

Probability and Mathematical Statistics: Final Project

Due on December 31, 2022 at 11:59am

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Problem 1 (Part 1: classical Bandit Algorithms)**Solution**

1,2. See jupyter notebook.

3. According to python simulation, the results are respectively:

results for epsilon-greedy Algorithm:

[0.69956534 0.50167567 0.39846044] with parameter: 0.1

[0.70064482 0.49688743 0.39858856] with parameter: 0.5

[0.70009067 0.49985083 0.39811366] with parameter: 0.9

results for UCB Algorithm:

[0.70088294 0.49274932 0.38448981] with parameter: 1

[0.7006747 0.49900798 0.39886189] with parameter: 5

[0.70135645 0.49877856 0.40023188] with parameter: 10

results for TS Algorithm:

[0.6995964 0.45887541 0.37554019] with parameter: [[1, 1], [1, 1], [1, 1]]

[0.68296885 0.4001996 0.362563] with parameter: [[601, 401], [401, 601], [2, 3]]

4.

5. In the case of all algorithms, more exploitation means that the total reward we get is larger, while more exploration means the estimated value of the reward of each arm is more accurate, but it also means that the total reward during our tests is less.

For the ϵ -greedy algorithm, the larger ϵ is, there is more chance of exploration than exploitation. So while ϵ grows, the gaps between the algorithm output and the oracle value decreases, but the sum of the reward of all trials is smaller.

As for the UCB Algorithm, the exploration-exploitation trade-off depends on the value of c . Similar to the ϵ -greedy algorithm, the larger c is, there is more exploration and less exploitation. As can be deduced from the data in the python simulation, for larger c , the estimated value for the third arm is more accurate. However, the estimated value for arms 1 and 2 becomes less accurate. After discussion, we believe this is because the larger c is, the more average the tests are. Consequently, there are more tests on the third arm (it has the smallest oracle value), but there are less test on the first and second arm.

The TS Algorithm is rather different from the previous ones. If the initial value of a and b we passed in is too small, then the result of the first few tests may influence the final result greatly, and the exploration may be so small that only a few dozen tests are on the second and third arm. If this happens, the result will be frightfully small. And after 200 independent trials of $N = 5000$, the final estimation still has a great gap with the oracle value. On the other hand, if the initial value of a and b we passed in is relatively large, this value of a and b may influence the final result greatly. As in the trial with parameter [[601, 401], [401, 601], [2, 3]], we can see that the final estimation of arm two is very close to $\frac{401}{401+601}$.

6.