Stevens Institute of Technology

SYS-601 Homework Cover Sheet

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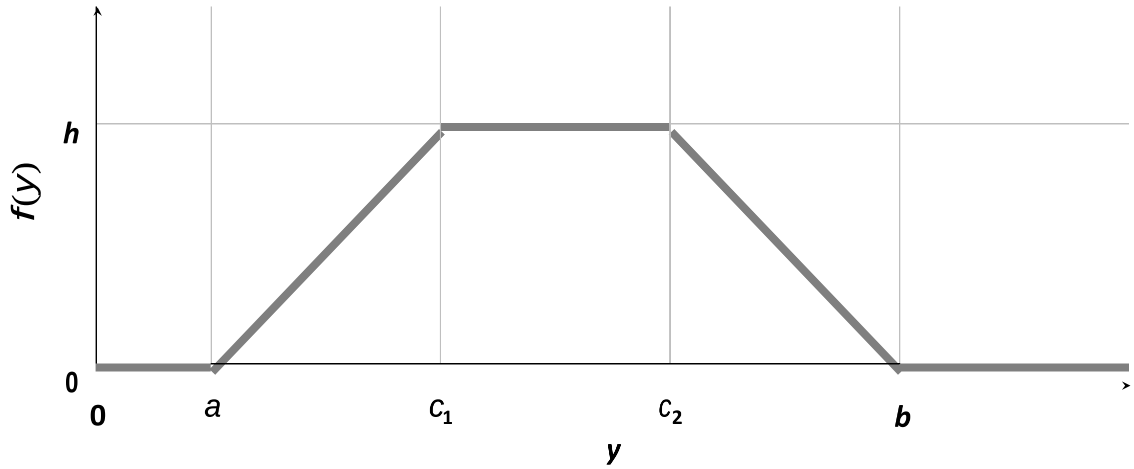
5.1 Trapezoidal Distribution

Consider a trapezoidal PDF with parameters $0 \le a \le c_1 \le c_2 \le b$ for the minimum value a, maximum value b, and transition points c_1 and c_2 between linear and constant segments:



(a) Using the property $\int_a^b f(y)dy = 1$, solve for h in terms of a, b, c_1, and c_2.
(*Hint:* write an equation for the area under the PDF, set equal to 1, and solve for h.)

⇨ We know that:
$\int_{-\infty}^{\infty} f(x)dx = 1$ and it is given that $\int_a^b f(y)\,dy = 1$
Area of trapezoid is: $\frac{Sum\ of\ parallel\ sides}{2}$ x $height$

Therefore.
$\int_a^b f(y)\,dy = 1 \quad \rightarrow \quad \frac{(c_2 - c_1) + (b-a)}{2}(h) = 1 \quad \rightarrow \quad h = \frac{2}{c_2 - c_1 + b - a}$

(b) Write an equation for the PDF $f(y)$ in terms of a, b, c_1 and c_2 for the ranges:
(i) $a < y \le c_1$ (*Hint:* verify $f(a) = 0$ and $f(c_1) = h$.)

⇨ f (y) = $\frac{h}{c_1 - a}$ x $(y-a)$ [formula of ramp equation]
Substituting value of h in this equation, we get

$$f(y) = \frac{2\,(y-a)}{(c_1 - a)(c_2 - c_1 + b - a)}$$

(ii) $c_1 < y \le c_2$

⇨ $f\,(y) = (c_2 - c_1)(h)$

$$= \frac{2(c_2 - c_1)}{c_2 - c_1 + b - a}$$

(iii) $c_2 < y \leq b$ (***Hint:*** verify f $(c_2) = h$ and f $(b) = 0$.)

⇨ $f\ (y) = \frac{h}{b - c_2}(b - y)$

$$= \frac{2\ (b - y)}{(b - c_2)(c_2 - c_1 + b - a)}$$

(c) Write an equation for the CDF $F\ (y)$ **in terms of** a, b, c_1 **and** c_2 **for the ranges:**

(i) $a < y \leq c_1$ (***Hint:*** **the area of the triangular region between** a **and** y.)

⇨ $F(y) = \frac{c_1 - a}{2} \text{ x } h$

$= \frac{c_1 - a}{2} \text{ x } \frac{2}{c_2 - c_1 + b - a}$

$$= \frac{c_1 - a}{c_2 - c_1 + b - a}$$

(ii) $c_1 < y \leq c_2$ (***Hint:*** $F(c_1)$ **plus the area of rectangular region between** c_1 **and** y.)

⇨ $F(y) = \frac{c_1 - a}{c_2 - c_1 + b - a} + [(c_2 - c_1) \text{ x } h]$

$= \frac{(c_1 - a) + [(c_2 - c_1)2]}{(c_2 - c_1 + b - a)}$

$$= \frac{2c_2 - c_1 - a}{(c_2 - c_1 + b - a)}$$

(iii) $c_2 < y \leq b$ (***Hint:*** $F\ (c_2)$ **plus the area of triangular region between** c_2 **and** y.)

