CHAPTER 14: METAHEURISTICS

14.1-1.

(a)

Tours	Distance	Tours	Distance
1-2-3-4-5-1	34	1-3-2-4-5-1	32
1-2-3-5-4-1	34	1-3-2-5-4-1	26
1-2-4-3-5-1	36	1-3-4-2-5-1	28
1-2-4-5-3-1	31	1-3-5-2-4-1	28
1-2-5-3-4-1	30	1-4-2-3-5-1	37
1-2-5-4-3-1	25	1-4-3-2-5-1	31

Optimal Solution: 1-2-5-4-3-1 (or the reverse 1-3-4-5-2-1)

(b) Start with the initial trial solution 1-2-3-4-5-1. There are three possible sub-tour reversals that improve upon this solution.

	1-2-3-4-5-1	Distance = 34
Reverse 2-3	1-3-2-4-5-1	Distance = 32
Reverse 2-3-4	1-4-3-2-5-1	Distance = 31
Reverse 3-4-5	1-2-5-4-3-1	Distance = 25

Choose 1-2-5-4-3-1 as the next trial solution. There is no sub-tour reversal that improves upon this solution. The tour 1-2-5-4-3-1 is optimal.

(c) Start with the initial trial solution 1-2-4-3-5-1. There are four possible sub-tour reversals that improve upon this solution.

	1-2-4-3-5-1	Distance = 36
Reverse 4-3	1-2-3-4-5-1	Distance = 34
Reverse 3-5	1-2-4-5-3-1	Distance = 31
Reverse 2-4-3	1-3-4-2-5-1	Distance = 28
Reverse 4-3-5	1-2-5-3-4-1	Distance = 30

Choose 1-3-4-2-5-1 as the next trial solution. There is only one possible sub-tour reversal that improves upon this solution.

	1-3-4-2-5-1	Distance = 28
Reverse 2-5	1-3-4-5-2-1	Distance = 25

Choose 1-3-4-5-2-1 as the next trial solution. There is no sub-tour reversal that improves upon this. The solution 1-3-4-5-2-1 is optimal.

(d) Start with the initial trial solution 1-4-2-3-5-1. There are five possible sub-tour reversals that improve upon this solution.

	1-4-2-3-5-1	Distance = 37
Reverse 2-4	1-2-4-3-5-1	Distance = 36
Reverse 2-3	1-4-3-2-5-1	Distance = 31
Reverse 3-5	1-4-2-5-3-1	Distance = 28
Reverse 4-2-3	1-3-2-4-5-1	Distance = 32
reverse 2-3-5	1-4-5-3-2-1	Distance = 34

Choose 1-4-2-5-3-1 as the next trial solution. There is only one possible sub-tour reversal that improves upon this solution.

	1-4-2-5-3-1	Distance = 28
Reverse 2-5	1-4-5-2-3-1	Distance = 26

Choose 1-4-5-2-3-1 as the next trial solution. There is one possible sub-tour reversal that improves upon this.

	1-4-5-2-3-1	Distance = 26
Reverse 4-5-2	1-2-5-4-3-1	Distance = 25

Choose 1-2-5-4-3-1 as the next trial solution. There is no sub-tour reversal that improves upon this. The solution 1-2-5-4-3-1 is optimal.

14.1-2.

(a) If the second reversal were chosen, the next trial solution would be 1-2-3-5-4-6-7-1 and there is no sub-tour reversal that gives an improvement.

(b) Start with the initial trial solution 1-2-4-5-6-7-3-1. There are two possible sub-tour reversals that improve upon this solution.

	1-2-4-5-6-7-3-1	Distance = 69
Reverse 5-6	1-2-4-6-5-7-3-1	Distance = 66
Reverse 2-4-5-6-7	1-7-6-5-4-2-3-1	Distance = 68

Choose 1-2-4-6-5-7-3-1 as the next trial solution. There is only one possible sub-tour reversal that improves upon this.

	1-2-4-6-5-7-3-1	Distance = 66
Reverse 5-7	1-2-4-6-7-5-3-1	Distance = 63

Choose 1-2-4-6-7-5-3-1 as the next trial solution. This is an optimal solution.

14.1-3.

