1. (6 points) Evaluate the following limits. Use $\infty, -\infty$ or DNE, as appropriate.

(a) $\lim\limits_{x\to -2} \dfrac{2x+(x+4)^2}{x^2+14x+24}$ (b) $\lim\limits_{x\to \pi^+} \dfrac{\sin\left(x+\frac{\pi}{6}\right)}{\sin(3x)}$

2. Given $f(x)=\begin{cases}\dfrac{\sin(7x)}{3x} & x\neq 0\\ b & x=0\end{cases}$

(a) (3 points) Determine $\lim\limits_{x\to 0}\dfrac{\sin(7x)}{3x}$

(b) (3 points) Determine $\lim\limits_{x\to \infty}\dfrac{\sin(7x)}{3x}$

(c) (1 point) For what value(s) of b is function $f(x)$ continuous for all x?

3. Given $f(x)=\begin{cases}\dfrac{\sqrt{4x^2-8x+4}-x}{2-x} & x<2\\ \dfrac{4}{x(x-4)} & x>2\end{cases}$

(a) (3 points) Evaluate $\lim\limits_{x\to -\infty} f(x)$

(b) (4 points) Evaluate $\lim\limits_{x\to 2} f(x)$

(c) (4 points) Determine and classify all points of discontinuity of f.

4. Let $f(x)=\dfrac{3x}{x+2}$.

(a) (4 points) Use the limit definition of the derivative to find $f'(x)$.

(b) (1 point) Find $f'(x)$ using derivative rules.

(c) (2 points) Find any points (x,y) where the line tangent to f has a slope of 6.

5. (16 points) Determine $\frac{dy}{dx}$ in each case. Do NOT simplify your answers.

(a) $y=\dfrac{3}{5x^6}-4(2^x)+\pi^3\csc(x)-\log_4(3+x)$

(b) $y=\dfrac{\sec(2x)-e^{x/3}}{(4x+5)^6}$

(c) $y=\tan^2\left(3x+\ln(\cos x)\right)$

(d) $y=7x(x^3+4x)^{1-x}$

6. (4 points) Given $e^{x+y}-x^2y^2=4x-10$, determine $\frac{dy}{dx}$.

7. (4 points) Find the x-values where the following function has horizontal tangents.

$f(x)=e^{\sin x}(2\sin x-3)$ on the interval $[0,\pi]$

8. (4 points) Find the absolute extrema of $f(x)=(x^2-1)^{2/3}$ on the interval $[0,3]$.

9. (5 points) At 4:00 pm, ship A is 10 km north of ship B. Ship A is travelling at 10 km/h due north. Ship B is travelling west at 20 km/h. Assuming the two ships maintain their speeds and directions, how fast is the distance between them changing at 6:00 pm?

10. (11 points) Consider the following function, along with its first and second derivatives.

$$f(x) = \frac{2 + x - x^2}{(x-1)^2}, \quad f'(x) = \frac{x-5}{(x-1)^3}, \quad f''(x) = \frac{2(7-x)}{(x-1)^4}$$

(a) Find the domain and intercepts of f.

(b) Find the vertical and horizontal asymptotes of f (if any).

(c) Find the intervals of increase/decrease of f.

(d) Find the local (relative) extrema of f (if any).

(e) Find the intervals of concavity of f.

(f) Find the points of inflection of f (if any).

(g) Sketch the graph of f.

11. (5 points) A company wants to design a box with a height of 4 metres that has a volume of 16 cubic metres. If the material for the base of the box costs \$2 per square metre and the material for the sides and the top costs \$3 per square metre, what are the dimensions of the box that minimize the total cost?

12. Evaluate the following integrals.

(a) (3 points) $\int \left(\frac{5}{x\sqrt{x}} + \frac{1}{5x} + e^x + e^5 \right) dx$

(b) (3 points) $\int \csc x \, (\tan^2 x - \sin x) \, dx$

(c) (2 points) $\int_1^4 \frac{x+1}{\sqrt{x}} \, dx$

13. Let the velocity of a particle that moves along a straight line be given by $v(t) = 3t^2 - 24t + 36$, where v is measured in metres per second.

(a) (1 point) When is the particle at rest?

(b) (4 points) Determine the total distance travelled over the first 3 seconds.

14. (4 points) Compute $\int_0^3 (x^2 - 3x + 4) \, dx$ as a limit of Riemann sums.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2} \right)^2$$

15. (3 points) Given $\int_2^5 [f(x) + 2g(x) - 3] \, dx = 7$ and $\int_5^2 f(x) = 4$, determine $\int_2^5 g(x) \, dx$.

ANSWERS

1. (a) $3/5$ (b) ∞

2. (a) $7/3$ (b) 0 (c) $7/3$

3. (a) 3 (b) -1

   (c) $x = 2$ (Removable dicontinuity)
   $x = 4$ (Infinite discontinuity)

4. (a/b) $f'(x) = \dfrac{6}{(x+2)^2}$ (c) $(-1,-3)$ and $(-3,9)$

5. (a) $\dfrac{dy}{dx} = \dfrac{-18}{5x^7} - 4(2^x)\ln(2) - \pi^3 \csc(x)\cot(x) - \dfrac{1}{(3+x)\ln(4)}$

   (b) $\dfrac{dy}{dx} = \dfrac{\left[2\sec(2x)\tan(2x) - \frac{1}{3}e^{x/3}\right](4x+5)^6 - \left[\sec(2x) - e^{x/3}\right]24(4x+5)^5}{(4x+5)^{12}}$

   (c) $\dfrac{dy}{dx} = 2\tan(3x + \ln(\cos x)) \cdot \sec^2(3x + \ln(\cos x)) \cdot \left[3 - \dfrac{\sin(x)}{\cos(x)}\right]$

   (d) $\dfrac{dy}{dx} = 7(x^3+4x)^{1-x} + 7x\left[(x^3+4x)^{1-x}\left(-\ln(x^3+4x) + (1-x)\cdot\dfrac{3x^2+4}{x^3+4x}\right)\right]$

6. $\dfrac{dy}{dx} = \dfrac{4 - e^{x+y} + 2xy^2}{e^{x+y} - 2x^2y}$

7. $x = \dfrac{\pi}{6}$, $x = \dfrac{\pi}{2}$, $x = \dfrac{5\pi}{6}$

8. Absolute max: 4 (at $x = 3$), Absolute min: 0 (at $x = 1$)

9. 22 km/h

10. (a) $x \in \mathbb{R} \setminus \{1\}$

(b) VA: $x = 1$, HA: $y = -1$

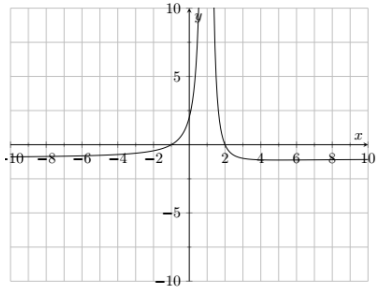
(c) Decreasing: $(1,5)$, Increasing: $(-\infty, 1) \cup (5, \infty)$

(d) Local minimum: $(5, -9/8)$

(e) Concave up: $(-\infty, 1) \cup (1, 7)$, Concave down: $(7, \infty)$

(f) Inflection point: $(7, -10/9)$

(g)



11. Height: 4m, Width: 2m, Length: 2m

12. (a) $\frac{-10}{\sqrt{x}} + \frac{\ln|x|}{5} + e^x + e^5 x + C$

(b) $\sec(x) - x + C$ (c) $\frac{20}{3}$

13. (a) $t = 2$ and $t = 6$ (b) 37 metres

14. $\frac{15}{2}$

15. 10