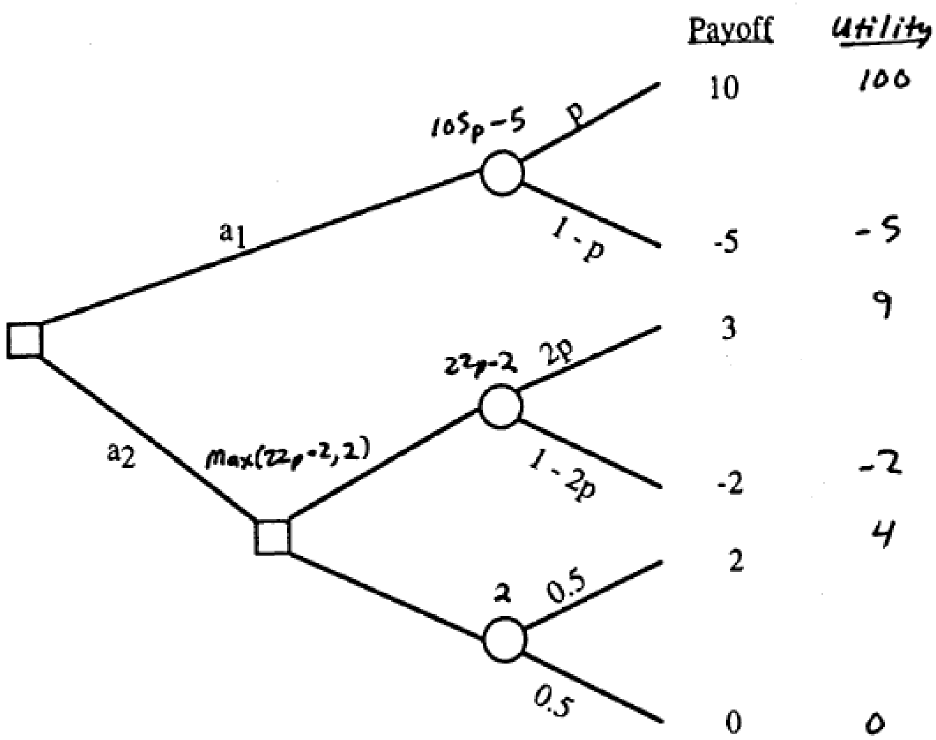
The optimal policy is to not test for disease A, but to treat disease A.





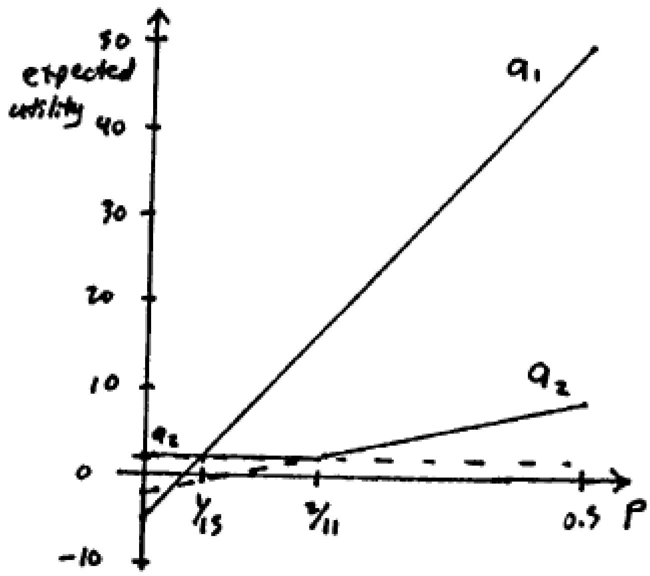
16.6-8.

(a)



At $p = 0.25$, $105p - 5 = 21.25$ and max $(22p - 2, 2) =$ max $(3.5, 2) = 3.5$, so A_1 is optimal.

(b)



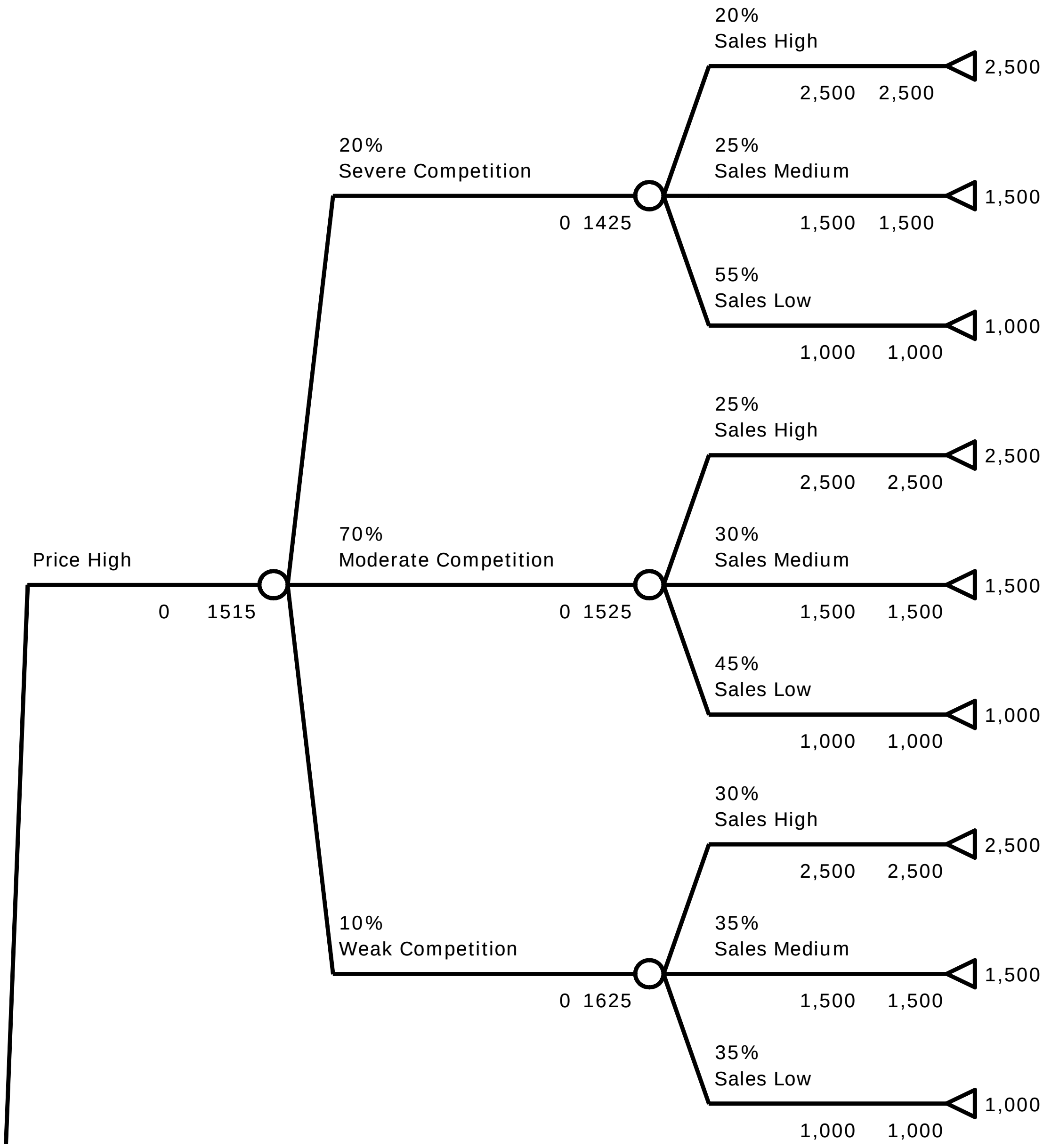
As can be seen on the graph, A_1 stays optimal for $1/15 \le p \le 0.5$.

