Stevens Institute of Technology

# SYS-611 Practice Exam A Solutions

## 1.1 Modeling and Simulation

(a) **Match** each problem (left) with the most appropriate analysis method (right).
*Use each option only once.*

| | | | | |
|---|---|---|---|---|
| How to arrange new furniture in an office. | (i) | (4) | (1) | Actual System |
| How to efficiently assemble a peanut butter and jelly sandwich. | (ii) | (1) | (2) | Analytical Model |
| How to predict the movement of crowds in a shopping mall. | (iii) | (5) | (3) | Conceptual Model |
| How to select a furnace for a house. | (iv) | (2) | (4) | Physical Model |
| How to explain the most significant factors contributing to healthcare costs. | (v) | (3) | (5) | Simulation Model |

(b) **Match** each type of simulation model (left) with the best description (right).
*Use each option only once.*

| | | | | |
|---|---|---|---|---|
| Conway's Game of Life. | (i) | (2) | (1) | Dynamic (Continuous) |
| Accumulation of liquid poured into a basin. | (ii) | (1) | (2) | Dynamic (Discrete) |
| Estimated profit for Dave's Candies. | (iii) | (3) | (3) | Static |

(c) X **True** or **False**: A stochastic model has at least one random variable.

(d) X **True** or **False**: Process generators are mathematical functions which generate random variables.

(e) **True** or X **False**: Selecting an appropriate modeling tool is the first step in the modeling and simulation process.

(f) **True** or X **False**: Verification and validation activities do the same thing.

(g) X **True** or **False**: Aleatory variability arises from natural variation.

(h) X **True** or **False**: Model state captures all of the information required to recreate a snapshot in time.

(i) **True** or X **False**: Model behavior describes the verification and validation plan for a simulation.

(j) **Describe** three elementary state variables and two state transition functions for a simulation model of a basketball game.

State variables: game time, shot clock time, points per team, number fouls, ...

State transition functions: time advancing by a second, scoring a basket, substituting a player, committing a foul, ...

# 1.2 Discrete Random Variables

While performing a study on dog walkers in Hoboken, you observe a total of $n = 100$ people walking dogs with the following frequencies for number of dogs walked per person ($D$):

| Number dogs ($d$): | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Frequency observed: | 65 | 15 | 5 | 5 | 10 |

(a) **Complete** the PMF and CDF values below based on these observations.

| $d$: | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $p(d)$: | 0.65 | 0.15 | 0.05 | 0.05 | 0.10 |
| $F(d)$: | 0.65 | 0.65+0.15=0.80 | 0.80+0.05=0.85 | 0.85+0.05=0.90 | 1.00 |

(b) **Compute** is the expected number of dogs walked per person.

$$E(D) = 0.65 \cdot 1 + 0.15 \cdot 2 + 0.05 \cdot 3 + 0.05 \cdot 4 + 0.10 \cdot 5 = 1.8$$

(c) **Sketch** the CDF in part (a) using a step chart in the space below.
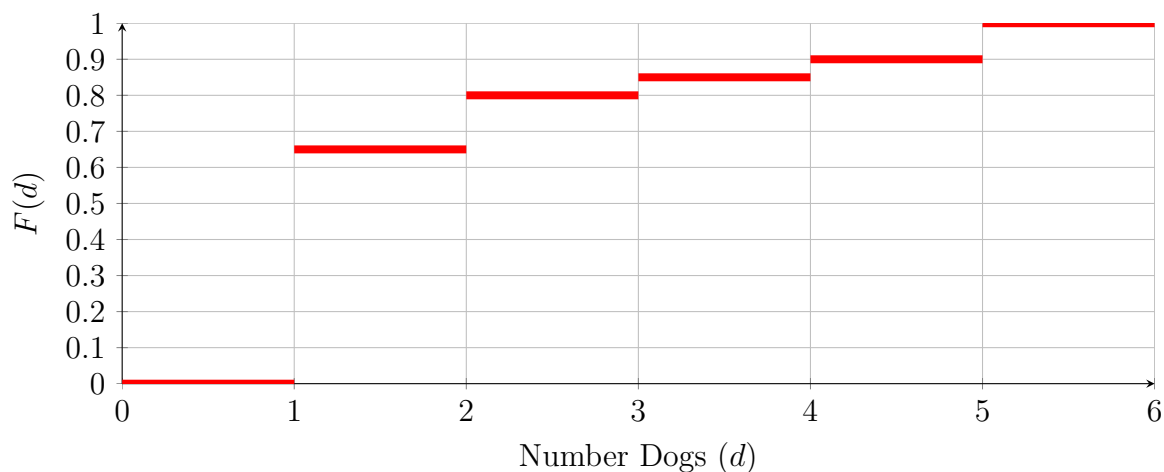


