SOLUTIONS

2.4 Approximating solutions of equations – Practice exercises

1. The size of a round pizza is described by its **diameter** (distance across). Assuming a 16-inch diameter pizza serves four people, and with a little geometry to help us out, we calculated that a pizza of diameter D inches serves P people where

$$P = .015625D^2$$

Story also appears in 3.3 #1 and 4.1 #3

(a) Confirm that a 16-inch pizza serves four people.

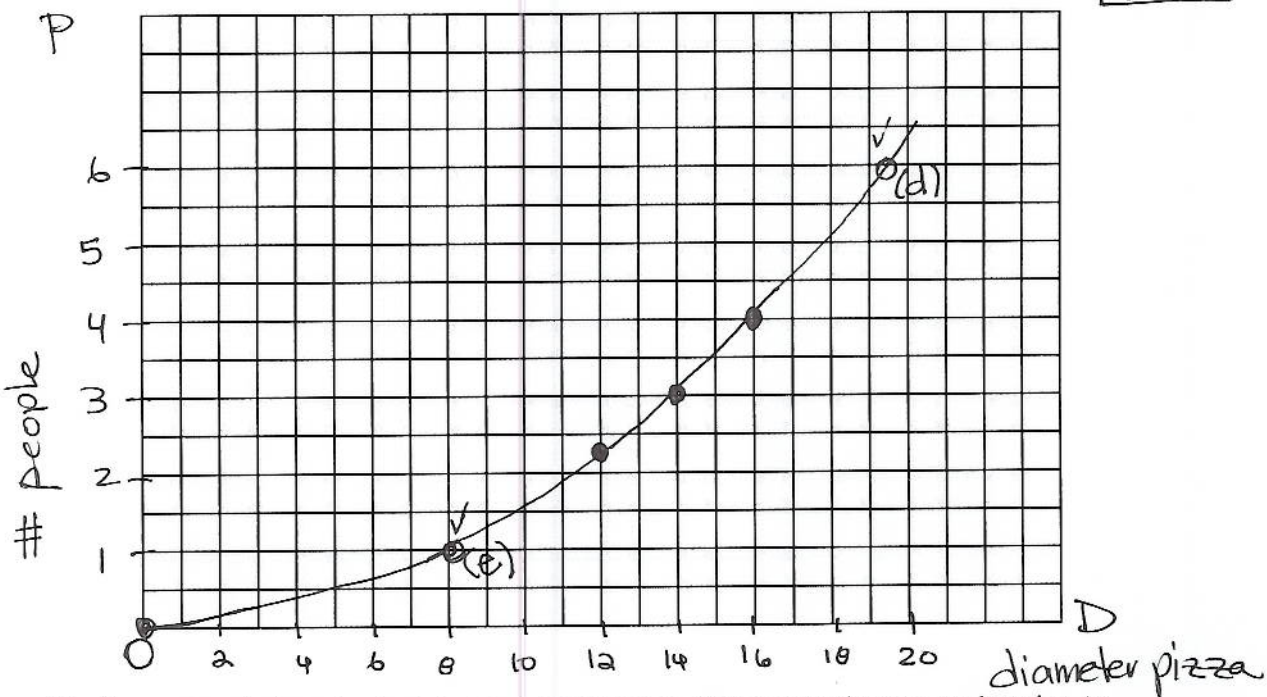
$P = .015625 * 16^2 = .015625 \times 16 \wedge 2 = 4$ people ✓

(b) How many people does a 12-inch pizza serve? A 14-inch pizza?

$P = .015625 \times 12 \wedge 2 = 2.25 \approx 2$ people

$P = .015625 \times 14 \wedge 2 = 3.0625 \approx 3$ people

(c) Graph the function. Include what happens when $D = 0$.



(d) A **personal** pizza is sized to serve one person. Use successive approximation to estimate the diameter of a personal pizza to the nearest inch.

D	0	12	8
P	0	2	1
vs 1	low	high	yes ☺

= 8 inches

(e) What diamter should an extra large pizza be to serve 6 people? Answer to the nearest 1/10 inch.

