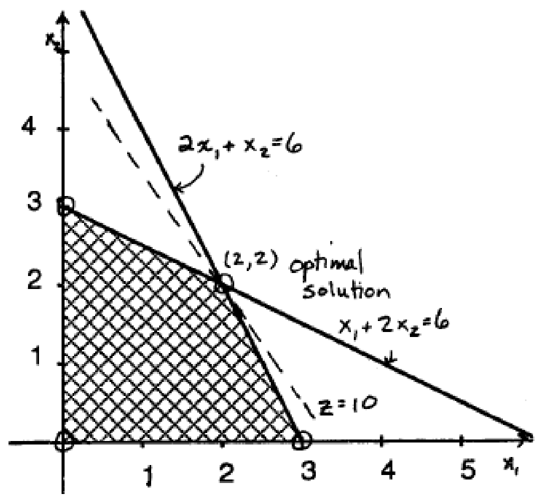
CHAPTER 5: THE THEORY OF THE SIMPLEX METHOD

5.1-1.

(a) Optimal Solution: $(x_1^*, x_2^*) = (2, 2)$ and $Z^* = 10$



(c) maximize $Z = 3x_1 + 2x_2$

subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 \quad &= 6 \\ x_1 + 2x_2 \quad + x_4 &= 6 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

(b) - (d)

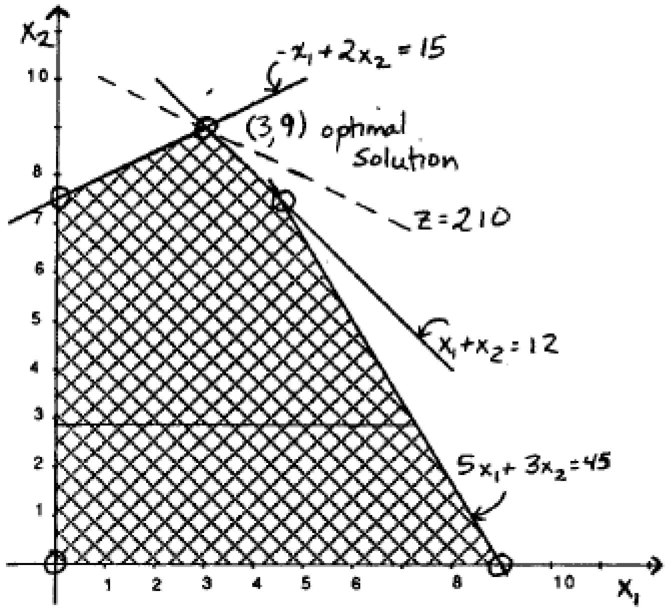
Defining Equations	CP	Feasible?	Basic Solution	Indicating Variables	Equations
$x_1 = 0$ $x_2 = 0$	$(0, 0)$	Yes	$(0, 0, 6, 6)$	x_1 x_2	$x_3 = 6$ $x_4 = 6$
$x_1 = 0$ $2x_1 + x_2 = 6$	$(0, 6)$	No	$(0, 6, 0, -6)$	x_1 x_3	$x_2 = 6$ $2x_2 + x_4 = 6$
$x_1 = 0$ $x_1 + 2x_2 = 6$	$(0, 3)$	Yes	$(0, 3, 3, 0)$	x_1 x_4	$x_2 + x_3 = 6$ $2x_2 = 6$
$x_2 = 0$ $2x_1 + x_2 = 6$	$(3, 0)$	Yes	$(3, 0, 0, 3)$	x_2 x_3	$2x_1 = 6$ $x_1 + x_4 = 6$
$x_2 = 0$ $x_1 + 2x_2 = 6$	$(6, 0)$	No	$(6, 0, -6, 0)$	x_2 x_4	$2x_1 + x_3 = 6$ $x_1 = 6$
$2x_1 + x_2 = 6$ $x_1 + 2x_2 = 6$	$(2, 2)$	Yes	$(2, 2, 0, 0)$	x_3 x_4	$2x_1 + x_2 = 6$ $x_1 + 2x_2 = 6$

(e)

Step	CPF Sol.'n	Deleted Defining Eq.	Added Defining Eq.	Deleted Ind.Var.	Added Ind.Var.
1	$(0, 0)$	$x_1 = 0$	$2x_1 + x_2 = 6$	x_1	x_3
2	$(3, 0)$	$x_2 = 0$	$x_1 + 2x_2 = 6$	x_2	x_4
3	$(2, 2)$ OPTIMAL				

5.1-2.

(a) Optimal Solution: $(x_1^*, x_2^*) = (3, 9)$ and $Z^* = 210$



(c) maximize $Z = 10x_1 + 20x_2$

subject to

$$
\begin{aligned}
-x_1 + 2x_2 + x_3 &= 15 \\
x_1 + x_2 + x_4 &= 12 \\
5x_1 + 3x_2 + x_5 &= 45 \\
x_1, x_2, x_3, x_4, x_5 &\geq 0
\end{aligned}
$$

(b) - (d)

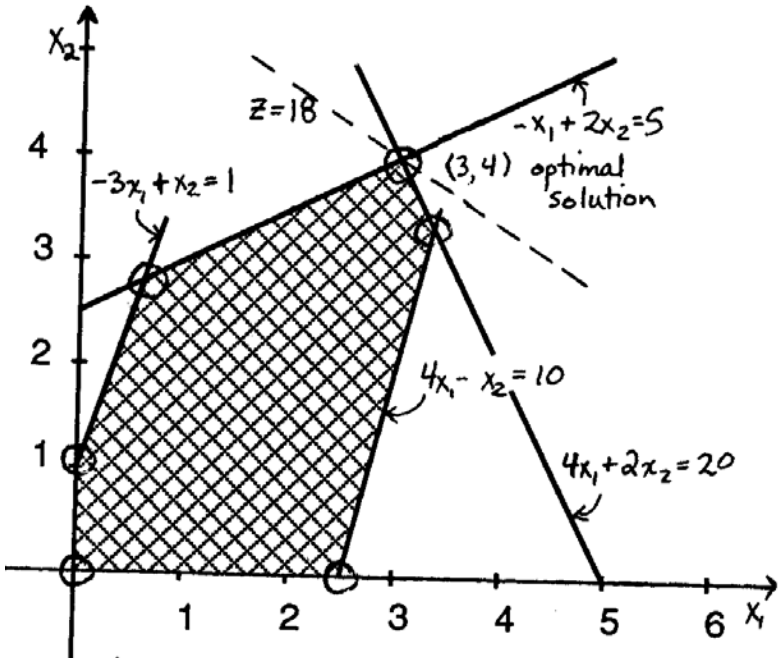
Defining Equations	CP	Feasible?	Basic Solution	Indicating Variables	Equations
$x_1 = 0$ $x_2 = 0$	(0 , 0)	Yes	(0,0,15,12,45)	x_1 x_2	$x_3 = 15$ $x_4 = 12$ $x_5 = 45$
$x_1 = 0$ $-x_1+2x_2 = 15$	(0 , 7.5)	Yes	(0,7.5,0,4.5,22.5)	x_1 x_3	$2x_2 = 15$ $x_2+x_4 = 12$ $3x_2+x_5 = 45$
$x_1 = 0$ $x_1+x_2 = 12$	(0 , 12)	No	(0,12,-9,0,9)	x_1 x_4	$2x_2+x_3 = 15$ $x_2+x_4 = 12$ $3x_2+x_5 = 45$
$x_1 = 0$ $5x_1+3x_2 = 45$	(0 , 15)	No	(0,15,-15,-3,0)	x_1 x_5	$2x_2+x_3 = 15$ $x_2+x_4 = 12$ $3x_2 = 45$
$x_2 = 0$ $-x_1+2x_2 = 15$	(-15 , 0)	No	(-15,0,0,3,120)	x_2 x_3	$x_1+x_4 = 12$ $-x_1 = 15$ $5x_1+x_5 = 45$
$x_2 = 0$ $x_1+x_2 = 12$	(12 , 0)	No	(12,0,27,0,-15)	x_2 x_4	$x_1 = 12$ $-x_1+x_3 = 15$ $5x_1+x_5 = 45$
$x_2 = 0$ $5x_1+3x_2 = 45$	(9 , 0)	Yes	(9,0,24,3,0)	x_2 x_5	$x_1+x_4 = 12$ $-x_1+x_3 = 15$ $5x_1 = 45$
$x_1+x_2 = 12$ $-x_1+2x_2 = 15$	(3 , 9)	Yes	(3,9,0,0,3)	x_3 x_4	$x_1+x_2 = 12$ $-x_1+2x_2 = 15$ $5x_1+3x_2+x_5 = 45$
$5x_1+3x_2 = 45$ $-x_1+2x_2 = 15$	(45/13 , 120/13)	No	(45/13,120/13,0,-19/13,0)	x_3 x_5	$x_1+x_2+x_4 = 12$ $-x_1+2x_2 = 15$ $5x_1+3x_2 = 45$
$x_1+x_2 = 12$ $5x_1+3x_2 = 45$	(4.5 , 7.5)	Yes	(4.5,7.5,3.5,0,0)	x_4 x_5	$x_1+x_2 = 12$ $-x_1+2x_2+x_4 = 15$ $5x_1+3x_2 = 45$

