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The Concept of Algorithm as an Interpretative Key of Modern Rationality

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Abstract

According to Ernst Cassirer, the transition from the concept of substance to that of mathematical function as a guide of knowledge coincided with the end of ancient and the beginning of modern theoretical thought. In the first part of this article we argue that a similar transition has also taken place in the practical sphere, where mathematical function occurs in one of its specific forms, which is that of the algorithm concept. In the second part we argue that with the rise of modernity the idea of substance and the related concepts of category and classification, which are deeply embedded in western culture, have not been totally supplanted by that of function. The intertwining of the concepts of substance and function has generated contradictory hybrids. These hybrids are used as a key for the understanding of the different repercussions of algorithmic logic on society in terms of social integration.

Keywords

modernity, rationality, sociological theory

Introduction

It is hard not to agree with Ernst Cassirer when he indicates the transition from the concept of *substance* to that of *function,* in the mathematical sense, as the gnoseological turning point marking the end of medieval theoretical thought and the beginning of modern thought

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(Cassirer, 1953). However, we believe that this interpretive key of modernity has not been exploited in all its potentiality. Notably, Cassirer's point of view has received scarce consideration in sociology. Since this discipline convincingly argues that theory and the social world are always closely related, it is surprising that it has ignored the mathematical concept of function as a possible interpretive category. The concept of function is an inexplicable 'blind spot' in the sociological view of the world.

The fact that numbers do not appear as such in many rational activities has likely contributed to this result. The notion that the 'logic of numbers' operates exclusively on numbers is misleading. In the second half of the last century, the theory of recursive functions has made it clear that the concept of calculation is very general and does not necessarily imply the manipulation of numerical symbols. As Church's thesis affirms, the class of algorithms coincides with the class of recursive functions, and the formal theory of numbers has shown that the latter coincides with a *specific* class of numerical functions[1] (Davis, 1958; Mendelson, 1964; Odifreddi, 1992). Hence, through the mediation of the concept of recursive function, it may be concluded that the class of algorithms coincides with that of this particular class of numeric functions and, therefore, that any algorithm is ultimately based on calculation by numbers.

Although theory tells us that algorithms can always be represented in terms of numerical functions, they do not necessarily operate on numbers. For example, the algorithm that places a list of words in alphabetical order apparently operates on objects (the words) that are not numbers. Once algorithms are applied to objects that apparently are not numbers, they take on a strongly practical connotation, going beyond the sphere of 'knowledge' to invade the sphere of 'action'. Thus, the logic of numerical functions is present in society not only as a cognitive language (i.e. as mathematics), but also as a performative one. The majority of manufacturing processes, the organization of services to 'citizen' and 'customer', and the myriad of 'clicks' that regulate our daily lives, are all inspired by algorithmic models. The logic of numeric functions enters the practical world, often unseen, and firmly takes root in everyday life and our consciousness.

We now need to go into a brief digression on recursive functions, to make clear what they are and highlight some of their features that are of special significance for our argumentation. In his popular book on the amazing properties of formal logic and the theory of computation, Hofstadter (1979) introduces the concept of recursion as the 'nesting of things within things and its variations'. It is like the phenomenon of the endless repetition of images reflected in two mirrors placed opposite one another. But what rules must a process comply with to be definable as recursive? And, most importantly, what is the principle underlying the concept of recursion?

Any recursive process can be reduced to three functions (see, for instance, Mendelson, 1964: 120–1; Odifreddi, 1992: 21–2). Two of these, called the *zero function* and the *projection function*, only serve the purpose of providing the first term of the process (initialization), whereas the third, called the *successor function*, 'builds' the process by yielding the 'successive' occurrence of a given value. The results generated by the successor function are purely logical constructions. The only content of each step is the action itself. For example, in set theory (Mendelson, 1964: 175) the concept of successor can be represented graphically by *bracketing* the sequence already enacted in the previous step. Thus, the successor of 0 (the empty set) is $\{0\}$. The successor of the latter is the set obtained by bracketing the previously generated elements, that is, $\{0,\{0\}\}$. The successor of this is $\{0,\{0\},\{0,\{0\}\}\}$, and so on. Therefore a recursive process, once initiated, generates nothing more than a repetition of the same operation. However, the results of this operation are not independent from one another, simply because they are embedded into one another. The intuitive image is that of an operation that operates on itself. We will be returning to this point in an important passage in our argumentation (see section 2). A further simple example would be that of natural numbers. Define the successor function as $s(n) = n + 1$ and the zero function as $z(n) = 0$ for each integer number n. It is then evident that, for instance, the recursive function $s(s(s(z(n))))$ computes the number 3, and it should be noted that both examples have the same logical structure.

Recursive functions are particularly important in defining and formalizing the notion of effective calculability.[2] A number is said to be effectively calculable if there exists a recursive function that computes it. In a celebrated paper, Turing (1937) gave an alternative characterization of calculable numbers by means of an abstract computational device now known as the Turing machine. Recursive functions and Turing machines are formalizations of the informal and intuitive notion of algorithm. It has been proved that these formalizations are equivalent, in the sense that if a number or a mathematical function is (effectively) computable with a Turing machine, then this number or function is recursive (Turing, 1937; Minsky, 1967: 169–98; Odifreddi, 1992: 97–101).

