Chapter 8: Planning and Learning with Tabular Methods

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Major Themes of Chapter 8

1. Unifying *planning* and *learning* methods
   - Model-based methods rely on planning
   - Model-free methods rely on learning
2. Benefits of planning in small, incremental steps
   - Key to efficient combination of planning and learning

Model of the Environment

- Anything the agent can use to predict the environment
- Used to mimic or *simulate* experience

1. *Distribution models*
    - Describe all possibilities and probabilities
    - Stronger but difficult to obtain
2. *Sample models*
    - Produce one possibility sampled according to the probabilities
    - Weaker but easier to obtain

Planning

- Process of using a model to produce / improve policy
- *State-space Planning*
  - Search through the state space for optimal policy
  - Includes RL approaches introduced until now
- *Plan-space Planning*
  - Search through space of plans
  - Not efficient in RL
  - ex) Evolutionary methods

$$\text{model} \xrightarrow{\text{planning}} \text{policy}$$

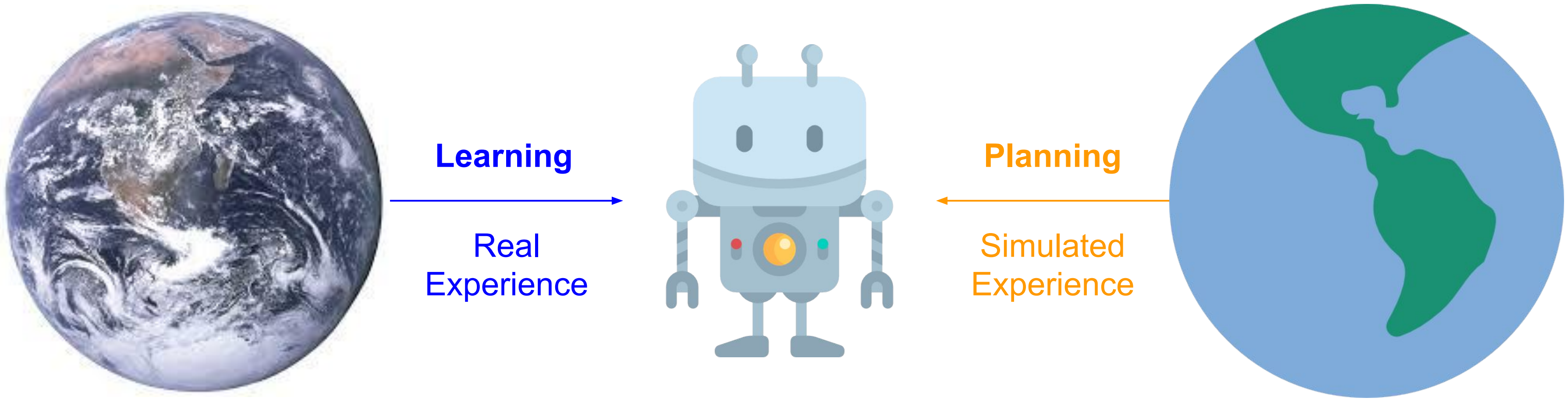
State-space Planning

- All state-space planning methods share common structure
  - Involves **computing value functions**
  - Compute value functions by **updates or backup operations**

- ex) Dynamic Programming

Relationship between Planning and Learning

- Both estimate value function by backup update operations
- *Planning* uses simulated experience from model
- *Learning* uses real experience from environment



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Random-sample one-step tabular Q-planning

- Planning and Learning is similar enough to transfer algorithms
- Same convergence guarantees as one-step tabular Q-learning
  - All states and actions selected infinitely many times
  - Step size decrease over time
- Need just the *sample model*

Random-sample one-step tabular Q-planning

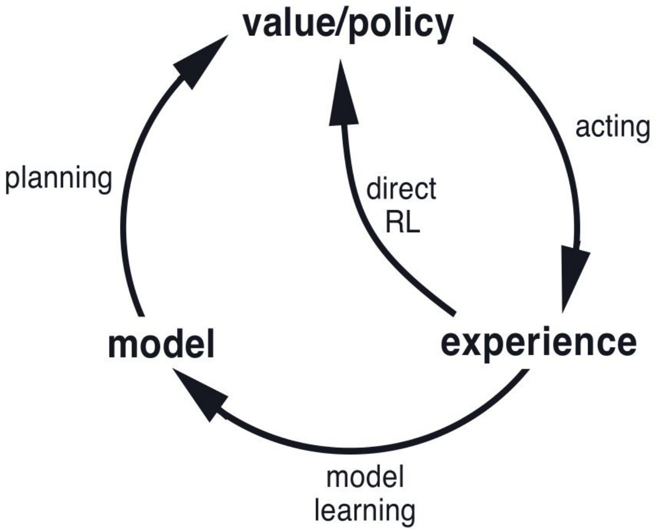
Random-sample one-step tabular Q-planning

Loop forever:

1. Select a state, $S \in \mathcal{S}$, and an action, $A \in \mathcal{A}(S)$, at random
2. Send S, A to a sample model, and obtain
   a sample next reward, R, and a sample next state, S'
3. Apply one-step tabular Q-learning to S, A, R, S':
   $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$

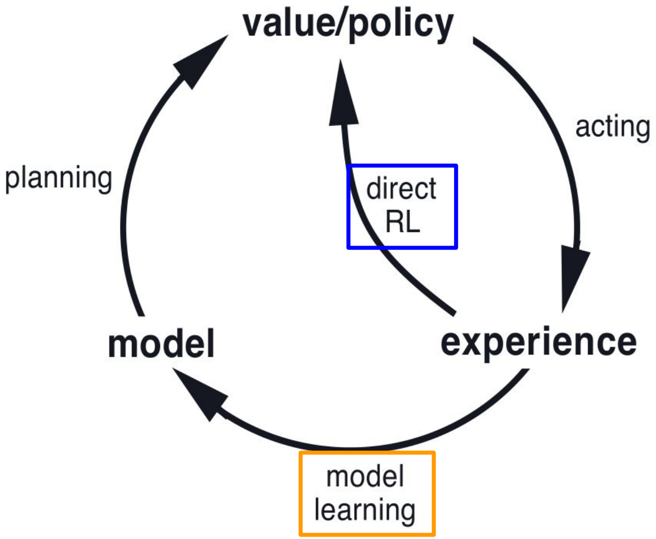
On-line Planning

- Need to incorporate new experience with planning
- Divide computational resources between decision making and model learning



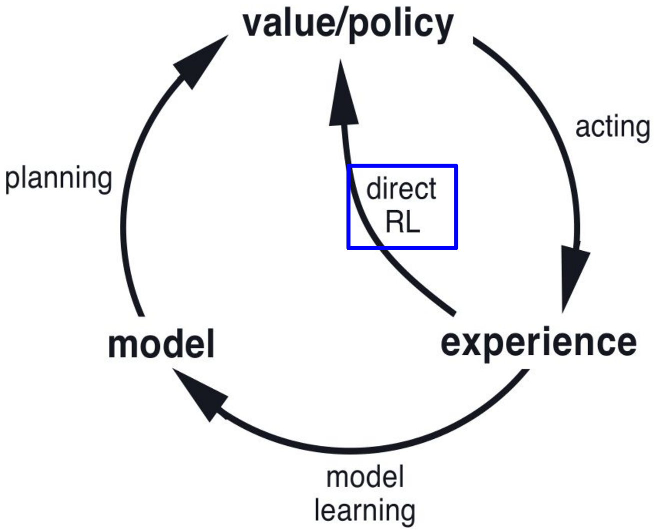
Roles of Real Experience

1. *Model learning*: Improve the model to increase accuracy
2. *Direct RL*: Directly improve the value function and policy



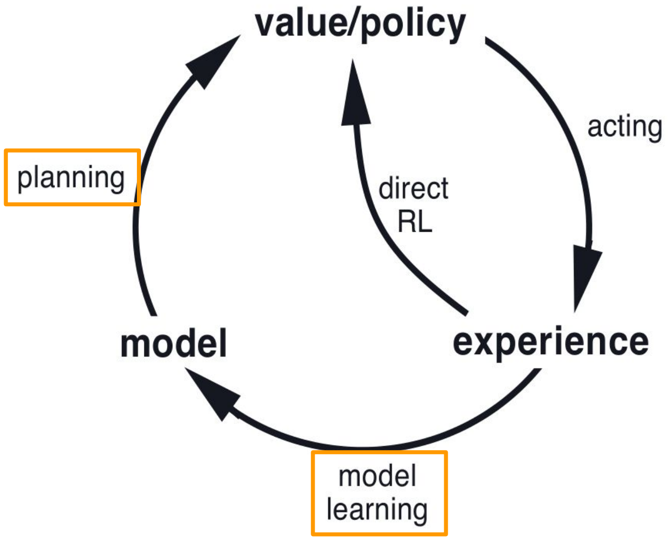
Direct Reinforcement Learning

- Improve value functions and policy with real experience
- Simpler and not affected by bias from model design