(d) **Generate** samples using the IVT method for the following numbers:

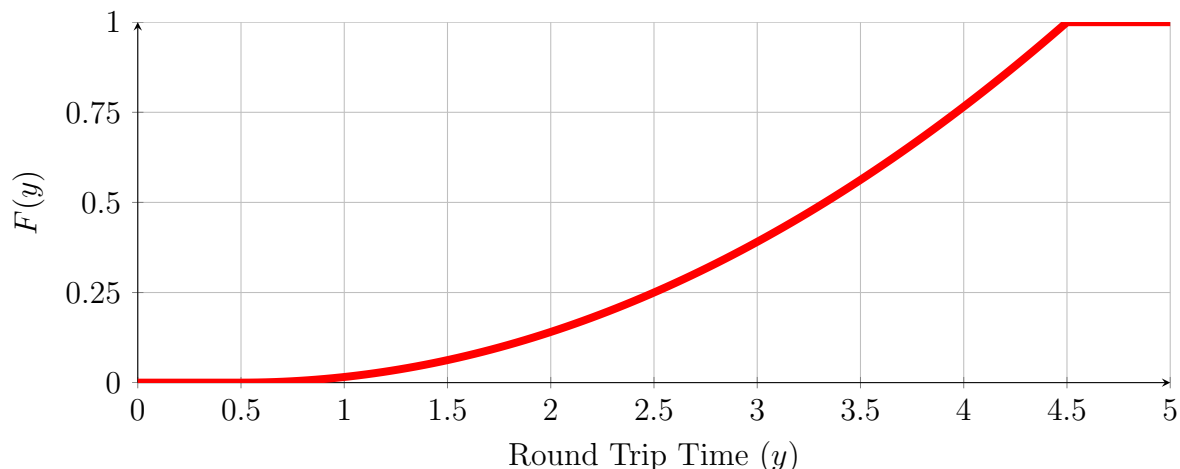| Random (0,1) | Generated Sample |
|---|---|
| 0.549 | 1 |
| 0.715 | 2 |
| 0.603 | 1 |
| 0.964 | 5 |

# 1.3 Continuous Random Variables

While performing a study on elevator service times in the Babbio Center, you hypothesize the following model for the round trip time ($Y$, in minutes) for an elevator car:

$$Y \sim \text{ramp\_up}(a = 0.5, b = 4.5), \qquad f(y) = \begin{cases} 0 & y < 0.5 \\ \frac{1}{8}(y - 0.5) & 0.5 \leq y < 4.5 \\ 0 & y \geq 4.5 \end{cases}$$

(a) **Write** an equation for the CDF for the round trip time below.

$$F(y) = \begin{cases} 0 & y < 0.5 \\ \left(\frac{y-0.5}{4.5-0.5}\right)^2 = \left(\frac{y-0.5}{4}\right)^2 & \text{if } 0.5 \leq y < 4.5 \\ 1 & y \geq 4.5 \end{cases}$$

(b) **Sketch** the CDF for the round trip time $Y$ using a line graph in the space below.



(c) **Generate** samples using the IVT method for the following numbers:

| Random (0,1) | Generated Sample |
|---|---|
| 0.25 | $0.5 + 4 \cdot \sqrt{0.25} = 0.5 + 4 \cdot 0.5 = 2.5$ |
| 0.64 | $0.5 + 4 \cdot \sqrt{0.64} = 0.5 + 4 \cdot 0.8 = 3.7$ |
| 0.81 | $0.5 + 4 \cdot \sqrt{0.81} = 0.5 + 4 \cdot 0.9 = 4.1$ |
| 0.36 | $0.5 + 4 \cdot \sqrt{0.36} = 0.5 + 4 \cdot 0.6 = 2.9$ |

(d) **Explain** how this model could be validated using a set of observations.

