NTNU
Kunnskap for en bedre verden

Department of Electrical Technology

TET 4565 -Kraftmarkeder og energisystemplanlegging

Project

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Date

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2.1 Motivation

Today, the power production has become more reliant on renewable power. The output from renewable energy sources such as wind and solar varies greatly with weather, hour of the day and other factors. This again means that the price of electricity may vary greatly throughout the day, depending on the supply and demand in the given hour. This offers an incentive for the consumers to use electricity when demand is low and reduce their load when demand is high. With the increasing number of electric home devices allowing for flexible usage, new opportunities open for scheduling electricity use. This will enable consumers to plan their use throughout the day, taking advantage of price variations.

For this project, it will be investigated how a household can optimise its electrical vehicle charging over two days. The dataset used is a household in Texas, Austin for June 1st and June 2nd. Due to the warm climate during summer, most households use air conditioning, thus making the electricity use in June high.

The EV is modelled with charging restrictions to avoid too much wear and tear on the battery. This means that the battery cannot fully charge within one hour, but need to be spread out throughout the day. It is assumed that the overall spending in the rest of the house is not affected by the electricity price.

Furthermore, the electricity demand for the next day is known, but the prices are not. Thus, the optimisation problem has a component of uncertainty. The next day's price becomes clear at midnight, and the system can adjust the schedule for EV charging for day two. This varies from a scenario with perfect information, where one can schedule optimally from the first hour.

Solving the optimisation problem may offer private households more knowledge as to how to use their electricity over several days, by taking advantage of the flexibility they have.

The questions that will be answered during the project are:

- How much can a household limit its peaks and troughs by optimizing EV charging?
- How will uncertainty affect the optimization problem?

2.2 Mathematical Formulation

The problem is formulated as an optimization problem to minimize the EV charging cost.

Sets:

$$
\begin{aligned}
T &= \{1, 2, \ldots, 48\} && \text{(Set of time periods, total of 48 hours)} \\
T_1 &= \{1, 2, \ldots, 24\} && \text{(Set of the first 24 hours)} \\
T_2 &= \{25, 26, \ldots, 48\} && \text{(Set of the last 24 hours)} \\
S &= \{1, 2, 3\} && \text{(Set of the 3 scenarios)}
\end{aligned}
$$

Parameters:

- $P_{\text{EV}}^{\text{max}}$: Maximum power rate of the EV charger (kW)
- $E_{\text{EV}}^{\text{required}}$: Total energy required by the EV over the period (kWh)
- $P_{\text{house}}(t)$: Household's power consumption at time t (kW)

- $p(t, s)$: Electricity price at time t (\$/kWh) in scenario s
- $\pi(s)$: Probability of scenario s

Variables:

- $P^{\text{EV1}}(t)$: Power used by the EV charger at time t (kW) in stage 1
- $P^{\text{EV2}}(t, s)$: Power used by the EV charger at time t (kW) in stage 2 and scenario s

Objective Function:

$$\min \sum_{t \in T_1} [P_{\text{house}}(t) + P^{\text{EV1}}(t)] \cdot p_s(t) + \sum_{s \in S} \pi_s \sum_{t \in T_2} [P_{\text{house}}(t) + P^{\text{EV2}}(t, s)] \cdot p_s(t, s)$$

Subject to:

1. **Energy Requirement Constraint:**

$$\sum_{t \in T_1} P^{\text{EV1}}(t) + \sum_{t \in T_2} P^{\text{EV2}}(t, s) = E_{\text{EV}}^{\text{required}}, \quad \forall s \in S$$

