Applied Algebra

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Chapter 1

Variables

Believe it or not, algebra is useful. Really useful. It's useful in later courses you might take in mathematics, statistics, science, social science, or business. But it's also useful in real life. A lot of what happens in the world around us is easier to understand using algebra. That's what this course is all about: using algebra to answer questions.

In this first chapter, we will introduce the key concepts of variable and function that help us translate between problems stated in words and the mathematics explaining the situation. We explain the important tools of units, tables, and graphs. We also describe how functions change and use the rate-of-change to approximate answers to questions. Throughout this chapter we keep a careful eye on evaluating the reasonableness of answers by connecting what we learn from algebra with our own life experience. After all, an answer to a real problem should make sense, right?

Some of our approach may feel very different to you. It is possibly quite different from what you've seen in mathematics classes before. It might take you a little time to get used to, but it will be worth it.

1.1 Variables and functions

Things change; like the price of gasoline, and just about every day it seems. What does it mean when the price of a gallon of gas drops from \$3.999/gal to \$3.699/gal? On one level that means it costs

$$\$3.999 - \$3.699 = \$.30 = 30¢$$

less for one gallon. Does this 30¢ truly matter?

Before we answer that question, are you wondering why there's that extra 9 at the end of the price? We might say a gallon costs \$3.99 but there's really a small 9 following it. Sometimes that 9 is raised up slightly on the gas station sign. You have to read the fine print. What it means is an extra $\frac{9}{10}$¢ for each gallon. So the true price of gas would be \$3.999. So gas costs a tiny bit more than you thought. Good grief.

For the record, the / symbol is short for "per". So when we write \$3.999/gal we mean \$3.999 per gallon. Sometimes we write this as a fraction instead: $\frac{\$3.999}{\text{gallon}}$.

Back to our question. Does 30¢ truly matter to us? Probably not. But, how often do you buy just one gallon of gas? Typically you might put five, or ten, or even twenty gallons of gas into the tank. We want to understand how the price of gasoline influences what it really costs us at the pump. To do that let's compare our costs when we buy ten gallons of gas. (There's no good reason for picking ten except that it's a nice number to work with.)

If gas costs \$3.999/gal and we buy 10 gallons, it costs

$$\frac{\$3.999}{\text{gallon}} * 10 \text{ gallons} = \$39.99.$$

In this text we use the $*$ symbol for multiplication because the standard $\times$ symbol looks too much like an x. Your calculator might use $*$ or $\times$ instead.

If gas drops to \$3.699/gal and we buy 10 gallons, it costs

$$\frac{\$3.699}{\text{gallon}} * 10 \text{ gals} = \$36.99$$

instead. That's \$3 less. That amount matters. I mean, for \$3 savings on gas you could buy something else. Especially when you take into account that it's \$3 savings every time you put 10 gallons in the tank.

Gas prices have been changing wildly, and along with them, the price of 10 gallons of gas. In mathematics things that change are called *variables*. The two variables we're focusing on in this story are

$$P = \text{the price of gasoline (\$/gal)}$$
$$C = \text{the total cost (\$)}$$

Notice that we gave each variable a letter name. It is helpful to just use a single letter chosen from the word it stands for. In our example, P stands for "price" and C stands for "cost". In this course we rarely use the letter x simply because so few words begin with x. Whenever we name a variable (P) we also describe in words what it represents (the price of gasoline), and we state what units it's measured in (\$/gal).

In talking about the relationship between these variables we might say "The cost depends on the price of gas." (C depends on P). That tells us that C is the *dependent variable* and P is the *independent variable*. In general, the variable we really care about is the dependent variable, in this case C the total amount of money it costs us. In some situations dependency can be viewed either way; there might not be one correct way to do it.

Given a choice, we usually assign dependence such that given a value of the independent variable, it is easy to calculate the corresponding value for the dependent variable. In our example it's easy to use the price per gallon, P, to figure out the total cost, C.

We can work backwards – from C to P – but it's not as easy. For example, suppose we buy 10 gallons of gas and it costs \$28.99. We can figure out that the price per gallon must be

$$P = \frac{\$28.99}{10 \text{ gallons}} = 28.99 \div 10 = \$2.899/\text{ gal}$$

Notice that in this example the fraction is shorthand for division.

There's another important way to express this dependence. We say "cost is a function of price" or that C is a *function* of P.

There's one more thing to think about when describing the values. From our experience we have a sense of what gas might cost. In my lifetime, I've seen gas prices as low as 35.9¢ /gallon in the 1960s to a high of \$4.099/gallon in 2008. This range of values sounds too specific, so it would sound better to say something more general like

"Gas prices are between \$0/gal and \$5/gal."

The mathematical shorthand for this sentence is

$$0 \leq P \leq 5$$

The inequality symbol $\leq$ is pronounced "less than or equal to". Formally, the range of realistic values of the independent variable is called the *domain* of P.

Similarly, in mathematics the things that do not change (at least not during the story) are called *constants*. The one variable in this story is that we're always buying 10 gallons of gas.

We can summarize all of this information as follows.

Constant: 10 gallons

Variables: P = the price of gasoline (\$/gal), indep, $0 \leq P \leq 5$

C = the total cost (\$), dep

Let's look at one more situation. The average price of gasoline in Minnesota in 2010 was \$2.900/gal. One year earlier, in 2009, it was \$2.149/gal. We might ask: by what percentage did the price increase this past year and, if it continued to increase at that rate each year, what would the price be in 2011 and 2012?

The price rose $\$2.900 - \$2.149 = \$.751$ per gallon. *Percent increase* is the proportion of the starting amount that was added, expressed as a percent. That is,

$$\text{percent increase} = \tfrac{\text{change in amount}}{\text{starting amount}}$$

In our situation we have

$$\frac{\text{change in amount}}{\text{starting amount}} = \frac{\$.751/\text{gal}}{\$2.149/\text{gal}} = .751 \div 2.149 = .34946487\ldots \approx .3494$$

Notice that we rounded off our answer. When we want the person reading our calculation to know that we mean approximately .3494, not exactly, we use the *approximately equal to* symbol $\approx$. We save the equal sign, $=$, for when we have not rounded off the number at all.

To find the percentage from the proportion (decimal), we multiply by 100%.

$$.3494 * \frac{100\%}{1} = .3494 * 100 = 34.94\%$$

One way to remember this conversion is that proportions and percentages work just like dollars and cents:

$$.3494 = 34.94\%$$

is just like

$$\$.3494 = 34.94¢$$

So the price of gas increased by approximately 34.94% last year.

If someone asks you would probably round off the answer even more and say that gas prices increased "about 35%". We are keeping more decimal places because we have more calculations to do.

At this rate, what would the price be the next year, in 2011? To answer that question we know the price in 2010 wss \$2.900/gal. In 2011, it would be 34.94% more, which means it would increase by

$$34.94\% \text{ of } \$2.900/\text{gal} = .3494 * 2.900 = 1.01326 \approx \$1.013/\text{gal}$$

The price in 2011 would therefore be

$$2.900 + 1.013 \approx \$3.913/\text{gal}$$

In case you're curious, there is a quicker way to calculate this answer:

$$2.900 * 1.3494 = 3.91326 \approx \$3.913/\text{gal}$$

Continuing as above, the following year, in 2012, it would be 34.94% more, which means 34.94% more than what it was in 2011. This time we add on 34.94% of the \$3.913/gal.

$$34.94\% \text{ of } \$3.913 = .3494 * 3.913 = 1.3672\ldots \approx \$1.367/\text{gal}$$

The total price in 2012 would therefore be

$$3.913 + 1.367 = \$5.280/\text{gal}$$

As before, we can calculate this answer in one step as:

$$3.913 * 1.3494 \approx \$5.280/\text{gal}$$

A lot of realistic problems involve percentages and so we'll use them often in this course.

There are two different types of exercises in this textbook. For the "Practice exercises," we have left space for you to write the solutions in the textbook. Think of them as additional examples. We provide full solutions to the Practice Exercises at the end of the textbook. For the regular "Exercises" you will need to write your solutions separately. We provide only the answers, not full solutions, to the Exercises at the end of the textbook.

Practice exercises

1. For each story, identify the variables and constants (if any). Include the units, realistic domain, and dependence.

   (a) The cost of holding a wedding reception at the Metropolitan Club is \$1,000 down and \$75 per person.

   (b) If I drive 60 mph (miles per hour), it takes me 20 minutes on the highway to get between exits, but when traffic is really bad it can take me an hour.

   (c) The sun set at 6:00 p.m. today and I heard on the radio that it sets about 2 minutes later each day this time of year.

   (d) Rent in the Riverside Neighborhood has increased 4.5% each year. Now rent in an apartment complex was \$600 per month.

2. Every morning Jill goes for a 45-minute walk.

   (a) Identify the variables and constants in this dependence. Include the units, realistic domain and range, and dependence.

   (b) If Jill walked 2.5 miles, how fast was she walking?

3. The employee-paid cost of health insurance has risen dramatically, increasing by 7% each year since 2003 when it cost $420/month.

   (a) Identify the variables and constants in this dependence. Include the units, realistic domain and range, and dependence.

   (b) If this rate of increase continues, when will or did the employee-paid cost pass $550/month?

4. The Roman Inn charges $19.95 for a "New York" pizza that's 16" diameter and $5.95 for a "personal" pizza that's 6" in diameter.

   (a) Identify the variables and constants in this dependence. Include the units, realistic domain and range, and dependence.

   (b) Approximately how much would you guess the Roman Inn charges for a "regular" pizza that's 12" in diameter?

Do you know ...

- What's the difference between a variable and a constant?
- How variables are named and their units specified?
- What we mean by function or dependence?
- How to distinguish the dependent from the independent variable?
- What's the (realistic) domain for a function?
- How to describe a range of values using an inequality?
- What notations are used for equal values and for and approximate values?
- How to calculate a percent increase?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. For each story, figure out an exact answer to the question if you can. If you can't find an exact answer, make a reasonable guess instead.

   (a) The cost of holding a wedding reception at the Metropolitan Club is \$1,000 down and \$75 per person. If the bill was for \$11,875, how many people were at the wedding dinner?

   (b) If I drive 60 mph (miles per hour), it takes me 20 minutes on the highway to get between exits, but when traffic is really bad it can take me an hour. How slow am I driving when traffic is really bad? *Hint: can you figure out the distance between exits?*

   (c) The sun set at 6:00 p.m. today and I heard on the radio that it sets about 2 minutes later each day this time of year. In how many days will the sun set at 7:30 p.m.?

   (d) Rent in the Riverside Neighborhood has increased 4.5% each year. Now rent in an apartment complex was \$600 per month. What will rent be in three years? *Hint: 4.5% = .045, don't forget the 0.*

6. The temperature was 40 degrees at noon yesterday but it dropped 3 degrees an hour in the afternoon.

   (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

   (b) When did the temperature drop below freezing (32 degrees)?

7. In 1990 the Lefèvre's property tax was \$450 but it doubled every year thereafter.

(a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

(b) What was the tax up to by 1994? *Note: the city did provide tax credit to offset this rapid increase.*

8. A phone call on Kyle's phonecard costs \$.48 for the first minute (connection fee) and \$.02/minute thereafter.

   (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

   (b) If Kyle used \$1.82 for a phone call, how long was the phone call?

9. A twenty pound bag of dog food costs \$12.49, but a five pound bag costs \$3.79.

   (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

   (b) What does a ten pound bag probably cost?

10. Social Security benefits have increased by 3% per year. In 1995 my grandmother's benefit was \$246.17/month.

    (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

    (b) When did her benefit pass \$300/month?

11. The bookstore charges 85¢ for a pack of gum.

    (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

    (b) How many packs can Kawena buy for \$3.00?

12. Gilberto's car was worth \$22,500 when he bought it new. Now it's ten years old and worth only \$7,500.

    (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

    (b) When will his car be practical worthless (under \$500)?

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.

- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

1.2 Tables and graphs

Lung cancer, chronic bronchitis, bad breath, stains on your clothes, and the expense. These are just a few of the consequences of smoking cigarettes. With what we know now about the dangers of smoking, are there more or fewer people smoking than there were ten years ago, fifty years ago, or even one hundred years ago?

A good way to look at these numbers is to compare the number of cigarettes smoked per person per year. How would we calculate that? Well, a person who doesn't smoke at all would count as 0 cigarettes per year. A person who smokes "a pack a day" would count as 7, 300 cigarettes smoked per year because

$$\frac{1 \text{ pack}}{\text{day}} * \frac{20 \text{ cigarettes}}{\text{pack}} * \frac{365 \text{ days}}{\text{year}} = \frac{7,300 \text{ cigarettes}}{\text{year}}$$

If we add up all the cigarettes smoked in one year and divide by the total number of people, we get the number of cigarettes per person per year. Get it?

Here's some information from the Center for Disease Control (http://www.cdc.gov) for the United States. There are more data on their web site, but this list of partial data is good enough for us to get started.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Cig/person/yr	54	128	649	1,452	2,374	3,247	3,908	4,192	3,933	2,967	1,128

What's changing are the number of cigarettes smoked per person per year and time. So, those are our variables. The smoking rate is a function of year, and it's what we care about, so it's the dependent variable. Time, as measured in years, is the independent variable. As before, it is convenient to measure time in years since 1900.

$$C = \text{smoking rate (cigarettes per person per year), dep}$$
$$Y = \text{time (years since 1900), indep}$$

Officially we should rewrite our table as:

Y	0	10	20	30	40	50	60	70	80	90	100
C	54	128	649	1,452	2,374	3,247	3,908	4,192	3,933	2,967	1,128

Notice that the variables are listed in the table, with the independent listed first (on top) and the dependent listed second (on bottom). There are two standard formats for tables used.

Horizontal table format:

independent variable				
dependent variable				

Vertical table format:

independent variable	dependent variable

In a table, the placement of the independent variable first (either top or left) and the dependent variable second (either bottom or right) is a mathematical convention. That's not when a bunch of math folks get together but a *mathematical convention* is a custom, practice, or standard used within the mathematical community. Though based on reason, it often involves some arbitrary choice, making it impossible to figure out. So, whenever some practice is introduced to you as a "convention", you will need to memorize it.

The years are listed from 1900 to 2000, but we might be interested in future projections perhaps through the year 2050. Remember that we are measuring time in years since 1900. Thus in 1900 we had $Y = 0$, in 2000 we had $Y = 100$, and to get to 2050 we'll need $Y = 150$. Our domain is $0 \le Y \le 150$.

The number of cigarettes smoked per person per year was never 0 in the years but we'll begin at 0 anyway. The largest number listed in the table is 4,192 but it might have been higher at some point so to be safe we'll say $0 \le C \le 4,500$.

$$C = \text{smoking rate (cigarettes per person per year), dep, } 0 \le C \le 4,500$$
$$Y = \text{time (years since 1900), indep, } 0 \le Y \le 150$$

We can tell a lot of information from this table. For example, how many cigarettes per person per year were smoked in 1980? The answer appears in the table, a whopping 3,933 cigarettes per person that year.

When did the consumption first pass 3,000? That answer does not appear in the table, but we can use the information in the table to make a good guess. In 1940 ($Y = 40$), there were an average of 2,374 cigarettes per person per year and by 1950 ($Y = 50$) there were 3,247. Somewhere between 1940 and 1950 the number first climbed above 3,000. More specifically, the number we're looking for (3,000) is closer to the 1950 figure (3,247) than to the 1940 figure (2,374). So, it would be reasonable to guess close to 1950. I'd say 1947. Of course, you might guess 1946 or 1948, or even 1949 and those would be good guesses too. Interestingly, the full table of data from the CDC shows that consumption first topped 3,000 as early as 1944. Here's an example where the history tells you more than the mathematics as cigarette consumption rose sharply during World War II.

When did the consumption drop below 3,000 again? This answer also does not appear in the table, but falls somewhere between 1980 ($Y = 80$), when consumption was 3,933, and 1990 ($Y = 90$), when consumption was 2,967. Here I'd guess just before 1990, say in 1989. This time that guess does agree with the full table of data from the CDC.

Tables are useful because they contain specific numbers, but it can be difficult to guess or see general trends. For that, a picture is worth a thousand words (or numbers, in this case).

SU: insert plotted points here (no connect the dots yet). Y-horiz count by 10s to 150 and C-vert count by 500s to 4500.

To draw this graph, we began by labeling the axes, with the independent variable (year Y) on the horizontal axis and the dependent variable (consumption C) on the vertical axis. That's another mathematical convention you'll need to learn.

SU: insert diagram like from old text about dep/indep variables (top old 1.3)

In any graph, we mark the axes in an even scale. That means each box or grid mark counts for the same amount. The domain for the cigarette function is $0 \leq Y \leq 150$ and so we needed to chose a scale that would fit all those values. Counting by 10s worked just fine. The range is $0 \leq C \leq 4,500$. Now counting by 10s would not have worked. Even to count by 100s we would have needed 45 grid marks, which wouldn't fit on the page either. Here we decided to count by 500s. In general, I like to count by round numbers (2s, 5s, 10s, etc.) because it makes guessing in between easier. Even if it would fit nicely to count by something like 300s, I'd rather use 200s or 500s instead. Similarly, you'll not see me count by 7s when 5s or 10s will work more easily. Officially the domain, range, and corresponding labels for the axes can start at any number, but I try to always start at 0. For now, I encourage you to do the same.

To plot each point we moved right to that Y-value, and then moved up to that C-value. The way the scale worked, the Y-values landed exactly on grid marks, but for the C-values we had to guess where to place the point. For example, in 1940 we had $Y = 40$ and $C = 2,374$. That C-value likes between 2,000 and 2,500, just above the un-marked half-way point of 2,250. In 1930 we had $Y = 30$ and $C = 1452$. That C-value lies between the grid lines for 1000 and 1500, but is almost equal to 1,500. We draw the point just a tiny bit below the gridline for 1,500. The point basically lies on that line.

To turn this plot of points into a graph, we need to draw in the curve showing the overall tendency. We can start by "connecting the dots" – drawing a line between each pair of points. But that isn't exactly right. As we can see, the values go up, level off, and then drop back down. It was probably more of a continuous trend and so the graph should be smoother. We can round off the corners to get a graph like this instead:

SU: graph with dots smoothly connected, but not exceeding the max.

When we draw in this smooth curve for the graph, what we are really doing is making a whole lot of guesses all at once. For example, earlier we guessed that consumption first passed 3,000 in around 1947. What does the graph show? If we look where $Y = 47$ the point is about at height $C = XX$.

SU: same graph, but now draw in line up from 47, then back left to the axis to show C value.

Can you tell from the graph what period of time $C > 3,400$? It looks like $x \leq Y \leq x$, or from years 19xx to 19xx, approximately.

SU: same graph, but now draw a horizontal line across at 3,400, then drop two verticals down to show the Y values.

Don't forget that when we drew in that curve it was really just a guess. We're sure about the points we plotted, but we're only guessing about where to draw the curve in. That means we're not sure about the other points. If we knew a lot more points we could have a more accurate graph. As I mentioned before there really is more data available from the CDC. Look at how the graph from the full data compares to our guess. In particular, notice the sharp increase during World War II (1939-1945) that we mentioned earlier.

SU draw same graph, but now superimpose the graph of the full CDC data.

For now I encourage you to draw graphs by hand, ideally on graph paper where it is easy to see the gridlines. At some point in time it would be good to learn how to use some technology to draw graphs too. Graphing calculators, spreadsheet programs (like Excel$^{\mathrm{TM}}$), or computer algebra systems (like Maple$^{\mathrm{TM}}$ or Mathematica$^{\mathrm{TM}}$) all draw graphs well. All the graphs in this text were done using Excel. But there's a reason I suggest you draw graphs by hand for now, and possibly for this whole course, even if you already know how to use the technology: you'll really understand graphs better.

There are four key steps in drawing a graph (by hand or by technology):

1. **Label** the axes.
2. **Scale** the axes.
3. **Plot** the points.
4. **Connect** the points smoothly.

SU: say something like this somewhere: Again, we could have done this calculation all at once as

$$1,000 + 75 * 200 = 16,000.$$

AND this too: but the units for the P was just "people". That may seem a little odd, but whenever you have a variable that counts something, like the number of people, the unit is what it counts.

Practice exercises

1. In 1962 my grandmother opened a savings account for me and put in \$2,000. Since then no-one has put in any money nor taken any money out, but the account earns interest each year. The table lists the account balance at various times since then. SU ADD 2010

year	1962	1970	1980	1990	2000
Y	0	8	18	28	38
B	\$2,000	\$3,187	\$5,709	\$10,223	\$18,308

(a) What are the variables in this story, denoted by Y and B in the table? Include the units, realistic domain, and dependence.

Use the table to answer the following questions:

(b) What was my account balance in 1970?

(c) When was my account balance \$10,223?

(d) Approximately when did my account balance equal \$7,000?

(e) What do you expect the account balance was in 2008? SU FIX

(f) When do you expect the account balance to be \$25,000?

(g) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points.

(h) Use the graph to check your answers to the questions. Modify your answers if necessary.

