CHAPTER 3: INTRODUCTION TO LINEAR PROGRAMMING

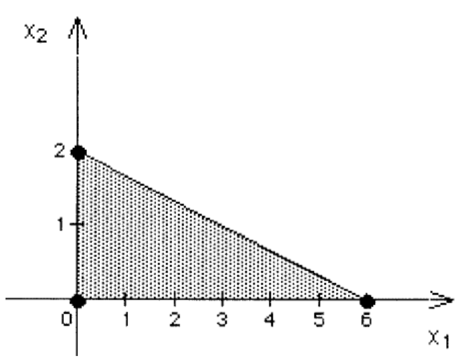
3.1-1.

Swift & Company solved a series of LP problems to identify an optimal production schedule. The first in this series is the scheduling model, which generates a shift-level schedule for a 28-day horizon. The objective is to minimize the difference of the total cost and the revenue. The total cost includes the operating costs and the penalties for shortage and capacity violation. The constraints include carcass availability, production, inventory and demand balance equations, and limits on the production and inventory. The second LP problem solved is that of capable-to-promise models. This is basically the same LP as the first one, but excludes coproduct and inventory. The third type of LP problem arises from the available-to-promise models. The objective is to maximize the total available production subject to production and inventory balance equations.

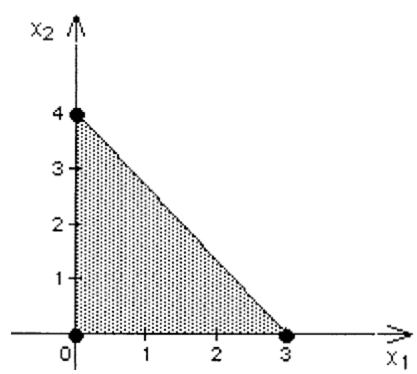
As a result of this study, the key performance measure, namely the weekly percent-sold position has increased by 22%. The company can now allocate resources to the production of required products rather than wasting them. The inventory resulting from this approach is much lower than what it used to be before. Since the resources are used effectively to satisfy the demand, the production is sold out. The company does not need to offer discounts as often as before. The customers order earlier to make sure that they can get what they want by the time they want. This in turn allows Swift to operate even more efficiently. The temporary storage costs are reduced by 90%. The customers are now more satisfied with Swift. With this study, Swift gained a considerable competitive advantage. The monetary benefits in the first years was $12.74 million, including the increase in the profit from optimizing the product mix, the decrease in the cost of lost sales, in the frequency of discount offers and in the number of lost customers. The main nonfinancial benefits are the increased reliability and a good reputation in the business.

3.1-2.

(a)



(b)



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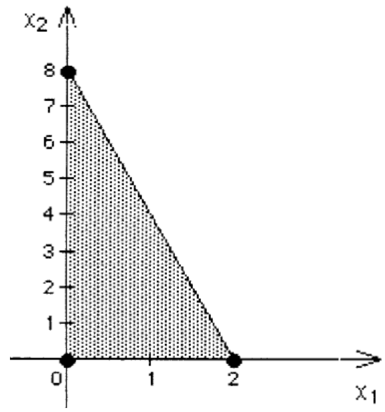

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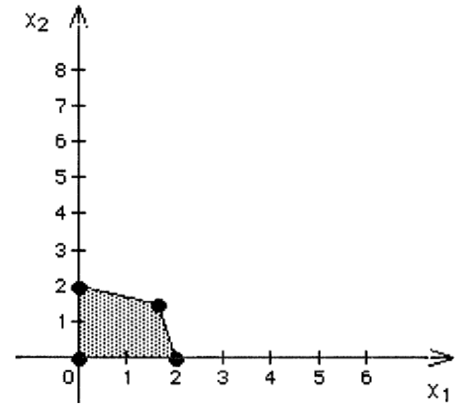
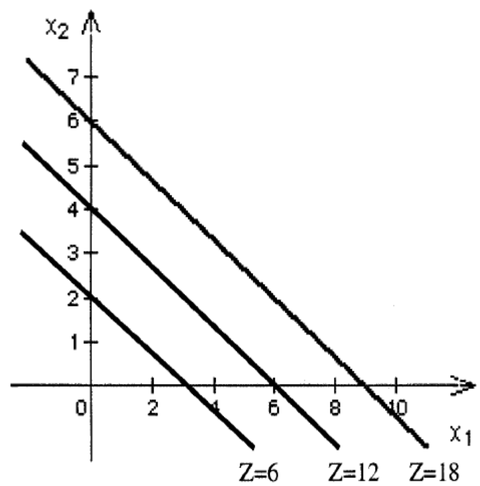


3.1-3.

(a)



(b)

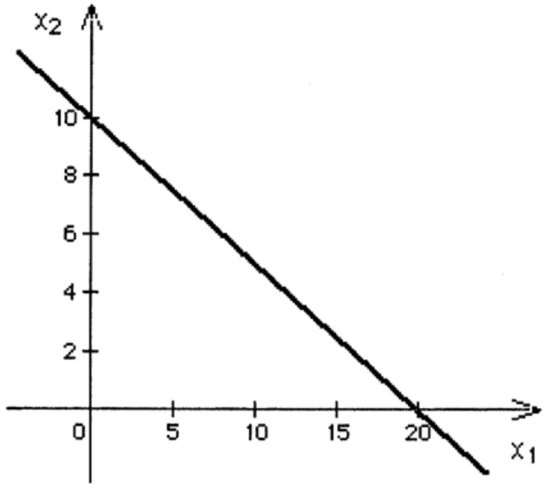
	Slope-Intercept Form	Slope	Intercept
$Z = 6$	$x_2 = -\frac{2}{3}x_1 + 2$	$-\frac{2}{3}$	2
$Z = 12$	$x_2 = -\frac{2}{3}x_1 + 4$	$-\frac{2}{3}$	4
$Z = 18$	$x_2 = -\frac{2}{3}x_1 + 6$	$-\frac{2}{3}$	6

3.1-4.

(a) $x_2 = -\frac{1}{2}x_1 + 10$

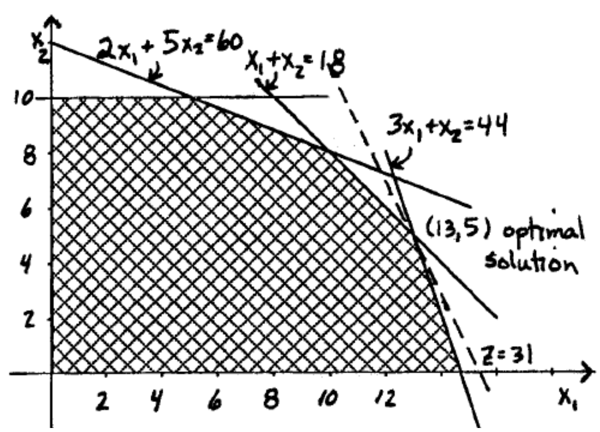
(b) The slope is $-1/2$, the x_2 intercept is 10.

(c)



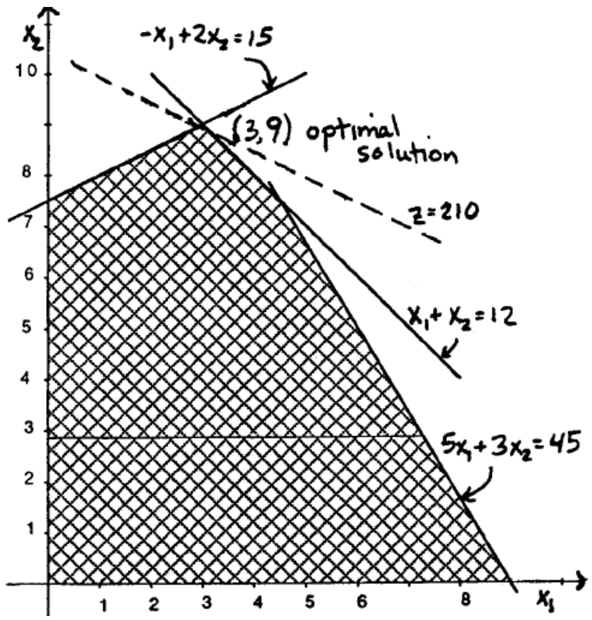
3.1-5.

Optimal Solution: $(x_1^*, x_2^*) = (13, 5)$ and $Z^* = 31$



3.1-6.

Optimal Solution: $(x_1^*, x_2^*) = (3, 9)$ and $Z^* = 210$



3.1-7.

(a) As in the Wyndor Glass Co. problem, we want to find the optimal levels of two activities that compete for limited resources. Let W be the number of wood-framed windows to produce and A be the number of aluminum-framed windows to produce. The data of the problem is summarized in the table below.

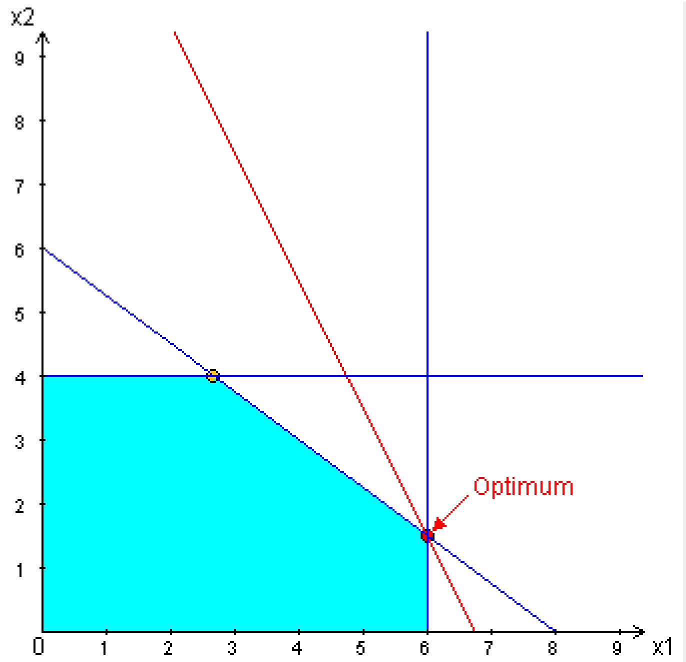
	Resource Usage per Unit of Activity		
Resource	**Wood-framed**	**Aluminum-framed**	**Available Amount**
Glass	6	8	48
Aluminum	0	1	4
Wood	1	0	6
Unit Profit	\$300	\$150	

(b) maximize $P = 300W + 150A$

subject to

$$\begin{aligned} 6W + 8A &\leq 48 \\ W &\leq 6 \\ A &\leq 4 \\ W, A &\geq 0 \end{aligned}$$

(c) Optimal Solution: $(W, A) = (x_1^*, x_2^*) = (6, 1.5)$ and $P^* = 2025$



(d) From Sensitivity Analysis in IOR Tutorial, the allowable range for the profit per wood-framed window is between 112.5 and infinity. As long as all the other parameters are fixed and the profit per wood-framed window is larger than \$112.50, the solution found in (c) stays optimal. Hence, when it is \$200 instead of \$300, it is still optimal to produce 6 wood-framed and 1.5 aluminum-framed windows and this results in a total profit of \$1425. However, when it is decreased to \$100, the optimal solution is to make 2.67 wood-framed and 4 aluminum-framed windows. The total profit in this case is \$866.67.

(e)

$$\begin{array}{llrl}
\text{maximize} & P = & 180W + 90A & \\
\text{subject to} & & 6W + 8A & \leq 48 \\
& & W \qquad & \leq 5 \\
& & A & \leq 4 \\
& & W, A \qquad & \geq 0
\end{array}$$

The optimal production schedule consists of 5 wood-framed and 2.25 aluminum-framed windows, with a total profit of \$1837.50.

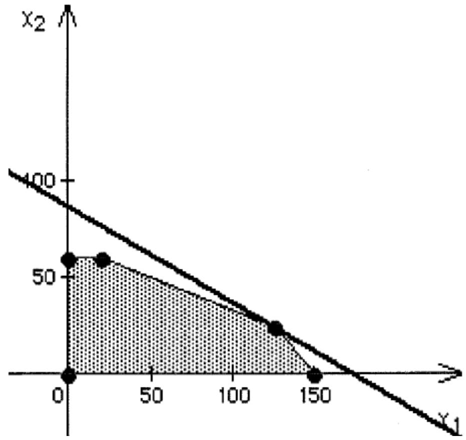
3.1-8.

(a) Let x_1 be the number of units of product 1 to produce and x_2 be the number of units of product 2 to produce. Then the problem can be formulated as follows:

$$\text{maximize } P = x_1 + 2x_2$$

$$\begin{array}{lrl}
\text{subject to} & 2x_1 + 3x_2 & \leq 200 \\
& 2x_1 + 2x_2 & \leq 300 \\
& x_2 & \leq 60 \\
& x_1, x_2 & \geq 0
\end{array}$$

(b) Optimal Solution: $(x_1^*, x_2^*) = (125, 25)$ and $P^* = 175$

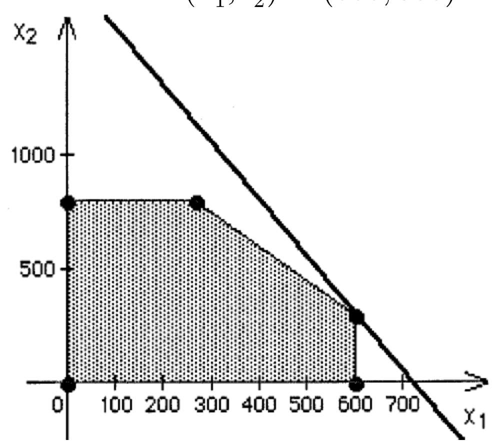


3.1-9.

(a) Let x_1 be the number of units on special risk insurance and x_2 be the number of units on mortgages.

$$\begin{array}{ll} \text{maximize} & z = 5x_1 + 2x_2 \\ \text{subject to} & 3x_1 + 2x_2 \leq 2400 \\ & \quad\quad\;\; x_2 \leq 800 \\ & 2x_1 \quad\quad\; \leq 1200 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

(b) Optimal Solution: $(x_1^*, x_2^*) = (600, 300)$ and $Z^* = 3600$



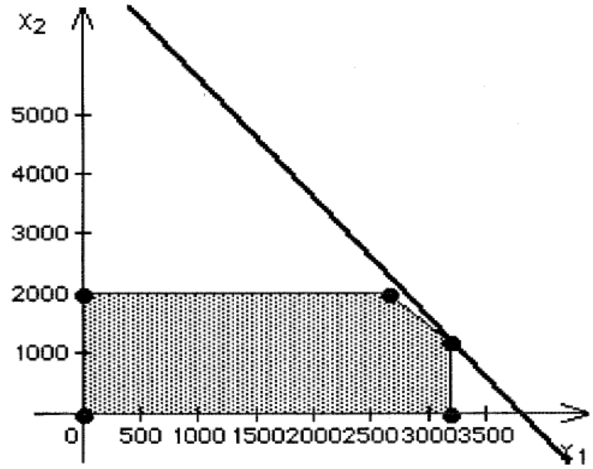
(c) The relevant two equations are $3x_1 + 2x_2 = 2400$ and $2x_1 = 1200$, so $x_1 = 600$ and $x_2 = \frac{1}{2}(2400 - 3x_1) = 300$, $z = 5x_1 + 2x_2 = 3600$.

3.1-10.

(a) maximize $P = 0.88H + 0.33B$

subject to
$$0.1B \leq 200$$
$$0.25H \leq 800$$
$$3H + 2B \leq 12{,}000$$
$$H, B \geq 0$$

(b) Optimal Solution: $(x_1^*, x_2^*) = (3200, 1200)$ and $P^* = 3212$



3.1-11.

