SCRATCH #4 (1)

$\boxed{\text{Need to write nicely}}$ ☆ → $\boxed{\text{See SCRATCH \#4 - ⊕ for how to handle "zero-mode"}}$

$$2\cos(ki+\theta) - \cos(k(i+1)+\theta) - \cos(k(i-1)+\theta) = \lambda \cos(ki+\theta)$$

$$2\cos(ki+\theta) - \cos(ki+\theta)\cos k + \cancel{\sin(ki+\theta)\sin(k)} - \cos(ki+\theta)\cos(-k) - \cancel{\sin(ki+\theta)\sin(-k)} = \lambda\cos(ki+\theta)$$

$$\boxed{\lambda = 2[1-\cos k]}$$

O.K.?

☆ $\boxed{\text{CLOSED CHAIN:}}$

for PERIODIC

$k = \frac{2\pi q}{N}$ $\qquad \frac{2\pi(N-1)}{N}$

$q = 0, \dots N-1$

$\cos(ki+\theta) \Rightarrow \cos(ki)$, $\sin(ki)$

$= \cos(ki)\cos\theta$

$+ \sin(ki)\sin\theta$

TWO BASIS FUNCTIONS.

$2\pi - \frac{2\pi q}{N}$

☆ $\boxed{\text{OPEN CHAINS}}$

$$\begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & \ddots & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{pmatrix}$$

$\cos(ki+\theta)$

Evaluate at first position...

$$\cos\theta - \cos(k+\theta) = 2[1-\cos k]\cos\theta$$

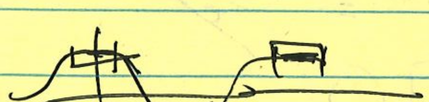
$$\cos\theta - \cos k\cos\theta + \sin k\sin\theta = 2\cos\theta - 2\cos k\cos\theta$$

$$\sin k\sin\theta = \cos\theta - \cos k\cos\theta$$

$$\boxed{\cos(\theta - k) = \cos\theta}$$

TRY $\theta = \left(\frac{\pi q}{N}\right)\left(\frac{1}{2}\right)$

$k = \frac{\pi q}{N}$ → WORKS... $\cos\left(\frac{\pi q}{2N} - \frac{\pi q}{N}\right)$



Check: $\sin\left[\frac{\pi q}{N} + \frac{\pi q}{2N}\right] = \cos$ $\qquad = \cos\left(\frac{\pi q}{2N}\right)$ ✓

OTHER BOUNDARY CONDITION SIDES

$\cos\left[\pi q + \frac{\pi q}{2N}\right] \stackrel{?}{=} \cos\left[\pi q - \frac{\pi q}{2N}\right]$ $\qquad \cos(\theta + (N-1)k) = \cos(\theta + Nk)$ $\qquad \cos\left[\frac{\pi q}{2N} + \pi q - \frac{\pi q}{N}\right] \stackrel{?}{=} \cos\left[\frac{\pi q}{2N} + \pi q\right]$

SCRATCH (2)

How to calculate determinant?

$$\prod_{q=1}^{N-1}\left[2-2\cos\left(\frac{2\pi q}{N}\right)\right]$$

Let $N=1$...

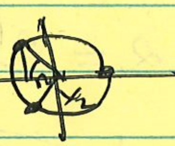
$2-2\cos(2\pi)=0$. o.k.

Let $N=2$...

$$2-2\cos\left(\frac{2\pi}{2}\right) = 2-2(-1)=4 \quad \text{o.k.}$$

Let $N=3$

$$\left[2-2\cos\left(\frac{2\pi}{3}\right)\right]\left[2-2\cos\left(\frac{4\pi}{3}\right)\right] = 3\times3 = 9 \quad \text{o.k.}$$



$$= 4\left[\sin\left(\frac{\pi}{3}\right)\right]^2 \cdot 4\left[\sin\left(\frac{2\pi}{3}\right)\right]^2$$

$$= 4\left(+\frac{\sqrt{3}}{2}\right)^2 \cdot 4\left(+\frac{\sqrt{3}}{2}\right)^2 = 9.$$

Let $N=4$

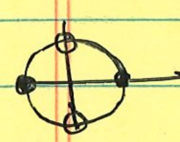
$$\left[2-2\cos\left(\frac{2\pi}{4}\right)\right]\left[2-2\cos\left(\frac{4\pi}{4}\right)\right]\left[2-2\cos\left(\frac{6\pi}{4}\right)\right] = 2\times4\times2 = 16$$

$$= \left[2\sin\left(\frac{\pi}{4}\right)\right]^2\left[2\sin\left(\frac{2\pi}{4}\right)\right]^2\left[2\sin\left(\frac{3\pi}{4}\right)\right]^2$$

$$= \left\{\left[2\cdot\frac{1}{\sqrt{2}}\right]\left[2\cdot1\right]\left[2\cdot\frac{1}{\sqrt{2}}\right]\right\}^2 = 16.$$

$$\left(e^{i\pi/4}-e^{-i\pi/4}\right)\left(e^{2i\pi/4}-e^{-2i\pi/4}\right)\left(e^{3i\pi/4}-e^{-3i\pi/4}\right)$$

$$= \Big[e^{i\frac{\pi}{4}[1+2+3]} + (-1)e^{i\frac{\pi}{4}[1+2-3]} + (-1)e^{i\frac{\pi}{4}[1-2+3]} + e^{i\frac{\pi}{4}[1-2-3]} + (-1)e^{i\frac{\pi}{4}[-1+2+3]} + e^{i\frac{\pi}{4}[-1+2-3]} + e^{i\frac{\pi}{4}[-1-2+3]} + (-1)e^{i\frac{\pi}{4}[-1-2-3]} \Big]$$



$$= \Big[e^{i\frac{\pi}{4}\cdot6} - \cancel{e^{i\frac{\pi}{4}\cdot0}} - e^{i\frac{\pi}{4}2} + e^{i\frac{\pi}{4}[-4]} - e^{i\frac{\pi}{4}[+4]} + e^{i\frac{\pi}{4}[-2]} - e^{i\frac{\pi}{4}[0]} - e^{i\frac{\pi}{4}[-6]} \Big] = 2\left[\sin\left(\frac{6\pi}{4}\right) - \sin\left(\frac{4\pi}{4}\right) - \sin\left(\frac{2\pi}{4}\right) + \sin(0)\right]$$

$$= 2[2] = 4 \quad \text{o.k.}$$

SCRATCH** - 3

Several N ?

