Engineering Acoustics/Reflection and transmission of planar waves

Specific acoustic impedance

Before discussing the reflection and transmission of planar waves, the relation between particle velocity and acoustic pressure is investigated.

$$\frac{\partial u}{\partial t} = \frac{-1}{\rho_o}\frac{\partial p}{\partial x}$$

The acoustic pressure and particle velocity can be described in complex form.

$$\mathbf{p} = \mathbf{P}e^{j(\omega t - Kx)}$$

$$\mathbf{u} = \mathbf{u_o}e^{j(\omega t - Kx)}$$

Differentiating and substituting,

$$-j\omega\mathbf{u} = \frac{-jK\mathbf{p}}{\rho_o} = \frac{-j\omega\mathbf{p}}{\rho_o c_o}$$

$$\mathbf{u} = \frac{\mathbf{p}}{\rho_o c_o}$$

The specific acoustic impedance for planar waves is defined.

$$\mathbf{z} = \frac{\mathbf{p}}{\mathbf{u}} = \rho_o c_o = r_o$$

Planar wave: Normal incidence

Consider an incident planar wave traveling in an infinite medium with specific impedance $r_1 = \rho_1 c_1$ which encounters the boundary between medium 1 and medium 2. Part of the wave is reflected back into medium 1 and the remaining part is transmitted to medium 2 with specific impedance $r_2 = \rho_2 c_2$. The pressure field in medium 1 is described by the sum of the incident and reflected components of the wave.

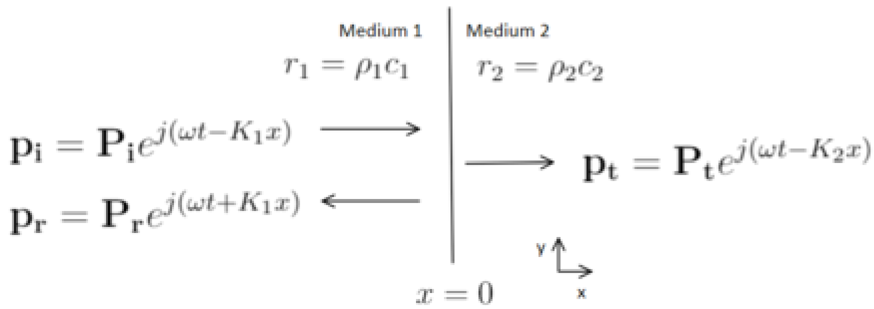
$$\mathbf{p_1} = \mathbf{p_i} + \mathbf{p_r} = \mathbf{P_i}e^{j(\omega t - K_1 x)} + \mathbf{P_r}e^{j(\omega t + K_1 x)}$$

The pressure field in medium 2 is composed only of the transmitted component of the wave.

$$\mathbf{p_2} = \mathbf{p_t} = \mathbf{P_t}e^{j(\omega t - K_2 x)}$$

Notice that the frequency of the wave remains constant across the boundary, however the specific acoustic impedance changes across the boundary. The propagation speed in each medium is different, so the wave number of each medium is also different. There are two boundary conditions to be satisfied:

1. The acoustic pressure must be continuous at the boundary
2. The particle velocity must be continuous at the boundary



Reflection and transmission of normally incident planar wave.

Imposition of the first boundary condition yields

$$\mathbf{p_1}(x=0) = \mathbf{p_2}(x=0),$$

$$\mathbf{P_i} + \mathbf{P_r} = \mathbf{P_t}.$$

Imposition of second boundary condition yields

$$\mathbf{u_1}(x=0) = \mathbf{u_2}(x=0),$$

$$\mathbf{u_i}(x=0) + \mathbf{u_r}(x=0) = \mathbf{u_t}(x=0),$$

and using the definition of specific impedance, the equations are expressed in terms of pressure

$$\frac{\mathbf{P_i}}{r_1} - \frac{\mathbf{P_r}}{r_1} = \frac{\mathbf{P_t}}{r_2}.$$

The pressure reflection coefficient is the ratio of the reflected acoustic pressure over the incident acoustic pressure, $\mathbf{R} = \frac{\mathbf{P_r}}{\mathbf{P_i}}$. The pressure transmission coefficient is the ratio of the transmitted acoustic pressure over the incident acoustic pressure, $\mathbf{T} = \frac{\mathbf{P_t}}{\mathbf{P_i}}$. The specific acoustic impedance ratio is also defined as: $\zeta = \frac{r_2}{r_1}$. Applying the above definitions, the boundary conditions can be rewritten as:

$$1 + \mathbf{R} = \mathbf{T}$$

$$1 - \mathbf{R} = \frac{\mathbf{T}}{\zeta}.$$

Solving for the pressure reflection coefficient yields:

$$\mathbf{R} = \mathbf{T} - 1 = \frac{\zeta - 1}{\zeta + 1} = \frac{r_2 - r_1}{r_2 + r_1}.$$

Solving for the pressure transmission coefficient yields:

$$\mathbf{T} = \mathbf{R} + 1 = \frac{2\zeta}{\zeta + 1} = \frac{2r_2}{r_2 + r_1}.$$

Solving for the specific acoustic impedance ratio yields:

$$\zeta = \frac{1 + \mathbf{R}}{1 - \mathbf{R}} = \frac{\mathbf{T}}{2 - \mathbf{T}}.$$

Case 1: Rigid boundary

Consider an incident planar wave that encounters a rigid boundary. This is the case if the specific impedance of medium 2 is significantly larger than the specific impedance of medium 1. Thus, the specific acoustic impedance ratio is very large, the reflection coefficient approaches 1 and the transmission coefficient approaches 2.

$$\mathbf{R} = 1 = \frac{\mathbf{P_r}}{\mathbf{P_i}} \Rightarrow \mathbf{P_r} = \mathbf{P_i} \Rightarrow \mathbf{u}(x=0) = 0$$

$$\mathbf{T} = 2 = \frac{\mathbf{P_t}}{\mathbf{P_i}} \Rightarrow \mathbf{P_t} = 2\mathbf{P_i} \Rightarrow \mathbf{p}(x=0) = 2\mathbf{P_i}$$

The amplitudes of the incident and reflected waves are equal. The reflected wave is in phase with the incident wave. The particle velocity at the boundary is zero. The acoustic pressure amplitude at the boundary is equal to twice the pressure amplitude of the incident wave and it is maximum.

Case 2: Resilient boundary

Consider an incident planar wave that encounters a resilient boundary. This is the case if the specific impedance of medium 2 is significantly smaller than the specific impedance of medium 1. Thus, the specific acoustic impedance ratio approaches zero, the reflection coefficient approaches -1 and the transmission coefficient approaches zero.

$$\mathbf{R} = -1 = \frac{\mathbf{P_r}}{\mathbf{P_i}} \Rightarrow \mathbf{P_r} = -\mathbf{P_i} \Rightarrow \mathbf{u}(x=0) = \frac{2\mathbf{P_i}}{r_1}$$

$$\mathbf{T} = 0 = \frac{\mathbf{P_t}}{\mathbf{P_i}} \Rightarrow \mathbf{P_t} = 0 \Rightarrow \mathbf{p}(x=0) = 0$$

The amplitudes of the incident and reflected waves are equal. The reflected wave is 180\degree out of phase with the incident wave. The particle velocity at the boundary is a maximum. The acoustic pressure at the boundary is zero.

Case 3: Equal impedance in both media

Consider two media with the same specific acoustic impedance so that the specific acoustic impedance ratio is unity, the reflection coefficient is zero and the transmission coefficient is unity. Therefore, the wave is not reflected, only transmitted. It behaves as if there was no boundary.