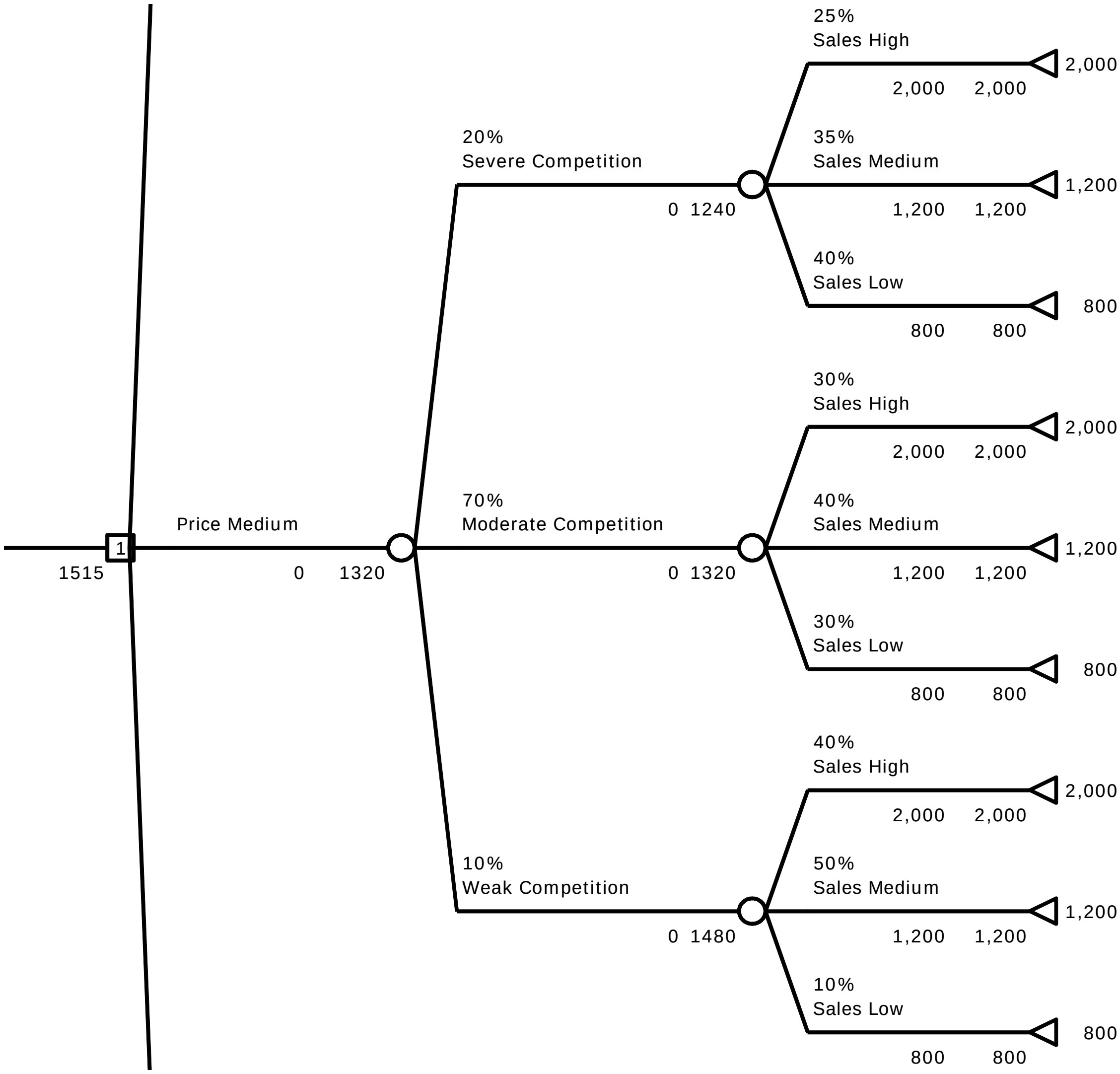
CASE 16.1 Brainy Business

(a) The decision alternatives are to price the product high ($50), medium ($40), or low ($30), or don't market the product at all. The possible states of nature are the demand could be high (50,000), medium (30,000), or low (20,000), which in turn depends upon the price, and whether the competition is severe, moderate, or weak. The various data are summarized in the spreadsheet below. The payoff table can be generated based on the results in the far right column.