D	16	20	18	19	19.5	19.7	19.6
P	4	6.25	5.0625	5.640	5.941	6.063	6.0025
vs 6	low	high	low	low	low	high	

≈ 19.6 inches

2. Suppose a car gas tank is designed to hold enough fuel to drive 350 miles. (That's fairly average.) A hybrid car with fuel efficiency of 50 miles per gallon (mpg) would only need a 7 gallon gas tank, but a recreational vehicle that gets only 10 mpg would need a 35 gallon gas tank. *Story also appears in 3.3 #3*

50 mpg * 7 gal = 350 mi
10 mpg * 35 gal = 350 mi

(a) Name the variables including units. The way the story is stated, the size tank is a function of the fuel efficiency.

G = size tank (gallons) ~ dep
F = fuel efficiency (mpg) ~ indep

(b) Write an equation describing this function.

$F * G = 350 \Longrightarrow \boxed{G = \frac{350}{F}}$

check: $\frac{350}{50} = 7$ ✓ $\quad \frac{350}{10} = 35$ ✓

(c) My Honda Accord's tank holds about 16 gallons. Approximate the corresponding fuel efficiency to one decimal place.

F	20	25	23	22	21	21.5	21.8	21.9
G	17.5	14	15.2	15.9	16.6	16.27	16.05	15.98
vs 16	high	low	low	low	high	high	high	low

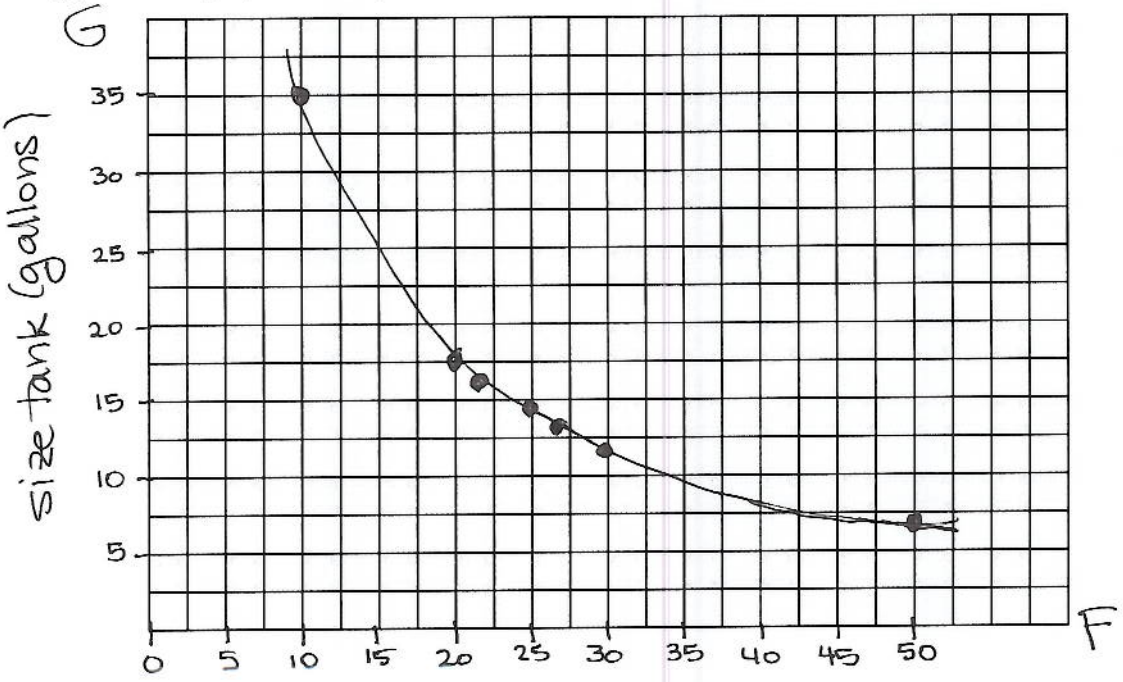
≈ 21.9 mpg

(d) My ex-husband's Honda Civic's tank holds only 13 gallons. Approximate the corresponding fuel efficiency to one decimal place.

