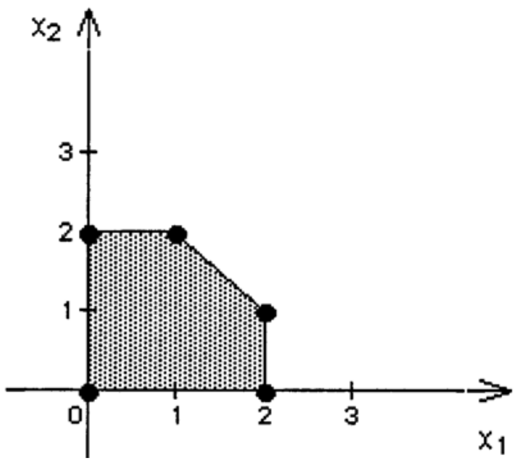
CHAPTER 4: SOLVING LINEAR PROGRAMMING PROBLEMS: THE SIMPLEX METHOD

4.1-1.

(a) Label the corner points as A, B, C, D, and E in the clockwise direction starting from $(0, 2)$.



(b)
A: $x_1 = 0$ and $x_2 = 2$
B: $x_2 = 2$ and $x_1 + x_2 = 3$
C: $x_1 + x_2 = 3$ and $x_1 = 2$
D: $x_1 = 2$ and $x_2 = 0$
E: $x_2 = 0$ and $x_1 = 0$

(c)
A: $(x_1,\ x_2) = (0, 2)$
B: $(x_1,\ x_2) = (1, 2)$
C: $(x_1,\ x_2) = (2, 1)$
D: $(x_1,\ x_2) = (2, 0)$
E: $(x_1,\ x_2) = (0, 0)$

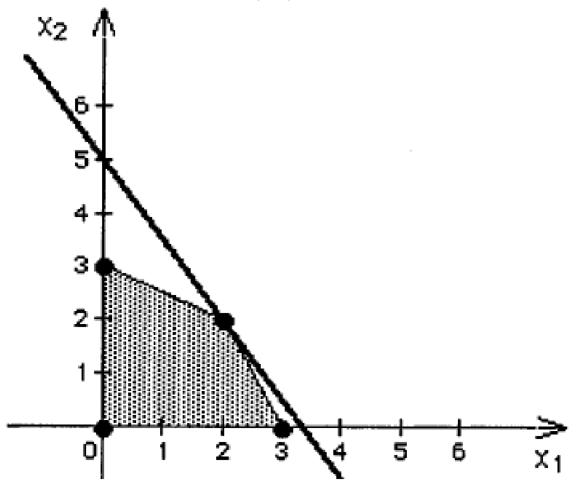
(d)

Corner Point	Adjacent Points
A	E, B
B	A, C
C	B, D
D	C, E
E	D, A

(e)
A and B: $x_2 = 2$
B and C: $x_1 + x_2 = 3$
C and D: $x_1 = 2$
D and E: $x_2 = 0$
E and A: $x_1 = 0$

4.1-2.

(a) Optimal solution: $(x_1^*, x_2^*) = (2, 2)$ with $Z^* = 10$



Label the corner points as A, B, C, and D in the clockwise direction starting from $(0, 3)$.

(b)

Corner Point	Corresponding Constraint Boundary Eq.s	
$A(0,3)$	$x_1 = 0$ and $x_1 + 2x_2 = 6$	$0 = 0$ and $0 + 2 \times 3 = 6$
$B(2,2)$	$x_1 + 2x_2 = 6$ and $2x_1 + x_2 = 6$	$2 + 2 \times 2 = 6$ and $2 \times 2 + 2 = 6$
$C(3,0)$	$2x_1 + x_2 = 6$ and $x_2 = 0$	$2 \times 3 + 0 = 6$ and $0 = 0$
$D(0,0)$	$x_1 = 0$ and $x_2 = 0$	$0 = 0$ and $0 = 0$

(c)

Corner Point	Adjacent Corner Points
$A(0,3)$	$D(0,0)$ and $B(2,2)$
$B(2,2)$	$A(0,3)$ and $C(3,0)$
$C(3,0)$	$B(2,2)$ and $D(0,0)$
$D(0,0)$	$C(3,0)$ and $A(0,3)$

(d) Optimal Solution: $(x_1^*, x_2^*) = (2, 2)$ with $Z^* = 10$

Corner Point (x_1, x_2)	Profit $= 3x_1 + 2x_2$
$A(0,3)$	6
$B(2,2)$	10
$C(3,0)$	9
$D(0,0)$	0

(e)

Corner Point	Profit	Next Step
$D(0,0)$	0	Check A and C.
$A(0,3)$	6	Move to C.
$C(3,0)$	9	Check B.
$B(2,2)$	10	Stop, B is optimal.*

* The next corner point is A, which has already been checked.

4.1-3.

(a)

Corner Point (A_1, A_2)	Profit $= 1,000A_1 + 2,000A_2$
$(0, 0)$	0
$(8, 0)$	$8,000$
$(6, 4)$	$14,000$
$(5, 5)$	$15,000$
$(0, 6.667)$	$13,333$

Optimal Solution: $(A_1^*, A_2^*) = (5, 5)$ with $Z^* = \$15,000$

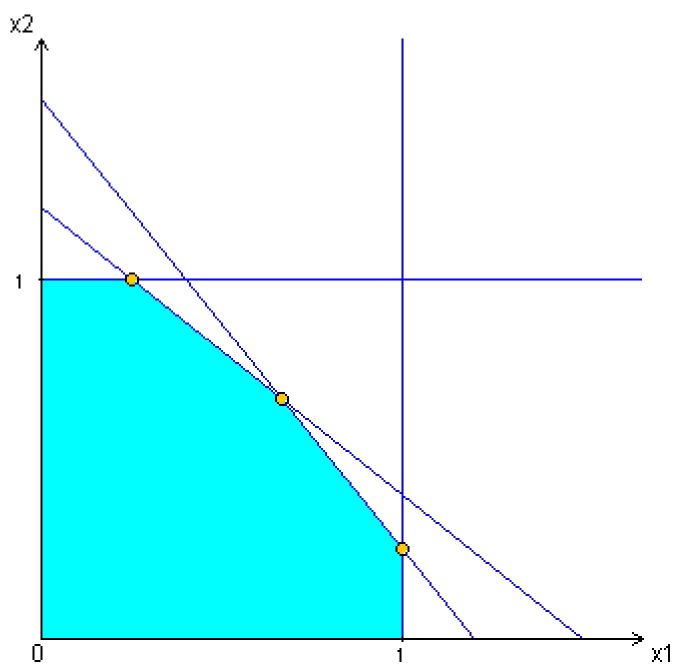
(b) Initiated at the origin, the simplex method can follow one of the two paths:

$(0, 0) \to (8, 0) \to (6, 4) \to (5, 5)$ or $(0, 0) \to (0, 6.7) \to (5, 5)$.

Consider the first path. The origin $(0, 0)$ is not optimal, since $(0, 6.7)$ and $(8, 0)$ are adjacent to $(0, 0)$, both are feasible and they have better objective values. $(8, 0)$ is not optimal because $(6, 4)$, which is adjacent to it, is feasible and better. $(5, 5)$ is optimal since both corner points that are adjacent to it are worse.

4.1-4.

(a)



(b)

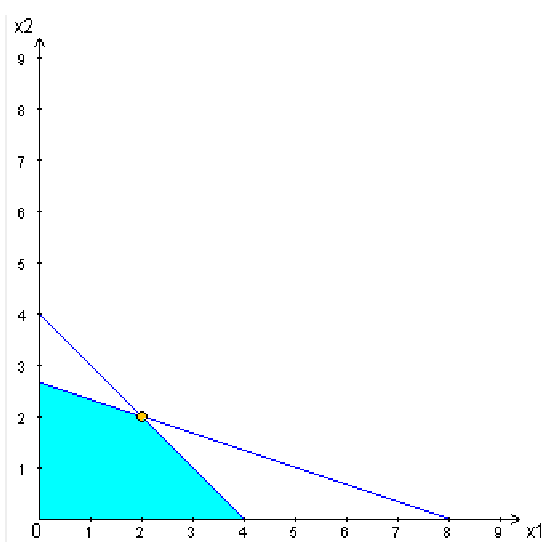
	CP Solution	Feasibility	Objective
A	$\left(0, \frac{3}{2}\right)$	Infeasible	6750
B	$\left(0, \frac{6}{5}\right)$	Infeasible	5400
C	$(0, 1)$	Feasible	4500
D	$\left(\frac{1}{4}, 1\right)$	Feasible	5625
E	$\left(\frac{2}{5}, 1\right)$	Infeasible	6300
F	$(1, 1)$	Infeasible	9000
G	$\left(\frac{2}{3}, \frac{2}{3}\right)$	Feasible	6000 *
H	$\left(1, \frac{2}{5}\right)$	Infeasible	6300
I	$\left(1, \frac{1}{4}\right)$	Feasible	5625
J	$(1, 0)$	Feasible	4500
K	$\left(\frac{6}{5}, 0\right)$	Infeasible	5400
L	$\left(\frac{3}{2}, 0\right)$	Infeasible	6750
M	$(0, 0)$	Feasible	0

The point G is optimal.

(c) Start at the origin M $= (0, 0)$. Both adjacent points C $= (1, 0)$ and J $= (0, 1)$ are feasible and have better objective values,so one can choose to move to either one of them. Suppose we choose C, which is not optimal since its adjacent CPF solution D is better. The other corner point that is adjacent to C is B, but it is infeasible, so move to D. Its adjacent G is feasible and better. The CPF solutions that are adjacent to G, namely D and I both have lower objective values. Hence, G is optimal. If one chooses to proceed to J instead of C after the starting point, then the simplex path follows the points M, J, I, G and using similar arguments, one obtains the optimality of G.

4.1-5.

(a)



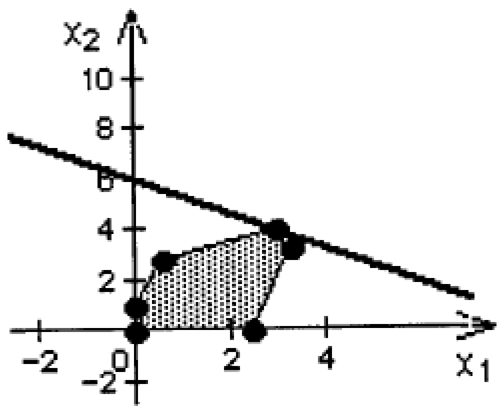
(b)

	CP Solution	Feasibility	Objective
A	$(0,4)$	Infeasible	8
B	$(0,\frac{8}{3})$	Feasible	$5\frac{1}{3}$
C	$(2,2)$	Feasible	6^*
D	$(4,0)$	Feasible	4
E	$(8,0)$	Infeasible	8
F	$(0,0)$	Feasible	0

The point C is optimal.

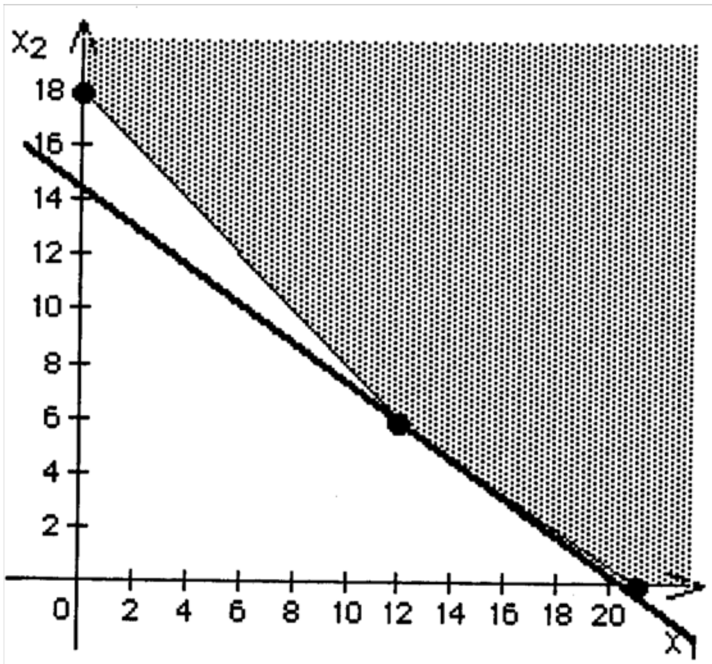
(c) The starting point F is not optimal, since B and D have better objective values. The objective value z increases faster along the edge FB ($5\frac{1}{3}/\frac{8}{3} = 2$) than along the edge FD ($4/4 = 1$), so we choose to move to point B. B is not optimal because the adjacent point C does better. Note that A is adjacent to B as well, but it is infeasible. C is optimal since the two CPF solutions adjacent to C, namely B and D have lower objective values.

4.1-6.



Corner Point	Profit $= 2x_1 + 3x_2$	Next Step
$(0,0)$	0	Check $(2.5,0)$ and $(0,1)$.
$(2.5,0)$	5	Move to $(2.5,0)$.
$(0,1)$	3	Check $(3.3333,3.333)$.
$(3.3333,3.333)$	16.667	Move to $(3,4)$. Check $(3,4)$.
$(3,4)$	18	Move to $(3,4)$. Check $(0.6,2.8)$.
$(0.6,2.8)$	9.6	Stop, $(3,4)$ is optimal.

4.1-7.



Corner Point	Cost $= 5x_1 + 7x_2$	Next Step
$(12, 6)$	102	Check $(21, 0)$ and $(0, 18)$.
$(21, 0)$	105	Stop, $(12, 6)$ is optimal.
$(0, 18)$	126	

4.1-8.

(a) TRUE. Use optimality test. In minimization problems, "better" means smaller. To see this, note that min $Z = -\max(-Z)$.

(b) FALSE. CPF solutions are not the only possible optimal solutions, there can be infinitely many optimal solutions. This is indeed the case when there are more than one optimal solution. For example, consider the problem

$$\begin{array}{lll} \text{maximize} & Z = & x_1 + x_2 \\ \text{subject to} & & x_1 + x_2 \leq 10 \\ & & x_1, x_2 \geq 0 \end{array}$$

where $Z^* = 10$, $x_1^* = k$ and $x_2^* = 10 - k$ with $k \in [0, 10]$ are all optimal solutions.

(c) TRUE. However, this is not always true. It is possible to have an unbounded feasible region where an entire ray with only one CPF solution is optimal.

4.1-9.

(a) The problem may not have an optimal solution.

(b) The optimality test checks whether the current corner point is optimal. The iterative step only moves to a new corner point.

(c) The simplex method can choose the origin as the initial corner point only when it is feasible.

(d) One of the adjacent points is likely to be better, not necessarily optimal.

(e) The simplex method only identifies the rate of improvement, not all the adjacent corner points.

4.2-1.

(a) Augmented form:

$$
\begin{array}{lrcl}
\text{maximize} & 4500x_1 + 4500x_2 & & \\
\text{subject to} & x_1 \qquad\qquad + \; x_3 \qquad\qquad\qquad\qquad & = & 1 \\
& x_2 \qquad\quad + \; x_4 \qquad\qquad\quad & = & 1 \\
& 5000x_1 + 4000x_2 \qquad\qquad\quad + \; x_5 \qquad & = & 6000 \\
& 400x_1 + \;\; 500x_2 \qquad\qquad\qquad\quad + \; x_6 & = & 600 \\
& x_1, x_2, x_3, x_4, x_5, x_6 & \geq & 0
\end{array}
$$

(b)

	CPF Solution	BF Solution	Nonbasic Variables	Basic Variables
A	$(0, 1)$	$(0, 1, 1, 0, 2000, 100)$	x_1, x_4	x_2, x_3, x_5, x_6
B	$(\frac{1}{4}, 1)$	$(\frac{1}{4}, 1, \frac{3}{4}, 0, 750, 0)$	x_4, x_6	x_1, x_2, x_3, x_5
C	$(\frac{2}{3}, \frac{2}{3})$	$(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0)$	x_5, x_6	x_1, x_2, x_3, x_4
D	$(1, \frac{1}{4})$	$(1, \frac{1}{4}, 0, \frac{3}{4}, 0, 75)$	x_3, x_5	x_1, x_2, x_4, x_6
E	$(1, 0)$	$(1, 0, 0, 1, 1000, 200)$	x_2, x_3	x_1, x_4, x_5, x_6
F	$(0, 0)$	$(0, 0, 1, 1, 6000, 600)$	x_1, x_2	x_3, x_4, x_5, x_6

(c) <u>BF Solution A:</u> Set $x_1 = x_4 = 0$ and solve

$$
\begin{aligned}
&x_3 = 1 \\
&x_2 = 1 \\
&4000x_2 + x_5 = 6000 \;\Rightarrow\; x_5 = 2000 \\
&500x_2 + x_6 = 600 \;\Rightarrow\; x_6 = 100
\end{aligned}
$$

<u>BF Solution B:</u> Set $x_4 = x_6 = 0$ and solve

$$
\begin{aligned}
&x_1 + x_3 = 1 \;\Rightarrow\; x_3 = 3/4 \\
&x_2 = 1 \\
&5000x_1 + 4000x_2 + x_5 = 6000 \;\Rightarrow\; x_5 = 750 \\
&400x_1 + 500x_2 = 600 \;\Rightarrow\; x_1 = 1/4
\end{aligned}
$$

<u>BF Solution C:</u> Set $x_5 = x_6 = 0$ and solve

$$
\begin{aligned}
&x_1 + x_3 = 1 \\
&x_2 + x_4 = 1 \\
&5000x_1 + 4000x_2 = 6000 \\
&400x_1 + 500x_2 = 600
\end{aligned}
$$

From the last two equations, $x_1 = x_2 = 2/3$ and from the first two, $x_3 = x_4 = 1/3$.

BF Solution D: Set $x_3 = x_5 = 0$ and solve

$$x_1 = 1$$
$$x_2 + x_4 = 1 \;\Rightarrow\; x_4 = 3/4$$
$$5000x_1 + 4000x_2 = 6000 \;\Rightarrow\; x_2 = 1/4$$
$$400x_1 + 500x_2 + x_6 = 600 \;\Rightarrow\; x_6 = 75$$

BF Solution E: Set $x_2 = x_3 = 0$ and solve

$$x_1 = 1$$
$$x_4 = 1$$
$$5000x_1 + x_5 = 6000 \;\Rightarrow\; x_5 = 1000$$
$$400x_1 + x_6 = 600 \;\Rightarrow\; x_6 = 200$$

BF Solution F: Set $x_1 = x_2 = 0$ and solve

$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = 6000$$
$$x_6 = 600$$

4.2-2.

(a) Augmented form:

$$\begin{array}{ll} \text{maximize} & x_1 + 2x_2 \\ \text{subject to} & \begin{array}{lcl} x_1 + 3x_2 + x_3 & = & 8 \\ x_1 + \;\; x_2 \qquad + x_4 & = & 4 \\ x_1, x_2, x_3, x_4 & \geq & 0 \end{array} \end{array}$$

(b)

	CPF Solution	BF Solution	Nonbasic Variables	Basic Variables
A	$(0,0)$	$(0,0,8,4)$	x_1, x_2	x_3, x_4
B	$(0, \frac{8}{3})$	$(0, \frac{8}{3}, 0, \frac{4}{3})$	x_1, x_3	x_2, x_4
C	$(2,2)$	$(2,2,0,0)$	x_3, x_4	x_1, x_2
D	$(4,0)$	$(4,0,4,0)$	x_2, x_4	x_1, x_3

(c) BF Solution A: Set $x_1 = x_2 = 0$ and solve

$$x_3 = 8$$
$$x_4 = 4$$

BF Solution B: Set $x_1 = x_3 = 0$ and solve

$$3x_2 = 8 \;\Rightarrow\; x_2 = 8/3$$
$$x_2 + x_4 = 4 \;\Rightarrow\; x_4 = 4/3$$

BF Solution C: Set $x_3 = x_4 = 0$ and solve

$$x_1 + 3x_2 = 8$$
$$x_1 + x_2 = 4$$

From these two equations, $x_1 = x_2 = 2$.

BF Solution D: Set $x_2 = x_4 = 0$ and solve

$$x_1 + x_3 = 8 \;\Rightarrow\; x_3 = 4$$
$$x_1 = 4$$

(d)

	CP Infeasible Sol.'n	Basic Infeasible Sol.'n	Nonbasic Var.'s	Basic Var.'s
E	$(0,4)$	$(0,4,-4,0)$	x_1, x_4	x_2, x_3
F	$(8,0)$	$(8,0,0,-4)$	x_2, x_3	x_1, x_4

(e) Basic Infeasible Solution E: Set $x_1 = x_4 = 0$ and solve

$$3x_2 + x_3 = 8 \;\Rightarrow\; x_3 = -4$$
$$x_2 = 4$$

Basic Infeasible Solution F: Set $x_2 = x_3 = 0$ and solve

$$x_1 = 8$$
$$x_1 + x_4 = 4 \;\Rightarrow\; x_4 = -4$$

4.3-1.

