Stevens Institute of Technology

SYS-601 Homework Cover Sheet

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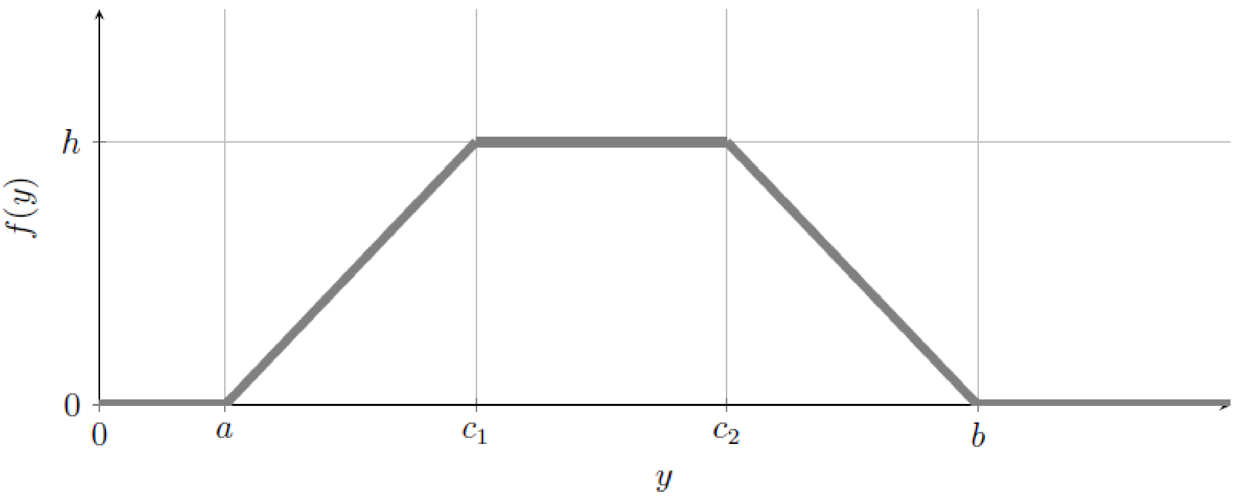
Stevens Institute of Technology

SYS-601 Homework #5

Due Feb. 26 2017

5.1 Trapezoidal Distribution [10 points]

Consider a trapezoidal PDF with parameters $0 \le a \le c_1 \le c_2 \le b$ for the minimum value a, maximum value b, and transition points c_1 and c_2 between linear and constant segments:



(a) 2 pts Using the property $\int_a^b f(y)dy = 1$, solve for h in terms of a, b, c1, and c2. (Hint: write an equation for the area under the PDF, set equal to 1, and solve for h.)

$$\int_{-\infty}^{+\infty} f(y)dy = 1 \quad \rightarrow \quad \int_a^b f(y)dy = 1 \quad \rightarrow \quad 0.5 * (c2 - c1 + b - a) * h = 1$$

$$\mathrm{h} = \frac{2}{c2 - c1 + b - a}$$

(b) 3 pts Write an equation for the PDF f(y) in terms of a, b, c1 and c2 for the ranges:

(i) a < y $\leqslant$ c1 (Hint: verify f(a) = 0 and f(c1) = h.)

$$f(y) = k1 * y + p1$$

$$f(a) = 0 \ \& \ f(c1) = h$$

$$\begin{cases} a * k1 + p1 = 0 \\ c1 * k1 + p1 = h \end{cases}$$

$$\begin{cases} k1 = \dfrac{h}{c1 - a} = \dfrac{2}{(c1 - a)(c2 - c1 + b - a)} \\ p1 = -\dfrac{a * h}{c1 - a} = -\dfrac{2a}{(c1 - a)(c2 - c1 + b - a)} \end{cases}$$

$$f(y) = \frac{2y - 2a}{(c1 - a)(c2 - c1 + b - a)} \quad (a < y \le c1)$$

(ii) c1 < y ⩽ c2

$$f(y) = \frac{2}{c2 - c1 + b - a} \quad (c1 < y \le c2)$$

(iii) c2 < y ⩽ b (Hint: verify f(c2) = h and f(b) = 0.)

$$f(y) = k2 * y + p2$$

$$f(c2) = h \;\; \& \;\; f(b) = 0$$

$$\begin{cases} c2 * k2 + p2 = h \\ b * k2 + p2 = 0 \end{cases}$$

$$\begin{cases} k2 = \dfrac{h}{c2 - b} = \dfrac{2}{(c2 - b)(c2 - c1 + b - a)} \\ p2 = -\dfrac{b * h}{c2 - b} = -\dfrac{2b}{(c2 - b)(c2 - c1 + b - a)} \end{cases}$$

$$f(y) = \frac{2y - 2b}{(c2 - b)(c2 - c1 + b - a)} \quad (c2 < y \le b)$$

(c) 3 pts Write an equation for the CDF F(y) in terms of a, b, c1 and c2 for the ranges:

(i) a < y ⩽ c1 (Hint: the area of the triangular region between a and y.)

$$F(y) = \int_{-\infty}^{y} f(i)di = 0.5 * (y - a) * \frac{2y - 2a}{(c1 - a)(c2 - c1 + b - a)}$$

$$F(y) = \frac{(y - a)^2}{(c1 - a)(c2 - c1 + b - a)} \quad (a < y \le c1)$$

(ii) c1 < y ⩽ c2 (Hint: F(c1) plus the area of rectangular region between c1 and y.)

$$F(y) = 0.5 * (c1 - a) * h + (y - c1) * h$$

$$F(y) = (y - 0.5a - 0.5c1) * h$$

$$F(y) = \frac{2(y - 0.5a - 0.5c1)}{c2 - c1 + b - a} \quad (c1 < y \le c2)$$

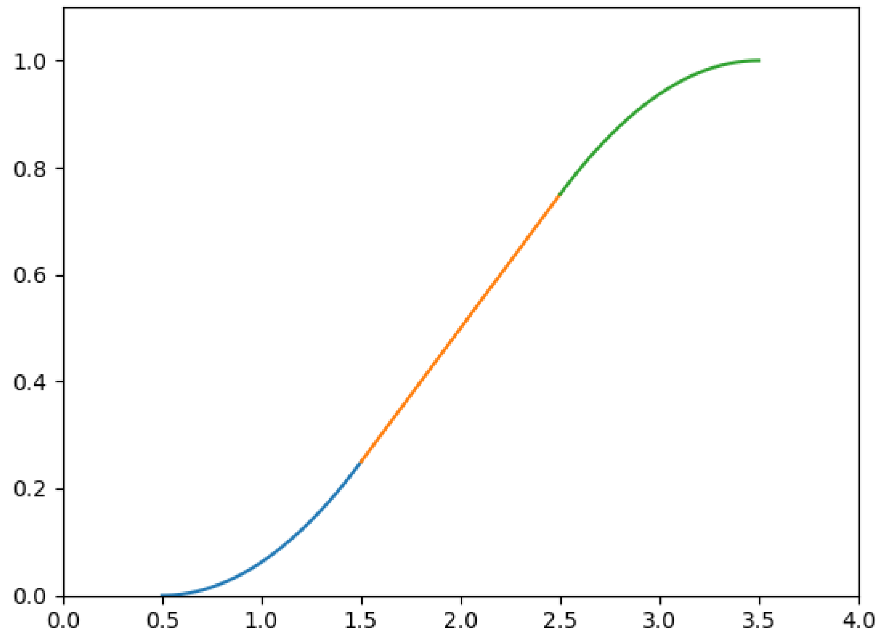
(iii) c2 < y ⩽ b (Hint: F(c2) plus the area of triangular region between c2 and y.)

$$F(y) = 1 - 0.5 * (b - y) * \frac{2y - 2b}{(c2 - b)(c2 - c1 + b - a)}$$

$$F(y) = 1 + \frac{(y - b)^2}{(c2 - b)(c2 - c1 + b - a)} \quad (c2 < y \le b)$$

(d) 2 pts Draw a sketch of the CDF F(y) for parameters a = 0:5; c1 = 1:5; c2 = 2:5; b = 3:5.

$$F(y) = \begin{cases} 0.25y^2 - 0.25y + 0.0625 & (0.5 < y \leq 1.5) \\ 0.5y - 0.5 & (1.5 < y \leq 2.5) \\ -0.25y^2 + 1.75y - 2.0625 & (2.5 < y \leq 3.5) \end{cases}$$



5.2 Cafe Java: Customer Inter-arrival [10 points]

Mathematically inclined customers arrive at Cafe Java following a Poisson process:

- There is a long-term average rate of λ = 2 customer arrivals per minute.
- The arrival rate is constant throughout the day.
- Customer arrivals are independent of each other.

