STEVENS INSTITUTE OF TECHNOLOGY

SYS-601 Exam #1 Solutions

1.1 Probability Theory [40 points]

The casino game "craps" is based on dice rolls. Its first phase is called the "come-out" roll which is based on the random variable X, the sum of two dice.

(a) 3 PTS Choose the distribution of the random variable X, the sum of two dice.

(A) Binomial
(B) Hypergeometric
(C) Normal
(D) Poisson
(E) Uniform
(F) X None of the Above

None of the named distributions above model the sum of two dice.

(b) 6 PTS Compute the probability mass function (PMF) values $p(x)$ below.

x:	2	3	4	5	6	7	8	9	10	11	12
$p(x)$:	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(c) 6 PTS There are three possible outcomes of the "come-out" roll derived from X:

- Rolling $X \in \{2, 3, 12\}$ is called "crapping out" (derived event C)
- Rolling $X \in \{7, 11\}$ is called a "natural" (derived event N)
- Rolling $X \in \{4, 5, 6, 8, 9, 10\}$ sets the "point" (derived event T)

Compute the following derived event probabilities of craps outcomes:

$$P(C) = p(2) + p(3) + p(12) = \frac{1+2+1}{36} = \frac{4}{36}$$

$$P(N) = p(7) + p(11) = \frac{6+2}{36} = \frac{8}{36}$$

$$P(T) = p(4) + p(5) + p(6) + p(8) + p(9) + p(10) = \frac{3+4+5+5+4+3}{36} = \frac{24}{36}$$

(d) 4 PTS Compute the probability of the derived event D which occurs when doubles are rolled, i.e. both dice have the same value (⚀ ⚀, ⚁ ⚁, etc.).

$$P(D) = 6 \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{6}{36}$$

(e) 12 PTS Complete the following probability matrix to identify the joint and marginal probabilities for events associated with a single craps roll:

Doubles Event	Craps Outcome			$\sum$
	C	N	T	
D	$\frac{1+1}{36} = \frac{2}{36}$	$\frac{0}{36}$	$\frac{1+1+1+1}{36} = \frac{4}{36}$	$P(D) = \frac{6}{36}$
$\neg D$	$\frac{2}{36}$	$\frac{6+2}{36} = \frac{8}{36}$	$\frac{2+4+4+4+4+2}{36} = \frac{20}{36}$	$P(\neg D) = \frac{30}{36}$
$\sum$	$P(C) = \frac{4}{36}$	$P(N) = \frac{8}{36}$	$P(T) = \frac{24}{36}$	

(f) 3 PTS Choose the best relationship between derived events C and N.

(A) Collectively Exhaustive
(B) Equally Likely
(C) Complementary
(D) X Mutually Exclusive
$P\{C \cap N\} = 0$
(E) None of the Above

(g) 3 PTS Choose the best relationship between derived events C and D.

(A) Collectively Exhaustive
(B) Equally Likely
(C) Complementary
(D) Mutually Exclusive
(E) X None of the Above

None of the above describe the relationship between C and D.

(h) 3 PTS Choose the best relationship between derived events N and D.

(A) Collectively Exhaustive
(B) Equally Likely
(C) Complementary
(D) X Mutually Exclusive
$P\{N \cap D\} = 0$
(E) None of the Above

1.2 Discrete Random Variables [40 points]

Castle Point Casino operates a high-stakes craps table. Management delivers a complementary beverage whenever a player “craps out” by rolling a 2, 3, or 12 on a “come-out” roll. On average, there are 36 “come-out” rolls per hour. The random variable Z counts the number of complementary beverages delivered in an hour, assumed to follow a Poisson distribution.

(a) 4 PTS Using results from problem 1.1 combined with information above, estimate λ, the average rate of complementary beverages delivered per hour.

$$\lambda = \frac{4}{36} \text{ beverages per roll} \cdot 36 \text{ rolls per hour} = 4 \text{ beverages per hour}$$