(e)

Step	CPF Sol.'n	Deleted Defining Eq.	Added Defining Eq.	Deleted Ind.Var.	Added Ind.Var.
1	$(0, 0)$	$x_2 = 0$	$-x_1 + 2x_2 = 15$	x_2	x_3
2	$(0, 7.5)$	$x_1 = 0$	$x_1 + x_2 = 12$	x_1	x_4
3	$(3, 9)$ OPTIMAL				

5.1-3.

(a) Optimal Solution: $(x_1^*, x_2^*) = (3, 4)$ and $Z^* = 18$



(b) The corner point $(3, 4)$ has the best objective value 18, so is optimal.

CPF Sol.'n	Defining Equations	BF Solution	NB Var.'s	z
$(0,0)$	$x_1 = 0, x_2 = 0$	$(0,0,1,20,10,5)$	x_1, x_2	0
$(0,1)$	$x_1 = 0, -3x_1 + x_2 = 1$	$(0,1,0,18,11,3)$	x_1, x_3	3
$(0.6, 2.8)$	$-3x_1 + x_2 = 1, -x_1 + 2x_2 = 5$	$(0.6, 2.8, 0, 12, 10.4, 0)$	x_3, x_6	9.6
$(3,4)$	$-x_1 + 2x_2 = 5, 4x_1 + 2x_2 = 20$	$(3,4,6,0,2,0)$	x_4, x_6	18
$(3.33, 3.33)$	$4x_1 + 2x_2 = 20, 4x_1 - x_2 = 10$	$(3.33, 3.33, 7.67, 0, 0, 1.67)$	x_4, x_5	16.67
$(2.5, 0)$	$4x_1 - x_2 = 10, x_2 = 0$	$(2.5, 0, 8.5, 10, 0, 7.5)$	x_2, x_5	5

(c) All sets yield a solution.

CP Infeas. Sol.'n	Defining Equations	Basic Infeas. Solutions	NB Var.'s
$(-\frac{1}{3}, 0)$	$-3x_1 + x_2 = 1, x_2 = 0$	$(-\frac{1}{3}, 0, 0, 21\frac{1}{3}, 11\frac{1}{3}, 4\frac{2}{3})$	x_2, x_3
$(-5, 0)$	$-x_1 + 2x_2 = 5, x_2 = 0$	$(-5, 0, -14, 40, 30, 0)$	x_2, x_6
$(0, 10)$	$4x_1 + 2x_2 = 20, x_1 = 0$	$(0, 10, -9, 0, 20, -15)$	x_1, x_4
$(0, \frac{5}{2})$	$-x_1 + 2x_2 = 5, x_1 = 0$	$(0, \frac{5}{2}, -\frac{3}{2}, 15, 12\frac{1}{2}, 0)$	x_1, x_6
$(\frac{9}{2}, \frac{32}{5})$	$4x_1 + 2x_2 = 20, -3x_1 + x_2 = 1$	$(\frac{9}{5}, \frac{32}{5}, 0, 0, \frac{46}{5}, -6)$	x_3, x_4
$(11, 34)$	$-3x_1 + x_2 = 1, 4x_1 - x_2 = 10$	$(11, 34, 0, -92, 0, -52)$	x_3, x_5
$(\frac{25}{7}, \frac{30}{7})$	$4x_1 - x_2 = 10, -x_1 + 2x_2 = 5$	$(\frac{25}{7}, \frac{30}{7}, \frac{52}{7}, -\frac{20}{7}, 0, 0)$	x_5, x_6
$(5, 0)$	$4x_1 + 2x_2 = 20, x_2 = 0$	$(5, 0, 16, 0, -10, 10)$	x_2, x_4
$(0, -10)$	$4x_1 - x_2 = 10, x_1 = 0$	$(0, -10, 11, 40, 0, 25)$	x_1, x_5

5.1-4.

(a) $(x_1, x_2, x_3) = (10, 0, 0)$

(b) $x_2 = 0, x_3 = 0, x_1 - x_2 + 2x_3 = 10$

5.1-5.

(a)

CPF Sol.'n	Defining Equations
$(0,0,0)$	$x_1=0, x_2=0, x_3=0$
$(4,0,0)$	$x_1=4, x_2=0, x_3=0$
$(4,2,0)$	$x_1=4, x_1+x_2=6, x_3=0$
$(2,4,0)$	$x_2=4, x_1+x_2=6, x_3=0$
$(0,4,0)$	$x_1=0, x_2=4, x_3=0$
$(0,4,2)$	$x_1=0, x_2=4, -x_1+2x_3=4$
$(2,4,3)$	$x_1+x_2=6, x_2=4, -x_1+2x_3=4$
$(4,2,4)$	$x_1+x_2=6, x_1=4, -x_1+2x_3=4$
$(4,0,4)$	$x_2=0, x_1=4, -x_1+2x_3=4$
$(0,0,2)$	$x_2=0, x_1=0, -x_1+2x_3=4$

(b) $x_1+x_2=6, x_2=4, -x_1+2x_3=4$

(c) $x_1=4, x_1=0, x_2=0 \Rightarrow$ inconsistent system

5.1-6.

(a) - (b)

Defining Equations	CP	Feas.?	Basic Solution	NB Var.'s
$x_1=0, x_2=0$	$(0,0)$	No	$(10,0,-10,6,-6)$	x_1, x_2
$x_1=0, 2x_1+x_2=10$	$(0,10)$	No	$(0,10,0,-14,4)$	x_1, x_3
$x_1=0, -3x_1+2x_2=6$	$(0,3)$	No	$(0,3,-7,0,-3)$	x_1, x_4
$x_1=0, x_1+x_2=6$	$(0,6)$	No	$(0,6,-4,-6,0)$	x_1, x_5
$x_2=0, 2x_1+x_2=10$	$(5,0)$	No	$(5,0,0,21,-11)$	x_2, x_3
$x_2=0, -3x_1+2x_2=6$	$(-2,0)$	No	$(-2,0,-14,0,-8)$	x_2, x_4
$x_2=0, x_1+x_2=6$	$(6,0)$	Yes	$(6,0,2,24,0)$	x_2, x_5
$2x_1+x_2=10, -3x_1+2x_2=6$	$(2,6)$	Yes	$(2,6,0,0,8)$	x_3, x_4
$2x_1+x_2=10, x_1+x_2=6$	$(4,2)$	Yes	$(4,2,0,14,0)$	x_3, x_5
$-3x_1+2x_2=6, x_1+x_2=6$	$(1.2,4.8)$	Yes	$(1.2,4.8,-2.8,0,0)$	x_4, x_5

5.1-7.

