In[1]:=
```
Get[FileNameJoin[{NotebookDirectory[],
    "OptimizationToolkit.m"}]];
Names["OptimizationToolkit`*"]
```

Out[2]=
```
{FactorExpression, Memoize,
 OptimizeDownValues, $DefaultExcludedForms}
```

Simple usage examples

In[3]:=
```
exp1 = ToSphericalCoordinates[{x, y, z}]
```

Out[3]= $\left\{\sqrt{x^2+y^2+z^2}, \text{ArcTan}\left[z, \sqrt{x^2+y^2}\right], \text{ArcTan}[x, y]\right\}$

The default output is in `HoldForm` to prevent evaluating and returning the original expression.

In[4]:=
```
FactorExpression[exp1]
```

Out[4]= $\text{Block}\Big[\{\$0, \$1\}, \$0 = x^2;$
$\$1 = y^2;$
$\left\{\sqrt{z^2+\$0+\$1}, \text{ArcTan}\left[z, \sqrt{\$0+\$1}\right], \text{ArcTan}[x, y]\right\}\Big]$

In[5]:=
```
exp2 = FromSphericalCoordinates[{r, θ, ϕ}]
```

Out[5]= $\{r \text{Cos}[\phi] \text{Sin}[\theta], r \text{Sin}[\theta] \text{Sin}[\phi], r \text{Cos}[\theta]\}$

In[6]:=
```
FactorExpression[exp2]
```

Out[6]= $\text{Block}\Big[\{\$2\}, \$2 = \text{Sin}[\theta];$
$\{r \$2 \text{Cos}[\phi], r \$2 \text{Sin}[\phi], r \text{Cos}[\theta]\}\Big]$

For large expressions, the factored form generally ends up being much smaller.

In[7]:=
```
exp3 = RotationTransform[θ, {xr, yr, zr}][{x, y, z}];
LeafCount[exp3]
```

Out[8]= 5563

In[9]:=
```
fexp3 = FactorExpression[exp3];
LeafCount[fexp3]
```

Out[10]= 919

Automatic optimization

Define a function:

In[11]:=
```
ClearAll[f];
f[r_, t_] :=
 Table[{r Cos[p] Sin[t], r Sin[t] Sin[p], r Cos[t]},
  {p, -Pi, Pi, Pi/4}]
f[r_, t_, n_] :=
 Table[{r Cos[p] Sin[t], r Sin[t] Sin[p], r Cos[t]},
  {p, -Pi, Pi, Pi/n}]
```

Check the definition:

In[14]:=
```
Definition[f]
```

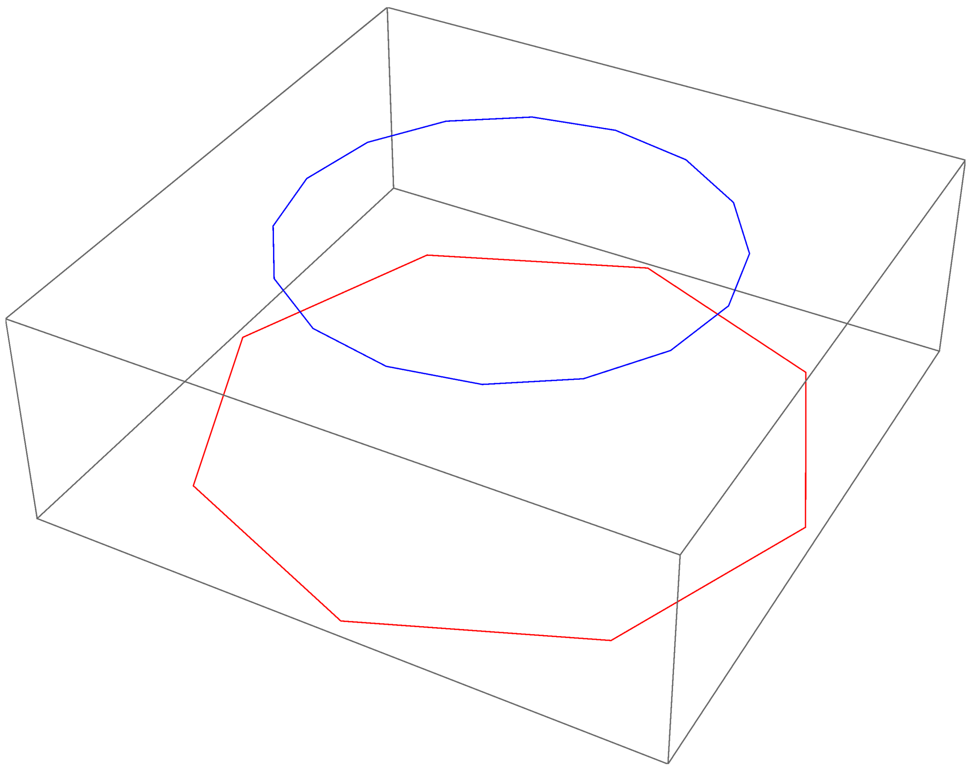
Out[14]=
```
f[r_, t_] :=
 Table[{r Cos[p] Sin[t], r Sin[t] Sin[p], r Cos[t]},
  {p, -π, π, π/4}]

f[r_, t_, n_] :=
 Table[{r Cos[p] Sin[t], r Sin[t] Sin[p], r Cos[t]},
  {p, -π, π, π/n}]
```

Example output:

In[15]:=
```
Graphics3D[{Red, Line[f[1, Pi/2]], Blue,
  Line[f[1, Pi/4, 8]]}]
```

Out[15]=



Use `OptimizeDownValues` to rewrite definitions of functions in an optimized form.