(a) Let x_i be the number of units of product i produced for $i = 1, 2, 3$.

$$\begin{array}{lrl} \text{maximize} & Z = & 50x_1 + 20x_2 + 25x_3 \\ \text{subject to} & & 9x_1 + 3x_2 + 5x_3 \leq 500 \\ & & 5x_1 + 4x_2 \leq 350 \\ & & 3x_1 + 2x_3 \leq 150 \\ & & x_3 \leq 20 \\ & & x_1, x_2, x_3 \geq 0 \end{array}$$

(b)

Solve Automatically by the Simplex Method:

Optimal Solution

Value of the
Objective Function: Z = 2904.7619

Variable	Value
X_1	26.1905
X_2	54.7619
X_3	20

Constraint	Slack or Surplus	Shadow Price
1	0	4.7619
2	0	1.42857
3	31.4286	0
4	0	1.19048

Sensitivity Analysis

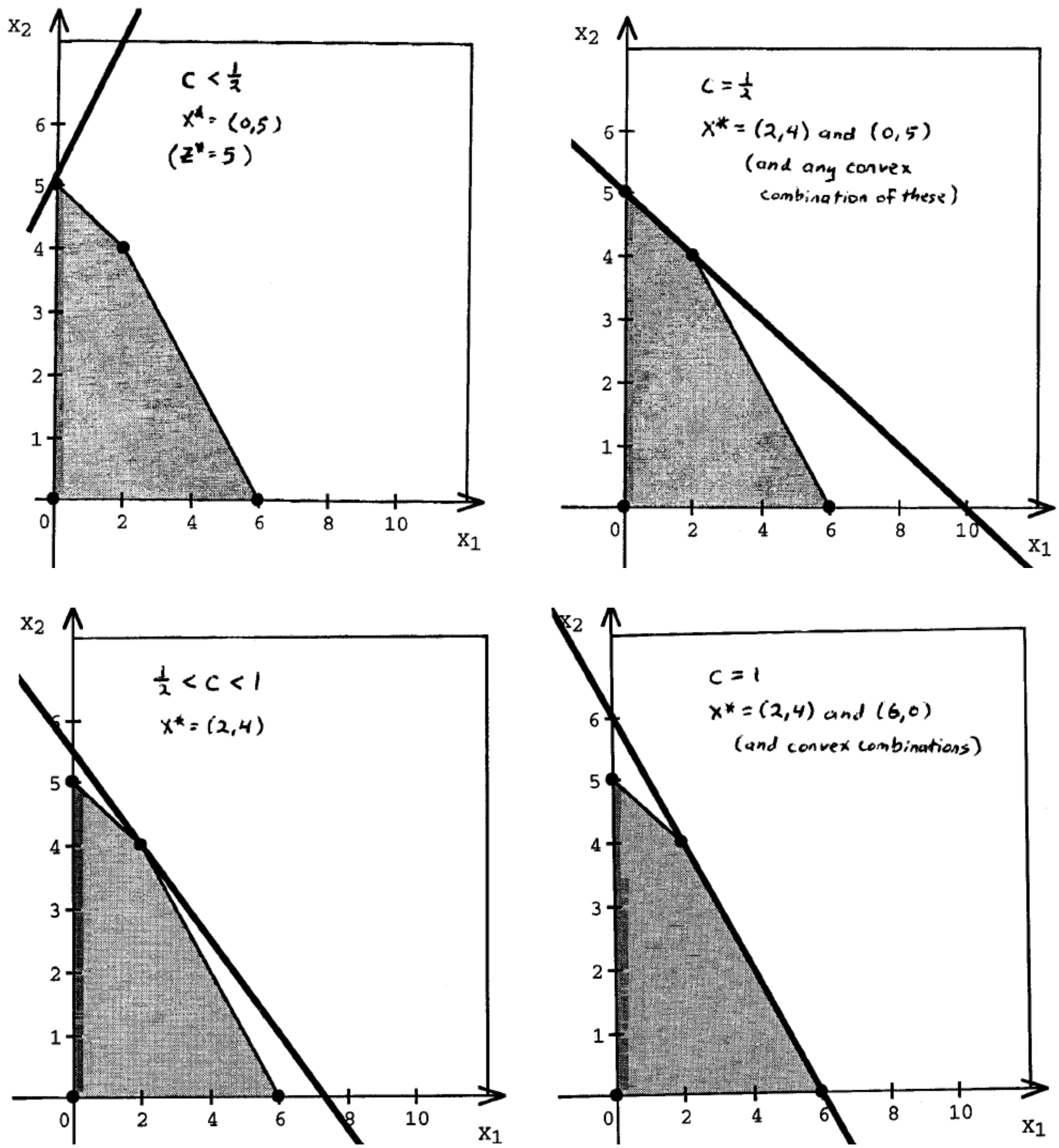
Objective Function Coefficient

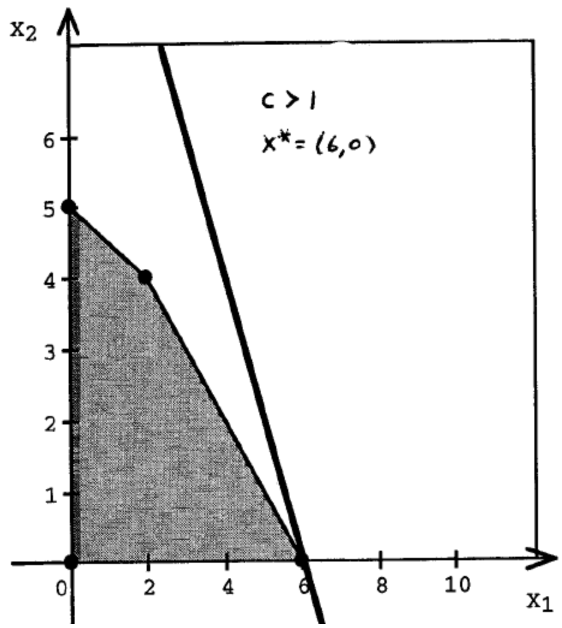
Current Value	Allowable Range To Stay Optimal Minimum	Maximum
50	25	51.25
20	19	40
25	23.8095	$+\infty$

Right Hand Sides

Current Value	Allowable Range To Stay Feasible Minimum	Maximum
500	362.5	555
350	276.667	533.333
150	118.571	$+\infty$
20	0	47.5

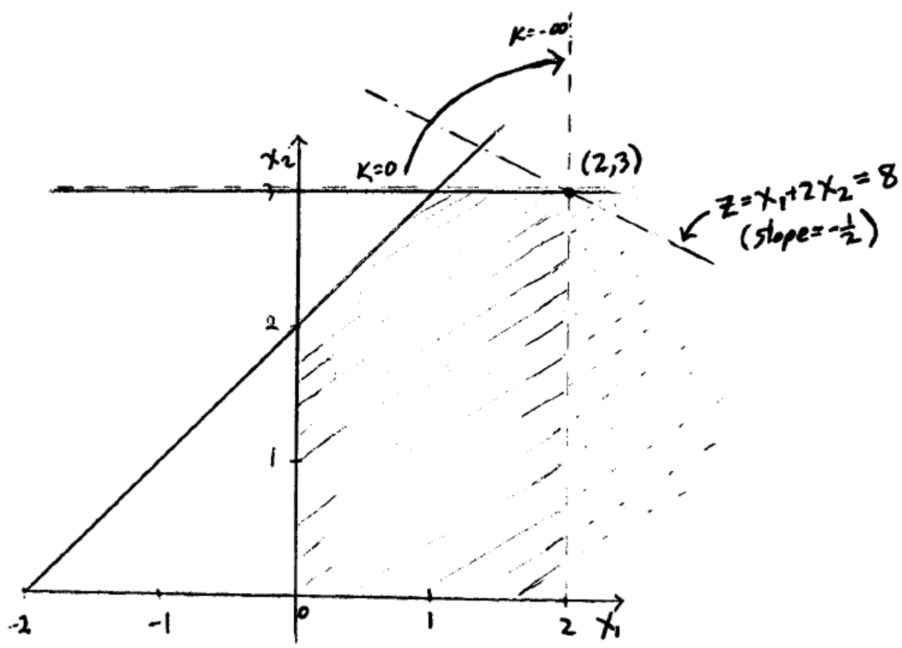
3.1-12.


c < 1/2
x* = (0,5)
(Z* = 5)
c = 1/2
x* = (2,4) and (0,5)
(and any convex combination of these)
1/2 < c < 1
x* = (2,4)
c = 1
x* = (2,4) and (6,0)
(and convex combinations)
x1
x2



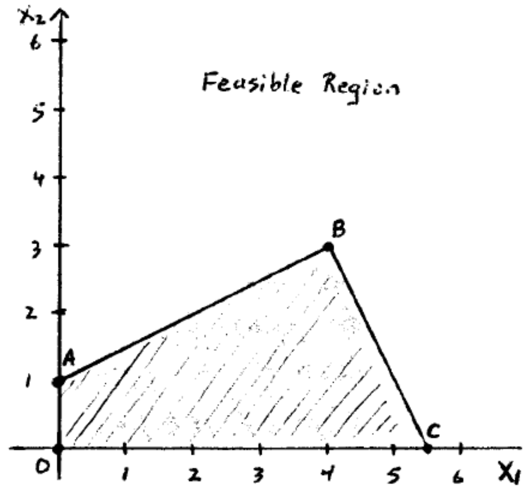
3.1-13.

First note that $(2,3)$ satisfies the three constraints, i.e., $(2,3)$ is always feasible for any value of k. Moreover, the third constraint is always binding at $(2,3)$, $kx_1 + x_2 = 2k + 3$. To check if $(2,3)$ is optimal, observe that changing k simply rotates the line that always passes through $(2,3)$. Rewriting this equation as $x_2 = -kx_1 + (2k+3)$, we see that the slope of the line is $-k$, and therefore, the slope ranges from 0 to $-\infty$.



As we can see, $(2,3)$ is optimal as long as the slope of the third constraint is less than the slope of the objective line, which is $-\frac{1}{2}$. If $k < \frac{1}{2}$, then we can increase the objective by traveling along the third constraint to the point $(2 + \frac{3}{k}, 0)$, which has an objective value of $2 + \frac{3}{k} > 8$ when $k < \frac{1}{2}$. For $k \geq \frac{1}{2}$, $(2, 3)$ is optimal.

3.1-14.



<u>Case 1:</u> $c_2 = 0$ (vertical objective line)

If $c_1 > 0$, the objective value increases as x_1 increases, so $x^* = (\frac{11}{2}, 0)$, point C.

If $c_1 < 0$, the opposite is true so that all the points on the line from $(0, 0)$ to $(0, 1)$, line $\overline{OA}$, are optimal.

If $c_1 = 0$, the objective function is $0x_1 + 0x_2 = 0$ and every feasible point is optimal.

<u>Case 2:</u> $c_2 > 0$ (objective line with slope $-\frac{c_1}{c_2}$)

If $-\frac{c_1}{c_2} > \frac{1}{2}$, $x^* = (0, 1)$, point A.

If $-\frac{c_1}{c_2} < -2$, $x^* = (\frac{11}{2}, 0)$, point C.

If $\frac{1}{2} > -\frac{c_1}{c_2} > -2$, $x^* = (4, 3)$, point B.

If $-\frac{c_1}{c_2} = \frac{1}{2}$, any point on the line $\overline{AB}$ is optimal. Similarly, if $-\frac{c_1}{c_2} = -2$, any point on the line $\overline{BC}$ is optimal.

<u>Case 3:</u> $c_2 < 0$ (objective line with slope $-\frac{c_1}{c_2}$, objective value increases as the line is shifted down)

If $-\frac{c_1}{c_2} > 0$, i.e., $c_1 > 0$, $x^* = (\frac{11}{2}, 0)$, point C.

If $-\frac{c_1}{c_2} < 0$, i.e., $c_1 < 0$, $x^* = (0, 0)$, point O.

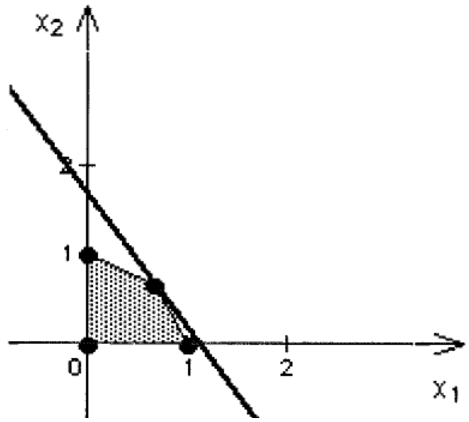
If $-\frac{c_1}{c_2} = 0$, i.e., $c_1 = 0$, x^* is any point on the line $\overline{OC}$.

3.2-1.

(a) maximize $P = 3A + 2B$

$$\begin{aligned} \text{subject to} \quad 2A + B &\leq 2 \\ A + 2B &\leq 2 \\ 3A + 3B &\leq 4 \\ A, B &\geq 0 \end{aligned}$$

(b) Optimal Solution: $(A, B) = (x_1^*, x_2^*) = (2/3, 2/3)$ and $P^* = 3.33$



(c) We have to solve $2A + B = 2$ and $A + 2B = 2$. By subtracting the second equation from the first one, we obtain $A - B = 0$, so $A = B$. Plugging this in the first equation, we get $2 = 2A + B = 3A$, hence $A = B = 2/3$.

3.2-2.

(a) TRUE (e.g., maximize $z = -x_1 + 4x_2$)

(b) TRUE (e.g., maximize $z = -x_1 + 3x_2$)

(c) FALSE (e.g., maximize $z = -x_1 - x_2$)

3.2-3.

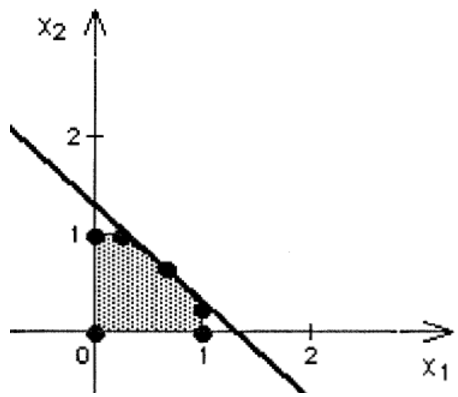
(a) As in the Wyndor Glass Co. problem, we want to find the optimal levels of two activities that compete for limited resources. Let x_1 and x_2 be the fraction purchased of the partnership in the first and second friends venture respectively.

	Resource Usage per Unit of Activity		
Resource	**1**	**2**	**Available Amount**
Fraction of partnership in 1st	1	0	1
Fraction of partnership in 2nd	0	1	1
Money	$\$10,000$	$8000	$\$12,000$
Summer work hours	400	500	600
Unit Profit	$9000	$9000	

(b) maximize $P = 9000x_1 + 9000x_2$

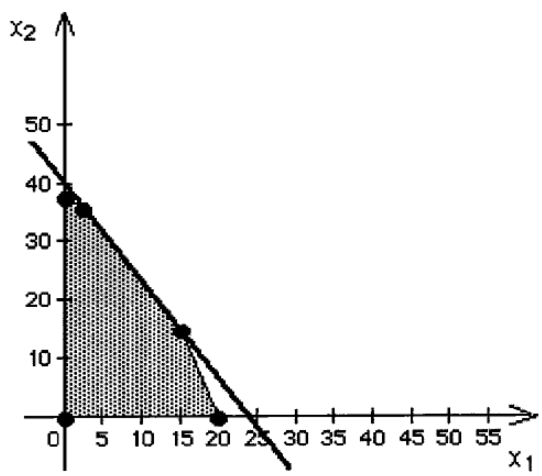
$$\begin{aligned} \text{subject to} \quad x_1 \qquad\qquad &\leq 1 \\ x_2 &\leq 1 \\ 10,000x_1 + 8000x_2 &\leq 12,000 \\ 400x_1 + 500x_2 &\leq 600 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(c) Optimal Solution: $(x_1^*, x_2^*) = (2/3, 2/3)$ and $P^* = 12,000$

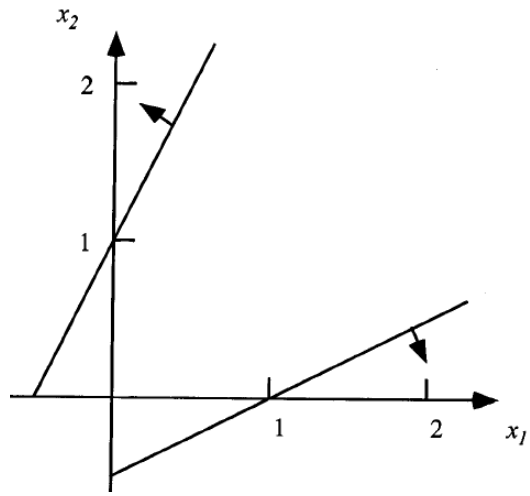


3.2-4.

Optimal Solutions: $(x_1^*, x_2^*) = (15, 15)$, $(2.5, 35.833)$ and all points lying on the line connecting these two points, $Z^* = 12,000$

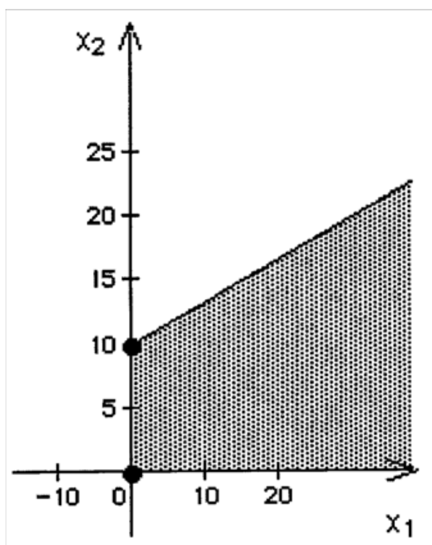


3.2-5.