(a) - (b)

Defining Equations	CP	Feas.?	Basic Solution	NB Var.'s
$x_1=0, x_2=0$	$(0,0)$	Yes	$(0,0,10,60,18,44)$	x_1, x_2
$x_1=0, x_2=10$	$(0,10)$	Yes	$(0,10,0,10,8,34)$	x_1, x_3
$x_1=0, 2x_1+5x_2=60$	$(0,12)$	No	$(0,12,-2,0,6,32)$	x_1, x_4
$x_1=0, x_1+x_2=18$	$(0,18)$	No	$(0,18,-8,-30,0,26)$	x_1, x_5
$x_1=0, 3x_1+x_2=44$	$(0,44)$	No	$(0,44,-34,-160,-26,0)$	x_1, x_6
$x_2=0, x_2=10$	No Solution			x_2, x_3
$x_2=0, 2x_1+5x_2=60$	$(30,0)$	No	$(30,0,10,0,-12,-46)$	x_2, x_4
$x_2=0, x_1+x_2=18$	$(18,0)$	No	$(18,0,10,24,0,-10)$	x_2, x_5
$x_2=0, 3x_1+x_2=44$	$(14.67,0)$	Yes	$(14.67,0,10,30.67,3.33,0)$	x_2, x_6
$x_2=10, 2x_1+5x_2=60$	$(5,10)$	Yes	$(5,10,0,0,3,19)$	x_3, x_4
$x_2=10, x_1+x_2=18$	$(8,10)$	No	$(8,10,0,-6,0,10)$	x_3, x_5
$x_2=10, 3x_1+x_2=44$	$(11.33,10)$	No	$(11.33,10,0,-12.67,-3.33,0)$	x_3, x_6
$2x_1+5x_2=60, x_1+x_2=18$	$(10,8)$	Yes	$(10,8,2,0,0,6)$	x_4, x_5
$2x_1+5x_2=60, 3x_1+x_2=44$	$(12.31,7.08)$	No	$(12.31,7.08,2.92,0,-1.38,0)$	x_4, x_6
$x_1+x_2=18, 3x_1+x_2=44$	$(13,5)$	Yes	$(13,5,5,9,0,0)$	x_5, x_6

5.1-8.

(a) If the feasible region is unbounded, then there may be no optimal solution.

(b) There may be multiple optimal solutions, in which case the weighted average of any optimal CPF solutions is optimal, too.

(c) An adjacent CPF solution may have an equal objective function value, then all the points that lie on the line segment between these two corner points are optimal.

5.1-9.

(a) FALSE. (p.5-10) Property 1: (a) If there is exactly one optimal solution, then it must be a CPF solution. (b) If there are multiple optimal solutions, then at least two of them must be adjacent CPF solutions. An optimal solution that is not a CPF solution can be obtained by taking a convex combination of two optimal CPF solutions.

(b) FALSE. (p.5-12) The number of CPF solutions is at most $\binom{m+n}{n} = \frac{(m+n)!}{m!n!}$.

(c) FALSE. (p.5-13) The adjacent CPF solution that has a better objective function value than the initial CPF solution may be adjacent to another CPF solution that has an even better objective function value.

5.1-10.

(a) TRUE. By Property 1(a), there must be multiple solutions, since this optimal solution is not a CPF solution. But then, there must be infinitely many optimal solutions, namely any convex combination of optimal solutions.

(b) TRUE. Any point x on the line segment connecting x^* and x^{**} can be expressed as $x = \alpha x^* + (1-\alpha)x^{**}$ with $\alpha \in [0,1]$. Both x^* and x^{**} have the optimal objective value Z^*. The objective function value at x is

$$Z = c^T(\alpha x^* + (1-\alpha)x^{**}) = \alpha Z^* + (1-\alpha)Z^* = Z^*,$$

so x is optimal. Since the feasible region is convex, any such point is feasible.

(c) FALSE. The simultaneous solution of any set of n constraint boundary equations may be infeasible or may not even exist.

5.1-11.

(a) TRUE. If there are no optimal solutions, then either the problem is infeasible or the objective value is unbounded (Chapter 3). The former is not the case by assumption of the problem. Also by assumption again, the feasible region is bounded, so the objective value is bounded, so the latter cannot be the case. Hence, there must be at least one optimal solution.

(b) FALSE. If a solution is optimal, it need not be a BF solution. A convex combination of two optimal BF solutions is optimal even though it is not a BF solution. This follows from Property 1, since BF solutions are CPF solutions.

(c) TRUE. Since BF solutions correspond to CPF solutions, this follows directly from Property 2.

5.1-12.

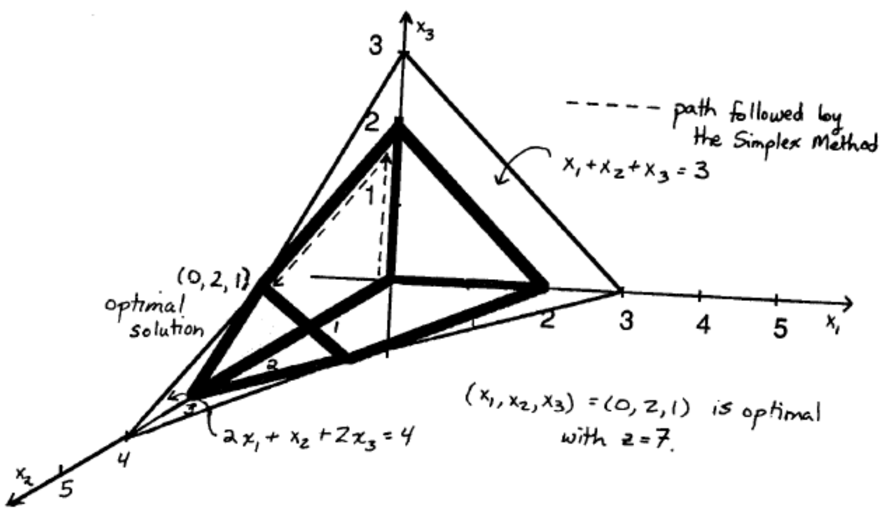
$x_1 = 0, 2x_1 + x_2 + 3x_3 = 60, 3x_1 + 3x_2 + 5x_3 = 120 \Rightarrow (x_1, x_2, x_3) = (0, 15, 15)$

5.1-13.

Since $x_2 > 0$ and $x_3 > 0$, $x_2 = 0$ and $x_3 = 0$ cannot be part of the three boundary equations, so the boundary equations are $x_1 = 0, 2x_1 + x_2 + x_3 = 20, 3x_1 + x_2 + 2x_3 = 30$. Then, the optimal solutions is $(x_1, x_2, x_3) = (0, 10, 10)$.

5.1-14.

(a)



(b) The simplex method follows this path because moving along the chosen edges provides the greatest increase in the objective value for a unit move in the chosen direction among all possible edges at each vertex/decision point.

(c)

Edge	Constraint Boundary Equations	End Points	Additional Constraints
1	$x_2 = 0, x_1 = 0$	$(0,0,0), (0,0,2)$	$x_3 = 0, 2x_1 + x_2 + 2x_3 = 4$
2	$2x_1 + x_2 + 2x_3 = 4, x_1 = 0$	$(0,0,2), (0,2,1)$	$x_2 = 0, x_1 + x_2 + x_3 = 3$

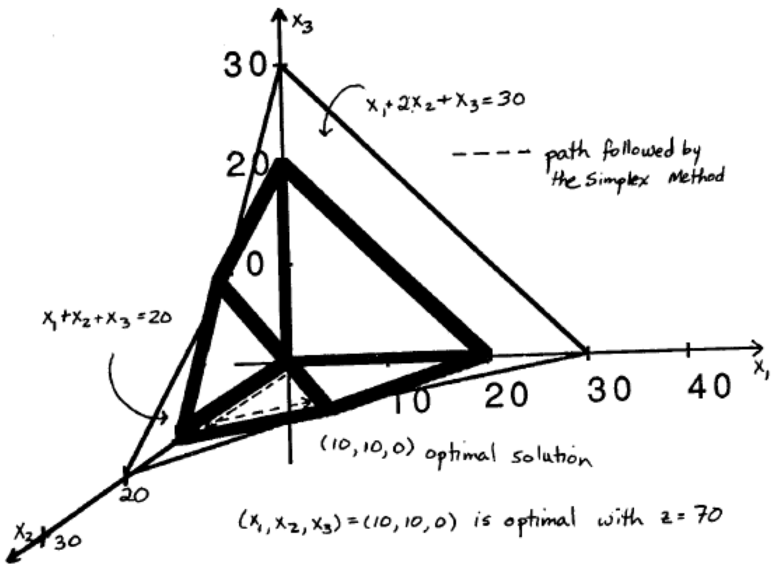
(d) - (e)

CP	Defining Equations	BF Solution	NB Var.'s
$(0,0,0)$	$x_1 = 0, x_2 = 0, x_3 = 0$	$(0,0,0,4,3)$	x_1, x_2, x_3
$(0,0,2)$	$x_1 = 0, x_2 = 0, 2x_1 + x_2 + 2x_3 = 4$	$(0,0,2,0,1)$	x_1, x_2, x_4
$(0,2,1)$	$x_1 = 0, 2x_1 + x_2 + 2x_3 = 4, x_1 + x_2 + x_3 = 3$	$(0,0,2,0,1)$	x_1, x_4, x_5

The nonbasic variables having value zero are equivalent to indicating variables. They indicate that their associated inequality constraints are actually equalities. The associated equalities are the defining equations.

