CHAPTER 9: THE TRANSPORTATION AND ASSIGNMENT PROBLEMS

9.1-1.

While growing continuously as a global company, Procter & Gamble faced the need to restructure for enhanced effectiveness. The goal was to optimize work processes and to minimize expenses while maintaining customer satisfaction. Lowered transportation costs due to changes in the trucking industry and reduced product packages suggested that the total transportation costs could be decreased. In the meantime, shorter product life cycles justified smaller number of plants. Consequently, P&G had to decide on where to locate the plants, what and how much to produce in each. This would be impossible without reviewing the distribution system. Hence, two problems for each product category needed to be solved: a distribution-location problem and a product-sourcing problem.

First, optimal distribution center (DC) locations and optimal customer assignments are found by solving an uncapacitated facility-location model. The objective in this problem is to minimize the total cost of transportation and supply while the primary restriction is to satisfy customer demand. Fixed costs involved in locating DCs are ignored. The total number of DCs is determined beforehand subjectively. The solution of this problem is an input to the product sourcing problem.

With fixed DC locations and their capacities, product sourcing is modeled as a transportation problem. Sources are plants, destinations are DCs and customers. The location and capacity of the plants are specified by the product-strategy teams. Decision variables are the amounts of demand at each destination to be met from each source. The objective is to minimize the total cost while satisfying the demand at each destination without exceeding the capacity of each source. The costs consist of manufacturing, warehousing and transportation costs. An out-of-kilter algorithm is used to solve this problem for each product category.

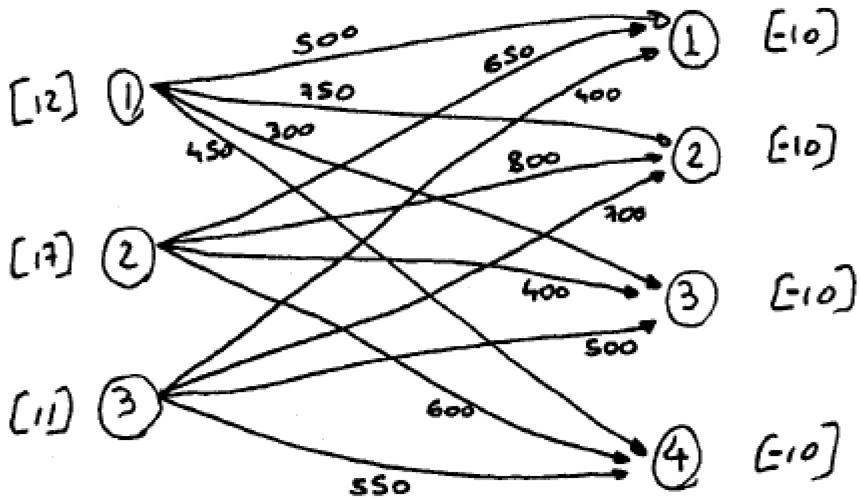
The benefits of this study included a reduction in the number of plants in North America by 20% and savings of over $200 million per year. The reduction in manufacturing costs, due to lowered number of plants and personnel coupled with improved efficiency of the supply chain, outweighs the increase in delivery costs. The gains from this study led P&G to making OR/MS a part of its decision-making process.

9.1-2.

(a)

		Unit Cost ($) Destination (Distribution Center)				
		1	2	3	4	Supply
Source (Plant)	1	500	750	300	450	12
	2	650	800	400	600	17
	3	400	700	500	550	11
Demand		10	10	10	10	

(b)



(c)

Shipments			Distribution Center					
		1	2	3	4	Total Shipped		Supply
	1	0	0	2	10	12	=	12
Plant	2	0	9	8	0	17	=	17
	3	10	1	0	0	11	=	11
Total Received		10	10	10	10			
		=	=	=	=			Total Cost
Demand		10	10	10	10			$20,200

9.1-3.

(a) Let x_1 and x_2 be the number of pints purchased from Dick today and tomorrow respectively, x_3 and x_4 be the number of pints purchased from Harry today and tomorrow respectively.

$$\text{Min } Z = 3X_1 + 2.7X_2 + 2.9X_3 + 2.8X_4$$

subject to

$$\begin{aligned}
1X_1 + 1X_2 + 0X_3 + 0X_4 &\le 5\\
0X_1 + 0X_2 + 1X_3 + 1X_4 &\le 4\\
1X_1 + 0X_2 + 1X_3 + 0X_4 &= 3\\
0X_1 + 1X_2 + 0X_3 + 1X_4 &\ge 4
\end{aligned}$$

and

$$X_1 \ge 0,\ X_2 \ge 0,\ X_3 \ge 0,\ X_4 \ge 0.$$

INITIAL TABLEAU

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	Right side
			-1M	-1M	-1M	-1M	1M					-7M
Z	0	-1	3	2.7	2.9	2.8	0	0	0	0	0	0
X6	1	0	1	1	0	0	0	1	0	0	0	5
X7	2	0	0	0	1	1	0	0	1	0	0	4
X8	3	0	1	0	1	0	0	0	0	1	0	3
X9	4	0	0	1	0	1	-1	0	0	0	1	4

(b)

	Destination 1 Today	2 Tomorrow	3 Dummy	Supply
Dick 1	3	2.7	0	5
Harry 2	2.9	2.8	0	4
Demand	3	4	2	

(c)

	Destination 1	2	3	Supply
Source 1		4	1	5
Source 2	3		1	4
Demand	3	4	2	Cost is 19.5

9.1-4.

(a)

		Cost Per Unit Distributed Destination product				
		1	2	3	dummy 4	Supply
Source plant	1	31	45	38	0	400
	2	29	41	35	0	600
	3	32	46	40	0	400
	4	28	42	1M	0	600
	5	29	43	1M	0	1000
Demand		600	1000	800	600	

(b)

		Destination				
		1	2	3	4	Supply
Source	1			200	200	400
	2			600		600
	3				400	400
	4	600				600
	5		1000			1000
Demand		600	1000	800	600	Cost is 88400

9.1-5.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D12	Bellingham Sacramento	0	15	464	1E+30	15
E12	Bellingham Salt Lake City	20	0	513	15	21
F12	Bellingham Rapid City	0	84	654	1E+30	84
G12	Bellingham Albuquerque	55	0	867	21	351
D13	Eugene Sacramento	80	0	352	15	1E+30
E13	Eugene Salt Lake City	45	0	416	21	15
F13	Eugene Rapid City	0	217	690	1E+30	217
G13	Eugene Albuquerque	0	21	791	1E+30	21
D14	Albert Lea Sacramento	0	728	995	1E+30	728
E14	Albert Lea Salt Lake City	0	351	682	1E+30	351
F14	Albert Lea Rapid City	70	0	388	84	1E+30
G14	Albert Lea Albuquerque	30	0	685	351	84

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D15	Total Received Sacramento	80	-418	80	45	0
E15	Total Received Salt Lake City	65	-354	65	55	0
F15	Total Received Rapid City	70	-297	70	30	0
G15	Total Received Albuquerque	85	0	85	0	1E+30
H12	Bellingham Total Shipped	75	867	75	0	55
H13	Eugene Total Shipped	125	770	125	0	45
H14	Albert Lea Total Shipped	100	685	100	0	30

		Range of Optimality Destination			
		Sacramento	Salt Lake City	Rapid City	Albuquerque
Source	Bellingham	449 to ∞	492 to 528	570 to ∞	516 to 888
	Eugene	-∞ to 367	401 to 437	473 to ∞	770 to ∞
	Albert Lea	267 to ∞	331 to ∞	-∞ to 472	601 to 1036

These ranges tell the management how much each individual cost can be changed without changing the optimal solution.

9.1-6.

(a) Introduce a dummy customer 5 to represent the excess amount sent to customer 3 and a dummy plant 4 to represent the units that are sold to, but not received by customers 4 and 5.

(a), (c), (d)

Unit Profit		Customer							
		1	2	3	4	Dummy			
	1	$800	$700	$500	$200	$500			
Plant	2	$500	$200	$100	$300	$100			
	3	$600	$400	$300	$500	$300			
	Dummy	($9,999)	($9,999)	($9,999)	$0	$0			
Shipments		Customer							
		1	2	3	4	Dummy	Total Shipped		Supply
	1	0	60	0	0	0	60	=	60
Plant	2	40	0	0	40	0	80	=	80
	3	0	0	20	20	0	40	=	40
	Dummy	0	0	0	0	60	60	=	60
Total Received		40	60	20	60	60			
		=	=	=	=	=			Total Profit
Commitment		40	60	20	60	60			$90,000

(b), (e)

Unit Cost		Customer							
		1	2	3	4	Dummy			
	1	($800)	($700)	($500)	($200)	($500)			
Plant	2	($500)	($200)	($100)	($300)	($100)			
	3	($600)	($400)	($300)	($500)	($300)			
	Dummy	$9,999	$9,999	$9,999	$0	$0			
Shipments		Customer							
		1	2	3	4	Dummy	Total Shipped		Supply
	1	0	60	0	0	0	60	=	60
Plant	2	40	0	0	40	0	80	=	80
	3	0	0	20	20	0	40	=	40
	Dummy	0	0	0	0	60	60	=	60
Total Received		40	60	20	60	60			
		=	=	=	=	=			Total Cost
Commitment		40	60	20	60	60			-$90,000

The profit is $90,000.

9.1-7.

(a)

		Distribution center				
		1	2	3	4 Dummy	Supply
Plant	1	800	700	400	0	50
	2	600	800	500	0	50
	Demand	20	20	20	40	

(b), (c)

Unit Cost		Distribution Center						
		1	2	3	Dummy			
Plant	A	$800	$700	$400	$0			
	B	$600	$800	$500	$0			
Shipments		Distribution Center						
		1	2	3	Dummy	Total Shipped		Supply
Plant	A	0	20	20	10	50	=	50
	B	20	0	0	30	50	=	50
Total Received		20	20	20	40			
		=	=	=	=			Total Cost
Demand		20	20	20	40			$34,000

9.1-8.

(a) Let destination $2i-1$ represent the demand of 10 at center i and destination $2i$ represent the extra demand up to 20 shipped to center $i=1,2,3$ in the parameter table below.

		Cost per unit distributed Destination							
		1	2	3	4	5	6	7 Dummy	Supply
plant	1	800	800	700	700	400	400	0	50
plant	2	600	600	800	800	500	500	0	50
dummy	3	1.00E+06	0	1.00E+06	0	1.00E+06	0	1.00E+06	30
	Demand	10	20	10	20	10	20	40	

(b), (c)

Unit Cost		Distribution Center									
		1	1 Extra	2	2 Extra	3	3 Extra	Dummy			
Plant	A	$800	$800	$700	$700	$400	$400	$0			
Plant	B	$600	$600	$800	$800	$500	$500	$0			
Dummy		$99,999	$0	$99,999	$0	$99,999	$0	$999,999			
Shipments		Distribution Center									
		1		2		3		Dummy	Total Shipped		Supply
Plant	A	0	0	10	0	10	20	10	50	=	50
Plant	B	10	10	0	0	0	0	30	50	=	50
Dummy		0	10	0	20	0	0	0	30	=	30
Total Received		10	20	10	20	10	20	40			
		=	=	=	=	=	=	=			Total Cost
Demand		10	20	10	20	10	20	40			$31,000

9.1-9.

(a) Let source $2i-1$ be regular time production and $2i$ be overtime production in month $i=1,2,3$. Let destination $2i-1$ represent the contracted sales for product 1 and $2i$ represent the contracted sales for product 2 in month $i=1,2,3$. Destination 7 is dummy.

Cost Per Unit Distributed (in $1,000's)

		Destination							
		1	2	3	4	5	6	7	Supply
Source	1	15	16	16	18	18	19	0	10
	2	18	20	19	22	21	23	0	3
	3	1M	1M	17	15	19	16	0	8
	4	1M	1M	20	18	22	19	0	2
	5	1M	1M	1M	1M	19	17	0	10
	6	1M	1M	1M	1M	22	22	0	3
Demand		5	3	3	5	4	4	12	

(b)

		Destination							
		1	2	3	4	5	6	7	Supply
Source	1	5	3	2					10
	2							3	3
	3			1	5		2		8
	4							2	2
	5					4	2	4	10
	6							3	3
Demand		5	3	3	5	4	4	12	Cost is 389,000

Hence, the total cost is $\$389,000$ and no overtime is necessary.

9.2-1.

(a) Vogel's approximation method would choose x_{21} as the first basic variable.

a)

		Destination				
		1	2	3	Supply	Row Difference
Source	1	6	3	5	4	2
	2	4	M	7	3	3 ←
	3	3	4	3	2	0
Demand		4	2	3		
Column Difference		1	1	2		

(b) Russell's approximation method would choose x_{12} as the first basic variable.

	Destination 1	Destination 2	Destination 3	Supply	Row Maximum
1	6 -6	3 -M-3	5 -8	4	6
2	4 -M-2	M -M	7 -M	3	M
3	3 -7	4 -M	3 -8	2	4
Demand	4	2	3		
Column Maximum	6	M	7		

(c) Initial BF solution using northwest corner rule:

4			4
0	2	1	3
		2	2
4	2	3	

9.2-2.

(a) Northwest Corner Rule

Source	Destination 1	2	3	4	5	Supply
1	2 / B / 4	4 / B / 0	6 / 0	5 / 0	7 / 0	4
2	7 / 0	6 / B / 4	3 / B / 2	M / B / 0	4 / 0	6
3	8 / 0	7 / 0	5 / 0	2 / B / 5	5 / B / 1	6
4	0 / 0	0 / 0	0 / 0	0 / 0	0 / B / 4	4
Demand	4	4	2	5	5	

Cost: 53

(b) Vogel's Approximation Method

Source	Destination 1	2	3	4	5	Supply
1	2 / B / 4	4 / 0	6 / 0	5 / 0	7 / 0	4
2	7 / 0	6 / 0	3 / B / 2	M / 0	4 / B / 4	6
3	8 / B / 0	7 / B / 0	5 / 0	2 / B / 5	5 / B / 1	6
4	0 / 0	0 / B / 4	0 / 0	0 / 0	0 / 0	4
Demand	4	4	2	5	5	

Cost: 45

(c) Russell's Approximation Method

Source	Destination 1	2	3	4	5	Supply	u[i]
1	2 / B / 4	4 / 2	6 / 7	5 / 8	7 / 7	4	-5
2	7 / 1	6 / 0	3 / B / 2	M / 1M- 1	4 / B / 4	6	-1
3	8 / 1	7 / B / 0	5 / 1	2 / B / 5	5 / B / 1	6	0
4	0 / B / 0	0 / B / 4	0 / 3	0 / 5	0 / 2	4	-7
Demand	4	4	2	5	5		
v[j]	7	7	4	2	5		

Cost: 45

Note that Vogel's and Russell's approximation methods return an optimal solution.

9.2-3.

(a) Northwest Corner Rule

Source	Destination 1	2	3	4	5	6	Supply
1	13 / 3	10 / 2	22	29	18	0	5
2	14	13 / 3	16 / 3	21	1M	0	6
3	3	0	1M / 1	11 / 5	6 / 1	0	7
4	18	9	19	23	11 / 4	0	4
5	30	24	34	36	28 / 1	0 / 2	3
Demand	3	5	4	5	6	2	

Cost: $M + 279$

(b) Vogel's Approximation Method

Source \ Destination	1	2	3	4	5	6	Supply
1	13	10 3	22	29	18 2	0	5
2	14 1	13 1	16 4	21	M	0	6
3	3 2	0	M	11 5	6	0	7
4	18	9	19	23	11 4	0	4
5	30	24 1	34	36	28	0 2	3
Demand	3	5	4	5	6	2	

Cost: 286

Arbitrarily breaking the tie differently returns the solution below with cost $M + 260$.

