Forecasting

Chapter Topics

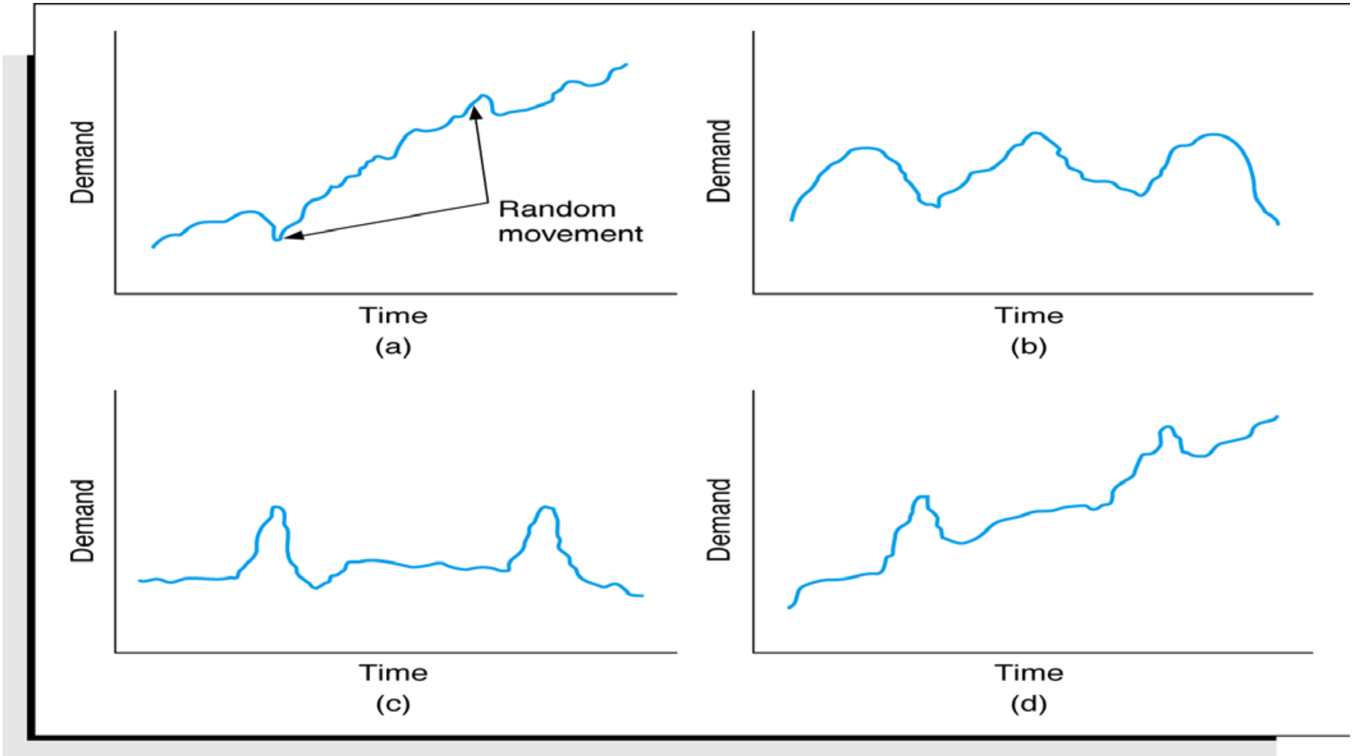
- Forecasting Components
- Time Series Methods
- Forecast Accuracy
- Time Series Forecasting Using Excel
- Time Series Forecasting Using QM for Windows
- Regression Methods

Forecasting Components

- A variety of forecasting methods are available for use depending on the *time frame* of the forecast and the existence of *patterns*
- Time Frames:
  - Short-range (one to two months)
  - Medium-range (two months to one or two years)
  - Long-range (more than one or two years)
- Patterns:
  - Trend
  - Random variations
  - Cycles
  - Seasonal pattern

Forecasting Components Patterns

- Trend - A long-term movement of the item being forecast
- Random variations - movements not predictable, follow no pattern
- Cycle - A movement, up or down, that repeats itself over a lengthy time span
- Seasonal pattern - Oscillating movement in demand that occurs periodically in the short run, and is repetitive



(a) Trend

(b) Cycle

(c) Seasonal

(d) Trend w/Season

Forecasting Components
Forecasting Methods

- **Qualitative Methods - Methods using judgment, expertise and opinion to make forecasts**

- **Times Series - Statistical techniques that use historical data to predict future behavior**

- **Regression Methods - Regression methods that attempt to develop a mathematical relationship between the item being forecast and factors that may cause it to behave the way it does**

Forecasting Components
Qualitative Methods

- "*Jury of executive opinion*," a qualitative technique, is the most common type of forecast for long-term strategic planning
  - Performed by individuals or groups within an organization, sometimes assisted by consultants and other experts, whose *judgments and opinions* are considered valid for the forecasting issue
  - Usually includes specialty functions such as marketing, engineering, purchasing, etc. in which individuals have *experience and knowledge* of the forecasted item
- Supporting techniques include the *Delphi Method, market research, surveys,* etc.

Time Series Methods
Overview

- Statistical techniques that make *use of historical data* collected over a long period of time
- Methods assume that *what has occurred in the past will continue* to occur in the future
- Forecasts based on only one factor *- time*

Time Series Methods
Moving Average

- Moving average uses values from the recent past to develop forecasts
- This *dampens or smoothes* random increases and decreases
- Useful for *forecasting relatively stable* items that do not display any trend or seasonal pattern
- Formula:

$$MA_n = \frac{\sum_{i=1}^{n} D_i}{n}$$

where:

$n =$ number of periods in the moving average

$D_i =$ data in period i

Time Series Methods
Moving Average Example

Acme Paper Clip Supply Company forecast of orders for the month of November

Month	Orders per Month
January	120
February	90
March	100
April	75
May	110
June	50
July	75
August	130
September	110
October	90
November	—

Time Series Methods
Moving Average Example

- Acme Paper Clip Supply Company forecast of orders for the month of November

Month	Orders per Month
January	120
February	90
March	100
April	75
May	110
June	50
July	75
August	130
September	110
October	90
November	—

- Three-month moving average:

$$MA_3 = \frac{\sum_{i=1}^{3} D_i}{3} = \frac{90+110+130}{3} = 110 \text{ orders}$$

- Five-month moving average:

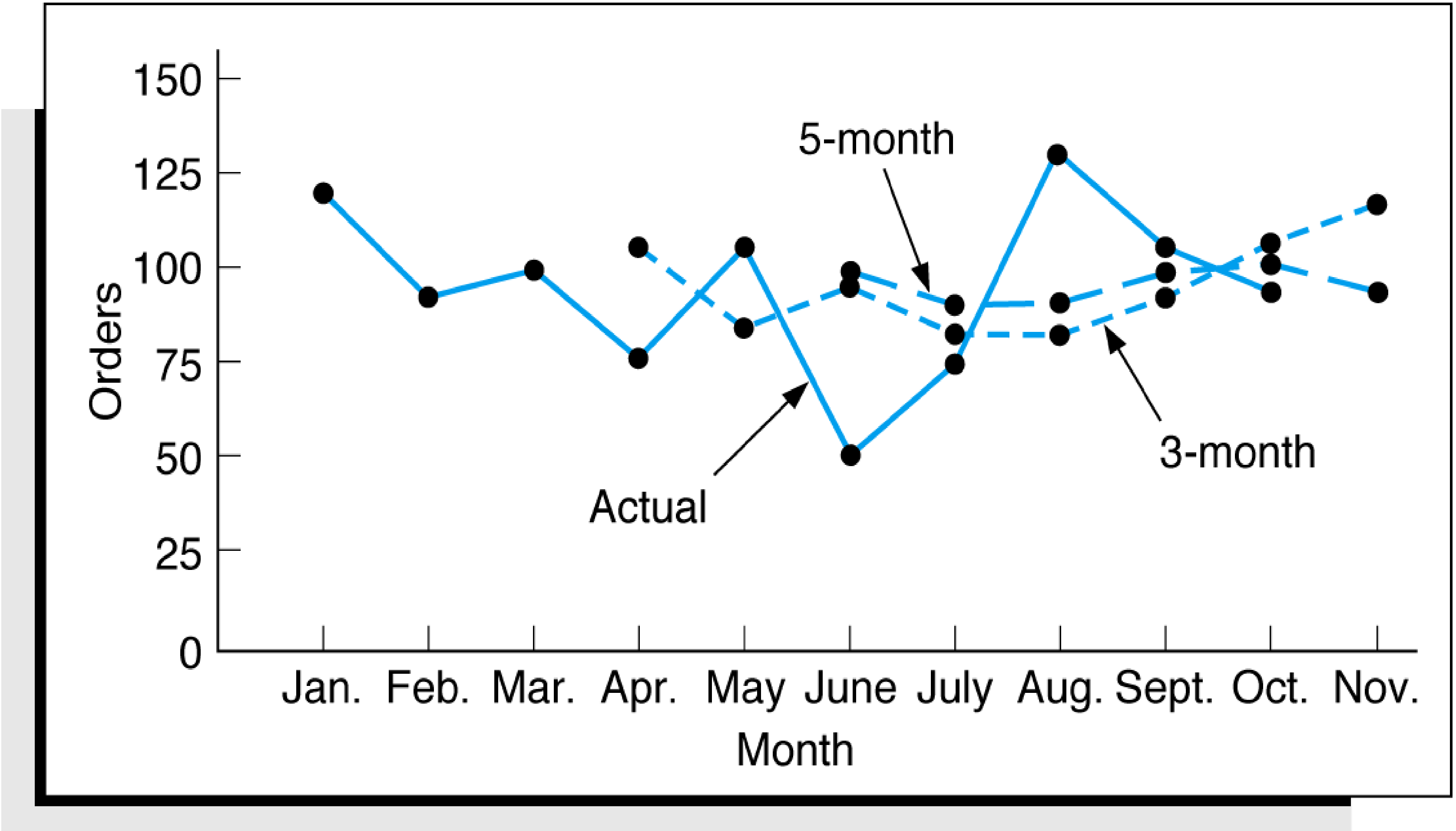
$$MA_5 = \frac{\sum_{i=1}^{5} D_i}{5} = \frac{90+110+130+75+50}{5} = 91 \text{ orders}$$