5.1-15.

(a)



(b) The simplex method follows this path because moving along the chosen edges provides the greatest increase in the objective value for a unit move in the chosen direction among all possible edges at each vertex/decision point.

(c)

Edge	Constraint Boundary Equations	End Points	Additional Constraints
1	$x_1 = 0, x_3 = 0$	$(0,0,0),(0,15,0)$	$x_2 = 0, x_1 + 2x_2 + x_3 = 30$
2	$x_3 = 0, x_1 + 2x_2 + x_3 = 30$	$(0,15,0),(10,10,0)$	$x_1 = 0, x_1 + x_2 + x_3 = 20$

(d) - (e)

CP	Defining Equations	BF Solution	NB Var.'s
$(0,0,0)$	$x_1 = 0, x_2 = 0, x_3 = 0$	$(0,0,0,20,30)$	x_1, x_2, x_3
$(0,15,0)$	$x_1 = 0, x_3 = 0, x_1 + 2x_2 + x_3 = 30$	$(0,15,0,5,0)$	x_1, x_3, x_5
$(10,10,0)$	$x_3 = 0, x_1 + 2x_2 + x_3 = 30, x_1 + x_2 + x_3 = 20$	$(10,10,0,0,0)$	x_3, x_4, x_5

The nonbasic variables having value zero are equivalent to indicating variables. They indicate that their associated inequality constraints are actually equalities. The associated equalities are the defining equations.

5.1-16.

(a) When the objective is to maximize $Z = x_3$, both corner points $(4,2,4)$ and $(4,0,4)$ are optimal, with $Z^* = 4$.

(b) When the objective is to maximize $Z = -x_1 + 2x_3$, all the corner points $(0,0,2)$, $(4,0,4)$, $(4,2,4)$, $(2,4,3)$ and $(0,4,2)$ are optimal, with $Z^* = 4$.

5.1-17.

(a) Geometrically, each constraint is a plane and the points that are feasible for a given (inequality) constraint form a half-space. The line segment defined by any two feasible points must lie entirely on the feasible side of the plane and therefore, all the points on the line segment are feasible, implying that the set of solutions for any one constraint is a convex set.

(b) Because the points in the feasible region of the LP problem satisfy all the constraints simultaneously, it must be the case that for any two feasible points, the points on the line segment joining them must also satisfy each constraint (from (a)). Hence, the set of solutions that satisfy all the constraints simultaneously is a convex set.

5.1-18.

To maximize $Z = 3x_1 + 4x_2 + 3x_3$, starting at the origin $(0, 0, 0)$, one first chooses to move to $(0, 4, 0)$ because this edge offers the best rate of improvement among all edges at the origin. From $(0, 4, 0)$, the edge that increases the objective function fastest is the one that connects to either $(0, 4, 2)$ or $(2, 4, 0)$. From either one these, the edge that gives the best rate of increase connects to $(2, 4, 3)$. Then, the only edge that provides an improvement in Z connects to the optimal solution $(4, 2, 4)$.

5.1-19.

(a)

Original Constraint	Boundary Equation	Indicating Variable
$x_1 \geq 0$	$x_1 = 0$	x_1
$x_2 \geq 0$	$x_2 = 0$	x_2
$x_3 \geq 0$	$x_3 = 0$	x_3
$x_1 + x_4 = 4$	$x_1 = 4$	x_4
$x_2 + x_5 = 4$	$x_2 = 4$	x_5
$x_1 + x_2 + x_6 = 6$	$x_1 + x_2 = 6$	x_6
$-x_1 + 2x_3 + x_7 = 4$	$-x_1 + 2x_3 = 4$	x_7

(b)

CPF Sol.'n	Defining Equations	BF Solution	NB Var.'s
$(2, 4, 3)$	$x_1 + x_2 = 6, x_2 = 4, -x_1 + 2x_3 = 4$	$(2, 4, 3, 2, 0, 0, 0)$	x_5, x_6, x_7
$(4, 2, 4)$	$x_1 + x_2 = 6, -x_1 + 2x_3 = 4, x_1 = 4$	$(4, 2, 4, 0, 2, 0, 0)$	x_4, x_6, x_7
$(0, 4, 2)$	$x_1 = 0, x_2 = 4, -x_1 + 2x_3 = 4$	$(0, 4, 2, 4, 0, 2, 0)$	x_1, x_5, x_7
$(2, 4, 0)$	$x_3 = 0, x_1 + x_2 = 6, x_2 = 4$	$(2, 4, 0, 2, 0, 0, 6)$	x_3, x_5, x_6

(c) Because the sets of defining equations of $(4, 2, 4)$, $(0, 4, 2)$ and $(2, 4, 0)$ differ from the set of defining equations of $(2, 4, 3)$ by only one equation, they are adjacent to $(2, 4, 3)$. On the other hand, the sets of defining equations of $(4, 2, 4)$, $(0, 4, 2)$ and $(2, 4, 0)$ differ by more than one equation, they are not adjacent to each other. The same statement is true if we substitute "nonbasic variables" for "defining equations" and "variable" for "equation."

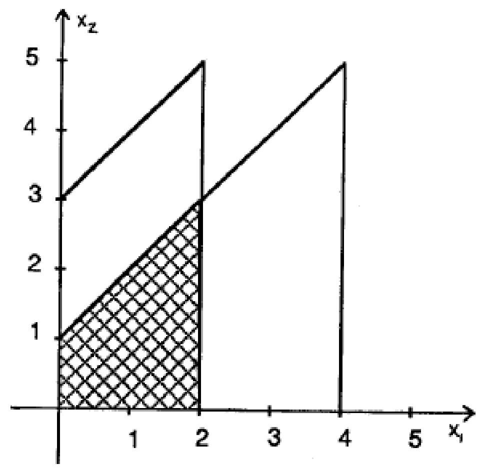
5.1-20.

(a) x_5 enters.

(b) x_4 leaves.

(c) $(4, 2, 4, 0, 2, 0, 0)$

5.1-21.



5.2-1.

(a) Optimal Solution: $\begin{pmatrix} x_3 \\ x_1 \\ x_5 \end{pmatrix} = B^{-1}b = \frac{1}{27}\begin{pmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{pmatrix}\begin{pmatrix} 180 \\ 270 \\ 180 \end{pmatrix} = \begin{pmatrix} 50 \\ 30 \\ 50 \end{pmatrix}$

$$Z = cx = \begin{pmatrix} 8 & 4 & 6 & 3 & 9 \end{pmatrix}\begin{pmatrix} 30 \\ 0 \\ 50 \\ 0 \\ 50 \end{pmatrix} = 990$$

(b) Shadow prices: $c_B B^{-1} = \frac{1}{27}\begin{pmatrix} 6 & 8 & 9 \end{pmatrix}\begin{pmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{pmatrix} = \begin{pmatrix} 1.33 \\ 1 \\ 2.67 \end{pmatrix}$

5.2-2.

$c = \begin{pmatrix} 5 & 8 & 7 & 4 & 6 & 0 & 0 \end{pmatrix}$, $A = \begin{pmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $x_B = \begin{pmatrix} x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$

$c_B = \begin{pmatrix} 0 & 0 \end{pmatrix}$, $-c = \begin{pmatrix} -5 & -8 & -7 & -4 & -6 & 0 & 0 \end{pmatrix}$, so x_2 enters.

Revised x_2 coefficients: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, so x_7 leaves.