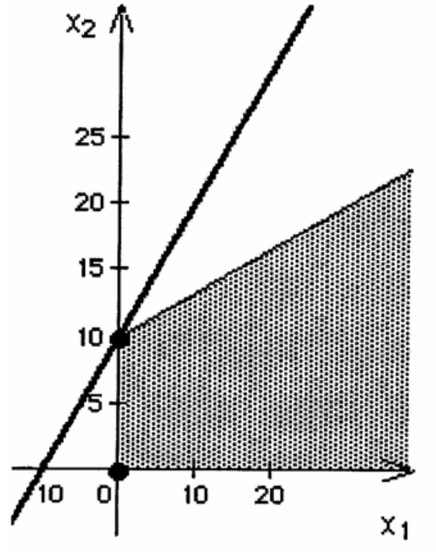


3.2-6.

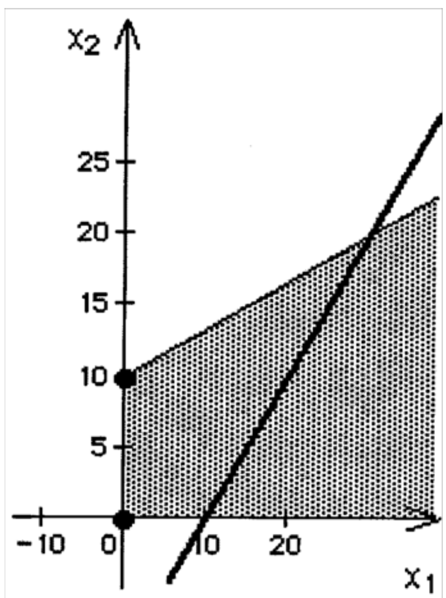
(a)



(b) Yes. Optimal solution: $(x_1^*, x_2^*) = (0, 10)$ and $Z^* = 10$



(c) No. The objective function value rises as the objective line is slid to the right and since this can be done forever, so there is no optimal solution.



(d) No, if there is no optimal solution even though there are feasible solutions, it means that the objective value can be made arbitrarily large. Such a case may arise if the data of the problem are not accurately determined. The objective coefficients may be chosen incorrectly or one or more constraints might have been ignored.

3.3-1.

<u>Proportionality:</u> It is fair to assume that the amount of work and money spent and the profit earned are directly proportional to the fraction of partnership purchased in either venture.

Additivity: The profit as well as time and money requirements for one venture should not affect neither the profit nor time and money requirements of the other venture. This assumption is reasonably satisfied.

Divisibility: Because both friends will allow purchase of any fraction of a full partnership, divisibility is a reasonable assumption.

Certainty: Because we do not know how accurate the profit estimates are, this is a more doubtful assumption. Sensitivity analysis should be done to take this into account.

3.3-2.

Proportionality: If either variable is fixed, the objective value grows proportionally to the increase in the other variable, so proportionality is reasonable.

Additivity: It is not a reasonable assumption, since the activities interact with each other. For example, the objective value at $(1, 1)$ is not equal to the sum of the objective values at $(0, 1)$ and $(1, 0)$.

Divisibility: It is not justified, since activity levels are not allowed to be fractional.

Certainty: It is reasonable, since the data provided is accurate.

3.4-1.

In this study, linear programming is used to improve prostate cancer treatments. The treatment planning problem is formulated as an MIP problem. The variables consist of binary variables that represent whether seeds were placed in a location or not and the continuous variables that denote the deviation of received dose from desired dose. The constraints involve the bounds on the dose to each anatomical structure and various physical constraints. Two models were studied. The first model aims at finding the maximum feasible subsystem with the binary variables while the second one minimizes a weighted sum of the dose deviations with the continuous variables.

With the new system, hundreds of millions of dollars are saved and treatment outcomes have been more reliable. The side effects of the treatment are considerably reduced and as a result of this, postoperation costs decreased. Since planning can now be done just before the operation, pretreatment costs decreased as well. The number of seeds required is reduced, so is the cost of procuring them. Both the quality of care and the quality of life after the operation are improved. The automated computerized system significantly eliminates the variability in quality. Moreover, the speed of the system allows the clinicians to efficiently handle disruptions.

3.4-2.

(a) Proportionality: OK, since beam effects on tissue types are proportional to beam strength.

Additivity: OK, since effects from multiple beams are additive.

Divisibility: OK, since beam strength can be fractional.

Certainty: Due to the complicated analysis required to estimate the data about radiation absorption in different tissue types, sensitivity analysis should be employed.

(b) Proportionality: OK, provided there is no setup cost associated with planting a crop.

Additivity: OK, as long as crops do not interact.

Divisibility: OK, since acres are divisible.

Certainty: OK, since the data can be accurately obtained.

(c) Proportionality: OK, setup costs were considered.

Additivity: OK, since there is no interaction.

Divisibility: OK, since methods can be assigned fractional levels.

Certainty: Data is hard to estimate, it could easily be uncertain, so sensitivity analysis is useful.

3.4-3.

(a) Reclaiming solid wastes

Proportionality: The amalgamation and treatment costs are unlikely to be proportional. They are more likely to involve setup costs, e.g., treating 1,000 lbs. of material does not cost the same as treating 10 lbs. of material 100 times.

Additivity: OK, although it is possible to have some interaction between treatments of materials, e.g., if A is treated after B, the machines do not need to be cleaned out.

Divisibility: OK, unless materials can only be bought or sold in batches, say, of 100 lbs.

Certainty: The selling/buying prices may change. The treatment and amalgamation costs are, most likely, crude estimates and may change.

(b) Personnel scheduling

Proportionality: OK, although some costs need not be proportional to the number of agents hired, e.g., benefits and working space.

Additivity: OK, although some costs may not be additive.

Divisibility: One cannot hire a fraction of an agent.

Certainty: The minimum number of agents needed may be uncertain. For example, 45 agents may be sufficient rather than 48 for a nominal fee. Another uncertainty is whether an agent does the same amount of work in every shift.

(c) Distributing goods through a distribution network

Proportionality: There is probably a setup cost for delivery, e.g., delivering 50 units one by one does probably cost much more than delivering all together at once.

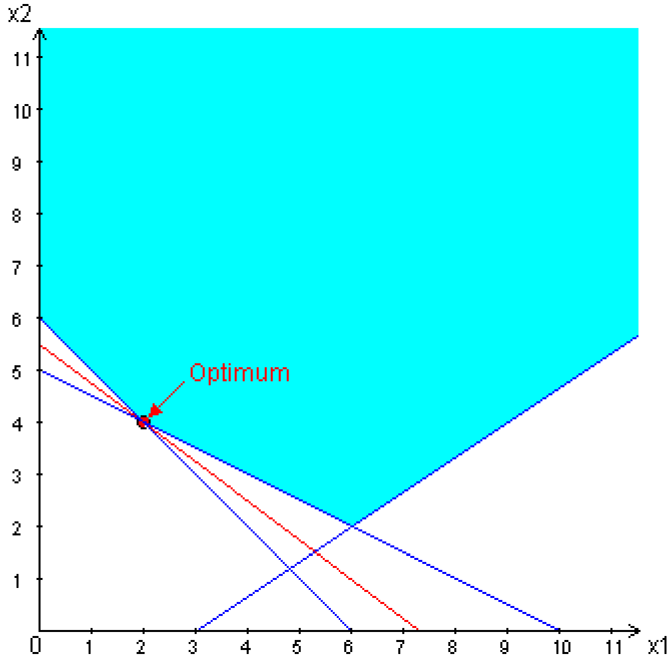
Additivity: OK, although it is possible to have two routes that can be combined to provide lower costs, e.g., $x_{\text{F2-DC}} = x_{\text{DC-W2}} = 50$, but the truck may be able to deliver 50 units directly from F2 to W2 without stopping at DC and hence saving some money. Another question is whether F1 and F2 produce equivalent units.

Divisibility: One cannot deliver a fraction of a unit.

Certainty: The shipping costs are probably approximations and are subject to change. The amounts produced may change as well.. Even the capacities may depend on available daily trucking force, weather and various other factors. Sensitivity analysis should be done to see the effects of uncertainty.

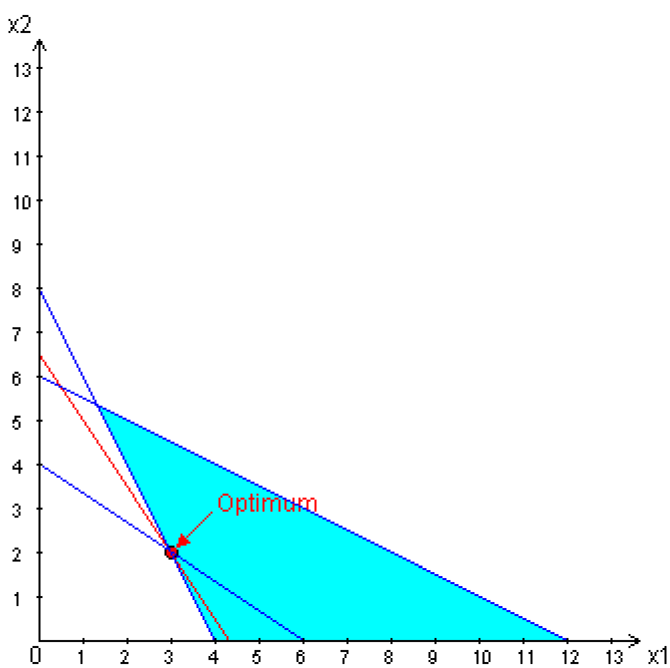
3.4-4.

Optimal Solution: $(x_1^*, x_2^*) = (2, 4)$ and $Z^* = 110$



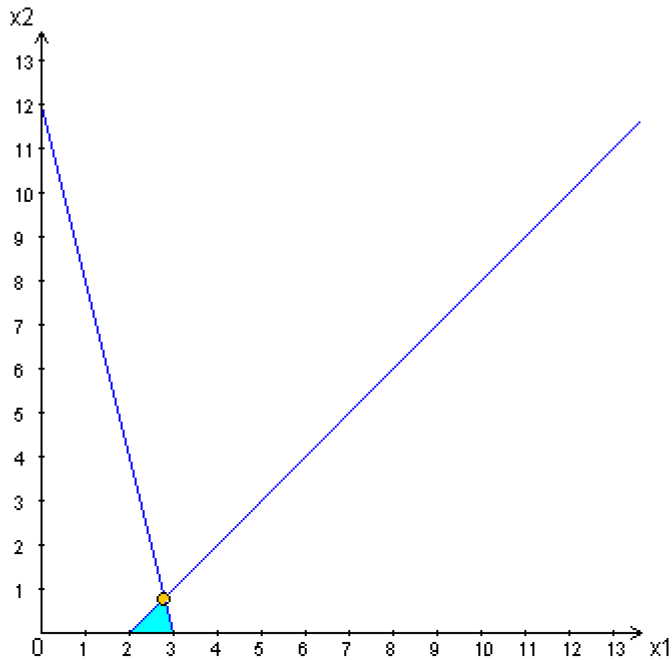
3.4-5.

Optimal Solution: $(x_1^*, x_2^*) = (3, 2)$ and $Z^* = 13$



3.4-6.

The feasible region can be represented as follows:

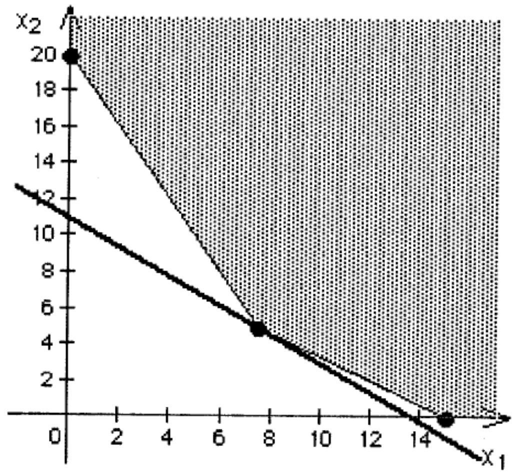


Given $c_2 = 2 > 0$, various cases that may arise are summarized in the following table:

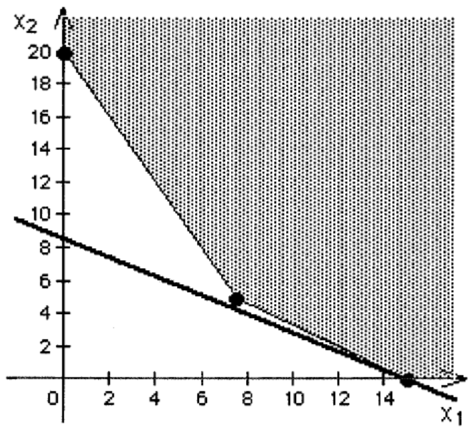
c_1	slope $= -\frac{c_1}{c_2}$	optimal solution (x_1^*, x_2^*)
$c_1 < -2$	$1 < -\frac{c_1}{c_2}$	$(2, 0)$
$c_1 = -2$	$-\frac{c_1}{c_2} = 1$	$(2, 0)$, $\left(\frac{14}{5}, \frac{4}{5}\right)$ and all points on the line connecting these two
$-2 < c_1 < 8$	$-4 < -\frac{c_1}{c_2} < 1$	$\left(\frac{14}{5}, \frac{4}{5}\right)$
$c_1 = 8$	$-\frac{c_1}{c_2} = -4$	$\left(\frac{14}{5}, \frac{4}{5}\right)$, $(3, 0)$ and all points on the line connecting these two
$8 < c_1$	$-\frac{c_1}{c_2} < -4$	$(3, 0)$

3.4-7.

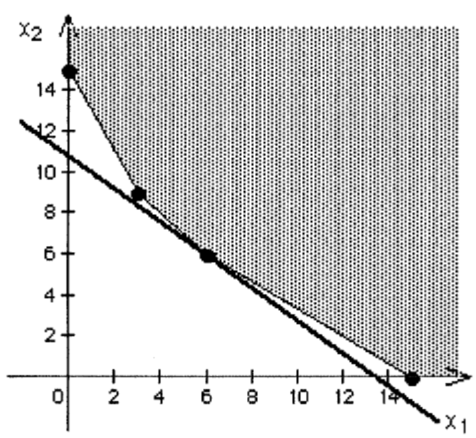
(a) Optimal Solution: $(x_1^*, x_2^*) = \left(7\frac{1}{2}, 5\right)$ and $C^* = 550$



(b) Optimal Solution: $(x_1^*, x_2^*) = (15, 0)$ and $C^* = 600$



(c) Optimal Solution: $(x_1^*, x_2^*) = (6, 6)$ and $C^* = 540$

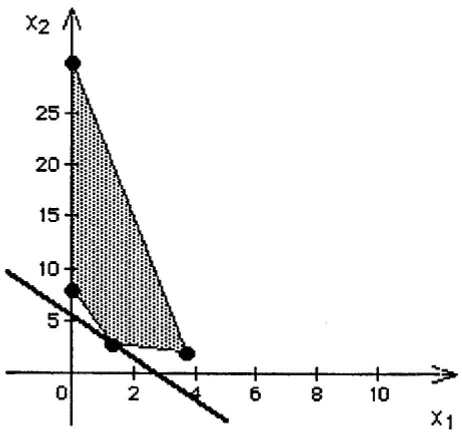


3.4-8.

(a) minimize $C = 8S + \ \ 4P$

subject to
$$\begin{aligned} 5S + 15P &\geq 50 \\ 20S + \ \ 5P &\geq 40 \\ 15S + \ \ 2P &\leq 60 \\ S, P &\geq 0 \end{aligned}$$

(b) Optimal Solution: $(S, P) = (x_1^*, x_2^*) = (1.3, 2.9)$ and $C^* = 21.82$



(c)

	Steak	Potatoes			
Cost per Serving	$8	$4			
	Grams of Ingredients per Serving		Totals		Requirement (g)
Carbohydrates	5	15	50	>=	50
Protein	20	5	40	>=	40
Fat	15	2	24.91	<=	60
					Total Cost
Solution	1.27	2.91			$21.82

3.4-9.