Time Series Methods
Moving Average

Month	Orders per Month	Three-Month Moving Average	Five-Month Moving Average
January	120	—	—
February	90	—	—
March	100	—	—
April	75	103.3	—
May	110	88.3	—
June	50	95.0	99.0
July	75	78.3	85.0
August	130	78.3	82.0
September	110	85.0	88.0
October	90	105.0	95.0
November	—	110.0	91.0

Three- and Five-Month Moving Averages

Time Series Methods
Moving Average



Three- and Five-Month Moving Averages

Time Series Methods
Moving Average

- *Longer-period moving averages react more slowly* to changes in demand than do shorter-period moving averages

- The appropriate number of periods to use often requires *trial-and-error experimentation*

- Moving average *does not react well to changes* (trends, seasonal effects, etc.) but it's easy to use and inexpensive

- Good for *short-term* forecasting

Time Series Methods
Weighted Moving Average

- **In a weighted moving average, weights are assigned to the** *most recent data*

$$WMA_n = \sum_{i=1}^{n} W_i D_i$$

where W_i = the weight for period i, between 0% and 100%

$\sum W_i = 1.00$

Example: Paper clip company weights 50% for October, 33% for September, 17% for August:

$$WMA_3 = \sum_{i=1}^{3} W_i D_i = (.50)(90)+(.33)(110)+(.17)(130) = 103.4 \text{ orders}$$

- **Determining precise weights and number of periods requires** *trial-and-error experimentation*

Time Series Methods
Exponential Smoothing

- Exponential smoothing usually *weights recent past data more strongly* than more distant data
- Two forms*: simple* exponential smoothing and *adjusted* exponential smoothing
- Simple exponential smoothing:

$$F_{t+1} = \alpha D_t + (1 - \alpha)F_t$$

where:

F_{t+1} = the forecast for the next period
D_t = actual demand in the present period
F_t = the previously determined forecast for the present period
α = a weighting factor (smoothing constant)

Time Series Methods
Exponential Smoothing

- The most commonly used values of α are b*etween 0.10 and 0.50*

- Determination of *α is usually judgmental and subjective* and often based on trial-and -error experimentation

Time Series Methods
Exponential Smoothing

Example: Acme Computer Services

- Exponential smoothing forecasts using smoothing constant of 0.30

- Forecast for period 2 (February):

  $F_2 = \alpha D_1 + (1- \alpha)F_1 = (.30)(37) + (.70)(37)$

  $= 37$ units

- Forecast for period 3 (March):

  $F_3 = \alpha D_2 + (1- \alpha)F_2 = (.30)(40) + (.70)(37)$

  $= 37.9$ units

Period	Month	Demand
1	January	37
2	February	40
3	March	41
4	April	37
5	May	45
6	June	50
7	July	43
8	August	47
9	September	56
10	October	52
11	November	55
12	December	54
13	January	—

Time Series Methods
Exponential Smoothing

Remember these values

			Forecast, F_{t+1}	
Period	Month	Demand	$\alpha = .30$	$\alpha = .50$
1	January	37	—	—
2	February	40	37.00	37.00
3	March	41	37.90	38.50
4	April	37	38.83	39.75
5	May	45	38.28	38.37
6	June	50	40.29	41.68
7	July	43	43.20	45.84
8	August	47	43.14	44.42
9	September	56	44.30	45.71
10	October	52	47.81	50.85
11	November	55	49.06	51.42
12	December	54	50.84	53.21
13	January	—	51.79	53.61

Exponential Smoothing Forecasts, $\alpha = 0.30$ and $\alpha = 0.50$

Time Series Methods
Exponential Smoothing

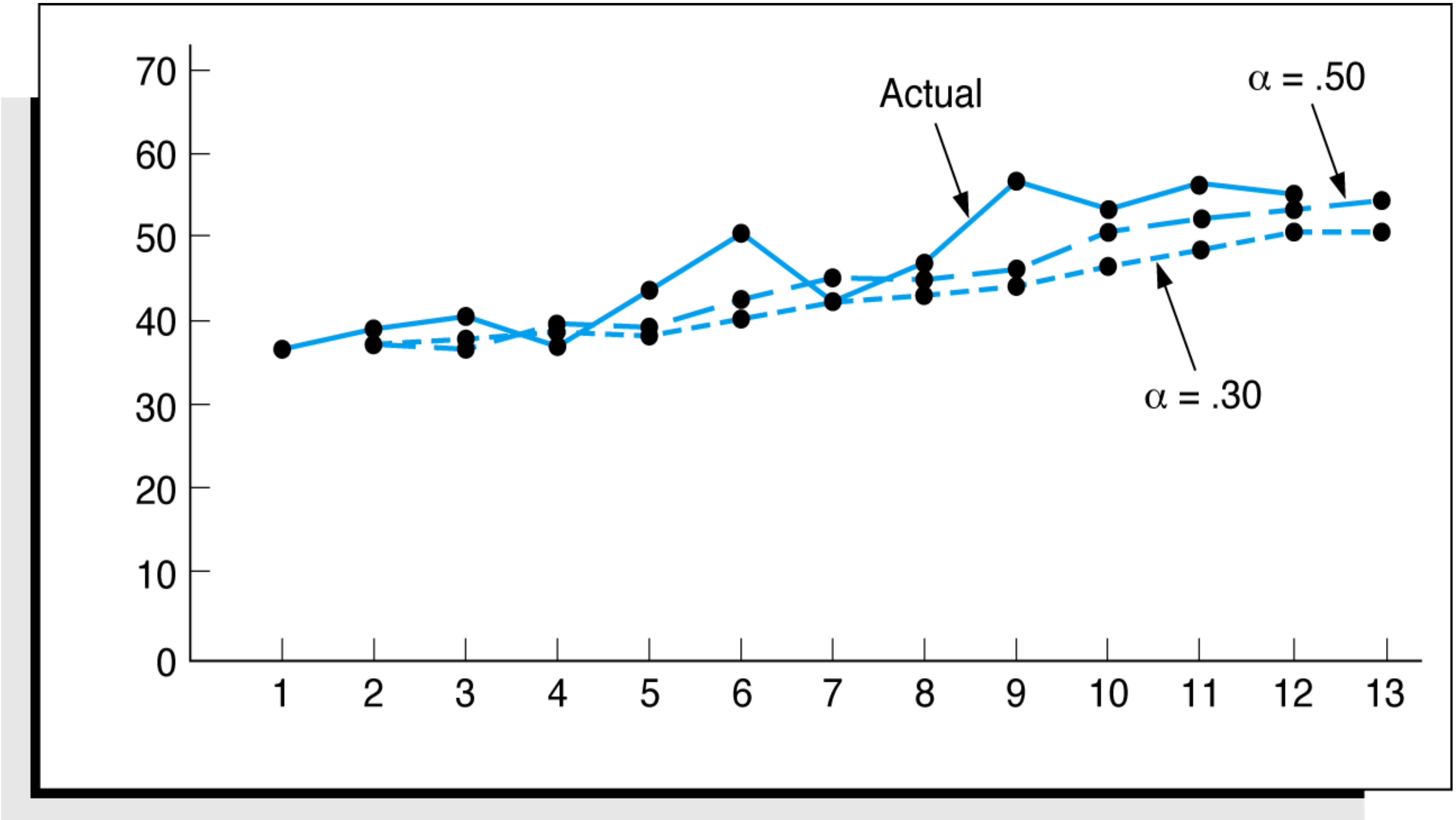
- The forecast that uses the higher smoothing constant (0.50) reacts more strongly to changes in demand than does the forecast with the lower constant (0.30)