2. How cold is it? In cold climates we know that it's not just the temperature, but also the wind, that governs how cold it feels outside. For example, at an air temperature of 10°F with no wind it feels like an honest 10°F. Cold but manageable. But add a 30 miles per hour wind and brrr, it feels like it's -12°F (12° below 0°F). At that temperature you can get frostbite in under an hour of exposure. We say the "wind chill" of 10°F with a 30 mph wind is -12°F. The National Weather Service lists the wind chill at 10°F air temperature for various wind speeds (from http://www.weather.gov/om/windchill/)

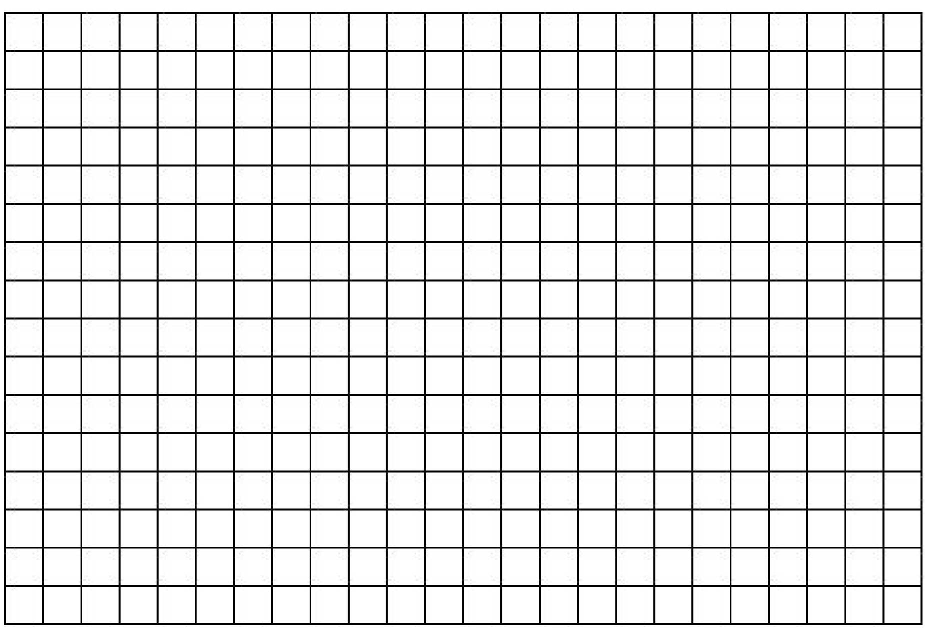
Wind (mph)	0	5	10	15	20	25	30	35	40	45	50	55	60
Wind chill (°F)	10	1	-4	-7	-9	-11	-12	-14	-15	-16	-17	-18	-19

(a) At an air temperature of 15°F, what's the wind chill when the wind is blowing 20 mph?

(b) A "cold advisory" is issued whenever the wind chill falls below 0°F. How fast does the wind need to be to issue a cold advisory?

(c) Between a wind chill of 0°F and -15°F, schools in our district are open but kids can't go outside for recess. What's the corresponding range of wind speeds?

(d) Draw a graph illustrating the dependence and use it to check your answers.

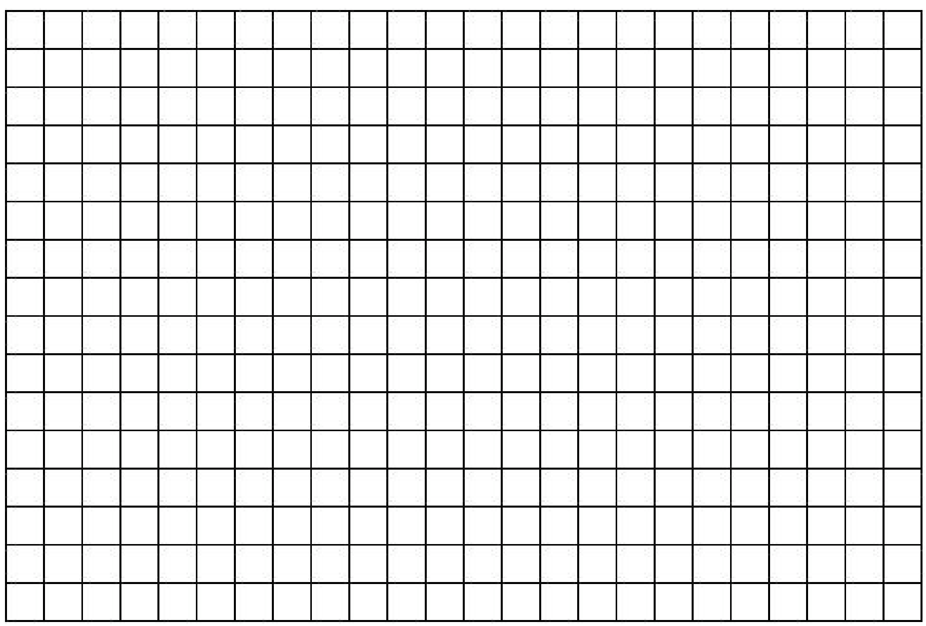


3. The cost of holding a wedding reception at the Metropolitan Club is \$1,000 down and \$75 per person.

   (a) Identify the variables and constants in this dependence. Include the units, realistic domain, and dependence.

   (b) Make a table of showing the cost for 20, 50, 75, 100, or 150 people.

   (c) Draw a detailed graph illustrating the dependence.

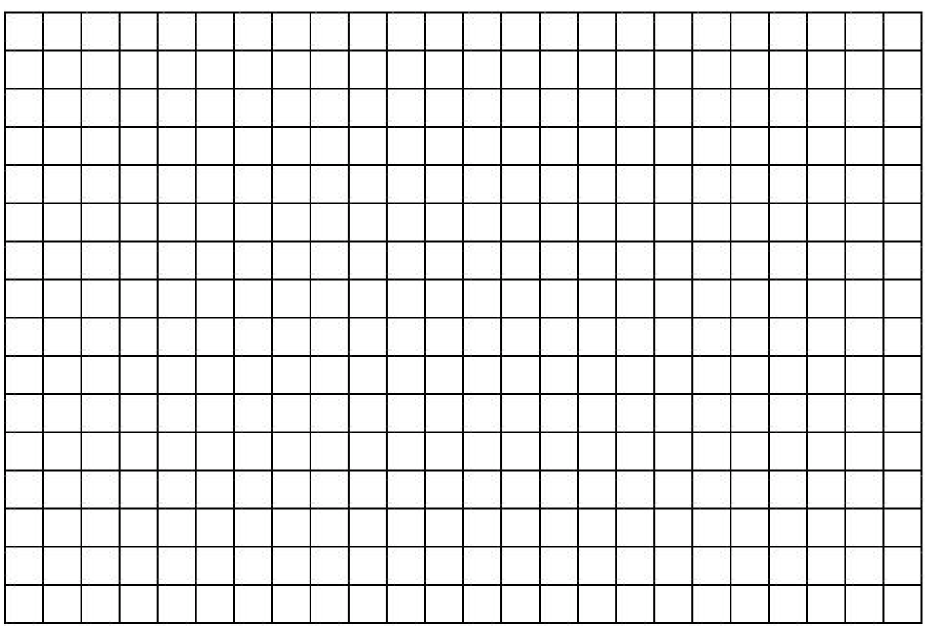


   (d) If our budget is \$10,000, how many people can we have at the reception? First estimate the answer from your table. Then confirm, or adjust, your answer based on your graph. Can you figure it out exactly from the story?

4. My neighborhood theater offers a "movie pass" for \$24.00. With it, tickets cost \$4.50 each all year long. Without the movie pass, tickets cost \$8.00 each.

   (a) If you're going to buy 5 movie tickets in a year, would you buy the pass?

   (b) How many tickets would you need to buy in a year to make the pass worth it? Make a table and graph to support your reasoning.



Do you know . . .

- Where the independent and dependent variables appear in a table and in a graph?
- How to guess values from a table or from a graph?
- How to make a graph from a table?
- What we mean by scaling an axis evenly?
- How to make a table and then a graph from a story?
- Why we draw in a smooth curve connecting the points?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. The table adapted from http://www.crh.noaa.gov shows the "heat index" as a function of humidity at an air temperature of 88°F. With up to about 40% humidity, 88°F feels like it's 88°F. But if the humidity rises to 60%, then it feels like it is 95°F; that is, the heat index is 95°F.

humidity (%)	50	60	70	85	90	95
heat index (°F)	91	95	100	110	113	117

All of the following questions refer to situations when the air temperature is 88°F.

(a) What is the heat index when the humidity level is 70%?

(b) At what humidity level does 88°F feel more like 98°F?

(c) Heat exhaustion is likely to occur when the heat index reaches 105°F. At what humidity level will heat exhaustion likely occur?

(d) The heat index is considered danger in the range from 105°F to 129°F. What range of humidity levels are considered dangerous?

(e) What do you think the heat index would be at 99% humidity?

(f) Identify the variables, including units, realistic domain, and dependence.

(g) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points.

(h) Use your graph to check your answers to the questions. Modify your answers if necessary.

6. Your local truck rental agency lists what it costs to rent a truck (for one day) based on the number of miles you drive the truck.

distance driven (miles)	50	100	150	200
rental cost ($)	37.50	55.00	72.50	90.00

Use the table to find or reasonably guess the answers to the following questions.

(a) How much does it cost to rent a truck if you drive it 100 miles?

(b) How many miles did you drive a truck costing $90.00 to rent?

(c) If you rent a truck and drive it 75 miles, how much do you think it will cost?

(d) If you rent a truck and drive it 10 miles, how much do you think it will cost?

(e) If you rent a truck and it costs $60.00, about how many miles was it driven?

(f) Identify the variables, including units, realistic domain, and dependence.

(g) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points.

(h) Use your graph to check your answers to the questions. Modify your answers if necessary.

7. The table lists the United Nations' estimates of population of the Earth, in billions, for select years since 1800. (from www.un.org/) *Their report was released in 1999 to coincide with the world population crossing the 6 billion mark.*

year	1800	1850	1900	1950	1970	1990	2000
population	0.98	1.26	1.65	2.52	3.70	5.270	6.06

Use the table to find or reasonably guess the answers to the following questions.

(a) What was the population of Earth in 1850?

(b) What do you think the population of Earth was in 1860?

(c) What do you think the population of Earth was in 1960?

(d) In what year do you think the population of Earth first exceeded 2 billion?

(e) In what year do you think the population of the world will exceed 7 billion?

(f) Identify the variables, including units, realistic domain, and dependence.

(g) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points.

(h) Use your graph to check your answers to the questions. Modify your answers if necessary.

8. The temperature was 40 degrees at noon yesterday but it dropped 3 degrees an hour in the afternoon.

(a) Make a table of reasonable values.

(b) Draw a graph illustrating the dependence

(c) According to your table and graph, when did the temperature drop below freezing (32 degrees)?

(d) Compare your answer to your estimate in Exercise 1.1.3

9. In 1990 the Lefèvre's property tax was \$450 but it doubled every year thereafter.

   (a) Make a table of reasonable values.

   (b) Draw a graph illustrating the dependence.

   (c) According to your table and graph, what was the tax up to by 1994?

   (d) Compare your answer to Exercise 1.1.4

10. A phone call on Kyle's phonecard costs \$.48 for the first minute (connection fee) and \$.02/minute thereafter.

    (a) Make a table of reasonable values.

    (b) Draw a graph illustrating the dependence.

    (c) According to your table and graph, if Kyle used \$1.82 for a phone call, how long was the phone call?

    (d) Compare your answer to Exercise 1.1.5

11. Social Security benefits have increased by 3% per year. In 1995 my grandmother's benefit was \$246.17/month.

    (a) Make a table of reasonable values.

    (b) Draw a graph illustrating the dependence.

    (c) According to your table and graph, when did her benefit pass \$300/month?

    (d) Compare your answer to Exercise 1.1.7

12. The bookstore charges 85¢ for a pack of gum.

    (a) Make a table of reasonable values.

    (b) Draw a graph illustrating the dependence.

    (c) According to your table and graph, how many packs can Kawena buy for \$3.00?

    (d) Compare your answer to Exercise 1.1.8

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.

- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

1.3 Rate of change (and interpolation)

A diver bounces on a 3-meter springboard. Up she goes. A summersault, a twist, then whoosh, into the water. The table shows the diver's height, measured as H meters above the water, as a function of time, T seconds.

T	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
H	3.00	3.88	4.38	4.48	4.20	3.52	2.45	1.00

In case you're wondering, 3 meters is nearly 10 feet up and the highest height listed, 4.48 meters, is close to 15 feet above the water. More on how we figured those numbers out in the next section, but thought you might like to know.

How fast is she moving? During the first 0.2 seconds, her height changes from 3.00 meters to 3.88 meters. She rose $3.88 - 3.00 = 0.88$ meters in that 0.2 seconds. Measured in meters per second, her speed is

$$\frac{0.88 \text{ meters}}{0.2 \text{ seconds}} = 4.4 \text{ meters/sec.}$$

Her speed is called the *rate of change*, and is calculated as

$$\text{Rate of change} = \frac{\text{change in height}}{\text{change in time}}$$

$$= \frac{3.88 - 3.00 \text{ meters}}{0.2 \text{ seconds}} = \frac{0.88 \text{ meters}}{0.2 \text{ seconds}} = 4.4 \text{ meters/sec.}$$

What about during the next 0.2 seconds? Does she move faster, slower, or the same? This time her height changed from 3.88 meters to 4.38 meters. In these 0.2 seconds she rose $4.38 - 3.88 = .50$ meters. That's less than before (since $0.50 < 0.88$), which means so she is going slower. Officially, we can calculate that her speed is

$$\text{Rate of change} = \frac{\text{change in height}}{\text{change in time}}$$

$$= \frac{4.38 - 3.88 \text{ meters}}{0.4 - 0.2 \text{ seconds}} = \frac{0.50 \text{ meters}}{0.2 \text{ seconds}} = 2.5 \text{ meters/sec.}$$

To calculate the rate of change we figured out the top of the fraction (0.50) and the bottom of the fraction (0.2), and then divided $0.50 \div 0.2 = 2.5$. There is a way to do the entire calculation at once on the calculator, but you need to use parentheses:

$$(4.38 - 3.88) \div (0.4 - 0.2) =$$

We need those parentheses to force the calculator to do the subtractions first and division second. The usual order of operations would do it the other way around: multiplication and division before addition and subtraction.

Notice that the top of our fraction is

height at 0.4 seconds - height at 0.2 seconds and the bottom of our fraction is

$$0.4 \text{ seconds - } 0.2 \text{ seconds}$$

It's important that they match up – the 0.4s first on top and bottom and the 0.2s second on top and bottom.

During the next time interval she's moving even slower.

$$\text{Rate of change} = \frac{4.48 - 4.38 \text{ meters}}{0.6 - 0.4 \text{ seconds}} = 0.5 \text{ meters/sec.}$$

And look what happens when we calculate her speed during the next time interval.

$$\text{Rate of change} = \frac{4.20 - 4.48 \text{ meters}}{0.8 - 0.6 \text{ seconds}} = \frac{-.28 \text{ meters}}{0.2 \text{ seconds}} = -1.4 \text{ meters/sec.}$$

What does a negative speed mean? During this time interval her height drops. She's headed down towards the water. Her speed is 1.4 meters/sec downward. The negative tells us her height is falling. We can add these speeds in our table.

T	0.0		0.2		0.4		0.6		0.8		1.0		1.2		1.4
H	3.00		3.88		4.38		4.48		4.20		3.52		2.45		1.00
speed		4.4		2.5		0.5		-1.4		-3.4		-5.35		-7.25	

To be perfectly correct, these are her "average" speeds over the interval. Instead of saying "rate of change" people will often say "average rate of change," but the formula is the same.

Over any interval where increasing the independent variable corresponds to an increase in the dependent variable, we say the function is *increasing*. The diver's height is increasing for $0 \leq T \leq 0.6$ seconds. It is possible that she continues to rise a little longer, but we can't tell from just the numbers in our table.

On the other hand, over any interval where increasing the dependent variable corresponds to a decrease in the dependent variable, we say the function is *decreasing*. The diver's height is decreasing for $0.8 \leq T \leq 1.4$ seconds. It is possible that her height starts decreasing sooner, and it certainly continues decreasing until she hits the water, but we don't know exactly when.

When does the diver's height stop increasing and start decreasing? When she's at the highest height, some time between 0.6 and 0.8 seconds into her dive. Before then her rate of change is positive. After that time her rate of change is negative. So, at the highest height her rate of change is probably equal to zero. Does that make sense? Think about watching a diver on film in very slow motion. Up, up she goes, then almost a pause at the top, and then down, down, into the water. At the top of her dive it's as if she stands still for an instant. That would correspond to zero speed.

We can use the rate of change to estimate values missing from the table. For example, let's guess her height at 0.3 seconds. During the time interval between 0.2 and 0.4 seconds, we figured out that her average speed was 2.5 meters/sec. From 0.2 to 0.3 is 0.1 extra seconds. During that 0.1 extra second, she goes about

$$0.1 \text{ extra seconds } * \frac{2.5 \text{ meters}}{\text{second}} = 0.25 \text{ extra meters},$$

so her height would be approximately

$$3.88 \text{ meters} + 0.25 \text{ extra meters} = 4.13 \text{ meters}.$$

We expected something in between 3.88 and 4.38 meters. In fact, since 0.3 was right in the middle of 0.2 and 0.4, we actually get the average $\frac{3.88+4.38}{2} = 4.13$. If it wasn't right in the middle we wouldn't get the average, but a more weighted average. By the way, we could do this estimate all at once as

$$\begin{aligned}
\text{Estimated value} &= \text{original value + extra independent variable } * \text{ rate of change} \\
&= 3.88 \text{ meters} + 0.1 \text{ extra seconds } * \frac{2.5 \text{ meters}}{\text{second}} \\
&= 3.88 \text{ meters} + 0.25 \text{ extra meters} \\
&\approx 4.13 \text{ meters}.
\end{aligned}$$

A photographer snapped a picture at exactly 1.03 seconds. How high was the diver then? We expect the answer to be just a little bit less than her height at 1.0 seconds, which was 3.52 meters, but not nearly as low as after 1.2 seconds, which was 2.45 meters. Let's see what the rate of change estimate is. First,

$$0.03 \text{ extra seconds } * \frac{-5.35 \text{ meters}}{\text{second}} = -0.1605 \text{ extra meters},$$

which means about 0.1605 lower. Her height would be approximately

$$3.52 \text{ meters} - 0.1605 \text{ extra meters} = 3.3595 \text{ meters} \approx 3.36 \text{ meters}.$$

As before we can calculate this estimate in one fell swoop as

$$\begin{aligned}
\text{Estimated value} &= 3.52 \text{ meters} + 0.03 \text{ extra seconds } * \frac{-5.35 \text{ meters}}{\text{second}} \\
&= 3.3595 \text{ meters} \\
&\approx 3.36 \text{ meters}.
\end{aligned}$$

Notice that we rounded off to two decimal places for our approximation because all the numbers in the table were rounded off. That's a reasonable answer, much closer to her height at 1.0 seconds (3.52 meters) than her height at 1.2 seconds (2.45 seconds).

How long is the diver in the air? At 1.4 seconds she's 1.00 meter up, so she must enter the water soon after that. We can use the rate of change to estimate her height after 1.5

and 1.6 seconds to see. We don't know the average speed past 1.4 seconds, so we'll just have to use the closest value we know, her speed was -7.25 meters/sec during the preceding interval. Bear in mind that we're really guessing about that, and so our estimate is even less accurate than usual. For both estimates we start with 1.00 meter at 1.4 seconds.

$$\begin{aligned}\text{Estimated height at 1.5 seconds} &= 1.00 \text{ meters} + 0.1 \text{ extra seconds } * \frac{-7.25 \text{ meters}}{\text{second}} \\ &= 0.275 \text{ meters} \\ &\approx 0.3 \text{ meters.}\end{aligned}$$

$$\begin{aligned}\text{Estimated height at 1.6 seconds} &= 1.00 \text{ meters} + 0.2 \text{ extra seconds } * \frac{-7.25 \text{ meters}}{\text{second}} \\ &= -0.45 \text{ meters} \\ &\Longrightarrow \text{already hit the water.}\end{aligned}$$

Here is a graph showing the diver's height. The variables are

$$T = \text{time (seconds), indep, } 0 \le T \le 1.6$$
$$H = \text{diver's height (meters), dep, } 0 \le H \le 5$$

As usual we drew in a smooth curve connecting the points, which illustrates our best guesses for the points we don't know. We also drew in the straight lines connecting each pair of points. As you can see, the first line segment is steepest – that's where the rate of change was 4.4 meters/sec. The next line segment was less steep – that's where the rate of change was less, down to 2.5 meters/sec. The third line segment is almost flat – that's where the rate of change was only 0.5 meters/sec.

We notice the same connection between the rate of change and steepness of the curve for the decreasing portion, only this time all the rate of changes are negative. The first downhill line segment is fairly flat – that's where the rate of change was -1.4 meters/sec. The next downhill line segment was much steeper – that's where the rate of change was -3.4 meters/sec. The next two downhill line segments were each steeper yet – this time with rates of change -5.35 and -7.25 meters/sec.

In each case we can visualize the rate of change as the steepness of the graph.

SHOULD WE INCLUDE THIS: When the rate of change is constant, the graph is a line and the function is called *linear*. OR MAYBE WITHIN THE PRACTICE PROBLEM ABOUT WEDDING?

DO WE DO ENOUGH INTERPOLATION IN THIS EXAMPLE – check old solutions from first version of 1.3Epsilon

Whenever we use the rate of change to estimate values it's as if we're assuming the rate of change is constant, at least for that interval of values. If the function really is linear and the graph is a line, then this estimate is excellent. If the function is pretty far off from linear and the graph curves a lot, then this estimate might not be as accurate. Something to keep in mind.

There's a formal name for what we're doing here. When we use the rate of change to estimate a number in between two numbers that we know it's called *linear interpolation.* When we estimate a number beyond what we know (smaller than the smallest number or larger than the largest number), it's called *linear extrapolation.* For both terms, the word "linear" reminds us that it only works perfectly for a line. In general interpolation is often a reasonable guess. Extrapolation can be pretty far off, but sometimes it's the only guess we have.

Sometimes we just want to know when a function is increasing, decreasing, at a maximum or minimum, how steep it is, or how much it seems to change. Even if there aren't specific numbers on the graph, we can sometimes learn a lot about a story. The next example looks at this sort of numberless graphs, sometimes called *qualitative* graphs.

