MESA Summer School 2019

GYRE in MESA

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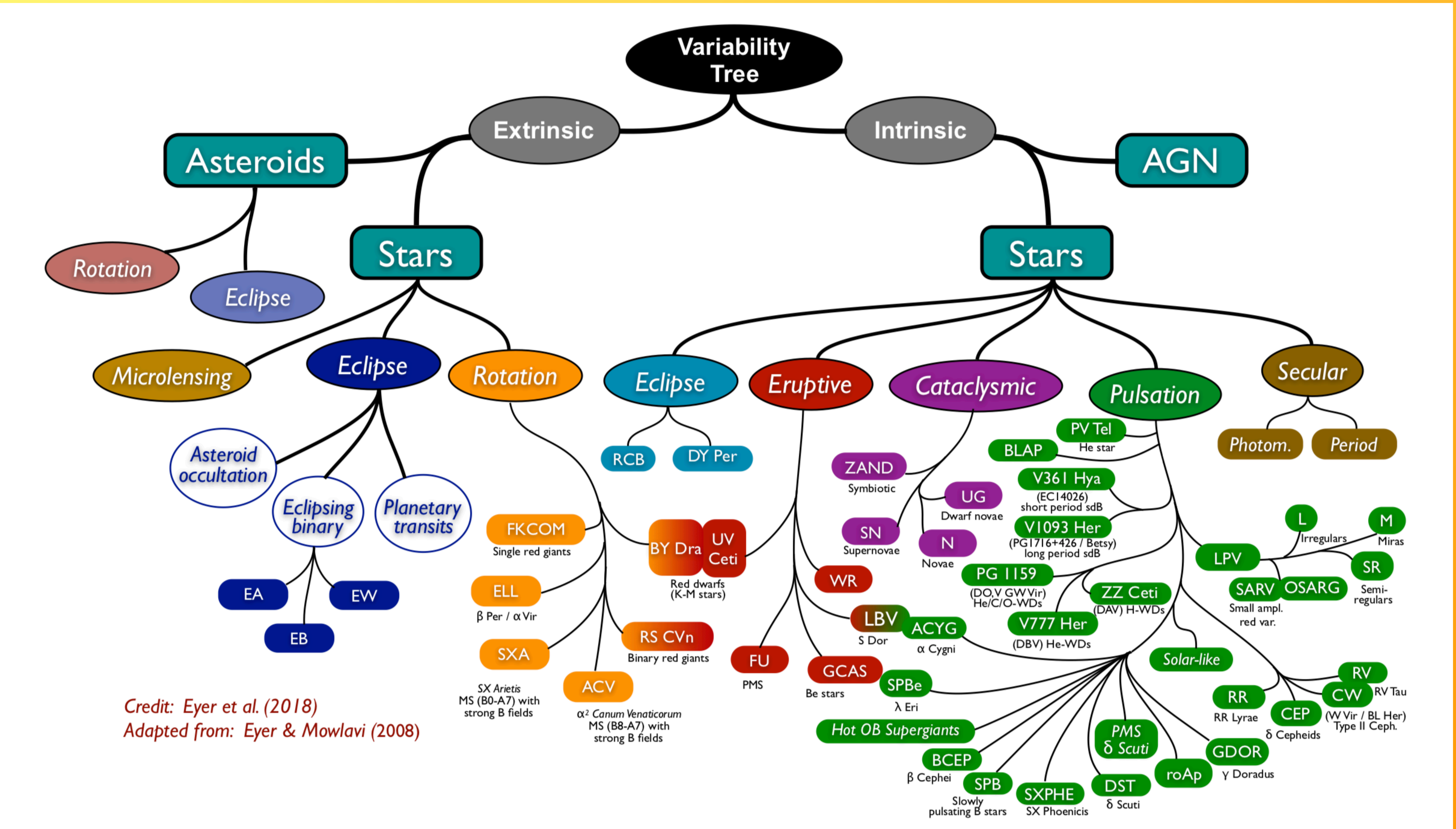
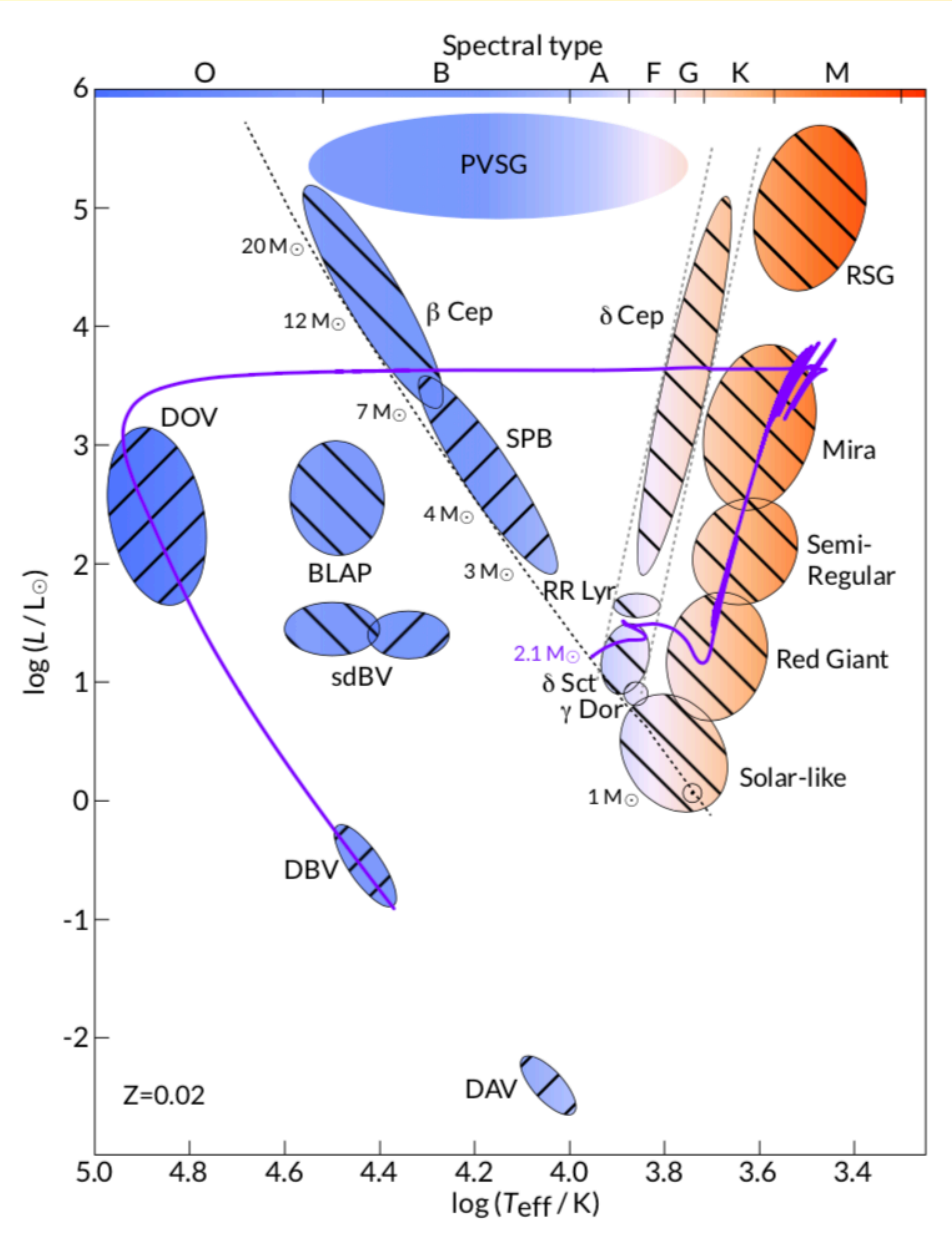
The Variable Sky