- Both *forecasts lag behind* actual demand

- Both forecasts tend to be consistently lower than actual demand

- *Low smoothing constants* are appropriate *for stable data* without trend; higher constants appropriate for data with trends

Time Series Methods
Exponential Smoothing



Exponential Smoothing Forecasts

Time Series Methods
Exponential Smoothing

- *Adjusted exponential smoothing:* exponential smoothing with a trend adjustment factor added

Formula:

$$AF_{t+1} = F_{t+1} + T_{t+1}$$

where: T = an exponentially smoothed trend factor

$$T_{t+1} = \beta(F_{t+1} - F_t) + (1 - \beta)T_t$$

T_t = the last period trend factor

β = smoothing constant for trend ($0 \leq \beta \leq 1$)

- Reflects the weight given to the most recent trend data
- *Determined subjectively*

Time Series Methods
Exponential Smoothing

$T_3 = \beta(F_3 - F_2) + (1 - \beta)T_2$

$= (.30)(38.5 - 37.0) + (.70)(0) = 0.45$

$AF_3 = F_3 + T_3 = 38.5 + 0.45 = 38.95$

Example: Acme Computer Services exponential smoothed forecasts with $\alpha = 0.50$ and $\beta = 0.30$

Period	Month	Demand	Forecast (F_{t+1})	Trend (T_{t+1})	Adjusted Forecasts (AF_{t+1})
1	January	37	37.00	—	—
2	February	40	37.00	0.00	37.00
3	March	41	38.50	0.45	38.95
4	April	37	39.75	0.69	40.44
5	May	45	38.37	0.07	38.44
6	June	50	41.68	1.04	42.73
7	July	43	45.84	1.97	47.82
8	August	47	44.42	0.95	45.37
9	September	56	45.71	1.05	46.76
10	October	52	50.85	2.28	53.13
11	November	55	51.42	1.76	53.19
12	December	54	53.21	1.77	54.98
13	January	—	53.61	1.36	54.96

$\alpha = 0.50$ and $\beta = 0.30$

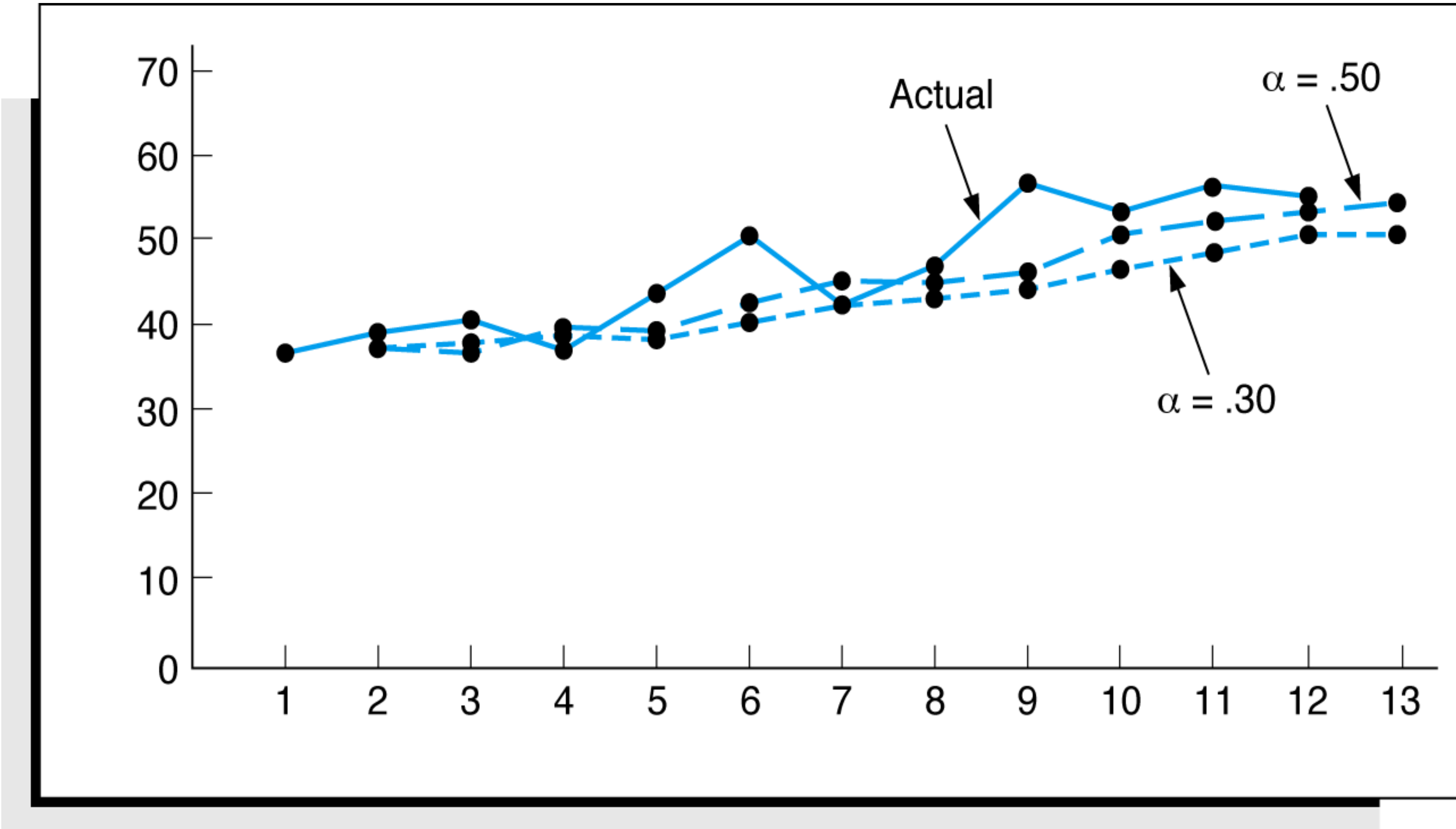
Adjusted Exponentially Smoothed Forecast Values

Time Series Methods
Exponential Smoothing

- **Adjusted forecast is consistently higher than the simple exponentially smoothed forecast**

