SUPPLEMENT 1 TO CHAPTER 18
DERIVATION OF THE OPTIMAL POLICY FOR THE STOCHASTIC SINGLE-PERIOD MODEL FOR PERISHABLE PRODUCTS

18S1-1.

$C(\underline{S}) = c\underline{S} + h\int_0^{\underline{S}}(\underline{S}-x)\underline{f}(x)dx + p\int_{\underline{S}}^{\infty}(x-\underline{S})\underline{f}(x)dx + kP\{D \geq \underline{S}\}$

D is uniformly distributed on $[a,b]$, so $P\{D \geq \underline{S}\} = \frac{b-\underline{S}}{b-a}$.

$C(\underline{S}) = c\underline{S} + k\frac{b-\underline{S}}{b-a} + L(\underline{S}) \Rightarrow \frac{dC(\underline{S})}{d\underline{S}} = c - \frac{k}{b-a} + h\underline{F}(\underline{S}) - p\{1-\underline{F}(\underline{S})] = 0$

$\Rightarrow \underline{F}(\underline{S}) = \frac{p+\frac{k}{b-a}-c}{p+h}$

Let $p = c+2, k = 14, h = -(c-1), a = 40, b = 60$.

$\Rightarrow \underline{F}(\underline{S}) = \frac{\underline{S}-40}{20} = \frac{2.7}{3} = 0.9 \Rightarrow \underline{S} = 58$

18S1-2.

(a)

$$C(\underline{I},\underline{S}) = c(\underline{S}-\underline{I}) + pP\{D > \underline{S}\} = c(\underline{S}-\underline{I}) + pe^{-\underline{S}}$$
$$\Rightarrow \frac{\partial C(\underline{I},\underline{S})}{\partial \underline{S}} = c - pe^{-\underline{S}} = 0 \Rightarrow \underline{S} = -\text{ln}\,(c/p)$$

Order up to $\underline{S}$ if $\underline{I} < \underline{S}$, do not order otherwise.

(b)

$$C(\underline{I},\underline{S}) = \begin{cases} K + c(\underline{S}-\underline{I}) + pe^{-\underline{S}} & \text{if } \underline{I} < \underline{S} \\ pe^{-\underline{I}} & \text{if } \underline{I} = \underline{S} \end{cases}$$

An (s,S) policy is optimal with $S = -\text{ln}\,(c/p)$ and s being the smallest value such that

$$cs + pe^{-s} = K - c\text{ln}\,(c/p) + c.$$