Event-Scheduling Discrete Event Simulation

SYS-611: Simulation and Modeling

Paul T. Grogan, Ph.D.
Assistant Professor
School of Systems and Enterprises

Agenda

1. Special Queuing Systems
2. Inventory Systems

Reading: S.M. Ross “The Discrete Event Simulation Approach,” Ch. 7 in *Simulation*, 5th Edition, 2013.

Special Queuing Systems

Customer Balking

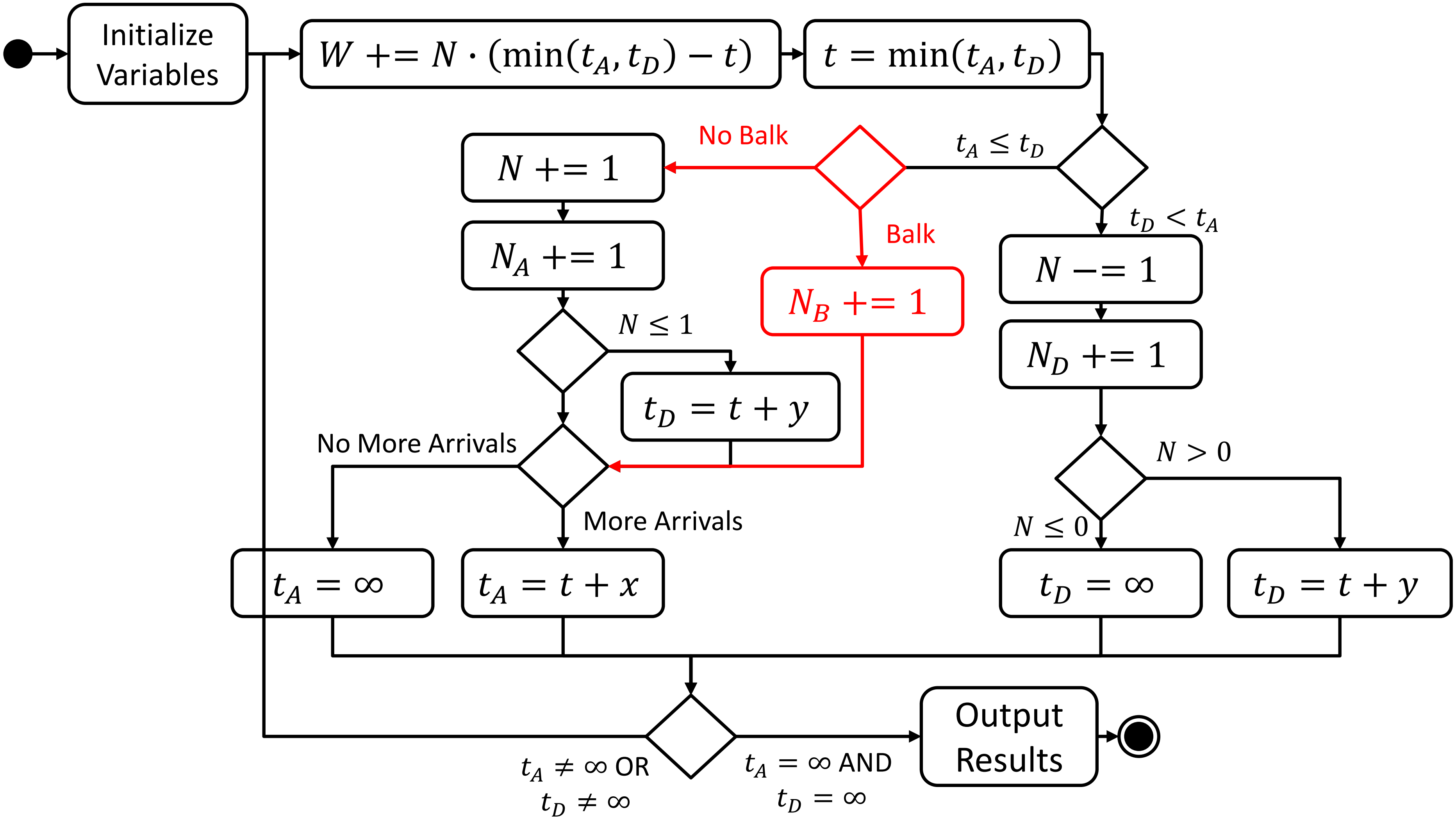
- Customers may balk (leave queuing system) if the queue is too long
- Example balking random variable PMF:

$$P(B) = \begin{cases} 0.5, & N > 5 \\ 0, & \text{otherwise} \end{cases}$$

- Balking process generator for r_B random (0,1):

$$B = \begin{cases} \text{Balk}, & \text{if } N > 5 \text{ and } r_B < 0.5 \\ \text{No Balk}, & \text{otherwise} \end{cases}$$

Balking Activity Diagram



Balking Sim (Excel)