Iteration 1: $B_{\text{new}}^{-1} = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -3/5 \\ 0 & 1/5 \end{pmatrix}$,

$$x_B = \begin{pmatrix} x_6 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & -3/5 \\ 0 & 1/5 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, c_B = \begin{pmatrix} 0 & 8 \end{pmatrix}$$

Revised row 0: $\begin{pmatrix} 0 & 8/5 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{pmatrix}$-$\begin{pmatrix} 5 & 8 & 7 & 4 & 6 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} -1/5 & 0 & -3/5 & -4/5 & -2/5 & 0 & 8/5 \end{pmatrix}$, so x_4 enters.

Revised x_4 coefficients: $\begin{pmatrix} 1 & -3/5 \\ 0 & 1/5 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4/5 \\ 2/5 \end{pmatrix}$, so x_6 leaves.

Iteration 2: $B_{\text{new}}^{-1} = \begin{pmatrix} 2 & 3 \\ 2 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} 5/4 & -3/4 \\ -1/2 & 1/2 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_4 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5/4 & -3/4 \\ -1/2 & 1/2 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 5/2 \\ 5 \end{pmatrix}, c_B = \begin{pmatrix} 4 & 8 \end{pmatrix}$$

Revised row 0: $\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{pmatrix}$-$\begin{pmatrix} 5 & 8 & 7 & 4 & 6 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$, so the current solution is optimal.

Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = (0, 5, 0, 5/2, 0)$ and $Z^* = 50$

5.2-3.

$c = \begin{pmatrix} 3 & 2 & 0 & 0 \end{pmatrix}$, $A = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$

CP $(0,0)$: $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $x_B = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$

Row 0: $\begin{pmatrix} -3 & -2 & 0 & 0 \end{pmatrix}$

CP $(3,0)$: $B = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 1/2 & 0 \\ -1/2 & 1 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_1 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ -1/2 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, c_B = \begin{pmatrix} 3 & 0 \end{pmatrix}$$

Row 0: $\begin{pmatrix} 3/2 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 2 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1/2 & 3/2 & 0 \end{pmatrix}$

CP $(2,2)$: $B = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, c_B = \begin{pmatrix} 3 & 2 \end{pmatrix}$$

Row 0: $\begin{pmatrix} 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 2 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1/3 & 1/3 \end{pmatrix}$

Optimal Solution: $(x_1^*, x_2^*) = (2, 2)$ and $Z^* = 10$

5.2-4.

$c = \begin{pmatrix} 1 & 2 & 0 & 0 \end{pmatrix}$, $A = \begin{pmatrix} 1 & 3 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $x_B = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

$c_B = \begin{pmatrix} 0 & 0 \end{pmatrix}$, Row 0: $\begin{pmatrix} -1 & -2 & 0 & 0 \end{pmatrix}$, so x_2 enters the basis.

Revised x_2 coefficients: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, so x_3 leaves the basis.

Iteration 1: $B_{\text{new}}^{-1} = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1/3 & 0 \\ -1/3 & 1 \end{pmatrix}$

$x_B = \begin{pmatrix} x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1/3 & 0 \\ -1/3 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 8/3 \\ 4/3 \end{pmatrix}$, $c_B = \begin{pmatrix} 2 & 0 \end{pmatrix}$

Revised row 0: $\begin{pmatrix} 2/3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 3 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} -1/3 & 0 & 2/3 & 0 \end{pmatrix}$, so x_1 enters the basis.

Revised x_1 coefficients: $\begin{pmatrix} 1/3 & 0 \\ -1/3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}$, so x_4 leaves.

Iteration 2: $B_{\text{new}}^{-1} = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix}$

$x_B = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix}\begin{pmatrix} 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $c_B = \begin{pmatrix} 2 & 1 \end{pmatrix}$

Revised row 0: $\begin{pmatrix} 1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 1 & 3 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 1/2 & 1/2 \end{pmatrix}$, so the current solution is optimal.

Optimal Solution: $(x_1^*, x_2^*) = (2, 2)$ and $Z^* = 6$

5.2-5.

$c = \begin{pmatrix} 5 & 4 & -1 & 3 & 0 & 0 \end{pmatrix}$, $A = \begin{pmatrix} 3 & 2 & -3 & 1 & 1 & 0 \\ 3 & 3 & 1 & 3 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 24 \\ 36 \end{pmatrix}$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $x_B = \begin{pmatrix} x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 24 \\ 36 \end{pmatrix} = \begin{pmatrix} 24 \\ 36 \end{pmatrix}$

$c_B = \begin{pmatrix} 0 & 0 \end{pmatrix}$, Row 0: $\begin{pmatrix} -5 & -4 & 1 & -3 & 0 & 0 \end{pmatrix}$, so x_1 enters the basis.

Revised x_1 coefficients: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$, so x_5 leaves the basis.

Iteration 1: $B_{\text{new}}^{-1} = \begin{pmatrix} 3 & 0 \\ 3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_1 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 24 \\ 36 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix}, \; c_B = \begin{pmatrix} 5 & 0 \end{pmatrix}$$

Revised row 0: $\begin{pmatrix} 5/3 & 0 \end{pmatrix}\begin{pmatrix} 3 & 2 & -3 & 1 & 1 & 0 \\ 3 & 3 & 1 & 3 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 5 & 4 & -1 & 3 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & -2/3 & -4 & -4/3 & 5/3 & 0 \end{pmatrix}$, so x_3 enters the basis.

Revised x_3 coefficients: $\begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, so x_6 leaves.

Iteration 2: $B_{\text{new}}^{-1} = \begin{pmatrix} 3 & -3 \\ 3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1/12 & 1/4 \\ -1/4 & 1/4 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/12 & 1/4 \\ -1/4 & 1/4 \end{pmatrix}\begin{pmatrix} 24 \\ 36 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \end{pmatrix}, \; c_B = \begin{pmatrix} 5 & -1 \end{pmatrix}$$

Revised row 0: $\begin{pmatrix} 2/3 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 & -3 & 1 & 1 & 0 \\ 3 & 3 & 1 & 3 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 5 & 4 & -1 & 3 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1/3 & 0 & 2/3 & 2/3 & 1 \end{pmatrix}$, so current solution is optimal.

Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (11, 0, 3, 0)$ and $Z^* = 52$

5.3-1.

(a) $B^{-1} = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$

Final constraint columns for (x_1, x_2, x_3):

$$B^{-1}A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 1 & 0 \\ 2 & 0 & 0 \\ 4 & 0 & 1 \end{pmatrix}$$

$c_B = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix}$

Final objective coefficients for (x_1, x_2, x_3):

$$c_B B^{-1}A - c = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 5 & 1 & 0 \\ 2 & 0 & 0 \\ 4 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \end{pmatrix}$$

Right-hand side:

$$B^{-1}b = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 14 \\ 5 \\ 11 \end{pmatrix} \text{ and } z = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 14 \\ 5 \\ 11 \end{pmatrix} = 8$$

Final tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	2	0	0	1	1	0	8
X2	1	0	5	1	0	1	3	0	14
X6	2	0	2	0	0	0	1	1	5
X3	3	0	4	0	1	1	2	0	11

(b) Defining equations: $2x_1 - 2x_2 + 3x_3 = 5, x_1 + x_2 - x_3 = 3, x_1 = 0$

5.3-2.