Source \ Destination	1	2	3	4	5	6	Supply
1	13	10 5	22	29	18	0	5
2	14	13 0	16 4	21 1	M 1	0	6
3	3 3	0	M	11 4	6	0	7
4	18	9	19	23	11 4	0	4
5	30	24	34	36	28 1	0 2	3
Demand	3	5	4	5	6	2	

(c) Russell's Approximation Method

Source \ Destination	1	2	3	4	5	6	Supply
1	13 2	10 3	22	29	18	0	5
2	14	13	16 4	21 2	M	0	6
3	3 1	0	M	11	6 6	0	7
4	18	9 2	19	23 2	11	0	4
5	30	24	34	36 1	28	0 2	3
Demand	3	5	4	5	6	2	

Cost: 301

9.2-4.

(a) All the supply and demand values are integers. By the integer solutions property, the resulting basic feasible solutions will be integral. All the supplies and demands are one, so the only possible values of the variables in a basic feasible solution are 0 and 1. The 1's indicate the assignment of a source to a destination.

(b) There are 7 basic variables in every basic feasible solution and 3 of them are degenerate.

(d) The variables are chosen in the order $x_{13}, x_{24}, x_{44}, x_{32}, x_{41}, x_{43}, x_{42}$.

	Destination					
	1	2	3	4	Supply	u[i]
1	7 / 0	4 / 0	1 B / 1	4 / 0	1	0
2	4 / 0	6 / 0	7 / 0	2 B / 1	1	0
3	8 / 0	5 B / 1	4 / 0	6 / 0	1	0
4	6 B / 1	7 B / 0	6 B / 0	3 B / 0	1	0
Demand	1	1	1	1		
v[j]	0	0	0	0		

(c) - (e)

(0)

	Destination 1	2	3	4	Supply	u[i]
1	7 L 1	4 -5	1 E -7	4 -1	1	2
2	4 P 0	6 P 1	7 2	2 0	1	-1
3	8 5	5 P 0	4 P 1	6 5	1	-2
4	6 1	7 0	6 B 0	3 B 1	1	0
Demand	1	1	1	1		
v[j]	5	7	6	3		

(1)

	Destination 1	2	3	4	Supply	u[i]
1	7 7	4 2	1 B 1	4 6	1	1
2	4 B 1	6 B 0	7 2	2 0	1	5
3	8 5	5 B 1	4 B 0	6 5	1	4
4	6 1	7 0	6 B 0	3 B 1	1	6
Demand	1	1	1	1		
v[j]	-1	1	0	-3		

Optimal assignment (source,destination): $(1,3)$, $(2,1)$, $(3,2)$, $(4,4)$, cost: 13

9.2-5.

				u_i
464 15	513 20	654 84	867 55	182
352 80	416 45	690 217	791 21	85
995 728	682 351	388 70	685 30	0
v_j: 267	331	388	685	

Cost: $\$152,535$, $c_{ij} - u_i - v_j \geq 0$ for all i and j, so the solution is optimal.

9.2-6.

```
       |              Destination                 |
  (0)  |   1  |   2  |   3  |   4  |   5  |Supply  u[i]
_______|______|______|______|______|______|____________
       |   8| |   6| |   3| |   7| |   5| |      |
   1   |----  |----  |---- B|----  |----  |      |
       |     1|     3|    20|     1|     2|    20|     2
       |______|______|______|______|______|      |
       |   5| |   M| |   8| |   4| |   7| |      |
   2   |---- P|----  |----  |---- P|----  |      |
       |    25| 1M- 1|     7|     5|     6|    30|     0
       |______|______|______|______|______|      |
       |   6| |   3| |   9| |   6| |   8| |      |
   3   |----  |---- P|----  |---- P|----  |      |
       |    -1|    25|     6|     5|     5|    30|     2
       |______|______|______|______|______|      |
       |   0| |   0| |   0| |   0| |   0| |      |
   4   |---- E|---- L|---- B|----  |---- B|      |
       |    -4|     0|     0|    -3|    20|    20|    -1
       |______|______|______|______|______|______|_____
Demand |    25    25    20    10    20|
       |______________________________|
  v[j] |     5     1     1     4     1|
```

(1)

	Destination 1	2	3	4	5	Supply	u[i]
1	8; 5	6; 7	3; B; 20	7; 5	5; 2	20	3
2	5; P; 25	M; 1M- 1	8; 3	4; P; 5	7; 2	30	5
3	6; E; -1	3; B; 25	9; 2	6; L; 5	8; 1	30	7
4	0; B; 0	0; 4	0; B; 0	0; 1	0; B; 20	20	0
Demand	25	25	20	10	20		
v[j]	0	-4	0	-1	0		

(2)

	Destination 1	2	3	4	5	Supply	u[i]
1	8; 5	6; 6	3; B; 20	7; 5	5; 2	20	3
2	5; B; 20	M; 1M- 2	8; 3	4; B; 10	7; 2	30	5
3	6; B; 5	3; B; 25	9; 3	6; 1	8; 2	30	6
4	0; B; 0	0; 3	0; B; 0	0; 1	0; B; 20	20	0
Demand	25	25	20	10	20		
v[j]	0	-3	0	-1	0		

The current solution is optimal: $x_{13} = 20, x_{21} = 20, x_{24} = 10, x_{31} = 5, x_{32} = 25$ and $x_{45} = 20$, with cost 305. The optimality condition $c_{ij} - u_i - v_j \geq 0$ for all i and j is met.

9.2-7.

(a) Northwest Corner Rule

(0)

Source	Destination 1	2	3	4	Supply	u_i
1	3 / 3	7 / 2	6 / 0	4 / 1	5	0
2	2 / 2	4 / 1 −	3 / 1 +	2 / 2	2	-3
3	4 / -1	3 / $\boxed{+}$	8 / 1 −	5 / 2	3	2
Demand	3	3	2	2		
v_j	3	7	6	3	Z = 48	

(1)

Source	Destination 1	2	3	4	Supply	u_i
1	3 / 3	7 / 2 −	6 / 0	4 / $\boxed{+}$ +	5	7
2	2 / 2	4 / 0	3 / 2	2 / -4	2	4
3	4 / 5	3 / 1 +	8 / 6	5 / 2 −	3	3
Demand	3	3	2	2		
v_j	-4	0	-1	2	Z = 42	

(2)

Source	Destination 1	2	3	4	Supply	u_i
1	3 / 3	7 / 5	6 / 5	4 / 2	5	0
2	2 / -3	4 / 0 −	3 / 2	2 / $\boxed{+}$ +	2	2
3	4 / 0	3 / 3 +	8 / 6	5 / 0 −	3	1
Demand	3	3	2	2		
v_j	3	2	1	4	Z = 32	

(3)

Source	Destination 1	2	3	4	Supply	u_i
1	3 / 3	7 / 1	6 / 1	4 / 2	5	2
2	2 / 1	4 / 0	3 / 2	2 / 0	2	0
3	4 / 4	3 / 3	8 / 6	5 / 4	3	-1
Demand	3	3	2	2		
v_j	1	4	3	2	Z = 32	

3 iterations are required to reach optimality.

(b) Vogel's Approximation Method

Source	Destination 1	2	3	4	Supply	u_i
1	3 / 3	7 / 0	6 / 0	4 / 2	5	0
2	2 / 2	4 / 0	3 / 2	2 / 1	2	-3
3	4 / 5	3 / 3	8 / 6	5 / 5	3	-4
Demand	3	3	2	2		
v_j	3	7	6	4	Z = 32	

The solution is optimal, no iteration of network simplex is needed.

(c) Russell's Approximation Method

Source	Destination 1	2	3	4	Supply	u_i
1	3 / 3	7 / 0	6 / 0	4 / 2	5	0
2	2 / 2	4 / 0	3 / 2	2 / 1	2	-3
3	4 / 5	3 / 3	8 / 6	5 / 5	3	-4
Demand	3	3	2	2		
v_j	3	7	6	4	Z = 32	

The solution is optimal, no iteration of network simplex is needed.

9.2-8.

(a)

		Unit Cost ($) Destination (Retail Outlet) 1	2	3	4	Supply
Source (Plant)	1	500	600	400	200	10
	2	200	900	100	300	20
	3	300	400	200	100	20
	4	200	100	300	200	10
Demand		20	10	10	20	

(b)

(0)	Destination 1	2	3	4	Supply	u[i]
1	5 P 10	6 -1M+ 3	4 -1M+ 3	2 E -1M+ 2	10	1M + 3
2	2 P 10	M L 10	1 -1M+ 3	3 -1M+ 6	20	1M
3	3 1M- 3	4 P 0	2 B 10	1 P 10	20	4
4	2 1M- 5	1 -4	3 0	2 B 10	10	5
Demand	20	10	10	20		
v[j]	-1M + 2	0	-2	-3		

(c)

(1)	Destination 1	2	3	4	Supply	u[i]
1	5 B 0	6 1	4 1	2 B 10	10	1
2	2 B 20	M 1M- 2	1 1	3 4	20	-2
3	3 -1	4 L 10	2 B 10	1 P 0	20	0
4	2 -3	1 E -4	3 0	2 P 10	10	1
Demand	20	10	10	20		
v[j]	4	4	2	1		

(2)

	Destination					
	1	2	3	4	Supply	u[i]
1	5 P 0	6 5	4 1	2 P 10	10	2
2	2 B 20	M 1M+ 2	1 1	3 4	20	-1
3	3 -1	4 4	2 B 10	1 B 10	20	1
4	2 E -3	1 B 10	3 0	2 L 0	10	2
Demand	20	10	10	20		
v[j]	3	-1	1	0		

(3)

	Destination					
	1	2	3	4	Supply	u[i]
1	5 L 0	6 2	4 1	2 P 10	10	5
2	2 B 20	M 1M- 1	1 1	3 4	20	2
3	3 E -1	4 1	2 B 10	1 P 10	20	4
4	2 B 0	1 B 10	3 3	2 3	10	2
Demand	20	10	10	20		
v[j]	0	-1	-2	-3		

```
(4)     |           Destination           |
        |   1   |   2   |   3   |   4    |Supply  u[i]
________|_______|_______|_______|________|____________
        |    5| |    6| |    4| |    2| |      |
      1 |----   |----   |----   |---- B|       |
        |      1|      3|      1|     10|     10|     1
        |_______|_______|_______|_______|      |
        |    2| |    M| |    1| |    3| |      |
      2 |---- B|----    |----   |----   |      |
        |     20| 1M- 1|      0|      3|     20|    -1
        |_______|_______|_______|_______|      |
        |    3| |    4| |    2| |    1| |      |
      3 |---- B|----    |---- B|---- B|       |
        |      0|      2|     10|     10|     20|     0
        |_______|_______|_______|_______|      |
        |    2| |    1| |    3| |    2| |      |
      4 |---- B|---- B|----    |----   |       |
        |      0|     10|      2|      2|     10|    -1
        |_______|_______|_______|_______|______|______
Demand  |   20      10      10      20|
        |_______________________________|
  v[j]  |    3       2       2       1|
        |                               |
```

Optimal Solution: $x_{14} = 10, x_{21} = 20, x_{33} = 10, x_{34} = 10, x_{42} = 10$, cost: \$100.

9.2-9.

(a) Since there is no limit on the electricity and natural gas available, let the supply of electricity be the sum of demands for electricity, water and space heating and the supply of natural gas be the sum of demands for water and space heating.

		Product			
	Electricity (1)	Water (2)	Space (3)	Dummy (4)	Supply
Electricity (1)	50	50	140	0	100
Natural Gas (2)	M	110	100	0	70
Solar Heater (3)	M	70	90	0	40
Demand	30	20	50	110	

(b), (c)

```
     |              Destination             |
 (0) |   1    |   2    |   3    |   4    |Supply   u[i]
_____|________|________|________|________|____________
     |   50|  |   90|  |   80|  |    0|  |        |
   1 |---- B|---- P|---- P|----    |        |
     |     20|     10|     30|    -30|     60|      0
     |________|________|________|________|        |
     |    M|  |   60|  |   50|  |    0|  |        |
   2 |----    |----    |---- L|---- P|        |
     | 1M-20|      0|      0|     40|     40|    -30
     |________|________|________|________|        |
     |    M|  |   30|  |   40|  |    0|  |        |
   3 |----    |---- E|----    |---- P|        |
     | 1M-20|    -30|    -10|     30|     30|    -30
     |________|________|________|________|________|______
Demand |     20       10       30       70|
     |____________________________________|
v[j] |     50       90       80       30|
     |                                    |
```

```
     |              Destination             |
 (1) |   1    |   2    |   3    |   4    |Supply   u[i]
_____|________|________|________|________|____________
     |   50|  |   90|  |   80|  |    0|  |        |
   1 |---- B|---- L|---- B|---- E|        |
     |     20|     10|     30|    -60|     60|      0
     |________|________|________|________|        |
     |    M|  |   60|  |   50|  |    0|  |        |
   2 |----    |----    |----    |---- B|        |
     | 1M+10|     30|     30|     40|     40|    -60
     |________|________|________|________|        |
     |    M|  |   30|  |   40|  |    0|  |        |
   3 |----    |---- P|----    |---- P|        |
     | 1M+10|      0|     20|     30|     30|    -60
     |________|________|________|________|________|______
Demand |     20       10       30       70|
     |____________________________________|
v[j] |     50       90       80       60|
     |                                    |
```

(2)

	Destination 1	2	3	4	Supply	u[i]
1	50 ---- B 20	90 ---- 60	80 ---- P 30	0 ---- P 10	60	0
2	M ---- 1M-50	60 ---- 30	50 ---- -30	0 ---- B 40	40	0
3	M ---- 1M-50	30 ---- B 10	40 ---- E -40	0 ---- L 20	30	0
Demand	20	10	30	70		
v[j]	50	30	80	0		

(3)

	Destination 1	2	3	4	Supply	u[i]
1	50 ---- B 20	90 ---- 20	80 ---- L 10	0 ---- P 30	60	0
2	M ---- 1M-50	60 ---- -10	50 ---- E -30	0 ---- P 40	40	0
3	M ---- 1M-10	30 ---- B 10	40 ---- B 20	0 ---- 40	30	-40
Demand	20	10	30	70		
v[j]	50	70	80	0		

(4)

	Destination 1	2	3	4	Supply	u[i]
1	50 ---- B 20	90 ---- 50	80 ---- 30	0 ---- B 40	60	0
2	M ---- 1M-50	60 ---- 20	50 ---- B 10	0 ---- B 30	40	0
3	M ---- 1M-40	30 ---- B 10	40 ---- B 20	0 ---- 10	30	-10
Demand	20	10	30	70		
v[j]	50	40	50	0		

The optimal solution is to meet 20 units of electricity with electricity, 10 units of space heating with natural gas, 10 units of water heating with solar heating, and 20 units of space heating with solar heating. The cost is $2,600.

(d), (e) The initial basic feasible solution provides the same optimal solution as in (c).