In this essay we shall not give a description of how a Turing machine can perform computations, but just underline the fact that a one-to-one correspondence can be established between the set of Turing machines and that of natural numbers. Now, since the set of real numbers is 'larger' than that of natural numbers, there are real numbers (the majority) that are left out of this one-to-one correspondence with Turing machines. This means that there are real numbers that are not effectively computable and therefore not recursive (Minsky, 1967: 157–68). An immediate implication of this result is that we cannot fill the continuum of a numerical axis with effectively calculable real numbers (integers, rational numbers, and some irrational numbers). Turing machines, and therefore recursive functions, only allow us to operate within the 'discrete'. To reach conclusions within the 'continuum', we need to abandon effective calculation in favour of 'analytical' calculus.[3]

Sciences that have developed their theoretical systems by applying the concept of function to continuous dimensions such as space, time and 'exchange value' do need to employ formulae that are not effectively calculable and, hence, not reproducible through algorithms. On the contrary, in practical activities, which involve computation of *discrete* quantities, computation processes are algorithmizable. This leads to the conclusion that *the logic of algorithms* (i.e. recursive functions) *is the specific form in which the concept of function occurs in everyday life*, as is the case with the calculation of real economic transactions or with algorithms used for the production and provision of many goods and services.

From the above discussion one can deduce the following two points, which constitute the main arguments of the present article: 1) if, as Cassirer argues, the transition from medieval to modern gnoseology consisted in the replacement of the concept of substance with that of function as a guide of knowledge, a similar transition must necessarily have occurred in society; 2) the concept of function has come to dominate the culture of the modern world, both on the theoretical and the practical level, but on the practical level it has done so in the specific form of recursive functions (algorithms). This implies that the hegemony of algorithmic logic in the performative and practical sphere, as part of the more general cultural hegemony of the concept of mathematical function, is not a distinctive trait of contemporaneity. Although its existence and principles have been theoretically identified only recently, at the cultural level it had already been incorporated by modern human beings through the dissemination of the bureaucratic organizational model, of machines, and of all kinds of practical applications of calculation.

It has thus become crucial to conduct an historical investigation to bring to light the process whereby the logic of the algorithm has prevailed in social practice, and the continuous interaction of this process with the more general concept of function. This is what we strive to do in the following two sections of the article. In the last section we will be dealing with the hybrids that 'abstract systems' have generated using the logic of classification and that of function, and the consequences these hybrids have had on social integration.

The Practical Origins of the Logic of Recursive Functions

All machines run an algorithm. One can say that they are the materialization of an algorithm, which in itself is a logical object. However, the preference that modernity accords to algorithmic procedures is not a *consequence* of the introduction of machines in production processes. The culture of mechanization did not arise as a 'superstructure' generated by the existence of machines. Rather, the opposite is true, in the sense that it was the rise of a mentality oriented towards process formalization that facilitated the designing of mechanical equipment and their spread.

Walker (1966: 591–2) has expressed this concept with the following interesting statement: men must themselves 'become mechanical' before they can realize the usefulness of introducing machines in their activities. Proof of this can be found in the fact that 'the first factories, great and small, employed hand workers and had no machines'. After all, Walker notes, 'the very word "manufacture" literally means "making by *hand*"'.

Walker contrasts his argument with Marx's determinism, stressing the 'cultural' root of process formalization against the hypothesis of its 'materialistic' origin. But actually Marx's (1952: 170) own analysis clearly highlights that manufacturing began as a kind of human mechanization. The origin of manufacture, Marx (1952: 164) says, is twofold, as it was organized according to two different logical schemes. One consists of 'the assemblage in one workshop under the control of a single capitalist of laborers belonging to various independent handicrafts, but through whose hands a given article must pass on its way to completion', whereas the other sees 'one capitalist employing simultaneously in one workshop a number of artificers who all do the same, or the same kind of work'. Even in this second case, what progressively emerges is the practice of isolating operations one from the other, sorting them in a logical sequence, and assigning to each worker (or group of workers) a particular segment of this sequence.

Marx (1952: 177), citing Ferguson, represents the workshop 'as an engine, the parts of which are men'. This human mechanization is what the 'system of machinery' of the Industrial Revolution of the 18th century is built upon. 'Detail workers' and their 'special tools' were replaced by 'detail machines', which complemented each other in the same way as the workers in the manufacture (Marx, 1952: 181–5). Thus, Marx argues that the integrated machine system of the modern factory has its roots in a specific form of organization of work, that is, in *the mechanization of production without the use of real machines*.

As Mantoux (1961: 89) observes, we must not attribute a descriptive but rather an explicative value to the relation observed by Marx between manufacturing and the machine system that arose in the Age of Steam. This relation is not an historical narration, but rather the expression of a logical and cultural connection. The factory, intended as the gathering of a number of workers 'in one workshop under the control of a single capitalist' (Marx, 1952: 164), was a rare phenomenon in the centuries that have preceded the Industrial Revolution. There certainly existed capitalistic enterprises of this kind – particularly in the 16th century (see, e.g., Patterson, 1957: 151–2) – but the form in which, in general, the proto-industrial factory occurred was that of the cottage industry (Mantoux, 1961: 67–8; Hobsbawm, 1996: 36–7), in which there was no concentration of labour in the same workshop. In the cottage industry, too, however, the model described by Marx was manifested in a certain division of work.

In the division of work in cottage industry, each craftsman's independent work yields a marketable product. This does not happen with the division of work in manufacturing industries, where 'the detail labourer produces no commodities', since 'it is only the common product of all the detail labourers that becomes a commodity' (Marx, 1952: 173). Marx, as is well known, traces the origin of this difference to the concentration of the means of production in the hands of a single capitalist, which occurs precisely with the transition from the cottage system to the manufacturing system.

Now, as Mantoux notes (1961: 56–62), this concentration – and the resulting alienation of commodities from their manufacturer – gradually emerged even within the cottage industry (domestic system). Let us take, for example, the textile industry in England, which was so important in both cottage industry and the Industrial Revolution. The worker, who is the owner of both the cottage and the loom, and who weaves the wool previously carded and spun by members of his family, is the owner of his product. Since the work of spinning is slower than that of weaving, he may be induced, in the case of an increase in demand, to buy wool already spun by others or to commission to others the spinning of wool which he has purchased. But this kind of division of labour is still within the domestic system. Both the spinner and the weaver produce a commodity. They are not 'detail workers'.