After the sudden decline of prices at the end of 1995, Samsung Electronics faced the urgent need to improve its noncompetitive cycle times. The project called SLIM (short cycle time and low inventory in manufacturing) was initiated to address this problem. As part of this project, floor-scheduling problem is formulated as a linear programming model. The goal is to identify the optimal values "for the release of new lots into the fab and for the release of initial WIP from every major manufacturing step in discrete periods, such as work days, out to a horizon defined by the user" [p. 71]. Additional variables are included to determine the route of these through alternative machines. The optimal values "minimize back-orders and finished-goods inventory" [p. 71] and satisfy capacity constraints and material flow equations. CPLEX was used to solved the linear programs.

With the implementation of SLIM, Samsung significantly reduced its cycle times and as a result of this increased its revenue by $1 billion (in five years) despite the decrease in selling prices. The market share increased from 18 to 22 percent. The utilization of machines was improved. The reduction in lead times enabled Samsung to forecast sales more accurately and so to carry less inventory. Shorter lead times also meant happier customers and a more efficient feedback mechanism, which allowed Samsung to respond to customer needs. Hence, SLIM did not only help Samsung to survive a crisis that drove many out of the business, but it did also provide a competitive advantage in the business.

4.3-2.

Optimal Solution: $(x_1^*, x_2^*) = \left(\frac{2}{3}, \frac{2}{3}\right)$, $Z^* = 6000$

$$
\begin{array}{lrcrcl}
\text{Max } Z = & 4500\, X_1 & + & 4500\, X_2 & & \\
\text{subject to} & & & & & \\
1) & 1\, X_1 & + & 0\, X_2 & \le & 1 \\
2) & 0\, X_1 & + & 1\, X_2 & \le & 1 \\
3) & 5000\, X_1 & + & 4000\, X_2 & \le & 6000 \\
4) & 400\, X_1 & + & 500\, X_2 & \le & 600 \\
\text{and} & & & & & \\
 & & & X_1 \ge 0,\ X_2 \ge 0. & &
\end{array}
$$

Solve Interactively by the Simplex Method:

$$
\begin{array}{llrrrrrrl}
0) & Z & -4500\, X_1 & -4500\, X_2 & +0\, X_3 & +0\, X_4 & +0\, X_5 & +0\, X_6 & = 0 \\
1) & & 1\, X_1 & +0\, X_2 & +1\, X_3 & +0\, X_4 & +0\, X_5 & +0\, X_6 & = 1 \\
2) & & 0\, X_1 & +1\, X_2 & +0\, X_3 & +1\, X_4 & +0\, X_5 & +0\, X_6 & = 1 \\
3) & & 5000\, X_1 & +4000\, X_2 & +0\, X_3 & +0\, X_4 & +1\, X_5 & +0\, X_6 & = 6000 \\
4) & & 400\, X_1 & +500\, X_2 & +0\, X_3 & +0\, X_4 & +0\, X_5 & +1\, X_6 & = 600
\end{array}
$$

$X_1 \ge 0,\ X_2 \ge 0,\ X_3 \ge 0,\ X_4 \ge 0,\ X_5 \ge 0,\ X_6 \ge 0.$

$$
\begin{array}{llrrrrrrl}
0) & Z+ & 0\, X_1 & -4500\, X_2 & +4500\, X_3 & +0\, X_4 & +0\, X_5 & +0\, X_6 & = 4500 \\
1) & & 1\, X_1 & +0\, X_2 & +1\, X_3 & +0\, X_4 & +0\, X_5 & +0\, X_6 & = 1 \\
2) & & 0\, X_1 & +1\, X_2 & +0\, X_3 & +1\, X_4 & +0\, X_5 & +0\, X_6 & = 1 \\
3) & & 0\, X_1 & +4000\, X_2 & -5000\, X_3 & +0\, X_4 & +1\, X_5 & +0\, X_6 & = 1000 \\
4) & & 0\, X_1 & +500\, X_2 & -400\, X_3 & +0\, X_4 & +0\, X_5 & +1\, X_6 & = 200
\end{array}
$$

$X_1 \ge 0,\ X_2 \ge 0,\ X_3 \ge 0,\ X_4 \ge 0,\ X_5 \ge 0,\ X_6 \ge 0.$

$$
\begin{array}{llrrrrrrl}
0) & Z+ & 0\, X_1 & +0\, X_2 & -1125\, X_3 & +0\, X_4 & +1.12\, X_5 & +0\, X_6 & = 5625 \\
1) & & 1\, X_1 & +0\, X_2 & +1\, X_3 & +0\, X_4 & +0\, X_5 & +0\, X_6 & = 1 \\
2) & & 0\, X_1 & +0\, X_2 & +1.25\, X_3 & +1\, X_4 & -2e{-4}\, X_5 & +0\, X_6 & = 0.75 \\
3) & & 0\, X_1 & +1\, X_2 & -1.25\, X_3 & +0\, X_4 & +2e{-4}\, X_5 & +0\, X_6 & = 0.25 \\
4) & & 0\, X_1 & +0\, X_2 & +225\, X_3 & +0\, X_4 & -0.12\, X_5 & +1\, X_6 & = 75
\end{array}
$$

$X_1 \ge 0,\ X_2 \ge 0,\ X_3 \ge 0,\ X_4 \ge 0,\ X_5 \ge 0,\ X_6 \ge 0.$

$$
\begin{array}{llrrrrrrl}
0) & Z+ & 0\, X_1 & +0\, X_2 & +0\, X_3 & +0\, X_4 & +0.5\, X_5 & +5\, X_6 & = 6000 \\
1) & & 1\, X_1 & +0\, X_2 & +0\, X_3 & +0\, X_4 & +6e{-4}\, X_5 & -4e{-3}\, X_6 & = 0.66667 \\
2) & & 0\, X_1 & +0\, X_2 & +0\, X_3 & +1\, X_4 & +4e{-4}\, X_5 & -6e{-3}\, X_6 & = 0.33333 \\
3) & & 0\, X_1 & +1\, X_2 & +0\, X_3 & +0\, X_4 & -4e{-4}\, X_5 & +6e{-3}\, X_6 & = 0.66667 \\
4) & & 0\, X_1 & +0\, X_2 & +1\, X_3 & +0\, X_4 & -6e{-4}\, X_5 & +4e{-3}\, X_6 & = 0.33333
\end{array}
$$

$X_1 \ge 0,\ X_2 \ge 0,\ X_3 \ge 0,\ X_4 \ge 0,\ X_5 \ge 0,\ X_6 \ge 0.$

4.3-3.

(a) maximize $Z = x_1 + 2x_2$

subject to

$$\begin{aligned} x_1 + 3x_2 + x_3 \qquad &= 8 \\ x_1 + \ \ x_2 \qquad + x_4 &= 4 \\ x_1, x_2, x_3, x_4 \qquad &\geq 0 \end{aligned}$$

Initialization: $x_1 = x_2 = 0 \ \Rightarrow x_3 = 8,\ x_4 = 4,\ z = x_1 + 2x_2 = 0$, is not optimal since the improvement rates are positive. Since it offers a rate of improvement of 2, choose to increase x_2, which becomes the entering basic variable for Iteration 1. Given $x_1 = 0$, the highest possible increase in x_2 is found by looking at:

$$x_3 = 8 - 3x_2 \geq 0 \ \Rightarrow \ x_2 \leq 8/3$$

$$x_4 = 4 - x_2 \geq 0 \ \Rightarrow \ x_2 \leq 4$$

The minimum of these two bounds is $8/3$, so x_2 can be raised to $8/3$ and $x_3 = 0$ leaves the basis. Using Gaussian elimination, we obtain:

$$\begin{aligned} Z = \ & \tfrac{1}{3}x_1 \qquad - \tfrac{2}{3}x_3 \qquad + \tfrac{16}{3} \\ & \tfrac{1}{3}x_1 + x_2 + \tfrac{1}{3}x_3 \qquad = \tfrac{8}{3} \\ & \tfrac{2}{3}x_1 \qquad - \tfrac{1}{3}x_3 + x_4 = \tfrac{4}{3} \\ & x_1, x_2, x_3, x_4 \qquad \geq 0 \end{aligned}$$

Again $\left(0, \frac{8}{3}, 0, \frac{4}{3}\right)$ is not optimal since the rate of improvement for x_1 is $\frac{1}{3} > 0$ and x_1 can be increased to 2. Consequently, x_4 becomes 0. By Gaussian elimination:

$$\begin{aligned} Z = \ & \qquad - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 + 6 \\ & \quad x_2 + \tfrac{1}{2}x_3 + \tfrac{1}{2}x_4 = 2 \\ & x_1 \qquad - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 = 2 \\ & x_1, x_2, x_3, x_4 \qquad \geq 0 \end{aligned}$$

The current solution is optimal, since increasing x_3 or x_4 would decrease the objective value. Hence $x^* = (2, 2, 0, 0)$, $Z^* = 6$.

(b) Optimal Solution: $\left(x_1^*, x_2^*\right) = \left(2, 2\right)$, $Z^* = 6$

Solve Interactively by the Simplex Method:

```
0) Z-   1 X1-   2 X2+   0 X3+   0 X4 = 0
1)      1 X1+   3 X2+   1 X3+   0 X4 = 8
2)      1 X1+   1 X2+   0 X3+   1 X4 = 4
```

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0.$

$$
\begin{array}{ll}
0) & Z - 0.33\,X_1 + 0\,X_2 + 0.67\,X_3 + 0\,X_4 = 5.33333 \\
1) & 0.333\,X_1 + 1\,X_2 + 0.33\,X_3 + 0\,X_4 = 2.66667 \\
2) & 0.667\,X_1 + 0\,X_2 - 0.33\,X_3 + 1\,X_4 = 1.33333
\end{array}
$$

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0.$

$$
\begin{array}{ll}
0) & Z + 0\,X_1 + 0\,X_2 + 0.5\,X_3 + 0.5\,X_4 = 6 \\
1) & 0\,X_1 + 1\,X_2 + 0.5\,X_3 - 0.5\,X_4 = 2 \\
2) & 1\,X_1 + 0\,X_2 - 0.5\,X_3 + 1.5\,X_4 = 2
\end{array}
$$

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0.$

(c) The solution is the same.

Value of the Objective Function: Z = 6

Variable	Value
X_1	2
X_2	2

Objective Function Coefficient

Current Value	Allowable Range To Stay Optimal: Minimum	Maximum
1	0.66667	2
2	1	3

4.3-4.

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 10, 6\frac{2}{3})$, $Z^* = 70$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	Right Side
Z	0	1	-4	-3	-6	0	0	0
X_4	1	0	3	1	3	1	0	30
X_5	2	0	2	2	3	0	1	40

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	Right Side
Z	0	1	2	-1	0	2	0	60
X_3	1	0	1	0.3333	1	0.3333	0	10
X_5	2	0	-1	1	0	-1	1	10

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	Right Side
Z	0	1	1	0	0	1	1	70
X_3	1	0	1.3333	0	1	0.6667	-0.333	6.66667
X_2	2	0	-1	1	0	-1	1	10

4.3-5.

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 1.58, 1.68)$, $Z^* = 9.89$

0) $Z - 1\ X_1 - 2\ X_2 - 4\ X_3 + 0\ X_4 + 0\ X_5 + 0\ X_6 = 0$
1) $3\ X_1 + 1\ X_2 + 5\ X_3 + 1\ X_4 + 0\ X_5 + 0\ X_6 = 10$
2) $1\ X_1 + 4\ X_2 + 1\ X_3 + 0\ X_4 + 1\ X_5 + 0\ X_6 = 8$
3) $2\ X_1 + 0\ X_2 + 2\ X_3 + 0\ X_4 + 0\ X_5 + 1\ X_6 = 7$

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0,\ X_5 \geq 0,\ X_6 \geq 0.$

0) $Z + 1.4\ X_1 - 1.2\ X_2 + 0\ X_3 + 0.8\ X_4 + 0\ X_5 + 0\ X_6 = 8$
1) $0.6\ X_1 + 0.2\ X_2 + 1\ X_3 + 0.2\ X_4 + 0\ X_5 + 0\ X_6 = 2$
2) $0.4\ X_1 + 3.8\ X_2 + 0\ X_3 - 0.2\ X_4 + 1\ X_5 + 0\ X_6 = 6$
3) $0.8\ X_1 - 0.4\ X_2 + 0\ X_3 - 0.4\ X_4 + 0\ X_5 + 1\ X_6 = 3$

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0,\ X_5 \geq 0,\ X_6 \geq 0.$

0) $Z + 1.53\ X_1 + 0\ X_2 + 0\ X_3 + 0.74\ X_4 + 0.32\ X_5 + 0\ X_6 = 9.89474$
1) $0.579\ X_1 + 0\ X_2 + 1\ X_3 + 0.21\ X_4 - 0.05\ X_5 + 0\ X_6 = 1.68421$
2) $0.105\ X_1 + 1\ X_2 + 0\ X_3 - 0.05\ X_4 + 0.26\ X_5 + 0\ X_6 = 1.57895$
3) $0.842\ X_1 + 0\ X_2 + 0\ X_3 - 0.42\ X_4 + 0.11\ X_5 + 1\ X_6 = 3.63158$

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0,\ X_5 \geq 0,\ X_6 \geq 0.$

4.3-6.

(a) The simplest adaptation of the simplex method is to force x_2 and x_3 into the basis at the earliest opportunity. One can also find the optimal solution directly by using Gaussian elimination.

(b) $Z = 5x_1 + 3x_2 + 4x_3$
$2x_1 + x_2 + x_3 + x_4 = 20$
$3x_1 + x_2 + 2x_3 + x_5 = 30$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

(i) Increase x_2 setting $x_1 = x_3 = 0$.
$x_4 = 20 - x_2 \geq 0 \Rightarrow x_2 \leq 20 \leftarrow$ minimum
$x_5 = 30 - x_2 \geq 0 \Rightarrow x_2 \leq 30$
Let $x_2 = 20$ and $x_4 = 0$.
$Z = -x_1 + x_3 - 3x_4 + 60$
$2x_1 + x_2 + x_3 + x_4 = 20$
$x_1 + x_3 - x_4 + x_5 = 10$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

(ii) Increase x_3 setting $x_1 = x_4 = 0$.
$x_2 = 20 - x_3 \geq 0 \Rightarrow x_3 \leq 20$
$x_5 = 10 - x_3 \geq 0 \Rightarrow x_2 \leq 10 \leftarrow$ minimum
Let $x_3 = 10$ and $x_5 = 0$.
$Z = -2x_1 - 2x_4 - x_5 + 70$
$x_1 + x_2 + 2x_4 - x_5 = 10$
$x_1 + x_3 - x_4 + x_5 = 10$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 10, 10)$ and $Z^* = 70$

4.3-7.

(a) Because $x_2 = 0$ in the optimal solution, the problem can be reduced to:

$$\begin{array}{lrl} \text{maximize} & Z = & 2x_1 + 3x_3 \\ \text{subject to} & & x_1 + 2x_3 \leq 30 \\ & & x_1 + x_3 \leq 24 \\ & & 3x_1 + 3x_3 \leq 60 \\ & & x_1, x_3 \geq 0 \end{array}$$

or equivalently

$$\begin{array}{lrl} \text{maximize} & z = & 2x_1 + 3x_3 \\ \text{subject to} & & x_1 + 2x_3 \leq 30 \\ & & x_1 + x_3 \leq 20 \\ & & x_1, x_3 \geq 0 \end{array}$$

Since $x_1 > 0$ and $x_3 > 0$ in the optimal solution, they should be basic variables in the optimal solution. Choosing these two as the first two entering basic variables will lead to an optimal solution. The leaving basic variables will be determined by the minimum ratio test.

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (10, 0, 10)$ and $Z^* = 50$

Basic			Coefficient of				
Variable	Eq.	Z	X1	X3	X4	X5	RHS
Z	0	1	-2	-3	0	0	0
X4	1	0	1	2	1	0	30
X5	2	0	1	1	0	1	20

Basic			Coefficient of				
Variable	Eq.	Z	X1	X3	X4	X5	RHS
Z	0	1	-0.5	0	1.5	0	45
X3	1	0	0.5	1	0.5	0	15
X5	2	0	0.5	0	-0.5	1	5

Basic			Coefficient of				
Variable	Eq.	Z	X1	X3	X4	X5	RHS
Z	0	1	0	0	1	1	50
X3	1	0	0	1	1	-1	10
X1	2	0	1	0	-1	2	10

4.3-8.

(a) FALSE. The simplex method's rule for choosing the entering basic variable is used because it gives the best rate of improvement for the objective value at the given corner point.

(b) TRUE. The simplex method's rule for choosing the leaving basic variable determines which basic variable drops to zero first as the entering basic variable is increased. Choosing any other one can cause this variable to become negative, so infeasible.

(c) FALSE. When the simplex method solves for the next BF solution, elementary algebraic operations are used to eliminate each basic variable from all but one equation (its equation) and to give it a coefficient of one in that equation.

4.4-1.

Optimal Solution: $(x_1^*, x_2^*) = (2/3, 2/3)$ and $Z^* = 6,000$

Solve Interactively by the Simplex Method:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-4500	-4500	0	0	0	0	0
X_3	1	0	1	0	1	0	0	0	1
X_4	2	0	0	1	0	1	0	0	1
X_5	3	0	5000	4000	0	0	1	0	6000
X_6	4	0	400	500	0	0	0	1	600

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	-4500	4500	0	0	0	4500
X_1	1	0	1	0	1	0	0	0	1
X_4	2	0	0	1	0	1	0	0	1
X_5	3	0	0	4000	-5000	0	1	0	1000
X_6	4	0	0	500	-400	0	0	1	200

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	0	-1125	0	1.125	0	5625
X_1	1	0	1	0	1	0	0	0	1
X_4	2	0	0	0	1.25	1	-2e-4	0	0.75
X_2	3	0	0	1	-1.25	0	0.0002	0	0.25
X_6	4	0	0	0	225	0	-0.125	1	75

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
			Coefficient of						
Z	0	1	0	0	0	0	0.5	5	6000
X_1	1	0	1	0	0	0	0.0006	-0.004	0.66667
X_4	2	0	0	0	0	1	0.0004	-0.006	0.33333
X_2	3	0	0	1	0	0	-4e-4	0.0056	0.66667
X_3	4	0	0	0	1	0	-6e-4	0.0044	0.33333

4.4-2.

Optimal Solution: $(x_1^*, x_2^*) = (2, 2)$ and $Z^* = 6$

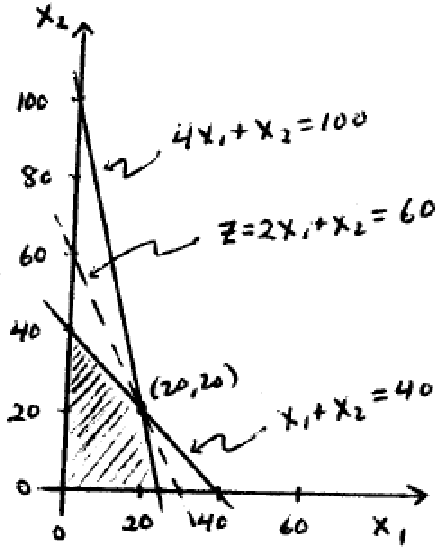
Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
			Coefficient of				
Z	0	1	-1	-2	0	0	0
X_3	1	0	1	3	1	0	8
X_4	2	0	1	1	0	1	4

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
			Coefficient of				
Z	0	1	-0.333	0	0.6667	0	5.33333
X_2	1	0	0.3333	1	0.3333	0	2.66667
X_4	2	0	0.6667	0	-0.333	1	1.33333

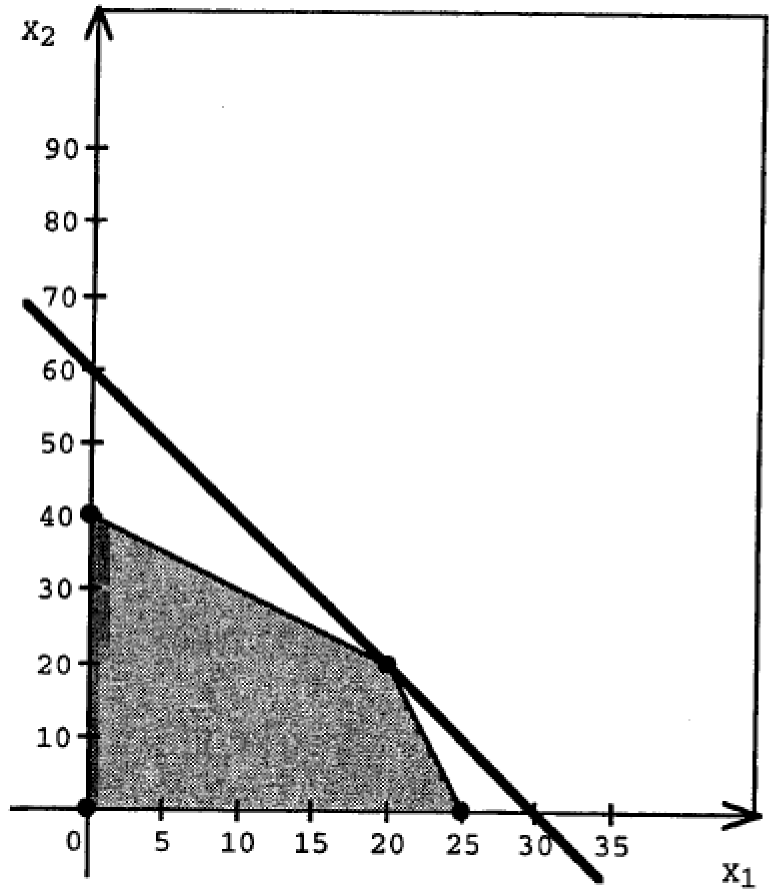
Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
			Coefficient of				
Z	0	1	0	0	0.5	0.5	6
X_2	1	0	0	1	0.5	-0.5	2
X_1	2	0	1	0	-0.5	1.5	2

4.4-3.

