Chapter 2: Multi-armed Bandits

Ryan Lee

One-armed Bandit

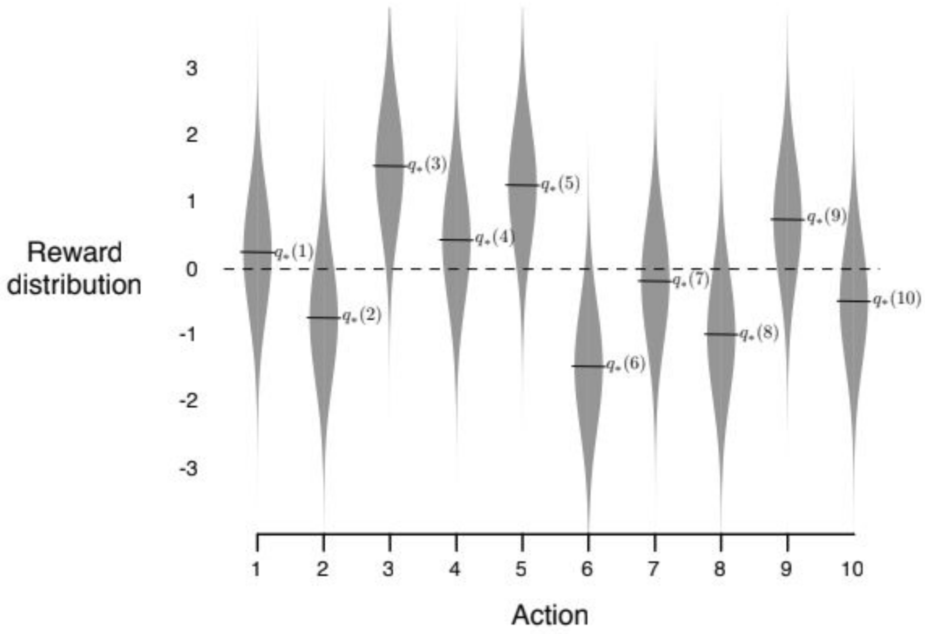
- Slot machine
- Each spin (action) is independent

Multi-armed Bandit problem

- Multiple slot machines to choose from
- Simplified setting to avoid complexities of RL problems
    - No observation
    - Action does not have delayed effect

10-armed Testbed

- 10 actions, 10 reward distributions
- Reward R_t chosen from stationary probability distributions



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Expected Reward

- Knowing expected reward trivializes the problem
- Estimate $q_*(a)$ with $Q_t(a)$

$$q_*(a) \doteq \mathbb{E}(R_t \mid A_t = a)$$

Sample-average

- Estimate $q_*(a)$ by averaging received rewards
- Default value (ex. 0) if action was never selected
- $Q_t(a)$ converges to $q_*(a)$ as denominator goes to infinity

$$Q_t(a) \doteq \frac{\text{sum of rewards when } a \text{ taken prior to } t}{\text{number of times } a \text{ taken prior to } t} = \frac{\sum_{i=1}^{t-1} R_i \cdot \mathbb{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbb{1}_{A_i=a}}$$

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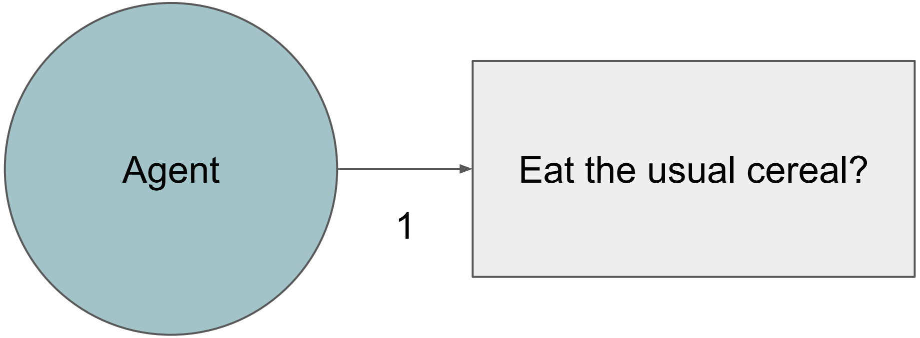
Greedy method

