Stevens Institute of Technology

SYS-601 Homework Cover Sheet

Date: 2/25/2018 **HW #:** HW #5

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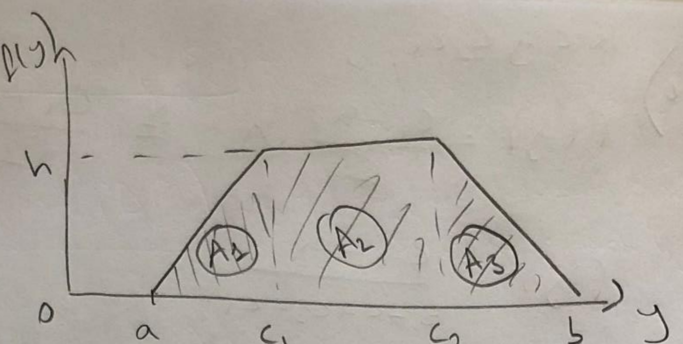
Collaborators: -

5.1

a) $f(y) = \int_0^b f(y)\,dy$



first calculate the areas ;

$A_1 = \frac{c_1 - a}{2} . h \qquad A_2 = (c_2 - c_1) . h \qquad A_3 = \frac{b - c_2}{2} . h$

Accordingly ;

$$f(y) = h\left[\left(\frac{c_1 - a}{2}\right) + (c_2 - c_1) + \left(\frac{b - c_2}{2}\right)\right] = 1$$

$$\frac{2}{h} = c_1 - a + 2c_2 - 2c_1 + b - c_2$$

$$\frac{2}{h} = b + c_2 - a - c_1 \Rightarrow \boxed{h = \frac{2}{b - a + c_2 - c_1}}$$

b) i) $a < y \leq c_1$;

$$f(y) = \frac{h}{c_1 - a} . (y - a) = \frac{2(y - a)}{(c_1 - a)(b - a + c_2 - c_1)}$$

ii) $c_1 < y \leq c_2$;

$$f(y) = h(c_2 - c_1) . (y - c_1) = \frac{2(c_2 - c_1)(y - c_1)}{b - a + c_2 - c_1}$$

iii) $c_2 < y \leq b$;

$$f(y) = \frac{h}{b - c_2}(y - c_2) = \frac{2(y - c_2)}{(b - c_2)(b - a + c_2 - c_1)}$$

c) i) $a < y \leq c_1$;

$$F(y) = \frac{c_1 - a}{2} \cdot h \doteq \frac{c_1 - a}{2} \cdot \frac{2}{b - a + c_2 - c_1}$$

$$F(y) = \frac{c_1 - a}{b - a + c_2 - c_1}$$

ii) $c_1 < y \leq c_2$;

$$F(y) = F(c_1) + (c_2 - c_1) \cdot h = \frac{c_1 - a}{b - a + c_2 - c_1} + \frac{2c_2 - 2c_1}{b - a + c_2 - c_1}$$

$$F(y) = \frac{2c_2 - c_1 - a}{b - a + c_2 - c_1}$$

iii) $c_2 < y \leq b$;

$$F(y) = F(c_2) + \frac{b - c_2}{2} \cdot h = \frac{2c_2 - c_1 - a}{b - a + c_2 - c_1} + \frac{b - c_2}{2} \cdot \frac{2}{b - a + c_2 - c_1}$$

$$F(y) = \frac{2c_2 - c_1 - a + b - c_2}{b - a + c_2 - c_1} = \frac{c_2 - c_1 - a + b}{b - a + c_2 - c_1}$$

d) • For in between $a < y \leq c_1$;

$$F(y) = \frac{c_1 - a}{b - a + c_2 - c_1} = \frac{1.5 - 0.5}{3.5 - 0.5 + 2.5 - 1.5} = \frac{1}{4} = 0.25$$

• For in between $c_1 < y \leq c_2$;

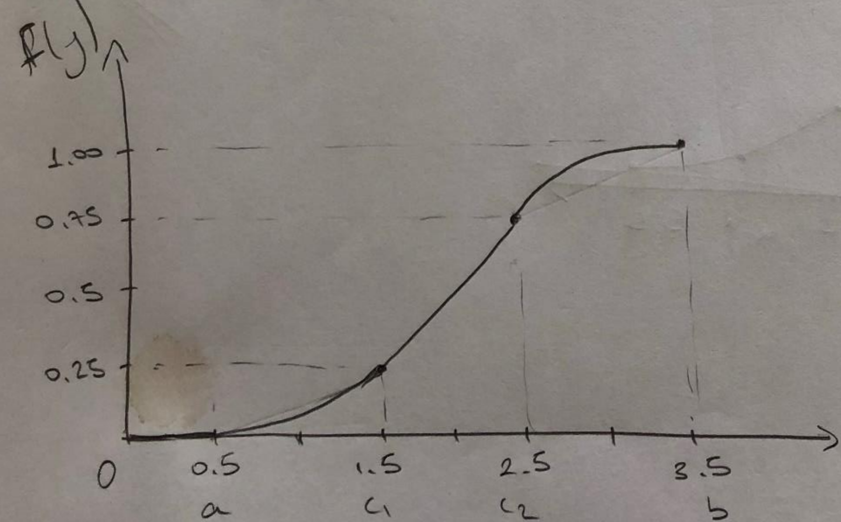
$$F(y) = \frac{2c_2 - c_1 - a}{b - a + c_2 - c_1} = \frac{2(2.5) - 1.5 - 0.5}{4} = \frac{5 - 2}{4}$$

