

STEVENS INSTITUTE OF TECHNOLOGY

SYS-601 Homework #4

Due Feb. 19 2018

Submit the following using the online submission system: 1) Completed assignment cover sheet, 2) Written responses in PDF format, 3) All saved models (e.g. .xlsx or .py files).

4.1 Derived Dice Rolls [5 points]

Use the `random.org`¹ generator to roll $N = 30$ samples of two virtual dice. The discrete random variable Z is a function of the left (Z_L) and right (Z_R) dice value as follows:

$$Z = \left\lceil \frac{Z_L}{2} \right\rceil - \left\lceil \frac{Z_R}{2} \right\rceil$$

where $\lceil \cdot \rceil$ is the mathematical ceiling function (i.e. round up to whole number).

- (a) 1 PT Write the number of observations for each value of $Z \in \{-2, -1, 0, 1, 2\}$.
- (b) 2 PTS Write the empirical probability mass function $P(z)$ in a tabular format.
- (c) 2 PTS Write the empirical cumulative distribution function $F(z)$ in a tabular format.

4.2 100 Year Floodplain [7 points]

The Federal Emergency Management Agency (FEMA) designated the city of East Biggs, California inside the 100-year floodplain assessment for Lake Oroville which means there is a 1% chance of a flood each year.

- (a) 2 PTS Assuming Y , the number of floods in East Biggs, follows a Poisson distribution, write an equation for $P(y)$, the probability of y floods over a 100 year time span.
- (b) 2 PTS Create a bar plot for the PMF of $P(y)$ above.
- (c) 2 PTS What is $P(y \geq 1)$, the probability of *at least* one flood in 100 years?
- (d) 1 PT What assumptions are required for a Poisson distribution assumed in (a)? Are these assumptions appropriate for events such as floods?

¹<https://www.random.org/dice/>

4.3 Super Bowl Coin Flips [8 points]

A traditional coin flip before the Super Bowl allows the winning team, either the American Football Conference (AFC) or National Football Conference (NFC), the choice to start on offense or defense. The attached file `superbowl.csv` contains results for the 52 Super Bowls played as of 2018.

- (a) 2 PTS Assuming the coin toss is fair (50% chance to win), write an equation for $P(x)$, the probability of winning exactly x tosses in 52 trials. (*Hint*: binomial distribution)
- (b) 2 PTS Create a bar plot for the PMF of $P(x)$ above.
- (c) 2 PTS Count how many coin tosses the NFC has won in the 52 Super Bowls (N). Where is this point on the PMF plot? Is this a typical outcome or an unusual outcome?
- (d) 2 PTS What is the probability of winning *at least* as many coin tosses as the NFC, $P(x \geq N)$, in 52 trials?