$$\prod_{q=1}^{N-1}\left[2 - 2\cos\left(\frac{2\pi q}{N}\right)\right]$$

$$= \prod_{q=1}^{N-1}\left[2\sin\left(\frac{\pi q}{N}\right)\right]^2$$

$$= \left\{\prod_{q=1}^{N-1}\left[e^{\frac{i\pi q}{N}} - e^{-\frac{i\pi q}{N}}\right]\right\}^2 \longrightarrow 2^{N-1} \text{ terms}$$

$$\stackrel{?}{=} \sum_{r=0}^{N(N-1)/2}\left[e^{\frac{i\pi q}{N}r} - e^{-\frac{i\pi q}{N}r}\right](-1)^{r} \quad ?$$

$\boxed{\text{How do they simplify?}}$

$1 \ldots N-1 = \frac{(N-1)N}{2}$

PROBABLY SHOULD WRITE OUT FOR N=5.

$$\left[e^{\frac{i\pi}{5}} - e^{-\frac{i\pi}{5}}\right]\left[e^{2\frac{i\pi}{5}} - e^{-\frac{2i\pi}{5}}\right]\left[e^{\frac{3i\pi}{5}} - e^{-\frac{3i\pi}{5}}\right]\left[e^{\frac{4i\pi}{5}} - e^{-\frac{4i\pi}{5}}\right]$$

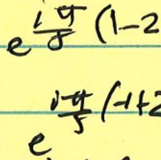
$$= \Big[e^{\frac{i\pi}{5}(1+2+3+4)} - e^{\frac{i\pi}{5}(1+2+3-4)} - e^{\frac{i\pi}{5}(1+2-3+4)} + e^{\frac{i\pi}{5}(1+2-3-4)}$$
$$- e^{\frac{i\pi}{5}(1-2+3+4)} + e^{\frac{i\pi}{5}(1-2+3-4)} + e^{\frac{i\pi}{5}(1-2-3+4)} - e^{\frac{i\pi}{5}(1-2-3-4)}$$
$$- e^{\frac{i\pi}{5}(-1+2+3+4)} + e^{\frac{i\pi}{5}(-1+2+3-4)} + e^{\frac{i\pi}{5}(-1+2-3+4)} - e^{\frac{i\pi}{5}(-1+2-3-4)}$$
$$+ e^{\frac{i\pi}{5}(-1-2+3+4)} - e^{\frac{i\pi}{5}(-1-2+3-4)} - e^{\frac{i\pi}{5}(-1-2-3+4)} + e^{\frac{i\pi}{5}(-1-2-3-4)}\Big]$$

$$= e^{\frac{i\pi}{5}\times 10} - e^{\frac{i\pi}{5}\times 2} - e^{\frac{i\pi}{5}(4)} + e^{\frac{i\pi}{5}(-4)}$$
$$- e^{\frac{i\pi}{5}\times 6} + e^{\frac{i\pi}{5}\cdot(-2)} + e^{\frac{i\pi}{5}\cdot 0} - e^{\frac{i\pi}{5}(-8)}$$
$$- e^{\frac{i\pi}{5}(+8)} + e^{\frac{i\pi}{5}(0)} + e^{\frac{i\pi}{5}(2)} - e^{\frac{i\pi}{5}(-6)}$$
$$+ e^{\frac{i\pi}{5}(4)} - e^{\frac{i\pi}{5}(-4)} - e^{\frac{i\pi}{5}(-2)} + e^{\frac{i\pi}{5}(-10)}$$

$$= 2\left[\cos\frac{\pi}{5}\cdot 10 - \cos\frac{\pi}{5}8 - \cos\frac{\pi}{5}6 + \overset{0}{\cos}\frac{\pi}{5}4 + \overset{0}{\cos}\frac{\pi}{5}2 + \cos\frac{\pi}{5}0\right]$$

$$= 2\left[1 - \cos\frac{8\pi}{5} - \cos\frac{6\pi}{5} + 1\right]$$

uhh...



SCRATCH #4 - (4)

How to handle zero eigenmode

$$Z = \int_{-L/2}^{L/2} \exp -\frac{1}{2\sigma^2} (x_0\, x_1 \ldots x_{N-1})\, A \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{pmatrix} \quad \boxed{dx_0 \ldots dx_{N}}$$

$$= \int \exp -\frac{1}{2\sigma^2} (v_0\, v_1 \ldots v_N) \begin{pmatrix} 0 & & \\ & \lambda_1 & \\ & & \ddots \\ & & & \lambda_{N-1} \end{pmatrix} \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_N \end{pmatrix} dv_0 \ldots dv_N$$

$$v_0 = \frac{-L}{2\sqrt{N}} \text{ to } \frac{L}{2}\sqrt{N}$$

$$\begin{pmatrix} x_0 \\ \vdots \\ x_{N-1} \end{pmatrix} = v_0 \hat{v}_0 + v_1 \hat{v}_1 + \ldots + v_{N-1} \hat{v}_{N-1}$$

VECTORS.

