1. Show $m(a+bx) = a + b \times m(x)$

$m(x) = \frac{1}{N}\sum_{i}^{N} x_i$

$m(a+bx) = \frac{1}{N}\sum_{i}^{N}(a+bx_i)$

$m(a+bx) = \frac{1}{N}\sum_{i}^{N} a + \frac{1}{N}\sum_{i}^{N} bx_i$

since $\sum_{i=1}^{N} a = Na$ we get:

$m(a+bx) = a + b \times \frac{1}{N}\sum_{i}^{N} x_i = a + b \times m(x)$

2. Show that $cov(x, a+by) = b \times cov(x,y)$

def cov:

$cov(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(y))$

transformation $a+by$:

$cov(x, a+by) = \frac{1}{N}\sum_{i}^{N}(x_i - m(x))(a+by_i) - m(a+by)$

results from pt 1:

$m(a+by) = a + b(m(y))$

substituting:

$cov(x, a+by) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(a+by_i - a - bm(y))$

→ a cancels out, factor B

$cov(x, a+by) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(b(y_i - m(y)))$

$cov(x, a+by) = b \times \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(y))$

$cov(x, a+by) = b \times cov(x,y)$

3. Show $cov(a+bX, a+bX) = b^2 cov(X,X)$ & $cov(X,X) = S^2$

$cov(a+bx, a+bx) = b \times cov(x, a+bx)$

applying result from pt 2:

$cov(a+bx, a+bx) = bx(bx\, cov(x,x)) = b^2 cov(x,x)$

cov x with x =

$cov(x,x) = s^2$

$\therefore cov(a+bx, a+bx) = b^2 s^2$

4.

median = middle value of ordered data set

if a non-decreasing transformation of $g(x)$ is applied, the trantormed values retain their order

If: $x \geq x'$ then $g(x) \geq g(x')$

median$(g(x)) = g($median$(x))$ → this holds true for quantiles or percentiles but not for IQR or range since those depend on differences not preserved under non-linear transformations

5. is $m(g(x))$ always $= g(m(x))$

→ No, because the mean value is affected by changes in scale and shape

If $g(x)) = x^2$

then

$m(x^2) = \frac{1}{N}\sum_{i=1}^{N} x_i^2 \neq m(x))^2$

→ $m(x^2) \neq (m(x))^2$ unless x is constant

→ while this holds true for linear transformations

it does not hold true for general transformations