(a) Let x_{ij} be the amount of space leased for $j = 1, \ldots, 6 - i$ months in month $i = 1, \ldots, 5$.

$$
\begin{array}{ll}
\text{minimize} & C = 650(x_{11} + x_{21} + x_{31} + x_{41} + x_{51}) \\
& \quad + 1000(x_{12} + x_{22} + x_{32} + x_{42}) + 1350(x_{13} + x_{23} + x_{33}) \\
& \quad + 1600(x_{14} + x_{24}) + 1900x_{15} \\
\text{subject to} & x_{11} + x_{12} + x_{13} + x_{14} + x_{15} \geq 30,000 \\
& x_{12} + x_{13} + x_{14} + x_{15} + x_{21} + x_{22} + x_{23} + x_{24} \geq 20,000 \\
& x_{13} + x_{14} + x_{15} + x_{22} + x_{23} + x_{24} + x_{31} + x_{32} + x_{33} \geq 40,000 \\
& x_{14} + x_{15} + x_{23} + x_{24} + x_{32} + x_{33} + x_{41} + x_{42} \geq 10,000 \\
& x_{15} + x_{24} + x_{33} + x_{42} + x_{51} \geq 50,000 \\
& x_{ij} \geq 0,\ j = 1, \ldots, 6 - i \text{ and } i = 1, \ldots, 5
\end{array}
$$

(b)

	1-1	1-2	1-3	1-4	1-5	2-1	2-2	2-3	2-4	3-1	3-2	3-3	4-1	4-2	5-1			
Unit Cost	$650	$1,000	$1,350	$1,600	$1,900	$650	$1,000	$1,350	$1,600	$650	$1,000	$1,350	$650	$1,000	$650			
																		Resource
Month	Contribution Toward Required Amount															Totals		Available
1	1	1	1	1	1											$30,000	>=	$30,000
2		1	1	1	1	1	1	1	1							$30,000	>=	$20,000
3			1	1	1		1	1	1	1	1	1				$40,000	>=	$40,000
4				1	1			1	1		1	1	1	1		$30,000	>=	$10,000
5					1				1			1		1	1	$50,000	>=	$50,000
																		Total Cost
Space Leased (sf)	0	0	0	0	30000	0	0	0	0	10000	0	0	0	0	20000			$76,500,000

3.4-10.

(a) Let f_1 = number of full-time consultants working the morning shift (8 a.m.-4 p.m.),
f_2 = number of full-time consultants working the afternoon shift (Noon-8 p.m.),
f_3 = number of full-time consultants working the evening shift (4 p.m.-midnight),
p_1 = number of part-time consultants working the first shift (8 a.m.-noon),
p_2 = number of part-time consultants working the second shift (Noon-4 p.m.),
p_3 = number of part-time consultants working the third shift (4 p.m.-8 p.m.),
p_4 = number of part-time consultants working the fourth shift (8 p.m.-midnight).

$$
\begin{array}{ll}
\text{minimize} & C = (40 \times 8)(f_1 + f_2 + f_3) + (30 \times 4)(p_1 + p_2 + p_3 + p_4) \\
\text{subject to} & f_1 + p_1 \geq 4 \\
& f_1 + f_2 + p_2 \geq 8 \\
& f_2 + f_3 + p_3 \geq 10 \\
& f_3 + p_4 \geq 6 \\
& f_1 \geq 2p_1 \\
& f_1 + f_2 \geq 2p_2 \\
& f_2 + f_3 \geq 2p_3 \\
& f_3 \geq 2p_4 \\
& f_1, f_2, f_3, p_1, p_2, p_3, p_4 \geq 0
\end{array}
$$

(b)

	FT1	FT2	FT3	PT1	PT2	PT3	PT5							
Unit Cost	$320	$320	$320	$120	$120	$120	$120							
										Minimum				2
Time of Day	Contribution Toward Required Amount							Totals		Required		FT		*PT
8am-Noon	1			1				4	>=	4		2.667	>=	2.667
Noon-4pm	1	1			1			8	>=	8		5.333	>=	5.333
4pm-8pm		1	1			1		10	>=	10		6.667	>=	6.667
8pm-Midnight			1				1	6	>=	6		4	>=	4
										Total Cost				
Number Hired	2.667	2.667	4	1.333	2.667	3.333	2			$4,107				

Note that the optimal solution has fractional components. If the number of consultants have to be integer, then the problem is an integer programming problem and the solution is $(3, 3, 4, 1, 2, 3, 2)$ with cost $\$4,160$.

3.4-11.

(a) Let x_{ij} be the number of units shipped from factory $i = 1, 2$ to customer $j = 1, 2, 3$.

minimize $\quad C = 600x_{11} + 800x_{12} + 700x_{13} + 400x_{21} + 900x_{22} + 600x_{23}$

subject to

$$
\begin{aligned}
x_{11} + x_{12} + x_{13} &= 400 \\
x_{21} + x_{22} + x_{23} &= 500 \\
x_{11} + x_{21} &= 300 \\
x_{12} + x_{22} &= 200 \\
x_{13} + x_{23} &= 400
\end{aligned}
$$

and $\quad x_{ij} \geq 0,\ i = 1, 2$ and $j = 1, 2, 3$

(b)

Shipping						
Cost	Customer 1	Customer 2	Customer 3			
Factory 1	$600	$800	$700			
Factory 2	$400	$900	$600			
Units						
Shipped	Customer 1	Customer 2	Customer 3			Output
Factory 1	0	200	200	400	=	400
Factory 2	300	0	200	500	=	500
	300	200	400			
	=	=	=			Total Cost
Order Size	300	200	400			$540,000

3.4-12.

(a)

$$
\begin{aligned}
A_1 + B_1 + R_1 &= 60,000 \\
A_2 + B_2 + C_2 + R_2 &= R_1 \\
A_3 + B_3 + R_3 &= R_2 + 1.40A_1 \\
A_4 + R_4 &= R_3 + 1.40A_2 + 1.70B_1 \\
D_5 + R_5 &= R_4 + 1.40A_3 + 1.70B_2
\end{aligned}
$$

(b) maximize $P = 1.40A_4 + 1.70B_3 + 1.90C_2 + 1.30D_5 + R_5$

subject to

$$A_1 + B_1 + R_1 = 60,000$$
$$A_2 + B_2 + C_2 - R_1 + R_2 = 0$$
$$-1.40A_1 + A_3 + B_3 - R_2 + R_3 = 0$$
$$-1.40A_2 + A_4 - 1.70B_1 - R_3 + R_4 = 0$$
$$-1.40A_3 - 1.70B_2 + D_5 - R_4 + R_5 = 0$$

and $\mathrm{A}_t, B_t, C_t, D_t, R_t \geq 0$

(c)

	A1	A2	A3	A4	B1	B2	B3	C2	D5	R1	R2	R3	R4	R5			
Unit Profit	0	0	0	1.4	0	0	1.7	1.9	1.3	0	0	0	0	1			
																	Required
Year	Contribution Toward Required Amount														Totals		Amount
1	1				1					1					$60,000	=	$60,000
2		1				1		1		-1	1				$0	=	$0
3	-1.4		1				1				-1	1			$0	=	$0
4		-1.4		1	-1.7							-1	1		$0	=	$0
5			-1.4			-1.7			1				-1	1	$0	=	$0
																	Total Profit
Amount Invested	$60,000	$0	$84,000	$0	$0	$0	$0	$0	$117,600	$0	$0	$0	$0	$0			$152,880

3.4-13.

(a) Let x_i be the amount of Alloy i used for $i = 1, 2, 3, 4, 5$.

minimize $C = 22x_1 + 20x_2 + 25x_3 + 24x_4 + 27x_5$

subject to

$$60x_1 + 25x_2 + 45x_3 + 20x_4 + 50x_5 = 40$$
$$10x_1 + 15x_2 + 45x_3 + 50x_4 + 40x_5 = 35$$
$$30x_1 + 60x_2 + 10x_3 + 30x_4 + 10x_5 = 25$$
$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

and $x_1, x_2, x_3, x_4, x_5 \geq 0$

(b)

	Alloy 1	Alloy 2	Alloy 3	Alloy 4	Alloy 5			
Cost per Pound	$22	$20	$25	$24	$27			
								Required
Requirement	Contribution Toward Required Amount					Totals		Amount
% tin	60	25	45	20	50	40	=	40
% zinc	10	15	45	50	45	35	=	35
% lead	30	60	10	30	10	25	=	25
% total	1	1	1	1	1	1	=	1
								Cost per Pound
Proportion	0.0435	0.2826	0.6739	0	0			$23.46

3.4-14.

(a) Let x_{ij} be the number of tons of cargo type $i = 1, 2, 3, 4$ stowed in compartment $j =$ F (front), C (center), B (back).

$$\begin{array}{ll} \text{maximize} & P = 320(x_{1F} + x_{1C} + x_{1B}) + 400(x_{2F} + x_{2C} + x_{2B}) \\ & \quad + 360(x_{3F} + x_{3C} + x_{3B}) + 290(x_{4F} + x_{4C} + x_{4B}) \\ \text{subject to} & x_{1F} + x_{2F} + x_{3F} + x_{4F} \leq 12 \\ & x_{1C} + x_{2C} + x_{3C} + x_{4C} \leq 18 \\ & x_{1B} + x_{2B} + x_{3B} + x_{4B} \leq 10 \\ & x_{1F} + x_{1C} + x_{1B} \leq 20 \\ & x_{2F} + x_{2C} + x_{2B} \leq 16 \\ & x_{3F} + x_{3C} + x_{3B} \leq 25 \\ & x_{4F} + x_{4C} + x_{4B} \leq 13 \\ & 500x_{1F} + 700x_{2F} + 600x_{3F} + 400x_{4F} \leq 7,000 \\ & 500x_{1C} + 700x_{2C} + 600x_{3C} + 400x_{4C} \leq 9,000 \\ & 500x_{1B} + 700x_{2B} + 600x_{3B} + 400x_{4B} \leq 5,000 \\ & \frac{1}{12}(x_{1F} + x_{2F} + x_{3F} + x_{4F}) - \frac{1}{18}(x_{1C} + x_{2C} + x_{3C} + x_{4C}) = 0 \\ & \frac{1}{12}(x_{1F} + x_{2F} + x_{3F} + x_{4F}) - \frac{1}{10}(x_{1B} + x_{2B} + x_{3B} + x_{4B}) = 0 \\ \text{and} & x_{1F}, x_{2F}, x_{3F}, x_{4F}, x_{1C}, x_{2C}, x_{3C}, x_{4C}, x_{1B}, x_{2B}, x_{3B}, x_{4B} \geq 0 \end{array}$$

(b)

	Cargo 1	Cargo 2	Cargo 3	Cargo 4						
Volume (cf/ton)	500	700	600	400						
Profit (per ton)	$320	$400	$360	$290						
Cargo					Total		Weight	Total		Volume
Placement (tons)	Cargo 1	Cargo 2	Cargo 3	Cargo 4	Weight		Capacity	Volume		Capacity
Front	0	0	11	1	12	<=	12	7,000	<=	7,000
Center	0	6	0	12	18	<=	18	9,000	<=	9,000
Back	10	0	0	0	10	<=	10	5,000	<=	5,000
Total	10	6	11	13						
	<=	<=	<=	<=			Total Profit			
Available (tons)	20	16	25	13			$13,330			
Percentage of Front Capacity	100%	=	100%	Percentage of Middle Capacity						
Percentage of Front Capacity	100%	=	100%	Percentage of Back Capacity						

3.4-15.

(a) Let x_{ij} be the number of hours operator i is assigned to work on day j for $i = KC$, DH, HB, SC, KS, NK and $j = M, Tu, W, Th, F$.

$$\begin{array}{lll}
\text{minimize} & Z = & 25(x_{KC,M} + x_{KC,W} + x_{KC,F}) + 26(x_{DH,Tu} + x_{DH,Th}) + \\
& & 24(x_{HB,M} + x_{HB,Tu} + x_{HB,W} + x_{HB,F}) + \\
& & 23(x_{SC,M} + x_{SC,Tu} + x_{SC,W} + x_{SC,F}) + \\
& & 28(x_{KS,M} + x_{KS,W} + x_{KS,Th}) + 30(x_{NK,Th} + x_{NK,F})
\end{array}$$

subject to

$$\begin{aligned}
& x_{KC,M} \leq 6,\ x_{KC,W} \leq 6,\ x_{KC,F} \leq 6 \\
& x_{DH,Tu} \leq 6,\ x_{DH,Th} \leq 6 \\
& x_{HB,M} \leq 4,\ x_{HB,Tu} \leq 8,\ x_{HB,W} \leq 4,\ x_{HB,F} \leq 4 \\
& x_{SC,M} \leq 5,\ x_{SC,Tu} \leq 5,\ x_{SC,W} \leq 5,\ x_{SC,F} \leq 5 \\
& x_{KS,M} \leq 3,\ x_{KS,W} \leq 3,\ x_{KS,Th} \leq 8 \\
& x_{NK,Th} \leq 6,\ x_{NK,F} \leq 2
\end{aligned}$$

$$\begin{aligned}
& x_{KC,M} + x_{KC,W} + x_{KC,F} \geq 8 \\
& x_{DH,Tu} + x_{DH,Th} \geq 8 \\
& x_{HB,M} + x_{HB,Tu} + x_{HB,W} + x_{HB,F} \geq 8 \\
& x_{SC,M} + x_{SC,Tu} + x_{SC,W} + x_{SC,F} \geq 8 \\
& x_{KS,M} + x_{KS,W} + x_{KS,Th} \geq 7 \\
& x_{NK,Th} + x_{NK,F} \geq 7
\end{aligned}$$

$$\begin{aligned}
& x_{KC,M} + x_{HB,M} + x_{SC,M} + x_{KS,M} = 14 \\
& x_{DH,Tu} + x_{HB,Tu} + x_{SC,Tu} = 14 \\
& x_{KC,W} + x_{HB,W} + x_{SC,W} + x_{KS,W} = 14 \\
& x_{DH,Th} + x_{HB,Th} + x_{NK,Th} = 14 \\
& x_{KC,F} + x_{HB,F} + x_{SC,F} + x_{NK,F} = 14
\end{aligned}$$

$x_{ij} \geq 0$ for all i, j.

(b)

		Hours Available							
	Wage Rate	Monday	Tuesday	Wednesday	Thursday	Friday			
K.C.	$10.00	6	0	6	0	6			
D.H.	$10.10	0	6	0	6	0			
H.B.	$9.90	4	8	4	0	4			
S.C.	$9.80	5	5	5	0	5			
K.S.	$10.80	3	0	3	8	0			
N.K.	$11.30	0	0	0	6	2			
							Hours		
	Hours Worked	Monday	Tuesday	Wednesday	Thursday	Friday	Worked		Output
	K.C.	2	0	4	0	3	9	>=	8
	D.H.	0	2	0	6	0	8	>=	8
	H.B.	4	7	4	0	4	19	>=	8
	S.C.	5	5	5	0	5	20	>=	8
	K.S.	3	0	1	3	0	7	>=	7
	N.K.	0	0	0	5	2	7	>=	7
	Hours Worked	14	14	14	14	14			
		=	=	=	=	=			Total Cost
	Hours Needed	14	14	14	14	14			$710
			Hours Worked	<=	Hours Available				

3.4-16.

(a) Let B = slices of bread, P = tablespoons of peanut butter, S = tablespoons of strawberry jelly, G = graham crackers, M = cups of milk, and J = cups of juice.

minimize $\quad C = 5B + 4P + 7S + 8G + 15M + 35J$

subject to
$$70B + 100P + 50S + 60G + 150M + 100J \geq 400$$
$$70B + 100P + 50S + 60G + 150M + 100J \leq 600$$
$$10B + 75P + 20G + 70M \leq 0.3(70B + 100P + 50S + 60G + 150M + 100J)$$
$$3S + 2M + 120J \geq 60$$
$$3B + 4P + G + 8M + J \geq 12$$
$$B = 2$$
$$P \geq 2S$$
$$M + J \geq 1$$

and $\quad B, P, S, G, M, J \geq 0$

(b)

		Peanut	Strawberry	Graham							
	Bread	Butter	Jelly	Cracker	Milk	Juice					
	(slice)	(tbsp.)	(tbsp.)	(tbsp.)	(cup)	(cup)					
Unit Cost (cents)	5	4	7	8	15	35					
							Level				
			Nutritional Contents				Achieved		Minimum		Maximum
Total Calories	70	100	50	60	150	100	400	>=	400	<=	600
Vitamin C (mg)	0	0	3	0	2	120	60	>=	60		
Protein (g)	3	4	0	1	8	1	13.949	>=	12		
Calories from Fat	10	75	0	20	70	0	120			<=	120
											30%
		Peanut	Strawberry	Graham							of Total Calories
	Bread	Butter	Jelly	Cracker	Milk	Juice					
	(slice)	(tbsp.)	(tbsp.)	(tbsp.)	(cup)	(cup)			Total Cost (cents/student)		
Contents (tbsp)	2	0.575	0.287	1.039	0.516	0.484			47.31		
	=										
	2										
	Peanut Butter	0.575	>=	0.575	2	Times Strawberry Jelly					
	Total Liquid	1	>=	1							

3.5-1.

Upon facing problems about juice logistics, Welch's formulated the juice logistics model (JLM), which is "an application of LP to a single-commodity network problem. The decision variables deal with the cost of transfers between plants, the cost of recipes, and carrying cost- all cost that are key to the common planning unit of tons" [p. 20]. The goal is to find the optimal grape juice quantities shipped to customers and transferred between plants over a 12-month horizon. The optimal quantities minimize the total cost, i.e., the sum of transportation, recipe and storage costs. They satisfy balance equations, bounds on the ratio of grape juice sold, and limits on total grape juice sold.

