CHAPTER 18: INVENTORY THEORY

18.3-1.

(a)

$$K = 15,\, h = 0.30,\, d = 30 \Rightarrow Q^* = \sqrt{\frac{(2)(30)(15)}{0.30}} = 54.77$$

$$t^* = Q^*/d = 1.83 \text{ months}$$

(b)

$$p = 3 \Rightarrow Q^* = \sqrt{\frac{2(30)(15)}{0.30}}\sqrt{\frac{3+0.30}{3}} = 57.45$$

$$S^* = \sqrt{\frac{2(30)(15)}{0.30}}\sqrt{\frac{3}{3+0.30}} = 52.22$$

$$t^* = Q^*/d = 1.91 \text{ months}$$

18.3-2.

(a)

$$K = 25,\, h = 0.05,\, d = 600 \Rightarrow Q^* = \sqrt{\frac{2(600)(25)}{0.05}} = 774.6$$

$$t^* = Q^*/d = 1.29 \text{ weeks}$$

(b)

$$p = 2 \Rightarrow Q^* = \sqrt{\frac{2(600)(25)}{0.05}}\sqrt{\frac{2+0.05}{2}} = 784.22$$

$$S^* = \sqrt{\frac{2(600)(25)}{0.05}}\sqrt{\frac{2}{2+0.05}} = 765.09$$

$$t^* = Q^*/d = 1.31 \text{ weeks}$$

18.3-3.

(a)

	Data				Results
d =	676	(demand/year)		Reorder Point	6.482191781
K =	$75	(setup cost)			
h =	$600.00	(unit holding cost)		Annual Setup Cost	$10,140.00
L =	3.5	(lead time in days)		Annual Holding Cost	$1,500.00
WD =	365	(working days/year)		Total Variable Cost	$11,640.00
	Decision				
Q =	5				

(b)

Q	Annual Setup Cost	Annual Holding Cost	Total Variable Cost
5	$10,140	$1,500	$11,640
7	$7,243	$2,100	$9,343
9	$5,633	$2,700	$8,333
11	$4,609	$3,300	$7,909
13	$3,900	$3,900	$7,800
15	$3,380	$4,500	$7,880
17	$2,982	$5,100	$8,082
19	$2,668	$5,700	$8,368
21	$2,414	$6,300	$8,714
23	$2,204	$6,900	$9,104
25	$2,028	$7,500	$9,528

(c)

	Data				Results
d =	676	(demand/year)		Reorder Point	6.482191781
K =	$75	(setup cost)			
h =	$600.00	(unit holding cost)		Annual Setup Cost	$3,900.00
L =	3.5	(lead time in days)		Annual Holding Cost	$3,900.00
WD =	365	(working days/year)		Total Variable Cost	$7,800.00
	Decision				
Q =	13				

(d)

	Data				Results
d =	676	(demand/year)		Reorder Point	6.482191781
K =	$75	(setup cost)			
h =	$600.00	(unit holding cost)		Annual Setup Cost	$3,900.00
L =	3.5	(lead time in days)		Annual Holding Cost	$3,900.00
WD =	365	(working days/year)		Total Variable Cost	$7,800.00
	Decision				
Q =	13	(optimal order quantity)			

The results are the same as those obtained in (c).

(e)

$$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2(75)(676)}{0.2(3000)}} = 13 \text{ computers purchased with each order}$$

(f)

Number of order per year: $\frac{D}{Q} = \frac{676}{13} = 52$

$$ROP = D(LT) = (13)\left(\frac{1}{2}\right) = 6.5 \text{ inventory level when each order is placed}$$

(g) The optimal policy reduces the total variable inventory cost by $\$3,840$ per year, which is a 33% reduction.

18.3-4.

(a)

	Data				Results
d =	120000	(demand/year)		Reorder Point	0
K =	$2,000	(setup cost)			
h =	$0.48	(unit holding cost)		Annual Setup Cost	$24,000.00
L =	0	(lead time in days)		Annual Holding Cost	$2,400.00
WD =	365	(working days/year)		Total Variable Cost	$26,400.00
	Decision				
Q =	10000				

(b)

Month	Q	Annual Setup Cost	Annual Holding Cost	Total Variable Cost
1	10,000	$24,000	$2,400	$26,400
2	20,000	$12,000	$4,800	$16,800
3	30,000	$8,000	$7,200	$15,200
4	40,000	$6,000	$9,600	$15,600
5	50,000	$4,800	$12,000	$16,800
6	60,000	$4,000	$14,400	$18,400
7	70,000	$3,429	$16,800	$20,229
8	80,000	$3,000	$19,200	$22,200
9	90,000	$2,667	$21,600	$24,267
10	100,000	$2,400	$26,400	$28,800

(c)

	Data				Results
d =	120000	(demand/year)		Reorder Point	0
K =	$2,000	(setup cost)			
h =	$0.48	(unit holding cost)		Annual Setup Cost	$7,589.47
L =	0	(lead time in days)		Annual Holding Cost	$7,589.47
WD =	365	(working days/year)		Total Variable Cost	$15,178.93
	Decision				
Q =	31622.78				

If Q is required to be integer:

	Data				Results
d =	120000	(demand/year)		Reorder Point	0
K =	$2,000	(setup cost)			
h =	$0.48	(unit holding cost)		Annual Setup Cost	$7,589.41
L =	0	(lead time in days)		Annual Holding Cost	$7,589.52
WD =	365	(working days/year)		Total Variable Cost	$15,178.93
	Decision				
Q =	31623				

(d)

	Data				Results
d =	120000	(demand/year)		Reorder Point	0
K =	$2,000	(setup cost)			
h =	$0.48	(unit holding cost)		Annual Setup Cost	$7,589.47
L =	0	(lead time in days)		Annual Holding Cost	$7,589.47
WD =	365	(working days/year)		Total Variable Cost	$15,178.93
	Decision				
Q =	31622.78	(optimal order quantity)			

The results are the same as those in (c).

(e)

$$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2(2{,}000)(10{,}000)}{0.04}} = 31,622.78 \text{ gallons purchased with each order}$$

18.3-5.

(a) Q^* will decrease by half.

(b) Q^* will double.

(c) Q^* remains the same.

(d) Q^* will double.

(e) Q^* remains the same.

18.3-6.

(a)

$$Q^* = \sqrt{\frac{2KD}{h}} \Rightarrow 50 = \sqrt{\frac{2(75)(50)}{h}} \Rightarrow h = \$3 \text{ per month,}$$

which is 15% of the acquisition cost.

	Data				Results
d =	600	(demand/year)		Reorder Point	0
K =	$75	(setup cost)			
h =	$36.00	(unit holding cost)		Annual Setup Cost	$900.00
L =	0	(lead time in days)		Annual Holding Cost	$900.00
WD =	365	(working days/year)		Total Variable Cost	$1,800.00
	Decision				
Q =	50				

(b) Optimal Order Quantity

	Data				Results
d =	600	(demand/year)		Reorder Point	0
K =	$75	(setup cost)			
h =	$48.00	(unit holding cost)		Annual Setup Cost	$1,039.23
L =	0	(lead time in days)		Annual Holding Cost	$1,039.23
WD =	365	(working days/year)		Total Variable Cost	$2,078.46
	Decision				
Q =	43.30				

Current Order Quantity

	Data				Results
d =	600	(demand/year)		Reorder Point	0
K =	$75	(setup cost)			
h =	$48.00	(unit holding cost)		Annual Setup Cost	$900.00
L =	0	(lead time in days)		Annual Holding Cost	$1,200.00
WD =	365	(working days/year)		Total Variable Cost	$2,100.00
	Decision				
Q =	50.00				

(c)

	Data				Results
d =	600	(demand/year)		Reorder Point	10
K =	$75	(setup cost)			
h =	$48.00	(unit holding cost)		Annual Setup Cost	$1,039.23
L =	5	(lead time in days)		Annual Holding Cost	$1,039.23
WD =	300	(working days/year)		Total Variable Cost	$2,078.46
	Decision				
Q =	43.30				

(d) $ROP = 5 + (50)(5/25) = 15$ hammers, which adds $5 \times \$4 = \$20$ to TVC every month, $\$240$ per year.

18.3-7.

$K = 12,000$, $h = 0.30$, $d = 8,000$, $p = 5$

$$Q^* = \sqrt{\frac{2(8000)(12000)}{0.30}}\sqrt{\frac{5+0.30}{5}} = 26,046$$

$$S^* = \sqrt{\frac{2(8000)(12000)}{0.30}}\sqrt{\frac{5}{5+0.30}} = 24,572$$

$t^* = Q^*/d = 3.26$ months

18.3-8.

(a)

	Data				Results
d =	6000	(demand/year)		Reorder Point	0
K =	$1,000	(setup cost)			
h =	$100.00	(unit holding cost)		Annual Setup Cost	$17,320.51
L =	0	(lead time in days)		Annual Holding Cost	$17,320.51
WD =	365	(working days/year)		Total Variable Cost	$34,641.02
	Decision				
Q =	346.41	(optimal order quantity)			

(b)

	Data				Results
d =	6000	(demand/year)		Max Inventory Level	268.33
K =	$1,000	(setup cost)			
h =	$100.00	(unit holding cost)		Annual Setup Cost	$13,416.41
p =	$150.00	(unit shortage cost)		Annual Holding Cost	$8,049.84
				Annual Shortage Cost	$5,366.56
	Decision			Total Variable Cost	$26,832.82
Q =	447.214	(optimal order quantity)			
S =	178.885	(optimal maximum shortage)			

18.3-9.

(a)

	Data				Results
d =	676	(demand/year)		Max Inventory Level	6.00
K =	$75	(setup cost)			
h =	$600.00	(unit holding cost)		Annual Setup Cost	$1,950.00
p =	$200.00	(unit shortage cost)		Annual Holding Cost	$415.38
				Annual Shortage Cost	$1,538.46
	Decision			Total Variable Cost	$3,903.85
Q =	26	(order quantity)			
S =	20	(maximum shortage)			

This TVC is almost half of the optimal value found for Problem 18.3-3.