With the expansion of markets, the merchant becomes the sole broker in the sale of commodities. By managing demand, he ends up managing production. He knows how much fabric he will need for future orders. He buys the raw material and delivers it to a number of workers involved in the various production stages (preparation, spinning, weaving and finishing). He asserts himself as the only possible medium not only for the sale of finished goods, but also for the transfer of products from one phase to the other and, therefore, from one manufacturer to the other. The cottage worker no longer has the freedom to produce finished goods, and finds himself confined to a phase of a process that he does not control. The remuneration of work increasingly takes on the form of a wage. Ownership of the means of production, work organization, and goods are alienated from the producer, who takes on the characteristics of Marx's 'detail labourer' (Mantoux, 1961: 56–62; Kriedte et al., 1981: 101–11; Bythell, 1983; Unwin, 1904).

Already in 'proto-industrial' Europe, workers whose job was not to produce a commodity but merely to complete a step in a process that took place over their heads became a widespread reality. We thus witness the spread of what we called ‘human mechanization’. But why is the analogy between the detail worker and a part of a machine pertinent? When Marx (1952: 170) says that ‘the collective labourer, formed by the combination of a number of detail labourers, is the machinery specially characteristic of the manufacturing period’, we understand very well what he means and feel the logical consistency of his statement. But why is it consistent? This is what we need to understand: what are the elements that make us accept that there is a similarity between the behaviour of a machine and that of a worker in the manufacturing industry, between animate and inanimate entities? We need to find out what mental or cultural scheme makes us conceive the two phenomena as something similar. This scheme must be related to our intuitive concept of mechanical processes and express a logic so obvious as to be implicit and go unnoticed.

The machine concept is strongly associated with the logic of recursive functions. As we mentioned in the introduction to the present article, the Theory of Computation, in its most famous formulation, is the study of the properties of an abstract machine, the Turing machine. One of the fundamental findings of this theory is that there is a logical equivalence between the algorithms executable by this machine and recursive functions (e.g. Davis, 1958; Odifreddi, 1992: 97–101). There are also other formulations of computation theory that are based on other kinds of abstract devices (for example Markov algorithms, Herbrand-Gödel systems, etc.). To indicate what these devices have in common on an *intuitive* ground, the literature often uses the concepts of ‘mechanical’ process or ‘computable’ process. However, even for these other devices it has been shown that the class of effectively executable processes coincides with that of recursive functions. It is therefore plausible to claim (Church’s thesis) that any computable (or ‘mechanical’) function is recursive, and vice versa (e.g. Mendelson, 1964: 227–8; Odifreddi 1992: 101–23). The theory thereby indicates that the common element of intuitive concepts such as ‘mechanical’, ‘computable’ and ‘algorithmic’ is recursion.

Now, as we have seen in the introduction, recursive functions, as a process, have their own foundational logic in the successor operator. We have also seen that the set-theoretic interpretation of the successor allows us to represent its logic in terms of a simple graphical rule: at each step, put in brackets what has already been bracketed in the previous steps. The detailed worker described by Marx has a function similar to that of the successor operator. He performs an elementary operation whose primary purpose is still that of performing the same operation. It is when we frame it in this type of logical model that a human action appears to us as ‘mechanical’, precisely because the logical foundation of mechanicalness is recursion. The crucial point *is not the repetition of the act* in itself. A craftsman who repeats a particular movement many times to produce a commodity does not bring to our mind the idea of someone who is acting mechanically. However, if we interpret his repetitive movements as an end in itself, then we think of his acting as mechanical. The crucial point in the idea of mechanical action, then, is that the *repetition is the aim of the action*, because it is then that the action becomes a step in a recursive process. Indeed, as we pointed out in the introduction, the intuitive definition of recursion is that of an operation whose aim is to operate on itself.

Effective Calculation as a Mediator between Practice and Theory

The culture of recursive processes does not just concern the world of organization and machines; it also plays an important role in mediating between empirics and theory. The concepts of money and value in market economy are an excellent case in point. Simmel (2004: 106–24) has made clear that the value of a commodity has a relational nature, and that money is the symbol that expresses that nature. But this relational nature of value only appears with the modern conception of money. In medieval economic theory, value was seen as embedded in the object to be exchanged. Individual entities in their concreteness were considered to be the essential foundation of reality and value was regarded as one of their attributes (Simmel, 2004: 124). In this conception, even the coin seemed to derive its value from the material it was made of (e.g. gold or silver), as was the case for all other goods. But when money came to be no longer identified with the coin-commodity, value emerged as an abstract concept (Simmel, 2004: 117–28).

With the increasing use of credit instruments to replace circulating money (see, e.g., Weatherford, 1997: 64–79), value become visibly independent of the material substrate. It is evident that value cannot reside in a piece of paper, but in the system of relations that the symbol (as a document with a signature) guarantees. However, since value finds a representation in money, which only speaks a quantitative language, it is placed on a different plane than that of the actual relations between economic actors, an abstract plane (Simmel, 2004: 118).

Summarizing, in the medieval conception money and material value *coincide*. The coin-commodity simultaneously bears one and the other. 'Modern money', instead, becomes *separate* from material value. In its new paper form, money retains its function as a medium of exchange, but is no longer a bearer of embedded value. The banknote is circulating money, but it is only a symbol of that value, which must reside elsewhere. As a result of this step, money and value are placed on different epistemological levels, respectively, that of everyday experience and that of abstraction.

The theoretical considerations of classical economy on exchange value can be regarded as aimed at defining value as an abstract dimension.