Pulsations in the HR Diagram

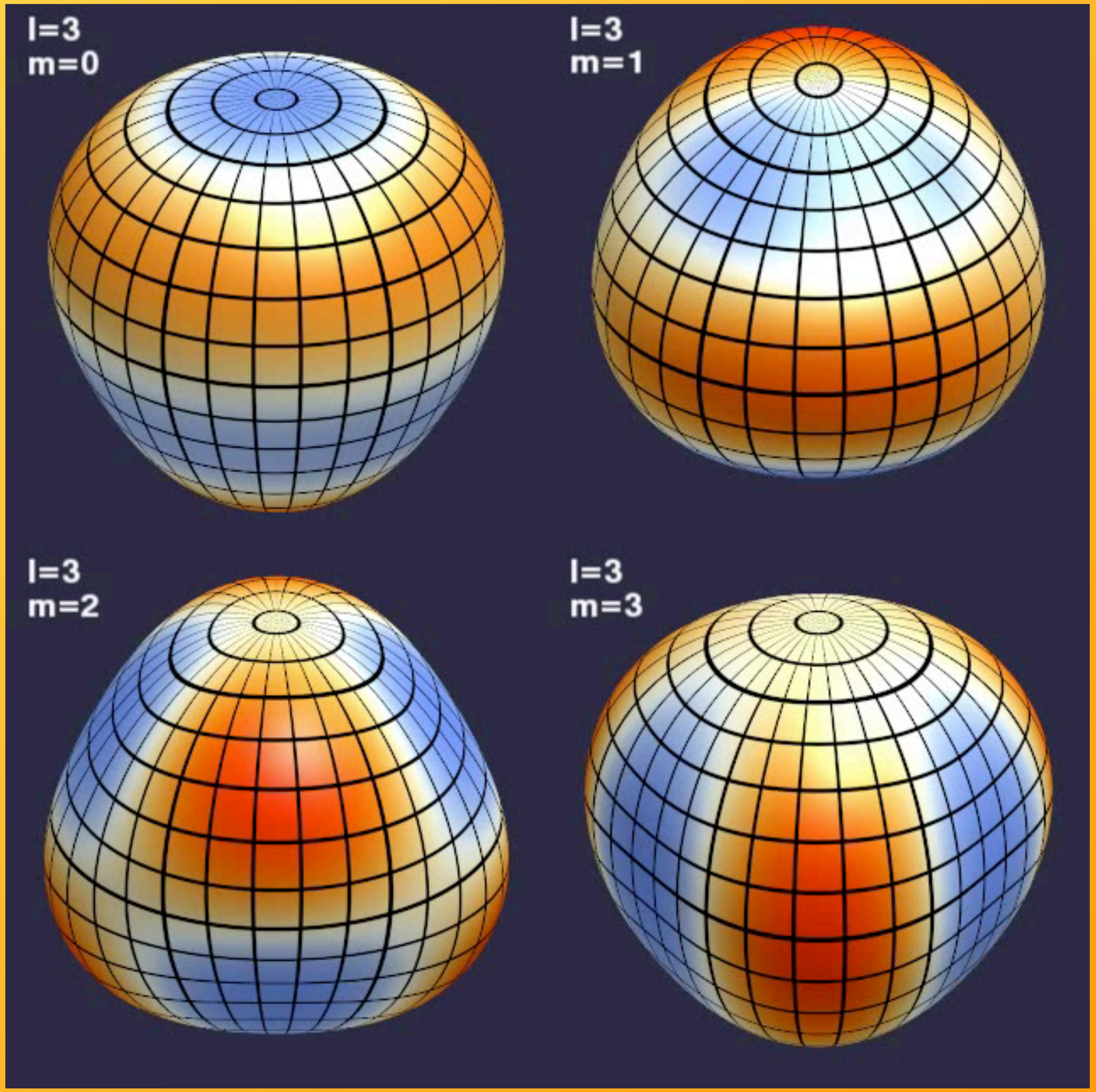


Paxton et al. (2019)

- Pulsation:(quasi-) periodic spectral/photometric variability intrinsic to the star
- Occurs in distinct regions of HR diagram (classes)
- Each class has unique variability characteristics (frequencies, amplitudes, light-curve shape, etc).

What are Stellar Pulsations?

- Excitation of one or more of the star's <u>normal modes of oscillation</u>
- Resulting perturbations to surface temperature, color, shape, velocity etc. produce the observed variability

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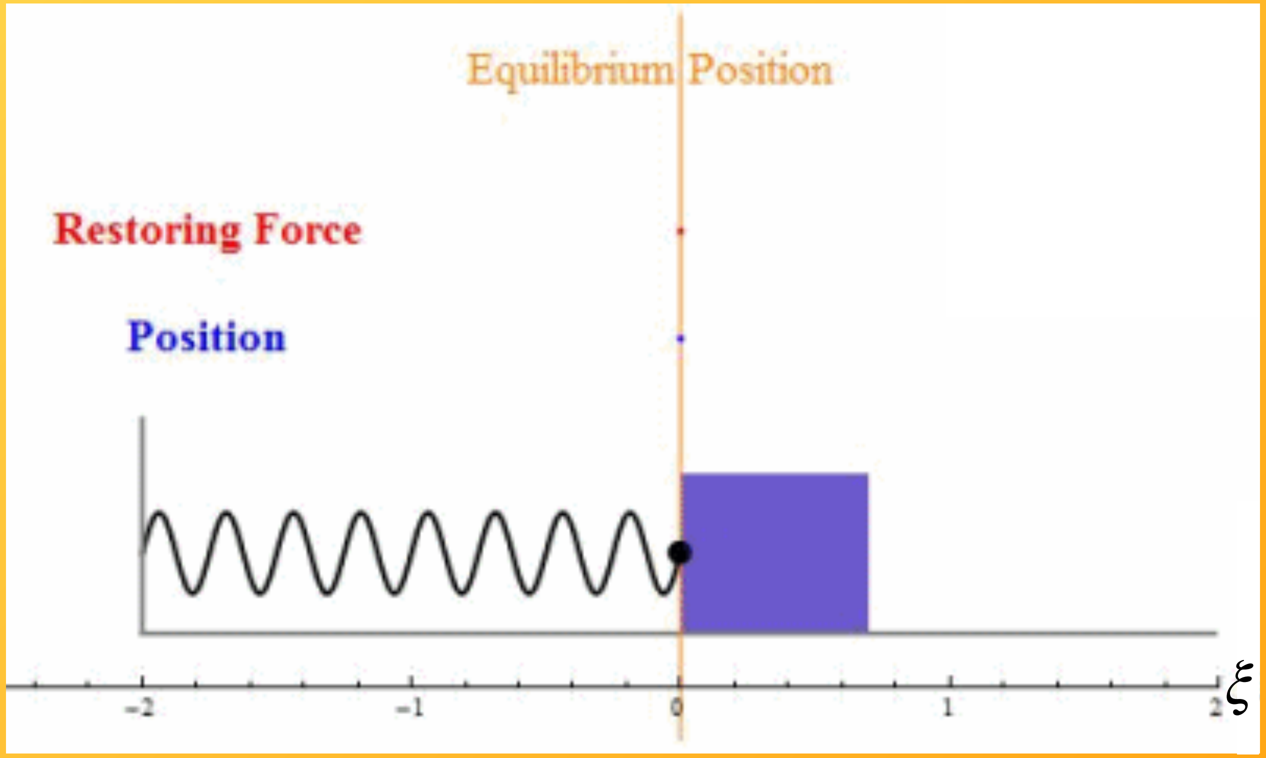
The Prototype Oscillator

- Two ingredients
  - restoring force to bring displaced element back toward equilibrium position
  - inertia to make the element overshoot equilibrium position



mass

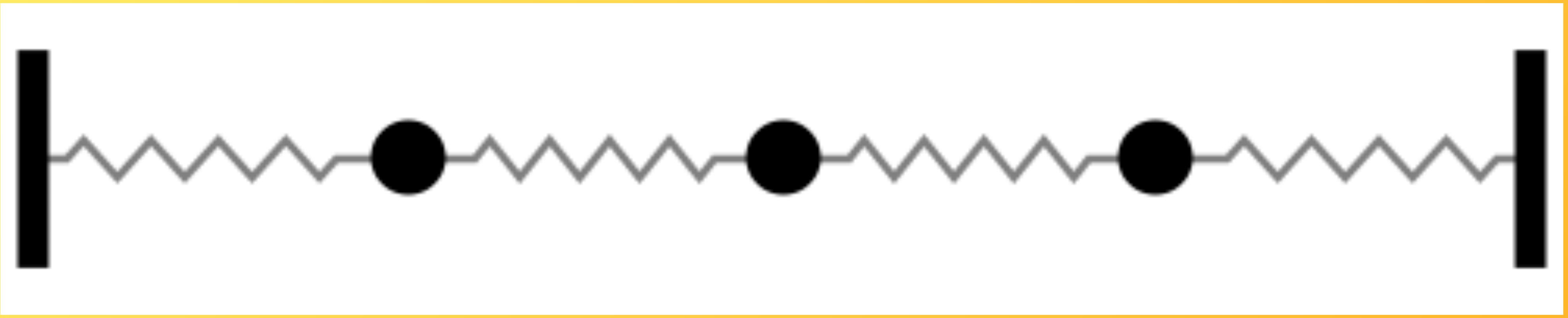
spring constant

$$m\frac{\partial^2\xi}{\partial t^2} = -\,q\,\xi$$

$$\xi(t) = A\mathrm{e}^{-\mathrm{i}\sigma t} \qquad \sigma = \sqrt{\frac{q}{m}}$$