(a)

Tours	Distance	Tours	Distance
1-2-3-4-5-6-1	64	1-2-6-3-4-5-1	69
1-2-3-4-6-5-1	59	1-5-2-3-4-6-1	56
1-2-3-6-4-5-1	67	1-5-2-4-3-6-1	61
1-2-4-4-6-5-1	64	1-6-2-3-4-5-1	63
1-2-5-4-3-6-1	67	1-6-3-2-4-5-1	66

Optimal Solution: 1-5-2-3-4-6-1 (or the reverse 1-6-4-3-5-2-1)

(b) Start with the initial trial solution 1-2-3-4-5-6-1. There are two possible sub-tour reversals that improve upon this solution.

	1-2-3-4-5-6-1	Distance = 64
Reverse 5-6	1-2-3-4-6-5-1	Distance = 59
Reverse 2-3-4-5	1-5-4-3-2-6-1	Distance = 63

Choose 1-2-3-4-6-5-1 as the next trial solution. There is no sub-tour reversal that improves upon this solution.

(c) Start with the initial trial solution 1-2-5-4-3-6-1. There are two possible sub-tour reversals that improve upon this solution.

	1-2-5-4-3-6-1	Distance = 67
Reverse 2-5	1-5-2-4-3-6-1	Distance = 61
Reverse 5-4-3	1-2-3-4-5-6-1	Distance = 64

Choose 1-5-2-4-3-6-1 as the next trial solution. There is no sub-tour reversal that improves upon this solution.

14.2-1.

Sears logistics services (SLS) provides delivery with its fleet of over 1,000 vehicles. Sears product services (SPS) offers home service with its fleet of 12,500 vehicles and technicians. A customer who asks for delivery or home service is given a day and a time window based on customer preferences and working schedule in the region where the customer is located. In either case, the goal is to generate efficient routes for the vehicles and to provide customers with accurate and convenient time windows while minimizing the operational costs. Both problems are instances of vehicle routing problem with time windows (VRPTW). A basic VRPTW determines routes for M vehicles, each starting at the depot and returning to the depot after visiting a subset of customers in some order. Every customer is visited by exactly one vehicle. The capacity constraints of the vehicles and the time windows imposed by customers should be met. The objective is to minimize the total cost. The problems faced by SLS and SPS differ from the basic VRPTW in that they include additional constraints. For instance, in the case of SPS, technicians' skills need to be considered in assigning service orders to them. In both cases, there may be restrictions on total route times and travel times between any two locations. Hence, the problem is a complex one and necessitates the use of a solution procedure that can provide good solutions in acceptable time.

To solve the problem, first an initial route is found for each vehicle, then unassigned stops are inserted into a route. This solution is improved using various local heuristic techniques. In order not to be stuck at local optima, the procedure is enhanced with tabu search technique. Once a stop in a route is relocated, the move is included in a tabu list and remains prohibited for a number of iterations unless the objective function value it offers exceeds the best value obtained up to that iteration.

Financial benefits of this study include $9 million in one-time savings and over $42 million in annual savings. The savings result from the reduction in travel times, mileage and routing times. Sears now offers more timely delivery of merchandise and home service, so more reliable customer service. The utilization of the fleets is improved. The routing process became faster and the facility, equipment and personnel costs related to routing decreased. Since the problem can be solved quickly, Sears can respond to disruptions and adjust its schedules more efficiently.

14.2-2.

Start with the initial trial solution with links AB, AC, AE, CD, which costs 232.

Iteration 1:

Add	Delete	Cost
BC	AB	138
	AC	246
BD	AB	56
	AC	164
	CD	268
DE	AC	152
	AE	240
	CD	256

Adding BD and deleting AB results in the lowest cost, so choose inserting links AC, AE, BD. CD. In fact, this is the optimal solution.

14.2-3.

Start with the initial trial solution with links AB, AD, BE, CD, which costs 390.

Iteration 1: Minimum local search

Add	Delete	Cost
AC	AD	185
	CD	275
CE	AB	275
	AD	180
	CD	270
	BE	365

Current solution: AB, BE, CD, CE.
Tabu list: CE

Iteration 2: Minimum local search

Add	Delete	Cost
DE	CD	95

Current solution: AB, BE, CE, DE
Tabu list: CE, DE

Iteration 3: Minimum local search

Add	Delete	Cost
AC	BE	75

The solution AB, AC, CE, DE is optimal.

14.2-4.

Start with the initial trial solution with links OA, AB, BC, BE, ED, DT, which costs 314.