This abstraction is the premise to a conception of value as a quantitative *continuum*. According to Adam Smith (1952: 13–14), the value of any commodity is equal to the amount of work that commodity can be exchanged with. But it is difficult to measure work. There may be, for example, 'more labour in an hour's hard work than in two hours' easy business'. The only way to quantify the work exchangeable for a commodity (and hence its value) would be to establish the money equivalent of that commodity. However, for various reasons, the monetary price of a commodity varies irregularly with respect to the amount of work required to produce it, and hence cannot be the true measure of the value of work. This amounts to saying that the work employed to produce a commodity represents its real price, while the money equivalent of that commodity is only its nominal price (Smith, 1952: 14–15).

So for Adam Smith value is not an exactly measurable quantity. On the one hand, there does not exist a direct measure of value as work incorporated in commodities. On the other, the value of money, although it approximates the value of work, cannot be identified with it, as it only corresponds to its 'nominal price'. Thus, in Adam Smith's work value already appears as an abstract dimension. Ricardo's criticism of Adam Smith places even more stress on the distance between the price of goods and the value of the work used to produce them. Notably, Ricardo argues that the cost of the labour incorporated in the price of goods varies with the rate of fixed capital used to produce them (1971: 79–86).

Marx (1952: 14–15) solves the issue of measuring the quantity of work as posed by Smith. Although there are qualitative differences in work, depending on the intensity of effort, competence and skills of the worker, what the market considers is the *work time socially necessary* to produce a commodity. The exchange value of a good is exactly proportional to the 'socially necessary' time required to produce it. Through this step, the classical economic concept of value becomes an abstract quantitative and continuous dimension. It is *abstract* because the concept of socially necessary work is abstract; it is *quantitative* and *continuous* because time, on which Marx bases value, is a uniform and continuous dimension. However, in the Marxian theoretical system the continuity of the dimension of value is not a necessary condition. The important thing is that there is proportionality between the amount of work and value. The mathematical expressions used by Marx do not go beyond fractional quantities and hence never involve irrational numbers (numbers that are not effectively computable). Reasoning in terms of numerical sets, we can say that Marx stays within the field of rational numbers, that is, within effective calculability.

In the neo-classical economic theory, instead, mathematical continuity becomes a necessary condition. The 'marginalist' conception of value (see esp. Jevons, 1970: 94–144; Walras, 1938: 85–95) postulates a 'utility function' of goods that must be assumed to be continuous. Value is the mathematical derivative of the utility function, and the derivative, as is known, requires the condition of continuity of the function to be differentiated. To affirm the validity of this condition in the case of the utility function, the cited neoclassical authors resort to an argument similar to that used by Marx to construe value as a one-dimensional quantity. According to Marx (1952: 15), value is defined by the abstract concept of 'average labour power of society'. In Jevons and Walras, it is the abstract concept of *social average* that generates continuity. Jevons (1970: 108) recognizes that it 'may seem absurd' to speak, for example, of infinitely small quantities of food consumption, 'but, when we consider the consumption of a nation as a whole, the consumption may well be conceived to increase or diminish by quantities which are, practically speaking, infinitely small compared with the whole consumption'. Similarly, Walras (1938: 70–1) uses convergence towards the average of the '*loi des grands nombres*' (large numbers regarding the market as a whole) to justify the continuity of the numeric functions to which he reduces exchange relations.

The scientific conception of value as a continuous dimension is therefore the result of an abstraction process. The first step in this process was the evolution of money into its modern form, which resulted in the separation of value from the material substratum of money. But modern money, in addition to representing values in an abstract dimension, is also a medium of exchange that actually passes from hand to hand. In this guise it can only be transferred in discrete quantities. That is to say that economic exchanges mediated by money actually involve only effectively calculable – and therefore recursive – numeric functions. The continuum – not recursive – conception of value was therefore mediated by recursive calculation of discrete values. A practical tool of economics was the condition for the construction of theoretical scientific analytical tools.

With the advent of modernity, the concept of function has not simply supplanted that of substance, which has been the guiding idea of ancient and premodern culture. In the next section, we will show how it is precisely the contradictory hybridizations between ideas connected to the concepts of substance and function that have diversified the effects of the culture of calculation in the past, and continue to do so today.

Calculation and Classification

Referring to a concept put forth by Weber, Giddens (1984: 152) stresses that double-entry accounting has been the instrument through which capital accounting has managed to compare the profitability of investments in spite of their dispersion over time and space as a result of the expansion of markets. This form of accounting transfers enterprises to different scenarios and times, as a sort of 'time machine', allowing each to estimate and verify profit margins. While the capitalist enterprise 'stacks' events temporally through double-entry bookkeeping – says Giddens – bureaucracy achieves a similar result by means of archives, which are records of the past and prescriptions for the future. Archives are not simple aids to procedures: they are instruments that ensure the continuity and regularity of the entire bureaucratic discipline.

Giddens's argument goes to the heart of a central problem for the analysis of modernity: the logical origin of the contiguity between capital and bureaucracy. But this contiguity cannot be explained, as Giddens does, by an equivalence between the time of capital accounting and that of archives. The logic of archives is that of a taxonomic classification, whose content is constituted by people, things and events intended as predetermined with respect to the archiving activity. In archiving, time is one of the attributes incorporated in the entity to be archived, and not a dimension of calculation. On the contrary, in Weber's concept of 'capital accounting', time is an independent variable of a numerical function:

> Capital accounting is the valuation and verification of opportunities for profit and of the success of profit-making activity by means of a valuation of the total assets (goods and money) of the enterprise at the beginning of a profit-making venture, and the comparison of this with a similar valuation of the assets still present and newly acquired, at the end of the process; in the case of a profit-making organization operating continuously, the same is done for an accounting period. (Weber, 1978: 91)

If we reduce bureaucracy to classification, we will be unable to see the common logic it shares with capital accounting. But the purpose of bureaucracy is not classification. Its aim is to make the activities of an organization 'calculable' through the adoption of formal rules that allow it to go, in a deterministic and unique way, from a point in the process to the next one (Weber, 1978: 975). Bureaucracy uses classification only for this purpose, namely to force material elements (people, things and events) to become compatible with the algorithmic processes. Classification is not its purpose. The first step in Taylor's 'scientific administration' (Taylor, 1911), for example, is the identification of individuals by properties useful to the process, precisely to minimize all accidental components of the person that are extraneous to the algorithmic operations to be performed. We now know – as we have seen in the previous sections – that algorithms represent calculations not in a metaphorical sense, but rather in the sense that they are equivalent to effective numerical calculations. Therefore, the contiguity between capital and bureaucracy derives primarily from the logic of numerical calculation, rather than from that of classification: on the one hand, capital represents its world in terms of numerical functions; on the other, bureaucracy aspires to transform the production of goods and services into algorithms, that is, calculable processes.