Notice that tables with constant iteration limits are “unrolled”, while those with parametrized limits are not.

In[16]:=
```
Quiet@OptimizeDownValues[f]
```

Out[16]=
```
{HoldPattern[f[r_, t_]] :> Block[
    {$107, $108, $109, $110, $111, $112, $113}, $107 = Sin[t];
    $108 = Cos[t];
    $109 = r $108;
    $110 = 1/Sqrt[2];
    $111 = r $107 $110;
    $112 = {-r $107, 0, $109};
    $113 = r $107;
    {$112, {-$111, -$111, $109}, {0, -$113, $109}, {$111,
      -$111, $109}, {$113, 0, $109}, {$111, $111, $109},
     {0, $113, $109}, {-$111, $111, $109}, $112}],
 HoldPattern[f[r_, t_, n_]] :> Block[{$114}, $114 = Sin[t];
    Table[{r Cos[p] $114, r $114 Sin[p], r Cos[t]},
     {p, -π, π, π/n}]]}
```

When the option “Rewrite” is False (default), the original definition is unchanged.

In[17]:=
```
Definition[f]
```

Out[17]=
```
f[r_, t_] :=
 Table[{r Cos[p] Sin[t], r Sin[t] Sin[p], r Cos[t]},
  {p, -π, π, π/4}]

f[r_, t_, n_] :=
 Table[{r Cos[p] Sin[t], r Sin[t] Sin[p], r Cos[t]},
  {p, -π, π, π/n}]
```

Use “Rewrite” set to True to redefine the function.

In[18]:=
```
Quiet@OptimizeDownValues[f, "Rewrite" → True]
```

Out[18]=
```
{HoldPattern[f[r_, t_]] :→ Block[
    {$115, $116, $117, $118, $119, $120, $121}, $115 = Sin[t];
    $116 = Cos[t];
    $117 = r $116;
    $118 = 1/√2;
    $119 = r $115 $118;
    $120 = {-r $115, 0, $117};
    $121 = r $115;
    {$120, {-$119, -$119, $117}, {0, -$121, $117}, {$119,
      -$119, $117}, {$121, 0, $117}, {$119, $119, $117},
     {0, $121, $117}, {-$119, $119, $117}, $120}],
 HoldPattern[f[r_, t_, n_]] :→ Block[{$122}, $122 = Sin[t];
    Table[{r Cos[p] $122, r $122 Sin[p], r Cos[t]},
     {p, -π, π, π/n}]]}
```

The function now has optimized defintions.