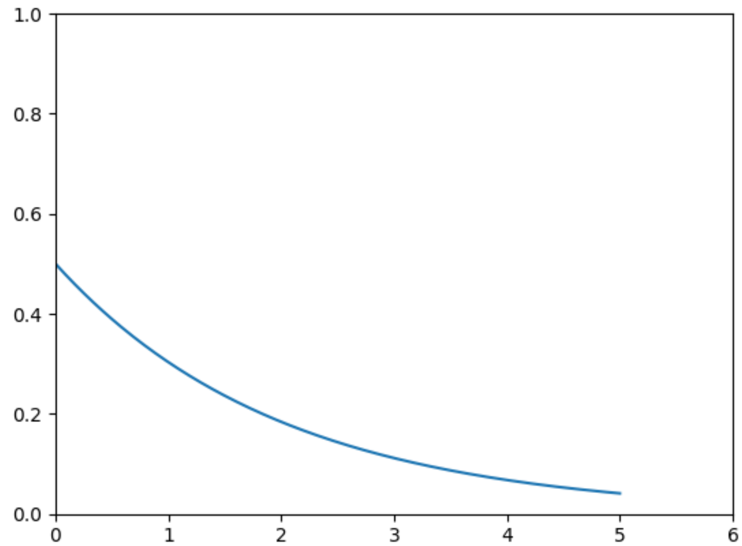
Under these assumptions, the inter-arrival time between customers is an exponentially-distributed random variable X with rate parameter λ :

$$X \sim \text{exponential}(\lambda)$$

(a) 1 pt Write an equation for the PDF f(x).

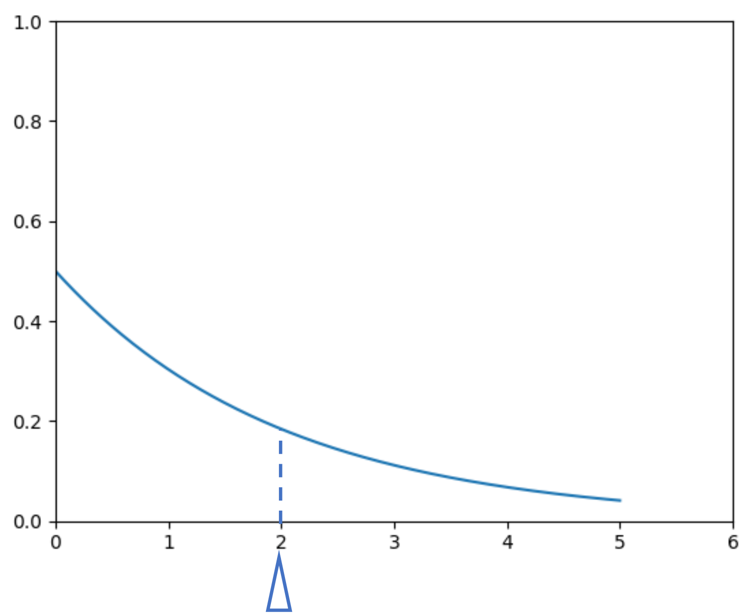
$$f(x) = 0.5\, e^{-0.5x}$$

(b) 2 pts Draw a sketch of the PDF f(x) for $0 \leqslant x \leqslant 5$.



(c) 1 pt Find the population mean $\mu = E[X]$ and mark on the PDF plot.

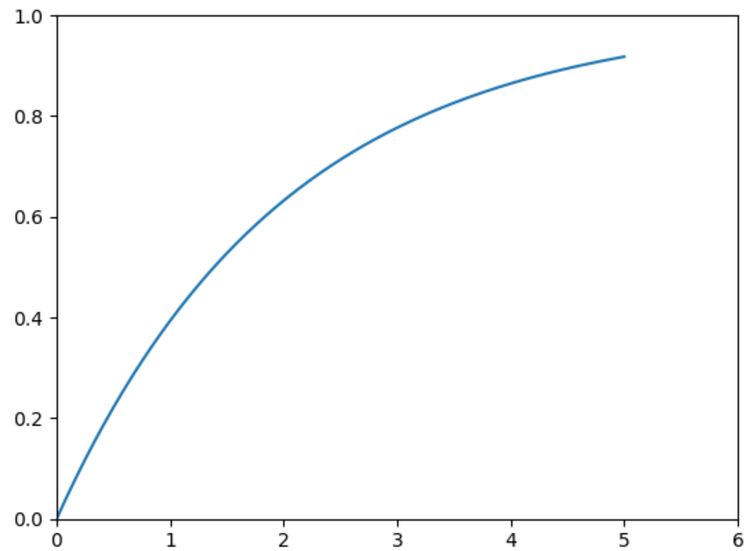
$$\mu = \frac{1}{\lambda} = 2$$



(d) 1 pt Write an equation for the CDF F(x).

$$F(x) = 1 - e^{-0.5x}$$

(e) 2 pts Draw a sketch of the CDF F(x) for $0 \leqslant x \leqslant 5$.



(f) 3 pts Evaluate or estimate the following quantities and mark on the CDF plot:

(i) 10th percentile inter-arrival time P10 (Hint: F(P10) = 0.10)

P10 = 0.21

(ii) Median inter-arrival time P50 (Hint: F(P50) = 0.50)

P50 = 1.386

(iii) 90th percentile inter-arrival time P90 (Hint: F(P90) = 0.90)

P90 = 4.605