Not distinguishing between the logic of calculation and the logic of classification can only lead to a partial and misleading view of modernity and its destiny. The classificatory order finds its gnoseological support in a 'realistic' view of the world.[4] As Cassirer (1953, 1999) makes clear, attention is focused on 'things' in themselves and not on the relation between the data of experience. Things are thought of as 'given', with their properties and attributes incorporated in them, and thus as possessing contents that are anterior and predetermined with respect to the way in which things can be ordered. Instead, the order produced by the logic of calculation, Cassirer goes on, generates the content of the datum in the very moment that it places the datum within an order. Its properties are the result of the ordering process, not vice versa.[5] The distance existing between classification and calculation is the same as that between 'substance' and 'function': in the first, the world is assumed; in the second, it constitutes itself.

If, as Cassirer has showed, the distinctive trait of modern thought is the concept of function, whereas ancient and premodern thought was based on the concept of substance, to identify modernity with classification is to mistake a feature that actually already existed in the past for a distinctive trait of the present. For example, Bauman (1995) has used the concept of classification as an interpretive key to the failure of the project of modernity to submit the world to a rational order created by man. His analysis revolves around the idea that classification always leaves 'ambivalent' zones, which reproduce themselves in a more persistent and destabilizing way the greater the effort is to eliminate them. But in reality, this limit does not concern calculation, where the reduction to formal representation has reached infinitesimal detail. The problem of the threshold separating an interval of values from another, which is most difficult in the case of the continuum, has been solved through the mathematical concept of 'limit'. For example, the point where the transition from the instant to the time interval occurs, or from stasis to movement, which has remained an unsolved paradox for classification ever since Greek thinkers first addressed it, can be exactly determined through the concept of 'derivative of a function'.

Even Lyotard's (1979) analysis of modernity appears to be incomplete. Lyotard poses the legitimation of knowledge as the central problem, and linguistic games as the model for analysis. In a nutshell, his key argument is the following: Among the various existing types of statements – denotative, prescriptive, performative, etc. – science only relies on sets of denotative ones, that is, on statements that indicate (denote) something (what is denoted) as given. But the linguistic game of science cannot legitimate its statements without having recourse to other types of statements. In fact, any denotation is merely *one* of many possible ways of signifying. In itself, there is nothing in denotation that makes it *the* way of signifying, the universally accepted representation of the datum. Science has hence always had a point of view in common with the world of ethics and politics. Linguistic games of the ethical and political type, which are performed at the scale of the collectivity as a whole and employ all types of statements, are what make possible the 'grand narratives' on which all that a society regards as valid, including science, is founded.

The shortcoming of this argument is that scientific theories do not contain exclusively denotative statements, but also tautological ones, which are self-evident and hence find their legitimation within themselves. Mathematical language is constituted by statements of this type. Only when one interprets mathematical data using intuitive metaphors (particles, waves, fields, etc.) does the argument gain all its strength and the scientific images of the world reveal all their 'narrative' relativity. This because a substantialist orientation still endures in those images, or, as Rorty (1999: 47–71) calls it, an 'essentialist' orientation. This orientation, however, is not the distinctive trait of modern science. The philosophical ideal of physics is the mathematical representation of the world (Galilei, 1980: 631–2), although the need to give an intuitive meaning to the mathematical frame of data has always induced scientists to introduce metaphysical, or rather ontological, elements in their theories. It remains doubtful and ultimately undecidable whether mass actually consists of the 'quantity of matter' (Mach, 1919: 216–22), or whether light is actually constituted by 'particles' or 'waves'. The only reference that remains certain and unaltered is the 'mathematical frame' of the data (Heisenberg, 1949: 1–12), that is, the component of scientific thought that is based on the mathematical function and not on its ontological interpretations (Totaro, 2010).

It is increasingly evident that the mathematical function regularly clashes with interpretations seeking to find an absolutely valid content for the physical relations it represents. Even the problem of the 'unification of fields', which constitutes the frontier of contemporary physics, seems to point in the same direction. In this research sector, one does find solutions, but only if one stays within the boundaries of mathematical abstraction, discarding the dimensions of space and time that we can visualize or imagine in favour of matrixes representing a six, ten or 11-dimensional space-time (Kaku, 1994; Green, 1999).

The same rebellion of the logic of the function against commixture with the logic of substance is observable in the practical world. The presumption that one could reduce ontological entities, individuals, to standardized ones through formal classification so that they could be included in algorithmic processes is what led to the crisis of the bureaucratic model. The 'system of rational action' (i.e. calculable acting) inevitably becomes part of a social matrix. Hence, the system is only one aspect of the structure of concrete interactions which, instead, involves individuals in their 'entirety' (Selznick, 1948: 25–6). Accordingly, formal administration schemes fail to adequately and completely take account of the concrete organization to which they relate. ‘Human relations’, which are beyond the technical plane, inevitably generate ‘informal groups’ (Blau, 1956; Blau and Scott, 1962) which exert a decisive influence on the actual functioning of social organization. Technocracy attempts to eliminate informal elements by exacerbating the formalization of processes, but thereby aggravates the problem rather than eliminating it. The more accentuated the formalization, the more the organization becomes blind to human relations and their dynamics. This leads to ‘vicious circles’ that determine the ‘structural inefficiency’ of the bureaucratic model (Crozier, 1963). Here too, as in physics, the concept of function, which is implicit in the algorithmic ideal of bureaucracy, clashes with the ontological element, the human factor. The obstinate application of the logic of calculation to the human factor causes the organization to wind up in a sort of loop, as happens with numerical functions when they are applied to an out-of-domain term.