Practice exercises

1. The table shows the costs for various sizes of sheet cakes available from the bakery.

Number of people	10	20	50
Price of cake ($)	$11.95	$19.95	$40.95

(a) On average, how much does sheet cake cost for each additional person if there are between 10 and 20 people?

(b) On average, how much does sheet cake cost for each additional person if there are between 20 and 50 people?

(c) Why do you think the average price per person drops?

The next table shows the costs for various orders of cupcakes available from the bakery.

Number of people	12	24	48
Price of cupcakes ($)	$6.95	$13.90	$27.80

(d) On average, how much does a cupcake cost for each additional person if there are between 12 and 24 people?

(e) On average, how much does a cupcake cost for each additional person if there are between 24 and 48 people?

(f) Why do you think the average price per person doesn't change?

(g) For 30 people, which is less expensive – sheet cake or cupcakes? *Explain.*

(h) For 18 people, which is less expensive – sheet cake or cupcakes? *Explain.*

2. As we read about in the last section the cost of holding a wedding reception at the Metropolitan Club is \$1,000 down and \$75 per person.

   (a) Calculate the extra cost, on average, for each additional person between 20 and 50 people.

   (b) Calculate the extra cost, on average, for each additional person between 75 and 100 people.

   (c) What do you notice?

   (d) Look back at the graph of the function. Explain why it is a line.

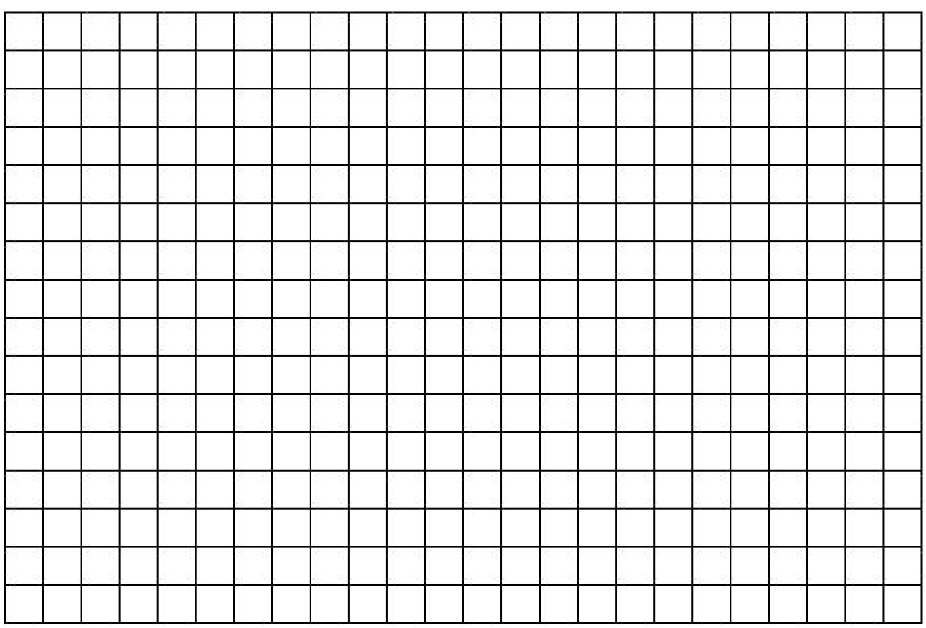
   (e) Is the cost function increasing, decreasing, or neither?

3. A twenty pound bag of dog food costs \$12.49, but a five pound bag costs \$3.79.

   (a) Calculate the rate of change. What does it mean in terms of the story?

   (b) Use the rate of change to estimate the cost of a 10 pound bag of dog food. Compare your answer to Exercise 1.1.6

   (c) Draw a graph illustrating the dependence using all three points calculated here. Do they fall on a line?



4. Suzanne's weight has certainly changed over the years. She sketched this graph to show how her weight has changed, but she deliberately didn't scale the axes because she didn't want to tell everyone her age and weight. The variables are

$$A = \text{Suzanne's age (years), indep, } 0 \leq T \leq ?$$
$$W = \text{Suzanne's weight (pounds), dep, } 0 \leq H \leq ?$$

Graph of Su's weight goes here

(a) Has she been gaining or losing weight lately?

(b) What might explain the two tall hill shapes in the graph?

(c) Did she ever get back to her pre-baby weight?

(d) After her last major weight loss, did she gain all the weight back?

Do you know ...

- How to calculate rate of change between two points?
- What the rate of change means in the story?
- How we can use the rate of change to estimate values?
- When a function is increasing or decreasing, and the connection to the rate of change?
- Why the rate of change is zero at the maximum (or minimum) value of a function?
- What the connection is between rate of change and the steepness of the graph?
- How to sketch or read trends from a qualitative graph?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. The tables in the previous exercise showed the cost of sheet cake and cupcakes.

   (a) Identify the variables, including units, realistic domain and range, and dependence.
   (b) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points for the sheet cake and another for the cupcakes.
   (c) Explain why the cupcake curve is a straight line, but the sheet cake curve is really curved.
   (d) According to your graph, which is less expensive for 30 people – sheet cake or cupcakes?
   (e) According to your graph, which is less expensive for 18 people – sheet cake or cupcakes?
   (f) Compare your answers to Exercise 1.3.1

6. It began to snow early one morning. At 7:00 a.m. there was no snow on the ground. By noon there was 2 inches of snow. By 3:00 p.m. there was a total of 4 inches.

   (a) Calculate the rate of snowfall during the time periods given – from 7:00 a.m.-noon and from noon-3:00 p.m.
   (b) During which time period, morning or afternoon, was the snow falling faster? *Explain.*
   (c) Estimate the amount of snowfall that was on the ground at 8:00 a.m., at 2:30 p.m., and at 4:00 p.m. *Explain your reasoning.*
   (d) Identify the variables, including units, realistic domain and range, and dependence.

(e) Is the snow cover increasing, decreasing, or neither?

7. After aerobics class was over, Katie measured her heart rate several times while she was cooling down. Right after class her heart rate was 178 beats per minute. Two minutes later, it had dropped to 153 beats per minute, and by ten minutes after class had ended it was down to 120 beats per minute.

   (a) Make a table showing how Katie's heart rate.

   (b) Identify the variables, including units, realistic domain and range, and dependence.

   (c) How quickly was Katie's heart rate dropping, on average, during the first two minutes following class? *Hint: the units are beats per minute per minute. Looks funny, but that's what it is.*

   (d) How quickly was Katie's heart rate dropping, on average, during the next time period?

   (e) Katie feels comfortable taking a shower once her heart rate is closer to normal, below 100. She usually waits 15 minutes after class. Do you think that's long enough? Explain.

   (f) Is Katie's heart rate increasing, decreasing, or neither?

8. The table shows Henry's weight as a baby.

Age (weeks)	0	12	15
Weight (pounds)	8	14	16

   (a) How much weight did Henry gain, on average, each week during his first 12 weeks?

   (b) During which time interval was Henry gaining weight faster? *Explain.*

   (c) What might you guess for Henry's weight at 20 weeks? *Explain your reasoning.*

   (d) Identify the variables, including units, realistic domain and range, and dependence.

   (e) Is Henry's weight increasing, decreasing, or neither?

9. Chaoxiang is considering investing in a certain company's stock. He looked up a table of values over the past 500 days. A few of those values are recorded in the table.

Day	0	50	75	150	200	300	350	400	450	500
Value ($)	29.00	24.75	23.19	20.75	21.00	26.00	30.75	37.00	44.75	54.00

   (a) Identify the variables, including units, realistic domain and range, and dependence.

   (b) Draw a detailed graph illustrating the dependence based on the points given in the table.

(c) Calculate the rate of change between 75 and 150 days. Sketch in the line segment connecting those two points on the graph. Was the stock price increasing or decreasing over that time period?

(d) Calculate the rate of change between 400 and 450 days. Sketch in the line segment connecting those two points on the graph. Was the stock price increasing or decreasing over that time period?

(e) Approximately when was the rate of change equal to zero? What does that tell us in terms of the story?

(f) Assuming current trends continue, do you think it's a good idea that Chaoxiang buy the stock now?

10. Your local truck rental agency lists what it costs to rent a truck (for one day) based on the number of miles you drive the truck.

distance driven (miles)	50	100	150	200
rental cost ($)	37.50	55.00	72.50	90.00

(a) Calculate the rate of change for each time period.

(b) Use the rate of change to estimate the cost of renting a truck to drive 75 miles. Compare your answer to Exercise 1.2.3

(c) Use the rate of change to estimate the cost of renting a truck to drive 10 miles. Compare your answer to Exercise 1.2.3

(d) Is truck rental pricing linear? *Explain.*

(e) Is the rental cost increasing, decreasing, or neither?

11. The table lists the United Nations' estimates of population of the Earth, in billions, for select years since 1800. (from www.un.org/) *Their report was released in 1999 to coincide with the world population crossing the 6 billion mark.*

year	1800	1850	1900	1950	1970	1990	2000
population	0.98	1.26	1.65	2.52	3.70	5.270	6.06

(a) Calculate the rate of change for each time period.

(b) Look back at your guess of when the population of the Earth first exceeded 2 billion. (Exercise 1.2.4) Use the rate of change to estimate the population of the Earth in that year. Is it close to 2 billion?

(c) Look back at your guess of when the population of the Earth first exceeded 7 billion. (Exercise 1.2.4) Use the rate of change to estimate the population of the Earth in that year. Is it close to 7 billion?

(d) Is the population function linear? *Explain.*

(e) Is the population increasing, decreasing, or neither?

12. Gilberto's car was worth \$22,500 when he bought it new. Now it's ten years old and worth only \$7,500.

    (a) Calculate the rate of change.

    (b) Look back at your guess of when when his car will be practical worthless (under \$500). (Exercise 1.1.9) Use the rate of change to estimate the the value of Gilberto's car in that year. Is it close to \$500?

    (c) That estimate is based on the assumption that the value of the car decreasing linearly. In fact, the value of a car drops sharply at first, and then more gradually over time. Given this new information, redraw a graph illustrating the dependence.

    (d) Add the line connecting the two given points to your graph. The rate of change estimate was based on that line. How does our estimate from the rate of change (line) compare to the estimate of the graph?

13. "My computer has crashed one time too many. Now it's really going to crash!" Tony proclaimed as he stood on the top of Science Building and hurled his computer up in the air – carefully checking first that no-one was walking by, of course. He watched it fall back down and then it crashed into thousands of pieces on the sidewalk. The graph shows how the height of Tony's computer changed over time.

    (a) Which point on the graph shows the highest height of the computer? What is the rate of change at that point?

    (b) Which point on the graph shows when the computer hit the ground? Was the rate of change positive or negative right before then?

    (c) Which point on the graph shows when Tony let the computer pass him on the way down?

14. The Greenrug company makes weather-resistant carpeting. The graph shows how their profit is a function of the amount of carpeting they sell each month.

(a) Which point on the graph shows the start-up costs each month?

(b) Which point on the graph shows the break-even point (when the company started to make money)?

(c) Which parts of the graph correspond to increasing profits?

(d) Which point on the graph shows the maximum profit?

(e) As the amount of carpet sold increase past the maximum profit point, what happens to the rate of change?

15. Sketch a qualitative graph showing each of the following:

(a) The temperature over the course of a year in Minnesota.

(b) The temperature over the course of a year in Las Vegas, Nevada.

(c) How your weight has changed in your lifetime.

(d) How the length of time it takes to get to school (or work, or the mall) depends on the time of day.

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012
Examples from the old 2.3 that could be here, or could wait until later.

A well-drilling company charges its customers by the number of feet it digs into the ground to hit water. A well that is 350 deep costs \$2,200 but a well that's only 200 feet deep only costs \$1,300.

1. Calculate the additional charge for each additional foot of depth.

2. What do you expect a 400 foot deep well would cost?

3. Identify the variables, including units, realistic domain and range, and dependence.

Sofía was been baking cookies for her church bake sale. Last night she made 12 batches of chocolate cookies; it took her 2 hours and 40 minutes from start to finish including making the dough and baking each batch. The night before she made 7 batches of chocolate cookies; it took her 1 hour, 40 minutes. Each time she first made the dough for all the batches at once, and then she baked the cookies one batch at a time.

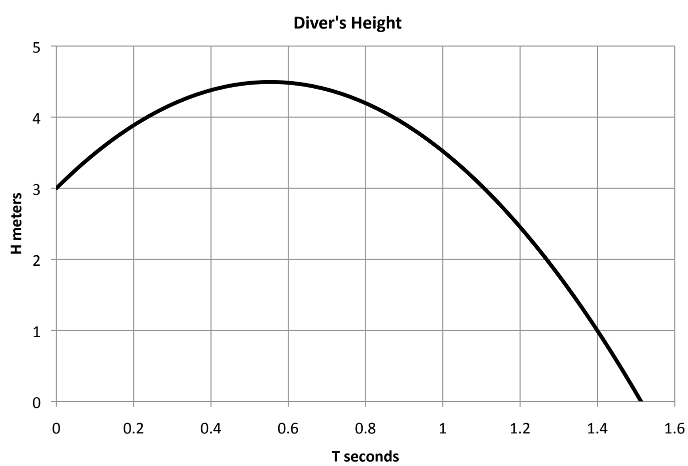
1. How long does each batch of cookies bake in the oven? *Hint: how much time does each additional batch of cookies add to the overall time?*

2. How long do you think it would take Sofía to prepare the dough and bake another 9 batches tonight?

3. If Sofía has only 2 hours, how many batches of cookies can she make?

4. Identify the variables, including units, realistic domain and range, and dependence.

4 in



1.4 Units

We know 5 city blocks and 5 miles are very different lengths to walk; \$5 and 5¢ are very different values of money; 5 minutes and 5 years are very different amounts of time to wait – even though all of these quantities are represented by the number 5. Every variable is measured in terms of some unit. Since there are often several different units available to use it is important when naming a variable to state which units we are choosing to measure it in.

In the last section we examined the height of a diver and her speed in the air. But, how high is 3 meters? How fast is 4.4 meters per second?

The metric unit of length called the *meter* is just over 3 feet (a yard). Let's use the conversion

$$1 \text{ meter} \approx 3.281 \text{ feet}.$$

We can convert

$$3 \text{ meters} * \frac{3.281 \text{ feet}}{1 \text{ meter}} = 9.843 \text{ feet} \approx 9.8 \text{ feet}.$$

Since our conversion is just approximate, we rounded off our calculation too.

See how the meters on the top and bottom cancel, leaving the units as feet? It might help to view it as

$$\frac{3 \cancel{\text{meters}}}{1} * \frac{3.281 \text{ feet}}{1 \cancel{\text{meter}}}.$$

Notice that we multiplied the quantity we were interested in (3 meters) by the fraction $\frac{3.281 \text{ feet}}{1 \text{ meter}}$. Since 3.281 feet and 1 meter are just two different ways of saying the same distance, the top and bottom of that fraction are equal. In other words,

$$\frac{3.281 \text{ feet}}{1 \text{ meter}} = 1.$$

A fraction where the top and bottom are equal quantities expressed in different units is sometimes called a *unit conversion fraction*. Because it's equal to 1, multiplying by the unit conversion fraction doesn't change the value, just the units.

One more thing to keep in mind when converting units: a few large things equals a lot of small things. Think calories here – instead of eating a few large cookies, you can eat a larger number of small cookies for the same caloric intake. So a small number of meters (3 meters) equalled a larger number of feet (9.8 feet). That might seem backwards, but that's how it works.

Of course, 9.843 feet might sound like a funny answer. We're much more used to a whole number of feet and then the fraction in inches. It's 9 feet and some number of inches. To figure out the inches we calculate

$$0.843 \text{ feet} * \frac{12 \text{ inches}}{1 \text{ foot}} = 10.116 \text{ inches} \approx 10 \text{ inches}.$$

So, the board is about 9 feet and 10 inches high, or 9'10" for short.

The highest height we had recorded for the diver was 4.48 meters. Now we know that's

$$4.48 \text{ meters} * \frac{3.281 \text{ feet}}{1 \text{ meter}} = 14.69888 \text{ feet} \approx 14.7 \text{ feet}.$$

In feet and inches, that's about 14 feet, 8 inches because

$$0.69888 \text{ feet} * \frac{12 \text{ inches}}{1 \text{ foot}} = 8.38656 \text{ inches} \approx 8 \text{ inches.}$$

What about the diver's speed? During the first 0.2 seconds we calculated her speed as 4.4 meters per second. How fast is that? We can certainly convert to feet per second.

$$\frac{4.4 \text{ meters}}{\text{second}} * \frac{3.281 \text{ feet}}{1 \text{ meter}} = \frac{14.4364 \text{ feet}}{\text{second}}.$$

Does that help us understand how fast she's going? Maybe a little. But, we're probably most familiar with speeds measured in miles per hour.

Let's convert to miles per hour. First,

$$\frac{14.4364 \text{ feet}}{\text{second}} * \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{866.184 \text{ feet}}{\text{minute}}.$$

The larger number makes sense here because she can go more feet in a minute than in just 1 second. Next,

$$\frac{866.184 \text{ feet}}{\text{minute}} * \frac{60 \text{ feet}}{1 \text{ hour}} = \frac{51,971.04 \text{ feet}}{\text{hour}}.$$

Again, the larger number makes sense because she can go more feet in an hour than in just 1 minute. Last,

$$\frac{51,971.04 \text{ feet}}{\text{hour}} * \frac{1 \text{ mile}}{5,280 \text{ feet}} = \frac{9.843 \text{ miles}}{\text{hour}} \approx 10 \text{ mph.}$$

This time we got a smaller number because she can go a lot fewer miles in an hour than feet.

We can do this entire calculation all at once. Notice how the units cancel to leave us with miles per hour (mph).

$$\frac{4.4 \cancel{\text{meters}}}{\cancel{\text{second}}} * \frac{3.281 \cancel{\text{feet}}}{1 \cancel{\text{meter}}} * \frac{60 \cancel{\text{seconds}}}{1 \cancel{\text{minute}}} * \frac{60 \cancel{\text{minutes}}}{1 \text{ hour}} * \frac{1 \text{ mile}}{5,280 \cancel{\text{feet}}} = 9.843 \text{ mph} \approx 10 \text{ mph.}$$

On the calculator we enter

$$4.4 \times 3.281 \times 60 \times 60 \div 5,280 =$$

Right before the diver hit the water she was going around 7.25 meters per second. How fast is that in mph?

$$\frac{7.25 \text{ meters}}{\text{second}} * \frac{3.281 \text{ feet}}{1 \text{ meter}} * \frac{60 \text{ seconds}}{1 \text{ minute}} * \frac{60 \text{ minutes}}{1 \text{ hour}} * \frac{1 \text{ mile}}{5,280 \text{ feet}} = 16.2185\ldots \approx 16 \text{ mph.}$$

On the calculator we enter

$$7.25 \times 3.281 \times 60 \times 60 \div 5,280 =$$

If you're having trouble setting up unit conversions, remember to write down the units so you can see how they cancel. If you can't remember a number to unit conversion, like 5280 feet for one mile, try a search engine on the Internet (like Google). A dictionary also has some conversions.

Practice exercises

1. Centimeters (cm), meters (m), and kilometers (km) measure length in the metric system. The conversions are approximately

$$1 \text{ inch} = 2.54 \text{ cm}$$
$$1 \text{ yard} = .914 \text{ m}$$
$$1 \text{ mile} = 1.61 \text{ km}$$

   (a) Which is longer: 1 inch or 1 centimeter?

   How long is 10 cm in inches?

   How many cm in a foot?

   (b) Which is longer: 1 yard or 1 meter?

   How long is 100 yards in meters?

   How many feet in 0.45 meters? *Note: 1 yard = 3 feet*

   (c) Which is longer: 1 mile or 1 kilometer?

   How many miles is a 5K run? *5K is short for 5 kilometers.*

   If the speed limit is 80 km/hr, how fast can you legally go in mph?

2. Authorities are tracking down the source of a pollution spill on a nearby river. They suspect that the local plant is inadvertently leaking waste water. Last week they found 35 minutes of waste water flow on Monday, 1 hour and 11 minutes on Tuesday, 1/4 hour on Wednesday (that's 0.25 hours in decimal), none on Thursday, and then 98 minutes Friday.

   (a) Convert units as needed to complete the following table showing each time in minutes, each time in hours, and each time in hours and minutes (H:MM format). *Hint: 15 minutes in H:MM format would be 0:15*

Day	Mon	Tue	Wed	Thu	Fri
Minutes	35			0	98
Hours			0.25		
H:MM		1:11			

   (b) Calculate the total waste water flow measured last week.

.

3. Baseball player Joe Mauer signed a multi-year contract with the Minnesota Twins for an average of $23 million per year. (And that doesn't include the income he gets from endorsements.)

   (a) What does Mauer's salary come to in dollars per hour? That means for every hour, waking or sleeping, all year long.

   (b) If Mauer were working a standard 40 hour work week for 50 weeks a year, what would his salary be, again in dollars per hour? *Hint: that's a total of 2,000 hours*

   (c) In a standard 162 game season, averaging about 2 hours and 51 minutes per game, assuming Joe plays every minute of every game, what does his salary come to in dollars per game minute? *Hint: calculate the total number of minutes*

4. Pramesh's new car used 20.5 gallons of gas for a 715 mile trip.

   (a) How many miles per gallon (mpg) does his car get?

   (b) At that rate, how many gallons of gas would Pramesh use on his 3,200 mile cross-country trip?

   (c) If gas costs \$3.799/gallon, how much will that trip cost for gas?

   (d) Make a table of values showing the cost for gas for Pramesh's trip if gas costs \$3.599/gal, \$3.599/gal, \$3.799/gal, or \$3.899/gal.

   (e) Identify the variables and constants (if any) suggested by the previous question. As always, include the units, realistic domain, and dependence.

Do you know . . .

- How to convert from one unit of measurement to another?
- What a unit conversion fraction is?
- Why multiplying by a unit conversion fraction doesn't change the amount, just the units?
- How to connect repeated conversions into one calculation?
- Which should be the larger number – the amount measured in a small unit, or the amount measured in a large unit?
- How many seconds in a minute, minutes in an hour, hours in a day, days in a year, inches in a foot, feet in a mile, and other common conversions? *Ask your instructor which you'll need to memorize and which will be given to you on quizzes and exams.*

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. Shannon recorded her study times each day. Convert each to hours. *That means into a decimal number of hours.*

   (a) 2:15 (2 hours and 15 minutes)
   (b) 10:20 (10 hours and 20 minutes)
   (c) 7:57 (7 hours and 57 minutes)

6. The production plant listed the following lifetimes for its very delicate electronic switches. Convert each into hours and minutes. *That means H:MM format.*

   (a) 4.18 hours
   (b) 19.50 hours
   (c) 12.05 hours
   (d) .18 hours

7. Convert each speed into miles per hour. Which of these might be how fast a car drives?

   (a) 100 km per hour
   (b) 1 million feet per minute
   (c) 88 feet per second
   (d) 5 cm per second.

8. (a) The football coach wants everyone to sprint three-quarters of a mile, up and back on the field which is labeled in yards. How many yards are in three-quarters of a mile?
   (b) The quilt pattern calls for .375 yards of calico fabric. How many feet is .375 yards?
   (c) The formula said that basil plants should be .35 feet tall a month after germinating. How many inches is .35 feet?
9. Sebastiao measured the following volumes of product in lab. Convert each of his measurement as indicated.
   (a) Convert 20 milliliters (ml) to ounces. *Use 1 ounce = 29.57 ml.*
   (b) Convert 1.3 liters to ounces. *Use 1 liter = 1,000 ml.*
   (c) Convert 1.3 liters to quarts. *Use 1 quart = 32 ounces.*
10. Some people say we should drink 8 glasses of water every day. If each of those glasses holds 8 ounces of water, how much water would we consume in 1 year? Give the answer in gallons. *1 gallon = 4 quarts and 1 quart = 32 ounces.*
11. The fuel efficiencies of several European cars are listed, below. Convert each quantity into miles per gallon. *Use 1 gallon = 4.546 liters.*
   (a) 8 km/liter
   (b) 7.6 km/liter
   (c) 9.1 km/liter
12. (a) How many seconds are there in a year (having 365 days)?
   (b) How many years and days are there in 1 million seconds?
   (c) Billy Bob wants to throw a party when he turns 1 billion seconds old. About how many years old will he be?
   (d) *Bonus question:* On what date were you or will you be 1 billion seconds old? Don't forget leap years!
13. The women's world record for the 5K in June 2008 was held by Tirunesh Dibaba of Ethiopia. She ran those 5 kilometers in 14 minutes, 11.15 seconds.
   (a) Convert her time into minutes.
   (b) How fast did she run, as measured in km/min?
   (c) Convert her speed to mph. *Pretty fast!*
14. On July 4, 2008 United States swimmer Michael Phelps set the world record for the 200 meter individual medley swimming it in 1 minute, 54.80 seconds.
   (a) Convert his time into minutes.
   (b) How fast did he swim, as measured in km/min?

(c) Convert his speed to mph.

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

1.5 The metric system and scientific notation*

INTRODUCTORY EXAMPLE

SU – look at old 3.1 and old 3.4 and also exercises that combined these with unit conversions. Maybe something about acid rain and pH?

OLD NOTES:

- The standard form of scientific notation. Converting between expanded decimal notation and scientific notation and comparing the relative size of numbers from their scientific notation.
- Converting units involving scientific notation such as millions, billions, and trillions and units involving standard metric prefixes.
- Imbedded topics: Exponents, order of operations, and the scientific calculator
- The definition of exponential notation (for integral powers). Recognizing the effect of multiplying by positive and negative powers of 10.
- Evaluating powers on a calculator, interpreting calculator display of numbers in scientific notation, and entering numbers in scientific notation into a calculator. [Formal properties of exponents and simplifying expressions are addressed in the exercises.]

Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

Practice exams on Variables

Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

Practice exam 1– version I

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

Practice exam 1 – version II

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

Chapter 2

Equations

For most of us the word "algebra" brings to mind equations, formulas, and all those symbols. One chapter into a book on algebra and we haven't seen any equations. What gives?

Remember, this course is all about using algebra to answer questions. Equations are going to be a very important part of that work. It turns out that equations are helpful algebraic tool for at least three reasons. First, equations provide a nice shorthand for describing a function – much quicker to write down than making an extensive table, careful graph, or describing the dependence in words. Second, equations help us categorize problems which, in turn, helps us know what to expect in that type of situation. Lastly, there are lots of powerful "symbolic" techniques we can use to solve problems when we have an equation.

So why haven't we used equations yet? Why did the first chapter focus on describing functions using words (verbal), tables (numeric), and graphs (graphical)? It turns out that there's one thing equations can't do – it's hard to tell from an equation whether an answer makes sense in the real world. If we just worked with equations we might find an answer calling for us to produce a negative number of tables or wait 300 years for an investment to reach our payoff level, or similar nonsense.

Even as we add equations to our list of tools for describing and working with functions, we will rely on words, numbers, and graphs to help evaluate the reasonableness of our answer. You will likely find it a good habit to use those tools to estimate the answer before using the equation as well doing the "reality check" after. Thus most problems will ask you to work with all of these modes.

In this chapter we introduce equations but taking a first look at the two most important types of equations – linear and exponential. Our emphasis will be on understanding where these equations arise and how to interpret them in context. Next, we work with a variety of equations learning how to use equations and general methods for approximating solutions to equations. In later chapters we will solve equations exactly (Chapter 3) and return to study linear and exponential equations each in greater depth (Chapters 4 and 5), so don't worry if we leave a few questions unresolved for now.

2.1 A first look at linear equations

SU: check that in unit conversions throughout the text you use $*$ in the set up line, and then $\times$ in the calculation line

Your sink, I'm sorry to say, is clogged. The bottle of drain opener didn't clear it out and you're expecting dinner guests in a few hours. Your brother-in-law has offered to help, but last time he tried he only made it worse. The plumber will charge you \$100 just to come to your house. In addition, there will be a charge of \$75 per hour for the service. If you decide to call the plumber, what will it cost?

The plumber's charge will depend on the amount of time it takes to unclog the sink. We can name these variables T for the time the plumber takes, measured in hours, and P for the total plumber's charge, measured in dollars. There are two important constants in this calculation: the \$100 trip charge and the \$75/hour rate.

Look at the the relationship between T and P by making a table of values. What will it cost if the plumber takes 1 hour, 2 hours, or 3 hours? If the plumber takes one hour, then he'll charge you \$100 for showing up and \$75 for the one hour of work. So, the total plumber's bill will be

$$\$100 + \$75 = \$175.$$

For two hours, there's still the \$100 charge, but also \$75 for each of the two hours. That's an additional charge of

$$2 \text{ hours} * \frac{\$75}{\text{hour}} = \$150.$$

So, the total plumber's bill will be

$$\$100 + 2 \text{ hours} * \frac{\$75}{\text{hour}} = \$100 + \$150 = \$250.$$

Remember that your calculator can do this entire calculation at once as

$$100 + 2 \times 75 = .$$

Let's hope it wouldn't take the plumber as long as three hours, but if it did, we can do a similar calculation. Add the fixed charge of \$100 to the additional charge of \$75 for each of the three hours. The plumber's bill would be

$$\$100 + 3 \text{ hours} * \frac{\$75}{\text{hour}} = \$100 + \$225 = \$325.$$

On the calculator we can just enter

$$100 + 3 \times 75 = .$$

What would it cost if the plumber takes only $\frac{1}{2}$ hour? Add the fixed charge of \$100 to the additional charge of \$75/hour for the $\frac{1}{2}$ hour. The additional charge would be half of \$75, which is \$37.50. The plumber's bill would be

$$\$100 + \frac{1}{2} \text{ hours} * \frac{\$75}{\text{hour}} = \$100 + .5 \text{ hours} * \frac{\$75}{\text{hour}} = \$100 + \$225 = \$325.$$

On the calculator we type

$$100 + .5 \times 75 = .$$

What would happen if the plumber was taking so long that before he got there you dumped another bottle of drain opener in the sink and that did the trick. But before you could call and cancel the plumber, wouldn't you know it, but there he was. What do you owe him for that 0 hours of work? Probably \$100. Unless your plumber is super sympathetic and tells you to "forget it." So, when $T = 0$ the bill is $P = 100$.

We can put these numbers in a table to describe how the plumber's bill is a function of the time.

T	0	$\frac{1}{2}$	1	2	3
P	100.00	137.50	175.00	250.00	325.00

Each time we knew how long the plumber spent and calculated the plumber's bill P by starting with the trip charge of \$100 and adding in \$75 times the number of hours. For example, for 3 hours we calculated

$$\$100 + 3 \text{ hours} * \frac{\$75}{\text{hour}} = \$325.$$

We have a name for the number of hours in general; it is T. So for T hours, we would calculate

$$\$100 + T \text{ hours} * \frac{\$75}{\text{hour}} = \$P.$$

See how we just put the T where the 3 was and the P in where the 325 was? We're just generalizing from our example. Drop the units and we have our equation. If the plumber works for T hours, then the cost is $\$P$ where

$$P = 100 + T * 75.$$

We started the equation "$P =$" because it is a convention to begin equations with the dependent variable, when possible.

An *equation* is a formula that shows how the value of the dependent variable (like P) depends on the value of the independent variable (like T). Usually an equation is in the form

$$\text{dep. var.} = \text{some formula involving the indep. var.}$$

An equation is another way to describe a function. It carries a lot of information in a few short symbols.

There is a mathematical convention that we write numbers before letters in an equation. So, instead of $T * 75$ we should write $75 * T$. There's a conventional shorthand for this product: when a number and letter are next to each other, it means that they are multiplied. So, instead of $75 * T$ we should write $75T$. Thus our equation is normally written as

$$P = 100 + 75T.$$

You'll have to remember the hidden multiplication when you're calculating.

If you wanted to write the equation as

$$P = 75T + 100,$$

that would be okay too. We can add the \$100 trip charge first, like we did in our examples, or at the end. Same answer.

Suppose the plumber shows up at your house and fixed the sink in 25 minutes. Whew! No sooner do you pay your bill than your first dinner guest arrives. How much do you owe the plumber? Notice that

$$25 \text{ minutes} * \frac{1 \text{ hour}}{60 \text{ minutes}} = 25 \div 60 = .4166\ldots \text{ hours.}$$

Therefore for 25 minutes we have $T \approx .4166$. Using our equation we get

$$P = 100 + 75T = 100 + 75 \times .4166 = 131.245 \approx \$131.25.$$

It was important that we rounded off our final answer because we had rounded off to get .4166 along the way. We could have done the entire calculation at once (avoiding the round off error) as

$$100 + 75 \times 25 \div 60 = 131.25.$$

If we plot the points from the table of values in a graph, we see that the points lie on a line.

SU need graph

Is this function linear? Remember, to be linear the function would need to have a constant rate of change. In this case we can calculate the rate of change between 1 hour and 2 hours as

$$\frac{\$250 - \$175}{2 \text{ hours} - 1 \text{ hour}} = \frac{\$75}{1 \text{ hour}} = \$75 \text{ per hour.}$$

Sure! We knew that. The plumber charges an extra \$75 for each extra hour he works. The rate of change is precisely \$75/hour. Because the rate of change is constant, the function is definitely linear. So, the graph should be a line as it appears.

Look back at our equation.

$$P = 100 + 75T.$$

This is the standard form of a linear equation

$$\text{dep. var.} = \text{starting amount} + \text{rate of change} * \text{indep. var.}$$

Notice our two variables are in our equation and there are two constants. Each constant has its own meaning. The first constant is 100 and it is measured in dollars. It is the trip charge, the fixed amount we would owe the plumber even if he does 0 hours work. In our standard form we refer to this quantity as the *starting value*, but it's official name is *intercept*. On the graph it's where the line crosses the vertical access. Think of a football player (running along the vertical axis) intercepting a pass (coming in the line). We can find the intercept from our equation by plugging in $T = 0$:

$$P = 100 + 75 \times 0 = 100.$$

The second constant is 75 and though its tempting to say it is measured in dollars, it is really measured in $ per hour. This number is the rate of change and in the context of linear equations it gets its own name too. Its called the *slope*. Since the rate of change measures the steepness of any curve or line, the word "slope", like mountain slope, makes sense.

In our plumber example the intercept was $100 and the slope was $75/hour. We can rewrite the standard form of a linear equation as

$$\text{dep. var.} = \text{intercept} + \text{slope} * \text{indep. var.}$$

In this text we use natural letters for the variables, but many other texts use "x" for the independent variable and "y" for the dependent variable. In that notation, a linear equation can be written in the form

$$y = mx + b$$

where m is the slope and b is the intercept. It is equally acceptable to write it $y = b + mx$ which is the format we tend to us in applied settings.

One more thing to note. In a situation modeled by a linear function it is possible that the slope is positive (increasing function) or negative (decreasing function). Similarly, the intercept could be positive (graph starts above horizontal axis) or negative (graph starts below the horizontal axis). There are even realistic problems where the intercept is zero (starts where the axes cross). While it is theoretically possible for the slope to be zero, that would mean the value of the dependent variable has 0 rate of change, which would mean that even though we though it was a variable, it really was a constant.

SU: where does this fit in? Here, and throughout the text, when writing out the calculator keys we underline the value we're plugging in so it's easy to see. SU – check that you do this in Sections 2.1 and 2.2. Is this something that should wait until 2.3 when we actually "evaluate"?

Practice exercises

1. At a local state university, the tuition each student pays is based on the number of credit hours that student takes plus fees. The university charges \$900 per credit hour plus a \$200 fee. The fee is paid once regardless of how many credits are taken.

   (a) Name the variables and write an equation relating them.

   (b) Find the slope and intercept and explain what each means in terms of the story.

   (c) Make a table of values showing the tuition cost for 3 credits, 12 credits, or 16 credits.

   At the local community college, the tuition each student pays is based only on the number of credits. The college charges \$245 per credit.

   (a) Using the same variables as before, write an equation relating them for the community college.

   (b) Find the slope and intercept and explain what each means in terms of the story.

   (c) Make a table of values showing the tuition cost for 3 credits, 12 credits, or 16 credits.

2. A truck hauling bags of grass seed pulls into a weigh station along the highway. In case you're curious, trucks are weighed to determine the amount of highway tax owed. This particular truck weighs 3,900 pounds when it's empty. Each bag of seed it carries weighs 4.2 pounds.

   For example, if truck is carrying 1000 bags of grass seed, then it would weigh

$$3,900 \text{ pounds } + \frac{4.2 \text{ pounds}}{\text{bag}} * 1000 \text{ bags} = 3900 + 4.2 \times 1000 = 8,100 \text{ pounds}$$

   In official trucking lingo, we'd say the "curb weight" of 3,900 pounds plus the "load weight" of 4,200 pounds results in a "gross weight" of 8,100 pounds. So, now you know.

   (a) Calculate the gross weight of the truck if it contains 2,000 bags of grass seed.

   (b) Identify the variables and constants (if any), including the units, realistic domain and range, and dependence.

   (c) Write an equation showing how the gross weight of the truck is a function of the number of bag seed it contains.

   (d) Find the slope and intercept, along with their units, and explain what each means in terms of the story.

   (e) How much does the truck weigh if it is carrying 12 pallets of grass seed bags, where each pallet holds 96 bags of seed?

3. The water in the local reservoir was 47 feet deep but over the past few months there's been so little rain that the depth has fallen 18 inches a week. Officials are worried that if dry conditions continue and the depth continues to fall, then the reservoir will not have enough water to supply the town.

   (a) Name the variables and write an equation relating them.

   (b) Find the slope and intercept, along with their units, and explain what each means in terms of the story.

   (c) Make a table of values showing the projected depth of the reservoir after 1 week, 5 weeks, 10 weeks, and 20 weeks if the current trend continues.

   (d) Draw a graph illustrating the function.

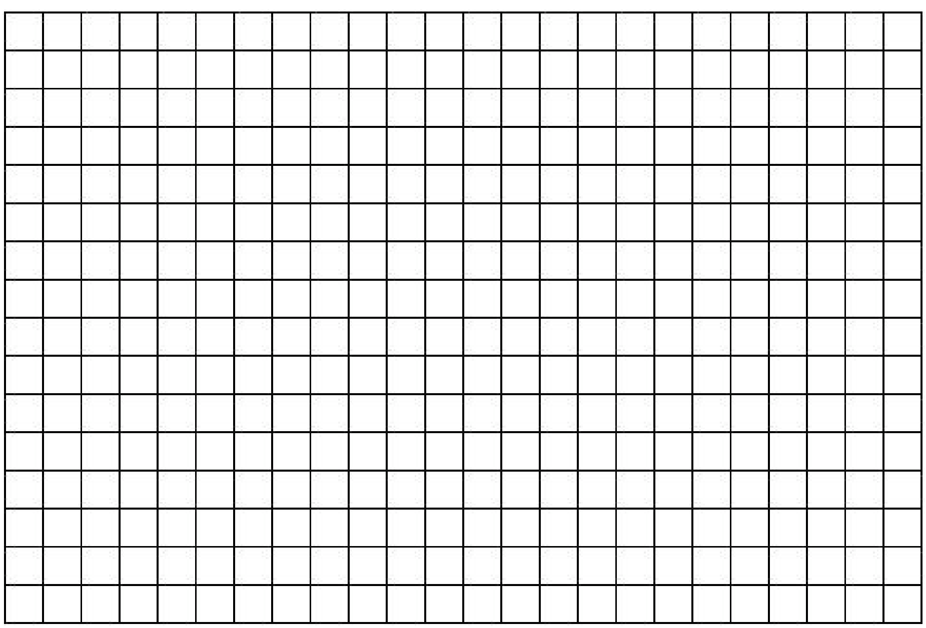
4. My bank account balance is -$1,200. Well, not really. I mean my bank lent me $1,200 through a line of credit on my account. We've agreed that I will pay back the interest I owe plus $250 each month until it's paid off. After that I plan to keep depositing the $250 per month to start saving some money.

   (a) Name the variables and write an equation relating them. Notice that we can completely ignore interest because I pay that in full each month.

   (b) Find the slope and intercept, along with their units, and explain what each means in terms of the story.

   (c) Make a table of values showing the projected account balance now, after 4 months, and at the end of a year.

   (d) Draw a graph showing how my account balance changed in a year's time.



Do you know ...

- How to generalize an example to find the equation of a function?
- Where the dependent variable is in the standard form of an equation?
- What the slope of a linear equation means in the story and what it tells us about the graph?
- What the intercept of a linear equation means in the story and what it tells us about the graph?
- Where the slope and intercept appear in the standard form of a linear equation?
- When an function is linear?
- How to plot negative numbers on a graph?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. The Torkelinsons want to dig a new well for water for their lake cabin. The company charges \$900 just to show up and then \$2 per foot to dig.
   (a) What would a 100 foot deep well cost?
   (b) Name the variables and write an equation relating them.
   (c) Identify the slope and intercept. Don't forget the units.
   (d) Make a table showing the total cost for a well 100, 250, or 400 feet deep.

6. It was very hot in Solana's office when she arrived – a steamy 87°F, and so she turned on the air conditioner. She knew that would cool her office down by around 5°F an hour.
   (a) What was the temperature in Solana's office 3 hours after she arrived?
   (b) Name the variables and write an equation relating them.
   (c) Identify the slope and intercept. Don't forget the units.
   (d) Make a small tables of values and use it to draw a graph showing the temperature in Solana's office that day.

7. Abduwali has just opened a restaurant. He spent \$82,000 to get started but hopes to earn back \$7,500 each month.
   (a) If all goes according to plan, will he have made money 10 months from now?
   (b) Name the variables and write an equation relating them.
   (c) Identify the slope and intercept. Don't forget the units.

(d) Make a small tables of values and use it to draw a graph showing Abduwali's profit.

8. When Kendrik walks on his treadmill, he burns 125 calories per hour.

   (a) How many calories will Kendrik burn if he walks 2.3 miles?
   (b) Name the variables and write an equation relating them.
   (c) Identify the slope and intercept. Don't forget the units.
   (d) Make a table showing the calories he burns walking 0, 1, 2, 3, or 4 miles.

9. Kathy is a single mom trying to raise her kids on the salary from her part-time job. She gets help from the state. Each month they give her \$300 credit on her EFT card that she can use to buy groceries. Every day after work Kathy stops at the corner store and buys \$10 worth of food, which is deducted from her EFT card.

   (a) How much does Kathy have left on her card at the end of the month (30 days)?
   (b) Name the variables and write an equation relating them.
   (c) Identify the slope and intercept. Don't forget the units.
   (d) How would the equation be different if she began with \$400 in credit and spent \$12 each day?

10. Write a linear equation for each of these stories we've seen before.

    (a) (Exercise 1.1 x) The cost of holding a wedding reception at the Metropolitan Club is \$1,000 down and \$75 per person.
    (b) (Exercise 1.1 x) The sun set at 6:00 p.m. today and I heard on the radio that it sets about 2 minutes later each day this time of year.
    (c) (Exercise 1.1 x) The temperature was 40 degrees at noon yesterday but it dropped 3 degrees an hour in the afternoon.
    (d) (Exercise 1.1 x) A phone call on Kyle's phonecard costs \$.48 for the first minute (connection fee) and \$.02/minute thereafter.
    (e) (Exercise 1.1 x) The bookstore charges 85¢ for a pack of gum.

