Lecture 2: Elements of Decision Problems – Structuring Decisions

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SYS 660

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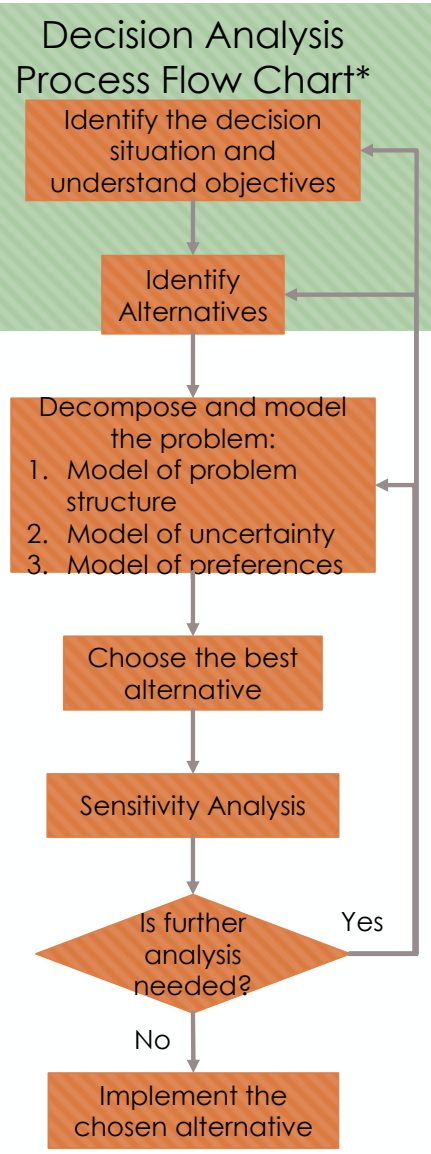
Today's Lecture

- Alaskan Pipelines – Challenger - Exercise
- The Decision Analysis Process
- Net Present Value
- Scales
- Structuring Decisions

The Decision Analysis Process

- Process is nominal.
- Real life is "messy".
- Jumping back and forth and iterating are common.
- Sometimes it is better to start with understanding the problem and other times it is better to start with understanding the objectives.
- The decision analysis process is a useful guide to bring some structure to what is often an ad hoc process.



*Clemen, R. T. and T. Reilly, 2014, *Making Hard Decisions*, 3rd Ed., Mason, OH: South-Western.

Understand the Decision Context

- What is the problem that you are trying to solve?
- This is not as trivial as it may seem. Solving the wrong problem is VERY common.
- Understanding your objectives and values may help you refine your understanding of the problem.
- What if your problem is that your software testing is finding too many bugs?
- One solution is to perform less software testing!
  - This actually happens…
- If your objective is to deliver a reliable software product, then you are solving the wrong problem!

Identify Alternatives

- "Don't jump to solutions!"
- Deliberately trying to generate creative alternatives may substantially improve your outcomes.
- This is especially critical in multi-stakeholders where the preferred solution is a "non-starter" with a particular stakeholder.
- Real life example:
  - An engineering team on an IT project produced an initial design that was far too expensive
  - The engineers had basically selected the "best" type of each component without considering other approaches
  - Consequently, the engineering team was given a design to cost target much lower
  - They found that by considering alternative equipment they could still meet all of the customer's requirements at a substantially lower cost

Model the Decision

- Major components:
  - Identify the relevant factors and relationships between them
  - Develop a model of the uncertainties involved
  - Develop a model of the decision maker(s) preferences
  - Evaluate each of the alternatives using the model
- The art of modeling is knowing what to simplify and how
- The models don't have to be mathematical, but mathematical models are more amenable to analysis
- The majority of this course will be spent on modeling decisions and risks

Sensitivity Analysis

- Sensitivity analysis is avoided at your peril
- What if a slight change in the value of a parameter triggers a completely different decision?
- Sensitivity analysis lets you understand how robust your solution is
- It also helps you identify areas that require more investigation
- For example, if sensitivity analysis reveals that your decision to move forward with a new product launch hinges on the likelihood of your competitor releasing a similar product, it is in your best interest to invest some effort in understanding your competitor's situation

More Analysis?

- The decision analysis process may result in:
  - A refined understanding of problems and objectives
  - The identification of additional alternatives
  - A shift in the decision maker's preferences
  - Areas that require more investigation
- Consequently, the process may take several iterations to reach a final decision
- Ideally, the result will be not only a good decision but also a well-supported one

Net Present Value

Time Value of Money

- Money today is worth more than money tomorrow.
- You could invest money today and receive a return on that investment tomorrow.
- For example, imagine that there is an investment opportunity that will cost you $1 today, but you will get $2 tomorrow, risk-free.
- If I were to offer you the choice of $1 today or $1 tomorrow, which would you prefer?
- In essence, you are concerned about the opportunity cost of deferring cash flows.

Time Value of Money

Accounts for time value of money when considering multiple cash flows over multiple years.

Two questions:

1- If I give you a dollar today, what will it be worth x years in the future? (compounding)

2- Given a dollar held or spent x years in the future, what is it worth in today's dollars (discounting)

Basic Compounding Formula

$FV = PV\ (1+i)^n$

Where:

FV = Future value

PV = Present value

i = Interest rate

n = Number of years

Compound Interest

- For discrete compounding:
- $FV = PV\left(1 + \frac{r}{m}\right)^{k}$
- where r is the annual interest rate, m is the compounding frequency per year, and k = t × m is the number of periods
- Note that because of compounding, the effective annual interest is greater than the nominal annual rate
- For example a 5% nominal annual interest rate compounded quarterly yields an effective annual interest rate of 5.1%
- The future value of the investment grows geometrically

Continuously Compounded Interest

- If we take the limit of discrete compounding
- $\lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^m = e^r$
- Thus, the formula for continuous compounding is
- $FV = PVe^{rt}$
- The value of an investment grows *exponentially* under continuously compounding interest

Basic Discounting Formula

$PV = FV[1/ (1+i)^n]$

Where:

FV = Future value

PV = Present value

i = Interest rate

n = Number of years

Present Value

- How much would a cash flow received in the future be worth today?
- All we have to do to find out is to solve the future value formula for present value
- $PV = FV\left(1 + \frac{r}{m}\right)^{-k}$
- $PV = FVe^{-rt}$
- The interest rate we used for this calculation is referred to as the discount rate
- For example, if we expect to receive $100 five years from now and the discount rate is 5% (effective annual), the present value is
- $\$100(1 + 0.05)^{-5} = \$78.35$

Think about this …

$1 today ⟶ $1.10 in 1 year

$1 today ⟵ $1.10 in 1 year

Assuming 10% interest

$0.91 today ⟷ $1 in 1 year

Discounting Assuming 10% Interest Rate

Year (n)	Compound factor $(1+i)^n$	Discount factor $1/(1+i)^n$
0	1.00	1.00
1	1.1	0.90
2	1.21	0.83
3	1.33	0.75
4	1.46	0.68
5	1.61	0.62
6	1.77	0.56

What is the purpose of discounting?