	A	B	C	D	E	F	G	H	I	J
1	Event	t	t_A	t_D	B	N	N_A	N_D	N_B	W
2	0	0	0.94	9999.00	FALSE	0	0	0	0	0.00
3	1	0.94	1.10	3.05	=AND(C3<=D3,F2>5,RAND()<0.5)				0	0.00
4	2	1.10	1.17	3.05	FALSE	2	2	0	0	0.17
5	3	1.17	1.35	3.05	FALSE	3	3	0	0	0.30
6	4	1.35	4.07	3.05	FALSE	4	4	0	0	0.83

Add new balking derived state + generator

	A	B	C	D	E	F	G	H	I	J
1	Event	t	t_A	t_D	B	N	N_A	N_D	N_B	W
2	0	0	0.94	9999.00	FALSE	0	0	0	0	0.00
3	1	0.94	1.10	3.05	FALSE	1	1	0	=IF(E3,I2+1,I2)	
4	2	1.10	1.17	3.05	FALSE	2	2	0	0	0.17
5	3	1.17	1.35	3.05	FALSE	3	3	0	0	0.30
6	4	1.35	4.07	3.05	FALSE	4	4	0	0	0.83

Add new balking counter variable

	A	B	C	D	E	F	G	H	I	J
1	Event	t	t_A	t_D	B	N	N_A	N_D	N_B	W
2	0	0	0.94	9999.00	FALSE	0	0	0	0	0.00
3	1	0.94	1.10	3.05	FALSE	=IF(E2,F2,IF(C2<=D2,F2+1,F2-1))				0.00
4	2	1.10	1.17	3.05	FALSE	2	2	0	0	0.17
5	3	1.17	1.35	3.05	FALSE	3	3	0	0	0.30
6	4	1.35	4.07	3.05	FALSE	4	4	0	0	0.83

Shortcut N update with IF check for balk

	A	B	C	D	E	F	G	H	I	J
1	Event	t	t_A	t_D	B	N	N_A	N_D	N_B	W
2	0	0	0.94	9999.00	FALSE	0	0	0	0	0.00
3	1	0.94	1.10	3.05	FALSE	1	=IF(E2,G2,IF(C2<=D2,G2+1,G2))			0.00
4	2	1.10	1.17	3.05	FALSE	2	2	0	0	0.17
5	3	1.17	1.35	3.05	FALSE	3	3	0	0	0.30
6	4	1.35	4.07	3.05	FALSE	4	4	0	0	0.83

Shortcut N_A update with IF check for balk

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Event	t	t_A	t_D	B	N	N_A	N_D	N_B	W		W_bar		
2	0	0	0.94	9999.00	FALSE	0	0	0	0	0.00		1.30		
3	1	0.94	1.10	=IF(OR(AND(C2<=D2,F2+1<=1,NOT(E2)),AND(D2<C2,F2-1>0)),B3-0.75*LN(1-RAND()),IF(AND(D2<C2,F2-1<=0),9999,D2))										
4	2	1.10	1.17	3.05	FALSE	2	2	0	0	0.17				
5	3	1.17	1.35	3.05	FALSE	3	3	0	0	0.30				
6	4	1.35	4.07	3.05	FALSE	4	4	0	0	0.83				

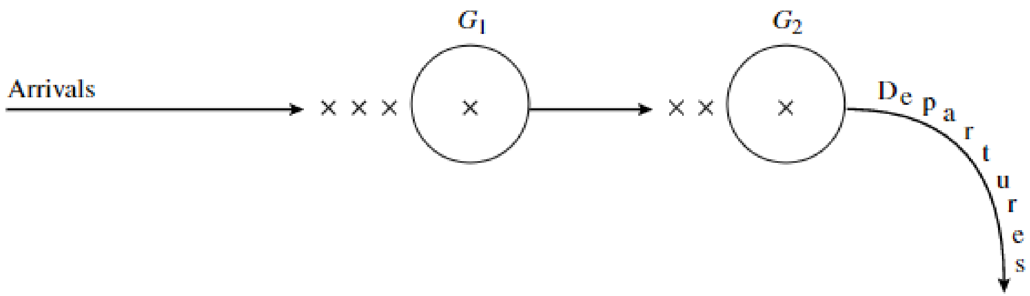
Edit t_D condition for NOT balk

Balking Sim (Python)

```
while (t_A < np.inf
       or t_D < np.inf):
  W += N*(min(t_A, t_D) - t)
  t  = min(t_A, t_D)
  if t_A <= t_D:
    if generate_b(N):
      N_B += 1
    else:
      N += 1
      N_A += 1
      if N <= 1:
        t_D = t + generate_y()
    t_A = (t + generate_x()
           if t < 1000 else np.inf)
...
```

```
def generate_b(N):
  r = np.random.rand()
  if N > 5 and r < 0.5
    return True
  else:
    return False
```

