★ SOLUTIONS ★

4.1 Modeling with linear equations – Practice exercises

1. A solar heating system costs approximately \$30,000 to install and \$150 per year to run. By comparison, a gas heating system costs approximately \$12,000 to install and \$700 per year to run. *Story also appears in 4.2 Exercises*

   (a) What is the total cost for installing and running a gas heating system for 30 years?

   \$12,000 + \$700/year × 30 years
   = 12000 + 700 × 30 = **\$33,000**

   don't forget to name the variables →

   (b) Write a linear equation showing how the total cost for a gas heating system depends on the number of years you run it.

   T = total cost heating system
   Y = time (years)
   T = 12000 + 700Y

   (c) Write a linear equation showing how the total cost for a solar heating system depends on the number of years you run it.

   T = 30,000 + 150Y

   (d) How many years of a solar heating system could you get for the cost of a gas heating system lasting 30 years (your answer to part (a))? Set up and solve an equation.

   $30,000 + 150Y = 33,000$
   $-30,000 \quad -30,000$
   $\frac{150Y}{150} = \frac{3,000}{150}$
   $Y =$ **20 years**

   [due to high installation cost 30 yrs gas = 20 yrs solar, so gas is less expensive.]

2. Since a very popular e-book reader was released in February 2009, the price has been decreasing at a constant rate. In fact, in February 2011, a blogger developed the following equation representing the price E of the e-book reader in the months M since it was released in February 2009.

$$E = 359 - 12M$$

(a) Make a table of values for the e-book reader price initially, 10 months, and 25 months since February 2009.

M=0

$359-12\times10=$

M	0	10	25
E	359	239	59

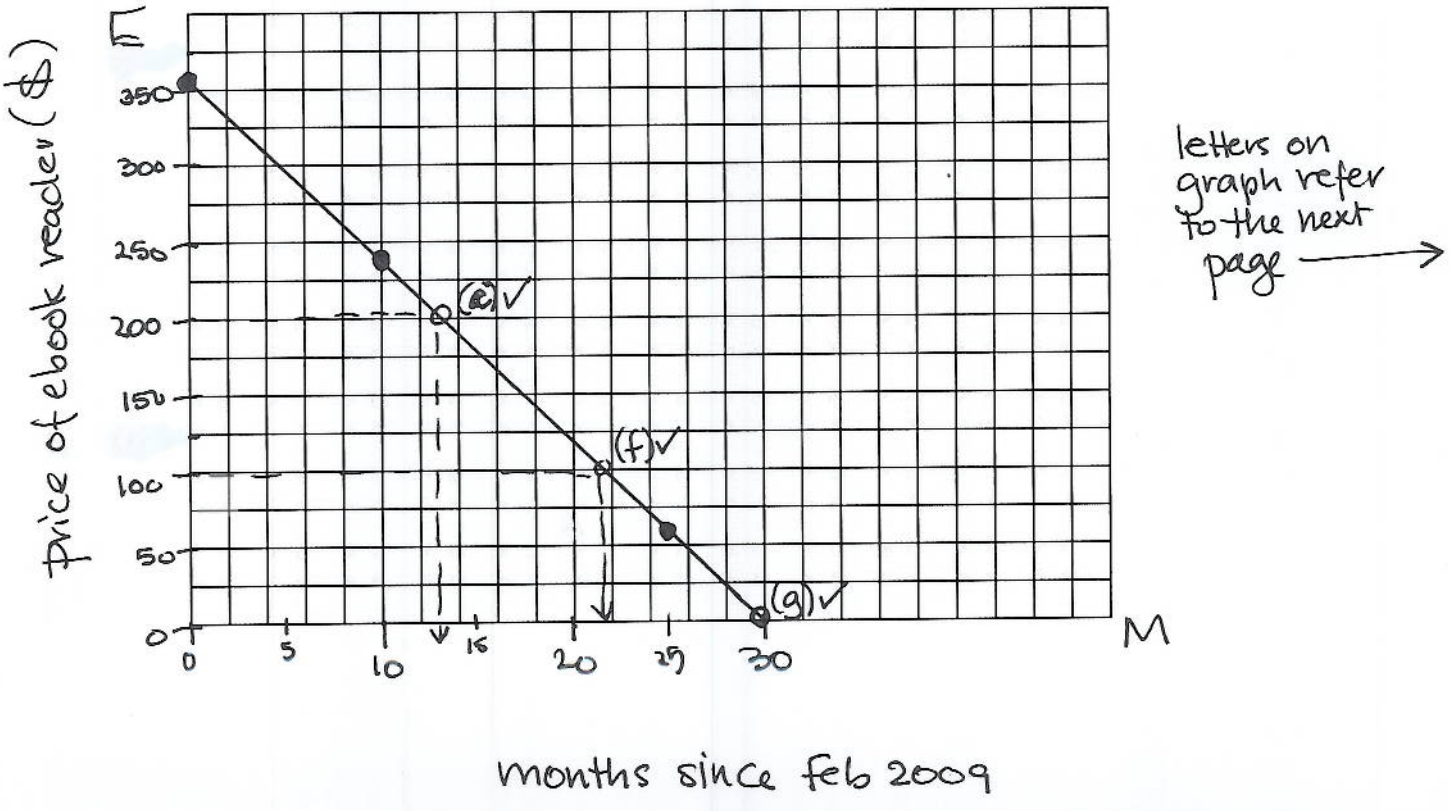
(b) What does the 359 mean in the story and what are its units?