Indirect Reinforcement Learning

- Improve value functions and policy by improving the model
- Achieve better policy with fewer environmental interactions



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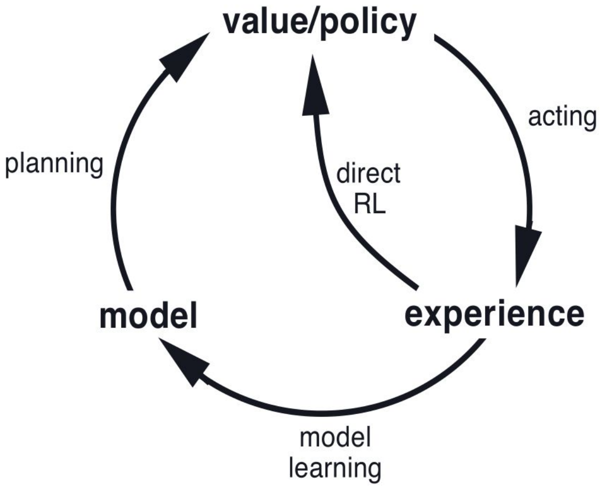
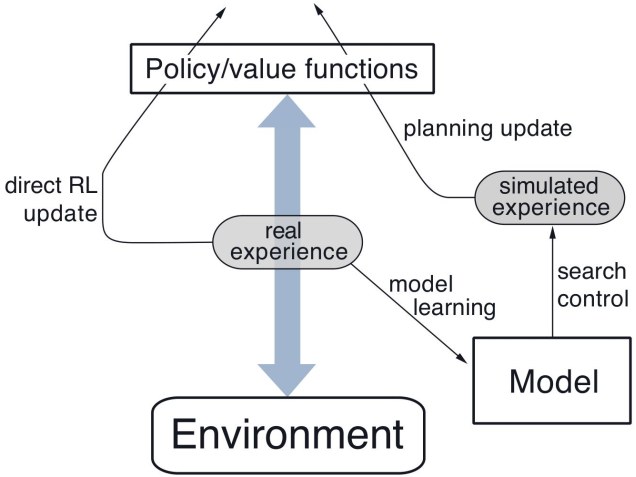
Dyna-Q

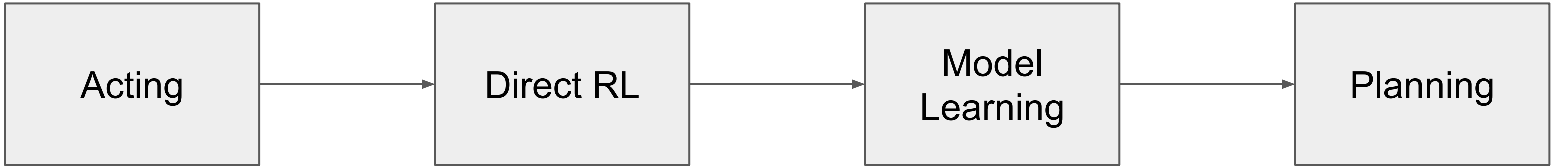
- Simple on-line planning agent with all processes
  - Planning: random-sample one-step tabular Q-planning
  - Direct RL: one-step tabular Q-learning
  - Model learning: Return last observed next state and reward as prediction





Dyna-Q: Implementation

- Order of process on a serial computer



- Can be parallelized to a *reactive, deliberative* agent
  - *reactive*: responding instantly to latest information
  - *deliberative:* always planning in the background
- Planning is most computationally intensive
  - Complete n iteration of Q-planning algorithm on each timestep

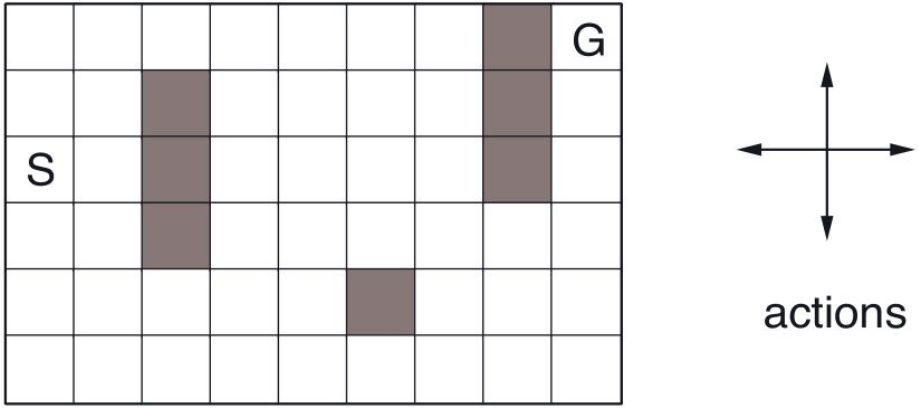
Dyna-Q: Pseudocode

Tabular Dyna-Q

Initialize $Q(s,a)$ and $Model(s,a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
Loop forever:
(a) $S \leftarrow$ current (nonterminal) state
(b) $A \leftarrow \varepsilon$-greedy(S,Q)
(c) Take action A; observe resultant reward, R, and state, S' — Acting
(d) $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$ — Direct RL
(e) $Model(S,A) \leftarrow R, S'$ (assuming deterministic environment) — Model Learning
(f) Loop repeat n times: — Planning
  $S \leftarrow$ random previously observed state
  $A \leftarrow$ random action previously taken in S
  $R, S' \leftarrow Model(S,A)$
  $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$

Dyna Maze Example

- Only reward is on reaching goal state (+1)
  - Takes long time for reward propagate



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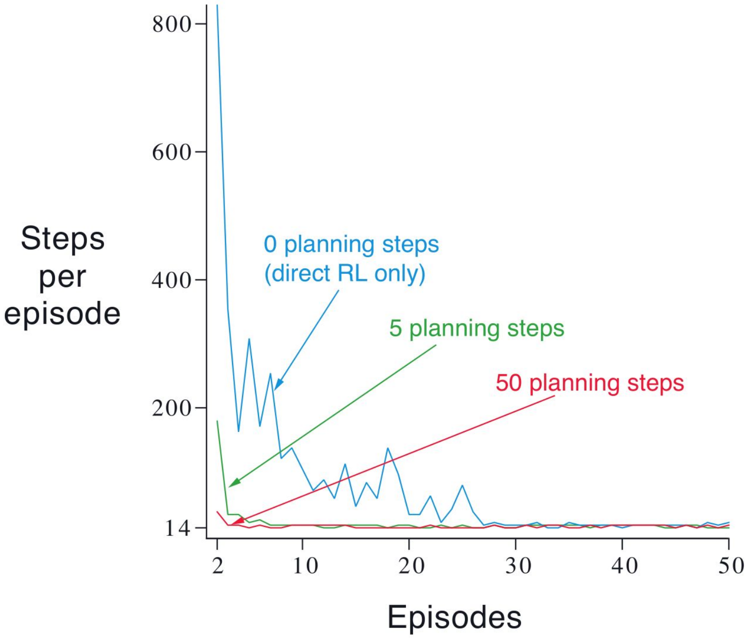
Dyna Maze Example: Result

- Much faster convergence to optimal policy



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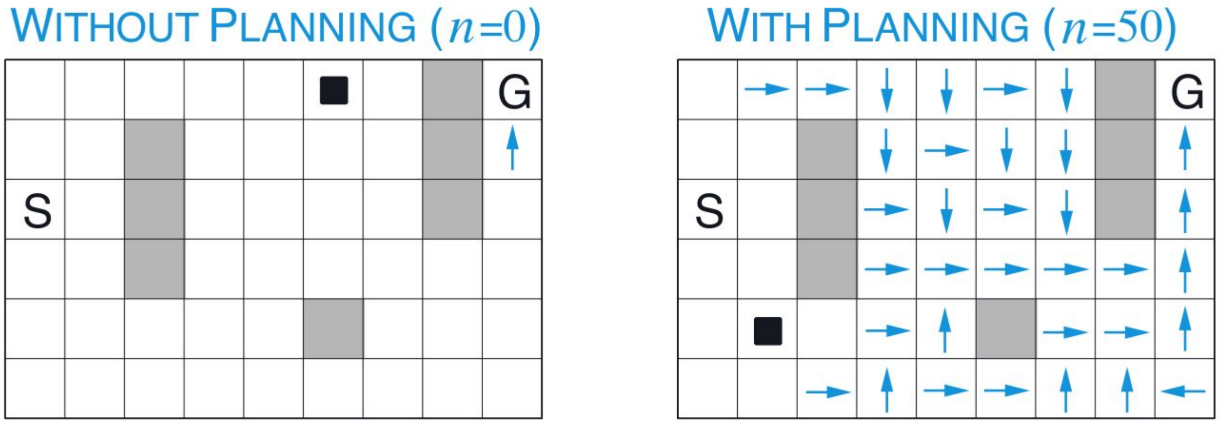
Dyna Maze Example: Intuition

