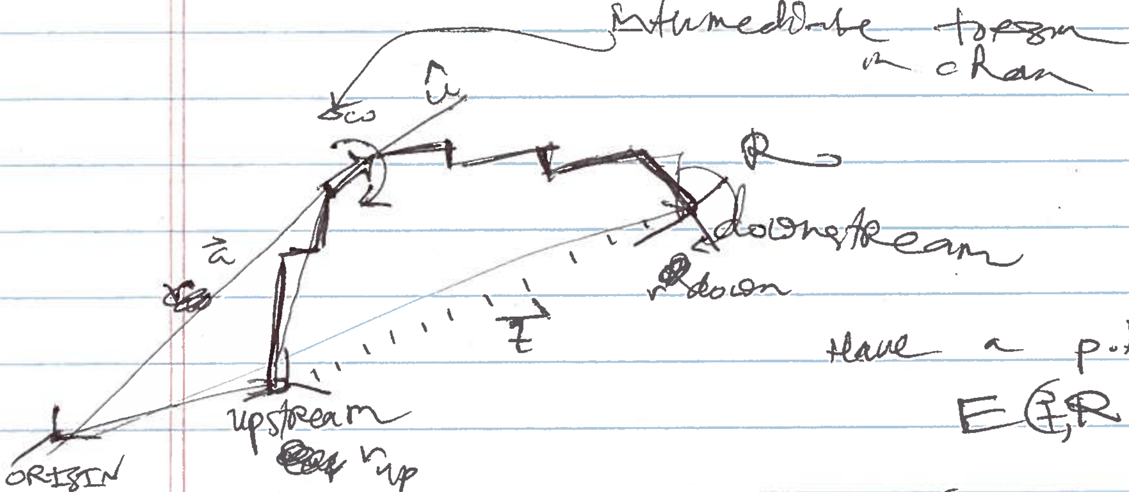
① 3-7-2017 (Summarizing work in Feb 2017), up to sign flips and inverses.

$\boxed{F_1, F_2 \text{ derivation for 6D potential}}$ ☆



have a 6D potential $E(\vec t, R)$.

what are F_1, F_2 in Abe, Go scheme?
i.e. how does energy change if we rotate by $\delta\omega$ around $\hat u$, sitting at $\vec a$?

$$R_{up}\,\vec t = \underbrace{\vec r_{down} - \vec r_{up}}_{\text{stride}} \qquad (1)$$

(MATRIX) rotation from global to local frame.

If upstream is root,

$$\vec r_{down}' = \vec r_{down} + \delta\omega\, \hat u \times (\vec r_{down} - \vec a)$$
$$\vec r_{up}' = \vec r_{up}$$

so

$$\vec t' = (R_{up}^{-1})\left[\vec r_{down} - \vec r_{up} + \delta\omega\left\{\hat u \times (\vec r_{down} - \vec a)\right\}\right] \qquad (2)$$

$$\delta E = (\vec\nabla_t E)\cdot \Delta t = \delta\omega\,(\vec\nabla_t E)\cdot R_{up}^{-1}\left[\hat u \times (\vec r_{down} - \vec a)\right]$$

$$= \hat\omega \cdot \underbrace{\left[\vec\nabla E \times \vec r_{down}\right]}_{\vec F_1} - \underbrace{\vec\nabla E \cdot \vec a}_{F_2}. \quad \text{if } R_{up} = I.$$

(2)

$$\delta E = \nabla_t E \cdot R_{up}^{-1}\left[\hat{\omega} \times (\vec{r}_{down} - \vec{a})\right]$$

↙ Rotate frame

$$= \nabla_t E \cdot \left[(R_{up}^{-1}\hat{\omega}) \times (\cancel{R}\,\vec{r}_{down})\right] - (\nabla_t E)\cdot (R_{up}^{-1}\cdot\vec{a})$$

$$= (R_{up}\nabla_t E)\cdot(\hat{\omega} \times \vec{r}_{down}) - (R_{up}\vec{\nabla}_t E)\cdot\vec{a}$$

(pre-rotate $\vec{\nabla}_t E$ instead of rotate $\vec{a}$)

$$= \boxed{\hat{\omega}\cdot\Big[\underbrace{(R_{up}\vec{\nabla}_t E)\times\vec{r}_{down}}_{\vec{F}_1}\Big] - \underbrace{(R_{up}\vec{\nabla}_t E)}_{\vec{F}_2}\cdot\vec{a}}$$

(3)

CHECK

o.k. what if downstream is Rot?

$$\vec{r}_{down}' = \vec{r}_{down}$$

$$\vec{r}_{up}' = \vec{r}_{up} - \Delta\omega\,\hat{u}\times(\vec{r}_{up} - \vec{a})$$

So:

$$\vec{f}' = (R_{up}^{-1})\left(\vec{r}_{down} - \vec{r}_{up} + \Delta\omega\,\hat{u}\times(\vec{r}_{up} - \vec{a})\right)$$

But look:

$$R_{up}' = R_{\Delta\omega} R_{up}$$

Rotation around $\hat{u}$ by $\Delta\omega$ [or is it $R_{up}R_{\Delta\omega}$? may not matter]

$$\vec{f} = R_{up}^{-1}\left(\vec{r}_{down} - \vec{r}_{up} + R_{\Delta\omega}(\vec{r}_{down} - \vec{r}_{up}) + \Delta\omega\,\hat{u}\times(\vec{r}_{up} - \vec{a})\right]$$

$$= R_{up}^{-1}\left[\vec{r}_{down} - \vec{r}_{up} + (\Delta\omega\,\hat{u})\times(\vec{r}_{down} - \vec{r}_{up}) + \Delta\omega\,\hat{u}\times(\vec{r}_{up} - \vec{a})\right]$$

$$= R_{up}^{-1}\left[\vec{r}_{down} - \vec{r}_{up} + \Delta\omega\,\hat{u}\times(\vec{r}_{down} - \vec{a})\right]$$

Same as (2) $\Rightarrow$ Same F_1, F_2.