



Home work 1 and 2

SUBMITTED 21 SEPT 2020

Home work 1

Calculate Porosity from Density

Given: ρ and S_w Logs

Input and Adjust : Grain Density ρ_0

Brine Density ρ_w

Hydrocarbon Density ρ_h

Calculate: Porosity ϕ

Plots : depth vs GR, Vp, Rho, porosity

Home work 1

Formulas of porosity from density

$$\phi = \frac{\rho_0 - \rho}{\rho_0 - \rho_f}$$

$$\rho_f = S_w \cdot \rho_w + (1 - S_w) \cdot \rho_{\text{gas}}$$

Home work 2

- (a) Make cross-plots of V_p/V_s versus porosity (shales and sandstone). Derive porosity from density assuming mineral density of quartz (2.65 gr/cm^3) and fluid density is brine of 1.08 gr/cm^3 . What can you say about the trends of shales versus sandstone?
- (b) Make cross-plots of K and μ versus porosity. Make theoretical plots with upper and lower bounds (Voigt, Reuss, Hill, Hashin-Shtrikman).
- (c) From (b) make cross-plots of V_p versus porosity (sandstone only). Make theoretical plots with upper and lower bounds (Voigt and Reuss)

Assume

$K_{\text{quartz}} = 37 \text{ GPa}$; $\mu_{\text{quartz}} = 44 \text{ GPa}$

$K_{\text{brine}} = 2.6 \text{ GPa}$; $\mu_{\text{brine}} = 0 \text{ GPa}$

Home work 2

Formulas of Voigt, Reuss Bounds, Hill Average
and Hashin-Shtrikman Bounds

Voigt and Reuss Bounds

On a strictly empirical basis one can imagine defining a power law average of the constituents

$$\overline{M}^{\alpha} = f_1 M_1^{\alpha} + f_2 M_2^{\alpha} + f_3 M_3^{\alpha} + \dots$$

where

\overline{M} = the effective modulus of the composite

M_i = the modulus of the i th constituent

f_i = the volume fraction of the i th constituent

α = a constant, generally between -1 and +1

Special cases are the Voigt average (an upper bound):

$$\overline{K}_V = f_Q K_Q + f_F K_F + f_C K_C \dots + f_W K_W + f_O K_O + f_G K_G$$

$$\overline{\mu}_V = f_Q \mu_Q + f_F \mu_F + f_C \mu_C \dots + f_W \mu_W + f_O \mu_O + f_G \mu_G$$

and the Reuss average (a lower bound):

$$\overline{K}_R^{-1} = f_Q K_Q^{-1} + f_F K_F^{-1} + f_C K_C^{-1} \dots + f_W K_W^{-1} + f_O K_O^{-1} + f_G K_G^{-1}$$

$$\overline{\mu}_R^{-1} = f_Q \mu_Q^{-1} + f_F \mu_F^{-1} + f_C \mu_C^{-1} \dots + f_W \mu_W^{-1} + f_O \mu_O^{-1} + f_G \mu_G^{-1}$$

Since these are upper and lower bounds, an estimate of the actual value is sometimes taken as the average of the two, known as the Voigt-Reuss-Hill average:

$$M_{VRH} = \frac{M_V + M_R}{2}$$

Hashin-Shtrikman Bounds

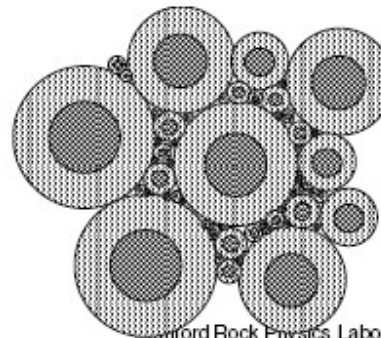
The narrowest possible bounds on moduli that we can estimate for an isotropic material, knowing only the volume fractions of the constituents, are the Hashin-Shtrikman bounds. (The Voigt-Reuss bounds are wider.) For a mixture of 2 materials:

$$K^{HS\pm} = K_1 + \frac{f_2}{(K_2 - K_1)^{-1} + f_1 \left(K_1 + \frac{4}{3} \mu_1 \right)^{-1}}$$
$$\mu^{HS\pm} = \mu_1 + \frac{f_2}{(\mu_2 - \mu_1)^{-1} + \frac{2 f_1 (K_1 + 2 \mu_1)}{5 \mu_1 \left(K_1 + \frac{4}{3} \mu_1 \right)}}$$

where subscript 1 = shell, 2 = sphere. f_1 and f_2 are volume fractions.

These give upper bounds when stiff material is K_1, μ_1 (shell) and lower bounds when soft material is K_1, μ_1 .

Interpretation of bulk modulus:



Home work 2

Table 3.1 - Mean density values (in $10^3 \text{ kg/m}^3 = \text{g/cm}^3$) for selected minerals; based on data from Olhoeft, Johnson 1989

Mineral	Density	Mineral	Density	Mineral	Density
Actinolite	3.200	Fluorite	3.179	Montmorillonite	2.608
Albite	2.620	Forsterite	3.213	Muscovite	2.831
Almandine	4.318	Galena	7.598	Natrolite	2.245
Andalusite	3.145	Glaucosite	2.300	Nepheline	2.623
Anhydrite	2.963	Graphite	2.267	Orthoclase	2.570
Anorthite	2.760	Gypsum	2.305	Phlogopite	2.784
Apatite	3.180	Glaucophane	3.200	Polyhalite	2.780
Aragonite	2.931	Halite	2.163	Pseudobrookite	4.390
Arsenopyrite	6.162	Hedenbergite	3.632	Pyrite	5.011
Augite	3.300	Hematite	5.275	CaAl-Pyroxene	3.360
Barite	4.480	Hornblende	3.080	Pyrrotite	4.610
Beryl	2.641	Illite	2.660	Quartz	2.648
Biotite	2.900	Ilmenite	4.788	Realgar	3.590
Calcite	2.710	Jacobsite	4.990	Rutile	4.245
Cassiterite	6.993	Kainite	2.130	Serpentine	2.600
Chalkopyrite	4.200	Kaolinite	2.594	Siderite	3.944
Chlorite	2.800	Kieserite	2.573	Sillimanite	3.241
Chromite	5.086	Kyanite	3.675	Sphalerite	4.089
Cordierite	2.508	Labradorite	2.710	Spinel	3.583
Danburite	3.000	Langbeinite	2.830	Sylvite	1.987
Diamond	3.515	Leucite	2.469	Talc	2.784
Diaspore	3.378	Maghemite	4.880	Titanomagnetite	4.776
Diopside	3.277	Magnesite	3.010	Tremolite	2.977
Dolomite	2.866	Magnetite	5.200	Trona	2.170

Home work 2

Table A.4.1 *Moduli, densities, and velocities of common minerals.*

Mineral	Bulk modulus (GPa)	Shear modulus (GPa)	Density (g/cm ³)	V _P (km/s)	V _S (km/s)	Poisson ratio	References
Olivines							
Forsterite	129.8	84.4	3.32	8.54	5.04	0.23	[1–3]
“Olivine”	130	80	3.32	8.45	4.91	0.24	[54]
Garnets							
Almandine	176.3	95.2	4.18	8.51	4.77	0.27	[1]
Zircon	19.8	19.7	4.56	3.18	2.08	0.13	[4, 7]
Epidotes							
Epidote	106.5	61.1	3.40	7.43	4.24	0.26	[9]
Dravite	102.1	78.7	3.05	8.24	5.08	0.19	[4–6]
Pyroxenes							
Diopside	111.2	63.7	3.31	7.70	4.39	0.26	[8, 9]
Augite	94.1	57.0	3.26	7.22	4.18	0.25	[9]
	13.5	24.1	3.26	3.74	2.72	0.06	[10]
Sheet silicates							
Muscovite	61.5	41.1	2.79	6.46	3.84	0.23	[11]
	42.9	22.2	2.79	5.10	2.82	0.28	[55]
	52.0	30.9	2.79	5.78	3.33	0.25	[24]
Phlogopite	58.5	40.1	2.80	6.33	3.79	0.22	[11]
	40.4	13.4	2.80	4.56	2.19	0.35	[55]
Biotite	59.7	42.3	3.05	6.17	3.73	0.21	[11]
	41.1	12.4	3.05	4.35	2.02	0.36	[55]
Clays							
Kaolinite	1.5	1.4	1.58	1.44	0.93	0.14	[10]