(O)	Destination 1	2	3	4	Supply	u[i]
1	50 B 20	90 50	80 30	0 B 40	60	0
2	M 1M-50	60 20	50 B 10	0 B 30	40	0
3	M 1M-40	30 B 10	40 B 20	0 10	30	-10
Demand	20	10	30	70		
v[j]	50	40	50	0		

(f) The initial basic feasible solution provides the same optimal solution as in (c).

Source	Destination 1	2	3	4	Supply	u_i
1	50 20	90 50	80 30	0 40	60	0
2	1M 1M-50	60 20	50 10	0 30	40	0
3	1M 1M-40	30 10	40 20	0 10	30	-10
Demand	20	10	30	70		
v_j	50	40	50	0	Z = 2600	

(g) The initial BF solution using Vogel's and Russell's methods provides the same optimal solution as in (c). The optimal solution obtained starting from each of the three rules is the same. (c) required four iterations of the transportation simplex while (d) and (f) required none..

9.2-10.

Vogel's Approximation Method

```
      |              Destination                   |
 (O)  |   1    |   2    |   3    |   4    |   5    |Supply  u[i]
 _____|________|________|________|________|________|___________
      |1.08|   |1.09|   |1.11|   |1.13|   |     0| |        |
   1  |---- B  |---- B  |---- B  |----    |----    |        |
      |     10 |     15 |      0 |  0.01  |  0.02  |    25  | -0.02
      |________|________|________|________|________|        |
      |     M| |1.11|   |1.13|   |1.14|   |     0| |        |
   2  |----    |----    |----    |---- P  |---- P  |        |
      | 1M- 1  |      0 |      0 |     15 |     20 |    35  |     0
      |________|________|________|________|________|        |
      |     M| |     M| | 1.1|   |1.11|   |     0| |        |
   3  |----    |----    |---- B  |---- B  |----    |        |
      | 1M- 1  | 1M- 1  |     25 |      5 |  0.03  |    30  | -0.03
      |________|________|________|________|________|        |
      |     M| |     M| |     M| |1.13|   |     0| |        |
   4  |----    |----    |----    |---- E  |---- L  |        |
      | 1M- 1  | 1M- 1  | 1M- 1  | -0.01  |     10 |    10  |     0
      |________|________|________|________|________|________|______
Demand|    10       15       25       20       30  |
      |____________________________________________|
  v[j]|    1.1     1.11     1.13     1.14       0  |
```

Optimal Solution:

Quantity	Production Month	Installation Month
10	1	1
15	1	2
5	2	4
25	3	3
5	3	4
10	4	4

This schedule incurs a cost of 77.3 million dollars.

9.2-11.

(a)

(0)

Source	Destination 1	2	3	4	Supply	u[i]
1	500 / P / 10	750 / P / 2	300 / -50	450 / 50	12	0
2	650 / 100	800 / P / 8	400 / P / 9	600 / 150	17	50
3	400 / E / -250	700 / -200	500 / L / 1	550 / B / 10	11	150
Demand	10	10	10	10		
v[j]	500	750	350	400		

(b)

(1)

Source	Destination 1	2	3	4	Supply	u[i]
1	500 / L / 9	750 / B / 3	300 / -50	450 / E / -200	12	0
2	650 / 100	800 / B / 7	400 / B / 10	600 / -100	17	50
3	400 / P / 1	700 / 50	500 / 250	550 / P / 10	11	-100
Demand	10	10	10	10		
v[j]	500	750	350	650		

(2)

	Destination 1	2	3	4	Supply	u[i]
1	500 ---- 200	750 ---- L 3	300 ---- E -50	450 ---- B 9	12	0
2	650 ---- 300	800 ---- P 7	400 ---- P 10	600 ---- 100	17	50
3	400 ---- B 10	700 ---- -150	500 ---- 50	550 ---- B 1	11	100
Demand	10	10	10	10		
v[j]	300	750	350	450		

(3)

	Destination 1	2	3	4	Supply	u[i]
1	500 ---- 200	750 ---- 50	300 ---- P 3	450 ---- P 9	12	0
2	650 ---- 250	800 ---- P 10	400 ---- P 7	600 ---- 50	17	100
3	400 ---- B 10	700 ---- E -100	500 ---- 100	550 ---- L 1	11	100
Demand	10	10	10	10		
v[j]	300	700	300	450		

```
(4)     |         Destination         |
        |  1   |  2   |  3   |  4   |Supply  u[i]
________|______|______|______|______|__________
        | 500| | 750| | 300| | 450| |      |
      1 |----  |----  |---- B|---- B|      |
        |   100|    50|     2|    10|    12|     0
        |______|______|______|______|      |
        | 650| | 800| | 400| | 600| |      |
      2 |----  |---- B|---- B|----  |      |
        |   150|     9|     8|    50|    17|   100
        |______|______|______|______|      |
        | 400| | 700| | 500| | 550| |      |
      3 |---- B|---- B|----  |----  |      |
        |    10|     1|   200|   100|    11|     0
        |______|______|______|______|______|_____
 Demand |   10     10     10     10|
        |__________________________|
   v[j] |  400    700    300    450|
```

The optimal solution is to send 2 shipments from plant 1 to center 3, 10 to center 4, 9 from plant 2 to center 2, 8 to center 3, 10 from plant 3 to center 1 and 1 to center 2. This has a total cost of $20, 200.

9.2-12.

```
(0)     |     Destination     |
        |  1   |  2   |  3   |Supply  u[i]
________|______|______|______|__________
        |  3|  | 2.7| |   0| |     |
      1 |---- P|---- P|----  |     |
        |     3|     2|   0.1|    5|     0
        |______|______|______|     |
        | 2.9| | 2.8| |   0| |     |
      2 |---- E|---- L|---- B|     |
        |  -0.2|     2|     2|    4|   0.1
        |______|______|______|_____|_____
 Demand |    3      4      2|
        |___________________|
   v[j] |    3    2.7   -0.1|
```

```
(1)     |     Destination     |
        |  1   |  2   |  3   |Supply  u[i]
________|______|______|______|__________
        |  3|  | 2.7| |   0| |     |
      1 |---- L|---- B|---- E|     |
        |     1|     4|  -0.1|    5|     0
        |______|______|______|     |
        | 2.9| | 2.8| |   0| |     |
      2 |---- P|----  |---- P|     |
        |     2|   0.2|     2|    4|  -0.1
        |______|______|______|_____|_____
 Demand |    3      4      2|
        |___________________|
   v[j] |    3    2.7    0.1|
```

```
        |     Destination     |
        |  1   |  2   |  3   |Supply  u[i]
________|______|______|______|__________
        |  3|  | 2.7| |   0| |     |
      1 |----  |---- B|---- B|     |
        |   0.1|     4|     1|    5|     0
        |______|______|______|     |
        | 2.9| | 2.8| |   0| |     |
      2 |---- B|----  |---- B|     |
        |     3|   0.1|     1|    4|     0
        |______|______|______|_____|_____
 Demand |    3      4      2|
        |___________________|
   v[j] |  2.9    2.7      0|
```

The optimal solution is to purchase 4 pints from Dick tomorrow and 3 pints from Harry today, with a cost $19.50.

9.2-13.

```
          |          Destination          |
          |   1    |   2    |   3    |   4    |Supply  u[i]
__________|________|________|________|________|______________
          |   41|  |   55|  |   48|  |    0|  |        |
    1     |---- B|----    |----    |----    |        |
          |   400|       2|-1M+47|      -1|   400|     41
          |________|________|________|________|        |
          |   39|  |   51|  |   45|  |    0|  |        |
    2     |---- B|---- P|---- E|----    |        |
          |   300|    300|-1M+46|       1|   600|     39
          |________|________|________|________|        |
          |   42|  |   56|  |   50|  |    0|  |        |
    3     |----    |---- B|----    |----    |        |
          |     -2|    400|-1M+46|      -4|   400|     44
          |________|________|________|________|        |
          |   38|  |   52|  |    M|  |    0|  |        |
    4     |----    |---- P|---- L|----    |        |
          |     -2|    300|    300|       0|   600|     40
          |________|________|________|________|        |
          |   39|  |   53|  |    M|  |    0|  |        |
    5     |----    |----    |---- B|---- B|        |
          |     -1|       1|    600|    400|  1000|     40
          |________|________|________|________|________|______
 Demand   |    700     1000      900      400|
__________|___________________________________|
   v[j]   |        0       12       1M     -40|
          |                       -  40       |  Z = 900M+81400

          |          Destination          |
          |   1    |   2    |   3    |   4    |Supply  u[i]
__________|________|________|________|________|______________
          |   41|  |   55|  |   48|  |    0|  |        |
    1     |---- B|----    |----    |----    |        |
          |   400|       2|       1| 1M-47|   400|      2
          |________|________|________|________|        |
          |   39|  |   51|  |   45|  |    0|  |        |
    2     |---- L|---- B|---- P|----    |        |
          |   300|       0|    300| 1M-45|   600|      0
          |________|________|________|________|        |
          |   42|  |   56|  |   50|  |    0|  |        |
    3     |----    |---- B|----    |----    |        |
          |     -2|    400|       0| 1M-50|   400|      5
          |________|________|________|________|        |
          |   38|  |   52|  |    M|  |    0|  |        |
    4     |----    |---- B|----    |----    |        |
          |     -2|    600| 1M-46| 1M-46|   600|      1
          |________|________|________|________|        |
          |   39|  |   53|  |    M|  |    0|  |        |
    5     |---- E|----    |---- P|---- B|        |
          |-1M+45|-1M+47|    600|    400|  1000|     1M
          |________|________|________|________|________|-   45
 Demand   |    700     1000      900      400|
__________|___________________________________|
   v[j]   |      39       51       45     -1M|
          |                            +  45|  Z = 600M+95200
```

```
          |        Destination          |
          |   1   |   2   |   3   |   4   |Supply  u[i]
__________|_______|_______|_______|_______|_____________
          |  41|  |  55|  |  48|  |   0|  |       |
    1     |---- B|----   |----   |----   |       |
          |   400|-1M+47|-1M+46|     -2|   400|      2
          |_______|_______|_______|_______|       |
          |  39|  |  51|  |  45|  |   0|  |       |
    2     |----   |---- L|---- P|----   |       |
          | 1M-45|      0|    600| 1M-45|   600|    -1M
          |_______|_______|_______|_______|       |+  45
          |  42|  |  56|  |  50|  |   0|  |       |
    3     |----   |---- B|----   |----   |       |
          | 1M-47|    400|      0| 1M-50|   400|    -1M
          |_______|_______|_______|_______|       |+  50
          |  38|  |  52|  |   M|  |   0|  |       |
    4     |----   |---- B|----   |----   |       |
          | 1M-47|    600| 1M-46| 1M-46|   600|    -1M
          |_______|_______|_______|_______|       |+  46
          |  39|  |  53|  |   M|  |   0|  |       |
    5     |---- B|---- E|---- P|---- B|       |
          |   300|-1M+47|    300|    400|  1000|      0
          |_______|_______|_______|_______|_______|______
Demand    |   700    1000     900     400|
__________|_____________________________|
   v[j]   |     39      1M      1M       0|
          |         +   6                 | Z = 300M+1E5
```

```
          |        Destination          |
          |   1   |   2   |   3   |   4   |Supply  u[i]
__________|_______|_______|_______|_______|_____________
          |  41|  |  55|  |  48|  |   0|  |       |
    1     |---- P|----   |---- E|----   |       |
          |   400|      0|-1M+46|     -2|   400|      2
          |_______|_______|_______|_______|       |
          |  39|  |  51|  |  45|  |   0|  |       |
    2     |----   |----   |---- B|----   |       |
          | 1M-45| 1M-47|    600| 1M-45|   600|    -1M
          |_______|_______|_______|_______|       |+  45
          |  42|  |  56|  |  50|  |   0|  |       |
    3     |----   |---- B|----   |----   |       |
          |      0|    400|-1M+47|     -3|   400|      3
          |_______|_______|_______|_______|       |
          |  38|  |  52|  |   M|  |   0|  |       |
    4     |----   |---- B|----   |----   |       |
          |      0|    600|      1|      1|   600|     -1
          |_______|_______|_______|_______|       |
          |  39|  |  53|  |   M|  |   0|  |       |
    5     |---- P|---- B|---- L|---- B|       |
          |   300|      0|    300|    400|  1000|      0
          |_______|_______|_______|_______|_______|______
Demand    |   700    1000     900     400|
__________|_____________________________|
   v[j]   |     39      53      1M       0|
          |                               | Z = 300M+1E5
```

```
         |          Destination          |
         |   1   |   2   |   3   |   4   |Supply  u[i]
_________|_______|_______|_______|_______|_____________
         |   41| |   55| |   48| |    0| |      |
    1    |----  B|----   |----  B|----   |      |
         |    100|      0|    300|     -2|   400|     2
         |_______|_______|_______|_______|      |
         |   39| |   51| |   45| |    0| |      |
    2    |----   |----   |----  B|----   |      |
         |      1|     -1|    600|      1|   600|    -1
         |_______|_______|_______|_______|      |
         |   42| |   56| |   50| |    0| |      |
    3    |----   |----  L|----   |----  E|      |
         |      0|    400|      1|     -3|   400|     3
         |_______|_______|_______|_______|      |
         |   38| |   52| |    M| |    0| |      |
    4    |----   |----  B|----   |----   |      |
         |      0|    600| 1M-45|      1|   600|    -1
         |_______|_______|_______|_______|      |
         |   39| |   53| |    M| |    0| |      |
    5    |----  B|----  P|----   |----  P|      |
         |    600|      0| 1M-46|    400|  1000|     0
         |_______|_______|_______|_______|______|______
Demand   |    700   1000     900     400|
_________|______________________________|
  v[j]   |     39     53      46       0|
         |                              | Z = 122500

         |          Destination          |
         |   1   |   2   |   3   |   4   |Supply  u[i]
_________|_______|_______|_______|_______|_____________
         |   41| |   55| |   48| |    0| |      |
    1    |----  P|----   |----  B|----  E|      |
         |    100|      0|    300|     -2|   400|     2
         |_______|_______|_______|_______|      |
         |   39| |   51| |   45| |    0| |      |
    2    |----   |----   |----  B|----   |      |
         |      1|     -1|    600|      1|   600|    -1
         |_______|_______|_______|_______|      |
         |   42| |   56| |   50| |    0| |      |
    3    |----   |----   |----   |----  B|      |
         |      3|      3|      4|    400|   400|     0
         |_______|_______|_______|_______|      |
         |   38| |   52| |    M| |    0| |      |
    4    |----   |----  B|----   |----   |      |
         |      0|    600| 1M-45|      1|   600|    -1
         |_______|_______|_______|_______|      |
         |   39| |   53| |    M| |    0| |      |
    5    |----  P|----  B|----   |----  L|      |
         |    600|    400| 1M-46|      0|  1000|     0
         |_______|_______|_______|_______|______|______
Demand   |    700   1000     900     400|
_________|______________________________|
  v[j]   |     39     53      46       0|
         |                              | Z = 121300
```

```
         |         Destination        |
         |   1   |   2   |   3   |   4   |Supply  u[i]
_________|_______|_______|_______|_______|_____________
         | 41|   | 55|   | 48|   |  0|   |       |
    1    |---- L|----   |---- P|---- B|       |
         |  100|      0|   300|      0|   400|      0
         |_______|_______|_______|_______|       |
         | 39|   | 51|   | 45|   |  0|   |       |
    2    |----   |---- E|---- P|----   |       |
         |      1|     -1|   600|      3|   600|     -3
         |_______|_______|_______|_______|       |
         | 42|   | 56|   | 50|   |  0|   |       |
    3    |----   |----   |----   |---- B|       |
         |      1|      1|      2|   400|   400|      0
         |_______|_______|_______|_______|       |
         | 38|   | 52|   |  M|   |  0|   |       |
    4    |----   |---- B|----   |----   |       |
         |      0|   600| 1M-45|      3|   600|     -3
         |_______|_______|_______|_______|       |
         | 39|   | 53|   |  M|   |  0|   |       |
    5    |---- P|---- P|----   |----   |       |
         |   600|   400| 1M-46|      2|  1000|     -2
         |_______|_______|_______|_______|_______|______
 Demand  |   700    1000     900     400|
_________|______________________________|
   v[j]  |     41      55      48      0|
         |                              |  Z = 121300
```

```
         |         Destination        |
         |   1   |   2   |   3   |   4   |Supply  u[i]
_________|_______|_______|_______|_______|_____________
         | 41|   | 55|   | 48|   |  0|   |       |
    1    |----   |----   |---- B|---- B|       |
         |      1|      1|   400|      0|   400|     54
         |_______|_______|_______|_______|       |
         | 39|   | 51|   | 45|   |  0|   |       |
    2    |----   |---- B|---- B|----   |       |
         |      2|   100|   500|      3|   600|     51
         |_______|_______|_______|_______|       |
         | 42|   | 56|   | 50|   |  0|   |       |
    3    |----   |----   |----   |---- B|       |
         |      2|      2|      2|   400|   400|     54
         |_______|_______|_______|_______|       |
         | 38|   | 52|   |  M|   |  0|   |       |
    4    |----   |---- B|----   |----   |       |
         |      0|   600| 1M-46|      2|   600|     52
         |_______|_______|_______|_______|       |
         | 39|   | 53|   |  M|   |  0|   |       |
    5    |---- B|---- B|----   |----   |       |
         |   700|   300| 1M-47|      1|  1000|     53
         |_______|_______|_______|_______|_______|______
 Demand  |   700    1000     900     400|
_________|______________________________|
   v[j]  |    -14       0      -6    -54|
         |                              |  Z = 121200
```