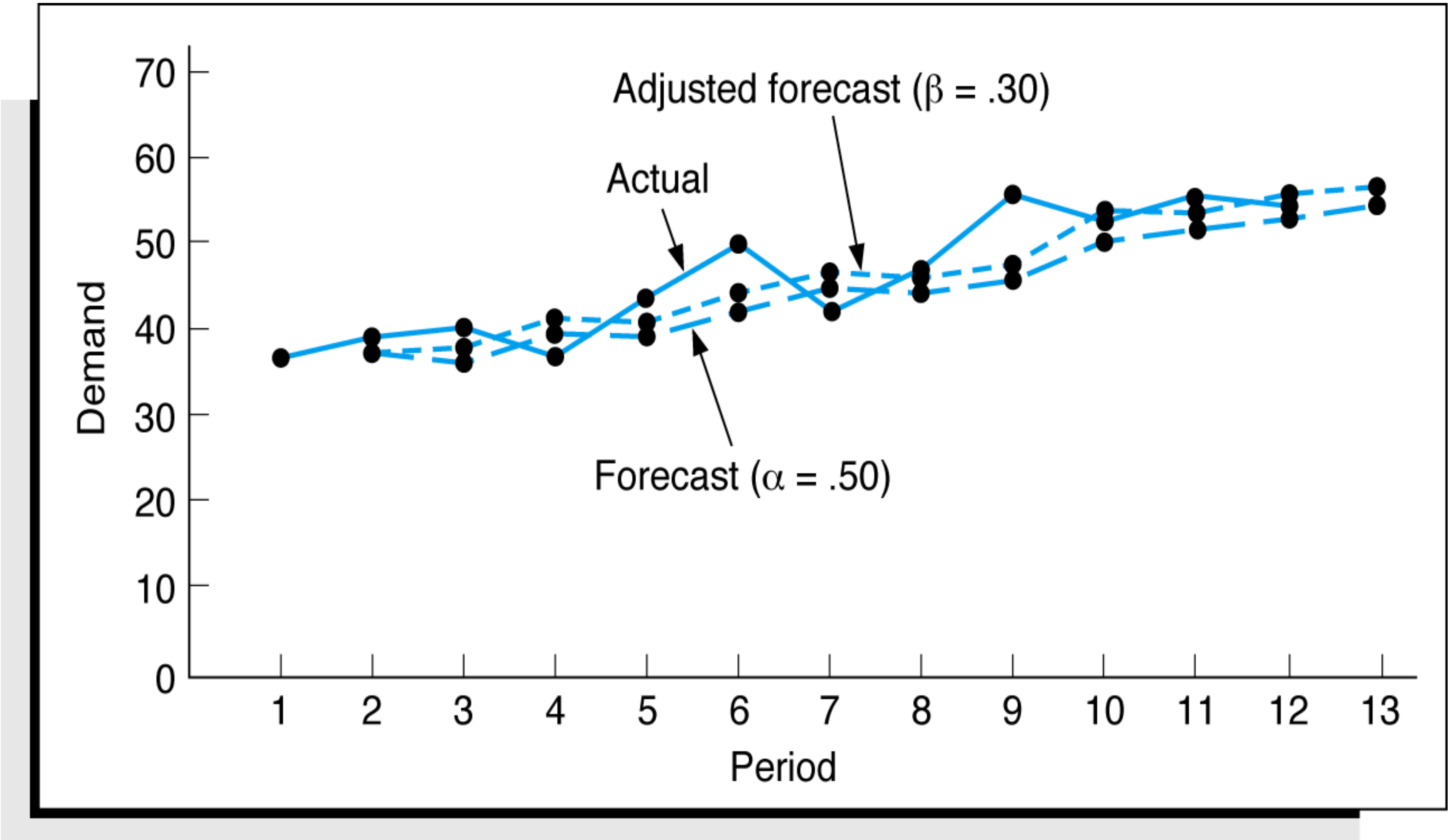
- **It is *more reflective of the generally increasing trend* of the data**

Time Series Methods
Exponential Smoothing (original F_t)



Exponential Smoothing Forecasts

Time Series Methods
Adjusted Exponential Smoothing



Adjusted Exponentially Smoothed Forecast

Time Series Methods
Linear Trend Line

- **When demand displays an obvious trend over time, a *least squares regression line* , or *linear trend line*, can be used to forecast**
- **Formula:**

$$y = a + bx$$

where:

$a =$ intercept (at period 0)
$b =$ slope of the line
$x =$ the time period
$y =$ forecast for demand for period x

$$b = \frac{\sum xy - n\overline{xy}}{\sum x^2 - n\bar{x}}$$

$$a = \bar{y} - b\bar{x}$$

where:

$n =$ number of periods

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{y} = \frac{\sum y}{n}$$

Time Series Methods
Linear Trend Line

Example: Acme Computer Services (data on next slide)

$$\bar{x} = \frac{78}{12} = 6.5 \qquad \bar{y} = \frac{557}{12} = 46.42$$

$$b = \frac{\sum xy - n\overline{xy}}{\sum x^2 - n\bar{x}^2} = \frac{3{,}867 - (12)(6.5)(46.42)}{650 - 12(6.5)^2} = 1.72$$

$$a = \bar{y} - b\bar{x} = 46.42 - (1.72)(6.5) = 35.2$$

$$y = 35.2 + 1.72x \ \text{ linear trend line}$$

Equation!

$$\text{for period 13, x} = 13,\ y = 35.2 + 1.72(13) = 57.56$$

Time Series Methods
Linear Trend Line

Least Squares Calculations

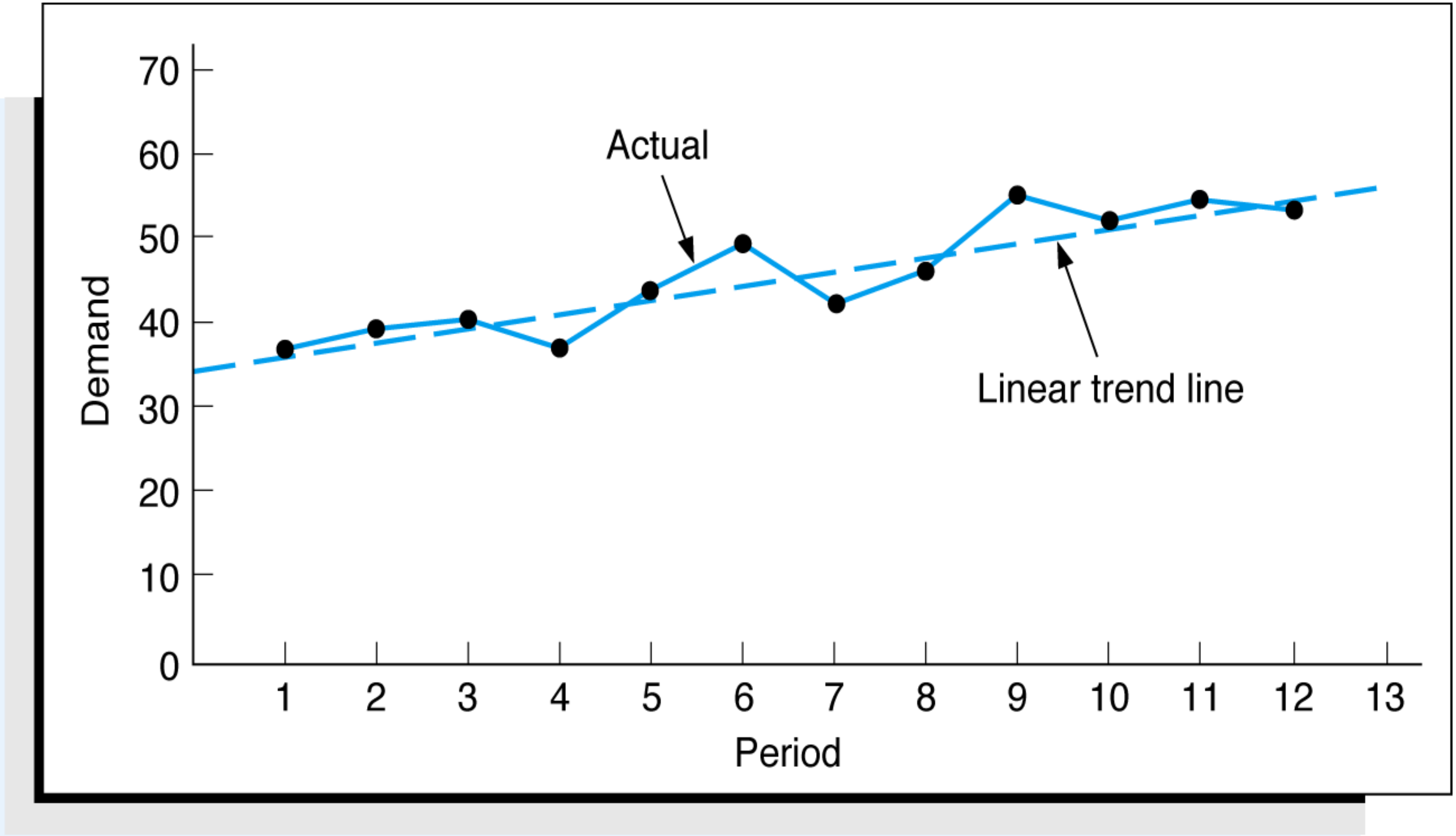
x (period)	y (demand)	xy	x^2
1	37	37	1
2	40	80	4
3	41	123	9
4	37	148	16
5	45	225	25
6	50	300	36
7	43	301	49
8	47	376	64
9	56	504	81
10	52	520	100
11	55	605	121
12	54	648	144
78	557	3,867	650

Time Series Methods
Linear Trend Line

- A *trend line does not adjust to a change in the trend* as does the exponential smoothing method