(b)

Q	Annual Setup Cost	Annual Holding Cost	Annual Shortage Cost	Total Variable Cost
	$1,950	$415	$1,538	$3,904
15	$3,380	$500	$2,667	$6,547
17	$2,982	$159	$2,353	$5,494
19	$2,668	$16	$2,105	$4,789
21	$2,414	$14	$1,905	$4,333
23	$2,204	$117	$1,739	$4,061
25	$2,028	$300	$1,600	$3,928
27	$1,878	$544	$1,481	$3,904
29	$1,748	$838	$1,379	$3,966
31	$1,635	$1,171	$1,290	$4,097
33	$1,536	$1,536	$1,212	$4,285
35	$1,449	$1,929	$1,143	$4,520

(c)

S	Annual Setup Cost	Annual Holding Cost	Annual Shortage Cost	Total Variable Cost
	$1,950	$415	$1,538	$3,904
10	$1,950	$2,954	$385	$5,288
12	$1,950	$2,262	$554	$4,765
14	$1,950	$1,662	$754	$4,365
16	$1,950	$1,154	$985	$4,088
18	$1,950	$738	$1,246	$3,935
20	$1,950	$415	$1,538	$3,904
22	$1,950	$185	$1,862	$3,996
24	$1,950	$46	$2,215	$4,212
26	$1,950	$0	$2,600	$4,550
28	$1,950	$46	$3,015	$5,012
30	$1,950	$185	$3,462	$5,596

18.3-10.

$\frac{p}{h}$	$Q^* = \sqrt{\frac{h+p}{p}}\sqrt{\frac{2KD}{h}}$	Maximum Inventory Level	Maximum Shortage
1/3	2,000	500	1,500
1	1,414	707	707
2	1,225	816	408
3	1,155	866	289
5	1,095	913	183
10	1,049	953	95

18.3-11.

(a)

Maximum inventory: $\frac{(b-a)Q}{b}$
Length of interval I: $\frac{Q}{b}$
Average inventory in interval I: $\frac{(b-a)Q}{2b}$
Length of interval II: $\frac{Q}{a} - \frac{Q}{b}$
Average inventory in interval II: $\frac{(b-a)Q}{2b}$

Average inventory per cycle: $\frac{(b-a)Q}{2b}$

Holding cost per cycle: $\frac{(b-a)Q}{2ah}$

$\Rightarrow T = -\frac{aK}{Q} + \frac{(b-a)hQ}{2b} + ac$

(b)

$$\frac{dT}{dQ} = -\frac{aK}{Q^2} + \frac{(b-a)h}{2b} = 0 \Rightarrow Q^* = \sqrt{\frac{2abk}{(b-a)h}}$$

18.3-12.

(a)

	Data								
D =	5200	(demand/year)							
K =	$50	(setup cost)							
I =	0.2	(inventory holding cost rate)							
N =	3	(number of discount categories)							
						Annual	Annual	Annual	Total
		Range of order quantities				Purchase	Setup	Holding	Variable
Category	Price	Lower Limit	Upper Limit	EOQ	Q*	Cost	Cost	Cost	Cost
1	$100.00	0	99	161	99	$520,000	$2,626	$990	$523,616
2	$95.00	100	499	165	165	$494,000	$1,572	$1,572	$497,143
3	$90.00	500	100000000	170	500	$468,000	$520	$4,500	$473,020
			Results						
		Optimal Q	500						
		Total Variable Cost	$473,020						

(b)

Orders placed per year: $\frac{D}{Q} = \frac{5200}{500} = 10.4$

Time interval between orders: $\frac{Q}{D} = \frac{500}{5200} = 0.096$ years ≈ 5 weeks

18.3-13.

(a)

	Data								
D =	365	(demand/year)							
K =	$5	(setup cost)							
I =	0.2	(inventory holding cost rate)							
N =	3	(number of discount categories)							
						Annual	Annual	Annual	Total
		Range of order quantities				Purchase	Setup	Holding	Variable
Category	Price	Lower Limit	Upper Limit	EOQ	Q*	Cost	Cost	Cost	Cost
1	$4.00	0	49	68	49	$1,460	$37	$20	$1,517
2	$3.90	50	99	68	68	$1,424	$27	$27	$1,477
3	$3.80	100	100000000	69	100	$1,387	$18	$38	$1,443
			Results						
		Optimal Q	100						
		Total Variable Cost	$1,443						

(b)

Orders placed per year: $\frac{D}{Q} = \frac{365}{100} = 3.65$

Time interval between orders: $\frac{Q}{D} = \frac{100}{365} = 0.274$ years ≈ 14.25 weeks

18.3-14.

(a)

Discount Category	$TVC = cD + K\frac{D}{Q} + h\frac{Q}{2}$
1	$TVC = (8.50)(400) + (80)\left(\frac{400}{Q}\right) + (0.2)(8.50)\left(\frac{Q}{2}\right)$
2	$TVC = (8.00)(400) + (80)\left(\frac{400}{Q}\right) + (0.2)(8.00)\left(\frac{Q}{2}\right)$
3	$TVC = (7.50)(400) + (80)\left(\frac{400}{Q}\right) + (0.2)(7.50)\left(\frac{Q}{2}\right)$

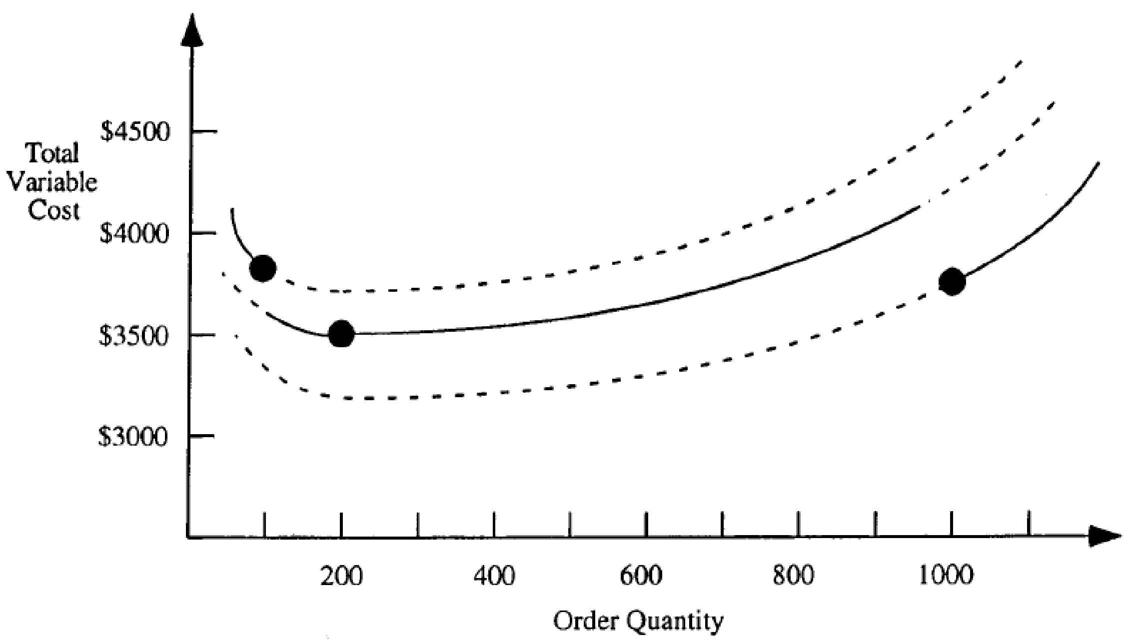
(b)

Discount Category	$Q^* = \sqrt{\frac{2KD}{h}}$
1	$Q^* = \sqrt{\frac{2(80)(400)}{0.2(8.50)}} = 194$
2	$Q^* = \sqrt{\frac{2(80)(400)}{0.2(8.00)}} = 200$
3	$Q^* = \sqrt{\frac{2(80)(400)}{0.2(7.50)}} = 207$

(c)

Discount Category	Feasible Q	$TVC = cD + K\frac{D}{Q} + h\frac{Q}{2}$
1	99	$\$3,807.38$
2	200	$\$3,520.00$
3	1000	$\$3,782.00$

(d)



(e) $Q^* = 200$ with a TVC of $\$3,520$

(f)

	Data								
D =	400	(demand/year)							
K =	$80	(setup cost)							
I =	0.2	(inventory holding cost rate)							
N =	3	(number of discount categories)							
						Annual	Annual	Annual	Total
		Range of order quantities				Purchase	Setup	Holding	Variable
Category	Price	Lower Limit	Upper Limit	EOQ	Q*	Cost	Cost	Cost	Cost
1	$8.50	0	99	194	99	$3,400	$323	$84	$3,807
2	$8.00	100	999	200	200	$3,200	$160	$160	$3,520
3	$7.50	1000	100000000	207	1000	$3,000	$32	$750	$3,782
			Results						
		Optimal Q	200						
		Total Variable Cost	$3,520						

(g) Since the value of Q that minimizes TVC for discount category 2 is feasible, this order quantity minimizes the annual setup and holding costs. Then, category 1 cannot have lower annual setup and holding costs. Furthermore, since the purchase price per case is higher for category 1, it cannot have lower purchasing costs. Hence, category 1 can be eliminated as a candidate for providing the optimal order quantity.

(h)

Orders placed per year: $\frac{D}{Q} = \frac{400}{200} = 2$

Time interval between orders: $\frac{Q}{D} = \frac{200}{400} = 0.5$ years $=$ 6 months

18.3-15.