Iteration 1: Minimum local search

Add	Delete	Cost
ET	DE	122

Current solution: OA, AB, BC, BE, ET, DT
Tabu list: ET

Iteration 2: Minimum local search

Add	Delete	Cost
CE	BC	23

The solution OA, AB, CE, BE, ET, DT is optimal.

14.2-5.

Traveling Salesman Problems:
Number of Cities: 5

City	1	2	3	4	5
1	0	8	4	8	11
2	8	0	5	6	3
3	4	5	0	4	7
4	8	6	4	0	6
5	11	3	7	6	0

Tabu Search Algorithm:

Initial trial solution: 1-2-4-3-5-1

Iteration	Trial Solution	Distance	Tabu List
0	1-2-4-3-5-1	36.0	
1	1-3-4-2-5-1	28.0	1-3,2-5
2	1-3-4-5-2-1	25.0	1-3,2-5,4-5,2-1
3	1-3-2-5-4-1	26.0	4-5,2-1,3-2,4-1
4	1-5-2-3-4-1	31.0	3-2,4-1,1-5,3-4
5	1-5-4-3-2-1	34.0	1-5,3-4,5-4,2-1

Best Distance = 25.0 Best Solution = 1-3-4-5-2-1

14.2-6.

Traveling Salesman Problems:
Number of Cities: 8

City	1	2	3	4	5	6	7	8
1	0	14	15	--	--	--	--	17
2	14	0	13	14	20	--	--	21
3	15	13	0	11	21	17	9	9
4	--	14	11	0	11	10	8	20
5	--	20	21	11	0	15	18	--
6	--	--	17	10	15	0	9	--
7	--	--	9	8	18	9	0	13
8	17	21	9	20	--	--	13	0

(a) Initial trial solution: 1-2-3-4-5-6-7-8-1

Iteration	Trial Solution	Distance	Tabu List
0	1-2-3-4-5-6-7-8-1	103.0	
1	1-3-2-4-5-6-7-8-1	107.0	1-3,2-4
2	1-3-8-7-6-5-4-2-1	100.0	1-3,2-4,3-8,2-1
3	1-8-3-7-6-5-4-2-1	98.0	3-8,2-1,1-8,3-7
4	1-8-3-7-6-4-5-2-1	99.0	1-8,3-7,6-4,5-2
5	1-8-3-7-4-6-5-2-1	102.0	6-4,5-2,7-4,6-5
6	1-8-3-4-7-6-5-2-1	103.0	7-4,6-5,3-4,7-6

Best Distance = 98.0 Best Solution = 1-8-3-7-6-5-4-2-1

(b) Initial trial solution: 1-2-5-6-7-4-8-3-1

Iteration	Trial Solution	Distance	Tabu List
0	1-2-5-6-7-4-8-3-1	110.0	
1	1-2-5-6-7-4-3-8-1	103.0	4-3,8-1
2	1-2-5-6-4-7-3-8-1	102.0	4-3,8-1,6-4,7-3
3	1-2-5-4-6-7-3-8-1	99.0	6-4,7-3,5-4,6-7
4	1-2-4-5-6-7-3-8-1	98.0	5-4,6-7,2-4,5-6
5	1-2-4-5-6-7-8-3-1	100.0	2-4,5-6,7-8,3-1
6	1-8-7-6-5-4-2-3-1	107.0	7-8,3-1,1-8,2-3
7	1-8-7-6-5-4-3-2-1	103.0	1-8,2-3,4-3,2-1

Best Distance = 98.0 Best Solution = 1-2-4-5-6-7-3-8-1

(c) Initial trial solution: 1-3-2-5-6-4-7-8-1

Iteration	Trial Solution	Distance	Tabu List
0	1-3-2-5-6-4-7-8-1	111.0	
1	1-3-8-7-4-6-5-2-1	104.0	3-8,2-1
2	1-3-8-7-6-4-5-2-1	101.0	3-8,2-1,7-6,4-5
3	1-8-3-7-6-4-5-2-1	99.0	7-6,4-5,1-8,3-7
4	1-8-3-7-6-5-4-2-1	98.0	1-8,3-7,6-5,4-2
5	1-8-3-7-5-6-4-2-1	106.0	6-5,4-2,7-5,6-4
6	1-3-8-7-5-6-4-2-1	108.0	7-5,6-4,1-3,8-7
7	1-3-8-7-4-6-5-2-1	104.0	1-3,8-7,7-4,5-2

Best Distance = 98.0 Best Solution = 1-8-3-7-6-5-4-2-1

14.2-7.