(a) Optimal Solution: $(x_1^*, x_2^*) = (20, 20)$ and $Z^* = 60$



(b) Optimal Solution: $(x_1^*, x_2^*) = (20, 20)$ and $Z^* = 60$



Corner Point	Z
$(20, 20)$	60^*
$(\ 0, 40)$	40
$(25,\ \ 0)$	50
$(\ 0,\ \ 0)$	0

(c) Iteration 1: $x_1 = x_2 = 0 \Rightarrow x_3 = 40$ and $x_4 = 100$ (slack variables)

Increase x_1, set $x_2 = 0$.

$x_3 = 40 - x_1 \geq 0 \Rightarrow x_1 \leq 40$

$x_4 = 100 - 4x_1 \geq 0 \Rightarrow x_1 \leq 25 \leftarrow$ minimum

Let $x_1 = 25$ and $x_4 = 0$.

$Z = \frac{1}{2}x_2 - \frac{1}{2}x_4 + 50$

$\frac{3}{4}x_2 + x_3 - \frac{1}{4}x_4 = 15$

$x_1 + \frac{1}{4}x_2 + \frac{1}{4}x_4 = 25$

$x_1, x_2, x_3, x_4 \geq 0$

Iteration 2: $(25, 0, 15, 0)$ is not optimal so increase x_2, set $x_4 = 0$.

$x_3 = 15 - \frac{3}{4}x_2 \geq 0 \Rightarrow x_2 \leq 20 \leftarrow$ minimum

$x_1 = 25 - \frac{1}{4}x_2 \geq 0 \Rightarrow x_2 \leq 100$

Let $x_2 = 20$ and $x_3 = 0$.

$Z = -\frac{2}{3}x_3 - \frac{1}{3}x_4 + 60$

$x_2 + \frac{4}{3}x_3 - \frac{1}{3}x_4 = 20$

$x_1 - \frac{1}{3}x_3 + \frac{1}{3}x_4 = 20$

$x_1, x_2, x_3, x_4 \geq 0$

Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (20, 20, 0, 0)$ and $Z^* = 60$

(d) Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (20, 20, 0, 0)$ and $Z^* = 60$

```
0)  Z-    2 X1-    1 X2+     0 X3+     0 X4  =  0
1)        1 X1+    1 X2+     1 X3+     0 X4  =  40
2)        4 X1+    1 X2+     0 X3+     1 X4  =  100

X1 ≥ 0,  X2 ≥ 0,  X3 ≥ 0,  X4 ≥ 0.

0)  Z+    0 X1- 0.5 X2+      0 X3+ 0.5 X4  =  50
1)        0 X1+0.75 X2+      1 X3-0.25 X4  =  15
2)        1 X1+0.25 X2+      0 X3+0.25 X4  =  25

X1 ≥ 0,  X2 ≥ 0,  X3 ≥ 0,  X4 ≥ 0.

0)  Z+    0 X1+     0 X2+0.67 X3+0.33 X4  =  60
1)        0 X1+     1 X2+1.33 X3-0.33 X4  =  20
2)        1 X1+     0 X2-0.33 X3+0.33 X4  =  20

X1 ≥ 0,  X2 ≥ 0,  X3 ≥ 0,  X4 ≥ 0.
```

(e) - (f)

Bas Var	Eq No	Coefficient of Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	-2	-1	0	0	0
X_3	1	0	1	1	1	0	40
X_4	2	0	4	1	0	1	100

The coefficients for x_1 and x_2 are negative so this solution is not optimal. Let x_1 enter the basis, since it offers largest improvement rate, so the column lying under x_1 will be the pivot column. To find out how much x_1 can be increased, use the ratio test:

x_3: $40/1 = 40$
x_4: $100/4 = 25 \leftarrow$ minimum,

so x_4 leaves the basis and its row is the pivot row.

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	0	-0.5	0	0.5	50
X_3	1	0	0	0.75	1	-0.25	15
X_1	2	0	1	0.25	0	0.25	25

The coefficient of x_2 is still negative, so this solution is not optimal. Let x_2 enter the basis, its column is the pivot column. To find out how much x_2 can be increased, use the ratio test:

x_3: $15/0.75 = 20 \leftarrow$ minimum
x_1: $25/0.25 = 100$,

so x_3 leaves the basis and its row is the pivot row.

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	0	0	0.6667	0.3333	60
X_2	1	0	0	1	1.3333	-0.333	20
X_1	2	0	1	0	-0.333	0.3333	20

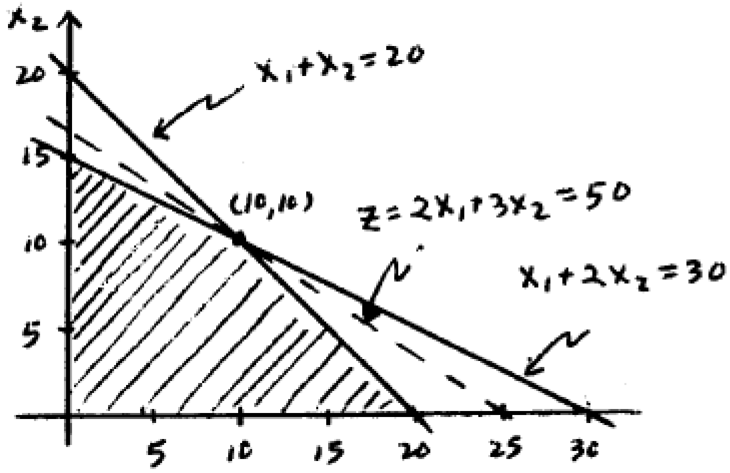
All the coefficients in the objective row are nonnegative, so the solution $(20, 20, 0, 0)$ is optimal with an objective value of 60.

(g)

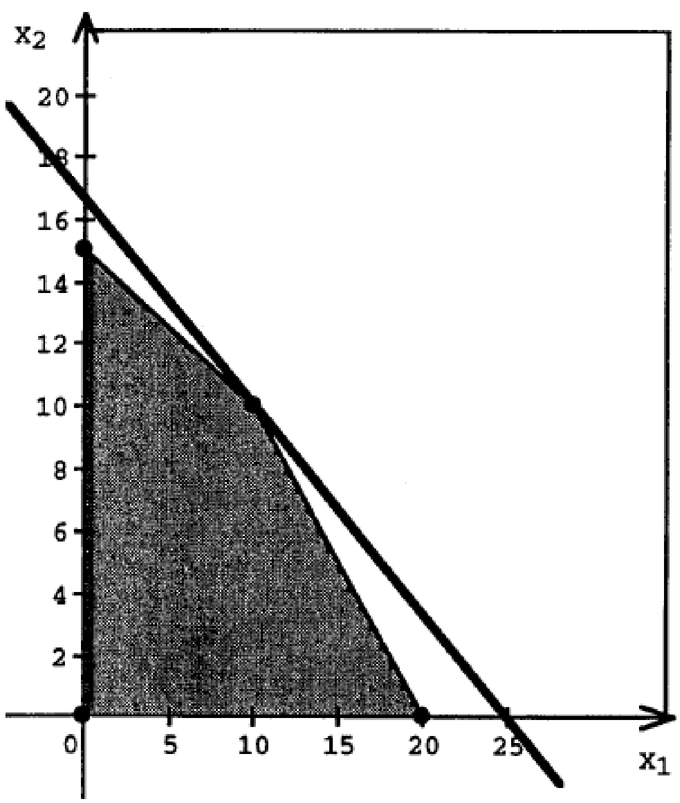
2	1	60		
20	20	← solution		
1	1	40	≤	40
4	1	100	≤	100

4.4-4.

(a) Optimal Solution: $(x_1^*, x_2^*) = (10, 10)$ and $Z^* = 50$



(b) Optimal Solution: $(x_1^*, x_2^*) = (10, 10)$ and $Z^* = 50$



Corner Point	Z
$(10, 10)$	50^*
$(\ 0, 15)$	45
$(20,\ \ 0)$	40
$(\ 0,\ \ 0)$	0

(c) Iteration 1: $x_1 = x_2 = 0 \Rightarrow x_3 = 30$ and $x_4 = 20$ (slack variables)

Increase x_2 and set $x_1 = 0$.

$x_3 = 30 - 2x_2 \geq 0 \Rightarrow x_2 \leq 15 \leftarrow$ minimum

$x_4 = 20 - x_2 \geq 0 \Rightarrow x_1 \leq 20$

Let $x_2 = 15$ and $x_3 = 0$.

$Z = \frac{1}{2}x_1 - \frac{3}{2}x_3 + 45$

$\frac{1}{2}x_1 + x_2 + \frac{1}{2}x_3 = 15$

$\frac{1}{2}x_1 - \frac{1}{2}x_3 + x_4 = 5$

$x_1, x_2, x_3, x_4 \geq 0$

Iteration 2: $(0, 15, 0, 5)$ is not optimal so increase x_1, set $x_3 = 0$.

$x_2 = 15 - \frac{1}{2}x_1 \geq 0 \Rightarrow x_1 \leq 30$

$x_4 = 5 - \frac{1}{2}x_1 \geq 0 \Rightarrow x_1 \leq 10 \leftarrow$ minimum

Let $x_1 = 10$ and $x_3 = 0$.

$Z = -x_3 - x_4 + 50$

$x_2 + x_3 - x_4 = 10$

$x_1 - x_3 + 2x_4 = 10$

$x_1, x_2, x_3, x_4 \geq 0$

Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (10, 10, 0, 0)$ and $Z^* = 50$

(d) Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (10, 10, 0, 0)$ and $Z^* = 50$

```
0) Z-   2 X1-   3 X2+   0 X3+   0 X4 = 0
1)      1 X1+   2 X2+   1 X3+   0 X4 = 30
2)      1 X1+   1 X2+   0 X3+   1 X4 = 20

X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0.

0) Z- 0.5 X1+   0 X2+ 1.5 X3+   0 X4 = 45
1)    0.5 X1+   1 X2+ 0.5 X3+   0 X4 = 15
2)    0.5 X1+   0 X2- 0.5 X3+   1 X4 = 5

X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0.

0) Z+   0 X1+   0 X2+   1 X3+   1 X4 = 50
1)      0 X1+   1 X2+   1 X3-   1 X4 = 10
2)      1 X1+   0 X2-   1 X3+   2 X4 = 10

X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0.
```

(e) - (f)

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	-2	-3	0	0	0
X_3	1	0	1	2	1	0	30
X_4	2	0	1	1	0	1	20

The coefficients for x_1 and x_2 are negative so this solution is not optimal. Let x_2 enter the basis, since it offers largest improvement rate, so the column lying under x_2 will be the pivot column. To find out how much x_1 can be increased, use the ratio test:

x_3: $30/2 = 15 \leftarrow$ minimum
x_4: $20/1 = 20$,

so x_3 leaves the basis and its row is the pivot row.

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	-0.5	0	1.5	0	45
X_2	1	0	0.5	1	0.5	0	15
X_4	2	0	0.5	0	-0.5	1	5

The coefficient of x_1 is still negative, so this solution is not optimal. Let x_1 enter the basis, its column is the pivot column. To find out how much x_1 can be increased, use the ratio test:

x_2: $15/0.5 = 30$
x_4: $5/0.5 = 10 \leftarrow$ minimum,

so x_4 leaves the basis and its row is the pivot row.

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	0	0	1	1	50
X_2	1	0	0	1	1	-1	10
X_1	2	0	1	0	-1	2	10

All the coefficients in the objective row are nonnegative, so the solution $(10, 10, 0, 0)$ is optimal with an objective value of 50.

(g)

	X1	X2			
Maximize	2	3			
			Totals		Limit
Constraint 1	1	2	30	<=	30
Constraint 2	1	1	20	<=	20
					Objective
Solution	10	10			50

4.4-5.

(a) Set $x_1 = x_2 = x_3 = 0$.

(0) $\quad Z - 2x_1 - 4x_2 - 3x_3 = 0$

(1) $\quad x_1 + 3x_2 + 2x_3 + x_4 = 80 \Rightarrow x_4 = 80$

(2) $\quad 3x_1 + 4x_2 + 2x_3 + x_5 = 60 \Rightarrow x_5 = 60$

(3) $\quad 2x_1 + x_2 + 2x_3 + x_6 = 40 \Rightarrow x_6 = 40$

Optimality Test: The coefficients of all nonbasic variables are negative, so the solution $(0, 0, 0, 80, 60, 40)$ is not optimal.

Choose x_2 as the entering basic variable, since it has the largest coefficient.

(1) $\quad x_1 + 3x_2 + 2x_3 + x_4 = 80 \Rightarrow x_4 = 80 - 3x_2 \Rightarrow x_2 \leq 26.67$

(2) $\quad 3x_1 + 4x_2 + 2x_3 + x_5 = 60 \Rightarrow x_5 = 60 - 4x_2 \Rightarrow x_2 \leq 15 \leftarrow$ minimum

(3) $\quad 2x_1 + x_2 + 2x_3 + x_6 = 40 \Rightarrow x_6 = 40 - x_2 \Rightarrow x_2 \leq 40$

We choose x_5 as the leaving basic variable. Set $x_1 = x_5 = x_3 = 0$.

(0) $\quad Z + x_1 - x_3 + x_5 = 60$

(1) $\quad -1.25x_1 + 0.5x_3 + x_4 - 0.75x_5 = 35 \Rightarrow x_4 = 35$

(2) $\quad 0.75x_1 + x_2 + 0.5x_3 - 0.25x_5 = 15 \Rightarrow x_2 = 15$

(3) $\quad 1.25x_1 + 1.5x_3 - 0.25x_5 + x_6 = 25 \Rightarrow x_6 = 25$

Optimality Test: The coefficient of x_3 is negative, so the solution $(0, 15, 0, 35, 0, 25)$ is not optimal.

Let x_3 be the entering basic variable.

(1) $\quad -1.25x_1 + 0.5x_3 + x_4 - 0.75x_5 = 35 \Rightarrow x_4 = 35 - 0.5x_3 \Rightarrow x_3 \leq 70$

(2) $\quad 0.75x_1 + x_2 + 0.5x_3 + 0.25x_5 = 15 \Rightarrow x_2 = 15 - 0.5x_3 \Rightarrow x_3 \leq 30$

(3) $\quad 1.25x_1 + 1.5x_3 - 0.25x_5 + x_6 = 25 \Rightarrow x_6 = 25 - 1.5x_3 \Rightarrow x_3 \leq 16.67 \leftarrow$ min

We choose x_6 as the leaving basic variable. Set $x_1 = x_5 = x_6 = 0$.

(0) $\quad Z + 1.83x_1 + 0.83x_5 + 0.67x_6 = 76.67$

(1) $\quad -1.67x_1 + x_4 - 0.67x_5 - 0.33x_6 = 26.67 \Rightarrow x_4 = 26.67$

(2) $\quad 0.33x_1 + x_2 + 0.33x_5 - 0.33x_6 = 6.67 \Rightarrow x_2 = 6.67$

(3) $\quad 0.83x_1 + x_3 - 0.17x_5 + 0.67x_6 = 16.67 \Rightarrow x_3 = 16.67$

Optimality Test: All of the coefficients are positive, so the solution $(0,\ 6.67,\ 16.67,\ 26.67,\ 0,\ 0)$ is optimal. $Z^* = 76.67$.

(b) Optimal solution: $(x_1^*, x_2^*, x_3^*) = (0, 6.67, 16.67)$ and $Z^* = 76.67$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	-2	-4	-3	0	0	0	0
X4	1	0	1	3	2	1	0	0	80
X5	2	0	3	4*	2	0	1	0	60
X6	3	0	2	1	2	0	0	1	40

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	1	0	-1	0	1	0	60
X4	1	0	-1.25	0	0.5	1	-0.75	0	35
X2	2	0	0.75	1	0.5	0	0.25	0	15
X6	3	0	1.25	0	1.5*	0	-0.25	1	25

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	1.833	0	0	0	0.833	0.667	76.67
X4	1	0	-1.67	0	0	1	-0.67	-0.33	26.67
X2	2	0	0.333	1	0	0	0.333	-0.33	6.667
X3	3	0	0.833	0	1	0	-0.17	0.667	16.67

(c) Excel Solver

	X1	X2	X3			
Maximize	2	4	3			
				Totals		Limit
Constraint 1	1	3	2	53.33	<=	80
Constraint 2	3	4	2	60	<=	60
Constraint 3	2	1	2	40	<=	40
						Objective
Solution	0	6.67	16.67			76.67

4.4-6.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, \frac{4}{3}, \frac{4}{3})$ and $Z^* = 14\frac{2}{3}$

$$\begin{array}{l}
0)\ Z - 3X_1 - 5X_2 - 6X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 = 0 \\
1)\ 2X_1 + 1X_2 + 1X_3 + 1X_4 + 0X_5 + 0X_6 + 0X_7 = 4 \\
2)\ 1X_1 + 2X_2 + 1X_3 + 0X_4 + 1X_5 + 0X_6 + 0X_7 = 4 \\
3)\ 1X_1 + 1X_2 + 2X_3 + 0X_4 + 0X_5 + 1X_6 + 0X_7 = 4 \\
4)\ 1X_1 + 1X_2 + 1X_3 + 0X_4 + 0X_5 + 0X_6 + 1X_7 = 3
\end{array}$$

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0,\ X_5 \geq 0,\ X_6 \geq 0,\ X_7 \geq 0.$

$$\begin{array}{l}
0)\ Z + 0X_1 - 2X_2 + 0X_3 + 0X_4 + 0X_5 + 3X_6 + 0X_7 = 12 \\
1)\ 1.5X_1 + 0.5X_2 + 0X_3 + 1X_4 + 0X_5 - 0.5X_6 + 0X_7 = 2 \\
2)\ 0.5X_1 + 1.5X_2 + 0X_3 + 0X_4 + 1X_5 - 0.5X_6 + 0X_7 = 2 \\
3)\ 0.5X_1 + 0.5X_2 + 1X_3 + 0X_4 + 0X_5 + 0.5X_6 + 0X_7 = 2 \\
4)\ 0.5X_1 + 0.5X_2 + 0X_3 + 0X_4 + 0X_5 - 0.5X_6 + 1X_7 = 1
\end{array}$$

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0,\ X_5 \geq 0,\ X_6 \geq 0,\ X_7 \geq 0.$

$$\begin{array}{l}
0)\ Z + 0.67X_1 + 0X_2 + 0X_3 + 0X_4 + 1.33X_5 + 2.33X_6 + 0X_7 = 14.6667 \\
1)\ 1.333X_1 + 0X_2 + 0X_3 + 1X_4 - 0.33X_5 - 0.33X_6 + 0X_7 = 1.33333 \\
2)\ 0.333X_1 + 1X_2 + 0X_3 + 0X_4 + 0.67X_5 - 0.33X_6 + 0X_7 = 1.33333 \\
3)\ 0.333X_1 + 0X_2 + 1X_3 + 0X_4 - 0.33X_5 + 0.67X_6 + 0X_7 = 1.33333 \\
4)\ 0.333X_1 + 0X_2 + 0X_3 + 0X_4 - 0.33X_5 - 0.33X_6 + 1X_7 = 0.33333
\end{array}$$

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0,\ X_5 \geq 0,\ X_6 \geq 0,\ X_7 \geq 0.$

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, \frac{4}{3}, \frac{4}{3})$ and $Z^* = 14\frac{2}{3}$

Bas Var	Eq No	Coefficient of: Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Right Side
Z	0	1	-3	-5	-6	0	0	0	0	0
X_4	1	0	2	1	1	1	0	0	0	4
X_5	2	0	1	2	1	0	1	0	0	4
X_6	3	0	1	1	2	0	0	1	0	4
X_7	4	0	1	1	1	0	0	0	1	3

Bas Var	Eq No	Coefficient of: Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Right Side
Z	0	1	0	-2	0	0	0	3	0	12
X_4	1	0	1.5	0.5	0	1	0	-0.5	0	2
X_5	2	0	0.5	1.5	0	0	1	-0.5	0	2
X_3	3	0	0.5	0.5	1	0	0	0.5	0	2
X_7	4	0	0.5	0.5	0	0	0	-0.5	1	1

Bas Var	Eq No	Coefficient of: Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Right Side
Z	0	1	0.6667	0	0	0	1.3333	2.3333	0	14.6667
X_4	1	0	1.3333	0	0	1	-0.333	-0.333	0	1.33333
X_2	2	0	0.3333	1	0	0	0.6667	-0.333	0	1.33333
X_3	3	0	0.3333	0	1	0	-0.333	0.6667	0	1.33333
X_7	4	0	0.3333	0	0	0	-0.333	-0.333	1	0.33333

(c)

	X1	X2	X3			
Maximize	3	5	6			
				Totals		Limit
Constraint 1	2	1	1	2.67	<=	4
Constraint 2	1	2	1	4	<=	4
Constraint 3	1	1	2	4	<=	4
Constraint 4	1	1	1	2.67	<=	3
						Objective
Solution	0	1.33	1.33			14.67

4.4-7.