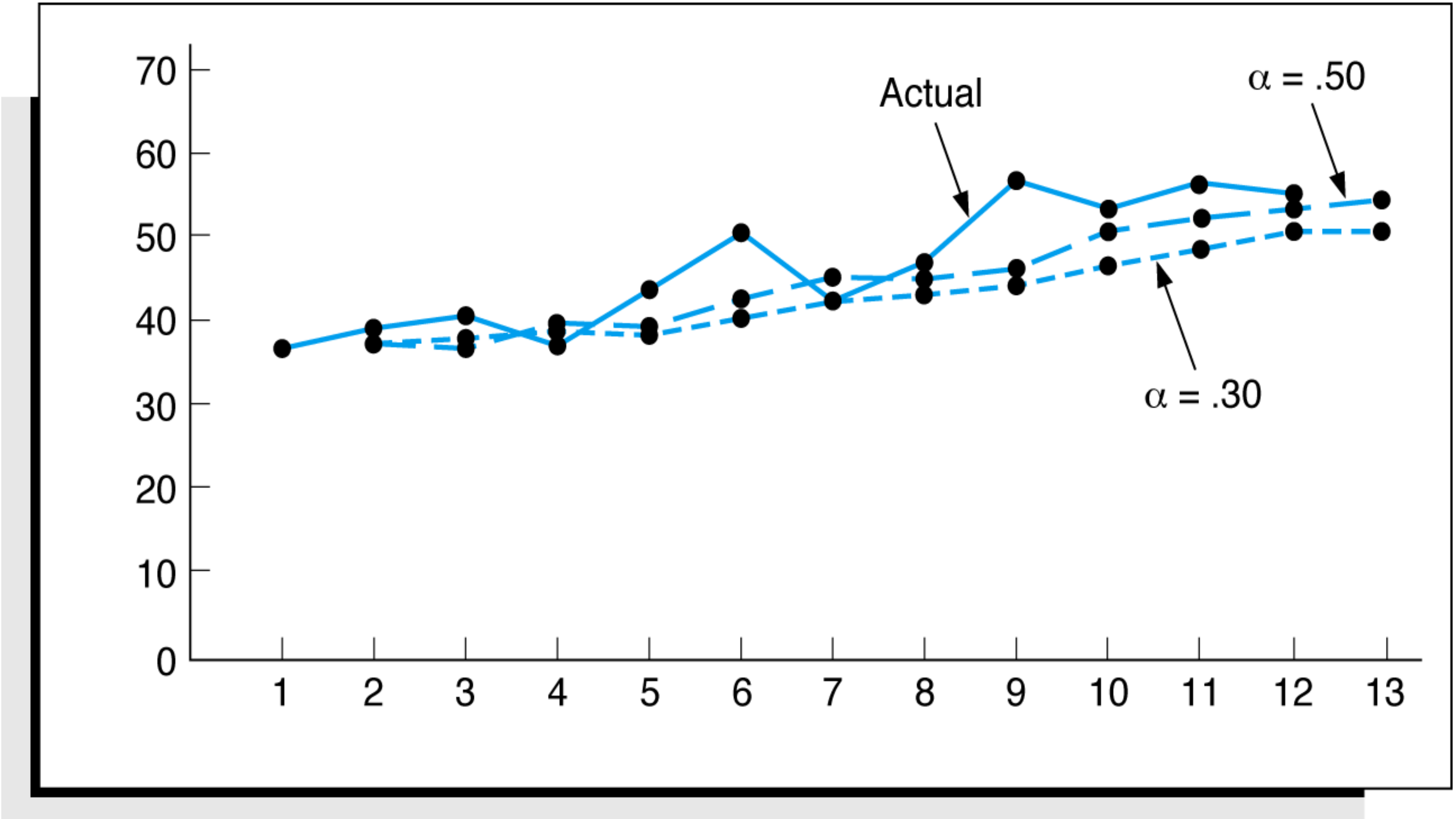
- This limits its use to *shorter time frames* in which trend will not change

Time Series Methods
Linear Trend



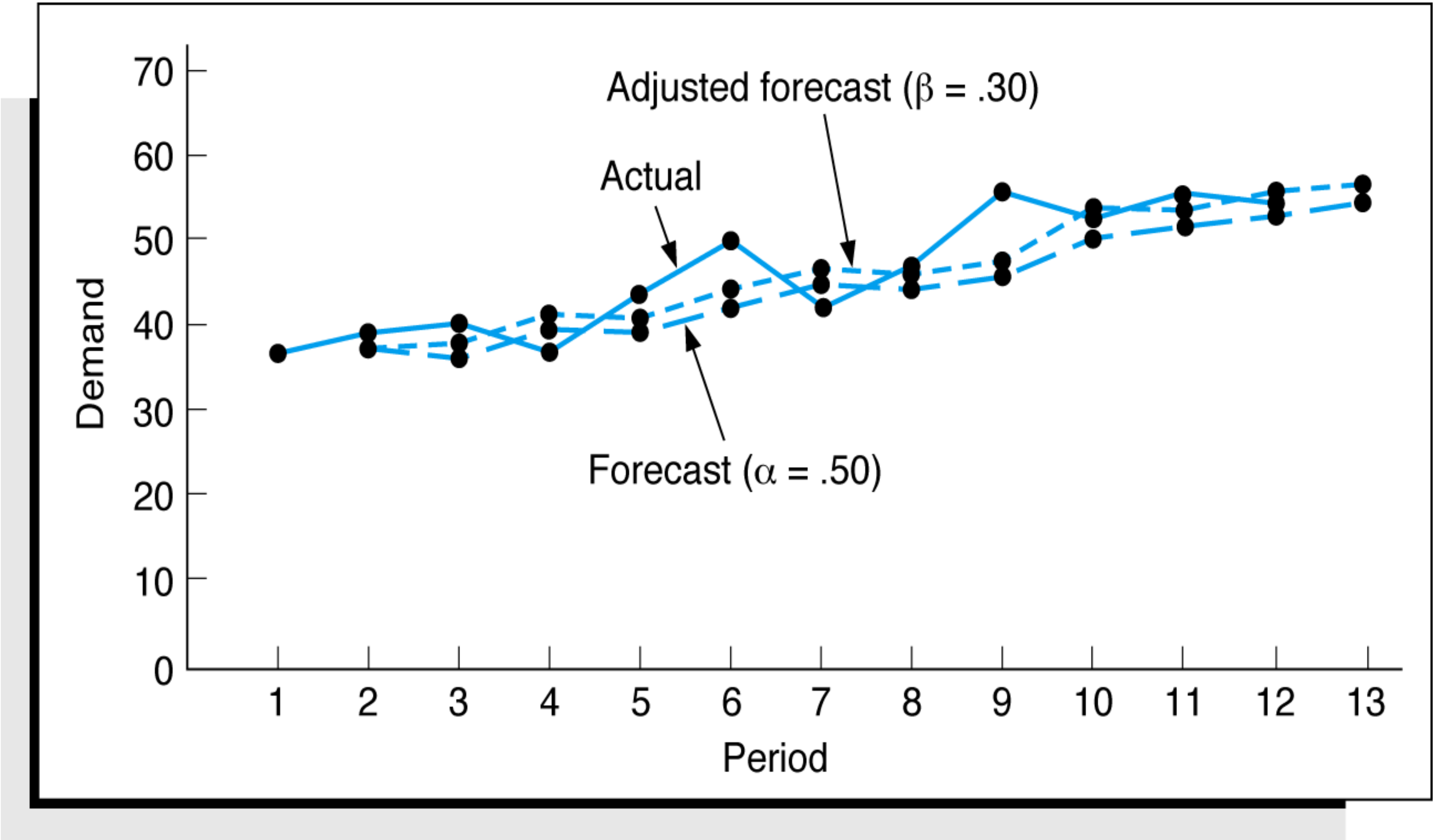
Linear Trend Line

Time Series Methods
Exponential Smoothing (for comparison)



Exponential Smoothing Forecasts

Time Series Methods
Adjusted Exponential Smoothing (for comparison)



Adjusted Exponentially Smoothed Forecast

Time Series Methods
Seasonal Adjustments

- A *seasonal pattern* is a repetitive up-and-down movement in demand
- Seasonal patterns can occur on a quarterly, monthly, weekly, or daily basis
- A *seasonally adjusted forecast* can be developed by multiplying the normal forecast by a seasonal factor
- A *seasonal factor* can be determined by dividing the actual demand for each seasonal period by total annual demand:

$$S_i = D_i / \sum D$$

Time Series Methods
Seasonal Adjustments

- Seasonal factors lie between zero and one and represent the portion of total annual demand assigned to each season

- Seasonal factors are multiplied by annual demand to provide adjusted forecasts for each period

Time Series Methods
Seasonal Adjustments

Example: Wishbone Farms

	Demand (1,000s)				
Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Total
1998	12.6	8.6	6.3	17.5	45.0
1999	14.1	10.3	7.5	18.2	50.1
2000	15.3	10.6	8.1	19.6	53.6
Total	42.0	29.5	21.9	55.3	148.7

Demand for Turkeys at Wishbone Farms

$S_1 = D_1 / \sum D = 42.0/148.7 = 0.28$

$S_2 = D_2 / \sum D = 29.5/148.7 = 0.20$

$S_3 = D_3 / \sum D = 21.9/148.7 = 0.15$

$S_4 = D_4 / \sum D = 55.3/148.7 = 0.37$

Time Series Methods - Seasonal Adjustments

- **Get the trend line for the data from Wishbone Farms**
- **We have three years of data, and we want to predict year 4**
- **We'll then apply the seasonal adjustments to the yearly demand predicted by the trend line**
- **Data for the regression:**

Year	Demand
1	45
2	50.1
3	53.6

Note: For most quarterly data, we would most often use dummy variables, but this particular problem is highlighting the use of seasonal indices, not dummy variables.