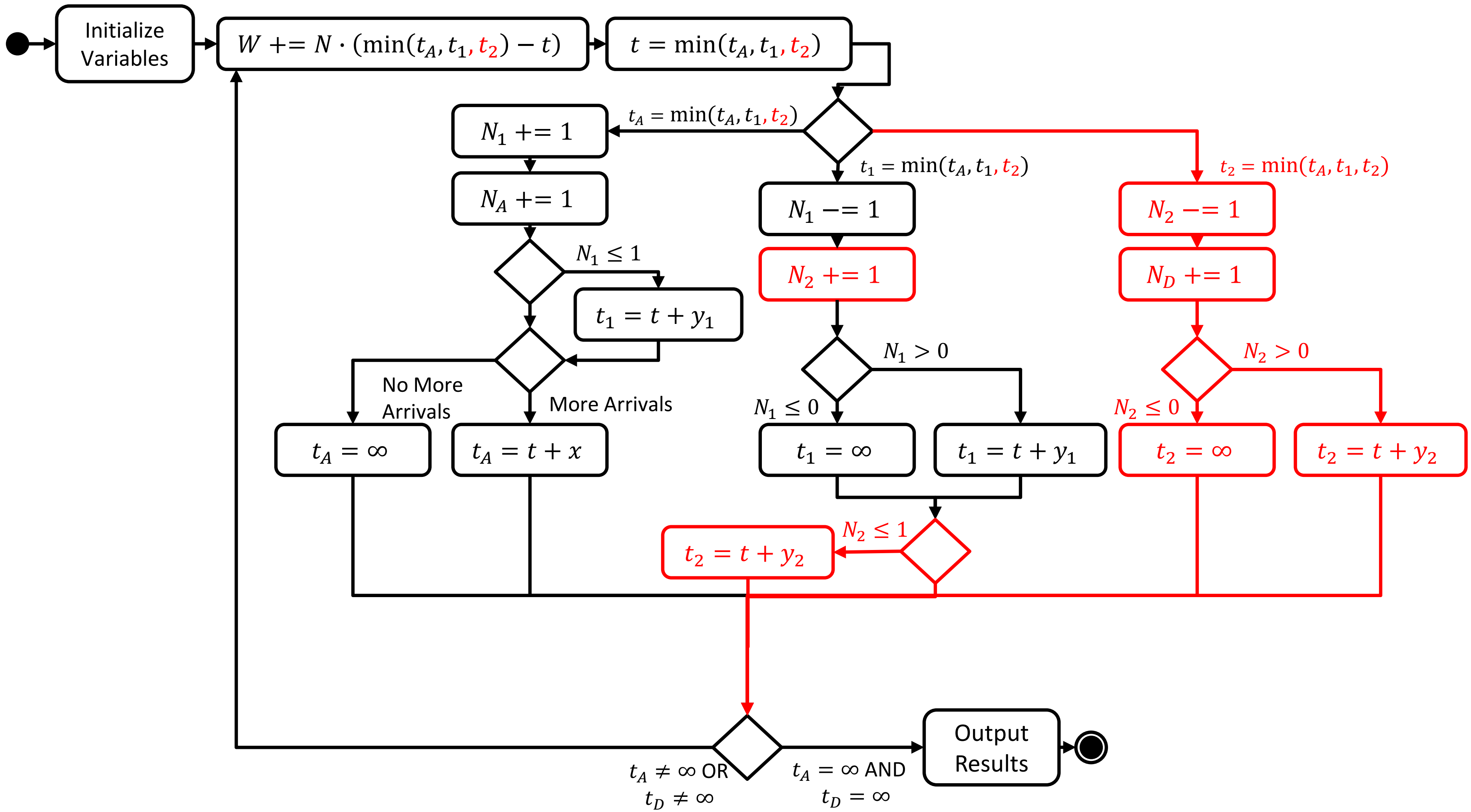
Queuing with Serial Servers



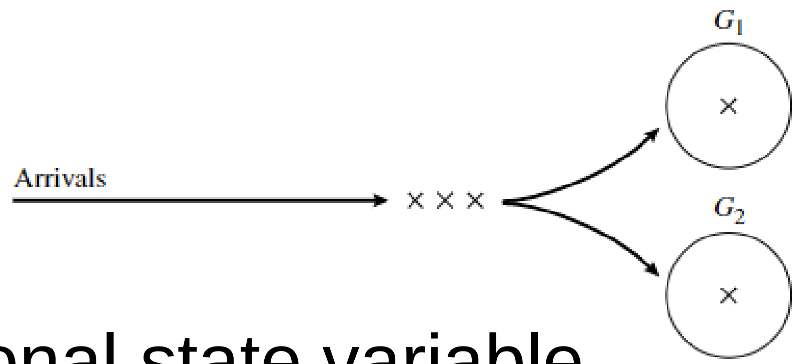
- Need additional state variable
  - N_1: customers waiting for server 1
  - N_2: customers waiting for server 2
- New events distinguish between:
  - Service complete for server 1 (t_1)
  - Service complete for server 2 (t_2)