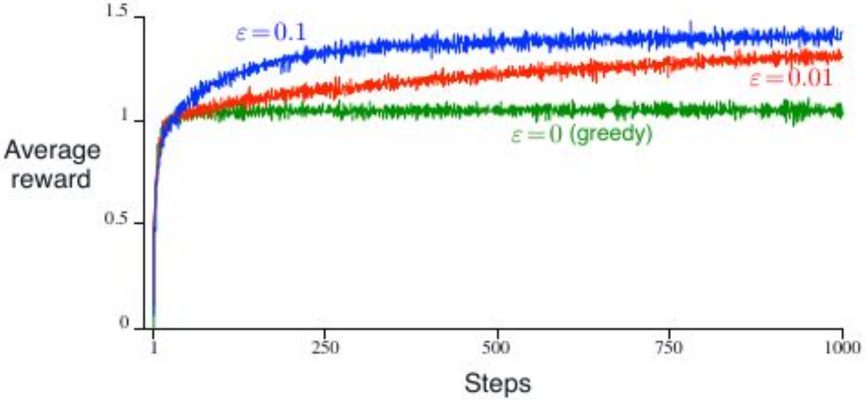
- Always select *greedily* : $A_t \doteq \mathrm{argmax}_a Q_t(a)$
- No exploration
- Often stuck in suboptimal actions

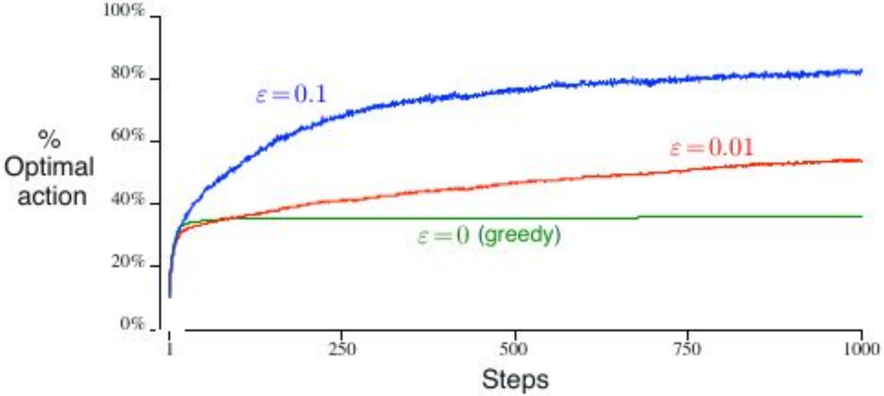


ε-greedy method

- Select random action with probability ε
- All $Q_t(a)$ converges to $q_*(a)$ as denominator goes to infinity

