

Homework 3 – Conditional Probability

3.1 Admissions Data

Consider the following dataset with 4486 college admission decisions in a frequency table with program, gender, and decision fields. Assume each person only applies to one program.

Program	Male (M)		Female (F)	
	Accepted (A)	Denied (D)	Accepted (A)	Denied (D)
P1	512	313	89	19
P2	313	207	17	8
P3	120	205	202	391
P4	138	279	131	244
P5	53	138	94	299
P6	22	351	24	317

Compute the following probabilities:

(a) Marginal probability of being accepted $P(A)$

- $P = 0.382$ or $\frac{3}{8}$

(b) Marginal probability of applying to program P1 $P(P1)$

- $P = 0.207$ or $\frac{1}{5}$

(c) Union probability of applying to either program P1 or P2 $P(P1 \cup P2)$

- $P = 0.329$ or $\frac{1}{3}$

(d) Joint probability of applying to program P1 and being male $P(P1 \cap M)$

- $P = 0.184$ or $\frac{1}{5}$

(e) Joint probability of applying to program P1 being accepted $P(P1 \cap A)$

- $P = 0.134$ or $\frac{1}{7}$

(f) Conditional probability of acceptance for females' $P(A|F)$ and males' $P(A|M)$

- $P(A|F) = 0.303$ or $\frac{1}{3}$
- $P(A|M) = 0.436$ or $\frac{3}{7}$

(g) Conditional probability of acceptance for females and males in each program:

(i) $P(A|F, P1)$ and $P(A|M, P1)$

- $P(A|F, P1) = 0.824$ or $\frac{5}{6}$
- $P(A|M, P1) = 0.620$ or $\frac{5}{8}$

(ii) $P(A|F, P2)$ and $P(A|M, P2)$

- $P(A|F, P2) = 0.68$ or $\frac{2}{3}$
- $P(A|M, P2) = 0.601$ or $\frac{3}{5}$

(iii) $P(A|F, P3)$ and $P(A|M, P3)$

- $P(A|F, P3) = 0.34$ or $\frac{1}{3}$
- $P(A|M, P3) = 0.369$ or $\frac{3}{8}$

(iv) $P(A|F, P4)$ and $P(A|M, P4)$

- $P(A|F, P4) = 0.349$ or $\frac{1}{3}$
- $P(A|M, P4) = 0.33$ or $\frac{1}{3}$

(v) $P(A|F, P5)$ and $P(A|M, P5)$

- $P(A|F, P5) = 0.239$ or $\frac{1}{4}$
- $P(A|M, P5) = 0.277$ or $\frac{2}{7}$

(vi) $P(A|F, P6)$ and $P(A|M, P6)$

- $P(A|F, P6) = 0.058$ or $\frac{1}{17}$
- $P(A|M, P6) = 0.07$ or $\frac{5}{71}$

(h) Based on results of (f) and (g) above, do gender and acceptance appear to be independent? Why may this result be important for an admissions committee?

Yes, the gender and acceptance appear to be independent. The percentage of Male and Female in acceptance is nearly the same. Having this result shows that there is no gender bias in selection. Assuming that acceptance is calculated on merit and not just for maintaining negligible gender bias.

3.2 Rock, Paper, Scissors

Play $N = 30$ rounds of “Rock-Paper-Scissors” and record the results of each game (X_i : your move and Y_i : your opponent’s move). Record your results in a 3 x 3 frequency table similar to the one below.

Note: you can play against a computer here: <http://www.nytimes.com/interactive/science/rock-paper-scissors.html>

Your Move (Xi)	Opponent's Move (Yi)		
	Rock (YR)	Paper (YP)	Scissors (YS)
Rock (XR)	2	3	4
Paper (XP)	6	3	4
Scissors (XS)	2	3	3

- (a) Compute the marginal probability of each opponent's move: $P(Y_R), P(Y_P), P(Y_S)$.
- $P(Y_R) = 0.33$
 - $P(Y_P) = 0.3$
 - $P(Y_S) = 0.36$
- (b) Write a mathematical expression using joint and union operators for and compute the probability of you winning a game.
- $P_{win} = (Y_P|X_S) \cup (Y_R|X_P) \cup (Y_S|X_R)$
- (c) Write a mathematical expression using joint and union operators for and compute the probability of tie game.
- $P_{tie} = (Y_P|X_P) \cup (Y_R|X_R) \cup (Y_S|X_S)$