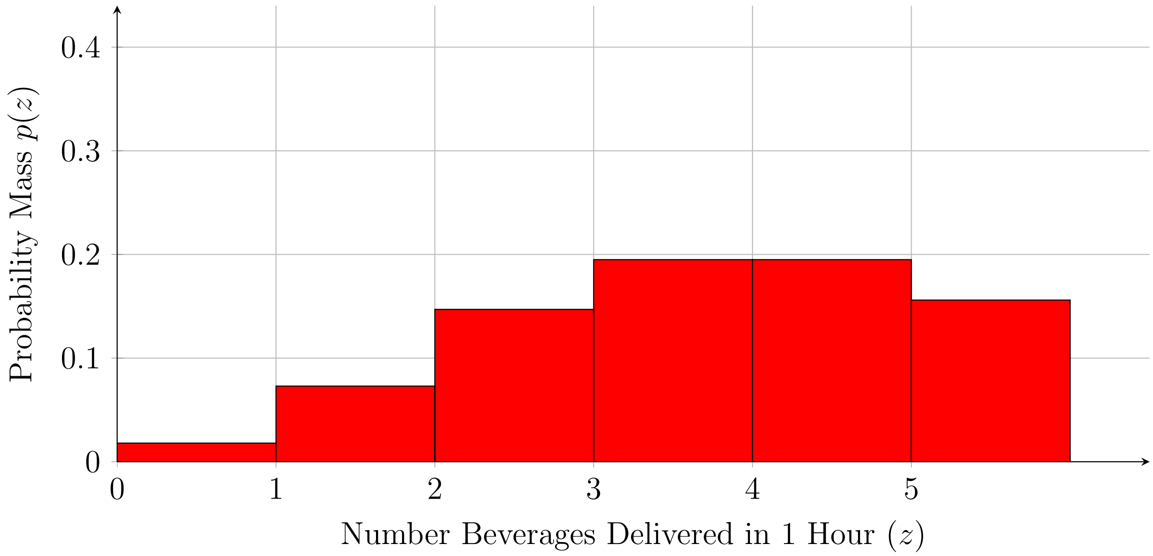
(b) 4 PTS Write an equation for the probability mass function $p(z)$.

$$p(z) = \frac{\lambda^z e^{-\lambda}}{z!} = \frac{4^z e^{-4}}{z!}$$

(c) 6 PTS Compute the probability mass function (PMF) values $p(z)$ below.

z:	0	1	2	3	4	5
$p(z)$:	0.018	0.073	0.147	0.195	0.195	0.156

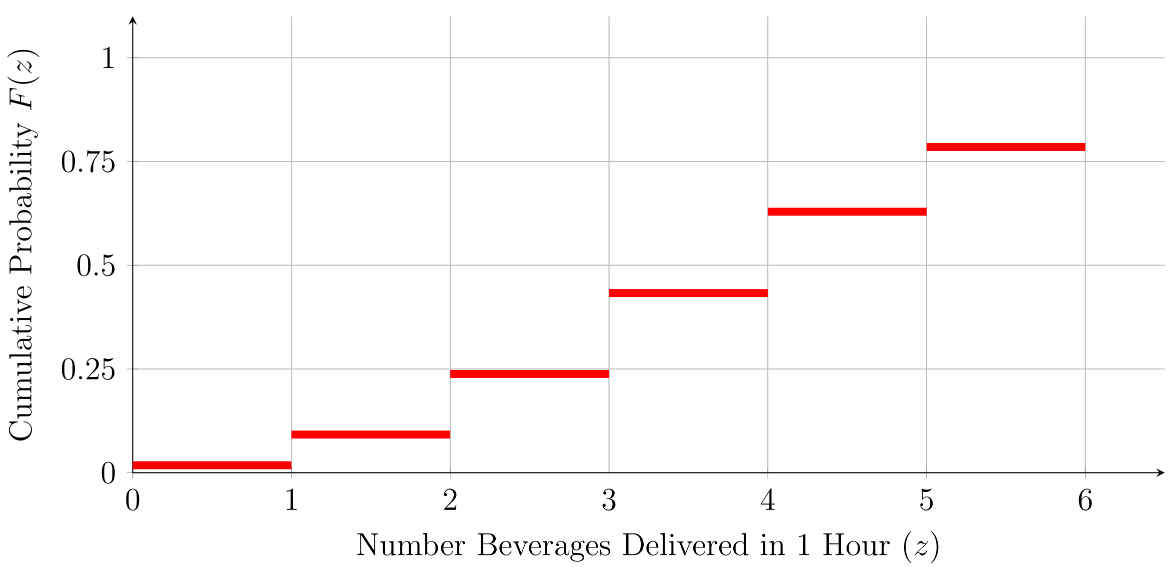
(d) 6 PTS Using the space below, sketch a PMF plot for $0 \le z \le 5$. Label the axes.