The contradictions of bureaucracy have become unsustainable for capital itself. Since the 1980s, ‘horizontal’ dynamic collaboration through interdependent adjustments of activities has become the hub of profitability (Porter, 1985). It has turned out that partnership carries an ‘added value’ (VAP), which is now the crucial element for a good position on the market (Johnston and Lawrence, 1988). As Lambert and Peppard (2000: 464–5) have pointed out, the true news is not teamwork, which was already experienced within the 1950s, but the fact that, while back then the focus of collaboration was the product, now it is the synchronization of activities through information and communication technologies involving geographically dispersed groups. This passage was part of the major transformation that led to ‘networked society’ (Castells, 2000), characterized by a higher social integration in the relation between algorithms and human actions.

There always existed a certain duality in the action of algorithmic ‘systems’ as regards the question of social integration. In other essays (Totaro, 2009, 2010), we proposed a distinction of these systems into two types. The first includes systems encouraging a physical ‘distancing’ of the subjects of the interaction, a rarefaction of community relations, but allowing the interaction to be restored on this distanced plane (Harvey, 1990; Giddens, 1990). Systems belonging to this type include those functioning as mediators between market agents, such as modern money (Simmel, 2004; Giddens, 1990), the ‘price system’ (von Hayek, 1945), accounting techniques (Crosby, 1998: 211–35) and systems capable of transforming geographical areas into ‘scientific-technological-informational environments’, thereby contributing in a major way to a generalization of the urban life style (Santos, 1993: 35–47). All of these are founded on effective calculation, although, as we have seen in the previous sections with regard to economic science, their theoretical modelling can take place in the sphere of not effectively calculable functions. Following the terminology of Giddens (1990: 21), we called these systems 'time-space distancing systems'.

The second type includes algorithms relative to bureaucracy and bureaucratization.[6] These algorithms hinder social interaction not because they physically distance subjects, but because they force them to abandon the plane of communication to connect them according to the formal rules of a process. Unlike the systems of the first type, they are incapable of offering any form of communicative interaction to replace the one they have taken away (Crozier, 1963). We call them 'systems of logical distancing' to stress that in these systems distancing is not physical, as in the former systems, but a result of the encapsulation of action within algorithmic steps. In sum, systems of time-space distancing physically distance individuals but do not interrupt their communicative interaction, whereas systems of logical distancing do not physically distance individuals but interrupt their communicative interaction.

This difference reflects the different position of the actors with respect to the algorithm. In the case of time-space distancing, effective calculation mediates between the actors and an element, time-space, which, since it possesses a physical basis, appears as something placed outside calculation, as an object of its possible manipulation. It thus stands as a tool used by human beings to solve the problem of interaction at a distance.

The bureaucratic algorithm, on the contrary, does not generate an external problem, but one that is intrinsic to the very logic of organizations based upon it. The encapsulation of individuals within algorithmic steps is implicit in the concept of bureaucracy. Thus, in this case the algorithm is the cause of the problem, and hence cannot be its solution. Human beings cannot use the bureaucratic algorithm to solve the problem because it is they who are the tools of the algorithm, not vice versa.

The crisis of the bureaucratic model and the formation of 'networked society' has moved large masses of men from a condition of logical distancing to one of time-space distancing. This is the reason for the current rise, both in the economic and the cultural world, of an interest in the precognitive component of human activity and its potential capability to transform the concept of efficacy and the means to achieve it (Thrift, 2008; Rifkin, 2009). When used as an instrument for communicative interaction, calculation allows men to enter systems with all the pre-discursive knowledge they have accumulated. This is the opposite of what happens in logical distancing systems, where, as we have seen, the precognitive and social matrix is regarded as a residue to be eliminated, as it does not appear to be formally classifiable and reducible to calculation units.

It is clear that in the network, too, there exists a trend to a more or less explicit classification of the user and his acts, intended to transform him into an executor of algorithmic sequences planned by various interest groups (Lanier, 2010). But the network nevertheless still retains the structural possibility of being an instrument of 'communicative action', exactly in the sense that Habermas (1984) assigned to the concept.

It would be very important to understand to what degree the incompatibility between calculation and classification, which we have illustrated in this section, has a gnoseological foundation or is merely a historical phenomenon. This would allow us, in fact, to give more or less credit to the ability of calculation, intended as a medium of interaction, to impose itself as hegemonic compared to the calculation intended as manipulation of human agents. The path to follow to investigate this point, in our opinion, is the one pointed out by Maturana (Maturana and Varela, 1998), that is, the exploration of the biological foundation of knowledge. The fact that autopoietic systems self-organize through recursive processes may have a significant connection with the subjects discussed by the present article. In particular, we think that interesting surprises might lie in store if we checked the theory of autopoietic systems against the question of the 'contradictory consistency' between the discrete, to which 'Maturana' recursive processes are condemned, and the continuous, which is intuitively and constantly in front of our eyes.

Conclusions

Cassirer has almost incontrovertibly demonstrated that the concept of function, in its mathematical sense of univocal relation, is the characteristic trait of modern rationality in the theoretical sphere. However, contemporary philosophy and sociology claim, we believe correctly, that there is no manifestation of theoretical knowledge that does not have a correlate in social relations and everyday praxis. In the present essay, we argue that the correlate of the concept of function is the logic of recursive functions.

