SOLUTIONS

2.5 Finance formulas – Practice exercises

Formulas referenced in the worksheets:

COMPOUND INTEREST FORMULA: $a = p\left(1 + \frac{r}{12}\right)^{12y}$

EQUIVALENT APR FORMULA: $\text{APR} = \left(1 + \frac{r}{12}\right)^{12} - 1$

FUTURE VALUE ANNUITY FORMULA: $a = p * \dfrac{\left(1 + \frac{r}{12}\right)^{12y} - 1}{\frac{r}{12}}$

LOAN PAYMENT FORMULA: $p = \dfrac{a * \frac{r}{12}}{1 - \left(1 + \frac{r}{12}\right)^{-12y}}$

where

a = account balance or loan amount ($)
p = initial deposit (principal), regular deposit, or regular payment ($)
y = time invested (years)
r = interest rate compounded monthly (as a decimal)

1. Use the indicated formulas to help Kiran figure out her finances.

   (a) Kiran deposited \$2,500 in a money market account that earned 7% interest compounded monthly. Use the COMPOUND INTEREST FORMULA to calculate her account balance after 4 years.

   $p = \$2500,\ r = \frac{7\%}{100} = .07,\ y = 4$ years

   $a = 2500\left(1 + \frac{.07}{12}\right)^{12\times 4}$

   $= 2500 \times (1 + .07 \div 12)\wedge(12\times 4) = 3305.1346\ldots$

   $\boxed{\approx \$3{,}305.13}$

   (b) What is the equivalent APR on Kiran's money market account? Use the EQUIVALENT APR FORMULA.

   $r = .07$

   $APR = \left(1 + \frac{.07}{12}\right)^{12} - 1 = (1 + .07 \div 12)\wedge 12 - 1 = .07229\ldots$

   $\approx \underbrace{.0723}_{\times 100\%} = \boxed{7.23\%}$

   (c) Kiran puts \$400 a month in her retirement account that amazingly also earns 7% interest compounded monthly. Use the FUTURE VALUE ANNUITY FORMULA to determine how much Kiran will have in her retirement account in 28 years.

   $p = \$400,\ r = .07,\ y = 28$ years

   $a = 400\,\dfrac{\left(1 + \frac{.07}{12}\right)^{12\times 28} - 1}{\frac{.07}{12}}$

   $= 400 \times \left((1 + .07 \div 12)\wedge(12\times 28) - 1\right) \div (.07 \div 12) = \boxed{\$415{,}475.29}$

   (d) Kiran would really like to buy a new hybrid car that sells for \$23,500. Sadly Kiran's credit rating is not very good, so the best the dealership offers is a loan at (you guessed it) 7% interest compouned monthly. Use the LOAN PAYMENT FORMULA to calculate her monthly car payments on a <u>six</u> year loan.

   $a = \$23{,}500,\ r = .07,\ y = 6$ years

   $p = \dfrac{23{,}500 \times \frac{.07}{12}}{1 - \left(1 + \frac{.07}{12}\right)^{-12\times 6}}$

   $= 23{,}500 \times .07 \div 12 \div \left(1 - (1 + .07 \div 12)\wedge((-)12\times 6)\right) =$

   $\boxed{\approx \$400.65}$

2. Tim and Josh are saving for their kids' college in fifteen years. The account pays the equivalent of 5.4% interest compounded monthly (taking into consideration various tax incentives).

   (a) Make a table showing how much they will have after fifteen years if every month they contribute \$100 vs. \$500 vs. \$1,000. Use the FUTURE VALUE ANNUITY FORMULA.

$$a = p * \frac{\left(1+\frac{.054}{12}\right)^{12\times 15} - 1}{\frac{.054}{12}}$$

$= p \times ((1+.054 \div 12)$
$\wedge (12\times 15)-1)$
$\div (.054 \div 12) =$

$r = \frac{5.4\%}{100\%} = .054$, $y = 15$ years

p	100	500	1000
a	\$27,640	\$138,203	\$276,406

   (b) Tim's parents decide to put \$15,000 into the account right now. How much will that add to the value in fifteen years? Use the COMPOUND INTEREST FORMULA.