(e) 6 PTS Compute the cumulative distribution function (CDF) values $F(z)$ below.

z:	0	1	2	3	4	5
$F(z)$:	0.018	0.092	0.238	0.433	0.629	0.785

(f) 6 PTS Using the space below, sketch a CDF plot for $0 \le z \le 5$. Label the axes.



(g) 4 PTS Compute the expected number of beverages delivered in an hour, $E[Z]$.

$$E[Z] = \lambda = 4$$

(h) 4 PTS Compute the probability of more than 4 beverages in an hour, $P\{Z > 4\}$.

$$P\{Z > 4\} = 1 - F(4) = 1 - 0.629 = 0.371$$

1.3 Descriptive Statistics [35 points]

The following dataset from Castle Point Casino records how many complementary beverages were delivered during $N = 20$ one-hour periods over a weekend.
Dataset A

2	4	3	3	3
2	6	5	4	8
3	1	4	3	5
4	5	1	6	6

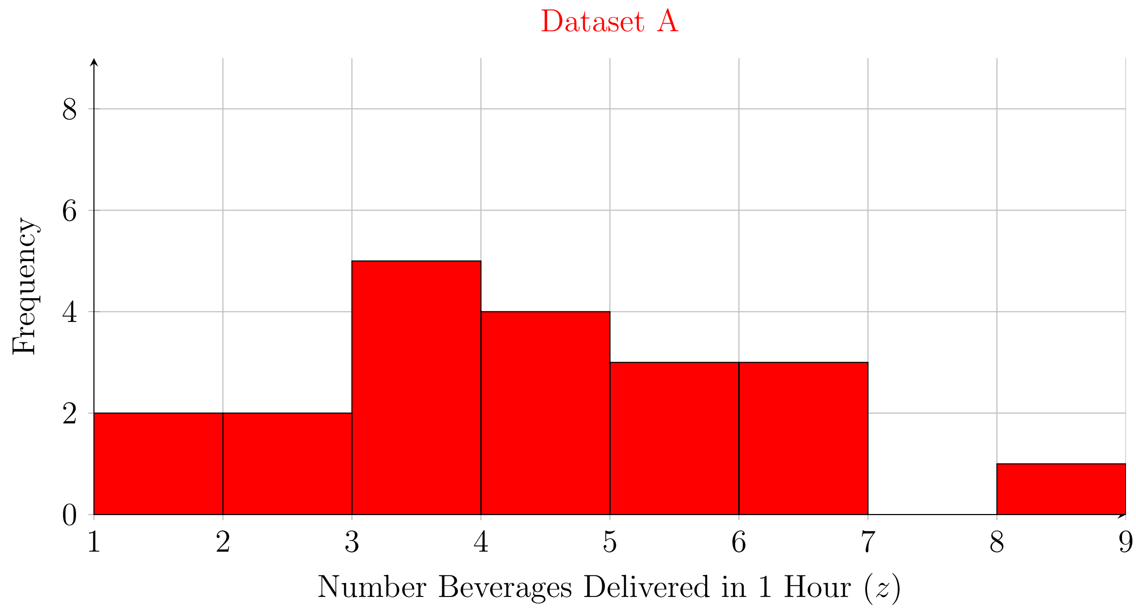
Dataset B

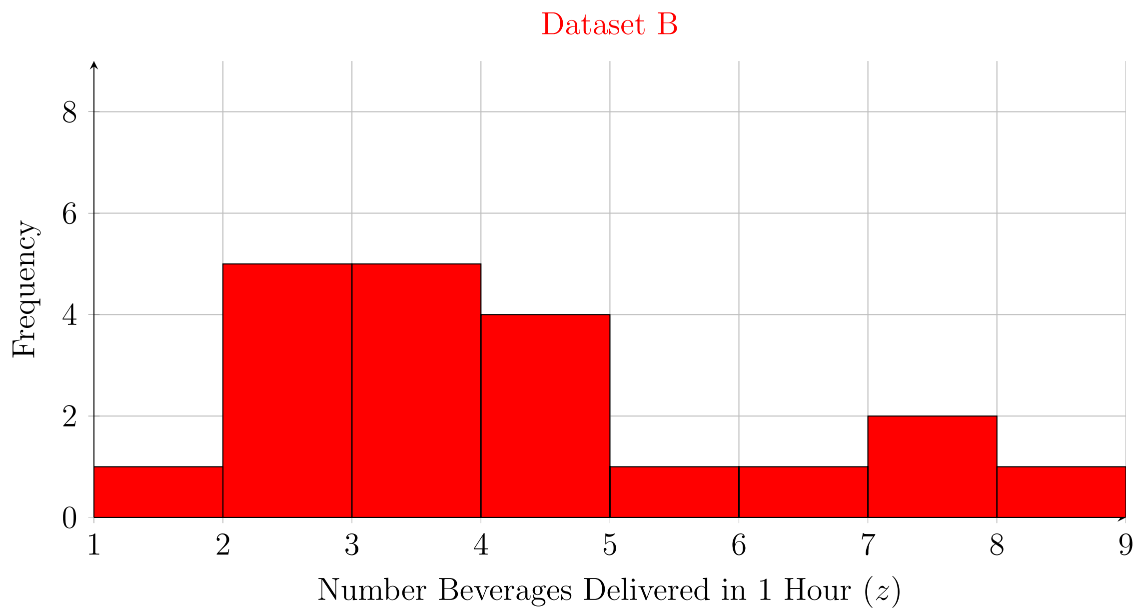
6	4	2	2	2
7	3	4	4	3
1	7	3	5	4
8	3	2	2	3

(a) 3 PTS Choose the best data type representing samples in this dataset.

(A) Nominal (B) Ordinal (C) Interval (D) X Ratio
0 is meaningful

(b) 8 PTS Using the space below, sketch a histogram of this dataset. Choose an appropriate bin size and label the axes.





(c) 4 PTS Compute the sample mean ($\bar{z}$) for this dataset.

$$A : \bar{z} = \frac{\sum_{i=1}^{20} z_i}{20} = 3.90 \qquad B : \bar{z} = \frac{\sum_{i=1}^{20} z_i}{20} = 3.75$$

(d) 4 PTS Compute the median (M_z) for this dataset.

$$A : M_z = 4 \qquad B : M_z = 3$$

(e) 4 PTS Compute the sample standard deviation (s_z) for this dataset.

$$A : s_z = \frac{\sum_{i=1}^{20}(z_i - \bar{z})^2}{20-1} = 1.80 \qquad B : s_z = \frac{\sum_{i=1}^{20}(z_i - \bar{z})^2}{20-1} = 1.94$$

(f) 4 PTS Compute the inter-quartile range (IQR) for this dataset.

$$A : \text{IQR} = Q_3 - Q_1 = 2.0 \qquad B : \text{IQR} = Q_3 - Q_1 = 2.25$$

