Stevens Institute of Technology

SYS-601 Homework #4 Solutions

Submit the following using the online submission system: 1) Completed assignment cover sheet, 2) Written responses in PDF format, 3) All saved models (e.g. .xlsx or .py files).

4.1 Derived Dice Rolls [5 points]

Use the `random.org`[1] generator to roll $N = 30$ samples of two virtual dice. The discrete random variable Z is a function of the left (Z_L) and right (Z_R) dice value as follows:

$$Z = \left\lceil \frac{Z_L}{2} \right\rceil - \left\lceil \frac{Z_R}{2} \right\rceil$$

where $\lceil \quad \rceil$ is the mathematical ceiling function (i.e. round up to whole number).

(a) 1 pt Write the number of observations for each value of $Z \in \{-2, -1, 0, 1, 2\}$.

(b) 2 pts Write the empirical probability mass function $P(z)$ in a tabular format.

(c) 2 pts Write the empirical cumulative distribution function $F(z)$ in a tabular format.

My results from $N = 30$ samples as follows.

z	-2	-1	0	1	2
Count	6	9	7	5	3
$p(z)$	$6/30 = 0.20$	$9/30 = 0.30$	$7/30 = 0.23$	$5/30 = 0.17$	$3/30 = 0.10$
$F(z)$	$6/30 = 0.20$	$15/30 = 0.50$	$22/30 = 0.73$	$27/30 = 0.90$	$30/30 = 1.00$

4.2 100 Year Floodplain [7 points]

The Federal Emergency Management Agency (FEMA) designated the city of East Biggs, California inside the 100-year floodplain assessment for Lake Oroville which means there is a 1% chance of a flood each year.

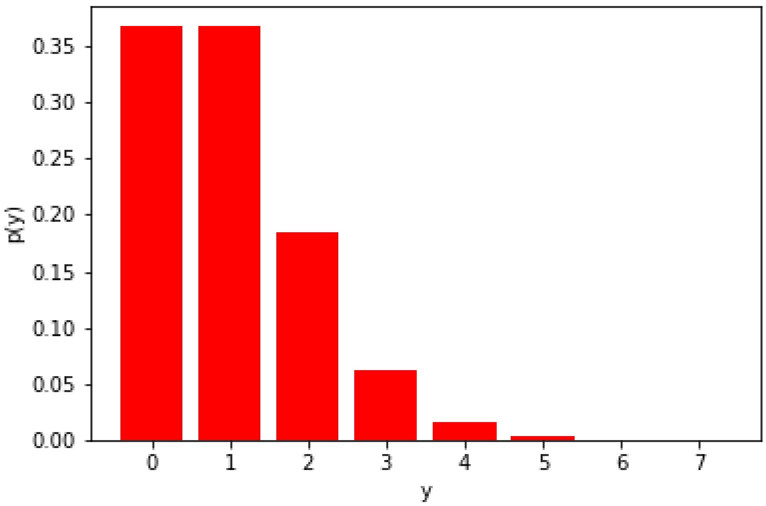
(a) 2 pts Assuming Y, the number of floods in East Biggs, follows a Poisson distribution, write an equation for $p(y)$, the probability of y floods over a 100 year time span.

Note: $\lambda = 1$ flood per 100 years.

$$p(y) = \frac{\lambda^y e^{-\lambda}}{y!} = \frac{1.0^y e^{-1.0}}{y!}$$

[1] `https://www.random.org/dice/`

(b) 2 PTS Create a bar plot for the PMF of $p(y)$ above.



(c) 2 PTS What is $P\{y \geq 1\}$, the probability of *at least* one flood in 100 years?

$$P\{y \geq 1\} = 1 - p(0) = 1 - \frac{1.0^0 e^{-1.0}}{0!} = 1 - e^{-1.0} = 0.63$$

(d) 1 PT What assumptions are required for a Poisson distribution assumed in (a)? Are these assumptions appropriate for events such as floods?

A Poisson distribution requires the following:

- Each event is independent of other events
- The average rate λ remains constant over time

First, floods may not be fully independent of each other. For example, the occurrence of a flood may likely *decrease* the probability of future floods (at least in the short term) because it releases stored-up water. However, if floods are sufficiently rare it is generally a reasonable assumption that they are independent because so much time passes between events.

Second, and more critically, it is not reasonable to assume the average rate λ remains constant over long time periods. Both the effect of flood control measures (e.g. dams) and effects of climate change are continuously changing the long-term "average" flood rate. Many authorities are now using more conservative estimates of the frequency of future natural disasters, especially weather-related, to reflect a changing environment.

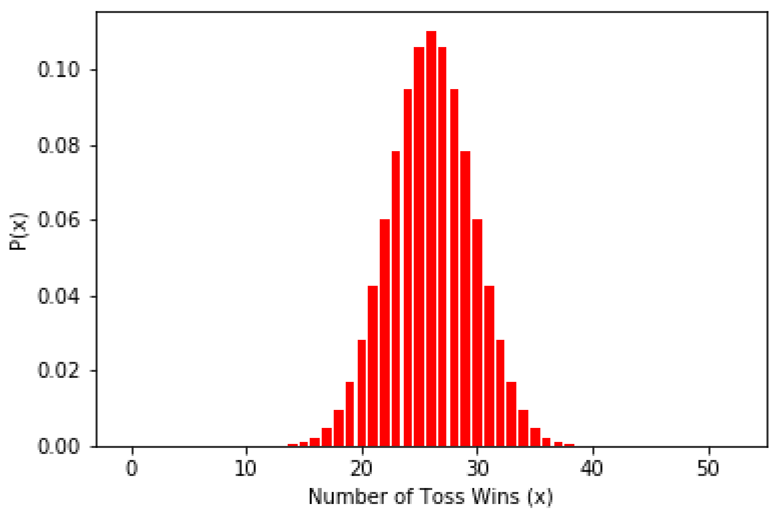
4.3 Super Bowl Coin Flips [8 points]

A traditional coin flip before the Super Bowl allows the winning team, either the American Football Conference (AFC) or National Football Conference (NFC), the choice to start on offense or defense. The attached file `superbowl.csv` contains results for the 52 Super Bowls played as of 2018.

(a) 2 PTS Assuming the coin toss is fair (50% chance to win), write an equation for $p(x)$, the probability of winning exactly x tosses in 52 trials. (*Hint:* binomial distribution)

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{52}{x} 0.5^x 0.5^{52-x}$$

(b) 2 PTS Create a bar plot for the PMF of $p(x)$ above.



(c) 2 PTS Count how many coin tosses the NFC has won in the 52 Super Bowls (N). Where is this point on the PMF plot? Is this a typical outcome or an unusual outcome?

The NFC won 35 of the 52 coin tosses. This is on the right tail of the distribution. It seems a little high compared to the expected number of wins (about 26).

(d) 2 PTS What is the probability of winning *at least* as many coin tosses as the NFC, $P\{x \geq N\}$, in 52 trials?

$$P\{X \geq 35\} = 1 - F(34) = \sum_{i=35}^{52} P(x) = 0.009$$

In other words, only in 9 out of 1000 cases will one conference win 35 or more tosses out of 52 attempts!