(a)

Discount Category	$TVC = cD + K\frac{D}{Q} + h\frac{Q}{2}$
1	$TVC = (1.00)(2400) + (4)\left(\frac{2400}{Q}\right) + (0.17)(1.00)\left(\frac{Q}{2}\right)$
2	$TVC = (0.95)(2400) + (4)\left(\frac{2400}{Q}\right) + (0.17)(0.95)\left(\frac{Q}{2}\right)$
3	$TVC = (0.90)(2400) + (4)\left(\frac{2400}{Q}\right) + (0.17)(0.90)\left(\frac{Q}{2}\right)$

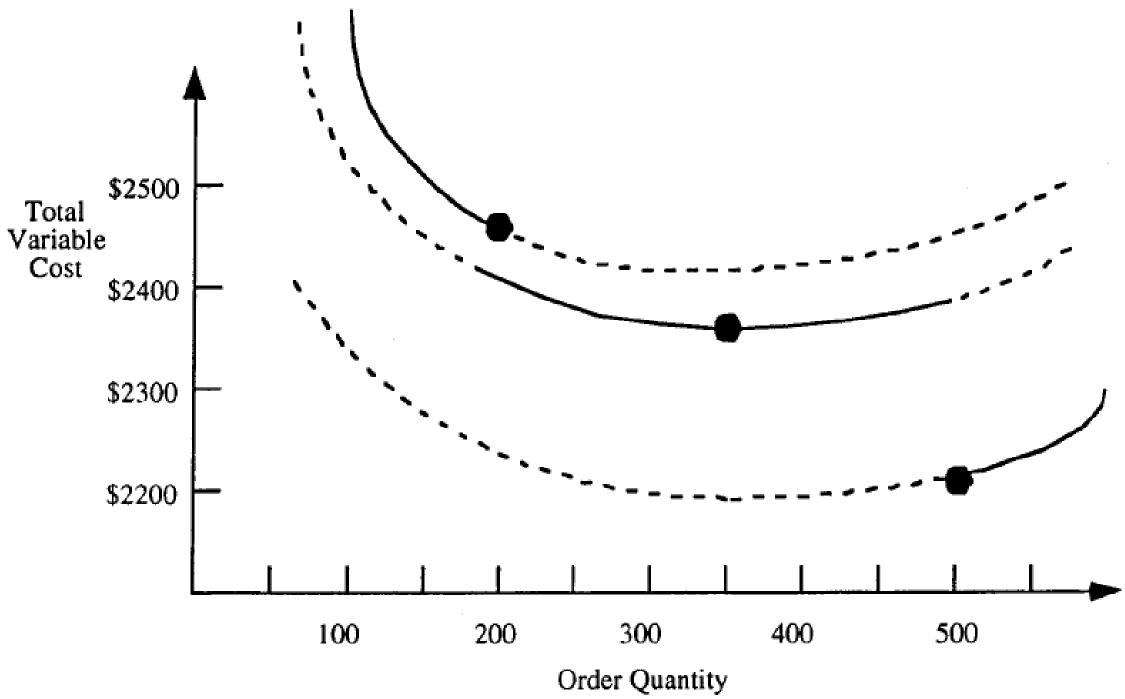
(b)

Discount Category	$Q^* = \sqrt{\frac{2KD}{h}}$
1	$Q^* = \sqrt{\frac{2(4)(2400)}{0.17(1.00)}} = 336$
2	$Q^* = \sqrt{\frac{2(4)(2400)}{0.17(0.95)}} = 345$
3	$Q^* = \sqrt{\frac{2(4)(2400)}{0.17(0.90)}} = 354$

(c)

Discount Category	Feasible Q	$TVC = cD + K\frac{D}{Q} + h\frac{Q}{2}$
1	199	$\$2,465.16$
2	345	$\$2,335.68$
3	500	$\$2,217.45$

(d)



(e) $Q^* = 500$ with a TVC of $\$2,217.45$

(f)

	Data								
D =	2400	(demand/year)							
K =	$4	(setup cost)							
I =	0.17	(inventory holding cost rate)							
N =	3	(number of discount categories)							
						Annual	Annual	Annual	Total
		Range of order quantities				Purchase	Setup	Holding	Variable
Category	Price	Lower Limit	Upper Limit	EOQ	Q*	Cost	Cost	Cost	Cost
1	$1.00	0	199	336	199	$2,400	$48	$17	$2,465
2	$0.95	200	499	345	345	$2,280	$28	$28	$2,336
3	$0.90	500	100000000	354	500	$2,160	$19	$38	$2,217
			Results						
		Optimal Q	500						
	Total Variable Cost		$2,217						

(g) Since the value of Q that minimizes TVC for discount category 2 is feasible, this order quantity minimizes the annual setup and holding costs. Then, category 1 cannot have lower annual setup and holding costs. Furthermore, since the purchase price per bag is higher for category 1, it cannot have lower purchasing costs. Hence, category 1 can be eliminated as a candidate for providing the optimal order quantity.

(h)

Orders placed per year: $\frac{D}{Q} = \frac{2400}{500} = 4.8$
Time interval between orders: $\frac{Q}{D} = \frac{500}{2400} = 0.21$ years ≈ 2.5 months

18.4-1.

$C_5 = 4 + 3 = 7$

$C_4^{(4)} = 7 + 4 + 2 = 13$; $C_4^{(5)} = 4 + 5 + 0.3(3) = 9.9$; $C_4 = \min\{13, 9.9\} = 9.9$

$C_3^{(3)} = 9.9 + 4 + 2 = 15.9$; $C_3^{(4)} = 7 + 4 + 4 + 0.3(2) = 15.6$;
$C_3^{(5)} = 4 + 7 + 0.3(2 + 6) = 13.4$; $C_3 = \min\{15.9, 15.6, 13.4\} = 13.4$

$C_2^{(2)} = 13.5 + 4 + 4 = 21.5$; $C_2^{(3)} = 9.9 + 4 + 6 + 0.3(2) = 20.5$;
$C_2^{(4)} = 7 + 4 + 8 + 0.3(2 + 4) = 20.8$; $C_2^{(5)} = 4 + 11 + 0.3(2 + 4 + 9) = 19.5$;
$C_2 = \min\{21.5, 20.5, 20.8, 19.5\} = 19.5$

$C_1^{(1)} = 19.5 + 4 + 2 = 25.5$; $C_1^{(2)} = 13.5 + 4 + 6 + 0.3(4) = 24.7$;
$C_1^{(3)} = 9.9 + 4 + 8 + 0.3(4 + 4) = 24.3$; $C_1^{(4)} = 7 + 4 + 10 + 0.3(4 + 4 + 6) = 25.2$;
$C_1^{(5)} = 4 + 13 + 0.3(4 + 4 + 6 + 12) = 24.8$;
$C_1 = \min\{25.5, 24.7, 24.3, 25.2, 24.8\} = 24.3$

The optimal production schedule is to produce 8 in the first month, and 5 in the fourth at a cost of $24.30.

18.4-2.

$C_4 = C_5 + 2 = 2$

$C_3^{(3)} = C_4 + 2 = 2 + 2 = 4$

$C_3^{(4)} = C_5 + 2 + 0.2(r_4) = 0 + 2 + 0.2(3) = 2.6$

$C_3 = \min\{4, 2.6\} = 2.6$

$C_2^{(2)} = C_3 + 2 = 2.6 + 2 = 4.6$

$C_2^{(3)} = C_4 + 2 + 0.2(r_3) = 2 + 2 + 0.2(4) = 4.8$

$C_2^{(4)} = C_5 + 2 + 0.2(r_3 + 2r_4) = 0 + 2 + 0.2(4 + 6) = 4$

$C_2 = \min\{4.6, 4.8, 4\} = 4$

$C_1^{(1)} = C_2 + 2 = 4 + 2 = 6$

$C_1^{(2)} = C_3 + 2 + 0.2(r_2) = 2.6 + 2 + 0.2(3) = 5.2$

$C_1^{(3)} = C_4 + 2 + 0.2(r_2 + 2r_3) = 2 + 2 + 0.2(3 + 8) = 6.2$

$C_1^{(4)} = C_5 + 2 + 0.2(r_2 + 2r_3 + 3r_4) = 0 + 2 + 0.2(3 + 8 + 9) = 6$

$C_1 = \min\{6, 5.2, 6.2, 6\} = 5.2$

The optimal production schedule is to produce 7 units in the first and third periods at a total variable cost of $5.2 million.

18.4-3.

x_4	z_4	$C_4^*(x_4)$	z_4^*
0	2	4	2
1	1	3	1
2	0	0	0

	$C_3(x_3, z_3)$							
x_3	0	1	2	3	4	5	$C_3^*(x_3)$	z_3^*
0	–	–	–	10.2	10.8	9.4	9.4	5
1	–	–	8.8	9.4	8.0	–	8.0	4
2	–	7.4	8.0	6.6	–	–	6.6	3
3	4.0	6.6	5.2	–	–	–	4.0	0
4	3.2	3.8	–	–	–	–	3.2	0
5	0.4	–	–	–	–	–	0.4	0

	$C_2(x_2, z_2)$								
x_2	0	1	2	3	4	5	6	7	$C_2^*(x_2)$	z_2^*
0	–	–	13.4	13.2	14.0	11.6	13.0	10.4	10.4	7
1	–	12.4	12.2	13.0	10.6	12.0	9.4	–	9.4	6
2	9.4	11.2	12.0	9.6	11.0	8.4	–	–	8.4	5
3	8.2	11.0	8.6	10.0	7.4	–	–	–	7.4	4
4	7.0	7.6	9.0	6.4	–	–	–	–	6.4	3
5	4.6	8.0	5.4	–	–	–	–	–	4.6	0
6	4.0	4.4	–	–	–	–	–	–	4.0	0
7	1.4	–	–	–	–	–	–	–	1.4	0

	$C_1(x_1, z_1)$									
x_1	3	4	5	6	7	8	9	10	$C_1^*(x_1)$	z_1^*
0	16.8	17.2	17.8	18.4	19.0	18.8	19.8	18.8	16.8	3

The optimal production schedule is to produce 3 units in period 1 and 7 units in period 2, with a cost of $16.8 million.

18.4-4.