Ross (2013), pp. 115-117

Serial Server Activity Diagram



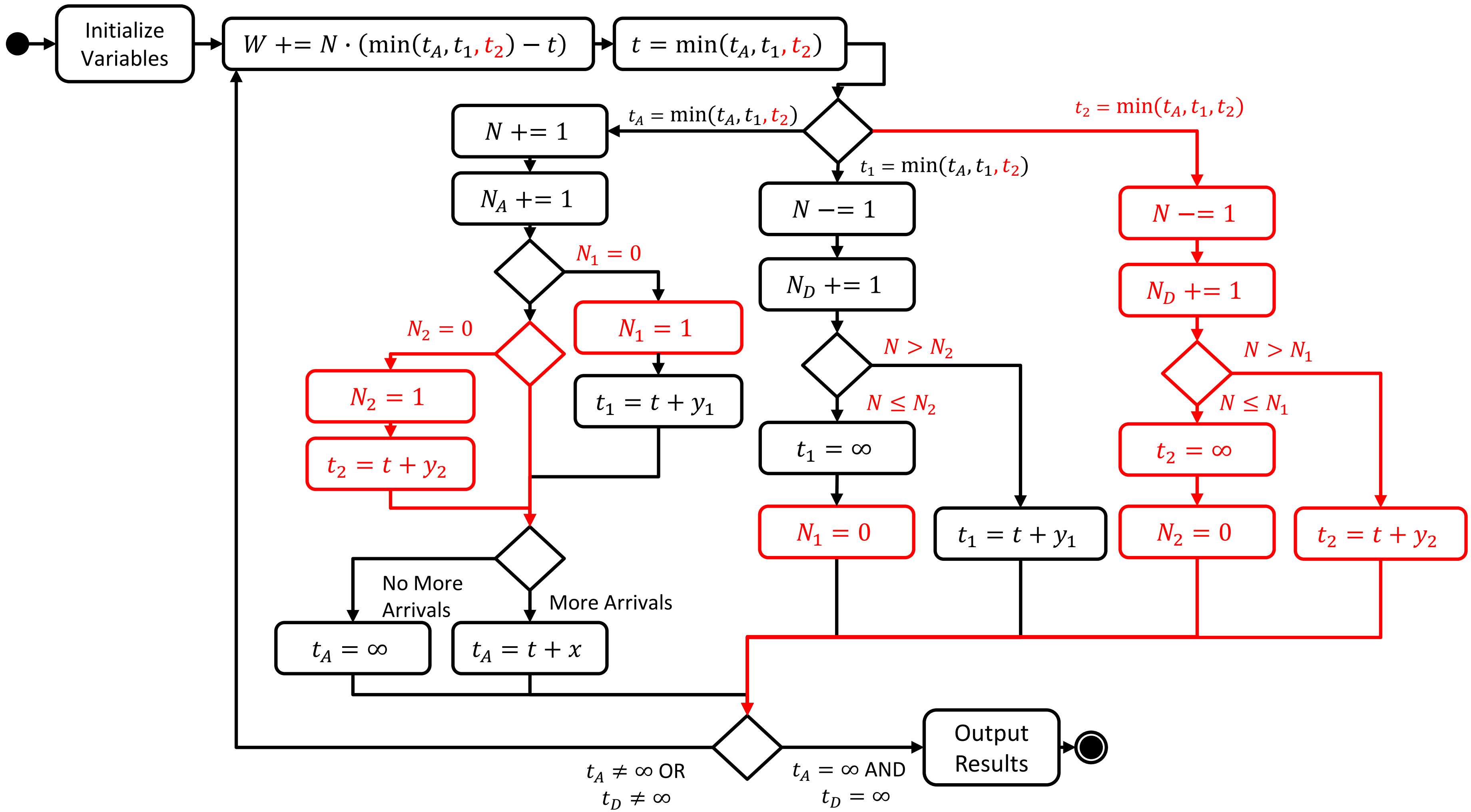
Queuing with Parallel Servers



- Need additional state variable
  - N: total number of customers
  - N_1: number of customers with server 1
  - N_2: number of customers with server 2
- New events distinguish between:
  - Service complete for server 1 (t_1)
  - Service complete for server 2 (t_2)

Ross (2013), pp. 117-120

Parallel Server Activity Diagram



Inventory Systems

Inventory Model

Ross (2013), pp. 120-122.

