1. Evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to -2} \frac{x^2-4x-12}{x^3-4x}$

(b) $\displaystyle\lim_{x\to 0} \frac{\tan(x)\sin(3x)}{x^2}$

(c) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{x^2-3x+1}}{2x-7}$

(d) $\displaystyle\lim_{x\to \frac{\pi}{2}^+} \frac{|x-\pi|}{\cos(x)}$

2. Let

$$f(x) = \begin{cases} \dfrac{\sqrt{k+x}-\sqrt{k+1}}{x-1} & \text{if } -1 \le x < 1, \\ \dfrac{1}{x^2+4x+3} & \text{if } x \ge 1. \end{cases}$$

Find the value of k such that f is continuous at $x = 1$.

3. Use the limit definition of the derivative to find $f'(x)$, where $f(x) = \dfrac{2}{x^2}$.

4. Find $\dfrac{dy}{dx}$ for each of the following. **Do not** simplify your answers.

(a) $y = \dfrac{5}{\sqrt[3]{x^2}} - 2^x + \log_3 x - \tan(3\pi/7)$

(b) $y = \cos^3\left(\dfrac{x^2-5x}{\ln x}\right)$

(c) $y = x^{\csc(2x)}$

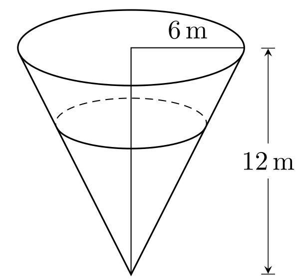
(d) $y = \sqrt{x^3-3x}\,(5-x)^4$

(e) $(x^2+y^2)^3 = x + 2y$

5. Find an equation of the tangent line to the curve $xy^3 + \tan(x+y) = \sqrt{3}$ at the point $(\pi/3, 0)$.

6. Find the 53[rd] derivative of $y = \cos(3x) + 5x^2$.

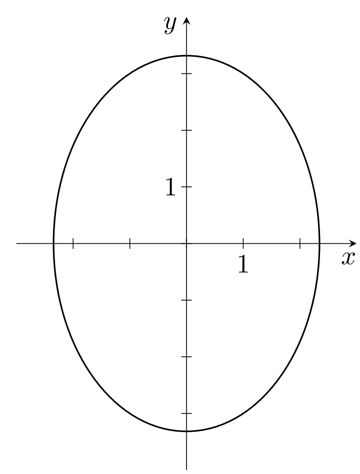
7. A reservoir has the shape of an inverted circular cone with base radius 6 meters and height 12 meters. Water is being poured into the reservoir at a rate of $18\pi\ \mathrm{m}^3/\mathrm{s}$. At what rate is the depth of the water increasing when the depth is 6 meters? (The volume of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)



8. Find the coordinates of the inflection point(s) of the function $f(x) = xe^{-2x}$.

9. Find the absolute maximum and absolute minimum values of $f(x) = \sqrt{x}(4x-3)$ on the interval $[0, 4]$.

10. Find the points on the ellipse $2x^2 + y^2 = 11$ that are farthest away from the point $(1, 0)$.



11. Consider the following function, along with its first and second derivatives.

$$f(x) = \sqrt[3]{2x(x-3)^2} \qquad f'(x) = \frac{2(x-1)}{\sqrt[3]{4x^2(x-3)}},$$

$$f''(x) = \frac{-4}{\sqrt[3]{4x^5(x-3)^4}}$$

(a) Find the domain and any intercepts of f.

(b) Find the vertical and horizontal asymptotes of f (if any).

(c) Find the intervals on which f is increasing or decreasing.

(d) Find the local (relative) extrema of f.

(e) Find the intervals of upward and downward concavity of f.

(f) Find all inflection points of f.

(g) On the next page, sketch the graph of f, labelling all intercepts, asymptotes, extrema, and points of inflection.

12. Evaluate $\displaystyle\int_0^4 (x^3 - 2)\,dx$ using the definition of the integral as a limit of Riemann sums.

13. Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{2 - \sqrt[3]{x^2} + \sqrt{2x}}{\sqrt{x}}\,dx$

(b) $\displaystyle\int_1^e \frac{3x^2 + x - 2}{x^2 + x}\,dx$

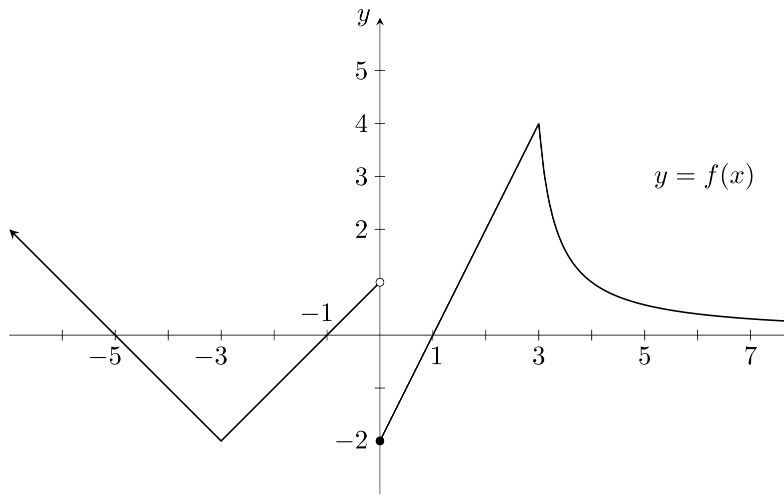
(c) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \csc^2 x(\tan^2 x \cos^3 x + \cos x)\,dx$