(a) $B^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$

Final constraint columns for (x_1, x_2, x_3, x_4):

$$B^{-1}A = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 4 & 2 & 1 & 1 \\ 3 & 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 2 & 0 & 3 & 1 \end{pmatrix}$$

$c_B = \begin{pmatrix} 3 & 2 \end{pmatrix}$

Final objective coefficients for (x_1, x_2, x_3, x_4):

$$c_B B^{-1}A - c = \begin{pmatrix} 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 & 0 \\ 2 & 0 & 3 & 1 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 2 & 0 \end{pmatrix}$$

Right-hand side:

$$B^{-1}b = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ and } Z = \begin{pmatrix} 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = 9$$

Final tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	3	0	2	0	1	1	9
X2	1	0	1	1	-1	0	1	-1	1
X4	2	0	2	0	3	1	-1	2	3

(b) Defining equations: $4x_1 + 2x_2 + x_3 + x_4 = 5, 3x_1 + x_2 + 2x_3 + x_4 = 4, x_1 = 0,$ $x_3 = 0$

5.3-3.

$$B^{-1}=\begin{pmatrix}1&1&2\\-2&0&4\\1&0&-1\end{pmatrix}$$

Final constraint columns for (x_1,x_2,x_3):

$$B^{-1}A=\begin{pmatrix}1&1&2\\-2&0&4\\1&0&-1\end{pmatrix}\begin{pmatrix}2&2&1/2\\-4&-2&-3/2\\1&2&1/2\end{pmatrix}=\begin{pmatrix}0&4&0\\0&4&1\\1&0&0\end{pmatrix}$$

$c_B=(\,0\quad 2\quad 6\,)$

Final objective coefficients for (x_1,x_2,x_3):

$$c_BB^{-1}A-c=(\,0\quad 2\quad 6\,)\begin{pmatrix}0&4&0\\0&4&1\\1&0&0\end{pmatrix}-(\,6\quad 1\quad 2\,)=(\,0\quad 7\quad 0\,)$$

Right-hand side:

$$B^{-1}b=\begin{pmatrix}1&1&2\\-2&0&4\\1&0&-1\end{pmatrix}\begin{pmatrix}2\\3\\1\end{pmatrix}=\begin{pmatrix}7\\0\\1\end{pmatrix}\text{ and }Z=(\,0\quad 2\quad 6\,)\begin{pmatrix}7\\0\\1\end{pmatrix}=6$$

Final tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	7	0	2	0	2	6
X5	1	0	0	4	0	1	1	2	7
X3	2	0	0	4	1	-2	0	4	0
X1	3	0	1	0	0	1	0	-1	1

(Coefficient of: X1 … X6)

5.3-4.

(a) $B^{-1}=\begin{pmatrix}3/16&-1/8&0&0\\-1/4&1/2&0&0\\-3/8&1/4&1&0\\0&0&0&1\end{pmatrix}$

Current constraint columns for (x_1,x_2,x_3):

$$B^{-1}A=\begin{pmatrix}3/16&-1/8&0&0\\-1/4&1/2&0&0\\-3/8&1/4&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}8&2&3\\4&3&0\\2&0&1\\0&0&1\end{pmatrix}=\begin{pmatrix}1&0&9/16\\0&1&-3/4\\0&0&-1/8\\0&0&1\end{pmatrix}$$

$c_B=(\,20\quad 6\quad 0\quad 0\,)$

Current objective coefficients for (x_1, x_2, x_3):

$$c_B B^{-1} A - c = \begin{pmatrix} 20 & 6 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 9/16 \\ 0 & 1 & -3/4 \\ 0 & 0 & -1/8 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 20 & 6 & 8 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -5/4 \end{pmatrix}$$

Right-hand side:

$$B^{-1} b = \begin{pmatrix} 3/16 & -1/8 & 0 & 0 \\ -1/4 & 1/2 & 0 & 0 \\ -3/8 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 200 \\ 100 \\ 50 \\ 20 \end{pmatrix} = \begin{pmatrix} 25 \\ 0 \\ 0 \\ 20 \end{pmatrix} \text{ and } Z = \begin{pmatrix} 20 & 6 & 0 & 0 \end{pmatrix} \begin{pmatrix} 25 \\ 0 \\ 0 \\ 20 \end{pmatrix} = 500$$

Current tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
Z	0	1	0	0	-1.25	2.25	0.5	0	0	500
X1	1	0	1	0	0.563	0.188	-0.13	0	0	25
X2	2	0	0	1	-0.75	-0.25	0.5	0	0	0
X6	3	0	0	0	-0.13	-0.38	0.25	1	0	0
X7	4	0	0	0	1	0	0	0	1	20

(b) The revised simplex method would generate the reduced costs for row 0 and then the revised column for x_3.

(c) Defining equations: $8x_1 + 2x_2 + 3x_3 = 200, 4x_1 + 3x_2 = 100$, $x_3 = 0$

Note that $2x_1 + x_3 = 50$ is also binding at the current solution.

5.3-5.

(a)

$$\begin{pmatrix} -c_1 & -c_2 & -c_3 & \vdots & 0 & 0 & \vdots & 0 \end{pmatrix} + \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 1 & 0 & \vdots & b \\ 2 & 1 & 3 & 0 & 1 & \vdots & 2b \end{pmatrix} = \begin{pmatrix} \frac{7}{16} & 0 & 0 & \frac{3}{5} & \frac{4}{5} & z^* \end{pmatrix}$$

$-c_1 + \frac{11}{5} = \frac{7}{16} \Rightarrow c_1 = \frac{3}{2}, -c_2 + 2 = 0 \Rightarrow c_2 = 2, -c_3 + 3 = 0 \Rightarrow c_3 = 3$

(b) $B^{-1} = \begin{pmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{pmatrix}$, $B^{-1} b = b^* \Leftrightarrow \begin{pmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{pmatrix} \begin{pmatrix} b \\ 2b \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \Leftrightarrow b = 5$

(c) Using (a): $Z^* = c_B b^* = \begin{pmatrix} c_2 & c_3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 11$

Using (b): $Z^* = \overline{c}_B b = \begin{pmatrix} 3/5 & 4/5 \end{pmatrix} \begin{pmatrix} b \\ 2b \end{pmatrix} = \begin{pmatrix} 3/5 & 4/5 \end{pmatrix} \begin{pmatrix} 5 \\ 10 \end{pmatrix} = 11$

5.3-6.

Iteration 1: Multiply row 2 by $5/2$ and add to row 0, i.e., premultiply A_0 by $(0 \quad 5/2 \quad 0)$ and add to row 0, where

$$A_0 = \begin{pmatrix} 1 & 0 & \vdots & 1 & 0 & 0 & \vdots & 4 \\ 0 & 2 & \vdots & 0 & 1 & 0 & \vdots & 12 \\ 3 & 2 & \vdots & 0 & 0 & 1 & \vdots & 18 \end{pmatrix}.$$

Iteration 2: Add row 3 to row 0, i.e., premultiply A_1 by $(0 \quad 0 \quad 1)$ and add to row 0, where

$$A_1 = \begin{pmatrix} 1 & 0 & \vdots & 1 & 0 & 0 & \vdots & 4 \\ 0 & 1 & \vdots & 0 & 1/2 & 0 & \vdots & 6 \\ 3 & 0 & \vdots & 0 & -1 & 1 & \vdots & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{pmatrix} A_0.$$

Therefore, the final row 0 is: initial row $0 + (0 \quad 5/2 \quad 0)A_0 + (0 \quad 0 \quad 1)A_1$,

$$= (-3 \quad -5 \quad \vdots \quad 0 \quad 0 \quad 0 \quad \vdots \quad 0) + \left[\begin{pmatrix} 0 & \frac{5}{2} & 0 \end{pmatrix} + (0 \quad 0 \quad 1)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{pmatrix}\right] A_0$$

$$= (-3 \quad -5 \quad \vdots \quad 0 \quad 0 \quad 0 \quad \vdots \quad 0) + \begin{pmatrix} 0 & \frac{3}{2} & 1 \end{pmatrix} A_0$$

5.3-7.

(a) Use the columns corresponding to artificial variables in exactly the same way as a slack variable would have been used. Note that the shadow price of this column may be positive or negative.

(b) For the reversed inequalities, use the negative of the column corresponding to the slack variable in exactly the same formulae. The artificial column may be discarded.

(c) Same as (b).

(d) No change, use slack and artificial variables as above.

5.3-8.

$$\begin{array}{lll} M\text{aximize} & Z = & 90x_1 + 70x_2 \qquad\qquad - Mx_5 \\ \text{subject to} & & 2x_1 + x_2 + x_3 \qquad\quad = 2 \\ & & x_1 - x_2 \qquad - x_4 + \quad x_5 = 2 \\ & & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}$$

Initial Tableau:

			Coefficient of					
BV	Eq	Z	x_1	x_2	x_3	x_4	x_5	RS
Z	0	1	$-90-M$	$-70+M$	0	M	0	$-2M$
x_3	1	0	2	1	1	0	0	2
x_5	2	0	2	-1	0	-1	1	2

The columns that will contain S^* for applying the fundamental insight in the final tableau are those associated with x_3 and x_5, since those columns form the 2×2 identity matrix in the initial tableau.

