

Transportation, Transshipment, and Assignment Problems

Topics

- The Transportation Model
- The Transshipment Model
- The Assignment Model

Overview

- Part of a class of LP problems known as network flow models
- Special mathematical features that permit very efficient, unique solution methods (variations of traditional simplex procedure)
- Text focuses on model formulation and solution with Excel and QM for Windows

The Transportation Model: Characteristics

- A product is transported from a number of sources to a number of destinations at the *minimum possible cost*
- Each *source is able to supply a fixed number* of units of the product, and *each destination has a fixed demand* for the product
- The linear programming model has *constraints for supply* at each source *and demand* at each destination
- All constraints are equalities in a balanced transportation model where supply equals demand
- Constraints contain inequalities in unbalanced models where supply does not equal demand

Transportation Model Example

Problem Definition and Data

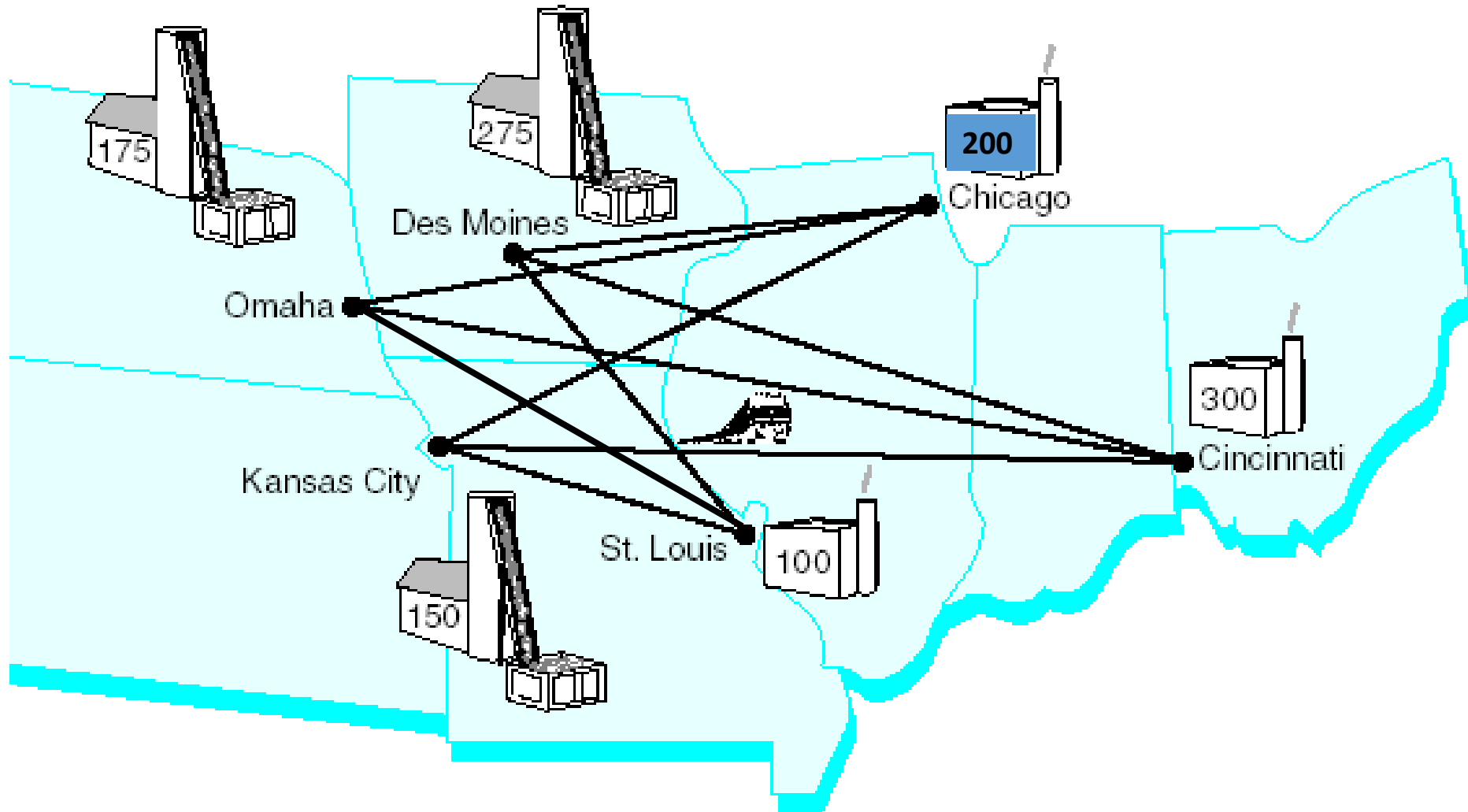
How many tons of wheat should we transport from each grain elevator to each mill on a monthly basis, in order to minimize the total cost of transportation?

<u>Grain Elevator</u>	<u>Supply</u>	<u>Mill</u>	<u>Demand</u>
1. Kansas City	150	A. Chicago	200
2. Omaha	175	B. St. Louis	100
3. Des Moines	275	C. Cincinnati	300
Total	600 tons	Total	600 tons

<u>Transport Cost from Grain Elevator to Mill (\$/ton)</u>			
<u>Grain Elevator</u>	<u>A. Chicago</u>	<u>B. St. Louis</u>	<u>C. Cincinnati</u>
1. Kansas City	\$ 6	\$ 8	\$ 10
2. Omaha	7	11	11
3. Des Moines	4	5	12