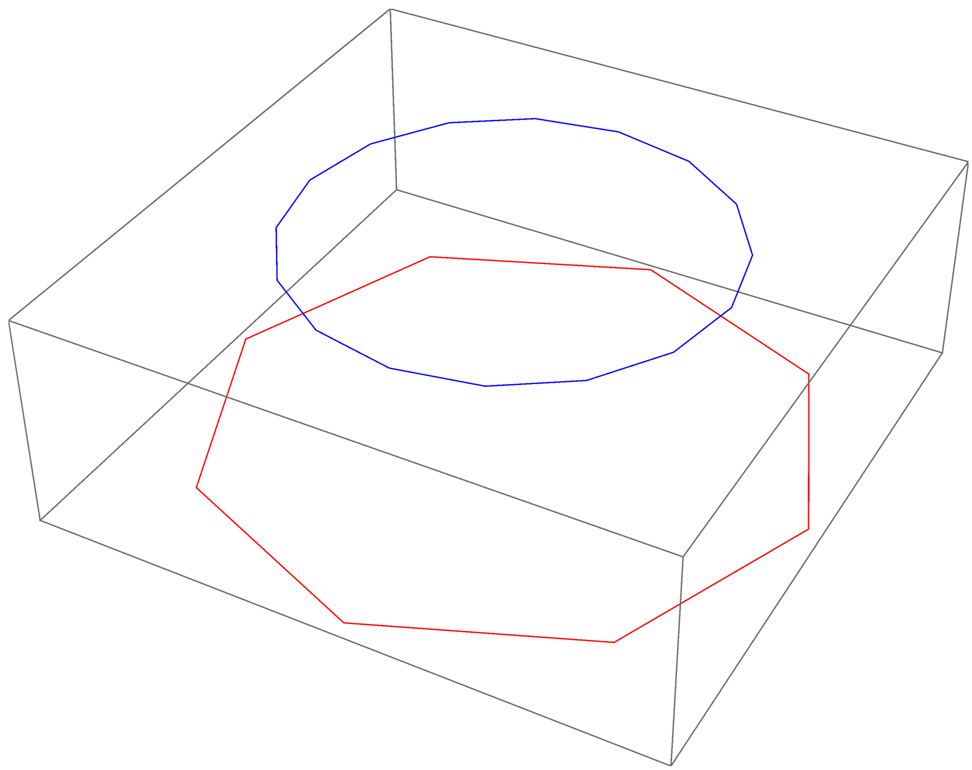
In[19]:=
```
Definition[f]
```

Out[19]=
```
f[r_, t_] := Block[
  {$115, $116, $117, $118, $119, $120, $121}, $115 = Sin[t];
  $116 = Cos[t];
  $117 = r $116;
  $118 = 1/Sqrt[2];
  $119 = r $115 $118;
  $120 = {-r $115, 0, $117};
  $121 = r $115;
  {$120, {-$119, -$119, $117}, {0, -$121, $117},
   {$119, -$119, $117}, {$121, 0, $117}, {$119, $119, $117},
   {0, $121, $117}, {-$119, $119, $117}, $120}]

f[r_, t_, n_] := Block[{$122}, $122 = Sin[t];
  Table[
   {r Cos[p] $122, r $122 Sin[p], r Cos[t]}, {p, -π, π, π/n}]]
```

In[20]:=
```
Graphics3D[{Red, Line[f[1, Pi/2]], Blue,
  Line[f[1, Pi/4, 8]]}]
```

Out[20]=



Other options: Memoization

In[21]:=
```
fibonacci[0] = 0;
fibonacci[1] = 1;
fibonacci[n_Integer] :=
  fibonacci[n - 1] + fibonacci[n - 2];
```

In[24]:=
```
AbsoluteTiming[fibonacci[30]]
```

Out[24]=
```
{3.63889, 832040}
```

In[25]:=
```
OptimizeDownValues[fibonacci, "Memoize" → True,
  "Rewrite" → True];
```

In[26]:=
```
AbsoluteTiming[fibonacci[30]]
```

Out[26]=
```
{0.000282578, 832040}
```

Memoization as a separate function

In[27]:=
```
factorial[0] = 1;
factorial[n : _Integer] := factorial[n - 1] * n;
```

In[29]:=
```
Memoize[factorial];
```

In[30]:=
```
Definition[factorial]
```

Out[30]=
```
factorial[0] := factorial[0] = 1

factorial[n_Integer] := factorial[n] = factorial[n - 1] n
```

Example of performance gains

Find a unit vector normal to a tetrahedron in 4D space.

In[31]:=

$$\text{nexp} = \text{Normalize}\left[\text{Det}\left[\begin{pmatrix} e1 & e2 & e3 & e4 \\ x2 - x1 & y2 - y1 & z2 - z1 & w2 - w1 \\ x3 - x1 & y3 - y1 & z3 - z1 & w3 - w1 \\ x4 - x1 & y4 - y1 & z4 - z1 & w4 - w1 \end{pmatrix}\right] \text{/.}\right.$$

```
    {e1 → {1, 0, 0, 0}, e2 → {0, 1, 0, 0}, e3 → {0, 0, 1, 0},
     e4 → {0, 0, 0, 1}}];
```

In[32]:=
```
sexp = Simplify[nexp,
   Union[Cases[nexp, _Symbol, Infinity]] ∈ Reals];
```

```
In[33]:= tetraNormal[{x1_, y1_, z1_, w1_}, {x2_, y2_, z2_, w2_},
           {x3_, y3_, z3_, w3_}, {x4_, y4_, z4_, w4_}] :=
          Evaluate[sexp]
```

Find the normals to each face of a pentachoron.

```
In[34]:= pentachoron = {{x1, y1, z1, w1}, {x2, y2, z2, w2},
            {x3, y3, z3, w3}, {x4, y4, z4, w4}, {x5, y5, z5, w5}};
         tetrahedra = Subsets[pentachoron, {4}];
         pentachoronFaceNormals = tetraNormal @@@ tetrahedra;
         LeafCount[pentachoronFaceNormals]

Out[37]= 10 586
```

Set up a function that evaluates the original expression.

```
In[38]:= test1[{{x1_, y1_, z1_, w1_}, {x2_, y2_, z2_, w2_},
            {x3_, y3_, z3_, w3_}, {x4_, y4_, z4_, w4_},
            {x5_, y5_, z5_, w5_}}] :=
          Evaluate[pentachoronFaceNormals]
```