Collect a set of observations and either apply the Komolgorov-Smirnov (K-S) or Chi-squared goodness-of-fit tests to perform a hypothesis test with a null hypothesis that the data come from the hypothesized distribution above.

3

# 1.4 Monte Carlo Simulation

Susan owns a Sushi restaurant and must place an order for tuna every day. Each tuna costs $50 and can produce 25 servings of sushi which sell for $5 apiece. Typical demands range between 100 and 200 servings of sushi per day. Any unused tuna must be disposed of at the end of the day. Help Susan study this problem with a Monte Carlo simulation.

(a) **What** is the primary random variable $(D)$ and its probability distribution?

Demand for sushi, $D \sim \text{uniform}(100, 200)$

(b) **Write** Susan's profit as a function of the primary random variable $D$ above and the number of tunas ordered $T$.

$$P(D, T) = \begin{cases} 5D - 50T & \text{if } D < 25T \\ 25 \cdot 5T - 50T = 75T & \text{otherwise} \end{cases}$$

(c) **Generate** 4 samples of the random variable in (a) using the IVT method with $d = 100 + 100r$ and the following random numbers: $r \in \{0.40, 0.10, 0.60, 0.80\}$. **Calculate** the profit for values $T \in \{4, 6, 8\}$ to complete the table below:

| $r$ | $d$ | $P(d, T = 4)$ | $P(d, T = 6)$ | $P(d, T = 8)$ |
|------|------|----------------|-----------------|-----------------|
| 0.40 | 140 | $75 \cdot 4 = 300$ | $5 \cdot 140 - 50 \cdot 6 = 400$ | $5 \cdot 140 - 50 \cdot 8 = 300$ |
| 0.10 | 110 | $75 \cdot 4 = 300$ | $5 \cdot 110 - 50 \cdot 6 = 250$ | $5 \cdot 110 - 50 \cdot 8 = 150$ |
| 0.60 | 160 | $75 \cdot 4 = 300$ | $75 \cdot 6 = 450$ | $5 \cdot 160 - 50 \cdot 8 = 400$ |
| 0.80 | 180 | $75 \cdot 4 = 300$ | $75 \cdot 6 = 450$ | $5 \cdot 180 - 50 \cdot 8 = 500$ |

(d) **Calculate** a 95% confidence interval for expected profit if ordering $T = 6$ tuna for a sample mean $\bar{p} = \$400$ and standard deviation $s_p = \$80$ after $n = 100$ samples.

$$\mu_p \in \bar{p} \pm z_{0.025} \frac{s_p}{\sqrt{N}} = 400 \pm 1.96 \cdot \frac{80}{\sqrt{100}} \approx [384, 416]$$

(e) **Calculate** how many samples would be required to narrow the 95% confidence interval for expected profit to within $1.00 based on the information in (d) above.

$$E = z_{0.025} \frac{s_p}{\sqrt{N}} \implies N = \left( \frac{z_{0.025} s_p}{E} \right)^2 = \left( \frac{1.96 \cdot 80}{1} \right)^2 \approx 160^2$$

## 1.5  Discrete Time Simulation

Wolfram's Rule 90 defines the following transition function for a 1D cellular automaton which shows the possible state transitions for state $q_i$ (the center cell) as a function of its neighbors $q_{i-1}$ and $q_{i+1}$.



*rule 90*
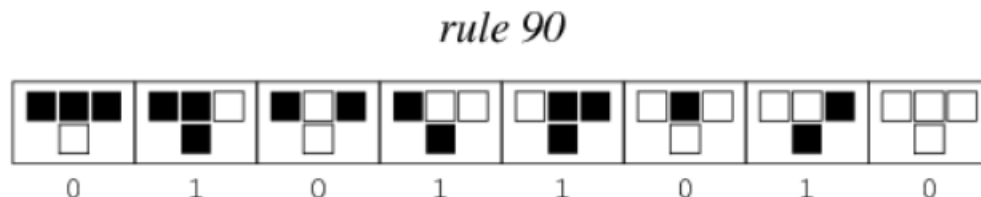
For example, the left-most case states that if $q_{i-1}$ (left), $q_i$ (center), and $q_{i+1}$ (right) are all 1, then $q_i$ (the center cell) should transition to 0 at the next time step.

