SUPPLEMENT 2 TO CHAPTER 18
STOCHASTIC PERIODIC-REVIEW MODELS

18S2-1.

(a) Single-period model with no setup cost:

Demand density is exponential with $\lambda = 25$. Per unit production/purchasing cost is $c = 10$. Per unit inventory holding cost is $h = 6$ and per unit shortage cost is $p = 15$. The optimal one-period inventory level is $S(0) = 6.79834$.

(b) Two-period model with no setup cost:

Demand density is exponential with $\lambda = 25$. Per unit production/purchasing cost is $c = 10$. Per unit inventory holding cost is $h = 6$ and per unit shortage cost is $p = 15$. The optimal two-period policy consists of the inventory levels $S_1(0) = 23.2932$ and $S_2(0) = 6.79834$.

18S2-2.

(a) Single-period model with no setup cost:

Demand density is uniform on $[0, 50]$. Per unit production/purchasing cost is $c = 10$. Per unit inventory holding cost is $h = 8$ and per unit shortage cost is $p = 15$. The optimal one-period inventory level is $S^* = 10.8696$. It is optimal to order up to S^* if the initial inventory is below S^* and not to order otherwise.

(b) Two-period model with no setup cost:

Demand density is uniform on $[0, 50]$. Per unit production/purchasing cost is $c = 10$. Per unit inventory holding cost is $h = 8$ and per unit shortage cost is $p = 15$. The optimal two-period policy consists of the inventory levels $S_1^* = 9.26156$ and $S_2^* = 10.8696$. It is optimal to order up to S_i^* if the initial inventory is below S_i^* in period i and not to order otherwise.

18S2-3.

Two-period model with no setup cost:

Demand density is exponential with $\lambda = 25$. Per unit production/purchasing cost is $c = 1$. Per unit inventory holding cost is $h = 0.25$ and per unit shortage cost is $p = 2$. The discount factor is 0.9. The optimal two-period policy is the same as the one for the infinite-period model, so consists of the inventory level $S(0) = 46.5188$.

18S2-4.

Two-period model with no setup cost:

Demand density is exponential with $\lambda = 25$. Per unit production/purchasing cost is $c = 1$. Per unit inventory holding cost is $h = 0.25$ and per unit shortage cost is $p = 2$. The optimal two-period policy consists of the inventory levels $S_1(0) = 36.521$ and $S_2(0) = 14.6947$.

18S2-5.

Infinite-period model with no setup cost:

Demand density is exponential with $\lambda = 25$. Per unit production/purchasing cost is $c = 1$. Per unit inventory holding cost is $h = 0.25$ and per unit shortage cost is $p = 2$. The discount factor is 0.9. The optimal policy consists of the inventory level $S(0) = 46.5188$.

18S2-6.

Infinite-period model with no setup cost:

Demand density is exponential with $\lambda = 1$. Per unit production/purchasing cost is $c = 2$. Per unit inventory holding cost is $h = 1$ and per unit shortage cost is $p = 5$. The discount factor is 0.95. The optimal policy consists of the inventory level $S(0) = 1.69645$.

18S2-7.

12-period model with no setup cost:

The answer is the same as in 18S2-6, so the optimal policy consists of the inventory level $S(0) = 1.69645$.

18S2-8.

Infinite-period model with no setup cost:

Demand density is uniform on $[2000, 3000]$. Per unit production/purchasing cost is $c = 150$. Per unit inventory holding cost is $h = 2$ and per unit shortage cost is $p = 30$. The discount factor is 0.9. The optimal policy consists of the inventory level $S(0) = 2,468.75$.

18S2-9.

Infinite-period model with no setup cost:

Demand density is exponential with $\lambda = 1000$. Per unit production/purchasing cost is $c = 80$. Per unit inventory holding cost is $h = 0.70$ and per unit shortage cost is $p = 2$. The discount factor is 0.998. The optimal policy consists of the inventory level $S(0) = 497$.

18S2-10.

$h = 0.3, p = 2.5$

$G(\underline{S}) = 0.3\int_0^{\underline{S}} \frac{(\underline{S}-\underline{x})}{25} e^{-\underline{x}/25} d\underline{x} + 2.5\int_{\underline{S}}^{\infty} \frac{(\underline{x}-\underline{S})}{25} e^{-\underline{x}/25} d\underline{x} = 0.3\underline{S} + 70e^{-\underline{S}/25} - 7.5$
$G'(\underline{S}) = 0.3 - 2.8e^{-\underline{S}/25} = 0 \Rightarrow \underline{S} = 55.84$
$G''(\underline{S}) = \frac{2.8}{25}e^{-\underline{S}/25} > 0 \Rightarrow \underline{S} = 55.84$ minimizes $G(\underline{S})$.

$G(k) = G(k+100) \Leftrightarrow 0.3k + 70e^{-k/25} = 0.3(k+100) + 70e^{-(k+100)/25}$

$\Leftrightarrow 70e^{-k/25}(1 - e^{-4}) = 30 \Leftrightarrow k = 20.72 \approx 21$

$k = 21 < \underline{S} = 55.84 < 121 = k + 100$ and $G(21) \approx G(121)$

Hence, the optimal policy is a $(k, Q) = (21, 100)$ policy.

18S2-11.

Since $c = 0$, the answer is identical to that for 18.S2-10, viz., $(k, Q) = (21, 100)$ is optimal.

18S2-12.

$L(\underline{S}) = \int_0^{\underline{S}} h(\underline{S} - \underline{x})\underline{f}(\underline{x})d\underline{x} + \int_{\underline{S}}^{\infty} p(\underline{x} - \underline{S})\underline{f}(\underline{x})d\underline{x}$

$\frac{dL(\underline{S})}{d\underline{S}} = \int_0^{\underline{S}} h\underline{f}(\underline{x})d\underline{x} + \int_{\underline{S}}^{\infty} -p\underline{f}(\underline{x})d\underline{x} = h\underline{F}(\underline{S}) - p[1 - \underline{F}(\underline{S})]$

$\frac{dL(\underline{S})}{d\underline{S}} + c(1-\alpha) = 0 \Rightarrow -p + p\underline{F}(\underline{S}) + h\underline{F}(\underline{S}) + c(1-\alpha) = 0$

$\Rightarrow \underline{F}(\underline{S}) = \frac{p - c(1-\alpha)}{p+h}$