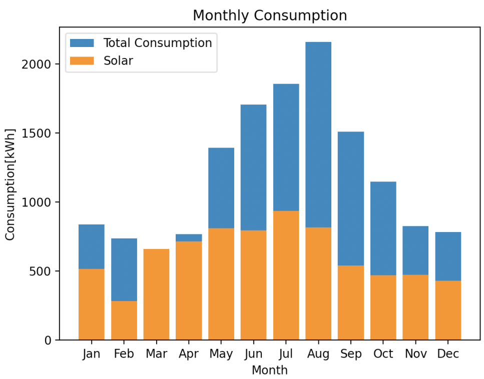
2. **EV Charger Power Limit:**

$$0 \leq P^{\text{EV1}}(t) \leq P_{\text{EV}}^{\max}, \quad \forall t \in T_1$$

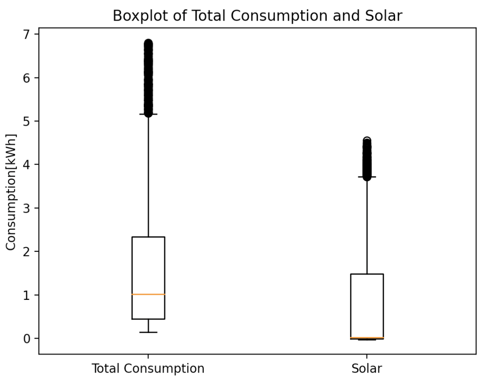
$$0 \leq P^{\text{EV2}}(t, s) \leq P_{\text{EV}}^{\max}, \quad \forall t \in T_2, \forall s \in S$$

2.3 Data Analysis

When analyzing the Austin household dataset, we created a new column called Total consumption, which is the sum of power from solar and the grid. By aggregating the consumption of solar and total consumption for each month for house nr. 10, we get the Monthly Consumption plot in Figure 1a. From the bar graph, there is clearly a seasonal trend in the power consumption. The consumption increases drastically in the summer months which might be because of an increase in air-conditioning used to cool down households in the Texan summer.



(a) Monthly Consumption

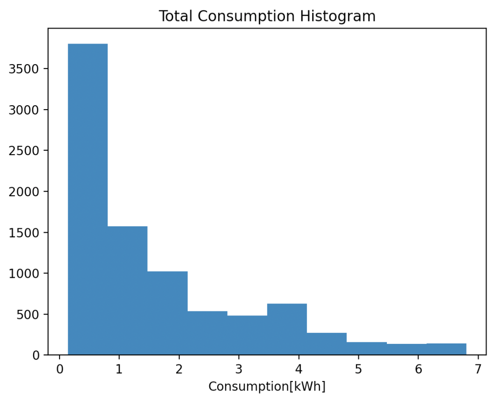


(b) Boxplot of Consumption and Solar

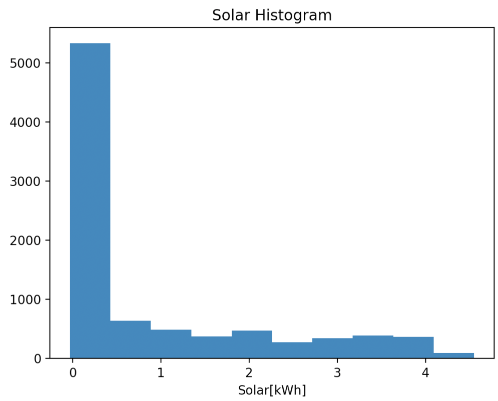
Figure 1: Solar and Total Consumption Comparison

In Figure 1b Total Consumption above, we can see how the data points in the total consumption column and the solar columns are spread. The orange line is the median value, the part in the box above the orange line is the third quartile, and the part in the box below the orange line is the second quartile. In the box is the middle 50% of the data. The black dots are the outliers, and what is between the box and the lines are the 1st and the 4th quartile. The total consumption box plot shows that the data has a few outliers and the solar box shows a very low median value, but still has a few high values.

Figures 2a and 2b show that there normally is low consumption and even low solar production, which makes sense because the total consumption is made up of solar. For solar, very high values are as common as medium-high values. The occurrence of specific consumption values gradually decreases as the consumption increases.



(a) Histogram Consumption



(b) Histogram Solar

Figure 2: Solar and Total Consumption Histograms

Figure 3 shows the total consumption grouped by the day of the week. The consumption is higher on Wednesday, Friday and Saturday, and lowest on Monday and Tuesday.

