Chapter 3: Finite Markov Decision Processes

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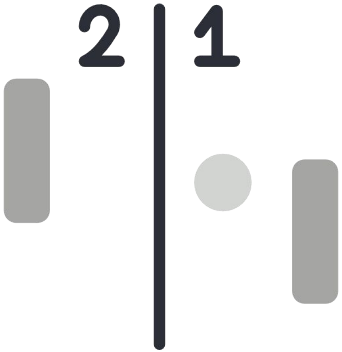
Markov Decision Process (MDP)

- Simplified, flexible reinforcement learning problem
- Consists of States $\mathcal{S}$, Actions $\mathcal{A}$, Rewards $\mathcal{R}$

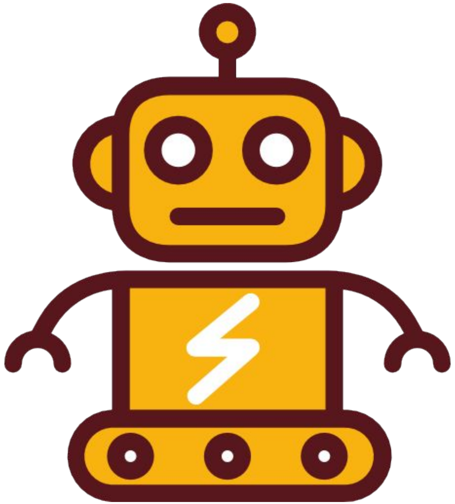


States
Info available to agent

Actions
Choice made by agent

Rewards
Basis for evaluating choices

Agent



The learner
Takes action

Environment

Everything outside the agent
Returns state and reward

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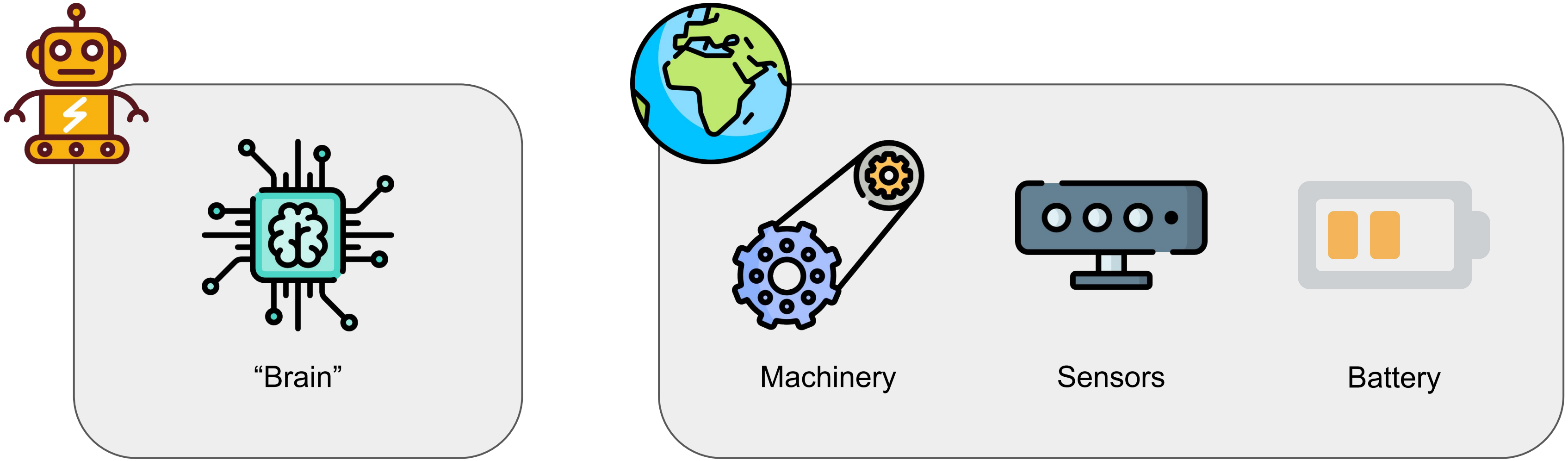
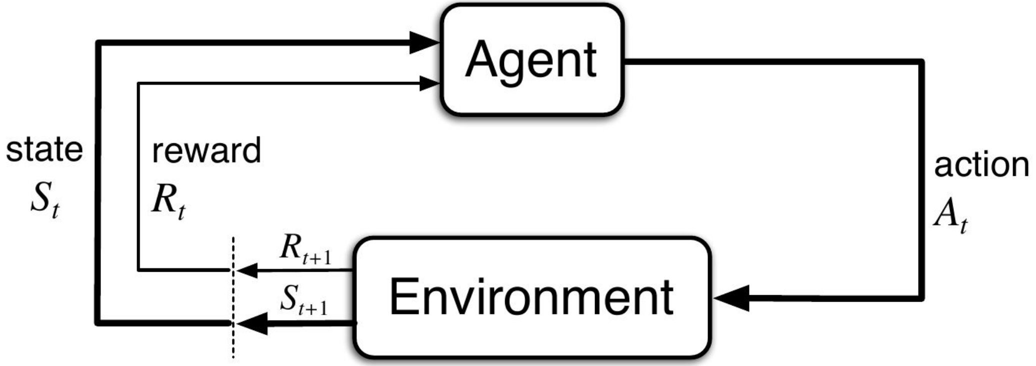
Agent-Environment Boundary