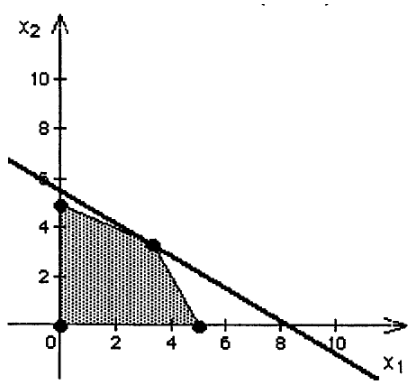
The JLM resulted in significant savings by preventing unprofitable decisions of the management. The savings in the first year of its implementation were over $130,000. Since the model can be run quickly, revising the decisions after observing the changes in the conditions is made easier. Thus, the flexibility of the system is improved. Moreover, the output helps the communication within the committee that is responsible for deciding on crop usage.

3.5-2.

(a) maximize $P = 20x_1 + 30x_2$

$$\begin{aligned} \text{subject to} \quad 2x_1 + x_2 &\leq 10 \\ 3x_1 + 3x_2 &\leq 20 \\ 2x_1 + 4x_2 &\leq 20 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(b) Optimal Solution: $(x_1^*, x_2^*) = \left(3\frac{1}{3}, 3\frac{1}{3}\right)$ and $P^* = 166.67$



(c), (e), (f)

	Activity 1	Activity 2			
Contribution per unit	$20	$30			
	Resource Usage		Resource		Resource
	per Unit of Activity		Used		Available
Resource 1	2	1	10	<=	10
Resource 2	3	3	20	<=	20
Resource 3	2	4	20	<=	20
	Activity 1	Activity 2			Total Contribution
Level of Activity	3.333	3.333			$166.67

(d)

(x_1, x_2)	Feasible?	P
$(2, 2)$	Yes	$\$100$
$(3, 3)$	Yes	$\$150$
$(2, 4)$	Yes	$\$160$ Best
$(4, 2)$	Yes	$\$140$
$(3, 4)$	No	
$(4, 3)$	No	

3.5-3.

(a) maximize $P = 50A + 40B + 30C$

subject to $0.02A + 0.03B + 0.05C \leq 40$

$0.05A + 0.02B + 0.04C \leq 40$

and $A, B, C \geq 0$

(b)

	Part A	Part B	Part C			
Unit Profit	$50	$40	$30			
				Hours		Hours
	Processing Time (hours per unit)			Used		Available
Machine 1	0.02	0.03	0.05	0	<=	40
Machine 2	0.05	0.02	0.04	0	<=	40
	Part A	Part B	Part C			Total Profit
Production						$0.00

(c) Many answers are possible.

(A, B, C)	Feasible?	P
$(500, 500, 300)$	No	
$(350, 1000, 0)$	Yes	$\$57,500$
$(400, 1000, 0)$	Yes	$\$60,000$ Best

(d)

	Part A	Part B	Part C			
Unit Profit	$50	$40	$30			
				Hours		Hours
	Processing Time (hours per unit)			Used		Available
Machine 1	0.02	0.03	0.05	40	<=	40
Machine 2	0.05	0.02	0.04	40	<=	40
	Part A	Part B	Part C			Total Profit
Production	363.636	1090.909	0			$61,818.18

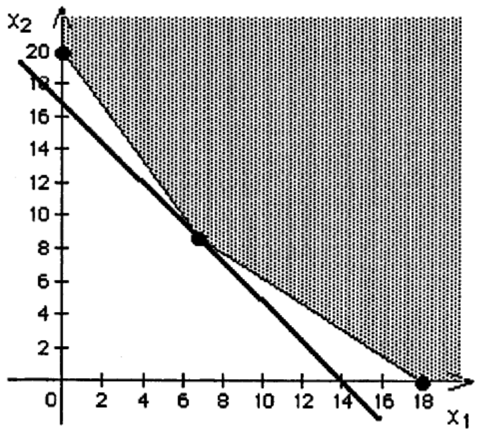
3.5-4.

(a) minimize $C = 60x_1 + 50x_2$

subject to
$$5x_1 + 3x_2 \geq 60$$
$$2x_1 + 2x_2 \geq 30$$
$$7x_1 + 9x_2 \geq 126$$

and $x_1, x_2 \geq 0$

(b) Optimal Solution: $(x_1^*, x_2^*) = (6.75, 8.75)$ and $C^* = 842.50$



(c), (e), (f)

	Activity 1	Activity 2			
Unit Cost	$60	$50			
					Minimum
	Benefit Contribution per		Level		Acceptable
	Unit of Each Activity		Achieved		Level
Benefit 1	5	3	60	>=	60
Benefit 2	2	2	31	>=	30
Benefit 3	7	9	126	>=	126
	Activity 1	Activity 2			Total Cost
Level of Activity	6.75	8.75			$842.50

(d)

(x_1, x_2)	Feasible?	C
$(7, 7)$	No	
$(7, 8)$	No	
$(8, 7)$	No	
$(8, 8)$	Yes	$880 Best
$(8, 9)$	Yes	$930
$(9, 8)$	Yes	$940

3.5-5.

(a) minimize $C = 2.10C + 1.80T + 1.50A$

subject to
$$90C + 20T + 40A \geq 200$$
$$30C + 80T + 60A \geq 180$$
$$10C + 20T + 60A \geq 150$$

and $C, T, A \geq 0$

(b), (e), (f)

	Corn	Tankage	Alfalfa			
Unit Cost	$2.10	$1.80	$1.50			
(per kg)						Minimum
				Level		Daily
	Nutritional Contents (per kg)			Achieved		Requirement
Carbohydrates	90	20	40	200	>=	200
Protein	30	80	60	180	>=	180
Vitamins	10	20	60	157.1429	>=	150
	Corn	Tankage	Alfalfa			Total Cost
Diet (kg)	1.143	0	2.429			$6.04

(c) $(x_1, x_2, x_3) = (1, 2, 2)$ is a feasible solution with a daily cost of $8.70. This diet will provide 210 kg of carbohydrates, 310 kg of protein, and 170 kg of vitamins daily.

(d) Answers will vary.

3.5-6.

(a) minimize $C = x_1 + x_2 + x_3$

subject to
$$2x_1 + x_2 + 0.5x_3 \geq 400$$
$$0.5x_1 + 0.5x_2 + x_3 \geq 100$$
$$1.5x_2 + 2x_3 \geq 300$$

and $x_1, x_2, x_3 \geq 0$

(b), (e), (f)

	Income per Unit of Asset ($million)			Cash Flow		Minimum
	Asset 1	Asset 2	Asset 3	Achieved		Required
Year 5	2	1	0.5	400	>=	400
Year 10	0.5	0.5	1	150	>=	100
Year 20	0	1.5	2	300	>=	300
						Total Cost
	Asset 1	Asset 2	Asset 3			($million)
Units Purchased	100	200	0			300

(c) $(x_1, x_2, x_3) = (100, 100, 200)$ is a feasible solution. This would generate $400 million in 5 years, $300 million in 10 years, and $550 million in 20 years. The total investment will be $400 million.

(d) Answers will vary.

3.6-1.

(a) In the following, the indices $i, j, k, l,$ and m refer to products, months, plants, processes and regions respectively. The decision variables are:

x_{ijklm} = amount of product i produced in month j in plant k using process l and to be sold in region m, and

s_{im} = amount of product i stored to be sold in March in region m.

The parameters of the problem are:

D_{ijm} = demand for product i in month j in region m,

c_{ikl} = unit production cost of product i in plant k using process l,

R_{ikl} = production rate of product i in plant k using process l,

p_i = selling price of product i,

T_{ikm} = transportation cost of product i product in plant k to be sold in region m,

A_j = days available for production in month j,

L = storage limit,

M_i = storage cost per unit of product i.

The objective is to maximize the total profit, which is the difference of the total revenue and the total cost. The total cost is the sum of the costs of production, inventory and transportation. Using the notation introduced, the objective is to maximize

$$\sum_i p_i\left(\sum_{j,k,l,m} x_{ijklm}\right) - \sum_{i,k,l} c_{ikl}\left(\sum_{j,m} x_{ijklm}\right) - \sum_i M_i\left(\sum_m s_{im}\right) - \sum_{i,k,m} T_{ikm}\left(\sum_{j,l} x_{ijklm}\right)$$

subject to the constraints

$$\sum_{k,l} x_{ijklm} - s_{im} \leq D_{ijm} \qquad \text{for } j = \text{February}; i = 1,2; m = 1,2$$

$$\sum_{k,l} x_{ijklm} + s_{im} \leq D_{ijm} \qquad \text{for } j = \text{March}; i = 1,2; m = 1,2$$

$$\sum_i s_{im} \leq L \qquad \text{for } m = 1,2$$

$$\sum_{i,l} \frac{1}{R_{ikl}}\left(\sum_m x_{ijklm}\right) \leq A_j \qquad \text{for } j = \text{February, March}; k = 1,2$$

$$x_{ijklm} \geq 0 \qquad \text{for } i,k,l,\ m = 1,2 \text{ and } j = \text{February, March}$$

(b)

Quantity produced to be sold in the same region with process 1

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	2400	2760	3200	3680

Quantity produced to be sold in the other region with process 1

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	0	0	0	0

Quantity produced to be sold in the same region with process 2

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	0	0	0	0

Quantity produced to be sold in the other region with process 2

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	0	0	0	0

Days available	
February	20
March	23

Storage Limit
1000

		Total Profit
Revenue	1348480	$333,680
Prod Cost	1014800	
Transp Cost	0	
Storage Cost	0	

Demand

Product	Region1 February	Region1 March	Region2 February	Region2 March
1	3600	6300	4900	4200
2	4500	5400	5100	6000

Production costs

Product	Plant 1 Process 1	Plant 1 Process 2	Plant2 Process 1	Plant2 Process 2
1	$ 62	$ 59	$ 61	$ 65
2	$ 78	$ 85	$ 89	$ 86

Production rates

Product	Plant 1 Process 1	Plant 1 Process 2	Plant2 Process 1	Plant2 Process 2
1	100	140	130	110
2	120	150	160	130

Transportation Costs

Region	Product 1: 1	Product 1: 2	Product 2: 1	Product 2: 2
1	0	9	0	7
2	9	0	7	0

Product	Revenue
1	$83
2	$112

Product	Storage cost
1	$ 3
2	$ 4

Stored quantity

Region	Product 1	Product 2
1	0	0
2	0	0

Demand satisfied

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	2400	2760	3200	3680

Capacity used

	Plant1	Plant2
Feb	20	20
Mar	23	23

	Amount stored
Plant1	0
Plant2	0

(c)

```
TITLE
    ManufacturingProblem;

INDEX
    product = (pr1,pr2);
    month = (feb,mar);
    plant = (pl1,pl2);
    process = (ps1,ps2);
    region = (r1,r2);

DATA
    demand[product,month,region] := (3600,4900,
                      6300,4200,
                      4500,5100,
                      5400,6000);
    days[month] := (20,23);
    storagecost[product] := (3,4);
    prodcost[product,plant,process] := (62,59,
                          61,65,
                          78,85,
                          89,86);
    rate[product,plant,process] := (100,140,
                     130,110,
                     120,150,
                     160,130);
    price[product] := (83,112);
    transpcost[product,plant,region] := (0,9,
                          9,0,
                          0,7,
                          7,0);

DECISION VARIABLES
    Volume[product,month,plant,process,region];
    Store[product,region];

MACRO

    Revenues := SUM(product,month,plant,process,region: price*Volume);
    ProductionCost := SUM(product,plant,process,month,region: prodcost*Volume);
    TransportationCost := SUM(product,plant,region,month,process: transpcost*Volume);
    StorageCost := SUM(product,region: storagecost*Store);
MODEL

    MAX  TotalProfit = Revenues - ProductionCost - TransportationCost - StorageCost;

SUBJECT TO
    SalesFeb[product,region,month] where(month=feb) : SUM(plant,process: Volume - Store) <= demand;
    SalesMar[product,region,month] where(month=mar) : SUM(plant,process: Volume + Store) <= demand;
    StorageLimit[region] : SUM(product: Store) <= 1000;
    Capacity[plant,month] : SUM(product,process,region: Volume/rate) <= days;

END
```

SOLUTION RESULT

Optimal solution found

MAX TotalPro = 333680.0000

MACROS

Macro Name	Values
Revenues	1348480.0000
ProductionCost	1014800.0000
TransportationCost	0.0000
StorageCost	0.0000

DECISION VARIABLES

VARIABLE Volume[product,month,plant,process,region] :

product	month	plant	process	region	Activity	Reduced Cost
pr1	feb	pl1	ps1	r1	0.0000	-19.8000
pr1	feb	pl1	ps1	r2	0.0000	-28.8000
pr1	feb	pl1	ps2	r1	0.0000	-5.1429
pr1	feb	pl1	ps2	r2	0.0000	-14.1429
pr1	feb	pl2	ps1	r1	0.0000	-15.3077
pr1	feb	pl2	ps1	r2	0.0000	-6.3077
pr1	feb	pl2	ps2	r1	0.0000	-24.4545
pr1	feb	pl2	ps2	r2	0.0000	-15.4545
pr1	mar	pl1	ps1	r1	0.0000	-19.8000
pr1	mar	pl1	ps1	r2	0.0000	-28.8000
pr1	mar	pl1	ps2	r1	0.0000	-5.1429
pr1	mar	pl1	ps2	r2	0.0000	-14.1429
pr1	mar	pl2	ps1	r1	0.0000	-15.3077
pr1	mar	pl2	ps1	r2	0.0000	-6.3077
pr1	mar	pl2	ps2	r1	0.0000	-24.4545
pr1	mar	pl2	ps2	r2	0.0000	-15.4545
pr2	feb	pl1	ps1	r1	2400.0000	0.0000
pr2	feb	pl1	ps1	r2	0.0000	-7.0000
pr2	feb	pl1	ps2	r1	0.0000	-0.2000
pr2	feb	pl1	ps2	r2	0.0000	-7.2000
pr2	feb	pl2	ps1	r1	0.0000	-7.0000
pr2	feb	pl2	ps1	r2	3200.0000	0.0000
pr2	feb	pl2	ps2	r1	0.0000	-9.3077
pr2	feb	pl2	ps2	r2	0.0000	-2.3077
pr2	mar	pl1	ps1	r1	2760.0000	0.0000
pr2	mar	pl1	ps1	r2	0.0000	-7.0000
pr2	mar	pl1	ps2	r1	0.0000	-0.2000
pr2	mar	pl1	ps2	r2	0.0000	-7.2000
pr2	mar	pl2	ps1	r1	0.0000	-7.0000
pr2	mar	pl2	ps1	r2	3680.0000	0.0000
pr2	mar	pl2	ps2	r1	0.0000	-9.3077
pr2	mar	pl2	ps2	r2	0.0000	-2.3077

VARIABLE Store[product,region] :

product	region	Activity	Reduced Cost
pr1	r1	0.0000	-3.0000
pr1	r2	0.0000	-3.0000
pr2	r1	0.0000	-4.0000
pr2	r2	0.0000	-4.0000