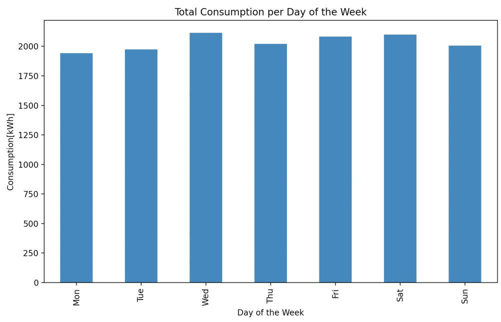


Figure 3: Total Consumption Grouped by Day of the Week

3 Part 2

In this part, three different variants of the problem are solved. The first variant is solving the problem using the expected value of all scenarios. The second variant is solving the different scenarios individually and the third is solving the problem as a stochastic problem where the first stage does not influence the second stage.

3.1 Expected Value

When solving the problem using the expected value for all scenarios, for each hour, the electricity used is multiplied by the probability of that scenario to create an expected dataset in the second stage. The first stage is already known. By using this dataset, the optimal charging pattern was found to minimize the electricity cost. The table below shows in which hour the charger was used and how much effect it used in KWh. The total cost of power from house consumption and EV charging was $5.12 over 48 hours. By looking at the table below, the EV charged from 1 am to 6 am the first day and 3 am to 4 am the second day. On average the prices are lowest during the night before the morning wake-up rush.

Hour	Charge
1	7.0
2	7.0
3	7.0
4	7.0
5	7.0
6	7.0
27	1.0
28	7.0

3.2 Individual Solution

In this variant of the problem, the scenarios are run individually. Since there are two stages of the problem, the first stage is the 1st of June prices in Austin 2018, and the second stage prices are the second day of June, July and August 2018 respectively. The table below shows in which hours the EV charger is used for the different scenarios. Scenarios one and three charge some on day one and some on day two, but some of the hours differ. In scenario 2, there are huge differences between the two days because the charger is fully charged within the first 10 hours, which leaves no charging for the second day. Comparing this variant to the expected value variant, hours 2, 3, 4 and 6 are common for them all, but other than that, the charger has to be used at sub-optimal hours for the different scenarios.

Hour	Scenario 1	Scenario 2	Scenario 3
0		7.0	
1		7.0	
2	7.0	7.0	7.0
3	7.0	7.0	7.0
4	7.0	7.0	7.0
5		7.0	
6	1.0	7.0	1.0
7		1.0	
25			7.0
26			7.0
27	7.0		7.0
28	7.0		7.0
29	7.0		
30	7.0		

3.3 Stochastic Nonanticipativity

For the stochastic nonanticipativity variant, the first 24 hours are known and there are three different scenarios for stage 2. The table below shows the EV charger consumption for stages 1 and 4 the three different scenarios in stage 2. The table shows that the optimal solution is to do most of the charging within the first 24 hours. This variant is somewhat similar to the expected value variant. They both charge the same amount of hours the first day, and hour 28, where the charger is used for all scenarios here, is the hour in stage 2 where 7 out of 8 KWh is used in the expected value variant. The objected value was calculated to be $5.12.

Hour	P_{EV} (kW)	Hour	Scenario 1 (kW)	Scenario 2 (kW)	Scenario 3 (kW)
0		24			
1	7.0	25			
2	7.0	26			
3	7.0	27			7.0
4	7.0	28	1.0	1.0	1.0
5	7.0	29		7.0	
6	7.0	30	7.0		
7		31			
8		32			
9		33			
10		34			
11		35			
12		36			
13		37			
14		38			
15		39			
16		40			
17		41			
18		42			
19		43			
20		44			
21		45			
22		46			
23		47			

Table 1: Hourly Power Values for Stochastic Nonanticipativity variant

4 Part 3

4.1 Mathematical Formulation of the Benders Decomposition Problem

Sets

$$
\begin{aligned}
T &= \{0, 1, \ldots, 47\} \quad \text{(Set of time periods, 48 hours)} \\
T_1 &= \{0, 1, \ldots, 23\} \quad \text{(Set of the first 24 hours)} \\
T_2 &= \{24, 25, \ldots, 47\} \quad \text{(Set of the last 24 hours)} \\
S &= \{1, 2, 3\} \quad \text{(Set of 3 scenarios)}
\end{aligned}
$$

