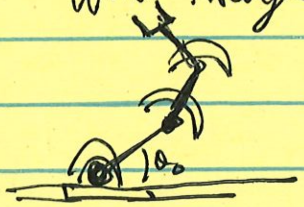
(7)

Walk through Wang, Chirikjian 2008 3-link manipulator with joint backlash.



$$\rho(r,\phi,\theta) = f(\theta-\theta_0)\,\underbrace{\delta(\phi-\theta)}_{\boxed{\text{so } \phi-\theta=0 - \text{no swivel.}}}\,\underbrace{\delta(r-L)/r}_{\boxed{\text{fixed length}}}$$

$$f(\theta,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\sum_{n=-\infty}^{\infty} e^{-(\theta-2\pi n)^2/2\sigma^2} \quad \text{o.k.}$$

$$= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} e^{-n^2\sigma^2/2} e^{in\theta} \quad \text{o.k.}$$

$$\hat{\rho}_{mn}(p) = \int \rho(r,\phi,\theta)\, u_{mn}(\overset{-1}{r,\phi,\theta};p)\; r\,dr\,d\phi\,d\theta$$

$$= \int \left[\frac{1}{2\pi}\sum_{n'=-\infty}^{\infty} e^{-n'^2\sigma^2/2} e^{in'[\theta-\theta_0]}\,\delta(\phi-\theta)\,\delta(r-L) \quad i^{n-m} e^{+i[n\theta+(m-n)\phi]} J_{m-n}(pr) \right] dr\,d\phi\,d\theta \quad \text{g? or } g^{-1}?$$

$$= i^{n-m} J_{m-n}(pL) \sum_{n'=-\infty}^{\infty} \int d\theta\; \underbrace{e^{-n'^2\sigma^2/2} e^{in'[\theta-\theta_0]} e^{in\theta}}_{e^{i(n+n')\theta}}$$

$$= i^{n-m} J_{m-n}(pL)\; e^{-n^2\sigma^2/2} e^{in\theta_0} \quad \text{o.k.}$$

If we convolve three times

$$\hat{\rho}'_{mn}(p) = \sum_{a,b} \hat{\rho}_{ma}(p)\,\hat{\rho}_{ab}(p)\,\hat{\rho}_{bn}(p)$$

– Chirikjian WANG SAMPLE $p = 0, 0.2, \ldots 300.0$ → get $\hat{\rho}'_{mn}(p)$.

How about integral over θ? [for plotting]

$$\rho(r,\phi) = \frac{1}{2\pi}\int_0^{2\pi} \rho(g)\,d\theta$$

$$= \sum_{m,n} \int_0^{2\pi} \hat{f}_{mn}(p)\; i^{n-m} e^{-i[n\theta+(m-n)\phi]} J_{n-m}(pr)\,p\,dp\,d\theta$$

$$= \int_{p=0}^{\infty} \sum_m \hat{f}_{m0}(p)\, i^{-m} e^{-im\phi} J_{-m}(pr)\,p\,dp$$

$$= \sum_{m\in\mathbb{Z}} i^{-m} e^{-im\phi} \underbrace{\int_0^{\infty} \hat{f}_{m0}(p)\, J_{-m}(pr)\,p\,dp}_{\to\, c_m(r)} \quad \text{ALMOST o.k.}$$

Do the NUMERICAL INTEGRAL FIRST.