Stevens Institute of Technology

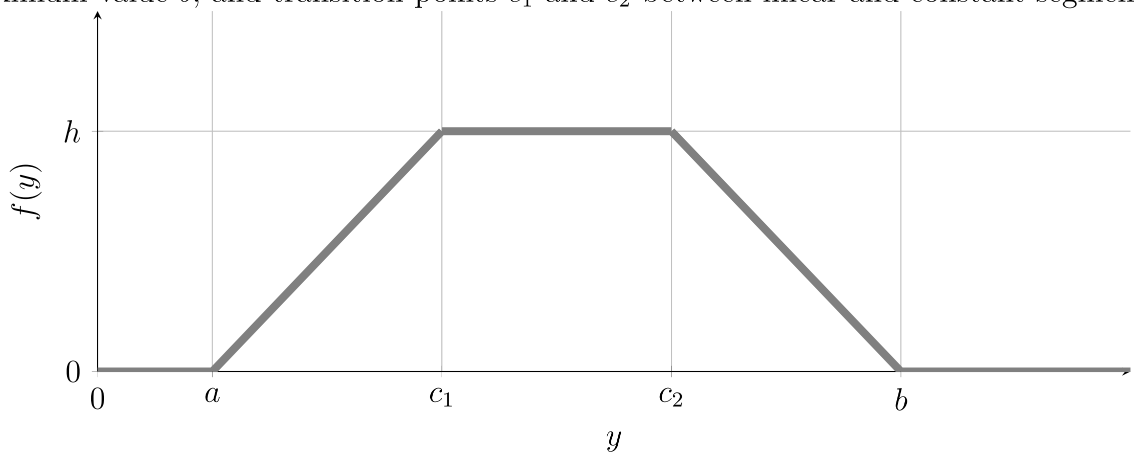
SYS-601 Homework #5

Due Feb. 26 2017

Submit the following using the online submission system: 1) Completed assignment cover sheet, 2) Written responses in PDF format, 3) All saved models (e.g. .xlsx or .py files).

5.1 Trapezoidal Distribution [10 points]

Consider a trapezoidal PDF with parameters $0 \leq a \leq c_1 \leq c_2 \leq b$ for the minimum value a, maximum value b, and transition points c_1 and c_2 between linear and constant segments:



(a) 2 pts Using the property $\int_a^b f(y)dy = 1$, solve for h in terms of a, b, c_1, and c_2. (*Hint:* write an equation for the area under the PDF, set equal to 1, and solve for h.)

(b) 3 pts Write an equation for the PDF $f(y)$ in terms of a, b, c_1 and c_2 for the ranges:

- (i) $a < y \leq c_1$ (*Hint:* verify $f(a) = 0$ and $f(c_1) = h$.)
- (ii) $c_1 < y \leq c_2$
- (iii) $c_2 < y \leq b$ (*Hint:* verify $f(c_2) = h$ and $f(b) = 0$.)

(c) 3 pts Write an equation for the CDF $F(y)$ in terms of a, b, c_1 and c_2 for the ranges:

- (i) $a < y \leq c_1$ (*Hint:* the area of the triangular region between a and y.)
- (ii) $c_1 < y \leq c_2$ (*Hint:* $F(c_1)$ plus the area of rectangular region between c_1 and y.)
- (iii) $c_2 < y \leq b$ (*Hint:* $F(c_2)$ plus the area of triangular region between c_2 and y.)

(d) 2 pts Draw a sketch of the CDF $F(y)$ for parameters $a = 0.5, c_1 = 1.5, c_2 = 2.5, b = 3.5$.

5.2 Café Java: Customer Inter-arrival [10 points]

Mathematically inclined customers arrive at Café Java following a Poisson process:

- There is a long-term average rate of $\lambda = 2$ customer arrivals per minute.
- The arrival rate is constant throughout the day.
- Customer arrivals are independent of each other.

Under these assumptions, the inter-arrival time between customers is an exponentially-distributed random variable X with *rate* parameter λ[1]:

$$X \sim \text{exponential}(\lambda)$$

(a) 1 PT Write an equation for the PDF $f(x)$.

(b) 2 PTS Draw a sketch of the PDF $f(x)$ for $0 \le x \le 5$.

(c) 1 PT Find the population mean $\mu = E[X]$ and mark on the PDF plot.

(d) 1 PT Write an equation for the CDF $F(x)$.

(e) 2 PTS Draw a sketch of the CDF $F(x)$ for $0 \le x \le 5$.

(f) 3 PTS Evaluate or estimate the following quantities and mark on the CDF plot:

(i) 10th percentile inter-arrival time P_{10} (*Hint*: $F(P_{10}) = 0.10$)

(ii) Median inter-arrival time P_{50} (*Hint*: $F(P_{50}) = 0.50$)

(iii) 90th percentile inter-arrival time P_{90} (*Hint*: $F(P_{90}) = 0.90$)

[1] Rate parameter λ is equivalent to *scale* parameter $1/\lambda$.