- Without planning, each episode adds only one step to the policy
- With planning, extensive policy is developed by the end of episode



Halfway through second episode

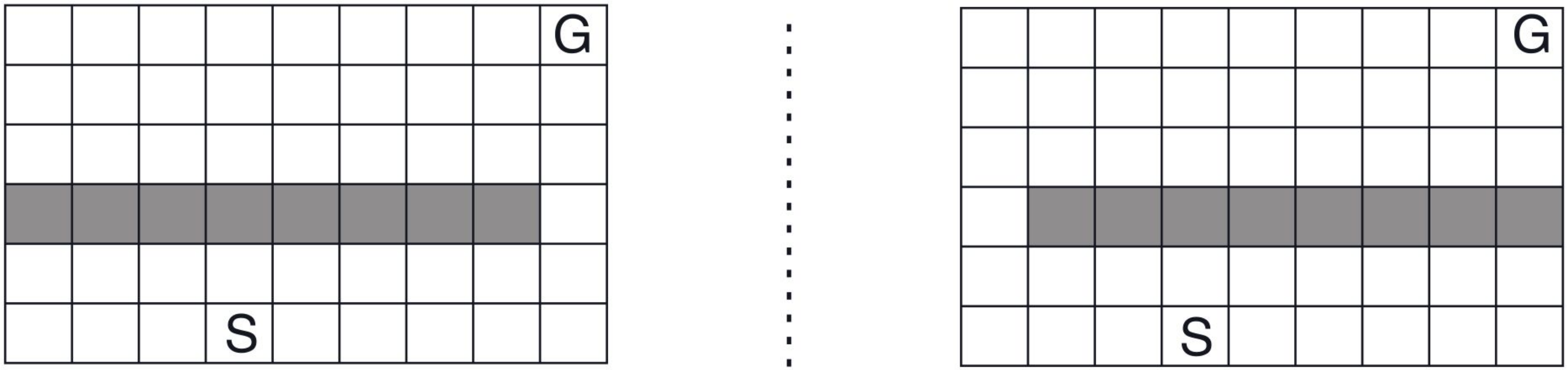
Black square ■ : location of the agent

Possibility of a Wrong Model

- Model can be wrong for various reasons:
  - Stochastic environment & limited number of samples
  - Function approximation
  - Environment has changed
- Can lead to suboptimal policy

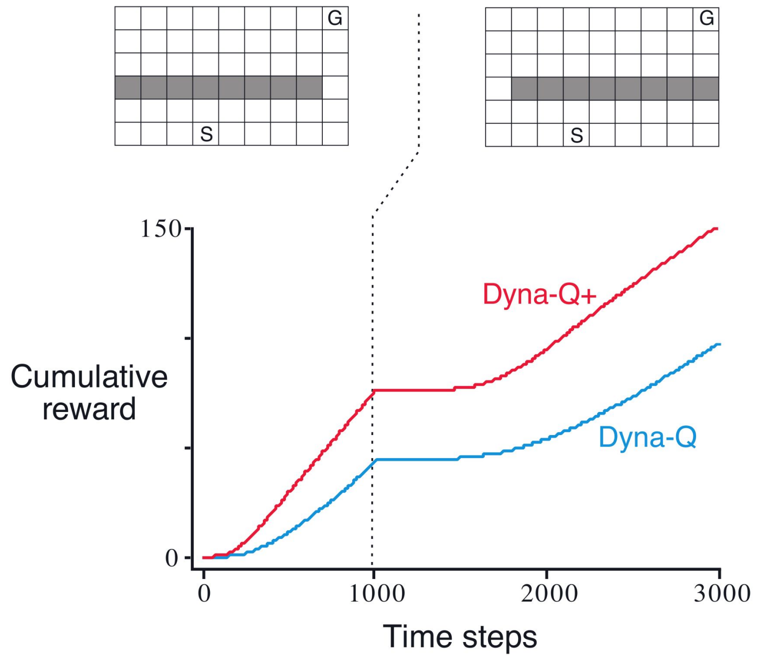
Optimistic Wrong Model: Blocking Maze Example

- Agent can correct modelling error for optimistic models
  - Agent attempts to “exploit” false opportunities
  - Agent discovers they do not exist



Environment changes after 1000 timesteps

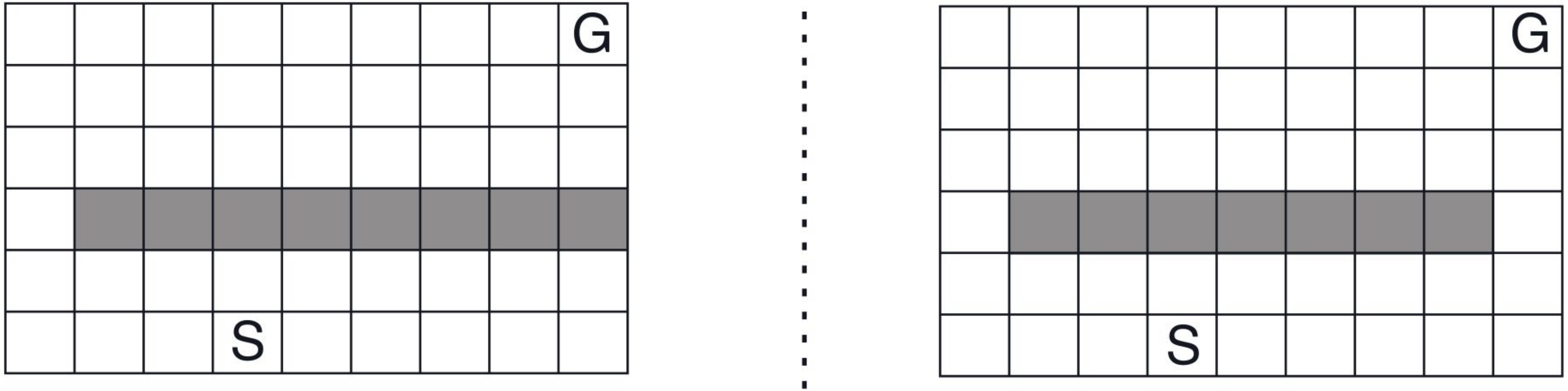
Optimistic Wrong Model: Blocking Maze Example



(Ignore Dyna-Q+ for now)

Pessimistic Wrong Model: Shortcut Maze Example

- Agent might never learn modelling error for optimistic models
  - Agent never realizes a better path exists
  - Unlikely even with an ε-greedy policy



Environment changes after 1000 timesteps

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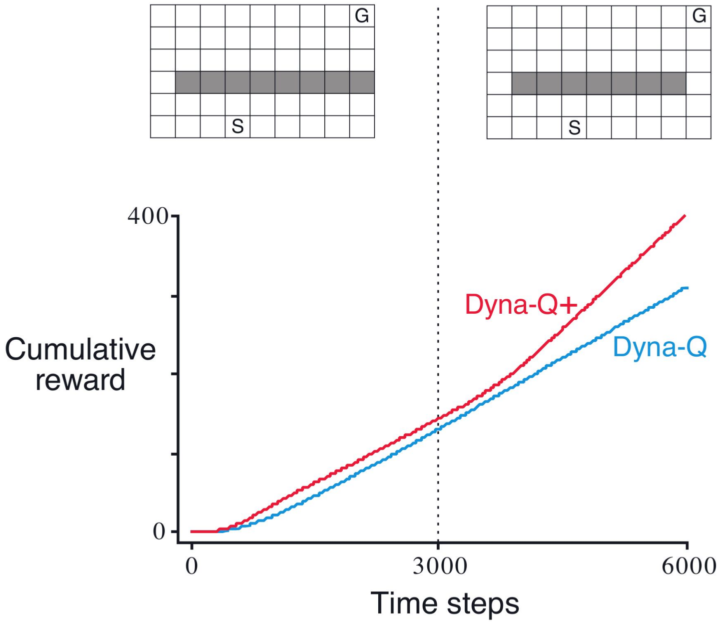
Pessimistic Wrong Model: Shortcut Maze Example



(Ignore Dyna-Q+ for now)

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Dyna-Q+

- Need to balance exploration and exploitation
- Keep track of elapsed time τ since last visit for each state-action pair
- Add bonus reward $\kappa\sqrt{\tau}$ during *planning*
- Allow untried state-action pair to be visited in *planning*