- **Regression Output:**

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.994281							
R Square	0.988594							
Adjusted R Square	0.977188							
Standard Error	0.653197							
Observations	3							
ANOVA								
	df	SS	MS	F	Significance F			
Regression	1	36.98	36.98	86.67188	0.068121			
Residual	1	0.426667	0.426667					
Total	2	37.40667						
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	40.96667	0.997775	41.05801	0.015502	28.28873	53.6446	28.28873	53.6446
X Variable 1	4.3	0.46188	9.309773	0.068121	-1.56874	10.16874	-1.56874	10.16874

Here's the resultant equation:

$$y = 40.97 + 4.30x$$

This will be used to forecast yearly demand for year 4

Time Series Methods
Seasonal Adjustments

- Multiply forecasted demand for entire year by seasonal factors to determine quarterly demand
- Forecast for entire year (trend line for data in table)

  $y = 40.97 + 4.30x \quad = 40.97 + 4.30(4) \quad = 58.17$

- Seasonally adjusted forecasts:

  $SF_1 = (S_1)(F_5) = (.28)(58.17) = 16.28$

  $SF_2 = (S_2)(F_5) = (.20)(58.17) = 11.63$

  $SF_3 = (S_3)(F_5) = (.15)(58.17) = 8.73$

  $SF_4 = (S_4)(F_5) = (.37)(58.17) = 21.53$

Forecast Accuracy Overview

- Forecasts *will always deviate* from actual values
- Difference between forecasts and actual values referred to as *forecast error*
- Would like forecast error to be as small as possible
- If error is large, either *technique* being used *is the wrong one*, or *parameters need adjusting*
- Measures of forecast errors:
  - Mean Absolute deviation (MAD)
  - Mean absolute percentage deviation (MAPD)
  - Cumulative error (E bar)
  - Average error, or bias (E)

Forecast Accuracy
Mean Absolute Deviation

- **MAD is the *average absolute difference* between the forecast and actual demand**
- **Most popular and simplest-to-use measures of forecast error**
- **Formula:**

$$MAD = \frac{\sum |D_t - F_t|}{n}$$

where:

t = the period number
D_t = demand in period t
F_t = the forecast for period t
n = the total number of periods

- **Can compare accuracies of different forecasts using MAD**

Forecast Accuracy
Mean Absolute Deviation

Period	Demand, D_t	Forecast, F_t, ($\alpha = .30$)	Error ($D_t - F_t$)	$\lvert D_t - F_t \rvert$
1	37	37.00	—	—
2	40	37.00	3.00	3.00
3	41	37.90	3.10	3.10
4	37	38.83	−1.83	1.83
5	45	38.28	6.72	6.72
6	50	40.29	9.71	9.71
7	43	43.20	−0.20	0.20
8	47	43.14	3.86	3.86
9	56	44.30	11.70	11.70
10	52	47.81	4.19	4.19
11	55	49.06	5.94	5.94
12	54	50.84	3.16	3.16
	520*		49.31	53.41

Computational Values for MAD and error

$$MAD = \frac{\sum \lvert D_t - F_t \rvert}{n} = \frac{53.41}{11} = 4.85$$

Forecast Accuracy
Mean Absolute Deviation

- The *lower the value of MAD* relative to the magnitude of the data, *the more accurate* the forecast

- When viewed *alone, MAD is difficult to assess*

- Must be considered in light of magnitude of the data

Forecast Accuracy
Mean Absolute Deviation

- Can be used to compare accuracy of different forecasting techniques working on the same set of demand data

- Exponential smoothing ($\alpha = .50$): MAD = 4.04
- Adjusted exponential smoothing ($\alpha = .50$, $\beta = .30$): MAD = 3.81
- Linear trend line: MAD = 2.29

- Linear trend line has lowest MAD; increasing α from .30 to .50 improved smoothed forecast

Forecast Accuracy
Mean Absolute Deviation

- A variation on MAD is the *mean absolute percent deviation* **(MAPD)**
- Measures *absolute error as a percentage of demand* rather than per period
- Eliminates problem of interpreting the measure of accuracy relative to the magnitude of the demand and forecast values
- Formula:

$$MAPD = \frac{\sum |D_t - F_t|}{\sum D_t} = \frac{53.41}{520} = .103 \text{ or } 10.3\%$$

Forecast Accuracy
Mean Absolute Deviation

MAPD for three forecasting techniques on Acme Computer data:

Exponential smoothing ($\alpha = .50$): MAPD = 8.5%

Adjusted exponential smoothing ($\alpha = .50$, $\beta = .30$): MAPD = 8.1%

Linear trend: MAPD = 4.9%

Forecast Accuracy
Cumulative Error

- *Cumulative error* is the sum of the forecast errors ($E = \sum e_t$)
- A relatively *large positive value indicates forecast is biased low*, a large negative value indicates forecast is biased high
- If preponderance of errors are positive, forecast is consistently low; and vice versa
- *Cumulative error for trend line is always almost zero*, and is therefore not a good measure for this method
- Cumulative error for Acme Computer Services can be read directly from Table 15.8 in book
- $E = \sum e_t = 49.31$ indicating forecasts are frequently below actual demand

Forecast Accuracy
Cumulative Error

- Cumulative error for pertinent forecasts:

  Exponential smoothing (α = .50): E = 33.21

  Adjusted exponential smoothing (α = .50, β =.30): E = 21.14

- *Average error (bias)* is the per period average of cumulative error
- Average error for exponential smoothing forecast:

$$\overline{E} = \frac{\sum e_t}{n} = \frac{49.31}{11} = 4.48$$

- A large positive value of average error indicates a forecast is biased low
- A large negative error indicates it is biased high

Forecast Accuracy
Example Forecasts by Different Measures

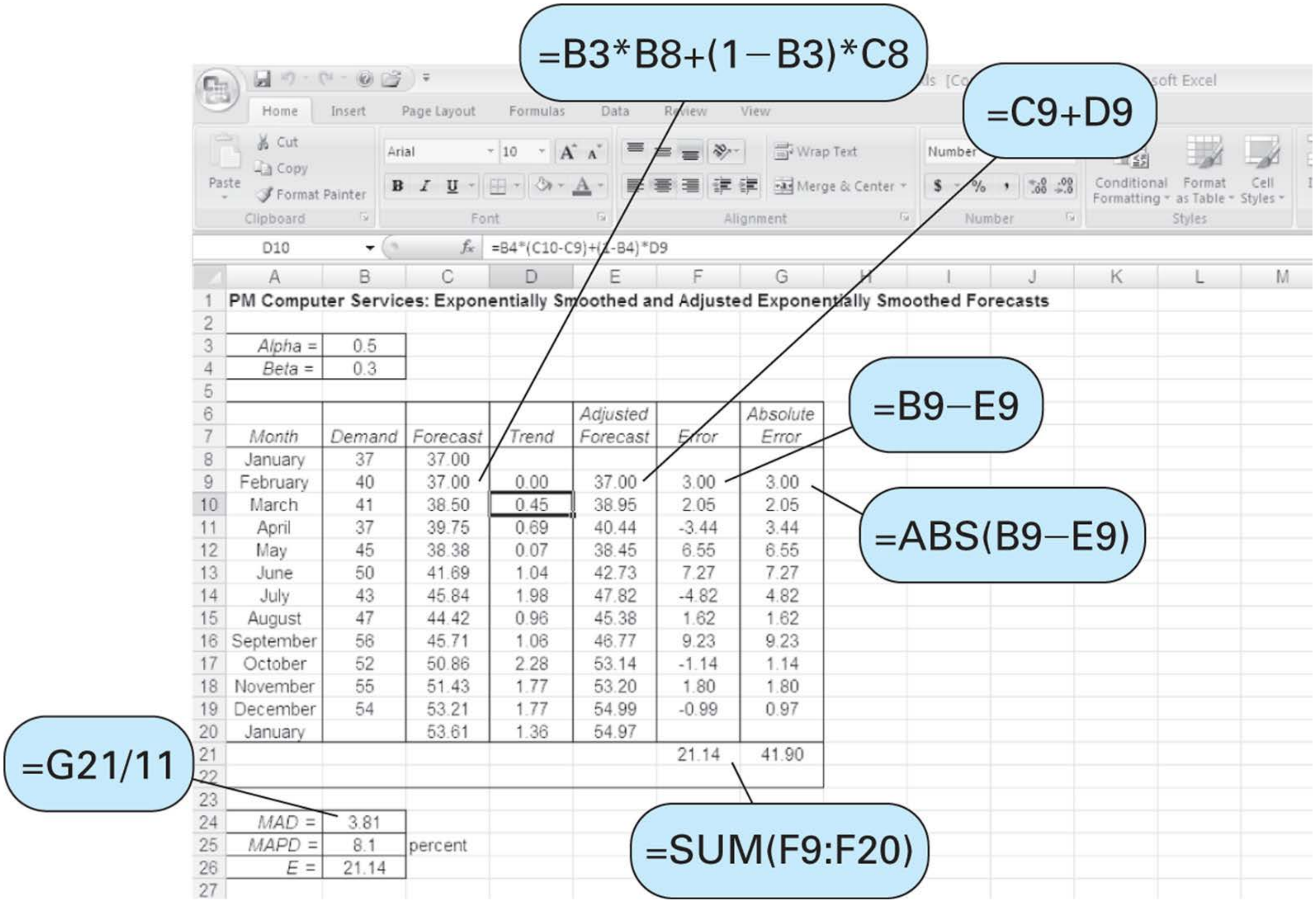
Forecast	*MAD*	*MAPD* (%)	*E*	$\bar{E}$
Exponential smoothing (α = .30)	4.85	10.3	49.31	4.48
Exponential smoothing (α = .50)	4.04	8.5	33.21	3.02
Adjusted exponential smoothing (α = .50, β = .30)	3.81	8.1	21.14	1.92
Linear trend line	2.29	4.9	—	—