$r = .054$, $y = 15$ years, $p = \$15{,}000$

$$a = 15{,}000\left(1+\frac{.054}{12}\right)^{12\times 15}$$
$= 15{,}000 \times (1+.054 \div 12) \wedge (12\times 15) =$

$\boxed{\approx \$33{,}657.41}$

3. Use the EQUIVALENT APR FORMULA to find the APR for each of the following published interest rates (compounded monthly) offered by recent credit card companies.

   (a) 9% $r = \frac{9\%}{100\%} = .09$ $\quad APR = \left(1+\frac{.09}{12}\right)^{12} - 1 = (1+.09 \div 12) \wedge 12 - 1 =$
   $= .093806\ldots \approx .0938 \xrightarrow{\times 100\%} = \boxed{9.38\%}$

   (b) 12.8% $r = \frac{12.8\%}{100\%} = .128$ $\quad APR = \left(1+\frac{.128}{12}\right)^{12} - 1 = (1+.128 \div 12) \wedge 12 - 1 =$
   $= .135782\ldots \approx .1358 \xrightarrow{\times 100\%} = \boxed{13.58\%}$

   (c) 20.19% $r = \frac{20.19\%}{100\%} = .2019$ $\quad APR = \left(1+\frac{.2019}{12}\right)^{12} - 1 = (1+.2019 \div 12) \wedge 12 - 1 =$
   $= .221671\ldots \approx .2217 \xrightarrow{\times 100\%} = \boxed{22.17\%}$

4. Cesar and Eliana are looking at three different houses to buy. The first, a large new townhouse, for \$240,000. The second, a small but charming bungalow, for \$260,000. The third, a large 2-story house down the block, for \$280,000.

   (a) Calculate the monthly payment for each house for a 30-year mortgage at 3.5% interest compounded monthly. Use the LOAN PAYMENT FORMULA.

$y = 30$ years

$r = \frac{3.5\%}{100\%} = .035$

Townhouse $a = \$240{,}000$

$$p = \frac{240{,}000 * \frac{.035}{12}}{1-\left(1+\frac{.035}{12}\right)^{-12\times 30}}$$

$= 240{,}000 \times .035 \div 12 \div (1-(1+.035\div 12)\wedge((-)12\times 30)) =$

$= 1{,}077.70725\ \boxed{\approx \$1{,}077.71}$

Bungalow $a = \$260{,}000$

$$p = \frac{260{,}000 * \frac{.035}{12}}{1-\left(1+\frac{.035}{12}\right)^{-12\times 30}}$$

$= 260{,}000 \times .035 \div 12 \div (1-(1+.035\div 12)\wedge((-)12\times 30)) =$

$= 1{,}167.5161\ldots\ \boxed{\approx \$1{,}167.52}$

2-Story $a = \$280{,}000$

$$p = \frac{280{,}000 * \frac{.035}{12}}{1-\left(1+\frac{.035}{12}\right)^{-12\times 30}}$$

$= 280{,}000 \times .035 \div 12 \div (1-(1+.035\div 12)\wedge((-)12\times 30)) =$

$= 1{,}257.3251\ldots\ \boxed{\approx \$1{,}257.33}$

   (b) Describe the effect on Cesar and Eliana's monthly payment of each \$20,000 increase in the house price at this interest rate.

$$\begin{array}{r} \$1{,}167.52 \\ -\ \$1{,}077.71 \\ \hline \$\ 89.81 \end{array}$$

$$\begin{array}{r} \$1{,}257.33 \\ -\ \$1{,}167.52 \\ \hline \$\ 89.81 \end{array}$$

$\approx \$90$/mo.

Each \$20,000 increase in price adds about \$90/month to their mortgage payment