Art Mathematics

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1 Notation

See [**?**] for this notation.

$<$ - Numeric less than.

$>$ - Numeric more than.

$\wedge$ - Logical and.

$\vee$ - Logical or.

$\not$ Logical not

$\in$ - Set membership.

$\cup$ - Set union.

$\cap$ - Set Intersection.

$\setminus$ - Set difference.

$\#$ - Set cardinality.

$=$ - Numeric or set equality.

$\sum$- Numeric or aesthetic sum.

(a, b) - An ordered pair. Also representable as a set [**?**]:
$\{\{a\}, \{a, b\}\}$
We use this notation when manipulating the elements of an ordered pair.

$\{a, b\}$ - A set. The set consists of the listed elements.

$\{A \mid B \bullet C\}$ - A set comprehension. The members of the set are the result of the expression C, which is applied to every value of the variables introduced in A for which B is true.

2 Ordered Pairs

2.1 The First And Second Items Of An Ordered Pair

An ordered pair can be represented as a set.

$p = (1, 2) = \{\{1\}, \{1, 2\}\}$

The first set of the pair is the same as the intersection of the first and second member sets.

$\{1\} \cap \{1, 2\} = \{1\}$

$\cap \{\{1\}, \{1, 2\}\} = \{1\}$

$\cap p = \cap \{\{1\}, \{1, 2\}\} = \{1\}$

The second set of the pair is the same as the union of the first and second member sets.

$\{1\} \cup \{1, 2\} = \{1, 2\}$

$\cup \{\{1\}, \{1, 2\}\} = \{1, 2\}$

$\cup p = \cap \{\{1\}, \{1, 2\}\} = \{, 21\}$

The first item of the pair is the only item of the intersection of the first and second member sets.

$X = (\cap p) \cap (\cup p) = \{1\} \cap \{1, 2\} = 1$

$x \in X$

$x = 1$

The second item of the pair is the difference of the second member set with the first member set.

$Y = (\cap p) \setminus (\cup p) = \{1, 2\} \setminus \{1\} = \{1\}$

$y \in Y$

$y = 2$

The sum of the pair is the sum of the second member set of the pair.

$z = \sum \cup p$

$Z = \cup \{\{1\}, \{1, 2\}\} = \{1, 2\}$

$z = \sum Z = 1 + 2 = 3$

2.2 Sets of Ordered Pairs

We can sum each ordered pair in a set.

$\{x :\in X \bullet \sum x\}$

We can get all the first items of a set of pairs

$S = \{(1, 2), (3, 4), (5, 6)\}$

$T = \{x :\in S \bullet \cup x\} = \{1, 3, 5\}$

And all the second items

$V = \{(1, 2), (3, 4), (5, 6)\}$

$W = \{x :\in S \bullet x\} = \{2, 4, 6\}$

Some items in some pairs may be the same.

$P = \{(1, 2), (1, 4), (5, 4)\}$

$Q = \{x :\in S \bullet \cup x\} = \{1, 1, 5\}$

So the cardinality may be different.

$\#S = \#T$

$\neg(\#P = \#Q)$

If the cardinality of the set of all the first items is equal to the cardinality of the set of pairs, each first item is unique.

$\#U = \#\ S$

If the cardinality of the set of all the first items is less than the cardinality of the set of pairs, not every first item is unique.

$\neg(\#R = \#P)$

We can get every member of a power set of ordered pairs with unique first items.

$A = \{\{(1, 2), (1, 4), (5, 4)\}, \{(1, 2), (1, 4), (5, 4)\}\}$

$B = \{a :\in A | \#a = (\# \{x \in a \bullet \cup x\})\}$

And we can sum that

$A = \{\{(1, 2), (1, 4), (5, 4)\}, \{(1, 2), (1, 4), (5, 4)\}\}$

$B = \{a \in A | \#a = (\# \{x \in a \bullet \cup x\})\#\}$

3 Colour

3.1 Primaries

$C = \{{\color{blue}\blacksquare}, {\color{red}\blacksquare}, {\color{yellow}\blacksquare}\}$
${\color{blue}\blacksquare} < {\color{red}\blacksquare} < {\color{yellow}\blacksquare}$

3.2 Secondaries

$C' = \{{\color{purple}\blacksquare}, {\color{green}\blacksquare}, {\color{orange}\blacksquare}\}$
${\color{purple}\blacksquare} < {\color{green}\blacksquare} < {\color{orange}\blacksquare}$

3.3 Tertiaries

$C'' = \{{\color{Goldenrod}\blacksquare}, {\color{RedOrange}\blacksquare}, {\color{Magenta}\blacksquare}, {\color{Violet}\blacksquare}, {\color{LimeGreen}\blacksquare}, {\color{Aquamarine}\blacksquare}\}$
${\color{Goldenrod}\blacksquare} < {\color{RedOrange}\blacksquare} < {\color{Magenta}\blacksquare} < {\color{Violet}\blacksquare} < {\color{LimeGreen}\blacksquare} < {\color{Aquamarine}\blacksquare}$

3.4 Achromatics

$\neg C = \{\blacksquare, {\color{gray}\blacksquare}, \square\}$
$\blacksquare < {\color{gray}\blacksquare} < \square$