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (1.5, 0.5, 0)$ and $Z^* = 2.5$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-2	1	-1	0	0	0	0
X_4	1	0	3	1	1	1	0	0	6
X_5	2	0	1	-1	2	0	1	0	1
X_6	3	0	1	1	-1	0	0	1	2

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	-1	3	0	2	0	2
X_4	1	0	0	4	-5	1	-3	0	3
X_1	2	0	1	-1	2	0	1	0	1
X_6	3	0	0	2	-3	0	-1	1	1

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	0	1.5	0	1.5	0.5	2.5
X_4	1	0	0	0	1	1	-1	-2	1
X_1	2	0	1	0	0.5	0	0.5	0.5	1.5
X_2	3	0	0	1	-1.5	0	-0.5	0.5	0.5

4.4-8.

Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (6\frac{2}{3}, 0, 36\frac{2}{3})$ and $Z^* = 66\frac{2}{3}$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	1	-1	-2	0	0	0	0
X4	1	0	1	2	-1	1	0	0	20
X5	2	0	-2	4	2	0	1	0	60
X6	3	0	2	3	1	0	0	1	50

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-1	3	0	0	1	0	60
X4	1	0	0	4	0	1	0.5	0	50
X3	2	0	-1	2	1	0	0.5	0	30
X6	3	0	3	1	0	0	-0.5	1	20

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	3.3333	0	0	0.8333	0.3333	66.6667
X4	1	0	0	4	0	1	0.5	0	50
X3	2	0	0	2.3333	1	0	0.3333	0.3333	36.6667
X1	3	0	1	0.3333	0	0	-0.167	0.3333	6.66667

4.5-1.

(a) TRUE. The ratio test tells how far the entering basic variable can be increased before one of the current basic variables drops below zero. If there is a tie for which variable should leave the basis, then both variables drop to zero at the same value of the entering basic variable. Since only one variable can become nonbasic in any iteration, the other will remain in the basis even though it will be zero.

(b) FALSE. If there is no leaving basic variable, then the solution is unbounded and the entering basic variable can be increased indefinitely.

(c) FALSE. All basic variables always have a coefficient of zero in row 0 of the final tableau.

(d) FALSE.

Example 1: maximize $x_1 - x_2$
subject to $x_1 - x_2 \leq 1$
$x_1, x_2 \geq 0$

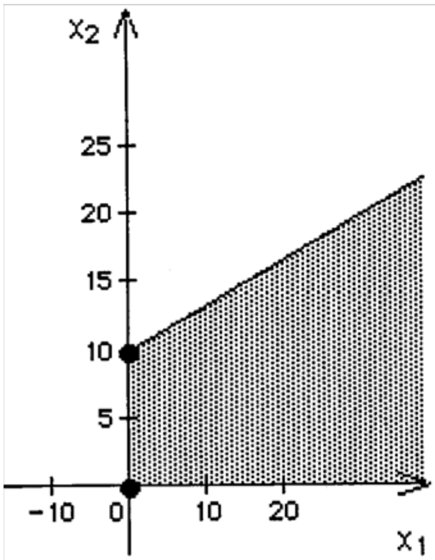
Clearly, any solution $(x_1^*, x_2^*) = (k+1, k)$ for $k \in [0, \infty)$ with $z^* = 1$ is optimal. The problem has infinitely many optimal solutions and the feasible region is not bounded.

Example 2: maximize $-x_1$
subject to $-x_1 - x_2 \leq 1$
$x_1, x_2 \geq 0$

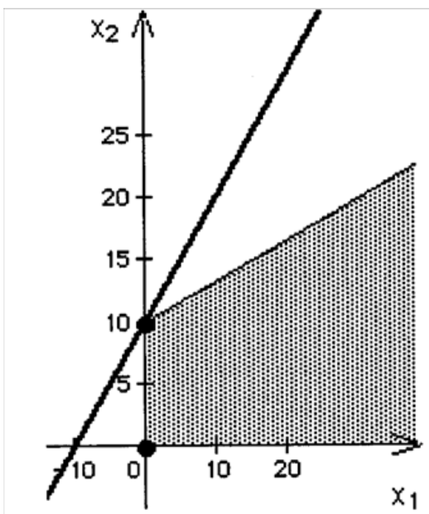
Any solution $(0, x_2^*)$ with $x_2 \geq 0$ is optimal.

4.5-2.

(a)



(b) Yes, the optimal solution is $(x_1^*, x_2^*) = (0, 10)$ with $Z^* = 10$.



(c) No, the objective function value is maximized by sliding the objective function line to the right. This can be done forever, so there is no optimal solution.

(d) No, there exist solutions that make the objective value arbitrarily large. This usually occurs when a constraint is left out of the model.

(e) Let the objective function be $Z = x_1 - x_2$. Then, the initial tableau is:

		Coefficient of					
BV	Eq.	Z	x_1	x_2	x_3	x_4	Right Side
Z	(0)	1	-1	1	0	0	0
x_3	(1)	0	-1	3	1	0	30
x_4	(2)	0	-3	1	0	1	30

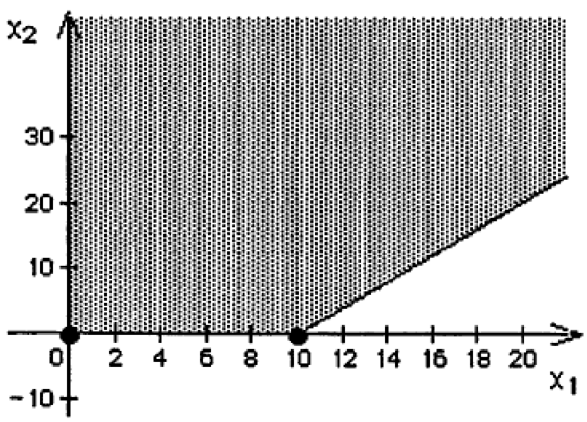
The pivot column, the column of x_1, has all negative elements, so Z is unbounded.

(f) The Solver tells that the Objective Cell values do not converge. There is no optimal solution because a better solution can always be found.

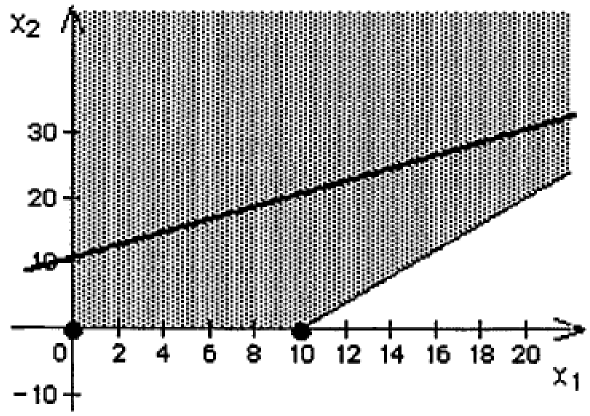
	X1	X2			
Maximize	1	-1			
			Totals		Limit
Constraint 1	-1	3	0	<=	30
Constraint 2	-3	1	0	<=	30
					Objective
Solution	0	0			0

4.5-3.

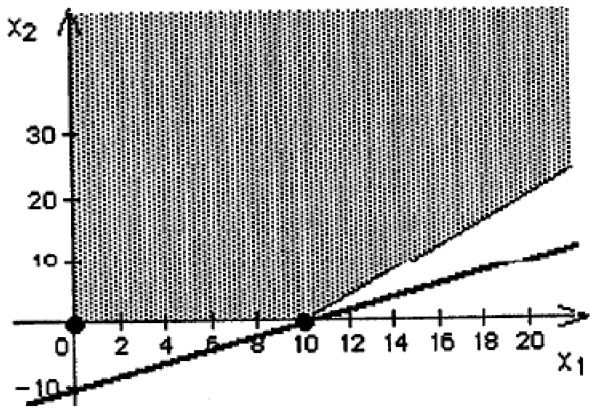
(a)



(b) No. the objective function value is maximized by sliding the objective function line upwards. This can be done forever, so there is no optimal solution.



(c) Yes, the optimal solution is $(x_1^*, x_2^*) = (10, 0)$ with $Z^* = 10$.



(d). No, there exist solutions that make z arbitrarily large. This usually occurs when a constraint is left out of the model.

(e) Let the objective function be $Z = -x_1 + x_2$. Then, the initial tableau is:

		Coefficient of					
BV	Eq.	Z	x_1	x_2	x_3	x_4	Right Side
Z	(0)	1	1	-1	0	0	0
x_3	(1)	0	2	-1	1	0	20
x_4	(2)	0	1	-2	0	1	20

The pivot column, the column of x_2, has all elements negative, so Z is unbounded.

(f) The Solver tells that the Objective Cell values do not converge. There is no optimal solution because a better solution can always be found.

	X1	X2			
Maximize	-1	1			
			Totals		Limit
Constraint 1	2	-1	0	<=	20
Constraint 2	1	-2	0	<=	20
					Objective
Solution	0	0			0

4.5-4.

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Right Side
Z	0	1	-5	-1	-3	-4	0	0	0	0
x_5	1	0	1	-2	4	3	1	0	0	20
x_6	2	0	-4	6	5	-4	0	1	0	40
x_7	3	0	2	-3	3	8	0	0	1	50

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Right Side
Z	0	1	0	-11	17	11	5	0	0	100
x_1	1	0	1	-2	4	3	1	0	0	20
x_6	2	0	0	-2	21	8	4	1	0	120
x_7	3	0	0	1	-5	2	-2	0	1	10

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Right Side
Z	0	1	0	0	-38	33	-17	0	11	210
x_1	1	0	1	0	-6	7	-3	0	2	40
x_6	2	0	0	0	11	12	0	1	2	140
x_2	3	0	0	1	-5	2	-2	0	1	10

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Right Side
Z	0	1	0	0	0	74.455	-17	3.4545	17.909	693.636
x_1	1	0	1	0	0	13.545	-3	0.5455	3.0909	116.364
x_3	2	0	0	0	1	1.0909	0	0.0909	0.1818	12.7273
x_2	3	0	0	1	0	7.4545	-2	0.4545	1.9091	73.6364

We can see from either the second or third iteration that because all of the constraint coefficients of x_5 are nonpositive, it can be increased without forcing any basic variable to zero. From the third iteration, $(116.364 + 3\theta, 73.6364 + 2\theta, 12.7273, 0)$ is feasible for any $\theta \geq 0$ and $Z = 693.636 + 17\theta$ is unbounded.

4.5-5.

(a) The constraints of any LP problem can be expressed in matrix notation as:

$$Ax = b, x \geq 0.$$

If $x^1, x^2, \ldots, x^N$ are feasible solutions and $x = \sum_{k=1}^{N} \alpha_k x^k$ with $\sum_{k=1}^{N} \alpha_k = 1$ and $\alpha_k \geq 0$ for $k = 1, \ldots, N$, then

$$Ax = A\sum_{k=1}^{N} \alpha_k x^k = \sum_{k=1}^{N} \alpha_k A x^k = \sum_{k=1}^{N} \alpha_k b = b,\ x = \sum_{k=1}^{N} \alpha_k x^k \geq 0,$$

so x is also a feasible solution.

(b) This follows immediately from (a), since basic feasible solutions are feasible solutions.

4.5-6.

(a) Suppose Z^* is the value of the objective function for an optimal solution and $x^1, x^2, \ldots, x^N$ are optimal BF solutions. From Problem 4.5-5, $x = \sum_{k=1}^{N} \alpha_k x^k$ is feasible for any choice of $\alpha_k \geq 0$ $(k = 1, \ldots, N)$ satisfying $\sum_{k=1}^{N} \alpha_k = 1$. The objective function value at x is:

$$c^T x = c^T \sum_{k=1}^{N} \alpha_k x^k = \sum_{k=1}^{N} \alpha_k c^T x^k = \sum_{k=1}^{N} \alpha_k Z^* = Z^*,$$

so x is also an optimal solution.

(b) Consider any feasible solution x that is not a weighted average of the optimal BF solutions. Since x is feasible, it must be a weighted average of the basic feasible solutions, which are not all optimal by assumption. Let $\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^L$ are the basic feasible solutions that are not optimal. Then,

$$x = \sum_{k=1}^{N} \alpha_k x^k + \sum_{i=1}^{L} \beta_i \overline{x}^i$$

where $\sum_{k=1}^{N} \alpha_k + \sum_{i=1}^{L} \beta_i = 1$, $\alpha_k \geq 0$ $(k = 1, \ldots, N)$, $\beta_i \geq 0$ $(i = 1, \ldots, L)$ and $\beta_i \neq 0$ for some i. The objective function value at x is:

$$c^T x = c^T \sum_{k=1}^{N} \alpha_k x^k + c^T \sum_{i=1}^{L} \beta_i \overline{x}^i = \sum_{k=1}^{N} \alpha_k c^T x^k + \sum_{i=1}^{L} \beta_i c^T \overline{x}^i.$$

Since $\overline{x}^i$ is not optimal, $c^T \overline{x}^i < Z^*$ for every i. Because there is at least one positive β_i and $c^T x^k = Z^*$,

$$c^T x < \left(\sum_{k=1}^{N} \alpha_k + \sum_{i=1}^{L} \beta_i \right) Z^* = Z^*.$$

Hence, x cannot be optimal.

4.5-7.

(a) $x_1 \leq 6$
$x_2 \leq 3$
$-x_1 + 3x_2 \leq 6$

(b)

Unit Profit (Prod.1)	Unit Profit (Prod.2)	Objective	Multiple Opt. Solutions
-1	3	$-x_1 + 3x_2$	line segment between $(0,2)$ & $(3,3)$
0	1	x_2	line segment between $(3,3)$ & $(6,3)$
1	0	x_1	line segment between $(6,3)$ & $(6,0)$
0	-1	$-x_2$	line segment between $(0,0)$ & $(6,0)$
-1	0	$-x_1$	line segment between $(0,0)$ & $(0,2)$

(c)

Corner Point (x_1, x_2)	Profit $= -x_1 + 2x_2$
$(0,0)$	0
$(0,2)$	4
$(3,3)$	3
$(6,3)$	0
$(6,0)$	-6

Optimal Solution: $(x_1^*, x_2^*) = (0,2)$ with $Z^* = 4$

(d)

Z	x_1	x_2	x_3	x_4	x_5	RS
1	1	-2	0	0	0	0
0	1	0	1	0	0	6
0	0	1	0	1	0	3
0	-1	[3]	0	0	1	6

$\rightarrow$

Z	x_1	x_2	x_3	x_4	x_5	RS
1	$1/3$	0	0	0	$2/3$	4
0	-1	0	1	0	0	6
0	$1/3$	0	0	1	$-1/3$	1
0	$-1/3$	1	0	0	$1/3$	2

So the unique optimal solution is $(x_1^*, x_2^*) = (0,2)$ with V$^* = 4$.

4.5-8.

Bas Var	Eq No	Coefficient of							Rignt Side
		Z	X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	-1	-1	-1	-1	0	0	0
X_5	1	0	1	1	0	0	1	0	3
X_6	2	0	0	0	1	1	0	1	2

Bas Var	Eq No	Coefficient of							Right Side
		Z	X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	0	0	-1	-1	1	0	3
X_1	1	0	1	1	0	0	1	0	3
X_6	2	0	0	0	1	1	0	1	2

Bas Var	Eq No	Coefficient of							Right Side
		Z	X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	0	0	0	0	1	1	5
X_1	1	0	1	1	0	0	1	0	3
X_3	2	0	0	0	1	1	0	1	2

Since the objective coefficients (row Z) for x_2 and x_4 are zero, we can pivot to get other optimal BF solutions.

Bas Var	Eq No	Coefficient of							Right Side
		Z	X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	0	0	0	0	1	1	5
X_2	1	0	1	1	0	0	1	0	3
X_3	2	0	0	0	1	1	0	1	2

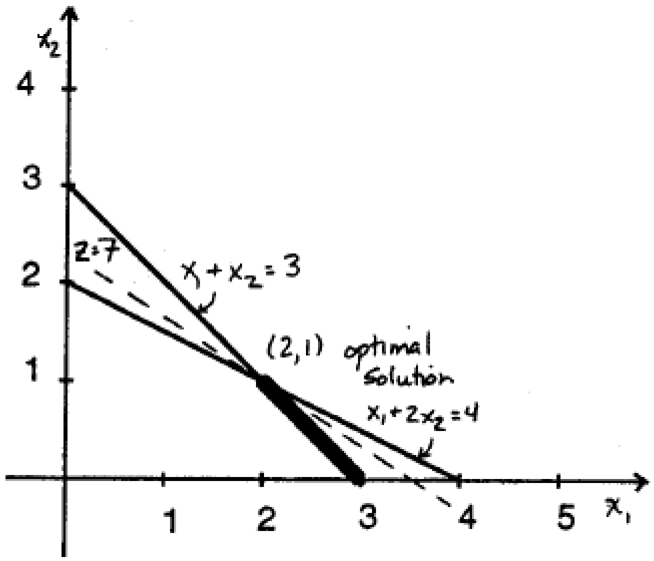
Bas Var	Eq No	Coefficient of							Right Side
		Z	X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	0	0	0	0	1	1	5
X_2	1	0	1	1	0	0	1	0	3
X_4	2	0	0	0	1	1	0	1	2

Bas Var	Eq No	Coefficient of							Right Side
		Z	X_1	X_2	X_3	X_4	X_5	X_6	
Z	0	1	0	0	0	0	1	1	5
X_1	1	0	1	1	0	0	1	0	3
X_4	2	0	0	0	1	1	0	1	2

Hence, the optimal BF solutions are $(3, 0, 2, 0)$, $(0, 3, 2, 0)$, $(0, 3, 0, 2)$, and $(3, 0, 0, 2)$, all with objective function value 5.

4.6-1.

(a) Optimal Solution: $(x_1^*, x_2^*) = (2, 1)$ and $Z^* = 7$



(b) Initial artificial BF solution: $(0, 0, 4, 3)$

Bas Var	Eq No	Z	x_1	x_2	x_3	$\bar{x}_4$	Right Side
Z	0	1	-1M -2	-1M -3	0	0	-3M
x_3	1	0	1	2	1	0	4
$\bar{x}_4$	2	0	1	1	0	1	3

(c) Optimal Solution: $(x_1^*, x_2^*) = (2, 1)$ and $Z^* = 7$

Bas Var	Eq No	Z	x_1	x_2	x_3	$\bar{x}_4$	Right Side
Z	0	1	-0.5M -0.5	0	0.5M +1.5	0	-1M +6
x_2	1	0	0.5	1	0.5	0	2
$\bar{x}_4$	2	0	0.5	0	-0.5	1	1

Bas Var	Eq No	Z	x_1	x_2	x_3	$\bar{x}_4$	Right Side
Z	0	1	0	0	1	1M +1	7
x_2	1	0	0	1	1	-1	1
x_1	2	0	1	0	-1	2	2

4.6-2.

(a) - (b) Initial artificial BF solution: $(0, 0, 0, 0, 300, 300)$

Bas Var	Eq No	Coefficient of							Right Side
		Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	
			-10M	-4M	-5M	-7M			
Z	0	1	-4	-2	-3	-5	0	0	-600M
$\bar{x}_5$	1	0	2	3	4	2	1	0	300
$\bar{x}_6$	2	0	8	1	1	5	0	1	300

Bas Var	Eq No	Coefficient of							Right Side
		Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	
				-2.75M	-3.75M	-0.75M		1.25M	-225M
Z	0	1	0	-1.5	-2.5	-2.5	0	+0.5	+150
$\bar{x}_5$	1	0	0	2.75	3.75	0.75	1	-0.25	225
x_1	2	0	1	0.125	0.125	0.625	0	0.125	37.5

Bas Var	Eq No	Coefficient of							Right Side
		Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	
							1M	1M	
Z	0	1	0	0.3333	0	-2	+0.667	+0.333	300
x_3	1	0	0	0.7333	1	0.2	0.2667	-0.067	60
x_1	2	0	1	0.0333	0	0.6	-0.033	0.1333	30

Bas Var	Eq No	Coefficient of							Right Side
		Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	
							1M	1M	
Z	0	1	3.3333	0.4444	0	0	+0.556	+0.778	400
x_3	1	0	-0.333	0.7222	1	0	0.2778	-0.111	50
x_4	2	0	1.6667	0.0556	0	1	-0.056	0.2222	50

Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 0, 50, 50)$ and $Z^* = 400$

(c) - (d) - (e) - (f) Initial artificial BF solution: $(0, 0, 0, 0, 300, 300)$

Phase 1:

Bas Var	Eq No	Coefficient of							Right Side
		Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	
Z	0	1	-10	-4	-5	-7	0	0	-600
$\bar{x}_5$	1	0	2	3	4	2	1	0	300
$\bar{x}_6$	2	0	8	1	1	5	0	1	300

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	-2.75	-3.75	-0.75	0	1.25	-225
X_5	1	0	0	2.75	3.75	0.75	1	-0.25	225
X_1	2	0	1	0.125	0.125	0.625	0	0.125	37.5

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	0	0	0	1	1	0
X_3	1	0	0	0.7333	1	0.2	0.2667	-0.067	60
X_1	2	0	1	0.0333	0	0.6	-0.033	0.1333	30

Phase 2:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	0	0.3333	0	-2	300
X_3	1	0	0	0.7333	1	0.2	60
X_1	2	0	1	0.0333	0	0.6	30

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	3.3333	0.4444	0	0	400
X_3	1	0	-0.333	0.7222	1	0	50
X_4	2	0	1.6667	0.0556	0	1	50

Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 0, 50, 50)$ and $Z^* = 400$

(g) The basic solutions of the two methods coincide. They are artificial BF solutions for the revised problem until both artificial variables x_5 and x_6 are driven out of the basis, which in the two-phase method is the end of Phase 1.