Paste the output of FactorExpression into another function.

```
In[39]:= Block[{$RecursionLimit = Infinity,
           $IterationLimit = Infinity},
          FactorExpression[pentachoronFaceNormals]]

In[40]:= (* output removed *)

In[41]:= test2[{{x1_, y1_, z1_, w1_}, {x2_, y2_, z2_, w2_},
            {x3_, y3_, z3_, w3_}, {x4_, y4_, z4_, w4_},
```

```
  {x5_, y5_, z5_, w5_}}] :=
Block[{$123, $124, $125, $126, $127, $128, $129, $130,
  $131, $132, $133, $134, $135, $136, $137, $138,
  $139, $140, $141, $142, $143, $144, $145, $146,
  $147, $148, $149, $150, $151, $152, $153, $154,
  $155, $156, $157, $158, $159, $160, $161, $162,
  $163, $164, $165, $166, $167, $168, $169, $170,
  $171, $172, $173, $174, $175, $176, $177, $178,
  $179, $180, $181, $182, $183, $184, $185, $186,
  $187, $188, $189, $190, $191, $192, $193, $194,
  $195, $196, $197, $198, $199, $200, $201, $202,
  $203, $204, $205, $206, $207, $208, $209, $210,
  $211, $212, $213, $214, $215, $216, $217, $218,
  $219, $220, $221, $222, $223, $224, $225, $226,
  $227, $228, $229, $230, $231, $232, $233, $234,
  $235, $236, $237, $238, $239, $240, $241, $242,
  $243, $244, $245, $246, $247, $248, $249, $250,
  $251, $252, $253, $254, $255, $256, $257, $258,
  $259, $260, $261, $262, $263, $264, $265, $266,
  $267, $268, $269, $270, $271, $272, $273, $274,
  $275, $276, $277, $278, $279, $280, $281, $282,
  $283, $284, $285, $286, $287, $288, $289, $290,
  $291, $292, $293, $294, $295, $296, $297, $298,
  $299, $300, $301, $302, $303, $304, $305, $306,
  $307, $308, $309, $310, $311, $312, $313, $314,
  $315, $316, $317, $318, $319, $320, $321, $322,
  $323, $324, $325, $326, $327, $328, $329, $330,
  $331, $332, $333, $334, $335, $336, $337, $338,
  $339, $340, $341, $342, $343, $344, $345, $346,
  $347, $348, $349, $350, $351, $352, $353, $354,
  $355, $356, $357, $358, $359, $360, $361, $362,
```

$363, $364, $365, $366, $367, $368, $369, $370,
$371, $372, $373, $374, $375, $376, $377, $378,
$379, $380, $381, $382, $383, $384, $385, $386,
$387, $388, $389, $390, $391, $392, $393, $394,
$395, $396, $397, $398, $399, $400, $401, $402,
$403, $404, $405, $406, $407, $408, $409, $410,
$411, $412, $413, $414}, $123 = z1 - z5;

```
$124 = y1 z2;
$125 = y2 z3;
$126 = y1 z5;
$127 = y4 z2;
$128 = y3 z1;
$129 = y4 z1;
$130 = y5 z2;
$131 = -y2 z1 - y3 z2 - y1 z3 + $124 + $125 + $128;
$132 = y2 $123;
$133 = -y5 z1 - $124 + $126 + $130 + $132;
$134 = y5 z3;
$135 = y3 z4;
$136 = x1 y2;
$137 = x1 z2;
$138 = x2 y3;
$139 = y1 - y5;
$140 = x1 y5;
$141 = z1 - z4;
$142 = x2 z3;
$143 = z2 - z5;
$144 = y1 z3;
$145 = x1 z5;
$146 = y1 z4;
$147 = w1 y3 z2;
$148 = w1 y4 z3;
```

$\$149 = w2\,\$144;$
$\$150 = w1\,x3\,y2;$
$\$151 = w2\,x1\,y3;$
$\$152 = w1\,x4\,y3;$
$\$153 = x4\,y2;$
$\$154 = x3\,y1;$
$\$155 = -x2\,y1 - x3\,y2 - x1\,y3 + \$136 + \$138 + \$154;$
$\$156 = w2\,y5\,z1;$
$\$157 = w1\,y2\,z5;$
$\$158 = w2\,y3\,z5;$
$\$159 = w2\,x5\,y1;$
$\$160 = w1\,x2\,y5;$
$\$161 = w2\,x3\,y5;$
$\$162 = x5\,y2;$
$\$163 = x2\,\$139;$
$\$164 = -x5\,y1 - \$136 + \$140 + \$162 + \$163;$
$\$165 = w1\,y5\,z4;$
$\$166 = y2\,z4;$
$\$167 = w1\,x5\,y4;$
$\$168 = x4\,y1;$
$\$169 = w3\,y4\,z5;$
$\$170 = w3\,x4\,y5;$
$\$171 = x5\,y3;$
$\$172 = x3\,y4;$
$\$173 = y2\,\$141;$
$\$174 = -\$124 + \$127 - \$129 + \$146 + \$173;$
$\$175 = x4\,z2;$
$\$176 = x3\,z1;$
$\$177 = x4\,z1;$
$\$178 = x5\,z2;$
$\$179 = -y2\,z1 + \$124 - \$127 + \$129 - \$146 + \$166;$
$\$180 = y3\,\$123;$

