1. Evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to -1} \frac{x^2-1}{x^2+3x+2}$

(b) $\displaystyle\lim_{x\to 1^+} \frac{\cos(\pi x/3)}{\ln(x)}$

(c) $\displaystyle\lim_{x\to -\infty} \frac{x-\sqrt{x^2-x+7}}{3x+7}$

(d) $\displaystyle\lim_{x\to 0} \frac{x^2-x}{\tan(x)}$

(e) $\displaystyle\lim_{x\to 0} \frac{\dfrac{1}{|2x-1|}-\dfrac{1}{|2x+1|}}{x}$

2. Consider the function $f(x)=\dfrac{3e^x+4}{e^x-2}$.

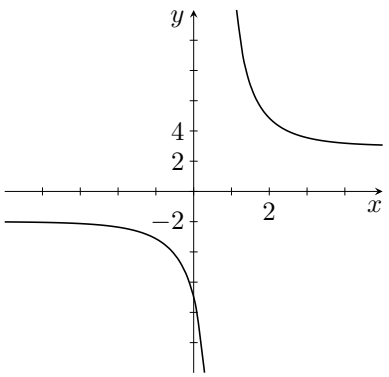
(a) Find the domain of f.

(b) Find the horizontal asymptotes of f.

(c) Find the vertical asymptote of f.

(d) Determine which one of the following graphs is the graph of f.

i.



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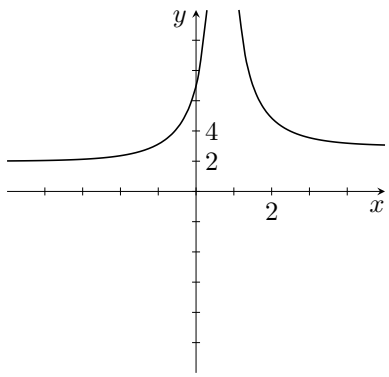

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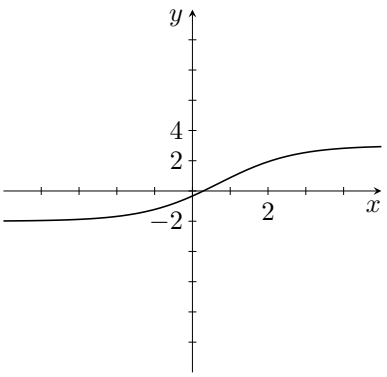

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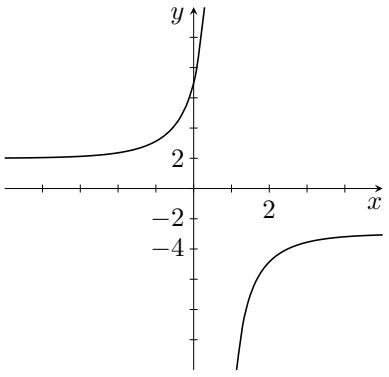

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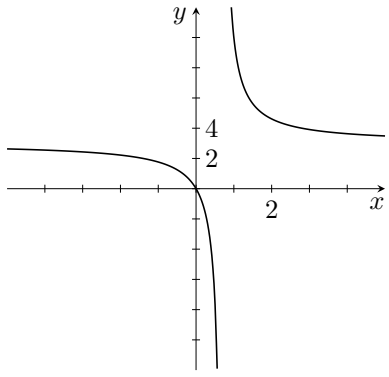


3. Find the value of a such that $f(x)$ is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{a\sin(ax) + x}{x} & \text{if } x < 0, \\ 2 & \text{if } x = 0, \\ 5e^x + 2a - a^2 & \text{if } x > 0. \end{cases}$$

4. Use the limit definition of the derivative to find $f'(x)$, where $f(x) = \sqrt{5 - 3x}$.

5. Find $\dfrac{dy}{dx}$ for each of the following. **Do not** simplify your answers.

(a) $y = \sqrt[5]{x^2} + 2^x + \log_2(x) - e^2$

(b) $y = \tan^3\left(x^2 \sec(x)\right)$

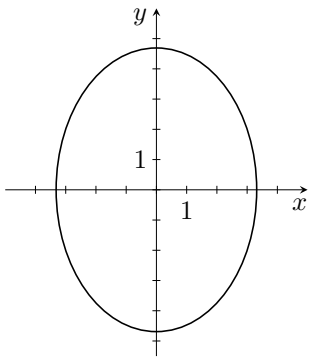
(c) $y = \dfrac{e^{2x} + \sin(x)}{\sqrt{x + \sqrt{x}}}$

(d) $y = (x + 1)^{\cos(x)}$

(e) $e^{xy} = (x + y)^5$

6. Use logarithmic differentiation to find the derivative of the function $y = \sqrt[4]{\dfrac{x^3 + 1}{7^x \tan(x)}}$.

7. Find the points on the ellipse $2x^2 + y^2 = 22$ where the tangent lines are parallel to the line $y = 3x + 8$.



8. Let $g(x) = \dfrac{f\big(\sin(x)\big)}{f'(x)}$. Find $g'(\pi/2)$ given that $f(1) = f'(\pi/2) = 2$ and $f''(\pi/2) = 3$.

9. Find the critical numbers of $f(x) = 3(2x-1)^{1/3}(x-2)^{2/3}$.

10. Let $f(x) = x^2 - 2\ln(x)$.

a) Find the domain of the function.

b) Find the interval(s) on which the function $f(x)$ is concave up.