(h)

	X1	X2	X3	X4			
Maximize	4	2	3	5			
					Totals		Limit
Constraint 1	2	3	4	2	300	<=	300
Constraint 2	8	1	1	5	300	<=	300
							Objective
Solution	0	0	50	50			400

4.6-3.

(a) maximize $\quad -Z = -2x_1 - 3x_2 - \ x_3$

subject to

$$\begin{aligned} -\ x_1 - 4x_2 - 2x_3 &\leq -8 \\ -3x_1 - 2x_2 \qquad\quad &\leq -6 \\ x_1, x_2, x_3 \qquad &\geq \ \ 0 \end{aligned}$$

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0.8, 1.8, 0)$ and $Z^* = 7$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	$\bar{X}_6$	$\bar{X}_7$	Right Side
Z	0	1	-4M +2	-6M +3	-2M +1	1M	1M	0	0	-14M
$\bar{X}_6$	1	0	1	4	2	-1	0	1	0	8
$\bar{X}_7$	2	0	3	2	0	0	-1	0	1	6

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	$\bar{X}_6$	$\bar{X}_7$	Right Side
Z	0	1	-2.5M +1.25	0	1M -0.5	-0.5M +0.75	1M	1.5M -0.75	0	-2M -6
X_2	1	0	0.25	1	0.5	-0.25	0	0.25	0	2
$\bar{X}_7$	2	0	2.5	0	-1	0.5	-1	-0.5	1	2

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	$\bar{X}_6$	$\bar{X}_7$	Right Side
Z	0	1	0	0	0	0.5	0.5	1M -0.5	1M -0.5	-7
X_2	1	0	0	1	0.6	-0.3	0.1	0.3	-0.1	1.8
X_1	2	0	1	0	-0.4	0.2	-0.4	-0.2	0.4	0.8

Pivoting x_3 for x_2 gives an alternate optimal BF solution, $(2, 0, 3)$.

(c) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0.8, 1.8, 0)$ and $Z^* = 7$

Phase 1:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	$\bar{X}_6$	$\bar{X}_7$	Right Side
Z	0	1	-4	-6	-2	1	1	0	0	-14
$\bar{X}_6$	1	0	1	4	2	-1	0	1	0	8
$\bar{X}_7$	2	0	3	2	0	0	-1	0	1	6

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	$\bar{X}_6$	$\bar{X}_7$	Right Side
Z	0	1	-2.5	0	1	-0.5	1	1.5	0	-2
X_2	1	0	0.25	1	0.5	-0.25	0	0.25	0	2
$\bar{X}_7$	2	0	2.5	0	-1	0.5	-1	-0.5	1	2

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	$\bar{X}_6$	$\bar{X}_7$	Right Side
Z	0	1	0	0	0	0	0	1	1	0
X_2	1	0	0	1	0.6	-0.3	0.1	0.3	-0.1	1.8
X_1	2	0	1	0	-0.4	0.2	-0.4	-0.2	0.4	0.8

Phase 2:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	Right Side
Z	0	1	0	0	-5e-20 ≈0	0.5	0.5	-7
X_2	1	0	0	1	0.6	-0.3	0.1	1.8
X_1	2	0	1	0	-0.4	0.2	-0.4	0.8

Pivoting x_3 for x_2 gives an alternate optimal BF solution, $(2, 0, 3)$.

(d) The basic solutions of the two methods coincide. They are artificial BF solutions for the revised problem until both artificial variables x_6 and x_7 are driven out of the basis, which in the two-phase method is the end of Phase 1.

(e)

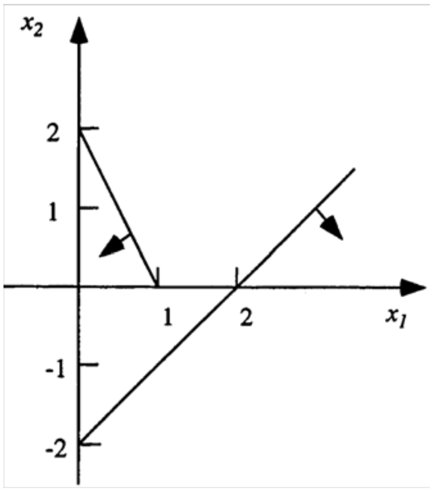
	X1	X2	X3			
Minimize	2	3	1			
				Totals		Limit
Constraint 1	1	4	2	8	>=	8
Constraint 2	3	2	0	6	>=	6
						Objective
Solution	0.8	1.8	0			7

4.6-4.

Once all artificial variables are driven out of the basis in a maximization (minimization) problem. Choosing an artificial variable to reenter the basis can only lower (raise) the objective function value by an arbitrarily large amount depending on M.

4.6-5.

(a)



(b) The Solver could not find a feasible solution.

	X1	X2			
Maximize	90	70			
			Totals		Limit
Constraint 1	2	1	2	<=	2
Constraint 2	1	-1	1	>=	2
					Objective
Solution	1	0			90

(c)

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	−90	−70	1.0e6	0	1.0e6	0
X1	1	0	2*	1	1	0	0	2
X1	2	0	1	−1	0	−1	1	2

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	−25	1.0e6	0	1.0e6	90
X1	1	0	1	0.5	0.5	0	0	1
X1	2	0	0	−1.5	−0.5	−1	1*	1

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	1.5e6	1.5e6	1.0e6	0	−1e6
X1	1	0	1	0.5	0.5	0	0	1
X5	2	0	0	−1.5	−0.5	−1	1	1

In the optimal solution, the artificial variable X_5 is basic and takes a positive value, so the problem has no feasible solutions.

(d)

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	1	0	1	0
X1	1	0	2*	1	1	0	0	2
X1	2	0	1	−1	0	−1	1	2

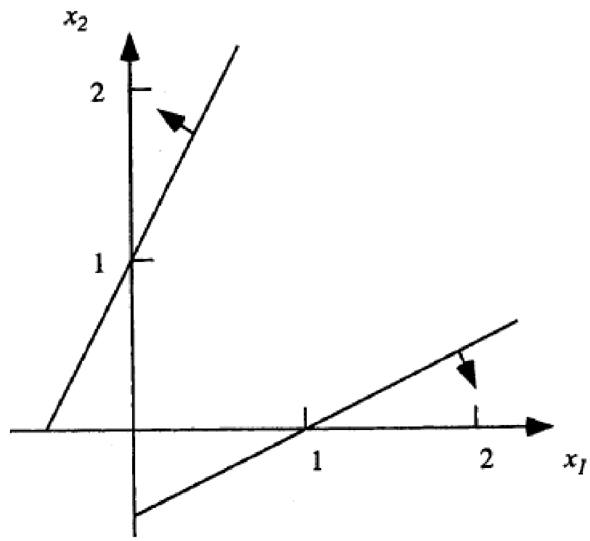
Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	1	0	1	0
X1	1	0	1	0.5	0.5	0	0	1
X1	2	0	0	−1.5	−0.5	−1	1*	1

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	1.5	1.5	1	0	−1
X1	1	0	1	0.5	0.5	0	0	1
X5	2	0	0	−1.5	−0.5	−1	1	1

Since the artificial variable X_5 is not zero in the optimal solution of Phase I Problem, the original model must have no feasible solutions.

4.6-6.

(a)



(b) The Solver could not find a feasible solution.

	X1	X2			
Unit Cost	5000	7000			
					Minimum
			Totals		Level
Benefit 1	-2	1	0	>=	1
Benefit 2	1	-2	0	>=	1
					Objective
Solution	0	0			0

(c)

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	5000	7000	0	0	1.0e6	1.0e6	0
X1	1	0	−2	1	−1	0	1	0	1
X1	2	0	1*	−2	0	−1	0	1	1

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	17000	0	5000	1.0e6	1.0e6	−5000
X1	1	0	0	−3	−1	−2	1	2	3
X1	2	0	1	−2	0	−1	0	1*	1

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	−1e6	2.0e6	0	1.0e6	1.0e6	0	−1e6
X1	1	0	−2	1	−1	0	1*	0	1
X6	2	0	1	−2	0	−1	0	1	1

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	1.0e6	1.0e6	1.0e6	1.0e6	0	0	−2e6
X5	1	0	−2	1	−1	0	1	0	1
X6	2	0	1	−2	0	−1	0	1	1

(d)

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	0	0	1	1	0
X1	1	0	−2	1	−1	0	1	0	1
X1	2	0	1*	−2	0	−1	0	1	1

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	0	0	1	1	0
X1	1	0	0	−3	−1	−2	1*	2	3
X1	2	0	1	−2	0	−1	0	1	1

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	3	1	2	0	−1	−3
X5	1	0	0	−3	−1	−2	1	2	3
X1	2	0	1	−2	0	−1	0	1*	1

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	1	1	1	1	0	0	−2
X5	1	0	−2	1	−1	0	1	0	1
X6	2	0	1	−2	0	−1	0	1	1

4.6-7.

(a) Initial artificial BF solution: $(0, 0, 0, 0, 20, 50)$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
			-3M	-2M	-2M				
Z	0	1	-2	-5	-3	1M	0	0	-70M
$\bar{X}_5$	1	0	1	-2	1	-1	1	0	20
$\bar{X}_6$	2	0	2	4	1	0	0	1	50

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 0, 50)$ and $Z^* = 150$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
				-8M	1M	-2M	3M		-10M
Z	0	1	0	-9	-1	-2	+2	0	+40
X_1	1	0	1	-2	1	-1	1	0	20
$\bar{X}_6$	2	0	0	8	-1	2	-2	1	10

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
							1M	1M	
Z	0	1	0	0	-2.125	0.25	-0.25	+1.125	51.25
X_1	1	0	1	0	0.75	-0.5	0.5	0.25	22.5
X_2	2	0	0	1	-0.125	0.25	-0.25	0.125	1.25

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
							1M	1M	
Z	0	1	2.8333	0	0	-1.167	+1.167	+1.833	115
X_3	1	0	1.3333	0	1	-0.667	0.6667	0.3333	30
X_2	2	0	0.1667	1	0	0.1667	-0.167	0.1667	5

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
								1M	
Z	0	1	4	7	0	0	1M	+3	150
X_3	1	0	2	4	1	0	0	1	50
X_4	2	0	1	6	0	1	-1	1	30

(c) Initial artificial BF solution: $(0, 0, 0, 0, 20, 50)$

Phase 1:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-3	-2	-2	1	0	0	-70
$\bar{X}_5$	1	0	1	-2	1	-1	1	0	20
$\bar{X}_6$	2	0	2	4	1	0	0	1	50

(d)

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	-8	1	-2	3	0	-10
X_1	1	0	1	-2	1	-1	1	0	20
$\bar{X}_6$	2	0	0	8	-1	2	-2	1	10

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	0	0	0	1	1	0
X_1	1	0	1	0	0.75	-0.5	0.5	0.25	22.5
X_2	2	0	0	1	-0.125	0.25	-0.25	0.125	1.25

(e) - (f) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 0, 50)$ and $Z^* = 150$

Phase 2:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	0	0	-2.125	0.25	51.25
X_1	1	0	1	0	0.75	-0.5	22.5
X_2	2	0	0	1	-0.125	0.25	1.25

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	2.8333	0	0	-1.167	115
X_3	1	0	1.3333	0	1	-0.667	30
X_2	2	0	0.1667	1	0	0.1667	5

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
Z	0	1	4	7	0	0	150
X_3	1	0	2	4	1	0	50
X_4	2	0	1	6	0	1	30

(g) The basic solutions of the two methods coincide. They are artificial basic feasible solutions for the revised problem until both artificial variables x_5 and x_6 are driven out of the basis, which in the two-phase method is the end of Phase 1.

(h)

	X1	X2	X3			
Maximize	2	5	3			
						Right-Hand
				Totals		Side
Constraint 1	1	-2	1	50	>=	20
Constraint 2	2	4	1	50	=	50
						Objective
Solution	0	0	50			150

4.6-8.

(a)

Phase 1:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-8	-4	-12	1	0	0	-700
$\bar{X}_5$	1	0	5	2	7	0	1	0	420
$\bar{X}_6$	2	0	3	2	5	-1	0	1	280

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-0.8	0.8	0	-1.4	0	2.4	-28
$\bar{X}_5$	1	0	0.8	-0.8	0	1.4	1	-1.4	28
X_3	2	0	0.6	0.4	1	-0.2	0	0.2	56

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	5e-20	1e-19	0	0	1	1	2e-18
X_4	1	0	0.5714	-0.571	0	1	0.7143	-1	20
X_3	2	0	0.7143	0.2857	1	0	0.1429	0	60

(b)

	0	0	0	0	1	1		0	Minimum Value
Variables	0	0	60	20	0	0			
								RHS	
Constraints	5	2	7	0	1	0	420 "="	420	
	3	2	5	-1	0	1	280 "="	280	

(c) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (35, 0, 35)$ and $Z^* = 175$

Phase 2:

Bas Var	Eq No	Z	X1	X2	X3	X4	Right Side
Z	0	1	-0.143	0.1429	0	0	-180
X4	1	0	0.5714	-0.571	0	1	20
X3	2	0	0.7143	0.2857	1	0	60

Bas Var	Eq No	Z	X1	X2	X3	X4	Right Side
Z	0	1	-1e-20 ≈0	8e-20 ≈0	0	0.25	-175
X1	1	0	1	-1	0	1.75	35
x3	2	0	0	1	1	-1.25	35

Pivoting x_2 into the basis for x_3 provides the alternative optimal BF solution $(70, 35, 0)$.

(d)

	X1	X2	X3			
Minimize	2	1	3			
						Right-Hand
				Totals		Side
Constraint 1	5	2	7	420	=	420
Constraint 2	3	2	5	280	>=	280
						Objective
Solution	70	35	0			175

4.6-9.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 15, 15)$ and $Z^* = 90$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-5M +3	-4M +2	-8M +4	1M	0	0	-180M
X5	1	0	2	1	3	0	1	0	60
X6	2	0	3	3	5	-1	0	1	120

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0.333M +0.333	-1.33M +0.667	0	1M	2.667M -1.333	0	-20M -80
X3	1	0	0.6667	0.3333	1	0	0.3333	0	20
X6	2	0	-0.333	1.3333	0	-1	-1.667	1	20

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
							1M	1M	
Z	0	1	0.5	0	0	0.5	-0.5	-0.5	-90
x_3	1	0	0.75	0	1	0.25	0.75	-0.25	15
x_2	2	0	-0.25	1	0	-0.75	-1.25	0.75	15

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 15, 15)$ and $Z^* = 90$

Phase 1:

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
$\underline{Z}$	0	1	-5	-4	-8	1	0	0	-180
$\bar{x}_5$	1	0	2	1	3	0	1	0	60
$\bar{x}_6$	2	0	3	3	5	-1	0	1	120

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
Z	0	1	0.3333	-1.333	0	1	2.6667	0	-20
x_3	1	0	0.6667	0.3333	1	0	0.3333	0	20
$\bar{x}_6$	2	0	-0.333	1.3333	0	-1	-1.667	1	20

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	Right Side
Z	0	1	-3e-20	0	0	0	1	1	0
x_3	1	0	0.75	0	1	0.25	0.75	-0.25	15
x_2	2	0	-0.25	1	0	-0.75	-1.25	0.75	15

Phase 2:

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	Right Side
Z	0	1	0.5	0	0	0.5	-90
x_3	1	0	0.75	0	1	0.25	15
x_2	2	0	-0.25	1	0	-0.75	15

(c) In both the Big-M method and the two-phase method, only the final tableau represents a feasible solution for the original problem.

(d)

	X1	X2	X3			
Minimize	3	2	4			
						Right-Hand
				Totals		Side
Constraint 1	2	1	3	60	=	60
Constraint 2	3	3	5	120	>=	120
						Objective
Solution	0	15	15			90

4.6-10.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (20, 30, 0)$ and $Z^* = 120$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-1M +3	2	-1M +7	1M	0	0	-20M
$\bar{X}_5$	1	0	-1	1	0	0	1	0	10
$\bar{X}_6$	2	0	2	-1	1	-1	0	1	10

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	-0.5M +3.5	-0.5M +5.5	0.5M +1.5	0	0.5M -1.5	-15M -15
$\bar{X}_5$	1	0	0	0.5	0.5	-0.5	1	0.5	15
X_1	2	0	1	-0.5	0.5	-0.5	0	0.5	5

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	0	2	5	1M -7	1M -5	-120
X_2	1	0	0	1	1	-1	2	1	30
X_1	2	0	1	0	1	-1	1	1	20

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (20, 30, 0)$ and $Z^* = 120$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	-1	0	-1	1	0	0	-20
$\bar{X}_5$	1	0	-1	1	0	0	1	0	10
$\bar{X}_6$	2	0	2	-1	1	-1	0	1	10

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	$\bar{X}_5$	$\bar{X}_6$	Right Side
Z	0	1	0	-0.5	-0.5	0.5	0	0.5	-15
$\bar{X}_5$	1	0	0	0.5	0.5	-0.5	1	0.5	15
X_1	2	0	1	-0.5	0.5	-0.5	0	0.5	5

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
			Coefficient of						
Z	0	1	0	0	0	0	1	1	0
X_2	1	0	0	1	1	-1	2	1	30
X_1	2	0	1	0	1	-1	1	1	20

Phase 2:

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	Right Side
			Coefficient of				
Z	0	1	0	0	2	5	-120
X_2	1	0	0	1	1	-1	30
X_1	2	0	1	0	1	-1	20

(c) Only the final tableau for the Big-M method and the two-phase method represent feasible solutions to the original problem.

(d)

	X1	X2	X3			
Minimize	3	2	7			
						Right-Hand
				Totals		Side
Constraint 1	-1	1	0	10	=	10
Constraint 2	2	-1	1	10	>=	10
						Objective
Solution	20	30	0			120

4.6-11.

(a) FALSE. The initial basic solution for the artificial model is not feasible for the original model.

(b) FALSE. If at least one of the artificial variables is not zero, then the real problem is infeasible.

(c) FALSE. The two methods are basically equivalent, so they should take the same number of iterations.

4.6-12.

(a) Substitute $x_1 = x_1^+ - x_1^-$, where both x_1^+ and x_1^- are nonnegative.

$$Maximize\ Z = x_1^+ - x_1^- + 4x_2 + 2x_3$$

$$\begin{array}{llr} \text{subject to} & 4x_1^+ - 4x_1^- + \ x_2 + x_3 & \leq 5 \\ & -x_1^+ + x_1^- + \ x_2 + 2x_3 & \leq 10 \\ & x_1^+, x_1^-, x_2, x_3 & \geq 0 \end{array}$$

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (-0.6, 10.8, 0)$ and $Z^* = 73.8$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-1	1	-4	-2	0	0	0
X5	1	0	4	-4	1	1	1	0	5
X6	2	0	-1	1	1	2	0	1	10

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	15	-15	0	2	4	0	20
X3	1	0	4	-4	1	1	1	0	5
X6	2	0	-5	5	0	1	-1	1	5

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	0	0	0	5	1	3	35
X3	1	0	0	0	1	1.8	0.2	0.8	9
X2	2	0	-1	1	0	0.2	-0.2	0.2	1

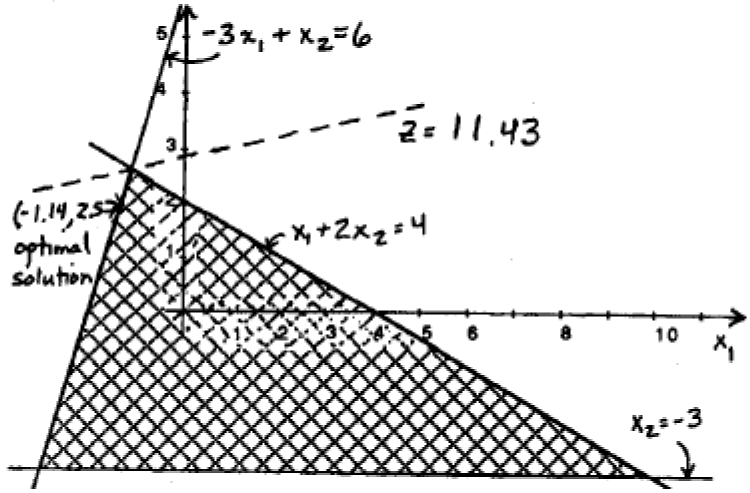
Note that x_1^+, x_1^-, x_2, and x_3 are renamed as X_1, X_2, X_3 and X_4 respectively.