$h = 2$

$$B(x_n, z_n) = \begin{cases} k_n + c_n z_n + 2\max\{0, z_n - 3\} + h(x_n + z_n - r_n) & \text{for } 0 < z_n \leq 4 \\ h(x_n - z_n) & \text{for } z_n = 0 \end{cases}$$

x_3	z_3	$C_3^*(x_3)$	z_3^*
0	4	47	4
1	3	36	3
2	2	27	2
3	1	18	1
4	0	4	0

	$C_2(x_2, z_2)$						
x_2	0	1	2	3	4	$C_2^*(x_2)$	z_2^*
0	–	–	–	87	90	87	3
1	–	–	77	78	83	77	2
2	–	67	68	71	76	67	1
3	47	58	61	64	64	47	0
4	38	51	54	52	–	38	0

	$C_1(x_1, z_1)$						
x_1	0	1	2	3	4	$C_1^*(x_1)$	z_1^*
0	87	92	92	82	85	82	3

The optimal production schedule is to produce 3 units in period 1 and 4 units in period 3, with a cost of \$82 thousand.

18.5-1.

Deere & Company uses inventory theory to determine optimal inventory levels ensuring product availability, on-time delivery, and customer satisfaction. In doing this, the multistage inventory planning and optimization (MIPO) tool developed by SmartOps is deployed. The underlying model is a stochastic, capacitated, multiechelon, multiproduct production and inventory model. In MIPO, the material flow in the supply chain is represented as an acyclic-directed graph. The recommended stock levels are found by minimizing the inventory costs among periodic-review replenishment policies with a certain service level. The demand is stochastic and its probability distribution is nonstationary over time. The latter allows to model seasonality of demand. The capacities and supply paths can be nonstationary. Lower bounds on service levels and other constraints can be encapsulated in the model. The main decision variables are safety stocks. Once the optimal stock levels are found, what-if analyses are performed to evaluate the impact of changes.

After the implementation of the results, on-time deliveries have increased from 63% to 92% with a 90% customer service level. The reduction in inventory provided a savings of \$890 million between 2001 and 2003 and a \$107 million increase in annual shareholder value added. Estimated savings by the end of 2004 exceed \$1 billion. The new system also allows Deere to reduce the amount of aged inventory and to offer customers newer models. This, in turn, avoids discounts and saves Deere over \$10 million per year. Other benefits from this study include enhanced manufacturing flexibility, improved service levels, accurate predictions, ability to respond to changes quickly and trust in the supply chain.

18.5-2.

$K_1 = \$15,000, K_2 = \$500, h_1 = \$20, h_2 = \$22, d = 5000$

Optimizing separately:

$$Q_2^* = \sqrt{\frac{2dK_2}{h_2}} = 477$$

$$C_2^* = \sqrt{2dK_2h_2} = \$10{,}488$$

$$n^* = \sqrt{\frac{K_1h_2}{K_2h_1}} = 5.74, \frac{n^*}{[n^*]} \leq \frac{[n^*]+1}{n^*} \Rightarrow n = 6$$

$$Q_1^* = nQ_2^* = 2860$$

$$C_1^* = \frac{dK_1}{nQ_2} + \frac{h_1(n-1)Q_2}{2} = \$50{,}055$$

$$C^* = C_1^* + C_2^* = \$60{,}543$$

Optimizing simultaneously:

$$e_1 = h_1 = 20, e_2 = h_2 - h_1 = 2$$

$$n^* = \sqrt{\frac{K_1e_2}{K_2e_1}} = 1.73, \frac{n^*}{[n^*]} > \frac{[n^*]+1}{n^*} \Rightarrow n = 2$$

$$Q_2^* = \sqrt{\frac{2d\left(\frac{K_1}{n}+K_2\right)}{ne_1+e_2}} = 1380$$

$$Q_1^* = nQ_2^* = 2760$$

$$C^* = \sqrt{2d\left(\frac{K_1}{n}+K_2\right)(ne_1+e_2)} = \$57{,}966$$

Quantity	Separate Optimization	Simultaneous Optimization
Q_2^*	477	1380
n^*	5.74	1.73
n	6	2
Q_1^*	2860	2760
C^*	\$60,543	$\$57,966$

The increase in the total variable cost per unit time if the results from separate optimization were to be used instead of the ones from simultaneous optimization is 3%.

18.5-3.

(a) $h_1 = \$25, h_2 = \$250, d = 2,500$

Quantity	$(\$25000, \$1000)$	$(\$10000, \$2500)$	$(\$5000, \$5000)$
Q_2^*	149	236	333
n^*	15	6	3
n	15	6	3
Q_1^*	2236	1414	1000

(b) $K_1 = \$10,000, K_2 = \$2500, d = 2,500$

Quantity	$(\$10, \$500)$	$(\$25, \$250)$	$(\$50, \$100)$
Q_2^*	160	236	500
n^*	14	6	2
n	14	6	2
Q_1^*	2236	1414	1000

(c) $K_1 = \$10,000, K_2 = \$2500, h_1 = \$25, h_2 = \$250$

Quantity	1000	2500	5000
Q_2^*	149	236	333
n^*	6	6	6
n	6	6	6
Q_1^*	894	1414	2000

18.5-4.

$K_1 = \$5,000, K_2 = \$200, h_1 = \$10, h_2 = \$11, d = 100$

Quantity	Separate Optimization (a)	Simultaneous Optimization (b)
Q_2^*	60	160
n^*	5.24	1.58
n	5	2
Q_1^*	302	321
C^*	3528	3367

(c) The decrease in the total variable cost per unit time C^* by using the approach in (b) rather than the one in (a) is 5%.

18.5-5.

$K_1 = \$50,000, K_2 = \$500, h_1 = \$50, h_2 = \$60, d = 500$

Quantity	Separate Optimization (a)	Simultaneous Optimization (b)
Q_2^*	91	249
C_2^*	5477	8469
n^*	10.95	4.47
n	11	4
Q_1^*	1004	995
C_1^*	47718	43780
C^*	53195	52249

(c) The assembly plant will lose money $(-\$2,992)$ by using the joint inventory policy obtained in (b) whereas the supplier will make money $(\$3,938)$ by doing so. One possible financial agreement between the supplier and the assembly plant is that the supplier will compensate for the loss of the plant so that the plant agrees to a supply contract inducing the inventory policy in (b). By using this policy instead of separately optimal ones, the total saving is $-\$2,992 + \$3,938 = \$946$.

18.5-6.

$K_1 = \$50,000, K_2 = \$2,000, K_3 = \$360, h_1 = \$1, h_2 = \$2, h_3 = \$10, d = 5,000$

Installation i	Solution of Relaxed Problem		Initial Solution of Revised Problem		Final Solution of Revised Problem	
	Q_i	C_i	Q_i^*	C_i	Q_i^*	C_i
1	10000	10000	9600	10008	9628	10007
2	2000	2000	2400	2033	2407	2034
3	300	2400	300	2400	301	2400
	$\underline{C}$ = 14400		C = 14442		$\overline{C}$ = 14442	

The cost $\overline{C}$ is about 0.29% above the optimal cost $\underline{C}$ of the relaxed problem. Since the latter is a lower bound on the optimal cost C^* of the original problem, the optimal cost $\overline{C}$ of the revised problem can exceed C^* at most by 0.29%.

18.5-7.

$K_1 = \$125,000, K_2 = \$20,000, K_3 = \$6,000, K_4 = \$10,000, K_5 = \$250$
$h_1 = \$2, h_2 = \$10, h_3 = \$15, h_4 = \$20, h_5 = \$30, d = 1,000$
$e_1 = \$2, e_2 = \$8, e_3 = \$5, e_4 = \$5, e_5 = \$10$

Since $(K_3/e_3) = 1200 < 2000 = (K_4/e_4)$, we need to merge the installation 3 and 4 as a new installation 3' with $K_{3'} = \$16,000$ and $e_{3'} = \$10$.

Installation i	Solution of Relaxed Problem		Initial Solution of Revised Problem		Final Solution of Revised Problem	
	Q_i	C_i	Q_i^*	C_i	Q_i^*	C_i
1	11180	22361	14311	23045	13954	22912
2	2236	17889	1789	18336	1744	18443
3' (3+4)	1789	17889	1789	17889	1744	17894
5	224	2236	224	2236	218	2237
	$\underline{C}$ = 60374		C = 61506		$\overline{C}$ = 61486	

The cost $\overline{C}$ is about 1.84% above the optimal cost $\underline{C}$ of the relaxed problem. Since the latter is a lower bound on the optimal cost C^* of the original problem, the optimal cost $\overline{C}$ of the revised problem can exceed C^* at most by 1.84%.

18.5-8.

$K_1 = \$1,000, K_2 = \$5, K_3 = \$75, K_4 = \$80$
$h_1 = \$0.5, h_2 = \$0.55, h_3 = \$3.55, h_4 = \$7.55, d = 4,000$

Installation *i*	Solution of Relaxed Problem		Initial Solution of Revised Problem		Final Solution of Revised Problem	
	Q_i	C_i	Q_i^*	C_i	Q_i^*	C_i
1	4000	2000	3200	2050	3200	2050
2	894	45	800	45	800	45
3	447	1342	400	1350	400	1350
4	400	1600	400	1600	400	1600
	$\underline{C}$ = 4986		C = 5045		$\overline{C}$ = 5045	

The cost $\overline{C}$ is about 1.18% above the optimal cost $\underline{C}$ of the relaxed problem. Since the latter is a lower bound on the optimal cost C^* of the original problem, the optimal cost $\overline{C}$ of the revised problem can exceed C^* at most by 1.18%.

18.5-9.