$\$181 = -y4\,z3 - \$128 + \$129 + \$135 + \$144 - \$146;$
$\$182 = -y5\,z1 + \$126 + \$134 - \$144 + \$180;$
$\$183 = y2\,z5;$
$\$184 = -y3\,z2 - y4\,z3 + \$125 + \$127 + \$135 - \$166;$
$\$185 = y3\,\$143;$
$\$186 = -\$125 - \$130 + \$134 + \$183 + \$185;$
$\$187 = x1\,y4\,z3;$
$\$188 = w1\,x3\,z2;$
$\$189 = y3\,\$137;$
$\$190 = w2\,x1\,z3;$
$\$191 = y1\,\$142;$
$\$192 = w1\,x4\,z3;$
$\$193 = -x2\,z1 - x3\,z2 - x1\,z3 + \$137 + \$142 + \$176;$
$\$194 = x2\,y5\,z1;$
$\$195 = w2\,x5\,z1;$
$\$196 = z5\,\$136;$
$\$197 = w1\,x2\,z5;$
$\$198 = z5\,\$138;$
$\$199 = w2\,x3\,z5;$
$\$200 = x2\,\$123;$
$\$201 = -x5\,z1 - \$137 + \$145 + \$178 + \$200;$
$\$202 = z4\,\$140;$
$\$203 = w1\,x5\,z4;$
$\$204 = z5\,\$172;$
$\$205 = w3\,x4\,z5;$
$\$206 = x5\,z3;$
$\$207 = x3\,z4;$
$\$208 = x1\,z3;$
$\$209 = x1\,z4;$
$\$210 = -z1 + z5;$
$\$211 = w2\,\$129;$
$\$212 = w4\,\$131;$

$\$213 = w2\,\$135;$
$\$214 = w1\,\$166;$
$\$215 = w3\,\$174;$
$\$216 = -w2\,y4\,z3 - w1\,\$125 - w1\,\$127 - w2\,\$128 - w1\,\$135 - w2\,\$146 + \$147 + \$148 + \$149 + \$211 + \$212 + \$213 + \$214 + \$215;$
$\$217 = w1\,x2\,y4;$
$\$218 = x2\,\$129;$
$\$219 = w1\,x2\,z4;$
$\$220 = w4\,\$155;$
$\$221 = x2\,\$135;$
$\$222 = w2\,\$177;$
$\$223 = y1 - y4;$
$\$224 = z4\,\$136;$
$\$225 = w2\,\$207;$
$\$226 = w2\,\$168;$
$\$227 = w2\,\$172;$
$\$228 = x1\,y4;$
$\$229 = x2\,\$223;$
$\$230 = -\$136 + \$153 - \$168 + \$228 + \$229;$
$\$231 = w3\,\$230;$
$\$232 = -w2\,x4\,y3 - w1\,\$138 + \$150 + \$151 + \$152 - w1\,\$153 - w2\,\$154 - w1\,\$172 + \$217 + \$220 + \$226 + \$227 - w2\,\$228 + \$231;$
$\$233 = w2\,x4\,z3;$
$\$234 = w1\,\$175;$
$\$235 = w2\,\$209;$
$\$236 = x1\,\$127;$
$\$237 = y4\,\$142;$
$\$238 = x2\,\$146;$
$\$239 = w5\,\$131;$
$\$240 = w3\,\$133;$
$\$241 = w1\,\$134;$

$\$242 = -w1\,y3\,z5 - w1\,\$125 - w2\,\$126 - w2\,\$128 - w1\,\$130 - w2\,\$134 + \$147 + \$149 + \$156 + \$157 + \$158 + \$239 + \$240 + \$241;$

$\$243 = w5\,\$155;$

$\$244 = x1\,\$134;$

$\$245 = w1\,\$206;$

$\$246 = w3\,\$164;$

$\$247 = w1\,\$171;$

$\$248 = -w1\,x3\,y5 - w1\,\$138 - w2\,\$140 + \$150 + \$151 - w2\,\$154 + \$159 + \$160 + \$161 - w1\,\$162 - w2\,\$171 + \$243 + \$246 + \$247;$

$\$249 = w1\,x3\,z5;$

$\$250 = y3\,\$145;$

$\$251 = w2\,y4\,z5;$

$\$252 = w1\,\$127;$

$\$253 = w4\,\$133;$

$\$254 = w2\,\$146;$

$\$255 = w5\,\$179;$

$\$256 = -w2\,y5\,z4 - w1\,y4\,z5 - w2\,\$126 - w1\,\$130 + \$156 + \$157 + \$165 - \$211 - \$214 + \$251 + \$252 + \$253 + \$254 + \$255;$

$\$257 = w2\,x4\,y5;$

$\$258 = w2\,x4\,z5;$

$\$259 = x2\,y4\,z5;$

$\$260 = w1\,\$153;$

$\$261 = x2\,z4;$

$\$262 = w4\,\$164;$

$\$263 = x2\,y4;$

$\$264 = w2\,\$228;$

$\$265 = -x2\,y1 + \$136 - \$153 + \$168 - \$228 + \$263;$

$\$266 = w5\,\$265;$

$\$267 = -w2\,x5\,y4 - w1\,x4\,y5 - w2\,\$140 + \$159 + \$160 - w1\,\$162 + \$167 - \$217 - \$226 + \$257 + \$260 + \$262 +$

