DSP/YPL

Homework 6 due 12/16 in class

1. Suppose $x[n] = \cos(\omega_0 n)$, for all n, be a discrete time sinusoid. Let $y[n] = x[n]$, for $n = 0, 1, \cdots, L-1$, and $y[n] = 0$ for $n < 0$ and $n \geq L$. Let $Y[k]$ be the N-point DFT of $y[n]$.

   (a) Sketch $|Y[k]|$ for $\omega_0 = \pi/2$, and $N = L = 100$.

   (b) Use the fact that $y[n]$ can be expressed as $x[n]h[n]$, where $h[n]$ is a rectangular window, to find an expression of $|Y(e^{j\omega})|$ in terms of ω_0 and the frequency response of a rectangular window.

   (c) Sketch $|Y(e^{j\omega})|$ using $|Y[k]|$ for $\omega_0 = \pi/3$, $L = 100$ and $N = 1024$.

   Note. The DTFT of $x[n]$ is $X(e^{j\omega}) = \pi\delta(\omega - \omega_0)$, for $0 \leq \omega \leq \pi$, i.e., an impulse. When we use a computer to compute the DTFT, truncation of $x[n]$ is needed as $x[n]$ is an infinite sequence. We would like to get a sketch of $X(e^{j\omega})$ through the DFT of $y[n]$. The DFT of $y[n]$ offers information of $X(e^{j\omega})$ but there are some effect due to finite N. In particular, the truncation of the sequence leads to smearing or broadening of the impulse, which is called windowing effect. The peak is narrower as N increases. This can be observed in the sketch of $|Y(k)|$.

2. (a) Let $H(z) = \sum_{n=0}^{M} h[n]z^{-n}$ be a filter with zeros at $z = -1, e^{j\pi/2}, e^{j\pi/4}, 0.8e^{j\pi 3/4}$. It is known that $h[n]$ has real coefficients and generalized linear phase. What is the minimum value of M? For this minimum value of M, mark the zeros of $H(z)$ on the z-plane.

   (b) Suppose $H(z)$ is a stable allpass filter with real coefficients. It is known that $H(z)$ has zeros at $z = -2, 2e^{j\pi/2}, 1.2e^{j\pi 3/4}$. What is the minimum order of $H(z)$? Determine $H(z)$ that has the minimum order.

3. Let

$$H(z) = \frac{1 - 2z^{-1} + 3z^{-2}}{1 - z^{-1} + 2z^{-2}}.$$

   (a) Find a causal and stable filter $F_1(z)$ that has the same magnitude response as $H(z)$.

   (b) Find a minimum phase filter $F_2(z)$ that has the same magnitude response as $H(z)$.

   (c) Find a causal and stable $G(z)$ so that when an input is applied, the output of $G(z)H(z)$ has the same magnitude response as the input.

4. **MATLAB**. Consider $x[n] = \cos(\omega_1 n) + \cos((\omega_1 + \Delta\omega)n)$ for $n = 0, 1, \cdots, N-1$ for some integer N and $x[n]$ is zero otherwise.

   (a) Let $\omega_1 = \pi/3$ and $\Delta\omega = \pi/10$. Find the smallest N such that the two sinusoids can be observed in the DFT of $x[n]$.

(b) Let $N = 100$. Find the smallest frequency spacing $\Delta\omega$ such that the two sinusoids are resolvable in the DFT of $x[n]$.

Note: Consider $y[n] = \cos(\omega_1 n) + \cos((\omega_1 + \Delta\omega)n)$ for all n. The Fourier transform of $y[n]$ is $Y(e^{j\omega}) = \pi\delta(\omega - \omega_1) + \pi\delta(\omega - (\omega_1 + \Delta\omega))$, for $0 \leq \omega \leq \pi$, i.e., two impulses. The sequence $x[n]$ contains only N samples of $y[n]$. The truncation of the sequence leads to windowing effect as mentioned in Problem 1 and the two impulses are broadened. When N is not large enough, we can not observe two peaks in $X[k]$. In this experiment, we get a thumb of rule for determining N so that two closely spaced sinusoids can be resolved. (All Matlab assignments should be accompanied by observations, or comments on why the plots are reasonable. Unexplained plots are not given credits.)