1. If $f(x)=2x\arctan 2x-\frac{1}{2}\ln\left(1+4x^2\right)+\arcsin\left(\frac{2}{3}\right)$, a) find $f'(x)$, and simplify your answer. **[4]**

b) evaluate $f'\left(\frac{1}{2}\right)$ **Ans:** $2\arctan 2x$, $\frac{\pi}{2}$

2. Evaluate the following limits; use the symbols $-\infty$ or $+\infty$ when appropriate:

3. a) $\lim\limits_{x\to\infty}\left(1+\frac{4}{x}\right)^{2x}$ **Ans:** e^8, 0, 18

[3]

b) $\lim\limits_{x\to 0^+} e^{-2/x}\ln x$ **[3]**

c) $\lim\limits_{x\to 0}\frac{e^{6x}-6x-1}{x^2}$ **[3]**

4.. Perform the operations: **Ans:**

a) $\int\frac{2x+1}{\sqrt{x-3}}dx$ $2\left[\frac{2}{3}(x-3)^{3/2}+7(x-3)^{1/2}\right]+C$ **[4]**

b) $\int\frac{9x-1}{(x-3)\left(x^2+4\right)}dx$ $2\ln|x-3|-\ln\left(x^2+4\right)+3/2\,\tan^{-1}\left(x/2\right)+C$ **[5]**

c) $\int x\sec^{-1}x\,dx$ $\frac{1}{2}\left(x^2\sec^{-1}x-\sqrt{x^2-1}\right)+C$ **[3]**

d) $\int_0^{\frac{\pi}{4}}\sin^3 2x\cos^4 2x\,dx$ $\frac{1}{35}$ **[5]**

e) $\int_0^{\frac{1}{2}}\frac{\arcsin x}{\sqrt{1-x^2}}dx$ $\frac{\pi^2}{72}$ **[3]**

f) $\int e^{3x}\sin x\,dx$ $\frac{e^{3x}}{10}(-\cos x+3\sin x)+C$ **[5]**

g) $\int\frac{1}{\sqrt{9x^2-16}}dx$ $\frac{1}{3}\left(\ln 3x+\sqrt{9x^2-16}\right)+C$ **[5]**

4. Evaluate the improper integral:

a) $\int_1^{\frac{2}{\sqrt{3}}}\frac{dx}{x\sqrt{x^2-1}}$ $\frac{\pi}{6}$ **[3]**

Ans:

b) $\displaystyle\int_4^{\infty} \frac{dx}{x\ln x}$ D **[4]**

5. Solve the differential equation satisfying the given conditions:

$2y\dfrac{dy}{dx} = y^2 - 1\ ,\ \ y > 0,\ y(0) = 2$ $\ln\left|\dfrac{y^2-1}{3}\right| = x$ **[4]**

6. Find the exact area of the region between the curves $y = \dfrac{2}{x} - 1$ and $y = 2 - x.$ **[4]**

Please draw a picture and shade the region. $\dfrac{3}{2} - \ln 4\ \ sq.\ units$

7. Let *R* be the region bounded by $y = \sin\left(x^2\right)$ and the x -axis on $\left[0, \sqrt{\pi}\,\right]$.

a) Find the exact volume of the solid obtained from revolving R about the *y*-axis. **[3]**

b) Set up, but **do not attempt to evaluate**, the integral required to find the volume of the solid obtained from revolving *R* about the *x*-axis. $2\pi\ cu.units$ $\displaystyle\int \pi \sin^2\left(x^2\right)dx$ **[2]**

8. Determine whether the sequence converges or diverges; if it converges, say what it converges to:

a) $\left\{1 + \cos\left[\ (2n+1)\dfrac{\pi}{2}\right]\right\}$ C to 1 **[1]**

b) $\left\{(-1)^n\,\dfrac{3n^2 + n - 2}{n^2}\right\}$ D : $\lim\limits_{n\to\infty} a_n\ dne$ **[1]**

9. Answer True (T) or False (F). If your answer is ‘False’ , give an example to illustrate. **[3]**

a) If $\lim\limits_{n\to\infty}\left|\,a_n\,\right| \neq 0,$ then $\lim\limits_{n\to\infty} a_n \neq 0$ _______________________ T **[1]**

b) If $\lim\limits_{n\to\infty} a_n = 0,$ then $\displaystyle\sum_{n=1}^{\infty} \sin a_n$ converges _________________ F ex. $a_n = \dfrac{1}{n}$ **[2]**

10. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty}\left(\frac{3^{n+1} + 2^n}{4^n}\right)$. Sum is 14 **[3]**

11. Classify each of the following series as convergent or divergent, and briefly justify your answer.

a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n^2}\right)$ D: $\sum \dfrac{1}{n} D,\ \sum \dfrac{1}{n^2} C$ **[3]**

b) $\sum_{n=1}^{\infty}\left(\frac{2n-e}{n^2}\right)^{2n}$ C by Root Test **[3]**

c) $\sum_{n=1}^{\infty}\frac{\sqrt{n^3-1}}{n^2+1}$ D by LCT with $\sum\frac{1}{\sqrt{n}}$ which D **[3]**

d) $\sum_{n=0}^{\infty}\frac{(n!)^2}{(2n)!}$ C by Ratio Test **[3]**

12. Classify each of the following series as absolutely convergent, conditionally convergent, or divergent. Briefly justify your answer.

a) $\sum_{n=1}^{\infty}(-1)^n\frac{\arctan n}{n^3+1}$ AC by LCT with $\sum\frac{1}{n^2}$ which C **[3]**

b) $\sum_{n=1}^{\infty}(-1)^n\cos\frac{1}{n}$ D by NTT **[2]**

c) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+1}$ Not AC but C By AST, hence CC **[3]**

13. Determine the radius and interval of convergence of the series:

$$\sum_{n=1}^{\infty}\frac{3^{n-1}(x+1)^n}{n\sqrt{n+1}}$$

$\left[-\frac{4}{3},-\frac{2}{3}\right]$ **[5]**

14. Let $f(x)=\ln(x+1)$. **[5]**

a) Write the first 5 non-zero terms of the Maclaurin series for *f*.

b) Find a formula for the nth term, and write the series in sigma notation.

$$x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\frac{1}{5}x^5\text{.........}$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$$