HW2

1. (a) $X(e^{jw}) = \sum_{k=-\infty}^{\infty} x[k]\, e^{-jwk}$

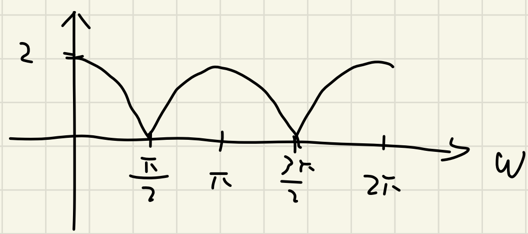
$$= x[1]e^{-jw} + x[-1]e^{jw} = e^{jw} + e^{-jw}$$

☆ $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ $\therefore e^{jw} + e^{-jw} = 2\cos w$

(b)

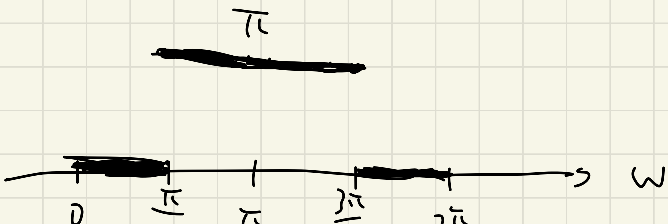
$X(e^{jw})$

$|X(e^{jw})|$



$$e^{j\angle X(e^{jw})} = \frac{X(e^{jw})}{|X(e^{jw})|}$$

$\angle X(e^{jw})$



3.

(a) $h[n] = \frac{1}{8}\left(\delta[n] + \delta[n-1] + \dots \delta[n-7]\right)$

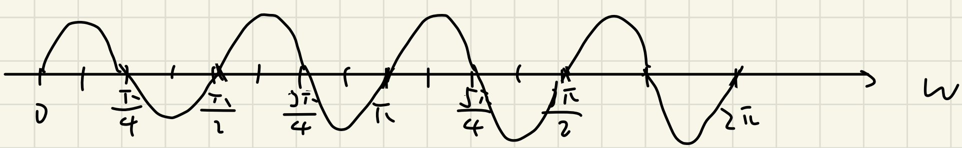
$$H(e^{jw}) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-jwk} = \frac{1}{8}\left(1 + e^{-jw} + e^{-j2w} + \dots e^{-j7w}\right)$$

$$= \frac{1}{8} \frac{1-e^{-j8w}}{1-e^{-jw}}$$

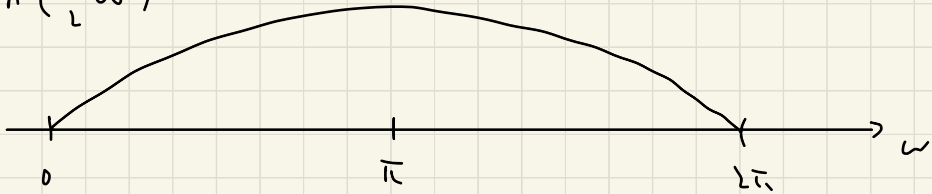
$$= \frac{1}{8} \frac{e^{-j4w}}{e^{-j\frac{w}{2}}} \frac{e^{j4w}+e^{-j4w}}{e^{j\frac{w}{2}}+e^{-j\frac{w}{2}}} = \frac{1}{8} e^{-j\frac{7}{2}w} \cdot \frac{\sin 4w}{\sin\frac{w}{2}}$$

$$|H(e^{jw})| = \left|\frac{1}{8} \frac{\sin 4w}{\sin\frac{w}{2}}\right|$$

$\sin(4w)$

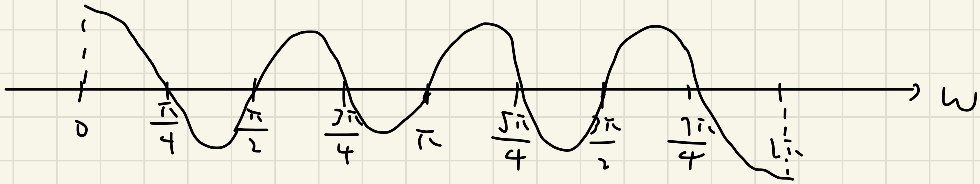


$\sin(\frac{1}{2}w)$

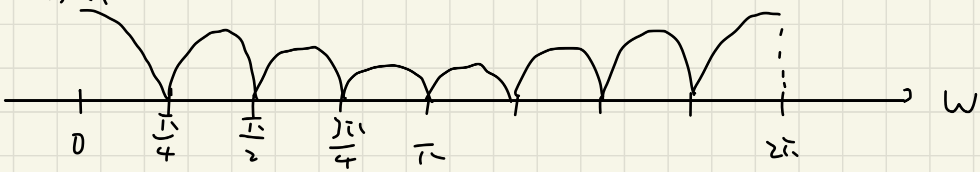


$$\lim_{w \to 0} \frac{1}{8} \frac{\sin 4w}{\sin \frac{w}{2}} = \frac{1}{8} \lim_{w \to 0} \frac{4\cos(4w)}{\frac{1}{2}\cos(\frac{w}{2})} = 1$$

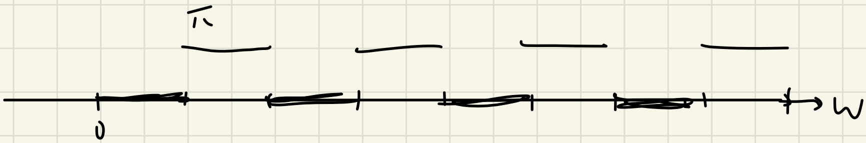
$$\frac{1}{8} \frac{\sin 4w}{\sin \frac{w}{2}}$$



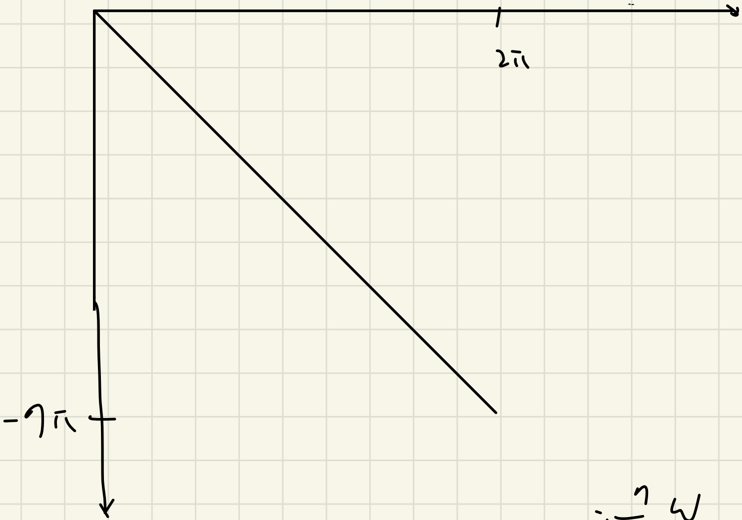
$$\left| \frac{1}{8} \frac{\sin 4w}{\sin(\frac{w}{2})} \right|$$



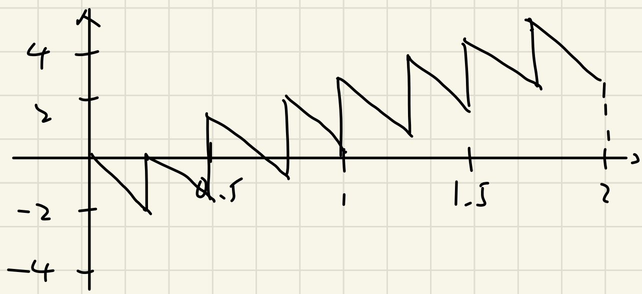
$$\angle \frac{1}{8} \frac{\sin 4w}{\sin \frac{w}{2}}$$



$$\angle e^{-j\frac{7}{2}w} = -\frac{7}{2}w$$



$$\angle H e^{jw} = \angle \frac{1}{8} \frac{\sin 4w}{\sin \frac{w}{2}} + \angle e^{-j\frac{7}{2}w}$$