11. The cost of vacation to Cork, Ireland from the Minneapolis/St. Paul airport for two people is given by the formula $C = 2828 + 310N$, where C is the total cost in U.S. dollars and N is the number of days.

    (a) What would it cost us to go on vacation for six days?
    (b) What might the number 2828 mean in terms of the story, and what are its units?
    (c) What might the number 310 mean in terms of the story, and what are its units?

12. Johannah figured out that the time T minutes that it takes her to warm up and then run M miles is given by the equation $T = 5 + 8M$.

    (a) What is the total time it takes Johannah to warm up and run 4 miles?

(b) What does the number 8 mean in terms of the story, and what are its units?

(c) What does the number 5 mean in terms of the story, and what are its units?

13. The altitude, A feet above ground, of an airplane M minutes after it begins its descent is given by the equation $A = 32,000 - 1,200M$.

    (a) At what altitude did the airplane begin its descent?

    (b) How fast is the airplane descending?

    (c) Will the plane be on the ground after 10 minutes? 20 minutes? 30 minutes? Display your calculations in a table.

14. Over the years the memory capacity of a desktop computer has been increasing rapidly. Some say that the memory capacity has doubled every decade since 1970, when typical capacity was 1,000 Mb (A *decade* is 10 years and Mb is short for "megabyte".)

    (a) Make a small table of values showing capacity in 1970, 1980, 1990, and 2000.

    (b) Calculate the rate of change. How do you know this function is not linear?

    (c) Draw a graph of this function. How does the graph confirm that the function is not linear?

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

2.2 A first look at exponential equations

Jocelyn got a job right out of college, as an administrative assistant earning \$28,000 a year. The position turned out to be a great fit for her, and after one year she was promoted to data analyst with a 15% raise. The next year Joceyln was promoted again, to senior data analyst along with a 21% raise. "Not bad," her friend Russell said, "a 36% raise in two years." But Jocelyn quickly corrected him. "Russ, it's even better than that! Over 39% raise."

To understand Jocelyn's calculation, we need to remember how percents work. We did a few examples in the last chapter, but since this section is all about percentage increase, let's review in a little more detail now. Luckily, the word "percent" is very descriptive. The "cent" part means "hundred," like 100 cents in a dollar or 100 years in a century. And, as usual, "per" means "for each." So the number 15% means 15 for each hundred. Written as a fraction it is $\frac{15}{100}$. As a decimal it's 0.15. Thus 15%, $\frac{15}{100}$, and 0.15 mean exactly the same thing. That is,

$$15\% = \frac{15}{100} = 15 \div 100 = 0.15$$

After the first year, Jocelyn's salary of \$28,000 was increased by 15%. That doesn't mean it was 0.15 more, but rather that it was 15% of \$28,000 more. To calculate 15% of \$28,000 we multiply using the decimal form to get

$$15\% \text{ of } \$28{,}000 = 0.15 \times 28000 = \$4{,}200$$

That's how much Jocelyn's raise was that first year. By adding that amount to the original salary we get

$$28000 + 4200 = 32200$$

After one year Jocelyn's salary was \$32,200.

After the second year, Jocelyn got a 21% raise. This means her rose by 21% from what it was just before the raise, that is, from the \$32,200. (The 21% does not refer back to the original \$28,000 value.) So, to calculate the increase, we take 21% of \$32,200, which is

$$21\% \text{ of } \$32{,}200 = 0.21 \times 32200 = \$6{,}762$$

By adding on this raise we get

$$32200 + 6762 = 38962$$

After the second year Jocelyn was earning \$38,962.

Since Jocelyn's original salary was \$28,000, the net increase in her salary is the difference

$$38962 - 28000 = \$10{,}962$$

The corresponding percentage increase was

$$\text{percentage increase} = \frac{\text{net increase}}{\text{original value}} = \frac{10962}{28000} = 0.3915 = 39.15\%$$

As Jocelyn said, that's over 39% increase.

What's going on here? Russell thought that 15% and 21% would be 36% because

$$15 + 21 = 36$$

The reason it doesn't work that was is that while the 15% is of the original \$28,000, the 21% was actually calculated on the \$32,200. So, we can't just combine percentages by adding.

There's a quicker way to calculate the percentage increase and to combine percentages. Notice that each time we figured out the value of Jocelyn's house, we did a two-step process. First, we calculated the amount of the increase, and second we found the new value by adding on. Notice that when we increase a number by 15%, then what we'll have at the end is the 100% we started with plus the 15% more. That is, we'll have 115% of what we started with. So we can just multiply by 1.15, which is 115% written in decimal. (Looks weird, works great.)

So, in our example, we can just do

$$28000 \times 1.15 = 32200$$

We can do the same thing for the next calculation

$$32200 \times 1.21 = 38962$$

Here we multiplied by 1.21 because after a 21% increase you'll have 121% of what you started with. And 121% in decimal form is just 1.21. A quicker way to calculate the growth factor is

$$1 + .21 = 1.21$$

or from the percentage

$$1 + 21 \div 100 = 1.21$$

.

There are a lot of important applications in which we'll consider repeated percentage increase. So, hang on to your hat because we can combine both of these parts together. In our example, we started with \$28,000. Then we multiplied by 1.15, which gave us \$32,200. And then we multiplied that answer by 1.21, to get our final answer of \$38,962. So really we just did

$$28000 \times 1.15 \times 1.21 = 38962$$

Same answer. A lot less effort.

And check it out,

$$1.15 \times 1.21 = 1.3915.$$

That's where the 39.15% is hidden. Cool.

There are names for the different numbers we are using. The percentage increase is called the ***growth rate*** and the number we multiply in the one-step method is called the ***growth factor***. For example, in calculating 15% increase, the growth rate is 15% and the growth factor is 1.15.

Jocelyn's most recent assignment has been analyzing information on rising health care costs. In 2007 the United States spent \$2.26 trillion on health care. For all you zero-philes that's

$$\$2.26 \times 10^{12} = 2.26 \times 10 \wedge 12 = \$2{,}260{,}000{,}000{,}000$$

Did your calculator leave the answer in scientific notation? We saw how to convert in Section SU FILL IN.

Health care costs were projected to increase at an average of 6.7% annually for the subsequent decade. Let's write an equation showing how health care costs are projected to grow. Our variables are

Constant: 6% increase

Variables: H = health care costs (\$trillions), dep, $0 \le H \le 10$

Y = time (years from 2007), indep, $0 \le Y \le 20$

We chose up to 20 years and up to \$10 trillion. Even though those numbers might not be realistic, they are definitely large enough.

Since

$$6.7\% = 6.7 \div 100 = .067$$

the growth factor is SU YOU ARE HERE!

That tells us that to find the effect of a 6% increase, we can just multiply by 1.06. Right now her salary is \$38,962. One year from now it would be

$$38962 \times 1.06 = \$41{,}299.72$$

Another year later she would earn

$$41299.72 \times 1.06 \approx \$43{,}777.70$$

And so on. For each year we multiply by another 1.06. For example, after ten more years her salary would be

$$38962 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \approx 69775.01$$

A more economical way to write and calculate this product is with an exponent. Jocelyn's salary after ten more years would be

$$38962 * 1.06^{10}$$

On a calculator we can do this calculation in one step as

$$38962 \times 1.06 \wedge 10 \approx \$69{,}775.01$$

The order of operations on the calculator does the power before the multiplication, which is exactly what we want.

Most calculators use the $\wedge$ symbol for exponents, as do most computer software packages. Two other notations calculators sometimes use are y^x or x^y. Sometimes that operation is accessible through the 2nd or shift key; something like SHIFT $\times$. If you're not sure, ask a classmate or your instructor.

We're so close to the equation now, we can smell it. We just found her salary after 10 years. It was

$$\$38{,}962 * 1.06^{10} \approx \$69,775.01$$

We can generalize to get the equation by putting in Y years (instead of 10) and S for the salary (instead of $69,775.01). When we do we get

$$\$38{,}962 * 1.06^{Y} = S$$

Rewriting the equation to begin with the dependent variable we get

$$S = 38962 * 1.06^{Y}$$

By the way, there are two other standard ways of writing this equation

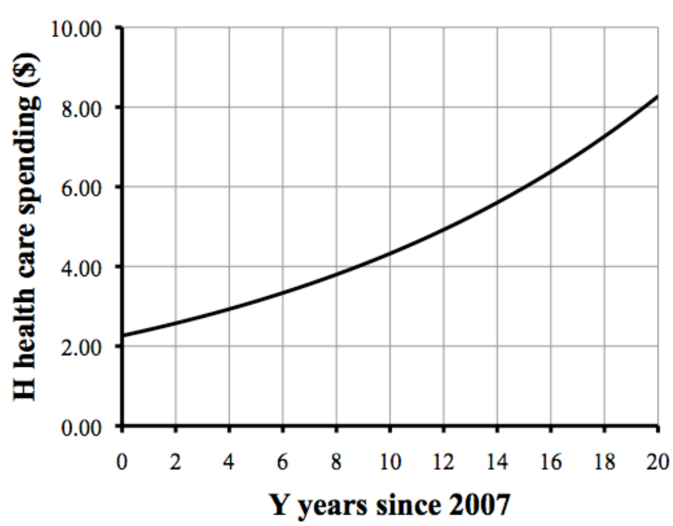
$$V = 38962(1.06)^{Y} \text{ or also } V = 38962\left(1.06^{Y}\right)$$

Our equation is called an ***exponential equation*** because the independent variable is in the exponent. More specifically, for our purposes an exponential equation has the form

$$\text{dep. var.} = \text{starting amount (growth factor)}^{\text{indep. var.}}$$

Exponential equations are not linear and so their graphs are not lines. Sometimes the graph of an exponential equation looks like a line, especially if you only plot a few points. So, be sure to plot at least 5 or more points to see the curve in the graph of an exponential equation. For example, here are five points and the graph. Admittedly the constant raise of 6% is not realistic, nor is the 20 year time period, but we need that much of the graph to see the curve.

Y	0	1	2	10	20
S	38,962	41,299.72	43,777.70	69,775.01	124,956.41



Another way to see that the function is not linear is to look at the rate of change. For example, let's see what happens when we use this table to estimate Jocelyn's salary after 15 years. First we might calculate the rate of change 10 and 20 years.

$$\text{Rate of change} = \frac{\text{change in salary}}{\text{change in time}} = \frac{\$124{,}956.41 - \$69{,}775.01}{20 \text{ years} - 10 \text{ years}}$$
$$= \frac{\$55{,}181.40...}{10 \text{ years}} \approx \$5{,}518.14/ \text{ year}$$

So on average Jocelyn's raise during those last 10 years would be $5,518.14 per year.

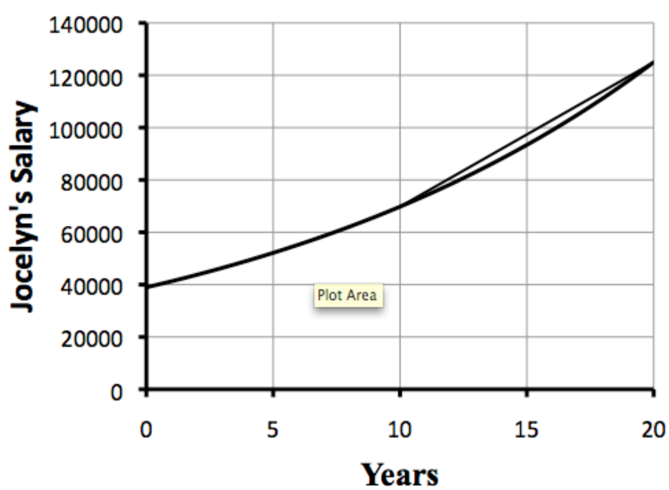
Next we would use the rate of change to interpolate. We want an estimate at 15 years, which is 5 years after the table value for 10 years. So we use the "original value" at 10 years and that there are 5 "extra years."

$$\begin{aligned}
\text{Estimated value} &= \text{original value} + \text{extra independent variable} * \text{rate of change} \\
&= \$69{,}775.01 + 5 \text{ extra years} \times \frac{\$5{,}518.14}{\text{year}} \\
&= 69775.01 + 5 \times 5518.14 \\
&\approx \$97{,}365.71
\end{aligned}$$

Compare this estimate to the value from our equation. After 15 years, our equation gives

$$38962 * 1.06^{15} = 38962 \times 1.06 \wedge 15 = \$93{,}374.70$$

The graph shows both our actual curve and the line from 10 to 20 years drawn in. Our interpolation found the salary of $97,365.71 which corresponds to the height of the line at 15 years. The actual value from our equation found the salary of $93,374.70 which corresponds to the height of the curve at 15 years. Since the line is above the curve there, the interpolation is more than the equation value.



Practice exercises

1. SU: find worksheet problem. No graph here, perhaps?

   (a) x

   (b) x

   (c) x

   (d) x

   (e) x

2. SU: update year and data! In 2006 there were about 5.2 million people living in the state of Minnesota. Predicted growth rates vary, perhaps around 4% per year.

   (a) Based on these figures, about how many people will be living in the state of Minnesota in 2010? In 2020?

   (b) Identify the variables and constants (if any), including the units, realistic domain and range, and dependence.

   (c) Write an equation showing how Minnesota's population is a function of the year.

   (d) Make a table of values showing the projected population every two years from 2006 to 2020.

   (e) Draw a graph illustrating the dependence.

3. Vladislav borrowed \$2,500 for the fall semester. Mention interest rate for the student loan 6.2%. Be sure to have rate of change calculated. *Often interest is applied monthly, but because he's not paying it back we're just using the equivalent annual rate, aka APR. More in section 6.x*

    (a) Questions here

    (b) x

    (c) x

    (d) x

    (e) No graph for this one.

4. Story here – perhaps the number of bacteria doubling? Say how it's 100% growth.

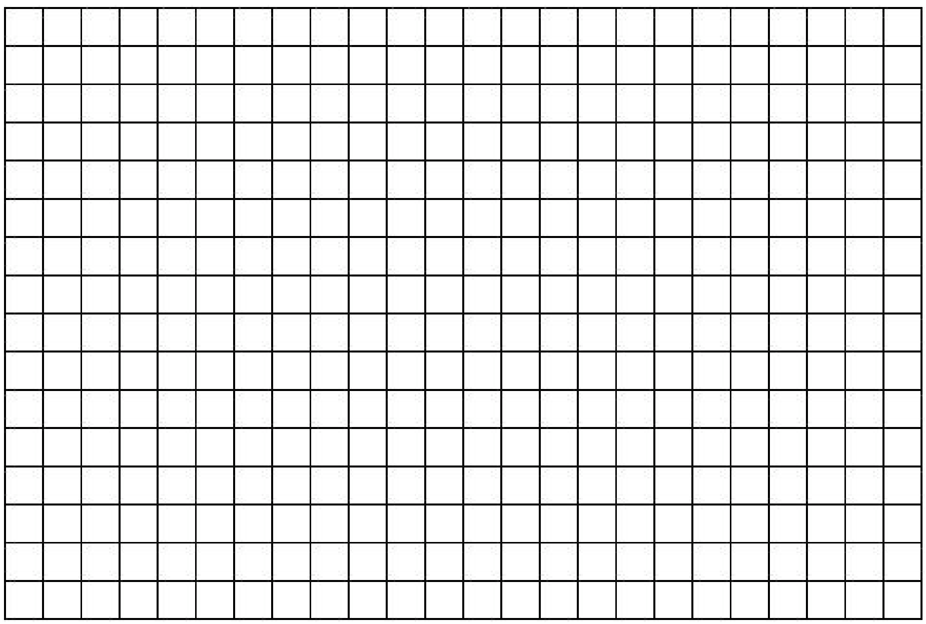
   (a) Questions here

   (b) x

   (c) x

   (d) x

   (e) Draw a graph illustrating the function



Do you know . . .

- What percent means and how to convert between percents and decimal?
- How to find the growth factor if you know the percent increase?
- How to calculate percent increase in one step?
- What makes an equation exponential?
- Where the starting value and growth factor appear in the standard form of an exponential equation?
- What the graph of an exponential function looks like.

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

SU: bring in exercises from 2.2 082503 and also look at 1.8 to be sure. Make sure there are rate of change problems here and interpolation. Be sure to use the word function occasionally and NOT range. Also, no approximations or solving yet.

SU: some of these problems should be shifted to 2.3 Using, 2.4 Approx, 3.3 Solving, or Ch. 5 on exponentials.

5. NAME has been comparing student loans. She plans to borrow $15,000 and will not be able to start payments for four years. Name the variables, including units, and write an equation for each annual percentage rate (APR) listed. SU this doesn't make sense? Do I want to change "four" to "a few"?

   (a) 5%

   (b) 12%

   (c) 2.4%

6. Estimates of SOMETHING are given by the following equations. SAY what variables mean. In each case identify the percentage change and calculate the value in 10 years. SU – do something like this in 2.1?

   (a) $A = 30(1.07)^Y$

   (b) $A = 30(1.017)^Y$

   (c) $A = 30(1.7)^Y$

7. Sales in our small business have grown very well from $138,495 in 2008, increasing 5.2% in 2009, increasing 6% in 2010, and 7.1% increase in 2011.

   - Calculate the sales in 2011. *Hint: find 2009, then 2010 first*
   - Draw a graph illustrating the dependence.
   - Calculate the rate of change for each year. Explain how to "see" the rate of change on the graph.
   - What does the rate of change estimate (linear interpolation) predict for 2012? Based on the graph do you think sales will be higher or lower than the rate of change prediction?

8. Mai's salary was $78,000 before she got a 6% raise. Now the economy was not doing as well and she got only a 1.5% raise this year.

   (a) What was her salary after the second raise?

   (b) Her colleague Deshawn started with a salary of $78,000 but did not get a raise the first year like Mai did. What percentage raise would he need now in order to have the same final salary as Mai?

   (c) Would Mai's salary have been the more than, less than, or the same as now if she had received the 1.5% raise first and then the 6% raise?

   (d) Which order would you rather have: 6% then 1.5% or 1.5% then 6%? Why?

9. There were 453 students at Elmwood Elementary school in 2010. The number of students is expected to increase by 4% each year. At that pace, how many students will there be in 2020?

10. In 1990 it was estimated that 2.5 million households watched reality television at least once a week. Executives predicted that number would increase by 7.2% each year. According to their estimates, how many millions of households watched reality television in 2000? In 2010? SU add real numbers??

11. My grandmother bought a set of sterling silverware for $800 in 1925. It has increased in value by 3% each year.

    (a) What was it worth in 1957 when she handed it down to my mother as a wedding present?
    (b) What was it worth in 1990 when my mother handed it down to me?
    (c) In 2003 I took out insurance on the silverware for up to $10,000 in value. Was that enough then? Is it still enough in 2010?

12. At a local college full-time tuition costs $37,000. Is continues to rise at 11% per year.

    - What do you expect the tuition to be in five years?
    - Name the variables, including units, and write an equation describing the dependence.
    - Make a table of values showing the tuition now, in 5 years, 10 years, 20 years, and 50 years (even though that's not realistic).
    - Draw a graph illustrating the function.

13. Repeat the previous problem assuming 10.5% increase per year instead. Add the graph to the previous graph.

14. At 8:00 p.m. after his first beer, Tom's blood alcohol content (BAC) was already up to 0.04. As Tom continued to drink, his BAC level rose 45% per hour.

    (a) Name the variables, including units, and write an equation illustrating how Tom's BAC is a function of time.
    (b) Make a table showing Tom's BAC at each hour from 8:00 p.m. to 2:00 a.m.
    (c) At a BAC of 0.10 it is illegal for Tom to drive. Approximately when does that happen?

15. SU: need more zobitz style problems. More interesting names and relevant stories

16. The Data One software company reported earnings of $42.7 billion in 2007. At that time executives projected 17% increase in earnings annually.

    (a) Name the variables and find an equation relating them.

(b) According to your equation, what would Data One's earnings be in 2015.

(c) If Data One reports earnings of \$78.1 billion in 2015, would you say the projected rate of 17% was too high or too low? Explain.

17. According to the U.S. Census, the population of the United States in 1990 was 248.7 million people.

(a) If the population increased 2% per year, what would it have been in the year 2000? In 2010?

(b) The actual population (as measured by the U.S. Census) was reported as 281.4 million people. Did the population increase at more than or less than the 2% predicted?

(c) Do you expect the reported population in 2010 to be higher or lower than your calculated value?

18. The total number of people with AIDS in the United States (A) can be approximated by the exponential function

$$A = 100,000(1.4)^t$$

where T is the number of years after 1989.

(a) What is the annual growth factor used in this equation? What is the corresponding percentage increase per year?

(b) According to this equation, how many people with AIDS were there in 1995? In 2005?

19. In a recent study of women who were 5'3" tall, the weight W of each woman measured in pounds was plotted against her age A years. These points fell approximately on the curve given by the formula

$$W = 90(1.012)^A$$

(a) According to this equation, what is the typical weight of 20 year old? A 40 year old?

(b) What is the significance of the number 1.012 in the equation?

(c) According to the equation, what value is W when $A = 0$? What does your answer mean in this context? Is is reasonable?

(d) According to the equation, what value is W when $A = 80$? What does your answer mean in this context? Is is reasonable?

(e) What might possibly be a reasonable range of values of women in this study?

20. The population of Buenos Aires, Argentina T years after 1950 is approximated by the equation

$$P = 5.0(1.026)^T$$

where the population P is measured in millions of people.

(a) Make a table of values showing the population of Buenos Aires every 20th year from 1950 to 2030.

(b) According to the equation, by what percentage has the population been increasing each year?

(c) By how many people has the population been increasing during each 20 year period? Add these calculations to your table. *Notice how this answer changes because the rate of change is not constant.*

(d) Draw a graph illustrating the function.

