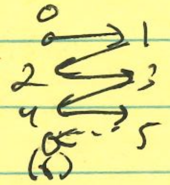
1

EARLY JUNE, 2019

$\boxed{\text{Toy Field} \to \text{1D CALCS OF } C_{eff}}$ 6 membered CHAIN

In 1D, 6 (N) membered chain → RING CLOSURE.



$$P_1(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-D)^2/2\sigma^2}$$

$$P_2(x) = \frac{1}{\sqrt{2\pi \cdot 2\sigma^2}} e^{-x^2/2\cdot 2\sigma^2}$$

$$P_3(x) = \frac{1}{\sqrt{2\pi \cdot 3\sigma^2}} e^{-\frac{x^2}{2(3\sigma^2)}}$$

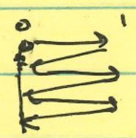
$$=$$

$$P_5(x) = \frac{1}{\sqrt{2\pi \cdot (5\sigma^2)}} e^{-\frac{(x-D)^2}{2(5\sigma^2)}}$$

$$P_6(x) = \frac{1}{\sqrt{2\pi(6\sigma^2)}} e^{-\frac{x^2}{2(6\sigma^2)}}$$

$$\int_{-\infty}^{\infty} P_6(x)\,\delta(x)\,dx = P_6(0) = \frac{1}{\sqrt{2\pi(6\sigma^2)}}.$$

Now calculate properties AFTER Ring closure,



$$P_0(x) = \delta(x) \leftarrow \text{TRANSLATE TO ORIGIN}$$

$$P_1(x) = \left[\frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-D)^2/2\sigma^2}\right]\left[\frac{1}{\sqrt{2\pi(5\sigma^2)}} e^{-\frac{(x-D)^2}{2(5\sigma^2)}}\right]\frac{1}{N}$$

$$= \frac{1}{\sqrt{2\pi \frac{5\sigma^2}{6}}} e^{-\frac{(x-D)^2}{2\sigma^2}\left(\frac{6}{5}\right)}$$

← Just based on normalization requirement.

BASED ON

$$P(x) \propto P_{\substack{\text{FORWARD}\\\text{PATH}}}(x)\; P_{\substack{\text{REVERSE}\\\text{PATH}}}(x)$$

$$P_2(x) = N \exp\left[-\frac{x^2}{2\sigma^2}\left[\frac{1}{2} + \frac{1}{4}\right]\right]$$

$$= \frac{1}{\sqrt{2\pi\left(\frac{4\sigma^2}{3}\right)}} \exp\left[-\frac{x^2}{2\sigma^2}\frac{3}{4}\right]$$

$$P_3(x) = N \exp\left[-\frac{(x-D)^2}{2\sigma^2}\left[\frac{1}{3} + \frac{1}{3}\right]\right]$$

$$= \frac{1}{\sqrt{2\pi \frac{3}{2}\sigma^2}} \exp\left[-\frac{(x-D)^2}{2\sigma^2}\cdot\frac{2}{3}\right]$$

$$P_4(x) = P_2(x)$$

$$P_5(x) = P_1(x).$$

TO CREATE SAMPLES,

Put X_0 → ORIGIN

Select X_1 from $P_1(x)$.

Then X_2 select from:

$$e^{-[x-(x_1-D)]^2/2\sigma^2} \; e^{-x^2/2(4\sigma^2)}$$

CROSS-checks to run on $P_1(x), \dots P_5(x)$

CODE UP BACKTRACK SAMPLE,

– Calculate $P_0(t), P_1(t) \dots P_5(t)$

Compare to analytical calculation.

Compare to numerical calculations

– based on MARKOV CHAIN Monte Carlo:

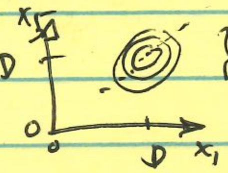
$\sum$ { backtrack $x_1, \dots x_5$, accept/reject BASED ON Metropolis-Hastings

– based on $P_F(x), P_R(x)$

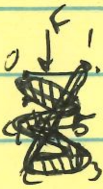
FROM simple sampling – then ~~take~~ products and check.

[This is analogous to Rangan's calculation of Ranging ends of Pum.]