- Anything the agent cannot **arbitrarily change** is part of the environment
    - Agent might still **know** everything about the environment
- Different boundaries for different purposes



Agent-Environment Interactions

1. Agent observes a state S_0
2. Agent takes action A_0
3. Agent receives reward R_1 and new state S_1
4. Agent takes another action A_1
5. Repeat



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Transition Probability

- Probability of reaching state s' and reward r by taking action a on state s
- **Fully describes the dynamics of a finite MDP**

$$p(s', r \mid s, a) := \Pr\{S_t = s', R_t = r \mid S_{t-1} = s, A_t = a\}$$

- Can deduce other properties of the environment

$$p(s' \mid s, a) := \Pr\{S_t = s' \mid S_{t-1} = s, A_t = a\} = \sum_{r \in \mathcal{R}} p(s', r \mid s, a)$$

Expected Rewards

- Expected reward of taking action a on state s

$$r(s,a) := \mathbb{E}[R_t \mid S_{t-1} = s, A_{t-1} = a] = \sum_{r \in \mathcal{R}} r \sum_{s' \in \mathcal{S}} p(s', r \mid s, a)$$

- Expected reward of arriving in state s' by taking action a on state s

$$r(s,a,s') := \mathbb{E}[R_t \mid S_{t-1} = s, A_{t-1} = a, S_t = s'] = \sum_{r \in \mathcal{R}} r \frac{p(s', r \mid s, a)}{p(s' \mid s, a)}$$

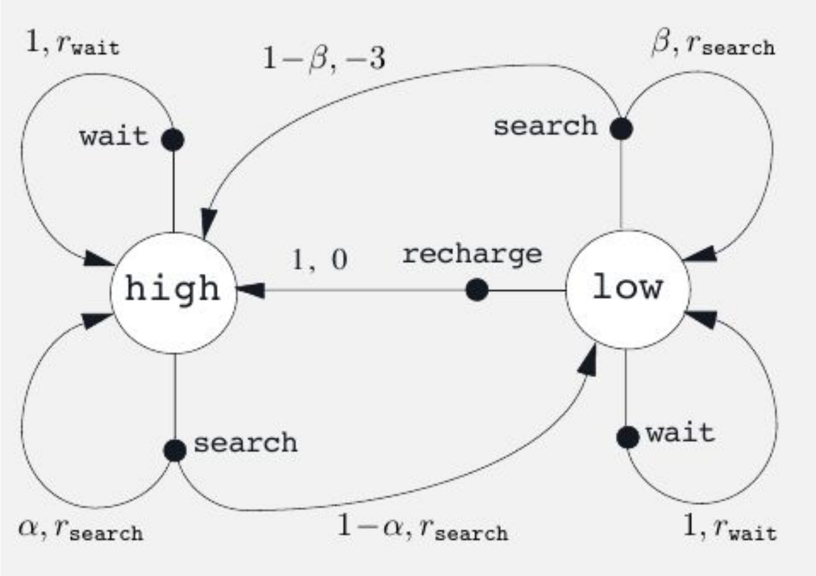
Recycling Robot Example

- States: Battery status (high or low)
- Actions
  - Search: High reward. Battery status can be lowered or depleted.
  - Wait: Low reward. Battery status does not change.
  - Recharge: No reward. Battery status changed to high.
- If battery is depleted, -3 reward and battery status changed to high.

