Lista 2 - CEQ

Pauline Andreia Ribeiro Aguirre

1. $\mu_0 = 34{,}994 \qquad \sigma_0 = 0{,}11$

$[34{,}745 \,;\, 35{,}183]$

$$PFE = P(Z < Z_{LIE}) + P(Z > Z_{LSE})$$

$$Z_{LIE} = \frac{34{,}745 - 34{,}994}{0{,}11} = -2{,}26$$

$$Z_{LSE} = \frac{35{,}183 - 34{,}994}{0{,}11} = 1{,}72$$

$$PFE = P(Z < -2{,}26) + P(Z > 1{,}72)$$

$$0{,}012 + 0{,}043$$

$$0{,}055 \longrightarrow \boxed{5{,}5\%}$$

$$C_{pk} = \min\left\{ \frac{LSE - \mu}{3\sigma} \,;\, \frac{\mu - LIE}{3\sigma} \right\} = \boxed{0{,}57}$$

$$\frac{35{,}183 - 34{,}994}{3 \cdot 0{,}11} = 0{,}57$$

$$\frac{34{,}994 - 34{,}745}{3 \cdot 0{,}11} = 0{,}75$$

$C_{pk} < 1$ o processo é incapaz

2. $LE = 135 \pm 4$ $\mu_0 = 133{,}70$ $\sigma = 2{,}02$

$$C_p = \frac{LSE - LIE}{6\sigma} \qquad C_{pk} = \min\left\{\frac{LSE-\mu}{3\sigma}, \frac{\mu - LIE}{3\sigma}\right\}$$

$$PFE = P\left(Z < \frac{131 - 133{,}70}{2{,}02}\right) + P\left(Z > \frac{139 - 133{,}70}{2{,}02}\right)$$

$$P(Z < -1{,}34) + P(Z > 2{,}62)$$

$$0{,}09 + 0{,}004$$

$$0{,}094 \longrightarrow \boxed{9{,}4\%}$$

$$C_p = \frac{139 - 131}{6 \cdot 2{,}02} = \boxed{0{,}66}$$

$$C_{pk} = \min\left\{\frac{139 - 133{,}70}{3 \cdot 2{,}02} ; \frac{133{,}70 - 131}{3 \cdot 2{,}02}\right\}$$

$$\simeq \min\{0{,}875 ; 0{,}446\} = \boxed{0{,}446}$$

3. $\mu_0 = 20$ $\sigma = 2{,}5$

$$PFE = P\left(Z < \frac{10{,}50 - 20}{2{,}5}\right) + P\left(\frac{25{,}5 - 20}{2{,}5}\right)$$

$$P(Z < -3{,}8) + P(Z > 2{,}2)$$

$$0 + 0{,}014 = \boxed{1{,}4\%}$$

$$C_p = \frac{LSE - LIE}{6\sigma} = \frac{15}{6 \cdot 2{,}5} = \boxed{1}$$

$$C_{pm} = \frac{LSE - LIE}{6\sqrt{\sigma^2 + (d-\mu)^2}} = \frac{15}{6\sqrt{2{,}5^2 + (18-20)^2}} = \frac{15}{19{,}21} = \boxed{0{,}781}$$

$$\hookrightarrow d = \frac{LSE + LIE}{2} = 18$$

$$C_{pk} = \min\left\{\underbrace{\frac{LSE - \mu}{3\sigma}}, \underbrace{\frac{\mu - LIE}{3\sigma}}\right\} = \boxed{0{,}733}$$

$$0{,}733 = \frac{25{,}5 - 20}{3 \cdot 2{,}5} \qquad \frac{20 - 10{,}5}{3 \cdot 2{,}5} = 1{,}27$$

O Cp é o menos confiável, pois é insensível a mudanças na média do processo.