Traveling Salesman Problems:
Number of Cities: 10

City	1	2	3	4	5	6	7	8	9	10
1	0	13	25	15	21	9	19	18	8	15
2	13	0	26	21	29	21	31	23	16	10
3	25	26	0	11	18	23	28	44	34	35
4	15	21	11	0	10	13	19	34	24	29
5	21	29	18	10	0	12	11	37	27	36
6	9	21	23	13	12	0	10	25	14	25
7	19	31	28	19	11	10	0	32	23	35
8	18	23	44	34	37	25	32	0	10	16
9	8	16	34	24	27	14	23	10	0	14
10	15	10	35	29	36	25	35	16	14	0

(a)

Iteration	Trial Solution	Distance	Tabu List
0	1-2-3-4-5-6-7-8-9-10-1	153.0	
1	1-9-8-7-6-5-4-3-2-10-1	144.0	1-9,2-10
2	1-9-8-10-2-3-4-5-6-7-1	132.0	1-9,2-10,8-10,7-1
3	1-9-8-10-2-3-4-5-7-6-1	121.0	8-10,7-1,5-7,6-1
4	1-9-8-10-2-4-3-5-7-6-1	124.0	5-7,6-1,2-4,3-5
5	1-8-9-10-2-4-3-5-7-6-1	132.0	2-4,3-5,1-8,9-10
6	1-8-9-6-7-5-3-4-2-10-1	138.0	1-8,9-10,9-6,10-1

Best Distance = 121.0 Best Solution = 1-9-8-10-2-3-4-5-7-6-1

(b)

Iteration	Trial Solution	Distance	Tabu List
0	1-3-4-5-7-6-9-8-10-2-1	130.0	
1	1-4-3-5-7-6-9-8-10-2-1	128.0	1-4,3-5
2	1-4-3-5-7-6-2-10-8-9-1	130.0	1-4,3-5,6-2,9-1
3	1-6-7-5-3-4-2-10-8-9-1	124.0	6-2,9-1,1-6,4-2
4	1-6-7-5-4-3-2-10-8-9-1	121.0	1-6,4-2,5-4,3-2
5	1-6-7-5-4-3-2-10-9-8-1	129.0	5-4,3-2,10-9,8-1
6	1-10-2-3-4-5-7-6-9-8-1	135.0	10-9,8-1,1-10,6-9
7	1-9-6-7-5-4-3-2-10-8-1	134.0	1-10,6-9,1-9,10-8

Best Distance = 121.0 Best Solution = 1-6-7-5-4-3-2-10-8-9-1

(c)

Iteration	Trial Solution	Distance	Tabu List
0	1-9-8-10-2-4-3-6-7-5-1	141.0	
1	1-9-8-10-2-4-3-5-7-6-1	124.0	3-5,6-1
2	1-9-8-10-2-3-4-5-7-6-1	121.0	3-5,6-1,2-3,4-5
3	1-8-9-10-2-3-4-5-7-6-1	129.0	2-3,4-5,1-8,9-10
4	1-8-9-6-7-5-4-3-2-10-1	135.0	1-8,9-10,9-6,10-1
5	1-8-10-2-3-4-5-7-6-9-1	134.0	9-6,10-1,8-10,9-1

Best Distance = 121.0 Best Solution = 1-9-8-10-2-3-4-5-7-6-1

14.3-1.

$Z_c = 30, T = 2$

(a) Maximization problem:
$Z_n = 29$, $x = (Z_n - Z_c)/T = -0.5$, P$\{$acceptance$\} = e^x = 0.607$
$Z_n = 34$, $Z_n > Z_c$, P$\{$acceptance$\} = 1$
$Z_n = 31$, $Z_n > Z_c$, P$\{$acceptance$\} = 1$
$Z_n = 24$, $x = (Z_n - Z_c)/T = -3$, P$\{$acceptance$\} = e^x = 0.05$

(b) Minimization problem:
$Z_n = 29$, $Z_n < Z_c$, P$\{$acceptance$\} = 1$
$Z_n = 34$, $x = (Z_c - Z_n)/T = -2$, P$\{$acceptance$\} = e^x = 0.135$
$Z_n = 31$, $x = (Z_c - Z_n)/T = -0.5$, P$\{$acceptance$\} = e^x = 0.607$
$Z_n = 24$, $Z_n < Z_c$, P$\{$acceptance$\} = 1$

14.3-2.