s	a	s'	$p(s' \mid s, a)$	$r(s, a, s')$
high	search	high	α	$r_{\texttt{search}}$
high	search	low	$1-\alpha$	$r_{\texttt{search}}$
low	search	high	$1-\beta$	-3
low	search	low	β	$r_{\texttt{search}}$
high	wait	high	1	$r_{\texttt{wait}}$
high	wait	low	0	$r_{\texttt{wait}}$
low	wait	high	0	$r_{\texttt{wait}}$
low	wait	low	1	$r_{\texttt{wait}}$
low	recharge	high	1	0
low	recharge	low	0	0

Transition Graph

- Graphical summary of MDP dynamics



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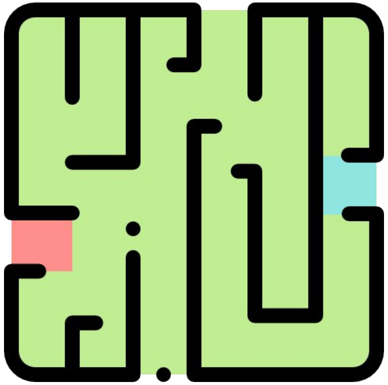
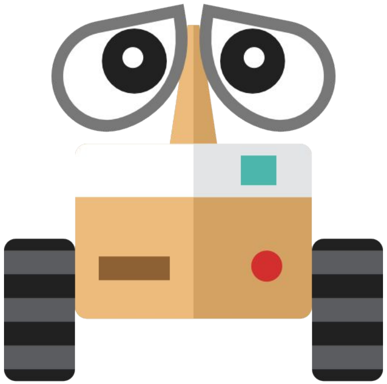
Designing Rewards

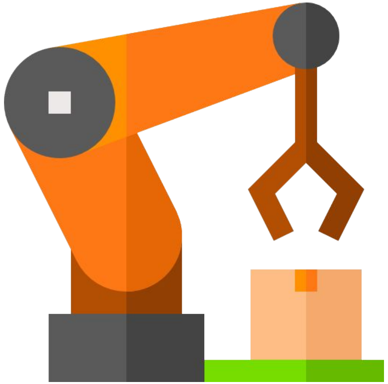
- Reward hypothesis
  - Goals and purposes can be represented by maximization of cumulative reward
- Tell *what* you want to achieve, not *how*



Always -1



Proportional to forward action



+1 for each box

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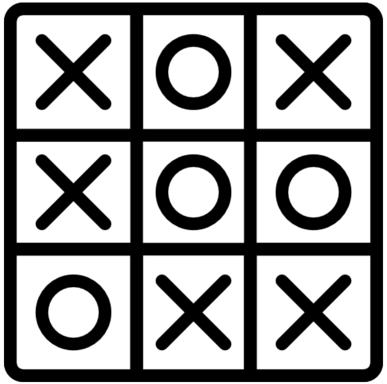
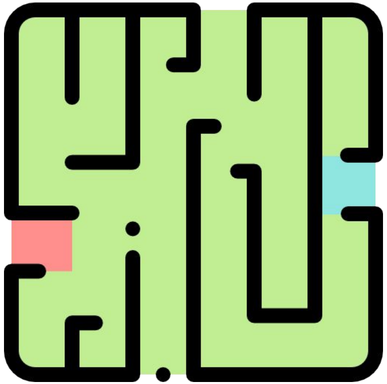
Episodic Tasks

- Interactions can be broken into *episodes*
- Episodes end in a special *terminal state*
- Each episode is independent