Greedy vs. ε-greedy





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Incremental Implementation

- Don't store reward for each step

$$Q_{n+1} = \frac{R_1 + R_2 + \ldots + R_n}{n}$$

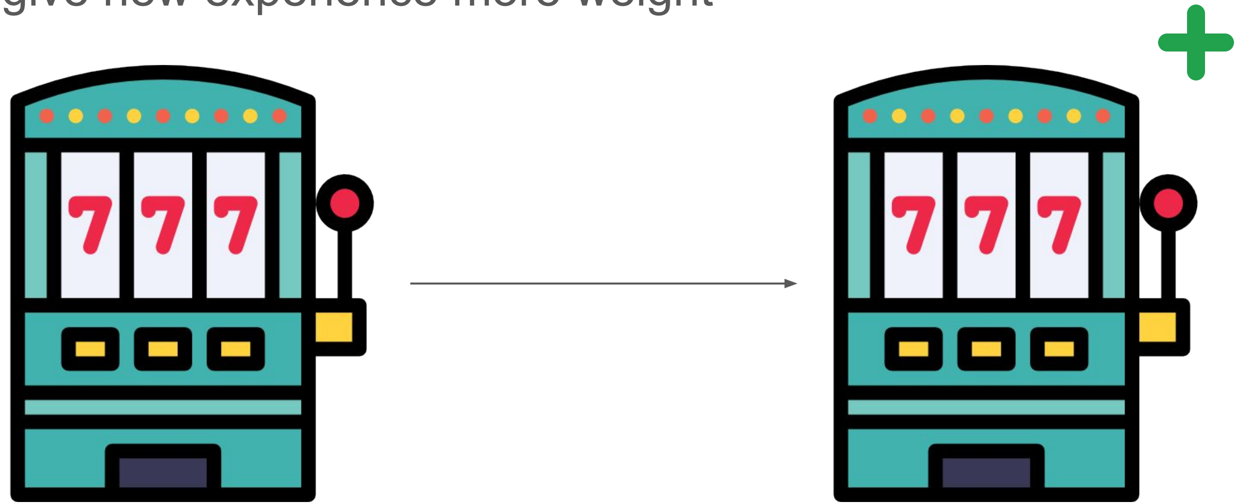
- Compute incrementally

$$Q_{n+1} = Q_n + \frac{1}{n}\Big[R_n - Q_n\Big]$$

$$NewEstimate \leftarrow OldEstimate \ + \ StepSize \ \big[Target - OldEstimate\big]$$

Nonstationary problem

- $q_*(a)$ changes over time
- Want to give new experience more weight



$$q_*(A_1 = a) \sim N(0, 1) \qquad\qquad q_*(A_2 = a) \sim N(3, 1)$$

Exponentially weighted average

- Constant step-size parameter α
- Give more weight to recent rewards

$$Q_{n+1} = Q_n + \alpha\big[R_n - Q_n\big]$$
$$= (1-\alpha)^n Q_1 + \sum_{i=1}^{n} \alpha(1-\alpha)^{n-i} R_i$$

Sample-average

$$Q_{n+1} = Q_n + \frac{1}{n}\left[R_n - Q_n\right]$$

- Guaranteed convergence
- Converge slowly: need tuning
- Seldomly used in applications