- Costly, but the computational curiosity is worth the cost

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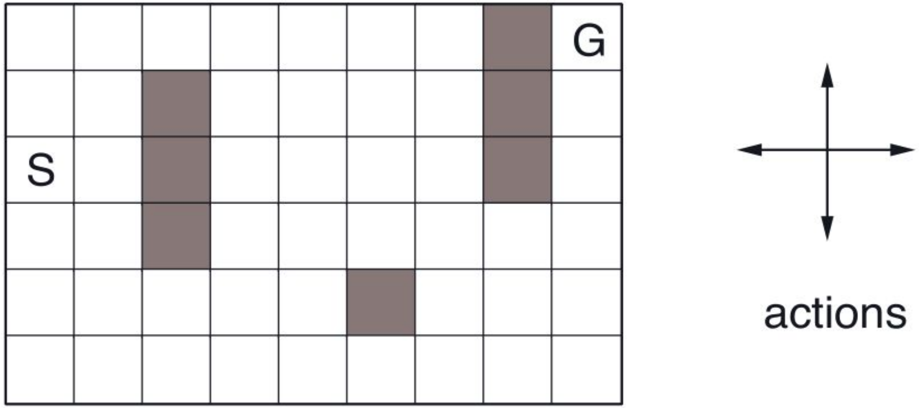
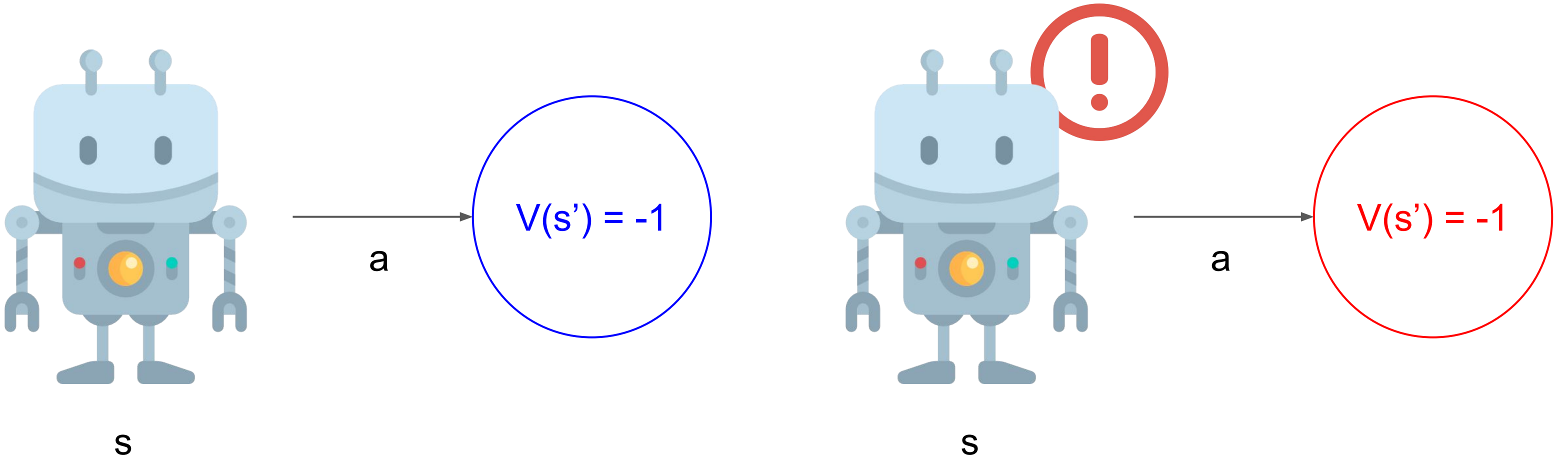
Prioritized Sweeping: Intuition

- *Focused* simulated transitions and updates can make planning more efficient
  - At the start of second episode, only the penultimate state has nonzero value estimate
  - Almost updates do nothing



Prioritized Sweeping: Backward Focusing

- Intuition: work backward from “goal states”
- Work back from any state whose value changed
  - Typically implies other states’ values also changed
- Update predecessor states of changed state



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Prioritized Sweeping

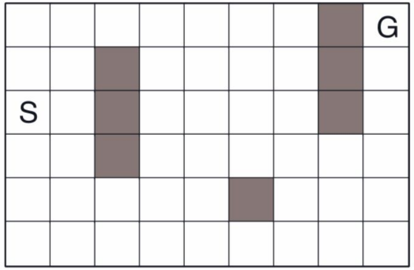
- Not all changes are equally useful
  - magnitude of change in value
  - transition probabilities
- Prioritize updates via *priority queue*
  - Pop max-priority pair and update
  - Insert all predecessor pairs with effect above some small threshold
    - (Only keep higher priority if already exists)
  - Repeat until quiescence

Prioritized Sweeping: Pseudocode

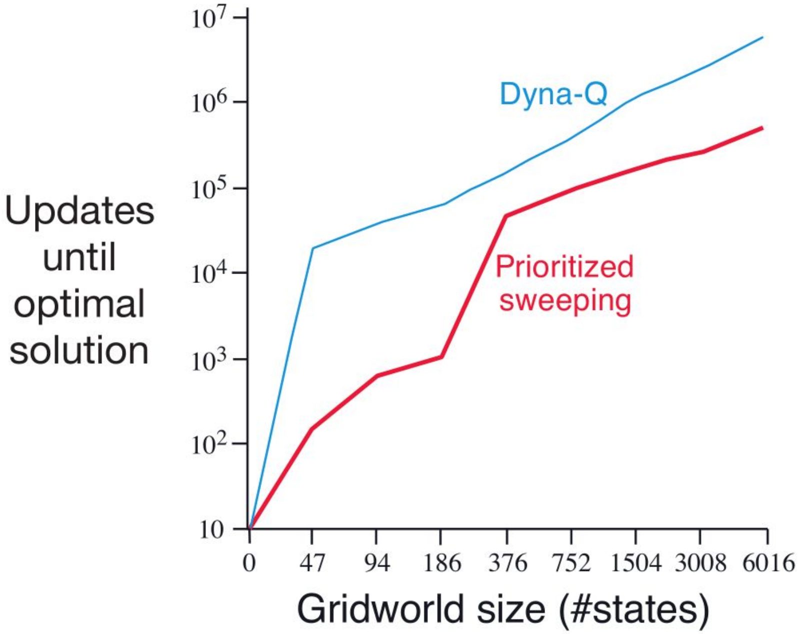
Prioritized sweeping for a deterministic environment

Initialize $Q(s, a)$, $Model(s, a)$, for all s, a, and $PQueue$ to empty
Loop forever:
(a) $S \leftarrow$ current (nonterminal) state
(b) $A \leftarrow policy(S, Q)$
(c) Take action A; observe resultant reward, R, and state, S'
(d) $Model(S, A) \leftarrow R, S'$
(e) $P \leftarrow |R + \gamma \max_a Q(S', a) - Q(S, A)|$.
(f) if $P > \theta$, then insert S, A into $PQueue$ with priority P
(g) Loop repeat n times, while $PQueue$ is not empty:
$S, A \leftarrow first(PQueue)$
$R, S' \leftarrow Model(S, A)$
$Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
Loop for all $\bar{S}, \bar{A}$ predicted to lead to S:
$\bar{R} \leftarrow$ predicted reward for $\bar{S}, \bar{A}, S$
$P \leftarrow |\bar{R} + \gamma \max_a Q(S, a) - Q(\bar{S}, \bar{A})|$.
if $P > \theta$ then insert $\bar{S}, \bar{A}$ into $PQueue$ with priority P

Prioritized Sweeping: Maze Example

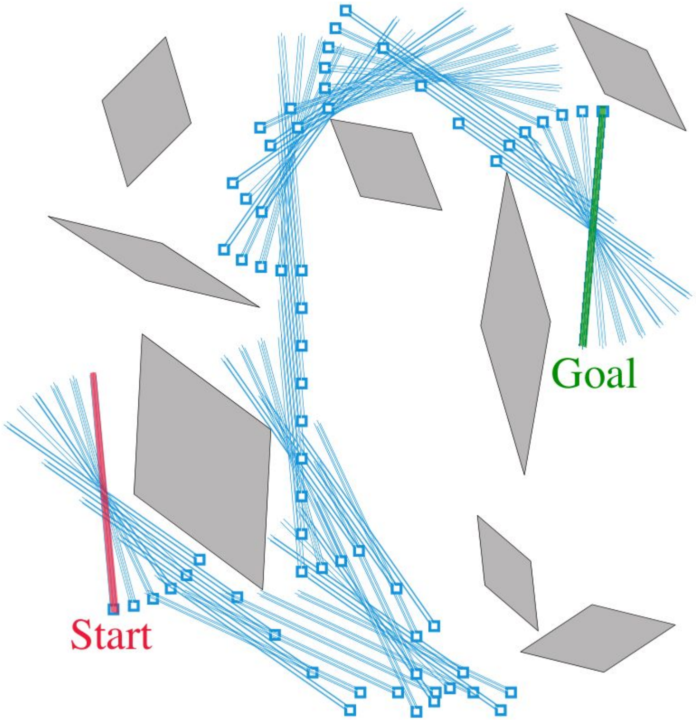


- Decisive advantage over unprioritized Dyna-Q



Prioritized Sweeping: Rod Maneuvering Example

- Maneuver rod around obstacles
    - 14400 potential states, 4 actions (translate, rotate)
- Too large to be solved without prioritization