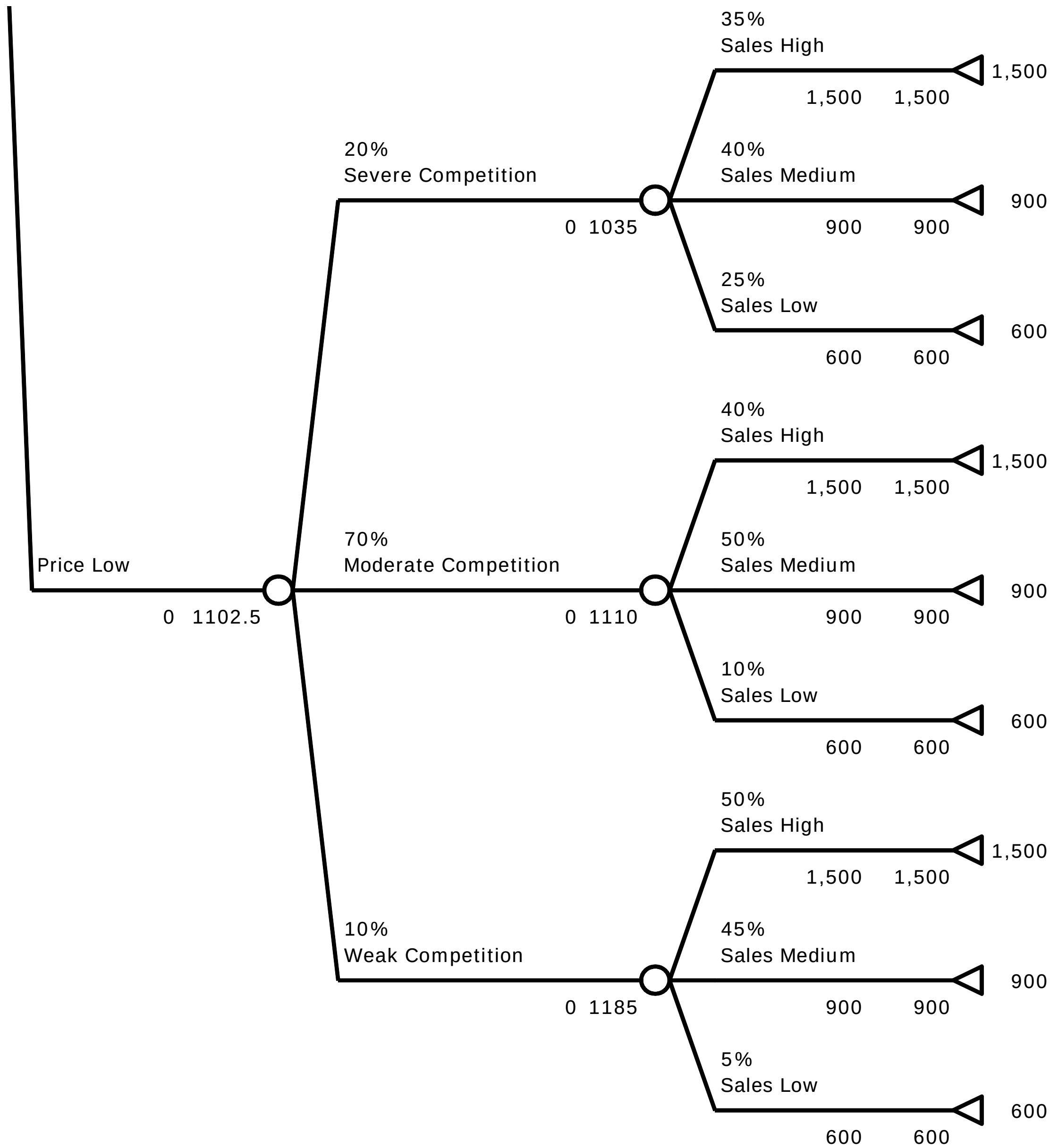
	Price			Severe	Moderate	Weak		
High	$50		Prior Probability	0.2	0.7	0.1		
Medium	$40							
Low	$30						Prior	Revenue
			High Price	Severe	Moderate	Weak	Probability	($thousands)
	Sales		Sales High	0.20	0.25	0.30	0.245	2,500
	(thousands)		Sales Medium	0.25	0.30	0.35	0.295	1,500
High	50		Sales Low	0.55	0.45	0.35	0.46	1,000
Medium	30							
Low	20		Medium Price	Severe	Moderate	Weak		
			Sales High	0.25	0.30	0.40	0.3	2,000
			Sales Medium	0.35	0.40	0.50	0.4	1,200
			Sales Low	0.40	0.30	0.10	0.3	800
			Low Price	Severe	Moderate	Weak		
			Sales High	0.35	0.40	0.50	0.4	1,500
			Sales Medium	0.40	0.50	0.45	0.475	900
			Sales Low	0.25	0.10	0.05	0.125	600

The decision tree for this probem follows (over three pages):





25%
Sales High
2,000 2,000
2,000
20%
Severe Competition
0 1240
35%
Sales Medium
1,200 1,200
1,200
40%
Sales Low
800 800
800
30%
Sales High
2,000 2,000
2,000
Price Medium
1
1515
0 1320
70%
Moderate Competition
0 1320
40%
Sales Medium
1,200 1,200
1,200
30%
Sales Low
800 800
800
40%
Sales High
2,000 2,000
2,000
10%
Weak Competition
0 1480
50%
Sales Medium
1,200 1,200
1,200
10%
Sales Low
800 800
800



(b) The scenario "moderate competition, sales of 30,000 units at a unit price of $30" has the largest total probability. Therefore, under the maximum likelihood criterion, Charlotte should price the product at $30.

To find out best maximin alternative, note that for a price of

$30: 20,000 units at a unit price $30 is the worst case,
$40: 20,000 units at a unit price $40 is the worst case,
$50: 20,000 units at a unit price $50 is the worst case.

The maximum of these three is for the price of $50, so it is optimal under the maximin criterion.

(c) As shown in the decision tree for part a (recall that decision trees assume Bayes' decision rule), Charlotte should charge the high price ($50), since this maximizes the expected revenue ($1.515 million). Alternatively, the expected revenues for each possible decision can be calculated directly as shown in the following spreadsheet.

	Price			Severe	Moderate	Weak			
High	$50		Prior Probability	0.2	0.7	0.1			
Medium	$40								Expected
Low	$30						Prior	Revenue	Revenue
			High Price	Severe	Moderate	Weak	Probability	($thousands)	($thousands)
	Sales		Sales High	0.20	0.25	0.30	0.245	2,500	
	(thousands)		Sales Medium	0.25	0.30	0.35	0.295	1,500	1,515
High	50		Sales Low	0.55	0.45	0.35	0.46	1,000	
Medium	30								
Low	20		Medium Price	Severe	Moderate	Weak			
			Sales High	0.25	0.30	0.40	0.3	2,000	
			Sales Medium	0.35	0.40	0.50	0.4	1,200	1,320
			Sales Low	0.40	0.30	0.10	0.3	800	
			Low Price	Severe	Moderate	Weak			
			Sales High	0.35	0.40	0.50	0.4	1,500	
			Sales Medium	0.40	0.50	0.45	0.475	900	1,102.5
			Sales Low	0.25	0.10	0.05	0.125	600	

(d) With more information from the marketing research company, the posterior probabilities for the state of competition can be found using the template for posterior probabilities as follows.

Data:		P(Finding | State)		
State of	Prior	Finding		
Nature	Probability	Predict Severe	Predict Moderate	Predict Weak
Severe	0.2	0.8	0.15	0.05
Moderate	0.7	0.15	0.8	0.05
Weak	0.1	0.03	0.07	0.9

Posterior		P(State | Finding)		
Probabilities:		State of Nature		
Finding	P(Finding)	Severe	Moderate	Weak
Predict Severe	0.268	0.597	0.392	0.011
Predict Moderate	0.597	0.050	0.938	0.012
Predict Weak	0.135	0.074	0.259	0.667

To keep the decision tree from becoming too unwieldy, we will break it into parts. The first three parts consider the situation after each possible prediction by the marketing research company. The decision tree from part a is reused with the only change being the prior probabilities of severe, moderate and weak competition used in part a are replaced by the appropriate posterior probabilities calculated above, depending upon the prediction of the marketing research company. For example, if the marketing research company predicts the competition will be severe, the probability of severe, moderate, and weak competition are 0.597, 0.392, and 0.011, respectively.

The optimal decision if the marketing research company predicts severe is to price high ($50), with expected revenue of $1.466 million.