Optimal Solution:

$x_{13} = x_{34} = 400, x_{22} = 100, x_{23} = 500, x_{42} = 600, x_{51} = 700, x_{52} = 300,$
Cost: $\$121,200$

9.2-14.

Using Russell's approximation method:

(a)

	Destination 1	2	3	4	5	Supply	u[i]
1	−800 L 40	−700 P 20	−500 100	−200 600	−500 300	60	−500
2	−500 E −200	−200 B 0	−100 B 20	−300 P 60	−100 200	80	0
3	−600 −100	−400 P 40	−300 0	−500 0	−300 200	40	−200
4	M 1M+ 0	M 1M−1e	M 1M−2e	0 B 0	0 B 60	60	300
Demand	40	60	20	60	60		
v[j]	−300	−200	−100	−300	−300		

Z = −82000

(b)

	Destination 1	2	3	4	5	Supply	u[i]
1	−800 P 40	−700 P 20	−500 −100	−200 400	−500 100	60	−300
2	−500 P 0	−200 200	−100 L 20	−300 B 60	−100 200	80	0
3	−600 −100	−400 P 40	−300 E −200	−500 −200	−300 0	40	0
4	M 1M+2e	M 1M+1e	M 1M−2e	0 B 0	0 B 60	60	300
Demand	40	60	20	60	60		
v[j]	−500	−400	−100	−300	−300		

Z = −82000

(2)

	Destination 1	2	3	4	5	Supply	u[i]
1	−800 P 20	−700 P 40	−500 100	−200 400	−500 100	60	0
2	−500 P 20	−200 200	−100 200	−300 P 60	−100 200	80	300
3	−600 −100	−400 L 20	−300 B 20	−500 E −200	−300 0	40	300
4	M 1M+2e	M 1M+1e	M 1M+ 0	0 B 0	0 B 60	60	600
Demand	40	60	20	60	60		
v[j]	−800	−700	−600	−600	−600		

Z = −86000

(3)

	Destination 1	2	3	4	5	Supply	u[i]
1	−800 B 0	−700 B 60	−500 −100	−200 400	−500 100	60	−600
2	−500 B 40	−200 200	−100 0	−300 B 40	−100 200	80	−300
3	−600 100	−400 200	−300 B 20	−500 B 20	−300 200	40	−500
4	M 1M+2e	M 1M+1e	M 1M−2e	0 B 0	0 B 60	60	0
Demand	40	60	20	60	60		
v[j]	−200	−100	200	0	0		

Z = −90000

The optimal solution is to ship 60 units from plant 1 to customer 2, 40 from plant 2 to customer 1, 40 from plant 2 to customer 4, 20 from plant 3 to customer 3 and 4. This offers a profit of $90,000.

9.2-15.

(a) - (b) - (c)

Using northwest corner rule:

(0)

	Destination 1	2	3	4	Supply	u[i]
1	800 P 20	700 B 20	400 P 10	0 100	50	−100
2	600 E −300	800 0	500 L 10	0 B 40	50	0
Demand	20	20	20	40		
v[j]	900	800	500	0		

Z = 39000

(1)

	Destination 1	2	3	4	Supply	u[i]
1	800 L 10	700 B 20	400 B 20	0 E −200	50	0
2	600 P 10	800 300	500 300	0 P 40	50	−200
Demand	20	20	20	40		
v[j]	800	700	400	200		

Z = 36000

(2)

	Destination 1	2	3	4	Supply	u[i]
1	800 200	700 B 20	400 B 20	0 B 10	50	0
2	600 B 20	800 100	500 100	0 B 30	50	0
Demand	20	20	20	40		
v[j]	600	700	400	0		

Z = 34000

With northwest corner rule, it took 22 seconds to find the initial BF solution and its objective value is 15% above the optimal cost. The two iterations took 48 seconds.

Using Vogel's approximation method:

(0)

	Destination 1	2	3	4	Supply	u[i]
1	800 / L / 10	700 / B / 20	400 / B / 20	0 / E / -200	50	20(
2	600 / P / 10	800 / 300	500 / 300	0 / P / 40	50	C
Demand	20	20	20	40		
v[j]	600	500	200	0		

Z = 36000

(1)

	Destination 1	2	3	4	Supply	u[i]
1	800 / 200	700 / B / 20	400 / B / 20	0 / B / 10	50	0
2	600 / B / 20	800 / 100	500 / 100	0 / B / 30	50	0
Demand	20	20	20	40		
v[j]	600	700	400	0		

Z = 34000

With Vogel's approximation method, it took 44 seconds to find the initial BF solution and its objective value is 6% above the optimal cost. One iteration took 28 seconds.

Using Russell's approximation method:

(0)

	Destination 1	2	3	4	Supply	u[i]
1	800 / 300	700 / 0	400 / P / 10	0 / P / 40	50	0
2	600 / B / 20	800 / B / 20	500 / L / 10	0 / E / -100	50	100
Demand	20	20	20	40		
v[j]	500	700	400	0		

Z = 37000

(1)

	Destination 1	2	3	4	Supply	u[i]
1	800 / 200	700 E / −100	400 B / 20	0 P / 30	50	0
2	600 B / 20	800 L / 20	500 / 100	0 P / 10	50	0
Demand	20	20	20	40		
v[j]	600	800	400	0	Z = 36000	

(2)

	Destination 1	2	3	4	Supply	u[i]
1	800 / 200	700 B / 20	400 B / 20	0 B / 10	50	0
2	600 B / 20	800 / 100	500 / 100	0 B / 30	50	0
Demand	20	20	20	40		
v[j]	600	700	400	0	Z = 34000	

With Russell's approximation method, it took 25 seconds to find the initial BF solution and its objective value is 9% above the optimal cost. The two iterations took 45 seconds.

Let x_0 denote the initial BF solution. The results are summarized in the following table.

Method	Time to Get x_0	Opt. Gap of x_0	No. Iter.'ns	Time Iter.'ns	Total Time
NW Corner	22 seconds	15%	2	48 seconds	70 seconds
Vogel's	44 seconds	6%	1	28 seconds	72 seconds
Russell's	25 seconds	9%	2	45 seconds	70 seconds

9.2-16.

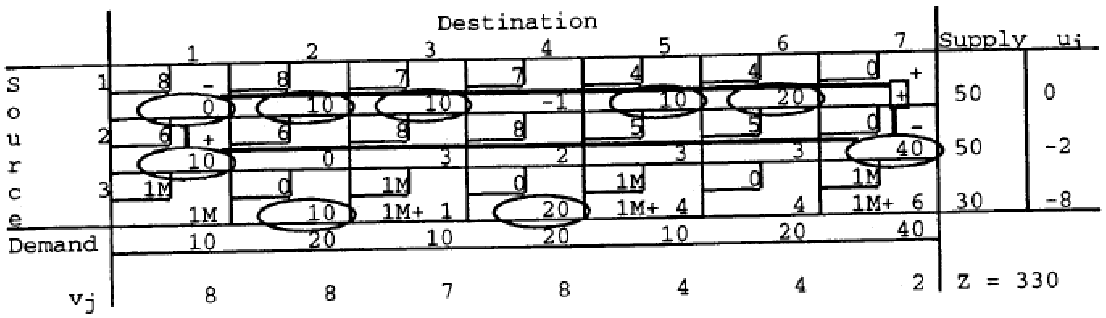
(a) - (b) - (c)

Using northwest corner rule:

Source	Destination 1	2	3	4	5	6	7	Supply	u_i
1	8 / 10	8 − / 20	7 / 10	7 + / 10	4 / 0	4 / 0	0 / 1	50	0
2	6 / −3	6 / −3	8 / 0	8 − / 10	5 / 10	5 / 20	0 + / 10	50	1
3	1M / −9	0 + / +	1M / −8	0 / −1M− 8	1M / −5	0 / −1M− 5	1M − / 30	30	1M +1
Demand	10	20	10	20	10	20	40		
v_j	8	8	7	7	4	4	−1	Z = 30M +610	

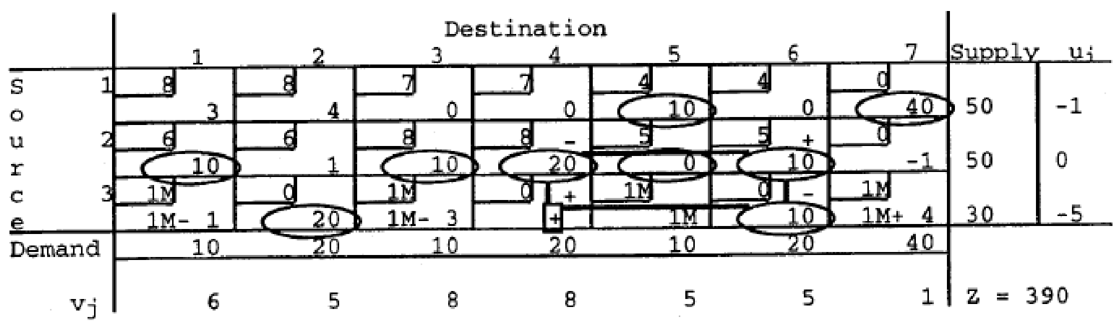
With northwest corner rule, it took 40 seconds to find the initial BF solution and its objective value is $M\%$ above the optimal cost. The seven iterations took 4 minutes.

Using Vogel's approximation method:



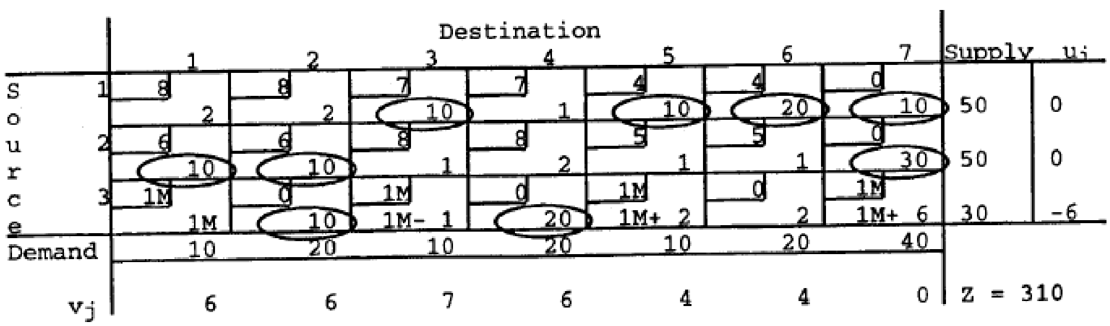
With Vogel's approximation method, it took 55 seconds to find the initial BF solution and its objective value is 6% above the optimal cost. The two iterations took 1 minute.

Using Russell's approximation method:



With Russell's approximation method, it took 63 seconds to find the initial BF solution and its objective value is 26% above the optimal cost. The five iterations took 2 minutes.

Optimal Solution: cost $31,000$



Let x_0 denote the initial BF solution. The results are summarized in the following table.

Method	Time to Get x_0	Opt. Gap of x_0	No. Iter.'ns	Time Iter.'ns	Total Time
NW Corner	40 seconds	$M\%$	7	4 minutes	280 seconds
Vogel's	55 seconds	6%	2	1 minute	115 seconds
Russell's	63 seconds	26%	5	2 minutes	183 seconds

9.2-17.

(a) Initial solution using northwest corner rule:

			u_i
	8 3	5 1	0
	6 -1	4 2	-1
v_j	8	5	

Final tableau: cost 35

			u_i
	8 1	5 3	0
	6 2	4 1	-2
v_j	8	5	

(b) minimize $8x_{11} + 5x_{12} + 6x_{21} + 4x_{22}$

subject to

$$\begin{aligned} x_{11} + x_{12} \phantom{+ x_{21} + x_{22}} &= 4 \\ x_{21} + x_{22} &= 2 \\ x_{11} + x_{21} &= 3 \\ x_{12} + x_{22} &= 3 \\ x_{11}, x_{12}, x_{21}, x_{22} &\geq 0 \end{aligned}$$

Iter.	B.V.	Eq. #	Z	x_{11}	x_{12}	x_{21}	x_{22}	w_1	w_2	w_3	w_4	RHS
0	Z	0	-1	8-2M	5-2M	6-2M	4-2M	0	0	0	0	-12M
	w_1	1	0	1	1	0	0	1	0	0	0	4
	w_2	2	0	0	0	1	1	0	1	0	0	2
	w_3	3	0	1	0	1	0	0	0	1	0	3
	w_4	4	0	0	1	0	1	0	0	0	1	3
4	Z	0	-1	0	0	0	1	2M-8	2M-6	0	3	-35
	x_{11}	1	0	1	0	0	-1	1	0	0	-1	1
	x_{21}	2	0	0	0	1	1	0	1	0	0	2
	w_3	3	0	0	0	0	0	-1	-1	1	1	0
	x_{12}	4	0	0	1	0	1	0	0	0	1	3

Hence, the transportation simplex method takes one iteration while the general simplex method takes four iterations. The computation times vary.

9.2-18.