21. Bus fares are up to \$1.40 per ride during rush hour. SU: realistic? Two different plans of increasing fares are being debated: 10¢per year or 3% per year.

(a) Make a table comparing these two plans over the next ten decade. *A decade is ten years.*

(b) As a city council representative, you want to support the plan that your constituents prefer. If most of your constituents ride the bus, which plan should you support?

(c) If most of your constituents are members of the same union as the bus drivers (who count on solid earnings from the bus company to keep their jobs), then which plan should you support?

(d) Which type of equation is being used in each plan?

SU: maybe this is too many exercises. If so, save some for Using equations or Approximating solutions or Solving equations later.

SU: check exercises on the word version of 2.1 are there more there? Also, are there problems in the current 2.1 where the equation is given and we ask to interpret the slope, intercept from the equation????

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

2.3 Using equations

The Cadillac Escalade is a cross between a sports utility vehicle (SUV) and luxury car. Either way, it's a big car. And it takes awhile to stop. One study showed that the 2010 Escalade traveling at 60 miles per hour takes about 144 feet to come to a complete stop from when you first hit the brakes.

In fact, the braking distance of any car depends on how fast it is going. If you were driving 30 mph on a residential street, then it wouldnt take nearly as far to stop, for example. We would like to be able to calculate the braking distances at other speeds.

Let S be the speed, in miles per hour, and let F be the braking distance that it takes to stop, in feet. Using the data and equations from physics, automobile analysts were able to determine that the equation relating these two variables is

$$F = 0.04S^2$$

Remember that the 0.04 written next to the S^2 means they are multiplied. We might equally well have written

$$F = 0.04 * S^2$$

You may be a little surprised to see the variable S squared or wonder what the number 0.04 means. This equation is definitely not easily figured out because it relies both on the data and knowledge of the physics involved. But, we can still work with this equation to find the braking distances at any speed. (If you must know, this equation is only approximate since things like tire and road conditions are a factor, but for what we want itll be good enough.)

By the way, although in the last couple of sections we were able to find the equation for linear or exponential functions by generalizing from examples, in reality there are many different mathematical and statistical techniques for finding equations. A scientist might use lab experiments and some theory to figure it out. An economist might recognize that the equation is from a particular family because of the underlying economics. A store manager might know from years of experience that a certain equation works well to predict sales. It can be comforting to know where an equation comes from but whether we find an equation for ourselves or get it from an expert, we can use it to answer our questions and make predictions.

Now that we have an equation we can calculate the braking distance for a Cadillac Escalade traveling 30, 50, 70 or 90 miles per hour. For 30 miles per hour, we have $S = 30$. So, we substitute 30 in place of the S in the equation to get

$$F = 0.04(30)^2 = 0.04 \times \underline{30} \wedge 2 = 36 \text{ feet}$$

At 30 mph, it will take the Cadillac Escalade 36 feet to stop. As we expected, it doesnt take nearly as far to stop as it did at 60 mph.

For the other speeds we do the same thing: substitute the appropriate value of S. At 50 mph, $S = 50$ and so

$$F = 0.04(50)^2 = 0.04 \times \underline{50} \wedge 2 = 100 \text{ feet}$$

At 70 mph, $S = 70$ and so

$$F = 0.04(70)^2 = 0.04 \times \underline{70} \wedge 2 = 196 \text{ feet}$$

At 90 mph, $S = 90$ and so

$$F = 0.04(90)^2 = 0.04 \times \underline{90} \wedge 2 = 324 \text{ feet}$$

What does our equation tell us when the speed is 0 mph? We evaluate at $S = 0$ and so

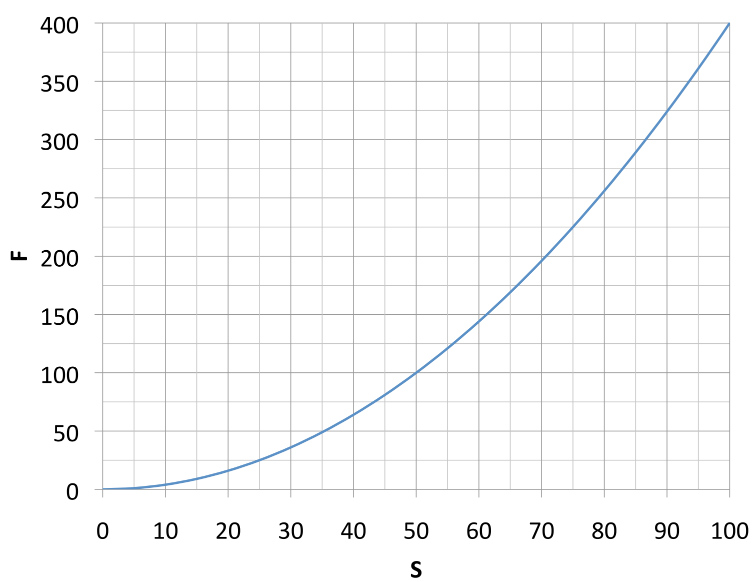
$$F = 0.04(0)^2 = 0.04 \times \underline{0} \wedge 2 = 0 \text{ feet}$$

Well, sure! If the car isn't moving, then it won't need any distance to stop.

A bit of terminology is useful here. When we substituted 30 in for S in the equation, we say we ***evaluated*** at $S = 30$. The verb evaluate is used when we know the value of the independent variable and we substitute it into the equation to find the value of the dependent variable.

We can summarize our data in a table and graph.

S	0	30	50	60	70	90
F	0.0	36	100	144	196	324



My neighbor Jeff happens to drive a 2010 Cadillac Escalade. The other day he almost was in an accident on the highway. Luckily no one was hurt, but he had to slam on the brakes to stop. The police report mentioned that the skid marks from Jeff's tires indicate that it took him 183 feet to stop. Jeff says he was not driving over the posted speed limit of 65 mph. Should we believe him?

We can see from the table that braking distance of 183 feet falls in between the 144 and 196 on our table which leads us to believe that Jeff was traveling faster than 60 mph and slower than 70 mph, but we can't answer the question yet. We can figure out if Jeff were driving at 65 mph, then his braking distance would have been

$$F = 0.04(65)^2 = 0.04 \times \underline{65}\wedge^2 = 169 \text{ feet}$$

That's less than the 183 feet Jeff took to stop. So, it appears that Jeff was driving faster than 65 mph.

But wait a minute. The braking distance is just the time it takes from when your foot hits the brake until the car is stopped. That distance doesn't take into account the reaction time – how long it takes between when see the situation and when your foot actually hits the brake. Suppose it takes 1.5 seconds to react. If we write D for the total stopping distance, measured in feet, then it turns out that a reasonable equation for D is

$$D = 1.5S + 0.04S^2$$

Something interesting to note about this equation is that the independent variable, S, appears twice. That means when we evaluate the equation we will need to plug in the value of S in two places. Let's do a few examples.

At 30 mph, $S = 30$ we have

$$D = 1.5(30) + 0.04(30)^2 = 1.5 \times \underline{30} + 0.04 \times \underline{30} \wedge 2 = 81 \text{ feet}$$

At 50 mph, $S = 30$ we have

$$D = 1.5(50) + 0.04(50)^2 = 1.5 \times \underline{50} + 0.04 \times \underline{50} \wedge 2 = 175 \text{ feet}$$

You can (and should) check the rest of the values in the table.

S	0	30	50	60	70	90
D	0	81	175	234	301	459

These numbers make us rethink Jeff's assertion. Given that he stopped in 183 feet, which is much less than the 234 feet it takes to stop at 60 mph, it looks like Jeff was driving less than 60 mph. If you like, calculate that at 65 mph (the speed limit), it would have taken Jeff

$$D = 1.5(65) + 0.04(65)^2 = 1.5 \times \underline{65} + 0.04 \times \underline{65} \wedge 2 = 266.5 \text{ feet}$$

Again, we should believe Jeff. And, be glad nobody was hurt.

Here's a graph of our new function

SU need graph here too.

Practice exercises

1. The amount of snow in a snowball, C cups, depends on the diameter (distance across) of the snowball, D inches according to the equation

$$C = 0.036D^3$$

   (a) How many cups of snow are needed to make a snowball that's 3 inches across?

   (b) How many cups of snow are needed to make a snowball that's 5 inches across?

   (c) How many cups of snow are needed to make a snowball that's 2 feet across?

   (d) Convert your answer to part c into gallons. *Recall that 1 gallon = 4 quarts, 1 quart = 4 cups*

2. Mom always said to sit close to the lamp when I was reading. The intensity of light L, measured in percentage (%) that you see from a lamp depends on your distance from the lamp, F feet as described by the formula

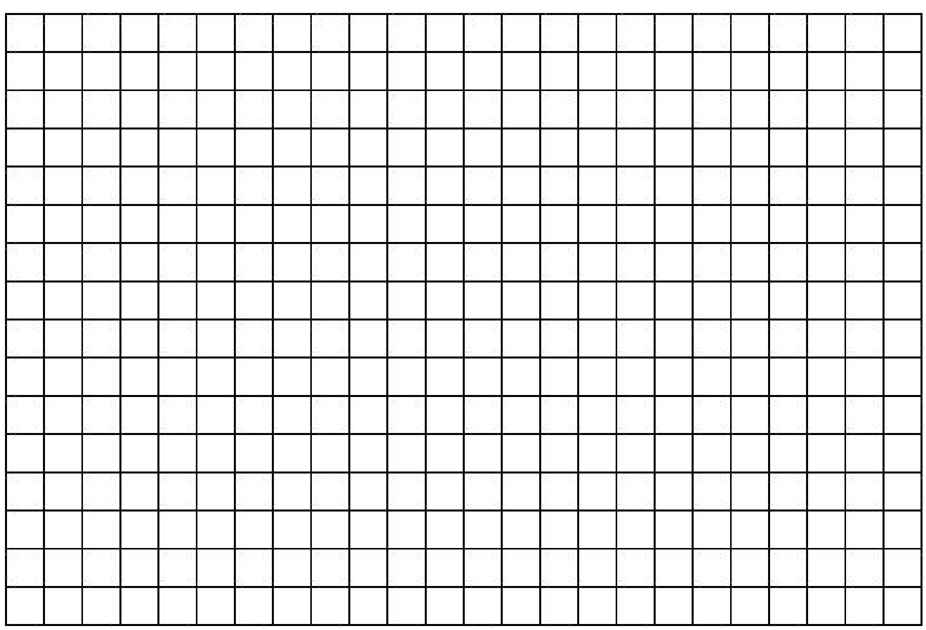
$$L = \frac{1}{F^2}$$

   (a) Calculate the density when I sit 1 foot, 2 feet, or 3 feet away.

   (b) The other day I was sitting 27 inches from the lamp. What was the intensity of the light there?

   (c) Calculate the rate of change of light intensity between 2 feet and 27 inches. What does that tell you in terms of the story?

   (d) Draw a graph illustrating the function



3. Our investment club has been tracking the value of each share of stock of a company over the past year. Using an econometrics software package, we were able to find the equation

$$V = 0.01W^3 - 0.95W^2 + 21W + 153$$

for $0 \leq W \leq 52$. This equation describes how the value of each share of stock $\$V$ is a function of the week W for the past year.

(a) Complete the following table of values (and check the values already entered):

W	0	13	26	39	52
V	153.00	287.42			82.28

(b) According to the table, what is the value of the stock now?

(c) According to the table, what was the value of the stock when the year began?

(d) According to the equation, if we bought some stock now and sold in in 10 weeks, would we make or lose money on the deal? *Hint: 10 weeks from now is not* $W = 10$.

(e) Draw a graph illustrating the dependence.

4. "Rose gold" is a mix gold and copper. Suppose we have 2 grams of an alloy that is equal parts gold and copper. We plan to add pure gold to it to create a lighter color. If we let

$$G = \text{gold added to alloy (grams)},\ P = \text{percentage of gold in alloy}$$

then

$$P = 100\left(\frac{1+G}{2+G}\right)$$

For example, if we don't add any gold then $G = 0$ and the percentage is

$$P = 100\left(\frac{1+0}{2+0}\right) = 100 \times (1+\underline{0}) \div (2+\underline{0}) = 50\%$$

as expected, since our original alloy is equal parts gold and copper.

(a) Calculate the percentage of gold in the alloy if we add 1 or 2 grams of pure gold.

(b) Is this function increasing, decreasing, or neither for $0 \leq G \leq 2$?

(c) Fill in the following table of values.

G	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
P	50.0		58.3		64.3			70.6	72.2		

(d) Draw a graph illustrating the function.

Do you know . . .

- Where equations come from?
- Where the dependent and independent variable (usually) are in an equation?
- What it means to evaluate?
- How to evaluate an equation when the independent variable occurs more than once?
- How to generate a table or graph from an equation?
- What graphs of different types of equations look like?
- ORDER of operations here – or where was it??

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

SU – check for (1) rate of change questions, (2) graphs, (3) follow up questions for 3.3 if power, 3.4 if a+b(x), and 2.4 otherwise
f power

5. If you drop a rock from a high place, the distance that it falls in T seconds is given by the equation $R = 16T^2$ where R is measured in feet.
   (a) How far does the rock fall in 2 seconds?
   (b) If you evaluate the equation at $T = 4$, what value of R do you get and what does it mean in the story?
   (c) Is the rock traveling faster during the first two seconds ($T = 0$ to $T = 2$) or during the second two seconds ($T = 2$ to $T = 4$)?
   (d) The rock was originally dropped from 300 feet above ground. Will it have hit the ground 4 seconds after it was dropped?
   (e) If you equation the equation at $T = 5$, what value of R do you get and what does it mean in the story?
   (f) If H represents the height of the rock after T seconds and still assuming that the rock was originally dropped from 300 feet above ground, write a new equation for H as a function of T.
   SU – follow up Solving Power

6. Mrs. Weber's cooking class came up with the equation

$$M = 7 + 4S + 1.2S^2$$

   to approximate the grilling time of a steak depending on its thickness. Here M is the number of minutes to grill the steak and S is the thickness of the steak (in inches).

(a) Evaluate the equation at $S = 0.25, 1.0, 1.5$, and 2.0.

(b) What do the answers you found mean in terms of cooking steak?

(c) Draw a graph showing how the cooking time depends on the thickness of steak.

(d) Your graph should show that the function is increasing. Explain how that makes sense in terms of the story.
SU – follow up in Approx Solns

7. Wynter has a pretty decent job. He is paid a salary of \$780 per week. His hours vary week-to-week. Even though Wynter is not paid by the hour, he can figure out what his hourly wage would be depending on the number of hours he works. For example, in a week where Wynter works 40 hours, he's earning the equivalent of \$19.50/hr because

$$\frac{\$780}{40 \text{ hours}} = 780 \div 40 = \$19.50/\text{hour}$$

(a) What's Wynter's equivalent hourly wage in a week when he works 50 hours? 60 hours?

(b) Name the variables and write an equation relating them.

(c) SU for later Wynter was complaining that things have been so busy lately at work that he's earning the equivalent of only \$x/hr. How many hours a week does that correspond to?
SU – follow up in Solving Power

8. Meagan just started a t-shirt printing company. To produce a particular pattern of t-shirt it costs her \$350 in materials and labor to set up a silkscreen and then \$7.50 for each shirt made to cover materials and printing. The average cost per t-shirt $\$C$ is a function of N, the number of t-shirts printed. The equation for this function is

$$C = \frac{350 + 7.50N}{N}$$

(a) Evaluate this formula when $N = 50$ and explain what the value of C you get means in the story.

(b) Explain in terms of the story why this function is decreasing. Sometimes this phenomena is referred to as "economy of scale."

(c) Make a table showing the average cost per t-shirt if Meagan makes 1, 20, 50, 100, or 300 t-shirts.

(d) Draw a graph of the function.
SU – follow up in Approx Solns
How many print to keep average cost at \$x? What if \$x instead. Do I also want to add on revenue here

9. One measure of the diversity of our news source is the count of the number of different daily newspapers in circulation. A reasonable equation estimating this count over the past century is

$$N = -0.0021T^3 + 0.34T^2 - 20T + 2,226$$

where N is the number of daily newspapers in circulation in the United States T years after 1900.

(a) Based on this equation, how many daily newspapers were in circulation in 1920? In 1955? In 1995? In 2010?

(b) During which period were the number of newspapers in circulation dropping faster: 1900 to 1920 or 1995 to 2010?

(c) Draw a graph illustrating the dependence.
SU – follow up in Approx Solns

10. Geoffrey plans to start cultivating orchids. He knows that they are fragile plants and estimates that there is a probability P that each plant will survive. He will consider his greehouse a success if at least nine of the orchids survive. According to probability theory, the probability Geoffrey's greenhouse will be a success S is given by the formula
$$S = 10P^9 - 9P^{10}$$
Technically this formula assumes that the success of the orchids are "independent" which might not be the case.

    (a) If the orchids are 100% perfect (with probability $P = 1$ of surviving), then what is the probability of a successful greenhouse? Explain how your answer is to be expected.

    (b) If the orchids are complete duds (with probability $P = 0$ of surviving), then what is the probability of a successful greenhouse? Explain how your answer is to be expected.

    (c) Make a table showing the probability of a successful greenhouse if the probability of each orchid surviving is $0, 0.2, 0.4, 0.6, 0.8,$ or 1.0.

    (d) Draw a graph of the data.
    SU – follow up in Approx Solns 90% change of success

11. Bill and Michelle SU BETTER NAMES borrowed money to buy a hour. To repay this loan (called a "mortgage"), they owe \$900 every month for 30 years. Based on this information and some mathematics of finance, after M payments are made (that is, after M months), Bill and Michelle will still owe \$$P$ where
$$P = 180,000 - 30,000(1.005)^M$$
The number P is sometimes referred to as the "payoff" amount since at any time Bill and Michelle can pay P and have "paid off" the loan. (By the way, this equation was calculated based on 6% interest compounded monthly. More on that later in the course. SU WHEN?)

    (a) How much did Bill and Michelle originally borrow to buy their house? What value of M did you to answer the question?

(b) Evaluate the equation at $M = 12$ and explain what the answer means in terms of the story.

(c) After making half the payments, how much money will Bill and Michelle still owe on the house? Will they have paid more or less (or exactly) half of the loan? *Hint: convert 30 years into months to find the total number of payments. Then divide by 2 to find the halfway point.*

(d) The very last month they don't actually pay the full \$900, just whatever balance is left on the loan. How much will that be? *Hint: they will have made all but one of the payments.*

SU – follow up in Solving Other

SU – do we want some other "Other" problems? Do we really want that section at all? If not, then maybe want to have some of this type in the Solving Exponential and Solving Power sections.

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012
TOSS:
SU – maybe just eliminate this one. Could put the thing about three second rule in for the escalade. The 2002 Chevrolet Tahoe 4WD will take about 158.1 feet to stop when traveling at 60 mph in normal highway conditions. Let S be the speed at which the vehicle is traveling, in miles per hour (mph), and D the distance it takes to stop, in feet. This information together with a little physics gives the equation

$$D = 0.0439S^2$$

1. According to this equation, how many feet does it take to stop the Tahoe when traveling 80 miles per hour?

2. In driver's training classes they teach the "two-second rule" for safety: you should follow no closer than two seconds behind the car in front of you. If you are traveling 80 miles per hour, how many feet can you travel in two seconds? *Hint: convert to feet per second, then multiply by two seconds.*

3. Compare your results from parts a and b of this problem to decide if the "two-second rule" is adequate for safety at 80 miles per hour. That is, if you are following two seconds behind the car in front of you and calamity strikes that car, will you be able to stop before hitting it?

4. Is the "two-second rule" adequate at 50 mph?

How fast water runs through a pipe is related to the length of the pipe by the equation $F = \frac{20}{L}$ where F is the flow rate, measured in feet/min, L is the length of pipe, measured in feet. SU does this even make sense physically? Should it be an inverse square related to the width? Something about force?

2.4 Approximating solutions of equations

Which country on Earth has the most people? If you guess China and India, in that order, you'd be right. And by a lot compared to other countries. Number three on the list in 2010 was the United States. Here are the numbers from the CIA Factbook:

Country	Population (2010)	Growth Rate
China	1,330,141,295	0.494%
India	1,173,108,018	1.376%
United States	310,232,863	0.97%

Notice that in comparison to China, the United States has only a fraction of the population. In fact, if we compare

$$\frac{\text{pop China}}{\text{pop U.S.}} = \frac{1,330,141,295}{310,232,863} = 4.287\ldots \approx 4.3$$

so the population of China is around 4.3 times that of the United States.

With the projected growth rates listed, when will the United States' population pass 400 million? When will China's population pass the 1.5 billion mark? Will India's pass it first? Let's tackle each question in turn.

We can measure the population in millions or billions. Either way we will have to convert some of the numbers. Let's use millions.

$$P = \text{population (measured in millions)}$$
$$Y = \text{year (years since 2010)}$$

The United States population began in 2010 at 310,232,863 which is approximately 310 million because

$$310,232,863 \text{ people} * \frac{\text{millions}}{1,000,000} = 310232863 \div 1000000 = 310.232863 \approx 310$$

Or, if you're in the mood to use scientific notation instead

$$310,232,863 \text{ people} * \frac{\text{millions}}{1 \times 10^6} = 310232863 \div (1 \times 10 \wedge 6) = 310.232863 \approx 310$$

Notice how we needed parentheses

As we saw in Section 2.2 SU CITE, since the United States population growth rate is 0.97%, each year we would have 100.97% of our previous population. That is, we multiply by the growth factor 1.0097. Then, our equation for the United States population is

$$P = 310(1.0097)^Y$$

Since we rounded off the population to the nearest million we will need to be sure to round off each of our calculations as well. Notice that we did not round off the growth factor of 1.0097 because that would lead to too much round off error.