4 Shape

$S = \{\cdot, \bigcirc, |, \triangle, \square\}$

$L = \{|, /, —, \backslash\}$

$A = \{), (, \frown, \smile\}$

5 Shape and Colour

$\square + {\color{blue}\blacksquare} = {\color{blue}\blacksquare}$
${\color{blue}\blacksquare} - {\color{blue}\blacksquare} = \square$
$\bigcirc + {\color{red}\blacksquare} = {\color{red}\bullet}$
${\color{red}\bullet} - {\color{red}\blacksquare} = \bigcirc$

6 Sol LeWitt

6.1 Straight lines in four directions & all their possible combinations

$L = \{|, /, —, \backslash\}$
$M = \mathcal{P}L$
$N = \{x \in M \bullet \sum x\}$

6.2 All one-, two-, three- & four part combinations of four transparent colours

$C = \{{\color{blue}\blacksquare}, {\color{red}\blacksquare}, {\color{yellow}\blacksquare}, \blacksquare\}$
$D = \mathcal{P}C$
$E = \{x \in D \bullet \sum x\}$

6.3 All one-, two-, three- & four part combinations of lines in four directions and in four colours

$C = \{{\color{red}\blacksquare}, {\color{yellow}\blacksquare}, {\color{blue}\blacksquare}, \blacksquare\}$
$L = \{|, , /, -\}$
$V = C \times L = \{({\color{red}\blacksquare}, |), \ldots (\blacksquare, -)\}$
$W = PV = \{\{({\color{red}\blacksquare}, |)\}, \ldots \{({\color{red}\blacksquare}, |), \ldots (\blacksquare, -)\}\}$
$X = \{x :\in W \mid \#x > 0 \#x < 5\}$
$Y = \{x :\in X \mid \#x = \# \cup (\cup x)\}$
$Z = \{x :\in Y, y :\in x \bullet \cup(\cup y)\}$

Alternative Z:

$Z = \{x :\in Y \bullet \cup(\cup x)\}$

6.4 Various lengths of representation for "All one-, two-, three- & four part combinations of lines in four directions and in four colours"

$3 : x \in X$

$4 : x :\in X$

$5 : \{x \in X\}, x :\in \mathcal{P}C$

$6 : \{x \in \mathcal{P}C\}, x \in X \bullet \sum x$

$7 : x :\in X \bullet \sum x, \{x :\in \mathcal{P}C\}, x \in \mathcal{P}C \bullet \sum x, \mathcal{P}\{kyrb\}$

$8 : \{x \in X \bullet \sum x\}$

$9 : \{x :\in X \bullet \sum x\}$

$10 : Y = \{x \in X \bullet \sum x\}$

$11 : Y = \{x :\in X \bullet \sum x\}, Y = \{x \in \mathcal{P}C \bullet \sum x\}$

$12 : Y = \{x :\in \mathcal{P}C \bullet \sum x\}$

$15 : x \in \mathcal{P}\{k, r, y, b\} \bullet \sum x, \{x :\in \mathcal{P}\{kryb\} \bullet \sum x\}$

7 Damien Hirst

7.1 Spot Paintings

$s(x) = \exists y | x = \frac{1}{2}(2y^2)$
$A = \mathcal{P}\left\{hsv\right\}$
$B = \{x \in A \mid s(\#x)\}$
$D = \{x \in B \bullet x + \bigcirc\}$

8 Godel Numbering

Godel Numbering, named after Austrian mathematician Kurt Godel, is the encoding of mathematical statements into numbers by some arbitrary scheme. The scheme used here is derived from that used by Douglas Hofstadter in "Godel, Escher, Bach: an Eternal Golden Braid" [**?**].

Several post-conceptual artists have used numbers as the content of their artworks. Godel Numbering a set representation of an artwork refers to this among other things.

8.1 Notation

Using a variant of Hofstadter's scheme:

$\{$	11
$\}$	12
,	13
$\vert$	21
$\bullet$	22
$=$	23
$\in$	31
$\mathcal{P}$	41
$\#$	42
$<$	43
$\sum$	44
x	51
C	61
D	74
E	75
$\blacksquare$	81
$\blacksquare$	82
$\blacksquare$	82
$\blacksquare$	84
hsv	89
0	91
S	92

8.2 All one-, two-, three- & four part combinations of four transparent colours

612311811382138313841312

74236141641

7523115131742142514392929292929122445112

or

115131118113821383138413124111811382138313841312214251439292929292912244 5112

9 Saville Colouring

Saville colouring, named here after British graphic designer Peter Saville, is the encoding of numeric or textual information into arbitrary sequences of colours. Such a scheme was used by Saville on a number of record covers for Factory Records in the 1980s [?], with the key being placed on the back cover of New Order's "Power, Corruption and Lies", 1983.

The colours Saville used to represent the numbers 1..9 are (in CMYK):

1	70C 40Y
2	100Y
3	50C 50M
4	60M 100Y
5	40C
6	40M
7	40M 80C
8	100M
9	100C

A more generic colour scheme would be:

1	Cyan
2	Magenta
3	Yellow
4	Black
5	Red
6	Green
7	Blue
8	Orange
9	Purple

Many artists have used arbitrary grids or other arrays of arbitrary colours as the content of their artworks. Saville Colouring a set representation of an artwork refers to this among other things.

10 Lines and Curves

References

[Dean] Neville Dean. *The Essence Of Computing: Discrete mathematics.* Prentice Hall, London, 1997.

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[Saville] Peter Saville. *Annotated artwork for the New Order album "Power Corruption and Lies".* Creative Review, January 2003 (Front Cover).

[Suppes] Patrick Suppes. *Axiomatic Set Theory.* Dover Publications Inc., New York, 1972.