Weighted average

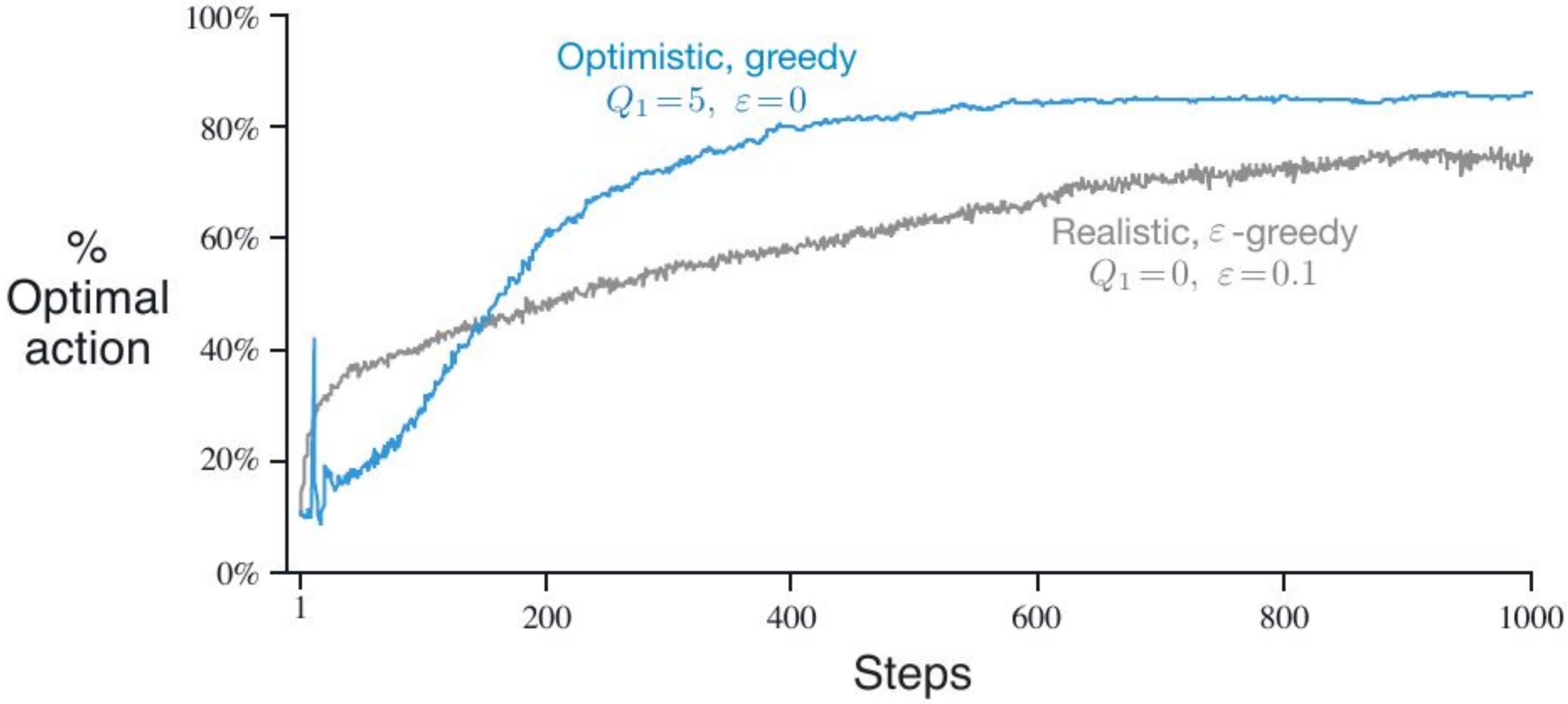
$$Q_{n+1} = Q_n + \alpha\left[R_n - Q_n\right]$$

- Never completely converges
- Desirable in nonstationary problems

Optimistic Initial Values

- Set initial action values optimistically (ex. +5)
- Temporarily encourage exploration
- Doesn't work in nonstationary problems

Optimistic Greedy vs. Realistic ε-greedy



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Upper Confidence Bound (UCB)

- Take into account each action's **potential** to be optimal
- Selected less → more potential
- Difficult to extend beyond multi-armed bandits

$$A_t \doteq \operatorname*{argmax}_a \left[Q_t(a) + c\sqrt{\frac{\ln t}{N_t(a)}} \right]$$