(a) **Write** the state transition function by specifying the conditions in terms of the neighboring states $q_{i-1}$ and $q_{i+1}$.

$$\delta(q_i) = \begin{cases} 1 & \text{if } q_{i-1} \neq q_{i+1} \\ 0 & \text{if } q_{i-1} = q_{i+1} \end{cases}$$

(b) **Complete** a manual simulation using the transition rule above to propagate the initial state of the cells below (black = 1, white = 0) forward by 5 steps.

| | $q_0$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ | $q_6$ | $q_7$ | $q_8$ | $q_9$ | $q_{10}$ | $q_{11}$ | $q_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t = 0$: | | | | | | | ■ | | | | | | |
| $t = 1$: | | | | | | ■ | | ■ | | | | | |
| $t = 2$: | | | | | ■ | | | | ■ | | | | |
| $t = 3$: | | | | ■ | | ■ | | ■ | | ■ | | | |
| $t = 4$: | | | ■ | | | | | | | | ■ | | |
| $t = 5$: | | ■ | | ■ | | | | | | ■ | | ■ | |

# 1.6 Continuous Time Simulation [15 points]

A bath tub has a faucet which has been adjusted to initially fill at 2 liters of water per minute and gradually decrease to turn off within $t = 2$ minutes. The tub also has a drain which removes 2 liters of water per minute. The volume of water in the tub $V(t)$ can be expressed with the following differential equation:

$$\frac{dV}{dt} = \begin{cases} 2 - 2t & \text{if } 0 \leq t \leq 2 \\ -2 & \text{if } t > 2 \end{cases}$$

(a) **Write** the state transition function in terms of $V(t)$, $\Delta t$, and $t$ using the Euler integration method.

$$\delta(V, \frac{dV}{dt}, \Delta t) = \begin{cases} V(t) + (2 - 2t)\Delta t & \text{if } 0 \leq t \leq 2 \\ V(t) - 2\Delta t & \text{if } t > 2 \end{cases}$$

(b) **Complete** a manual simulation to propagate the initial state forward by 5 steps with $V(0) = 10$ liters and $\Delta t = 0.5$ minutes.

| $t$ | $V(t)$ | $dV/dt$ | $\delta(V, \frac{dV}{dt}, \Delta t) = V(t + \Delta t)$ |
|-----|--------|---------|------------------------------------------------------|
| 0 | 10 | $2 - 2 \cdot 0 = 2$ | $10 + 2 \cdot 0.5 = 11.0$ |
| 0.5 | 11.0 | $2 - 2 \cdot 0.5 = 1$ | $11.0 + 1 \cdot 0.5 = 11.5$ |
| 1.0 | 11.5 | $2 - 2 \cdot 1.0 = 0$ | $11.5 + 0 \cdot 0.5 = 11.5$ |
| 1.5 | 11.5 | $2 - 2 \cdot 1.5 = -1$ | $11.5 - 1 \cdot 0.5 = 11.0$ |
| 2.0 | 11.0 | $2 - 2 \cdot 2.0 = -2$ | $11.0 - 2 \cdot 0.5 = 10.0$ |

(c) **Sketch** the simulated volume of water over time using the space below.
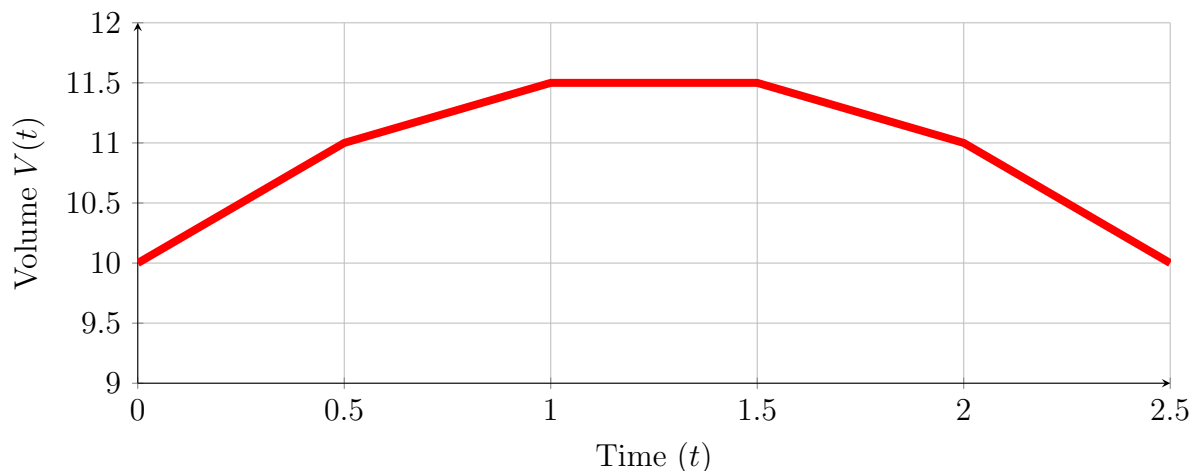
## 1.7 Discrete Event Simulation

