Checkpoint: Traces and level curves

Consider the functions $C(x,y) = 2x^2 + y^2$ and $S(x,y,z) = x^2 + y^2 + z^2$. (Notice that S is, like, a 4-dimensional function – three inputs and one output.)

(a) Write equations and sketch graphs for the $y = -2$, $y = 0$, and $y = 2$ traces of C.

(b) Write equations and sketch graphs for the $x = -2$, $x = 0$, and $x = 2$ traces of C.

(c) What can you say about the general shape of x-traces of this function? How about y-traces? How are they similar and how are they different?

(d) Write equations and sketch graphs for the level curves of C at 0, 2, and 4.

(e) What can you say about the general shape of the contours of this function?

(f) Why didn't I ask about contours at -2 or -4?

(g) Write an equation for the contour of S at 1.

(h) What's this equation?
(Hint: you can graph it in CalcPlot3D if you use the "implicit surface" option.)

(i) Maybe now I can't call this a "level curve" but instead I should call it a "level surface." Why?

Checkpoint: Partial derivatives

The gas law for a fixed mass m of an ideal gas at absolute temperature T in Kelvins, pressure P in pascals, and volume V in liters is $PV = mRT$, where R is the gas constant.

- Compute each of the partial derivatives below,
- assign them the correct units,
- and say a sentence or two about what each one means. (Be careful to think about what is *constant* and what is *changing*.)

(a) $\dfrac{\partial P}{\partial V}$

(b) $\dfrac{\partial V}{\partial T}$

(c) $\dfrac{\partial T}{\partial P}$

(Bonus question for the physics-knowers: Why does the sign of each one make sense?)

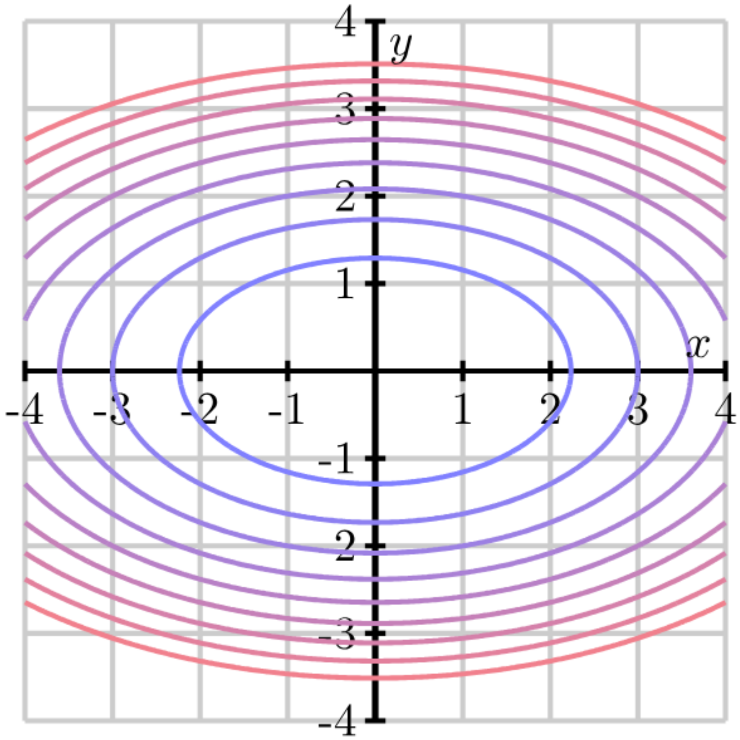
Checkpoint: Differentials slash linear approximation slash tangent planes

Wolfram Alpha tells me that $\sqrt{(4.98)^2 - (3.03)^2} = 3.9521513128927642311893057433005702498114436228955$
This is, however, way too precise to be useful. Show how to compute a reasonable estimate of this value **without a calculator** – I bet your answer will agree with the actual value in at least the first two or three decimal places.
Hints:

- Consider the function $f(x,y) = \sqrt{x^2 - y^2}$ and think about the title of this checkpoint.
- Do you remember how to multiply decimals by hand? To find $1.5 * 0.2$, you can compute $1.5 * 2$ or $15 * 2$, then move the decimal point in your result an appropriate number of places.

Checkpoint: The gradient

Consider the function $f(x,y) = x^2 + 3y^2$; here's a contour plot of the function but with the contours deliberately not labeled.



(a) Compute $\nabla f(x,y)$.

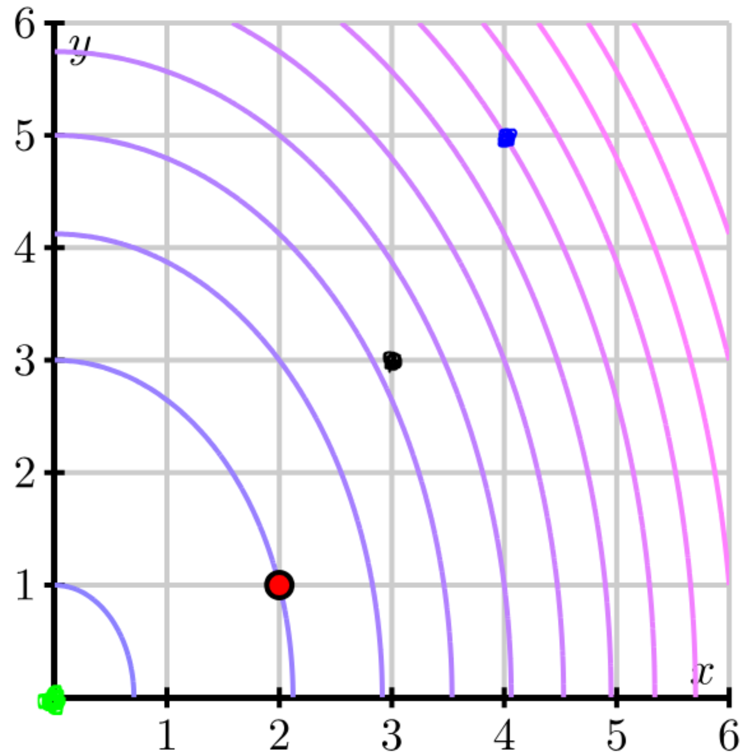
(b) At the point $(1,2)$, what is the direction of greatest increase?

(c) Draw a vector on the contour plot showing the direction of greatest increase.

(d) Find the slope of f in this direction.

Checkpoint: Directional derivatives

The function $T(x,y) = 30 - x^2 - y^2$ measures the temperature, in degrees Celsius, at a given point (x,y) on a heated metal plate, where x and y are measured in centimeters. An ant is walking on this plate in a straight line from the point $(1,2)$ to the point $(4,5)$. Here's a contour plot with some relevant points highlighted.



(a) How fast is the temperature under the ant's feet changing at the beginning of its journey? Give units.

(b) How fast is the temperature under the ant's feet changing when it's halfway through its journey? (Hint: if you draw this out you'll see that this is at the point $(3,3)$.)

(c) The ant reaches $(4,5)$ and decides to walk directly back to the origin. Now how fast is the temperature under the ant's feet changing?

Checkpoint: Optimization

The temperature T of the disk $x^2+y^2 \leq 1$ is given by $T(x,y) = 2x^2 - 3y^2 - 2x$. Find the hottest and coldest points of the disk. A contour plot of this function in these boundaries is shown below.