

STEVENS INSTITUTE OF TECHNOLOGY

# SYS-601 Homework Cover Sheet

Date:

HW #:

Author:

Collaborators:

## Homework 4

## 4.1 Derived Dice Rolls

Use the random.org generator to roll  $N = 30$  samples of two virtual dice. The discrete random variable  $Z$  is a function of the left ( $Z_L$ ) and right ( $Z_R$ ) dice value as follows:

$$Z = \left\lceil \frac{Z_L}{2} \right\rceil - \left\lceil \frac{Z_R}{2} \right\rceil$$

Where  $\lceil \cdot \rceil$  is the mathematical ceiling function (i.e. round up to whole number).

(a) Write the number of observations for each value of  $Z \in \{-2, -1, 0, 1, 2\}$ .

Dice Roll		Z Values		Ceiling Value		Z
Dice 1	Dice 2	$Z_L / 2$	$Z_R / 2$	$Z_L / 2$	$Z_R / 2$	
6	1	3	0.5	3	1	2
3	3	1.5	1.5	2	2	0
1	3	0.5	1.5	1	2	-1
1	2	0.5	1	1	1	0
4	5	2	2.5	2	3	-1
2	1	1	0.5	1	1	0
3	1	1.5	0.5	2	1	1
1	2	0.5	1	1	1	0
1	2	0.5	1	1	1	0
1	6	0.5	3	1	3	-2
5	6	2.5	3	3	3	0
1	2	0.5	1	1	1	0
4	4	2	2	2	2	0
3	1	1.5	0.5	2	1	1
2	1	1	0.5	1	1	0
4	6	2	3	2	3	-1
5	2	2.5	1	3	1	2
4	3	2	1.5	2	2	0
1	3	0.5	1.5	1	2	-1
2	1	1	0.5	1	1	0
2	5	1	2.5	1	3	-2
1	6	0.5	3	1	3	-2
1	1	0.5	0.5	1	1	0
4	5	2	2.5	2	3	-1
5	6	2.5	3	3	3	0
2	2	1	1	1	1	0
6	6	3	3	3	3	0
5	6	2.5	3	3	3	0
2	2	1	1	1	1	0
2	3	1	1.5	1	2	-1

Z Value	Count of Z
-2	3
-1	6
0	17
1	2
2	2
<b>Grand Total</b>	<b>30</b>

- If a 6 or 5 and 1 appears on the dice roll the Z value is the highest (2), the lowest when 1 and 6 or 5 appears (-2)
- If consecutive numbers appear the Z value is 0.
- If numbers appear with a step up of 2 or 3 the value is -1 or 1 depending on the 1<sup>st</sup> dice roll.

**(b) Write the empirical probability mass function  $P(z)$  in a tabular format.**

Z Values	Count of Z	Probability
-2	3	0.1
-1	6	0.2
0	17	0.566666667
1	2	0.066666667
2	2	0.066666667
<b>Grand Total</b>	<b>30</b>	<b>1</b>

**(c) Write the empirical cumulative distribution function  $F(z)$  in a tabular format.**

Z Value (z)	Count of Z	Probability P (z)	CDF F (z)
-3	0	0	0
-2	3	0.1	0.1
-1	6	0.2	0.3
0	17	0.566666667	0.866666667
1	2	0.066666667	0.933333333
2	2	0.066666667	1
3	0	0	1