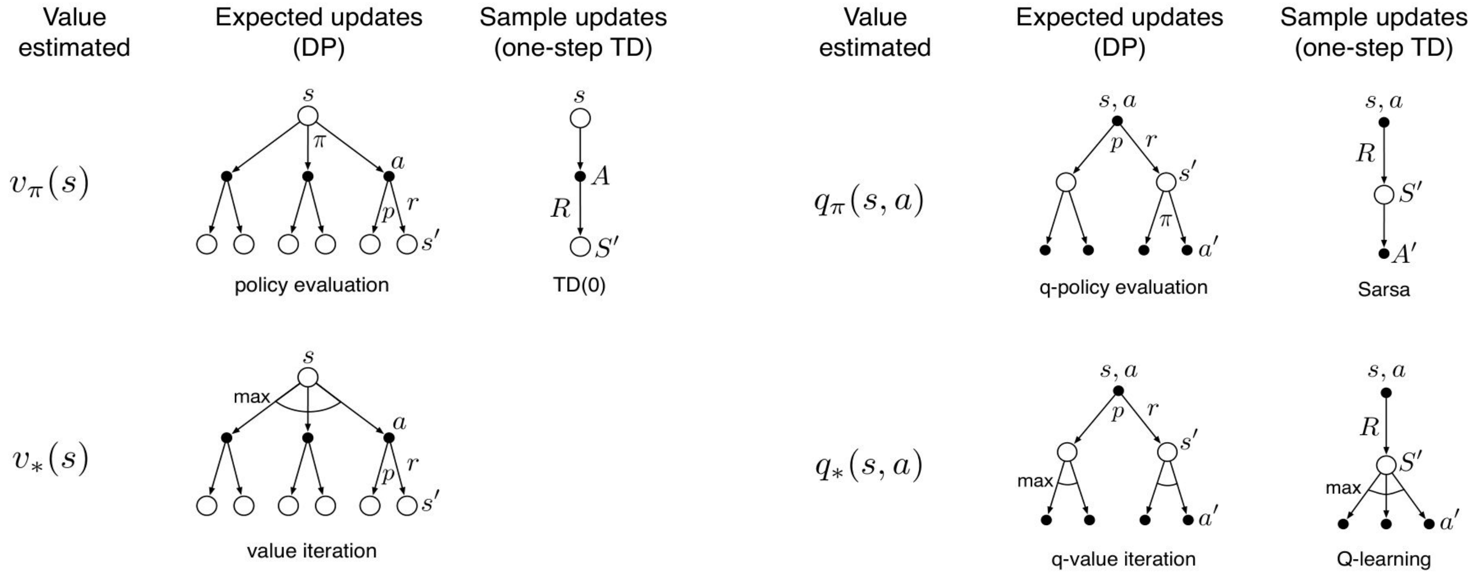


Prioritized Sweeping: Limitations

- Stochastic environment
  - Use expected update instead of sample updates
  - Can waste computation on low-probability transitions
- How about sample updates?

Expected vs Sample Updates

- Recurring theme throughout RL
- Any can be used for planning



Expected vs Sample Updates

- Expected updates yields better estimate

$$Q(s,a) \leftarrow \sum_{s',r} \hat{p}(s',r|s,a)\Big[r + \gamma \max_{a'} Q(s',a')\Big].$$

- Sample updates requires less computation

$$Q(s,a) \leftarrow Q(s,a) + \alpha\Big[R + \gamma \max_{a'} Q(S',a') - Q(s,a)\Big],$$

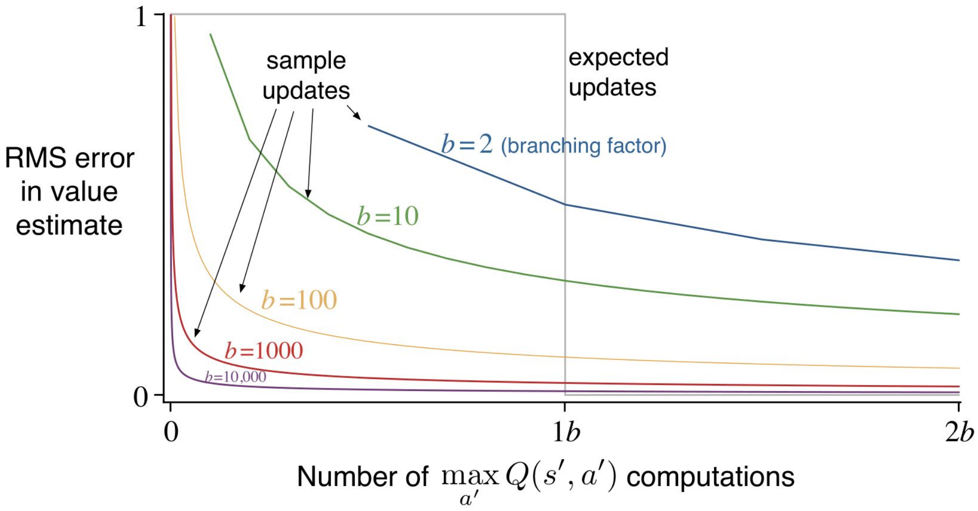
Expected vs Sample Updates: Computation

- *Branching factor d*: number of possible next states
    - Determines computation needed for expected update
    - T(Expected update) ≈ d * T(Sample update)

- If enough time for expected update, resulting estimate is usually better
    - No sampling error
- If not enough time, sample update can at least somewhat improve estimate
    - Smaller steps

Expected vs Sample Updates: Analysis

- Sample updates reduce error according to $\sqrt{\frac{b-1}{bt}}$
- Does not account sample update having better estimate of successor states



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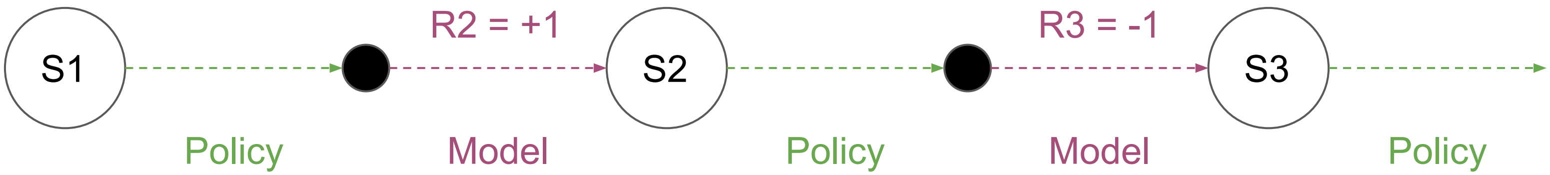
Distributing Updates

1. Dynamic Programming
    - Sweep through entire state space (or state-action space)
2. Dyna-Q
    - Sample uniformly

- Both suffers from updating irrelevant states most of the time

Trajectory Sampling

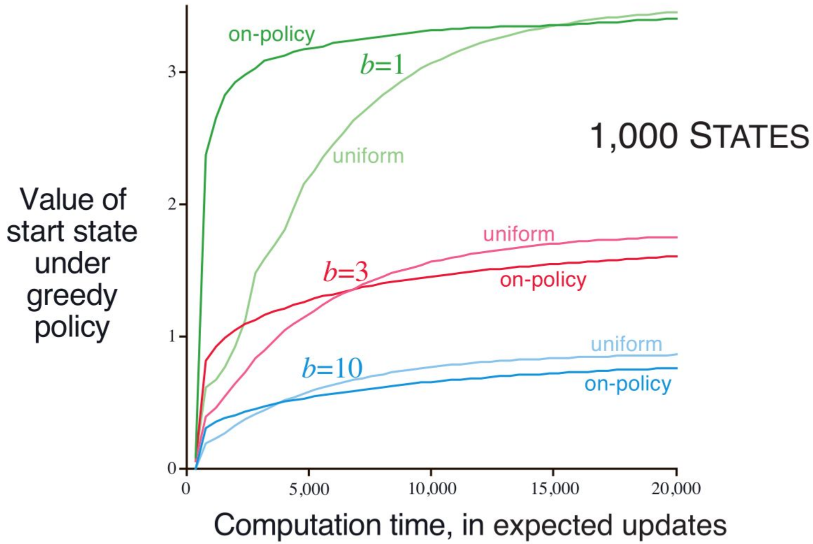
- Gather experience by sampling explicit individual trajectories
    - Sample state transitions and rewards from the model
    - Sample actions from a **distribution**



- Effects of **on-policy distribution**
    - Can ignore vast, uninteresting parts of the space
    - Significantly advantageous when function approximation is used