5.3-9.

(a) $B^{-1} = \begin{pmatrix} 3/10 & -1/10 \\ -2/10 & 2/5 \end{pmatrix}$

Final constraint columns for $(x_1, x_2, x_3, x_4, x_6)$:

$$B^{-1}A = \begin{pmatrix} 3/10 & -1/10 \\ -2/10 & 2/5 \end{pmatrix}\begin{pmatrix} 1 & 4 & 2 & -1 & 0 \\ 3 & 2 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 3/5 & -3/10 & 1/10 \\ 1 & 0 & -2/5 & 2/10 & -2/5 \end{pmatrix}$$

$$c_B = (\,-6M+3 \quad -4M+2\,)$$

Final objective coefficients for $(x_1, x_2, x_3, x_4, x_6)$:

$$-c_B B^{-1}A + c = -\,(\,-6M+3 \quad -4M+2\,)\begin{pmatrix} 0 & 1 & 3/5 & -3/10 & 1/10 \\ 1 & 0 & -2/5 & 2/10 & -2/5 \end{pmatrix}$$

$$+\,(\,-4M{+}2 \quad -6M{+}3 \quad -2M{+}2 \quad M \quad M\,) = (\,0 \quad 0 \quad 1 \quad 1/2 \quad 1/2\,)$$

Right-hand side:

$$B^{-1}b = \begin{pmatrix} 3/10 & -1/10 \\ -2/10 & 2/5 \end{pmatrix}\begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 9/5 \\ 4/5 \end{pmatrix}$$

$$z = -14M + c_B x_B = -14M + (\,-6M+3 \quad -4M+2\,)\begin{pmatrix} 9/5 \\ 4/5 \end{pmatrix} = 7$$

Final tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X_6	$\bar{X}_5$	$\bar{X}_7$	Right side
								1M	1M	
Z	0	-1	0	0	1	0.5	0.5	-0.5	-0.5	-7
X2	1	0	0	1	0.6	-0.3	0.1	0.3	-0.1	1.8
X1	2	0	1	0	-0.4	0.2	-0.4	-0.2	0.4	0.8

(b) The constraints in the original tableau can be expressed as $(\,A \;\vdots\; I \;\vdots\; I \;\vdots\; b\,)$ with the second identity matrix corresponding to the artificial variables. Premultiply this matrix by M to get:

$$(\,A^* \;\vdots\; S^* \;\vdots\; L^* \;\vdots\; b^*) = \mathrm{M}(\,A \;\vdots\; I \;\vdots\; I \;\vdots\; b\,) = (\,\mathrm{M}A \;\vdots\; \mathrm{M} \;\vdots\; \mathrm{M} \;\vdots\; \mathrm{M}b\,),$$

where $\mathrm{M} = S^* = L^* = \begin{pmatrix} 3/10 & -1/10 \\ -1/5 & 2/5 \end{pmatrix}$.

Original row 0: $t = (\,c{+}e^T A\mathrm{M} \;\vdots\; \mathrm{M}e^T \;\vdots\; 0 \;\vdots\; -\mathrm{M}e^T b\,)$

Final tableau: $t^* = t + v^T(\,A \;\vdots\; I \;\vdots\; I \;\vdots\; b\,)$

$= (\,Z^*{+}c \;\vdots\; -y^* \;\vdots\; \mathrm{M}e^T{-}y^* \;\vdots\; Z^*\,)$

$= (\,c{+}e^T A\mathrm{M} \;\vdots\; \mathrm{M}e^T \;\vdots\; 0 \;\vdots\; -\mathrm{M}e^T b\,) + v^T(\,A \;\vdots\; I \;\vdots\; I \;\vdots\; b\,)$

$= (\,c{+}e^T A\mathrm{M}{+}v^T A \;\vdots\; \mathrm{M}e^T{+}v \;\vdots\; v \;\vdots\; -\mathrm{M}e^T b + v^T b\,)$

Hence, $v = -y^* + \mathrm{M}e^T = \left(-\frac{1}{2}{+}\mathrm{M} \quad -\frac{1}{2}{+}\mathrm{M}\right)$.

(c) $t = (\,2 \quad 3 \quad 2 \quad 0 \quad \mathrm{M} \quad 0 \quad \mathrm{M} \quad 0\,) = (\,c \;\vdots\; 0 \;\vdots\; \mathrm{M}e^T \;\vdots\; \mathrm{M}e^T b\,)$

$t^* = t + v^T(\,A \;\vdots\; I \;\vdots\; I \;\vdots\; b\,)$

$= (\,Z^*{+}c \;\vdots\; -y^* \;\vdots\; \mathrm{M}e^T{-}y^* \;\vdots\; Z^*\,)$

$= (\,c \;\vdots\; 0 \;\vdots\; \mathrm{M}e^T \;\vdots\; \mathrm{M}e^T b\,) + v^T(\,A \;\vdots\; I \;\vdots\; I \;\vdots\; b\,)$

$= (\,c{+}v^T A \;\vdots\; v \;\vdots\; \mathrm{M}e^T{+}v \;\vdots\; \mathrm{M}e^T b + v^T b\,)$

Hence, $v = -y^* = \left(-\frac{1}{2} \quad -\frac{1}{2}\right)$.

(d) Defining equations: $x = \mathrm{M}b \Leftrightarrow \mathrm{M}^{-1}x = b$

$$x_1 + 4x_2 + 2x_3 = 8, 3x_1 + 2x_2 = 6, x_3 = 0$$

5.3-10.

(a) $\quad -2x_1 + 2x_2 + x_3 + x_4 \qquad = 10$ (i)

$\qquad 3x_1 + \; x_2 - x_3 \qquad + x_5 = 20$ (ii)

Multiply (i) by 1.5 and add to (ii).

$$4x_2 + \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 + x_5 = 35 \text{ (iii)}$$

Divide (*) by -2 and add to (iii).

$$x_1 + 3x_2 + x_4 + x_5 = 30 \text{ (iv)}$$

Multiply (iii) by 2.

$$8x_2 + x_3 + 3x_4 + 2x_5 = 70 \text{ (v)}$$

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (30, 0, 70)$ and $Z^* = 230$

(b) $\quad$ (original objective)-3(iv) $-$ 2(v)

$$\begin{aligned} &3x_1 + 7x_2 + 2x_3 \\ &-3x_1 - 9x_2 - 3x_3 \qquad - 3x_5 \\ &\qquad -16x_2 - 2x_3 - 6x_4 - 4x_5 \\ \Rightarrow &\qquad -18x_2 - 3x_3 - 6x_4 - 7x_5 \end{aligned}$$

Hence, the shadow prices are 9 and 7.

(c) Defining equations: $-2x_1 + 2x_2 + x_3 = 10, 3x_1 + x_2 - x_3 = 20, x_2 = 0$

(d) $B = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}, B^{-1} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}, x_B = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 10 \\ 20 \end{pmatrix} = \begin{pmatrix} 30 \\ 70 \end{pmatrix}$

$$y^* = \begin{pmatrix} 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 9 & 7 \end{pmatrix}$$

Revised row 0: $\begin{pmatrix} 9 & 7 \end{pmatrix}\begin{pmatrix} -2 & 2 & 1 & 1 & 0 \\ 3 & 1 & -1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 7 & 2 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 18 & 0 & 9 & 7 \end{pmatrix}$,

so the current solution is optimal.

(e) Final tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	18	0	9	7	230
X1	1	0	1	3	0	1	1	30
X3	2	0	0	8	1	3	2	70

5.4-1.

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Revised x_2 coefficients: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

x_2 enters and x_7 leaves.

Iteration 1: $\eta = \begin{pmatrix} -\frac{a_{12}}{a_{22}} \\ \frac{1}{a_{22}} \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} \\ \frac{1}{5} \end{pmatrix}$

$B_{\text{new}}^{-1} = \begin{pmatrix} 1 & -\frac{3}{5} \\ 0 & \frac{1}{5} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{3}{5} \\ 0 & \frac{1}{5} \end{pmatrix}$

Revised x_4 coefficients: $\begin{pmatrix} 1 & -\frac{3}{5} \\ 0 & \frac{1}{5} \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{2}{5} \end{pmatrix}$

x_4 enters and x_6 leaves.

