Stevens Institute of Technology

SYS-601 Homework #3 Solutions

3.1 Admissions Data [14 points]

Consider the following dataset with 4486 college admission decisions in a frequency table with program, gender, and decision fields. Assume each person only applies to one program.

Program	Male (M)		Female (F)	
	Accepted (A)	Denied (D)	Accepted (A)	Denied (D)
P1	512	313	89	19
P2	313	207	17	8
P3	120	205	202	391
P4	138	279	131	244
P5	53	138	94	299
P6	22	351	24	317

Compute the following probabilities:

(a) 1 PT Marginal probability of being accepted $P(A)$

$P(A) = \frac{N_A}{N} = \frac{512+313+120+138+53+22+89+17+202+131+94+24}{4486} = \frac{1715}{4486} = 0.382$

(b) 1 PT Marginal probability of applying to program P1 $P(P1)$

$P(P1) = \frac{N_{P1}}{N} = \frac{512+313+89+19}{4486} = \frac{933}{4486} = 0.208$

(c) 1 PT Union probability of applying to either program P1 or P2 $P(P1 \cup P2)$

The special law of addition applies because $P1$ and $P2$ are mutually exclusive (students only apply to one program).

$P(P1 \cup P2) = P(P1) + P(P2) = \frac{512+313+89+19}{4486} + \frac{313+207+17+8}{4486} = \frac{1478}{4486} = 0.329$

(d) 1 PT Joint probability of applying to program P1 and being male $P(P1 \cap M)$

$P(P1 \cap M) = \frac{N_{P1 \cap M}}{N} = \frac{512+313}{4486} = \frac{825}{4486} = 0.184$

(e) 1 PT Joint probability of applying to program P1 being accepted $P(P1 \cap A)$

$P(P1 \cap A) = \frac{N_{P1 \cap A}}{N} = \frac{512+89}{4486} = \frac{601}{4486} = 0.134$

(f) 1 PT Conditional probability of acceptance for females $P(A|F)$ and males $P(A|M)$

Using the law of conditional probability:

$P(A|F) = \frac{P(A \cap F)}{P(F)} = \frac{N_{A \cap F}}{N_F} = \frac{89+17+202+131+94+24}{89+19+17+8+202+391+131+244+94+299+24+317} = \frac{557}{1835} = 0.304$

$P(A|M) = \frac{P(A \cap M)}{P(M)} = \frac{N_{A \cap M}}{N_M} = \frac{512+313+120+138+53+22}{512+313+313+207+120+205+138+279+53+138+22+351} = \frac{1158}{2651} = 0.437$

(g) 6 PTS Conditional probability of acceptance for females and males in each program:

(i) $P(A|F,P1)$ and $P(A|M,P1)$
$P(A|F,P1) = \frac{N_{A\cap F\cap P1}}{N(F\cap P1)} = \frac{89}{89+19} = 0.824$
$P(A|M,P1) = \frac{N_{A\cap M\cap P1}}{N(M\cap P1)} = \frac{512}{512+313} = 0.621$

(ii) $P(A|F,P2)$ and $P(A|M,P2)$
$P(A|F,P2) = \frac{N_{A\cap F\cap P2}}{N(F\cap P2)} = \frac{17}{17+8} = 0.680$
$P(A|M,P2) = \frac{N_{A\cap M\cap P2}}{N(M\cap P2)} = \frac{313}{313+207} = 0.602$

(iii) $P(A|F,P3)$ and $P(A|M,P3)$
$P(A|F,P3) = \frac{N_{A\cap F\cap P3}}{N(F\cap P3)} = \frac{202}{202+391} = 0.341$
$P(A|M,P3) = \frac{N_{A\cap M\cap P3}}{N(M\cap P3)} = \frac{120}{120+205} = 0.369$