intercept → The e-book originally cost $359. ($)

(c) What does the 12 mean in the story and what are its units?

slope → The e-book drops $12/mo in price. ($/month)

(d) Draw a graph illustrating the dependence.



letters on graph refer to the next page →

The problem continues ...

(e) Approximately how many months after February 2009 ~~is~~ was the price of the e-book reader expected to be down to $200? Set up and solve an equation.

$$359 - 12M = 200$$
$$-359 \qquad -359$$
$$\frac{-12M}{-12} = \frac{-159}{-12}$$
$$M = 13.25$$

after 13 months → March 2010

(f) Sareth decided ~~she will~~ to purchase a e-book reader ~~if~~ when the price ~~falls~~ fell below $100. When ~~will~~ did the price fall below that level? Set up and solve an inequality.

$$359 - 12M < 100$$
$$-359 \qquad -359$$
$$\frac{-12M}{-12} < \frac{-259}{-12}$$

remember: ÷ negative ⇒ switch < to >

$$M > 21.58$$

after 22 months → Dec. 2010

(g) If you can believe what you read in blogs, the manufacturer will soon be giving away the e-book reader for free, since they make money on the e-book sales themselves. When would that happen, according to our equation? Set up and solve an equation.

$E = 0$

$$359 - 12M = 0$$
$$-359 \qquad -359$$
$$\frac{-12M}{-12} = \frac{-359}{12}$$
$$M = 29.916...$$

around 30 months → August 2011.

Check answers on our graph!

3. Can you tell from the table which of these functions are linear? Use the rate of change to help you decide. Remember that these numbers may have been rounded.

(a) Savings bonds from grandpa. *Story also appears in 1.2 #1 and 5.3 #1*

Year	1962	1970	1980	1990	2000	2010
Value bond ($)	200.00	318.77	570.87	1,022.34	1,830.85	3,278.77

(2) $(570.87-318.77)\div(1980-1970)=25.217 \approx \$25.22/\text{yr}$

(1) rate of change (slope) $=\frac{\text{change dep}}{\text{change indep}}=\frac{\$318.77-\$200}{1970-1962}=\frac{\$118.77}{8\text{ years}}$

$=(\$318.77-200)\div(1970-1962)=14.84625\approx\$14.85/\text{yr}$

$\Rightarrow$ rate of change is different $\Rightarrow$ **NOT linear**

(b) Wind chill at 10°F. *Story also appears in 1.2 #2 and 2.1 Exercises*

Wind (mph)	0	10	20	30	40
Wind chill (°F)	10	-4	-9	-12	-15

(1) (2)

–OR– compare drop in °F for each 10 mph increase. Drops $-14, -13, \ldots$

(1) $\text{ROC}=\frac{-4-10}{10-0}=(-4-10)\div(10-0)=-1.4\ ^\circ\text{F/mph}$

(2) $\text{ROC}=\frac{-9-4}{20-10}=(-9-4)\div(20-10)=-1.3\ ^\circ\text{F/mph}$

$\Rightarrow$ rate of change is different $\Rightarrow$ **NOT linear**

(c) Pizza. *Story also appears in 2.4 #1 and 3.3 #1*

Size (inches)	8	14	16
People	1	3	4

(1) (2)

(1) $\text{ROC}=\frac{3-1\text{ people}}{14-8\text{ inches}}=(3-1)\div(14-8)=.33\ldots\approx.33\text{ people/inch}$

(2) $\text{ROC}=\frac{4-3\text{ people}}{16-14\text{ inches}}=(4-3)\div(16-14)=.5\text{ people/inch}\Rightarrow$ **NOT linear**

(d) Water in the reservoir. *Story also appears in 2.1 #2 and 3.2 Exercises*

Week	1	5	10	20
Depth (feet)	45.5	39.5	32	17

(1) (2) (3)

∴ guessing this one was linear because parts (a)-(c) were not

(1) $\text{ROC}=\frac{39.5-45.5\text{ feet}}{5-1\text{ years}}=(39.5-45.5)\div(5-1)=-1.5\text{ feet/year}$

(2) $\text{ROC}=\frac{32-39.5\text{ feet}}{10-5\text{ years}}=(32-39.5)\div(10-5)=-1.5\text{ feet/year}$ SAME!