Transportation Model Example

Transportation Network Routes



Network of Transportation Routes for Wheat Shipments

Transportation Model Example

Model Formulation

x_{ij} = tons of wheat from each grain elevator, $i, i = 1, 2, 3$, to each mill $j, j = A, B, C$

$$\text{Minimize } Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} \\ + 4x_{3A} + 5x_{3B} + 12x_{3C}$$

subject to:

$$x_{1A} + x_{1B} + x_{1C} = 150$$

$$x_{2A} + x_{2B} + x_{2C} = 175$$

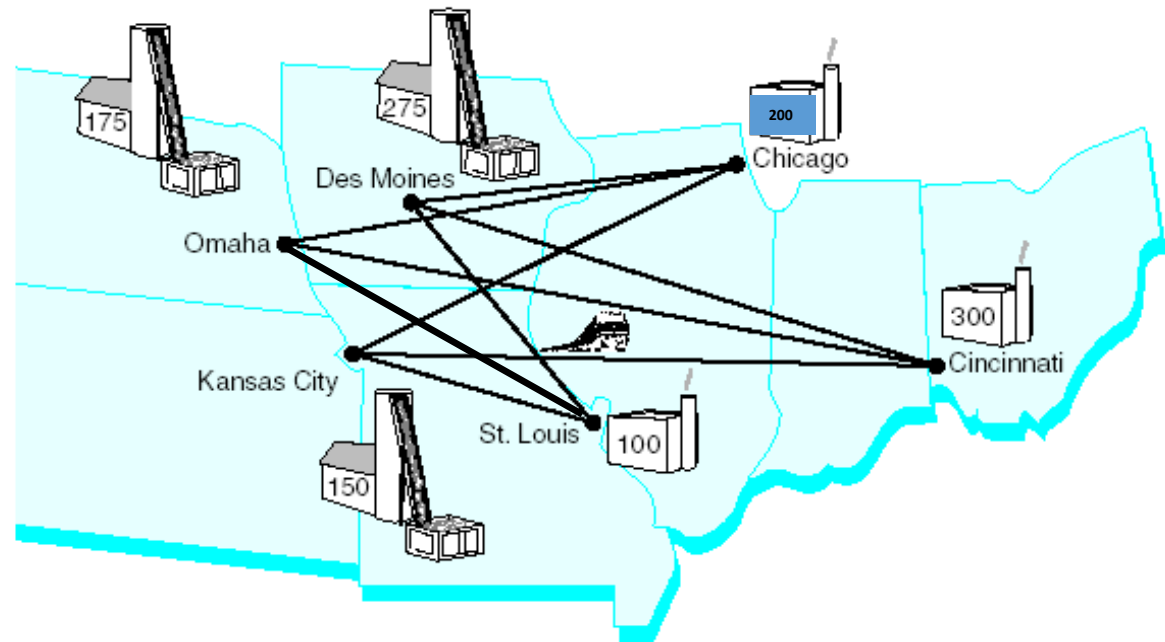
$$x_{3A} + x_{3B} + x_{3C} = 275$$

$$x_{1A} + x_{2A} + x_{3A} = 200$$

$$x_{1B} + x_{2B} + x_{3B} = 100$$

$$x_{1C} + x_{2C} + x_{3C} = 300$$

$$x_{ij} \geq 0$$



Transportation Model Solution with Excel

Objective function

Exhibit6.1.xls [Compatibility Mode] - Microsoft Excel

Home Insert Page Layout Formulas Data Review View

Font: Arial, 10, Bold, Italic, Underline, Text Color, Background Color, Wrap Text, Merge & Center, Alignment, Number, Styles, Cells, Editing

Formula Bar: C10 =SUMPRODUCT(C5:E7,K5:M7)

The Wheat Shipping Example							Shipping costs (\$/ton)			
Grain Elevators	Chicago	St. Louis	Cincinnati	Supply	Grain Shipped		Grain Elevators	Chicago	St. Louis	Cincinnati
Kansas City				150	0		Kansas City	6	8	10
Omaha				175	0		Omaha	7	11	11
Des Moines				275	0		Des Moines	4	5	12
Demand	200	100	300	600						
Grain Shipped	0	0	0							
Cost =	0									

$=D5+D6+D7$

$=C7+D7+E7$

Decision variables
in cells C5:E7

Cost array in
cells K5:M7

Transportation Model Example

Solution with Excel

Solver Parameters

Set Target Cell:

Equal To: ☐ Max ☒ Min ☐ Value of:

By Changing Cells:

Subject to the Constraints:

Demand
constraints

Supply
constraints

Exhibit6.1.xls [Compatibility Mode] - Microsoft Excel

Home Insert Page Layout Formulas **Data** Review View

From Access From Web From Text From Other Sources Existing Connections

Get External Data

Refresh All Connections Properties Edit Links

Connections

Sort Filter Clear Reapply Advanced

Sort & Filter

Text to Columns Remove Duplicates Data Validation Consolidate What-If Analysis

Data Tools

C10 \sum =SUMPRODUCT(C5:E7,K5:M7)