Finished when the game ends



Finished when the agent is out of the maze

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Return for Episodic Tasks

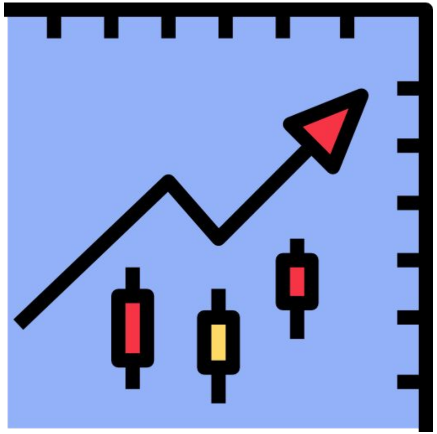
- Sum of rewards from time step t
- Time of termination: T

$$G_t = R_{t+1} + R_{t+2} + \ldots R_T$$

$$G_t = \sum_{k=t+1}^{T} R_k$$

Continuing Tasks

- Cannot be naturally broken into episodes
- Goes on without limit



Stock Trading

Return for Continuing Tasks

- Sum of rewards is almost always infinite
- Need to *discount* future rewards by factor $0 \leq \gamma < 1$
  - If $\gamma = 0$, the return only considers immediate reward (*myopic*)

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} \dots$$

$$G_t = \sum_{k=t+1}^{\infty} \gamma^{k-t-1} R_k$$

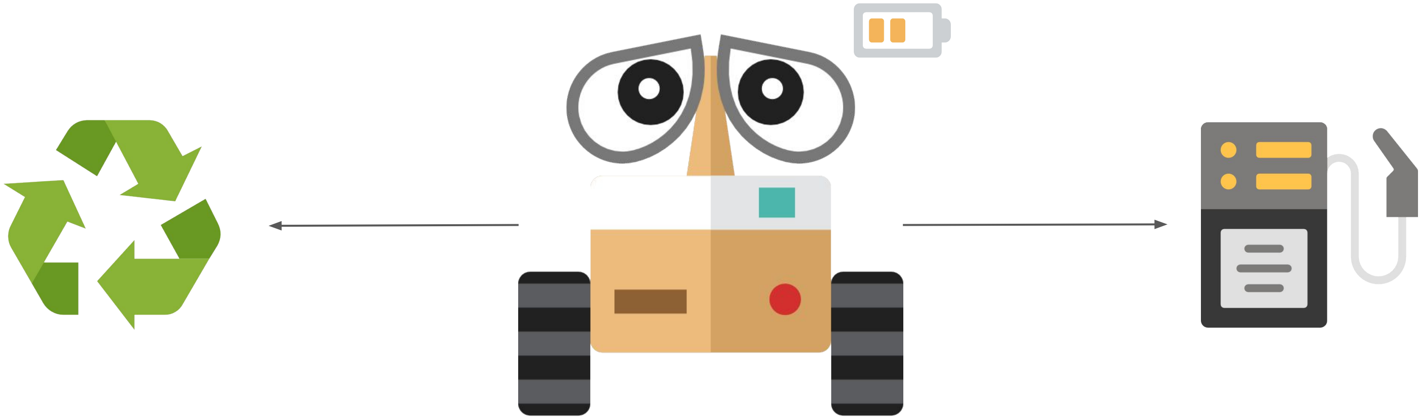
Unified Notation for Return

- Cumulative reward
- T can be a finite number or infinity
- Future rewards can be *discounted* with factor γ
    - If $T = \infty$, then γ must be less than 1.

$$G_t := \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

Policy

- Mapping from states to probabilities of selecting each possible action
- $\pi(a \mid s)$: Probability of selecting action a in state s



State-value function

- Expected return from state s and following policy π

$$
\begin{aligned}
v_{\pi} &:= \mathbb{E}_{\pi}[G_t \mid S_t = s] \\
&:= \mathbb{E}_{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s\right]
\end{aligned}
$$