How about also calculate & visualize $P(x_1, x_5)$ BACKTRACK SAMPLES + numerical samples. $\overset{?}{=} \int P(t_1, x_1, x_0) P(x_2 - t_1, t_0 - t_2) P(x_3 - x_2, x_2 - t_3) P(x_4 - \frac{1}{3} x_3, x_3 - t_4) P(x_5 - t_4, t_5 - t_4) P(x_0 - t_5, t_4 - t_5)$



} there should be a weak correlation – WHAT IS it?



$$P(x_5 | x_1) \propto e^{-(x_5 - D)^2 / 2\sigma^2} \, e^{-(x_5 - x_1)^2 / 2[4\sigma^2]}$$

$$P(x_1, x_5) = P(x_1) P(x_5 | x_1) \propto \left[e^{-\frac{(x_5 - D)^2}{2\sigma^2}} \, e^{-\frac{(x_5 - x_1)^2}{2(4\sigma^2)}} \right]$$

$\overset{?}{\longrightarrow} e^{-\frac{(x_1 - D)^2}{2\sigma^2} \cancel{(5/6)}}$ o.k. – let's expand

$$\propto \exp\left[-\frac{1}{2\sigma^2}\right] \left[(x_5 - D)^2 + \frac{(x_5 - x_1)^2}{4} + \cancel{\frac{(x_1 - D)^2}{5}} \right]$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\right) \left[\Delta x_5^2 + \frac{1}{4}(\Delta x_5 - \Delta x_1)^2 + \Delta x_1^2 \right]$$ where $\Delta x_1 = x_1 - D$, $\Delta x_5 = x_5 - D$.

Check: $P(x_1) = \int P(x_1, x_5) \, d\Delta x_5 = \int \exp\left[-\frac{1}{2\sigma^2}\right] \left[\frac{5}{4} \Delta x_5^2 - \frac{1}{2} \Delta x_1 \Delta x_5 + \frac{5}{4} \Delta x_1^2 \right] d\Delta x_5$

$= \int \exp\left[-\frac{1}{2\sigma^2}\right] \left[\frac{5}{4} \left(\Delta x_5 - \frac{1}{5} \Delta x_1\right)^2 - \frac{1}{20} \Delta x_1^2 + \frac{5}{4} \Delta x_1^2 \right]$

$\propto \exp\left[-\frac{1}{2\sigma^2}\right] \left[\frac{24}{20} = \frac{6}{5} \Delta x_1^2 \right]$. $\boxed{OK}$

③

Calculate σ^2 for $x_1 - x_0$, $x_5 - x_0$

$$p(x_1, x_5) \sim \exp\left[-\frac{1}{2\sigma^2}\right]\left[\Delta x_5^2 + \frac{1}{4}(\Delta x_5 - \Delta x_1)^2 + \Delta x_1^2\right]$$

$$\cancel{\langle \Delta x_5^2 \rangle = \sqrt{\tfrac{4}{5}}\sigma^2 ?} \qquad \leftarrow \quad p(x_1, x_5) \sim \exp\left(-\frac{1}{2\sigma^2}\right)\left[\frac{5}{4}\left(\Delta x_5 - \frac{1}{5}\Delta x_1\right)^2 + \frac{5}{5}\Delta x_1^2\right]$$

$$\cancel{\langle \Delta x_1^2 \rangle = \sqrt{\tfrac{4}{5}}\sigma ?}$$

$$\langle \Delta x_5^2 \rangle = \int \Delta x_5^2 \exp\left(-\frac{1}{2\sigma^2}\right)\left(\Delta x_5^2 + \frac{1}{4}(\Delta x_5 - \Delta x_1)^2 + \Delta x_1^2\right) d\Delta x_5 \, d\Delta x_1 / N$$

$$= \int [\;\;] \exp\left(-\frac{1}{2\sigma^2}\right)\left(\frac{5}{4}A^2 + \frac{5}{5}\Delta x_1^2\right] dA \, d\Delta x_1$$

$$A^2 + \frac{2}{5}\Delta x_1 A + \frac{1}{25}\Delta x_1^2$$

$$\overset{?}{=} \int \left[\frac{4}{5}\sigma^2 + \frac{1}{25}\Delta x_1^2\right] e^{-\frac{1}{2\sigma^2}\left[\frac{6}{5}\Delta x_1^2\right]} d\Delta x_1 \qquad A = \Delta x_5 - \frac{1}{5}\Delta x_1$$

$$\Delta x_5 = A + \frac{1}{5}\Delta x_1$$

$$= \left[\frac{4}{5}\sigma^2 + \frac{1}{25} \times \frac{5}{6}\sigma^2\right]$$

$$= \left[\frac{4}{5} + \frac{1}{30}\right]\sigma^2$$

$$= \frac{5}{6}\sigma^2. \qquad \left(\text{Should be } \frac{5}{6}\sigma^2\right)$$