(c)

	X1	X2	X3			
Maximize	1	4	2			
						Right-Hand
				Totals		Side
Constraint 1	4	1	1	5	<=	5
Constraint 2	-1	1	2	10	<=	10
						Objective
Solution	-1	9	0			35
		>=	>=			
		0	0			

4.6-13.

(a) Optimal Solution: $(x_1^*, x_2^*) = (-1.14, 2.57)$ and $Z^* = 11.43$



(b) Let $x_{1,\text{OLD}} = x_1 - x_2$ and $x_{2,\text{OLD}} + 3 = x_3$.

$$\begin{array}{lrl} \text{maximize } Z = & -x_1 + x_2 + 4x_3 - 12 & \\ \text{subject to} & -3x_1 + 3x_2 + x_3 & \leq 9 \\ & x_1 - x_2 + 2x_3 & \leq 10 \\ & x_1, x_2, x_3 & \geq 0 \end{array}$$

(c) Optimal Solution: $(x_1^*, x_2^*) = (-1.14, 2.57)$ and $Z^* = 11.43$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right Side
Z	0	1	1	-1	-4	0	0	0
X4	1	0	-3	3	1	1	0	9
X5	2	0	1	-1	2	0	1	10

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right Side
Z	0	1	3	-3	0	0	2	20
X4	1	0	-3.5	3.5	0	1	-0.5	4
X3	2	0	0.5	-0.5	1	0	0.5	5

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right Side
Z	0	1	0	0	0	0.8571	1.5714	23.4286
X2	1	0	-1	1	0	0.2857	-0.143	1.14286
X3	2	0	0	0	1	0.1429	0.4286	5.57143

Optimal solution for the revised problem: $(0, 1.14, 5.57)$ with $Z^* = 23.43$

4.6-14.

(a) Let $x_{1,\text{OLD}} = x_1 - x_2$, $x_{2,\text{OLD}} = x_3 - x_4$, and $x_{3,\text{OLD}} = x_5 - x_6$.

$$
\begin{array}{llr}
\text{maximize } Z = -x_1 + x_2 + 2x_3 - 2x_4 + x_5 - x_6 & & \\
\text{subject to} \qquad\qquad 3x_3 - 3x_4 + x_5 - x_6 & \leq & 120 \\
x_1 - x_2 - x_3 + x_4 - 4x_5 + 4x_6 & \leq & 80 \\
-3x_1 + 3x_2 + x_3 - x_4 + 2x_5 - 2x_6 & \leq & 100 \\
x_1, x_2, x_3, x_4, x_5, x_6 & \geq & 0
\end{array}
$$

(b)

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	Right Side
Z	0	1	1	-1	-2	2	-1	1	0	0	0	0
X7	1	0	0	0	3	-3	1	-1	1	0	0	120
X8	2	0	1	-1	-1	1	-4	4	0	1	0	80
X9	3	0	-3	3	1	-1	2	-2	0	0	1	100

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	Right Side
Z	0	1	1	-1	0	0	-0.33	0.333	0.667	0	0	80
X3	1	0	0	0	1	-1	0.333	-0.33	0.333	0	0	40
X8	2	0	1	-1	0	0	-3.67	3.667	0.333	1	0	120
X9	3	0	-3	3	0	0	1.667	-1.67	-0.33	0	1	60

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	Right Side
Z	0	1	0	0	0	0	0.222	-0.22	0.556	0	0.333	100
X3	1	0	0	0	1	-1	0.333	-0.33	0.333	0	0	40
X8	2	0	0	0	0	0	-3.11	3.111	0.222	1	0.333	140
X2	3	0	-1	1	0	0	0.556	-0.56	-0.11	0	0.333	20

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	Right Side
Z	0	1	0	0	0	0	0	0	0.571	0.071	0.357	110
X3	1	0	0	0	1	-1	3e-20	-3e-2	0.357	0.107	0.036	55
X6	2	0	0	0	0	0	-1	1	0.071	0.321	0.107	45
X2	3	0	-1	1	0	0	0	0	-0.07	0.179	0.393	45

Optimal solution for the revised problem: $(0, 45, 55, 0, 0, 45)$

Optimal solution for the original problem: $(x_1^*, x_2^*, x_3^*) = (-45, 55, -45)$ and $Z^* = 110$

(c)

	X1	X2	X3			
Maximize	-1	2	1			
						Right-Hand
				Totals		Side
Constraint 1	0	3	1	120	<=	120
Constraint 2	1	-1	-4	80	<=	80
Constraint 3	-3	1	2	100	<=	100
						Objective
Solution	-45	55	-45			110

4.6-15.

(a) In order to decrease the objective function value in the simplex method, choose the nonbasic variable that has the (largest) positive coefficient in the objective row, as the entering basic variable. The ratio test is conducted the same way as in the maximization problem to determine the leaving basic variable.

(b) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (11.67, 0, 17.5)$ and $Z^* = 122$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
			3M	8M	6M	-1M	-1M			-140
Z	0	1	-3	-8	-5	0	0	0	0	0
X6	1	0	0	3	4	-1	0	1	0	70
X7	2	0	3	5*	2	0	-1	0	1	70

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
			-1.8M		2.8M	-1M	0.6M		-1.6M	28 M
Z	0	1	1.8	0	-1.8	0	-1.6	0	1.6	112
X6	1	0	-1.8	0	2.8*	-1	0.6	1	-0.6	28
X2	2	0	0.6	1	0.4	0	-0.2	0	0.2	14

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
								-1M	-1M	
Z	0	1	0.64	0	0	-0.643	-1.214	0.64	1.21	130
X3	1	0	-0.64	0	1	-0.36	0.214	0.357	-0.21	10
X2	2	0	0.857*	1	0	0.143	-0.29	-0.14	0.286	10

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
								-1M	-1M	
Z	0	1	0	-0.75	0	-0.75	1	0.75	1	122
X3	1	0	0	0.75	1	-0.25	0	0.25	0	17.5
X1	2	0	1	1.167	0	0.167	-0.33	-0.17	0.333	11.67

4.6-16.

(a) maximize $Z = -2x_1 + 2x_2 + x_3 - 4x_4 + 3x_5$

subject to

$$
\begin{aligned}
x_1 - x_2 + x_3 + 3x_4 - x_5 &\leq 4 \\
-x_1 + x_2 + x_4 - x_5 &\leq 1 \\
2x_1 - 2x_2 + x_3 &\leq 2 \\
x_1 - x_2 + 2x_3 + x_4 + 2x_5 &= 2 \\
x_1, x_2, x_3, x_4, x_5 &\geq 0
\end{aligned}
$$

(b)

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	Right side
			-1M	1M	-2M	-1M	-2M					-2 M
Z	0	1	2	-2	-1	4	-3	0	0	0	0	0
X6	1	0	1	-1	1	3	2	1	0	0	0	4
X7	2	0	-1	1	0	1	-1	0	1	0	0	1
X8	3	0	2	-2	1	0	0	0	0	1	0	2
X9	4	0	1	-1	2	1	2*	0	0	0	1	2

(c)

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	Right side
Z	0	1	-1	1	-2	-1	-2	0	0	0	0	-2

(d)

	X1	X2	X3	X4			
Maximize	-2	1	-4	3			
							Right-Hand
					Totals		Side
Constraint 1	1	1	3	2	2	<=	4
Constraint 2	1	0	-1	1	-1	>=	-1
Constraint 3	2	1	0	0	-8	<=	2
Constraint 4	1	2	1	2	2	=	2
							Objective
Solution	-4	0	0	3			17
		>=	>=	>=			
		0	0	0			

4.6-17.

Reformulation:

$$\begin{aligned} \text{maximize} \quad & Z = 4x_1 + 5x_2 + 3x_3 \\ \text{subject to} \quad & x_1 + x_2 + 2x_3 - x_4 + \overline{x}_7 = 20 \\ & 15x_1 + 6x_2 - 5x_3 + x_5 = 50 \\ & x_1 + 3x_2 + 5x_3 + x_6 = 30 \\ & x_1, x_2, x_3, x_4, x_5, x_6, \overline{x}_7 \geq 0 \end{aligned}$$

Phase 1:

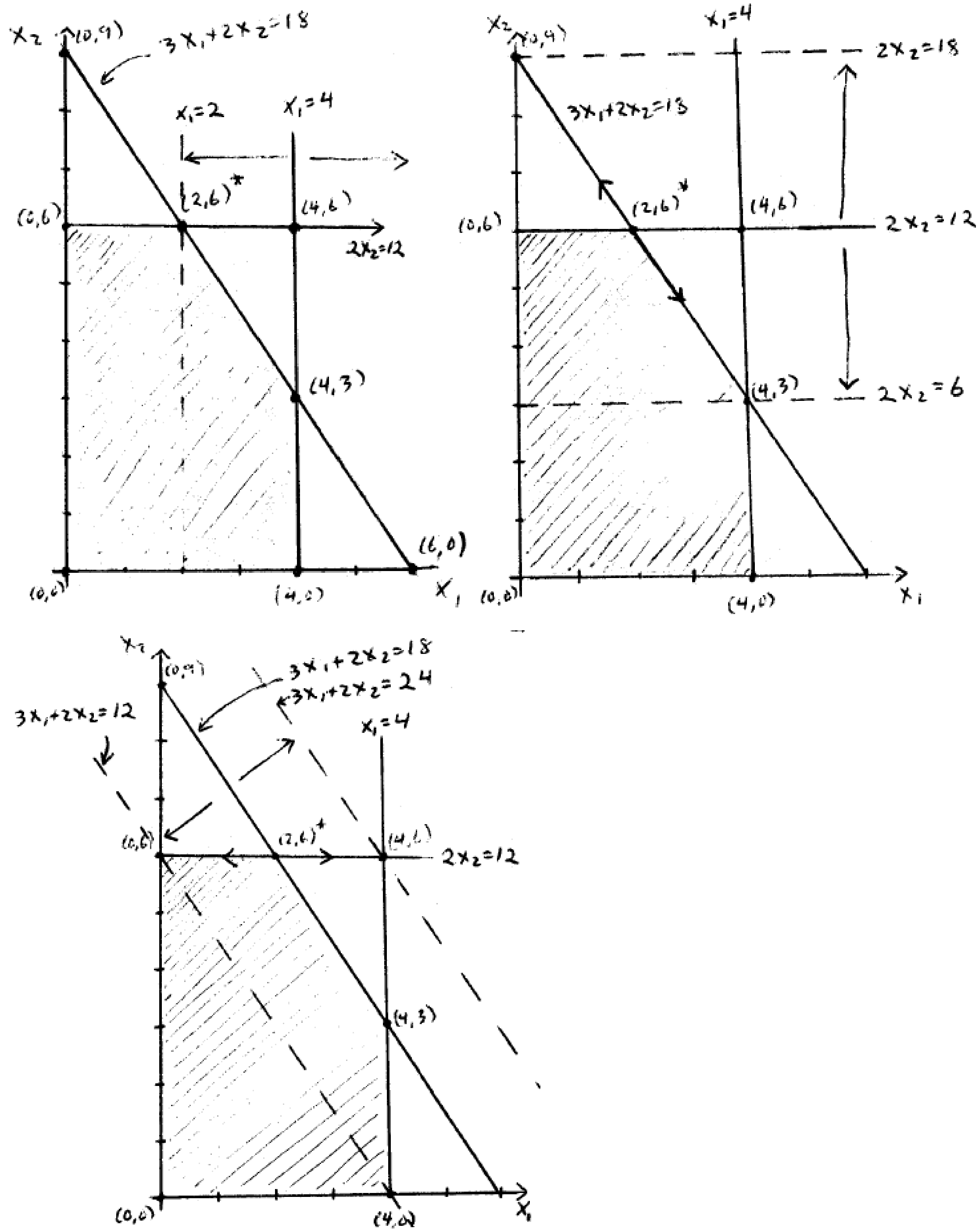
Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	$\overline{x}_7$	Right Side
Z	0	1	-1	-1	-2	1	0	0	0	-20
$\overline{x}_7$	1	0	1	1	2	-1	0	0	1	20
x_5	2	0	15	6	-5	0	1	0	0	50
x_6	3	0	1	3	5	0	0	1	0	30

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	$\overline{x}_7$	Right Side
Z	0	1	-0.6	0.2	0	1	0	0.4	0	-8
$\overline{x}_7$	1	0	0.6	-0.2	0	-1	0	-0.4	1	8
x_5	2	0	16	9	0	0	1	1	0	80
x_3	3	0	0.2	0.6	1	0	0	0.2	0	6

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	$\overline{x}_7$	Right Side
Z	0	1	0	0.5375	0	1	0.0375	0.4375	0	-5
$\overline{x}_7$	1	0	0	-0.538	0	-1	-0.038	-0.438	1	5
x_1	2	0	1	0.5625	0	0	0.0625	0.0625	0	5
x_3	3	0	0	0.4875	1	0	-0.013	0.1875	0	5

Since this is the optimal tableau for Phase 1 and the artificial variable $\overline{x}_7 = 5 > 0$, the problem is infeasible.

4.7-1.



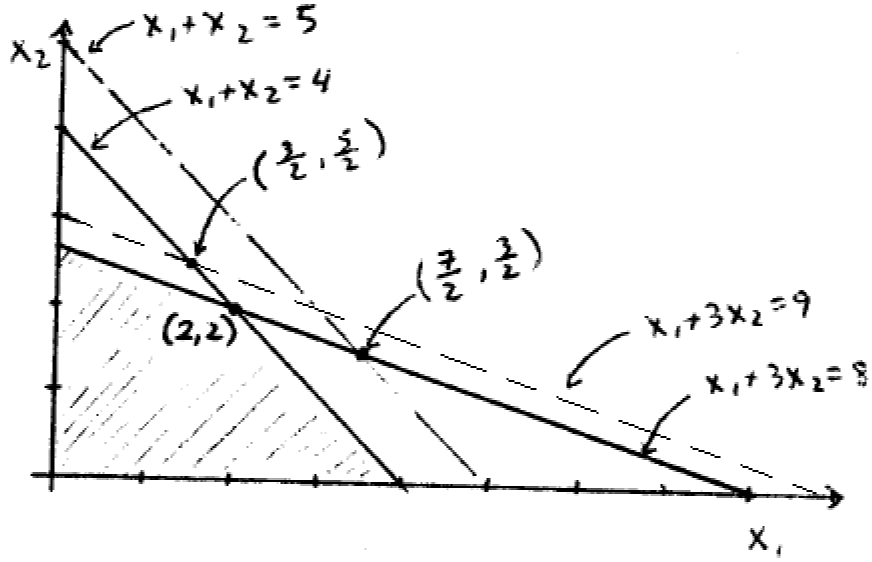
The CP solution $(2, 6)$ remains feasible and optimal if the constraint $x_1 \leq 4$ is changed to $x_1 \leq k$ with $2 \leq k < \infty$. However, if $k < 2$, then this solution ceases to be feasible and the optimal solution becomes $(k, 6)$. This agrees with the allowable range (allowable increase: 1E+30, allowable decrease: 4) for this constraint given in Figure 4.10.

Now, suppose instead that the constraint $2x_2 \leq 12$ is replaced by $2x_2 \leq k$. Then, the intersection of the lines $2x_2 = k$ and $3x_1 + 2x_2 = 18$ can be expressed as $((18-k)/3, k/2)$. This CP solution is feasible as long as $0 \leq x_1 \leq 4$ or equivalently $6 \leq k \leq 18$. In that case, provided that the objective function is the same, this solution is optimal. Hence, the right-hand side of this constraint can be increased or decreased by 6.

If the third constraint is $3x_1 + 2x_2 \leq k$, then the CP solution determined by this and $2x_2 \leq 12$ becomes $((k-12)/3, 6)$. This point is feasible and optimal as long as $0 \leq x_1 \leq 4$ or equivalently $12 \leq k \leq 24$, so the allowable change for this constraint is also ± 6, as given in Figure 4.10.

4.7-2.

(a)

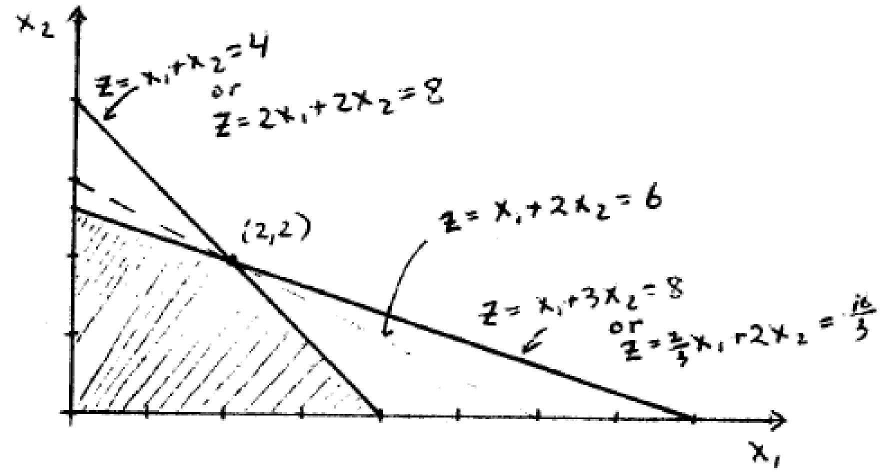


Constraint (1): $x_1 + 3x_2 \leq 8$: $x_1 + 3x_2 = 8 \Rightarrow x_1 = x_2 = 2$ and $Z = 6$
$x_1 + 3x_2 = 9 \Rightarrow x_1 = 3/2$, $x_2 = 5/2$ and $Z = 13/2$
$\Delta Z = 13/2 - 6 = 1/2 = y_1^*$

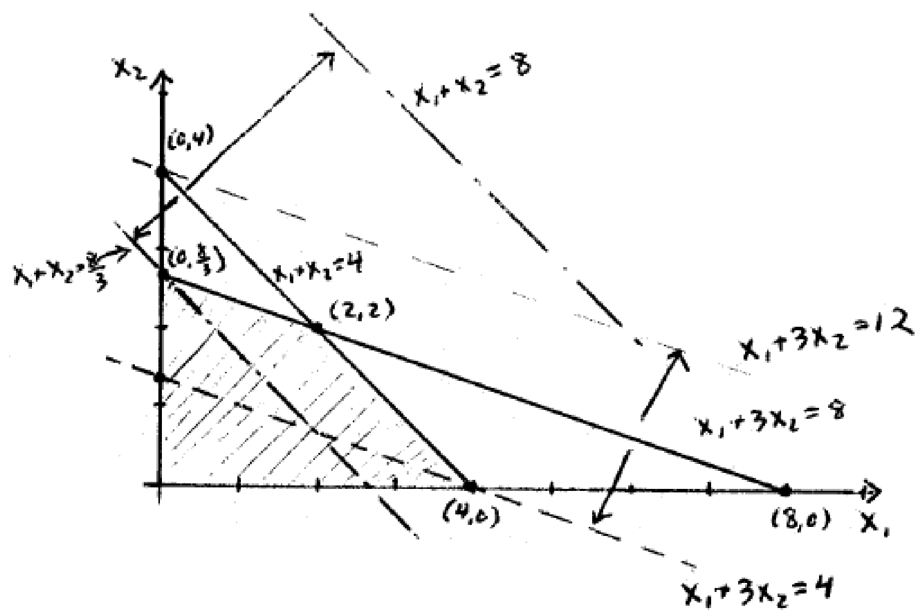
Constraint (2): $x_1 + x_2 \leq 4$: $x_1 + x_2 = 4 \Rightarrow x_1 = x_2 = 2$ and $Z = 6$
$x_1 + x_2 = 5 \Rightarrow x_1 = 7/2$, $x_2 = 3/2$ and $Z = 13/2$
$\Delta Z = 13/2 - 6 = 1/2 = y_2^*$

(b) From (a), we see that the right-hand sides $b_1 = 8$ and $b_2 = 4$ are sensitive parameters. The graph in part (a) shows that both constraints are active (binding) at the optimal solution, so all the coefficients $a_{11} = 1$, $a_{12} = 3$, $a_{21} = 1$, and $a_{22} = 1$ are sensitive parameters, too. As will be seen in (c), the objective coefficients $c_1 = 1$ and $c_2 = 2$ are not sensitive parameters.

(c) Observe that the optimal solution remains the same for $2/3 \leq c_1 \leq 2$ (with $c_2 = 2$ fixed) and $1 \leq c_2 \leq 3$ (with $c_1 = 1$ fixed)



(d) The dashed lines "- - -" in the graph below suggest that the CP solution ranges from $(4,0)$ to $(0,4)$ when $4 \leq b_1 \leq 12$. Outside this range, the CP solution becomes infeasible. The dashed lines "- · -" represent the second constraint for different right-hand side values. They suggest that the CP solution ranges from $(0,8/3)$ to $(0,8)$ when $8/3 \leq b_2 \leq 8$. Hence, the allowable ranges are $4 \leq b_1 \leq 12$ and $8/3 \leq b_2 \leq 8$.