Because of the randomness in the algorithm, the output will vary.

14.3-3.

(a) Initial trial solution: 1-2-3-4-5-1, $Z_c = 34$, $T_1 = 0.2$*$Z_c = 6.8$

0.0000 - 0.3332	Sub-tour begins in slot 2.
0.3333 - 0.6666	Sub-tour begins in slot 3.
0.6667 - 0.9999	Sub-tour begins in slot 4.

The random number is 0.09656: choose a sub-tour that begins in slot 2. The sub-tour needs to end either in slot 3 or slot 4.

0.0000 - 0.4999	Sub-tour ends in slot 3.
0.5000 - 0.9999	Sub-tour ends in slot 4.

The random number is 0.96657: choose a sub-tour that ends in slot 4.
Reverse 2-3-4 to obtain the new solution 1-4-3-2-5-1, $Z_n = 31$. Since $Z_n < Z_c$, accept this solution as the next trial solution.

(b) Because of the randomness in the algorithm, the output will vary.

14.3-4.

Because of the randomness in the algorithm, the output will vary.

14.3-5.

Because of the randomness in the algorithm, the output will vary.

14.3-6.

Because of the randomness in the algorithm, the output will vary.

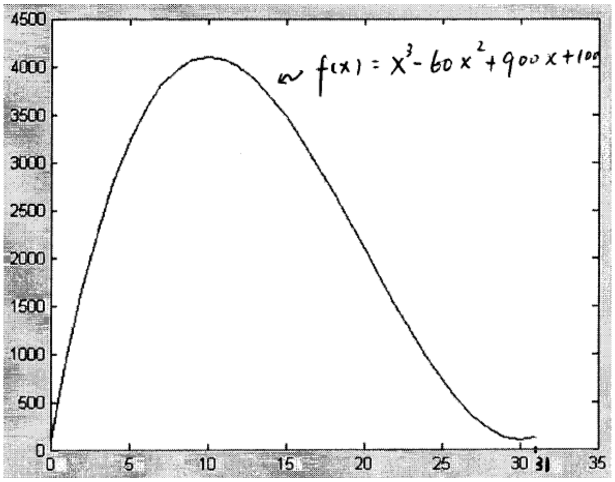
14.3-7.

(a) $f(x) = x^3 - 60x^2 + 900x + 100$
$f'(x) = 3x^2 - 120x + 900$ and $f''(x) = 6x - 120$

Stationary Points: $f'(x^*) = 0 \Rightarrow x^*$is either 10 or 30 (stationary points of f).
$f''(10) = -60 < 0 \Rightarrow x^* = 10$ is a local maximum.
$f''(30) = 60 > 0 \Rightarrow x^* = 30$ is a local minimum.

End Points: $f'(0) = 900 > 0 \Rightarrow x = 0$ is a local minimum.
$f'(31) = 63 > 0 \Rightarrow x = 31$ is a local minimum.

(b)



(c) $x = 15.5$, $f(x) = Z_c = 3558.9$, $T = 0.2Z_c = 671.775$
$L = 0$, $U = 31$, $\sigma = (U - L)/6 = 5.167$

The random number obtained from Table 20.3 is 0.09656. From Appendix 5,

P$\{Y \leq -1.315\} \simeq 0.09656$,

with Y a standard Normal random variable, $N(0, 5.167) = -1.315 \cdot 5.167 = -6.79$.

$x = 15.5 + N(0, 5.167) = 8.71$, $Z_n = f(x) = 4047.6$

Since $Z_n > Z_c$, accept $x = 8.71$ as the next trial solution.

(d) Because of the randomness in the algorithm, the output will vary.

14.3-8.

The nonconvex problem is to:

maximize	$0.5x^5 - 6x^4 + 24.5x^3 - 39x^2 + 20x$
subject to	$0 \leq x \leq 5$.