$$F(y) = \frac{3}{4} = 0.75$$

• for in between $c_2 < y \leq b$;

$$F(y) = \frac{c_2 - c_1 - a + b}{b - a + c_2 - c_1} = \frac{2.5 - 1.5 - 0.5 + 3.5}{4} = \frac{6 - 2}{4}$$

$$F(y) = \frac{4}{4} = 1$$



* In between a and c_1, the graph is increasing parabolic curve since f(y) function is positive increasing.

* In between c_1 and c_2 the graph is linear increasing since f(y) has "0" slope.

* In between c_2 and b, the graph is decreasing parabolic curve since f(y) function is positive decreasing.

5.2

a) $P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \Rightarrow f(x) = \frac{2^x \cdot e^{-2}}{x!}$

b) ① $f(0) = 0$

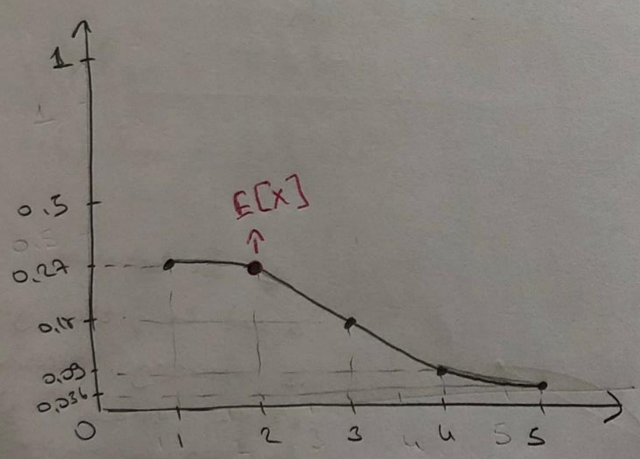
② $f(1) = \frac{2^1 \cdot \frac{1}{(2.72)^2}}{1} = \frac{2}{7.398} = 0.27$

③ $f(2) = \frac{2^2 \times \frac{1}{7.398}}{2} = \frac{\frac{4}{7.398}}{2} = 0.27$

④ $f(3) = \frac{2^3 \times \frac{1}{7.398}}{3\times2\times1} = \frac{\frac{4}{7.398}}{3} = 0.18$

⑤ $f(4) = \frac{2^4 \times \frac{1}{7.398}}{4\times3\times2\times1} = \frac{\frac{2}{7.398}}{3} = 0.09$

⑥ $f(5) = \frac{2^5 \times \frac{1}{7.398}}{5\times4\times3\times2\times1} = \frac{\frac{4}{7.398}}{3\times3} = 0.036$



c) For Poisson;

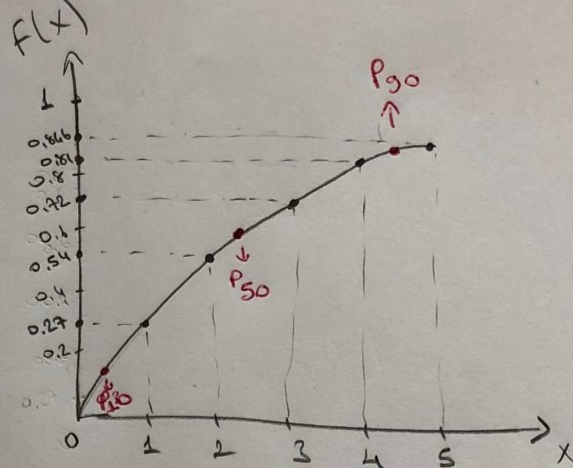
$\mu = E[x] = \lambda$

$E[X] = 2$

d) $F(x) = \sum_{i=0}^{x} f(x) = \sum_{i=0}^{5} \frac{2^x \cdot e^{-2}}{x!}$

e)

x	f(x)	F(x)
0	0	0
1	0.27	0.27
2	0.27	0.54
3	0.18	0.72
4	0.09	0.81
5	0.036	0.846



f) from excel, according to the CDF (F(x)) values;

i) $P_{10} = 0.135$

ii) $P_{50} = 0.63$

iii) $P_{90} = 0.828$