	Price			Severe	Moderate	Weak		
High	$50		Prior Probability	0.597	0.392	0.011		
Medium	$40							
Low	$30						Prior	Revenue
			High Price	Severe	Moderate	Weak	Probability	($thousands)
	Sales		Sales High	0.20	0.25	0.30	0.220709	2,500
	(thousands)		Sales Medium	0.25	0.30	0.35	0.270709	1,500
High	50		Sales Low	0.55	0.45	0.35	0.5085821	1,000
Medium	30							
Low	20		Medium Price	Severe	Moderate	Weak		
			Sales High	0.25	0.30	0.40	0.2712687	2,000
			Sales Medium	0.35	0.40	0.50	0.3712687	1,200
			Sales Low	0.40	0.30	0.10	0.3574627	800
			Low Price	Severe	Moderate	Weak		
			Sales High	0.35	0.40	0.50	0.3712687	1,500
			Sales Medium	0.40	0.50	0.45	0.4397388	900
			Sales Low	0.25	0.10	0.05	0.1889925	600
			Optimal Decision	Price High				
			Expected Revenue	1466.42				
				($thousands)				

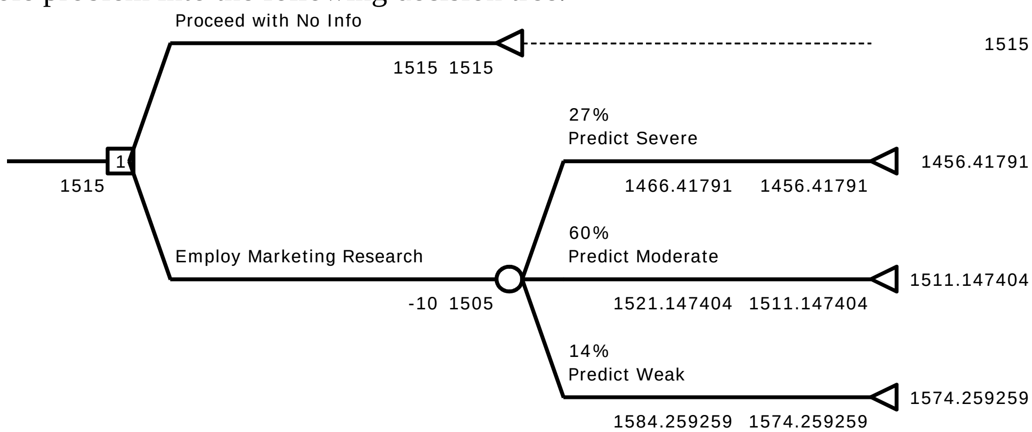
The optimal decision if the marketing research company predicts moderate competition is to price high ($50), with expected revenue of $1.521 million.

	Price			Severe	Moderate	Weak		
High	$50		Prior Probability	0.050	0.938	0.012		
Medium	$40							
Low	$30						Prior	Revenue
			High Price	Severe	Moderate	Weak	Probability	($thousands)
	Sales		Sales High	0.20	0.25	0.30	0.2480737	2,500
	(thousands)		Sales Medium	0.25	0.30	0.35	0.2980737	1,500
High	50		Sales Low	0.55	0.45	0.35	0.4538526	1,000
Medium	30							
Low	20		Medium Price	Severe	Moderate	Weak		
			Sales High	0.25	0.30	0.40	0.29866	2,000
			Sales Medium	0.35	0.40	0.50	0.39866	1,200
			Sales Low	0.40	0.30	0.10	0.3026801	800
			Low Price	Severe	Moderate	Weak		
			Sales High	0.35	0.40	0.50	0.39866	1,500
			Sales Medium	0.40	0.50	0.45	0.4943886	900
			Sales Low	0.25	0.10	0.05	0.1069514	600
			Optimal Decision	Price High				
			Expected Revenue	1521.15				
				($thousands)				

The optimal decision if the marketing research company predicts weak competition is to price high ($50), with expected revenue of $1.584 million.

	Price			Severe	Moderate	Weak		
High	$50		Prior Probability	0.074	0.259	0.667		
Medium	$40							
Low	$30						Prior	Revenue
			High Price	Severe	Moderate	Weak	Probability	($thousands)
	Sales		Sales High	0.20	0.25	0.30	0.2796296	2,500
	(thousands)		Sales Medium	0.25	0.30	0.35	0.3296296	1,500
High	50		Sales Low	0.55	0.45	0.35	0.3907407	1,000
Medium	30							
Low	20		Medium Price	Severe	Moderate	Weak		
			Sales High	0.25	0.30	0.40	0.362963	2,000
			Sales Medium	0.35	0.40	0.50	0.462963	1,200
			Sales Low	0.40	0.30	0.10	0.1740741	800
			Low Price	Severe	Moderate	Weak		
			Sales High	0.35	0.40	0.50	0.462963	1,500
			Sales Medium	0.40	0.50	0.45	0.4592593	900
			Sales Low	0.25	0.10	0.05	0.0777778	600
			Optimal Decision	Price High				
			Expected Revenue	1584.26				
				($thousands)				

Then, incorporating the expected payoff with each possible prediction by the marketing company, along with the expected revenue without information from part a, we combine the whole problem into the following decision tree.



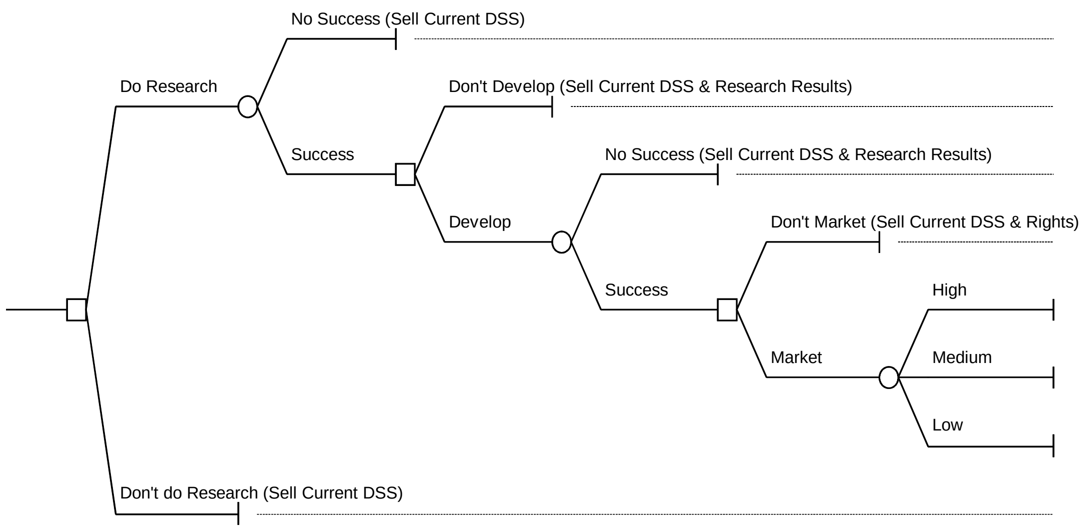
Charlotte should not purchase the services of the market research company. The information is not worth anything since it does not affect the decision. Regardless of the prediction, the optimal policy is to set the price at $50.