b. $PFE = 0{,}01$

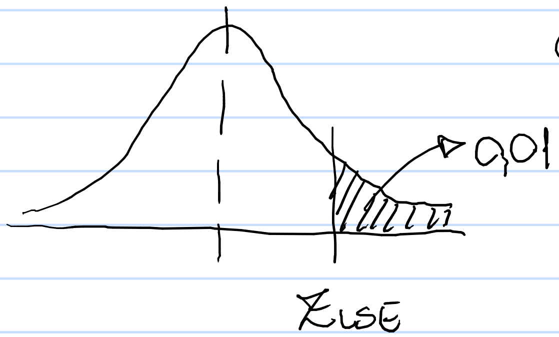
$$PFE = \underbrace{P\left(z < \frac{10{,}5 - 20}{\sigma}\right)} + P\left(z > \frac{25{,}5 - 20}{\sigma}\right)$$

quando $\sigma = 2{,}5 \longrightarrow P(z < z_{LIE}) \approx 0$

se $\sigma \downarrow$ então $z_{LIE} \uparrow$ portantanto $P(z < z_{LIE}) \approx 0$

Assim procuramos um σ que atenda:

$$P\left(z > \frac{25{,}5 - 20}{\sigma}\right) = 0{,}01$$



Olhando a tabela Z, concluímos:

$$Z_{LSE} = 2,33$$

$$\frac{25,5 - 20}{\sigma} = 2,33$$

$$5,5 = 2,33\,\sigma$$

redução de 2,50 para 2,36 $\longleftarrow$ $\boxed{\sigma = 2,36}$

4. $\mu_0 = \overline{\overline{X}} = \frac{\Sigma \overline{X}}{m} = 1000,285$

$$\sigma_0 = \frac{\overline{R}}{d_2} = 2,15$$

a. $Z_{LIE} = \frac{994 - 1000}{2} = -3$

$$Z_{LSE} = \frac{1006 - 1000}{2} = 3$$

$$PFE = P(z < -3) + P(z > 3) = 0,0027 \rightarrow \boxed{0,27\%}$$

$$C_p = \frac{LSE - LIE}{6.\sigma} = \frac{12}{6.2} = \boxed{1,00}$$

$$C_{pk} = \min\left\{\underbrace{\frac{LSE - \mu}{3\sigma}}_{1 = \frac{1006 - 1000}{3.2}}, \underbrace{\frac{\mu - LIE}{3\sigma}}_{\frac{1000 - 994}{3.2} = 1}\right\} = \boxed{1,00}$$

b. $\mu_1 = 1002 \qquad \sigma_0 = 2{,}0$

$$Z_{LIE} = \frac{994 - 1002}{2} = -4$$

$$Z_{LSE} = \frac{1006 - 1002}{2} = 2$$

$$PFE = P(z < -4) + P(z > 2) = 0{,}02275 \rightarrow \boxed{2{,}28\%}$$

$$C_p = \frac{LSE - LIE}{6\sigma} = \frac{12}{6 \cdot 2} = \boxed{1{,}00}$$

$$C_{pK} = \min\left\{ \underbrace{\frac{LSE - \mu}{3\sigma}}_{0{,}\bar{6} = \frac{1006 - 1002}{3 \cdot 2}}, \underbrace{\frac{\mu - LIE}{3\sigma}}_{\frac{1002 - 994}{3 \cdot 2} = 1{,}\bar{3}} \right\} = \boxed{0{,}67}$$

c. $\mu_1 = 1002 \qquad \sigma_1 = 4$

$$Z_{LIE} = \frac{994 - 1002}{4} = -2$$

$$Z_{LSE} = \frac{1006 - 1002}{4} = 1$$

$$PFE = P(z < - \quad) + P(z > \quad) = 0{,}1815 \rightarrow \boxed{18{,}15\%}$$

$$C_p = \frac{LSE - LIE}{6\sigma} = \frac{12}{6 \cdot 4} = \boxed{0{,}5}$$

$$C_{pk} = \min\left\{\underbrace{\frac{LSE-\mu}{3\sigma}}, \underbrace{\frac{\mu-LIE}{3\sigma}}\right\} = \boxed{0{,}33}$$

$$\frac{1}{3} = \frac{1006-1002}{3.4} \qquad \frac{1002-994}{3.4} = \frac{2}{3}$$