Let $z_1 = x_1 - 10,$
$z_2 = x_1 + x_2 - 25,$
$z_3 = x_1 + x_2 + x_3 - 50,$
$z_4 = x_1 + x_2 + x_3 + x_4 - 70.$

$$
\begin{array}{ll}
\text{minimize} & 1.08x_1 + 1.11x_2 + 1.10x_3 + 1.13x_4 + 0.15(z_1 + z_2 + z_3 + z_4) \\
\text{subject to} & x_1 \qquad\qquad\qquad\qquad - z_1 = 10 \\
& x_1 + x_2 \qquad\qquad\qquad - z_2 = 25 \\
& x_1 + x_2 + x_3 \qquad\qquad - z_3 = 50 \\
& x_1 + x_2 + x_3 + x_4 \qquad - z_4 = 70 \\
& 0 \leq x_1 \leq 25 \\
& 0 \leq x_2 \leq 35 \\
& 0 \leq x_3 \leq 30 \\
& 0 \leq x_4 \leq 10 \\
& z_1, z_2, z_3, z_4 \geq 0
\end{array}
$$

Initial simplex tableau:

B.V.	Eq.#	Z	x_1	x_2	x_3	x_4	z_1	z_2	z_3	z_4	w_1	w_2	w_3	w_4	y_1	y_2	y_3	y_4	RHS
Z	0	-1	-4M+1.06	-3M+1.11	-2M+1.1	-M+1.13	M+.015	M+.015	M+.015	M+.015	0	0	0	0	0	0	0	0	-155M
w_1	1		1				-1				1								10
w_2	2		1	1				-1				1							25
w_3	3		1	1	1				-1				1						50
w_4	4		1	1	1	1				-1				1					70
y_1	5		1												1				25
y_2	6			1												1			35
y_3	7				1												1		30
y_4	8					1												1	10

Simplex tableau: 16 variables and 8 constraints

Transportation tableau: 20 variables and 9 constraints

Even though the transportation tableau is larger, it requires less work than the simplex tableau.

9.2-19.

If we multiply the demand constraints by -1, each constraint column will have exactly two nonzero entries, one -1 and one $+1$. Summing all these constraints gives the equality:

$$0x = \sum \text{supplies} - \sum \text{demands} = 0,$$

since the total supply equals the total demand. Hence, there is a redundant constraint.

9.2-20.

In the initialization step, after selecting the next basic variable, the allocation made is equal to either the (remaining) supply or demand for that row or column. Since these quantities are known to be integer, the allocation will be integer.

Given a current BF solution that is integer, step 3 of an iteration adds and subtracts, around the chain-reaction cycle, the current value of the leaving basic variable. Since we know this is an integer, and all the other basic variables on the cycle began with integer values, the new BF solution must be all integer.

During the initialization step, we can select the next basic variable for allocation arbitrarily from among the rows and columns not already eliminated. Thus, by altering our selections, we can construct any BF solution as our initial one. Because we have shown that the initialization step gives integer solutions, all BF solutions must be integer.

9.2-21.

(a) Let x_{ij} be the number of tons hauled from pit $i = 1, 2$ (North, South) to site $j = 1, 2, 3$.

minimize $\quad 400x_{11} + 490x_{12} + 460x_{13} + 600x_{21} + 530x_{22} + 560x_{23}$

subject to

$$\begin{aligned}
&x_{11} + x_{12} + x_{13} \leq 18 \\
&x_{21} + x_{22} + x_{23} \leq 14 \\
&x_{11} + x_{21} = 10 \\
&x_{12} + x_{22} = 5 \\
&x_{13} + x_{23} = 10 \\
&x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0
\end{aligned}$$

Initial tableau:

Bas Var	Eq No	Z	X11	X12	X13	X21	X22	X23	X7	X8	X9	X10	X11	Right side
Z	0	-1	-M+400	-M+490	-M+460	-M+600	-M+530	-M+560	0	0	0	0	0	-25M
X7	1	0	1	1	1	0	0	0	1	0	0	0	0	18
X8	2	0	0	0	0	1	1	1	0	1	0	0	0	14
X9	3	0	1	0	0	1*	0	0	0	0	1	0	0	10
X1	4	0	0	1	0	0	1	0	0	0	0	1	0	5
X1	5	0	0	0	1	0	0	1	0	0	0	0	1	10

(b) This table is much smaller than the simplex tableau and it stores the same information.

Cost Per Unit Distributed

		Destination				
		1	2	3	4	Supply
Source	1	400	490	460	0	18
	2	600	530	560	0	14
Demand		10	5	10	7	

(c) The solution is not optimal, since $c_{13} - u_1 - v_3 = -100$.

```
          |         Destination        |
          |   1   |   2   |   3   |   4   |Supply  u[i]
__________|_______|_______|_______|_______|_____________
          | 400|  | 490|  | 460|  |   0|  |       |
       1  |---- B|---- B|----   |---- B|       |
          |    10|     5|  -100|     3|    18|      0
          |_______|_______|_______|_______|       |
          | 600|  | 530|  | 560|  |   0|  |       |
       2  |----   |----   |---- B|---- B|       |
          |   200|    40|    10|     4|    14|      0
          |_______|_______|_______|_______|_______|______
  Demand  |    10      5     10      7|
__________|___________________________|
    v[j]  |   400    490    560      0|
          |                           | Z = 12050
```

(d)

```
          |         Destination        |
          |   1   |   2   |   3   |   4   |Supply  u[i]
__________|_______|_______|_______|_______|_____________
          | 400|  | 490|  | 460|  |   0|  |       |
       1  |---- B|---- B|---- B|----   |       |
          |    10|     5|     3|     0|    18|      0
          |_______|_______|_______|_______|       |
          | 600|  | 530|  | 560|  |   0|  |       |
       2  |----   |----   |---- B|---- B|       |
          |     0|     0|     7|     7|    14|      0
          |_______|_______|_______|_______|_______|______
  Demand  |    10      5     10      7|
__________|___________________________|
    v[j]  |     0      0      0      0|
          |                           | Z = 11750
```

```
          |         Destination        |
          |   1   |   2   |   3   |   4   |Supply  u[i]
__________|_______|_______|_______|_______|_____________
          | 400|  | 490|  | 460|  |   0|  |       |
       1  |---- B|---- L|---- P|----   |       |
          |    10|     5|     3|   100|    18|      0
          |_______|_______|_______|_______|       |
          | 600|  | 530|  | 560|  |   0|  |       |
       2  |----   |---- E|---- P|---- B|       |
          |   100|   -60|     7|     7|    14|    100
          |_______|_______|_______|_______|_______|______
  Demand  |    10      5     10      7|
__________|___________________________|
    v[j]  |   400    490    460   -100|
          |                           | Z = 11750
```

```
          |          Destination          |
          |   1   |   2   |   3   |   4   |Supply  u[i]
__________|_______|_______|_______|_______|______________
          | 400|  | 490|  | 460|  |   0|  |       |
        1 |---- B|----   |---- B|----   |       |
          |    10|     60|      8|    100|     18|  -100
          |_______|_______|_______|_______|       |
          | 600|  | 530|  | 560|  |   0|  |       |
        2 |----   |---- B|---- B|---- B|       |
          |   100|      5|      2|      7|     14|      0
          |_______|_______|_______|_______|_______|______
   Demand |    10       5      10       7|
__________|_______________________________|
     v[j] |   500     530     560       0|
          |                               | Z = 11450
```

The optimal solution is to haul 10 tons from the north pit to site 1 and 8 tons to site 3, 5 tons from the south pit to site 2 and 2 tons from the south pit to site 3. This incurs a cost of $\$11,450$.

(e) From the reduced costs $(c_{ij} - u_i - v_j)$ in the final tableau, we see that

$$\Delta c_{12} \geq -60 \quad \Rightarrow c_{12} \geq 430$$

$$\Delta c_{21} \geq -100 \;\; \Rightarrow c_{21} \geq 500.$$

If the contractor can negotiate a hauling cost per ton of 130 or less from the north pit to site 2, or of 80 or less from the south pit to site 1, a new solution using these options would give a cost at least as small as the current optimal cost $\$11,450$.

9.2-22.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C11	Colombo River Berdoo	0	0	160	1E+30	0
D11	Colombo River Los Devils	5	0	130	20	1E+30
E11	Colombo River San Go	0	10	220	1E+30	10
F11	Colombo River Hollyglass	0	0	170	0	20
C12	Sacron River Berdoo	2	0	140	0	1E+30
D12	Sacron River Los Devils	0	20	130	1E+30	20
E12	Sacron River San Go	2.5	0	190	10	10
F12	Sacron River Hollyglass	1.5	0	150	20	0
C13	Calorie River Berdoo	0	10	190	1E+30	10
D13	Calorie River Los Devils	0	50	200	1E+30	50
E13	Calorie River San Go	1.5	0	230	10	20
F13	Calorie River Hollyglass	0	-190	0	1E+30	190

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C14	Total To City Berdoo	2	180	2	2.5	1.5
D14	Total To City Los Devils	5	150	5	0	1.5
E14	Total To City San Go	4	230	4	3.5	1.5
F14	Total To City Hollyglass	1.5	190	1.5	2.5	1.5
G11	Colombo River From River	5	-20	5	1.5	0
G12	Sacron River From River	6	-40	6	1.5	2.5
G13	Calorie River From River	1.5	0	5	1E+30	3.5

(a) The optimal solution would change because the decrease of \$30 million is outside the allowable decrease of \$20 million.

(b) The optimal solution would remain the same, since the allowable increase is ∞.

(c) By the 100% rule for simultaneous changes, the optimal solution must remain the same.

$$C_{CS}: \quad \$230 \rightarrow \$215 \quad \text{\% of allowable decrease} = 100\left(\tfrac{230-215}{20}\right) = 75\%$$

$$C_{SL}: \quad \$130 \rightarrow \$145 \quad \text{\% of allowable decrease} = 100\left(\tfrac{130-145}{\infty}\right) = 0\%$$

These sum up to 75%.

(d) By the 100% rule for simultaneous changes, the shadow prices may or may not remain valid.

$$C_S: \quad \$6 \rightarrow \$5.5 \quad \text{\% of allowable decrease} = 100\left(\tfrac{6-5.5}{2.5}\right) = 20\%$$

$$C_H: \quad \$1.5 \rightarrow \$1 \quad \text{\% of allowable decrease} = 100\left(\tfrac{1.5-1}{0}\right) = \infty\%$$

These sum up to ∞%.

9.2-23.

(a) $\Delta c_{34} = -3 \Rightarrow \Delta(c_{34} - u_3 - v_4)^* = -3$, $(c_{34} - u_3 - v_4)^* = -2$

Iteration 3		Destination 1	2	3	4	5	Supply	u_i
	1	16 +4	16 +4	13 (50)	22 +7	17 +2	50	−7
	2	14 +2	14 +2	13 (20)	19 +4	15 (40)	60	−7
Source	3	19 (30)	19 (20)	20 (0)	20 −2	M M − 22	50	0
	4(D)	M M + 3	0 +3	M M + 2	0 (30)	0 (20)	50	−22
Demand		30	20	70	30	60	Z = 2460	
v_j		19	19	20	22	22		

The current feasible solution is feasible, but not optimal.

(b) $\Delta c_{23} = 3 \Rightarrow \Delta(c_{23} - u_2 - v_3)^* = 3$

We can revise the tableau by changing u_2 from -7 to $-7 + 3 = -4$. This causes v_5 to change to $22 - 3 = 19$, u_4 to $-22 + 3 = -19$, and v_4 to $22 - 3 = 19$.

Δ(reduced cost x_{41}) $= \Delta$(reduced cost x_{42}) $= \Delta$(reduced cost x_{43}) $= -\Delta u_4 = -3$

Δ(reduced cost x_{34}) $= \Delta$(reduced cost x_{14}) $= -\Delta v_4 = 3$

Δ(reduced cost x_{35}) $= \Delta$(reduced cost x_{15}) $= -\Delta v_5 = 3$

Iteration 3	Destination 1	2	3	4	5	Supply	u_i
Source 1	16 / +4	16 / +4	13 / 50	22 / 10	17 / 5	50	−7
2	14 / +2	14 / +2	16 / 20	19 / +4	15 / 40	60	−4
3	19 / 30	19 / 20	20 / 0	23 / 4	M / M−19	50	0
4(D)	M / M	0 / 0	M / M−1	0 / 30	0 / 20	50	−19
Demand	30	20	70	30	60	$Z = 2460$	
v_j	19	19	20	19	19		

The basic solution remains feasible and optimal.

(c) $\Delta s_2 = -10, \Delta d_5 = 10 \Rightarrow \Delta x_{25} = 10$

Iteration 3	Destination 1	2	3	4	5	Supply	u_i
Source 1	16 / +4	16 / +4	13 / 50	22 / +7	17 / +2	50	−7
2	14 / +2	14 / +2	13 / 20	19 / +4	15 / 30	50	−7
3	19 / 30	19 / 20	20 / 0	23 / +1	M / M − 22	50	0
4(D)	M / M + 3	0 / +3	M / M + 2	0 / 30	0 / 20	50	−22
Demand	30	20	70	30	50	$Z = 2460$	
v_j	19	19	20	22	22		

The basic solution remains feasible and optimal.

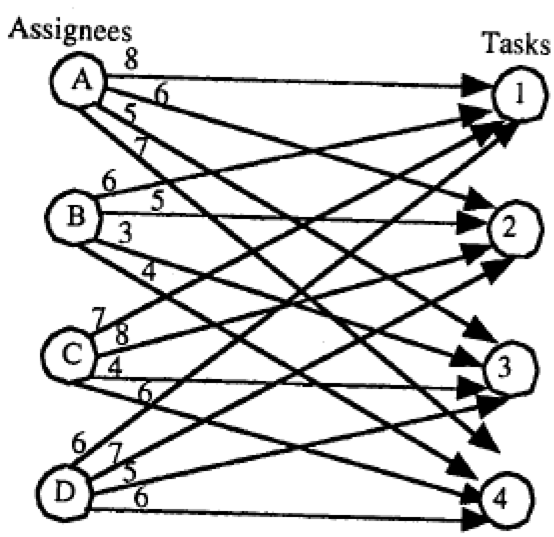
(d) $\Delta s_2 = \Delta d_2 = 20 \Rightarrow \Delta x_{23} = \Delta x_{32} = 20$ and $\Delta x_{33} = -20$

Iteration 3	Destination 1	2	3	4	5	Supply	u_i
Source 1	16 +4	16 +4	13 (50)	22 +7	17 +2	50	−7
2	14 +2	14 +2	13 (40)	19 +4	15 (40)	80	−7
3	19 (30)	19 (40)	20 (−20)	23 +1	M M − 22	50	0
4(D)	M M + 3	0 +3	M M + 2	0 (30)	0 (20)	50	−22
Demand	30	40	70	30	60	Z = 2460	
v_j	19	19	20	22	22		

This solution satisfies the optimality criterion, but it is infeasible.

9.3-1.

(a)



(b)

		Unit Cost ($) Task 1	2	3	4	Supply
Assignee	A	8	6	5	7	1
	B	6	5	3	4	1
	C	7	8	4	6	1
	D	6	7	5	6	1
Demand		1	1	1	1	

(c), (d)

Unit Cost			Task					
		1	2	3	4			
	A	$8	$6	$5	$7			
Assignee	B	$6	$5	$3	$4			
	C	$7	$8	$4	$6			
	D	$6	$7	$5	$6			
Assignments			Task			Total		
		1	2	3	4	Assignments		Supply
	A	0	1	0	0	1	=	1
Assignee	B	0	0	0	1	1	=	1
	C	0	0	1	0	1	=	1
	D	1	0	0	0	1	=	1
Total Assigned		1	1	1	1			
		=	=	=	=			Total Cost
Demand		1	1	1	1			$20

9.3-2.