(NB Take real part of ξ to get physical displacement)

Coupled Oscillators



$$m\frac{\partial^2 \xi_1}{\partial t^2} = -q(\xi_2 - 2\xi_1)$$

$$m\frac{\partial^2 \xi_2}{\partial t^2} = -q(\xi_3 - 2\xi_2 + \xi_1)$$

$$m\frac{\partial^2 \xi_2}{\partial t^2} = -q(-2\xi_3 + \xi_2)$$

Many Coupled Oscillators

$$m\frac{\partial^2 \xi_j}{\partial t^2} = -q\Delta x^2\left(\frac{\xi_{j+1} - 2\xi_j + \xi_{j-1}}{\Delta x^2}\right)$$

$$\lim_{\Delta x \to 0}\left(\frac{\xi_{j+1} - 2\xi_j + \xi_{j-1}}{\Delta x^2}\right) = \frac{\partial^2 \xi}{\partial x^2}$$

$$\frac{\partial^2 \xi}{\partial t^2} = -\frac{T}{\mu}\frac{\partial^2 \xi}{\partial x^2}$$

$$\begin{cases} T = \lim_{\Delta x \to 0} q\Delta x \\ \mu = \lim_{\Delta x \to 0} \frac{m}{\Delta x} \end{cases}$$

The Stretched String Wave Equation

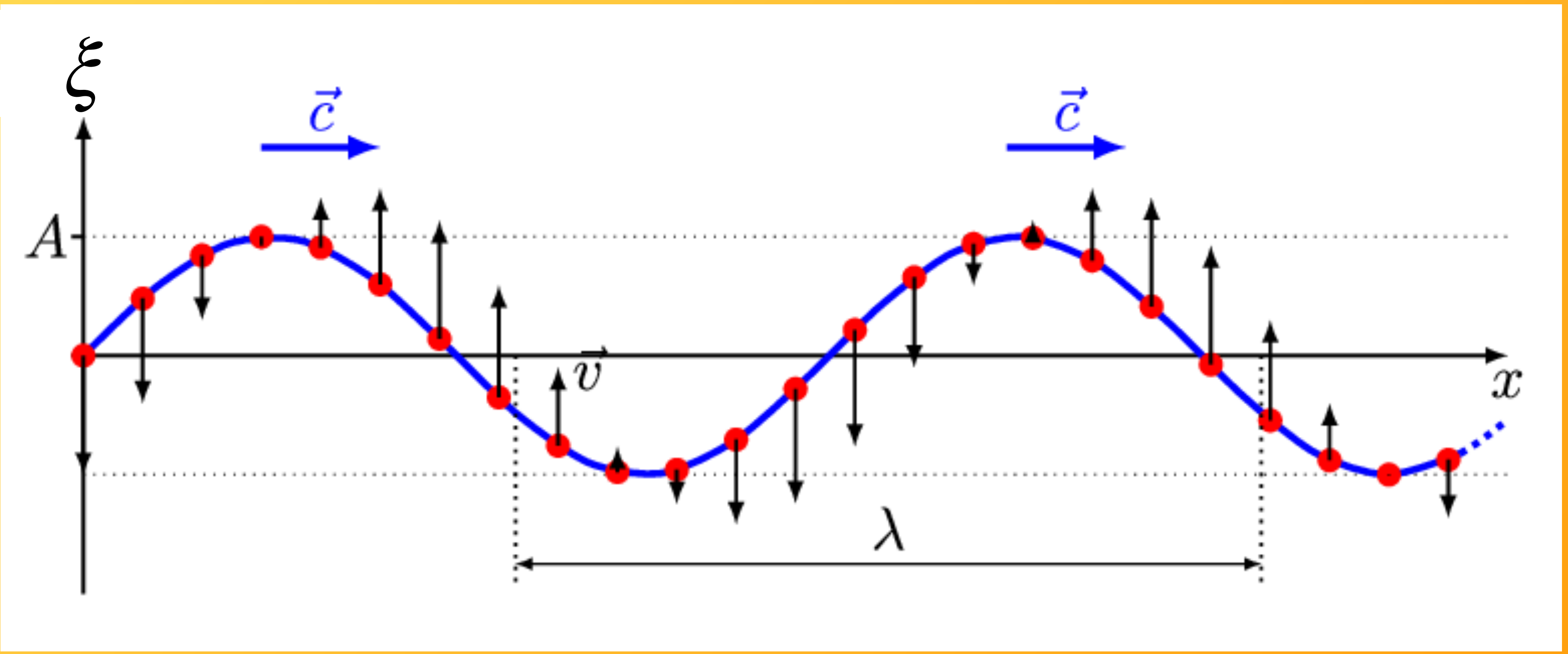
$$\frac{\partial^2 \xi}{\partial t^2} = -\frac{T}{\mu}\frac{\partial^2 \xi}{\partial x^2}$$

T: tension

μ: mass/unit length

$$\xi(x,t) = A\mathrm{e}^{\mathrm{i}(kx-\sigma t)} \qquad \sigma = \sqrt{\frac{\mu}{T}}k$$

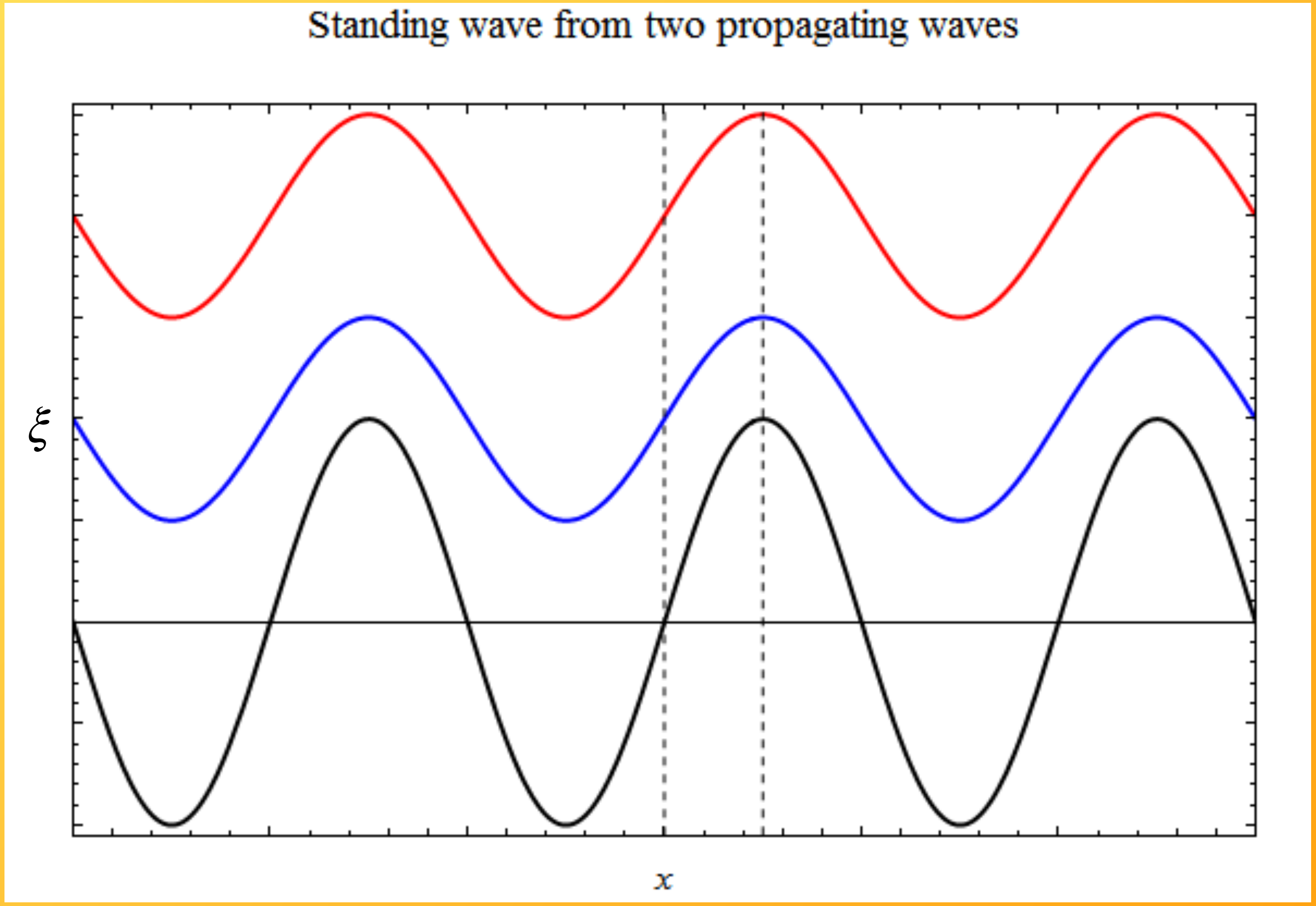
Propagating waves with wavelength $\lambda = 2\pi/k$ and speed $c = \sqrt{\mu/T}$