Our first question asks when the U.S. population will pass 400 million. Let's try to figure out the answer by guessing. Since we're not sure where to start, let's see what the equation projects for 2011:

$$310(1.0097)^1 = 310 \times 1.0097 \wedge \underline{1} = 313.007 \approx 313 \text{ million}$$

Hardly budged. We'll have to make a much larger guess to get up near 400 million.

How about in 10 years, in the year 2020? The equation gives us

$$310(1.0097)^{10} = 310 \times 1.0097 \wedge \underline{10} \approx 342 \text{ million}$$

Still much less than 400 million.

By the way, if we had used the precise population of 310.232863 million people, we would estimate 341.67355263010 million here instead of 341.41709 million. As you can see these two numbers differ after the decimal point. That's why it's important to round off our answer. Even then, we probably should have 342 instead of 341 – but let's not be that picky about it.

Let's try 20 years, in the year 2030.

$$310(1.0097)^{20} = 310 \times 1.0097 \wedge \underline{20} \approx 376 \text{ million}$$

Still less.

This is going slowly. We would really like to find a point at which the equation gives us more than 400 million. Then we can work backwards from there to narrow things down. How about 50 years?

$$310(1.0097)^{50} = 310 \times 1.0097 \wedge \underline{50} \approx 502 \text{ million}$$

That's too much, but the good news is now we know the answer is between 20 years and 50 years.

Let's summarize what we have so far in a table. Notice how we've added a third row to keep track of our progress for our goal.

Y	0	1	10	20	50						
P	310	313	341	376	502						
vs. 400	low	low	low	low	high						

We know the answer is between 20 and 50 years, and it seems closer to 20, so let's guess 30 years which gives $\approx$ 414 million as you can check. That means the answer is between 20 years (a bit low) and 30 years (a bit high), so let's split the difference and guess 25 years which gives $\approx$ 395 million. Ooooh, we're getting close. It's between 25 and 30 years, and likely closer to 25 so let's guess 27 years which gives $\approx$ 402 million. Would 26 years have been enough? That gives $\approx$ 398 million, not quite enough. Let's add these numbers to our table.

Y	0	1	10	20	50	30	25	27	26
P	310	313	341	376	502	414	395	402	398
vs. 400	low	low	low	low	high	high	low	a bit high	a bit low

According to our equation, the population of the United States should pass 400 million in about 27 years, which would be in the year 2037. By the way, if you're not sure how we got the year 2037, it works to add the numbers

$$2010 + 27 \text{ years } = 2037$$

SU graph with points labelled here, maybe horizontal line at value 400 and the words high and low written into their regions?

The overall strategy we used here is called ***successive approximation***. That's just a fancy way of saying "guess-and-check". It's called "successive" because we're trying to get a closer guess each time. Typically once we have a value that's too big and one that's too small, we guess a value in between (for example, their average). This sort of splitting the difference method of guessing is a rough version of the ***bisection method***. Now you know.

You might be surprised that you're supposed to guess the answer at this point in the course. I mean, in the beginning of the course we didn't have equations, just tables and graphs, and so guessing was all we had to work with. But now we have actual equations, right? In previous courses your instructor or textbook might have emphasized getting the "exact" answer.

Here's why it's different in this course. First, in almost every story in this book the numbers in the problem are approximations, or at least rounded off. If you start with approximations, no matter how exact your mathematics is, the answers will still be approximate.

Second, even if our numbers started out precisely exact, chances are that the equation is only approximating reality. Do we really know what the population growth rate will be in the U.S. over the next fifty years? And, if the equation is just approximate, then no matter how exact the numbers or the mathematics, the answer will again still be approximate.

Last, and this is good news – we really just want approximations. Do you really need to know that working out will burn 427.2889 calories? Isn't 430 calories close enough?

In previous mathematics courses you may have seen ways to solve equations "exactly," and we will talk about those methods in the next chapter of this text. It is true that successive approximations can taking a long time and, because of that, is a bit annoying. Solving techniques we'll learn later are much, much quicker.

But there are two important reasons for using successive approximations. First, the method of successive approximations works in most situations for any type of equation. Solving methods that we will see later on just work for one type of an equation or another

– one technique for linear equations, a different technique for exponential equations, etc. That's a lot of different methods to know.

Second, even if you're going to use a formal equation-solving technique to solve a problem it's a good habit to guess-and-check a bit first to make sure your answer is reasonable. It is easy to make mistakes when using those formal techniques. There's a rather famous quote here that I like, often falsely attributed to the famous economist John Maynard Keynes, but reportedly from logician Carveth Read in 1898.

> It is better to be vaguely right than exactly wrong.

Something to think about.

Okay, enough digression. Let's finish up our example. In 2010 the population of China was reported to be 1,330,141,295 and growing at 0.494% each year. Converting to millions we get the original population of approximately 1,330 million people. The growth rate of 0.494% means each year there will be 100.494% of the previous year and, so, the growth factor should be 1.00494. So the equation for China's population is

$$P = 1,330(1.00494)^Y$$

We are curious when China's population will exceed 1.5 billion. Oh shoot. That's billions and we need millions to use our equation. We'll have to convert. Maybe we should have used billions all along, but at this point we're kinda committed to millions, so here goes.

$$1.5 \text{ billion } * \frac{1 \times 10^9}{1 \text{ billion}} * \frac{1 \text{ million}}{1 \times 10^6} = 1.5 \times 10 \wedge 9 \div 10 \wedge 6 = 1,500$$

Does that make sense?

$$1.5 \text{ billion } = 1,500,000,000 \text{ and } 1,500 \text{ million} = 1,500,000,000$$

Okay.

Let's start with a guess of 30 years because that worked well last time

$$P = 1,330(1.00494)^{30} = 1330 \times 1.00494 \wedge 30 \approx 1,527$$

Since that turns out to be higher than we're looking for (though not too much higher), we can guess in between values as before. Here's our work displayed in a table.

Y	0	30	20	25	27	26
P	1,330	1,527	1,468	1,490	1,504	1,497
vs. 1,500	low	high	low	low	a bit high	a bit low

Since

$$2010 + 26 \text{ years } = 2036$$

we estimate that the population of China should pass 1.5 billion in the year 2036.

Will India get there first? Remember India's population in 2010 was 1,173,108,018 and growing at 1.376% per year. The equation for India's population is approximately

$$P = 1173(1.01376)^Y$$

Notice where the decimal point is in our growth factor. Since 1.376% increase means there will be 101.376% of the previous year, our growth factor is 1.01376 as in our equation.

Here's a table of our guesses for India's population.

Y	0	30	20	15	19	18
P	1,173	1,720	1,542	1,440	1,521	1,500
vs. 1500	low	high	high	low	a bit high	yes!

It looks like India's population will reach 1.5 billion by the year 2028 – eight years before China.

Which raises a new question. When will India's population pass China's? Let's put our guess together in a table. We need to keep our equations straight:

$$\textbf{China: } P = 1,330(1.00494)^Y \quad \text{and} \quad \textbf{India: } P = 1,173(1.01376)^Y$$

Y	0	20	10	15	13	14
China: P	**1,330**	1,468	**1,397**	1,432	**1,418**	**1,425**
India: P	1,173	**1,542**	1,345	**1,440**	1,401	1,420
larger	China	India	China	India	China	China

It appears that the population of India will pass the population of China around the year 2025.

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- Questions?

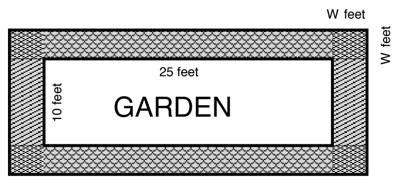
If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

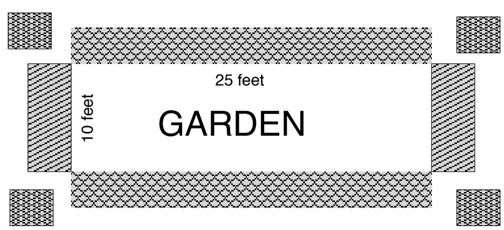
5. xx

   (a) xxx

6. Urban gardens are catching on. What was once an abandoned lot down the block is now houses a thriving vegetable and berry garden for the neighborhood that's 10 feet wide and 25 feet long. One of the neighbors works for a landscaper who has volunteered to donate some gravel to make a path around the garden. Typically such a path would be 3 inches deep and the same width all around. Let's write $W =$ width of path (feet) and $G =$ amount of gravel (cubic feet).



   (a) To find the amount of gravel needed, imagine the path being broken into pieces.



   If each of the four corner squares is W feet by W feet, each of the two long segment is 25 feet by W feet, and each of the two short segment is 10 feet by W feet, what is the total area of the path?

   (b) The path is 3 inches deep. Convert 3 inches into feet.

   (c) The amount (volume) of gravel they need is the product of the total area and the depth. Multiply your answers and simplify to get

$$G = W^2 + 17.5W$$

   Even if you can't get this equation, use it to do the rest of this problem.

   (d) How many cubic feet of gravel would they need to make the path 2 feet wide? 3 feet wide? 4 feet wide?

   (e) If the neighbor donates 60 cubic feet of gravel, how wide a path can they build? Report your answer to two decimal places.

   (f) Convert your answer to feet and inches. Do you think that's a wide enough path?

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

SU – check PROJECTION not PREDICTION (latter assumes human behavior) SU – check that all succ approx has the vs. line in the table

2.5 Finance formulas*

Addison is trying to figure out her finances – finding a good investment for her tax refund, saving for a down payment on a house, and dealing with her student loans. While there are various online tools that will "do the math" for her, Addison would really like to work out the formulas for herself.

First that tax refund. What a relief, $1,040 back this year. Much as Addison is tempted to catch the next flight to Cancun play on the beach, she knows she should save that money. Her local bank offers her two choices: a savings account paying 1.2% interest compounded monthly or a 3-year certificate of deposit paying 3.0% interest compounded monthly.

Compounded monthly means that the bank will pay her 1/12th of the annual interest each month, and then use that new balance in computing her interest in the month that follows.

For example, the savings account will pay Addison $\frac{1.2\%}{12} = 0.1\%$ each month. As we know from our study of exponential equations, to increase by 0.1% we multiply the balance each month by the monthly growth factor $1 + \frac{0.1}{100} = 1.001$.

Since we know the monthly growth factor, it would make sense to measure time in months. It is customary, however, to measure time in years instead. No big deal. To get the number of months we just multiply by 12. So, after Y years, we have $12Y$ months.

With those subtleties in mind, Addison's account balance of $\$A$ is given by the equation

$$A = 1040(1.001)^{12Y}$$

After one year, Addison would have

$$1040(1.001)^{12} = 1040 \times 1.001 \wedge 12 = 1052.54887\ldots \approx \$1052.55$$

After three years, she would have a not terribly impressive

$$1040(1.001)^{12*3} = 1040 \times 1.001 \wedge (12 \times 3) = 1078.10269\ldots \approx \$1078.10$$

This is complicated. It is probably easier to work with a general formula rather than having to figure it out the equation each time. The general formula that gives an account balance when interest is compounded monthly is

MONTHLY COMPOUND INTEREST FORMULA

$$A = P\left(1 + \frac{r}{12}\right)^{12Y}$$

where the variables are

$$A = \text{account balance (\$)}$$
$$Y = \text{time invested (years)}$$

and the constants we have to plug in are

$$P = \text{initial deposit or "principal" (\$)}$$
$$r = \text{interest rate compounded monthly (as a decimal)}$$

For Addison's savings account we have $p = \$1,040$ and $r = 1.2\% = 0.012$, so we get

$$A = 1040\left(1 + \frac{.012}{12}\right)^{12Y}$$

Since $1 + \frac{.012}{12} = 1.001$, we have the same equation as before. If we use this general form of the equation to find her account balance after one year we have

$$1040\left(1 + \frac{.012}{12}\right)^{12*1} = 1040 \times (1 + .012 \div 12) \wedge (12 \times \underline{1}) = 1052.54887\ldots \approx \$1052.55$$

and after three years we have

$$1040\left(1 + \frac{.012}{12}\right)^{12*3} = 1040 \times (1 + .012 \div 12) \wedge (12 \times \underline{3}) = 1078.10269\ldots \approx \$1078.10$$

as before. And for something new, after six years Addison's balance will be

$$1040\left(1 + \frac{.012}{12}\right)^{12*6} = 1040 \times (1 + .012 \div 12) \wedge (12 \times \underline{6}) = \$1117.60135 \approx \$1117.60$$

But wait. Addison might want to choose that certificate of deposit instead. That pays 3.0% interest compounded monthly, so now $r = 3.0\% = .03$. Equipped with our fancy new formula, we calculate that this account balance would be given by

$$A = 1040\left(1 + \frac{.03}{12}\right)^{12Y}$$

After one year Addison would have

$$1040\left(1 + \frac{.03}{12}\right)^{12*1} = 1040 \times (1 + .03 \div 12) \wedge (12 \times \underline{1}) = 1071.6326\ldots \approx \$1071.63$$

after three years she would have

$$1040\left(1 + \frac{.03}{12}\right)^{12*3} = 1040 \times (1 + .03 \div 12) \wedge (12 \times \underline{3}) = 1137.81346\ldots \approx \$1137.81$$

and after six years

$$1040\left(1 + \frac{.03}{12}\right)^{12*6} = 1040 \times (1 + .03 \div 12) \wedge (12 \times \underline{6}) = \$1244.82641\ldots \approx \$1244.83$$

It looks like the certificate of deposit is a clear winner, but there is a catch. If Addison wants her money before the three year term is up, she'll lose all (or most) of the interest earned. Ouch. So Addison should decide not only based on what the accounts pay – \$1,078.10 after 3 years in the savings account versus \$1,137.81 after three years in the certificate of deposit (a whopping \$59.71 more) – but also on whether she is comfortable leaving the money alone for three years or not.

Unimpressed by the \$59.71 difference after three years and uncomfortable locking her money in for that long, Addison decides on the savings account. When she reads the account information carefully she is surprised to see the account pays "1.207% APR." What does that mean?

APR stands for "annual percentage rate". It means that 1.2% interest compounded monthly has the same net effect as paying 1.207% at the end of each year. Where does that number come from? Imagine \$1 in the account. Silly, yes, but watch what we learn. After one year the balance would be

$$1\left(1+\frac{.012}{12}\right)^{12*1} = (1+.012\div 12)\wedge 12 = 1.01206622\ldots$$

That tells us the annual growth factor is $1.01206622\ldots \approx 1.01207$ which corresponds to an annual growth rate of $.01207 = 1.207\%$ APR. There's a formula for this too.

Equivalent Annual Percentage Rate (APR) Formula

$$\text{APR } = \left(1+\frac{r}{12}\right)^{12} - 1$$

To check our work, use $r = 1.2\% = .012$ to get

$$\text{APR } = (1+\underline{.012}\div 12)\wedge 12 - 1 = .01206622 \approx .01207 = 1.207\%$$

All this thinking about savings reminds Addison that she wants to own her own place someday. She promised herself that she would start putting away some money each month to save for a down payment on a house, or maybe condo. Living back in her parent's house rent-free, postponing buying her first car, and bringing lunch from home most days leaves Addison nearly \$1,000 per month to save. Her bank offers a special savings account paying 4.2% compounded monthly if she commits to depositing \$1,000 every month.

Suppose Addison deposits \$1,000 to open the account at the end of the month. At the end of the next month, the account adds $\frac{4.2\%}{12} = .35\%$ so we multiply by $1+\frac{.35}{100} = 1.0035$ to add in the interest and add another \$1,000 deposit, bringing the balance to

$$1000(1.0035) + 1000 = \$2003.50$$

At the end of the third month, we add another .35% interest (this time on the \$2003.50) and then add another \$1,000 deposit to get

$$2003.50(1.0035) + 1000 = \$3010.51$$

Any sequence of regular deposits is called an **annuity**. The general formula that gives the account balance for an annuity is

Value Monthly Deposits Formula

$$A = P\,\frac{\left(1+\frac{r}{12}\right)^{12Y} - 1}{\frac{r}{12}}$$

where the variables are

$$A = \text{account balance (\$)}$$
$$Y = \text{time invested (years)}$$

and the constants we have to plug in are

$$P = \text{regular deposit (\$)}$$
$$r = \text{interest rate compounded monthly (as a decimal)}$$

In Addison's situation, we have $P = \$1000$ and $r = 4.2\% = .042$ so the equation becomes

$$A = 1000\,\frac{\left(1+\frac{.042}{12}\right)^{12Y}-1}{\frac{.042}{12}}$$

Let's check our calculations for the end of three months. Remembering that time is measured in years, we convert

$$3 \text{ months } = 3 \cancel{\text{months}}\,\frac{1 \text{ year}}{12 \cancel{\text{months}}} = \frac{3}{12} \text{ years} = .25 \text{ years}$$

That means $Y = .25$ and so

$$A = 1000\,\frac{\left(1+\frac{.042}{12}\right)^{12*.25}-1}{\frac{.042}{12}}$$

$$= 1000 \times ((1 + .042 \div 12) \wedge (12 \times \underline{.25}) - 1) \div (.042 \div 12)$$

$$= 3010.5123\ldots \approx \$3010.51$$

as before. Notice how we need parentheses not only where they appear in the formula, but also around the entire numerator (top) of the fraction, around the entire denominator (bottom) of the fraction, and around the exponent. That's going to take some practice to get used to.

Let's practice by finding her balance after two years. Now $Y = 2$ so

$$A = 1000\,\frac{\left(1+\frac{.042}{12}\right)^{12*2}-1}{\frac{.042}{12}}$$

$$= 1000 \times ((1 + .042 \div 12) \wedge (12 \times \underline{2}) - 1) \div (.042 \div 12)$$

$$= 24991.2560\ldots \approx \$24{,}991.26$$

Wow. She'll be buying her own house in no time.

Oh, but wait, there's those looming student loans. Addison currently owes $16,700 at 5.75% interest compounded monthly. She's ready to start paying it back every month, which means this loan repayment is another example of an annuity. The general formula that gives the monthly payment due for an annuity is

MONTHLY LOAN PAYMENT FORMULA

$$P = \frac{A * \frac{r}{12}}{1 - \left(1 + \frac{r}{12}\right)^{-12Y}}$$

where the variables are

$$P = \text{regular payment (\$)}$$
$$Y = \text{time invested (years)}$$

and the constants we have to plug in are

$$A = \text{loan amount (\$)}$$
$$r = \text{interest rate compounded monthly (as a decimal)}$$

In Addison's situation, $A = 16700$ and $r = 5.75\% = .0575$ so the equation is

$$P = \frac{16700 * \frac{.0575}{12}}{1 - \left(1 + \frac{.0575}{12}\right)^{-12Y}}$$

If she wants to pay back the loan in two years ($Y = 2$), she will need to pay

$$P = \frac{16700 * \frac{.0575}{12}}{1 - \left(1 + \frac{.0575}{12}\right)^{-12*\underline{2}}}$$

$$= (16700 \times .0575 \div 12) \div (1 - (1 + .0575 \div 12) \wedge (-12 \times \underline{2}))$$

$$= 738.2744\ldots \approx \$738.27$$

That \$738.27 per month payment on her student loan won't leave much money for her to save for that down payment.

If she takes the full five years to pay back the loan (as allowed in her loan agreement), then we plug in $Y = 5$ to get her new monthly payment of

$$P = \frac{16700 * \frac{.0575}{12}}{1 - \left(1 + \frac{.0575}{12}\right)^{-12*\underline{5}}}$$

$$= (16700 \times .0575 \div 12) \div (1 - (1 + .0575 \div 12) \wedge (-12 \times \underline{5}))$$

$$= 320.9200\ldots \approx \$320.92$$

That's more like it, \$320.92 per month.

Exercises

1. Use the indicated formulas to help Bill figure out his finances.

   (a) Bill deposited \$2,500 in a money market account that earned 7% interest compounded monthly. Use the MONTHLY COMPOUND INTEREST FORMULA to calculate his account balance after 4 years.

   (b) What is the equivalent APR on Bill's money market account? Use the EQUIVALENT APR FORMULA.

   (c) Bill puts \$400 a month in his retirement account that amazingly also earns 7% interest compounded monthly. Use the VALUE MONTHLY DEPOSITS FORMULA to determine how much Bill will have in his retirement account in 28 years.

   (d) Bill would really like to buy a new hybrid car that sells for \$23,500. Sadly Bill's credit rating is not very good, so the best the dealership offers is a loan at – you guessed it – 7% interest compouned monthly. Use the MONTHLY LOAN PAYMENT FORMULA to calculate his monthly car payments on a six year loan.

2. If you invest \$35,000 for three years, how much will you have if it earns each of the following rates compounded monthly? *Use the* MONTHLY COMPOUND INTEREST FORMULA.

   (a) 6%

   (b) 11%

   (c) 1.9%

3. Use the EQUIVALENT APR FORMULA to find the APR for each of the following published interest rates (compounded monthly) offered by recent credit card companies.

   (a) 9%

   (b) 12.8%

   (c) 20.19%

4. How much will Addison be able to save for a down payment now that she has to pay off her student loans? Assume she can deposit \$1,000- \$320.92 = \$679.08 per month into that special savings account paying 4.2% compounded monthly for two years. Use the VALUE MONTHLY DEPOSITS FORMULA. What if she saves for five years instead?

5. Dwight and Rhianna owe \$192,000 on their house.

   (a) They are offered a chance to refinance at 3.5% compounded monthly on a 30-year mortgage loan. Calculate the corresponding monthly payment be (ignoring taxes, insurance, escrow, etc.) using the MONTHLY LOAN PAYMENT FORMULA.

   (b) Or, they can refinance at 3.25% compounded monthly on a 15-year mortgage. Calculate the new corrresponding monthly payment (ignoring taxes, insurance, escrow, etc.) using the MONTHLY LOAN PAYMENT FORMULA.