$\$264 + \$266;$
$\$268 = w2\, x5\, z4;$
$\$269 = y5\, \$261;$
$\$270 = w3\, y5\, z1;$
$\$271 = w1\, y3\, z5;$
$\$272 = w3\, \$146;$
$\$273 = w5\, \$181;$
$\$274 = w4\, \$182;$
$\$275 = -w3\, y5\, z4 - w1\, y4\, z5 - w3\, \$126 - w3\, \$129 - w1\, \$135 + \$148 + \$165 + \$169 - \$241 + \$270 + \$271 + \$272 + \$273 + \$274;$
$\$276 = w3\, x5\, y1;$
$\$277 = w1\, x3\, y5;$
$\$278 = x3\, \$139;$
$\$279 = x1\, y3;$
$\$280 = w3\, \$228;$
$\$281 = -x5\, y1 + \$140 + \$171 + \$278 - \$279;$
$\$282 = -x4\, y3 - \$154 + \$168 + \$172 - \$228 + \$279;$
$\$283 = w4\, \$281;$
$\$284 = w5\, \$282;$
$\$285 = -w3\, x5\, y4 - w1\, x4\, y5 - w3\, \$140 + \$152 + \$167 - w3\, \$168 + \$170 - w1\, \$172 - \$247 + \$276 + \$277 + \$280 + \$283 + \$284;$
$\$286 = w2\, y4\, z3;$
$\$287 = w2\, y5\, z4;$
$\$288 = w3\, \$130;$
$\$289 = w3\, \$166;$
$\$290 = w5\, \$184;$
$\$291 = w4\, \$186;$
$\$292 = -w3\, y5\, z4 - w3\, \$127 - w2\, \$134 + \$158 + \$169 - w3\, \$183 - \$213 - \$251 + \$286 + \$287 + \$288 + \$289 + \$290 + \$291;$
$\$293 = w2\, x4\, y3;$

$\$294 = w2\,x5\,y4;$

$\$295 = w3\,\$162;$

$\$296 = y2 - y5;$

$\$297 = -x3\,y2 - x4\,y3 + \$138 + \$153 + \$172 - \$263;$

$\$298 = w3\,\$263;$

$\$299 = x2\,y5;$

$\$300 = w5\,\$297;$

$\$301 = x3\,\$296;$

$\$302 = -\$138 - \$162 + \$171 + \$299 + \$301;$

$\$303 = w4\,\$302;$

$$\$304 = -w3\,x5\,y4 - w3\,\$153 + \$161 + \$170 - w2\,\$171 - \$227 - \$257 + \$293 + \$294 + \$295 + \$298 - w3\,\$299 + \$300 + \$303;$$

$\$305 = \$216^2;$

$\$306 = \$232^2;$

$\$307 = w4\,\$193;$

$\$308 = x4\,\$131;$

$\$309 = x2\,\$141;$

$\$310 = x3\,\$174;$

$\$311 = -\$137 + \$175 - \$177 + \$209 + \$309;$

$$\$312 = -x1\,\$125 - x2\,\$128 - x1\,\$135 + \$187 + \$189 + \$191 + \$218 + \$221 + \$224 - \$236 - \$237 - \$238 + \$308 + \$310;$$

$\$313 = w3\,\$311;$

$\$314 = \$312^2;$

$$\$315 = -w1\,\$142 - w2\,\$176 + \$188 + \$190 + \$192 - w1\,\$207 + \$219 + \$222 + \$225 - \$233 - \$234 - \$235 + \$307 + \$313;$$

$\$316 = \$315^2;$

$\$317 = \$305 + \$306 + \$314 + \$316;$

$$\$318 = \frac{1}{\sqrt{\$317}};$$

$\$319 = \$242^2;$

$\$320 = \$248^2;$

$\$321 = w5\ \$193;$

$\$322 = x5\ \$131;$

$\$323 = w3\ \$201;$

$\$324 = x3\ \$133;$

$\$325 = -w1\ \$142 - w2\ \$145 - w2\ \$176 - w1\ \$178 + \$188 + \$190 + \$195 + \$197 + \$199 - w2\ \$206 + \$245 - \$249 + \$321 + \$323;$

$\$326 = -x1\ \$125 - x2\ \$126 - x2\ \$128 - x1\ \$130 - x2\ \$134 + \$189 + \$191 + \$194 + \$196 + \$198 + \$244 - \$250 + \$322 + \$324;$

$\$327 = \$325^2;$

$\$328 = \$326^2;$

$\$329 = \$319 + \$320 + \$327 + \$328;$

$\$330 = \frac{1}{\sqrt{\$329}};$

$\$331 = \$256^2;$

$\$332 = \$267^2;$

$\$333 = w4\ \$201;$

$\$334 = x4\ \$133;$

$\$335 = -x2\ z1 + \$137 - \$175 + \$177 - \$209 + \$261;$

$\$336 = x5\ \$179;$

$\$337 = w5\ \$335;$

$\$338 = -x2\ \$126 - x1\ \$130 - y4\ \$145 + \$194 + \$196 + \$202 - \$218 - \$224 + \$236 + \$238 + \$259 - \$269 + \$334 + \$336;$