Standing Waves

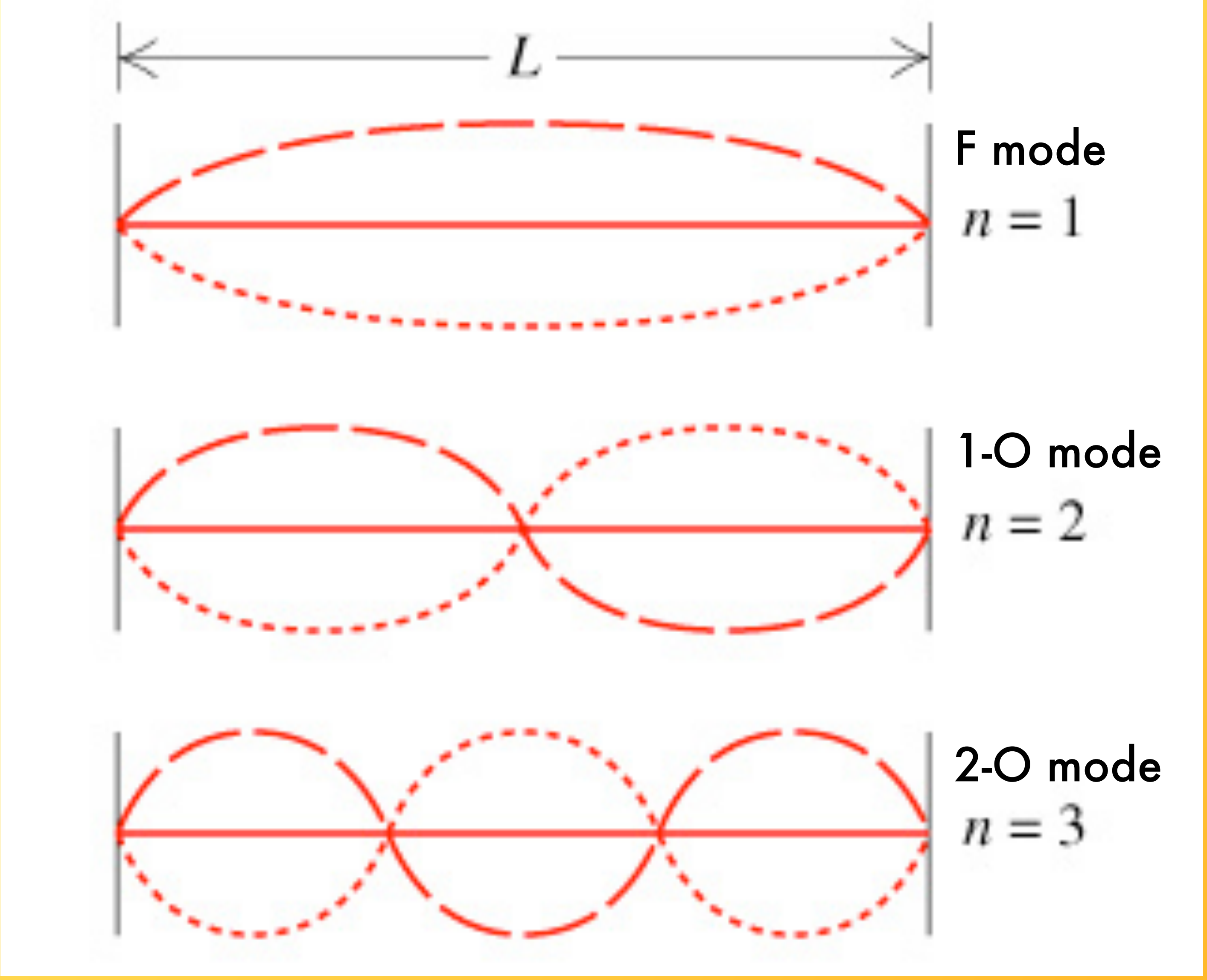
Two waves superposed with the same frequency, but opposite amplitudes & wavenumbers

$$\xi(x,t) = A\left[e^{i(kx-\sigma t)} - e^{i(-kx-\sigma t)}\right] = A\sin(kx)\,e^{-i\sigma t}$$



Standing Waves on Clamped String

The clamped endpoints must correspond to nodes of standing-wave solutions



$$\xi(x,t) = A\sin(kx)\,\mathrm{e}^{-\mathrm{i}\sigma t}$$

$$\left.\begin{aligned}\xi(0,t) &= 0\\ \xi(L,t) &= 0\end{aligned}\right\} \rightarrow \sin(kL) = 0$$

$$kL = n\pi \qquad (n = 1,2,\ldots)$$

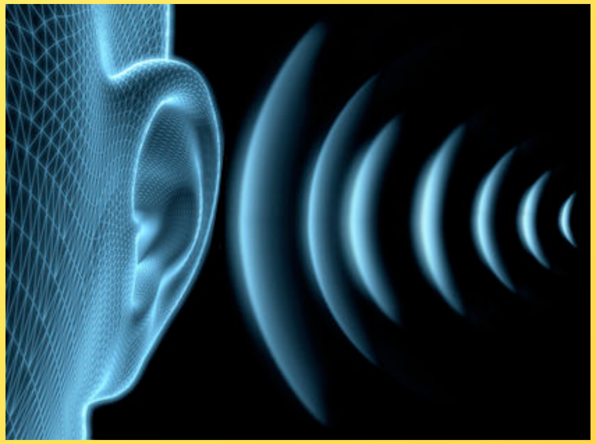
$$\sigma = ck = n\frac{c\pi}{L}$$

...a discrete set of normal modes, indexed by an integer order n and characterized by a distinct frequency and an associated wavefunction