(d)

```
MODEL:

SETS:
PRODUCT/PR1 PR2/: PRICE, STORAGECOST;
MONTH/FEB MAR/: DAYS;
PLANT/PL1 PL2/;
PROCESS/PS1 PS2/;
REGION/R1 R2/;
LINK1(PRODUCT,MONTH,PLANT,PROCESS,REGION): VAR;
LINK2(PRODUCT,MONTH,REGION): DEMAND;
LINK3(PRODUCT,PLANT,PROCESS): PRODCOST;
LINK4(PRODUCT,PLANT,PROCESS): RATE;
LINK5(PRODUCT,REGION): STORE;
LINK6(PRODUCT,PLANT,REGION): TRANSPCOST;
ENDSETS

!OBJECTIVE FUNCTION;
MAX = @SUM(PRODUCT(I): PRICE(I)*@SUM(MONTH(J): @SUM(PLANT(K): @SUM(PROCESS(L):
@SUM(REGION(M): VAR(I,J,K,L,M)))))) - @SUM(LINK3(I,K,L): PRODCOST(I,K,L)*@SUM(MONTH(J):
@SUM(REGION(M): VAR(I,J,K,L,M)))) - @SUM(PRODUCT(I): STORAGECOST(I)*@SUM(REGION(M):
STORE(I,M))) - @SUM(LINK6(I,K,M): TRANSPCOST(I,K,M)*@SUM(MONTH(J): @SUM(PROCESS(L):
VAR(I,J,K,L,M))));

!CONSTRAINTS;
@FOR(PRODUCT(I): @FOR(REGION(M): @SUM(PLANT(K): @SUM(PROCESS(L): VAR(I,FEB,K,L,M))) -
STORE(I,M) <= DEMAND(I,FEB,M)));
@FOR(PRODUCT(I): @FOR(REGION(M): @SUM(PLANT(K): @SUM(PROCESS(L): VAR(I,MAR,K,L,M))) +
STORE(I,M) <= DEMAND(I,MAR,M)));
@FOR(REGION(M): @SUM(PRODUCT(I): STORE(I,M))<=1000);
@FOR(PLANT(K): @FOR(MONTH(J): @SUM(PRODUCT(I): @SUM(PROCESS(L):
(1/RATE(I,K,L))*@SUM(REGION(M): VAR(I,J,K,L,M)))) <= DAYS(J)));

!DATA PART;
DATA:
DEMAND = 3600 4900
         6300 4200
         4500 5100
         5400 6000;
DAYS = 20 23;
STORAGECOST = 3 4;
PRODCOST = 62 59
           61 65
           78 85
           89 86;
RATE = 100 140
       130 110
       120 150
       160 130;
PRICE = 83 112;
TRANSPCOST = 0 9
             9 0
             0 7
             7 0;
ENDDATA
END
```

```
Global optimal solution found at step:              8
Objective value:                             333680.0

                    Variable              Value

        VAR( P1, FEB, P1, P1, R1)         0.0000000
        VAR( P1, FEB, P1, P1, R2)         0.0000000
        VAR( P1, FEB, P1, P2, R1)         0.0000000
        VAR( P1, FEB, P1, P2, R2)         0.0000000
        VAR( P1, FEB, P2, P1, R1)         0.0000000
        VAR( P1, FEB, P2, P1, R2)         0.0000000
        VAR( P1, FEB, P2, P2, R1)         0.0000000
        VAR( P1, FEB, P2, P2, R2)         0.0000000
        VAR( P1, MAR, P1, P1, R1)         0.0000000
        VAR( P1, MAR, P1, P1, R2)         0.0000000
        VAR( P1, MAR, P1, P2, R1)         0.0000000
        VAR( P1, MAR, P1, P2, R2)         0.0000000
        VAR( P1, MAR, P2, P1, R1)         0.0000000
        VAR( P1, MAR, P2, P1, R2)         0.0000000
        VAR( P1, MAR, P2, P2, R1)         0.0000000
        VAR( P1, MAR, P2, P2, R2)         0.0000000
        VAR( P2, FEB, P1, P1, R1)          2400.000
        VAR( P2, FEB, P1, P1, R2)         0.0000000
        VAR( P2, FEB, P1, P2, R1)         0.0000000
        VAR( P2, FEB, P1, P2, R2)         0.0000000
        VAR( P2, FEB, P2, P1, R1)         0.0000000
        VAR( P2, FEB, P2, P1, R2)          3200.000
        VAR( P2, FEB, P2, P2, R1)         0.0000000
        VAR( P2, FEB, P2, P2, R2)         0.0000000
        VAR( P2, MAR, P1, P1, R1)          2760.000
        VAR( P2, MAR, P1, P1, R2)         0.0000000
        VAR( P2, MAR, P1, P2, R1)         0.0000000
        VAR( P2, MAR, P1, P2, R2)         0.0000000
        VAR( P2, MAR, P2, P1, R1)         0.0000000
        VAR( P2, MAR, P2, P1, R2)          3680.000
        VAR( P2, MAR, P2, P2, R1)         0.0000000
        VAR( P2, MAR, P2, P2, R2)         0.0000000

         STORE( P1, R1)        0.0000000
         STORE( P1, R2)        0.0000000
         STORE( P2, R1)        0.0000000
         STORE( P2, R2)        0.0000000
```

3.6-2.

(a)

```
MAX

    50x1+20x2+25x3 ;

SUBJECT TO

    9x1+3x2+5x3<=500 ;
    5x1+4x2<=350;
    3x1+2x3<=150;
    x3<=20;

END
```

Variable Name	Activity
x1	26.1905
x2	54.7619
x3	20.0000

(b)

```
max = 50*x1+20*x2+25*x3;
9*x1+3*x2+5*x3<=500;
5*x1+4*x2<=350;
3*x1+2*x3<=150;
x3<=20;
x1>=0; x2>=0; x3>=0;
```

```
Global optimal solution found at step:          4
Objective value:                         2904.762

                    Variable           Value
                          X1        26.19048
                          X2        54.76190
                          X3        20.00000
```

3.6-3.

(a)

```
TITLE
    TransportationProblem;

INDEX
    supply = (Wh1,Wh2);
    dest   = (C1,C2,C3);

DATA
    MaxCapacity[supply]  := (400,500);
    Required[dest]       := (300,200,400);

    ShippingCost[supply,dest] := (600,800,700,
                 400,900,600);


 DECISION VARIABLES
    VolumeShipped[supply,dest]  ->  ""


 MODEL

    MIN  TotalCost = SUM(supply,dest: ShippingCost * VolumeShipped);

 SUBJECT TO

    Capacity[supply] : SUM(dest: VolumeShipped)    =  MaxCapacity ;
    Demand[dest]     : SUM(supply: VolumeShipped)  =  Required ;

 END
```

(b)

```
MODEL:

SETS:
      FACTORIES /F1 F2/: CAPACITY;
      CUSTOMERS /C1 C2 C3/: DEMAND;
      LINKS(FACTORIES, CUSTOMERS): COST, VOLUME;
ENDSETS
[OBJECTIVE] MIN = @SUM(LINKS(I,J):COST(I,J)*VOLUME(I,J));
!DEMAND CONSTRAINTS;
@FOR(CUSTOMERS(J): @SUM(FACTORIES(I): VOLUME(I,J))=DEMAND(J));
!SUPPLY CONSTRAINTS;
@FOR(FACTORIES(I): @SUM(CUSTOMERS(J):VOLUME(I,J))=CAPACITY(I));
!HERE IS THE DATA;
DATA:
CAPACITY = 400 500;
DEMAND = 300 200 400;
COST = 600 800 700
       400 200 400;
ENDDATA
END
```

```
Global optimal solution found at step:          2
Objective value:                         410000.0

              Variable            Value

          VOLUME( F1, C1)      300.0000
          VOLUME( F1, C2)     0.0000000
          VOLUME( F1, C3)      100.0000
          VOLUME( F2, C1)     0.0000000
          VOLUME( F2, C2)      200.0000
          VOLUME( F2, C3)      300.0000
```

3.6-4.

(a)

```
TITLE
    TransportationProblem;

INDEX
    student  = (KC,OH,HB,SC,KS,NK);
    day = (M,TU,W,TH,F);

DATA

    Wage[student]        :=(10,10.1,9.9,9.8,10.8,11.3);
    Gender[student]         := (0,0,0,0,1,1);
    Available[student,day] := (6,0,6,0,6,
                   0,6,0,6,0
                   4,8,4,0,4
                   5,5,5,0,5
                   3,0,3,8,0
                   0,0,0,6,2);
```

```
DECISION VARIABLES

    Work[student,day]   ->   ""

MODEL

    MIN  TotalCost = SUM(student,day: Wage * Work);

SUBJECT TO

    TimeConstraint[student,day] : Work <= Available ;
    MinimumWork0[student] where(Gender=0) : SUM(day: Work) >=8 ;
    MinimumWork1[student] where(Gender=1) : SUM(day: Work) >=7 ;
    AlwaysOpen[day] : SUM(student: Work) = 14 ;
END
```

```
                              MIN TotalCos =        709.6000

VARIABLE Work[student,day] :
```

student	day	Activity
KC	M	4.0000
KC	TU	0.0000
KC	W	2.0000
KC	TH	0.0000
KC	F	3.0000
OH	M	0.0000
OH	TU	2.0000
OH	W	0.0000
OH	TH	6.0000
OH	F	0.0000
HB	M	4.0000
HB	TU	7.0000
HB	W	4.0000
HB	TH	0.0000
HB	F	4.0000
SC	M	5.0000
SC	TU	5.0000
SC	W	5.0000
SC	TH	0.0000
SC	F	5.0000
KS	M	1.0000
KS	TU	0.0000
KS	W	3.0000
KS	TH	3.0000
KS	F	0.0000
NK	M	0.0000
NK	TU	0.0000
NK	W	0.0000
NK	TH	5.0000
NK	F	2.0000

(b)

```
MODEL:

SETS:
      STUDENTS /KC OH HB SC KS NK/: WAGE, GENDER;
      DAYS /M TU W TH F/;
      LINKS(STUDENTS, DAYS): AVAILABLE, WORK;
ENDSETS

[OBJECTIVE] MIN = @SUM(LINKS(I,J):WAGE(I)*WORK(I,J));

!TIME CONSTRAINTS;
@FOR(LINKS(I,J): WORK(I,J)<=AVAILABLE(I,J));

!MINIMUM WORK CONSTRAINTS;
@FOR(STUDENTS(I)|GENDER(I) #EQ# 0: @SUM(LINKS(I,J):WORK(I,J))>=8);
@FOR(STUDENTS(I)|GENDER(I) #EQ# 1: @SUM(LINKS(I,J):WORK(I,J))>=7);

!ALWAYS OPEN CONSTRAINTS;
@FOR(DAYS(J): @SUM(LINKS(I,J): WORK(I,J))=14);
!HERE IS THE DATA;
DATA:
WAGE = 10 10.1 9.9 9.8 10.8 11.3;
GENDER = 0 0 0 0 1 1;
AVAILABLE=6 0 6 0 6
          0 6 0 6 0
          4 8 4 0 4
          5 5 5 0 5
          3 0 3 8 0
          0 0 0 6 2;
ENDDATA
END
```

WORK(KC, M)	2.000000	WORK(SC, M)	5.000000
WORK(KC, TU)	0.0000000	WORK(SC, TU)	5.000000
WORK(KC, W)	3.000000	WORK(SC, W)	5.000000
WORK(KC, TH)	0.0000000	WORK(SC, TH)	0.0000000
WORK(KC, F)	4.000000	WORK(SC, F)	5.000000
WORK(OH, M)	0.0000000	WORK(KS, M)	3.000000
WORK(OH, TU)	2.000000	WORK(KS, TU)	0.0000000
WORK(OH, W)	0.0000000	WORK(KS, W)	2.000000
WORK(OH, TH)	6.000000	WORK(KS, TH)	2.000000
WORK(OH, F)	0.0000000	WORK(KS, F)	0.0000000
WORK(HB, M)	4.000000	WORK(NK, M)	0.0000000
WORK(HB, TU)	7.000000	WORK(NK, TU)	0.0000000
WORK(HB, W)	4.000000	WORK(NK, W)	0.0000000
WORK(HB, TH)	0.0000000	WORK(NK, TH)	6.000000
WORK(HB, F)	4.000000	WORK(NK, F)	1.000000

3.6-5.

(a)

```
MODEL

    MIN  84c+72t+60a;

SUBJECT TO

90c+20t+40a>=200;
30c+80t+60a>=180;
10c+20t+60a>=150;

END
□
```

```
SOLUTION RESULT

  Optimal solution found

    MIN Z        =        241.7143


DECISION VARIABLES

PLAIN VARIABLES

   Variable Name              Activity
  -------------------------------------
   c                            1.1429
   t                            0.0000
   a                            2.4286
  -------------------------------------
```

(b)

```
[OBJECTIVE] MIN = 84*C+72*T+60*A;

!CONSTRAINTS;

90*C+20*T+40*A>=200;
30*C+80*T+60*A>=180;
10*C+20*T+60*A>=150;
```

```
Global optimal solution found at step:            8
Objective value:                           241.7143

                    Variable           Value
                           C        1.142857
                           T       0.0000000
                           A        2.428571
```

3.6-6.

(a)

```
MODEL

    MIN  x1+x2+x3;

SUBJECT TO

2x1+x2+0.5x3>=400;
0.5x1+0.5x2+x3>=100;
1.5x2+2x3>=300;

END
□
```

```
SOLUTION RESULT

Optimal solution found

MIN Z          =        300.0000

Variable Name              Activity
------------------------------------
x1                         100.0000
x2                         200.0000
x3                           0.0000
------------------------------------
```

(b)

```
[OBJECTIVE] MIN = X+Y+Z;

!CONSTRAINTS;

2*X+Y+0.5*Z>=400;
0.5*X+0.5*Y+Z>=100;
1.5*Y+2*Z>=300;
```

```
Global optimal solution found at step:            8
Objective value:                           300.0000

                  Variable           Value
                         X        100.0000
                         Y        200.0000
                         Z       0.0000000
```

```
Global optimal solution found at step:           21
Objective value:                           709.6000
```

3.6-7.

(a) The problem is to choose the amount of paper type k to be produced on machine type l at paper mill k and to be shipped to customer j, which we can represent as x_{ijkl} for $i = 1, \ldots, 10$; $j = 1, \ldots, 1000$; $k = 1, \ldots, 5$ and $l = 1, 2, 3$. The objective is to minimize

$$\sum_{i,k,l} P_{ikl} \left(\sum_j x_{ijkl} \right) + \sum_{i,j,k} T_{ijk} \left(\sum_l x_{ijkl} \right)$$

subject to

$\sum_{i,l} x_{ijkl}$	$\leq \; D_{jk}$	for $j = 1, \ldots, 1000$; $k = 1, \ldots, 5$	DEMAND
$\sum_{k,l} r_{klm} \left(\sum_j x_{ijkl} \right)$	$\leq \; R_{im}$	for $i = 1, \ldots, 10$; $m = 1, 2, 3, 4$	RAW MATERIAL
$\sum_k c_{kl} \left(\sum_j x_{ijkl} \right)$	$\leq \; C_{il}$	for $i = 1, \ldots, 10$; $l = 1, 2, 3$	CAPACITY
x_{ijkl}	$\geq \; 0$	for $i = 1, \ldots, 10$; $j = 1, \ldots, 1000$; $k = 1, \ldots, 5$; $l = 1, 2, 3$	

Note that $\sum_l x_{ijkl}$ is the total amount of paper type k shipped to customer j from paper mill i and $\sum_j x_{ijkl}$ is the total amount of paper type k made on machine type l at paper mill i.

(b) $1000*5 + 10*4 + 10*3 = 5,070$ functional constraints

$10*1000*5*3 = 150,000$ decision variables

(c)

```
TITLE
    PaperManufacturing;

INDEX
    mill = 1..10;
    customer = 1..1000;
    machine = 1..3;
    material = 1..4;
    paper = 1..5;

DATA
    Required[customer,paper] = DATAFILE(Required.dat);
    Rate1[paper,machine,material] = DATAFILE(Rate1.dat);
    RawMaterial[mill,material] = DATAFILE(RawMaterial.dat);
    Rate2[paper,machine] = DATAFILE(Rate2.dat);
    MaxCapacity[mill,machine] = DATAFILE(MaxCapacity.dat);
    ProdCost[mill,paper,machine] = DATAFILE(ProdCost);
    TranspCost[mill,customer,paper] = DATAFILE(TranspCost);

DECISION VARIABLES
    Quantity[mill,customer,machine,paper]  -> ""
```

```
MODEL

    MIN  TotalCost = SUM(mill,customer,machine,paper: ProdCost * Quantity)
        + SUM(mill,customer,machine,paper: TranspCost * Quantity);

SUBJECT TO

    Demand[customer,paper] : SUM(mill,machine: Quantity) >= Required ;
    Supply[mill,material] : SUM(customer,paper,machine: Rate1 * Quantity) <= RawMaterial;
    Capacity[mill,machine] : SUM(customer,paper: Rate2 * Quantity) < MaxCapacity ;
END
□
```

(d)

```
MODEL:

SETS:
MILLS /1..10/;
CUSTOMERS /1..1000/;
MACHINES /1..3/;
MATERIALS /1..4/;
PAPER /1..5/;
LINK1(CUSTOMERS,PAPER): DEMAND;
LINK2(PAPER, MACHINES, MATERIALS): RATE1;
LINK3(MILLS, MATERIALS): CAPACITY1;
LINK4(PAPER, MACHINES): RATE2;
LINK5(MILLS, MACHINES): CAPACITY2;
LINK6(MILLS, PAPER, MACHINES): PROD_COST;
LINK7(MILLS, CUSTOMERS, PAPER): TRANSP_COST;
LINK8(MILLS, CUSTOMERS, PAPER, MACHINES): QUANTITY;
ENDSETS

!OBJECTIVE IS TO MINIMIZE PRODUCTION COST + TRANSPORTATION COST;
MIN = @SUM(LINK6(I,K,L):PROD_COST(I,K,L) * @SUM(CUSTOMERS(J): QUANTITY(I,J,K,L))) +
      @SUM(LINK7(I,J,K):TRANSP_COST * @SUM(MACHINES(L): QUANTITY(I,J,K,L)));

!DEMAND CONSTRAINTS;
@FOR(LINK1(J,K): @SUM(MILLS(I): @SUM(MACHINES(L): QUANTITY(I,J,K,L)))>= DEMAND(J,K));

!RAW MATERIALS SUPPLY CONSTRAINTS;
@FOR(LINK3(I,M): @SUM(PAPER(K): @SUM(MACHINES(L): RATE1(K,L,M)*@SUM(CUSTOMERS(J):
QUANTITY(I,J,K,L)))) <= CAPACITY1(I,M));

!CAPACITY SUPPLY CONSTRAINTS;
@FOR(LINK5(I,L): @SUM(PAPER(K): RATE2(K,L) * @SUM(CUSTOMERS(J): QUANTITY(I,J,K,L))) <=
CAPACITY2(I,L));

!READ DATA FROM AN EXCEL FILE;
DATA:
DEMAND, RATE1, CAPACITY1, RATE2, CAPACITY2, PROD_COST, TRANSP_COST =
@WKX('C:\LINGO\DATA.WK4','DEMAND','RATE1','CAPACITY1','RATE2','CAPACITY2','PROD_COST','TRA
NSP_COST');
ENDDATA
END
```

3.6-8

Answers will vary.