(b) $x[n] = e^{-j\frac{\pi}{5}n}$ $\quad h[n] = \frac{1}{8}(\delta[n] + \delta[n-1] + \dots \delta[n-7])$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x[n-k] = \frac{1}{8}\left(e^{-j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}(n-1)} + \dots e^{-j\frac{\pi}{5}(n-7)}\right)$$

$$X(e^{jw}) = 2\pi\delta(w - \frac{\pi}{5})$$

$$Y(e^{jw}) = X(e^{jw})H(e^{jw}) = 2\pi\delta(w - \frac{\pi}{5}) \cdot \left(\frac{1}{8}\frac{1-e^{-j8w}}{1-e^{-jw}}\right)$$

$$= \frac{\pi}{4}\frac{1-e^{-j\frac{8\pi}{5}}}{1-e^{-j\frac{\pi}{5}}} \cdot \delta(w - \frac{\pi}{5})$$

$x[n] = \cos(\frac{\pi n}{5})$, $\quad h[n] = \frac{1}{8}(\delta[n] + \delta[n-1] + \dots \delta[n-7])$

$$y(n) = \sum_{k=-\infty}^{\infty} h[k]x(n-k) = \frac{1}{8}\left(\cos\frac{\pi}{5}n + \cos\frac{\pi}{5}(n-1) + \dots \cos\frac{\pi}{5}(n-7)\right)$$

$$\cos(\frac{\pi n}{5}) = \frac{e^{j\frac{\pi}{5}} + e^{-j\frac{\pi}{5}}}{2}, \quad X(e^{jw}) = \pi(\delta(w-\frac{\pi}{5}) + \delta(w+\frac{\pi}{5}))$$

$$Y(e^{jw}) = \frac{\pi}{8}\frac{1-e^{-j\frac{8\pi}{5}}}{1-e^{-j\frac{\pi}{5}}}\delta(w-\frac{\pi}{5}) + \frac{\pi}{8}\frac{1-e^{j\frac{8\pi}{5}}}{1-e^{j\frac{\pi}{5}}}\delta(w+\frac{\pi}{5})$$

(d)

$$Y(e^{jw}) = \frac{\pi}{4}\delta(w-w_0)\left(\frac{1-e^{-j8w_0}}{1-e^{-jw_0}}\right)$$