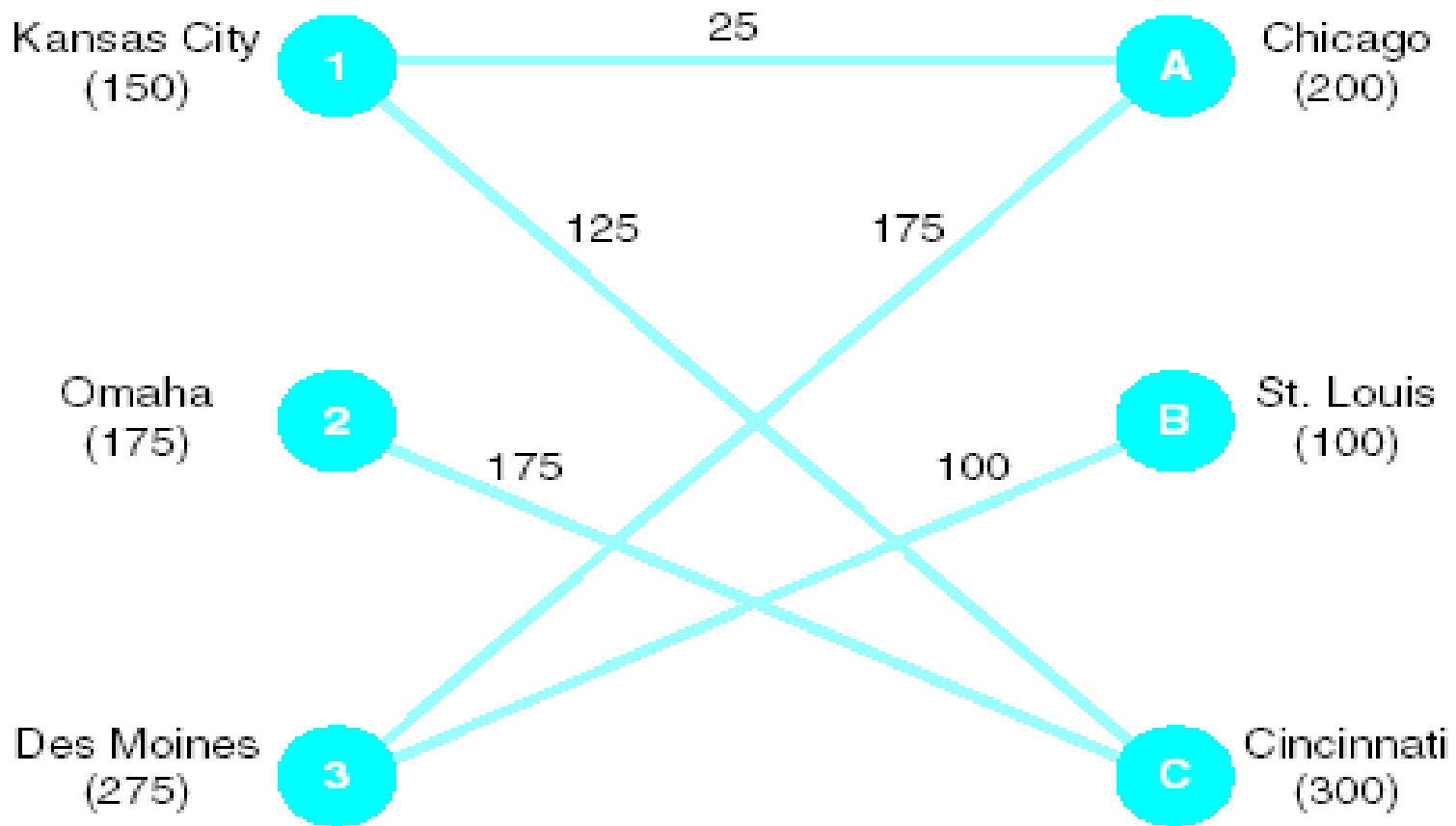
The Wheat Shipping Example

Shipping costs (\$/ton)

			Mills		Grain				Mills		
Grain Elevators	Chicago	St. Louis	Cincinnati	Supply	Shipped	Grain Elevators	Chicago	St. Louis	Cincinnati		
Kansas City	25	0	125	150	150	Kansas City	6	8	10		
Omaha	0	0	175	175	175	Omaha	7	11	11		
Des Moines	175	100	0	275	275	Des Moines	4	5	12		
Demand	200	100	300	600							
Grain Shipped	200	100	300								
Cost =	4525										

Transportation Model Example

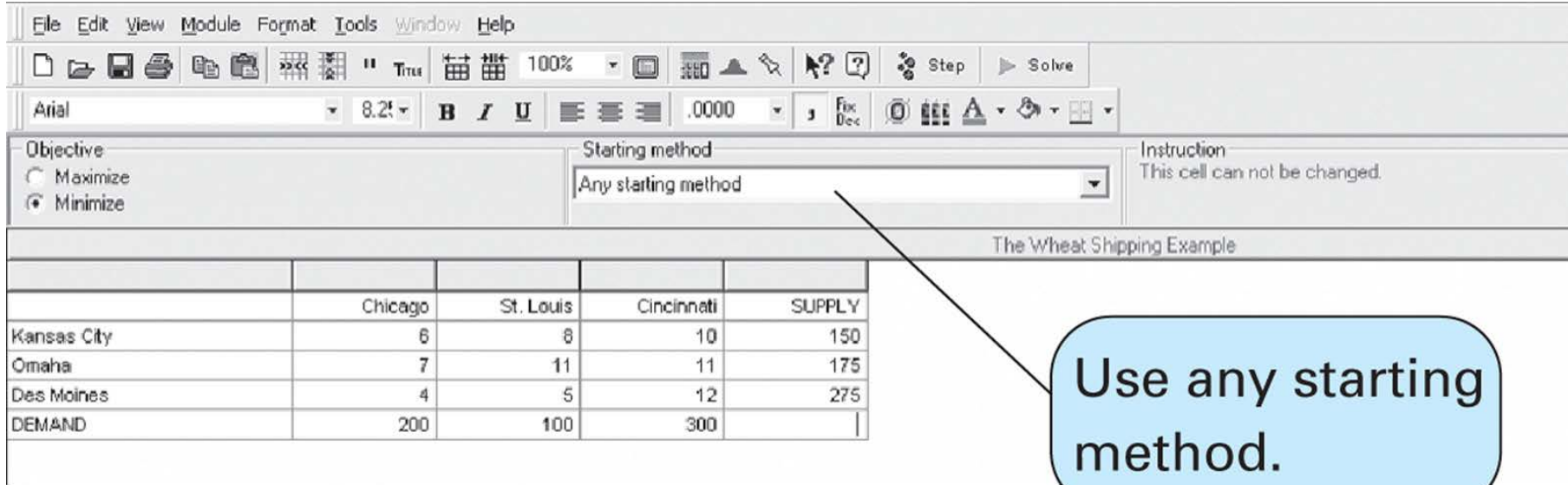
Solution with Excel



Transportation Network Solution

Transportation Model Example

Solution with QM for Windows



File Edit View Module Format Tools Window Help

100%

Objective
☐ Maximize
☒ Minimize

Starting method
 Any starting method

Instruction
 This cell can not be changed.