$$\hat{v}_0 = \frac{1}{\sqrt{N}}(1, 1, \ldots 1)$$

So... $$Z = \frac{1}{(\sqrt{2\pi})^N} \left(\frac{L}{\sqrt{N}} \right) \frac{1}{\sqrt{\det A}}$$

up to zero eigenmode.

$$Z_{open} = \frac{L\, \sigma^{N-1}}{\sqrt{2\pi}^{N-1}} \frac{\sqrt{N}}{\sqrt{N}} = \frac{L\, \sigma^{N-1}}{(\sqrt{2\pi})^{N-1}}$$

$$Z_{closed} = \frac{L}{\sqrt{2\pi}^{N-1}} \sigma^{N-1} \frac{\sqrt{N}}{\sqrt{N^2}} = \frac{L}{(\sqrt{2\pi})^{N-1}} \frac{\sigma^{N-1}}{\sqrt{N}}.$$

o.k.

Alternatively, imagine trapping each $x_0, x_1, \ldots x_{N-1}$ in gaussian with stdev $\sigma_0\ [\to \infty]$

$$Z = \int_{-\infty}^{\infty} \ldots\ dx_0 \ldots dx_{N-1}$$

$$A = \frac{1}{2\sigma^2} \begin{bmatrix} 2 + (\sigma/\sigma_0)^2 & -1 & & & -1 \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ -1 & & & -1 & 2 + (\sigma/\sigma_0)^2 \end{bmatrix}$$

$$\lambda_q = 2\left[1 - \cos\left(\frac{2\pi q_j}{N}\right)\right] + (\sigma/\sigma_0)^2$$

$$\lambda_{q=0} \to (\sigma/\sigma_0)^2$$

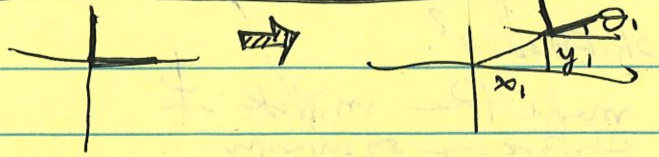
~~closed~~

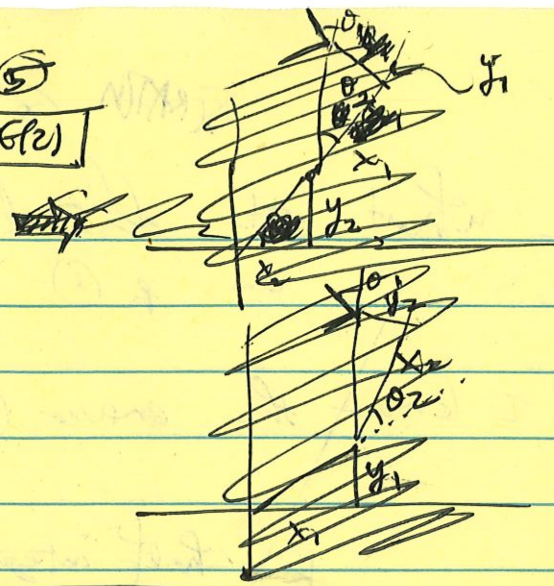
$$Z_{OPEN} = \frac{\sigma_0\, \sigma^{N-1}}{\sqrt{2\pi}\, (\sqrt{2\pi})^{N-1}} \cdot \frac{1}{\sqrt{N}}$$

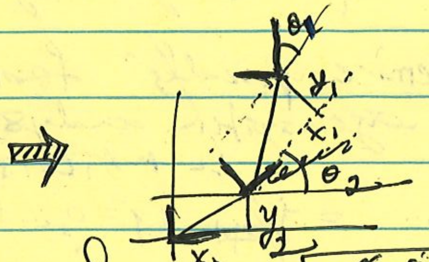
$$Z_{CLOSED} = \frac{\sigma_0}{\sqrt{2\pi}} \frac{\sigma^{N-1}}{(\sqrt{2\pi})^{N-1}} \cdot \frac{1}{N}.$$

SCRATCH ** - 5

PROPAGATION OF GAUSSIANS IN SE(2)







Homogeneous transform

$\boxed{\text{OOPS - sign is wrong} \rightarrow \text{Fix...}}$

$$H_1 = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & x_1 \\ +\sin\theta_1 & \cos\theta_1 & y_1 \\ 0 & 0 & 1 \end{pmatrix} \qquad H_1 \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} = \begin{pmatrix} a\cos\theta_1 - b\sin\theta_1 + x_1 \\ +a\sin\theta_1 + b\cos\theta_1 + y_1 \\ 1 \end{pmatrix} \text{ ok.}$$

$$\frac{\partial H_1}{\partial x} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

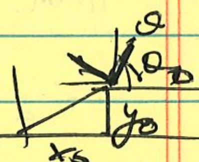
$$\frac{\partial H_1}{\partial y} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{\partial h}{\partial \theta} = \begin{pmatrix} -\sin\theta & -\cos\theta & 0 \\ +\cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

~~Imagine H_0~~

Lets try & propagate a gaussian...