You want to study the average waiting time at the NJ Transit 126 bus stop at Washington and 7th Street. In this system, passengers arrive and gather at the bus stop until a bus arrives and takes them all away (assume there is always capacity on the bus for all passengers). (*Hint:* this problem is similar to a queuing model.)

(a) **Describe** the system state variable $N$ which describes the state of the bus stop.

$N$ : The number of passengers waiting at the bus stop.

(b) **Describe** the two events $t_P$, $t_B$ which must be scheduled in this simulation.

$t_P$ : The time of the next passenger arrival.

$t_B$ : The time of the next bus arrival.

(c) **Describe** statistical counters required to find the average waiting time $W/N_P$.

$W$ : The total passenger waiting time accumulated.

$N_P$ : The total number of passengers who waited at the bus stop.

(d) **Complete** a manual simulation for the first 7 events using the following values for the passenger inter-arrival time $(X)$ and bus inter-arrival time $(Y)$.

$$X \in \{\cancel{1.6}, 1.4, 2.5, 2.4, 2.8, 3.3, 4.8\} \qquad Y \in \{\cancel{16.5}, 9.1\}$$

| $t$ | $N$ | $t_P$ | $t_B$ | $W$ | $N_P$ |
|------|-----|----------------------|----------------------|-------------------------------|-------|
| 0    | 0   | 1.6                  | 16.5                 | 0                             | 0     |
| 1.6  | 1   | $1.6 + 1.4 = 3.0$    | 16.5                 | 0                             | 1     |
| 3.0  | 2   | $3.0 + 2.5 = 5.5$    | 16.5                 | $0 + 1 \cdot 1.4 = 1.4$       | 2     |
| 5.5  | 3   | $5.5 + 2.4 = 7.9$    | 16.5                 | $1.4 + 2 \cdot 2.5 = 6.4$     | 3     |
| 7.9  | 4   | $7.9 + 2.8 = 10.7$   | 16.5                 | $6.4 + 3 \cdot 2.4 = 13.6$    | 4     |
| 10.7 | 5   | $10.7 + 3.3 = 14.0$  | 16.5                 | $13.6 + 4 \cdot 2.8 = 24.8$   | 5     |
| 14.0 | 6   | $14.0 + 4.8 = 18.8$  | 16.5                 | $24.8 + 5 \cdot 3.3 = 41.3$   | 6     |
| 16.5 | 0   | 18.8                 | $16.5 + 9.1 = 25.6$  | $41.3 + 6 \cdot 2.5 = 56.3$   | 6     |

>>>>>> **Note: Part (e) on next page!** <<<<<<

(e) **Complete** the blocks in the activity diagram / flow chart below to describe the state transition function logic for how the variables $t$, $N$, $t_P$, $t_B$, $W$, and $N_P$ are updated in a simulation. (*Hint*: explain how the table rows in (d) are filled).

$$\downarrow$$

$$— \; t_P \leq t_B \; — \; \diamond \; — \; t_B < t_P \; —$$

| | |
|---|---|
| $\downarrow$ | $\downarrow$ |
| $W = W + (t_P - t) \cdot N$ | $W = W + (t_B - t) \cdot N$ |
| $\downarrow$ | |
| $N = N + 1$ | $N = 0$ |
| $\downarrow$ | |
| $N_P = N_P + 1$ | |
| $\downarrow$ | |
| $t = t_P$ | $t = t_B$ |
| $\downarrow$ | $\downarrow$ |
| $t_P = t + X$ | $t_B = t + Y$ |

$$\downarrow$$