REVISIT ON NEXT PAGE → o.k., found fix

Alternatively, do integral over x_1 first.

$$\langle \Delta x_5^2 \rangle = \int \Delta x_5^2 \exp\left[-\frac{1}{2}\right]\left[\frac{6}{5}\Delta x_5^2\right] d x_5$$

$$= \frac{5}{6}\sigma^2$$

$$\langle \Delta x_1^2 \rangle = \frac{5}{6}\sigma^2$$

④

$$\langle \Delta x_5^2 \rangle = \frac{\int dx_1 dx_5\, \Delta x_5^2 \exp\left[-\frac{1}{2\sigma^2}\right]\left[\Delta x_1^2 + \frac{1}{4}(\Delta x_1 - \Delta x_5)^2 + \Delta x_5^2\right]}{\int dx_1 dx_5 \exp\left[-\frac{1}{2\sigma^2}\right]\left[\Delta x_1^2 + \frac{1}{4}(\Delta x_1 - \Delta x_5)^2 + \Delta x_5^2\right]}$$

N

complete the SQUARE

Compute N

$$\langle \Delta x_5^2 \rangle \text{ DENOMINATOR } N = \int dx_1 dx_5 \exp\left[-\frac{1}{2\sigma^2}\right]\left[\frac{5}{4}\left(\Delta x_1 - \frac{1}{5}\Delta x_5\right)^2 + \frac{6}{5}\Delta x_5^2\right]$$

$\boxed{\text{suggests } \sigma^2 = \frac{1}{5}}$

$$= \int dx_5 \exp\left(-\frac{1}{2}\right)\left[\frac{6}{5}\Delta x_5^2\right]$$

$$\frac{1}{20} + \frac{6}{5} = \frac{25}{20}$$

$$\times \sqrt{\frac{4}{5}}\sqrt{2\pi}\,\sigma$$

$$= \sqrt{\frac{5}{6}} \times \sqrt{\frac{4}{5}}\, 2\pi\sigma^2 .$$

now compute $\langle \Delta x_5^2 \rangle$, $\langle \Delta x_1^2 \rangle$, similarly completing the SQUARE.

$$\langle \Delta x_5^2 \rangle \text{ NUMERATOR } = \int dx_1 dx_5\, \Delta x_5^2 \exp\left(-\frac{1}{2\sigma^2}\right)\left[\frac{5}{4}(a^2) + \frac{6}{5}\Delta x_5^2\right]$$

$a = \Delta x_1 - \frac{1}{5}\Delta x_5$

$dx_1 dx_5 = da\, dx_5$

$$= \sqrt{\frac{4}{5}}\sqrt{2\pi}\,\sigma \int (d\Delta x_5)\Delta x_5^2 \exp\left(-\frac{1}{2\sigma^2}\right)\frac{6}{5}\Delta x_5^2$$

$$= \sqrt{\frac{4}{5}} \times \sqrt{\frac{5}{6}} \times 2\pi\sigma^2 \times \left(\frac{5}{6}\sigma^2\right)$$

$$\langle \Delta x_5^2 \rangle = \frac{5}{6}\sigma^2$$

Now try other integral First.

$$\langle \Delta x_5^2 \rangle = \frac{1}{N}\int dx_1 dx_5\, \Delta x_5^2 \exp\left(-\frac{1}{2\sigma^2}\right)\left[\frac{5}{4}(a^2) + \frac{6}{5}\Delta x_1^2\right]$$

$a = \Delta x_5 - \frac{1}{5}\Delta x_1$

$$= \frac{1}{N}\int da\, dx_1 \left(a^2 + \frac{1}{25}\Delta x_1^2 + \frac{2}{5}a\Delta x_1\right) \exp\left(-\frac{1}{2\sigma^2}\right)\left[\frac{5}{4}a^2 + \frac{6}{5}\Delta x_1^2\right]$$