Discounting is used to adjust future, or actual dollars expended or earned/saved over multiple years to constant or base-year dollars (typically today's dollar). This helps a decision maker with fair comparison of all revenues and costs for all alternatives being considered.

Application

Imagine winning the State lottery and being faced with this choice:

- Take a lump sum payout (today) of $50 million
- Take a twenty-year annuity of $5 million per year for 20 years
- Assume 10% discount rate

Application

- $1,000 investment (up front)
- Net cash inflow of $500 per year over the following 5 years
- Cost of capital is 12% (also known as discount rate, required rate of return)

Net Present Value

- If we have a series of cash flows, both positive and negative, we can use the discount rate to calculate the Net Present Value (NPV)
- Let x_k be the cash flow in period k

$$NPV = \sum_{k=0}^{n} \frac{x_k}{\left(1 + \frac{r}{m}\right)^k}$$

$$NPV = \sum_{k=0}^{n} x(t_k) e^{-rt_k}$$

Net Present Value

- Net Present Value (NPV) is critical for investment decisions
- Generally speaking, given a set of possible investments, you should choose the investment that has the highest NPV
- NPV is a standard criterion for business decision making
- The catch, of course, is setting the right discount rate
- Determining the appropriate discount rate is non-trivial and it is a key aspect of financial theory
- We will discuss discount rates more when we discuss financial risk

Measurement Scales

Measurement Scales

- Nominal
  - Items can be grouped but there is no relationship between the groups
  - For example, grouping items by color: red, green, blue, yellow, etc.
- Ordinal
  - Items can grouped by order, but we cannot say how much greater one group is than another
  - For example, alphabetical order: A, B, C, D or star ratings of restaurants (2-star, 3-star, etc.)

Measurement Scales

- Interval
  - Items can be measured quantitatively, but only the differences between measurements are meaningful
  - For example, altitude above sea level
- Ratio
  - Both differences and ratios are meaningful. the scale has a true zero point
  - For example, height and weight

Mathematical Rules

- The nominal scale is considered qualitative while the remainder are considered quantitative
- You cannot perform arithmetical operations on ordinal scales
  - The difference between ordinal numbers is NOT meaningful
  - Performing arithmetical operations on ordinal numbers is a very common mistake in decision and risk analysis
- You can only perform addition and subtraction on interval scales
  - Ratios of interval scaled numbers are NOT meaningful
- You can perform all arithmetic operations on ratio scaled numbers
  - Since ratios are meaningful, you can perform addition, subtraction, multiplication, and division

Interval Scale Example

- Imagine that you are standing 10 meters below sea level
- Imagine that your friend is standing 10 meters above sea level
- This means that your friend is 20 meters higher than you
- However, your friend is not -1 times higher than you
- Since altitude above sea level does not have a true zero point, the intervals are meaningful but the ratios are not
- This is an example of an interval scale

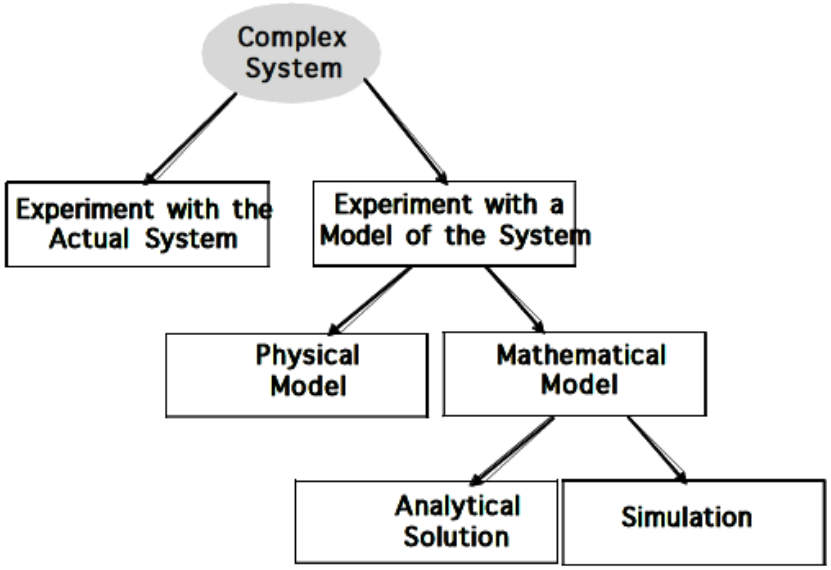
Ratio scale example

- Imagine that you measure the temperatures of two different containers of liquid
- The temperature of the first container is 100K and the temperature of the second container is 200K
  - Both are very cold!
- You can say that the temperature of the second container is 100K greater than the first
- You can also say that the second container is twice as hot as the first
- Temperature in Kelvin has a true zero point so ratios are meaningful
- This is an example of a ratio scale

Modeling a Decision

Model

A model is a physical, mathematical or logical representation of a system, entity, phenomenon, or process. There is no movement in a model. Think of a plastic replica of an airplane or a car, or a mathematical equation that predicts the probability of an event occurring.



Source: Farr -2007 – Chapter 1

Modeling Decisions

1. Value
2. Objective

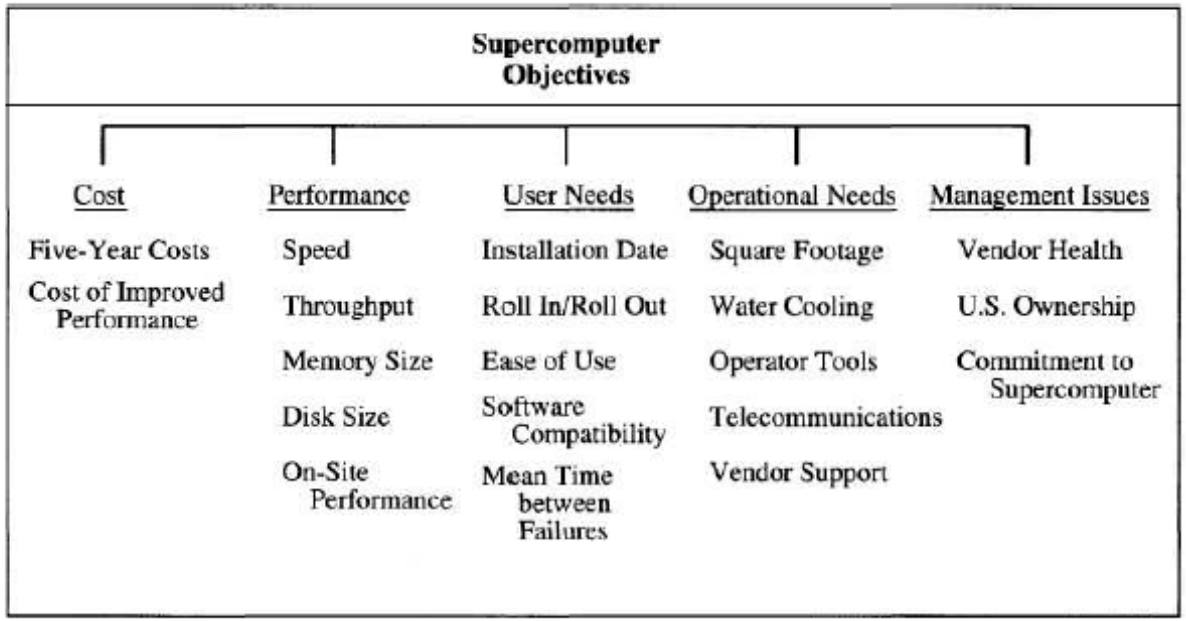
An objective is a specific thing that you want to achieve.