(a) Ships are assignees and ports are assignments.

(b)

Assignee	Task A	B	C	D
1	500	400	600	700
2	600	600	700	500
3	700	500	700	600
4	500	400	600	600

Optimal Solution:

```
Task A is assigned to Assignee 1
Task D is assigned to Assignee 2
Task B is assigned to Assignee 3
Task C is assigned to Assignee 4
```

This incurs a cost of $2, 100.

(c)

	Destination 1	2	3	4	Supply
1	500	400	600	700	1
2	600	600	700	500	1
3	700	500	700	600	1
4	500	400	600	600	1
Demand	1	1	1	1	

(d) - (e)

(0)	Destination 1	2	3	4	Supply	u[i]
1	500 ---- P 1	400 ---- P 0	600 ---- 100	700 ---- 300	1	0
2	600 ---- -100	600 ---- P 1	700 ---- P 0	500 ---- -100	1	200
3	700 ---- 0	500 ---- -100	700 ---- L 1	600 ---- P 0	1	200
4	500 ---- E -200	400 ---- -200	600 ---- -100	600 ---- P 1	1	200
Demand	1	1	1	1		
v[j]	500	400	500	400		

(1)

	Destination 1	2	3	4	Supply	u[i]
1	500 / P / 0	400 / P / 1	600 / 100	700 / 100	1	0
2	600 / -100	600 / L / 0	700 / B / 1	500 / E / -300	1	200
3	700 / 200	500 / 100	700 / 200	600 / B / 1	1	0
4	500 / P / 1	400 / 0	600 / 100	600 / P / 0	1	0
Demand	1	1	1	1		
v[j]	500	400	500	600		

(2)

	Destination 1	2	3	4	Supply	u[i]
1	500 / B / 0	400 / B / 1	600 / -200	700 / 100	1	600
2	600 / 200	600 / 300	700 / P / 1	500 / P / 0	1	500
3	700 / 200	500 / 100	700 / -100	600 / B / 1	1	600
4	500 / B / 1	400 / 0	600 / E / -200	600 / L / 0	1	600
Demand	1	1	1	1		
v[j]	-100	-200	200	0		

(3)

	Destination 1	2	3	4	Supply	u[i]
1	500 P 0	400 P 1	600 0	700 300	1	0
2	600 0	600 100	700 L 1	500 P 0	1	100
3	700 0	500 E -100	700 -100	600 P 1	1	200
4	500 P 1	400 0	600 P 0	600 200	1	0
Demand	1	1	1	1		
v[j]	500	400	600	400		

(4)

	Destination 1	2	3	4	Supply	u[i]
1	500 B 1	400 B 0	600 0	700 200	1	0
2	600 100	600 200	700 100	500 B 1	1	0
3	700 100	500 B 1	700 0	600 B 0	1	100
4	500 B 0	400 0	600 B 1	600 100	1	0
Demand	1	1	1	1		
v[j]	500	400	600	500		

One optimal assignment is: $(1,1),(2,4),(3,2),(4,3)$, where the first entry is ship and the second port.

(f) Continuing to pivot where reduced costs are zero:

(5)

	Destination 1	2	3	4	Supply	u[i]
1	500 B 1	400 B 0	600 0	700 200	1	400
2	600 100	600 200	700 100	500 B 1	1	400
3	700 100	500 P 1	700 E 0	600 B 0	1	500
4	500 0	400 P 0	600 L 1	600 100	1	400
Demand	1	1	1	1		
v[j]	100	0	200	200		

Alternative optimal matching: $(1,1), (2,4), (3,3), (4,2)$

(6)

	Destination 1	2	3	4	Supply	u[i]
1	500 L 1	400 P 0	600 0	700 200	1	-100
2	600 100	600 200	700 100	500 B 1	1	-100
3	700 100	500 B 0	700 B 1	600 B 0	1	0
4	500 E 0	400 P 1	600 0	600 100	1	-100
Demand	1	1	1	1		
v[j]	600	500	700	600		

Alternative optimal matching: $(1,2), (2,4), (3,3), (4,1)$

(7)

	Destination 1	2	3	4	Supply	u[i]
1	500 ---- 0	400 ---- P 1	600 ---- E 0	700 ---- 200	1	-100
2	600 ---- 100	600 ---- 200	700 ---- 100	500 ---- B 1	1	-100
3	700 ---- 100	500 ---- P 0	700 ---- L 1	600 ---- B 0	1	0
4	500 ---- B 1	400 ---- B 0	600 ---- 0	600 ---- 100	1	-100
Demand	1	1	1	1		
v[j]	600	500	700	600		

Alternative optimal matching: $(1,3),(2,4),(3,2),(4,1)$

9.3-3.

(a)

a)

Cost Table

Assignee	Task 1	2	3	4	5
1	7440	18000	12160	0	0
2	6960	16400	11200	0	0
3	7680	18400	12800	0	0
4	6720	16800	1M	0	0
5	6960	17200	1M	0	0

(b) The optimal cost is $34,960.

Assignee	Task 1	2	3	4	5
1				X	
2			X		
3					X
4		X			
5	X				

Cost is
34960

(c)

	Product					
	1	2	3	4	5	Supply
Plant 1	74.4	180	121.6	0	0	1
2	69.6	164	112	0	0	1
3	76.8	184	128	0	0	1
4	67.2	168	M	0	0	1
5	69.6	172	M	0	0	1
Demand	1	1	1	1	1	

(d)

(0)	1	2	3	4	5	Supply	u[i]
1	74.4 ---- 4.8	180 ---- 9.6	122 ---- 3.2	0 ---- 0	0 ---- B 1	1	6.4
2	69.6 ---- 6.4	164 ---- B 0	112 ---- B 1	0 ---- 6.4	0 ---- 6.4	1	0
3	76.8 ---- 7.2	184 ---- 13.6	128 ---- 9.6	0 ---- B 1	0 ---- B 0	1	6.4
4	67.2 ---- B 0	168 ---- B 1	M ---- 1M-e3	0 ---- 2.4	0 ---- 2.4	1	4
5	69.6 ---- B 1	172 ---- 1.6	M ---- 1M-e3	0 ---- 0	0 ---- B 0	1	6.4
Demand	1	1	1	1	1		
v[j]	63.2	164	112	-6.4	-6.4		

The initial solution from Vogel's approximation method is optimal. Plant 2 produces product 3, plant 4 produces product 2, plant 5 produces product 1. This incurs a cost of $34,960.

9.3-4.

(a) After adding a dummy stroke, which everyone can swim in zero seconds, the problem becomes that of assigning 5 swimmers to 5 strokes. The optimal solution turns out to be the following: David swims the backstroke, Tony swims the breaststroke, Chris swims the butterfly, and Carl swims the freestyle.

	Task					
	Carl	Chris	David	Tony	Ken	
Assignee	A	B	C	D	E	Row Min
Back 1	37.7	32.9	33.8	37	35.4	32.9
Breast 2	43.4	33.1	42.2	34.7	41.8	33.1
Fly 3	33.3	28.5	38.9	30.4	33.6	28.5
Free 4	29.2	26.4	29.6	28.5	31.1	26.4
Dummy 5	0	0	0	0	0	0

(b) Cost: 126.2

```
Task C is assigned to Assignee 1
Task D is assigned to Assignee 2
Task B is assigned to Assignee 3
Task A is assigned to Assignee 4
Task E is assigned to Assignee 5
```

9.3-5.

(a)

		Product					
		1	2	3	4	5	Supply
Plant	1	820	810	840	960	0	2
	2	800	870	M	920	0	2
	3	740	900	810	840	0	1
Demand		1	1	1	1	1	

(b) - (c)

	Destination						
	1	2	3	4	5	Supply	u[i]
1	820 / 20	810 B / 1	840 B / 1	960 / 40	0 B / 0	2	0
2	800 B / 1	870 / 60	M / 1M-e3	920 B / 0	0 B / 1	2	0
3	740 / 20	900 / 170	810 / 50	840 B / 1	0 / 80	1	-80
Demand	1	1	1	1	1		
v[j]	800	810	840	920	0		

Since all the reduced costs are nonnegative, this solution is optimal.

(d)

	Product 1	2	3	4	5	Supply
Plant 1	820	810	840	960	0	1
2	820	810	840	960	0	1
3	800	870	M	920	0	1
4	800	870	M	920	0	1
5	740	900	810	840	M	1
Demand	1	1	1	1	1	

This is identical to the table in (a) except that plants 1 and 2 have been split into two plants each.

(e)

	Destination 1	2	3	4	5	Supply	u[i]
1	820 / 20	810 B / 1	840 / 0	960 / 40	0 / 0	1	0
2	820 / 20	810 B / 0	840 B / 1	960 / 40	0 B / 0	1	0
3	800 B / 1	870 / 60	M / 1M-e3	920 / 0	0 / 0	1	0
4	800 B / 0	870 / 60	M / 1M-e3	920 B / 0	0 B / 1	1	0
5	740 / 20	900 / 170	810 / 50	840 B / 1	M / 1M+80	1	-80
Demand	1	1	1	1	1		
v[j]	800	810	840	920	0		

The basic feasible solution for the transformed problem above corresponds to that given in part (c).

9.3-6.

0	Destination 1	2	3	4	Supply	u[i]
1	13 / 7	16 / 8	12 / 8	11 B / 1	1	-9
2	15 B / 1	M / 1M-17	13 P / 0	20 L / 0	1	0
3	5 B / 0	7 B / 1	10 / 7	6 / -4	1	-10
4	0 / -2	0 / -4	0 P / 1	0 E / -7	1	-13
Demand	1	1	1	1		
v[j]	15	17	13	20		

(1)	Destination 1	2	3	4	Supply	u[i]
1	13 / 0	16 / 1	12 / 1	11 B / 1	1	-2
2	15 L / 1	M / 1M-17	13 P / 0	20 / 7	1	0
3	5 P / 0	7 P / 1	10 / 7	6 / 3	1	-10
4	0 / -2	0 E / -4	0 P / 1	0 B / 0	1	-13
Demand	1	1	1	1		
v[j]	15	17	13	13		

(2)

Source	Destination 1	Destination 2	Destination 3	Destination 4	Supply	u[i]
1	13 / 4	16 / 5	12 / 1	11 / B / 1	1	11
2	15 / 4	M / 1M-13	13 / B / 1	20 / 7	1	13
3	5 / B / 1	7 / L / 0	10 / 3	6 / E / -1	1	7
4	0 / 2	0 / P / 1	0 / B / 0	0 / P / 0	1	0
Demand	1	1	1	1		
v[j]	-2	0	0	0		

(3)

Source	Destination 1	Destination 2	Destination 3	Destination 4	Supply	u[i]
1	13 / 3	16 / 5	12 / 1	11 / B / 1	1	11
2	15 / 3	M / 1M-13	13 / B / 1	20 / 7	1	13
3	5 / B / 1	7 / 1	10 / 4	6 / B / 0	1	6
4	0 / 1	0 / B / 1	0 / B / 0	0 / B / 0	1	0
Demand	1	1	1	1		
v[j]	-1	0	0	0		

This solution corresponds to that given in Section 9.3; although the set of basic variables is different, the values of the variables are the same.

9.3-7.

(a) Let assignees 1 and 2 represent plant A, assignees 3 and 4 represent plant B, and the tasks be the distribution centers.

Cost Table

		Task 1	2	3	dummy 4
Assignee	1	8000	14000	12000	0
	2	8000	14000	12000	0
	3	6000	16000	15000	0
	4	6000	16000	15000	0

(b) Cost: $32,000$

		Task 1	2	3	4
Assignee	1		X		
	2			X	
	3	X			
	4				X

(c)

Cost Per Unit Distributed

		Destination 1	2	3	4	Supply
Source	1	8000	14000	12000	0	1
	2	8000	14000	12000	0	1
	3	6000	16000	15000	0	1
	4	6000	16000	15000	0	1
Demand		1	1	1	1	

(d)

		Destination 1	2	3	4	Supply
Source	1		1			1
	2			1		1
	3	1				1
	4				1	1
Demand		1	1	1	1	Cost is 32000

(e)

Cost Per Unit Distributed

		Destination 1	2	3	4	Supply
Source	1	8000	14000	12000	0	2
	2	6000	16000	15000	0	2
Demand		1	1	1	1	

(f)

		Destination				
		1	2	3	4	Supply
	1		1	1		2
Source	2	1			1	2
Demand		1	1	1	1	Cost is 32000

9.3-8.

(a)

		Task		
		1	2	3
	1			X
Assignee	2	X		
	3		X	

Cost
10

(b)

Cost Per Unit Distributed

		Destination			
		1	2	3	Supply
	1	5	7	4	1
Source	2	3	6	5	1
	3	2	3	4	1
Demand		1	1	1	

(c)

		Destination			
		1	2	3	Supply
	1			1	1
Source	2	1			1
	3		1		1
Demand		1	1	1	Cost is 10

(d) A transportation problem of size $m \times n$ has $m+n-1$ basic variables. Since $m = n$ for the assignment problem, there are $2(3) - 1 = 5$ basic variables, but only 3 assignments. Thus, 2 basic variables are degenerate, they equal zero. Assignment problems are always highly degenerate. This can be seen using the interactive routine in the OR Courseware.

(e) x_{A1}, x_{A2}, x_{B2} and one of (x_{B3}, x_{C3}) are nonbasic, too. x_{C1} and one of (x_{B3}, x_{C3}) are basic and equal zero.

Dual variables:

				u_i
	5 +3	7 +4	4 1	0
	3 1	6 +2	5 0	1
	2 0	3 1	4 0	0
v_j	2	3	4	

Looking at $c_{ij} - u_i - v_j$, we see that the allowable ranges for this solution to stay optimal are: $c_{A1} \geq 2, c_{A2} \geq 3, c_{B1} \geq 4, c_{B2} \geq 5$.

9.3-9.

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{subject to} \quad & \sum_{j=1}^{n} x_{ij} = 1 && \text{for } i = 1, 2, \ldots, n \\ & \sum_{i=1}^{n} x_{ij} = 1 && \text{for } j = 1, 2, \ldots, n \\ & x_{ij} \geq 0 && \text{for } i, j = 1, 2, \ldots, n \end{aligned}$$

The table of constraint coefficients is identical to that for the transportation problem (Table 9.6). The assignment problem has a more special structure because $m = n$ and $s_i = d_i = 1$ for every i.

9.4-1.

Start with:

5	4	6	7
6	6	7	5
7	5	7	6
5	4	6	6

Subtract the minimum element from each element in the column and continue the algorithm.

$$
\begin{array}{|cccc|}
\hline
0 & 0 & 0 & 2 \\
1 & 2 & 1 & 0 \\
2 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 \\
\hline
\end{array}
\quad
\begin{array}{|cccc|}
\hline
0 & 0 & \boxed{0} & 3 \\
0 & 1 & 0 & \boxed{0} \\
1 & \boxed{0} & 0 & 1 \\
\boxed{0} & 0 & 0 & 2 \\
\hline
\end{array}
$$

One optimal solution is to assign ships $(1, 2, 3, 4)$ to ports $(3, 4, 2, 1)$, with cost 21.