- Imagine H_0 is centre of transform
- take samples (x, y, θ)
  - BACK-TRANSFORM H_0 (i.e., compute $\Delta x, \Delta y, \Delta\theta$ in local frame)
  - TRACE co-variances.



$$H_1 \cdot H_2 = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & x_1 \\ +\sin\theta_1 & \cos\theta_1 & y_1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & x_2 \\ +\sin\theta_2 & \cos\theta_2 & y_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & x_2\cos\theta_1 - y_2\sin\theta_1 + x_1 \\ +\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & +x_2\sin\theta_1 + y_2\cos\theta_1 + y_1 \\ 0 & 0 & 1 \end{pmatrix} \qquad (1)$$

Hmm note that above corresponds to applying H_2 first, then H_1 — makes sense if coordinates are column vectors

↳ BUT REVERSED FROM DIAGRAM ABOVE

SCRATCH 4/4 - 6

$$p(\Delta x, \Delta y, \Delta\theta) = \exp\left[(\Delta x \;\; \Delta y \;\; \Delta\theta) \begin{pmatrix} A_{xx} & A_{xy} & A_{x\theta} \\ A_{yx} & A_{yy} & A_{y\theta} \\ A_{\theta x} & A_{\theta y} & A_{\theta\theta} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{pmatrix} \right] \frac{1}{2} \times \underbrace{\frac{1}{(2\pi)^{3/2}\sqrt{\det A}}}_{\text{Normalization factor.}}$$

← Imagine we've Rotated into coordinate frame

Need $\Delta x, \Delta y, \Delta\theta$ when $\Delta x_1, \Delta y_1, \Delta\theta_1, \Delta x_2, \Delta y_2, \Delta\theta_2$, change.

First figure out inverse transform... $H_1 \circ H_2 \rightarrow$ figure out $H_1 \rightarrow H_2^{-1}$

$$\boxed{\theta_1 = -\theta_2}$$

$$x_1 = -x_2\cos\theta_1 + y_2\sin\theta_1 = -x_2\cos\theta_2 - y_2\sin\theta_2$$

$$y_1 = -x_2\sin\theta_1 - y_2\cos\theta_1 = +x_2\sin\theta_2 - y_2\cos\theta_2$$

$$\boxed{\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & +\sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} -x_2 \\ -y_2 \end{pmatrix} = R_{-\theta_2}\vec{b}_2}$$

$$(H_2 \circ H_1) = \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & x_1\cos\theta_2 - y_1\sin\theta_2 + x_2 \\ +\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & +x_1\sin\theta_2 + y_1\cos\theta_2 + y_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(H_2 \circ H_1)^{-1} = \begin{pmatrix} \cos(\theta_1+\theta_2) & +\sin(\theta_1+\theta_2) & -x_1\cos\theta_1 - y_1\sin\theta_1 - x_2\cos(\theta_1+\theta_2) - y_2\sin(\theta_1+\theta_2) \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & +x_1\sin\theta_1 - y_1\cos\theta_1 + x_2\sin(\theta_1+\theta_2) - y_2\cos(\theta_1+\theta_2) \\ 0 & 0 & 1 \end{pmatrix}$$

x for $(H_2 \cdot H_1)^{-1}$

$$-[x_1\cos\theta_2 - y_1\sin\theta_2 + x_2]\cos(\theta_1+\theta_2)$$
$$-[+x_1\sin\theta_2 + y_1\cos\theta_2 + y_2]\sin(\theta_1+\theta_2)$$

$$= -x_1[\cos\theta_2\cos(\theta_1+\theta_2) + \sin\theta_2\sin(\theta_1+\theta_2)]$$
$$+ y_1[\sin\theta_2\cos(\theta_1+\theta_2) - \cos\theta_2\sin(\theta_1+\theta_2)]$$
$$- x_2\cos(\theta_1+\theta_2) - y_2\sin(\theta_1+\theta_2)$$

$$= -x_1\cos(+\theta_1)$$
$$+ y_1\sin(-\theta_1)$$
$$- x_2\cos(\theta_1+\theta_2) - y_2\sin(\theta_1+\theta_2)$$

$$= -x_1\cos\theta_1 - y_1\sin\theta_1 - x_2\cos(\theta_1+\theta_2) - y_2\sin(\theta_1+\theta_2)$$

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y for $(H_2 \cdot H_1)^{-1}$

$$= +[x_1\cos\theta_2 - y_1\sin\theta_2 + x_2]\sin(\theta_1+\theta_2)$$
$$- [+x_1\sin\theta_2 + y_1\cos\theta_2 + y_2]\cos[\theta_1+\theta_2]$$

$$= +x_1\sin\theta_1 - y_1\cos\theta_1 + x_2\sin(\theta_1+\theta_2) - y_2\cos(\theta_1+\theta_2)$$

SCRATCH** – ①

what happens to H if applied after local movement (Δt):

$$H = H_{g(\Delta x,0,0)} \begin{pmatrix} \cos\theta & -\sin\theta & x \\ +\sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \Delta t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{c|c} R_\theta & \begin{matrix} \Delta t\cos\theta + x \\ +\Delta t\sin\theta + y \end{matrix} \\ \hline 0\ 0 & 1 \end{array}\right)$$

$$H^{-1}(H\, g(\Delta t,0,0)) = g(\Delta t, 0, 0)$$

$$(H_2 \cdot H_1)^{-1}(H_2 \cdot H_1(x_1 + \Delta x_1)) = \begin{pmatrix} \cos(\theta_1+\theta_2) & +\sin(\theta_1+\theta_2) & -x_1\cos\theta_1 - y_1\sin\theta_1 - x_2\cos(\theta_1+\theta_2) + y_2\sin(\theta_1+\theta_2) \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & +x_1\sin\theta_1 - y_1\cos\theta_1 + x_2\sin(\theta_1+\theta_2) - y_2\cos(\theta_1+\theta_2) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & \Delta x_1 \cos\theta_2 \ldots \\ +\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & +\Delta x_1 \sin\theta_2 \ldots \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & \Delta x_1 [\cos\theta_2 \cos(\theta_1+\theta_2) + \sin\theta_2 \sin(\theta_1+\theta_2)] \\ 0 & 1 & \Delta x_1 [-\cos\theta_2 \sin(\theta_1+\theta_2) + \sin\theta_2 \cos(\theta_1+\theta_2)] \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta x_1 \cos\theta_1 \\ 0 & 1 & -\Delta x_1 \sin\theta_1 \\ 0 & 0 & 1 \end{pmatrix}$$