CASE 16.2 Smart Steering Support

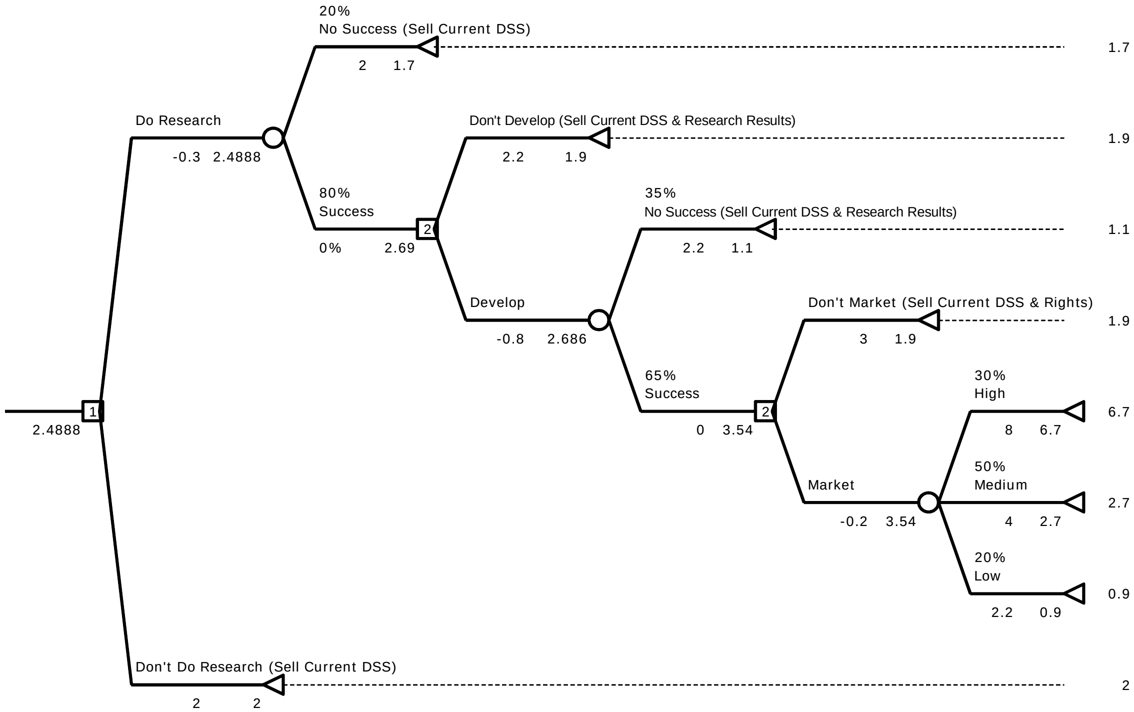
(a) The available data are summarized in the table.

	Costs		Probability
	($million)		of Success
Research	0.3		0.8
Development	0.8		0.65
Marketing	0.2		
	Revenues from R&D		
	($million)		
Sell Product Rights	1		
Sell Research Results	0.2		
Sell Current DSS	2		
	Sales		
	Revenue		
	($million)		Probability
High	8		0.3
Medium	4		0.5
Low	2.2		0.2

(b) The basic decision tree is shown below.

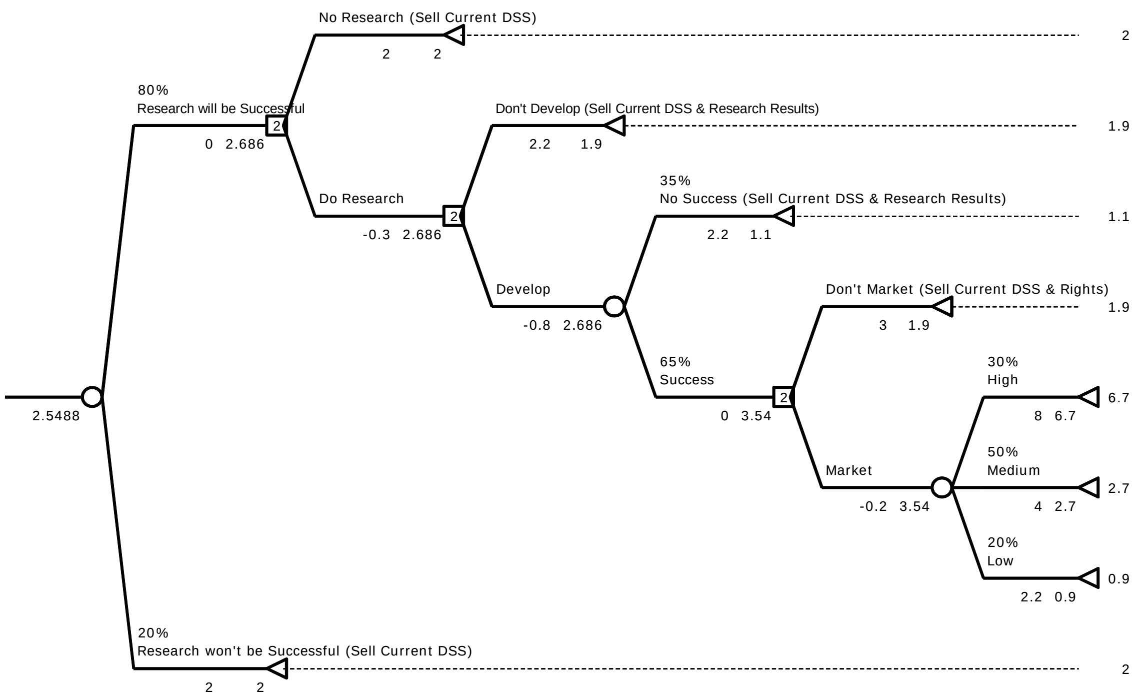


(c) The decision tree displays all the expected payoffs and probabilities.

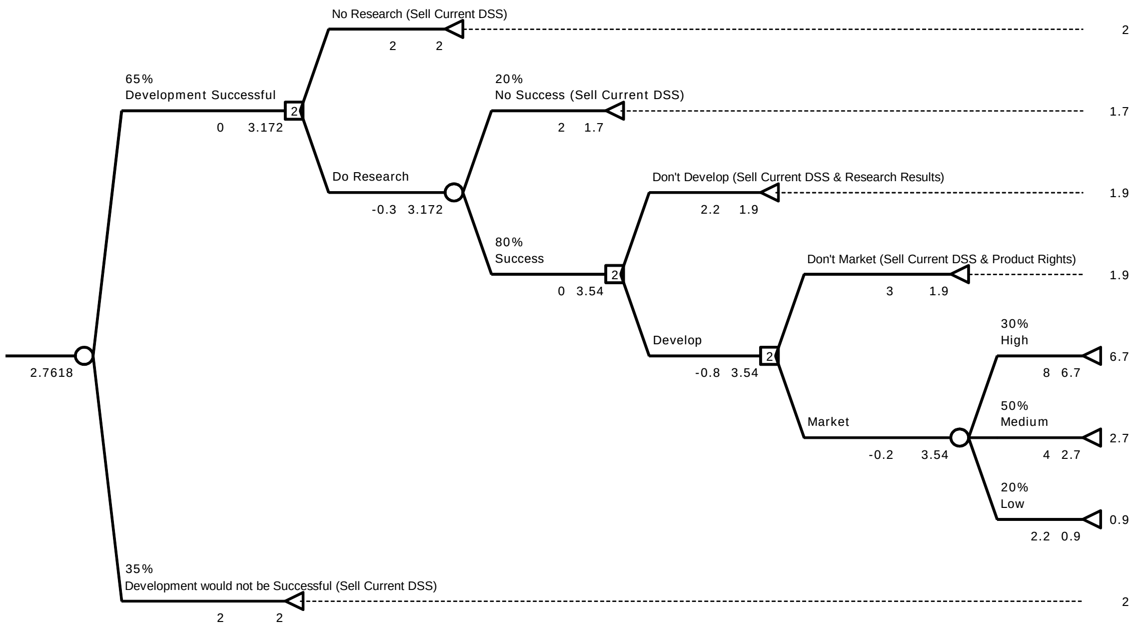


(d) The best course of action is to do the research project. The expected payoff is $\$2.489$ million.

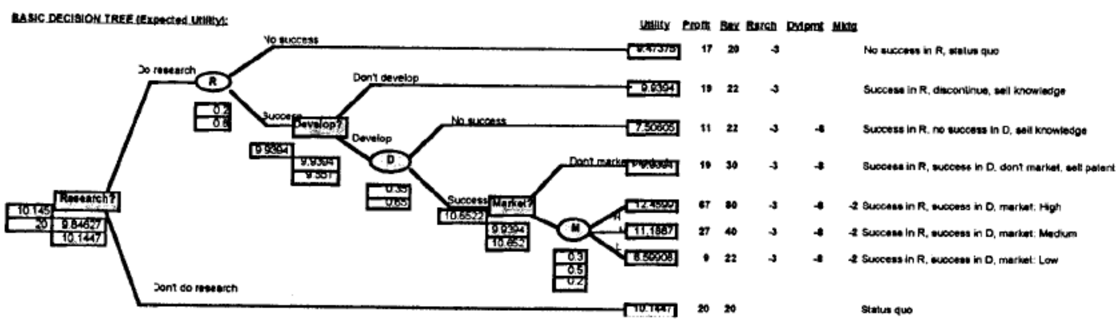
(e) The decision tree with perfect information on research is displayed. The expected value in this case equals $\$2.549$ million. The difference between the expected values with and without information is $\$60,000$, which is the value of perfect information on research.



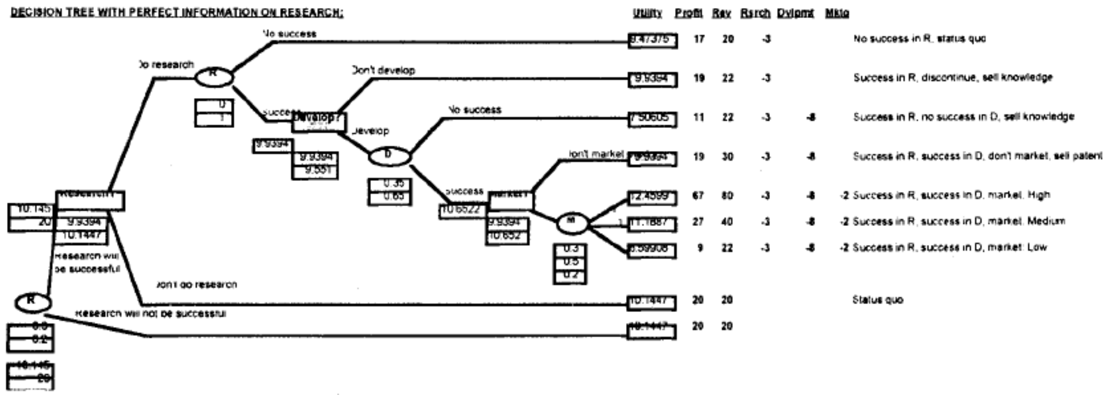
(f) The decision tree with perfect information on development is displayed. The expected value in this case equals \$$2.762$ million. The difference between the expected values with and without information is \$$273,000$, which is the value of perfect information on development



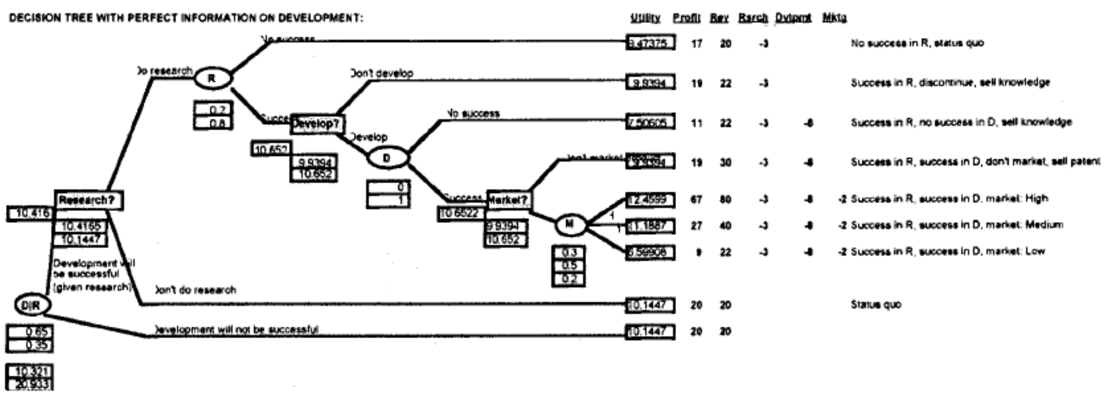
(g) - (h) - (i) The decision tree with expected utilities is displayed. The expected utilities are calculated in the following way: for each of the outcome branches of the decision tree (e.g., profit of \$$6,700,000$), the corresponding utility is computed (e.g., 12.45992). Once this is done, the expected utilities are calculated. The best course of action is to not do research (expected utility of 10.14469 vs. 9.846267 in the case of doing research).



(j) The expected utility for perfect information on research equals 9.939397, which is still less than the expected utility of not doing research (10.14469). Therefore, the best course of action is to not do research, implying a value of zero for perfect information on the outcome of the research effort.

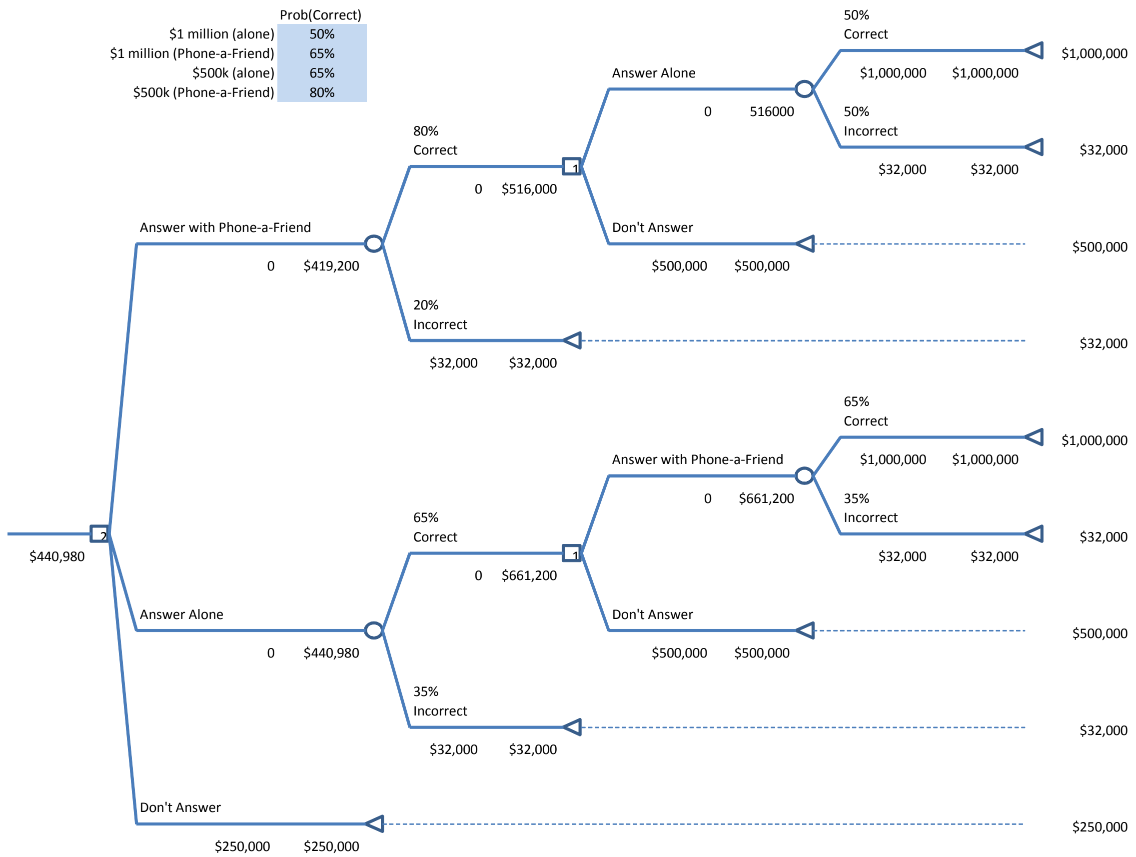


(k) The expected utility for perfect information on development equals 10.321347, which is more than the expected utility without information (10.14469). The value of perfect information on development is the difference between the inverses of these two utility values, $\mathrm{U}^{-1}(10.321347) - \mathrm{U}^{-1}(10.14469) = 20.93274 - 20 = 0.93274$. The value of perfect information on the outcome of the development effort is $\$93.274$.



CASE 16.3 Who Wants to be a Millionaire

(a) The course of action that maximizes the expected payoff is to answer \$500,000 question alone. If you get the question correct, then use the phone-a-friend lifeline to help answer \$1 million question. The expected payoff is \$440,980.



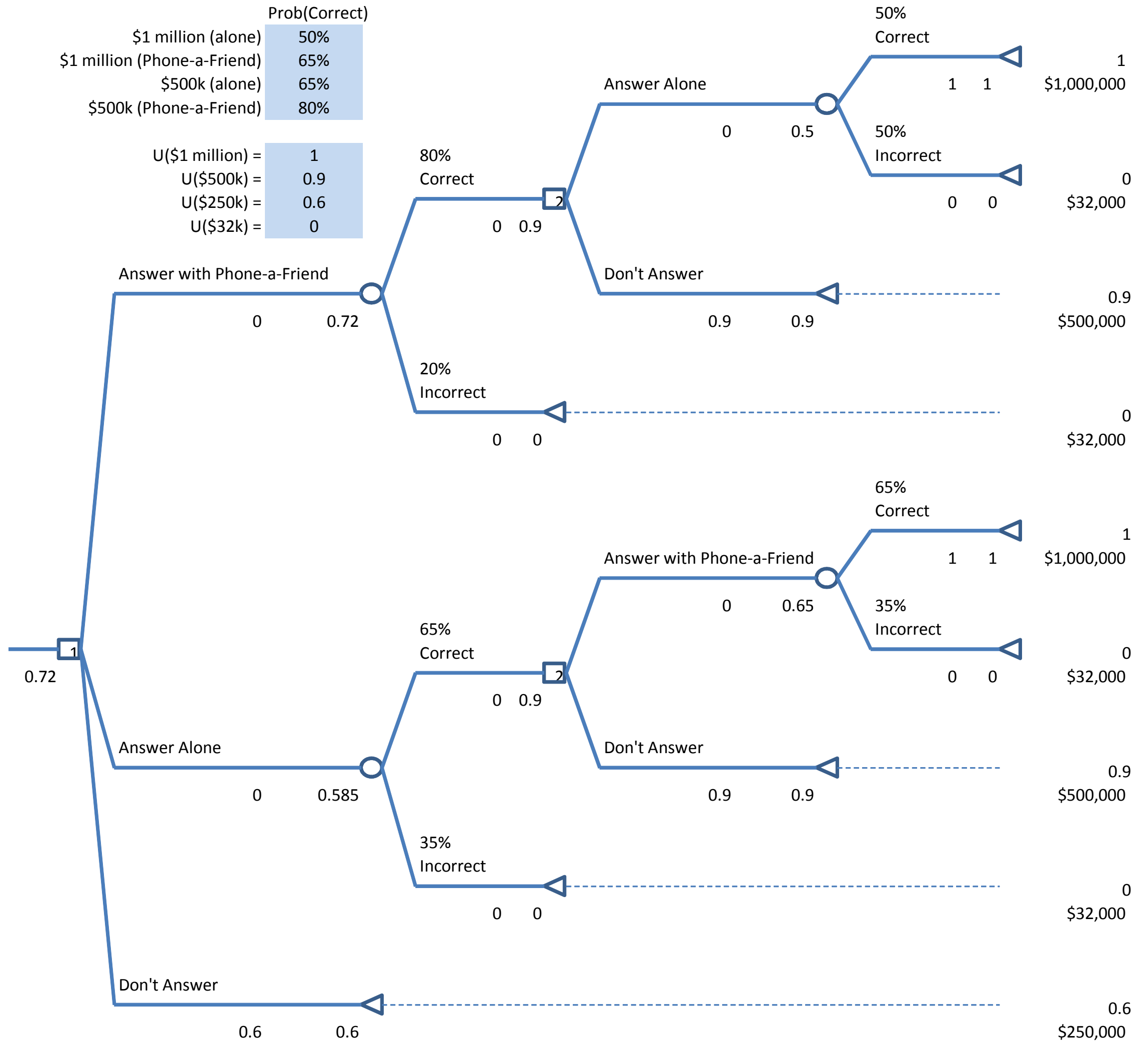
(b) Answers will vary depending on your level of risk aversion. One possible solution is obtained by setting

$$U(\textit{Maximum}) = U(\$1 \text{ million}) = 1 \text{ and } U(\textit{Minimum}) = U(\$32{,}000) = 0.$$

If getting \$250,000 for sure is equivalent to a 60% chance of getting \$1 million vs. a 40% chance of getting \$32,000, then $U(\$250{,}000) = p = 0.6$.