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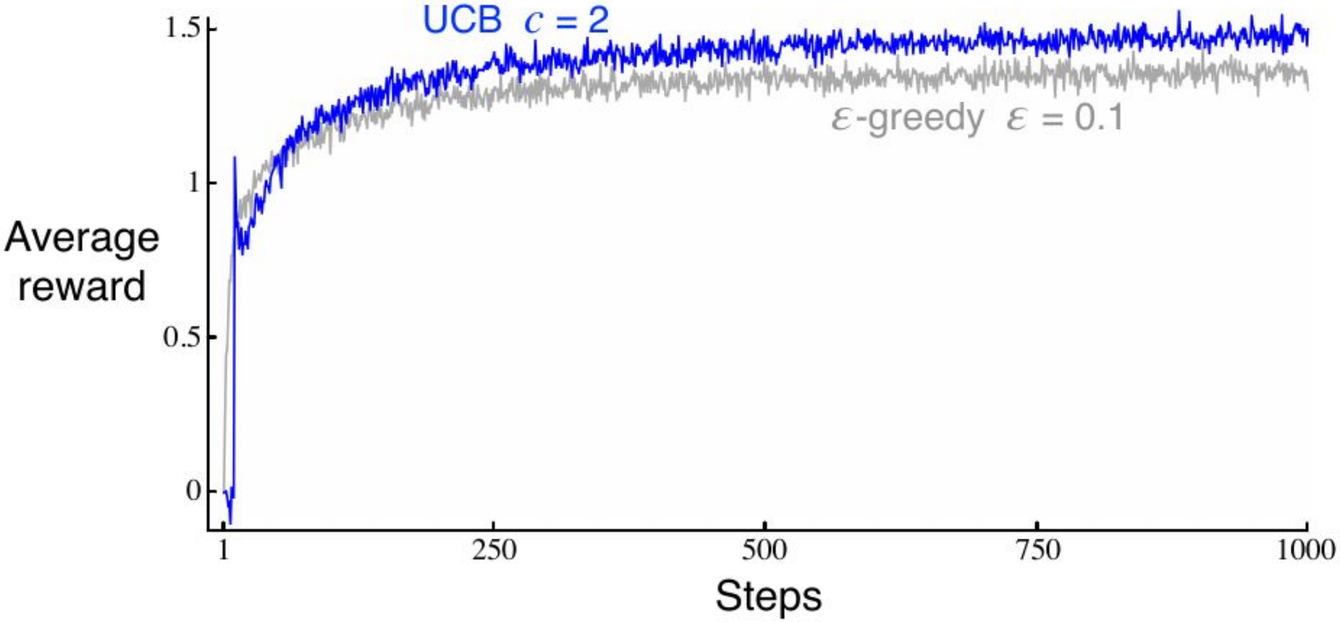
UCB vs. ε-greedy



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Gradient Bandit Algorithms

- Learn a numerical preference $H_t(a)$ for each action
- Convert to probability with softmax:

$$\pi_t(a) = \frac{e^{H_t(a)}}{\sum_{b \in \mathcal{A}} e^{H_t(b)}}$$

Gradient Bandit: Stochastic Gradient Descent

- Update preference $H_t(a)$ with SGD

$$
\begin{aligned}
H_{t+1}(A_t) &= H_t(A_t) \;+ \alpha(R_t - \bar{R}_t)(1 - \pi_t(A_t)) \\
H_{t+1}(a) &= H_t(a) \quad - \alpha(R_t - \bar{R}_t)\pi_t(a) \qquad\qquad \text{for all } a \neq A_t
\end{aligned}
$$

- Baseline $\bar{R}_t$: average of all rewards $R_1, R_2, \ldots, R_t$
  - Increase probability if reward is above baseline
  - Decrease probability if reward is below baseline

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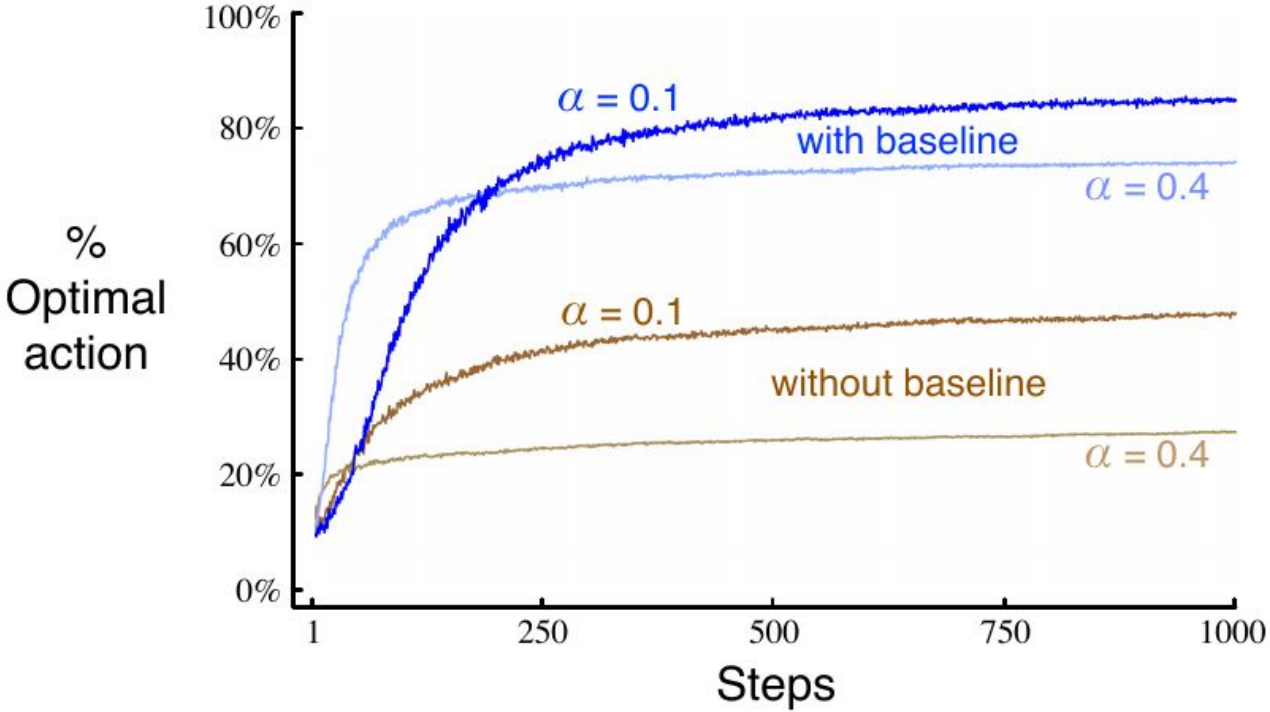
Gradient Bandit: Results