⇨ $F(y) = \frac{2c_2 - c_1 - a}{(c_2 - c_1 + b - a)} + [\frac{(b - c_2)}{2} \text{ x } \frac{2}{c_2 - c_1 + b - a}]$

$$= \frac{c_2 - c_1 + b - a}{(c_2 - c_1 + b - a)}$$

$$= 1$$

(d) Draw a sketch of the CDF $F(y)$ for parameters $a = 0.5, c_1 = 1.5, c_2 = 2.5, b = 3.5$.

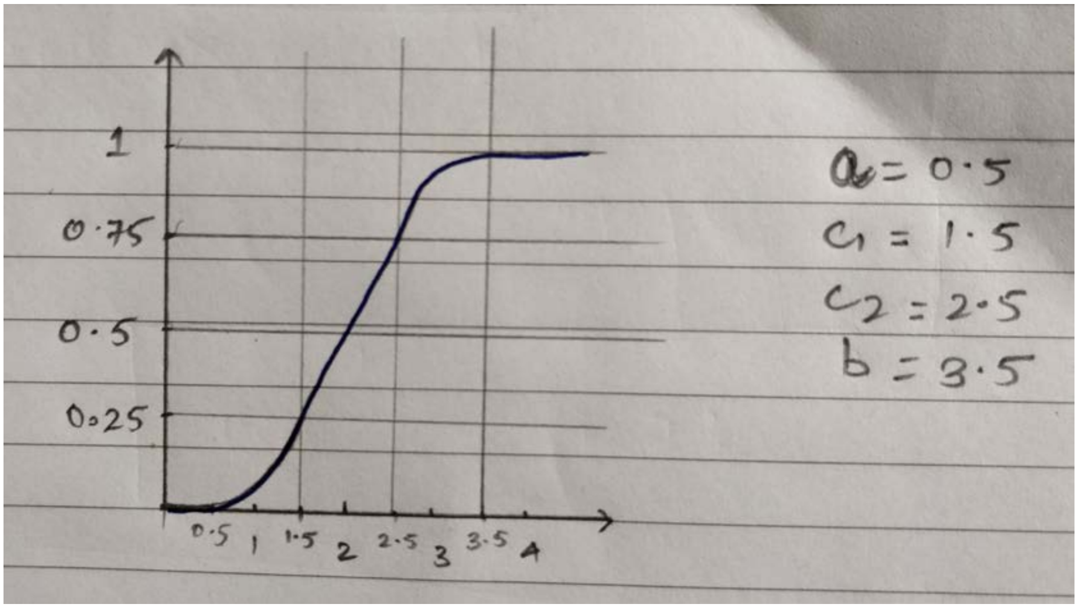


Figure 1 CDF plot

Y	F(y)
0	0
1.5	0.25
2.5	0.75
3.5	1
4	1

Table 1 CDF Values

5.2 Café Java: Customer Inter-arrival

Mathematically inclined customers arrive at Café Java following a Poisson process:

- **There is a long-term average rate of λ = 2 customer arrivals per minute.0.**
- **The arrival rate is constant throughout the day.**
- **Customer arrivals are independent of each other.**

Under these assumptions, the inter-arrival time between customers is an exponentially-distributed random variable *X* with *rate* parameter λ[1]:

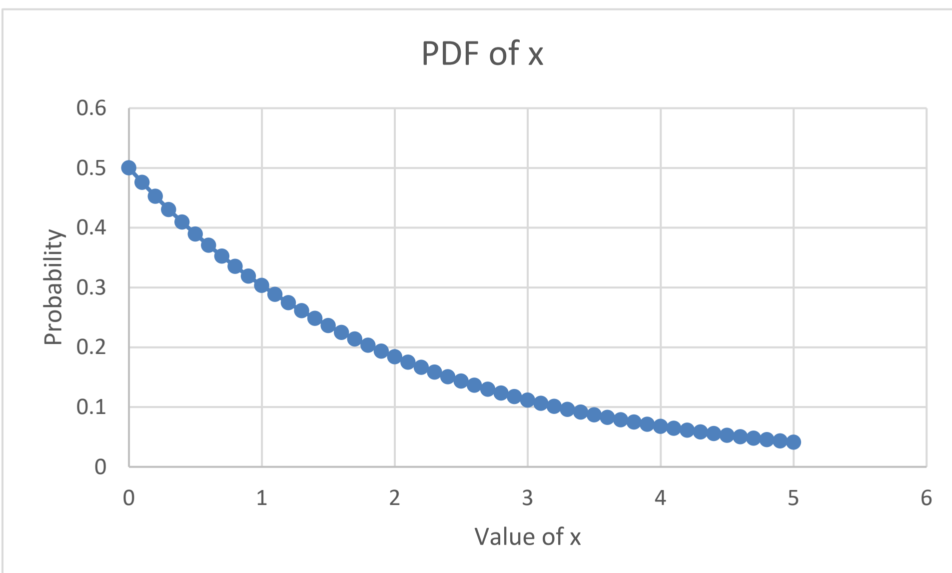
$$X \sim \text{exponential}(\lambda)$$

(a) Write an equation for the PDF *f* (*x*).

