DSP/YPL

Homework 1 due in class

1. Consider the 5-point moving average system.
   (a) Is the system LTI? Justify your answer. If so, identify the impulse response.
   (b) Let the input be $x[n] = \delta[n] + 0.5\delta[n-1] + \delta[n-2]$. Plot the output $y[n]$ using the two interpretations of convolution.
   (c) Let the input be $x[n] = 0.5^n u[n-1]$. Determine the output.

2. (a) Consider a system whose output $y[n]$ is related to its input $x[n]$ by $y[n] = \begin{cases} x[n/2], & \text{for even } n \\ 0, & \text{otherwise.} \end{cases}$ Is the system linear? Is it time invariant?
   (b) Suppose an LTI system has impulse response $h[n] = u[n]$. Is the system causal? Is it BIBO stable? Express the output $y[n]$ in terms of the input $x[n]$.
   (c) Suppose the lengths of two sequences are respectively N and M. What is the length of their convolution?

3. Consider the 4-point moving average system.
   (a) Suppose the input $x[n]$ is periodic with period N. Is the output always a periodic sequence with period N?
   (b) Suppose the input is $x[n] = \cos(n\pi/2)$. Determine the output $y[n]$. (Simplify the expression as much as possible).
   (c) Determine the output $y[n]$ when the input is a complex exponential $x[n] = e^{j2n\pi/N}$ for some integer N. Show that the output is of the form $y[n] = cx[n]$. Determine the constant c. (This means that output is also a complex exponential.)

4. MATLAB Let $h[n]$ be a M-pt moving average system with input $x[n]$ and output $y[n]$. Plot the output for the following cases by writing a matlab code to compute the convolutional sum. (Useful matlab command: `plot, stem`)

   Note: All Matlab assignments should be accompanied by observations, or comments on why the plots are reasonable. Unexplained plots are not given credits.
   (a) Plot $h[n]$ for $M = 8$.
   (b) Let $M = 8$ and $x[n] = \cos(n\pi/2)$ for all n. ($x[n]$ is an infinite sequence. Explain how you compute the output using matlab and why.) Also compute the output by hand and compare with the matlab plot.
   (c) Consider the input $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $n = 0, 1, \cdots, 50$ and $x[n]$ is zero otherwise. Suppose $M = 8$ and the output of $h[n]$ is $y[n]$. Consider another system with impulse response $g[n] = (-1)^n h[n]$. Suppose the output is $w[n]$ when $x[n]$ is the input. Observe the difference between the outputs of the two systems and explain.