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"Gulf clays" (Han) ^a	25	9	2.55	3.81	1.88	0.34	[50, 53]
"Gulf clays" (Tosaya) ^a	21	7	2.6	3.41	1.64	0.35	[49, 53]
Mixed clays ^a				3.40	1.60		[49]
				3.41	1.63		[50]
Montmorillonite- illite mixture ^a				3.60	1.85		[51]
Illite ^a				4.32	2.54		[52]
Framework silicates							
Perthite	46.7	23.63	2.54	5.55	3.05	0.28	[54]
Plagioclase feldspar (Albite)	75.6	25.6	2.63	6.46	3.12	0.35	[10]
"Average" feldspar	37.5	15.0	2.62	4.68	2.39	0.32	
Quartz	37	44.0	2.65	6.05	4.09	0.08	[54]
	36.6	45.0	2.65	6.04	4.12	0.06	[14–16]
	36.5	45.6	2.65	6.06	4.15	0.06	[44]
	37.9	44.3	2.65	6.05	4.09	0.08	[47]
Quartz with clay (Han)	39	33.0	2.65	5.59	3.52	0.17	[50, 53]
Oxides							
Corundum	252.9	162.1	3.99	10.84	6.37	0.24	[17, 18]
Hematite	100.2	95.2	5.24	6.58	3.51	0.14	[19, 20]
	154.1	77.4	5.24	7.01	3.84	0.28	[10, 12]
Rutile	217.1	108.1	4.26	9.21	5.04	0.29	[21, 22]
Spinel	203.1	116.1	3.63	9.93	5.65	0.26	[1]

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Table A.4.1 (cont.)

Mineral	Bulk modulus (GPa)	Shear modulus (GPa)	Density (g/cm ³)	V_p (km/s)	V_s (km/s)	Poisson ratio	References
Magnetite	161.4	91.4	5.20	7.38	4.19	0.26	[4, 23, 24]
	59.2	18.7	4.81	4.18	1.97	0.36	[10]
Hydroxides							
Limonite	60.1	31.3	3.55	5.36	2.97	0.28	[10]
Sulfides							
Pyrite	147.4	132.5	4.93	8.10	5.18	0.15	[25]
	138.6	109.8	4.81	7.70	4.78	0.19	[10]
Pyrrhotite	53.8	34.7	4.55	4.69	2.76	0.23	[10]
Sphalerite	75.2	32.3	4.08	5.38	2.81	0.31	[26, 27]
Sulfates							
Barite	54.5	23.8	4.51	4.37	2.30	0.31	[14]
	58.9	22.8	4.43	4.49	2.27	0.33	[28]
	53.0	22.3	4.50	4.29	2.22	0.32	[7]
Celestite	81.9	21.4	3.96	5.28	2.33	0.38	[4]
	82.5	12.9	3.95	5.02	1.81	0.43	[28]
Anhydrite	56.1	29.1	2.98	5.64	3.13	0.28	[30]
	62.1	33.6	2.96	6.01	3.37	0.27	[48]
Gypsum	42.5	15.7	2.35	5.80			[29, 56, 57]
Polyhalite			2.78	5.30			[31]
Carbonates							
Calcite	76.8	32.0	2.71	6.64	3.44	0.32	[14]
	63.7	31.7	2.70	6.26	3.42	0.29	[32]
	70.2	29.0	2.71	6.34	3.27	0.32	[33]
	74.8	30.6	2.71	6.53	3.36	0.32	[43]
	68.3	28.4	2.71	6.26	3.24	0.32	[44]
Siderite	123.7	51.0	3.96	6.96	3.59	0.32	[34]