3.7-1.

Answers will vary.

3.7-2.

Answers will vary.

Case 3.1

a) In this case, we have two decision variables: the number of Family Thrillseekers we should assemble and the number of Classy Cruisers we should assemble. We also have the following three constraints:

1. The plant has a maximum of 48,000 labor hours.
2. The plant has a maximum of 20,000 doors available.
3. The number of Cruisers we should assemble must be less than or equal to 3,500.

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	48,000	<=	48,000
7	Doors	4	2	20,000	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	3,800	2,400			$26,640,000
12			<=			
13	Demand		3,500			

	D
4	Resources
5	Used
6	=SUMPRODUCT(B6:C6,Production)
7	=SUMPRODUCT(B7:C7,Production)

Range Name	Cells
ClassyCruisers	C11
Demand	C13
Production	B11:C11
ResourcesAvailable	F6:F7
ResourcesUsed	D6:D7
TotalProfit	F11
UnitProfit	B3:C3

	F
10	Total Profit
11	=SUMPRODUCT(UnitProfit,Production)

Solver Parameters
Set Objective Cell: TotalProfit
To: Max
By Changing Variable Cells:
Production
Subject to the Constraints:
ClassyCruisers <= Demand
ResourcesUsed <= Resources Available
Solver Options:
Make Variables Nonnegative

Solving Method: Simplex LP

Rachel's plant should assemble 3,800 Thrillseekers and 2,400 Cruisers to obtain a maximum profit of $26,640,000.

b) In part (a) above, we observed that the Cruiser demand constraint was not binding. Therefore, raising the demand for the Cruiser will not change the optimal solution. The marketing campaign should not be undertaken.

c) The new value of the right-hand side of the labor constraint becomes 48,000 * 1.25 = 60,000 labor hours. All formulas and Solver settings used in part (a) remain the same.

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	56,250	<=	60,000
7	Doors	4	2	20,000	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	3,250	3,500			$30,600,000
12			<=			
13	Demand		3,500			

Rachel's plant should now assemble 3,250 Thrillseekers and 3,500 Cruisers to achieve a maximum profit of $30,600,000.

d) Using overtime labor increases the profit by $30,600,000 – $26,640,000 = $3,960,000. Rachel should therefore be willing to pay at most $3,960,000 extra for overtime labor beyond regular time rates.

e) The value of the right-hand side of the Cruiser demand constraint is 3,500 * 1.20 = 4,200 cars. The value of the right-hand side of the labor hour constraint is 48,000 * 1.25 = 60,000 hours. All formulas and Solver settings used in part (a) remain the same. Ignoring the costs of the advertising campaign and overtime labor,

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	60,000	<=	60,000
7	Doors	4	2	20,000	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	3,000	4,000			$32,400,000
12			<=			
13	Demand		4,200			

Rachel's plant should produce 3,000 Thrillseekers and 4,000 Cruisers for a maximum profit of $32,400,000. This profit excludes the costs of advertising and using overtime labor.

f) The advertising campaign costs $500,000. In the solution to part (e) above, we used the maximum overtime labor available, and the maximum use of overtime labor costs $1,600,000. Thus, our solution in part (e) required an extra $500,000 + $1,600,000 = $2,100,000. We perform the following cost/benefit analysis:

Profit in part (e):	$32,400,000
– Advertising and overtime costs:	$ 2,100,000
	$30,300,000

We compare the $30,300,000 profit with the $26,640,000 profit obtained in part (a) and conclude that the decision to run the advertising campaign and use overtime labor is a very wise, profitable decision.

g) Because we consider this question independently, the values of the right-hand sides for the Cruiser demand constraint and the labor hour constraint are the same as those in part (a). We now change the profit for the Thrillseeker from $3,600 to $2,800 in the problem formulation. All formulas and Solver settings used in part (a) remain the same.

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$2,800	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	48,000	<=	48,000
7	Doors	4	2	14,500	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	1,875	3,500			$24,150,000
12			<=			
13	Demand		3,500			

Rachel's plant should assemble 1,875 Thrillseekers and 3,500 Cruisers to obtain a maximum profit of $24,150,000.

h) Because we consider this question independently, the profit for the Thrillseeker remains the same as the profit specified in part (a). The labor hour constraint changes. Each Thrillseeker now requires 7.5 hours for assembly. All formulas and Solver settings used in part (a) remain the same.

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	7.5	10.5	48,000	<=	48,000
7	Doors	4	2	13,000	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	1,500	3,500			$24,300,000
12			<=			
13	Demand		3,500			

Rachel's plant should assemble 1,500 Thrillseekers and 3,500 Cruisers for a maximum profit of $24,300,000.

i) Because we consider this question independently, we use the problem formulation used in part (a). In this problem, however, the number of Cruisers assembled has to be strictly equal to the total demand. The formulas used in the problem formulation remain the same as those used in part (a).

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	48,000	<=	48,000
7	Doors	4	2	14,500	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	1,875	3,500			$25,650,000
12			=			
13	Demand		3,500			

The new profit is $25,650,000, which is $26,640,000 – $25,650,000 = $990,000 less than the profit obtained in part (a). This decrease in profit is less than $2,000,000, so Rachel should meet the full demand for the Cruiser.

j) We now combine the new considerations described in parts (f), (g), and (h). In part (f), we decided to use both the advertising campaign and the overtime labor. The advertising campaign raises the demand for the Cruiser to 4,200 sedans, and the overtime labor increases the labor hour capacity of the plant to 60,000 labor hours. In part (g), we decreased the profit generated by a Thrillseeker to $2,800. In part (h), we increased the time to assemble a Thrillseeker to 7.5 hours. The formulas and Solver settings used for this problem are the same as those used in part (a).

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$2,800	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	7.5	10.5	60,000	<=	60,000
7	Doors	4	2	16,880	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	2,120	4,200			$28,616,000
12			<=			
13	Demand		4,200			

Rachel's plant should assemble 2,120 Thrillseekers and 4,200 Cruisers for a maximum profit of $28,616,000 – $2,100,000 = $26,516,000.

Case 3.2

a) We want to determine the amount of potatoes and green beans Maria should purchase to minimize ingredient costs. We have two decision variables: the amount (in pounds) of potatoes Maria should purchase and the amount (in pounds) of green beans Maria should purchase. We also have constraints on nutrition, taste, and weight.

<u>Nutrition Constraints</u>

1. We first need to ensure that the dish has 180 grams of protein. We are told that 100 grams of potatoes have 1.5 grams of protein and 10 ounces of green beans have 5.67 grams of protein. Since we have decided to measure our decision variables in pounds, however, we need to determine the grams of protein in one pound of each ingredient.

We perform the following conversion for potatoes:

$$\left(\frac{1.5 \text{ g protein}}{100 \text{ g potatoes}}\right)\left(\frac{28.35 \text{ g}}{1 \text{ oz.}}\right)\left(\frac{16 \text{ oz.}}{1 \text{ lb.}}\right) = \frac{6.804 \text{ g protein}}{1 \text{ lb. of potatoes}}$$

We perform the following conversion for green beans:

$$\left(\frac{5.67 \text{ g protein}}{10 \text{ oz. green beans}}\right)\left(\frac{16 \text{ oz.}}{1 \text{ lb.}}\right) = \frac{9.072 \text{ g protein}}{1 \text{ lb. of green beans}}$$

2. We next need to ensure that the dish has 80 milligrams of iron. We are told that 100 grams of potatoes have 0.3 milligrams of iron and 10 ounces of green beans have 3.402 milligrams of iron. Since we have decided to measure our decision variables in pounds, however, we need to determine the milligrams of iron in one pound of each ingredient.

We perform the following conversion for potatoes:

$$\left(\frac{0.3 \text{ mg iron}}{100\text{g potatoes}}\right)\left(\frac{28.35 \text{ g}}{1 \text{ oz.}}\right)\left(\frac{16 \text{ oz.}}{1 \text{ lb.}}\right) = \frac{1.361 \text{ mg iron}}{1 \text{ lb. of potatoes}}$$

We perform the following conversion for green beans:

$$\left(\frac{3.402 \text{ mg iron}}{10 \text{ oz. green beans}}\right)\left(\frac{16 \text{ oz.}}{1 \text{ lb.}}\right) = \frac{5.443 \text{ mg iron}}{1 \text{ lb. of green beans}}$$

3. We next need to ensure that the dish has 1,050 milligrams of vitamin C. We are told that 100 grams of potatoes have 12 milligrams of vitamin C and 10 ounces of green beans have 28.35 milligrams of vitamin C. Since we have decided to measure our decision variables in pounds, however, we need to determine the milligrams of vitamin C in one pound of each ingredient.

We perform the following conversion for potatoes:

$$\left(\frac{\text{12 mg Vitamin C}}{\text{100g potatoes}}\right)\left(\frac{\text{28.35 g}}{\text{1 oz.}}\right)\left(\frac{\text{16 oz.}}{\text{1 lb.}}\right) = \frac{\text{54.432 mg Vitamin C}}{\text{1 lb. of potatoes}}$$

We perform the following conversion for green beans:

$$\left(\frac{\text{28.35 mg Vitamin C}}{\text{10 oz. green beans}}\right)\left(\frac{\text{16 oz.}}{\text{1 lb.}}\right) = \frac{\text{45.36 mg Vitamin C}}{\text{1 lb. of green beans}}$$

<u>Taste Constraint</u>

Edson requires that the casserole contain at least a six to five ratio in the weight of potatoes to green beans. We have:

$$\frac{\text{pounds of potatoes}}{\text{pounds of green beans}} \geq \frac{6}{5}$$

5 (pounds of potatoes) ≥ 6 (pounds of green beans)

<u>Weight Constraint</u>

Finally, Maria requires a minimum of 10 kilograms of potatoes and green beans together. Because we measure potatoes and green beans in pounds, we must perform the following conversion:

$$\text{10 kg of potatoes and green beans}\left(\frac{\text{1000 g}}{\text{1 kg}}\right)\left(\frac{\text{1 lb}}{\text{453.6 g}}\right)$$

$$= \text{22.046 lb of potatoes and green beans}$$

	A	B	C	D	E	F	G
1			Potatoes	Green Beans			
2		Unit Cost (per lb.)	$0.40	$1.00			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	9.072	194.87	>=	180
6		Iron (mg)	1.361	5.443	80.00	>=	80
7		Vitamin C (mg)	54.432	45.36	1,251.27	>=	1,050
8							
9			Potatoes	Green Beans	Total Weight		Total Cost
10		Quantity (lb.)	13.57	11.31	25		$16.73
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	67.833	>=	67.833	6	Times Green Beans

	E
3	Total
4	Nutrition
5	=SUMPRODUCT(C5:D5,Quantity)
6	=SUMPRODUCT(C6:D6,Quantity)
7	=SUMPRODUCT(C7:D7,Quantity)
8	
9	Total Weight
10	=SUM(Quantity)

Range Name	Cells
BeanRatio	E15
MinimumWeight	E12
NutritionalRequirement	G5:G7
PotatoRatio	C15
Quantity	C10:D10
TotalCost	G10
TotalNutrition	E5:E7
TotalWeight	E10
UnitCost	C2:D2

	G
9	Total Cost
10	=SUMPRODUCT(UnitCost,Quantity)

	A	B	C	D	E	F	G
14			Taste Constraint:				
15	5	Times Potatoes	=A15*C10	>=	=F15*D10	6	Times Green Beans

Solver Parameters

Set Objective Cell: TotalCost

To: Min

By Changing Variable Cells:

Quantity

Subject to the Constraints:

PotatoRatio >= BeanRation

TotalNutrition >= NutritionalRequirement

TotalWeight <= MinimumWeight

Solver Options:

Make Variables Nonnegative

Solving Method: Simplex LP

Maria should purchase 13.57 lb. of potatoes and 11.31 lb. of green beans to obtain a minimum cost of $16.73.

b) The taste constraint changes. The new constraint is now.

$$\frac{\text{pounds of potatoes}}{\text{pounds of green beans}} \geq \frac{1}{2}$$

$$2\ (\text{pounds of potatoes}) \geq 1\ (\text{pounds of green beans})$$

The formulas and Solver settings used to solve the problem remain the same as part (a).

	A	B	C	D	E	F	G
1			Potatoes	Green Beans			
2		Unit Cost (per lb.)	$0.40	$1.00			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	9.072	180.00	>=	180
6		Iron (mg)	1.361	5.443	80.00	>=	80
7		Vitamin C (mg)	54.432	45.36	1,110.00	>=	1,050
8							
9			Potatoes	Green Beans	Total Weight		Total Cost
10		Quantity (lb.)	10.29	12.13	22		$16.24
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	2	Times Potatoes	20.576	>=	12.125	1	Times Green Beans

Maria should purchase 10.29 lb. of potatoes and 12.13 lb. of green beans to obtain a minimum cost of $16.24.

c) The right-hand side of the iron constraint changes from 80 mg to 65 mg. The formulas and Solver settings used in the problem remain the same as in part (a).

	A	B	C	D	E	F	G
1			Potatoes	Green Beans			
2		Unit Cost (per lb.)	$0.40	$1.00			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	9.072	180.00	>=	180
6		Iron (mg)	1.361	5.443	65.00	>=	65
7		Vitamin C (mg)	54.432	45.36	1,222.51	>=	1,050
8							
9			Potatoes	Green Beans	Total Weight		Total Cost
10		Quantity (lb.)	15.80	7.99	24		$14.31
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	79.001	>=	47.947	6	Times Green Beans

Maria should purchase 15.80 lb. of potatoes and 7.99 lb. of green beans to obtain a minimum cost of $14.31.

d) The iron requirement remains 65 mg. We need to change the price per pound of green beans from $1.00 per pound to $0.50 per pound. The formulas and Solver settings used in the problem remain the same as in part (a).

	A	B	C	D	E	F	G
1			Potatoes	Green Beans			
2		Unit Cost (per lb.)	$0.40	$0.50			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	9.072	180.00	>=	180
6		Iron (mg)	1.361	5.443	73.90	>=	65
7		Vitamin C (mg)	54.432	45.36	1,155.79	>=	1,050
8							
9			Potatoes	Green Beans	Total Weight		Total Cost
10		Quantity (lb.)	12.53	10.44	23		$10.23
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	62.657	>=	62.657	6	Times Green Beans

Maria should purchase 12.53 lb. of potatoes and 10.44 lb. of green beans to obtain a minimum cost of $10.23.

e) We still have two decision variables: one variable to represent the amount (in pounds) of potatoes Maria should purchase and one variable to represent the amount (in pounds) of lima beans Maria should purchase. To determine the grams of protein in one pound of lima beans, we perform the following conversion:

$$\left(\frac{22.68 \text{ g protein}}{10 \text{ oz. lima beans}}\right)\left(\frac{16 \text{ oz.}}{1 \text{ lb.}}\right) = \frac{36.288 \text{ g protein}}{1 \text{ lb. of lima beans}}$$

To determine the milligrams of iron in one pound of lima beans, we perform the following conversion:

$$\left(\frac{6.804 \text{ mg iron}}{10 \text{ oz. lima beans}}\right)\left(\frac{16 \text{ oz.}}{1 \text{ lb.}}\right) = \frac{10.886 \text{ mg iron}}{1 \text{ lb. of lima beans}}$$

Lima beans contain no vitamin C, so we do not have to perform a measurement conversion for vitamin C.

We change the decision variable from green beans to lima beans and insert the new parameters for protein, iron, vitamin C, and cost. The formulas and Solver settings used in the problem remain the same as in part (a).