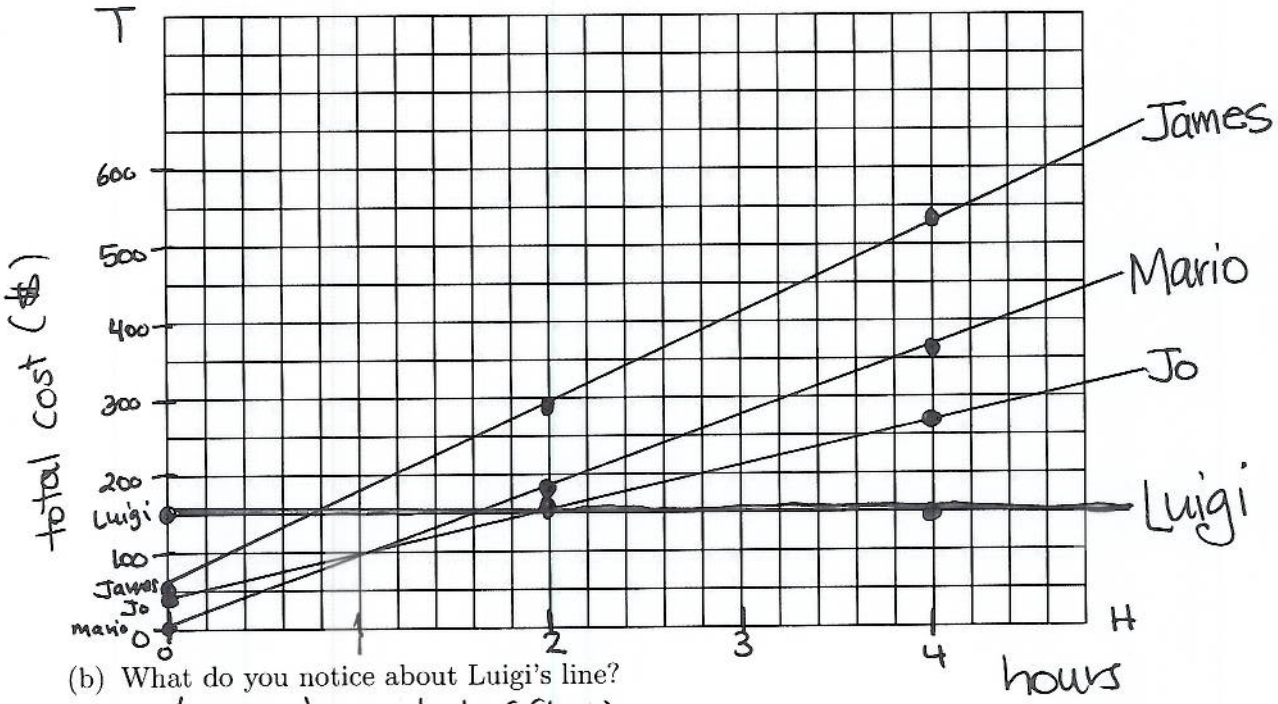
(3) $\text{ROC}=\frac{17-32\text{ feet}}{20-10\text{ years}}=(17-32)\div(20-10)=-1.5\text{ feet/year}$ SAME!

$\Rightarrow$ rate of change is constant $\Rightarrow$ **Linear**

4. Plumbers are really expensive, so I've been shopping around. James charges \$50 to show up plus \$120 per hour. Jo is just getting started in the business. She charges \$45 to show up plus \$55 per hour. Mario advertises "no trip charge" but his hourly rate is \$90 per hour. Not to be outdone, Luigi offers to unclog any drain for \$150, no matter how long it takes. For each plumber, the table lists the corresponding equation and several points. In each equation, the plumber charges \$$P$ for T hours of work. *Story also appears in 2.1 Exercises*

Plumber	James	Jo	Mario	Luigi
Equation	$P = 50 + 120T$	$P = 45 + 55T$	$P = 90T$	$P = 150$
0 hours	\$50	\$45	\$0	\$150
2 hours	\$290	\$155	\$180	\$150
4 hours	\$530	\$265	\$360	\$150

(a) Use the points given to plot each of the four lines on the same set of axes. Label each line with the plumber's name.



(b) What do you notice about Luigi's line?

It's horizontal (flat).

(c) List the plumbers in order from steepest to least steep line. What does that mean in terms of the story?

slope → James, Mario, Jo, Luigi most \$/hr → least \$/hr

(d) Now list the plumbers in order from smallest to largest intercept of their line. What does that mean in terms of the story?

intercept → Mario, Jo, James, Luigi least trip charge \$ → most trip charge \$