$dx_5 dx_5 = da\, dx_1$

$$= \frac{1}{N}\sqrt{\frac{4}{5}} \times \sqrt{\frac{5}{6}} \times 2\pi\sigma^2 \left[\frac{4}{5}\sigma^2 + \frac{1}{25}\left(\frac{5}{6}\right)\sigma^2\right]$$

$$\frac{24}{30} + \frac{1}{30} \to \frac{25}{30}$$

$$= \sqrt{\frac{4}{5}}\sqrt{\frac{5}{6}}\left(\frac{5}{6}\sigma^2\right)\Big/ N \quad \text{O.K.}$$

$$= \frac{5}{6}\sigma^2$$

$$\langle \Delta x_1 \Delta x_5 \rangle = \frac{1}{N} \int \Delta x_1 \Delta x_5 \exp\left[-\frac{1}{2\sigma^2}\right]\left[\frac{5}{4}\left(\Delta x_1 - \frac{1}{5}\Delta x_5\right)^2 + \frac{6}{5}\Delta x_5^2\right] d\Delta x_1 d\Delta x_5$$

$a = \Delta x_1 - \frac{1}{5}\Delta x_5$

will cancel out

$$= \frac{1}{N}\int da\, d\Delta x_5 \, [\cancel{a} + \frac{1}{5}\Delta x_5]\Delta x_5 \exp\left(-\frac{1}{2\sigma^2}\right)\left[\frac{5}{4}a^2 + \frac{6}{5}\Delta x_5^2\right]$$

$$= \frac{1}{5} \times \frac{5}{6}\sigma^2 = \frac{1}{6}\sigma^2$$

$$\frac{\langle \Delta x_1 \Delta x_5 \rangle}{\sqrt{\langle \Delta x_1^2 \rangle \langle \Delta x_5^2 \rangle}} = \frac{\frac{1}{6}\sigma^2}{\frac{5}{6}\sigma^2} = \frac{1}{5} \qquad \boxed{\text{O.K.}}$$

Quicker way to get there? $f(\Delta x_1, \Delta x_5) \sim e^{-\frac{1}{2\sigma^2}(\Delta x_1, \Delta x_5)\begin{pmatrix} \frac{5}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{5}{4} \end{pmatrix}\begin{pmatrix} \Delta x_1 \\ \Delta x_5 \end{pmatrix}}$

$$\begin{pmatrix} 5 & -1 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} \quad \end{pmatrix}$$

Now the question is … how do we do the Fourier Transform that includes ORIENTATION as well as translation?

Suppose we have

$$P(s, x)$$

s: ± 1 ; x: position

Expand in BASIS SET = r, q ← wavevector

r: symmetric/antisymmetric?

$$P(s,x) = \sum_{r=\pm 1} \int \tilde{p}(r,q)\, e^{2\pi i q x}\,(sr)\, dq$$

$$= \cancel{\int \tilde{p}(r,q)\, e^{2\pi i q x}\, s\, dq}$$

$$\begin{cases} \int dq \left[\tilde{p}(+1,q) + \tilde{p}(-1,q) \right] e^{2\pi i q x} & s=1 \\ \int dq \left[\tilde{p}(+1,q) - \tilde{p}(-1,q) \right] e^{2\pi i q x} & s=-1 \end{cases}$$

$$= \int dq \left[\tilde{p}(+1,q) + s\,\tilde{p}(-1,q) \right] e^{2\pi i q x}$$

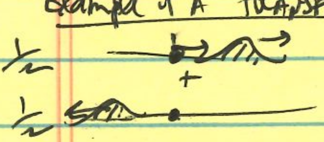
$$\sum r\, \tilde{p}(r,q)$$

$$e^{i\pi r}$$

q, r

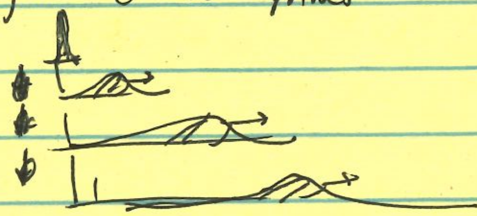
Hmm

Example of A TRANSFORM:

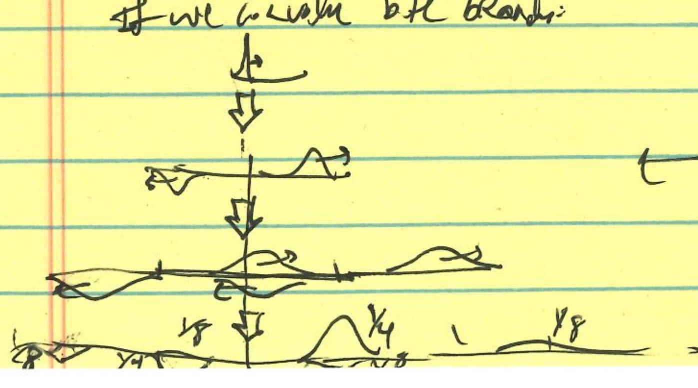


PAIRS ⟶ If we just convolve PAIRS:

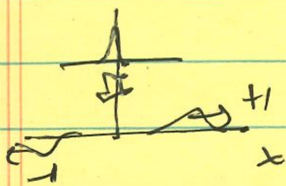
"REFLECT"

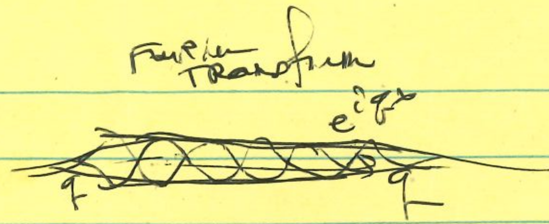


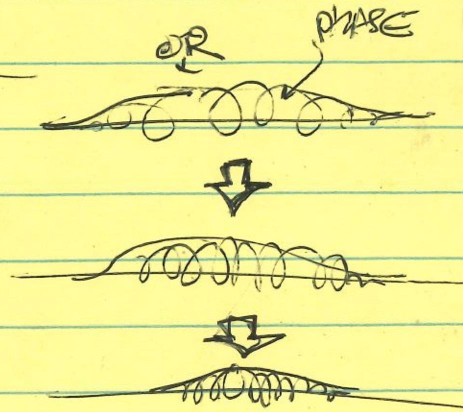
If we convolve both branches:

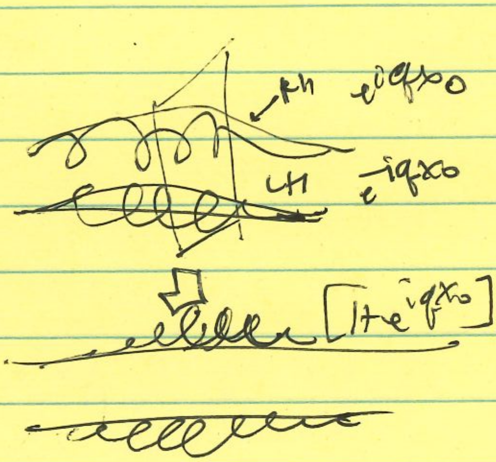


← The Question is: AFTER WHAT TRANSFORM DOES THIS CONVOLUTION OPERATION look like a multiplication?









$$\left(a e^{iqx_0} + b e^{-iqx_0}\right)\left(a e^{iqx_0} + b e^{-iqx_0}\right)$$
$$= a^2 e^{2iqx_0} + 2ab\, e^{-iqx_0} + b^2 e^{-2iqx_0}$$

$$a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$b = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

I made some separate notes on Fourier Transform Analysis in SE(1), but didn't achieve satisfying conclusion.

15 JUNE(?) 2019

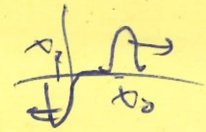
on Road to Salt Wieliczka Salt Mines, Poland

Forward 2

$$\begin{pmatrix} e^{ikx_0} \\ e^{-ikx_1} \end{pmatrix} \begin{matrix} \leftarrow \text{FORWARD} \\ \leftarrow \text{BACKWARD} \end{matrix}$$

1 8

$$\begin{pmatrix} \delta(x_0) \\ \delta(x_1) \end{pmatrix} \begin{matrix} \leftarrow \text{FAKE FORWARD} \\ \leftarrow \text{FAKE BACKWARD} \end{matrix}$$



Imagine transform that just moves each direction FORWARD by Δx_0 ...