(a) $x = 2.5$, $f(x) = Z_c = 3.5156$, $T = 0.2Z_c = 0.7031$

$L = 0$, $U = 5$, $\sigma = (U - L)/6 = 0.8333$

The random number obtained from Table 20.3 is 0.09656. From Appendix 5,

P$\{Y \leq -1.315\} \simeq 0.09656$,

with Y a standard Normal random variable, $N(0, 0.8333) = -1.315 \cdot 0.8333 = -1.0958$.

$x = 2.5 + N(0, 0.8333) = 1.4042$, $Z_n = f(x) = -1.5782$

Since $(Z_n - Z_c)/T = -7.2488$, the probability of accepting $x = 1.4042$ as the next trial solution is P$\{\text{acceptance}\} = e^{-7.2488} = 0.00071$. From Table 20.3, the next random number is $0.96657 > 0.00071$, so we reject $x = 1.4042$ as the next trial solution.

(b) Because of the randomness in the algorithm, the output will vary.

14.3-9.

(a) $x = 25$, $f(x) = Z_c = 6,640,625$, $T = 0.2Z_c =$ 1,328,125

$L = 0$, $U = 50$, $\sigma = (U - L)/6 = 8.333$

The random number obtained from Table 20.3 is 0.09656. From Appendix 5,

P$\{Y \leq -1.315\} \simeq 0.09656$,

with Y a standard Normal random variable, $N(0, 8.333) = -1.315 \cdot 8.333 = -10.958$.

$x = 25 + N(0, 8.333) = 14.042$, $Z_n = f(x) = 7,995,655$

Since $Z_n > Z_c$, accept the new solution.

(b) Because of the randomness in the algorithm, the output will vary.

14.3-10.

(a) $(x_1, x_2) = (18, 25)$, $f(x_1, x_2) = Z_c = 133,509.5$, $T = 0.2Z_c = 26,701.9$

$L = (0, 0)$, $U = (36, 50)$

$\sigma_1 = (36 - 0)/6 = 6$

$\sigma_2 = (50 - 0)/6 = 8.333$

The random number obtained from Table 20.3 is 0.09656. From Appendix 5,

P$\{Y \leq -1.315\} \simeq 0.09656$,

with Y a standard Normal random variable,

$N(0, 6) = -1.315 \cdot 6 = -7.89$

$x_1 = 18 + N(0, 6) = 10.11$

$N(0, 8.333) = -1.315 \cdot 8.333 = -10.958$

$x_2 = 25 + N(0, 8.333) = 14.042$

This solution is feasible.

$Z_n = f(x) = -107,467$

Since $(Z_n - Z_c)/T = -9.0247$, the probability of accepting this solution as the next trial solution is P$\{$acceptance$\} = e^{-9.0247} = 0.00012$. From Table 20.3, the next random number is $0.96657 > 0.00012$, so we reject $(10.11, 14.042)$ as the next trial solution.

(b) Because of the randomness in the algorithm, the output will vary.

14.4-1.

(a) P1: 010011 and
P2: 100101

Only the last digits agree, the children then become:

C1: xxxxx1 and
C2: xxxxx1,

where x represents the unknown digits. Random numbers are used to identify these unknown digits and let random numbers:

0.00000 - 0.49999 correspond to x = 0,
0.50000 - 0.99999 correspond to x = 1.

Starting from the front of the top row of Table 20.3, the first 10 random numbers are: 0.09656, 0.96657, 0.64842, 0.49222, 0.49506, 0.10145, 0.48455, 0.23505, 0.90430, 0.04180. The corresponding digits are: 0,1,1,0,0,0,0,0,1,0. The children then become:

C1: 011001 and
C2: 000101.

Next, we consider the possibility of mutations. The probability of a mutation in any generation is set at 0.1, and let random numbers

0.00000 - 0.09999 correspond to a mutation,
0.10000 - 0.99999 correspond to no mutation.

Starting from the second row of Table 20.3, we obtain the next 12 random numbers. Accordingly, the 8^{th} and 11^{th} ones correspond to a mutation, so the final conclusion is that the two children are

C1: 011001 and
C2: 010111.

(b) P1: 000010 and
P2: 001101

The first and second digits agree, the children then become:

C1: 00xxxx and
C2: 00xxxx,

where x represents the unknown digits. Random numbers are used to identify these unknown digits and let random numbers:

0.00000 - 0.49999 correspond to x = 0,
0.50000 - 0.99999 correspond to x = 1.