- Stock products which sell for $r = 100$ each
- Customer inter-arrival time

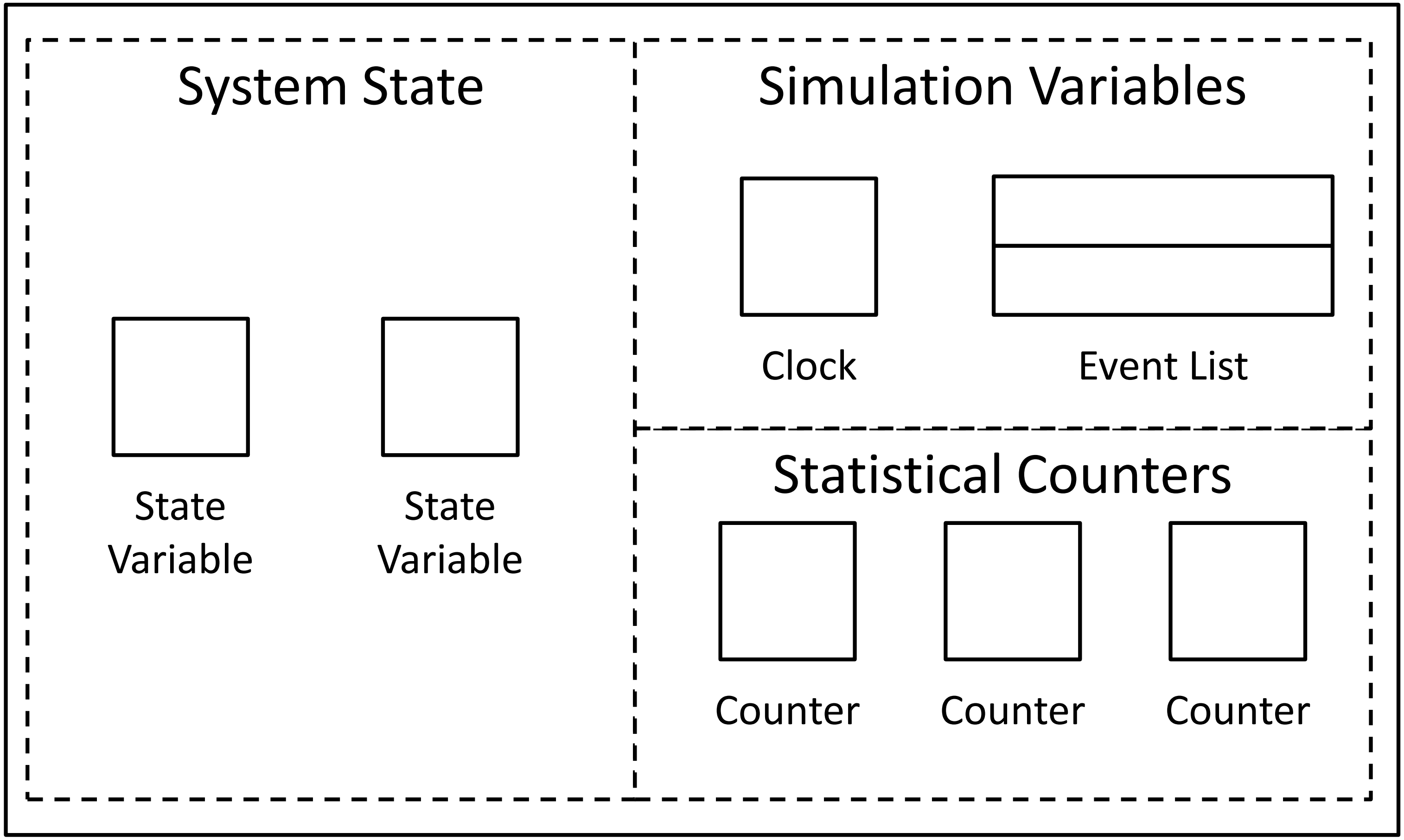
$$d \sim \text{exponential}(\lambda = 5)$$

- Each customer demands products (can only sell stock)