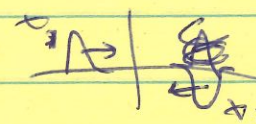
$$\begin{pmatrix} e^{ik\Delta x_0} & 0 \\ 0 & e^{-ik\Delta x_0} \end{pmatrix}$$

check:

$$\begin{pmatrix} e^{ik\Delta x_0} & 0 \\ 0 & e^{-ik\Delta x_0} \end{pmatrix}\begin{pmatrix} e^{ikx_0} \\ e^{-ikx_1} \end{pmatrix} = \begin{pmatrix} e^{ik(\Delta x_0 + x_0)} \\ e^{-ik(x_1 + \Delta x_0)} \end{pmatrix}$$

Reverse directions

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} e^{ikx_0} \\ e^{-ikx_1} \end{pmatrix} = \begin{pmatrix} e^{-ikx_1} \\ e^{ikx_0} \end{pmatrix}$$



Could also set directions as:

$\theta = 0, 1$

$e^{i\pi\theta} = \boxed{1, -1}$

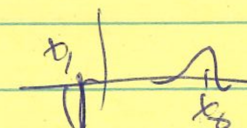
$\delta(0) \leftrightarrow e^{i\theta(0)}$

$\delta(\pi) \leftrightarrow e^{i\theta(\pi)}$

Forward?

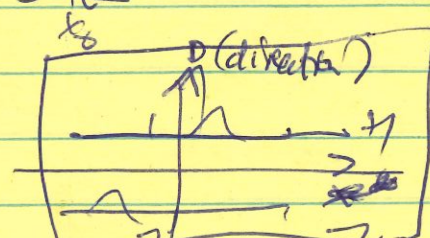
$$e^{+ikx_0} + e^{i\pi\theta} e^{-ikx_1}$$

$[\theta = 0, 1]$



Reversal: multiply by $e^{i\pi\theta}$

$$e^{i\pi\theta} e^{+ikx_0} + \cancel{e^{2\pi i\theta}} e^{-ikx_1}$$



② ⑤

$\boxed{\text{BASIS FUNCTIONS}} \overset{?}{=} \begin{pmatrix} +e^{ikx} \\ +e^{-ikx} \end{pmatrix}, \begin{pmatrix} +e^{ikx} \\ -e^{-ikx} \end{pmatrix}$

$$P(x) = \begin{pmatrix} A(x) \\ B(x) \end{pmatrix}$$

$$\tilde{P}_+(k) = \int dx \begin{pmatrix} e^{ikx} & e^{-ikx} \end{pmatrix} \begin{pmatrix} A(x) \\ B(x) \end{pmatrix}$$

$$= \int dx \left[e^{ikx} A(x) + e^{-ikx} B(x) \right]$$

$$= \tilde{A}(k) + \tilde{B}(k)$$

$$\tilde{P}_-(k) = \int dx \begin{pmatrix} e^{ikx} & -e^{-ikx} \end{pmatrix} \begin{pmatrix} A(x) \\ B(x) \end{pmatrix} = \tilde{A}(k) - \tilde{B}(k)$$

BASIS TRANSFORMATION:

$$\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$e^{ix\varepsilon} = e^{ix \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}} = \begin{pmatrix} e^{ix} & 0 \\ 0 & e^{-ix} \end{pmatrix}$$

$$P(x) = \int \left[\tilde{P}_+(k) \begin{pmatrix} e^{ikx} \\ e^{-ikx} \end{pmatrix} + \tilde{P}_-(k) \begin{pmatrix} +e^{ikx} \\ -e^{-ikx} \end{pmatrix} \right] dk$$

move FORWARD Δx $\begin{pmatrix} e^{ik\Delta x} & 0 \\ 0 & e^{-ik\Delta x} \end{pmatrix}$ ... YES, $P'(x) = \begin{pmatrix} e^{ik\Delta x} & 0 \\ 0 & e^{-ik\Delta x} \end{pmatrix} P(x)$

$$= \tilde{P}_+(k) \begin{pmatrix} e^{ik(x+\Delta x)} \\ e^{-ik(x+\Delta x)} \end{pmatrix} + \tilde{P}_-(k) \begin{pmatrix} e^{ik(x+\Delta x)} \\ -e^{-ik(x+\Delta x)} \end{pmatrix}$$

REFLECT AND move Δx $\begin{pmatrix} 0 & e^{iq} \\ & \end{pmatrix}$

?

= ?

See next page.