(e)

	1	2		6	Optimal Value
Variables	2	2			
				RHS	
Constraints	1	3	8 <=	8	
	1	1	4 <=	4	

Adjustable Cells

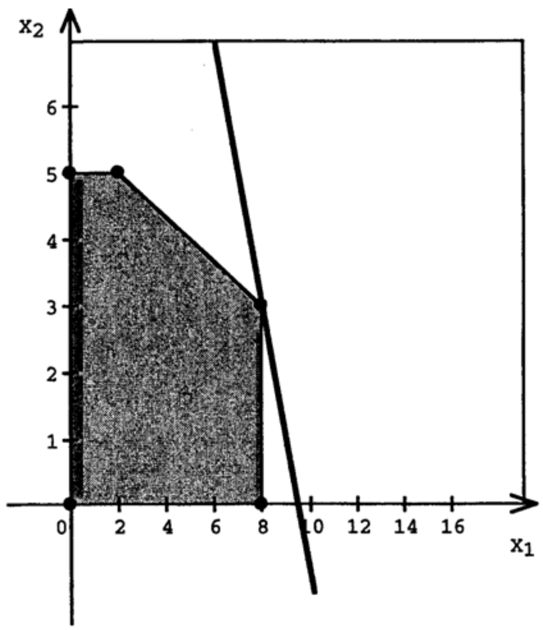
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2		2	0	1	1	0.333333
$C$2		2	0	2	1	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$4		8	0.5	8	4	4
$E$5		4	0.5	4	4	1.333333

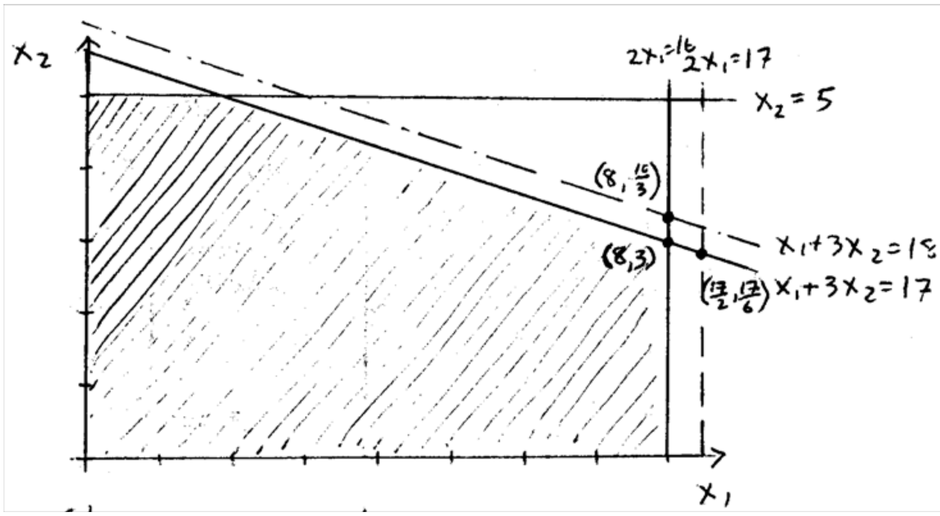
4.7-3.

(a) Optimal Solution: $(x_1^*, x_2^*) = (8, 3)$ and $Z^* = 38$



Corner Point	Z
$(8,3)$	$38*$
$(8,0)$	32
$(2,5)$	18
$(0,5)$	10
$(0,0)$	0

(b)



Increasing resource 1 to 17 units increases Z to $4(8.5)+2(2.83)=39.67$, so $\Delta Z = y_1^* = 1.67$.
Increasing resource 2 to 18 units increases Z to $4(8)+2(3.33)=38.33$, so $\Delta Z = y_2^* = 0.67$.
The third constraint is not binding, so $y_3^* = 0$.

(c) To increase Z by 15, resource 1 should be increased by $\frac{15}{y_1^*} = \frac{15}{1.67} \approx 9$. Solving the LP problem with resource 1 set to $16+9=25$ returns the result $Z=53$.

4.7-4.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0.5, 0, 4.5)$ and $Z^* = 14$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-1	7	-3	0	0	0	0
X4	1	0	2	1	-1	1	0	0	4
X5	2	0	4	-3	0	0	1	0	2
X6	3	0	-3	2	1	0	0	1	3

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right Side
Z	0	1	-10	13	0	0	0	3	9
X4	1	0	-1	3	0	1	0	1	7
X5	2	0	4	-3	0	0	1	0	2
X3	3	0	-3	2	1	0	0	1	3

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	5.5	0	0	2.5	3	14
X_4	1	0	0	2.25	0	1	0.25	1	7.5
X_1	2	0	1	-0.75	0	0	0.25	0	0.5
X_3	3	0	0	-0.25	1	0	0.75	1	4.5

(b) The shadow prices for the three resources are given by the reduced costs (in the objective function) for the corresponding slack variables. These values are circled in the table above. The shadow prices for resources 1, 2 and 3 are 0, 2.5 and 3 respectively. They represent the rate at which the objective function value z increases as the corresponding resource is increased. For instance, increasing resource 3 by one unit increases Z by 3, provided that no other constraints cause any trouble.

(c)

	X1	X2	X3			
Maximize	1	-7	3			
						Right-Hand
				Totals		Side
Constraint 1	2	1	-1	-3.5	<=	4
Constraint 2	4	-3	0	2	<=	2
Constraint 3	-3	2	1	3	<=	3
						Objective
Solution	0.5	0	4.5			14

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$10	Solution X1	0.5	0	1	7.33333	10
\$C\$10	Solution X2	0	-5.5	-7	5.5	1E+30
\$D\$10	Solution X3	4.5	0	3	22	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$5	Constraint 1 Totals	-3.5	0	4	1E+30	7.5
\$E\$6	Constraint 2 Totals	2	2.5	2	1E+30	2
\$E\$7	Constraint 3 Totals	3	3	3	1E+30	4.5

4.7-5.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 1, 3)$ and $Z^* = 7$

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	0	1	-2	2	-3	0	0	0	0
x_4	1	0	-1	1	1	1	0	0	4
x_5	2	0	2	-1	1	0	1	0	2
x_6	3	0	1	1	3	0	0	1	12

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	0	1	4	-1	0	0	3	0	6
x_4	1	0	-3	2	0	1	-1	0	2
x_3	2	0	2	-1	1	0	1	0	2
x_6	3	0	-5	4	0	0	-3	1	6

Bas Var	Eq No	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	0	1	2.5	0	0	0.5	2.5	0	7
x_2	1	0	-1.5	1	0	0.5	-0.5	0	1
x_3	2	0	0.5	0	1	0.5	0.5	0	3
x_6	3	0	1	0	0	-2	-1	1	2

(b) The shadow prices are $y_1^* = 0.5$, $y_2^* = 2.5$ and $y_3^* = 0$. They are the marginal values of resources 1, 2 and 3 respectively.

(c)

	X1	X2	X3			
Maximize	2	-2	3			
						Right-Hand
				Totals		Side
Constraint 1	-1	1	1	4	<=	4
Constraint 2	2	-1	1	2	<=	2
Constraint 3	1	1	3	10	<=	12
						Objective
Solution	0	1	3			7

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$10	Solution X1	0	-2.5	2	2.5	1E+30
\$C\$10	Solution X2	1	0	-2	1.6667	1
\$D\$10	Solution X3	3	0	3	1E+30	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$5	Constraint 1 Totals	4	0.5	4	1	2
\$E\$6	Constraint 2 Totals	2	2.5	2	2	6
\$E\$7	Constraint 3 Totals	10	0	12	1E+30	2

4.7-6.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (11, 0, 3, 0)$ and $Z^* = 52$

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-5	-2	1	-3	0	0	0
X_5	1	0	3	2	-3	1	1	0	24
X_6	2	0	3	3	1	3	0	1	36

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	1.3333	-4	-1.333	1.6667	0	40
X_1	1	0	1	0.6667	-1	0.3333	0.3333	0	8
X_6	2	0	0	1	4	2	-1	1	12

Bas Var	Eq No	Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	2.3333	0	0.6667	0.6667	1	52
X_1	1	0	1	0.9167	0	0.8333	0.0833	0.25	11
X_3	2	0	0	0.25	1	0.5	-0.25	0.25	3

(b) The shadow prices are $y_1^* = 0.6667$ and $y_2^* = 1$. They are the marginal values of resources 1 and 2 respectively.

(c)

	X1	X2	X3	X4			
Maximize	5	4	-1	3			
							Right-Hand
					Totals		Side
Resource 1	3	2	-3	1	24	<=	24
Resource 2	3	3	1	3	36	<=	36
							Objective
Solution	11	0	3	0			52

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Solution X1	11	0	5	1E+30	0.3636
$C$9	Solution X2	0	-0.33333	4	0.33333	1E+30
$D$9	Solution X3	3	0	-1	2.66667	1.33333
$E$9	Solution X4	0	-0.66667	3	0.66667	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$5	Resource 1 Totals	24	0.66667	24	12	132
$F$6	Resource 2 Totals	36	1	36	1E+30	12

4.9-1.

Linear Programming Model:

Number of Decision Variables: 2

Number of Functional Constraints: 4

Max Z = 4500 X1 + 4500 X2

subject to

1) 1 X1 + 0 X2 <= 1

2) 0 X1 + 1 X2 <= 1

3) 5000 X1 + 4000 X2 <= 6000

4) 400 X1 + 500 X2 <= 600

and

X1 >= 0, X2 >= 0.

Solve Automatically by the Interior Point Algorithm:

(X1, X2) = (0.1, 0.2) and Alpha = 0.5

It.	X1	X2	Z
0	0.1	0.2	1350
1	0.1999	0.58008	3509.91
2	0.26144	0.76085	4600.3
3	0.33761	0.81491	5186.35
4	0.40279	0.82027	5503.76
5	0.4661	0.79837	5690.12
6	0.56345	0.73487	5842.42
7	0.62351	0.69021	5911.71
8	0.6511	0.67092	5949.09
9	0.66172	0.66525	5971.35
10	0.66487	0.66511	5984.91
11	0.66582	0.66582	5992.4
12	0.66624	0.66624	5996.2
13	0.66646	0.66646	5998.1
14	0.66656	0.66656	5999.05
15	0.66661	0.66661	5999.52

4.9-2.

```
The linear programming problem is:

Number of Decision Variables:      2

Number of Functional Constraints:  2

Max Z =       1 X1 +    2 X2

subject to

 1)           1 X1 +    3 X2 <=       8

 2)           1 X1 +    1 X2 <=       4

and

        X1 >= 0, X2 >= 0.
Solve Automatically by the Interior Point Algorithm:

(X1, X2) = (0.1, 0.2) and Alpha = 0.5
```

It.	X1	X2	Z
0	0.1	0.2	0.5
1	0.24587	1.36804	2.98196
2	0.25651	1.97283	4.20217
3	0.26482	2.27423	4.81327
4	0.28233	2.42047	5.12328
5	0.32398	2.48263	5.28924
6	0.43489	2.48368	5.40225
7	0.82513	2.37261	5.57036
8	1.4229	2.17597	5.77485
9	1.72185	2.07758	5.87702
10	1.86959	2.03012	5.92984
11	1.94077	2.00909	5.95894
12	1.97327	2.00166	5.97659
13	1.98735	2.00011	5.98758
14	1.99373	2	5.99373
15	1.99687	2	5.99687

Case 4.1

a) The fixed design and fashion costs are sunk costs and therefore should not be considered when setting the production now in July. Since the velvet shirts have a positive contribution to covering the sunk costs, they should be produced or at least considered for production according to the linear programming model. Had Ted raised these concerns before any fixed costs were made, then he would have been correct to advise against designing and producing the shirts. With a contribution of $22 and a demand of 6000 units, maximum expected profit will be only $132,000. This amount will not be enough to cover the $500,000 in fixed costs directly attributable to this product.

b) The linear programming spreadsheet model for this problem is shown below.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1													Button-			
2			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			
3			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			
4		Price	$300	$450	$180	$120	$270	$320	$350	$130	$75	$200	$120			
5		L&M Cost	$160	$150	$100	$60	$120	$140	$175	$60	$40	$160	$90			
6		Material Cost	$30.00	$90.00	$19.50	$6.50	$6.75	$24.75	$39.00	$3.75	$1.25	$18.00	$3.38			
7		Net Contribution	$110.00	$210.00	$60.50	$53.50	$143.25	$155.25	$136.00	$66.25	$33.75	$22.00	$26.63			
8																
9		Cost of												Material		Material
10		Material						Material Requirements						Used		Available
11	Wool	$9.00	3					2.5						25,100	<=	45,000
12	Acetate	$1.50	2				1.5	1.5	2					28,000	<=	28,000
13	Cashmere	$60.00		1.5										6,000	<=	9,000
14	Silk	$13.00			1.5	0.5								18,000	<=	18,000
15	Rayon	$2.25					2						1.5	30,000	<=	30,000
16	Velvet	$12.00							3			1.5		9,000	<=	20,000
17	Cotton	$2.50								1.5	0.5			30,000	<=	30,000
18																
19													Button-			
20			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			Total
21			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			Contribution
22		Items Produced	4,200	4,000	7,000	15,000	8,067	5,000	0	0	60,000	6,000	9,244			$6,862,933
23			<=	<=	<=	<=		<=	<=			<=			Fixed Cost	$8,960,000
24		Demand Forecast	7,000	4,000	12,000	15,000		5,000	5,500			6,000			Total Profit	-$2,097,067
25			>=				>=	>=								
26		Minimum Production	4,200				2,800	3,000		Also:						
27			60%					60%		Silk Camisole >= Silk Blouse						
28			of demand					of demand		Cotton Miniskirt >= Cotton Sweater						

	B	C	D
6	Material Cost	=SUMPRODUCT(CostOfMaterial,C11:C17)	=SUMPRODUCT(CostOfMaterial,D11:D17)
7	Net Contribution	=Price-LMCost-MaterialCost	=Price-LMCost-MaterialCost

Range Name	Cells
CostOfMaterial	B11:B17
FixedCost	P23
ItemsProduced	C22:M22
LMCost	C5:M5
MaterialAvailable	P11:P17
MaterialCost	C6:M6
MaterialRequirements	C11:M17
MaterialUsed	N11:N17
NetContribution	C7:M7
Price	C4:M4
TotalContribution	P22
TotalProfit	P24

	N
9	Material
10	Used
11	=SUMPRODUCT(C11:M11,ItemsProduced)
12	=SUMPRODUCT(C12:M12,ItemsProduced)
13	=SUMPRODUCT(C13:M13,ItemsProduced)
14	=SUMPRODUCT(C14:M14,ItemsProduced)
15	=SUMPRODUCT(C15:M15,ItemsProduced)
16	=SUMPRODUCT(C16:M16,ItemsProduced)
17	=SUMPRODUCT(C17:M17,ItemsProduced)

	O	P
20		Total
21		Contribution
22		=SUMPRODUCT(NetContribution,ItemsProduced)
23	Fixed Cost	8960000
24	Total Profit	=TotalContribution-FixedCost

TrendLine should produce 4,200 Wool Slacks, 4,000 Cashmere Sweaters, 7,000 Silk Blouses, 15,000 Silk Camisoles, 8,067 Tailored Skirts, 5,000 Wool Blazers, 40,000 Cotton Minis, 6,000 Velvet Shirts, and 9,244 Button-Down Blouses. The total net contribution of all clothing items is $6,862,933. However, with the total fixed cost of $860,000 + 3($2,700,000) or $8,960,000, TrendLines actually loses $2,097,067.

c) If velvet cannot be sent back to the textile wholesaler, then the whole quantity will be considered as a sunk cost and therefore added to the fixed costs. The objective function coefficients of items using velvet will no longer include the material cost. The net contribution of the velvet pants and shirts are now $175 and $40, respectively. The revised spreadsheet model is as follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1													Button-			
2			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			
3			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			
4		Price	$300	$450	$180	$120	$270	$320	$350	$130	$75	$200	$120			
5		L&M Cost	$160	$150	$100	$60	$120	$140	$175	$60	$40	$160	$90			
6		Material Cost	$30.00	$90.00	$19.50	$6.50	$6.75	$24.75		$3.75	$1.25		$3.38			
7		Net Contribution	$110.00	$210.00	$60.50	$53.50	$143.25	$155.25	$175.00	$66.25	$33.75	$40.00	$26.63			
8																
9		Cost of												Material		Material
10		Material						Material Requirements						Used		Available
11	Wool	$9.00	3					2.5						25,100	<=	45,000
12	Acetate	$1.50	2				1.5	1.5	2					28,000	<=	28,000
13	Cashmere	$60.00		1.5										6,000	<=	9,000
14	Silk	$13.00			1.5	0.5								18,000	<=	18,000
15	Rayon	$2.25					2						1.5	30,000	<=	30,000
16	Velvet	$12.00							3			1.5		20,000	<=	20,000
17	Cotton	$2.50								1.5	0.5			30,000	<=	30,000
18																
19													Button-			
20			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			Total
21			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			Contribution
22		Items Produced	4,200	4,000	7,000	15,000	3,178	5,000	3,667	0	60,000	6,000	15,763			$7,085,822
23			<=	<=	<=	<=		<=	<=			<=			Original Fixed Cost	$8,960,000
24		Demand Forecast	7,000	4,000	12,000	15,000		5,000	5,500			6,000			Velvet Sunk Cost	$240,000
25			>=				>=	>=							Total Profit	-$2,114,178
26		Minimum Production	4,200				2,800	3,000		Also:						
27			60%					60%		Silk Camisole >= Silk Blouse						
28			of demand					of demand		Cotton Miniskirt >= Cotton Sweater						

	O	P
20		Total
21		Contribution
22		=SUMPRODUCT(NetContribution,ItemsProduced)
23	Original Fixed Cost	8960000
24	Velvet Sunk Cost	=B16*P16
25	Total Profit	=TotalContribution-FixedCost-VelvetSunkCost

The production plan changes considerably. TrendLines should produce 3,178 tailored skirts (down from 8,067), 3,667 velvet pants (up from 0), 60,000 cotton minis (up from 40,000), and 15,763 button-down blouses (up from 9,244). The production decisions for all other items are unaffected by the change. The total net contribution of all clothing items equals $840,000 + $1,226,00 + $ 2,025,000 + $2,983,822.22 = $7,085,822. The sunk costs now include the material cost for velvet and totals $9,200,000. The loss now equals $2,114,178.

d) When TrendLines cannot return the velvet to the wholesaler, the costs for velvet cannot be recovered. These cost are no longer variable cost but now are sunk cost. As a consequence the increased net contribution of the velvet items makes them more attractive to produce. This way the revenues from selling these items can contribute to the recovery of at least some of the fixed costs. Instead of zero TrendLines now produces 3,667 velvet pants. These pants also require some acetate and thus their production affects the production plan for all other items. Since it is not optimal to make full use of the ordered velvet in part (b) it comes as no surprise that the loss in part (c) is even bigger than in part (b).

e) The unit contribution of a wool blazer changes to $75.25.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1													Button-			
2			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			
3			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			
4		Price	$300	$450	$180	$120	$270	$320	$350	$130	$75	$200	$120			
5		L&M Cost	$160	$150	$100	$60	$120	$220	$175	$60	$40	$160	$90			
6		Material Cost	$30.00	$90.00	$19.50	$6.50	$6.75	$24.75	$39.00	$3.75	$1.25	$18.00	$3.38			
7		Net Contribution	$110.00	$210.00	$60.50	$53.50	$143.25	$75.25	$136.00	$66.25	$33.75	$22.00	$26.63			
8																
9		Cost of												Material		Material
10		Material						Material Requirements						Used		Available
11	Wool	$9.00	3					2.5						20,100	<=	45,000
12	Acetate	$1.50	2				1.5	1.5	2					28,000	<=	28,000
13	Cashmere	$60.00		1.5										6,000	<=	9,000
14	Silk	$13.00			1.5	0.5								18,000	<=	18,000
15	Rayon	$2.25					2						1.5	30,000	<=	30,000
16	Velvet	$12.00							3			1.5		9,000	<=	20,000
17	Cotton	$2.50								1.5	0.5			30,000	<=	30,000
18																
19													Button-			
20			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			Total
21			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			Contribution
22		Items Produced	4,200	4,000	7,000	15,000	10,067	3,000	0	0	60,000	6,000	6,578			$6,527,933
23			<=	<=	<=	<=		<=	<=			<=		Fixed Cost		$8,960,000
24		Demand Forecast	7,000	4,000	12,000	15,000		5,000	5,500			6,000		Total Profit		-$2,432,067
25			>=				>=	>=								
26		Minimum Production	4,200				2,800	3,000		Also:						
27			60%					60%		Silk Camisole >= Silk Blouse						
28			of demand					of demand		Cotton Miniskirt >= Cotton Sweater						

TrendLines should produce 10,067 skirts (up from 8,067), the minimum of 3,000 wool blazers (down from 5,000), and 6,578 button-down blouses (down from 9,244). The production decisions for all other items are unaffected by the change. The total net contribution of all clothing items is $6,527,933.33. The total loss is $2,432,067.