F	25	30	27	26	26.5	26.8	26.9
G	14	11.66	12.96	13.46	13.21	13.06	13.01
vs 13	high	low	low	high	high	high	high

≈ 26.9 mpg

(e) Draw a graph showing how the size tank depends on the fuel efficiency



assorted values:

F	G
50	7
10	35
20	17.5
21.9	16
25	14
30	11.7
26.9	13

3. Monty hopes to grow orchids but they are fragile plants. He will consider his greenhouse a success if at least nine of the ten orchids survive. Assuming each orchid survives independently with probability P, the probability his greenhouse is a success, G, is given by

$$G = 10P^9 - 9P^{10}$$ *Story also appears in 2.3 #1*

(a) Monty can buy orchids guaranteed to have a probability .8 of survival each. Is that enough to give probability .8 of a successful greenhouse?

$P=.8$ $G = 10*.8^9 - 9*.8^{10}$

$= 10x.8\wedge 9 - 9x.8\wedge 10$

$\approx .3758\ldots \approx .38 < .8$ **No**

(b) What quality of orchids would Monty need to have probability .8 of a successful greenhouse? *Answer to two decimal places.*

P	.8	.9	.95	.92	.91
G	.3858	.7361	.9139	.8121	.7745
vs.8	low	low	high	high	low

$P \approx .92$

(c) What quality of orchids would Monty need to have probability .95 of a successful greenhouse? *Answer to three decimal places.*

P	.95	.98	.96	.97	.965	.964	.963
G	.9139	.9038	.9418	.9654	.9542	.9510	.9494
vs.95	low	high	low	high	high	high	low

$P \approx .963$

4. After China, India, and the United States, the next five most populous countries (in 2011) are Indonesia, Brazil, Pakistan, Nigeria, and Bangladesh. Their projected growth rates and corresponding equation are listed below. Here Q is the population measured in millions and Y is the years since 2011. Source: CIA Factbook

4th	Indonesia	pop. 248 million	growth rate 1.04%	$Q = 248 * 1.0104^Y$
5th	Brazil	pop. 205 million	growth rate 1.10%	$Q = 205 * 1.0110^Y$
6th	Pakistan	pop. 190 million	growth rate 1.55%	$Q = 190 * 1.0155^Y$
7th	Nigeria	pop. 170 million	growth rate 2.55%	$Q = 170 * 1.0255^Y$
8th	Bangladesh	pop. 161 million	growth rate 1.58%	$Q = 161 * 1.0158^Y$

(a) Which of these countries is projected to have the largest population in 2020? In 2030? In 2050?

Y = 2020−2011 = 9 year	Y = 19 2030	Y = 39 2050
Indonesia 272.2	301.9	371.3
Brazil 226.2	252.4	314.1
Pakistan 218.2	254.5	346.2
Nigeria 213.2	274.3	453.9
Bangladesh 165.4	216.9	296.7

Indonesia in 2020 and still in 2030.

Nigeria in 2050

(b) Explain why Bangladesh's population will not overtake Nigeria's, assuming these projections are accurate.

Bangladesh's population is smaller than Nigeria's and it's growing slower, so it can never overtake Nigeria's population.

(c) Approximately when will Brazil's population top 500 million? Will Nigeria get there first? Display your work in a table.

Brazil

Y	39	50	100	70	80	85	83	82	81
pop	314.1	354.2	612.2	440.9	491.9	519.5	508.3	502.7	497
vs 500	low	low	high	low	low	high	high	high	low

$Y \approx 82$

2011 + 82 = in year 2093

$Y \approx 43$

Nigeria

Yes, will get there sooner

Y	39	50	45	42	43	2011 + 43
pop	453.9	598.7	527.9	489.5	501.9	in year
vs 500	low	high	high	low	high	2054