$K_1 = \$60,000, K_2 = \$6,000, K_3 = \$400, h_1 = \$3, h_2 = \$7, h_3 = \$9, d = 10,000$

Installation *i*	Solution of Relaxed Problem		Initial Solution of Revised Problem		Final Solution of Revised Problem	
	Q_i	C_i	Q_i^*	C_i	Q_i^*	C_i
1	20000	60000	16000	61500	20257	60005
2	5477	21909	4000	23000	5064	21976
3	2000	4000	2000	4000	2532	4112
	$\underline{C}$ = 85909		C = 88500		$\overline{C}$ = 86093	

The cost $\overline{C}$ is about 0.21% above the optimal cost $\underline{C}$ of the relaxed problem. Since the latter is a lower bound on the optimal cost C^* of the original problem, the optimal cost $\overline{C}$ of the revised problem can exceed C^* at most by 0.21%.

18.6-1.

(a)

$$Q = \sqrt{\frac{h+p}{p}}\sqrt{\frac{2KD}{h}} = \sqrt{\frac{3000+1000}{1000}}\sqrt{\frac{2(1500)(900)}{3000}} = 60$$

(b) $R = \mu + K_L\sigma = 50 + 0.675(15) = 60$

(c)

	Data				Results
d =	900	(average demand/unit time)		Q =	60
K =	\$1,500	(setup cost)		R =	60
h =	\$3,000.00	(unit holding cost)			
p =	\$1,000	(unit shortage cost)			
L =	0.75	(service level)			
Demand During Lead Time					
Distribution	Normal				
mean =	50				
stand. dev. =	15				

(d) Safety Stock: $R - \text{mean} = 60 - 50 = 10$

(e) If demand during the delivery time exceeds the order quantity 60, then the reorder point will be hit again before the order arrives, triggering another order.

18.6-2.

(a)

$$Q = \sqrt{\frac{2DK}{h}}\sqrt{\frac{h+p}{p}} = \sqrt{\frac{2(40)(40)}{8}}\sqrt{\frac{8+1}{1}} = 60$$

$$R = a + L(b-a) = 5 + 0.8(15-5) = 13$$

(b)

	Data				Results
d =	40	(average demand/unit time)		Q =	60
K =	$40	(setup cost)		R =	13
h =	8	(unit holding cost)			
p =	1	(unit shortage cost)			
L =	0.8	(service level)			
Demand During Lead Time					
Distribution	Uniform				
a =	5	(lower endpoint)			
b =	15	(upper endpoint)			

(c) Average number of orders per year: $(40)(12)/60 = 8$
Probability of a stock-out before the order is received: $1 - 0.8 = 0.2$
Average number of stock-outs per year: $8(0.2) = 1.6$

18.6-3.

(a)

	Case 1	Case 2	Case 3	Case 4
L	$h = \$1, \sigma = 1$	$h = \$100, \sigma = 1$	$h = \$1, \sigma = 100$	$h = \$100, \sigma = 100$
0.5	0	0	0	0
0.75	0.675	67.5	67.5	6750
0.9	1.282	128.2	128.2	12,820
0.95	1.645	164.5	164.5	16,450
0.99	2.327	232.7	232.7	23,270
0.999	3.098	309.8	309.8	30,980

(b)

	Case 1	Case 2	Case 3	Case 4
ΔL	$h = \$1, \sigma = 1$	$h = \$100, \sigma = 1$	$h = \$1, \sigma = 100$	$h = \$100, \sigma = 100$
0.5	0.675	67.5	67.5	6750
0.15	0.607	60.7	60.7	6070
0.05	0.363	36.3	36.3	3630
0.04	0.682	68.2	68.2	6820
0.009	0.771	77.1	77.1	7710

(c) As the service level gets higher, increasing the service level further costs more for smaller increases. Thus, there will be diminishing returns when raising the service level further and further. A manager should balance the cost of the safety stock with the cost of stock-outs to determine the best service level.

18.6-4.

(a) $C = hK_L\sigma = (100)(1.282)(100) = \$12,820$

(b) $\sigma = \sqrt{d}\sigma_1 \Rightarrow 100 = \sqrt{4}\sigma_1 \Rightarrow \sigma_1 = 50$

If the lead time were one day: $C = hK_L\sigma_1 = (100)(1.282)(50) = \$6,410$. This is a 50% reduction in the cost of the safety stock.

(c) $\sigma = \sqrt{d}\sigma_1 = \sqrt{8}(50) = 141.4$, $C = hK_L\sigma_1 = (100)(1.282)(141.4) = \$18,127$

This is a 41% increase in the cost of the safety stock.

(d) The lead time would need to quadruple to 16 days.

18.6-5.

(a) The safety stock drops to zero.

(b) The safety stock decreases.

(c) The safety stock remains the same for a given service level. However, with higher shortage costs, there will be an incentive to increase the service level, which induces a higher level of safety stock.

(d) The safety stock increases.

(e) The safety stock doubles.

(f) The safety stock doubles.

18.6-6.

(a) Ground Chuck

	Data				Results
d =	26000	(average demand/unit time)		Q =	2,183
K =	$25	(setup cost)		R =	145
h =	$0.30	(unit holding cost)			
p =	$3	(unit shortage cost)			
L =	0.95	(service level)			
Demand During Lead Time					
Distribution	Uniform				
a =	50	(lower endpoint)			
b =	150	(upper endpoint)			

Chuck Wagon

	Data				Results
d =	26000	(average demand/unit time)		Q =	6,175
K =	$200	(setup cost)		R =	829
h =	$0.30	(unit holding cost)			
p =	$3	(unit shortage cost)			
L =	0.95	(service level)			
Demand During Lead Time					
Distribution	Normal				
mean =	500				
stand. dev. =	200				

(b) Ground Chuck: $R = a + L(b-a) = 50 + 0.95(150-50) = 145$

Chuck Wagon: $R = \mu + K_L\sigma = 500 + 1.645(200) = 829$

(c) Ground Chuck: safety stock $R -$ mean $= 145 - 100 = 45$

Chuck Wagon: safety stock $R -$ mean $= 829 - 500 = 329$

(d) Ground Chuck:

Annual average holding cost: $(0.30)\left(\frac{45+(2183+45)}{2}\right) = \$340.95$

Chuck Wagon:

Annual average holding cost: $(0.30)\left(\frac{329+(6175+329)}{2}\right) = \$3,416.50$

(e) Ground Chuck:

Annual shipping cost: $K\left(\frac{D}{Q}\right) = 25\left(\frac{26{,}000}{2183}\right) = \$297.76$

Annual purchasing cost: $(26,000)(1.49) = \$38,740$

Average annual acquisition cost: $\$297.76 + \$38,740 = \$39,037.76$

Chuck Wagon:

$$\text{Annual shipping cost: } K\left(\frac{D}{Q}\right) + 0.10D = 200\left(\frac{26{,}000}{6175}\right) + 0.10(26,000)$$
$$= \$3442.11$$

Annual purchasing cost: $(26,000)(1.35) = \$35,100$

Average annual acquisition cost: $\$3442.11 + \$35,100 = \$38,542.11$

(f) Ground Chuck: $\$340.95 + \$39,037.76 = \$39,378.71$

Chuck Wagon: $\$3,416.50 + \$38,542.11 = \$41,958.61$

Jed should choose Ground Chuck as their supplier.

(g) If Jed would like to use the beef within a month of receiving it, then Ground Chuck is the best choice. The order quantity with Ground Chuck is roughly one month's supply whereas with Chuck Wagon, it is roughly three months' supply.

18.7-1.

In this study, inventory theory is applied to the three-echelon distribution problem faced by Time Inc., the largest magazine publisher in the US. For each issue of each magazine, Time Inc. needs to solve three subproblems. The first is to determine the total number D of copies to be printed and shipped. The second is to find an allocation $D_1, \ldots, D_N$ of these D copies among N wholesalers. The third subproblem is to decide on the distribution d_{ij} of D_j copies among n_j retailers of wholesaler j for every j. Complicated cost and revenue structures, timing and constraints on available information complicate these problems. The overall objective is to maximize the expected total profit. The problem is solved backwards by using readily available results from the literature of newsvendor problem under ideal conditions. The solution found is then adjusted to incorporate deviations from the ideal.

To solve the store-level allocation problem, first the distribution of demand is estimated using statistical analysis. If $F(k|\mu_{ij})$ is the probability that the demand in store i of wholesaler j is at least k, then the optimal allocation to this retailer is determined as $d_{ij} = F^{-1}(\lambda|\mu_{ij})$ or the best approximation to this. With this allocation, the probability of selling out is $1-\lambda$ for each store and the solution satisfies $\sum_i d_{ij} = D_i$. Similarly, the wholesaler-level allocation is found from the equation $m_j(D_j) = m$, where $m_j(\,\cdot\,)$ is the probability that wholesaler j will sell the last copy shipped and m is chosen such that $\sum_j D_j = D$. Finally, a lower bound on the national print order is determined from $M(D_0) = c/r$, where $M(\,\cdot\,)$ is the probability of selling the last copy printed and shipped, c and r are the marginal cost and revenue respectively. Because of the complications in identifying c and r, Time Inc. aims at producing more than D_0.

The new system increased Time Inc.'s annual profits by over \$3.5 million. The benefits include improvement of wholesaler and retailer allocations, and increase of sales stimulation effect by over 1%.

18.7-2.

F$(S^*) = \frac{S^*-200}{100} = \frac{p-c}{p+h} = \frac{25-18}{25+0.1} \Rightarrow S^* = 100\left(2+\frac{7}{25.1}\right) \approx 228$

18.7-3.

(a) Freddie's most profitable alternative is to order 16 copies.

	State of Nature				Expected
Alternative	15	16	17	18	Payoff
Order 15 copies	15	15	15	15	\$15.00
Order 16 copies	14	16	16	16	\$15.20 Maximum
Order 17 copies	13	15	17	17	\$15.00
Order 18 copies	12	14	16	18	\$14.20
Prior Probability	0.4	0.2	0.3	0.1	

(b) Freddie's most profitable alternative is to order 16 copies.