11. Find the absolute maximum and the absolute minimum of $f(x) = \ln(4x^4 + x^2 + 1)$ on the interval $[-1, 1]$.

12. Consider the following function, along with its first and second derivatives.

$$f(x) = \frac{3}{2}\sqrt[3]{x^2} - x = \sqrt[3]{x^2}\left(\frac{3}{2} - \sqrt[3]{x}\right) \qquad f'(x) = \frac{1-\sqrt[3]{x}}{\sqrt[3]{x}} \qquad f''(x) = \frac{-1}{3\sqrt[3]{x^4}}$$

(a) Find the domain and any intercepts of f.

(b) Find the vertical and horizontal asymptotes of f (if any).

(c) Find the intervals on which f is increasing or decreasing.

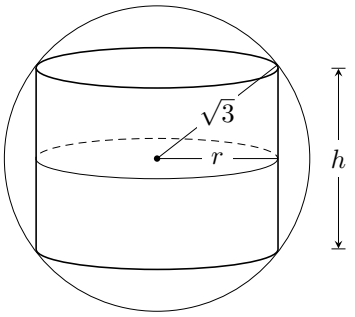
(d) Find the local (relative) extrema of f.

(e) Find the intervals of upward and downward concavity of f.

(f) Find all inflection points of f.

(g) On the next page, sketch the graph of f, labelling all intercepts, asymptotes, extrema, and points of inflection.

13. A right circular cylinder is inscribed in a sphere of radius $\sqrt{3}$. Find the largest possible volume of such a cylinder ($V = \pi r^2 h$).



14. Evaluate each of the following integrals.

(a) $\displaystyle\int_1^4 \frac{x^2+\sqrt{x}}{x}\,dx$

(b) $\displaystyle\int \left(\frac{3}{\sqrt[3]{x^2}}+e+\frac{1}{x}\right)dx$

(c) $\displaystyle\int \big(\sec^2(x)+\csc^3(x)\big)\sin(x)\,dx$

15. Find the position function $s(t)$ if the acceleration is given by $a(t)=3\sin(t)-\cos(t)$, where $s(0)=4$ and $s(\pi)=4$.

16. Consider the function $f(x)=\displaystyle\int_2^x \frac{\sin(t)}{t+\sin(t)}\,dt$.

(a) Find $f(2)$.

(b) Find $f'(x)$.

(c) Show that $f(x)$ has a horizontal tangent line at $x=\pi$.

17. Evaluate $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{e^{i/n}}{n}$ by expressing this limit as a definite integral over the interval $[0,1]$.

Answers:

1) a)-2, b) ∞, c) $2/3$, d) -1, e) 4; 2) a) $x\in\mathbb{R}\setminus\{\ln 2\}$, b) $y=3$ and $y=-2$, c) $x=\ln 2$, d) i.

3) $a=-1$; 4) $f'(x)=\dfrac{-3}{2\sqrt{5-3x}}$; 5) a) $y'=2/5x^{-3/5}+2^x\ln 2+\dfrac{1}{x\ln 2}$,

b) $y'=3\tan^2(x^2\sec(x))\sec^2(x^2\sec(x))\big(2x\sec(x)+x^2\sec(x)\tan(x)\big)$,

c) $y'=\dfrac{(2e^{2x}+\cos(x))\sqrt{x+\sqrt{x}}-(e^{2x}+\sin(x))\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}}{x+\sqrt{x}}$,

d) $y'=(x+1)^{\cos(x)}\Big(\dfrac{\cos x}{1+x}-\sin x\ln(1+x)\Big)$,

e) $y'=\dfrac{xy+y^2-5}{5-x^2-xy}$

6) $y'=\dfrac{1}{4}\sqrt[4]{\dfrac{x^3+1}{7^x\tan(x)}}\left[\dfrac{3x^2}{x^3+1}-\ln 7-\dfrac{1}{\sin x\cos x}\right]$.

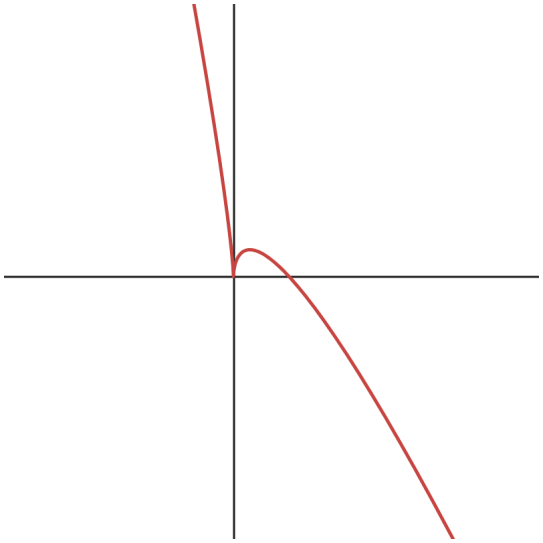
7) $(3,-2)$ and $(-3,2)$; 8) $-3/2$; 9) $x=1/2$, $x=1$ and $x=2$; 10) a) $x\in(0,\infty)$, b) concave up on its domain;

11) Absolute maximum: $f(-1)=f(1)=\ln 6$, absolute minimum $f(0)=0$; 12) a) domain: $\mathbb{R}$,

x-intercept: $(0,0)$ and $(27/8,0)$, y -intercept $(0,0)$, b) None, c)decreasing : $(-\infty,0)$ and $(1,\infty)$,

increasing: $(0,1)$, d) local minimum at $(0,0)$, local maximum at $(1,1/2)$, e) C.D. on the whole domain,

f) no inflection points;



13) $V_{max}=4\pi$; 14) a)19/2, b) $9\sqrt[3]{x}+ex+\ln|x|+C$, c)$\sec(x)-\cot(x)+C$;

15) $-3\sin t+\cos t+\frac{2}{\pi}x+3$; 16) a) 0, b)$f'(x)=\dfrac{\sin(x)}{x+\sin(x)}$, c)$f'(\pi)=0$; 17) $e-1$.