An individual's objectives taken together make up his or her values.

A person's values are the reason for making decisions in the first place!

Boeing Example - Objectives

Figure 2.1
Objectives for Boeing's supercomputer.



Structuring Decisions

- List your objectives
- Prioritize into fundamentals vs. means

Figure 3.1
A fundamental-objectives hierarchy. *Source:* Keeney (1992, p. 70).

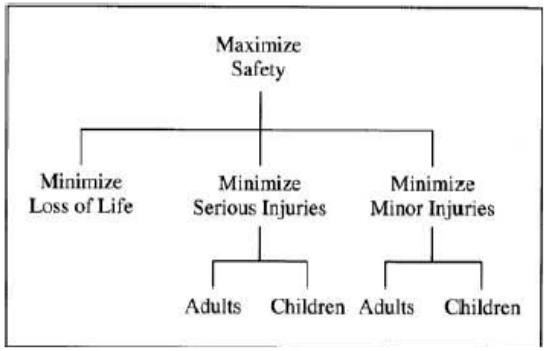
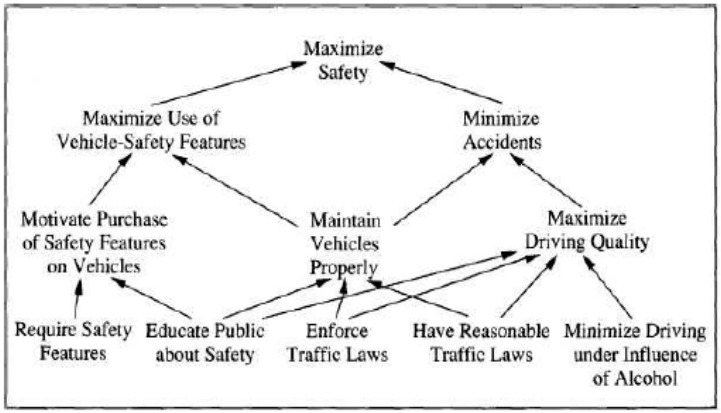
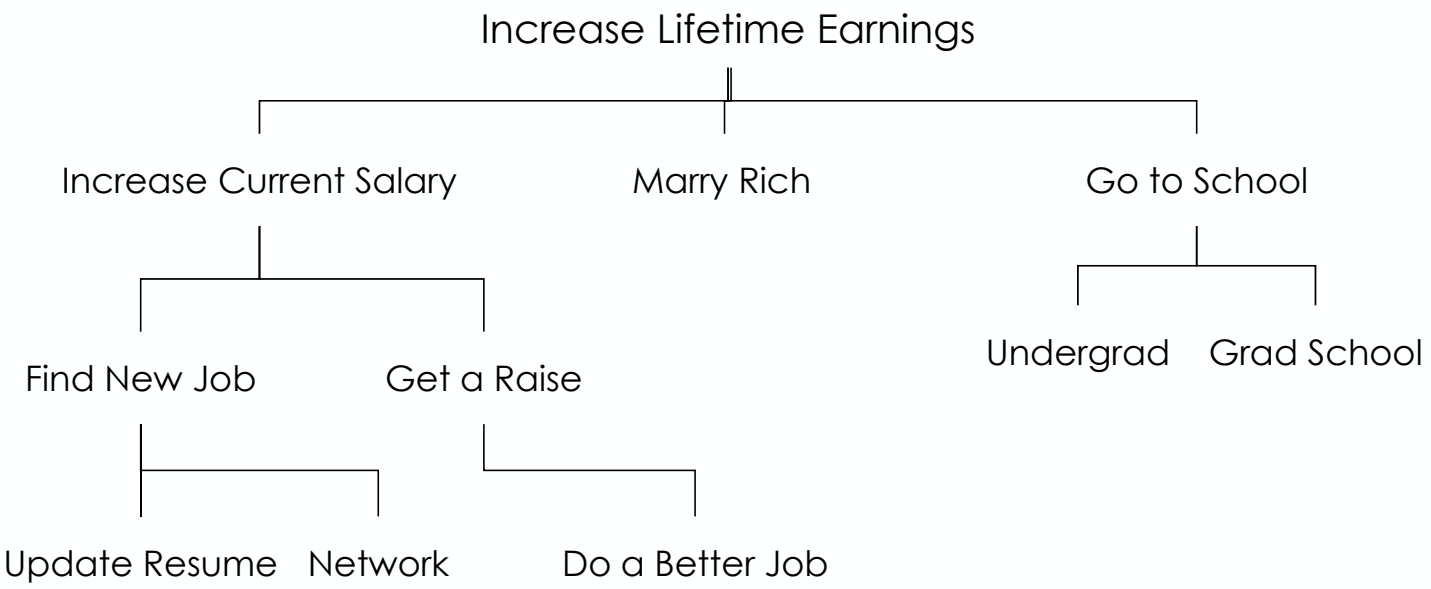


Figure 3.2
A means-objectives network. *Source:* Keeney, (1992, p. 70).



Fundamental Objectives Hierarchy



Structuring a Decision

- All about the objectives (what you want to achieve)

- Decision: choice between options
  There is always an option, including status quo
  Waiting for more information also an option

- Uncertainty: always exists
  Outcomes: possible results of uncertain events
  Many uncertain events lead to complexity

Structuring a Decision

- Elements of smart choices
- Separating means and fundamental objectives
- Getting the decision context right
- Generating alternatives
- Influence diagrams and decision trees

Elements of Smart Choices

- *Problem:* Work on the right decision problem
- *Objectives:* Specify your objectives
- *Alternatives:* Create imaginative alternatives
- *Consequences:* Understand the consequences
- *Tradeoffs:* Grapple with your tradeoffs
- *Uncertainty:* Clarify your uncertainties
- *Risk Tolerance:* Think hard about your tolerance
- *Linked Decisions:* Consider linked decisions

*Hammond, J.S., Keeney, R.L., & Raiffa, H. (1999). Smart Choices: A Practical Guide to Making Better Decisions. Boston, MA: Harvard Business School Press.