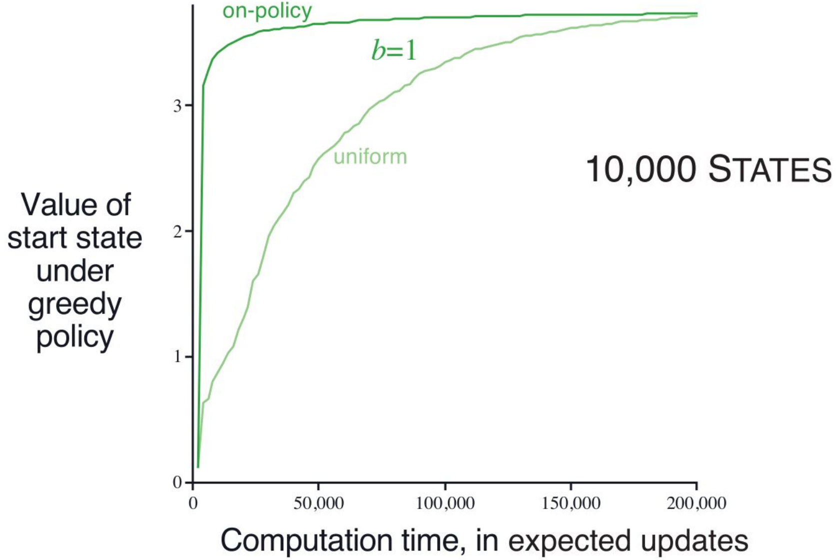
Trajectory Sampling: On-policy Distribution Example

- Faster planning initially, but retarded in the long run
- Better when branching factor *b* is small (can focus on just few states)



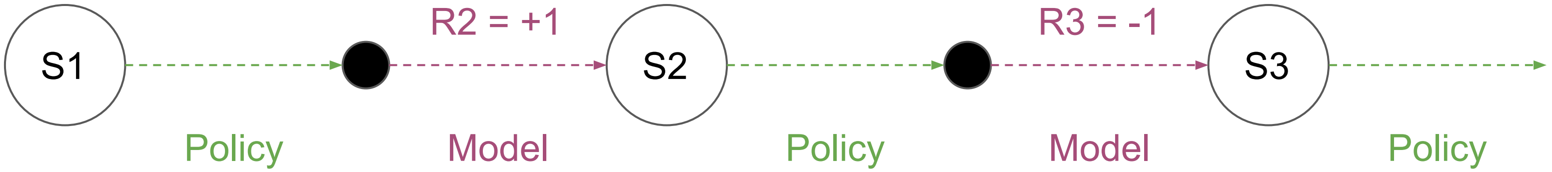
Trajectory Sampling: On-policy Distribution Example

- Long-lasting advantage when state space is large
    - Focusing on states have bigger impact when state space is large



Real-time Dynamic Programming (RTDP)

- On-policy trajectory-sampling value iteration algorithm
- Gather real or simulated trajectories
  - *Asynchronous DP:* nonsystematic sweeps

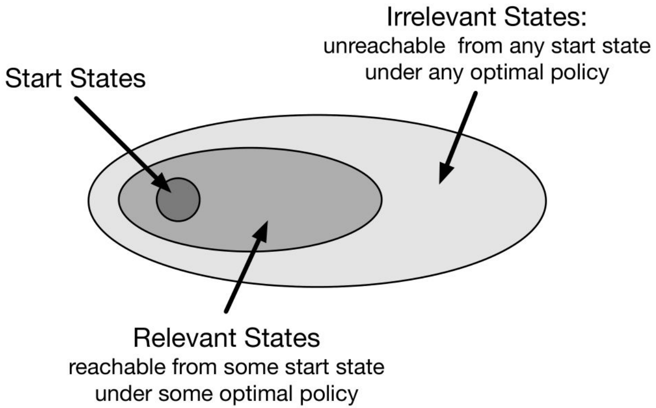


- Update with value iteration

$$\begin{aligned} v_{k+1}(s) &\doteq \max_a \mathbb{E}[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s, A_t = a] \\ &= \max_a \sum_{s',r} p(s', r \mid s, a) \Big[r + \gamma v_k(s') \Big], \end{aligned}$$

RTDP: Prediction and Control

- Prediction problem: skip any state not reachable by policy
- Control problem: find the *optimal partial policy*
  - A policy that is optimal for relevant states but arbitrary for other states
  - Finding such policy requires visiting all (s, a) infinitely many times



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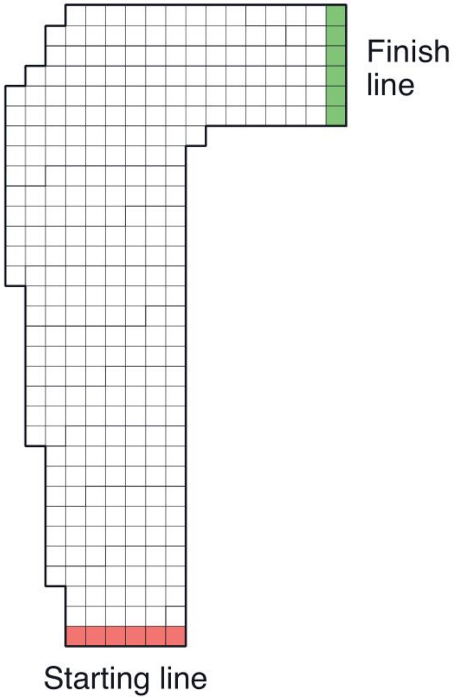
Stochastic Optimal Path Problems

- Conditions
  - Undiscounted episodic task with absorbing goal states
  - Initial value of every goal state is 0
  - At least one policy can definitively reach a goal state from any starting state
  - All rewards from non-goal states are negative
  - All initial values are optimistic
- Guarantees
  - RTDP does not need to visit all (s, a) infinite times to find *optimal partial policy*

Stochastic Optimal Path Problem: Racetrack

- 9115 reachable states, 599 relevant
- RTDP needs half the update of DP
  - Visits almost all states at least once
  - Focuses to relevant states quickly



	DP	RTDP
Average computation to convergence	28 sweeps	4000 episodes
Average number of updates to convergence	252,784	127,600
Average number of updates per episode	—	31.9
% of states updated $\leq$ 100 times	—	98.45
% of states updated $\leq$ 10 times	—	80.51
% of states updated 0 times	—	3.18

DP vs. RTDP: Checking Convergence

- DP: Update with exhaustive sweeps until Δv is sufficiently small
    - Unaware of policy performance until value function has converged
    - Could lead to overcomputation
- RTDP: Update with trajectories
    - Check policy performance via trajectories
    - Detect convergence earlier than DP

Background Planning vs. Decision-Time Planning

- *Background Planning*
  - Gradually improve policy or value function
  - Not focused on the current state
  - Better when low-latency action selection is required
- *Decision-Time Planning*
  - Select single action through planning
  - Focused on the current state
  - Typically discard value / policy used in planning after each action selection
  - Most useful when fast responses are not required

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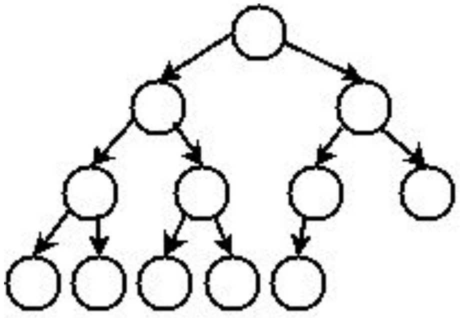
Heuristic Search

- Generate tree of possible continuations for each encountered states
  - Compute best action with the search tree
  - More computation is needed
  - Slower response time
- Value function can be held constant or updated
- Works best with perfect model and imperfect Q

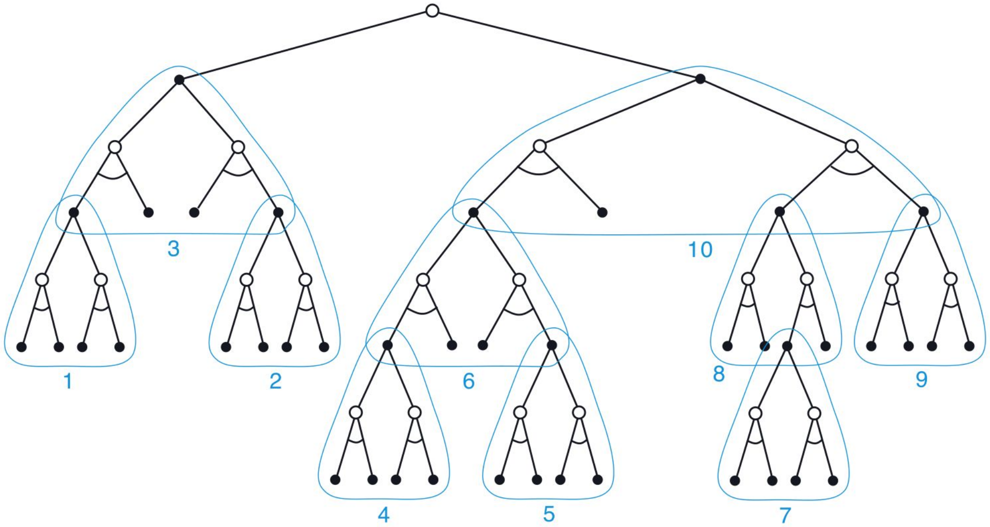


Heuristic Search: Focusing States

- Focus on states that might immediate follow the current state
    - Computations: Generate tree with current state as head
    - Memory: Store estimates only for relevant states
- Particularly efficient when state space is large (ex. chess)