Iteration 2: $\eta = \begin{pmatrix} \frac{1}{a'_{11}} \\ -\frac{a'_{24}}{a'_{11}} \end{pmatrix} = \begin{pmatrix} \frac{5}{4} \\ -\frac{1}{2} \end{pmatrix}$

$B_{\text{new}}^{-1} = \begin{pmatrix} \frac{5}{4} & 0 \\ -\frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 & -\frac{3}{5} \\ 0 & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & -\frac{3}{4} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

5.4-2.

$c = (1 \quad 2 \quad 4 \quad 0 \quad 0 \quad 0)$, $A = \begin{pmatrix} 3 & 1 & 5 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$x_B = \begin{pmatrix} x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}$

$c_B = (0 \quad 0 \quad 0)$, Row 0: $(-1 \quad -2 \quad -4 \quad 0 \quad 0 \quad 0)$

x_3 enters the basis.

Revised x_3 coefficients: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$

x_4 leaves the basis.

Iteration 1: $\eta = \begin{pmatrix} \frac{1}{5} \\ -\frac{1}{5} \\ -\frac{2}{5} \end{pmatrix}$

$$B_{\text{new}}^{-1} = \begin{pmatrix} \frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & 1 & 0 \\ -\frac{2}{5} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & 1 & 0 \\ -\frac{2}{5} & 0 & 1 \end{pmatrix}$$

$$x_B = \begin{pmatrix} x_3 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & 1 & 0 \\ -\frac{2}{5} & 0 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$$

$c_B = \begin{pmatrix} 4 & 0 & 0 \end{pmatrix}$

Revised row 0:

$$\begin{pmatrix} \frac{4}{5} & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 & 1 & 5 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 4 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{7}{5} & -\frac{6}{5} & 0 & \frac{4}{5} & 0 & 0 \end{pmatrix}$$

x_2 enters the basis.

Revised x_2 coefficients: $\begin{pmatrix} \frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & 1 & 0 \\ -\frac{2}{5} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} \\ \frac{19}{5} \\ -\frac{2}{5} \end{pmatrix}$

x_5 leaves.

Iteration 2: $\eta = \begin{pmatrix} -\frac{1}{19} \\ \frac{5}{19} \\ \frac{2}{19} \end{pmatrix}$

$$B_{\text{new}}^{-1} = \begin{pmatrix} 1 & -\frac{1}{19} & 0 \\ 0 & \frac{5}{19} & 0 \\ 0 & \frac{2}{19} & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & 1 & 0 \\ -\frac{2}{5} & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{4}{19} & -\frac{1}{19} & 0 \\ -\frac{1}{19} & \frac{5}{19} & 0 \\ -\frac{8}{19} & \frac{2}{19} & 1 \end{pmatrix}$$

$$x_B = \begin{pmatrix} x_3 \\ x_2 \\ x_6 \end{pmatrix} = \begin{pmatrix} \frac{4}{19} & -\frac{1}{19} & 0 \\ -\frac{1}{19} & \frac{5}{19} & 0 \\ -\frac{8}{19} & \frac{2}{19} & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix} = \begin{pmatrix} \frac{32}{19} \\ \frac{30}{19} \\ \frac{69}{19} \end{pmatrix}$$

$c_B = \begin{pmatrix} 4 & 2 & 0 \end{pmatrix}$

Revised row 0:

$$\begin{pmatrix} \frac{14}{19} & \frac{6}{19} & 0 \end{pmatrix} \begin{pmatrix} 3 & 1 & 5 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 4 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{29}{19} & 0 & 0 & \frac{14}{19} & \frac{6}{19} & 0 \end{pmatrix}$$

The current solution is optimal.

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = \left(0, \frac{30}{19}, \frac{32}{19}\right)$ and $Z^* = \frac{188}{19}$

5.4-3.

$$c = \begin{pmatrix} 2 & -2 & 3 & 0 & 0 & 0 \end{pmatrix}, A = \begin{pmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix}$$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix}$$

$c_B = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$, Row 0: $\begin{pmatrix} -2 & 2 & -3 & 0 & 0 & 0 \end{pmatrix}$

x_3 enters the basis.

Revised x_3 coefficients: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

x_5 leaves the basis.

Iteration 1: $\eta = \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix}$, $B_{\text{new}}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_4 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$$

$c_B = \begin{pmatrix} 0 & 3 & 0 \end{pmatrix}$

Revised row 0:

$$\begin{pmatrix} 0 & 3 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & -2 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -1 & 0 & 0 & 3 & 0 \end{pmatrix}$$

x_2 enters the basis.

Revised x_2 coefficients: $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$

x_4 leaves.

Iteration 2: $\eta = \begin{pmatrix} 1/2 \\ 1/2 \\ -2 \end{pmatrix}$, $B_{\text{new}}^{-1} = \begin{pmatrix} 1/2 & -1/2 & 0 \\ 1/2 & 1/2 & 0 \\ -2 & -1 & 1 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 & 0 \\ 1/2 & 1/2 & 0 \\ -2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$c_B = \begin{pmatrix} -2 & 3 & 0 \end{pmatrix}$

Revised row 0:

$$\begin{pmatrix} 1/2 & 5/2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & -2 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5/2 & 0 & 0 & 1/2 & 5/2 & 0 \end{pmatrix}$$

The current solution is optimal.

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 1, 3)$ and $Z^* = 7$

5.4-4.

$c = (\,10 \quad 20 \quad 0 \quad 0 \quad 0\,)$, $A = \begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 5 & 3 & 0 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 15 \\ 12 \\ 45 \end{pmatrix}$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

$$x_B = \begin{pmatrix} x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 15 \\ 12 \\ 45 \end{pmatrix} = \begin{pmatrix} 15 \\ 12 \\ 45 \end{pmatrix}$$

$c_B = (\,0 \quad 0 \quad 0\,)$, Row 0: $(\,-10 \quad -20 \quad 0 \quad 0 \quad 0 \quad 0\,)$

x_2 enters the basis.

Revised x_2 coefficients: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

x_3 leaves the basis.

Iteration 1: $\eta = \begin{pmatrix} 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$, $B_{\text{new}}^{-1} = \begin{pmatrix} 1/2 & 0 & 0 \\ -1/2 & 1 & 0 \\ -3/2 & 0 & 1 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_2 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 0 \\ -1/2 & 1 & 0 \\ -3/2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 15 \\ 12 \\ 45 \end{pmatrix} = \begin{pmatrix} 7.5 \\ 4.5 \\ 22.5 \end{pmatrix}$$

$c_B = (\,20 \quad 0 \quad 0\,)$

Revised row 0:

$$(\,10 \quad 0 \quad 0\,) \begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 5 & 3 & 0 & 0 & 1 \end{pmatrix} - (\,10 \quad 20 \quad 0 \quad 0 \quad 0\,)$$

$= (\,-20 \quad 0 \quad 10 \quad 0 \quad 0\,)$

x_1 enters the basis.

Revised x_1 coefficients: $\begin{pmatrix} 1/2 & 0 & 0 \\ -1/2 & 1 & 0 \\ -3/2 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 3/2 \\ 13/2 \end{pmatrix}$

x_4 leaves.

Iteration 2: $\eta = \begin{pmatrix} 1/3 \\ 2/3 \\ -13/3 \end{pmatrix}$, $B_{\text{new}}^{-1} = \begin{pmatrix} 1/3 & 1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 2/3 & -13/3 & 1 \end{pmatrix}$

$$x_B = \begin{pmatrix} x_2 \\ x_1 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 2/3 & -13/3 & 1 \end{pmatrix} \begin{pmatrix} 15 \\ 12 \\ 45 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ 3 \end{pmatrix}$$

$c_B = \begin{pmatrix} 20 & 10 & 0 \end{pmatrix}$

Revised row 0:

$$\begin{pmatrix} 10/3 & 40/3 & 0 \end{pmatrix} \begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 5 & 3 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 10 & 20 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 10/3 & 40/3 & 0 \end{pmatrix}$$

The current solution is optimal.

Optimal Solution: $(x_1^*, x_2^*) = (3, 9)$ and $Z^* = 210$