	State of Nature				Expected
Alternative	15	16	17	18	Cost
Order 15 copies	0	1	2	3	\$1.10
Order 16 copies	1	0	1	2	\$0.90 Minimum
Order 17 copies	2	1	0	1	\$1.10
Order 18 copies	3	2	1	0	\$1.90
Prior Probability	0.4	0.2	0.3	0.1	

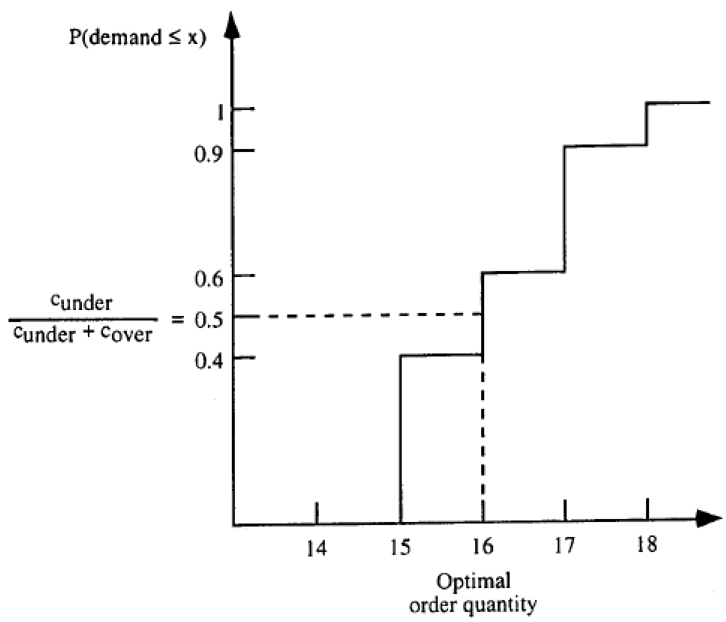
(c)

Alternative	Service Level
Order 15 copies	0.4
Order 16 copies	0.6
Order 17 copies	0.9
Order 18 copies	1

Optimal service level: $\frac{C_{\text{under}}}{C_{\text{under}}+C_{\text{over}}} = \frac{1}{1+1} = 0.5$

Freddie should order 16 copies.

(d)



18.7-4.

(a) $C_{\text{under}} = \$3 - \$1 = \$2$, $C_{\text{over}} = \$1 - \$0.50 = \$0.50$

(b) Prepare 4 doughnuts everyday to minimize the costs.

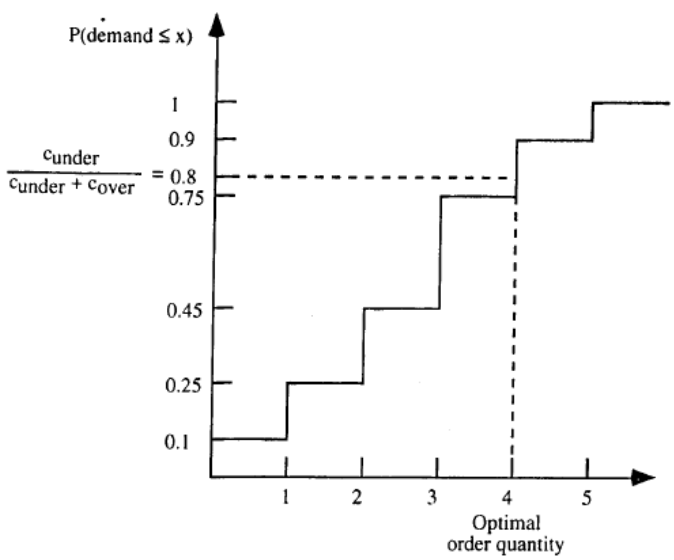
Alternative	State of Nature (Purchase Requests) 0	1	2	3	4	5	Expected Cost	
Make 0	0.0	2.0	4.0	6.0	8.0	10.0	5.10	
Make 1	0.5	0.0	2.0	4.0	6.0	8.0	3.35	
Make 2	1.0	0.5	0.0	2.0	4.0	6.0	1.98	
Make 3	1.5	1.0	0.5	0.0	2.0	4.0	1.10	
Make 4	2.0	1.5	1.0	0.5	0.0	2.0	0.98	Minimum
Make 5	2.5	2.0	1.5	1.0	0.5	0.0	1.23	
Prior Probability	0.1	0.15	0.2	0.3	0.15	0.1		

(c)

Alternative	Service Level
Make 0	0.1
Make 1	0.25
Make 2	0.45
Make 3	0.75
Make 4	0.9
Make 5	1

Optimal service level: $\frac{C_{\text{under}}}{C_{\text{under}}+C_{\text{over}}} = \frac{2}{2+0.5} = 0.8$

Prepare 4 doughnuts everyday.



(d) The probability of running short is $1 - 0.9 = 10\%$.

(e) Before 5 doughnuts are prepared, the optimal service level needs to exceed 0.9. Let g be the cost of lost customer goodwill. Then $C_{\text{under}} = 2 + g$.

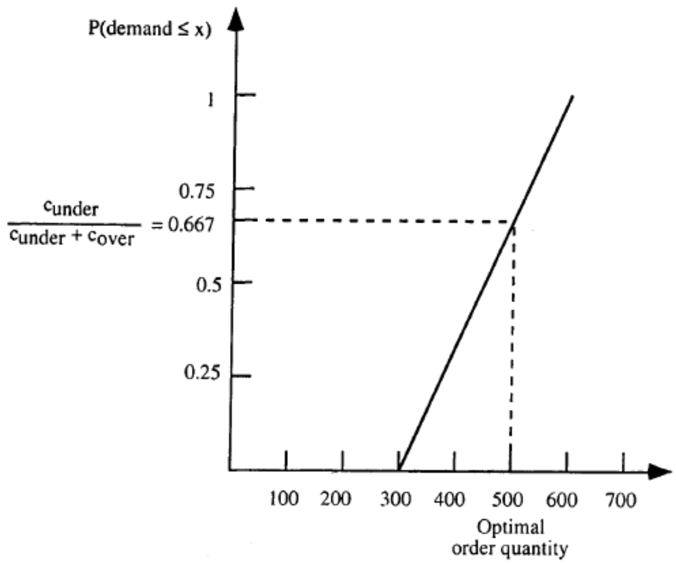
$$\frac{C_{\text{under}}}{C_{\text{under}}+C_{\text{over}}} > 0.9 \Leftrightarrow \frac{2+g}{2+g+0.5} > 0.9 \Leftrightarrow g > 2.50$$

The goodwill cost should be at least $\$2.50$ before 5 doughnuts are prepared.

18.7-5.

(a) Optimal service level: $\frac{C_{\text{under}}}{C_{\text{under}}+C_{\text{over}}} = \frac{1}{1+0.5} = 0.667$

(b)



(c) $Q^* = 300 + 0.667(600 - 300) = 500$

(d) The probability of running short is $1 - 0.667 = 33.3\%$.

(e) Optimal service level:

$$\frac{C_{\text{under}}}{C_{\text{under}}+C_{\text{over}}} = \frac{1+1.5}{1+1.5+0.5} = 0.833$$

$$Q^* = 300 + 0.833(600 - 300) = 550$$

The probability of running short is $1 - 0.833 = 16.7\%$.

18.7-6.

(a) Revenue (with shortages): $500(3) = \$1,500$

(b) Average number of loaves sold (without shortages): $300 + \frac{500-300}{2} = 400$

Average daily revenue (without shortages): $400(3.00) = \$1,200$

(c) With shortages: $1,500 \times 0.333 = \$500$

Without shortages: $1,200 \times 0.667 = \$800$

Average daily revenue over all days: $\$500 + \$800 = \$1,300$

(d) Average number of loves not sold: $\frac{200-0}{2} = 100$

Average number of day-old loaves obtained over all days: $100 \times 0.667 = 66.7$

Average daily revenue from day-old loaves: $66.7(1.50) = \$100$

(e) Average total daily revenue: $\$1,300 + \$100 = \$1,400$

Average daily profit: $\$1,400 - \$2(500) = \$400$

(f) Average daily profit with 600 loaves: $3(450) - 2(600) + 1.50(150) = \$375$

(g) Average daily profit with 550 loaves: $375+\frac{400-375}{2}=\$387.50$

(h) Average size of shortage with 550 loaves: $\frac{600-550}{2}=25$ loaves

Average daily shortage over all days: $25\times 0.167=4.167$

Average daily cost of lost goodwill: $4.167\times 1.50=\$6.25$

Average daily profit with 550 loaves and lost goodwill: $\$387.50-\$6.25=\$381.25$

(i) Average size of shortage with 500 loaves: $\frac{100-0}{2}=50$ loaves

Average daily shortage over all days: $50\times 0.333=16.67$

Average daily cost of lost goodwill: $16.67\times 1.50=\$25$

Average daily profit with 500 loaves and lost goodwill: $\$400-\$25=\$375$

18.7-7.

(a) $Q^{*}=a+(\text{service level})(b-a)=a+(0.667)(75)=a+50$

(b) Probability of incurring shortage: $1-0.667=33.3\%$ (same as in 18.7-4)

(c) Maximum shortage: $b-(a+50)=25$

Maximum number of loaves that will not be sold: 50

The corresponding numbers for 18.7-5 are 100 and 200 respectively, which are four times the amounts in this problem.

(d) The average daily costs of underordering and overordering for the new plan are 25% of the original costs, so it is quite valuable to obtain as much information as possible about the demand before placing the final order for a perishable product.

(e) $Q^{*}=a+(\text{service level})(b-a)=a+(0.833)(75)=a+62.5$

Probability of incurring shortage: $1-0.833=16.67\%$

Maximum shortage: $b-(a+62.5)=12.5$

Maximum number of loaves that will not be sold: 62.5

18.7-8.

(a)

$$S^{*}=-\lambda\ln\left(\frac{c+h}{p+h}\right)=-50\ln\left(\frac{1000+300}{10000+300}\right)\approx 103$$

(b) $C(y)=c(y-I)+L(y)=cy-cI+L(y)$

Taking the derivative with respect to y, the term involving the initial inventory I vanishes, so the optimal policy is the same as in (a), i.e., to order up to 103 or equivalently to order $103-23=80$ parts.