(iv) $P(A|F,P4)$ and $P(A|M,P4)$
$P(A|F,P4) = \frac{N_{A\cap F\cap P4}}{N(F\cap P4)} = \frac{131}{131+244} = 0.349$
$P(A|M,P4) = \frac{N_{A\cap M\cap P4}}{N(M\cap P4)} = \frac{138}{138+279} = 0.331$

(v) $P(A|F,P5)$ and $P(A|M,P5)$
$P(A|F,P5) = \frac{N_{A\cap F\cap P5}}{N(F\cap P5)} = \frac{94}{94+299} = 0.239$
$P(A|M,P5) = \frac{N_{A\cap M\cap P5}}{N(M\cap P5)} = \frac{53}{53+138} = 0.277$

(vi) $P(A|F,P6)$ and $P(A|M,P6)$
$P(A|F,P6) = \frac{N_{A\cap F\cap P6}}{N(F\cap P6)} = \frac{24}{24+317} = 0.070$
$P(A|M,P6) = \frac{N_{A\cap M\cap P6}}{N(M\cap P6)} = \frac{22}{22+351} = 0.059$

(h) 2 PTS Based on results of (f) and (g) above, do gender and acceptance appear to be independent? Why may this result be important for an admissions committee?

Part (f) suggests acceptance and gender are *not* independent: males have a 13% higher chance of acceptance on the whole. Part (g) also suggests acceptance and gender are *not* independent for some programs: females have a 20% higher chance of acceptance in P1 and 8% higher chance in P2 (acceptance rates are within 5% for all other programs which suggest acceptance and gender are independent for these programs).

These results seem contradictory: on one hand, it appears females are biased against acceptance on a whole, but in several programs they are biased in favor of acceptance. This is an example of "Simpson's Paradox" the data is from a classic example from Bickel et al. (1975) using 1973 admissions data from University of California Berkeley.

3.2 Rock, Paper, Scissors [6 points]

Play $N = 30$ rounds of "Rock-Paper-Scissors" and record the results of each game (X_i: your move and Y_i: your opponent's move). Record your results in a 3×3 frequency table similar to the one below. Note: you can play against a computer here:
`http://www.nytimes.com/interactive/science/rock-paper-scissors.html`

	Opponent's Move (Y_i)		
Your Move (X_i)	Rock (Y_R)	Paper (Y_P)	Scissors (Y_S)
Rock (X_R)	6	3	1
Paper (X_P)	4	3	5
Scissors (X_S)	4	1	3

(a) 2 PTS Compute the marginal probability of each opponent's move: $P(Y_R), P(Y_P), P(Y_S)$.

$$P(Y_R) = \frac{6+4+4}{30} = \frac{14}{30} = 0.467$$
$$P(Y_P) = \frac{3+3+1}{30} = \frac{7}{30} = 0.233$$
$$P(Y_S) = \frac{1+5+3}{30} = \frac{9}{30} = 0.300$$

(b) 2 PTS Write a mathematical expression using joint and union operators for and compute the probability of you winning a game.

$$\begin{aligned} P(W) &= P\left((X_R \cap Y_S) \cup (X_P \cap Y_R) \cup (X_S \cap Y_P)\right) \\ &= P\left(X_R \cap Y_S\right) + P\left(X_P \cap Y_R\right) + P\left(X_S \cap Y_P\right) \\ &= \frac{1}{30} + \frac{4}{30} + \frac{1}{30} = \frac{6}{30} = 0.200 \text{ (not so good!)} \end{aligned}$$

(c) 2 PTS Write a mathematical expression using joint and union operators for and compute the probability of a tie game.

$$\begin{aligned} P(T) &= P\left((X_R \cap Y_R) \cup (X_P \cap Y_P) \cup (X_S \cap Y_S)\right) \\ &= P\left(X_R \cap Y_R\right) + P\left(X_P \cap Y_P\right) + P\left(X_S \cap Y_S\right) \\ &= \frac{6}{30} + \frac{3}{30} + \frac{3}{30} = \frac{12}{30} = 0.400 \end{aligned}$$