Identifying Objectives

- Write down all the concerns you hope to address through your decision
- Convert your concerns into succinct objectives
- Separate ends from means to establish your fundamental objectives
- Clarify what you mean by each objective
- Test your objectives to see if they capture your interests

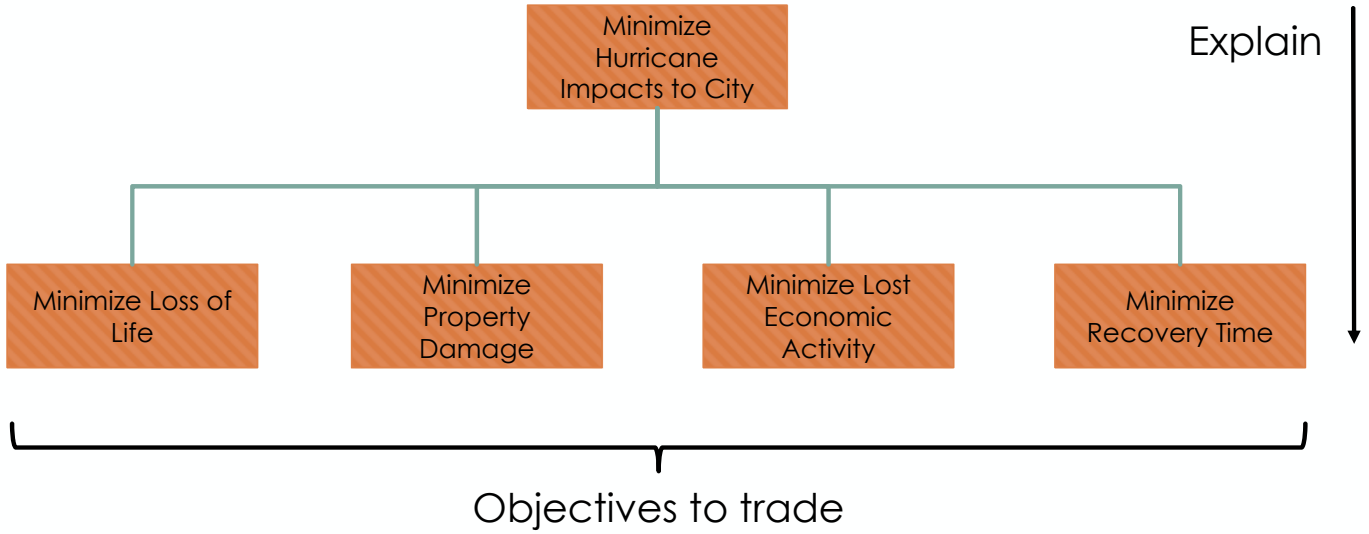
Separating Ends and Means

- What is wrong with using a "means" objective in place of an "ends" objective?
  - Risk of solving the wrong problem
  - Risk of artificially restricting the set of alternatives
- However, exploring means and their relationship to ends can generate a tremendous amount of insight
- In some circumstances, a means objective may be substituted for an ends of objective because it may be more amenable to measurement
  - Should be a conscious decision that is based on an thorough understanding of the risks

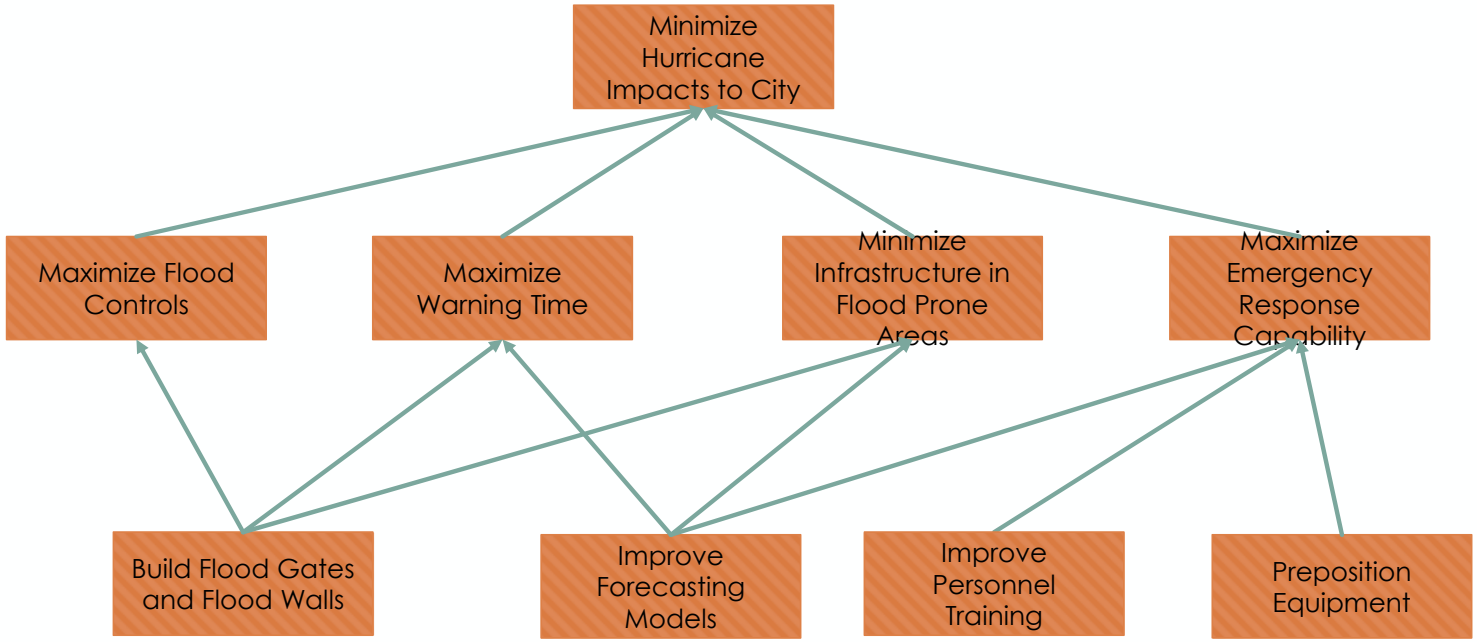
Example: Improving Decision Making

- *Ends Objective:* Improve your decision making at work
- *Means Objective:* Get an 'A' in SYS 660
- Solving the wrong problem:
  - You might be able to get an 'A' in the class by cheating, but then you would not understand decision analysis well enough to your improve decision making at work
- Restricting the set of alternatives
  - Taking SYS 660 may not be the only way to improve decision making at work
  - Other alternatives: find a mentor at work who is an effective decision maker, read books on decision analysis on your own, hire a consultant, etc.
- Taking SYS 660 may still turn out to be the best alternative, but you don't want to artificially restrict the set of potential alternatives
  - For example: What if you took a decision analysis course in the past, and you just need to brush up? Perhaps reading some books would be a good alternative.

Fundamental Objectives Hierarchy



Means Network



Identifying Means and Fundamental Objectives

	Fundamental Objectives	**Means Objectives**
To Move: Ask:	Downward in the Hierarchy: What do you mean by that?	Away from the Fundamental Objectives: How could you achieve this?
To Move: Ask:	Upward in the Hierarchy: Of what more general objective is this an aspect?	Toward Fundamental Objectives: Why is that important? (WITI)

What is the Decision Context?