(c) $\mathrm{P}\{D\leq S\}=\mathrm{F}(S)=1-e^{-\frac{S}{50}}=0.9\Rightarrow S=-50\ln(0.1)\approx 115$

(d)

$$\frac{p-c}{p+h}=0.9\Rightarrow\frac{p-1000}{p+300}=0.9\Rightarrow p=\$12,700$$

18.7-9.

(a) Optimal service level:

$$\frac{C_{\text{under}}}{C_{\text{under}}+C_{\text{over}}} = \frac{3000}{3000+1000} = 0.75$$

(b) $Q = \mu + K_L\sigma = 50 + (0.675)(15) = 60$

18.7-10.

$L(y) = \frac{1}{20}\left[\int_0^y (y-x)dx + 3\int_y^{20}(x-y)dx\right] = \frac{y^2}{10} - 3y + 30$

$cy + L(y) = 2y + \frac{y^2}{10} - 3y + 30 = \frac{y^2}{10} - y + 30$

Taking the derivative with respect to y: $\frac{y}{5} - 1 = 0 \Rightarrow S = 5$. We could have used the result P$\{D \leq S\} = (p-c)/(p+h)$ directly:

$$\text{P}\{D \leq S\} = S/20 = (p-c)/(p+h) = (3-2)/(3+1) = 0.25 \Rightarrow S = 5.$$

$$C(s) = K + cS + L(S) \Rightarrow cs + L(s) = K + cS + L(S)$$

$$\Rightarrow \frac{s^2}{10} - s + 30 = 1.50 + \frac{5^2}{10} - 5 + 30 \Rightarrow \frac{s^2}{10} - s + 1 = 0$$

$$\Rightarrow s = 5 - \sqrt{15} \approx 1.13$$

The $(s,S) = (1.13, 5)$ policy is optimal.

18.7-11.

Single-period model with a setup cost:

Demand density is exponential with $\lambda = 25$. Per unit production/purchasing cost is $c = 1$. Per unit inventory holding cost is $h = 0.4$ and per unit shortage cost is $p = 1.5$. The setup cost is $K = 10$. The optimal policy is an (s,S) policy with $s = -11.e3$ and $S = 7.63454$.

18.8-1.

In each case, $L = 200, p_1 = \$1000$ and D has a normal distribution with mean 60 and standard deviation 20.

$p_2 = \$300 \Rightarrow F(x^*) = 1 - \frac{p_2}{p_1} = 0.7 \Rightarrow \ x^* \ = 60 + K_{0.3}(20) = 60 + 0.52(20) = 70.4$

When the discount fare is \$300, 70 seats should be reserved for class 1 customers and the request to make a sale to the class 2 customer should be accepted if there are 71 or more seats remaining.

$p_2 = \$400 \Rightarrow F(x^*) = 1 - \frac{p_2}{p_1} = 0.6 \Rightarrow \ x^* \ = 60 + K_{0.4}(20) = 60 + 0.25(20) = 65$

$p_2 = \$500 \Rightarrow F(x^*) = 1 - \frac{p_2}{p_1} = 0.5 \Rightarrow \ x^* \ = 60 + K_{0.5}(20) = 60 + 0(20) = 60$

$p_2 = \$600 \Rightarrow F(x^*) = 1 - \frac{p_2}{p_1} = 0.4 \Rightarrow \ x^* \ = 60 + K_{0.6}(20)$

$\qquad = 60 - K_{0.4}(20) = 60 - 0.25(20) = 55$

As the discount fare increases, the optimal number x^* of reservation slots for class 1 customers decreases.

18.8-2.

The capacity L is 1000, the price p_1 paid by luxury-seeking customers is $\$20,000$ and the discount fare is $p_2 = \$12,000$. The demand D by luxury-seeking customers has a normal distribution with mean 400 and standard deviation 100.

$$F(x^*) = 1 - \tfrac{p_2}{p_1} = 0.4$$
$$\Rightarrow\ x^* = 400 + K_{0.6}(100) = 400 - K_{0.4}(100) = 400 - 0.25(100) = 150$$
$$\Rightarrow\ L - x^* = 1000 - 150 = 850$$

Hence, the maximum number of cabins that should be sold at the discount fare is 850.

18.8-3.

$L = 100, p_1 = 300, p_2 = 100$

The demand D for full-fare tickets has a uniform distribution on integers between 31 and 50.

$$p_2 \leq p_1 P(D \geq x^*) \Leftrightarrow \tfrac{p_2}{p_1} = \tfrac{1}{3} \leq \tfrac{50 - x^* + 1}{20} \Leftrightarrow x^* \leq 51 - \tfrac{20}{3} = 44.33$$

$$p_2 > p_1 P(D \geq x^* + 1) \Leftrightarrow \tfrac{p_2}{p_1} = \tfrac{1}{3} > \tfrac{50 - x^*}{20} \Leftrightarrow x^* > 50 - \tfrac{20}{3} = 43.33$$

Thus $x^* = 44$ slots should be reserved to full-fare customers.

18.8-4.

$L = 150, p = 0.8, r = \$300, s = \$1500$

$$P\{D(n^*) \geq 150\} = \tfrac{r}{sp} = 0.25$$

$D(n)$ is normally distributed with mean $0.8n$ and standard deviation $0.4\sqrt{n}$.

$$K_{0.25} = \tfrac{150 - 0.8n}{0.4\sqrt{n}} \Rightarrow 0.67 = \tfrac{150 - 0.8n}{0.4\sqrt{n}} \Rightarrow 0.8n + 0.268\sqrt{n} - 150 = 0$$

$$\Rightarrow \sqrt{n} = \tfrac{-0.268 + \sqrt{(0.268)^2 - 4(0.8)(-150)}}{1.6} = 13.527 \Rightarrow n^* = (13.527)^2 = 183$$

We chose the smallest integer that is greater than $(13.527)^2$ to determine n^*. Hence, the number of reservations to accept for this flight is 183.

18.8-5.

$L = 125, r = \$250, s = 300 + 300 = \$600$

Finding the optimal overbooking requires finding the smallest integer n with $\Delta E(P(n))$ nonpositive.

$$\Delta E(P(n)) \quad = 250 - 600\left[\sum_{d=126}^{n} (d - 125)[P\{D(n+1) = d\} - P\{D(n) = d\}]\right]$$

Let X denote the random variable associated with no-shows.

$$\begin{aligned} \Delta E(P(n)) \quad &= 250 - 600\left[\sum_{k=0}^{n-126} (n - k - 125)[P\{X = k+1\} - P\{X = k\}]\right] \\ &= 250 - 600\left[\sum_{k=0}^{n-126} P\{X = k+1\}\right] = 250 - 600P\{X \leq n - 125\} \end{aligned}$$

Then the problem is to find the smallest n such that

$P\{X \le n - 125\} \ge \frac{250}{600} = 0.417.$

x	0	1	2	3	4	5	6	7	8	9
$P\{X \le x\}$	0	0.05	0.15	0.25	0.4	0.6	0.75	0.85	0.95	1

From the cumulative distribution of X, n^* is found to be $125 + 5 = 130$, so 5 reservations can be accepted in addition to the capacity.

18.8-6.

$L = 3, p = 0.5, r = \$1000, s = \$5000$

To determine the optimal number of reservations to accept, we need to find the smallest integer n such that

$$spP\{D(n) \ge 3\} \ge r \Leftrightarrow P\{D(n) \ge 3\} \ge 0.4 \Leftrightarrow P\{D(n) \le 2\} \le 0.6$$

$$\Leftrightarrow \left[\binom{n}{0} + \binom{n}{1} + \binom{n}{2}\right] 0.5^n \le 0.6$$

$$\Leftrightarrow (n^2 + n + 2)0.5^{n+1} \le 0.6$$

A first guess can be $n = 6$, since then the average number of customers with reservation and who actually show up is $L = 3$. It satisfies

$(6^2 + 6 + 2)0.5^7 < 0.6,$

so $spP\{D(6) \ge 3\} \ge r$. This suggests $n^* \le 6$. Now consider $n = 5$.

$(5^2 + 5 + 2)0.5^6 < 0.6,$

so $spP\{D(5) \ge 3\} \ge r$. Then $n^* \le 5$. For $n = 4$,

$(4^2 + 4 + 2)0.5^5 > 0.6,$

so $spP\{D(4) \ge 3\} < r$. Hence the optimal number of reservations to accept is 5.

18.8-7.

$L = 100, p = 0.9, r = \$3000, s = \$20000$

$P\{D(n^*) \ge 100\} = \frac{r}{sp} = \frac{1}{6} \approx 0.167$

$D(n^*)$ is normally distributed with mean $0.9n$ and standard deviation $0.3\sqrt{n}$.

$K_{0.167} = \frac{100-0.9n}{0.3\sqrt{n}} \Rightarrow 0.97 = \frac{100-0.9n}{0.3\sqrt{n}}$

$\Rightarrow \sqrt{n} = \frac{-0.291+\sqrt{(0.291)^2-4(0.9)(-100)}}{1.8} \approx 10.38 \Rightarrow n^* = (10.38)^2 = 108$

The number of reservations to accept is 108, so 8 reservations should be overbooked.

18.8-8.

Answers will vary.

18.9-1.

Answers will vary.

18.9-2.

Answers will vary.

CASES

CASE 18.1 Brushing Up on Inventory Control

(a) Robert's problem can be solved using the basic EOQ model, with the data:

$$D = 12(250) = 3,000,\ K = 18.75/3 = 6.25,$$

$$h = 0.12(1.25) = 0.15,\ L = 0,\ WD = 12(30) = 360$$

	Data				Results
D =	3,000	(demand/year)		Reorder Point	0
K =	$6.25	(setup cost)			
h =	$0.15	(unit holding cost)		Annual Setup Cost	$37.50
L =	0	(lead time in days)		Annual Holding Cost	$37.50
WD =	360	(working days/year)		Total Variable Cost	$75.00
	Decision				
Q =	500	(optimal order quantity)			

Robert should order 500 toothbrushes 6 times per year.

(b) EOQ model with $L = 6$ days

	Data				Results
D =	3,000	(demand/year)		Reorder Point	41.7
K =	$6.25	(setup cost)			
h =	$0.15	(unit holding cost)		Annual Setup Cost	$37.50
L =	5	(lead time in days)		Annual Holding Cost	$37.50
WD =	360	(working days/year)		Total Variable Cost	$75.00
	Decision				
Q =	500	(optimal order quantity)			

Whenever the inventory drops down to 50, Robert should place an order for 500 toothbrushes. He needs to place 6 orders per year.

(c) Planned shortages with $p = \$1.50$/unit

	Data				Results
D =	3,000	(demand/year)		Max Inventory Level	476.7
K =	$6.25	(setup cost)			
h =	$0.15	(unit holding cost)		Annual Setup Cost	$35.75
p =	$1.50	(unit shortage cost)		Annual Holding Cost	$32.50
				Annual Shortage Cost	$3.25
	Decision			Total Variable Cost	$71.51
Q =	524	(optimal order quantity)			
S =	48	(optimal maximum shortage)			

Robert should order about 524 toothbrushes. Since the lead time is 6 days, the reorder point is $-47.67 + 6(3000/360) = 2.33$. The maximum shortage size is approximately 48.

(d) Two extreme cases: $p = \$0.85$/unit and $p = \$25$/unit

	Data				Results
D =	3,000	(demand/year)		Max Inventory Level	461.0
K =	$6.25	(setup cost)			
h =	$0.15	(unit holding cost)		Annual Setup Cost	$34.57
p =	$0.85	(unit shortage cost)		Annual Holding Cost	$29.39
				Annual Shortage Cost	$5.19
	Decision			Total Variable Cost	$69.15
Q =	542	(optimal order quantity)			
S =	81	(optimal maximum shortage)			

The reorder point when $p = \$0.85$/unit is $-81 + 6(3000/360) = -31$.

	Data				Results
D =	3,000	(demand/year)		Max Inventory Level	498.5
K =	$6.25	(setup cost)			
h =	$0.15	(unit holding cost)		Annual Setup Cost	$37.39
p =	$25.00	(unit shortage cost)		Annual Holding Cost	$37.17
				Annual Shortage Cost	$0.22
	Decision			Total Variable Cost	$74.78
Q =	501	(optimal order quantity)			
S =	3	(optimal maximum shortage)			

The reorder point when $p = \$25$/unit is $-3 + 6(3000/360) = 47$. This suggests that as the shortage cost increases, the reorder point increases.

(e) EOQ model with quantity discounts, with three prices $\$1.25$, $\$1.15$ and $\$1.00$ and holding cost rate $I = 0.12$.

	Data								
D =	3,000	(demand/year)							
K =	$6.25	(setup cost)							
I =	0.12	(inventory holding cost rate)							
N =	3	(number of discount categories)							
						Annual	Annual	Annual	Total
		Range of order quantities				Purchase	Setup	Holding	Variable
Category	Price	Lower Limit	Upper Limit	EOQ	Q*	Cost	Cost	Cost	Cost
1	$1.25	0	499	500	499	$3,750	$38	$37	$3,825
2	$1.15	500	999	521	521	$3,450	$36	$36	$3,522
3	$1.00	1,000	10,000,000	559	1,000	$3,000	$19	$60	$3,079
			Results						
		Optimal Q	1,000						
	Total Variable Cost		$3,079						

The optimal order quantity is $Q = 1,000$ and Robert should order 3 times a year.

CASE 18.2 TNT: Tackling Newsboy's Teaching

For the analysis of this case, we use the template for perishable products.

(a) First we need to determine the optimal service level for Howie. The unit sale price is $5, the unit purchase cost is $3, and the unit salvage value is $0.5 \times \$3 = \$1$.

	Data			Results
Unit Sales Price	$5		Cost of Overordering	$2
Unit Purchase Cost	$3		Cost of Underordering	$2
Unit Salvage Value	$1		Optimal Service Level	0.5

Since Talia assumes that the demand is uniformly distributed between 120 and 420 sets, Howie should order $120 + 0.5 \times 300 = 270$ sets.

(b) If Leisure Limited refunds 75% of the purchase cost, then the unit salvage value for a returned set becomes $0.75 \times \$3 - \$0.5 = \$1.75$. We determine the new optimal service level.

	Data			Results
Unit Sales Price	$5		Cost of Overordering	$1
Unit Purchase Cost	$3		Cost of Underordering	$2
Unit Salvage Value	$1.75		Optimal Service Level	0.615

The order quantity is now $120 + 0.615 \times 300 = 304.5$. Note that Howie can now order more sets at one time than he could under the scenario of part (a) because he is not punished as severely as before when he fails to sell all sets.

When the refund is 25%, the unit salvage cost is $\$0.25$.

	Data			Results
Unit Sales Price	$5		Cost of Overordering	$3
Unit Purchase Cost	$3		Cost of Underordering	$2
Unit Salvage Value	$0.25		Optimal Service Level	0.421

Consequently, the order quantity is reduced to $120 + 0.421 \times 300 = 246.3$. In this case, Howie should purchase fewer sets at one time (compared to previous scenarios), since he is punished more severely for failing to sell all the sets.

(c) The unit sale price is now $6 and there is a 50% refund on returned firecracker sets.

	Data			Results
Unit Sales Price	$6		Cost of Overordering	$2
Unit Purchase Cost	$3		Cost of Underordering	$3
Unit Salvage Value	$1		Optimal Service Level	0.6

However, if Howie raises the price of a firecracker set, one would expect a decrease in the demand for his sets, so Talia should not use the same uniform demand distribution that she used for her previous calculations of the optimal order quantity.

(d) Talia's strategy for estimating the demand is overly simplistic. She makes the very simplifying assumption that the demand is uniformly distributed between 120 and 420 sets. However, she does not take into account that the demand depends on the price of a firecracker set. She should expect that stands charging less than the average price of $\$5$ per set typically sell more sets than stands charging more. Talia should call Buddy again to try to obtain more detailed information such as the range of sales and the average sale of stands charging $\$5$ or $\$6$ per set.

Talia should also reevaluate her assumption that the demand is uniformly distributed. She should check how her forecasts change if she uses other demand distribution like normal distribution.

CASE 18.3 Jettisoning Surplus Stock

(a) We can use Excel to compute the sample mean and variance.

Observations													Mean	Std. Dev.
25	31	18	22	40	19	38	21	25	36	34	28	27	28	7.29154

Hence, the sample mean is 28 and the sample variance is $7.29154^2 \approx 53.1667$.

(b) Based on the findings of Scarlett Windermere, American Aerospace can use an (R, Q) policy for the inventory of part 10003487. The assumptions of the model are satisfied.

1- The part is a stable product.

2- Its inventory level is under continuous review.

3- While the production of the part itself has no lead time, it is typically delayed by the lead time of 1.5 months of the little steel part. Assume the lead time is 1.5 months.

4- The demand for the part is the same as for the jet engine MX332, since it is used only for this particular engine. Hence, assume that the demand is approximately normally distributed with mean 28 and variance 53.1667.

5- Excess demand is backlogged.

6- There is a fixed setup cost $K = \$5,800$, a holding cost $h = \$750$ and a shortage cost $p = \$3,250$.

Note that the average demand per year is $12 \times 28 = 336$, the average demand during the lead time is $1.5 \times 28 = 42$ and it has a standard deviation of $1.5 \times 7.29154 = 10.93732$.

	Data				Results
D =	336	(average demand/unit time)		Q =	80
K =	$5,800	(setup cost)		R =	53
h =	$750	(unit holding cost)			
p =	$3,250	(unit shortage cost)			
L =	0.85	(service level)			
Demand During Lead Time					
Distribution	Normal				
mean =	42				
stand. dev. =	10.9				

American Aerospace should implement the (R, Q) policy with $R = 53$ and $Q = 80$.

(c) The average inventory just before an order arrives is $53 - 42 = 11$ and the one just after an order has arrived is $11 + 80 = 91$. Then, the average inventory is $(11 + 91)/2 = 51$, with an average holding cost of $51(750) = \$38,250$ per year. The average number of setups in a year is $336/80 = 4.2$, with a resulting average setup cost of $4.2(5,800) = \$24,360$ per year.

(d) The new service level is $L = 0.95$.

	Data				Results
D =	336	(average demand/unit time)		Q =	80
K =	$5,800	(setup cost)		R =	60
h =	$750	(unit holding cost)			
p =	$3,250	(unit shortage cost)			
L =	0.95	(service level)			
Demand During Lead Time					
Distribution	Normal				
mean =	42				
stand. dev. =	10.9				

$R = 60$ and $Q = 80$. The average inventory just before an order arrives is $80 - 60 = 20$ and just after an order has arrived is $20 + 80 = 100$, so the average inventory is 60 and the resulting average inventory holding cost is $60(750) = \$45,000$ per year. Note that the average holding cost has increased substantially. This is a consequence of increasing the safety stock to 20 from 11. The average number of setups per year is still 4.2 and the average setup cost is $\$24,360$ per year.

(e) Scarlett's independent analysis of the stationary part 10003487 can be justified since there is only one jet engine that needs this part and this part appears to be the bottleneck in the production process. However, in general, a stationary part is used for several jet engines, so the demand for stationary parts depends on the demand for several jet engines and a stock-out in one stationary part affects the demand for other parts. These interdependencies cannot be captured by an independent analysis of each part; therefore, Scarlett's approach is most likely to result in rather inaccurate inventory policies for many other stationary parts.

(f) Scarlett could try to forecast the demand for jet engines based on sales data from previous years.