A company is planning its advertising strategy for next year for its three major products

Since the three products are quite different, each advertising effort will focus on a single product

In units of millions of dollars, a total of 6 is available for advertising next year, where the advertising expenditure for each product must be an integer greater than or equal to 1

The vice-president for marketing has established the objective: Determine how much to spend on each product in order to maximize total sales

The following table gives the estimated increase in sales (in appropriate units) for the different advertising expenditures

Use dynamic programming to solve this problem

Advertising	Product		
Expenditure	1	2	3
1	7	4	6
2	10	8	9
3	14	11	13
4	17	14	15

Since the decisions to be made are the advertising expenditures on the three products, the stages for a dynamic programming formulation of this problem correspond to the three products.

When making the decision for a particular product, the essential information is the amount of the advertising budget still remaining, so this becomes the current state in this formulation

x_n = the advertising dollars (in millions) spent on product n
s_n = the amount of advertising budget remaining
$p_n x_n$ = the increase in sales of product n when x_n million dollars are spent on product n, as given by the table in the problem statement

Using the dynamic programming notation presented in the text, the recursive relationship for this problem is:

$$f_n^*(s_n) = \max_{1 \le x_n \le s_n} \left\{ p_n(x_n) + f_{n+1}^*(s_n - x_n) \right\}$$

- The solution procedure now starts at the end (stage 3) and moves backward stage by stage.

Advertising Expenditure	Product 1	2	3
1	7	4	6
2	10	8	9
3	14	11	13
4	17	14	15

For n=3 This is the "end" of the problem

s_3	$f_3^*(s_3)$	x_3^*
1	6	1
2	9	2
3	13	3
4	15	4

If you spend 3 in stage 2 below (green column), you can only have spent 1 or 2 in stage 3, and would have either 1 or 2 remaining for stage 1.
(We're actually working backwards, to stage 1, so the remaining allocated budget is really being summed to the total allocated amount of 6 million)
If you spend 4 in stage 2 below (red column), you can only have spent 1 in stage 3, because you must have 1 left for stage 1, right?

Original table of expected returns

Advertising Expenditure	Product 1	Product 2	Product 3
1	7	4	6
2	10	8	9
3	14	11	13
4	17	14	15

For n=2 This is considered the middle of the problem

s_2 \ x_2	$f_2(s_2, x_2) = p_2(s_2) + f_3^*(s_2 - x_2)$: 1	2	3	4	Maximum $f_2^*(s_2)$	Which x_2 generates the max $f_2^*(s_2)$ x_2^*
2	4+6				4+6=10	1
3	4+9	8+6			8+6=14	2
4	4+13	8+9	11+6		17	1, 2, 3
5	4+15	8+13	11+9	14+6	8+13=21	2

For $n = 2$ This is considered the middle of the problem

x_2 / s_2	$f_2(s_2, x_2) = p_2(s_2) + f_3^*(s_2 - x_2)$: 1	2	3	4	Maximum $f_2^*(s_2)$	Which x_2 generates the max $f_2^*(s_2)$ x_2^*
2	4+6				4+6=10	1
3	4+9	8+6			8+6=14	2
4	4+13	8+9	11+6		17	1, 2, 3
5	4+15	8+13	11+9	14+6	8+13=21	2

I put these arrows here to show source of #.

This column represents the amount of remaining budget, so:

- if 4 is spent in stage 3, then there is 2 left
- if 3 is spent in stage 3, then there is 3 left
- if 2 is spent in stage 3, then there is 4 left
- if 1 is spent in stage 3, then there is 5 left

Original table of expected returns

Advertising Expenditure	Product 1	2	3
1	7	4	6
2	10	8	9
3	14	11	13
4	17	14	15

For $n = 1$ This is considered the "beginning" of the problem – you have a complete budget to allocate – although the problem constraint says that all stages must have 1 million allocated, you actually have 6 million at the beginning of the problem. I would've accepted it if you had put 4 here, but 6 is accurate, and consistent.

x_1 / s_1	$f_1(s_1, x_1) = p_1(s_1) + f_2^*(s_1 - x_1)$: 1	2	3	4	$f_1^*(s_1)$	x_1^*
6	21+7	17+10	14+14	10+17	28	1, 3

Since $s_2 = 6 - x_1^*$ and $s_3 = s_2 - x_2^*$, there are two optimal plans as given in the table below.

Optimal plan	x_1^*	x_2^*	x_3^*	**Total Sales**
1	1	2	3	**28**
2	3	2	1	**28**