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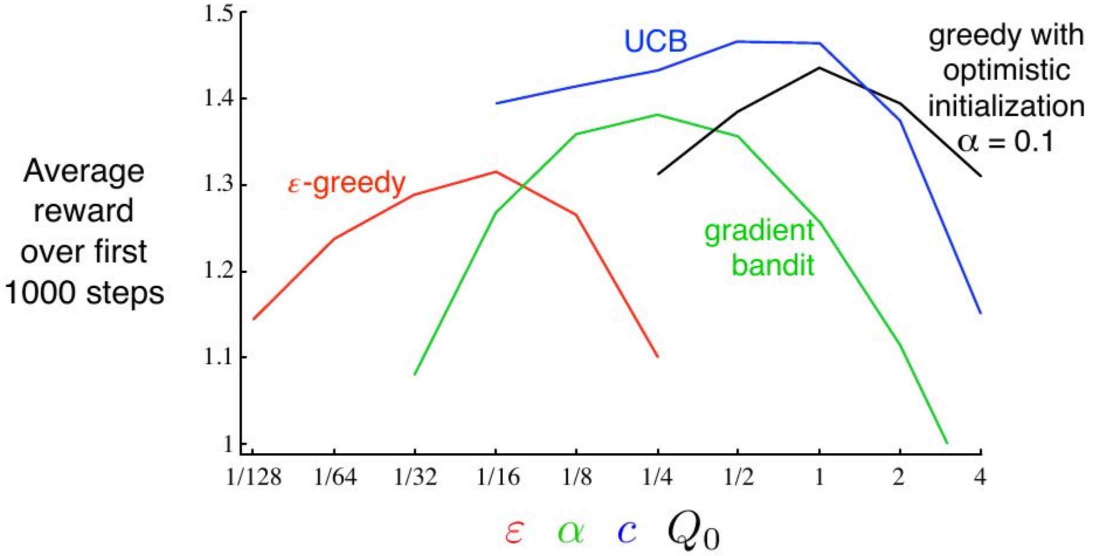
Parameter Study

- Check performance in best setting
- Check hyperparameter sensitivity



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Associative Search (Contextual Bandit)

- Observe some *context* that can help decision
- Intermediate between multi-armed bandit and full RL problem
  - Need to learn a **policy** to *associate* observations and actions
  - Each action only affects immediate reward

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Thank you!

Original content from

- Reinforcement Learning: An Introduction by Sutton and Barto

You can find more content in

- github.com/seungjaeryanlee
- www.endtoend.ai

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