similarly

$$(H_2 \cdot H_1)^{-1}(H_2 \cdot H_1(\cdot, y_1 + \Delta y_1, \theta)) = \begin{pmatrix} 1 & 0 & \Delta y_1 [-\sin\theta_2 \cos(\theta_1+\theta_2) + \cos\theta_2 \sin(\theta_1+\theta_2)] \\ 0 & 1 & \Delta y_1 [\sin\theta_2 \sin(\theta_1+\theta_2) + \cos\theta_2 \cos(\theta_1+\theta_2)] \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & +\Delta y_1 \sin\theta_1 \\ 0 & 1 & \Delta y_1 \cos\theta_1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(H_2 \cdot H_1)^{-1}(H_2 \cdot H_1(\theta_1 + \Delta\theta_1)) = \begin{pmatrix} \cos(\theta_1+\theta_2) & +\sin(\theta_1+\theta_2) & \cdots \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & \cdots \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\sin(\theta_1+\theta_2)\Delta\theta_1 & -\cos(\theta_1+\theta_2)\Delta\theta_1 & \cdots \\ +\cos(\theta_1+\theta_2)\Delta\theta_1 & -\sin(\theta_1+\theta_2)\Delta\theta_1 & \cdots \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\Delta\theta_1 & 0 \\ +\Delta\theta_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{\text{cool}}$ oh wait actually ... $\cos[\theta_1+\theta_2]$

$$(H_2 \cdot H_1)^{-1}(H_2\, g(\Delta x_2) \cdot H_1) = \begin{pmatrix} 1 & 0 & \Delta x_2 \cos(\theta_1+\theta_2) \\ 0 & 1 & -\Delta x_2 \sin(\theta_1+\theta_2) \\ 0 & 0 & 1 \end{pmatrix}$$

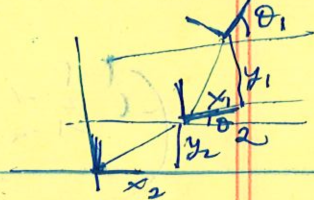
$$(H_2 \cdot H_1)^{-1}(H_2\, g(\Delta y_2)\, H_1) = \begin{pmatrix} 1 & 0 & +\Delta y_2 \sin(\theta_1+\theta_2) \\ 0 & 1 & \Delta y_2 \cos(\theta_1+\theta_2) \\ 0 & 0 & 1 \end{pmatrix}$$

$$(H_2 \cdot H_1)^{-1}(H_2(\cdots \theta_2 + \Delta\theta_2)\, H_1) = \begin{pmatrix} \cos(\theta_1+\theta_2) & +\sin(\theta_1+\theta_2) & \cdots \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & \cdots \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\sin(\theta_1+\theta_2)\Delta\theta_2 & -\cos(\theta_1+\theta_2)\Delta\theta_2 & -x_1\sin\theta_2 \Delta\theta_2 - y_1\cos\theta_2 \Delta\theta_2 \cdots \\ +\cos(\theta_1+\theta_2)\Delta\theta_2 & -\sin(\theta_1+\theta_2)\Delta\theta_2 & +x_1\cos\theta_2 \Delta\theta_2 - y_1\sin\theta_2 \Delta\theta_2 \cdots \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\Delta\theta_2 & \Delta\theta_2(-x_1\sin\theta_2 \cos(\theta_1+\theta_2) + x_1\sin(\theta_1+\theta_2)\cos\theta_2 - y_1\cos\theta_2\cos(\theta_1+\theta_2) - y_1\sin\theta_2 \sin(\theta_1+\theta_2)) \\ +\Delta\theta_2 & 1 & \Delta\theta_2(+x_1\sin\theta_2\sin(\theta_1+\theta_2) + x_1\cos(\theta_1+\theta_2)\cos\theta_2 + y_1\cos\theta_2\sin(\theta_1+\theta_2) - y_1\sin\theta_2\cos(\theta_1+\theta_2)) \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\Delta\theta_2 & \Delta\theta_2 [x_1\sin\theta_1 - y_1\cos\theta_1] \\ \Delta\theta_2 & 1 & \Delta\theta_2 [x_1\cos\theta_1 + y_1\sin\theta_1] \\ 0 & 0 & 1 \end{pmatrix}$$

SCRATCH** - 8



local coordinates in frame after total transformation

$$\begin{pmatrix}\Delta x_{TOT}\\ \Delta y_{TOT}\\ \Delta\theta_{TOT}\end{pmatrix} = \underbrace{\begin{pmatrix}\cos\theta_1 & +\sin\theta_1 & 0\\ -\sin\theta_1 & \cos\theta_1 & 0\\ 0 & 0 & 1\end{pmatrix}}_{\frac{\partial v_{TOT}}{\partial v_1}}\begin{pmatrix}\Delta x_1\\ \Delta y_1\\ \Delta\theta_1\end{pmatrix}$$

$$\begin{pmatrix}\Delta x_{TOT}\\ \Delta y_{TOT}\\ \Delta\theta_{TOT}\end{pmatrix} = \begin{pmatrix}\cos(\theta_1+\theta_2) & +\sin(\theta_1+\theta_2) & x_1\sin\theta_1 - y_1\cos\theta_1\\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & +x\cos\theta_1 + y_1\sin\theta_1\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta x_2\\ \Delta y_2\\ \Delta\theta_2\end{pmatrix}$$

$$\int_{\substack{\Delta x_{TOT}\\ \Delta y_{TOT}\\ \Delta\theta_{TOT}}} (\Delta x_{TOT}\ \Delta y_{TOT}\ \Delta\theta_{TOT})\ e^{-(\Delta x_1^{local}, \Delta y_1^{local}, \Delta\theta_1^{local})\begin{pmatrix}\sigma^2_{xx} & \sigma^2_{xy} & \sigma^2_{x\theta}\\ \sigma^2_{yx} & \sigma^2_{yy} & \sigma^2_{y\theta}\\ \sigma^2_{\theta x} & \sigma^2_{\theta y} & \sigma^2_{\theta\theta}\end{pmatrix}^{-1}\begin{pmatrix}\Delta x_1^{local}\\ \Delta y_1^{local}\\ \Delta\theta_1^{local}\end{pmatrix}} \frac{1}{N_\Omega} d\Delta x_{TOT}\, d\Delta y_{TOT}\, d\Delta\theta_{TOT}$$

local coordinate after total transformation $H_2 \circ H_1$

normalization

OUTER PRODUCT.