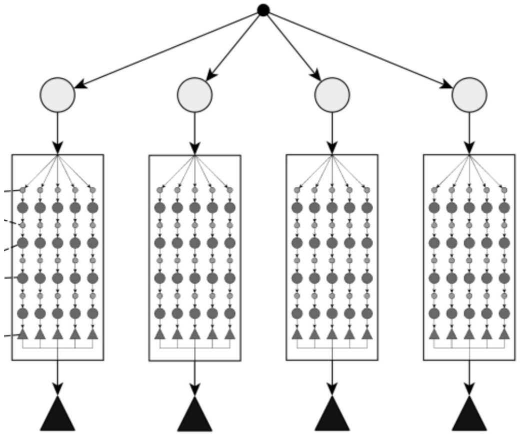
Heuristic Search: Focusing Updates

- Focus distribution of *updates* on the current state
  - Construct search tree
  - Perform one-step updates from bottom-up



Rollout Algorithms

- Estimate Q by averaging simulated trajectories from a *rollout policy*
- Choose action with highest Q
- Does not compute Q for all states / actions (unlike MC control)
- Not a *learning* algorithm since values and policies are not stored



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Rollout Algorithms: Policy

- Satisfies the *policy improvement theorem* for the policy π
  - Same as one step of the policy iteration algorithm

$$v_{\pi'}(s) = q_\pi(s, a) \geq v_\pi(s)$$

- Rollout seeks to improve upon the default policy, not to find the optimal policy
  - Better default policy → Better estimates → Better policy from rollout algorithm

Rollout Algorithms: Time Constraints

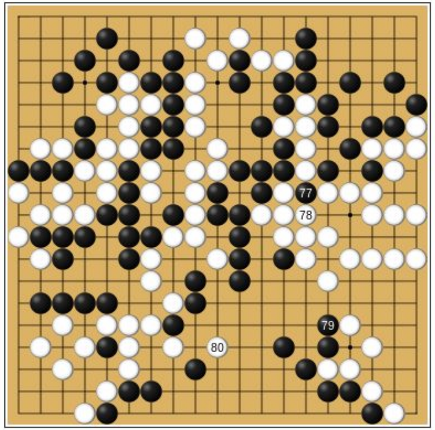
- Good rollout policies need a lot of trajectories
- Rollout algorithms often have strict time constraints

Possible Mitigations

1. Run trajectories on separate processors
2. Truncate simulated trajectories before termination
    - Use stored evaluation function
3. Prune unlikely candidate actions

Monte Carlo Tree Search (MCTS)

- Rollout algorithm with *directed simulations*
  - Accumulate value estimates in a tree structure
  - Direct simulations toward more high-rewarding trajectories
- Behind successes in Go



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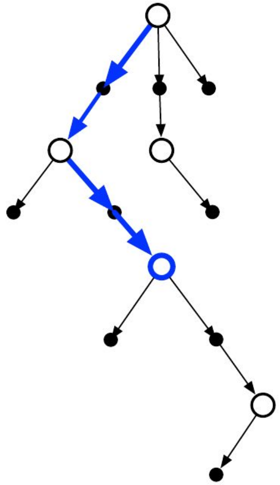
Monte Carlo Tree Search: Algorithm

- Repeat until termination:
    a. **Selection**: Select beginning of trajectory
    b. **Expansion**: Expand tree
    c. **Simulation**: Simulate an episode
    d. **Backup**: Update values
- Select action with some criteria
    a. Largest action value
    b. Largest visit count

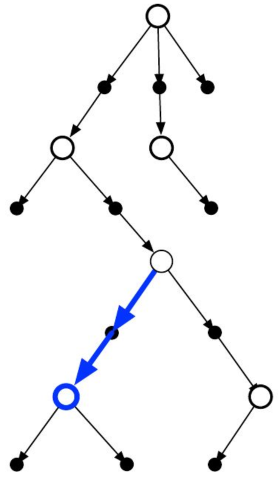
Monte Carlo Tree Search: Selection

- Start at the root node
- Traverse down the tree to select a leaf node



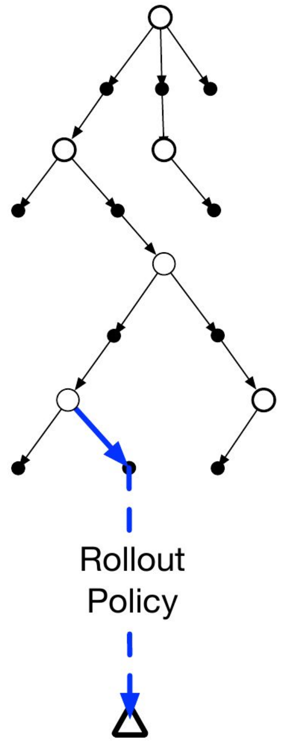
Monte Carlo Tree Search: Expansion

- Expand the selected leaf node
  - Add one or more child nodes via unexplored actions



Monte Carlo Tree Search: Simulation

- From leaf node or new child node, simulate a complete episode
- Generates a Monte Carlo trial
    - Selected first by the tree policy
    - Selected beyond the tree by the rollout policy



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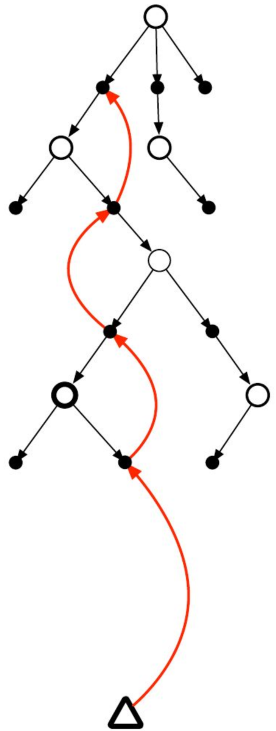
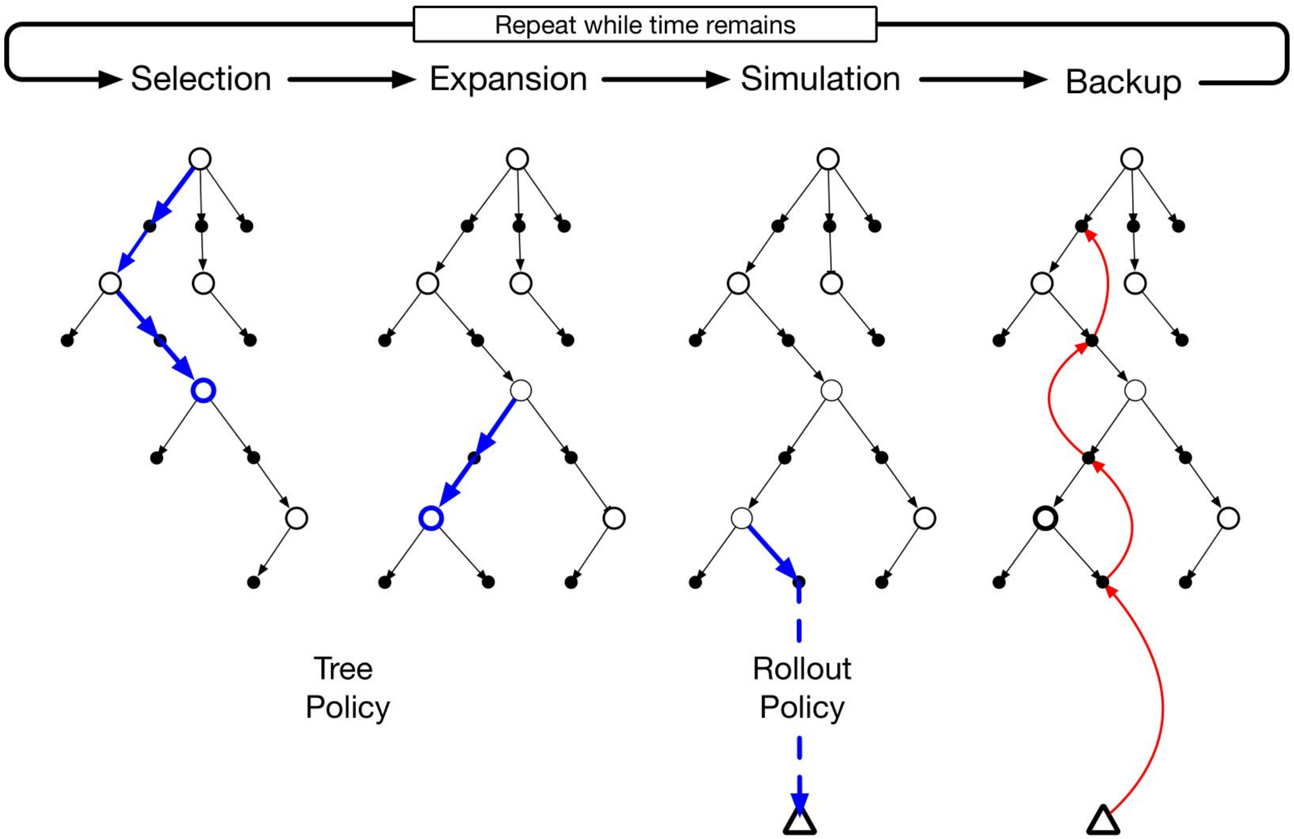
Monte Carlo Tree Search: Backup

