Stevens Institute of Technology

SYS-601 Homework Cover Sheet

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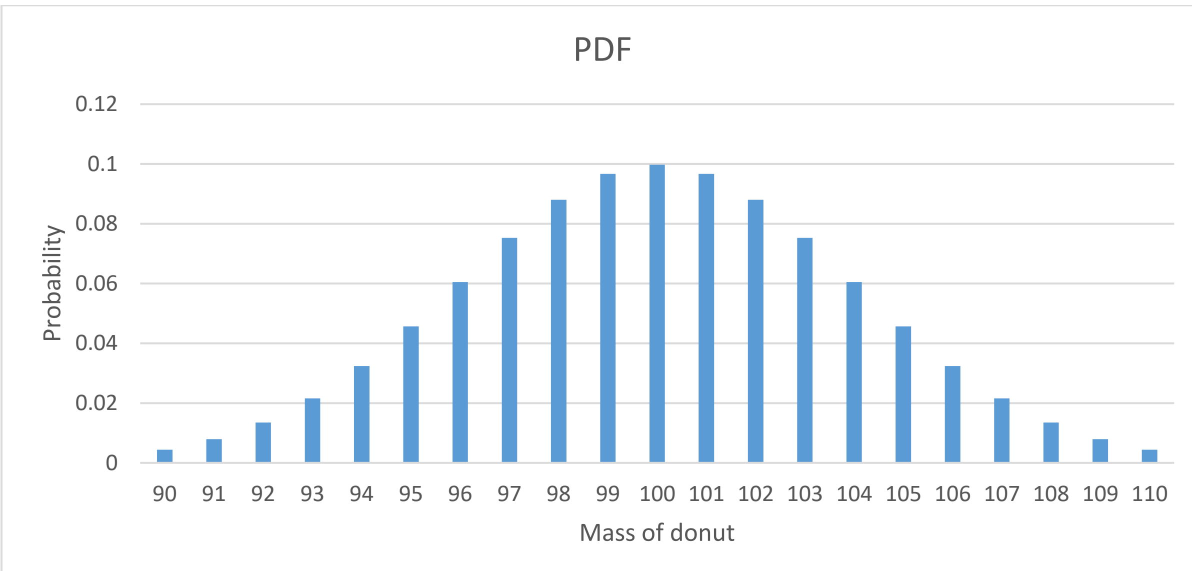
Homework 6

6.1 Donut Inference I

Castle Point Bakery (CPB) makes delicious donuts. The high-tech production line is set up to make donuts with mean mass μ = 100 grams and standard deviation σ = 4 grams. Assume this information is accurate and donut mass follows a normal distribution. Compute the following for a single (N = 1) donut:

(a) A PDF plot for X, the mass of a CPB donut.

Solution:



(b) The 5th percentile CPB donut mass.

Solution:

N = 1

$$F_{\bar{x}}^{-1}(i) = F_{norm}^{-1}\left(\frac{i}{100}, \mu_x, \sigma_x\right)$$

$$\mu_x = \mu = 100$$

$$\sigma_x = \frac{\sigma}{\sqrt{N}} = 4$$

$$\therefore F_{\bar{x}}^{-1}(0.05) = F_{norm}^{-1}(0.05, 100, 2)$$

$$= 93.42$$

(c) The 95th percentile CPB donut mass.

Solution:

$F_{\bar{x}}^{-1}(0.95) = F_{norm}^{-1}(0.95, 100, 2)$

$= 106.57$

(d) The probability a CPB donut mass is ≤ 90 grams.

Solution:

$F_x(90) = F_{norm}(90, 100, 2)$

= 0.006

(e) The probability a CPB donut mass is ≥ 110 grams.

Solution:

$1 - F_x(110) = 1 - F_{norm}(110, 100, 2)$

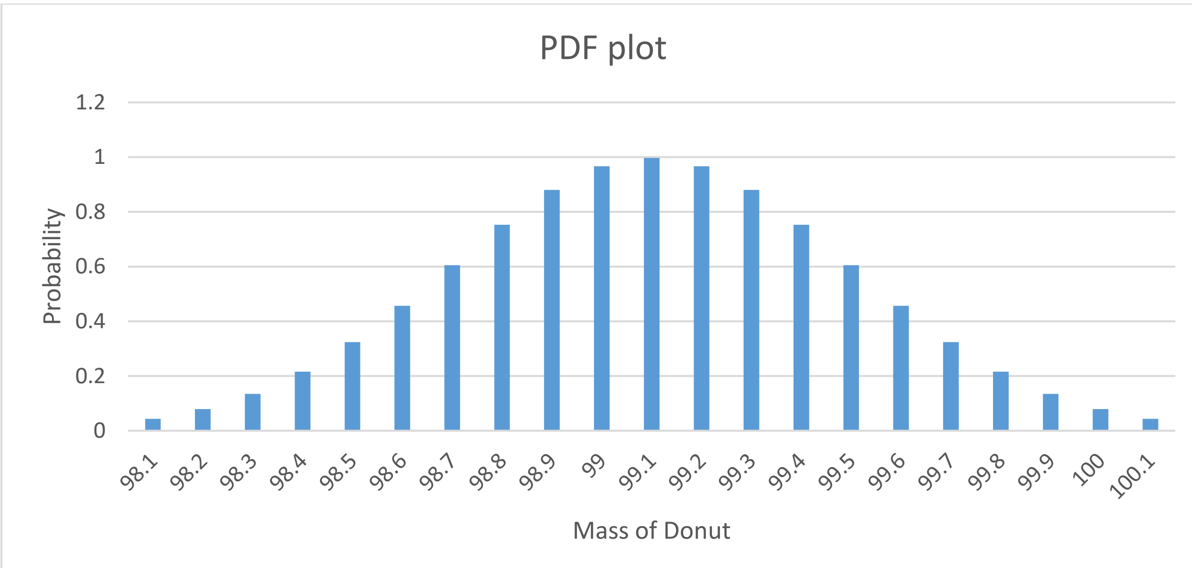
= 0.0062

6.2 Donut Inference II

Hoboken-Os (H-Os) also produces delicious donuts. After N = 100 visits you have collected sample data showing the average donut mass to be $\bar{y}$ = 99.1 grams. Assume H-Os has the same standard deviation as CPB (σ = 4 grams). Based on this data, compute the following:

(a) A PDF plot for $\bar{Y}$, the mean mass for N = 100 H-Os donut samples.

Solution:



(b) The 5th percentile mean mass for N = 100 H-Os donut samples.

Solution:

$$F_{\bar{x}}^{-1}(0.05) = F_{norm}^{-1}(0.05, 99.1, 0.4)$$

$$= 98.44$$

(c) The 95th percentile mean mass for N = 100 H-Os donut samples.

Solution:

$$F_{\bar{x}}^{-1}(0.95) = F_{norm}^{-1}(0.95, 99.1, 0.4)$$

$$= 99.76$$

(d) The probability the mean mass for N = 100 H-Os donut samples is ≤ 100 grams.

Solution:

$$F_x(100) = F_{norm}(100, 99.1, 0.4)$$

= 0.9877

(e) A 95% confidence interval for the mean mass of H-Os donuts.

Solution:

$Z_{0.025} = -1.96$

$Z_{0.975} = 1.96$

Considering $\overline{x} = 99.1$

$$\frac{\sigma}{\sqrt{N}} = 0.4$$

$$\mu \in \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{N}}$$

= 99.1 ± (1.96 x 0.4)

= 99.1 ± 0.783

= [98.31, 99.88]

Considering 100 samples, the population mean will fall in the range [98.31, 99.88] 95% of the time.

6.3 Spring Break Recovery

The binary random variable X measures the event of rolling a sum of 7 or 11 from a pair of dice (1 for rolling 7 or 11; 0 for anything else).

Using a random dice generator, collect at least N = 30 samples for X.

Sample Space

Sample Space			SUM		Binary Equ
1	5		6		0
4	5		9		0
5	1		6		0
3	3		6		0
3	1		4		0
3	5		8		0
2	5		7		1
3	2		5		0
1	5		6		0
6	4		10		0
6	5		11		1
1	1		2		0
1	4		5		0
2	6		8		0
4	2		6		0
2	2		4		0
4	2		6		0
2	6		8		0
4	3		7		1
4	4		8		0
4	2		6		0
2	4		6		0
4	4		8		0
5	4		9		0
3	6		9		0
1	1		2		0
5	6		11		1
2	4		6		0
3	3		6		0
6	2		8		0

(a) Compute the sample mean $\overline{x}$.

Solution:

Mean $\overline{x} = 0.1\dot{3}$

(b) Compute the sample standard deviation s_x and use it as an estimate of σ.

Solution:

Standard Deviation [Sample] = 0.345

$\sigma = \sqrt{\frac{\sum(x_i-\mu)^2}{N}}$ [Calculation in Excel]

= 0.3399

(c) Compute the 95% confidence interval for the population mean, i.e. the true probability of rolling a 7 or 11.

Solution:

$Z_{0.025} = -1.96$

$Z_{0.975} = 1.96$

Considering $\overline{x} = 0.13$

$$\frac{\sigma}{\sqrt{N}} = \frac{0.3399}{\sqrt{30}} = 0.062$$

$$\mu \in \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{N}}$$

= 0.133 ± (1.96 x 0.062)

= 0.133 ± 0.121

= [0.011, 0.254]

Considering 100 samples, the population mean will fall in the range [0.011, 0.254] 95% of the time.

(d) Estimate how many samples would be required to reduce the 95% confidence interval to a maximum error $|\overline{x} - \mu| = 0.01$.

Solution:

$$N = \left(\frac{z_{\alpha/2} \times \sigma}{E}\right)^2$$

$$= \left(\frac{1.96 \times 0.062}{0.01}\right)^2$$

= 147.67 ≈ 148 samples

(e) Was the true probability of rolling a 7 or 11 within your 95% confidence interval? How often do “mistakes" happen?

Solution:

No, the probability didn’t fall under the 95% confidence interval. That may be due to the number of samples recorded for this experiment. I got a minimum error of 25% in 3 tries for this experiment.