Comparison of Forecasts for Acme Computer Services

<u>Results consistent for all forecasts:</u>

- Larger value of alpha is preferable
- Adjusted forecast is more accurate than exponential smoothing
- Linear trend is more accurate than all the others

Time Series Forecasting Using Excel



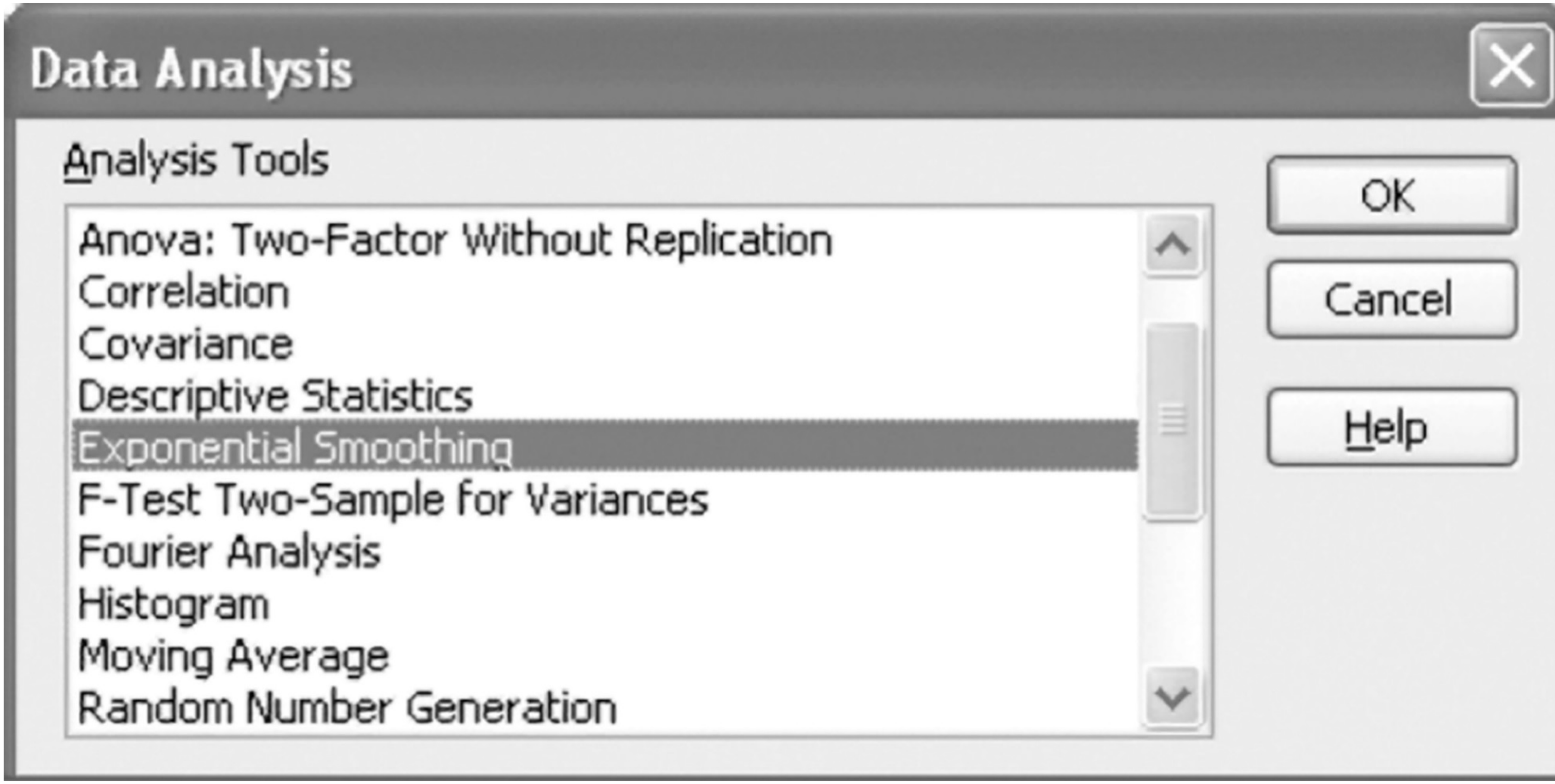
PM Computer Services: Exponentially Smoothed and Adjusted Exponentially Smoothed Forecasts

Alpha =	0.5
Beta =	0.3

Month	Demand	Forecast	Trend	Adjusted Forecast	Error	Absolute Error
January	37	37.00				
February	40	37.00	0.00	37.00	3.00	3.00
March	41	38.50	0.45	38.95	2.05	2.05
April	37	39.75	0.69	40.44	-3.44	3.44
May	45	38.38	0.07	38.45	6.55	6.55
June	50	41.69	1.04	42.73	7.27	7.27
July	43	45.84	1.98	47.82	-4.82	4.82
August	47	44.42	0.96	45.38	1.62	1.62
September	56	45.71	1.06	46.77	9.23	9.23
October	52	50.86	2.28	53.14	-1.14	1.14
November	55	51.43	1.77	53.20	1.80	1.80
December	54	53.21	1.77	54.99	-0.99	0.97
January		53.61	1.36	54.97		
					21.14	41.90