Recursive functions are logically equivalent to effectively calculable numeric functions, on the one hand, and algorithmic processes, on the other. This means that effective calculable numeric functions and algorithmic processes are based on the same logic, that of recursive functions. Algorithmic processes can be intuitively understood as operations that can manipulate non-numeric objects. Such is the case, as we have seen in detail, for labour in the 'manufacturing organization' and, in general, in the bureaucratic organization. In modern civilization, the logic of recursive functions is, often unconsciously, the main organizational and cognitive inspiration of production processes, and also, as we shall argue in other studies, of consumption practices.

Recursive functions are often applied to objects that are explicitly numerical, as in the case of the measurement and processing of the experimental data of the physical sciences, or of 'monetary accounting'.

Here, as we have seen when we analysed the relation between money and value, they are in a dialectic relation with non-recursive functions, as used in abstract theoretical thought. Mathematical functions reunite under the same logic the most representative macrophenomena of modernity, viz., bureaucracy, the market and science, but it is the recursive function that mediates between them. The recursive function not only informs the algorithms of bureaucracy but also finds application in the market, in capital and in science, when the need arises for effective calculation. Non-recursive functions remain outside the practical world, whereas recursive ones, besides being used in everyday and performative life, also play a fundamental role in theory. There could be no analytical calculation without effective calculation, since the former exploits the properties of the latter. While the concept of mathematical function in general allows a unified understanding of modern rationality, recursive functions are what make the unification possible.

Of course, the concept of function has not, with the rise of modernity, totally and instantaneously supplanted that of substance. Indeed, the concept of substance and the related concepts of category and classification are deeply embedded in western culture and are, possibly, necessary conditions for human knowledge. They have not only survived in modern culture but have intertwined with the hegemonic idea of mathematical function to generate contradictory hybrids. The most significant among these is the hybrid between the concept of classification and that of function, on which bureaucratic logic is based.

The notion that one can reduce individuals and objects to undifferentiated units by reuniting them into the same class, a class distinguished by a given property that is useful to the bureaucratic algorithm, has generated an insurmountable problem. It has made bureaucracy ideologically blind and powerless when confronted with communicative interaction, which is inseparable from human action. This has determined the crisis and obsolescence of the bureaucratic organizational model.

The systems that, instead, have represented the instruments for overcoming distances in space and time have had a different sort of impact. While they have indeed undermined community relations by supporting the markets' need for expansion and physically distancing their actors, they have also allowed social interaction to continue on this distanced plane.

In itself, there is nothing contradictory in the concept of classification. On the contrary, it is indispensable for intuition. However, it implies a 'realistic' and substantialist conception that may clash with the logic of the mathematical function. This occurs when one uses classification to reduce the ontological element to a calculation unit. It is what happened with bureaucratic systems, but also in modern physics, when the application of a substantialist classification – heat as 'caloric', light as corpuscles or waves, electricity as a fluid, etc. – reduced the intuitive datum to a single undifferentiated and quantitative element to give a univocal and absolute interpretation of the mathematical datum.

These contradictions are overcome – possibly to resurface at other levels, as Nicolescu (1996) argues – when logical and calculation instruments are used as instruments for communication. The discussion of this issue as specifically regards physics as a science goes beyond the scope of this article, but we would like to take leave of the reader with the following reflection: Isn't the principle of complementarity whereby Bohr (1928) solves the wave-particle dualism precisely a passage from a conception of the mathematical datum as a 'description' of phenomena – a recording of nature as it is – to one that sees the mathematical datum as an instrument to communicate a logically coherent interpretation of sensible experience?

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Notes

1. Namely, the class of numerical functions that operate on positive integers according to the 'Peano axioms' (Mendelson, 1964: 102 et seq.). It is important to stress that a numerical function is such that both the set of objects that it, so to speak, manipulates ('domain of the function') and the set of possible results of such manipulations ('co-domain of the function') are constituted by numbers.
2. In computability theory, '*effective'* and '*mechanical'* can be considered as equivalent and strictly related to the intuitive notion of algorithm.
3. There have been interesting and ingenious proposals for extending computability to the continuum of real numbers. One possibility is the recursion theory of real numbers developed by Moore (1996). Unfortunately, this theory appears to be somehow unphysical, because there are instances in which variables diverge to infinity. This means that a physical realization of a computer based on that theory may easily run out of resources. In any case, it has been argued that even models of analog computing need discreteness and finiteness conditions that can be strongly motivated on physical grounds (Trautteur and Tamburrini, 2007). It should be emphasized that the connection between computability issues (discrete versus continuum, for instance) and the way in which we develop our views and understanding of the real world is penetrating many aspects of the scientific and cultural debate (Cooper and Sorbi, 2011).
4. Classification is the methodological correlate of the 'generic concept', which is the nodal point both of Aristotle's logic and of his metaphysics. Through classification, Aristotelian science orders the elements of nature and deduces their essential properties (Cassirer, 1953: 3–26).

5. For instance, a point in the Cartesian plane acquires all the properties of a given curve – bowed upward or downward, concave or convex, etc. – only when the coordinates of that point are regarded as connected according to the rule expressed by the curve function. The properties do not belong to the point itself, but to the ordering according to which we think of the point.
6. As one of the authors of the present essay has argued elsewhere, the phenomenon of 'consumption' is itself an instance of bureaucratization (Totaro, 2009, 2010).

References

Bauman Z (1995) *Modernity and Ambivalence*. Cambridge: Polity Press.

Blau PM (1956) *Bureaucracy in Modern Society*. New York: Random House.

Blau PM and Scott RW (1962) *Formal Organization*. San Francisco: Chandler Publishing.

Bohr N (1928) The quantum postulate and the recent development of the atomic theory. *Nature* 121(Supplement): 580–590.

Bourdieu P (1984) *Distinction*. London: Routledge.