	A	B	C	D	E	F	G
1			Potatoes	Lima Beans			
2		Unit Cost (per lb.)	$0.40	$0.60			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	36.288	260.41	>=	180
6		Iron (mg)	1.361	10.886	65.00	>=	65
7		Vitamin C (mg)	54.432	0	1,050.00	>=	1,050
8							
9			Potatoes	Lima Beans	Total Weight		Total Cost
10		Quantity (lb.)	19.29	3.56	23		$9.85
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	96.451	>=	21.356	6	Times Lima Beans

Maria should purchase 19.29 lb. of potatoes and 3.56 lb. of lima beans to obtain a minimum cost of $9.85.

f) Edson takes pride in the taste of his casserole, and the optimal solution from above does not seem to preserve the taste of the casserole. First, Maria forces Edson to use lima beans instead of green beans, and lima beans are not an ingredient in Edson's original recipe. Second, although Edson places no upper limit on the ratio of potatoes to beans, the above recipe uses an over five to one ratio of potatoes to beans. This ratio seems unreasonable since such a large amount of potatoes will overpower the taste of beans in the recipe.

g) We only need to change the values on the right-hand side of the iron and vitamin C constraints. The formulas and Solver settings used in the problem remain the same as in part (a). The values used in the new problem formulation and solution follow.

	A	B	C	D	E	F	G
1			Potatoes	Lima Beans			
2		Unit Cost (per lb.)	$0.40	$0.60			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	36.288	428.58	>=	180
6		Iron (mg)	1.361	10.886	120.00	>=	120
7		Vitamin C (mg)	54.432	0	685.72	>=	500
8							
9			Potatoes	Lima Beans	Total Weight		Total Cost
10		Quantity (lb.)	12.60	9.45	22		$10.71
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	62.988	>=	56.690	6	Times Lima Beans

Maria should purchase 12.60 lb. of potatoes and 9.45 lb. of lima beans to obtain a minimum cost of $10.71.

Case 3.3

a) The number of operators that the hospital needs to staff the call center during each two-hour shift can be found in the following table:

	A	B	C	D	E	F
1			Average	Average	English	Spanish
2		Average	Calls/hour	Calls/hour	Speaking	Speaking
3		Number	from English	from Spanish	Agents	Agents
4	Work Shift	of Calls	Speakers	Speakers	Needed	Needed
5	7am-9am	40	32	8	6	2
6	9am-11am	85	68	17	12	3
7	11am-1pm	70	56	14	10	3
8	1pm-3pm	95	76	19	13	4
9	3pm-5pm	80	64	16	11	3
10	5pm-7pm	35	28	7	5	2
11	7pm-9pm	10	8	2	2	1
12						
13	Percent English Speakers		80%			
14						
15	Calls Handled per hour		6			

For example, the average number of phone calls per hour during the shift from 7am to 9am equals 40. Since, on average, 80% of all phone calls are from English speakers, there is an average number of 32 phone calls per hour from English speakers during that shift. Since one operator takes, on average, 6 phone calls per hour, the hospital needs 32/6 = 5.333 English-speaking operators during that shift. The hospital cannot employ fractions of an operator and so needs 6 English-speaking operators for the shift from 7am to 9am.

b) The problems of determining how many Spanish-speaking operators and English-speaking operators Lenny needs to hire to begin each shift are independent. Therefore we can formulate two smaller linear programming models instead of one large model. We are going to have one model for the scheduling of the Spanish-speaking operators and another one for the scheduling of the English-speaking operators.

Lenny wants to minimize the operating costs while answering all phone calls. For the given scheduling problem we make the assumption that the only operating costs are the wages of the employees for the hours that they answer phone calls. The wages for the hours during which they perform paperwork are paid by other cost centers. Moreover, it does not matter for the callers whether an operator starts his or her work day with phone calls or with paperwork. For example, we do not need to distinguish between operators who start their day answering phone calls at 9am and operators who start their day with paperwork at 7am, because both groups of operators will be answering phone calls at the same time. And only this time matters for the analysis of Lenny's problem.

We define the decision variables according to the time when the employees have their first shift of answering phone calls. For the scheduling problem of the English-speaking operators we have 7 decision variables. First, we have 5 decision variables for full-time employees.

The number of operators having their first shift on the phone from 7am to 9am.
The number of operators having their first shift on the phone from 9am to 11am.
The number of operators having their first shift on the phone from 11am to 1pm.
The number of operators having their first shift on the phone from 1pm to 3pm.
The number of operators having their first shift on the phone from 3pm to 5pm.

In addition, we define 2 decision variables for part-time employees.

The number of part-time operators having their first shift from 3pm to 5pm.
The number of part-time operators having their first shift from 5pm to 7pm.

The unit cost coefficients in the objective function are the wages operators earn while they answer phone calls. All operators who have their first shift on the phone from 7am to 9am, 9am to 11am, or 11am to 1pm finish their work on the phone before 5pm. They earn 4*$10 = $40 during their time answering phone calls. All operators who have their first shift on the phone from 1pm to 3pm or 3pm to 5pm have one shift on the phone before 5pm and another one after 5pm. They earn 2*$10+2*$12 = $44 during their time answering phone calls. The second group of part-time operators, those having their first shift from 5pm to 7pm, earn 4*$12 = $48 during their time answering phone calls.

There are 7 constraints, one for each two-hour shift during which phone calls need to be answered. The right-hand sides for these constraints are the number of operators needed to ensure that all phone calls get answered in a timely manner. On the left-hand side we determine the number of operators on the phone during any given shift. For example, during the 11am to 1pm shift the total number of operators answering phone calls equals the sum of the number of operators who started answering calls at 7am and are currently in their second shift of the day and the number of operators who started answering calls at 11am.

The following spreadsheet describes the entire problem formulation for the English-speaking employees:

	A	B	C	D	E	F	G	H	I	J	K
1	**English**	Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
2	**Speaking**	on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
3		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
4		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			
5	Unit Cost	$40	$40	$40	$44	$44	$44	$48			
6									Total		Agents
7	Work Shift?								Working		Needed
8	7am-9am	1	0	0	0	0	0	0	6	>=	6
9	9am-11am	0	1	0	0	0	0	0	13	>=	12
10	11am-1pm	1	0	1	0	0	0	0	10	>=	10
11	1pm-3pm	0	1	0	1	0	0	0	13	>=	13
12	3pm-5pm	0	0	1	0	1	1	0	11	>=	11
13	5pm-7pm	0	0	0	1	0	1	1	5	>=	5
14	7pm-9pm	0	0	0	0	1	0	1	2	>=	2
15											
16		Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
17		on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
18		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
19		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			Total Cost
20	Number Working	6	13	4	0	2	5	0			$1,228

	I
6	Total
7	Working
8	=SUMPRODUCT(B8:H8,NumberWorking)
9	=SUMPRODUCT(B9:H9,NumberWorking)
10	=SUMPRODUCT(B10:H10,NumberWorking)
11	=SUMPRODUCT(B11:H11,NumberWorking)
12	=SUMPRODUCT(B12:H12,NumberWorking)
13	=SUMPRODUCT(B13:H13,NumberWorking)
14	=SUMPRODUCT(B14:H14,NumberWorking)

	K
19	Total Cost
20	=SUMPRODUCT(UnitCost,NumberWorking)

Range Name	Cells
AgentsNeeded	K8:K14
NumberWorking	B20:H20
TotalCost	K20
TotalWorking	I8:I14
UnitCost	B5:H5

Solver Parameters

Set Objective Cell: TotalCost

To: Min

By Changing Variable Cells:

NumberWorking

Subject to the Constraints:

TotalWorking >=

AgentsNeeded
Solver Options:
Make Variables Nonnegative
Solving Method: Simplex LP

The linear programming model for the Spanish-speaking employees can be developed in a similar fashion.

	A	B	C	D	E	F	G	H	I
1	**Spanish**	Full-Time	Full-Time	Full-Time	Full-Time	Full-Time			
2	**Speaking**	on Phone	on Phone	on Phone	on Phone	on Phone			
3		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm			
4		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm			
5	Unit Cost	$40	$40	$40	$44	$48			
6							Total		Agents
7	Work Shift?						Working		Needed
8	7am-9am	1	0	0	0	0	2	>=	2
9	9am-11am	0	1	0	0	0	3	>=	3
10	11am-1pm	1	0	1	0	0	4	>=	3
11	1pm-3pm	0	1	0	1	0	5	>=	4
12	3pm-5pm	0	0	1	0	1	3	>=	3
13	5pm-7pm	0	0	0	1	0	2	>=	2
14	7pm-9pm	0	0	0	0	1	1	>=	1
15									
16		Full-Time	Full-Time	Full-Time	Full-Time	Full-Time			
17		on Phone	on Phone	on Phone	on Phone	on Phone			
18		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm			
19		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm			Total Cost
20	Number Working	2	3	2	2	1			$416

c) Lenny should hire 25 full-time English-speaking operators. Of these operators, 6 have their first phone shift from 7am to 9am, 13 from 9am to 11am, 4 from 11am to 1pm, and 2 from 3pm to 5pm. Lenny should also hire 5 part-time operators who start their work at 3pm. In addition, Lenny should hire 10 Spanish-speaking operators. Of these operators, 2 have their first shift on the phone from 7am to 9am, 3 from 9am to 11am, 2 from 11am to 1pm and 1pm to 3pm, and 1 from 3pm to 5pm. The total (wage) cost of running the calling center equals $1640 per day.

d) The restriction that Lenny can find only one English-speaking operator who wants to start work at 1pm affects only the linear programming model for English-speaking operators. This restriction does not put a bound on the number of operators who start their first phone shift at 1pm because those operators can start work at 11am with paperwork. However, this restriction does put an upper bound on the number of operators having their first phone shift from 3pm to 5pm. The new worksheet appears as follows.

	A	B	C	D	E	F	G	H	I	J	K
1	**English**	Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
2	**Speaking**	on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
3		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
4		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			
5	Unit Cost	$40	$40	$40	$44	$44	$44	$48			
6									Total		Agents
7	Work Shift?								Working		Needed
8	7am-9am	1	0	0	0	0	0	0	6	>=	6
9	9am-11am	0	1	0	0	0	0	0	13	>=	12
10	11am-1pm	1	0	1	0	0	0	0	12	>=	10
11	1pm-3pm	0	1	0	1	0	0	0	13	>=	13
12	3pm-5pm	0	0	1	0	1	1	0	11	>=	11
13	5pm-7pm	0	0	0	1	0	1	1	5	>=	5
14	7pm-9pm	0	0	0	0	1	0	1	2	>=	2
15											
16		Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
17		on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
18		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
19		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			Total Cost
20	Number Working	6	13	6	0	1	4	1			$1,268
21						<=					
22						1					

Lenny should hire 26 full-time English-speaking operators. Of these operators, 6 have their first phone shift from 7am to 9am, 13 from 9am to 11am, 6 from 11am to 1pm, and 1 from 3pm to 5pm. Lenny should also hire 4 part-time operators who start their work at 3pm and 1 part-time operator starting work at 5pm. The hiring of Spanish-speaking operators is unaffected. The new total (wage) costs equal $1680 per day.

e) For each hour, we need to divide the average number of calls per hour by the average processing speed, which is 6 calls per hour. The number of bilingual operators that the hospital needs to staff the call center during each two-hour shift can be found in the following table:

	A	B	C
1		Average	
2		Number	Agents
3	Work Shift	of Calls	Needed
4	7am-9am	40	7
5	9am-11am	85	15
6	11am-1pm	70	12
7	1pm-3pm	95	16
8	3pm-5pm	80	14
9	5pm-7pm	35	6
10	7pm-9pm	10	2
11			
12		Calls Handled per hour	6

f) The linear programming model for Lenny's scheduling problem can be found in the same way as before, only that now all operators are bilingual. (The formulas and the solver dialog box are identical to those in part (b).)

	A	B	C	D	E	F	G	H	I	J	K
1	**Bilingual**	Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
2		on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
3		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
4		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			
5	Unit Cost	$40	$40	$40	$44	$44	$44	$48			
6									Total		Agents
7	Work Shift?								Working		Needed
8	7am-9am	1	0	0	0	0	0	0	7	>=	7
9	9am-11am	0	1	0	0	0	0	0	16	>=	15
10	11am-1pm	1	0	1	0	0	0	0	13	>=	12
11	1pm-3pm	0	1	0	1	0	0	0	16	>=	16
12	3pm-5pm	0	0	1	0	1	1	0	14	>=	14
13	5pm-7pm	0	0	0	1	0	1	1	6	>=	6
14	7pm-9pm	0	0	0	0	1	0	1	2	>=	2
15											
16		Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
17		on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
18		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
19		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			Total Cost
20	Number Working	7	16	6	0	2	6	0			$1,512

Lenny should hire 31 full-time bilingual operators. Of these operators, 7 have their first phone shift from 7am to 9am, 16 from 9am to 11am, 6 from 11am to 1pm, and 2 from 3pm to 5pm. Lenny should also hire 6 part-time operators who start their work at 3pm. The total (wage) cost of running the calling center equals $1512 per day.

g) The total cost of part (f) is $1512 per day; the total cost of part (b) is $1640. Lenny could pay an additional $1640-$1512 = $128 in total wages to the bilingual operators without increasing the total operating cost beyond those for the scenario with only monolingual operators. The increase of $128 represents a percentage increase of 128/1512 = 8.47%.

h) Creative Chaos Consultants has made the assumption that the number of phone calls is independent of the day of the week. But maybe the number of phone calls is very different on a Monday than it is on a Friday. So instead of using the same number of average phone calls for every day of the week, it might be more appropriate to determine whether the day of the week affects the demand for phone operators. As a result Lenny might need to hire more part-time employees for some days with an increased calling volume.

   Similarly, Lenny might want to take a closer look at the length of the shifts he has scheduled. Using shorter shift periods would allow him to “fine tune” his calling centers and make it more responsive to demand fluctuations.

   Lenny should investigate why operators are able to answer only 6 phone calls per hour. Maybe additional training of the operators could enable them to answer phone calls quicker and so increase the number of phone calls they are able to answer in an hour.

   Finally, Lenny should investigate whether it is possible to have employees switching back and forth between paperwork and answering phone calls. During slow times phone operators could do some paperwork while they are sitting next to a phone, while in times of sudden large call volumes employees who are scheduled to do paperwork could quickly switch to answering phone calls.

   Lenny might also want to think about the installation of an automated answering system that gives callers a menu of selections. Depending upon the caller’s selection, the call is routed to an operator who specializes in answering questions about that selection.

Case 3.4

a) In this case, the decisions to be made are

TV = number of commercials on television
M = number of advertisements in magazines
SS = number of advertisements in Sunday supplements

The resulting linear programming model is
Maximize Exposures = 1,300 TV + 600 M + 500 SS
subject to

Resource Constraints

$300\ TV + 150\ M + 100\ SS \leq 4{,}000$ (ad budget in \$1,000s)
$90\ TV + 30\ M + 40\ SS \leq 1{,}000$ (planning budget in \$1,000s)
$TV \leq 5$ (television spots available)

Benefits Constraints:

$1.2\ TV + 0.1\ M \geq 5$ (millions of young children)
$0.5\ TV + 0.2\ M + 0.2\ S \geq 5$ (millions of parents)

Fixed-Requirement Constraints:

$40\ TV + 120\ SS = 5$ (coupon budget in \$1,000s)

Nonnegativity Constraints:

$TV \geq 0,\ M \geq 0,\ S \geq 0.$

The linear programming spreadsheet solution is shown below.

	TV Spots	Magazine Ads	SS Ads			
Exposures per Ad (thousands)	1,300	600	500			
		Cost per Ad ($thousands)		Budget Spent		Budget Available
Ad Budget	300	150	100	3,775	<=	4,000
Planning Budget	90	30	40	1,000	<=	1,000
		Number Reached per Ad (millions)		Total Reached		Minimum Acceptable
Young Children	1.2	0.1	0	5	>=	5
Parents of Young Children	0.5	0.2	0.2	5.85	>=	5
	TV Spots	Magazine Ads	SS Ads	Total Redeemed		Required Amount
Coupon Redemption per Ad ($thousands)	0	40	120	1,490	=	1,490
						Total Exposures
	TV Spots	Magazine Ads	SS Ads			(thousands)
Number of Ads	3	14	7.75			16,175
	<=					
Maximum TV Spots	5					

b) The violations of the four assumptions of LP:

(1) **Proportionality assumption:** the advertisement cost may not be proportional to number of commercials on television or number of advertisements in magzines. The marginal cost for additional commercial can decrease.

(2) **Additivity assumption:** This assumption can be violated for benefit constraints because it states that there is no overlap between people who see the commercial on television or see the advertisements in magzine or Sunday supplements.

(3) **Divisibility assumption:** The decision variables in this case are number of commercial on TV or advertisements in magzines and Sunday supplements of major newspapers. Naturally, these variables should take on integer values.

(4) **Certainty assumption:** Since this LP model is formulated to select some future courses of actions, the parameters used in this case, such as Exposures per Ad or Number Reached per Ad, are based on a prediction of future situation, which inevitably introduces some degree of uncertainty.

c) Since none of the assumptions appear to be badly violated, LP is reasonable at least as a first approximation. Later models, such as IP or NLP can provide some refinement.