Waves in Stars

Stars are gravitationally stratified ⟶ two types of waves

Acoustic (Pressure) Waves



Local Dispersion Relation

$$\sigma = c\sqrt{k_{\rm r}^2 + k_{\rm h}^2}$$

Sound Speed

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

Gravity (Buoyancy) Waves

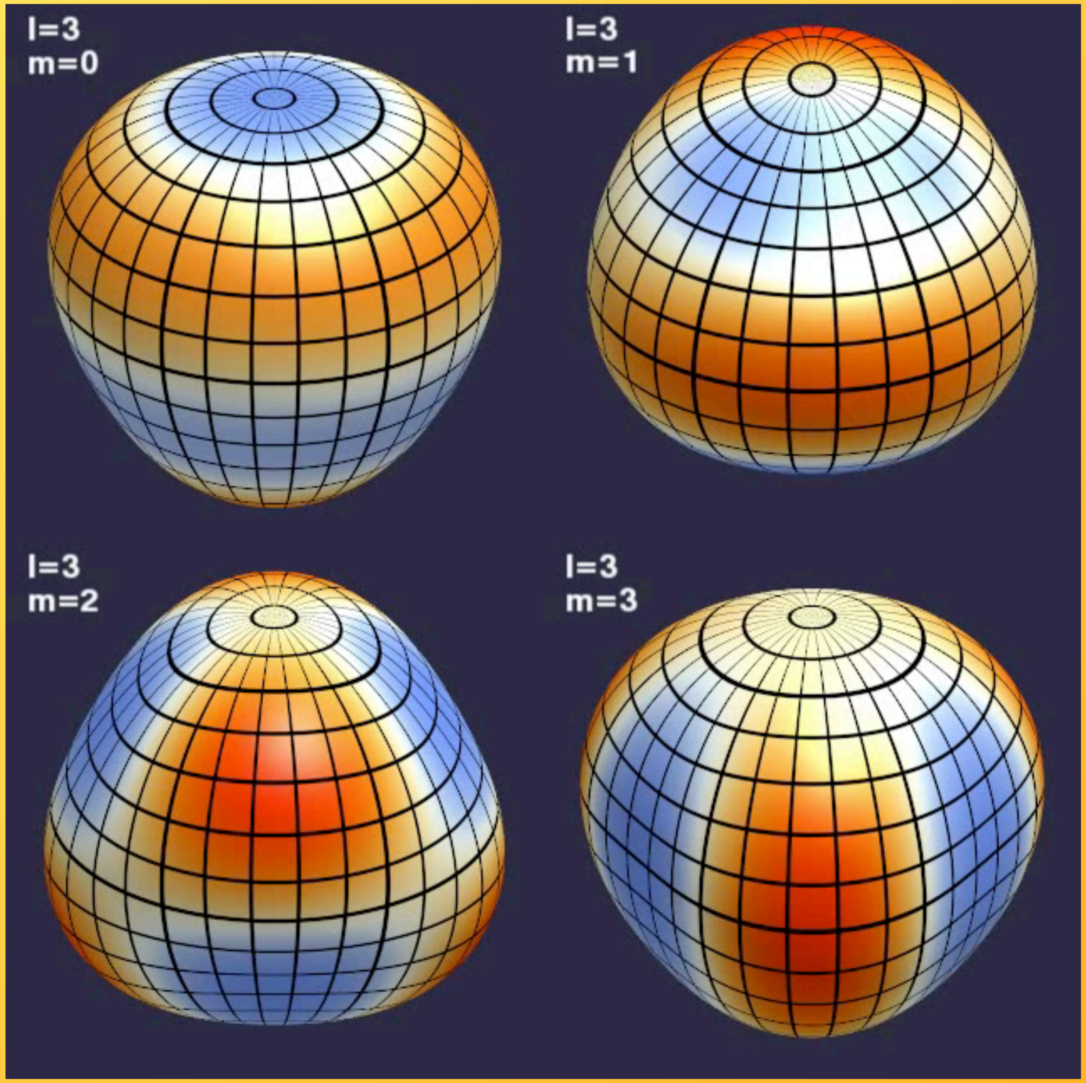
Local Dispersion Relation

$$\sigma = N\sqrt{\frac{k_{\rm h}^2}{k_{\rm r}^2 + k_{\rm h}^2}}$$

Brunt-Väisälä Frequency

$$N = \sqrt{\frac{g}{r}\left(\frac{1}{\gamma}\frac{{\rm d}\ln P}{{\rm d}\ln r} - \frac{{\rm d}\ln \rho}{{\rm d}\ln r}\right)}$$

Stellar Oscillation Modes



$$\xi_r(r,\theta,\phi,t) = \xi_r(r)\, Y_\ell^m(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\sigma t}$$

Radial wavefunction

Spherical harmonic
(harmonic degree ℓ, azmuthal order m)

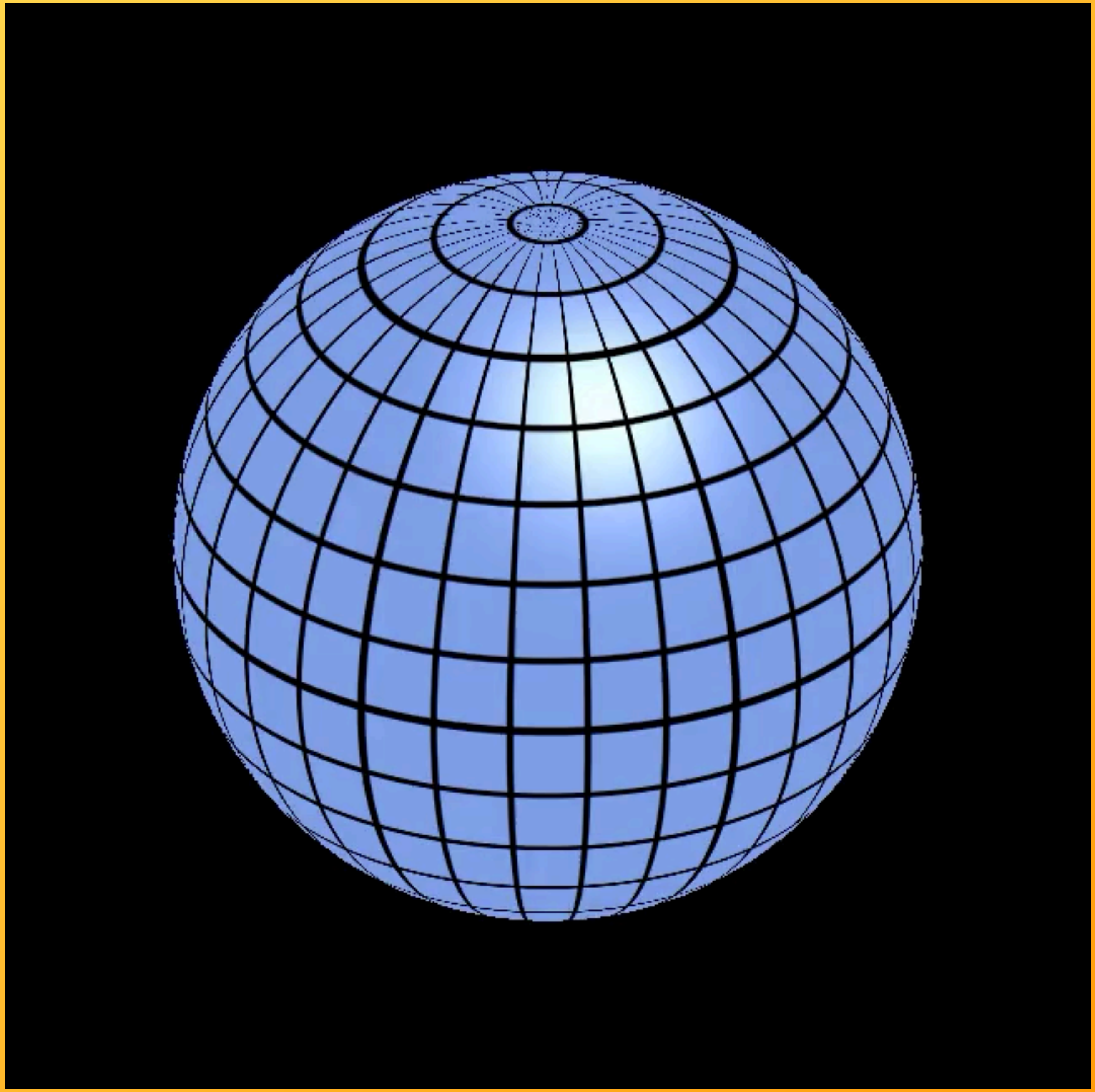
Frequency

Simplest Case: Radial Modes

$$\left.\begin{aligned} \ell &= 0 \\ m &= 0 \end{aligned}\right\} \rightarrow Y_\ell^m = 1$$

$$\xi_r(r, \theta, \phi, t) = \xi_r(r)\,\mathrm{e}^{-\mathrm{i}\sigma t}$$

Motion is spherically symmetric



Quantization Condition

On a stretched string...

$$kL = n\pi$$

In a star...

boundary phase term

$$\int_0^R k_r \, \mathrm{d}r \approx (n + \alpha)\pi$$

Radial Mode Frequencies

Dispersion Relaiton ($k_{\mathrm{h}} = 0$)