Action-value function

- Expected return from taking action a in state s and following policy π

$$q_\pi := \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a]$$

$$:= \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s, A_t = a\right]$$

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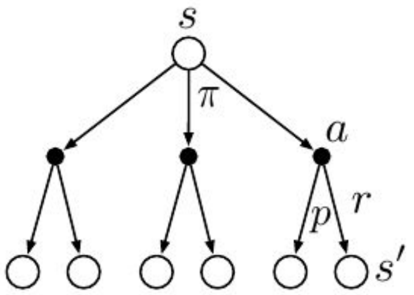
Bellman Equation



- Recursive relationship between $v_\pi(s)$ and $v_\pi(s')$

$$v_\pi(s) = \sum_a \pi(a \mid s) \sum_{s',r} p(s', r \mid s, a) \left[r + \gamma v_\pi(s') \right]$$

Optimal Policies π_* and Value Functions v_*, q_*

- For any policy π, $v_{\pi_*}(s) \geq v_\pi(s)$ for all states s
- There can be multiple optimal policies
- All optimal policies share same optimal value functions:

$$v_*(s) \doteq \max_\pi v_\pi(s)$$

$$q_*(s, a) \doteq \max_\pi q_\pi(s, a)$$

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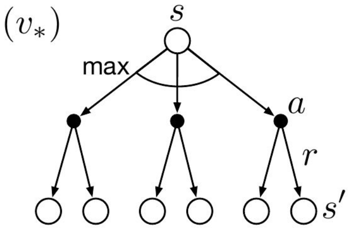
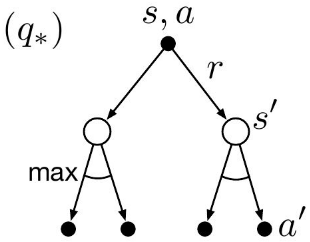
Bellman Optimality Equation

- Bellman Equation for optimal policies



$$v_*(s) = \max_a \sum_{s',r} p(s', r \mid s, a) \left[r + \gamma v_*(s') \right]$$



$$q_*(s, a) = \sum_{s',r} p(s', r \mid s, a) \left[r + \gamma \max_{a'} q_*(s', a') \right]$$

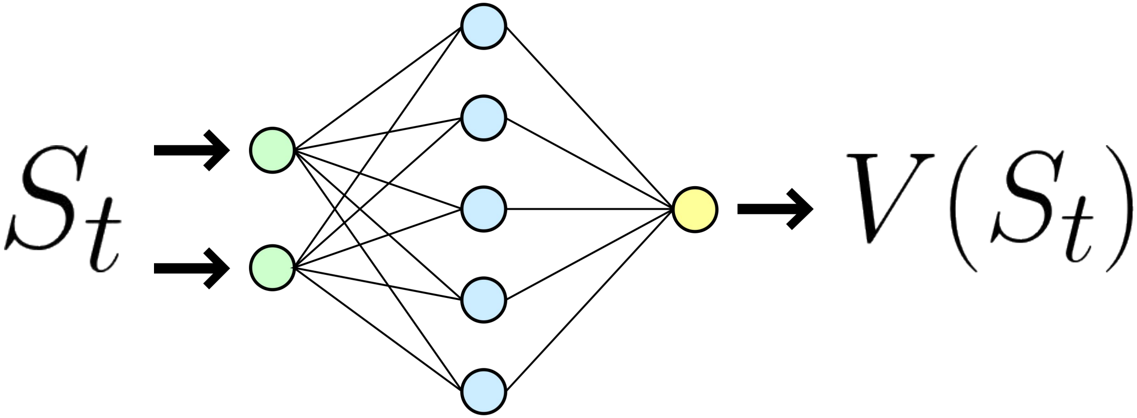
Solving Bellman Optimality Equation

- Linear system: $|\mathcal{S}|$ equations, $|\mathcal{S}|$ unknowns
- Possible to find the exact optimal policy
- Impractical in most environments
    - Need to know the dynamics of the environment
    - Need extreme computational power
    - Need Markov property

→ In most cases, approximation is the best possible solution.

Approximation

- Does not require complete knowledge of environment
- Less memory and computational power needed
- Can focus learning on frequently encountered states



Thank you!

Original content from

- Reinforcement Learning: An Introduction by Sutton and Barto

You can find more content in

- github.com/seungjaeryanlee
- www.endtoend.ai

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