Let's work this out again:

$$P(x) = \begin{pmatrix} A(x) \\ B(x) \end{pmatrix} = \int_{-\infty}^{\infty} \left[\tilde{P}_+(k) \frac{1}{\sqrt{2}} \begin{pmatrix} e^{ikx} \\ e^{-ikx} \end{pmatrix} + \tilde{P}_-(k) \frac{1}{\sqrt{2}} \begin{pmatrix} e^{ikx} \\ -e^{-ikx} \end{pmatrix} \right] dk$$

$A(x)$ ← FORWARD ~~POINTING~~

$B(x)$ ← BACKWARD POINTING

Check: PROPAGATION IN DIRECTION OF POINTING IS simply multiplication ~~by~~ by $e^{ik\Delta x}$:

$$P'(x) = \int \left[\tilde{P}_+(k) \frac{1}{\sqrt{2}} \begin{pmatrix} e^{ik(x+\Delta x)} \\ e^{-ik(x-\Delta x)} \end{pmatrix} + \frac{\tilde{P}_-(k)}{\sqrt{2}} \begin{pmatrix} e^{ik(x+\Delta x)} \\ -e^{-ik(x-\Delta x)} \end{pmatrix} \right] dk$$

$$= \begin{pmatrix} A(x+\Delta x) \\ B(x-\Delta x) \end{pmatrix}$$

Check: REFLECT POINTING DIRECTION IS $F_x \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\tilde{P}_+(k)$, $-\tilde{P}_-(k)$

$k \to -k$ ~~complex conjugation~~

$$P'(x) = \int_{-\infty}^{\infty} \left[\frac{\tilde{P}_+(k)}{\sqrt{2}} \begin{pmatrix} e^{-ik(x)} \\ e^{ik(x)} \end{pmatrix} + \frac{\tilde{P}_-(k)}{\sqrt{2}} \begin{pmatrix} -e^{ik(x)} \\ -e^{ik(x)} \end{pmatrix} \right] dk$$

$$\overset{\boxed{k \to -k}}{=} \begin{pmatrix} B(x) \\ A(x) \end{pmatrix}$$

So... lets check two rod convolutions

FORWARD

BACKWARD

One transform:

$$e^{ikx_0 - \sigma^2 k^2/2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} e^{-\sigma^2 k^2/2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Uh wait.. NOT WORKING → the dual Fourier transform THE FLIP? → Need complex conjugation? or $k \to -k$.

$F_{flip}\, P_{+/-}(k) = P_{+/-}(-k) = P_{+/-}^{*}(k)$ (I think)

$F_{flip}\begin{pmatrix}0&1\\1&0\end{pmatrix} e^{ikx-\sigma^2k^2/2}$ $F_{flip}\begin{pmatrix}0&1\\1&0\end{pmatrix} e^{ikx-\sigma^2k^2/2}$

$F_{flip}\begin{pmatrix}0&1\\1&0\end{pmatrix} e^{ikx-\sigma^2k^2/2}$ $\begin{pmatrix}0 & e^{-ikx-\sigma^2k^2/2}\\ e^{-ikx-\sigma^2k^2/2} & 0\end{pmatrix}$

$= e^{-\sigma^2k^2}\begin{pmatrix}1&0\\0&1\end{pmatrix}$

$\boxed{\text{Looks o.k.}}$

$$\left\{\frac{1}{2}\left[I + P\begin{pmatrix}0&1\\1&0\end{pmatrix}\right]e^{ikx}e^{-k^2\sigma^2/2}\right\}^n$$

$\downarrow$

$$\sum_{m=1}^{n}\frac{1}{2^n}\binom{n}{m}e^{-nk^2\sigma^2/2}\left[\underset{x}{I} + \underset{x}{F}\right]$$

$(\ )\ (\ )$ ... ...

- might work out o.k.
- The F^{*} thing is not very clear - mixes k and $-k$?

How about <u>instead</u> $\tilde{P}_{-}(k) \to -\tilde{P}_{-}(k)$?