$\$339 = -w1\ x4\ z5 - w2\ \$145 - w1\ \$178 + \$195 + \$197 + \$203 - \$219 - \$222 + \$234 + \$235 + \$258 - \$268 + \$333 + \$337;$

$\$340 = \$338^2;$

$\$341 = \$339^2;$

$\$342 = \$331 + \$332 + \$340 + \$341;$

$$\$343 = \frac{1}{\sqrt{\$342}};$$

$\$344 = w3\,x5\,z1;$

$\$345 = x3\,\$146;$

$\$346 = \$275^2;$

$\$347 = \$285^2;$

$\$348 = x3\,\$123;$

$\$349 = y5\,\$176;$

$\$350 = -x4\,z3 - \$176 + \$177 + \$207 + \$208 - \$209;$

$\$351 = x5\,\$181;$

$\$352 = w3\,\$209;$

$\$353 = x4\,\$182;$

$\$354 = -x5\,z1 + \$145 + \$206 - \$208 + \$348;$

$\$355 = -x3\,\$126 - x3\,\$129 - x1\,\$135 - y4\,\$145 + \$187 + \$202 + \$204 - y5\,\$207 - \$244 + \$250 + \$345 + \$349 + \$351 + \$353;$

$\$356 = w5\,\$350;$

$\$357 = \$355^2;$

$\$358 = w4\,\$354;$

$\$359 = -w3\,x5\,z4 - w1\,x4\,z5 - w3\,\$145 - w3\,\$177 + \$192 + \$203 + \$205 - w1\,\$207 - \$245 + \$249 + \$344 + \$352 + \$356 + \$358;$

$\$360 = \$359^2;$

$\$361 = \$346 + \$347 + \$357 + \$360;$

$$\$362 = \frac{1}{\sqrt{\$361}};$$

$\$363 = w3\,\$178;$

$\$364 = x3\,\$130;$

$\$365 = \$292^2;$

$\$366 = \$304^2;$

$\$367 = x2\, z5;$

$\$368 = x3\, \$166;$

$\$369 = -x3\, z2 - x4\, z3 + \$142 + \$175 + \$207 - \$261;$

$\$370 = x5\, \$184;$

$\$371 = w3\, \$261;$

$\$372 = x4\, \$186;$

$\$373 = x3\, \$143;$

$\$374 = -x3\, \$127 - x2\, \$134 - x3\, \$183 + \$198 + \$204 - y5\, \$207 - \$221 + \$237 - \$259 + \$269 + \$364 + \$368 + \$370 + \$372;$

$\$375 = w5\, \$369;$

$\$376 = \$374^2;$

$\$377 = -\$142 - \$178 + \$206 + \$367 + \$373;$

$\$378 = w4\, \$377;$

$\$379 = -w3\, x5\, z4 - w3\, \$175 + \$199 + \$205 - w2\, \$206 - \$225 + \$233 - \$258 + \$268 + \$363 - w3\, \$367 + \$371 + \$375 + \$378;$

$\$380 = \$379^2;$

$\$381 = \$365 + \$366 + \$376 + \$380;$

$$\$382 = \frac{1}{\sqrt{\$381}};$$

$\$383 = x2\, z1;$

$\$384 = x3\, z2;$

$\$385 = y2\, z1;$

$\$386 = y3\, z2;$

$\$387 = x5\, z1;$

$\$388 = y5\, z1;$

$\$389 = w1\, \$142;$

$\$390 = w2\, \$176;$

$\$391 = w1\, \$207;$

$\$392 = -z1 + z4;$

```
$393 = -$137 - $142 - $176 + $208 + $383 + $384;
$394 = x1 $125;
$395 = x2 $128;
$396 = x1 $135;
$397 = -$124 - $125 - $128 + $144 + $385 + $386;
$398 = w2 $145;
$399 = w1 $178;
$400 = w2 $206;
$401 = x2 $210;
$402 = $137 - $145 - $178 + $387 + $401;
$403 = x2 $126;
$404 = x1 $130;
$405 = x2 $134;
$406 = y2 $210;
$407 = $124 - $126 - $130 + $388 + $406;
$408 = w1 x4 z5;
$409 = y4 $145;
$410 = w3 x5 z4;
$411 = x4 z3;
$412 = y4 z3;
$413 = y5 $207;
$414 = -z2 + z5;
{{$216 $318,
  $318 (-$188 - $190 - $192 - $219 - $222 - $225 + $233 +
     $234 + $235 + $389 + $390 + $391 +
     w3 ($137 - $175 + $177 - $209 + x2 $392) + w4 $393),
  $232 $318,
  $318 (-$187 - $189 - $191 - $218 - $221 - $224 + $236 +
     $237 + $238 + x3 ($124 - $127 + $129 - $146 + y2 $392) +
     $394 + $395 + $396 + x4 $397)},
 {$242 $330,
```

