

**Final checkpoint!**

## Scorecard!

Learning target:	P1	P2	L1	L2	L3	L4
Your confidence level before starting (0-5):						
Your confidence level after the quiz (0-5):						
The mark you earned on this attempt:	Success! Try again!	Success! Try again!	Success! Try again!	Success! Try again!	Success! Try again!	Success! Try again!
Learning target:	G1	G2	G3	G4		
Your confidence level before starting (0-5):						
Your confidence level after the quiz (0-5):						
The mark you earned on this attempt:	Success! Try again!	Success! Try again!	Success! Try again!	Success! Try again!		
Learning target:	C1	C2	C3	C4	Q1	Q3
Your confidence level before starting (0-5):						
Your confidence level after the quiz (0-5):						
The mark you earned on this attempt:	Success! Try again!	Success! Try again!	Success! Try again!	Success! Try again!	Success! Try again!	Success! Try again!

Have fun and do your best! I believe in u ♥

Answer each of the counting questions on this page. Explain why your approach is correct.

- (a) How many different bags of 20 Skittles are possible, assuming we care about how many of each of the 5 “flavors” are in the bag?
- (b) How many stacks of 9 coins are possible, using pennies, nickels, dimes, and quarters?
- (c) How many bit strings of length 12 have weight either 5 or 6?
- (d) How many 5-letter words can you make from the letters  $\{a, b, c, d, e, f, g\}$  without using any letter more than once?
- (e) How many ways are there to divide 15 people into “red team,” “blue team,” and “green team,” so that each team has 5 people?

**Learning target C2, version 1**

You are hosting a party for 50 people. As a conscientious host, you've asked all of your guests what they are allergic to: shellfish (S), peanuts (P), and/or broccoli (B). Here are the results:

Allergen	S	P	B	Both S and P	Both S and B	Both P and B	S, P, and B
Number	14	12	12	4	5	3	1

How many of your 50 guests are **not** allergic to anything?

**Learning target C4, version 1**

Give a *combinatorial* proof of the fact that each row of Pascal's triangle adds up to a power of 2:

$$\binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n} = 2^n.$$

Remember, a combinatorial proof is one in which you ask a counting question, answer it in two different ways, and then conclude that the two answers must be the same. I have provided some structure hints.

Consider the counting question: \_\_\_\_\_

One way that we can answer this question is:

Therefore, one answer to this counting question is  $\binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n}$ .

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A second way that we can answer this question is:

Therefore, another answer to this same counting question is  $2^n$ .

(Now write your conclusion here!)

**Learning target Q1 and Q3, version 1**

The abandoned field behind your house is home to a large prairie dog colony. Each week the size of the colony triples, before 4 prairie dogs sadly die. Let  $(a_n)_{n \geq 1}$  be the sequence giving the number of prairie dogs in the colony after the  $n$ th week (after the tripling followed by the death of 4). After the first week, there are 5 prairie dogs (so  $a_1 = 5$ ).

- (a) Write down a recurrence relation to describe  $a_n$  in terms of  $a_{n-1}$ .

Briefly explain why your formula is correct.

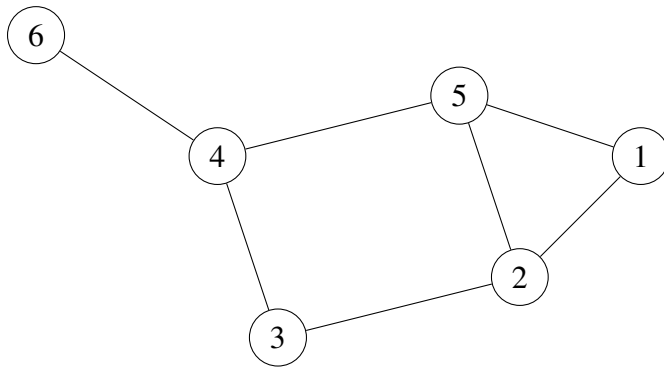
- (b) Give a careful proof by mathematical induction that  $a_n = 3^n + 2$ .

Base case:

Inductive step:

**Learning target G3 and G4, version 2**

Consider the graph below.



**(G3)** Is the graph a tree, complete, bipartite, planar, or have an Euler circuit? Explain.

**(G4)** What is the chromatic number of this graph? Illustrate by providing a proper coloring.

**Learning target Q3, version 2**

The following is true for all integers  $n \geq 1$ . Prove it using mathematical induction.

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

Base case:

Inductive step: