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THE INNOVATION UNIVERSITY

Transportation, Transshipment, and Assignment Problems

Topics

- The Transportation Model
- The Transshipment Model
- The Assignment Model

Overview

- Part of a class of LP problems known as network flow models
- Special mathematical features that permit very efficient, unique solution methods (variations of traditional simplex procedure)
- Text focuses on model formulation and solution with Excel and QM for Windows

The Transportation Model: Characteristics

- A product is transported from a number of sources to a number of destinations at the ***minimum possible cost***
- Each ***source is able to supply a fixed number*** of units of the product, and ***each destination has a fixed demand*** for the product
- The linear programming model has ***constraints for supply*** at each source ***and demand*** at each destination
- All constraints are equalities in a balanced transportation model where supply equals demand
- Constraints contain inequalities in unbalanced models where supply does not equal demand

Transportation Model Example
Problem Definition and Data

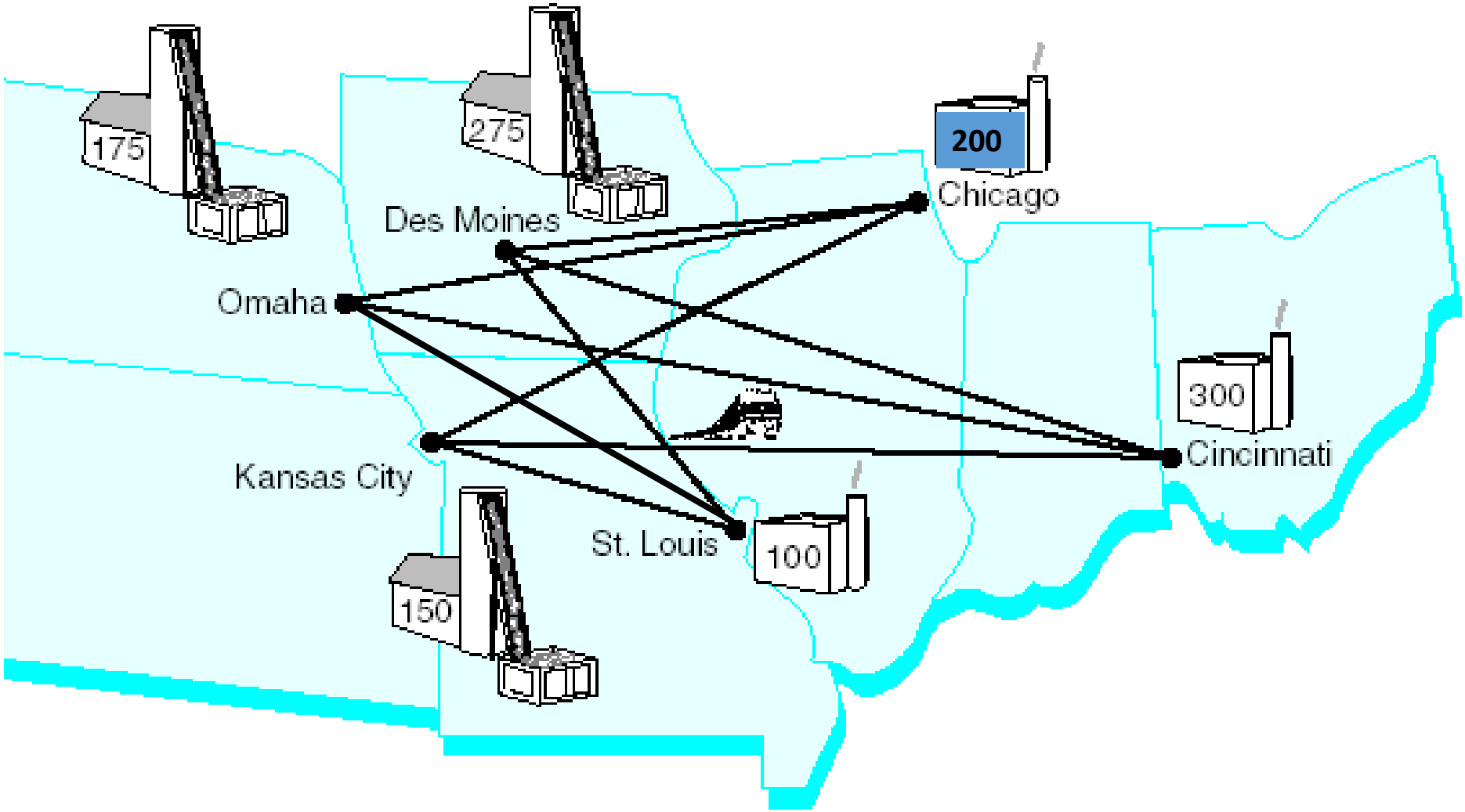
How many tons of wheat should we transport from each grain elevator to each mill on a monthly basis, in order to minimize the total cost of transportation?

Grain Elevator	Supply	Mill	Demand
1. Kansas City	150	A. Chicago	200
2. Omaha	175	B. St. Louis	100
3. Des Moines	275	C. Cincinnati	300
Total	600 tons	Total	600 tons

Transport Cost from Grain Elevator to Mill ($/ton)

Grain Elevator	A. Chicago	B. St. Louis	C. Cincinnati
1. Kansas City	$ 6	$ 8	$ 10
2. Omaha	7	11	11
3. Des Moines	4	5	12

Transportation Model Example
Transportation Network Routes



Network of Transportation Routes for Wheat Shipments

Transportation Model Example
Model Formulation

x_{ij} = tons of wheat from each grain elevator, i, i = 1, 2, 3, to each mill j, j = A,B,C

Minimize $Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$

subject to:

$x_{1A} + x_{1B} + x_{1C} = 150$

$x_{2A} + x_{2B} + x_{2C} = 175$

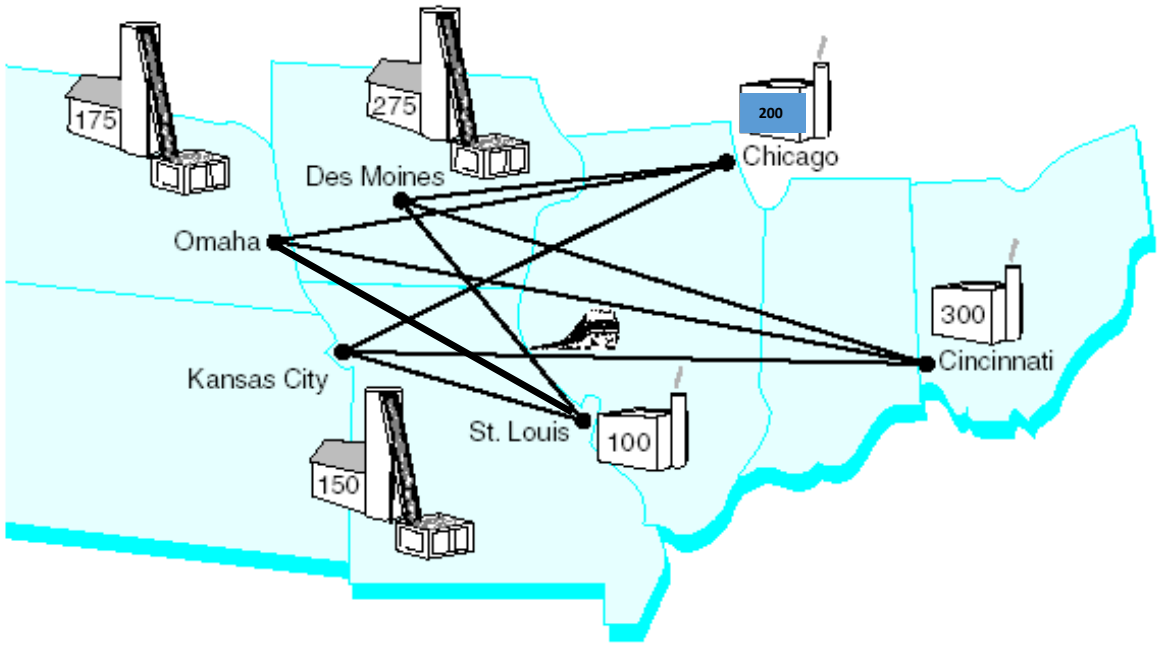
$x_{3A} + x_{3B} + x_{3C} = 275$

$x_{1A} + x_{2A} + x_{3A} = 200$

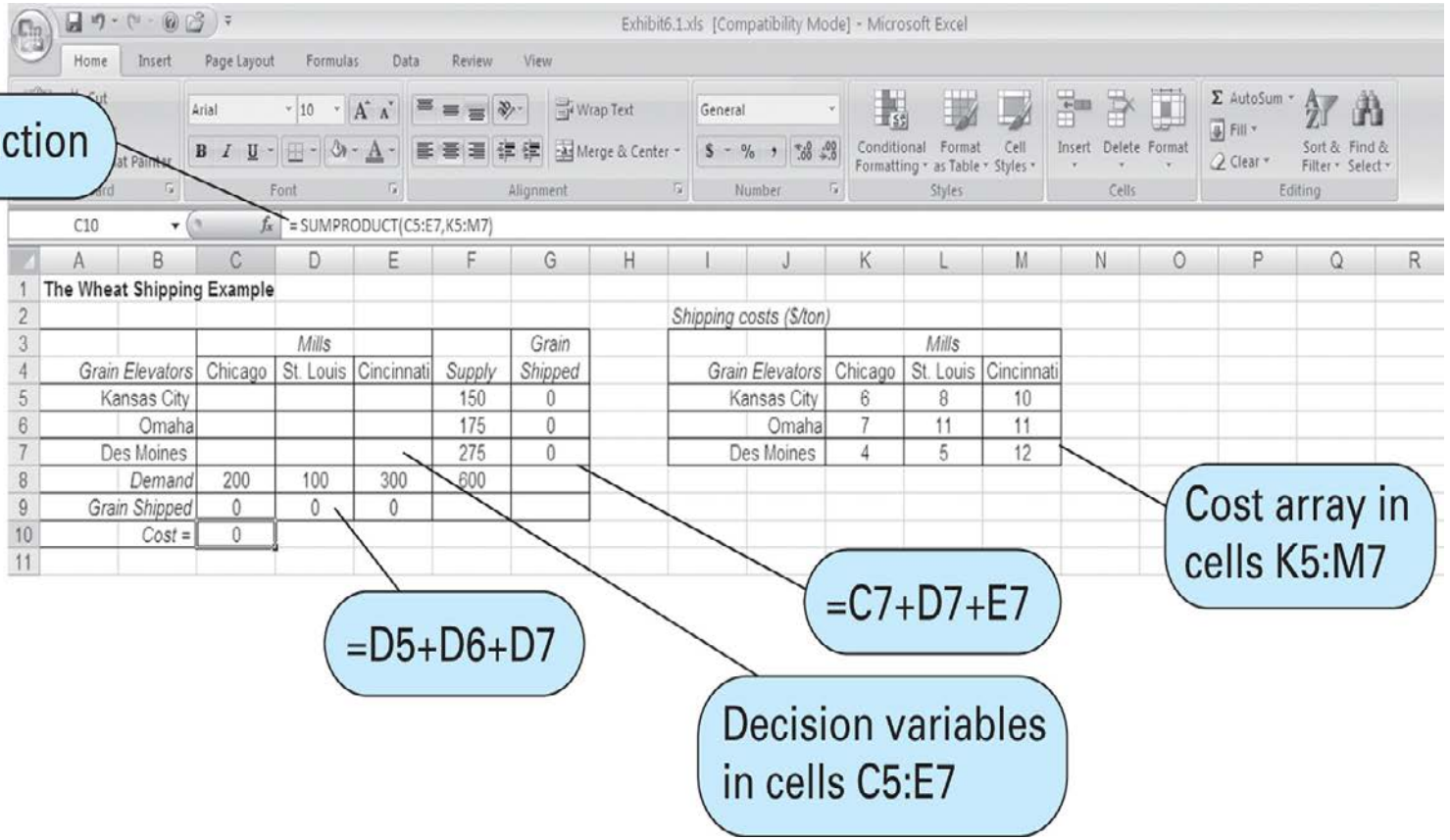
$x_{1B} + x_{2B} + x_{3B} = 100$

$x_{1C} + x_{2C} + x_{3C} = 300$

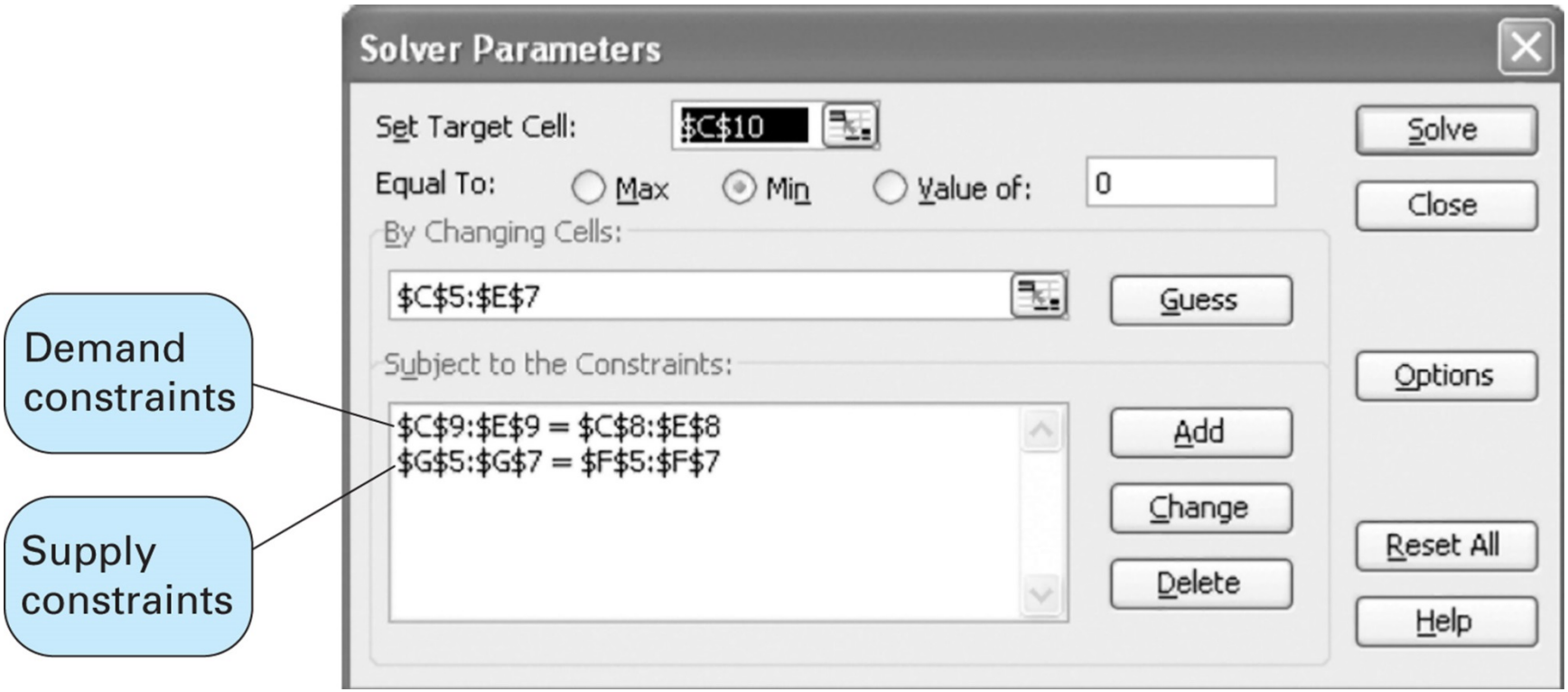
$x_{ij} \geq 0$



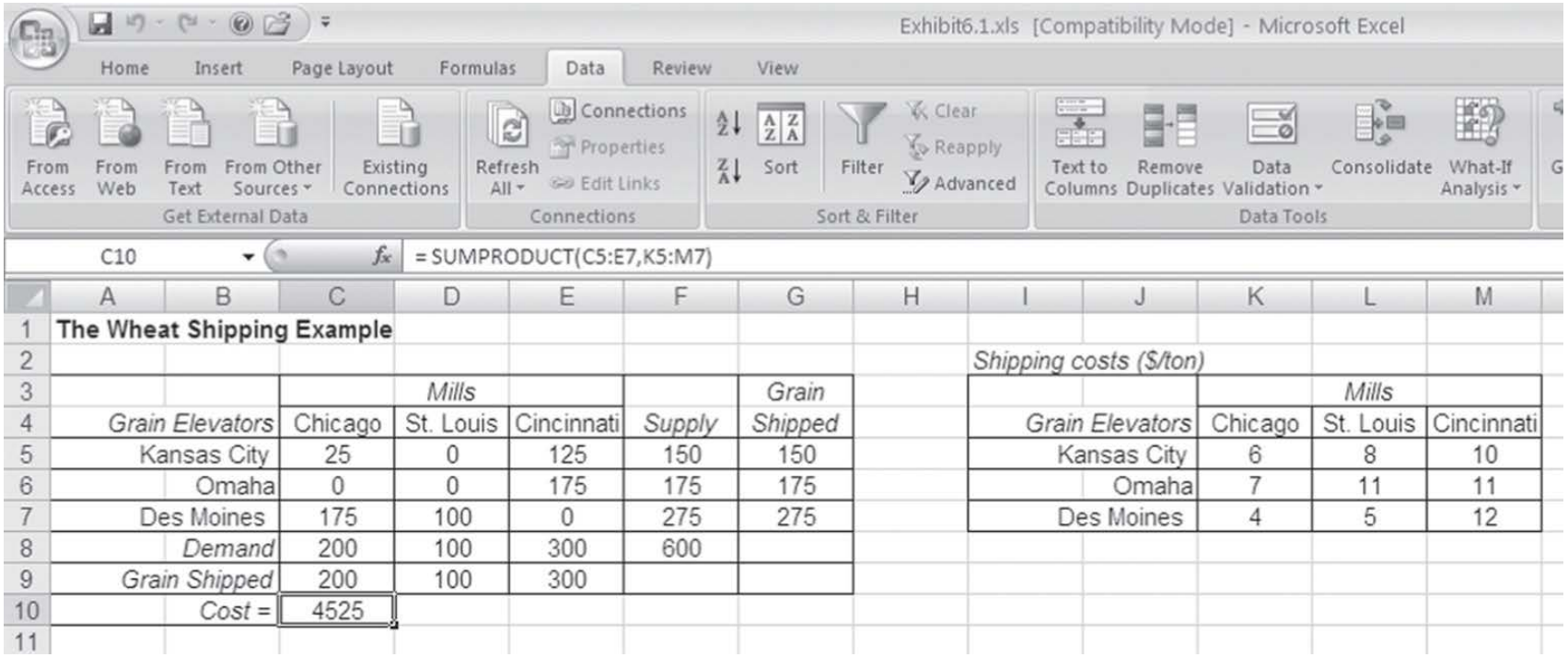
Transportation Model Solution with Excel



Transportation Model Example Solution with Excel



Transportation Model Example Solution with Excel



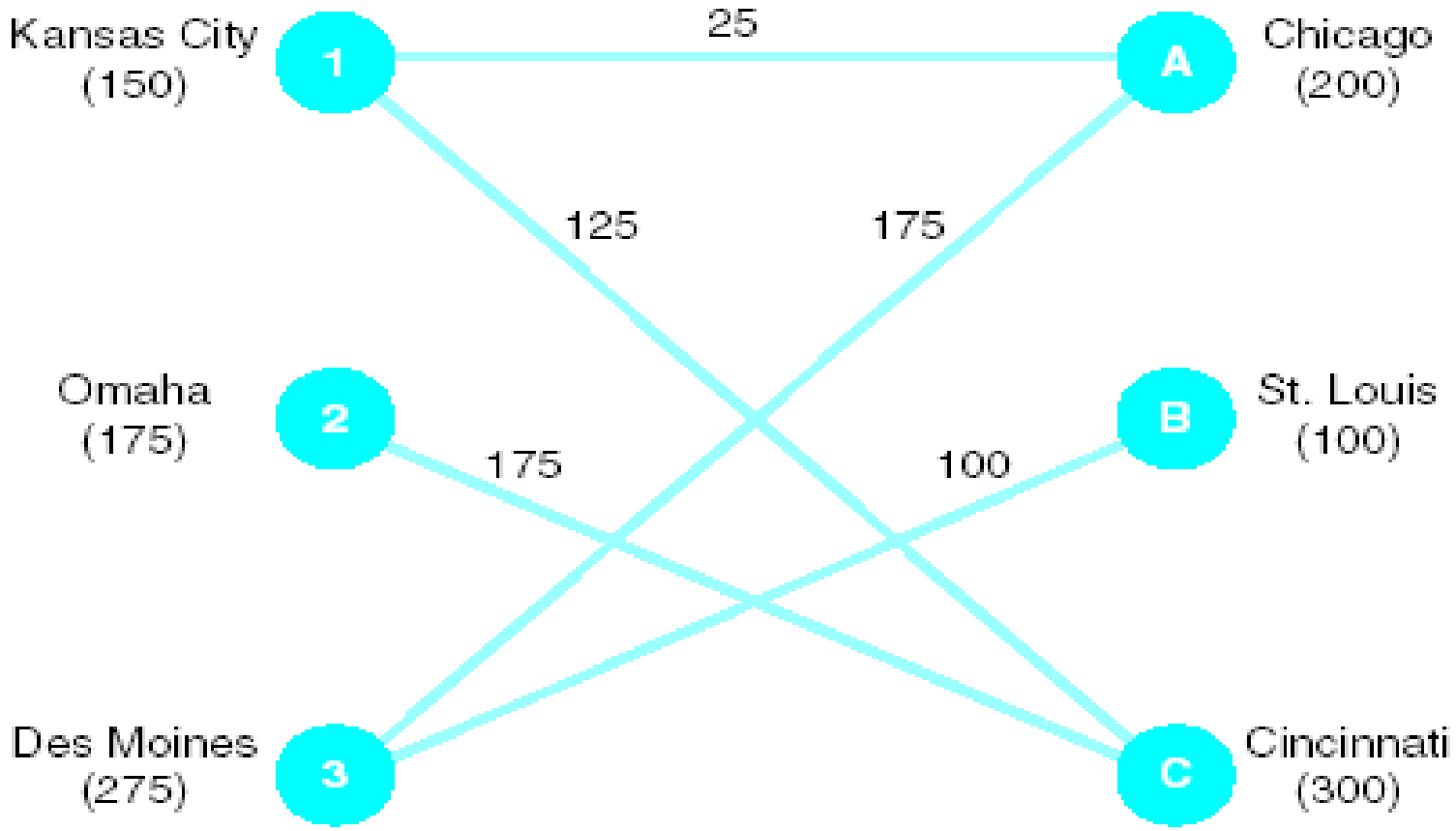
The Wheat Shipping Example

		Mills			Grain
Grain Elevators	Chicago	St. Louis	Cincinnati	*Supply*	*Shipped*
Kansas City	25	0	125	150	150
Omaha	0	0	175	175	175
Des Moines	175	100	0	275	275
Demand	200	100	300	600	
Grain Shipped	200	100	300		
Cost =	4525				

Shipping costs ($/ton)

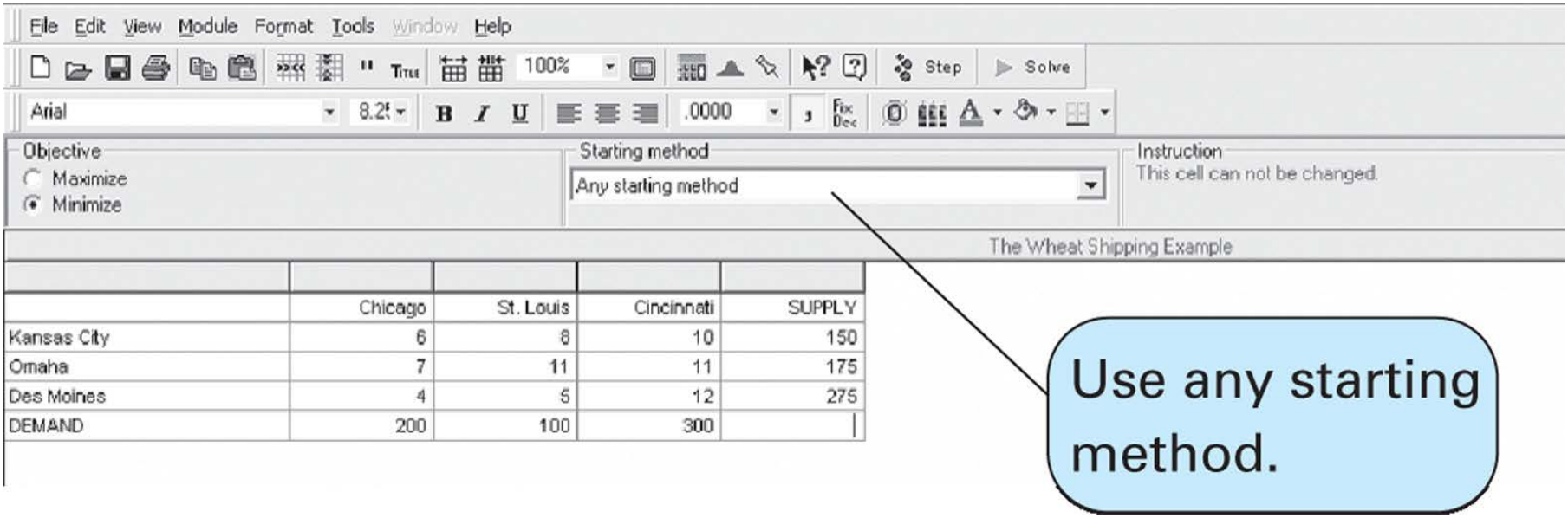
	Mills		
Grain Elevators	Chicago	St. Louis	Cincinnati
Kansas City	6	8	10
Omaha	7	11	11
Des Moines	4	5	12

Transportation Model Example Solution with Excel



Transportation Network Solution

Transportation Model Example Solution with QM for Windows

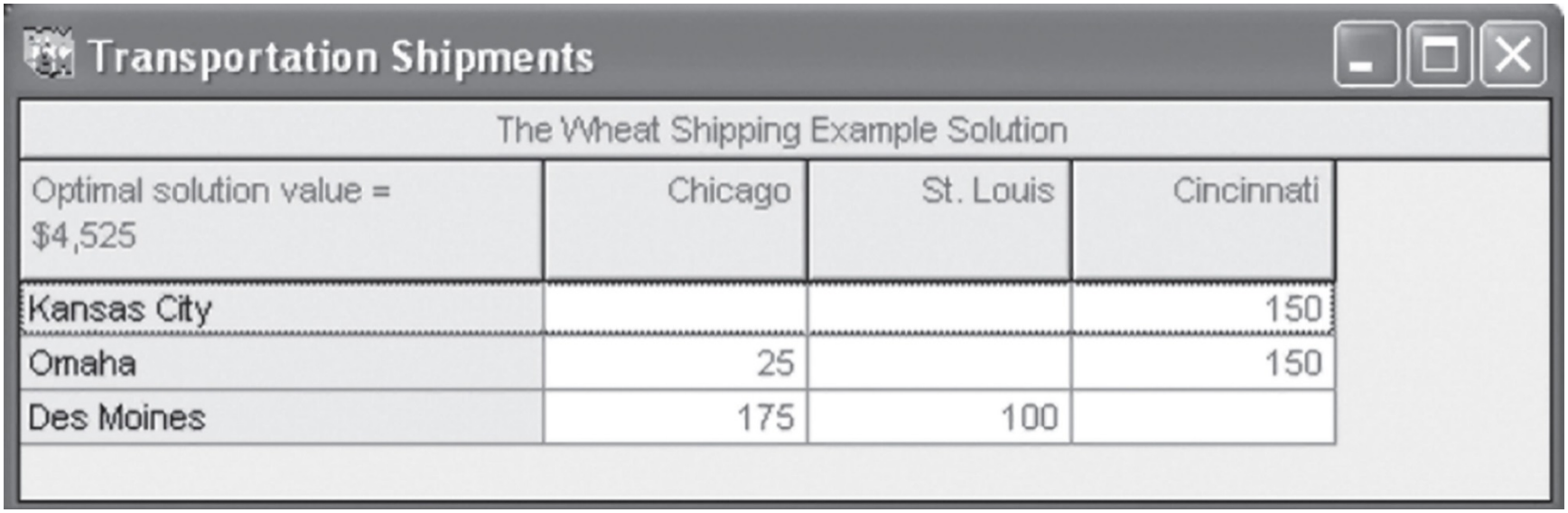


	Chicago	St. Louis	Cincinnati	SUPPLY
Kansas City	6	8	10	150
Omaha	7	11	11	175
Des Moines	4	5	12	275
DEMAND	200	100	300	

For more information:

http://www.scribd.com/doc/49602895/30/Northwest-Corner-Method

Transportation Model Example Solution with QM for Windows



Transportation Model Example Solution with QM for Windows

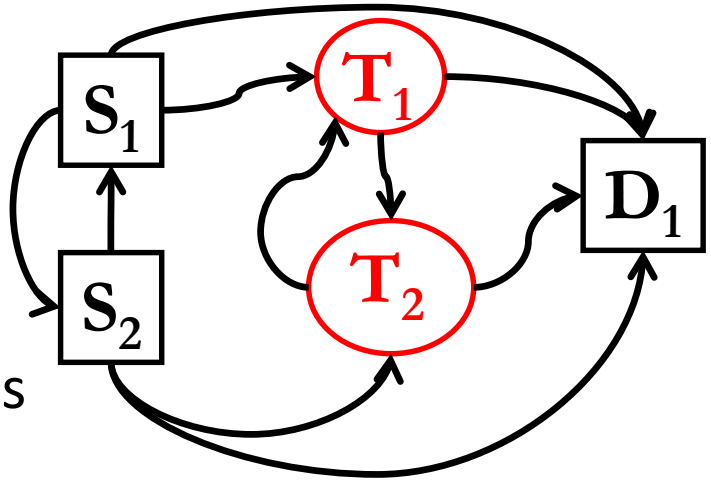
Shipping list

The Wheat Shipping Example Solution

From	To	Shipment	Cost per unit	Shipment cost
Kansas City	Cincinnati	150	10	1,500
Omaha	Chicago	25	7	175
Omaha	Cincinnati	150	11	1,650
Des Moines	Chicago	175	4	700
Des Moines	St. Louis	100	5	500

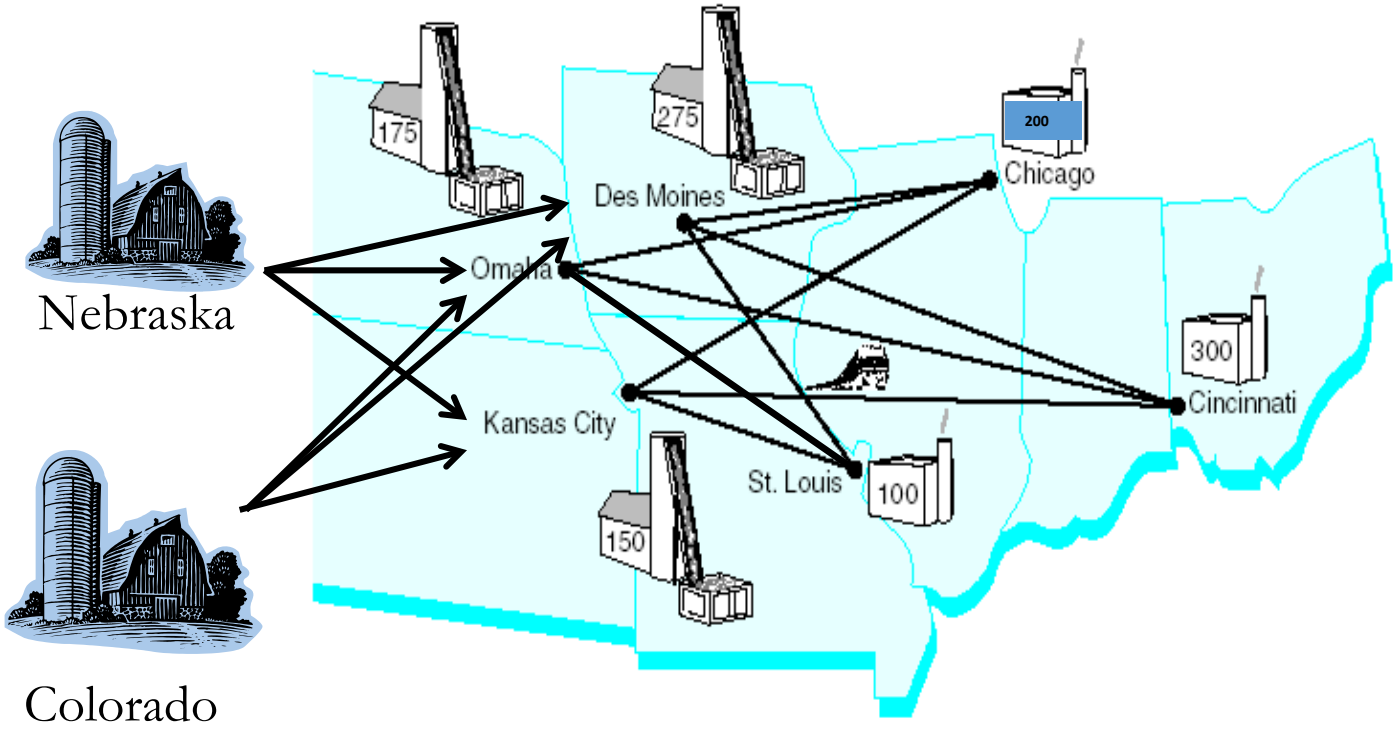
The Transshipment Model Characteristics

- Extension of the transportation model
- Intermediate transshipment points are added between the sources and destinations
- Items may be transported from:
  - Sources through transshipment points to destinations
  - One source to another
  - One transshipment point to another
  - One destination to another
  - Directly from sources to destinations
  - Some combination of these



Transshipment Model Example
Problem Definition and Data

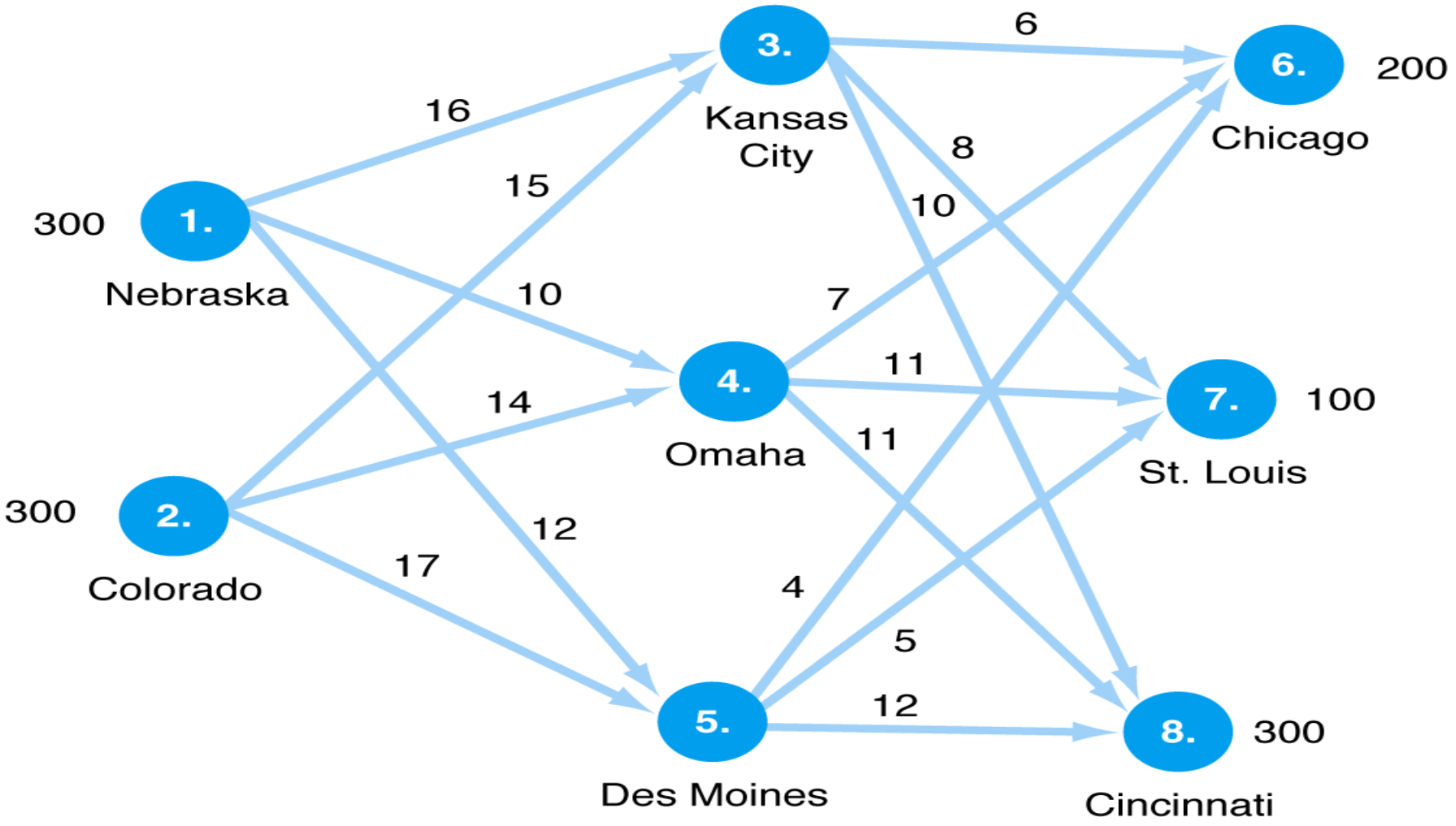
Let's extend the earlier transportation example by adding intermediate transshipment points between sources and destinations



Shipping Costs

	Grain Elevator		
Farm	**3. Kansas City**	**4. Omaha**	**5. Des Moines**
1. Nebraska	**$16**	**10**	**12**
2. Colorado	**15**	**14**	**17**

Transshipment Model Example
Transshipment Network Routes



Network of Transshipment Routes

Transshipment Model Example
Model Formulation

x_{ij} = # of tons of wheat from farm i to grain elevator j, and
# of tons of wheat from grain elevator i to mill j

$$\text{Minimize } Z = \$16x_{13} + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24} + 17x_{25} + 6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47} + 11x_{48} + 4x_{56} + 5x_{57} + 12x_{58}$$

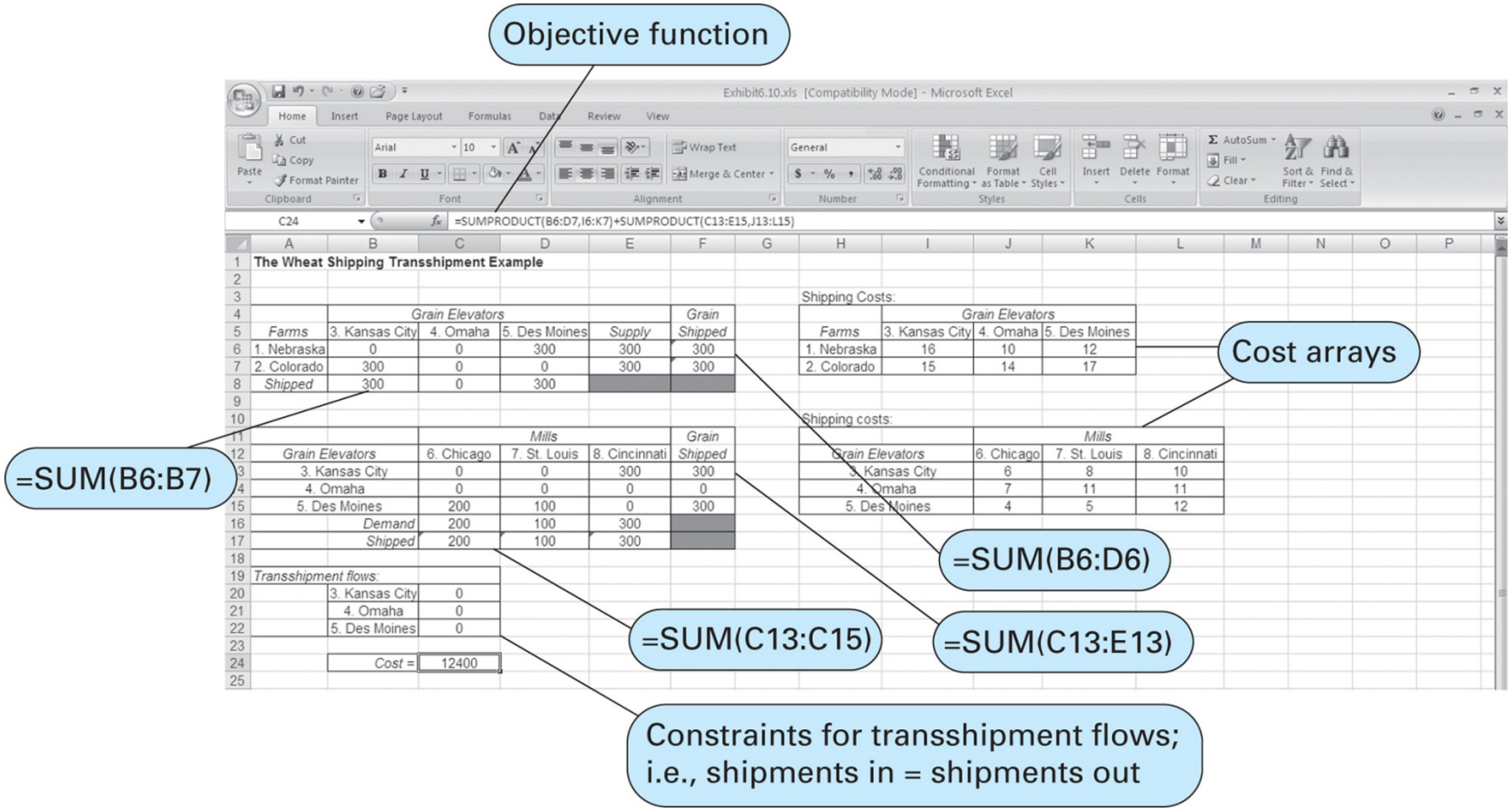
subject to:

$$x_{13} + x_{14} + x_{15} = 300 \quad \text{Nebraska Farm}$$
$$x_{23} + x_{24} + x_{25} = 300 \quad \text{Colorado Farm}$$
$$x_{36} + x_{46} + x_{56} = 200 \quad \text{Chicago Demand}$$
$$x_{37} + x_{47} + x_{57} = 100 \quad \text{St. Louis Demand}$$
$$x_{38} + x_{48} + x_{58} = 300 \quad \text{Cincinnati Demand}$$
$$x_{13} + x_{23} - x_{36} - x_{37} - x_{38} = 0$$
$$x_{14} + x_{24} - x_{46} - x_{47} - x_{48} = 0$$
$$x_{15} + x_{25} - x_{56} - x_{57} - x_{58} = 0$$

Balancing of transshipment nodes, to ensure no hold-up – "What comes in must go out!"

$$x_{ij} \geq 0$$

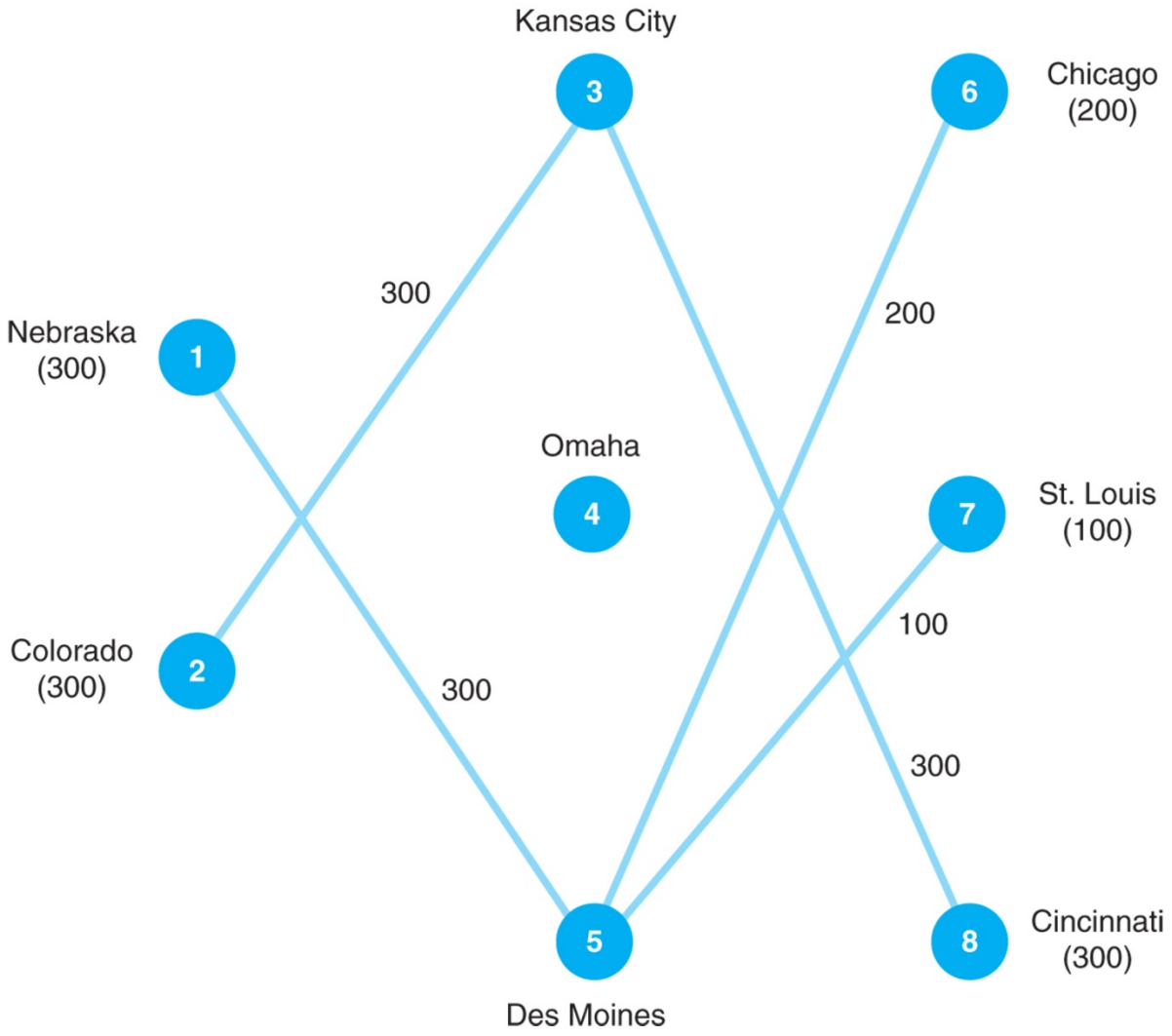
Transshipment Model Example Solution with Excel



Transshipment Model Example Solution with Excel

Transshipment constraints in cells C20:C22

Transshipment Model Example
Network Solution: Wheat Shipping



The Assignment Model Characteristics

- Special form of linear programming model similar to the transportation model
- ***Supply*** at each source ***and demand*** at each destination ***limited to one unit***
- In a balanced model supply equals demand
- In an unbalanced model, supply does not equal demand

Assignment Model Example
Problem Definition and Data

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Teamwork – Assignment Problem

Can you determine an optimal solution by inspection?

Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Assignment Model Example
Model Formulation

x_{ij} = assignment of officials team i to game j [binary variable]

Minimize Z = $210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC}$

subject to:

$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$
$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$
$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$
$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$

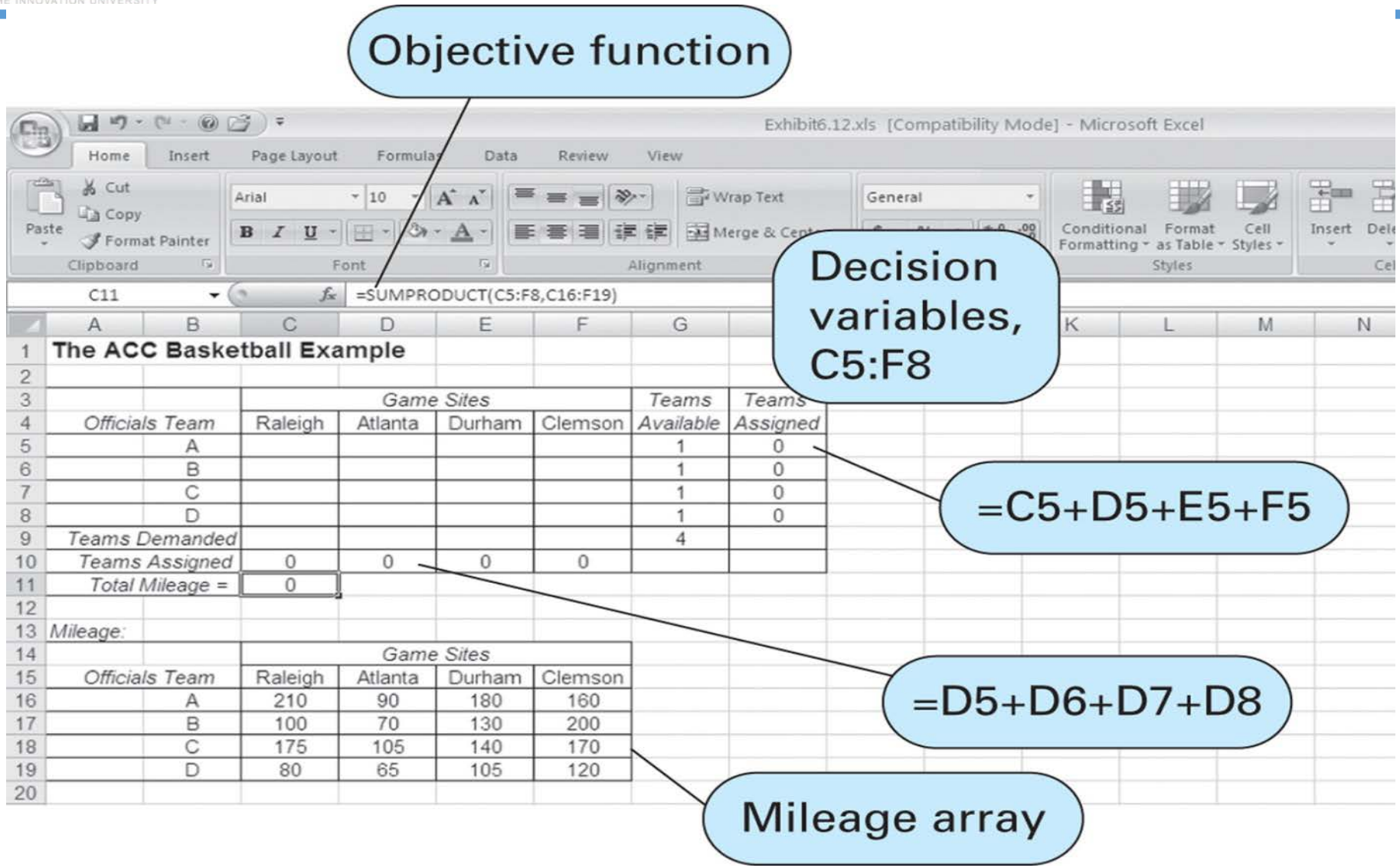
Any one team of officials can only be at one game

$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$
$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$
$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$
$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$

Requirement of one team (and only one team) of officials at each game

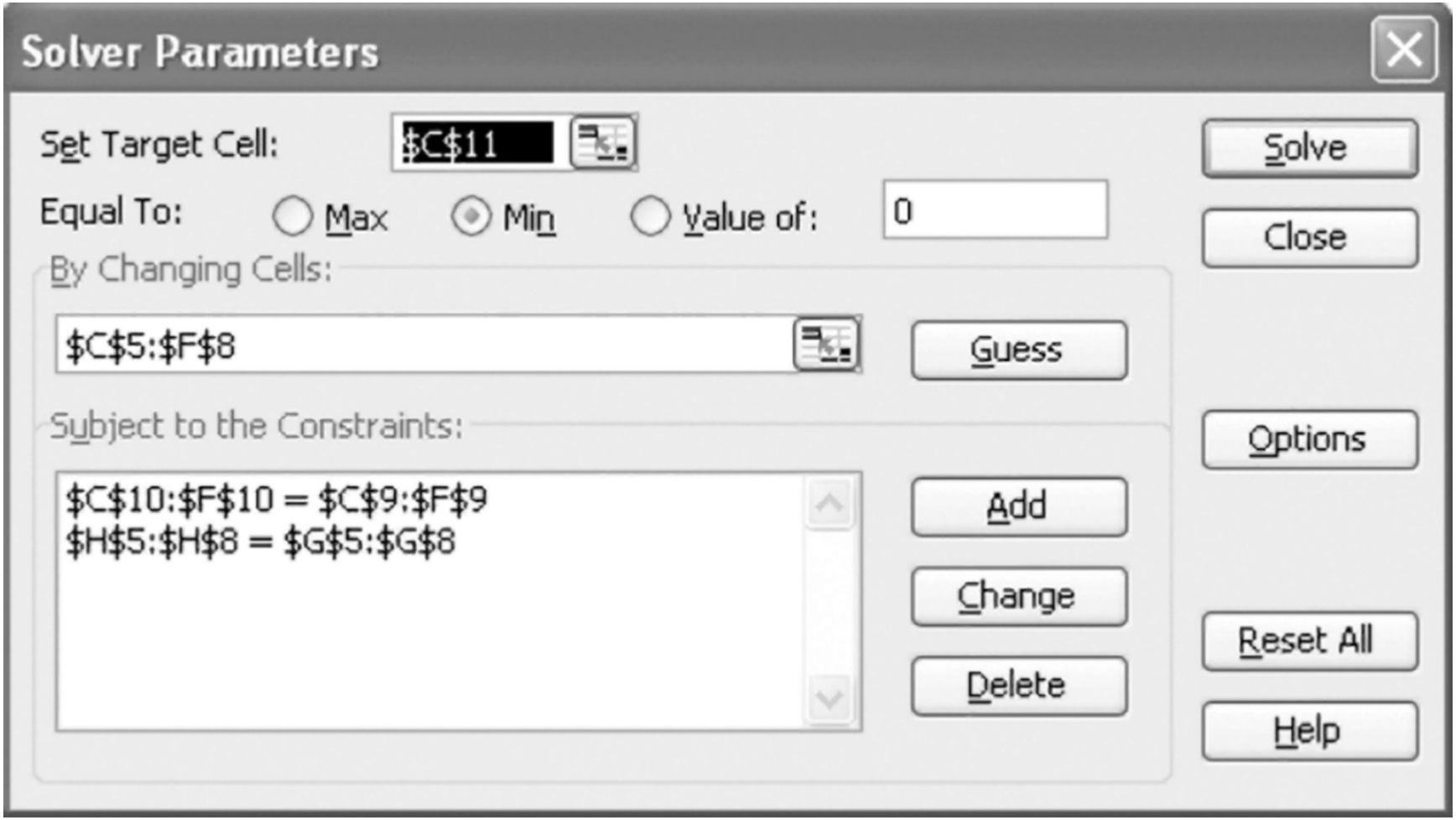
$x_{ij} = 0, 1$

Assignment Model Example Solution with Excel



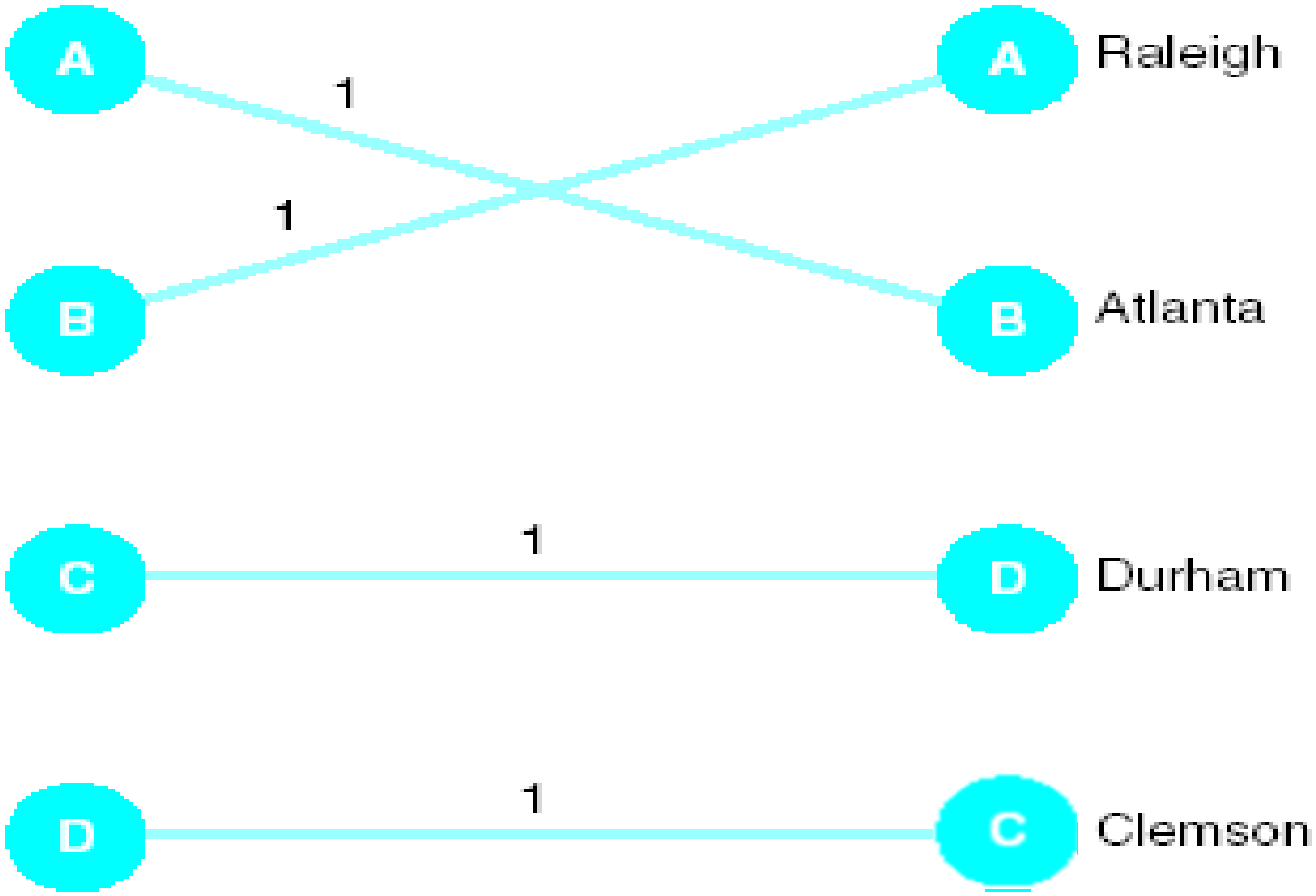
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**The ACC Basketball Example**													
2														
3			*Game Sites*				*Teams*	*Teams*						
4	*Officials Team*		Raleigh	Atlanta	Durham	Clemson	*Available*	*Assigned*						
5		A					1	0						
6		B					1	0						
7		C					1	0						
8		D					1	0						
9	*Teams Demanded*						4							
10	*Teams Assigned*		0	0	0	0								
11	*Total Mileage =*		0											
12														
13	*Mileage:*													
14			*Game Sites*											
15	*Officials Team*		Raleigh	Atlanta	Durham	Clemson								
16		A	210	90	180	160								
17		B	100	70	130	200								
18		C	175	105	140	170								
19		D	80	65	105	120								
20														

Assignment Model Example Solution with Excel



Assignment Model Example Solution with Excel

Assignment Model Example
Assignment Network Solution



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Assignment Model Example Solution with QM for Windows

Assignment Model Example Solution with QM for Windows

Assignments

ACC Basketball Example Solution

Optimal cost = $450	Raleigh	Atlanta	Durham	Clemson
A	210.	Assign 90	180.	160.
B	Assign 100	70.	130.	200.
C	175.	105.	Assign 140	170.
D	80.	65.	105.	Assign 120

Teamwork - Transportation Problem

Acme Bolts (a sub-division of Acme Enterprises) needs to transport tons of its bolts ("Better Bolts for a Better Tomorrow") from manufacturing plants to various construction sites at the lowest possible cost. There is no storage available at the plants, and demand is sometimes overstated by the construction sites. (The costs below are in $/ton)

	Construction site			
Plant	A	B	C	Supply (tons)
1	$ 8	$ 5	$ 6	120
2	15	10	12	80
3	3	9	10	80
Demand (tons)	150	70	100	

Example Problem Solution
Model Formulation

x_{ij} = # of tons of bolts shipped from plant i to construction site j

Minimize Z = $\$8x_{1A} + 5x_{1B} + 6x_{1C} + 15x_{2A} + 10x_{2B} + 12x_{2C}$
$+3x_{3A} + 9x_{3B} + 10x_{3C}$

subject to:

$x_{1A} + x_{1B} + x_{1C} = 120$
$x_{2A} + x_{2B} + x_{2C} = 80$
$x_{3A} + x_{3B} + x_{3C} = 80$
$x_{1A} + x_{2A} + x_{3A} \leq 150$
$x_{1B} + x_{2B} + x_{3B} \leq 70$
$x_{1C} + x_{2C} + x_{3C} \leq 100$
$x_{ij} \geq 0$

The "=" comes from the verbiage of the problem…no storage space at the plants

The ≤ also comes from the verbiage…demand is sometimes overstated by the construction sites.

Example Problem Solution
Solution with Excel

	A	B	C	D	E	F	G	H	I	J	K
1											
2		Site A	B	C	Supply		Decision Valriables			Supply Used	
3	Plant 1	8	5	6	120		30	0	90	120	
4	2	15	10	12	80		0	70	10	80	
5	3	3	9	10	80		80	0	0	80	
6	Demand	150	70	100		Demand Met	110	70	100		
7											
8					Total Cost	1840					
9											
10											