If getting \$500,000 for sure is equivalent to a 90% chance of getting \$1 million vs. a 10% chance of getting \$32,000, then $U(\$500{,}000) = p = 0.9$.

(c) With the utilities derived in part (b), the decision changes to using the phone-a-friend lifeline to help answer the $500,000 question, and then walk away.



CASE 16.4 University Toys and the Business Professor Action Figures

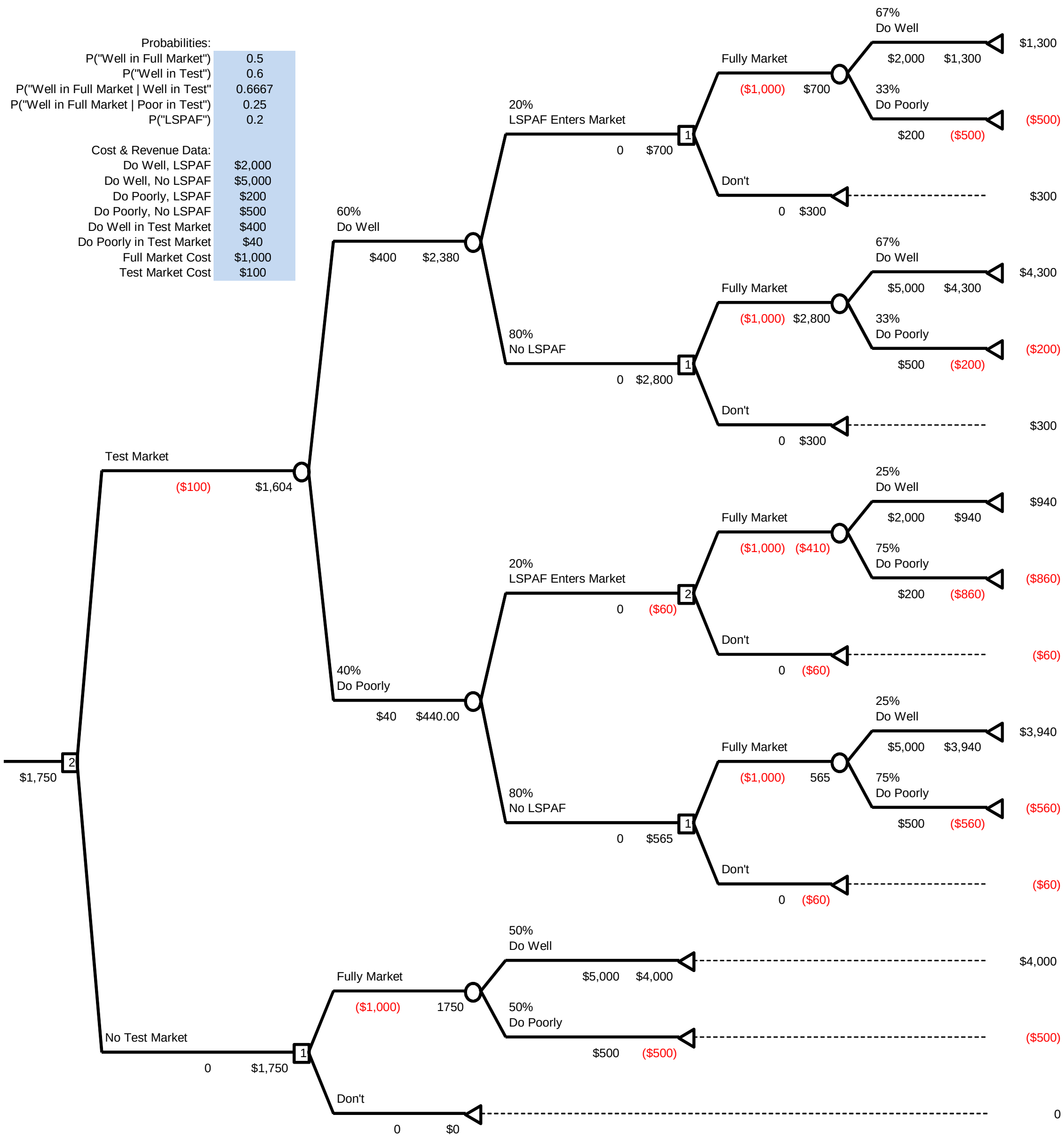
(a)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Well in Test	Poor in Test
Well in Full Market	0.5	0.8	0.2
Poor in Full Market	0.5	0.4	0.6

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Well in Full Market	Poor in Full Market
Well in Test	0.6	0.6667	0.3333
Poor in Test	0.4	0.25	0.75

(b) The best course of action is to skip the test market, and immediately market the product fully. The expected payoff is $1750.

Probabilities:	
P("Well in Full Market")	0.5
P("Well in Test")	0.6
P("Well in Full Market | Well in Test"	0.6667
P("Well in Full Market | Poor in Test")	0.25
P("LSPAF")	0.2
Cost & Revenue Data:	
Do Well, LSPAF	$2,000
Do Well, No LSPAF	$5,000
Do Poorly, LSPAF	$200
Do Poorly, No LSPAF	$500
Do Well in Test Market	$400
Do Poorly in Test Market	$40
Full Market Cost	$1,000
Test Market Cost	$100



(c) If the probability that the LSPAFs enter the market before the test marketing would be completed increases this would make the test market even less desirable, so it would still not be worthwhile to do. However, if the probability decreases, this would make the test market more desirable. It might reach the point where the test market is worthwhile.

(d) Let p denote the probability that the LSPAFs will enter and EP the expected payoff.

p	EP	Test Market?
	\$1,750	No
0.0	\$1,906	Yes
0.1	\$1,755	Yes
0.2	\$1,750	No
0.3	\$1,750	No
0.4	\$1,750	No
0.5	\$1,750	No
0.6	\$1,750	No
0.7	\$1,750	No
0.8	\$1,750	No
0.9	\$1,750	No
1.0	\$1,750	No

(e) It is better to perform the test market if the probability that the LSPAFs will enter the market is 10% or less. It is better to skip the test market if this probability is greater than 10%.