$$= \int \left(\frac{\partial v_{TOT}}{\partial v_1}\begin{pmatrix}\Delta x_1\\ \Delta y_1\\ \Delta\theta_1\end{pmatrix}(\Delta x_1, \Delta y_1, \Delta\theta_1)\left(\frac{\partial v_{TOT}}{\partial v_1}\right)^T\right) e^{-(\Delta x_1, \Delta y_1, \Delta\theta_1)\Sigma^{-1}\begin{pmatrix}\Delta x_1\\ \Delta y_1\\ \Delta\theta_1\end{pmatrix}} \frac{1}{N} d\Delta x\, d\Delta y\, d\Delta\theta$$

Assuming $\det\left|\frac{\partial v_{TOT}}{\partial v_1}\right| = 1$

$$= \begin{pmatrix}\cos\theta_1 & +\sin\theta_1 & 0\\ -\sin\theta_1 & \cos\theta_1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\sigma^2_{xx} & \sigma^2_{xy} & \sigma^2_{x\theta}\\ \sigma^2_{yx} & \sigma^2_{yy} & \sigma^2_{y\theta}\\ \sigma^2_{\theta x} & \sigma^2_{\theta y} & \sigma^2_{\theta\theta}\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta & 0\\ +\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$$

- OR WAIT → Need to BACK CONVERT/ROTATE?

$$\begin{pmatrix}\Delta x_1^{local}\\ \Delta y_1^{local}\\ \Delta\theta_1^{local}\end{pmatrix} = \begin{pmatrix}\cos\theta_1 & +\sin\theta_1 & 0\\ -\sin\theta_1 & \cos\theta_1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta x_1\\ \Delta y_1\\ \Delta\theta_1\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\Delta x_{TOT}\\ \Delta y_{TOT}\\ \Delta\theta_{TOT}\end{pmatrix} = \begin{pmatrix}\Delta x_1^{local}\\ \Delta y_1^{local}\\ \Delta\theta_1^{local}\end{pmatrix} \quad \text{good.}$$

- But FOR x_2, y_2, θ_2 more interesting...

$$\begin{pmatrix}\Delta x_{TOT}\\ \Delta y_{TOT}\\ \Delta\theta_{TOT}\end{pmatrix} = \begin{pmatrix}\cos(\theta_1+\theta_2) & +\sin(\theta_1+\theta_2) & x_1\sin\theta_1 - y_1\cos\theta_1\\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & +x_1\cos\theta_1 + y_1\sin\theta_1\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\theta_2 & -\sin\theta_2 & 0\\ +\sin\theta_2 & \cos\theta_2 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta x_2^{local}\\ \Delta y_2^{local}\\ \Delta\theta_2^{local}\end{pmatrix}$$

$$= \underbrace{\begin{pmatrix}\cos\theta_1 & +\sin\theta_1 & x_1\sin\theta_1 - y_1\cos\theta_1\\ -\sin\theta_1 & \cos\theta_1 & +x_1\cos\theta_1 + y_1\sin\theta_1\\ 0 & 0 & 1\end{pmatrix}}\begin{pmatrix}\Delta x_2^{local}\\ \Delta y_2^{local}\\ \Delta\theta_2^{local}\end{pmatrix}$$

$$\begin{pmatrix}\cos\theta_1 & -\sin\theta_1 & x_1\sin\theta_1 - y_1\cos\theta_1\\ \sin\theta_1 & \cos\theta_1 & -x_1\cos\theta_1 + y_1\sin\theta_1\\ 0 & 0 & 1\end{pmatrix}$$

BASICALLY THIS IS INVERSE (?) OF H_1 - Converts Local coordinates in intermediate frame (2) to final frame via transform.

SCRATCH

Let's do Rigid Rotation $\to \pi/2$ (so').

$$H = \left(\begin{array}{cc|c} 0 & -1 & 1 \\ 1 & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

$$H^2 = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$H^3 = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ -1 & 0 & 1 \\ \hline 0 & 0 & 1 \end{array}\right)$$

$$H^4 = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ good.

(*) oops... actually this transform rotates 90° then translates along x. For clockwise-way, need to rotate along 90° then shift along y: 8

→ see p. (11)

Now imagine we have a rigid rotator...

$\langle \Delta y^2 \rangle = 0$
$\langle \Delta x^2 \rangle = \sigma^2$
$\langle \Delta x \Delta\theta \rangle = \sigma^2$
$\langle \Delta\theta^2 \rangle = \sigma^2$
} IN LOCAL FRAME

check: $\langle(\Delta x - \Delta\theta)^2\rangle = 2\sigma^2 - 2\sigma^2 = 0$. yes.