## 4.2 100 Year Floodplain

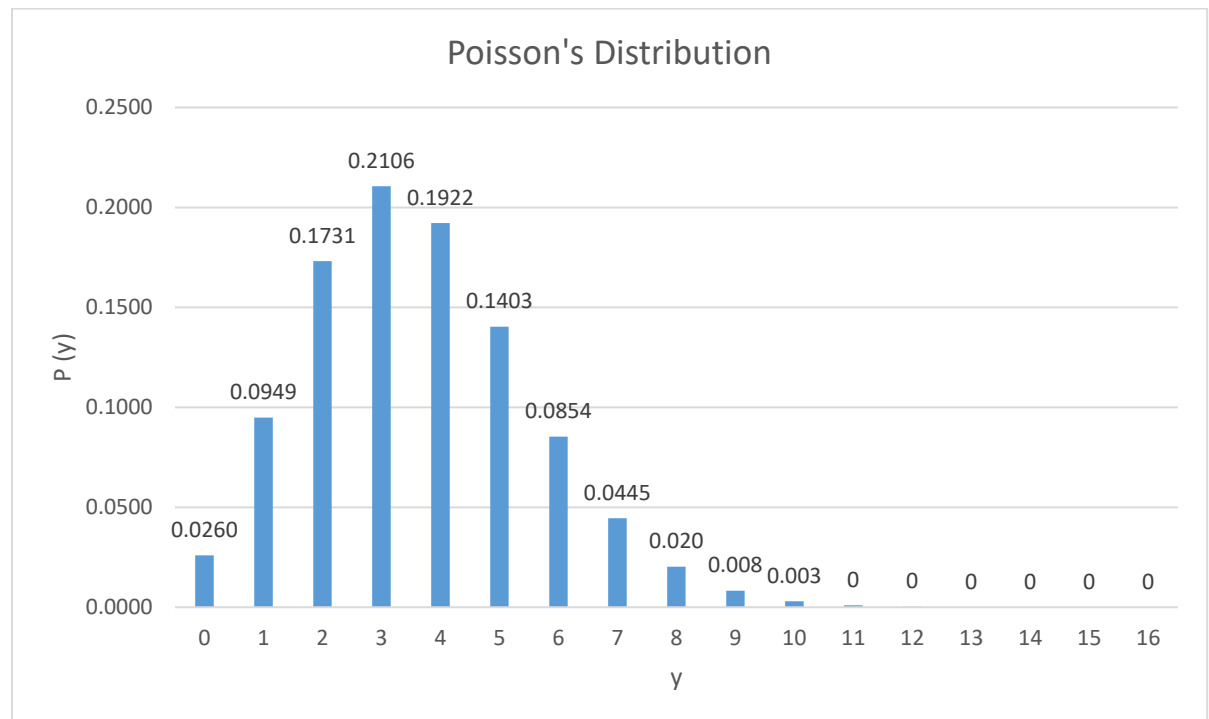
The Federal Emergency Management Agency (FEMA) designated the city of East Biggs, California inside the 100-year floodplain assessment for Lake Oroville which means there is a 1% chance of a flood each year.

(a) Assuming  $Y$ , the number of floods in East Biggs, follows a Poisson distribution, write an equation for  $P(y)$ , the probability of  $y$  floods over a 100 year time span.

- Number of floods in East Biggs =  $P(y)$
- Average rate of flood occurring =  $\lambda = 1\%$  per year = 3.65
- Therefore, Poisson's equation becomes

$$P(y) = \frac{(3 \cdot 65^y)(e^{-3 \cdot 65})}{y!}$$

(b) Create a bar plot for the PMF of  $P(y)$  above.



(c) What is  $P(y \geq 1)$ , the probability of *at least* one flood in 100 years?

- $P(y \geq 1) = 1 - P(y = 0)$   
 $= 1 - 0.0260$   
 $= 0.974$

**(d) What assumptions are required for a Poisson distribution assumed in (a)? Are these assumptions appropriate for events such as floods?**

- The average rate of 3.65 chance of flood in a year is assumed. Floods are a natural calamity and can occur any time. Also the structural stability could shift in its lifecycle, proportionally changing the probability of an occurrences of floods.