9.4-2.

Subtract the minimum element in each row from each element in the row and continue the algorithm.

$$
\begin{array}{|ccccc|}
\hline
4.8 & 0 & 0.9 & 4.1 & 25 \\
10.3 & 0 & 9.1 & 1.6 & 8.7 \\
4.8 & 0 & 10.4 & 1.9 & 5.1 \\
2.8 & 0 & 3.2 & 2.1 & 4.7 \\
0 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
\quad
\begin{array}{|ccccc|}
\hline
3.9 & 0 & 0 & 3.2 & 1.6 \\
9.4 & 0 & 8.2 & 0.7 & 7.8 \\
3.9 & 0 & 9.5 & 1 & 4.2 \\
1.9 & 0 & 2.3 & 1.2 & 3.6 \\
0 & 0.9 & 0 & 0 & 0 \\
\hline
\end{array}
$$

$$
\begin{array}{|ccccc|}
\hline
3.9 & 0.7 & 0 & 3.2 & 1.6 \\
8.7 & 0 & 7.5 & 0 & 7.1 \\
3.2 & 0 & 8.8 & 0.3 & 3.5 \\
1.2 & 0 & 1.6 & 0.5 & 2.9 \\
0 & 1.6 & 0 & 0 & 0 \\
\hline
\end{array}
\quad
\begin{array}{|ccccc|}
\hline
2.7 & 0.7 & \boxed{0} & 3.2 & 0.4 \\
7.5 & 0 & 7.5 & \boxed{0} & 5.9 \\
2.0 & \boxed{0} & 8.8 & 0.3 & 2.3 \\
\boxed{0} & 0 & 1.6 & 0.5 & 1.7 \\
0 & 2.8 & 1.2 & 1.2 & \boxed{0} \\
\hline
\end{array}
$$

One optimal solution is that David swims the backstroke, Tony the breaststroke, Chris the butterfly and Carl the freestyle. The total time is 126.2.

9.4-3.

Cost: $3,260$

$$\begin{pmatrix} 0 & 0 & 30 & 120 & 0 \\ 100 & 0 & 30 & 120 & 0 \\ 80 & 60 & M & 80 & 0 \\ 80 & 60 & M & 80 & 0 \\ 20 & 90 & 0 & 0 & M \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 30 & 120 & 60 \\ 100 & 0 & 30 & 120 & 60 \\ 20 & 0 & M & 20 & 0 \\ 20 & 0 & M & 20 & 0 \\ 20 & 90 & 0 & 0 & M \end{pmatrix}$$

$$\begin{pmatrix} \boxed{0} & 0 & 10 & 100 & 60 \\ 100 & \boxed{0} & 10 & 100 & 60 \\ 20 & 0 & M & \boxed{0} & 0 \\ 20 & 0 & M & 0 & \boxed{0} \\ 40 & 110 & \boxed{0} & 0 & M \end{pmatrix}$$

9.4-4.

Subtract the minimum element in each row from every element in the row and continue the algorithm. This gives an optimal solution with cost 12.

$$\begin{pmatrix} M & 1 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} M & \boxed{0} & 0 \\ 2 & 1 & \boxed{0} \\ \boxed{0} & 0 & 1 \end{pmatrix}$$

9.4-5.

Subtract the minimum element in each column from every element in the column and continue the algorithm.

$$\begin{pmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 4 & 0 \\ 2 & 1 & 1 & 3 \\ 1 & 1 & 3 & 0 \end{pmatrix} \quad \begin{pmatrix} 3 & 0 & \boxed{0} & 2 \\ \boxed{0} & 2 & 4 & 1 \\ 1 & \boxed{0} & 0 & 3 \\ 0 & 0 & 2 & \boxed{0} \end{pmatrix}$$

An optimal assignment is $(A,3)$, $(B,1)$, $(C,2)$, $(D,4)$, with cost 3.

9.4-6. Subtracting the minimum element in each row from each element in the row, and then continuing the algorithm, we get:

$$
\begin{bmatrix} 0 & 2 & 1 & 1 \\ 3 & 0 & 1 & 2 \\ 0 & 3 & 2 & 0 \\ 2 & 0 & 1 & 4 \end{bmatrix}
\qquad
\begin{bmatrix} \boxed{0} & 2 & 0 & 0 \\ 3 & 0 & \boxed{0} & 1 \\ 1 & 4 & 2 & \boxed{0} \\ 2 & \boxed{0} & 0 & 3 \end{bmatrix}
$$

An optimal assignment is (A, 1), (B, 3), (C, 4), (D, 2). Cost = 16.information is needed to determine this.

Case 9.1

Option 1 (Shipping by Rail):

	A	B	C	D	E	F	G	H	I
1	**Shipping Cost ($thousands)**								
2		Market 1	Market 2	Market 3	Market 4	Market 5			
3	Source 1	61	72	45	55	66			
4	Source 2	69	78	60	49	56			
5	Source 3	59	66	63	61	47			
6									
7									
8	**Shipment Quantity (million board feet)**						Total		Total
9		Market 1	Market 2	Market 3	Market 4	Market 5	Shipped		Available
10	Source 1	6	0	9	0	0	15	=	15
11	Source 2	2	0	0	10	8	20	=	20
12	Source 3	3	12	0	0	0	15	=	15
13	Total Received	11	12	9	10	8			
14		=	=	=	=	=			Total Cost ($thousands)
15	Total To Sell	11	12	9	10	8			2816

Option 2 (Shipping by Ship):

	A	B	C	D	E	F	G	H	I
1	**Shipping Cost ($thousands)**								
2		Market 1	Market 2	Market 3	Market 4	Market 5			
3	Source 1	31	38	24	**55**	35			**bold** = rail cost
4	Source 2	36	43	28	24	31			(only rail feasible)
5	Source 3	**59**	33	36	32	26			
6									
7	**Ship Investment ($thousand)**								
8		Market 1	Market 2	Market 3	Market 4	Market 5			
9	Source 1	275	303	238	**0**	285			
10	Source 2	293	318	270	250	265			
11	Source 3	**0**	283	275	268	240			
12									
13	**Equivalent Annual Cost ($thousands)**								Equivalent Annual
14		Market 1	Market 2	Market 3	Market 4	Market 5			Investment Cost Factor
15	Source 1	58.5	68.3	47.8	55	63.5			10%
16	Source 2	65.3	74.8	55	49	57.5			
17	Source 3	59	61.3	63.5	58.8	50			
18									
19	**Shipment Quantity (million board-feet)**						Total		Total
20		Market 1	Market 2	Market 3	Market 4	Market 5	Shipped		Available
21	Source 1	6	0	9	0	0	15	=	15
22	Source 2	5	0	0	10	5	20	=	20
23	Source 3	0	12	0	0	3	15	=	15
24	Total Received	11	12	9	10	8			
25		=	=	=	=	=			Total Cost ($thousands)
26	Total To Sell	11	12	9	10	8			2770.8

Option 3 (Shipping by Best Available for each Route):

	A	B	C	D	E	F	G	H	I
1	**Shipping Cost (Rail) ($thousands)**								
2		Market 1	Market 2	Market 3	Market 4	Market 5			
3	Source 1	61	72	45	55	66			
4	Source 2	69	78	60	49	56			
5	Source 3	59	66	63	61	47			
6									
7	**Shipping Cost (Ship) ($thousands)**								
8		Market 1	Market 2	Market 3	Market 4	Market 5			
9	Source 1	31	38	24	**55**	35			
10	Source 2	36	43	28	24	31			
11	Source 3	**59**	33	36	32	26			
12									
13	**Ship Investment ($thousands)**								
14		Market 1	Market 2	Market 3	Market 4	Market 5			
15	Source 1	275	303	238	**0**	285			
16	Source 2	293	318	270	250	265			
17	Source 3	**0**	283	275	268	240			
18									
19	**Equivalent Annual Cost ($thousands)**								Equivalent Annual
20		Market 1	Market 2	Market 3	Market 4	Market 5			Investment Cost Factor
21	Source 1	58.5	68.3	47.8	55	63.5			10%
22	Source 2	65.3	74.8	55	49	57.5			
23	Source 3	59	61.3	63.5	58.8	50			
24									
25	**Annual Cost (Best Method) ($thousands)**								
26		Market 1	Market 2	Market 3	Market 4	Market 5			
27	Source 1	58.5	68.3	45	55	63.5			
28	Source 2	65.3	74.8	55	49	56			
29	Source 3	59	61.3	63	58.8	47			
30									
31	**Shipment Quantity (million board feet)**						Total		Total
32		Market 1	Market 2	Market 3	Market 4	Market 5	Shipped		Available
33	Source 1	6	0	9	0	0	15	=	15
34	Source 2	5	0	0	10	5	20	=	20
35	Source 3	0	12	0	0	3	15	=	15
36	Total Received	11	12	9	10	8			
37		=	=	=	=	=			Total Cost ($000)
38	Total To Sell	11	12	9	10	8			2729.1
39									
40	**Method of Shipment**								
41		Market 1	Market 2	Market 3	Market 4	Market 5			
42	Source 1	Ship		Rail					
43	Source 2	Ship			Ship	Rail			
44	Source 3		Ship			Rail			

When comparing the three options, it is best to use the combination plan, while shipping entirely by rail leads to the highest costs.

If costs of shipping by water are expected to rise considerably more than for shipping by rail, stay with rail and use Option 1. If the reverse is true, then use Option 2. If the cost comparisons will remain roughly the same, use Option 3. Option 3 is clearly the most feasible but may not be chosen if it is too logistically cumbersome. More knowledge of the situation is necessary to determine this.

Case 9.2

a) \$20 million is saved in comparison with the results in Figure 6.13 by shipping 20 million fewer barrels to Charleston and 20 million more to St. Louis.

	B	C	D	E	F	G	H	I	J
3			Refineries						
4	**Unit Cost (\$millions)**		New Orleans	Charleston	Seattle	St. Louis			
5		Texas	2	4	5	1			
6	Oil	California	5	5	3	4			
7	Fields	Alaska	5	7	3	7			
8		Middle East	2	3	5	4			
9									
10									
11	**Shipment Quantity**		Refineries						
12	**(millions of barrels)**		New Orleans	Charleston	Seattle	St. Louis	Total Shipped		Supply
13		Texas	0	0	0	80	80	=	80
14	Oil	California	0	0	0	60	60	=	60
15	Fields	Alaska	20	0	80	0	100	=	100
16		Middle East	80	40	0	0	120	=	120
17		Total Received	100	40	80	140			
18			<=	<=	<=	<=			Total Cost
19		Capacity	100	60	80	150			(\$millions)
20									940

b) \$40 million is saved in comparison with the results in Figure 6.17.

	B	C	D	E	F	G	H	I	J
3			Distribution Center						
4	**Unit Cost (\$millions)**		Pittsburgh	Atlanta	Kansas City	San Francisco			
5		New Orleans	6.5	5.5	6	8			
6	Refineries	Charleston	7	5	4	7			
7		Seattle	7	8	4	3			
8		St. Louis	4	3	1	5			
9									
10									
11	**Shipment Quantity**		Distribution Center						
12	**(millions of barrels)**		Pittsburgh	Atlanta	Kansas City	San Francisco	Total Shipped		Supply
13		New Orleans	60	40	0	0	100	=	100
14	Refineries	Charleston	0	40	0	0	40	=	40
15		Seattle	0	0	0	80	80	=	80
16		St. Louis	40	0	80	20	140	=	140
17		Total Received	100	80	80	100			
18			=	=	=	=			Total Cost
19		Demand	100	80	80	100			(\$millions)
20									1,390

The cost of shipping both crude oil and finished product under this plan is \$940 million + \$1,390 million = \$2,330 million or \$2.33 billion — a savings of \$60 million compared to the original results in Table 6.20.

c) $35 million is saved in comparison with the results in part (b).
$75 million is saved in comparison with the results in Figure 6.17.

	B	C	D	E	F	G	H	I	J
3			Distribution Center						
4	**Unit Cost ($millions)**		Pittsburgh	Atlanta	Kansas City	San Francisco			
5		New Orleans	6.5	5.5	6	8			
6	Refineries	Charleston	7	5	4	7			
7		Seattle	7	8	4	3			
8		St. Louis	4	3	1	5			
9									
10									
11	**Shipment Quantity**		Distribution Center						
12	**(millions of barrels)**		Pittsburgh	Atlanta	Kansas City	San Francisco	Total Shipped		Capacity
13		New Orleans	50	20	0	0	70	<=	100
14	Refineries	Charleston	0	60	0	0	60	<=	60
15		Seattle	0	0	0	80	80	<=	80
16		St. Louis	50	0	80	20	150	<=	150
17		Total Received	100	80	80	100			
18			=	=	=	=			Total Cost
19		Demand	100	80	80	100			($millions)
20									1,355

d) This solution costs $40 million more than the solution in part (a).
This solution costs $20 million more than the solution is Figure 6.13.

	B	C	D	E	F	G	H	I	J
3			Refineries						
4	**Unit Cost ($millions)**		New Orleans	Charleston	Seattle	St. Louis			
5		Texas	2	4	5	1			
6	Oil	California	5	5	3	4			
7	Fields	Alaska	5	7	3	7			
8		Middle East	2	3	5	4			
9									
10									
11	**Shipment Quantity**		Refineries						
12	**(millions of barrels)**		New Orleans	Charleston	Seattle	St. Louis	Total Shipped		Supply
13		Texas	0	0	0	80	80	=	80
14	Oil	California	0	0	0	60	60	=	60
15	Fields	Alaska	10	0	80	10	100	=	100
16		Middle East	60	60	0	0	120	=	120
17		Total Received	70	60	80	150			
18			=	=	=	=			Total Cost
19		Demand	70	60	80	150			($millions)
20									980

The total cost of shipping both crude oil and finished product under this plan is $1,355 million + $980 million = $2,335 million or $2.335 billion. This is $5 million more than the cost of the combined total obtained in part (b), but $55 million less than the total in Table 6.20.

e) The two transportation problems (shipping to refineries and shipping to distributions centers) are combined into a single model. The amount shipped to the refineries is constrained to be no more than capacity: TotalReceived(D16:G16) ≤ Capacity(D18:G18). The total shipped out of the refineries is constrained to equal the total amount shipped in: ShippedOut(H31:H34) = ShippedIn(J31:J34). The goal is to minimize the total combined cost (in J45) which is the sum of the two intermediate costs (in J20 and J39).