5. $LIE = 12{,}00$ $\qquad$ $LSE = 24{,}00$

$C_{pk} = 0{,}80$ $\qquad$ $C_{pm} = 0{,}857$

$$C_{pk} = \min\left\{\frac{LSE-\mu}{3\sigma}, \frac{\mu-LIE}{3\sigma}\right\} = 0{,}8$$

$$\frac{24-\mu}{3\sigma} = 0{,}8 \quad \text{ou} \quad \frac{\mu-12}{3\sigma} = 0{,}8$$

$$\sigma = (24-\mu)/3.0{,}8 \qquad \sigma = (\mu-12)/3.0{,}8$$

$$C_{pm} = \frac{LSE-LIE}{6\sqrt{\sigma^2+(d-\mu)^2}} \qquad d = \frac{LSE+LIE}{2}$$

$$d = 18$$

$$= \frac{\cancel{12}\;2}{\cancel{6}\sqrt{\sigma^2+(18-\mu)^2}} = 0{,}857$$

$$= \frac{2}{0{,}857} = \sqrt{\sigma^2+(18-\mu)^2}$$

$$= 5{,}45 = \sigma^2 + (18-\mu)^2$$

$$5{,}45 = \sigma^2 + (18-\mu)^2 \qquad 2{,}4^2 = 5{,}76$$

$1^{\underline{o}}$: se $\sigma = (24-\mu)/2{,}4$

$$5{,}45 = \left(\frac{24-\mu}{2{,}4}\right)^2 + 324 - 36\mu + \mu^2$$

$$31{,}4 = (24-\mu)^2 + 1866{,}24 - 207{,}36\mu + 5{,}76\mu^2$$

$$31{,}4 = 576 - 48\mu + \mu^2 + 1866{,}24 - 207{,}36\mu + 5{,}76\mu^2$$

$$6{,}76\mu^2 - 255{,}36\mu + 2410{,}84 = 0$$

$$\frac{255{,}36 \pm \sqrt{\Delta}}{2 \cdot 6{,}76} \rightarrow \frac{255{,}36 \pm 4{,}43}{13{,}52} \rightarrow \boxed{\mu_1 = 18{,}56} \quad \boxed{\mu_2 = 19{,}21}$$

$$\Delta = (-255{,}36)^2 - 4 \cdot 2410{,}84 \cdot 6{,}76 = 19{,}616$$

$$\sigma_0 = \frac{\mu - 12}{2{,}4} = \frac{19{,}2 - 12}{2{,}4} = \boxed{3}$$

$$Z_{LIE} = \frac{12 - 19{,}2}{3} = -2{,}4 \qquad Z_{LSE} = \frac{24 - 19{,}2}{3} = 1{,}6$$

$$PFE = P(Z < -2{,}4) + P(Z > 1{,}6)$$

$$0{,}0082 + 0{,}0548$$

$$0{,}063 \longrightarrow \boxed{6{,}3\%}$$

6. $LIE = 88 \qquad LSE = 112$

$C_p = 1,00 \qquad C_{pm} = 0,80$

$$C_p = \frac{LSE - LIE}{6\sigma} = 1$$

$$LSE - LIE = 6\sigma$$

$$112 - 88 = 6\sigma \longrightarrow 24 = 6\sigma$$

$$\boxed{\sigma = 4}$$

$$C_{pm} = \frac{LSE - LIE}{6\sqrt{\sigma^2 + (d-\mu)^2}} \qquad d = \frac{LSE + LIE}{2}$$

$$d = 100$$

$$= \frac{\overbrace{(112 - 88)}^{\cancel{24}\,4}}{\cancel{6}\sqrt{4^2 + (100-\mu)^2}} = 0,8$$

$$\frac{4}{0,8} = \sqrt{16 + 100^2 - 200\mu + \mu^2}$$

$$25 = 10016 - 200\mu + \mu^2$$

$$\mu^2 - 200\mu + 9991 = 0$$

$$\frac{200 \pm \sqrt{\Delta}}{2} \rightarrow \frac{200 \pm 6}{2} \qquad \boxed{\begin{array}{l}\mu_1 = 97 \\ \mu_2 = 103\end{array}}$$

$$\Delta = 200^2 - 4 \cdot 9991 = 36$$

$$Z_{LIE} = \frac{88-97}{4} = -2{,}25$$

$$Z_{LSE} = \frac{112-97}{4} = 3{,}75$$

$$PFE = P(Z < -2{,}25) + P(Z > 3{,}75)$$

$$0{,}0122 \quad + \quad 0$$

$$0{,}0122 \longrightarrow \boxed{1{,}22\,\%}$$

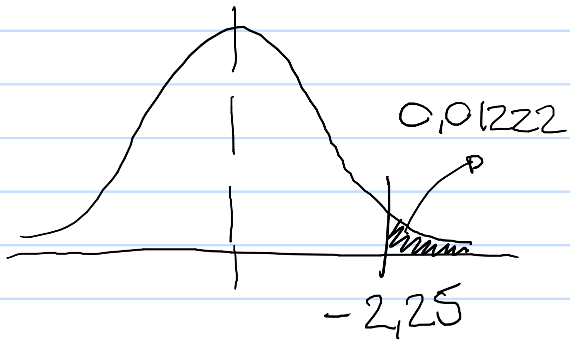
7.