The Wheat Shipping Example

	Chicago	St. Louis	Cincinnati	SUPPLY
Kansas City	6	8	10	150
Omaha	7	11	11	175
Des Moines	4	5	12	275
DEMAND	200	100	300	

Use any starting method.

For more information:

<http://www.scribd.com/doc/49602895/30/Northwest-Corner-Method>

Transportation Model Example Solution with QM for Windows

Transportation Shipments			
The Wheat Shipping Example Solution			
Optimal solution value = \$4,525	Chicago	St. Louis	Cincinnati
Kansas City			150
Omaha	25		150
Des Moines	175	100	

Transportation Model Example Solution with QM for Windows

Shipping list

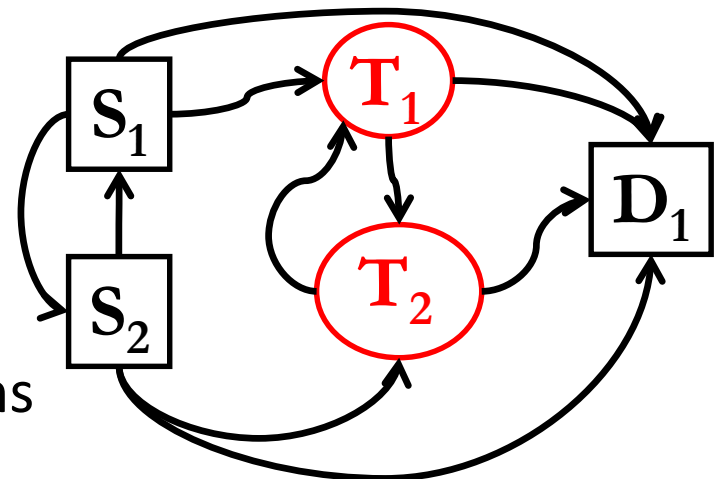
The Wheat Shipping Example Solution

From	To	Shipment	Cost per unit	Shipment cost
Kansas City	Cincinnati	150	10	1,500
Omaha	Chicago	25	7	175
Omaha	Cincinnati	150	11	1,650
Des Moines	Chicago	175	4	700
Des Moines	St. Louis	100	5	500

The Transshipment Model

Characteristics

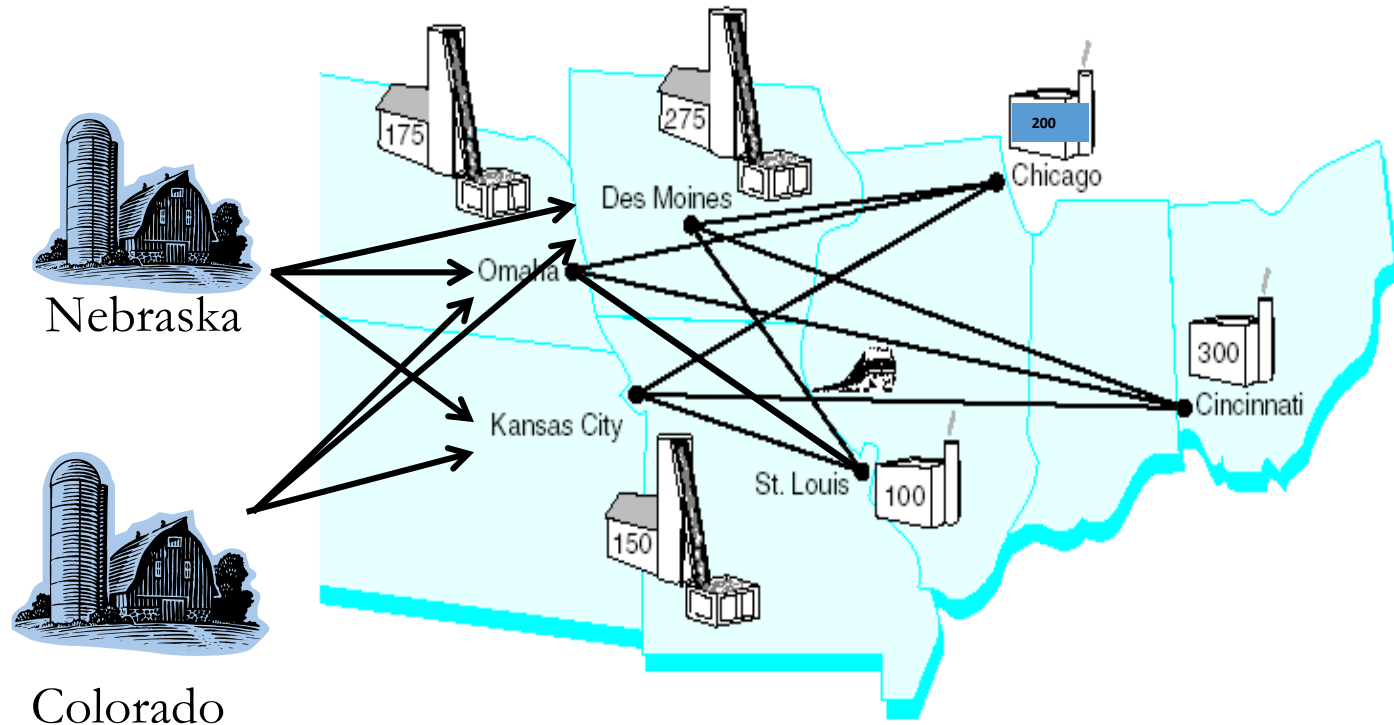
- Extension of the transportation model
- Intermediate transshipment points are added between the sources and destinations
- Items may be transported from:
 - Sources through transshipment points to destinations
 - One source to another
 - One transshipment point to another
 - One destination to another
 - Directly from sources to destinations
 - Some combination of these