⇨ $\lambda = \frac{1}{2}$ customer per minute [inter arrival rate]

$\therefore\ p(x) = 0.5e^{-0.5x}$

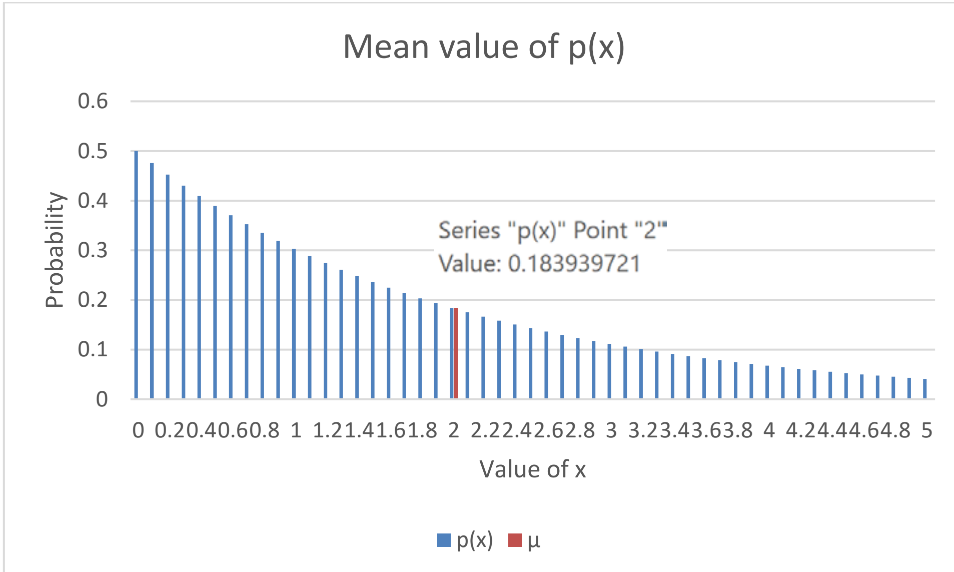
(b) Draw a sketch of the PDF *f* (*x*) for 0 ≤ *x* ≤ 5.



Graph 1 PDF of x

(c) Find the population mean $\mu = E[X]$ and mark on the PDF plot.

⇨ Mean $\mu = \frac{1}{\lambda} = \frac{1}{0.5} = 2$

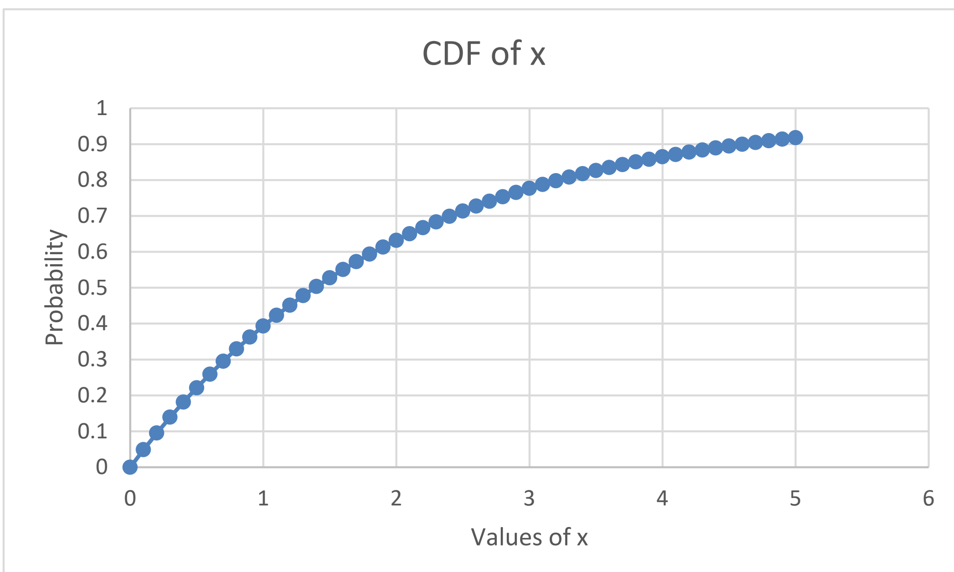


Graph 2 Mean value in p(x)

(d) Write an equation for the CDF *F*(*x*).

⇨ $F(x) = 1 - e^{-0.5x}$

(e) Draw a sketch of the CDF *F* (*x*) for 0 ≤ *x* ≤ 5.



Graph 3 CDF of x

(f) Evaluate or estimate the following quantities and mark on the CDF plot:

(i) 10th percentile inter-arrival time P_{10} (*Hint* : *F* (P_{10}) = 0.10)

➔ $0.1 = 1 - e^{\frac{x}{2}} \approx 0.2107$

(ii) Median inter-arrival time P_{50} (*Hint* : *F* (P_{50}) = 0.50)

➔ $0.5 = 1 - e^{\frac{x}{2}} \approx 1.3862$

(iii) 90th percentile inter-arrival time P_{90} (*Hint* : *F* (P_{90}) = 0.90)

➔ $0.9 = 1 - e^{\frac{x}{2}} \approx \ 4.6051$