$$\frac{88-\mu}{4} = Z_{LIE}$$

$$\frac{112-\mu}{4} = Z_{LSE}$$

$$PFE = P(Z < Z_{LIE}) + P(Z > Z_{LSE}) = 0{,}01222$$

$$\frac{88-\mu}{4} = -2{,}25$$



$$88-\mu = -9$$

$$\boxed{\mu = 97}$$

$$\frac{\overbrace{112-88}^{\cancel{24}\ 4}}{\cancel{6}\sqrt{\sigma^2 + (100-\mu)^2}} = 0{,}8$$

$$25 = \sigma^2 + (100-97)^2$$

$$25 - 9 = \sigma^2 \longrightarrow \sigma^2 = 16 \rightarrow \boxed{\sigma = 4}$$

8. $LIC = 96{,}16$ $\quad LSC = 107{,}84$

$\sigma = 4{,}00 \quad LIE = 91{,}00 \quad LSE = 115{,}00$

$$LIC = \mu_0 - 3 \cdot \frac{\sigma}{\sqrt{n}} = 96{,}16 \qquad \sigma_{\bar{x}} = \frac{4}{2} = 2$$

$$\mu_0 - 3 \cdot \frac{4}{\sqrt{4}} = 96{,}16$$

$$\mu_0 = 96{,}16 + 6 = 102{,}16$$

$$\alpha = P\left(Z > \frac{107{,}84 - 102{,}16}{2}\right) + P\left(Z < \frac{96{,}16 - 102{,}16}{2}\right)$$

$$= P(Z > 2{,}84) + P(Z < -3)$$

$$= 0{,}0022 + 0{,}0013$$

$$= 0{,}0035 \longrightarrow \boxed{0{,}35\%}$$

$$Z_{LIE} = \frac{91 - 102{,}16}{4} = -2{,}79$$

$$Z_{LSE} = \frac{115 - 102{,}16}{4} = 3{,}21$$

$$PFE = P(Z < -2{,}79) + P(Z > 3{,}21)$$

$$= 0{,}0026 + 0{,}0006$$

$$= 0{,}0036 \longrightarrow \boxed{0{,}36\%}$$

$$Cpm = \frac{115-91}{6\sqrt{16+(103-\mu)^2}}$$

$$d = \frac{115+91}{2}$$

$$d = 103$$

$$= \frac{\cancel{24}\ 4}{\cancel{6}\sqrt{16+(103-102{,}16)^2}}$$

$$= \frac{4}{\sqrt{16{,}7056}} = \boxed{0{,}9787}$$

→ Desajustado

$$\delta = \frac{104-102{,}16}{4} = 0{,}46$$

$$Pd = P(z < -3 + 0{,}46\sqrt{4}) + P(z < -3 - 0{,}46\sqrt{4})$$

$$= P(z < -2{,}08) + P(z < -3{,}92)$$

$$= 0{,}0188 + 0{,}0000 = 0{,}0188$$

$$Pd = 1-\beta \rightarrow 0{,}0188 = 1-\beta \longrightarrow \beta = 0{,}9812$$

$$z_{LIE} = \frac{91-104}{4} = -3{,}25$$

$$z_{LSE} = \frac{115-104}{4} = 2{,}75$$

$$PFE = P(z < -3{,}26) + P(z > 2{,}75)$$

$$= 0{,}0006 + 0{,}00298$$

$$= 0{,}0036 \longrightarrow \boxed{0{,}36\%}$$

$$C_{pm} = \frac{115 - 91}{6\sqrt{16 + (103 - \mu)^2}}$$

$$d = \frac{115 + 91}{2}$$

$$d = 103$$

$$= \frac{\cancel{24}^{\,4}}{\cancel{6}\sqrt{16 + (103 - 104)^2}}$$

$$= \frac{4}{\sqrt{17}} = \boxed{0,97}$$