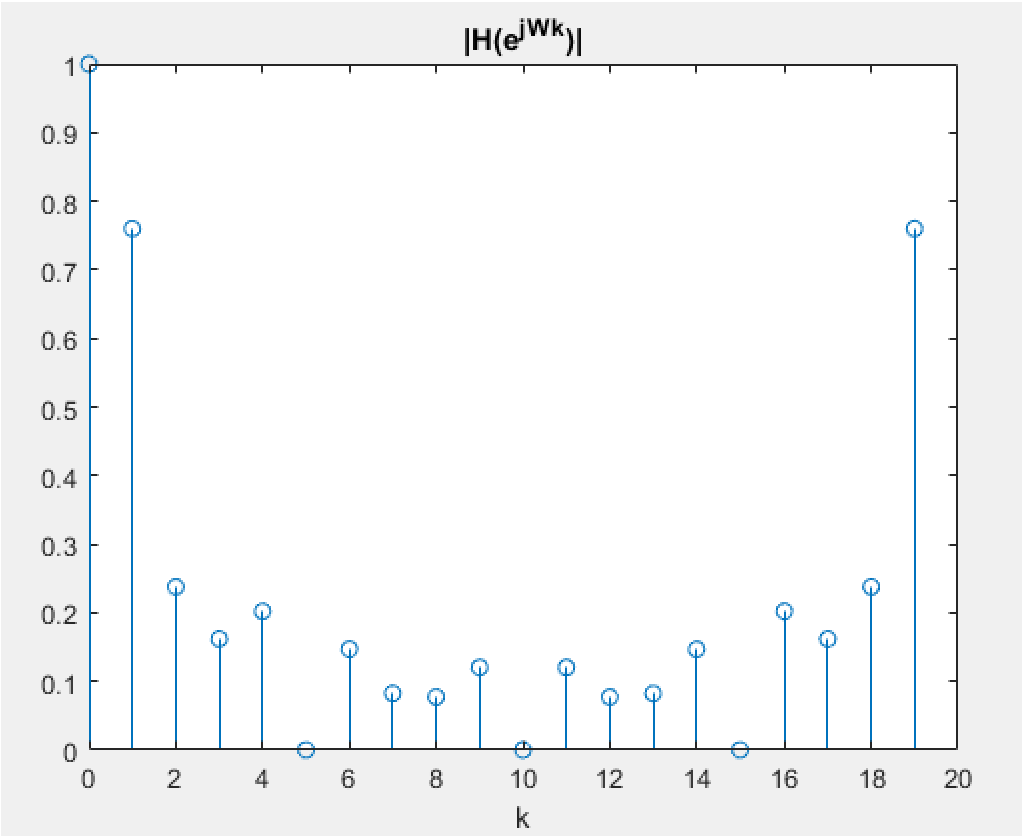
$w_0=2k\pi$時，因羅必達法則，$Y(e^{jw})\neq 0$

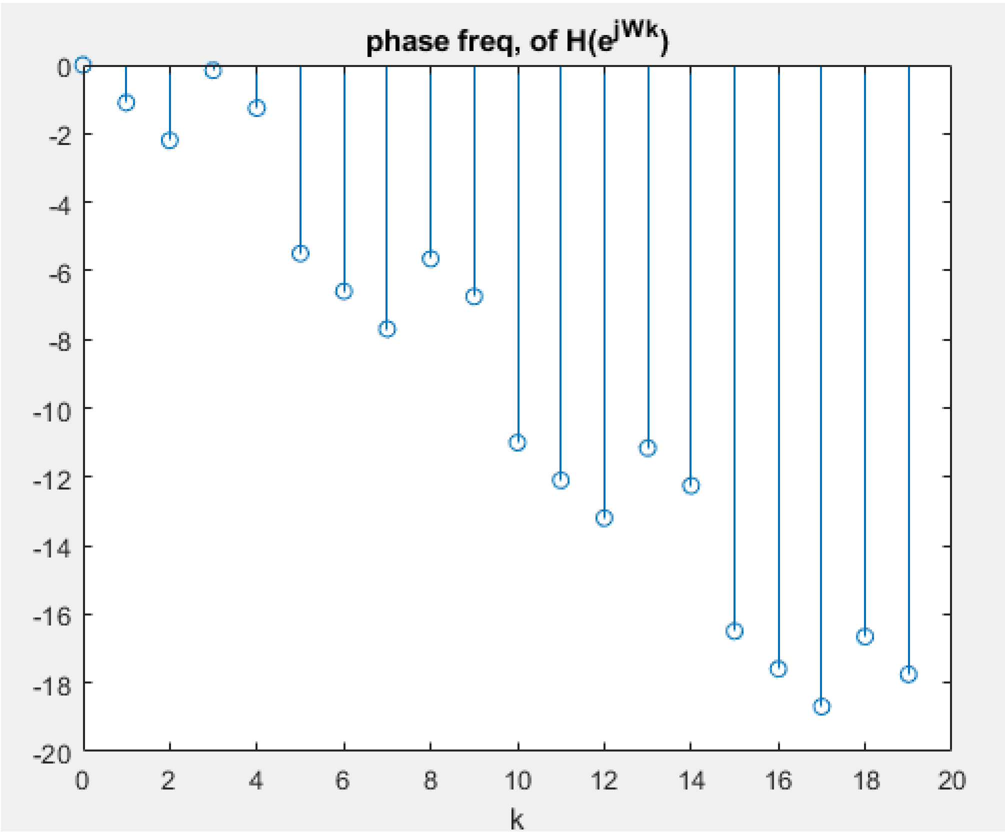
當 $w_0=\frac{k\pi}{4}$，且 $w_0\neq 2k\pi$，$k=0,1,2,\ldots$ 時，$y[n]=0$

ex：$w_0=\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, \frac{9\pi}{4}, \ldots$

4.(a) 利用 matlab 計算，$H(e^{jw})=2.5$

與3.(a)的差異：matlab是將0到2π的圖形取20個點做圖，而3.(a)則是連續型做圖


|H(e^jWk)|
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 2 4 6 8 10 12 14 16 18 20
k


phase freq, of H(e^jWk)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
0 2 4 6 8 10 12 14 16 18 20
k