Answers

1. $\frac{dy}{dx} = \frac{1-t}{t}$; $\frac{d^2y}{dx^2} = -\frac{1}{2t^3}$
   H.T. at $(-3,1)$ when $t = 1$
   $A = \int_0^2 (4t^2 - 2t^3)dt$ and $\mathcal{L} = 2\int_0^2 \sqrt{2t^2 - 2t + 1}dt$

2. Points of intersection: $(3, \pi/3)$, $(3, 5\pi/3)$ and the pole.
   $A = 2\left(\frac{1}{2}\int_0^{\pi/3} 4(1+\cos\theta)^2 d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} 36\cos^2\theta d\theta\right)$
   $\mathcal{L} = 2\int_0^\pi \sqrt{4(1+\cos\theta)^2 + 4\sin^2\theta}d\theta = 8\int_0^\pi \cos(\theta/2)d\theta = 16$

3. $\mathcal{L} = 2$; $a_T = 6t$; $a_N = \sqrt{6}$; $\kappa(t) = \frac{\sqrt{6}}{(3t^2+1)^2}$ and $\kappa(1) = \frac{\sqrt{6}}{16}$.

4. (a) Note $x^2 + y^2 = z^2$ and $z = t$. So the curve spirals upward on the boundary of the cone $x^2 + y^2 = z^2$
   (b) $x^2 + y^2 - z^2 = 9$, and $z \geq 0$ Hyperboloid of one sheet, top part only.
   (c) Three parabolas, $x = y^2$, $x = y^2 + 1$ and $x = y^2 - 1$.

5. (a) Maximum rate of change$=\|\nabla w(3,-2,1)\| = 2\sqrt{41}$ in the direction of $\nabla w(3,-2,1)$, or in the direction of the unit vector $\frac{1}{\sqrt{41}}\langle 6,2,1\rangle$.
   (b) Its direction vector $\vec{v}$ is parallel to $\nabla F(3,-2,1) \times \nabla G(3,-2,1)$ where $F(x,y,z) = x^2 + 4y^2 + 2z^2$ and $G(x,y,z) = x^2 + y^2 - 2z^2$.
   $L:\ \langle x,y,z\rangle = \langle 3,-2,1\rangle + t\langle 10,6,9\rangle;\quad t \in \mathbf{R}$

6. $(-1,-2)$ and $(-1,2)$ are saddle points; $(\sqrt{5},0)$ is a local minimum while $(-\sqrt{5},0)$ is a local maximum.

7. (i) $\frac{\partial f}{\partial y} = \frac{x^2}{y}$ and $\frac{\partial^2 f}{\partial x \partial y} = \frac{2x}{y}$
   (ii) $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}(2s) + \frac{\partial z}{\partial y}(2r)$
   $\frac{\partial^2 z}{\partial r \partial s} = 4rs\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x^2}\right) + 4(r^2+s^2)\frac{\partial^2 z}{\partial y \partial x} + 2\frac{\partial z}{\partial y}$
   (iii) Let $F(x,y,z) = e^{xz} + \tan(yz) - xz^2$. Then
   $$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{z(z - e^{xz})}{xe^{xz} + y\sec^2(yz) - 2xz}$$

8. (a) $I = \frac{1}{4}(\sqrt{2} - 1)$ (Change the order of integration)
   (b) $I = \int_0^{\pi/4}\int_1^2 r dr d\theta$

9. $V = \int_0^\pi \int_0^{4\sin\theta} \int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} r dz dr d\theta$

10. (b) $V = \int_0^2 \int_0^{2-x} (4 - x^2) dy dx$

11. (a) $I = \int_0^{2\pi}\int_0^2\int_{r^2}^4 (r^2\cos\theta\sin\theta) r dz dr d\theta$
    (b) $I = \int_0^{2\pi}\int_0^{\arctan(1/2)}\int_0^{4/\cos\phi} (\rho^2\sin^2\phi\cos\theta\sin\theta)\rho^2\sin\phi d\rho d\phi d\theta +$
    $\int_0^{2\pi}\int_{\arctan(1/2)}^{\pi/2}\int_0^{\cot\phi\csc\phi} (\rho^2\sin^2\phi\cos\theta\sin\theta)\rho^2\sin\phi d\rho d\phi d\theta$

12. $\sqrt{4+x^3} = 2(1 + \frac{x^3}{4})^{1/2} = 2(1 + \frac{1}{2}(x^3/4) + \sum_{n=2}^\infty \frac{(-1)^{(n-1)}(1)(3)\cdots(2n-3)x^{3n}}{2^{3n}n!})$
    $\int_0^t \sqrt{4+x^3} = 2(t + \frac{t^4}{32} - \frac{t^7}{2^7(7)} + \frac{t^{10}}{2^{10}(10)} - \cdots)$
    $\int_0^{0.5} \sqrt{4+x^3} \simeq 1 + \frac{1}{2^8} - \frac{1}{2^{13}(7)} \simeq 1.003889$
    $|error| \leq \frac{1}{2^{19}(10)} = 0.2 \times 10^{-6}$

13.

$$\lim_{x\to 0} \frac{x^2(\frac{1}{2} - \frac{x^2}{4!} + \cdots)}{x^2(1 + x + \frac{x^2}{2!} + \cdots)} = \frac{1}{2}$$

14. (a) $T_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$

$R_3(x) = \frac{-15(x-4)^4}{16(4!)z^{7/2}}$

(b) $T_3(4.1) = 2 + \frac{1}{4}(0.1) - \frac{1}{64}(0.1)^2 + \frac{1}{512}(0.1)^3 \simeq 2.0248457$

$|R_2(4.1)| \leq \frac{(15)(0.1)^4}{16(4!)(4^{7/2})} = \frac{(15)(0.1)^4}{2^{11}(4!)} \simeq 3.0518 \times 10^{-8}$ (since $4 < z < 4.1$)

15. (a) $f^{(6)}(0) = \frac{6!}{3} = 240$ (b) $\ln(3/2)$

16. (i) Starting with the geometric series one can show $\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} nx^n$ with $R = 1$

(ii)

$$\frac{1}{2x+5} = (1/9)\left(\frac{1}{1+(2/9)(x-2)}\right) = (1/9)\sum_{n=0}^{\infty}(-2/9)^n(x-2)^n = \sum_{n=0}^{\infty}\frac{(-2)^n(x-2)^n}{3^{2n+2}}$$

where $R = 9/2$