- The decision context involves the scope and circumstances that the surround the decision
- Changing the context can have an impact on the fundamental objectives hierarchy and the means network
- Broadening the scope may:
  - Convert some fundamental objectives to means objectives
  - Open up additional alternatives
- Consider the hurricane example: What would happen if we broadened the scope to the entire nation and/or all natural disasters?
- While broadening the scope may open up additional alternatives, do we have the authority and resources to make decisions at that level?

Do you have the Right Context?

- Does the context really capture the situation at hand?
  - Are you addressing the right problem?
  - Example: Improving hurricane warning time might not do much to reduce property damage or lost economic activity
- Do you have the authority to make the decision?
  - If you don't have the authority, you need to narrow the context
  - Example: If you are the mayor of New York City, you would not have the authority to improve federal disaster response
- Do you have the time and resources to make the decision?
  - A decision with a broad scope may require more time and resources than you have
  - Example: If you expect a hurricane in three days, improving flood control infrastructure is not a viable option

*Clemen, R. T. and T. Reilly, 2014, *Making Hard Decisions*, 3rd Ed., Mason, OH: South-Western.

Generating Alternatives

- Use your objectives – ask "How?"
- Challenge constraints
- Set high aspirations
- Do your own thinking first
- Learn from experience
- Ask others for suggestions
- Give your subconscious time to operate
- Create alternatives first, evaluate them later
- Never stop looking for alternatives

Choices & Consequences

- The real world is uncertain:
  - Good choices can have bad consequences
  - Poor choices can have good consequences
- Fundamental attribution errors:
  - If I make a decision that results in a negative outcome, it is because I had bad luck
  - If you make a decision that results in a negative outcome, it is because you are incompetent
  - If I make a decision that results in a positive outcome, it is because I am talented
  - If you make a decision that results in a positive outcome, it is because you are lucky

Building Consequence Tables

- Mentally put yourself into the future
- Create a free-form description of the consequences of each alternative
- Eliminate any clearly inferior alternatives
- Organize descriptions of any remaining alternatives into a consequences table

Example: Consequence Table

- Buying a new car:

	Model A	Model B	Model C	Model D	Model E
Purchase Price	$15k	$20k	$35k	$25k	$30k
Annual Operating Cost	$1.5k	$1.7k	$2.5k	$2.2k	$3k
Seating	5	5	8	6	4
Trunk Capacity	12	14	20	16	10
Appearance	Low	Med	Low	Med	High

Overarching Issues

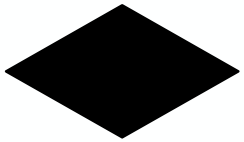
- Uncertainty – What might happen?
- Attributes – What matters?
- Stakeholders – Who matters?
- Support – How to inform?

Elements of an Influence Diagram

Represents a decision that needs to be made

Represents a mathematical calculation, a constant, or an intermediate consequence

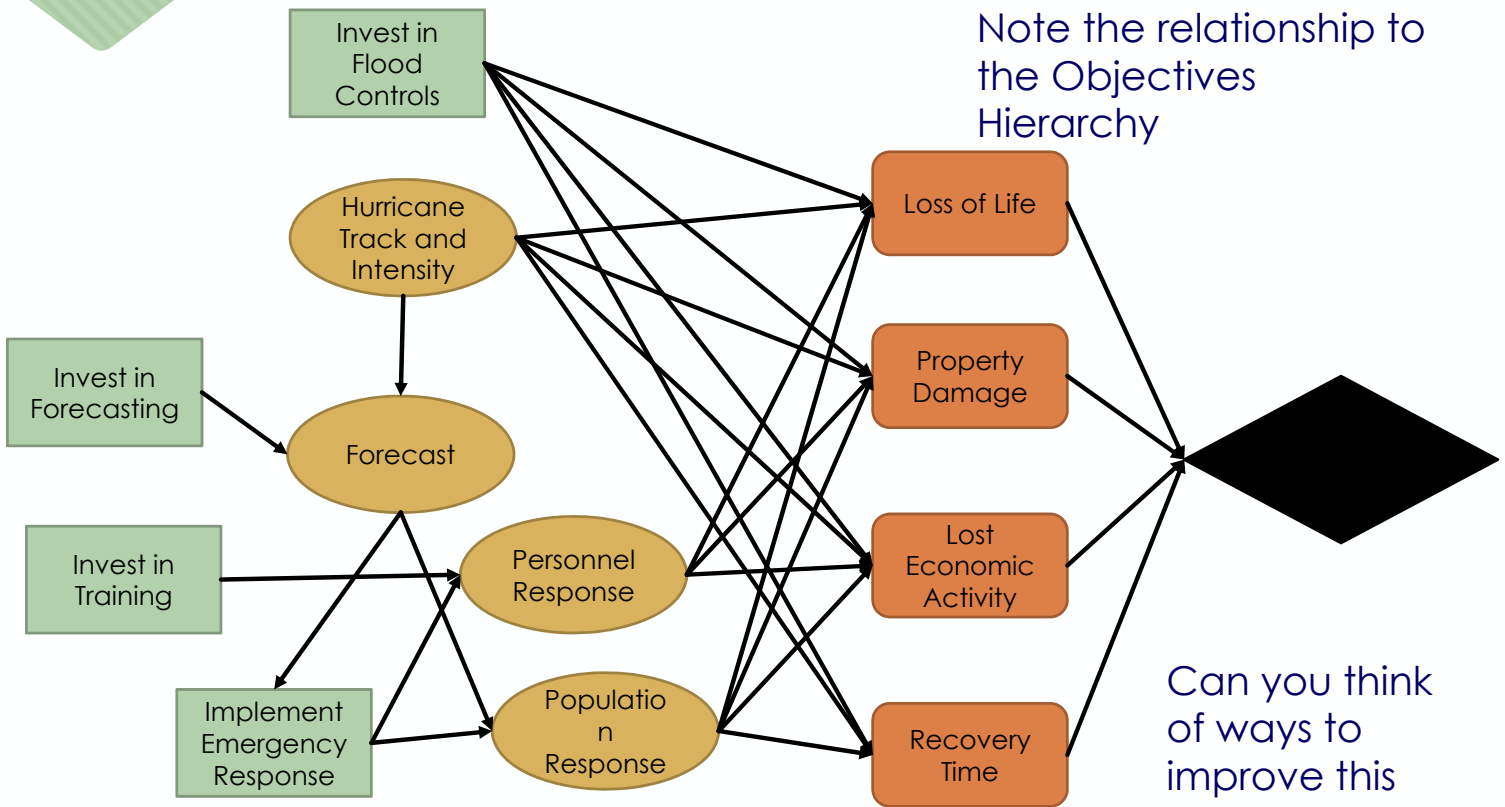
Represents an uncertain event



Represents a final outcome or a payoff function

Represents either the influence of one node on another or the sequence of one node before the other

