(5) 1. Find Maclaurin series for the following functions, using known series and state their radii of convergence:

(a) $f(x) = (8 + x^2)^{2/3}$

(b) $g(x) = \sin x \cos x$

(6) 2. Let $g(x) = \displaystyle\int_0^x \frac{\ln(1+t)}{t}\,dt$, if $x \neq 0$ and $g(0) = 0$.

(a) Find the Maclaurin series for $g(x)$; express your answer in $\sum$ form and state the radius of convergence.

(b) Find $g(0.2)$ correct to 3 decimal places.

(c) Find $g^{(7)}(0)$.

(7) 3. For the function $f(x) = x^2 e^{x-1}$:

(a) Find the third degree polynomial $T_3(x)$ centered at $a = 1$ and an expression for the remainder $R_3(x)$.

(b) Use $T_3(x)$ to approximate $f(1/2)$.

(c) Estimate the maximum error of your approximation using Taylor's inequality or Lagrange's form of the remainder.

(6) 4. Given the curve $\mathcal{C}$ having parametric equations: $x = -(t^3 + 3t)$, $y = t^2$

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. Simplify your answers.

(b) Find, if any, intercepts and points on $\mathcal{C}$ where the tangent line is vertical or horizontal.

(c) Sketch the graph of $\mathcal{C}$ showing the orientation of the curve.

(d) Set up, **but do not evaluate**, an integral needed to find the area of the region bounded by $\mathcal{C}$, the y-axis and $y = 4$.

(8) 5. Given the polar curves $r_1 = 1 - 2\cos\theta$ and $r_2 = 1 - \cos\theta$, do the following:

(a) Sketch both graphs on the same axes.

(b) Find all the points of intersection for $\theta \in [0, 2\pi]$.

(c) Set up, **but do not evaluate,** the integral needed to find the area of the region inside r_1 and outside r_2.

(d) Find the length of r_2.

(10) 6. Let $\mathcal{C}$ be the space curve represented by $\mathbf{r}(t) = \langle e^t, \sqrt{2}\, t, e^{-t} \rangle$.

(a) Find a set of the parametric equations for the tangent line to $\mathcal{C}$ at $P(1, 0, 1)$.

(b) Find an equation (in $ax + by + cz = d$ form) of the normal plane of $\mathcal{C}$ at $P(1, 0, 1)$.

(c) Find the length of the curve for $0 \leq t \leq 1$.

(d) Find the curvature at any point.

(e) Find the tangential and normal components of the acceleration vector (a_T and a_N) at any point.

(6) 7. Sketch and give the name of the following surfaces:

(a) $y - z^2 = 0$

(b) $x^2 + 9z^2 - 3y^2 + 9 = 0$

(c) $\rho = \cos\phi$

(4) 8. Find the limit if it exists or show that it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\cos^2(y)}{4x^2 + 3y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

(7) 9. Let $f(x, y, z) = x^3y^4z^2$ and $P(1, 1, 1)$.

(a) Find the direction in which the maximum rate of change of f at P occurs.

(b) What is the maximum rate of change?

(c) Find an equation (in $ax + by + cz = d$ form) of the tangent plane to the level surface $x^3y^4z^2 = 1$ at the point P.

(d) If $x^3y^4z^2 = 1$, find $\dfrac{\partial z}{\partial y}$.

(e) Find the directional derivative of f at P in the direction $\overrightarrow{PQ}$ where $Q(3, 2, 5)$.

(f) Show that the space curve $\mathbf{r}(t) = \langle -t^2 + 2, 1, t^3\rangle$ is tangent to the level surface $x^3y^4z^2 = 1$ at $P(1, 1, 1)$.

(3) 10. Let $f(x, y, z) = \sqrt{xyz}$.

(a) Find the total differential of f.

(b) Approximate $f(1.9, 2.02, 4.05)$ using the differential of f.

(3) 11. Find a set of parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $2x^2 + y^2 + z^2 = 76$ at the point $(2, 2, 8)$.

(4) 12. Let $z = f(u, v)$ where $u = x^2 + y^2$ and $v = x - y$. Find $\dfrac{\partial^2 z}{\partial x^2}$.

(5) 13. Find and classify the critical points of $f(x, y) = y^2x - yx^2 + xy$.

(5) 14. Use the method of Lagrange multipliers to find the maximum and minimum of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.

(8) 15. Evaluate

(a) $\displaystyle\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dydx$

(b) $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{4 - y^2 - x^2}\, dydx$

(3) 16. Rewrite the integral $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} dzdydx$ in the order $dxdydz$ (**do not evaluate**).

(5) 17. Sketch the region below the paraboloid $z = x^2 + y^2$, above the xy-plane and inside the cylinder $x^2 + y^2 = 2x$. Set up (**do not evaluate**) triple integrals needed to find its volume in

(a) cartesian coordinates

(b) cylindrical coordinates

(5) 18. Show that the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 2az$ where $a > 0$ has volume $V = \pi a^3$.