14. Consider the function $f(x) = \displaystyle\int_3^{\sqrt{x}} e^{-t^2}\,dt$.

(a) Find $f(9)$.

(b) Find $f'(9)$.

15. Given the graph of f below, evaluate each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate.



(a) $\displaystyle\lim_{x\to 0^-} f(x)$

(b) $\displaystyle\lim_{x\to\infty} \cos\big(f(x)\big)$

(c) $f'(3)$

(d) $f(0)$

(e) $\displaystyle\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

(f) $\displaystyle\int_{-3}^{1} f(x)\,dx$

In the following two questions circle the correct answer:

(g) The product $f(4)\,f'(4)\,f''(4)$ is

i. greater than 0;

ii. less than 0;

iii. equal to 0.

(h) At $x = -3$, the *derivative* of $f(x)$ has

i. an infinite discontinuity;

ii. a jump discontinuity;

iii. a removable discontinuity;

iv. none of the above since $f'(x)$ is continuous at $x = -3$.

Answers

1. (a) -1,

(b) 3,

(c) $-1/2$,

(d) $-\infty$.

2. $k = 15$

3. $\displaystyle\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = -\frac{4}{x^3}$

4. (a) $y' = -\dfrac{10}{3}x^{-5/3} - 2^x\ln 2 + \dfrac{1}{x\ln 3}$,

(b) $y'=-3\cos^2\left(\frac{x^2-5x}{\ln x}\right)\sin\left(\frac{x^2-5x}{\ln x}\right)\frac{(2x-5)\ln x-x+5}{\ln^2 x}$,

(c) $y'=x^{\csc(2x)}\left[-2\csc(2x)\cot(2x)\ln x+\frac{\csc(2x)}{x}\right]$,

(d) $y' = \dfrac{(3x^2-3)(5-x)^4}{2\sqrt{x^3-3x}} - 4\sqrt{x^3-3x}(5-x)^3$,

(e) $y' = \dfrac{6x(x^2+y^2)^2-1}{2-6y(x^2+y^2)^2}$.

5. $y = -x + \pi/3$.

6. $y^{(53)} = -3^{53}\sin(3x)$.

7. $\dfrac{dh}{dt} = 2$ m/s.

8. $(1, e^{-2})$.

9. $f(1/4) = -1$ absolute minimum,
$f(4) = 26$ absolute maximum.

10. $(-1, 3)$ and $(-1, -3)$.

11. (a) $\mathbb{R}$; $(0,0), (3,0)$

(b) no horizontal nor vertical asymptotes;

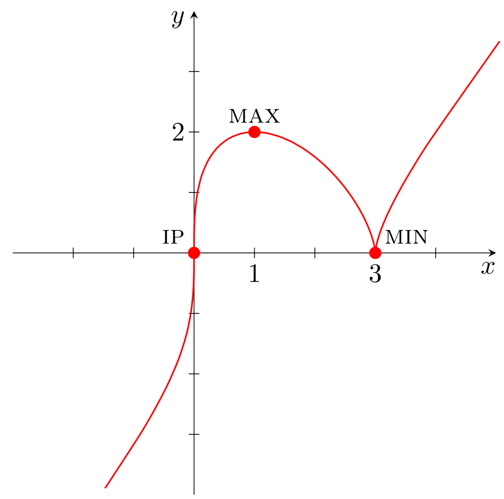
(c) increasing on $(-\infty, 1)$ and $(3, \infty)$, decreasing on $(1, 3)$.

(d) local max. $(1, 2)$, local min. $(3, 0)$

(e) concave up on $(-\infty, 0)$; concave down on $(0, 3)$ and $(3, \infty)$

(f) $(0, 0)$.

(g)



12. 56.

13. (a) $4x^{1/2} - 6/7x^{7/6} + \sqrt{2}x + C$;

(b) $3e - 5$;

(c) $\dfrac{-2+3\sqrt{2}}{2\sqrt{2}} = \dfrac{-\sqrt{2}+3}{2}$;

14. (a) 0

(b) $e^{-9}/6$;

15. (a) 1;

(b) 1

(c) D.N.E;

(d) -2

(e) 2

(f) $-5/2$

(g) ii

(h) ii