Example Influence Diagram



Common Mistakes

- Treating an influence diagram like a flow chart
  - Everything does not need to be sequenced
  - View the influence diagram as snapshot of what is known at one instance of time
- Including many chance nodes pointing to the primary decision node
  - Will the decision maker really know the results of all of these uncertain events before the decision is made?
- Creating cycles
  - While feedback may exist in the dynamic environment, it doesn't fit in the snapshot view

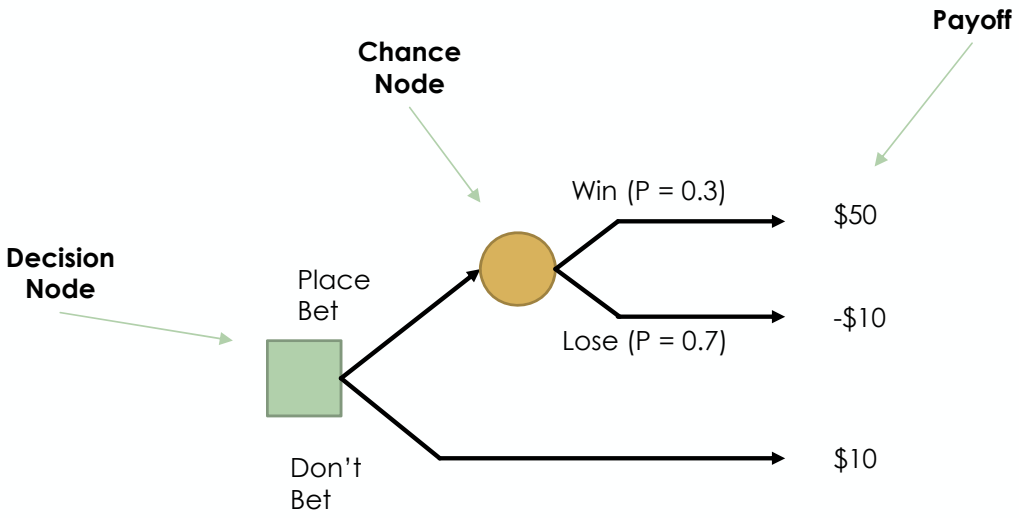
When to stop

- As with any modeling effort, several iterations are often necessary
  - The very act of trying to construct the model will improve your understanding of the decision
- Apply Occam's razor
  - Try to get the simplest model that still captures all of the essential elements of the decision
  - All things being equal, simpler models have fewer ways to go wrong
- Sensitivity analysis can help
  - If a model element has little impact on the decision, it can probably be dropped

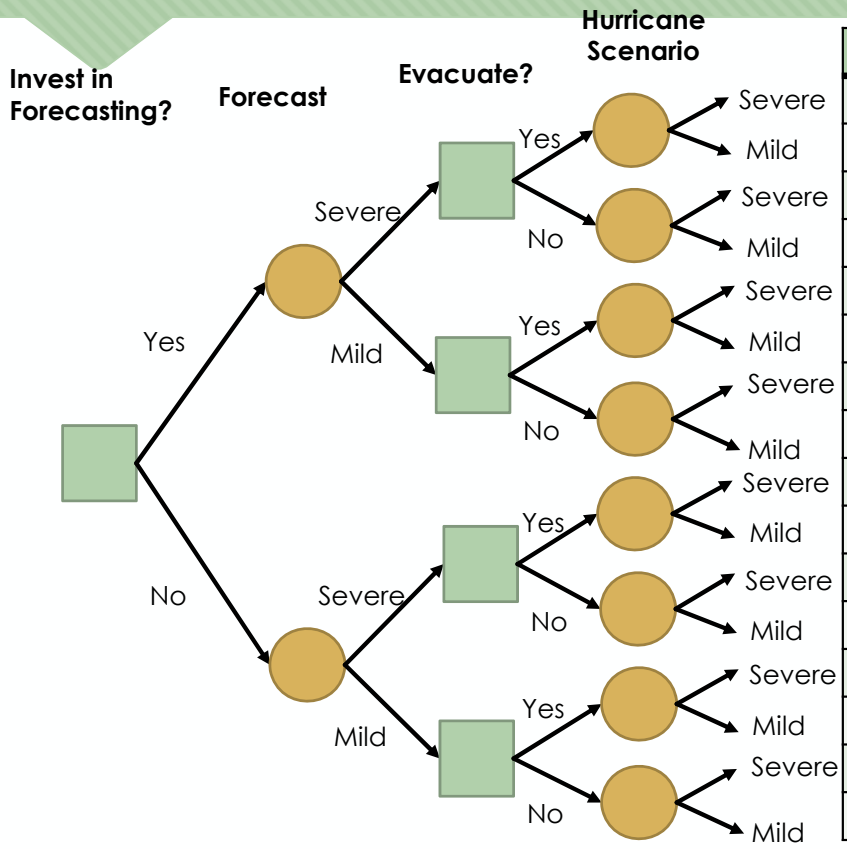
Characteristics of a Decision Tree

- All decisions and events are sequenced
- All possible decision options are exhaustively included
- All possible outcomes for chance events are exhaustively included
- All possible combinations of outcomes against the objectives hierarchy are represented by the branches at the end of the tree
- A decision tree is a form of dynamic program and can be solved recursively to find the "optimal" policy

Elements of a Decision Tree



Example Decision Tree



Loss of Life	Damage	Activity Loss
Low	Med	High
Low	Low	Med
High	High	High
Low	Low	Low
Low	Med	High
Low	Low	Med
High	High	High
Low	Low	Low
Low	Med	High
Low	Low	Med
High	High	High
Low	Low	Low
Low	Med	High
Low	Low	Med
High	High	High
Low	Low	Low

Comparison

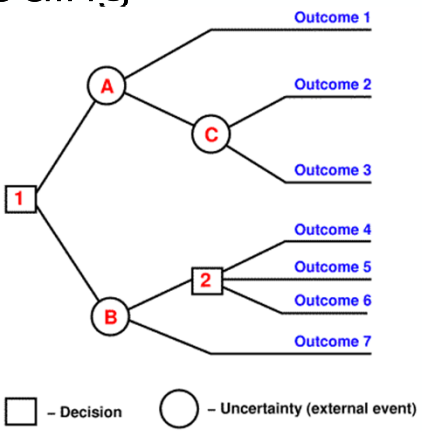
- Properly constructed influence diagrams and decision trees are isomorphic
  - It is possible to convert one into the other
- Decision trees contain more detail but have a tendency to "explode" graphically
- Each has advantages and disadvantages
  - Influence diagrams may be better for communicating with others
  - Decision trees may be better for detailed analysis like sensitivity analysis
- One possible approach is to start with an influence diagram to explore the decision context and then convert it to a decision tree for detailed analysis

Solving decision Trees

- Decision trees are a form of dynamic programming
- They can be solved by "rolling back" the tree
- This means starting at end of the tree and working back to the start
- In other words, solve the last decision first for each possible situation
- This determines the expected outcome for each of those situations
- Repeat the process for the next to last decision and so on until you reach the first decision
- The result will be an optimal policy for each possible path through the tree

Decision Trees

A decision tree is a decision support tool that uses a tree-like graph or model of decisions and their possible consequences, including chance event outcomes, resource costs, and utility.