Starting from the front of the top row of Table 20.3, the first 8 random numbers correspond to digits: 0,1,1,0,0,0,0,0. The children then become:

C1: 000110 and
C2: 000000.

Next, we consider the possibility of mutations. The probability of a mutation in any generation is set at 0.1, and let random numbers

0.00000 - 0.09999 correspond to a mutation,
0.10000 - 0.99999 correspond to no mutation.

Use Table 20.3 to obtain the next 12 random numbers. Accordingly, the 2^{nd} and 10^{th} ones correspond to a mutation, so the final conclusion is that the two children are

C1: 010110 and
C2: 000100.

(c) P1: 100000 and
P2: 101000

All but the third digits agree, the children then become:

C1: 10x000 and
C2: 10x000,

where x represents the unknown digits. Random numbers are used to identify these unknown digits and let random numbers:

0.00000 - 0.49999 correspond to x = 0,
0.50000 - 0.99999 correspond to x = 1.

Starting from the front of the top row of Table 20.3, the first 2 random numbers correspond to digits: 0,1. The children then become:

C1: 100000 and
C2: 101000.

Next, we consider the possibility of mutations. The probability of a mutation in any generation is set at 0.1, and let random numbers

0.00000 - 0.09999 correspond to a mutation,
0.10000 - 0.99999 correspond to no mutation.

Use Table 20.3 to obtain the next 12 random numbers. Accordingly, only the 8^{th} one corresponds to a mutation, so the final conclusion is that the two children are

C1: 100000 and
C2: 111000.

14.4-2.

(a)	P1:	1-2-3-4-7-6-5-8-1 and
	P2:	1-5-3-6-7-8-2-4-1

Start from city 1.
Possible links: 1-2, 1-8, 1-5, 1-4

Random numbers:	0.09656	choose 1-2
	0.96657	no mutation

Start from city 2. Current tour: 1-2
Possible links: 2-3, 2-8, 2-4

Random numbers:	0.64842	choose 2-8
	0.49222	no mutation

Start from city 8. Current tour: 1-2-8
Possible links: 8-5, 8-7

Random numbers:	0.49506	choose 8-5
	0.10145	no mutation

Start from city 5. Current tour: 1-2-8-5
Possible links: 5-6, 5-3

Random numbers:	0.48455	choose 5-6
	0.23505	no mutation

Start from city 6. Current tour: 1-2-8-5-6
Possible links: 6-7, 6-7, 6-3

Random numbers:	0.90430	choose 6-3
	0.04180	mutation

Reject 6-3 and consider all other possible links: 6-4, 6-7

Random numbers:	0.24712	choose 6-4

Start from city 4. Current tour: 1-2-8-5-6-4
Possible links: 4-3, 4-7

Random numbers:	0.55799	choose 4-7
	0.60857	no mutation

The only remaining city is 3. Hence, C1 = 1-2-8-5-6-4-7-3-1.

(b)	P1:	1-6-4-7-3-8-2-5-1 and
	P2:	1-2-5-3-6-8-4-7-1

Start from city 1.
Possible links: 1-6, 1-5, 1-2, 1-7

Random numbers:	0.09656	choose 1-6
	0.96657	no mutation

Start from city 6. Current tour: 1-6
Possible links: 6-4, 6-3, 6-8

Random numbers:	0.64842	choose 6-3
	0.49222	no mutation

Start from city 3. Current tour: 1-6-3
Possible links: 3-7, 3-8, 3-5

Random numbers:	0.49506	choose 3-8
	0.10145	no mutation

Start from city 8. Current tour: 1-6-3-8
Possible links: 8-2, 8-4

Random numbers:	0.48455	choose 8-2
	0.23505	no mutation

Start from city 2. Current tour: 1-6-3-8-2
Possible links: 2-5

Random numbers:	0.04180	mutation

Reject 2-5 and consider all other possible links: 2-4, 2-7

Random numbers:	0.24712	choose 2-4

Start from city 4. Current tour: 1-6-3-8-2-4
Possible links: 4-7

Random numbers:	0.60857	no mutation

The only remaining city is 5. Hence, C1 = 1-6-3-8-2-4-7-5-1.