Bythell D (1983) Cottage industry and the factory system. *History Today* 33(4): 17–23.

Cassirer E (1953) *Substance and Function and Einstein's Theory of Relativity*. New York: Dover.

Cassirer E (1999) *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit, Vol. 1*. Hamburg: Meiner.

Castells M (2000) *The Rise of the Network Society*. Oxford: Blackwell.

Cooper SB and Sorbi A (eds) (2011) *Computability in Context*. London: Imperial College Press.

Crosby AW (1998) *The Measure of Reality*. Cambridge: Cambridge University Press.

Crozier M (1963) *Le Phénomène bureaucratique*. Paris: Seuil.

Davis M (1958) *Computability and Unsolvability*. New York: McGraw-Hill.

Galilei G (1980) *Opere, Vol. 1*. Torino: Utet.

Giddens A (1984) *The Constitution of Society: Outline of the Theory of Structuration*. Cambridge: Polity Press.

Giddens A (1990) *The Consequences of Modernity*. Cambridge: Polity Press.

Green B (1999) *The Elegant Universe*. New York: Norton.

Habermas J (1984) *The Theory of Communicative Action: Reason and Rationality of Society*. Boston: Beacon Press.

Harvey D (1990) *The Condition of Postmodernity*. Cambridge: Blackwell.

Hayek FA von (1945) The use of knowledge in society. *The American Economic Review* 4(35): 519–530.

Heisenberg W (1949) *The Physical Principles of the Quantum Theory*. New York: Dover.

Hobsbawm E (1996) *The Age of Revolution, 1789–1848*. New York: Vintage Books.

Hofstadter DR (1979) *Gödel, Escher, Bach: An Eternal Golden Braid*. New York: Basic Books.

Jevons WS (1970) *The Theory of Political Economy*. Middlesex: Penguin.

Johnston R and Lawrence PR (1988) Beyond vertical integration: The rise of the value-adding partnership. *Harvard Business Review* 66(4): 94–101.
Kaku M (1994) *Hyperspace*. Oxford: Oxford University Press.
Kriedte P, Medick H and Schlumbohm J (1981) *Industrialization before Industrialization*. Cambridge: Cambridge University Press.
Lambert R and Peppard J (2000) The information technology–organizational design relationship. In: Galliers RD, Leidner DE and Baker BSH (eds) *Challenges and Strategies in Managing Information Systems*. Oxford: Butterworth Heinemann.
Lanier J (2010) *You Are Not a Gadget*. New York: A. Knopf.
Lyotard J (1979) *La condition postmoderne*. Paris: Les Editions de Minuit.
Mach E (1919) *The Science of Mechanics*. Chicago: Open Court Publishing.
Mantoux P (1961) *The Industrial Revolution in the Eighteenth Century: An Outline of the Beginnings of the Modern Factory System in England*. London: Jonathan Cape.
Marx K (1952) *Capital*. Chicago: Encyclopaedia Britannica.
Maturana HR and Varela FJ (1998) *The Tree of Knowledge*. Boston: Shambhala Publications.
Mendelson E (1964) *Introduction to Mathematical Logic*. Princeton, NJ: D. Van Nostrand.
Minsky ML (1967) *Computation: Finite and Infinite Machines*. Englewood Cliffs, NJ: Prentice-Hall.
Moore C (1996) Recursion theory on the reals and continuous-time computation. *Theoretical Computer Science* 162(1): 23–44.
Nicolescu B (1996) *La transdisciplinarité*. Monaco: Èdition du Rocher.
Odifreddi PG (1992) *Classical Recursion Theory*. Amsterdam: Elsevier.
Patterson R (1957) Spinning and weaving. In: Singer C, et al. (eds) *A History of Technology, Vol. III*. Oxford: Oxford University Press.
Porter ME (1985) *Competitive Advantage*. New York: Free Press.
Ricardo D (1971) *On the Principles of Political Economy, and Taxation*. Middlesex: Penguin.
Rifkin J (2009) *The Empathic Civilization*. New York: Tarcher.
Rorty R (1999) *Philosophy and Social Hope*. New York: Penguin Books.
Santos M (1993) *A urbanização brasileira*. São Paulo: Hucitec.
Selznick P (1948) Foundations of the theory of organization. *American Sociological Review* 13(1): 25–35.
Simmel G (2004) *The Philosophy of Money*. London: Routledge & Kegan Paul.
Smith A (1952) *An Inquiry into the Nature and Causes of the Wealth of Nations*. Chicago: Encyclopaedia Britannica.
Taylor FW (1911) *The Principles of Scientific Management*. New York: Harper & Brothers.
Thrift N (2008) *Non-representational Theory*. New York: Routledge.
Totaro P (2009) Le origini logiche della disaggregazione sociale. *Rassegna Italiana di Sociologia* 50(2): 227–250.
Totaro P (2010) *A cultura do cálculo e a desagregação social*. PhD thesis. Available at: http://biblioteca.asav.org.br/vinculos/tede/PaoloTotaroCienciasSociais.pdf (accessed 13 September 2013).
Trautteur G and Tamburrini G (2007) A note on discreteness and virtuality in analog computing. *Theoretical Computer Science* 371(1–2): 106–114.

Turing AM (1937) On computable numbers, with an application to the Entscheidungsproblem. *Proceedings of the London Mathematical Society* s2–42(1): 230–265.

Unwin G (1904) *Industrial Organization in the Sixteenth and Seventeenth Centuries*. Oxford: The Clarendon Press.

Walker PG (1966) The origins of the machine age. *History Today*, 16 September: 591–600.

Walras L (1938) *Abrégé des éléments d'économie politique pure*. Paris: Librairie Générale de Droitet de Jurisprudence.

Weatherford J (1997) *The History of Money*. New York: Three Rivers Press.

Weber M (1978) *Economy and Society*. Berkeley: University of California Press.

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