Source: What is a decision tree algorithm? – Dictionary - Wikipedia

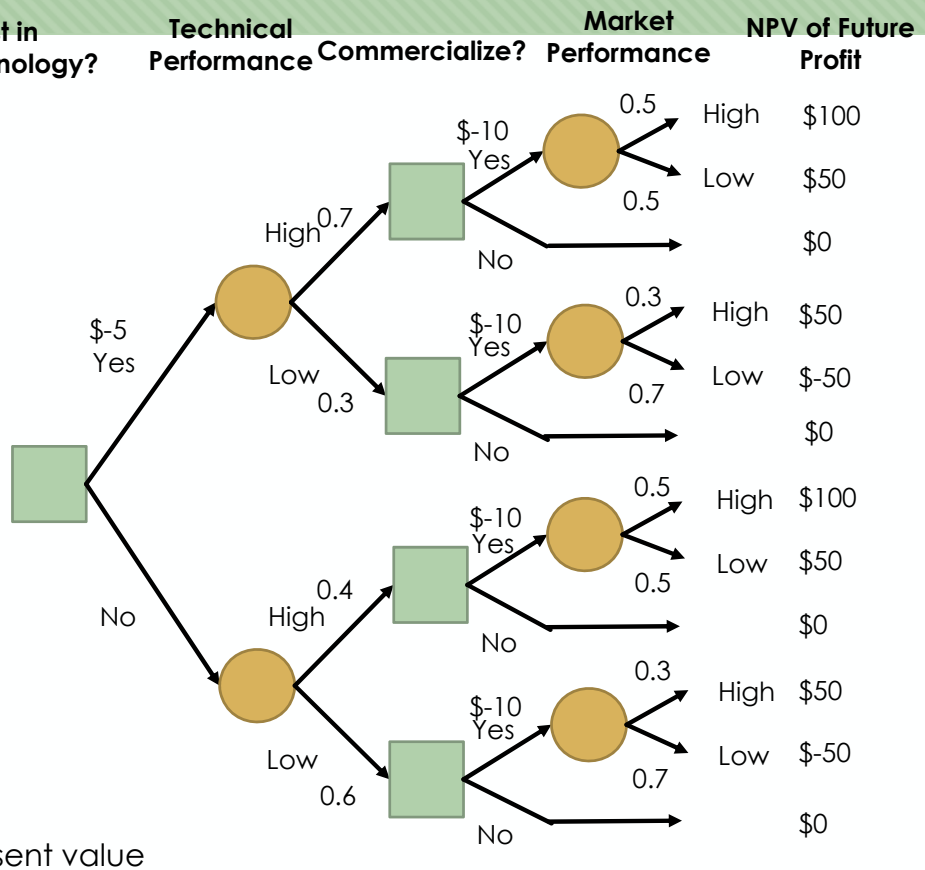
Expected Value

Expected value is calculated as the sum of all possible values each multiplied by the probability of its occurrence.

- Outcome: $x_1, x_2, x_3, \ldots.$
- Probability: $p_1, p_2, p_3, \ldots.$
- Expected Value: $x_1 p_{1+} x_2 p_{2+} x_3 p_{3+} \ldots.$

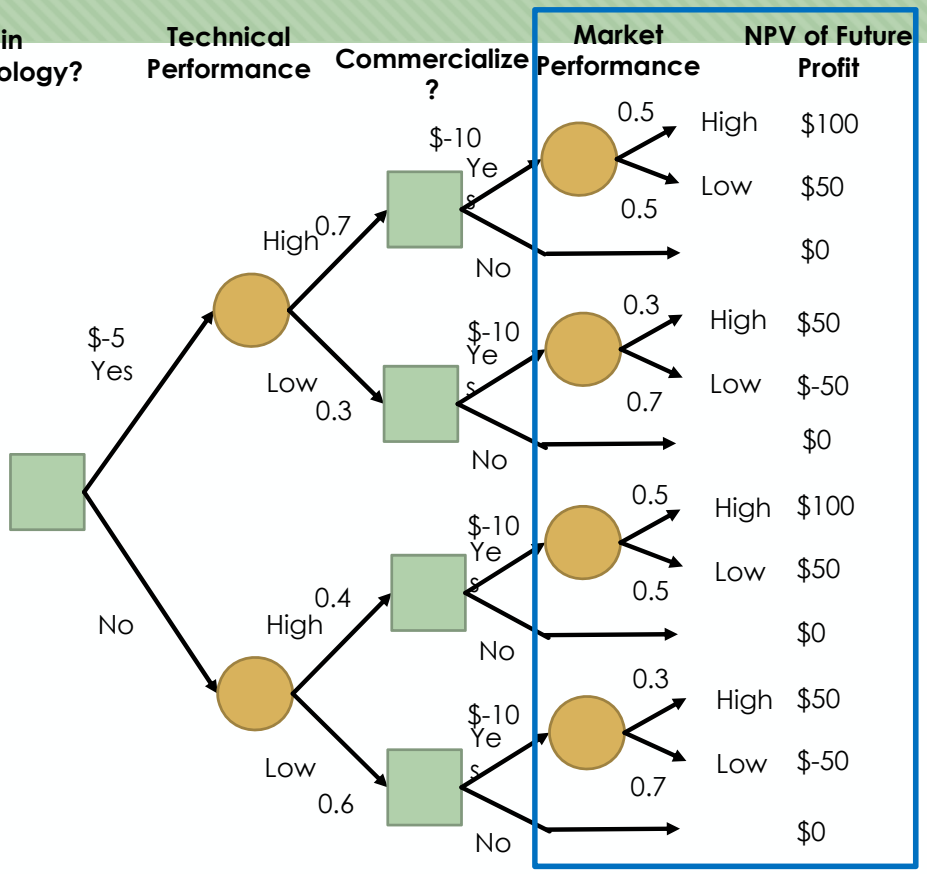
Decision trees and expected values can serve as effective tools for your decision analysis.