	A	B	C	D	E	F	G	H	I	J
1	**Shipping to Refineries**									
2					Refineries					
3		**Unit Cost ($millions)**		New Orleans	Charleston	Seattle	St. Louis			
4			Texas	2	4	5	1			
5		Oil	California	5	5	3	4			
6		Fields	Alaska	5	7	3	7			
7			Middle East	2	3	5	4			
8										
9										
10		**Shipment Quantity**			Refineries					
11		**(millions of barrels)**		New Orleans	Charleston	Seattle	St. Louis	Total Shipped		Supply
12			Texas	0	0	0	80	80	=	80
13		Oil	California	0	0	0	60	60	=	60
14		Fields	Alaska	20	0	80	0	100	=	100
15			Middle East	80	30	0	10	120	=	120
16			Total Received	100	30	80	150			
17				<=	<=	<=	<=			Cost
18			Capacity	100	60	80	150			(Oil Fields --> Refineries)
19										($millions)
20	**Shipping to Distribution Centers**									950
21					Distribution Center					
22		**Unit Cost ($millions)**		Pittsburgh	Atlanta	Kansas City	San Francisco			
23			New Orleans	6.5	5.5	6	8			
24		Refineries	Charleston	7	5	4	7			
25			Seattle	7	8	4	3			
26			St. Louis	4	3	1	5			
27										
28										
29		**Shipment Quantity**			Distribution Center					
30		**(millions of barrels)**		Pittsburgh	Atlanta	Kansas City	San Francisco	Shipped Out		Shipped In
31			New Orleans	100	0	0	0	100	=	100
32		Refineries	Charleston	0	10	0	20	30	=	30
33			Seattle	0	0	0	80	80	=	80
34			St. Louis	0	70	80	0	150	=	150
35			Total Received	100	80	80	100			
36				=	=	=	=			Cost
37			Demand	100	80	80	100			(Refineries --> D.C.'s)
38										($millions)
39										1,370
40										
41										Combined
42										Total
43										Cost
44										($millions)
45										2,320

The total combined cost is $2,320 million or $2.32 billion, which is $10 million less than in part (b), $15 million less than in part (d), and $70 million less than in Table 6.20.

f) If the Los Angeles refinery is chosen instead, then the combined shipping cost is $2,450 million.

	A	B	C	D	E	F	G	H	I	J
1	**Shipping to Refineries**									
2						Refineries				
3		**Unit Cost ($millions)**		New Orleans	Charleston	Seattle	Los Angeles			
4			Texas	2	4	5	3			
5		Oil	California	5	5	3	1			
6		Fields	Alaska	5	7	3	4			
7			Middle East	2	3	5	4			
8										
9										
10		**Shipment Quantity**				Refineries				
11		**(millions of barrels)**		New Orleans	Charleston	Seattle	St. Louis	Total Shipped		Supply
12			Texas	40	0	0	40	80	=	80
13		Oil	California	0	0	0	60	60	=	60
14		Fields	Alaska	0	0	80	20	100	=	100
15			Middle East	60	60	0	0	120	=	120
16			Total Received	100	60	80	120			
17				<=	<=	<=	<=			Cost
18			Capacity	100	60	80	150			(Oil Fields --> Refineries)
19										($millions)
20	**Shipping to Distribution Centers**									880
21						Distribution Center				
22		**Unit Cost ($millions)**		Pittsburgh	Atlanta	Kansas City	San Francisco			
23			New Orleans	6.5	5.5	6	8			
24		Refineries	Charleston	7	5	4	7			
25			Seattle	7	8	4	3			
26			Los Angeles	8	6	3	2			
27										
28										
29		**Shipment Quantity**				Distribution Center				
30		**(millions of barrels)**		Pittsburgh	Atlanta	Kansas City	San Francisco	Shipped Out		Shipped In
31			New Orleans	80	20	0	0	100	=	100
32		Refineries	Charleston	0	60	0	0	60	=	60
33			Seattle	20	0	60	0	80	=	80
34			St. Louis	0	0	20	100	120	=	120
35			Total Received	100	80	80	100			
36				=	=	=	=			Cost
37			Demand	100	80	80	100			(Refineries --> D.C.'s)
38										($millions)
39										1,570
40										
41										Combined
42										Total
43										Cost
44										($millions)
45										2,450

If the Galveston refinery is chosen instead, then the combined shipping cost is $2,470 million.

	A	B	C	D	E	F	G	H	I	J
1	**Shipping to Refineries**									
2				Refineries						
3		**Unit Cost ($millions)**		New Orleans	Charleston	Seattle	Galveston			
4			Texas	2	4	5	1			
5		Oil	California	5	5	3	3			
6		Fields	Alaska	5	7	3	5			
7			Middle East	2	3	5	3			
8										
9										
10		**Shipment Quantity**		Refineries						
11		**(millions of barrels)**		New Orleans	Charleston	Seattle	Galveston	Total Shipped		Supply
12			Texas	0	0	0	80	80	=	80
13		Oil	California	0	0	0	60	60	=	60
14		Fields	Alaska	10	0	80	10	100	=	100
15			Middle East	90	30	0	0	120	=	120
16			Total Received	100	30	80	150			
17				<=	<=	<=	<=			Cost
18			Capacity	100	60	80	150			(Oil Fields --> Refineries)
19										($millions)
20	**Shipping to Distribution Centers**									870
21				Distribution Center						
22		**Unit Cost ($millions)**		Pittsburgh	Atlanta	Kansas City	San Francisco			
23			New Orleans	6.5	5.5	6	8			
24		Refineries	Charleston	7	5	4	7			
25			Seattle	7	8	4	3			
26			Galveston	5	4	3	6			
27										
28										
29		**Shipment Quantity**		Distribution Center						
30		**(millions of barrels)**		Pittsburgh	Atlanta	Kansas City	San Francisco	Shipped Out		Shipped In
31			New Orleans	100	0	0	0	100	=	100
32		Refineries	Charleston	0	0	30	0	30	=	30
33			Seattle	0	0	0	80	80	=	80
34			Galveston	0	80	50	20	150	=	150
35			Total Received	100	80	80	100			
36				=	=	=	=			Cost
37			Demand	100	80	80	100			(Refineries --> D.C.'s)
38										($millions)
39										1,600
40										
41										Combined
42										Total
43										Cost
44										($millions)
45										2,470

Site	Total Cost of Shipping Crude Oil	Total Cost of Shipping Finished Product	Operating Cost for New Refinery	Total Variable Cost
Los Angeles	$880 million	$1.57 billion	$620 million	$3.07 billion
Galveston	870 million	1.60 billion	570 million	3.12 billion
St. Louis	950 million	1.37 billion	530 million	2.92 billion

g) Answers will vary.

Case 9.3

a) Assign one scientist to each of the five projects to maximize the total number of bid points.

	A	B	C	D	E	F	G	H	I
1	Bid	Project Up	Project Stable	Project Choice	Project Hope	Project Release			
2	Dr. Kvaal	100	400	200	200	100			
3	Dr. Zuner	0	200	800	0	0			
4	Dr. Tsai	100	100	100	100	600			
5	Dr. Mickey	267	153	99	451	30			
6	Dr. Rollins	100	33	33	34	800			
7									
8							Total		
9	Assignment	Project Up	Project Stable	Project Choice	Project Hope	Project Release	Assignments		Supply
10	Dr. Kvaal	0	1	0	0	0	1	=	1
11	Dr. Zuner	0	0	1	0	0	1	=	1
12	Dr. Tsai	1	0	0	0	0	1	=	1
13	Dr. Mickey	0	0	0	1	0	1	=	1
14	Dr. Rollins	0	0	0	0	1	1	=	1
15	Total Assigned	1	1	1	1	1			
16		=	=	=	=	=			Total Bid Points
17	Demand	1	1	1	1	1			2551

To maximize the scientists preferences you want to assign Dr. Tsai to lead project Up, Dr. Kvaal to lead project Stable, Dr. Zuner to lead project Choice, Dr. Mickey to lead project Hope, and Dr. Rollins to lead project Release.

b) Since there are only four assignees, we introduce a dummy assignee with preferences of –1. The task that gets assigned the dummy assignee will not be done.

	A	B	C	D	E	F	G	H	I
1	Bid	Project Up	Project Stable	Project Choice	Project Hope	Project Release			
2	Dr. Kvaal	100	400	200	200	100			
3	Dr. Zuner	0	200	800	0	0			
4	Dr. Tsai	100	100	100	100	600			
5	Dr. Mickey	267	153	99	451	30			
6	Dummy	-1	-1	-1	-1	-1			
7									
8							Total		
9	Assignment	Project Up	Project Stable	Project Choice	Project Hope	Project Release	Assignments		Supply
10	Dr. Kvaal	0	1	0	0	0	1	=	1
11	Dr. Zuner	0	0	1	0	0	1	=	1
12	Dr. Tsai	0	0	0	0	1	1	=	1
13	Dr. Mickey	0	0	0	1	0	1	=	1
14	Dummy	1	0	0	0	0	1	=	1
15	Total Assigned	1	1	1	1	1			
16		=	=	=	=	=			Total Bid Points
17	Demand	1	1	1	1	1			2250

Project Up would not be done.

c) Since two of the assignees can do two tasks, we need to double them. We include assignees Zuner-1, Zuner-2, Mickey-1, and Mickey-2 into the problem. In order to have an equal number of assignees and tasks we also need to include one dummy task. In order to ensure that neither Dr. Kvall nor Dr. Tsai can get assigned the dummy task and thus no project, we insert a large negative number as their point bid for the dummy project.

	A	B	C	D	E	F	G	H	I	J
1	**Bid**	Project Up	Project Stable	Project Choice	Project Hope	Project Release	Dummy			
2	Dr. Kvaal	100	400	200	200	100	-10000			
3	Dr. Zuner -1	0	200	800	0	0	-1			
4	Dr. Zuner -2	0	200	800	0	0	-1			
5	Dr. Tsai	100	100	100	100	600	-10000			
6	Dr. Mickey -1	267	153	99	451	30	-1			
7	Dr. Mickey -2	267	153	99	451	30	-1			
8										
9								Total		
10	**Assignment**	Project Up	Project Stable	Project Choice	Project Hope		Project Release	Assignments		Supply
11	Dr. Kvaal	0	1	0	0	0	0	1	=	1
12	Dr. Zuner -1	0	0	0	0	0	1	1	=	1
13	Dr. Zuner -2	0	0	1	0	0	0	1	=	1
14	Dr. Tsai	0	0	0	0	1	0	1	=	1
15	Dr. Mickey -1	0	0	0	1	0	0	1	=	1
16	Dr. Mickey -2	1	0	0	0	0	0	1	=	1
17	Total Assigned	1	1	1	1	1	1			
18		=	=	=	=	=	=			Total Bid Points
19	Demand	1	1	1	1	1	1			2517

Dr. Kvaal leads project Stable, Dr. Zuner leads project Choice, Dr. Tsai leads project Release, and Dr. Mickey leads the projects Hope and Up.

d) Under the new bids of Dr. Zuner the assignment does not change:

	A	B	C	D	E	F	G	H	I	J
1	**Bid**	Project Up	Project Stable	Project Choice	Project Hope	Project Release	Dummy			
2	Dr. Kvaal	100	400	200	200	100	-10000			
3	Dr. Zuner -1	20	450	451	39	40	-1			
4	Dr. Zuner -2	20	450	451	39	40	-1			
5	Dr. Tsai	100	100	100	100	600	-10000			
6	Dr. Mickey -1	267	153	99	451	30	-1			
7	Dr. Mickey -2	267	153	99	451	30	-1			
8										
9								Total		
10	**Assignment**	Project Up	Project Stable	Project Choice	Project Hope		Project Release	Assignments		Supply
11	Dr. Kvaal	0	1	0	0	0	0	1	=	1
12	Dr. Zuner -1	0	0	0	0	0	1	1	=	1
13	Dr. Zuner -2	0	0	1	0	0	0	1	=	1
14	Dr. Tsai	0	0	0	0	1	0	1	=	1
15	Dr. Mickey -1	0	0	0	1	0	0	1	=	1
16	Dr. Mickey -2	1	0	0	0	0	0	1	=	1
17	Total Assigned	1	1	1	1	1	1			
18		=	=	=	=	=	=			Total Bid Points
19	Demand	1	1	1	1	1	1			2168

e) Certainly Dr. Zuner could be disappointed that she is not assigned to project Stable, especially when she expressed a higher preference for that project than the scientist assigned. The optimal solution maximizes the preferences overall, but individual scientists may be disappointed. We should therefore make sure to communicate the reasoning behind the assignments to the scientists.

f) Whenever a scientist cannot lead a particular project we use a large negative number as the point bid.

	A	B	C	D	E	F	G	H	I
1	**Bid**	Project Up	Project Stable	Project Choice	Project Hope	Project Release			
2	Dr. Kvaal	86	343	171	-10000	-10000			
3	Dr. Zuner	0	200	800	0	0			
4	Dr. Tsai	100	100	100	100	600			
5	Dr. Mickey	300	-10000	125	-10000	175			
6	Dr. Rollins	-10000	50	50	100	600			
7									
8							Total		
9	**Assignment**	Project Up	Project Stable	Project Choice	Project Hope	Project Release	Assignments		Supply
10	Dr. Kvaal	0	1	0	0	0	1	=	1
11	Dr. Zuner	0	0	1	0	0	1	=	1
12	Dr. Tsai	0	0	0	0	1	1	=	1
13	Dr. Mickey	1	0	0	0	0	1	=	1
14	Dr. Rollins	0	0	0	1	0	1	=	1
15	Total Assigned	1	1	1	1	1			
16		=	=	=	=	=			Total Bid Points
17	Demand	1	1	1	1	1			2143

Dr. Kvaal leads project Stable, Dr. Zuner leads project Choice, Dr. Tsai leads project Release, Dr. Mickey leads project Up, and Dr. Rollins leads project Hope.

g) When we want to assign two assignees to the same task we need to duplicate that task.

	A	B	C	D	E	F	G	H	I	J	K
1	**Bid**	Project Up	Project Stable	Project Choice	Hope-A	Hope-B	Release-A	Release-B			
2	Dr. Kvaal	86	343	171	-10000	-10000	-10000	-10000			
3	Dr. Zuner	0	200	800	0	0	0	0			
4	Dr. Tsai	100	100	100	100	100	600	600			
5	Dr. Mickey	300	-10000	125	-10000	-10000	175	175			
6	Dr. Rollins	-10000	50	50	100	100	600	600			
7	Dr. Arriaga	250	250	0	250	250	250	250			
8	Dr. Santos	111	1	0	333	333	555	555			
9											
10									Total		
11	**Assignment**	Project Up	Project Stable	Project Choice	Hope-A	Hope-B	Release-A	Release-B	Assignments		Supply
12	Dr. Kvaal	0	1	0	0	0	0	0	1	=	1
13	Dr. Zuner	0	0	1	0	0	0	0	1	=	1
14	Dr. Tsai	0	0	0	0	0	1	0	1	=	1
15	Dr. Mickey	1	0	0	0	0	0	0	1	=	1
16	Dr. Rollins	0	0	0	0	0	0	1	1	=	1
17	Dr. Arriaga	0	0	0	0	1	0	0	1	=	1
18	Dr. Santos	0	0	0	1	0	0	0	1	=	1
19	Total Assigned	1	1	1	1	1	1	1			
20		=	=	=	=	=	=	=			Total Bid Points
21	Demand	1	1	1	1	1	1	1			3226

Project Up is led by Dr. Mickey, Stable by Dr. Kvaal, Choice by Dr. Zuner, Hope by Dr. Arriaga and Dr. Santos, and Release by Dr. Tsai and Dr. Rollins.

h) No. Maximizing overall preferences does not maximize individual preferences. Scientists who do not get their first choice may become resentful and therefore lack the motivation to lead their assigned project. For example, in the optimal solution of part (g), Dr. Santos clearly elected project Release as his first choice, but he was assigned to lead project Hope.

In addition, maximizing preferences ignores other considerations that should be factored into the assignment decision. For example, the scientist with the highest preference for a project may not be the scientist most qualified to lead the project.