So, $\Sigma^2 = \begin{pmatrix} \sigma^2 & 0 & \sigma^2 \\ 0 & 0 & 0 \\ \sigma^2 & 0 & \sigma^2 \end{pmatrix} = \sigma^2 \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

After H, $\Sigma^2 = \sigma^2 \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

After H^2, $\Sigma^2 = \sigma^2 \, \mathrm{Adj} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \mathrm{Adj}^T = \begin{pmatrix} 0 & +1 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}}$

$$\frac{\partial v_{TOT}}{\partial v^{(2)}_{LOCAL}} = \mathrm{ADJ} = \begin{pmatrix} \cos\theta_1 & +\sin\theta_1 & x_1\sin\theta_1 - y_1\cos\theta_1 \\ -\sin\theta_1 & \cos\theta_1 & +x_1\cos\theta_1 + y_1\sin\theta_1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & +1 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\theta_1 = \pi/2$, $\cos\theta_1 = 0$
$x_1 = 1$, $\sin\theta_1 = 1$
$y_1 = 0$

check $\frac{\partial v_{TOT}}{\partial v_{LOCAL}} = \mathrm{ADJ}\ \mathrm{ADJ}$ $= \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ FAIL

Hmm, maybe be OK.

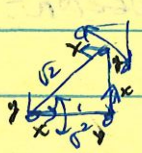
re-derive expression on previous page.

$$\int \begin{pmatrix} \Delta x^{TOT} \\ \Delta y^{TOT} \\ \Delta\theta^{TOT} \end{pmatrix}_{LOCAL} (\Delta x\ \Delta y\ \Delta\theta)^{TOT}_{LOCAL} \frac{1}{N} e^{-(\Delta x^{(1)}\ \Delta y^{(1)}\ \Delta\theta^{(1)})_{LOCAL}\Sigma^{-2}\begin{pmatrix}\Delta x^{(1)} \\ \Delta y^{(1)} \\ \Delta\theta^{(1)}\end{pmatrix}_{LOCAL}} d\Delta x^{(2)} d\Delta y^{(2)} d\Delta\theta^{(2)}_{LOCAL}$$

$$= \int \left[\frac{\partial\theta^{TOT}_{LOCAL}}{\partial\theta^{(2)}_{LOCAL}}\right]\begin{pmatrix}\Delta x^{(1)} \\ \Delta y^{(1)} \\ \Delta\theta^{(1)}\end{pmatrix}(\Delta x^{(2)}\ \Delta y^{(2)}\ \Delta\theta^{(1)})_{LOCAL}\left[\frac{\partial\theta^{TOT}_{LOCAL}}{\partial\theta^{2}_{LOCAL}}\right]^T e^{-\cdots} d\Delta x^{(1)} d\Delta y^{(1)} d\Delta\theta^{(1)}_{LOCAL}$$

$$= \left[\frac{\partial v^{TOT}}{\partial v^{(1)}}\right]_{LOCAL} \Sigma^2 \left[\frac{\partial v^{TOT}}{\partial v}\right]^T_{LOCAL}$$

Check H^2...



First represent GLOBAL FRAME...

$\langle \Delta x^2 \rangle = \sqrt{2}\sqrt{2}/2 = 1$
$\langle \Delta y^2 \rangle = \sqrt{2}/\sqrt{2} = 1$
$\langle \Delta x \Delta y \rangle = -1$
$\langle \Delta x \Delta\theta \rangle = -1$
$\langle \Delta y \Delta\theta \rangle = +1$
$\langle \Delta\theta^2 \rangle = 1$

then convert to local frame by rotation $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\Delta x' = \Delta y$
$\Delta y' = -\Delta x$
$\Delta\theta' = \Delta\theta$

$\langle \Delta x'^2 \rangle = 1$
$\langle \Delta y'^2 \rangle = 1$
$\langle \Delta x' \Delta y' \rangle = 1$
$\langle \Delta x' \Delta\theta' \rangle = +1$
$\langle \Delta y' \Delta\theta' \rangle = 1$
$\langle \Delta\theta'^2 \rangle = 1$
$\Rightarrow \boxed{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}}$

yes.

Hmm... check order with. opposite

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$
2, 4

SCRATCH** - 10

After one operation,

$$\Sigma^2 = \sigma^2\begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix}$$

After two operations,

$$\Sigma^2 = \underbrace{\sigma^2\begin{pmatrix}1&-1&1\\-1&1&-1\\1&-1&1\end{pmatrix}}_{\text{Rotation of FRAME 2}} + \underbrace{\sigma^2\begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix}}_{\text{Rotation of FRAME 1}} = \sigma^2\begin{pmatrix}2&-1&2\\-1&1&-1\\2&-1&2\end{pmatrix}$$

[See prev. page SCRATCH**-9]

After three operations

$$\Sigma^2 = \underbrace{\sigma^2\begin{pmatrix}0&0&0\\0&1&-1\\0&-1&1\end{pmatrix}}_{\text{Rotation of FRAME3 [At origin]}} + \sigma^2\begin{pmatrix}2&-1&2\\-1&1&-1\\2&-1&2\end{pmatrix} = \sigma^2\begin{pmatrix}2&-1&2\\-1&2&-2\\2&-2&3\end{pmatrix}$$

$$\frac{\partial U_{tot}}{\partial U^{(3)}_{Local}} = \begin{pmatrix}\cos\theta_{12} & +\sin\theta_{12} & x_1\sin\theta_{12} - y_{12}\cos\theta_{12}\\ -\sin\theta_{12} & \cos\theta_{12} & +x_{12}\cos\theta_{12} + y_{12}\sin\theta_{12}\\ 0 & 0 & 1\end{pmatrix} = \begin{pmatrix}-1&0&+1\\0&-1&-1\\0&0&1\end{pmatrix}$$

$h_2 h_1 = h^2 = \begin{pmatrix}-1&0&1\\0&-1&1\\0&0&1\end{pmatrix}$

$x_2 = 1$, $y_2 = 1$, $\theta_2 = \pi$, $\cos\theta_{12} = -1$, $\sin\theta_{12} = 0$

$$\text{so } \Sigma^2_{3\to 1,2,3} = \begin{pmatrix}-1&0&+1\\0&-1&-1\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix}\begin{pmatrix}-1&0&0\\0&-1&0\\+1&-1&1\end{pmatrix} = \begin{pmatrix}0&0&0\\-1&0&-1\\1&0&1\end{pmatrix}\begin{pmatrix}-1&0&0\\0&-1&0\\+1&-1&1\end{pmatrix} = \begin{pmatrix}0&0&0\\0&1&-1\\0&-1&1\end{pmatrix}$$

yes - makes some intuitive sense.