Example: Invest in Technology



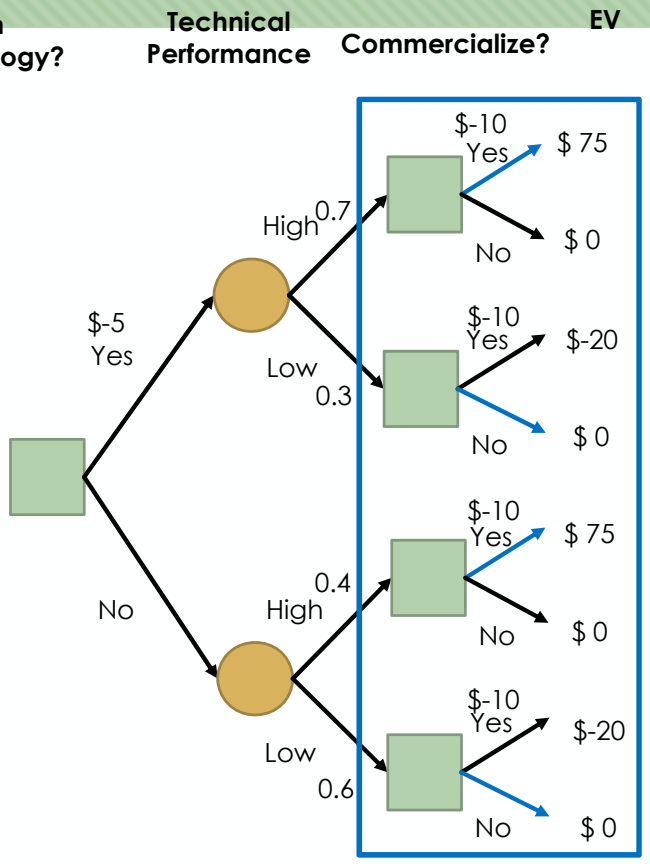
Assumption: All $ values are in present value

Example: Invest in Technology



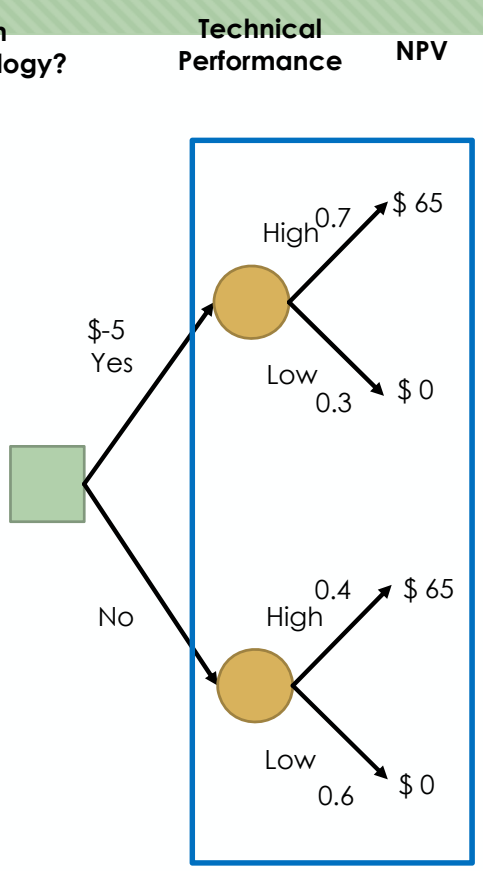
Calculate the expected values of the outcomes

Example: Invest in Technology



Select the best option at each decision node

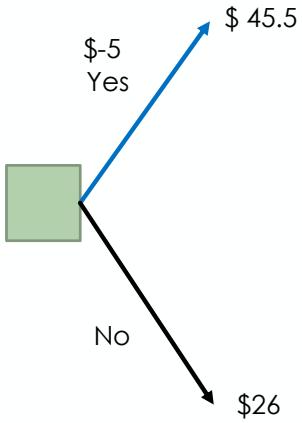
Example: Invest in Technology



Calculate the expected values of the outcomes

Example: Invest in Technology

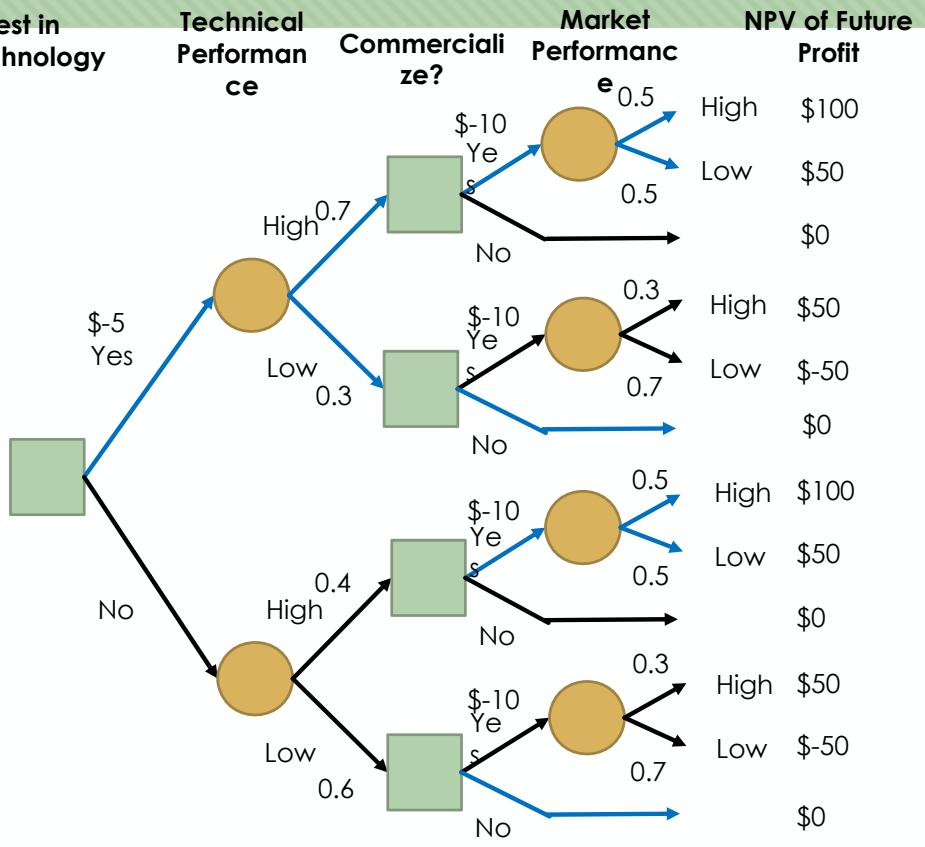
Invest in Technology? **EV**



Select the best option

You should invest in the technology for an expected NPV of $40.5

Example: Optimal Policy



Objective Measurement Scales

- Not every objective can be measured in dollars
- Ideally, you would define one numerical measurement scale for each objective
  - These are called attributes
  - Think back to the last lecture, what type of measurement scale would we prefer?
- What if there is no natural measurement scale for a particular objective?
- Option1: Choose a surrogate measure
  - E.g., maximize student test scores in place of maximize student learning
- Option 2: Construct an ordinal attribute scale

Dealing with Multiple Objectives

- What happens when a decision has multiple objectives and each is assessed on a different measurement scales?
- We call these objectives non-commensurate
- Making tradeoffs among non-commensurate objectives is the core mission of decision analysis
- We will discuss how to do this for deterministic outcomes in the next lecture and for stochastic outcomes in the lectures on Multi-Attribute Utility Theory (MAUT)
- We can even deal with an ordinal attribute

Before Lecture 3:

- Read Chapters 3 and 4.
- Homework 1 is due next week.