Calculus II Science exam, Fall 2023

1. (30 points) Evaluate the following integrals.

   (a) $\displaystyle\int \frac{x+21}{(x+1)(x^2+9)}\,dx$

   (b) $\displaystyle\int_1^5 \frac{x+2}{\sqrt{2x-1}}\,dx$

   (c) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-25}}$

   (d) $\displaystyle\int e^{\sqrt{2x+1}}dx$

   (e) $\displaystyle\int \sin^5 x\,dx$

   (f) $\displaystyle\int \ln(x^2+1)\,dx$

2. (6 points) Evaluate the following limits. If using l'Hospital's rule, justify why it may be used.

   (a) $\displaystyle\lim_{x\to 0^+} \frac{(\ln x)^2}{1+x^{-1}}$

   (b) $\displaystyle\lim_{x\to\infty} \left(\frac{x}{x+3}\right)^{2x}$

3. (10 points) Evaluate the improper integral.

   (a) $\displaystyle\int_0^2 \frac{1}{(2x-1)^2}\,dx$

   (b) $\displaystyle\int_0^\infty \frac{e^{-x}}{e^{-x}+1}\,dx$

4. (5 points) Find the area of the region bounded by $y=\sqrt{8x}$ and $y=x^2$.

5. (5 points) Let R be the region bounded by the graphs of $x=4y-y^2$ and $x=0$. Set up, but do not evaluate, an integral for the volume of the the solid obtained by rotating R about

   (a) the y-axis

   (b) the line $y=5$

6. (5 points) Solve the differential equation $y'=\dfrac{x}{ye^x}$ given $y(-1)=-2$.

7. (5 points) The rate of decay at time t (in hours) of a radioactive substance N is proportional to the amount of substance present. 6g remain after two hours and 1.2g remain after another hour. Solve a differential equation to express N as a function of t.

8. (5 points) Find the limit of the sequence or explain why the sequence diverges.

   (a) $a_n = \dfrac{\arcsin(2/n)}{\arcsin(3/n)}$

   (b) $a_n = \ln(en+1)-\ln(n+5)$

9. (4 points) Find the sum or explain why the series diverges.

   (a) $\displaystyle\sum_{n=3}^{\infty}\left(\frac{1}{n+3}-\frac{1}{n+5}\right)$

   (b) $\displaystyle\sum_{n=1}^{\infty} 3^{1-n}4^{n+1}$

10. (8 points) Determine whether the series converges or diverges.

    (a) $\displaystyle\sum_{n=1}^{\infty}\frac{n+3^n}{n+4^n}$

    (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1+\cos^2 n}{1+n^3}$

    (c) $\displaystyle\sum_{n=1}^{\infty}\sin\left(\frac{\pi n}{2n+1}\right)$

11. (7 points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

    (a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{(\arctan(n))^n}$

    (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n}}{2n-1}$

12. (5 points) Find the radius and interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(x+3)^n}{n4^n}$.

13. (5 points) Find the Taylor Series expansion for $f(x)=\dfrac{1}{2-x}$ centered at $x=5$.

ANSWERS

1. (a) $2\ln(|x+1|) - \ln(x^2+9) + \arctan\left(\dfrac{x}{3}\right) + C$
   (b) $28/3$
   (c) $\dfrac{\sqrt{x^2-25}}{25x} + C$
   (d) $e^{\sqrt{2x+1}}\left(\sqrt{2x+1} - 1\right) + C$
   (e) $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$
   (f) $x\ln(x^2+1) - 2x + 2\arctan x + C$
2. (a) 0
   (b) e^{-6}
3. (a) divergent
   (b) $\ln 2$
4. $8/3$
5. (a) $\displaystyle\int_0^4 \pi(4y-y^2)^2\,dy$
   (b) $\displaystyle\int_0^4 2\pi(5-y)(4y-y^2)\,dy$
6. $y = -\sqrt{2(2 - xe^{-x} - e^{-x})}$
7. $N(t) = 150(0.2)^t$
8. (a) $2/3$
   (b) 1
9. (a) $13/42$
   (b) divergent
10. (a) convergent
    (b) convergent
    (c) divergent
11. (a) absolutely convergent
    (b) conditionally convergent
12. $\text{IOC} = [-7, 1),\ R = 4$
13. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{3^{n+1}}$