MAD =	3.81	
MAPD =	8.1	percent
E =	21.14	

Time Series Forecasting Using Excel

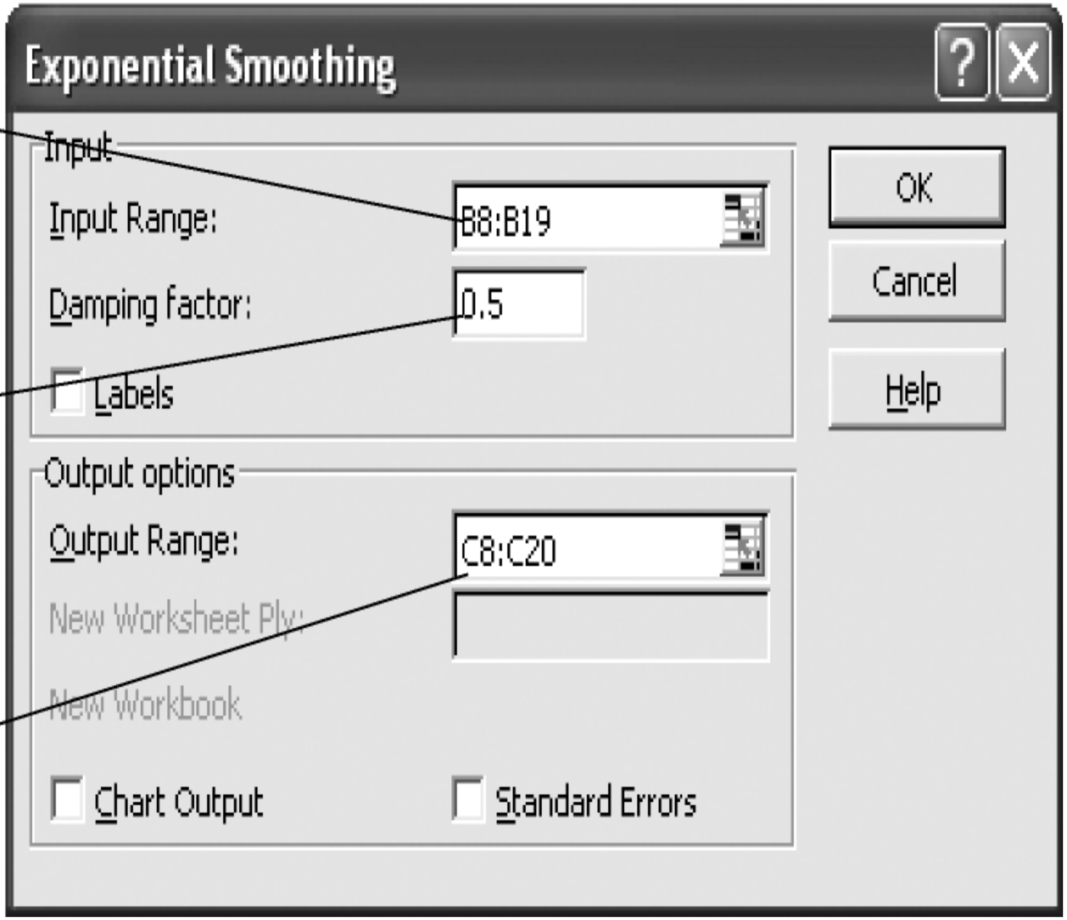


Time Series Forecasting Using Excel

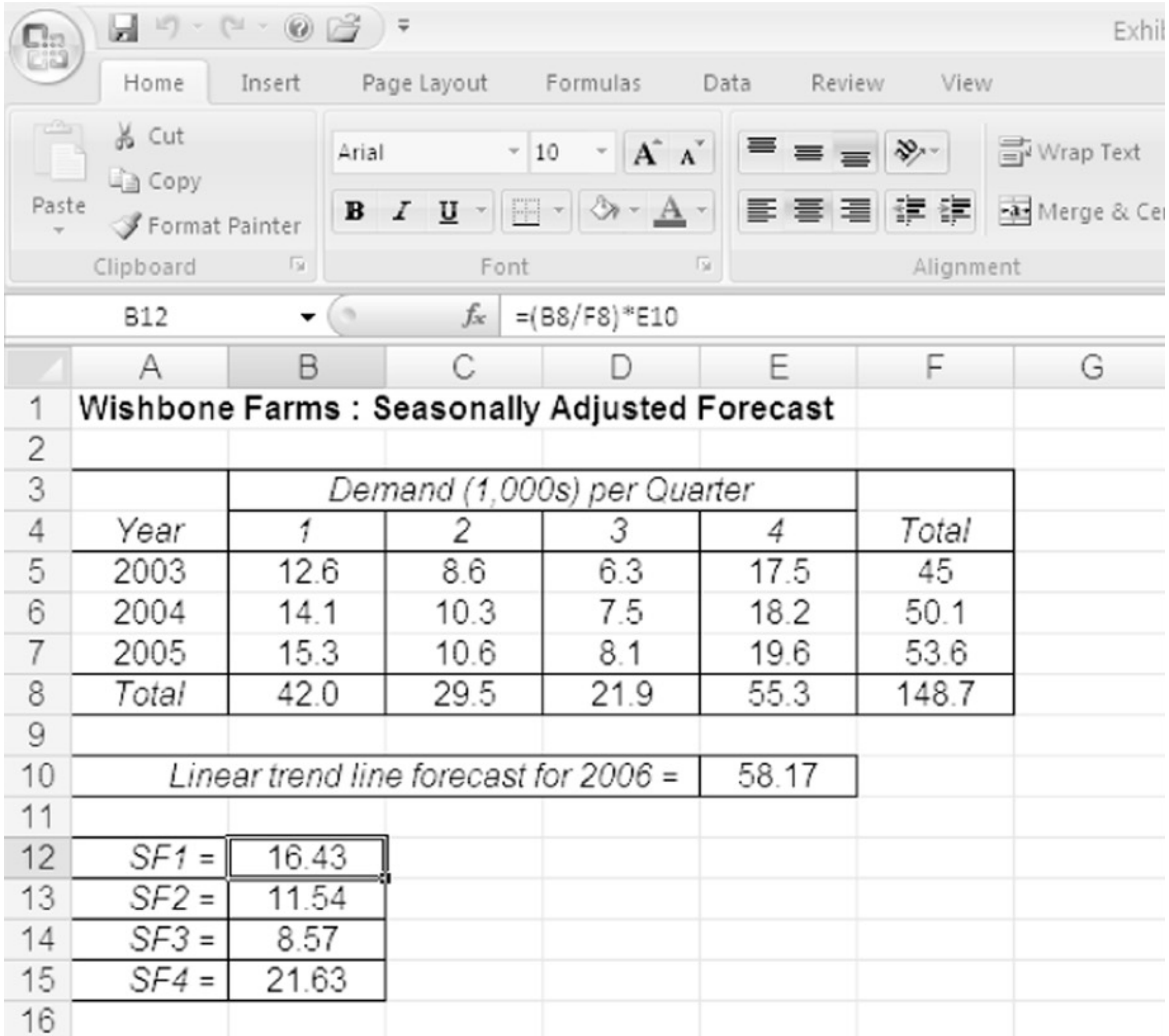
Demand values

$\alpha = 0.5$

Cells in which the forecasted values will be placed



Time Series Forecasting Using Excel



	A	B	C	D	E	F	G
1	**Wishbone Farms : Seasonally Adjusted Forecast**						
2							
3		*Demand (1,000s) per Quarter*					
4	*Year*	*1*	*2*	*3*	*4*	*Total*	
5	2003	12.6	8.6	6.3	17.5	45	
6	2004	14.1	10.3	7.5	18.2	50.1	
7	2005	15.3	10.6	8.1	19.6	53.6	
8	*Total*	42.0	29.5	21.9	55.3	148.7	
9							
10	*Linear trend line forecast for 2006 =*				58.17		
11							
12	*SF1 =*	16.43					
13	*SF2 =*	11.54					
14	*SF3 =*	8.57					
15	*SF4 =*	21.63					
16							

Time Series Forecasting
Solution with QM for Windows

Details and Error Analysis

PM Computer Services Example Solution

	Demand(y)	Forecast	Error	|Error|	Error^2	|Pct Error|
1	37					
2	40	37	3	3	9	.075
3	41	37.9	3.1	3.1	9.61	.0756
4	37	38.83	-1.83	1.83	3.3489	.0495
5	45	38.281	6.719	6.719	45.1449	.1493
6	50	40.2967	9.7033	9.7033	94.154	.1941
7	43	43.2077	-.2077	.2077	.0431	.0048
8	47	43.1454	3.8546	3.8546	14.8581	.082
9	56	44.3018	11.6982	11.6982	136.8486	.2089
10	52	47.8112	4.1888	4.1888	17.5457	.0806
11	55	49.0679	5.9321	5.9321	35.1902	.1079
12	54	50.8475	3.1525	3.1525	9.9382	.0584
TOTALS	557		49.3108	53.3862	375.6818	1.086
AVERAGE	46.4167		4.4828	4.8533	34.1529	.0987
Next period forecast		51.7933	(Bias)	(MAD)	(MSE)	(MAPE)
				Std err	6.4608	

Time Series Forecasting Solution with QM for Windows

Forecasting Results

PM Computer Services Example Summary

Measure	Value	Future Period	Forecast
Error Measures		13	57.6212
Bias (Mean Error)	0	14	59.345
MAD (Mean Absolute Deviation)	2.2892	15	61.0688
MSE (Mean Squared Error)	8.6672	16	62.7925
Standard Error (denom=n-2=10)	3.225	17	64.5163
MAPE (Mean Absolute Percent Error)	.0499	18	66.2401
Regression line		19	67.9639
Demand(y) = 35.21213		20	69.6876
+ 1.7238 * Time(x)		21	71.4114
Statistics		22	73.1352
Correlation coefficient	.8963	23	74.859
Coefficient of determination (r^2)	.8034	24	76.5827
		25	78.3065
		26	80.0303

Problem Time!
Computer Software Firm

- For data below, develop an
  - exponential smoothing forecast $\alpha = 0.40$
  - adjusted exponential smoothing forecast $\alpha = 0.40$, $\beta = 0.20$
    - Adjusted for trend
- Compare the accuracy of the forecasts using MAD and cumulative error

Period	Units
1	56
2	61
3	55
4	70
5	66
6	65
7	72
8	75