$$
\begin{aligned}
&\$330\,(-\$188-\$190-\$195-\$197-\$199-\$245+\$249+\\
&\quad \$389+\$390+w5\,\$393+\$398+\$399+\$400+w3\,\$402),\\
&\$248\,\$330,\\
&\$330\,(-\$189-\$191-\$194-\$196-\$198-\$244+\$250+\\
&\quad \$394+\$395+x5\,\$397+\$403+\$404+\$405+x3\,\$407)\},\\
&\{\$256\,\$343,\\
&\$343\,(-\$195-\$197-\$203+\$219+\$222-\$234-\$235-\\
&\quad \$258+\$268+\\
&\quad w5\,(-\$137+\$175-\$177+\$209-\$261+\$383)+\\
&\quad \$398+\$399+w4\,\$402+\$408),\ \$267\,\$343,\\
&\$343\,(-\$194-\$196-\$202+\$218+\$224-\$236-\$238-\\
&\quad \$259+\$269+\\
&\quad x5\,(-\$124+\$127-\$129+\$146-\$166+\$385)+\\
&\quad \$403+\$404+x4\,\$407+\$409)\},\\
&\{\$275\,\$362,\\
&\$362\,(w3\,\$145+w3\,\$177-\$192-\$203-\$205+\$245-\\
&\quad \$249-\$344-\$352+\\
&\quad w4\,(-\$145-\$206+\$208+x3\,\$210+\$387)+\$391+\\
&\quad \$408+\$410+\\
&\quad w5\,(\$176-\$177-\$207-\$208+\$209+\$411)),\\
&\$285\,\$362,\\
&\$362\,(x3\,\$126+x3\,\$129-\$187-\$202-\$204+\$244-\\
&\quad \$250-\$345-\$349+\\
&\quad x4\,(-\$126-\$134+\$144+y3\,\$210+\$388)+\$396+\\
&\quad \$409+x5\,(\$128-\$129-\$135-\$144+\$146+\$412)+\\
&\quad \$413)\},\\
&\{\$292\,\$382,\\
&\$382\,(w3\,\$175-\$199-\$205+\$225-\$233+\$258-\\
&\quad \$268-\$363+w3\,\$367-\$371+\$400+\$410+
\end{aligned}
$$

```
      w5 (-$142 - $175 - $207 + $261 + $384 + $411) +
      w4 ($142 + $178 - $206 - $367 + x3 $414)),
  $304 $382,
  $382 (x3 $127 + x3 $183 - $198 - $204 + $221 - $237 +
      $259 - $269 - $364 - $368 + $405 +
      x5 (-$125 - $127 - $135 + $166 + $386 + $412) +
      $413 + x4 ($125 + $130 - $134 - $183 + y3 $414))}}]
```

The “Output” option allows a compiled function to be returned instead of a Block in HoldForm. This is currently experimental and assumes all symbols are Real.

In[42]:=
```
cExp =
 Block[{$RecursionLimit = Infinity,
   $IterationLimit = Infinity},
  FactorExpression[pentachoronFaceNormals,
   "Output" → CompiledFunction]]
```

Out[42]=
```
CompiledFunction[
  Argument count: 20
  Argument types:
    {_Real, _Real, _Real, _Real, _Real, _Real, _Real, _Real, _R
]
```

In[43]:=
```
test3[{{x1_, y1_, z1_, w1_}, {x2_, y2_, z2_, w2_},
    {x3_, y3_, z3_, w3_}, {x4_, y4_, z4_, w4_},
    {x5_, y5_, z5_, w5_}}] :=
 cExp[w1, w2, w3, w4, w5, x1, x2, x3, x4, x5, y1, y2,
  y3, y4, y5, z1, z2, z3, z4, z5]
```

Run some timing tests.

```
In[44]:= testParams = RandomReal[{-1, 1}, {5, 4}];

In[45]:= {t1, res1} =
           SetPrecision[RepeatedTiming[test1[testParams], 3],
            $MachinePrecision];
         t1

Out[46]= 0.002212249823404273

In[47]:= {t2, res2} =
           SetPrecision[RepeatedTiming[test2[testParams], 3],
            $MachinePrecision];
         t2

Out[48]= 0.0007356430387412677

In[49]:= {t3, res3} =
           SetPrecision[RepeatedTiming[test3[testParams], 3],
            $MachinePrecision];
         t3

Out[50]= 0.00002148350944493357
```

Using FactorExpression can yield modest performance gains (about 3x faster in this example). For much larger expressions, the performance increase can get rather ridiculous (several orders of magnitude).

```
In[51]:= t1 / t2

Out[51]= 3.007232729598821

In[52]:= t1 / t3

Out[52]= 102.9743221923169
```

The results are also correct in case you were wondering.

```
In[53]:= Chop@Total[Abs[Flatten[res1 - res2]]]

Out[53]= 0

In[54]:= Chop@Total[Abs[Flatten[res1 - res3]]]

Out[54]= 0
```