Acoustic waves: $\sigma^2 = c^2 k_{\mathrm{r}}^2$

Gravity waves: $\sigma^2 = 0$

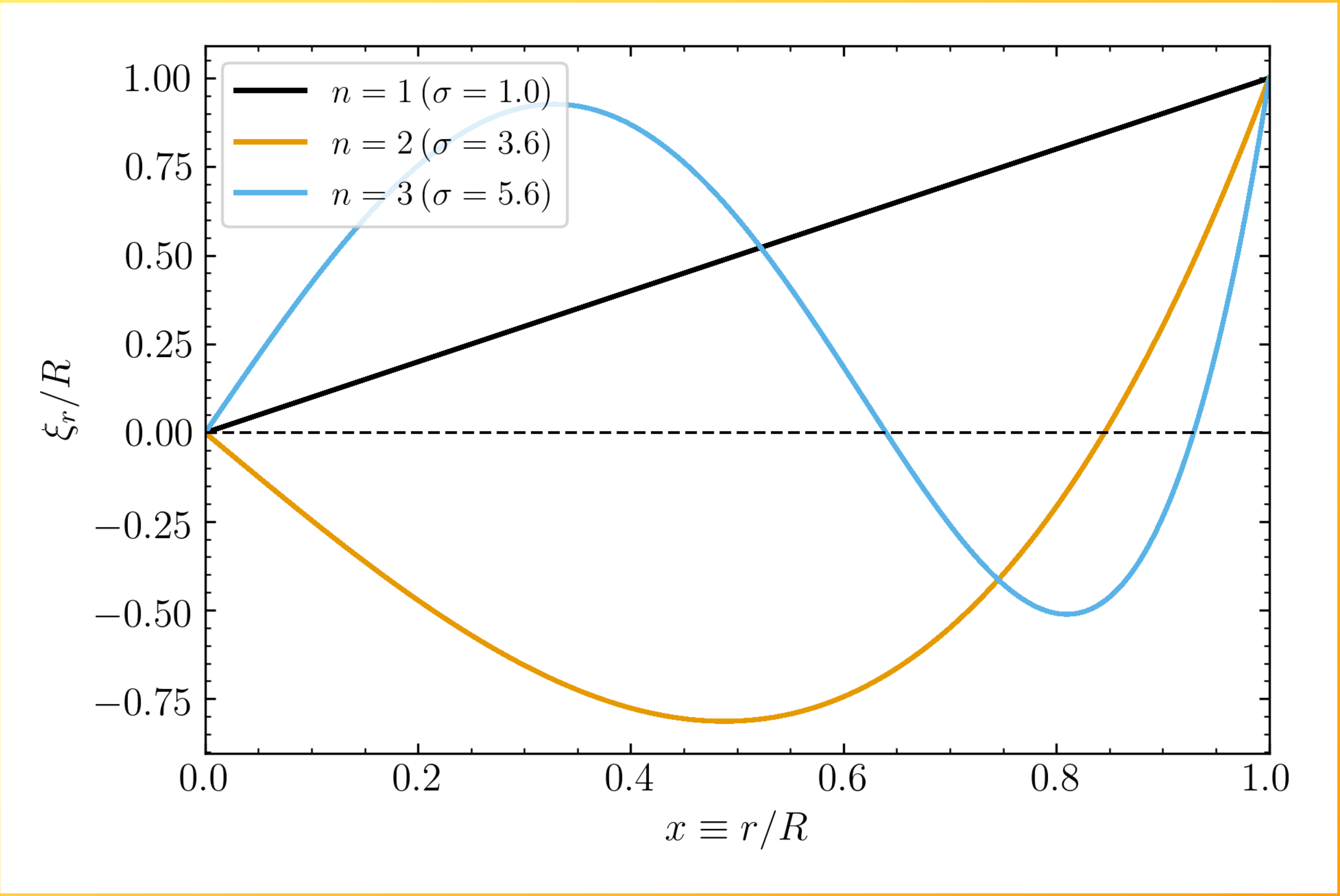
Quantization Condition

$$\int_0^R k_{\mathrm{r}}\,\mathrm{d}r \approx (n+\alpha)\pi$$

$$\sigma \int_0^R \frac{\mathrm{d}r}{c} \approx (n+\alpha)\pi$$

$$\sigma \approx \pi(n+\alpha)\Bigg[\underbrace{\int_0^R \frac{\mathrm{d}r}{c}}_{\tau_{\mathrm{dyn}} \approx \sqrt{\frac{R^3}{GM}}}\Bigg]^{-1}$$

Radial Eigenfunctions



For a homogeneous compressible sphere ($\gamma = 5/3$)

GYRE

- A companion software instrument to MESA
- Takes an input stellar model and calculates
  - mode frequencies σ
  - radial displacement wavefunctions ξ_{r}
  - plus lots more
- Bundled with MESA (see $MESA_DIR/gyre)
- For latest release, and full documentation, visit https://bitbucket.org/rhdtownsend/gyre/wiki/

Three Ways to Run GYRE

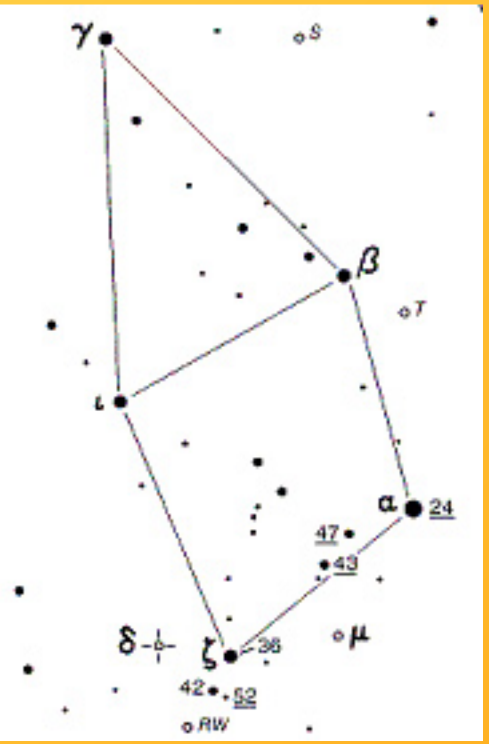
- Stand-alone
  - Write stellar MESA models to disk (in special format)
  - Post-process these models using GYRE
- Within MESA's *astero* module
  - Runs GYRE after selected evolution timesteps
  - Compares frequencies vs observations
  - Repeats evolution with different parameter (e.g., mass, metallicity) until convergence
- Using the GYRE-in-MESA hooks

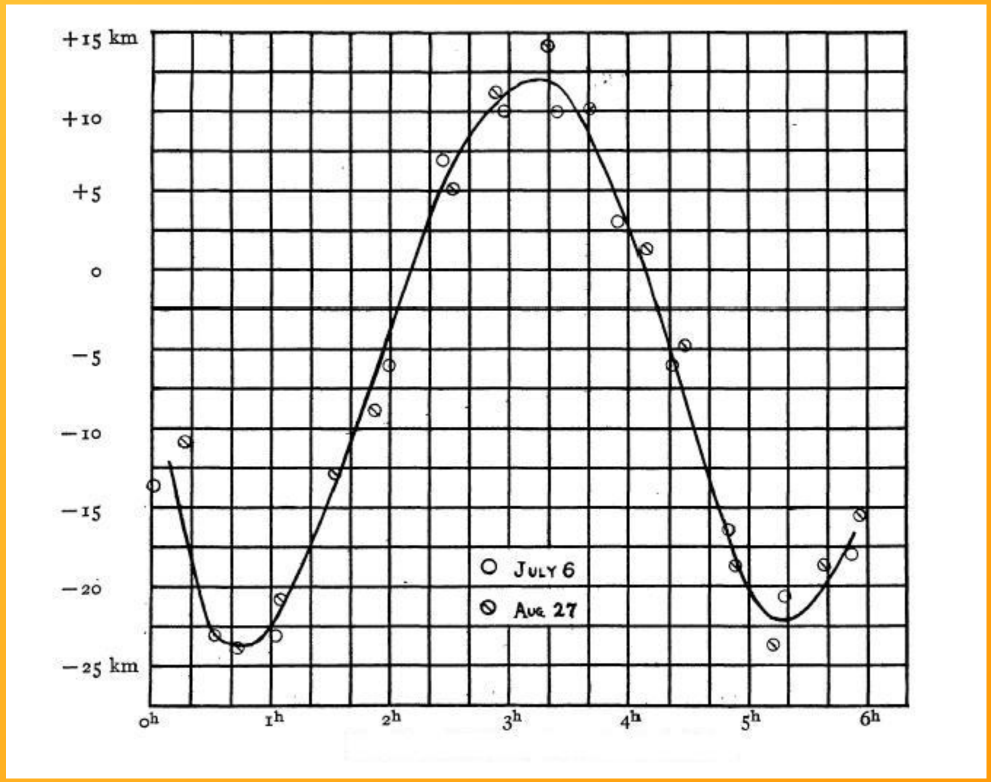
GYRE-in-MESA

- Provides a set of routines to control GYRE within `run_star_extras.f90`
  - Initialize GYRE
  - Pass current MESA model to GYRE
  - Ask GYRE to find modes
- For each mode found, a user-defined callback routine performs on-the-fly analysis
- GYRE parameters are specified in `gyre.in` file

Exploring β Cephei Stars

- Spectral types B0-B3
- Masses $\sim 8\,M_\odot$ and up
- Photometric & spectral variations
- Periods 2-12 hours
- Low-n acoustic modes (both radial and non-radial)





Lab Overview

- MiniLab 1
  - evolve $15\,M_{\odot}$ β Cephei model from pre-MS to TAMS
  - run GYRE-in-MESA during ZAMS-to-TAMS phase
- Minilab 2
  - plot frequencies of F and 1-O radial modes
- Minilab 3
  - plot radial displacement wavefunctions of modes
- Maxilab
  - explore process responsible for exciting the modes
  - create theoretical instability strip for modes

Let's Get Going!

https://rhdtownsend.github.io/mesa-summer-school-2019/

What Excites Pulsations?

