Homework 5 due 11/25 in class

1. Let $x[n]$ be an N-point sequence and $x[n] = e^{j2\pi n/N}$, for $n = 0, 1, \cdots, N-1$ and $\tilde{x}[n]$ be the periodic extension of $x[n]$.
   (a) Apply N-point DFT on $x[n]$ to obtain $X[k]$ for $k = 0, 1, \cdots, N-1$.
   (b) Find $\tilde{X}(e^{j\omega})$, the DTFT of $\tilde{x}[n]$.
   (c) Determine the N-point DFT of $y[n] = \cos(2\pi n/N)$, for $n = 0, 1, \cdots, N-1$.

2. Let $x[n] = \cos(\pi n/3)$ for $n = 0, 1, \cdots, 5$ and $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$.
   (a) Find the 6-point circular convolution of $x[n]$ and $h[n]$.
   (b) Determine $X[k]$ and $H[k]$, the 6-point DFT of $x[n]$ and $h[n]$, respectively. Apply 6-point IDFT on $Y[k] = H[k]X[k]$ to obtain $y[n]$. Is $y[n]$ the same as that obtained in (a)?

3. Let $x[n]$ be a sequence of length N and $x[n] = 0$ for $n < 0$ and $n \geq N$. Suppose $X[k]$ is the N-point DFT of $x[n]$. Let $y[n] = x[n-1]$ for all n.
   (a) Express $\tilde{Y}[k]$ in terms of $X[k]$.
   (b) Let $w[n]$ be a sequence of length N and $w[n] = 0$ for $n < 0$ and $n \geq N$. Suppose $W(e^{j\omega}) = X(e^{j(\omega - 2\pi/N)})$. Express $w[n]$ in terms of $x[n]$.
   (c) Let $Z[k] = \tilde{Y}[k]$ for $k = 0, 1, \cdots, N-1$ and let $z[n]$ be the N-point IDFT of $Z[k]$, for $n = 0, 1, \cdots N-1$. Express $z[n]$ in terms of $x[n]$.

4. **MATLAB**. In the previous Matlab problem, you were asked to find the fundamental frequency of a recording sampled from a continuous time signal $x_c(t)$. Use the same recording for (a)-(b). Let $y[n]$ be a part of the recording that is approximately periodic with around 10 periods. Say the length is N. Suppose the fundamental period is P.

   Note: All Matlab assignments should be accompanied by observations, or comments on why the plots are reasonable. Unexplained plots are not given credits.

   (a) Apply N-point DFT on $y[n]$ using the formula $Y[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$ for $k = 0, 1, \cdots, N-1$. Plot $|Y(e^{j\omega})|$ and $\angle Y(e^{j\omega})$ for the range $|\omega| \leq \pi$ using $|Y[k]|$ and $\angle Y[k]$. (Two useful MATLAB commands: `abs`, `angle`)
   (b) Use (a) to plot $|Y_r(j\Omega)|$, where $Y_r(j\Omega)$ is the Fourier transform of the continuous time signal $Y_r(t)$ reconstructed from $y[n]$.
   (c) Explain how we can use (b) to obtain the fundamental frequency of $x_c(t)$.