Parameters

$$
\begin{aligned}
P_{\text{EV}}^{\max} &: \text{Maximum power rate of the EV charger in kW} \\
E_{\text{EV}}^{\text{req}} &: \text{Total energy required by the EV over the period in kWh} \\
P_{\text{house}}[t] &: \text{Household's power consumption at time } t \text{ (kW)} \\
p1[t] &: \text{Electricity price at time } t \in T1 \text{ (\$/kWh) for the first stage} \\
p2[s,t] &: \text{Electricity price in scenario } s \in S \text{ at time } t \in T2 \text{ (\$/kWh) for the second stage} \\
\pi[s] &: \text{Probability of scenario } s \in S
\end{aligned}
$$

Parameters for Cuts

$$\hat{x}^c = \sum_{t\in T1} P_{\mathrm{EV}}[t]^c$$
$$\Phi^c = Q\left(\hat{x}^c\right)$$
$$\Lambda^c = \sum_{s\in S} \lambda_s$$

Decision Variables

First-stage variables:

$$P_{\mathrm{EV}}[t] \geq 0 \quad \forall t \in T1 \quad \text{(EV charging power at time } t \text{ in the first stage)}$$
$$\alpha : \text{Expected second-stage cost}$$

Second-stage variables (for each scenario s):

$$P_{\mathrm{EV}}[s,t] \geq 0 \quad \forall s \in S \quad \forall t \in T2 \quad \text{(EV charging power in scenario } s \text{ at time } t \text{ in the second stage)}$$

First-Stage Problem

$$\begin{aligned}
\min_{P_{\mathrm{EV}}[t],\alpha} \quad & \sum_{t\in T1} p1[t]\left(P_{\mathrm{house}}[t] + P_{\mathrm{EV}}[t]\right) + \alpha \\
\text{s.t.} \quad & P_{\mathrm{EV}}[t] \leq P_{\mathrm{EV}}^{\max}, \quad \forall t \in T1 \\
& \alpha \geq \Phi^c + \Lambda^c \left(\sum_{t\in T1} P_{\mathrm{EV}}[t] - \hat{x}^c\right), \quad \forall c \in C
\end{aligned}$$

Second-Stage Problem for Scenario s

$$\begin{aligned}
Q: \min_{P_{\mathrm{EV}}[s,t]} \quad & \sum_{s\in S} \pi_s \sum_{t\in T2} p2[s,t]\left(P_{\mathrm{house}}[t] + P_{\mathrm{EV}}[s,t]\right) \\
\text{s.t.} \quad & P_{\mathrm{EV}}[s,t] \leq P_{\mathrm{EV}}^{\max}, \quad \forall s \in S \quad \forall t \in T2 \\
& \sum_{t\in T2} P_{\mathrm{EV}}[s,t] + \hat{x} = E_{\mathrm{EV}}^{\mathrm{req}} \quad \forall s \in S
\end{aligned}$$

4.2 Benders Decomposition Result

The benders decomposition problem was solved with 20 iterations, which resulted in a difference between upper bound and lower bound of $2 * 10^{-10}$. The final objective function was \$5.12, and the charging hours can be seen in the table below.

The solution is exactly the same as the stochastic nonanticipativity variant. This is as expected because both the methods handle distribution between hours in the scenarios independently from another in stage 2. In stage 1 the charging has to be decided before the scenarios are known, so it has to be the same regardless of which scenario happens.

5 Conclusion

Hour	P_{EV} **(kW)**	**Hour**	**Scenario 1 (kW)**	**Scenario 2 (kW)**	**Scenario 3 (kW)**
0		24			
1	7.0	25			
2	7.0	26			
3	7.0	27			7.0
4	7.0	28	1.0	1.0	1.0
5	7.0	29		7.0	
6	7.0	30	7.0		
7		31			
8		32			
9		33			
10		34			
11		35			
12		36			
13		37			
14		38			
15		39			
16		40			
17		41			
18		42			
19		43			
20		44			
21		45			
22		46			
23		47			

Table 2: Hourly Power Values for P_{EV} for Benders Decomposition solution

Appendix