All swings *can* swing; but only some swings *do* swing

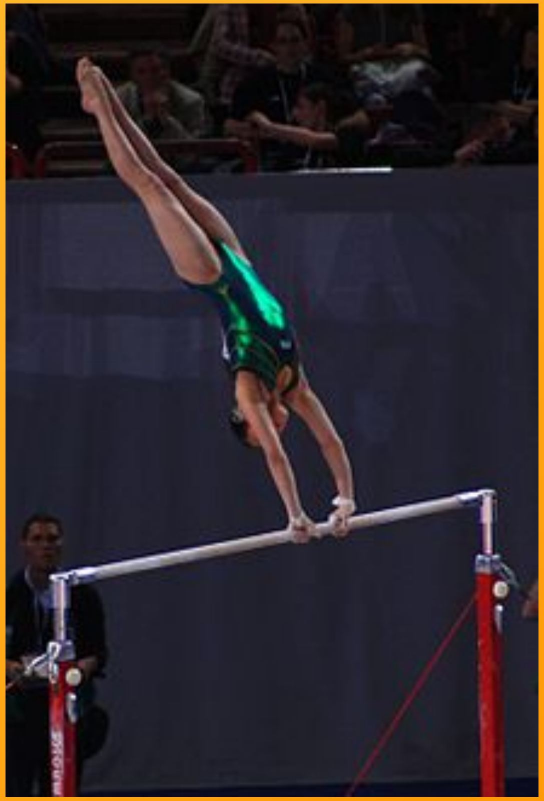
Four Excitation Mechanisms

Stochastic

Parametric

Forced

Convection



The Parametric Oscillator

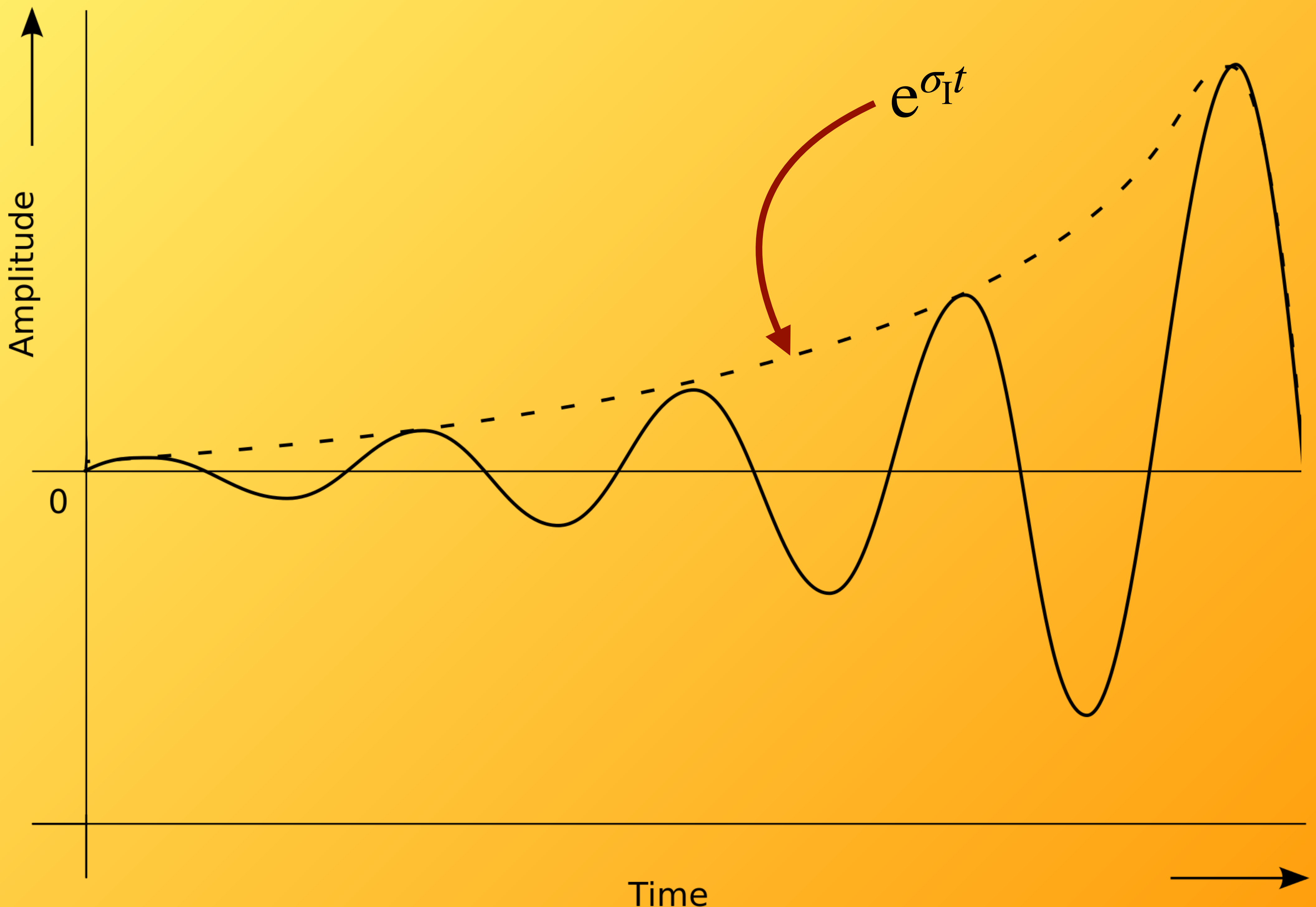
$$m\frac{\partial^2 \xi}{\partial t^2} = -\,q\,[1+\epsilon\sin(2\sigma_{\mathrm{p}}t)]\,\xi$$

If pumping frequency σ_{p} equals the natural frequency $\sqrt{q/m}$ and $\epsilon \ll 1$then...

$$\xi(t) = A\,\mathrm{e}^{-\mathrm{i}\sigma t} = A\,\mathrm{e}^{-\mathrm{i}\sigma_R t}\,\mathrm{e}^{\sigma_{\mathrm{I}} t}$$

$$\sigma_{\mathrm{R}} = \sqrt{\frac{q}{m}} \qquad \sigma_{\mathrm{I}} = \sqrt{\frac{q}{m}}\,\frac{\epsilon}{4}$$