After four operations

$$\Sigma^2 = \sigma^2\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix} + \sigma^2\begin{pmatrix}2&-1&2\\-1&2&-2\\2&-2&3\end{pmatrix} = \boxed{\begin{array}{c|c} \begin{array}{ccc}\Delta x & \Delta y & \Delta\theta\end{array} & \\ \sigma^2\begin{pmatrix}2&-1&2\\-1&2&-2\\2&-2&4\end{pmatrix} & \begin{array}{c}\Delta x\\ \Delta y\\ \Delta\theta\end{array}\end{array}}$$

$h^3 = \left(\begin{array}{cc|c}0&1&0\\-1&0&1\\0&0&1\end{array}\right)$

$$\frac{\partial U_{tot}}{\partial U^{(4)}_{Local}} = \begin{pmatrix}\cos\theta_{123} & +\sin\theta_{123} & x_{123}\sin\theta_{123} - y_{123}\cos\theta_{123}\\ -\sin\theta_{123} & \cos\theta_{123} & +x_{123}\cos\theta_{123} + y_{123}\sin\theta_{123}\\ 0&0&1\end{pmatrix} = \begin{pmatrix}0&-1&0\\+1&0&-1\\0&0&1\end{pmatrix}$$

$x_{123} = 0$, $y_{123} = 1$, $\theta_{123} = \frac{3\pi}{2}$, $\cos\theta_{123} = 0$, $\sin\theta_{123} = -1$

$$\Sigma^2_{4\to 1,2,3,4} = \begin{pmatrix}0&-1&0\\+1&0&-1\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix}\begin{pmatrix}0&+1&0\\-1&0&0\\0&-1&1\end{pmatrix} = \begin{pmatrix}0&0&0\\0&0&0\\1&0&1\end{pmatrix}\begin{pmatrix}0&+1&0\\-1&0&0\\0&-1&1\end{pmatrix} = \begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}-9&1&0\\1&0&-1\\0&0&1\end{pmatrix}\begin{pmatrix}0&-2&1\\0&0&0\\0&-2&1\end{pmatrix} = \begin{pmatrix}0&0&0\\0&4&-2\\0&-2&1\end{pmatrix}$$

Hmm does not look right. Should be $\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$

NOTE: Looks like ~~mistake in $\frac{\partial U_{tot}}{\partial U^{(4)}_{Local}}$~~

$$\cancel{\begin{pmatrix}0&1&0\\-1&0&+1\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix}\begin{pmatrix}0&-1&0\\1&0&0\\0&+1&1\end{pmatrix} = \begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}\begin{pmatrix}0&-1&0\\1&0&0\\0&1&1\end{pmatrix} = \begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}}$$

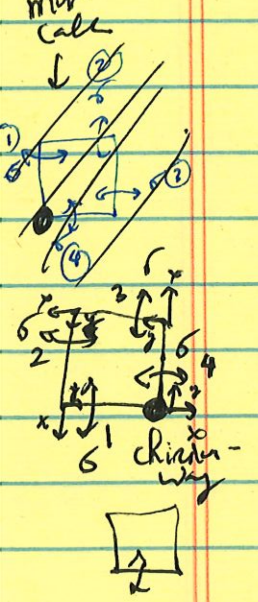
NUMERICALLY IN MATLAB ($\sigma = 0.01$ radians)

$$\Rightarrow \begin{pmatrix}2&1&2\\1&2&2\\2&2&4\end{pmatrix}$$ ** yes :D see next page for GTSAM

see page

SCRATCH — 11

Uh I almost got it, but I'm getting thrown off by sign errors.
In MATLAB... Actual Rigid Rotation:



$$\Sigma = \overset{1}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}} + \overset{2}{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}} + \overset{3}{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}} + \overset{4}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \Bigg\}$$ contribution to variance

$$= \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 4 \end{pmatrix}$$

Ah, in p. 5 - 10 above I accidentally screwed up "starting" FRAME.

~~Let's recompute using actual rigid rotation~~

I could convert BACK FROM my calculation (2-4) to Rigid Rotation [Chirikjian-Wang, 2005]

by rotating $\Delta x_{cw} = +\Delta y^{my}$, $\Delta y_{cw} = -\Delta x^{my}$ (2.)

~~$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -2 \\ 2 & -2 & 4 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$~~

$\langle \Delta x^2_{cw} \rangle = \langle \Delta y^2_{my} \rangle$

$\langle \Delta y^2_{cw} \rangle = \langle \Delta x^2_{my} \rangle$

$\langle \Delta x_{cw} \Delta y_{cw} \rangle = -\langle \Delta x_{my} \Delta y_{my} \rangle$

$\langle \Delta \theta_{cw} \Delta y_{cw} \rangle = -\langle \Delta \theta_{my} \Delta x_{my} \rangle$

$\langle \Delta \theta_{cw} \Delta x_{cw} \rangle = \langle \Delta \theta_{my} \Delta y_{my} \rangle$

So, $\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -2 \\ 2 & -2 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & +1 & 2 \\ +1 & 2 & +2 \\ 2 & +2 & 4 \end{pmatrix}$. $\boxed{\text{AGREEMENT!}}$

Would be better to work through this all and exactly follow Rigid-Rotator...