f) The available acetate changes from 28,000 to 38,000 square yards. The resulting spreadsheet solution is shown below.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1													Button-			
2			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			
3			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			
4		Price	$300	$450	$180	$120	$270	$320	$350	$130	$75	$200	$120			
5		L&M Cost	$160	$150	$100	$60	$120	$140	$175	$60	$40	$160	$90			
6		Material Cost	$30.00	$90.00	$19.50	$6.50	$6.75	$24.75	$39.00	$3.75	$1.25	$18.00	$3.38			
7		Net Contribution	$110.00	$210.00	$60.50	$53.50	$143.25	$155.25	$136.00	$66.25	$33.75	$22.00	$26.63			
8																
9		Cost of												Material		Material
10		Material						Material Requirements						Used		Available
11	Wool	$9.00	3					2.5						25,100	<=	45,000
12	Acetate	$1.50	2				1.5	1.5	2					38,000	<=	38,000
13	Cashmere	$60.00		1.5										6,000	<=	9,000
14	Silk	$13.00			1.5	0.5								18,000	<=	18,000
15	Rayon	$2.25					2						1.5	30,000	<=	30,000
16	Velvet	$12.00							3			1.5		9,000	<=	20,000
17	Cotton	$2.50								1.5	0.5			30,000	<=	30,000
18																
19													Button-			
20			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			Total
21			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			Contribution
22		Items Produced	4,200	4,000	7,000	15,000	14,733	5,000	0	0	60,000	6,000	356			$7,581,267
23			<=	<=	<=	<=		<=	<=			<=			Fixed Cost	$8,960,000
24		Demand Forecast	7,000	4,000	12,000	15,000		5,000	5,500			6,000			Total Profit	-$1,378,733
25			>=				>=	>=								
26		Minimum Production	4,200				2,800	3,000		Also:						
27			60%					60%		Silk Camisole >= Silk Blouse						
28			of demand					of demand		Cotton Miniskirt >= Cotton Sweater						

TrendLines should produce 14,733 skirts (up from 8,067) and 356 button-down blouses (down from 9,244). The production decisions for all other items are unaffected by the change. The total net contribution of all clothing items is $7,581,267. The loss is $1,378,733.

g) We need to include new decision variables representing the number of clothing items that are sold during the November sale. The new spreadsheet model is shown below.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1													Button-			
2			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			
3			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			
4		Price	$300	$450	$180	$120	$270	$320	$350	$130	$75	$200	$120			
5		L&M Cost	$160	$150	$100	$60	$120	$140	$175	$60	$40	$160	$90			
6		Material Cost	$30.00	$90.00	$19.50	$6.50	$6.75	$24.75	$39.00	$3.75	$1.25	$18.00	$3.38			
7		Net Contribution (Sept-Oct)	$110.00	$210.00	$60.50	$53.50	$143.25	$155.25	$136.00	$66.25	$33.75	$22.00	$26.63			
8																
9		Nov Discount	40%													
10		Price (Nov)	$180	$270	$108	$72	$162	$192	$210	$78	$45	$120	$72			
11		Net Contribution (Nov)	-$10.00	$30.00	-$11.50	$5.50	$35.25	$27.25	-$4.00	$14.25	$3.75	-$58.00	-$21.38			
12																
13		Cost of												Material		Material
14		Material						Material Requirements						Used		Available
15	Wool	$9.00	3					2.5						25,100	<=	45,000
16	Acetate	$1.50	2				1.5	1.5	2					28,000	<=	28,000
17	Cashmere	$60.00		1.5										9,000	<=	9,000
18	Silk	$13.00			1.5	0.5								18,000	<=	18,000
19	Rayon	$2.25					2						1.5	30,000	<=	30,000
20	Velvet	$12.00							3			1.5		9,000	<=	20,000
21	Cotton	$2.50								1.5	0.5			30,000	<=	30,000
22																
23													Button-			
24			Wool	Cashmere	Silk	Silk	Tailored	Wool	Velvet	Cotton	Cotton	Velvet	Down			Total
25			Slacks	Sweater	Blouse	Camisole	Skirt	Blazer	Pants	Sweater	Miniskirt	Shirt	Blouse			Contribution
26		Sept-Oct Sales	4,200	4,000	7,000	15,000	8,067	5,000	0	0	60,000	6,000	9,244			$6,922,933
27			<=	<=	<=	<=		<=	<=			<=			Fixed Cost	$8,960,000
28		Demand Forecast	7,000	4,000	12,000	15,000		5,000	5,500			6,000			Total Profit	-$2,037,067
29																
30		Nov Sales	0	2,000	0	0	0	0	0	0	0	0	0			
31		Total Sales	4,200	6,000	7,000	15,000	8,067	5,000	0	0	60,000	6,000	9,244			
32			>=				>=	>=								
33		Minimum Production	4,200				2,800	3,000		Also:						
34			60%					60%		Silk Camisole >= Silk Blouse						
35			of demand					of demand		Cotton Miniskirt >= Cotton Sweater						

	B	C	D
9	Nov Discount	0.4	
10	Price (Nov)	=(1-NovDiscount)*Price	=(1-NovDiscount)*Price
11	Net Contribution (Nov)	=PriceNov-LMCost-MaterialCost	=PriceNov-LMCost-MaterialCost

Range Name	Cells
CostOfMaterial	B15:B21
FixedCost	P27
LMCost	C5:M5
MaterialAvailable	P15:P21
MaterialCost	C6:M6
MaterialRequirements	C15:M21
MaterialUsed	N15:N21
NetContribution	C7:M7
NetContributionNov	C11:M11
NovDiscount	C9
NovSales	C30:M30
Price	C4:M4
PriceNov	C10:M10
SeptOctSales	C26:M26
TotalContribution	P26
TotalProfit	P28
TotalSales	C31:M31

	N
13	Material
14	Used
15	=SUMPRODUCT(C15:M15,TotalSales)
16	=SUMPRODUCT(C16:M16,TotalSales)
17	=SUMPRODUCT(C17:M17,TotalSales)
18	=SUMPRODUCT(C18:M18,TotalSales)
19	=SUMPRODUCT(C19:M19,TotalSales)
20	=SUMPRODUCT(C20:M20,TotalSales)
21	=SUMPRODUCT(C21:M21,TotalSales)

	O	P
24		Total
25		Contribution
26		=SUMPRODUCT(NetContribution,SeptOctSales)+SUMPRODUCT(NetContributionNov,NovSales)
27	Fixed Cost	8960000
28	Total Profit	=TotalContribution-FixedCost

It only pays to produce 2,000 more Cashmere sweaters. The production plan for all other items is the same as in part (b). The sale of the Cashmere sweaters increases the total net contribution by $60,000 to $6,922,933, and reduces the loss to $2,037,066.67.

Case 4.2

a) We define 12 decision variables, one for each age group surveyed in each region. Rob's restrictions are easily modeled as constraints. For example, his condition that at least 20 percent of the surveyed customers have to be from the first age group requires that the sum of the variables for the age group "18 to 25" across all three regions is at least 400. All his other requirements are modeled similarly. Finally, the sum of all variables has to equal 2000, because that is the number of customers Rob wants to have interviewed.

	A	B	C	D	E	F	G	H	I	J
1	**Cost of Survey**			Age Group						
2			18 to 25	26 to 40	41 to 50	51 and over				
3		Silicon Valley	$4.75	$6.50	$6.50	$5.00				
4	Region	Big Cities	$5.25	$5.75	$6.25	$6.25				
5		Small Towns	$6.50	$7.50	$7.50	$7.25				
6										Percentage
7				Age Group			Total		Required	Required
8	Number to Survey		18 to 25	26 to 40	41 to 50	51 and over	in Region		in Region	in Region
9		Silicon Valley	600	0	0	300	900	>=	300	15%
10	Region	Big Cities	150	550	0	0	700	>=	700	35%
11		Small Towns	100	0	300	0	400	>=	400	20%
12		Total in A.G.	850	550	300	300				
13			>=	>=	>=	>=			Total Surveys	2000
14		Required in A.G.	400	550	300	300				=
15		Percentage Required in A.G.	20%	27.5%	15%	15%			Required Surveys	2000
16										
17									Total Cost	$11,200
18										
19									Profit Margin	15%
20									Bid	$12,880

Range Name	Cells
CostOfSurvey	C3:F5
NumberToSurvey	C9:F11
PercentageRequiredInAG	C15:F15
PercentageRequiredInRegion	J9:J11
RequiredInAG	C14:F14
RequiredInRegion	I9:I11
RequiredSurveys	J15
TotalCost	J17
TotalInAG	C12:F12
TotalInRegion	G9:G11
TotalSurveys	J13

	G	H	I
7	Total		Required
8	in Region		in Region
9	=SUM(C9:F9)	>=	=J9*RequiredSurveys
10	=SUM(C10:F10)	>=	=J10*RequiredSurveys
11	=SUM(C11:F11)	>=	=J11*RequiredSurveys

	B	C	D
12	Total in A.G.	=SUM(C9:C11)	=SUM(D9:D11)
13		>=	>=
14	Required in A.G.	=C15*RequiredSurveys	=D15*RequiredSurveys

	I	J
13	Total Surveys	=SUM(NumberToSurvey)
14		=
15	Required Surveys	2000
16		
17	Total Cost	=SUMPRODUCT(CostOfSurvey,NumberToSurvey)

The cost of conducting the survey meeting all constraints imposed by AmeriBank incurs cost of $11,200. The mix of customers is displayed in the spreadsheet above. Note that there are multiple optimal solutions that all lead to a total cost of $11,200.

b) Sophisticated Surveys will submit a bid of (1.15)($11,200) = $12,880.

c) We need to include the new lower-bound constraint (Minimum to Survey in C19:F21) on all variables: NumberToSurvey (C9:F11) ≥ MinimumToSurvey (C19:F21)

	A	B	C	D	E	F	G	H	I	J
1	**Cost of Survey**		Age Group							
2			18 to 25	26 to 40	41 to 50	51 and over				
3		Silicon Valley	$4.75	$6.50	$6.50	$5.00				
4	Region	Big Cities	$5.25	$5.75	$6.25	$6.25				
5		Small Towns	$6.50	$7.50	$7.50	$7.25				
6										Percentage
7			Age Group				Total		Required	Required
8	Number to Survey		18 to 25	26 to 40	41 to 50	51 and over	in Region		in Region	in Region
9		Silicon Valley	600	50	50	200	900	>=	300	15%
10	Region	Big Cities	50	450	150	50	700	>=	700	35%
11		Small Towns	200	50	100	50	400	>=	400	20%
12		Total in A.G.	850	550	300	300				
13			>=	>=	>=	>=			Total Surveys	2000
14		Required in A.G.	400	550	300	300				=
15		Percentage Required in A.G.	20%	27.5%	15%	15%			Required Surveys	2000
16										
17			Age Group						Total Cost	$11,388
18	Minimum to Survey		18 to 25	26 to 40	41 to 50	51 and over				
19		Silicon Valley	50	50	50	50			Profit Margin	15%
20	Region	Big Cities	50	50	50	50			Bid	$13,096
21		Small Towns	50	50	50	50				
22										
23			(Number to Survey >= Minimum to Survey)							

The new requirement increases the bid to $13,096.

d) We include upper bounds on the total number of people surveyed in Silicon Valley and from the age group of 18 to 25 year-olds: G9 ≤ MaxInSiliconValley (L9) and C12 ≤ MaxIn18to25 (C17).

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Cost of Survey**		Age Group									
2			18 to 25	26 to 40	41 to 50	51 and over						
3		Silicon Valley	$4.75	$6.50	$6.50	$5.00						
4	Region	Big Cities	$5.25	$5.75	$6.25	$6.25						
5		Small Towns	$6.50	$7.50	$7.50	$7.25						
6										Percentage		Max in
7			Age Group				Total		Required	Required		Silicon
8	Number to Survey		18 to 25	26 to 40	41 to 50	51 and over	in Region		in Region	in Region		Valley
9		Silicon Valley	100	50	50	450	650	>=	300	15%	<=	650
10	Region	Big Cities	400	450	50	50	950	>=	700	35%		
11		Small Towns	100	50	200	50	400	>=	400	20%		
12		Total in A.G.	600	550	300	550						
13			>=	>=	>=	>=			Total Surveys	2000		
14		Required in A.G.	400	550	300	300				=		
15		Percentage Required in A.G.	20%	27.5%	15%	15%			Required Surveys	2000		
16			<=									
17		MaxIn18to25	600						Total Cost	$11,575		
18												
19			Age Group						Profit Margin	15%		
20	Minimum to Survey		18 to 25	26 to 40	41 to 50	51 and over			Bid	$13,311		
21		Silicon Valley	50	50	50	50						
22	Region	Big Cities	50	50	50	50						
23		Small Towns	50	50	50	50						
24												
25			(Number to Survey >= Minimum to Survey)									

The new requirements increase the bid to $13,311.

e) The three cost factors for the age group "18 to 25" are changed.

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Cost of Survey**		Age Group									
2			18 to 25	26 to 40	41 to 50	51 and over						
3		Silicon Valley	$6.50	$6.50	$6.50	$5.00						
4	Region	Big Cities	$6.75	$5.75	$6.25	$6.25						
5		Small Towns	$7.00	$7.50	$7.50	$7.25						
6										Percentage		Max in
7			Age Group				Total		Required	Required		Silicon
8	Number to Survey		18 to 25	26 to 40	41 to 50	51 and over	in Region		in Region	in Region		Valley
9		Silicon Valley	50	50	50	500	650	>=	300	15%	<=	650
10	Region	Big Cities	100	600	200	50	950	>=	700	35%		
11		Small Towns	250	50	50	50	400	>=	400	20%		
12		Total in A.G.	400	700	300	600						
13			>=	>=	>=	>=			Total Surveys	2000		
14		Required in A.G.	400	550	300	300				=		
15		Percentage Required in A.G.	20%	27.5%	15%	15%			Required Surveys	2000		
16			<=									
17		MaxIn18to25	600						Total Cost	$12,025		
18												
19			Age Group						Profit Margin	15%		
20	Minimum to Survey		18 to 25	26 to 40	41 to 50	51 and over			Bid	$13,829		
21		Silicon Valley	50	50	50	50						
22	Region	Big Cities	50	50	50	50						
23		Small Towns	50	50	50	50						
24												
25			(Number to Survey >= Minimum to Survey)									

With the new cost factors the bid increases to $13,829.

f) We eliminate all lower and upper bounds on the age groups and regions and replace them with Rob's strict requirements. These requirements also ensure that exactly 2000 people are surveyed so that we can drop that constraint too.

	A	B	C	D	E	F	G	H	I	J
1	**Cost of Survey**			Age Group						
2			18 to 25	26 to 40	41 to 50	51 and over				
3		Silicon Valley	$6.50	$6.50	$6.50	$5.00			Required Surveys	2,000
4	Region	Big Cities	$6.75	$5.75	$6.25	$6.25				
5		Small Towns	$7.00	$7.50	$7.50	$7.25				
6										Percentage
7				Age Group			Total		Required	Required
8	Number to Survey		18 to 25	26 to 40	41 to 50	51 and over	in Region		in Region	in Region
9		Silicon Valley	50	50	50	250	400	=	400	20%
10	Region	Big Cities	50	600	300	50	1000	=	1000	50%
11		Small Towns	400	50	50	100	600	=	600	30%
12		Total in A.G.	500	700	400	400				
13			=	=	=	=			Total Cost	$12,475
14		Required in A.G.	500	700	400	400				
15		Percentage Required in A.G.	25%	35%	20%	20%			Profit Margin	15%
16									Bid	$14,346
17				Age Group						
18	Minimum to Survey		18 to 25	26 to 40	41 to 50	51 and over				
19		Silicon Valley	50	50	50	50				
20	Region	Big Cities	50	50	50	50				
21		Small Towns	50	50	50	50				
22										
23			(Number to Survey ≥ Minimum to Survey)							

Rob's strict requirements increase the cost of the survey by $450. The new bid of Sophisticated Surveys is $14,346.25.

Case 4.3

a & b)

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th		Bussing Cost ($/Student)	
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$300	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$300	$200
7	4	28%	40%	32%	$200	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$500	$300	$0
10							
11							
12	**Solution:**		Number of Students Assigned		Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	0	450	0	450	=	450
15	Area 2	0	422.22	177.78	600	=	600
16	Area 3	0	227.78	322.22	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	366.67	0	133.33	500	=	500
19	Area 6	83.33	0	366.67	450	=	450
20	Total In School	800	1,100	1,000			
21		<=	<=	<=			Total
22	Capacity	900	1,100	1,000			Bussing
23							Cost
24							$555,556
25	**Grade Constraints:**						
26		240	330	300	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	269.33	368.56	339.11			
29	7th Graders	288.00	362.11	300.89			
30	8th Graders	242.67	369.33	360.00			
31		<=	<=	<=			
32		288	396	360	36%	of total in school	

Range Name	Cells
BussingCost	E4:G9
Capacity	B22:D22
NumberOfStudents	G14:G19
PercentageInGrade	B4:D9
Solution	B14:D19
TotalBussingCost	G24
TotalFromArea	E14:E19
TotalInSchool	B20:D20

	E
12	Total
13	From Area
14	=SUM(B14:D14)
15	=SUM(B15:D15)
16	=SUM(B16:D16)
17	=SUM(B17:D17)
18	=SUM(B18:D18)
19	=SUM(B19:D19)

	G
21	Total
22	Bussing
23	Cost
24	=SUMPRODUCT(BussingCost,Solution)

	A	B	C	D
20	Total In School	=SUM(B14:B19)	=SUM(C14:C19)	=SUM(D14:D19)

	A	B	C	D	E
25	**Grade Constraints:**				
26		=$E$26*TotalInSchool	=$E$26*TotalInSchool	=$E$26*TotalInSchool	0.3
27		<=	<=	<=	
28	6th Graders	=SUMPRODUCT(B14:B19,B4:B9)	=SUMPRODUCT(C14:C19,B4:B9)	=SUMPRODUCT(D14:D19,B4:B9)	
29	7th Graders	=SUMPRODUCT(B14:B19,C4:C9)	=SUMPRODUCT(C14:C19,C4:C9)	=SUMPRODUCT(C4:C9,D14:D19)	
30	8th Graders	=SUMPRODUCT(B14:B19,D4:D9)	=SUMPRODUCT(C14:C19,D4:D9)	=SUMPRODUCT(D14:D19,D4:D9)	
31		<=	<=	<=	
32		=$E$32*TotalInSchool	=$E$32*TotalInSchool	=$E$32*TotalInSchool	0.36

c) The recommendation to the school board is to assign students to schools as shown in the above solution section of the spreadsheet. Quantities that are not integers must be rounded since partial students cannot be sent.

d) The following solution decreases total bussing costs by over $135,000 but violates the grade constraints that were imposed. Solutions will vary and those than satisfy the grade constraints will increase the total bussing costs.

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$300	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$300	$200
7	4	28%	40%	32%	$200	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$500	$300	$0
10							
11							
12	**Solution:**	Number of Students Assigned			Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	0	450	0	450	=	450
15	Area 2	0	600	0	600	=	600
16	Area 3	0	0	550	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	500	0	0	500	=	500
19	Area 6	0	0	450	450	=	450
20	Total In School	850	1,050	1,000			
21		<=	<=	<=			Total
22	Capacity	900	1,100	1,000			Bussing
23							Cost
24							$420,000
25	**Grade Constraints:**						
26		255	315	300	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	293.00	366.00	318.00			
29	7th Graders	310.00	339.00	302.00			
30	8th Graders	247.00	345.00	380.00			
31		<=	<=	<=			
32		306	378	360	36%	of total in school	

e) The number of students assigned from each area to each school changes to the solution shown below and the total bussing cost is reduced by almost $162,000.

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$300	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$300	$0
7	4	28%	40%	32%	$0	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$500	$300	$0
10							
11							
12	**Solution:**	Number of Students Assigned			Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	0	450	0	450	=	450
15	Area 2	0	600	0	600	=	600
16	Area 3	0	0	550	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	318.18	0	181.82	500	=	500
19	Area 6	131.82	50	268.18	450	=	450
20	Total In School	800	1,100	1,000			
21		<=	<=	<=			Total
22	Capacity	900	1,100	1,000			Bussing
23							Cost
24							$393,636
25	**Grade Constraints:**						
26		240	330	300	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	266.91	383.00	327.09			
29	7th Graders	285.09	353.00	312.91			
30	8th Graders	248.00	364.00	360.00			
31		<=	<=	<=			
32		288	396	360	36%	of total in school	

f) The number of students assigned from each area to each school changes to the solution shown below and the total bussing cost is reduced by over $215,000.

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$0	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$0	$0
7	4	28%	40%	32%	$0	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$500	$0	$0
10							
11							
12	**Solution:**	Number of Students Assigned			Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	38.71	411.29	0	450	=	450
15	Area 2	0	236.56	363.44	600	=	600
16	Area 3	0	77.96	472.04	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	435.48	0	64.52	500	=	500
19	Area 6	75.81	374.19	0	450	=	450
20	Total In School	900	1,100	900			
21		<=	<=	<=			Total
22	Capacity	900	1,100	1,000			Bussing
23							Cost
24							$340,054
25	**Grade Constraints:**						
26		270	330	270	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	306.00	369.75	301.25			
29	7th Graders	324.00	352.25	274.75			
30	8th Graders	270.00	378.00	324.00			
31		<=	<=	<=			
32		324	396	324	36%	of total in school	

g)

Option	Cost	# students walking 1 to 1.5 miles	# students walking more than 1.5 miles
current	$555,556	0	0
1	$393,636	900	0
2	$340,054	900	491

h) Answers will vary.