- Update or Initialize values of nodes traversed in *tree policy*
    - No values saved for the rollout policy beyond the tree




Repeat while time remains
Selection
Expansion
Simulation
Backup
Tree
Policy
Rollout
Policy

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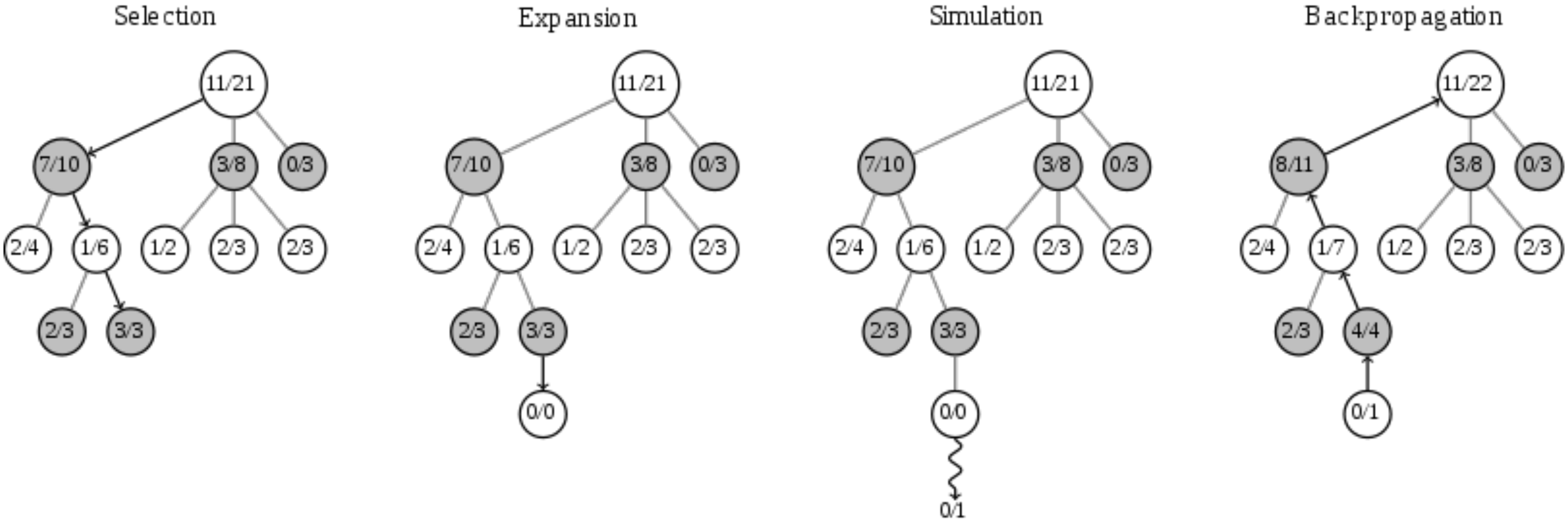

https://en.wikipedia.org/wiki/Monte_Carlo_tree_search

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Monte Carlo Tree Search: Insight

- Can use online, incremental, sample-based methods
- Can focus MC trials on segments with high-return trajectories
- Can efficiently grow a partial value table
  - Does not need to save all values
  - Does not need function approximation

Summary of Chapter 8

- Planning requires a *model* of the environment
  - *Distribution model* consists of transition probabilities
  - *Sample model* produces single transitions and rewards
- Planning and Learning share many similarities
  - Any learning method can be converted to planning method
- Planning can vary in *size* of updates
  - ex) 1-step sample updates
- Planning can vary in *distribution* of updates
  - ex) Prioritized sweeping, On-policy trajectory sampling, RTDP
- Planning can *focus forward* from pertinent states
  - Decision-time planning
  - ex) Heuristic search, Rollout algorithms, MCTS

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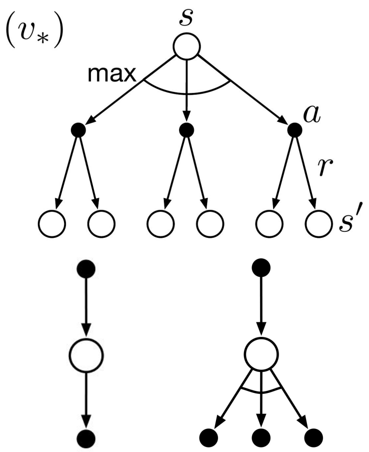
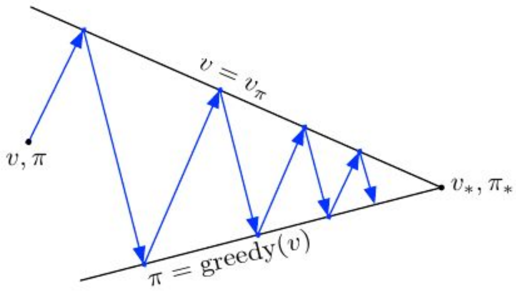
Summary of Part I

- Three underlying ideas:
  a. Estimate value functions
  b. Back up values along actual / possible state trajectories
  c. Use Generalized Policy Iteration (GPI)
     - Keep approximate value function and policy
     - Use one to improve another

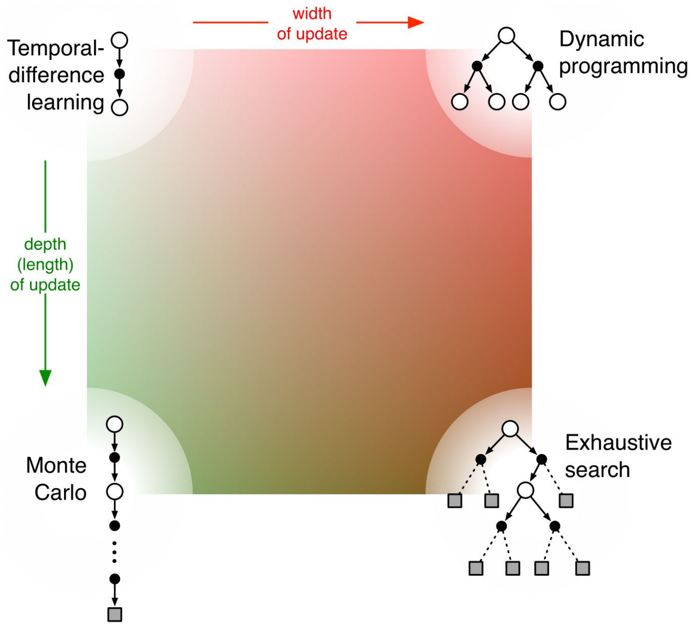
$$V_\pi \rightarrow v_\pi$$





Summary of Part I: Dimensions

- Three important dimensions:
  a. Sample update vs Expected update
     - Sample update: Using a sample trajectory
     - Expected update: Using the distribution model
  b. Depth of updates: degree of bootstrapping
  c. On-policy vs Off-policy methods



- One undiscussed important dimension:
  Function Approximation

Summary of Part I: Other Dimensions

- Episodic vs. Continuing returns
- Discounted vs. Undiscounted returns
- Action values vs. State values vs. Afterstate values
- Exploration methods
    - ε-greedy, optimistic initialization, softmax, UCB
- Synchronous vs. Asynchronous updates
- Real vs. Simulated experience
- Location, Timing, and Memory of updates
    - Which state or state-action pair to update in model-based methods?
    - Should updates be part of selected actions, or only afterward?
    - How long should the updated values be retained?

Thank you!

Original content from

- Reinforcement Learning: An Introduction by Sutton and Barto

You can find more content in

- github.com/seungjaeryanlee
- www.endtoend.ai