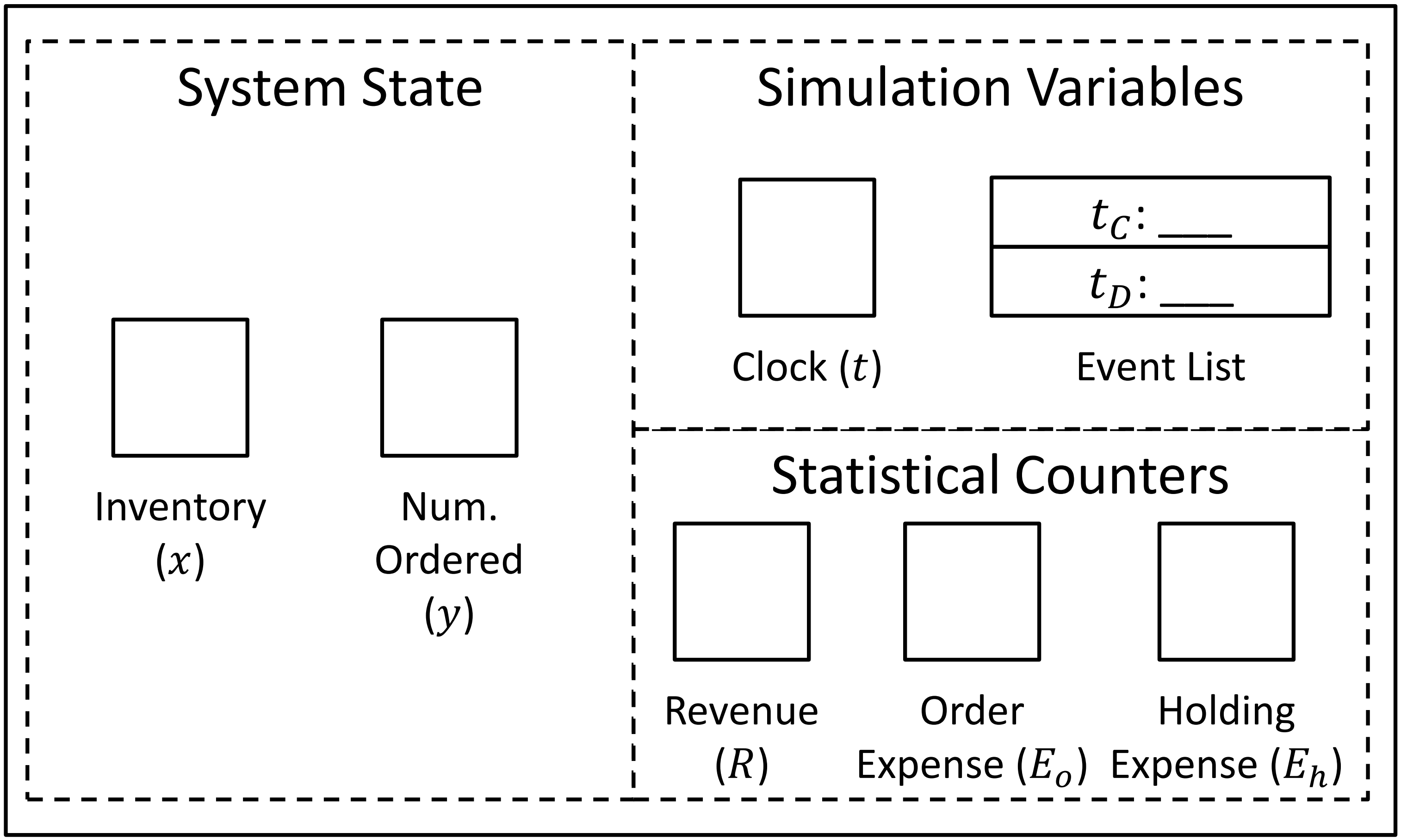
$$D \sim \text{uniform}(1,4)$$

- Order policy: when inventory is $x < Q$, place an order for $y = S - x$ (only one outstanding order at a time)
- Costs $c(y) = 50 \cdot y$ to order y units
- Delay of $L = 2$ days until delivery
- Holding cost of $h = 2$ per item per day

Inventory Model Structure?



Inventory Model Structure



Initialize Simulation

$S = 20, Q = 15$

Inter-arrival times: 0.02, 0.18, 0.18, 0.38

Demands: 1, 1, 4, 4

System State

20

Inventory (x)

0

Num. Ordered (y)

Simulation Variables

0

Clock (t)

t_C: 0.02

t_D: ∞

Event List

Statistical Counters

0

Revenue (R)

0

Order Expense (E_o)

0

Holding Expense (E_h)

Customer @ $t = 0.02$

$s = 20, Q = 15$

Inter-arrival times: ~~0.02~~, 0.18, 0.18, 0.38

Demands: 1, 1, 4, 4

System State

Inventory (x)	Num. Ordered (y)
~~20~~ 19	0

Simulation Variables

Clock (t): ~~0.00~~ 0.02

Event List:
- t_C: ~~0.02~~ 0.20
- t_D: ∞

Statistical Counters

Revenue (R)	Order Expense (E_o)	Holding Expense (E_h)
100	0	~~0~~ 0.73

Customer @ $t = 0.20$

$s = 20, \mathbf{Q} = 15$

Inter-arrival times: ~~0.02~~, ~~0.18~~, 0.18, 0.38 Demands: ~~1~~, 1, 4, 4

System State

~~19~~
18

Inventory (x)

0

Num. Ordered (y)

Simulation Variables

~~0.02~~
0.20

Clock (t)

t_C: ~~0.20~~ 0.38

$t_D: \infty$

Event List

Statistical Counters

~~100~~
200

Revenue (R)

0

Order Expense (E_o)

~~0.73~~
7.56

Holding Expense (E_h)

Customer @ $t = 0.38$ $\quad s = 20, Q = 15$

Inter-arrival times: ~~0.02~~, ~~0.18~~, ~~0.18~~, 0.38 Demands: ~~1~~, ~~1~~, 4, 4

System State

Inventory (x)	Num. Ordered (y)
~~18~~ 14	~~0~~ 6

Simulation Variables

Clock (t): ~~0.20~~ 0.38

Event List:
- t_C: ~~0.20~~ 0.76
- t_D: ~~∞~~ 2.38

Statistical Counters

Revenue (R)	Order Expense (E_o)	Holding Expense (E_h)
~~200~~ 600	~~0~~ 300	~~7.56~~ 14.2