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→ Did not complete – REVISIT LATER

Harmonic mode analysis: First, do two-point:

$$\uparrow\downarrow \quad Z = \int \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \frac{e^{-y^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \delta(x-y)\, dx\, dy$$

$$= \int \frac{e^{-x^2/\sigma^2}}{\sqrt{2\pi}\sigma} dx = \frac{\sqrt{2\pi(\sigma^2/2)}}{2\pi\sigma^2} = \boxed{\frac{1}{\sqrt{4\pi}\sigma}}$$

$$e^{\frac{2\pi i k}{2}}$$

$$A \quad \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix}$$

$$?\ 2(1-\cos\pi)$$

Compared to no-delta function:

$$Z = \int \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi}\sigma} \frac{e^{-y^2/2\sigma^2}}{\sqrt{2\pi}\sigma^2} dx\, dy = 1.$$

$$Z = \int \frac{(\sqrt{2\pi})^{N-1}}{(\sqrt{2\pi})^{N}} \frac{\det[\gamma_1\sigma^2]}{\det\begin{bmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{\sigma^2} \end{bmatrix}}$$

$$\int \exp\left(-\frac{1}{2\sigma^2}\right)\left[a^2+b^2+c^2+d^2+e^2+(-a-b-c-d-e)^2\right] da\, db\, dc\, dd\, de$$

$$= \int \exp\left(-\frac{1}{2\sigma^2}\right)\left[(a\ b\ c\ d\ e)\begin{pmatrix} 2 & 1 & 1 & \cdots & 1 \\ & 2 & 2 & & \vdots \\ \vdots & & & 2 & \\ 1 & \cdots & & & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix}\right]$$

Alternatively,

$$Z = \int \exp\left(-\frac{1}{2\sigma^2}\right)\left[(x_0\ x_1\ x_2\ x_3\ x_4\ x_5)\underbrace{\begin{pmatrix} 2 & -1 & & & & -1 \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & -1 & 2 & -1 & \\ & & & -1 & 2 & -1 \\ -1 & & & & -1 & 2 \end{pmatrix}}_{A}\begin{pmatrix} x_0 \\ \vdots \\ x_5 \end{pmatrix}\right]$$

normal mode

$$A\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 0\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$A\begin{pmatrix} \\ \\ \end{pmatrix} \quad e^{2\pi i n/6} \quad e^{2\pi i n 4/6} \quad e^{2\pi i n 5/6}$$

$n = 0, 1, \ldots 5.$

check: $2e^{2\pi i n/6} - e^{2\pi i n/6}$

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$$A \begin{pmatrix} 1 \\ e^{2\pi i \gamma/6} \\ \vdots \\ e^{2\pi i \cdot 5\gamma/6} \end{pmatrix} = 2\left[1 - \cos\left(\frac{2\pi\gamma}{6}\right)\right] \begin{pmatrix} 1 \\ e^{2\pi i \gamma/6} \\ \vdots \\ e^{2\pi i \cdot 5\gamma/6} \end{pmatrix}$$

Interestingly...

$$Z = \underset{\text{zero-mode}}{L} \times \underset{\text{5 vibrational modes}}{\left(\frac{\sigma}{\sqrt{2\pi}}\right)^5} \times \frac{1}{\sqrt{2^5 [1-\cos(\frac{2\pi}{6})][1-\cos(\frac{2\pi \cdot 2}{6})][1-\cos(\frac{2\pi \cdot 3}{6})][1-\cos(\frac{2\pi \cdot 4}{6})][1-\cos(\frac{2\pi \cdot 5}{6})]}}$$

$Z_{\text{CLOSED}} = \cos\frac{2\pi}{6} = \frac{1}{2}$

$$= L\left(\frac{\sigma}{\sqrt{2\pi}}\right)^5 \frac{1}{\sqrt{2^5 \left(\frac{1}{2}\right)^2 \cdot \left(\frac{3}{2}\right)^2 \cdot 2}}$$

$$= L\left(\frac{\sigma}{\sqrt{2\pi}}\right)^5 \frac{1}{(3 \cdot 2)} = \frac{L}{(\sqrt{2\pi})^5 \cdot 6}\,\sigma^5$$

~~Z_{OPEN}~~ ? $Z_{\text{OPEN}} = L \cdot (\sqrt{2\pi}\,\sigma)^4$

$$Z_{\text{CLOSED}} / Z_{\text{OPEN}} = \frac{1}{\sqrt{2\pi}\,\sigma \times 6}$$

? O.K.? NO... should be $\frac{1}{\sqrt{2\pi}\,6\,\sigma}$ → $\sqrt{6}$ not 6.

How to resolve!

Go into MATLAB → . calculate determinants in matrices above.

. calculate $\prod_{m=1}^{N-1}\left(1-\cos\frac{2\pi m}{N}\right)$

the mode analysis to estimate res. locations too. & ARM differences