### 4.3 Super Bowl Coin Flips

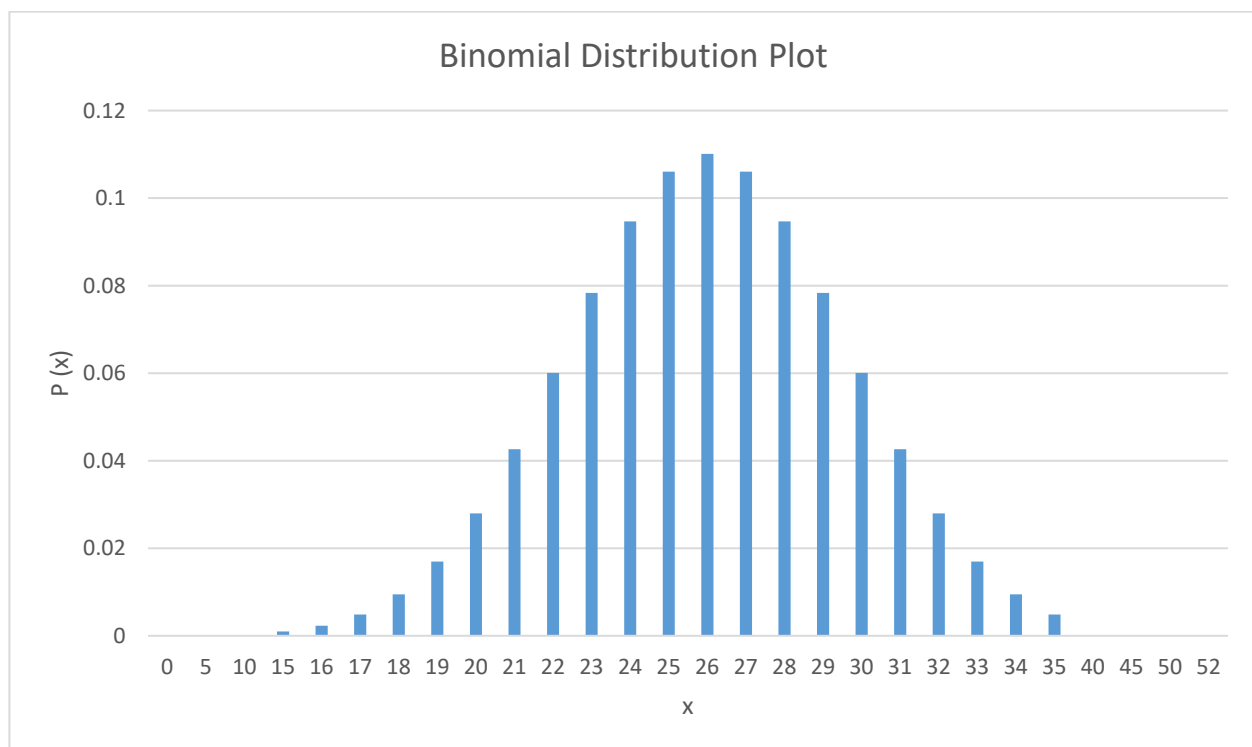
A traditional coin flip before the Super Bowl allows the winning team, either the American Football Conference (AFC) or National Football Conference (NFC), the choice to start on offense or defense. The attached file superbowl.csv contains results for the 52 Super Bowls played as of 2018.

**(a) Assuming the coin toss is fair (50% chance to win), write an equation for  $P(x)$ , the probability of winning exactly  $x$  tosses in 52 trials. (*Hint: binomial distribution*)**

- Number of trials ( $n$ ) = 52
- Probability of an event trial ( $p$ ) =  $\frac{1}{2}$  [because coin flip]
- Therefore Binomial expression equates to:

$$P(x) = \binom{52}{x} (0.5)^x (0.5)^{(52-x)}$$

**(b) Create a bar plot for the PMF of  $P(x)$  above.**



**(c) Count how many coin tosses the NFC has won in the 52 Super Bowls ( $N$ ). Where is this point on the PMF plot? Is this a typical outcome or an unusual outcome?**

- The NFC has won 35 out of 52 coin tosses. The probability of such an event occurring is 0.0048 (calculated in Excel). It is a highly unlikely event.

**(d) What is the probability of winning *at least* as many coin tosses as the NFC,  $P(x \geq N)$ , in 52 trials?**

- $P(x \geq 35) = \sum_{x=35}^{x=52} P(x)$   
= 0.008766618 [calculation in Excel]