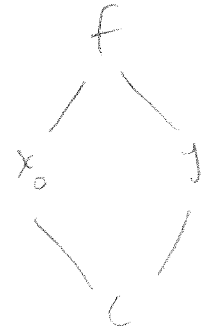
PS #9 BONUS PROBLEM



so $\dfrac{\partial f}{\partial c} = \dfrac{\partial f}{\partial x_0}\dfrac{\partial x_0}{\partial c} + \dfrac{\partial f}{\partial y_0}\dfrac{\partial y_0}{\partial c}$

NOTE A similar thing is true of g:
$$\frac{\partial g}{\partial c} = \frac{\partial g}{\partial x_0}\frac{\partial x_0}{\partial c} + \frac{\partial g}{\partial y_0}\frac{\partial y_0}{\partial c}$$

a) Since $\nabla f = \lambda \nabla g$ at (x_0, y_0),

$$\langle f_x(x_0,y_0), f_y(x_0,y_0)\rangle = \lambda\cdot\langle g_x(x_0,y_0), g_y(x_0,y_0)\rangle$$

Equating components,

$$f_x(x_0,y_0) = \lambda\cdot g_x(x_0,y_0) \quad , \quad \text{and} \quad f_y(x_0,y_0) = \lambda\cdot g_y(x_0,y_0)$$

In other words, at the point (x_0, y_0),

$$\frac{\partial f}{\partial x_0} = \lambda\cdot\frac{\partial g}{\partial x_0} \quad \text{and} \quad \frac{\partial f}{\partial y_0} = \lambda\cdot\frac{\partial g}{\partial y_0}$$

Therefore,
$$\frac{\partial f}{\partial c} = \frac{\partial f}{\partial x_0}\frac{\partial x_0}{\partial c} + \frac{\partial f}{\partial y_0}\frac{\partial y_0}{\partial c}$$
$$= \left(\lambda\frac{\partial g}{\partial x_0}\right)\frac{\partial x_0}{\partial c} + \left(\lambda\frac{\partial g}{\partial y_0}\right)\frac{\partial y_0}{\partial c} = \lambda\left(\frac{\partial g}{\partial x_0}\frac{\partial x_0}{\partial c} + \frac{\partial g}{\partial y_0}\frac{\partial y_0}{\partial c}\right)$$
$$= \lambda\cdot\frac{\partial g}{\partial c}.$$

b) Since $g(x,y) = c$,
$$\frac{\partial g}{\partial c} = 1 \qquad \text{so} \quad \frac{\partial f}{\partial c} = \lambda\cdot\frac{\partial g}{\partial c} = \lambda\cdot 1 = \lambda$$

c) If c increases by 1 — that is, if we can bump up length + girth to <u>109</u> instead of 108 — we'd get another <u>324</u> cubic inches of volume.

d) c has decreased by 2, so f should decrease by about 2λ.

$$236 - 2\cdot\lambda = 236 - 30 = 206$$