(c) P1: 1-5-7-4-6-2-3-8-1 and
P2: 1-3-7-2-5-6-8-4-1

Start from city 1.
Possible links: 1-5, 1-8, 1-3, 1-4

Random numbers:	0.09656	choose 1-5
	0.96657	no mutation

Start from city 5. Current tour: 1-5
Possible links: 5-7, 5-2, 5-6

Random numbers:	0.64842	choose 5-2
	0.49222	no mutation

Start from city 2. Current tour: 1-5-2
Possible links: 2-6, 2-3, 2-7

Random numbers:	0.49506	choose 2-3
	0.10145	no mutation

Start from city 3. Current tour: 1-5-2-3
Possible links: 3-8, 3-7

Random numbers:	0.48455	choose 3-8
	0.23505	no mutation

Start from city 8. Current tour: 1-5-2-3-8
Possible links: 8-6, 8-4

Random numbers:	0.90430	choose 8-4
	0.04189	mutation

Reject 8-4 and consider all other possible links: 8-6, 8-7

Random numbers:	0.24712	choose 8-6

Start from city 6. Current tour: 1-5-2-3-8-6
Possible links: 6-4

Random numbers:	0.55799	choose 6-4
	0.60857	no mutation

The only remaining city is 7. Hence, C1 = 1-5-2-3-8-6-4-7-1.

14.4-3.

(a) Because of the randomness in the algorithm, the output will vary.

(b) Because of the randomness in the algorithm, the output will vary.

14.4-4.

Integer nonlinear programming: maximize $f(x) = x^3 - 60x^2 + 900$
subject to $0 \le x \le 31$

(a)

Iter.	Best Solution	Fitness
1	(0)	900.0

Iteration 1
Population:

Member	Population	Solution	Fitness
1	(00000)	(0)	900.0
2	(00001)	(1)	841.0
3	(00100)	(4)	4.0
4	(00110)	(6)	-1044.0
5	(01010)	(10)	-4100.0
6	(01110)	(14)	-8116.0
7	(10111)	(23)	-18673.0
8	(11010)	(26)	-22084.0
9	(11100)	(28)	-24188.0
10	(11101)	(29)	-25171.0

Children:

Member	Parents	Children	Solution	Fitness
5	(01010)	([1][1]010)	(26)	-22084.0
3	(00100)	(01100)	(12)	-6012.0
4	(00110)	(0011[1])	(7)	-1697.0
6	(01110)	(00110)	(6)	-1044.0
2	(00001)	([1]00[0]0)	(16)	-10364.0
8	(11010)	(11000)	(24)	-19836.0

(b) Because of the randomness in the algorithm, the output will vary.

14.4-5.

Because of the randomness in the algorithm, the output will vary.

14.4-6.

Because of the randomness in the algorithm, the output will vary.

14.4-7.

(a) Because of the randomness in the algorithm, the output will vary.

(b) Because of the randomness in the algorithm, the output will vary.

14.4-8.

(a) Genetic Algorithm

Iteration	Best Solution	Fitness
1	1-2-5-4-3-1	24.0

Iteration 1:

Member	Population	Fitness
1	1-4-5-3-2-1	33.0
2	1-2-5-4-3-1	24.0
3	1-3-2-4-5-1	31.0
4	1-4-5-3-2-1	33.0
5	1-2-3-4-5-1	33.0
6	1-3-4-2-5-1	28.0
7	1-5-2-3-4-1	31.0
8	1-4-2-3-5-1	37.0
9	1-5-2-3-4-1	31.0
10	1-5-2-4-3-1	28.0

Member	Children	Fitness
10	1-3-4-5-2-1	24.0
3	1-3-2-4-5-1	31.0
5	1-2-3-4-5-1	33.0
1	1-2-3-5-4-1	33.0
2	1-5-2-4-3-1	28.0
8	1-4-3-2-5-1	31.0

(b) Because of the randomness in the algorithm, the output will vary.

14.4-9.

Because of the randomness in the algorithm, the output will vary.

14.4-10.

Because of the randomness in the algorithm, the output will vary.

14.4-11.

Answers will vary.

14.5-1.

See the solution for Problem 14.2-6(a) for the output from the basic tabu search algorithm. Because of the randomness in the basic simulated annealing and genetic algorithms, their outputs will vary.

14.5-2.

See the solution for Problem 14.2-7(a) for the output from the basic tabu search algorithm. Because of the randomness in the basic simulated annealing and genetic algorithms, their outputs will vary.