Transshipment Model Example

Problem Definition and Data

Let's extend the earlier transportation example by adding intermediate transshipment points between sources and destinations

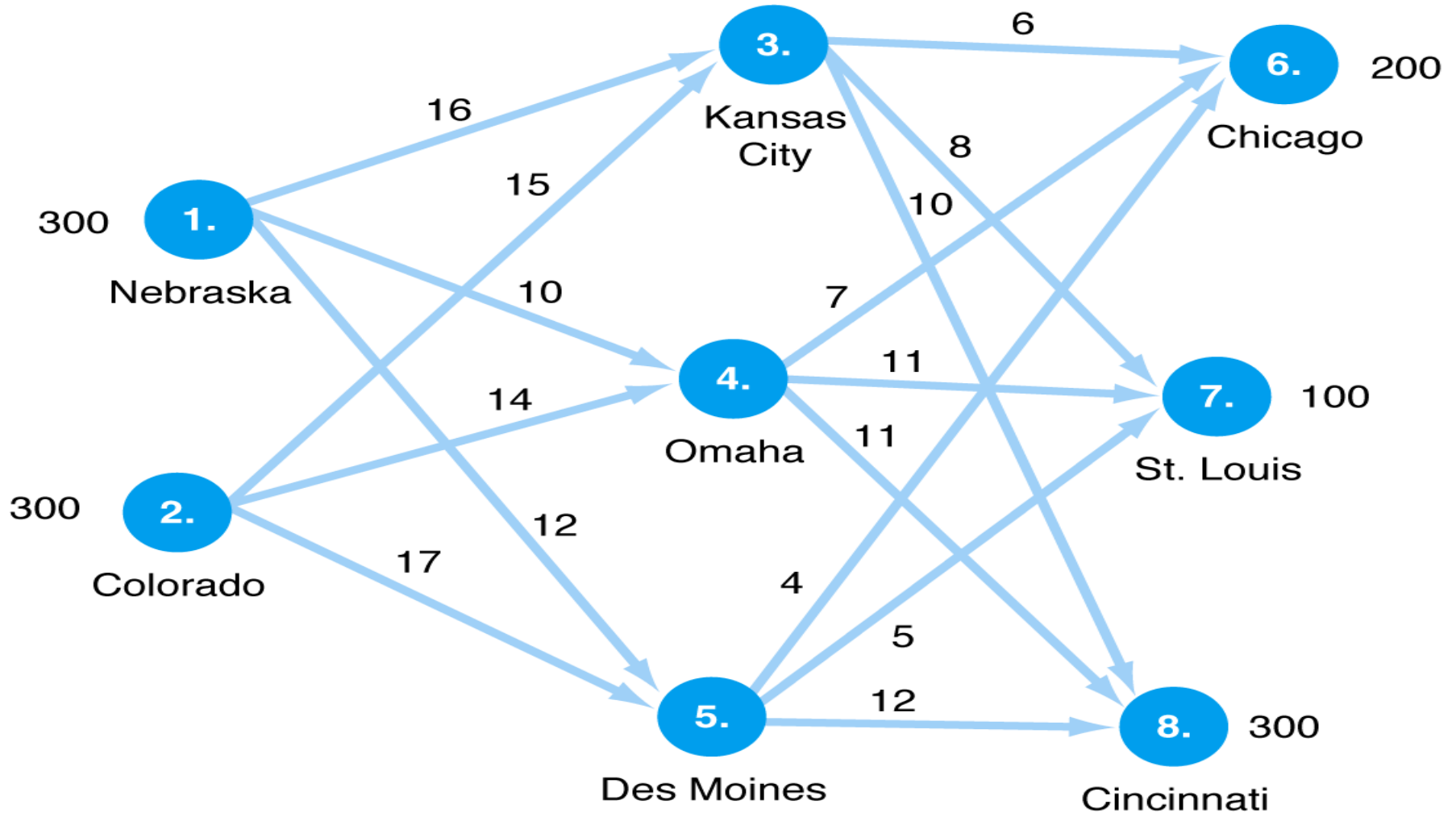


Shipping Costs

Farm	<u>Grain Elevator</u>		
	3. Kansas City	4. Omaha	5. Des Moines
1. Nebraska	\$16	10	12
2. Colorado	15	14	17

Transshipment Model Example

Transshipment Network Routes



Network of Transshipment Routes

Transshipment Model Example

Model Formulation

x_{ij} = # of tons of wheat from farm i to grain elevator j , and
 # of tons of wheat from grain elevator i to mill j

$$\begin{aligned} \text{Minimize } Z = & \$16x_{13} + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24} \\ & + 17x_{25} + 6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47} \\ & + 11x_{48} + 4x_{56} + 5x_{57} + 12x_{58} \end{aligned}$$

subject to:

$$x_{13} + x_{14} + x_{15} = 300$$

Nebraska Farm

$$x_{23} + x_{24} + x_{25} = 300$$

Colorado Farm

$$x_{36} + x_{46} + x_{56} = 200$$

Chicago Demand

$$x_{37} + x_{47} + x_{57} = 100$$

St. Louis Demand

$$x_{38} + x_{48} + x_{58} = 300$$

Cincinnati Demand

$$x_{13} + x_{23} - x_{36} - x_{37} - x_{38} = 0$$

$$x_{14} + x_{24} - x_{46} - x_{47} - x_{48} = 0$$

$$x_{15} + x_{25} - x_{56} - x_{57} - x_{58} = 0$$

$$x_{ij} \geq 0$$

Balancing of transshipment nodes,
 to ensure no hold-up – “What
 comes in must go out!”