Instability of Parametric Oscillator



Energetics of Parametric Oscillator

Over one cycle, the change in the system energy E is given by the “work”

$$W = 4\pi \frac{\sigma_{\mathrm{I}}}{\sigma_{\mathrm{R}}} E$$

The energy gain comes from whatever mechanism is doing the pumping

Pumping Stellar Pulsations

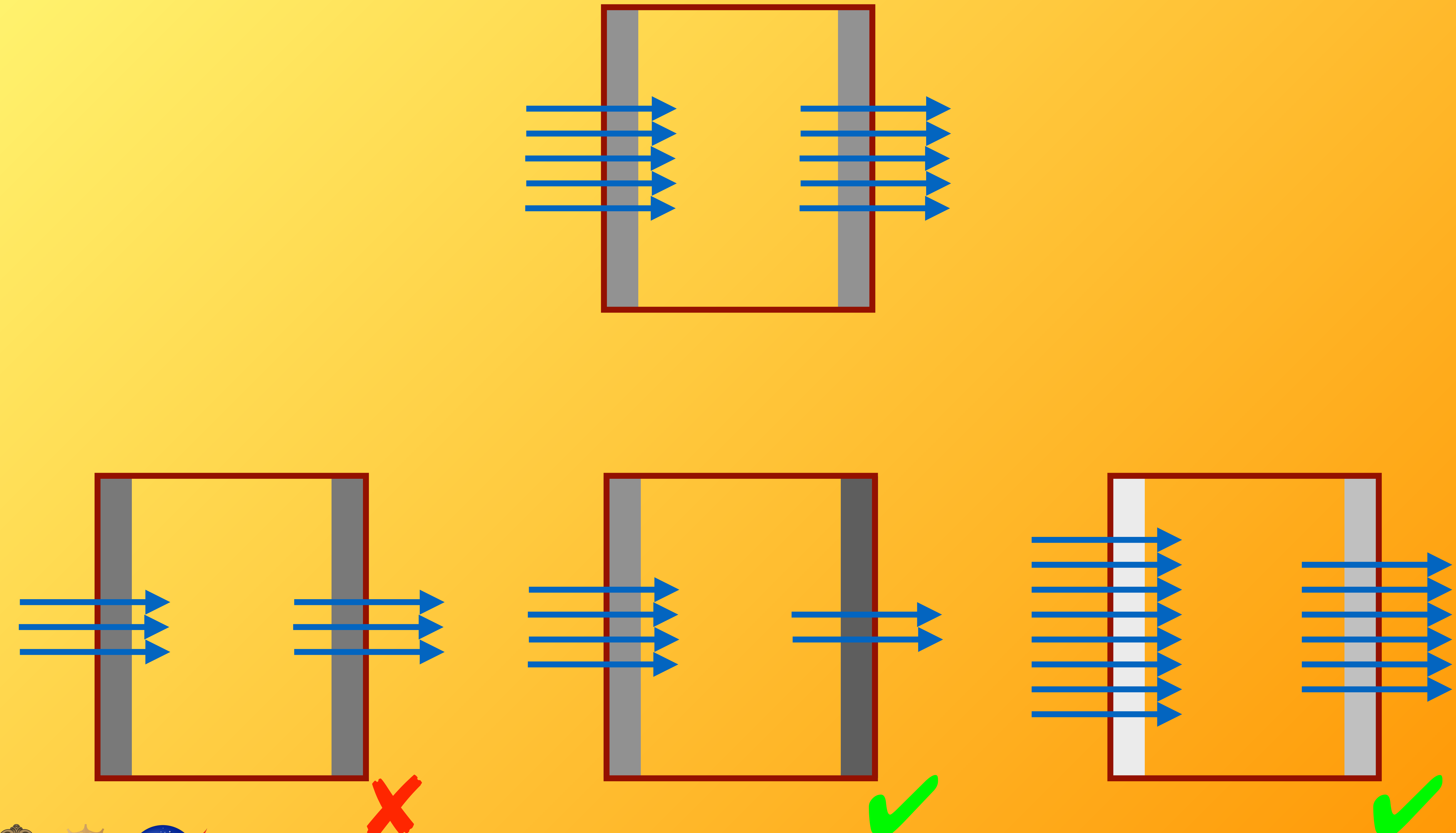
To increase the restoring force at extrema, raise (lower) the pressure at maximum compression (expansion)

To raise (lower) the pressure, add (remove) heat

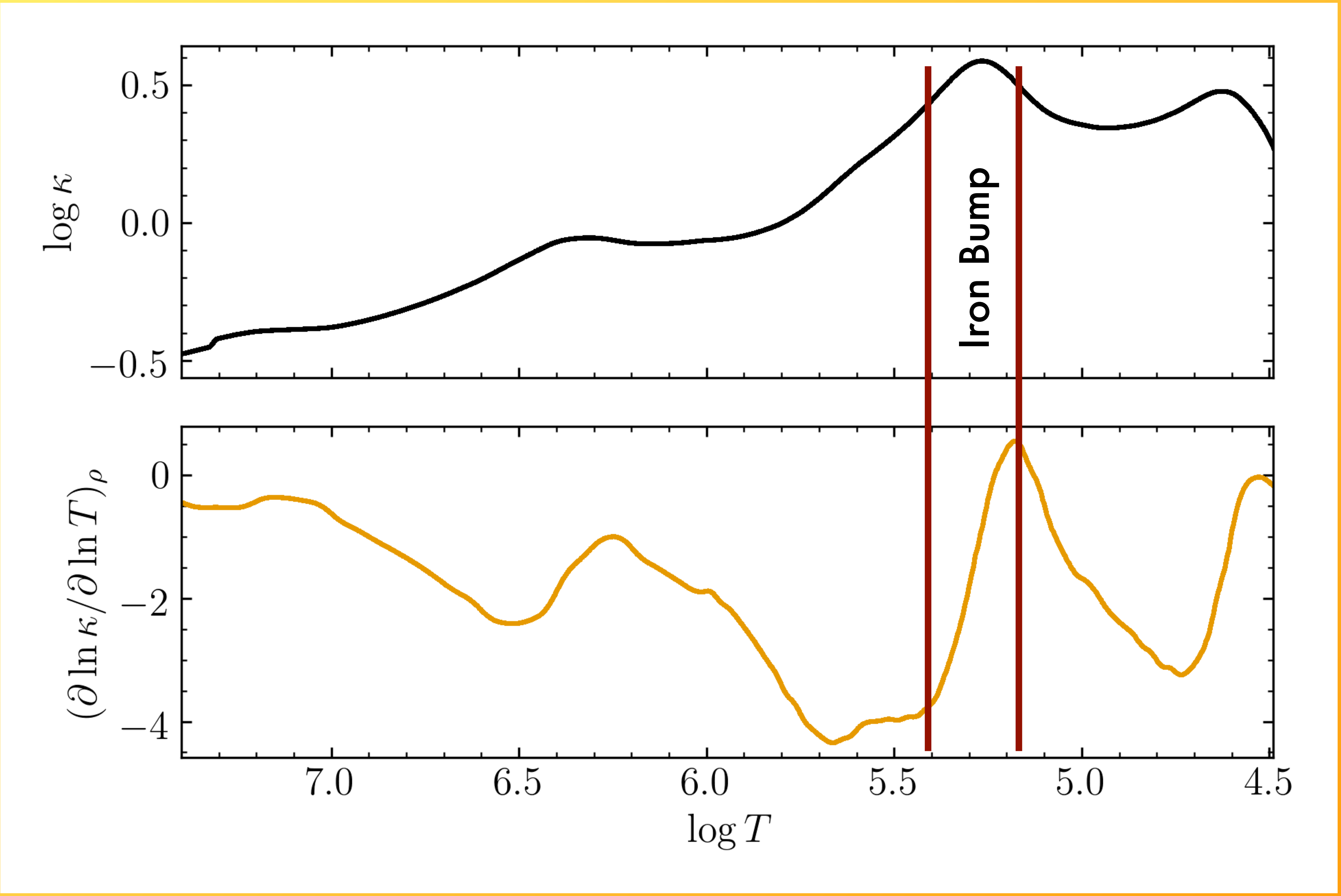
"Excess heat must be added to matter when at a high temperature and withdrawn at a low temperature. We require, in fact, something corresponding to the valve-mechanism of a heat engine."

Eddington (1926)

The Kappa Mechanism: An Opacity Valve



Opacity in Massive Stars



$15\,M_\odot$ model, $X_\mathrm{c} \approx 0.6$

The Kappa Mechanism in GYRE

- Turn on non-adiabatic effects (to include the interaction between pulsation & heat flow)
- Resulting pulsation frequencies are complex
  - $\sigma_{\rm I} > 0$ indicates instability (growth)
  - $\sigma_{\rm I} < 0$ indicates decay (damping)
- The differential work $\mathrm{d}W/\mathrm{d}x$ indicates which regions of the star contribute toward driving or damping

$$W = 4\pi \frac{\sigma_{\rm I}}{\sigma_{\rm R}} E = \int_0^1 \frac{\mathrm{d}W}{\mathrm{d}x}\,\mathrm{d}x \qquad (x \equiv r/R)$$

MaxiLab

- Turn on non-adiabatic effects in GYRE
- Print out/plot growth rates $\sigma_{\rm I}$, find where radial modes become unstable
- Plot differential work $\mathrm{d}W/\mathrm{d}x$ to explore driving
- Crowd project
  - Repeat runs for other masses / metallicities
  - Store $T_{\rm eff}$, $\log L$ boundaries of instability
  - Map out radial-mode instability strip for β Cephei stars

Go for It!

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