Dolomite	94.9	45.0	2.87	7.34	3.96	0.30	[35]
	69.4	51.6	2.88	6.93	4.23	0.20	[13]
	76.4	49.7	2.87	7.05	4.16	0.23	[45]
Aragonite	44.8	38.8	2.92	5.75	3.64	0.16	[19, 20, 36]
Natronite	52.6	31.6	2.54	6.11	3.53	0.26	[53, 54]
Phosphates							
Hydroxyapatite	83.9	60.7	3.22	7.15	4.34	0.21	[4]
Fluorapatite	86.5	46.6	3.21	6.80	3.81	0.27	[37]
Fluorite	86.4	41.8	3.18	6.68	3.62	0.29	[38, 39]
Halite	24.8	14.9	2.16	4.55	2.63	0.25	[14, 40–42]
			2.16	4.50	2.59		[46]
Sylvite	17.4	9.4	1.99	3.88	2.18	0.27	[40]
Organic							
Kerogen	2.9	2.7	1.3	2.25	1.45	0.14	[53, 54]
Zeolites							
Narolite	46.6	28.0	2.25	6.11	3.53	0.25	[53, 54]

Note:

* Clay velocities were interpreted by extrapolating empirical relations for mixed lithologies to 100% clay (Castagna *et al.*, 1993).

References: [1] Verma (1960); [2] Graham and Barsch (1969); [3] Kumazawa and Anderson (1969); [4] Hearmon (1956); [5] Mason (1950); [6] Voigt (1890); [7] Huntington (1958); [8] Ryzhova *et al.* (1966); [9] Alexandrov *et al.* (1964); [10] Woeber *et al.* (1963); [11] Alexandrov and Ryzhova (1961a); [12] Wyllie *et al.* (1956); [13] *Log Interpretation Charts* (1984); [14] Simmons (1965); [15] Mason (1943); [16] Koga *et al.* (1958); [17] Wachtman *et al.* (1960); [18] Bernstein (1963); [19] Hearmon (1946); [20] Voigt (1907); [21] Birch (1960a); [22] Joshi and Mitra (1960); [23] Doraiswami (1947); [24] Alexandrov and Ryzhova (1961b); [25] Simmons and Birch (1963); [26] Einspruch and Manning (1963); [27] Berlincourt *et al.* (1963); [28] Seshagiri Rao (1951); [29] Tixier and Alger (1967); [30] Schwerdtner *et al.* (1965); [31] *Formation Evaluation Data Handbook* (1982); [32] Bhimasenacher (1945); [33] Peselnick and Robie (1963); [34] Christensen (1972); [35] Humbert and Plique (1972); [36] Birch (1960b); [37] Yoon and Newnham (1969); [38] Bergmann (1954); [39] Huffman and Norwood (1960); [40] Spangenburg and Haussuhl (1957); [41] Lazarus (1949); [42] Papadakis (1963); [43] Dandekar (1968); [44] Anderson and Lieberman (1966); [45] Nur and Simmonds (1969b); [46] Birch (1966); [47] McSkimin *et al.* (1965); [48] Rafavich *et al.* (1984); [49] Tosaya (1982); [50] Han *et al.* (1986); [51] Castagna *et al.* (1985); [52] Eastwood and Castagna (1986); [53] Blangy (1992); [54] Carmichael (1989); [55] Ellis *et al.* (1988); [56] Choy *et al.* (1979); [57] Bhalla *et al.* (1984).