Transshipment Model Example

Solution with Excel

Objective function

Exhibit6.10.xls [Compatibility Mode] - Microsoft Excel

Home Insert Page Layout Formulas Data Review View

Clipboard Font Alignment Number Styles Cells Editing

C24 =SUMPRODUCT(B6:D7,I6:K7)+SUMPRODUCT(C13:E15,J13:L15)

The Wheat Shipping Transshipment Example						
Grain Elevators						
Farms	3. Kansas City	4. Omaha	5. Des Moines	Supply	Grain Shipped	
1. Nebraska	0	0	300	300	300	
2. Colorado	300	0	0	300	300	
Shipped	300	0	300			
Mills						
Grain Elevators	6. Chicago	7. St. Louis	8. Cincinnati	Grain Shipped		
3. Kansas City	0	0	300	300		
4. Omaha	0	0	0	0		
5. Des Moines	200	100	0	300		
Demand	200	100	300			
Shipped	200	100	300			
Transshipment flows:						
	3. Kansas City	0				
	4. Omaha	0				
	5. Des Moines	0				
Cost =	12400					

Shipping Costs:

Farms	3. Kansas City	4. Omaha	5. Des Moines
1. Nebraska	16	10	12
2. Colorado	15	14	17

Shipping costs:

Grain Elevators	6. Chicago	7. St. Louis	8. Cincinnati
3. Kansas City	6	8	10
4. Omaha	7	11	11
5. Des Moines	4	5	12

=SUM(B6:B7)

=SUM(B6:D6)

=SUM(C13:C15)

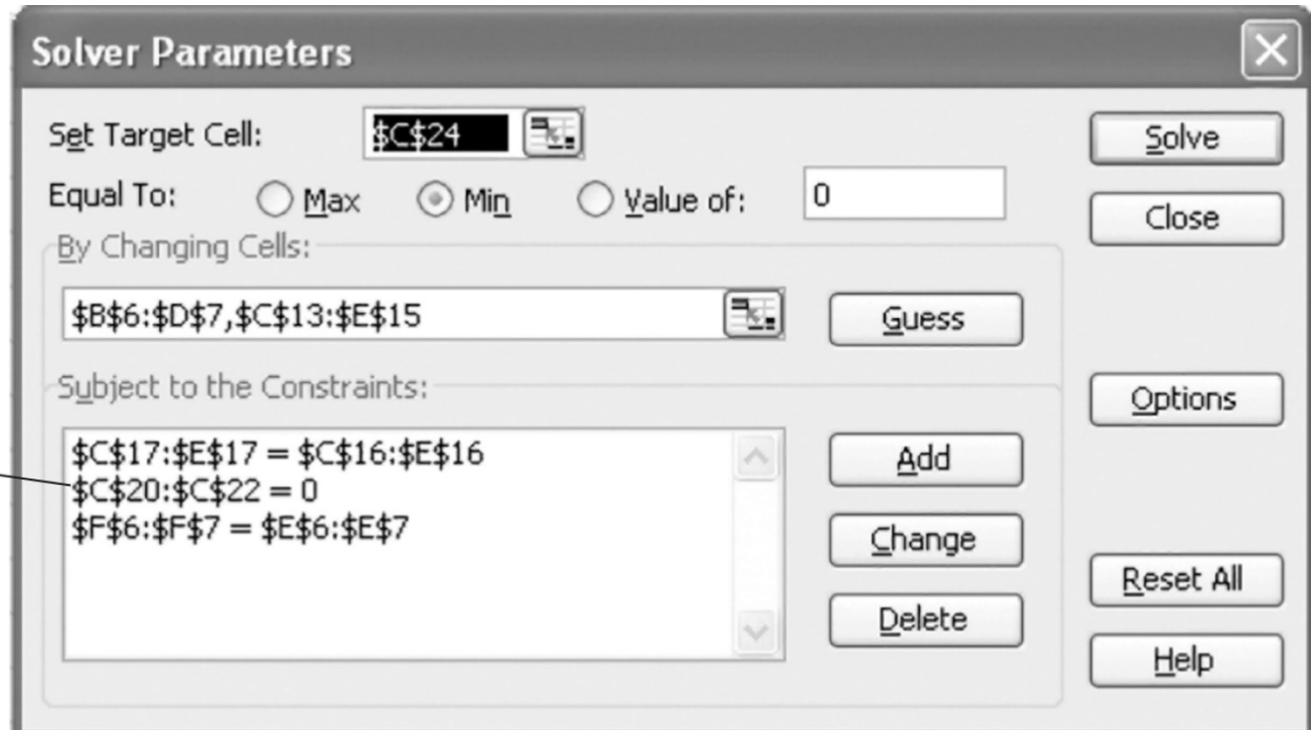
=SUM(C13:E13)

Cost arrays

Constraints for transshipment flows;
i.e., shipments in = shipments out

Transshipment Model Example Solution with Excel

Transshipment
constraints in
cells C20:C22



Solver Parameters

Set Target Cell:

Equal To: ☐ Max ☒ Min ☐ Value of:

By Changing Cells:

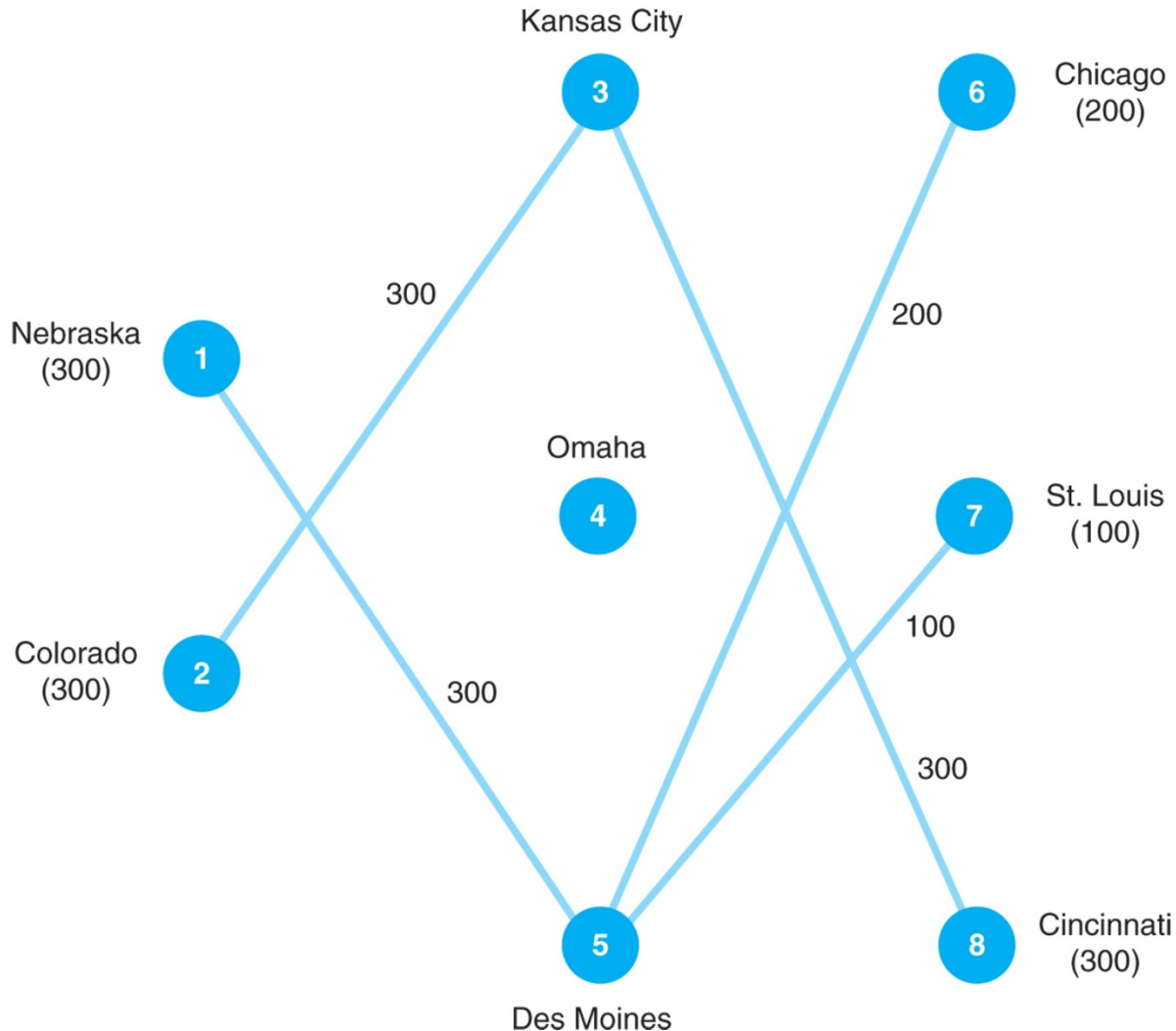
Subject to the Constraints:

-
-
-

Buttons: Solve, Close, Options, Add, Change, Delete, Reset All, Help

Transshipment Model Example

Network Solution: Wheat Shipping



The Assignment Model Characteristics

- Special form of linear programming model similar to the transportation model
- *Supply* at each source *and demand* at each destination *limited to one unit*
- In a balanced model supply equals demand
- In an unbalanced model, supply does not equal demand

Assignment Model Example

Problem Definition and Data

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

Officials	Game Sites			
	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Teamwork – Assignment Problem

Can you determine an optimal solution by inspection?

Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

Officials	Game Sites			
	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Assignment Model Example

Model Formulation

x_{ij} = assignment of officials team i to game j [binary variable]

$$\begin{aligned} \text{Minimize } Z = & 210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} \\ & + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} \\ & + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC} \end{aligned}$$

subject to:

$$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$$

$$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$$

$$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$$

$$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$$

$$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$$

$$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$$

$$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$$

$$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$$

$$x_{ij} = 0, 1$$

Any one team of
officials can only be at
one game

Requirement of one
team (and only one
team) of officials at
each game

Assignment Model Example

Solution with Excel

Objective function

Exhibit6.12.xls [Compatibility Mode] - Microsoft Excel

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Clipboard Font Alignment Styles

C11 =SUMPRODUCT(C5:F8,C16:F19)

The ACC Basketball Example							
		Game Sites				Teams Available	Teams Assigned
Officials Team		Raleigh	Atlanta	Durham	Clemson		
A						1	0
B						1	0
C						1	0
D						1	0
Teams Demanded						4	
Teams Assigned		0	0	0	0		
Total Mileage =		0					

Mileage:					
		Game Sites			
Officials Team		Raleigh	Atlanta	Durham	Clemson
A		210	90	180	160
B		100	70	130	200
C		175	105	140	170
D		80	65	105	120

Decision variables, C5:F8

=C5+D5+E5+F5

=D5+D6+D7+D8

Mileage array

Assignment Model Example Solution with Excel

Solver Parameters [X]

Set Target Cell: [icon]

Equal To: ☐ Max ☒ Min ☐ Value of:

By Changing Cells: [icon]

Subject to the Constraints:

\$C\$10:\$F\$10 = \$C\$9:\$F\$9

\$H\$5:\$H\$8 = \$G\$5:\$G\$8

↑

↓

Add

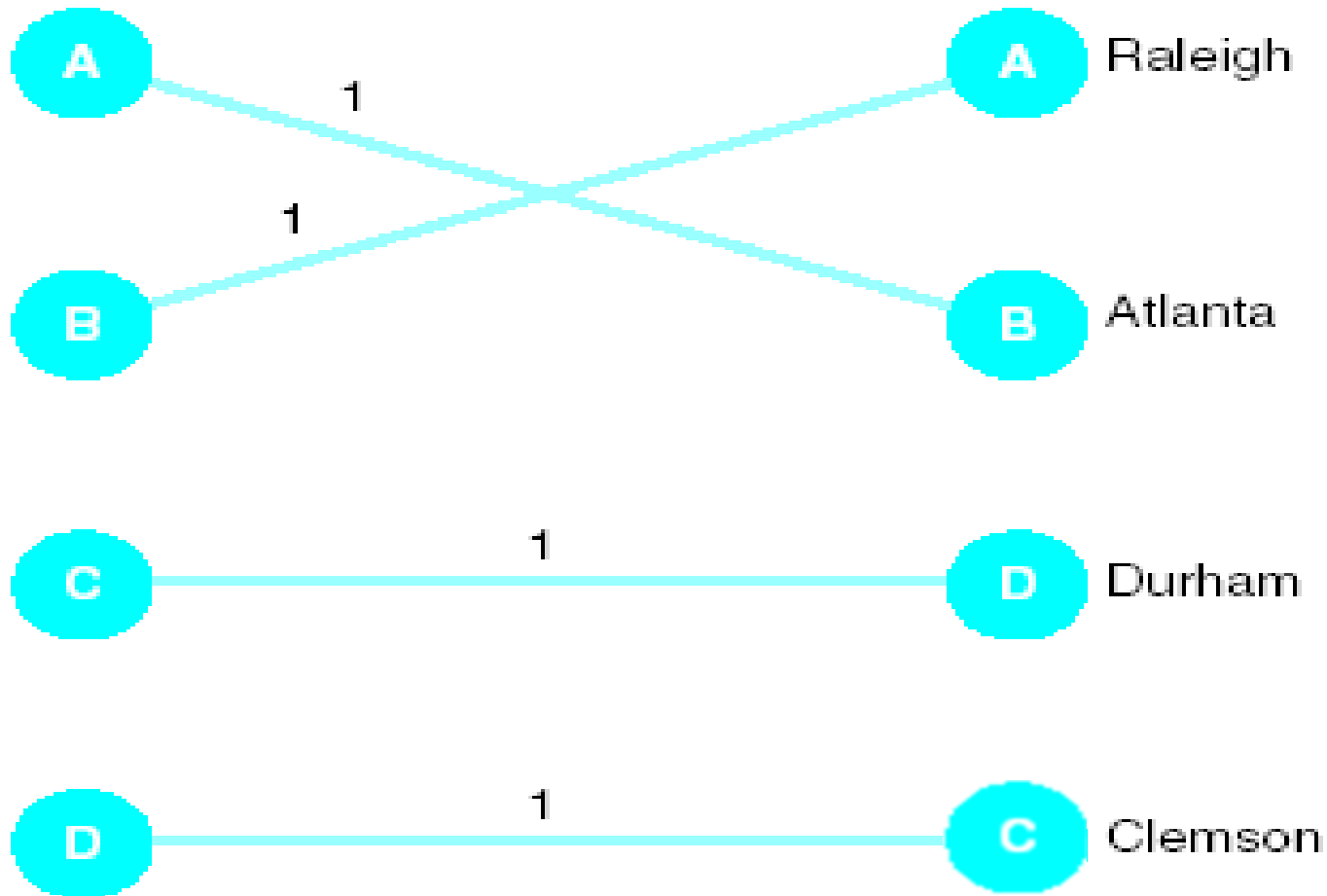
Change

Delete

Assignment Model Example Solution with Excel

Assignment Model Example

Assignment Network Solution



Assignment Model Example Solution with QM for Windows

Assignment Model Example

Solution with QM for Windows

Assignments

ACC Basketball Example Solution

Optimal cost = \$450	Raleigh	Atlanta	Durham	Clemson
A	210.	Assign 90	180.	160.
B	Assign 100	70.	130.	200.
C	175.	105.	Assign 140	170.
D	80.	65.	105.	Assign 120

Teamwork - Transportation Problem

Acme Bolts (a sub-division of Acme Enterprises) needs to transport tons of its bolts (“Better Bolts for a Better Tomorrow”) from manufacturing plants to various construction sites at the lowest possible cost. There is no storage available at the plants, and demand is sometimes overstated by the construction sites. (The costs below are in \$/ton)

Plant	Construction site			Supply (tons)
	A	B	C	
1	\$ 8	\$ 5	\$ 6	120
2	15	10	12	80
3	3	9	10	80
Demand (tons)	150	70	100	

Example Problem Solution

Model Formulation

x_{ij} = # of tons of bolts shipped from plant i to construction site j

$$\text{Minimize } Z = \$8x_{1A} + 5x_{1B} + 6x_{1C} + 15x_{2A} + 10x_{2B} + 12x_{2C} \\ + 3x_{3A} + 9x_{3B} + 10x_{3C}$$

subject to:

$$x_{1A} + x_{1B} + x_{1C} = 120$$

$$x_{2A} + x_{2B} + x_{2C} = 80$$

$$x_{3A} + x_{3B} + x_{3C} = 80$$

$$x_{1A} + x_{2A} + x_{3A} \leq 150$$

$$x_{1B} + x_{2B} + x_{3B} \leq 70$$

$$x_{1C} + x_{2C} + x_{3C} \leq 100$$

$$x_{ij} \geq 0$$

The “=” comes from the verbiage of the problem...no storage space at the plants

The \leq also comes from the verbiage...demand is sometimes overstated by the construction sites.