Customer @ $t = 0.76$

$s = 20, \mathbf{Q} = 15$

Inter-arrival times: ~~0.02~~, ~~0.18~~, ~~0.18~~, ~~0.38~~

Demands: ~~1~~, ~~1~~, 4, 4

System State

Inventory (x): ~~14~~ 10

Num. Ordered (y): 6

Simulation Variables

Clock (t): ~~0.38~~ 0.76

Event List:
- t_C: ~~0.20~~ ∞
- t_D: ~~∞~~ 2.38

Statistical Counters

Revenue (R): ~~600~~ 1000

Order Expense (E_o): 300

Holding Expense (E_h): ~~14.2~~ 24.7

Delivery @ $t = 2.38$

$S = 20, \mathbf{Q} = 15$

Inter-arrival times: ~~0.02~~, ~~0.18~~, ~~0.18~~, ~~0.38~~

Demands: ~~1~~, ~~1~~, 4, 4

System State

Inventory (x)	Num. Ordered (y)
~~10~~ 16	~~6~~ 0

Simulation Variables

Clock (t): ~~0.76~~ 2.38

Event List:

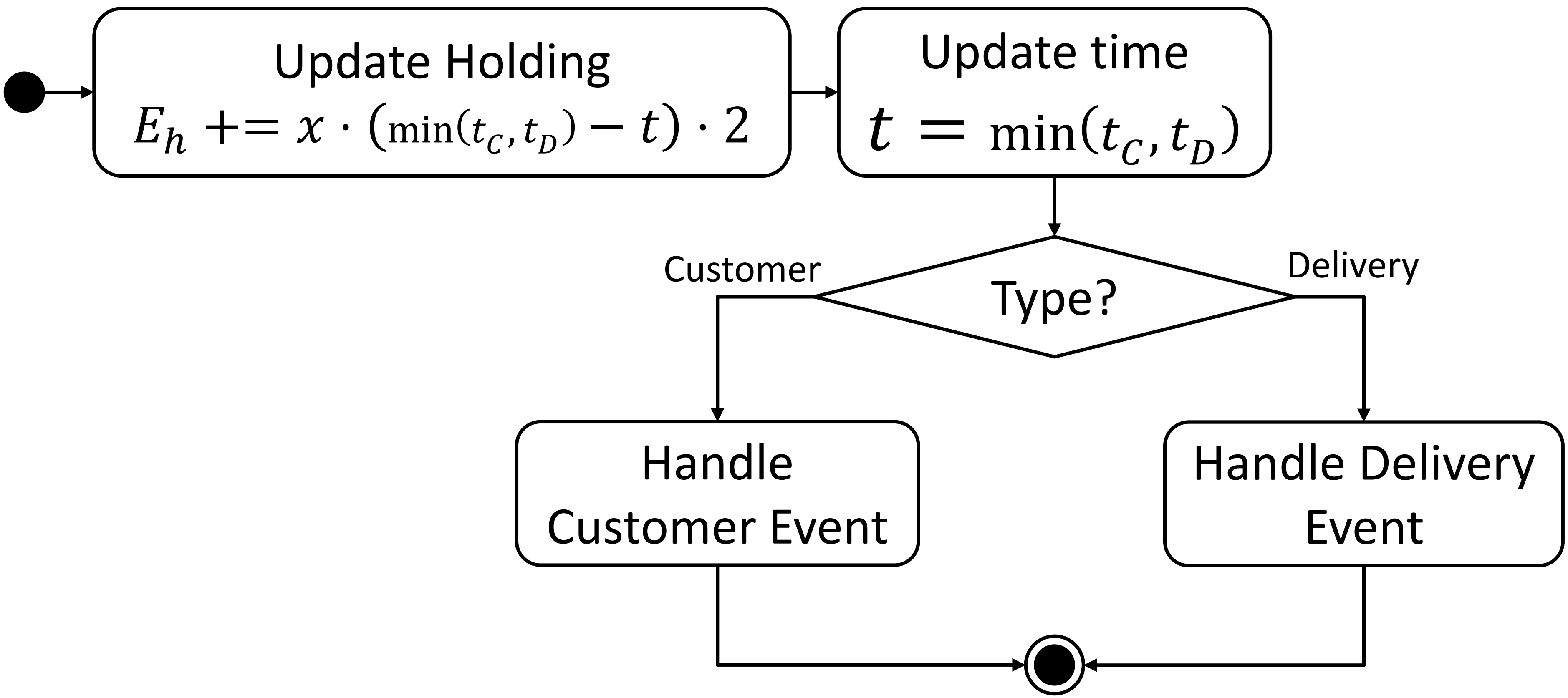
$t_C: \infty$

t_D: ~~2.38~~ ∞

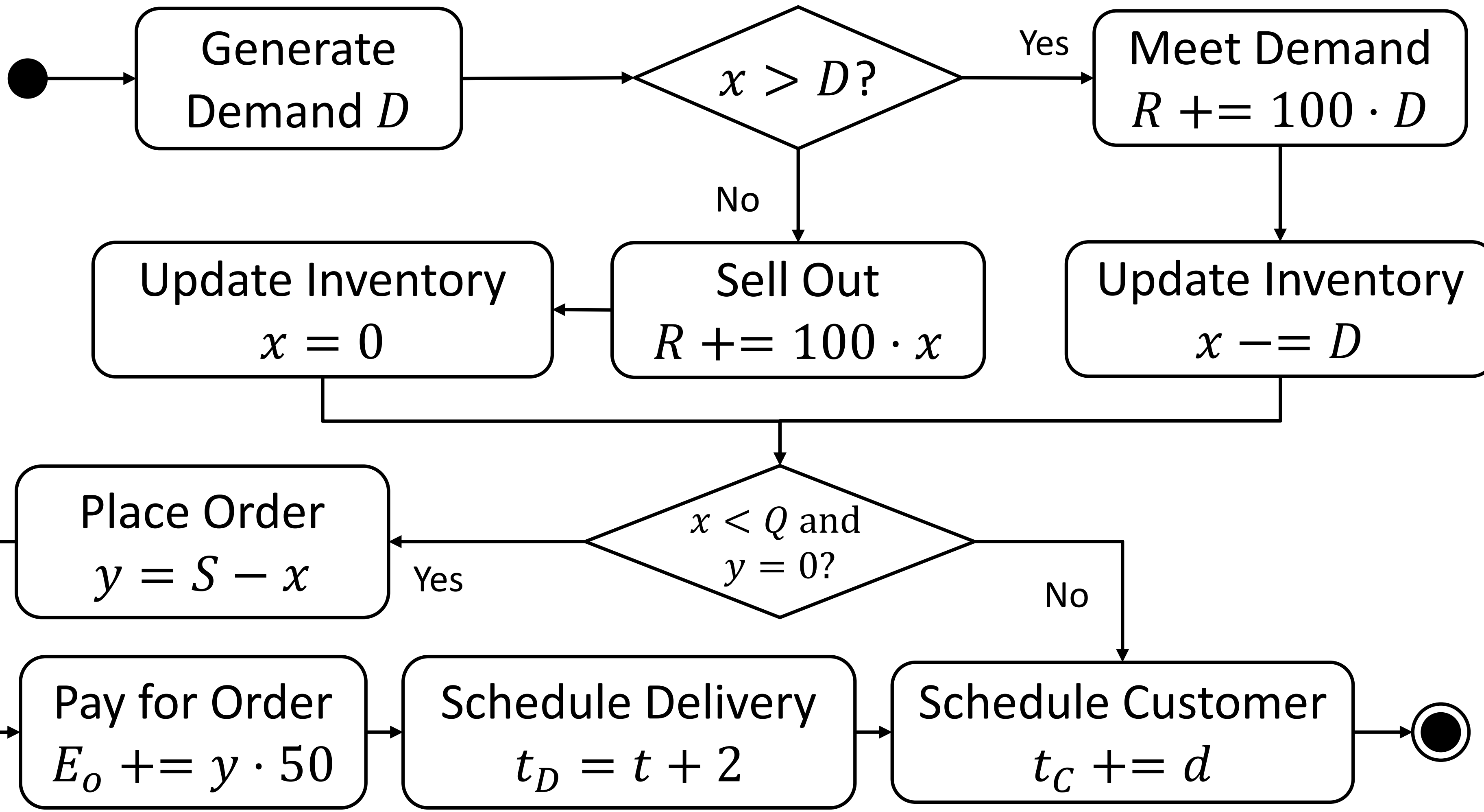
Statistical Counters

Revenue (R)	Order Expense (E_o)	Holding Expense (E_h)
1000	300	~~24.7~~ 57.1

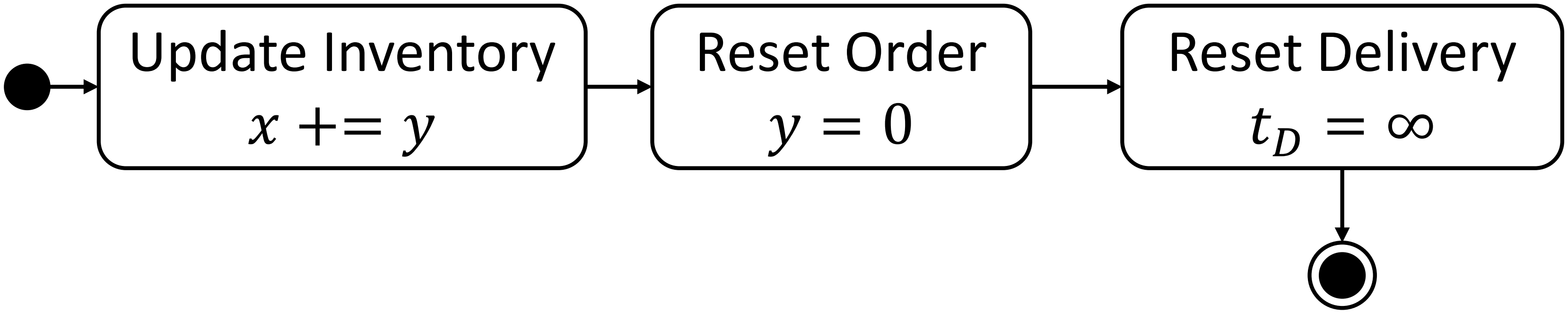
Advance Time / Handle Event



Handle Customer Event



Handle Delivery Event



Inventory Activity Diagram