(g) 4 PTS In your own words, briefly explain the difference between sample statistics ($\bar{z}, s_z$) and population statistics (μ_z, σ_z).

Sample statistics describe data from a finite number of observations. Population statistics describe the entire set of all possible data.

(h) 4 PTS In your own words, briefly explain why median and IQR may be more descriptive compared to mean and range for some datasets.

Median and IQR are based on the order of data, rather than the absolute value, making them robust to extreme values that are very large or small compared to others.

1.4 Continuous Random Variables [35 points]

The random variable Y measures the amount of time (in hours) between complementary beverages delivered at Castle Point Casino. Assume Y follows an exponential distribution with rate parameter λ previously found in question 1.2(a).

(a) 4 PTS Write an equation for the probability density function $f(y)$.

$$f(y) = \lambda e^{-\lambda y} = 4e^{-4y}$$

(b) 6 PTS Using the space below, sketch a PDF plot for $0 \leq y \leq 2$. Label the axes.

(c) 3 PTS Compute the expected time between beverage deliveries, $E[Y]$.

$$E[Y] = \frac{1}{\lambda} = \frac{1}{4} \text{ hours}$$

(d) 4 PTS Write an equation for the cumulative density function $F(y)$.

$$F(y) = 1 - e^{-\lambda y} = 1 - e^{-4y}$$

(e) 6 PTS Using the space below, sketch a CDF plot for $0 \leq y \leq 2$. Label the axes.

(f) 4 PTS Compute the median time between beverage deliveries, M_y.

$$M_y = F^{-1}(0.50) = \frac{-\ln(1-0.50)}{4} = 0.173 \text{ hours}$$

(g) 4 PTS Compute the 5th percentile time between beverage deliveries, P_5.

$$P_5 = F^{-1}(0.05) = \frac{-\ln(1-0.05)}{4} = 0.013 \text{ hours}$$

(h) 4 PTS Compute the 95th percentile time between beverage deliveries, P_{95}.

$$P_{95} = F^{-1}(0.95) = \frac{-\ln(1-0.95)}{4} = 0.749 \text{ hours}$$