6. Make a table showing the balance now, after 1 year, after 5 years, and after 12 years if $10,000 sits in a certificate of deposit earning 4.77% interest compounded monthly. *Use the* MONTHLY COMPOUND INTEREST FORMULA.

7. Sarah and Alison are saving for their kids' college in fifteen years. The account pays the equivalent of 5.4% interest compounded monthly (taking into consideration various tax incentives). Make a table showing how much they will have after fifteen years if every month they contribute $100, $500, or $1,000. Use the VALUE MONTHLY DEPOSITS FORMULA.

8. Cesar and Eliana are looking at three different houses to buy. The first, a large new townhouse, for \$240,000. The second, a small but charming bungalow, for \$260,000. The third, a large story house down the block, for \$280,000.

   (a) Calculate the monthly payment for each house for a 30-year mortgage at 3.5% interest compounded monthly. Use the MONTHLY LOAN PAYMENT FORMULA.

   (b) Describe the affect of paying \$20,000 more at this interest rate.

Answers to Exercises

1. (a) \$3,305.13 (b) 7.23% (c) \$415,475 (d) \$400.65

2. (a) \$41,883 (b) \$48,610 (c) \$37,051

3. (a) 9.38% (b) 12.58% (c) 22.17%

4. For 2 years: \$16,971 and for 5 years \$45,251.

5. (a) \$862.17 (b) \$1,349.12

6.

Y	0	1	6	12
A	10,000.00	10487.57	12,687.44	17,705.06

7.

P	100	500	1000
A	27,640.60	138,203.01	276,406.03

8. (a)

A	240,000	260,000	280,000
P	1,077.71	1,167.52	1,257.33

(b) Increases monthly payment by \$89.81

Practice exams on Equations

Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

Practice exam 2– version I

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

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1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

Practice exam 2 – version II

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

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1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

Chapter 3

Solving equations

Yada yada

3.1 Solving linear equations

Your kitchen sink keeps getting clogged. Very annoying. Last time the plumber was able to fix it pretty quickly, well ahead of when your dinner guests were due. But now the sink is clogged again. This time when the plumber comes and unclogs the sink, he suggests replacing the trap and a few other things that were causing the problem. You are pretty tired of it clogging up and tell him to "go ahead." While you're glad that the sink works when he's done, you're a bit shocked when his bill arrives a few days later for \$278.75. Does that seem right?

Remember our plumber charged \$100 for just showing up and then \$75 per hour for the service. Using the variables T for the time the plumber takes, measured in hours, and P for the total plumber's charge, measured in dollars, we found that the equation was

$$P = 100 + 75T$$

Let's figure out how many hours of work would add up to a bill of \$278.75. Our first approach might be to look at a table. From earlier we had

T	0	$\frac{1}{2}$	1	2	3
P	100.00	137.50	175.00	250.00	325.00

Since \$278.75 is between \$250.00 and \$325.00, we see that the time must be between 2 and 3 hours. You remember the plumber being there over 2 hours, so this is certainly a reasonable answer. Well, a lot of money, but mathematically it makes sense.

Still curious, you'd like to know exactly how many hours and minutes he worked. We could use successive approximations.For example, for 2.5 hours the bill would have been

$$100 + 75 \times \underline{2.5} = \$287.50$$

which is more than our bill. Continuing to guess and check, and displaying our work in a table, we get

T	2	3	2.5	2.3	2.4	2.35	2.37	2.38
P	250.00	325.00	287.50	272.50	280.00	276.25	277.75	278.50
vs. 278.75	low	high	high	low	high	low	low	close enough

So now we know that the plumber took approximately 2.38 hours.

Converting units we calculate

$$.38 \text{ hours} * \frac{60 \text{ minutes}}{1 \text{ hour}} = .38 \times 60 = 22.8 \approx 23 \text{ minutes}$$

The plumber took about 2 hours, 23 minutes. Thinking back, the plumber had arrived around 10:30 in the morning and stayed past lunch, probably until around 1:00 p.m. That's about right.

Wait a minute! We could have figured this out much more quickly. If the bill was \$278.75, we know the first \$100 was the trip charge. That leaves

$$\$278.75 - \$100.00 = \$178.75$$

in hourly charges. At \$75 per hour that comes to

$$\$178.75 * \frac{1 \text{ hour}}{\$75} = 178.75 \div 75 = 2.388\ldots \approx 2.388 \text{ hours}$$

which comes to around 2 hours, 23 minutes as before. See how we used the \$75/hour as a unit conversion here? Very clever.

That worked well. But, can we figure out this sort of calculation in other problems? What is the general method we're using? Can we write down our method in an organized fashion so that someone else could follow our thinking here? Turns out there is a formal way to show this calculation, called ***symbolically solving the equation***. Officially *any* method of getting a solution to an equation is considered solving the equation, but in the rest of this book, and in most places that use algebra, when we refer to "solving the equation" we mean *symbolically*.

Here's how it works. We want to figure out when $P = 278.75$. We know from our equation that $P = 100 + 75T$. Replace P by $100 + 75T$ in the equation to get

$$100 + 75T = 278.75$$

Remember that the equal sign indicates that the two sides are the same number. On the left-hand side we have $100 + 75T$. On the right-hand side we have 278.75. Looks different, but same thing. That is, we're looking for the value of T that makes these two sides the same number.

The first thing we did to figure out the answer was subtract the \$100 trip charge. In this formal method, we can subtract 100 from each side of our equation. I mean, if the left-hand side and the right-hand side are the same number, then we sure better get the same answer when we take away 100 from each side, right? When we subtract 100 from each side we get

$$\begin{aligned} \cancel{100} + 75T &= 278.75 \\ -\cancel{100} \quad &\quad -100 \end{aligned}$$

which simplifies to

$$75T = 178.75$$

because the $+100$ and -100 cancelled.

The next thing we did to figure out the answer was divide by the \$75/hour charge. In this formal method, we can divide both side of our equation by 75. Again, if the left-hand side and right-hand side are the same number, then we will definitely get the same answer when we divide by 75. Here goes:

$$\frac{\cancel{75}T}{\cancel{75}} = \frac{178.75}{75}$$

Notice that we wrote the division in fraction form (instead of using $\div$). To understand why the 75's cancelled, remember that $75T$ is short for $75 * T$ and so

$$\frac{75T}{75} = \frac{75 * T}{75} = 75 \times T \div 75 = T$$

because the $\times 75$ and $\div 75$ cancelled.

So we have

$$T = \frac{178.75}{75} = 178.75 \div 75 = 2.388\ldots$$

as before. Yet again our answer is around 2 hours, 23 minutes.

Let's practice working with this symbolic way of solving equations. Suppose instead the plumber went to my neighbor's house and billed her for \$160. How long did the plumber work at my neighbor's? As before, we begin with our equation

$$P = 100 + 75T$$

and we are looking for $P = 160$. Putting these together we get

$$100 + 75T = 160$$

Then, we subtract 100 from both sides

$$\begin{aligned} \cancel{100} + 75T &= 160 \\ -\cancel{100} \quad & \quad -100 \end{aligned}$$

which simplifes to

$$75T = 60$$

Last, we divide both sides by 75 to get

$$\frac{\cancel{75}T}{\cancel{75}} = \frac{60}{75}$$

which simplifies to

$$T = 60 \div 75 = 0.8 \text{ hours}$$

We have solved the equation, but it would make more sense to report our answer in minutes so we convert

$$0.8 \text{ hours} * \frac{60 \text{ minutes}}{1\text{hour}} = 0.8 \times 60 = 48 \text{ minutes}$$

The plumber worked for 48 minutes at my neighbor's house.

Let's quick check this answer. Since T is measured in hours we need to go back and use $T = 0.8$, not 48 which is in minutes. Evaluating in our original equation we get

$$P = 100 + 75 \times \underline{0.8} = 160$$

Yes!

You might be wondering how we knew to subtract the 100 first and then later divide by 75. In this particular situation we had figured it out already and knew it made sense to take the \$100 right off the top. But, in general, how would we know? It turns out that when solving an equation we do the operations in the *opposite* order from evaluating:

When evaluating, first do $\times, \div$, and then do $+, -$

When solving, first do $+, -$, and then do $\times, \div$

When we solve more complicated equations we will see that this reverse order of operations for solving works more generally.

Practice exercises

1. In the United States temperatures for everyday things like the weather or cooking are given in Fahrenheit, denoted °F. In this system, water freezes into ice at 32°F and boils into steam at 212°F. A common setting for room temperature is 68°F whereas average human body temperature is around 98.6°F. And, most importantly, chocolate brownies bake at 350°F.

   In the sciences, medicine, and most other countries, temperatures are measured in Celsius, denoted °C, instead. For comparison's sake, it's useful to know that water freezes at 0°C and boils at 100°C. Room temperature is closer to 20°C whereas now average human body temperature is around 37°C. And those brownies?

   A common conversion is given by the equation

   $$F = 1.8C + 32$$

   where F is the temperature measured in Fahrenheit and C is the temperature measured in Celsius.

   (a) Evaluate the equation at $C = 0, 100, 20,$ and 37 and check that you get the same answers as stated in the problem.

   (b) At what temperature Celsius do chocolate brownies bake? Solve the equation (and that means *symbolically*) to find the answer.

   (c) You're planning a trip to Norway over Christmas and have heard it's will be around 10°C. What sort of jacket will you need? Convert to Fahrenheit to decide.

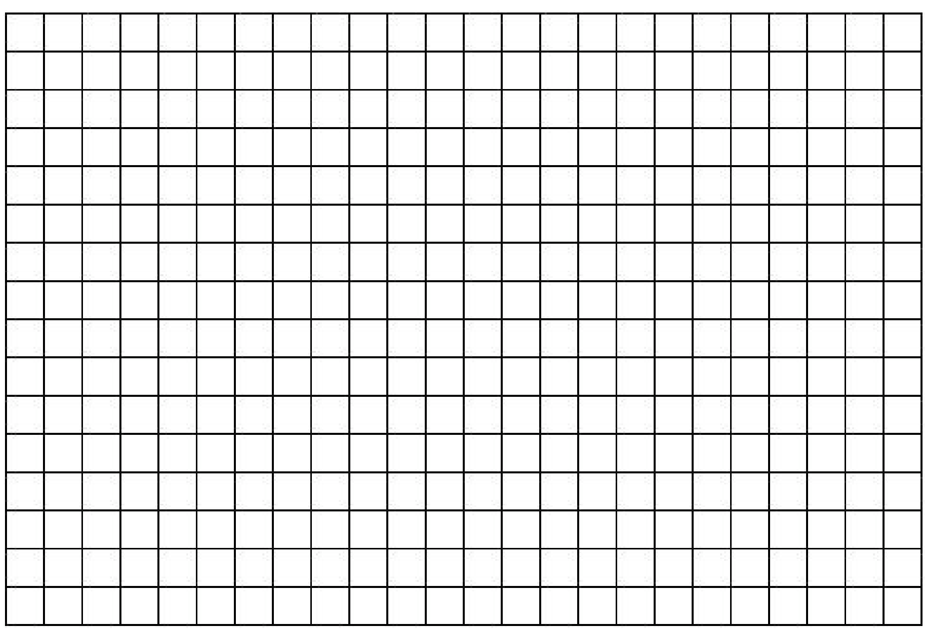
2. The altitude, A feet above ground, of an airplane M minutes after it begins its descent is given by the equation $A = 32,000 - 1,200M$. Answer each question by evaluating or setting up and solving an equation.

   (a) At what altitude does the plane begin its descent?

   (b) What is the plane's altitude 8 minutes into its descent?

   (c) How many minutes into its descent is the plane at 20,000 feet in altitude? *Set up and solve an equation.*

   (d) Draw a graph illustrating the function.



3. Lizbeth wants to send her mom truffles for mother's day. It cost $\$C$ to send a box of T truffles where

$$C = 1.90T + 7.95$$

(a) Make a table of values showing the charges for a box of 8 truffles, 12 truffles, or 30 truffles.

(b) Draw a graph illustrating the cost of sending truffles.

(c) If Lizbeth was charged \$53.55 for the box of truffles she sent her mom, how many truffles were there? *Set up and solve an equation to answer the question.*

4. SU given one equation, solve to find various values. Or, solve for same value in several equations.

Do you know . . .

- How to check that a solution is correct using the equation?
- When you solve an equation (as opposed to just evaluating)?
- How to solve a linear equation?
- Why we "do the same thing to both sides" of an equation when solving?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. A truck hauling bags of grass seed pulls into a weigh station along the highway. Trucks are weighed, by the way, to determine the amount of highway tax owed. This particular truck weighs 3,900 pounds when it's empty. Each bag of seed it carries weighs 4.2 pounds.
   (a) Name the variables and write an equation describing the function.
   (b) Set up and solve an equation to determine the number of bags of grass seed being carried by the truck with gross weight of 16,500 pounds.
   (c) Check your answer.
6. It was very hot in Solana's office when she arrived – a steamy 87°F, and so she turned on the air conditioner. She knew that would cool her office down by around 5°F an hour.
   (a) Name the variables and write an equation relating them.
   (b) How long will it take for Solana's office to reach the comfortable temperature of 72°F? *Set up and solve an equation to answer the question.*
7. Johannah figured out that the time T minutes that it takes her to warm up and then run M miles is given by the equation $T = 5 + 8M$.
   (a) Make a table showing Johannah's time if she runs 2 miles, 5 miles, or 10 miles.
   (b) Draw a graph illustrating the function. *Include 0 miles on your graph too.*
   (c) How far can Johannah run if she has an hour? *Set up and solve an equation to answer the question.*
8. Abduwali has just opened a restaurant. He spent \$82,000 to get started but hopes to earn back \$7,500 each month.
   (a) Name the variables and write an equation relating them.
   (b) Set up and solve an equation to determine how long it will take Abduwali to "break even" (that means make a profit of \$0)?

(c) Set up and solve an equation to determine how long it will take Abduwali to earn \$100,000.

9. When Kendrik walks on his treadmill, he burns 125 calories per hour.

   (a) Name the variables and write an equation relating them.

   (b) Set up and solve an equation to calculate how far Kendrik has to walk to burn 400 calories.

10. Lizbeth decides to send her mom jelly beans for mother's day instead. It cost $\$C$ to send a box of J ounces of jelly beans where

$$C = 0.56J + 7.95$$

Evaluate or set up and solve an equations – whichever is appropriate, to answer each question.

   (a) What will it cost Lizbeth to send her mother 10 ounces of jelly beans?

   (b) If Lizbeth is charged \$15.79, how many ounces did she send?

   (c) Lizbeth's budget is \$25. Can she afford to send two pounds of jelly beans? *Recall there are 16 ounces in a pound.*

   (d) How many pounds of jelly beans can Lizbeth afford to send, staying within her \$25 budget?

11. The more expensive something is, the less likely we are to buy it. Well, if we have a choice. For example, when blueberries are in the peak of season, they cost about \$2.50 per pint at my neighborhood farmer's market and demand is approximately 180 pints. (That means, people want to buy about 180 pints at that price.) We approximate that the demand D, measured in millions of pints, depends on the price, measured in dollars, as described by the equation $D = 305 - 50P$.

   (a) How many pints of blueberries are in demand when the price rises to \$3.19 per pint?

   (b) Make a table of values showing the demand for blueberries at the following prices: \$2.00/pint, \$2.25/pint, \$2.50/pint, \$2.75/pint, \$3.00/pint, \$3.25/pint, \$3.50/pint.

   (c) Draw a graph illustrating the function.

   (d) It's been a great week for blueberries and there are 240 pints to be sold at my neighborhood farmer's. What price should the farmer charge for her blueberries in order to sell them all? *Estimate your answer from the graph. Then set up and solve an equation to answer the question.*

12. The water in the local reservoir was 47 feet deep but over the past few months there's been so little rain that the depth has fallen 18 inches a week. Officials are worried that if dry conditions continue and the depth continues to fall, then the reservoir will not have enough water to supply the town.

(a) Name the variables and write an equation describing the function.

(b) Set up and solve an equation to determine how many weeks it took for the depth of water in the local reservoir to fall to 20 feet.

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

3.2 Solving linear inequalities

We are considering hiring the plumber who fixed our kitchen sink to work on our bathroom, which also needs a lot of work. Suppose we can't go over a \$500 budget. How much time will that buy us, assuming the plumber can do all the work in one trip to the house?

As before, the plumber charges \$75/hour plus a trip charge of \$100. The equation is $P = 100 + 75T$ where T is the time, measured in hours, and P is the total bill, measured in dollars.

In this situation we want to know when

$$P \leq 500$$

so we have an inequality instead of an equation. Replacing P by $100+75T$ in the inequality we get

$$100 + 75T \leq 500$$

We're looking for values of T that make the left-hand side a number that's smaller than, or maybe as large as, 500, but no larger.

To solve this inequality we begin the same way as when we were solving the equation, by subtracting 100 from both sides to get

$$\begin{array}{rcl} \cancel{100} + 75T & \leq & 500 \\ -\cancel{100} & & -100 \end{array}$$

which simplifies to

$$75T \leq 400$$

To understand why the inequality stays the same when we subtract the same number from both sides, think of the inequality as

$$\text{“little”} \leq \text{“big”}$$

If one number is littler than the other, the same will be true when we take away equal amounts. So, for example, say that you have more cash than I do and we each pay cash to buy a ticket to go see a movie – maybe you have \$21 but I only have \$18. Afterwards, it will still be true that you have more cash than I do – if each ticket costs \$12 then you will have \$9 left but I will only have \$6. I mean, we each will have less cash than before we bought the ticket, but you still have more than I do. Which is why you should buy the popcorn. But I digress.

Back to our example. We had

$$75T \leq 400$$

Just like when we solved our equation the next step is to divide both sides by 75 to get

$$\frac{\cancel{75}T}{\cancel{75}} \leq \frac{400}{75}$$

which simplifies to

$$T \leq 400 \div 75 = 5.33\ldots$$

Converting we find 5.33... hours is 5 hours and 20 minutes. Thus we want

$$T \leq 5 \text{ hours, } 20 \text{ minutes}$$

We can afford up to 5 hours and 20 minutes of the plumber's time.

To understand why the inequality stays the same when we divided each side by the same number, think again of the inequality as

$$\text{“little”} \leq \text{“big”}$$

If one number is littler than the other, the same will be true when we divide both amounts by the same number. So, for example, suppose we decided against buying popcorn so that that you have \$9 left from the movie and I only had \$6. While we're making up stories, suppose we each have three children, who want some money for treats. We each divide our remaining cash among our three children, respectively. Yours each get $\$9 \div 3 = \$3$ but mine only get $\$6 \div 3 = \$2$. So mine get less than yours.

There is a little bit of caution when solving inequalities. When symbolically solving an equation, any operation you do to both sides preserves the equality – start with equal amounts, do same thing to each, end with equal amounts. But, when symbolically solving an inequality, only some operations you do to both sides preserve the inequality – add or subtract from both sides, multiply or divide both sides by the same (positive) number. But other operations can reverse the inequality. For example, we can swap sides of an equation, but if we swap sides of an inequality then the direction of the sign reverses. If we had some problem where, say, $12 \leq C$ we could rewrite that inequality as $C \geq 12$. In each case, C is "little" and 12 is "big". Make sense? Negative numbers and fancier operations, neither of which we'll see here, may also cause the sign to reverse. Nothing to worry about in this course, but just so you know.

Inequalities are a very useful notation for indicating "between". So, for example, suppose we were willing to spend *between* \$400 and \$500 on plumber's fees for the bathroom remodel project. That is, we want $P \geq 400$ and $P \leq 500$. We can express this concept mathematically by writing

$$400 \leq P \leq 500$$

which is read

$$\text{“}P \text{ is between 400 and 500 (inclusive)”}$$

The word "inclusive" indicates that we're allowing $P = 400$ or $P = 500$.

Anyway, we can solve this chain of inequalities all at once using the same steps as before but now being sure to do the same thing to all *three* sides. "*Three* sides?" you ask. Yes, "three," I confirm. Watch how this works.

Start with

$$400 \leq P \leq 500$$

Use that $P = 100 + 75T$ to get

$$400 \leq 100 + 75T \leq 500$$

Subtract 100 from each of the three sides to get

$$\begin{array}{ccccc} 400 & \leq & \cancel{100} + 75\text{T} & \leq & 500 \\ \text{-100} & & \text{-}\cancel{100} & & \text{-100} \end{array}$$

which simplifies to

$$300 \leq 75T \leq 400$$

Next, divide all three sides by 75 to get

$$\frac{300}{75} \quad \leq \quad \frac{\cancel{75}T}{\cancel{75}} \quad \leq \quad \frac{400}{75}$$

which simplifies to

$$300 \div 75 \leq T \leq 400 \div 75$$

or, best yet,

$$4 \leq T \leq 5.33\ldots$$

To stay between \$400 and \$500, we can afford between 4 hours and 5 hours, 20 minutes.

Practice exercises

1. Straightforward inequality – less than, more than (no between).

2. In the United States temperatures for everyday things like the weather or cooking are given in Fahrenheit. In the sciences, medicine, and most other countries, temperatures are measured in Celsius, denoted °C, instead. A common conversion is given by the equation

$$F = 1.8C + 32$$

where F is the temperature measured in Fahrenheit and C is the temperature measured in Celsius.

(a) You're planning a trip to Norway over Christmas. You want to explain to your Norwegian hosts that back in Minnesota this time of year temperatures can range between -20°F and 40°F. Express this range in Celsius instead. *Show how to set up and solve a chain of inequalities to find the answer.*

(b) Your Norwegian hosts ask about the temperature in Minnesota during the summer. You explain that summer temperatures typically range from 55°F and 105°F. Express this range in Celsius instead. *Show how to set up and solve a chain of inequalities to find the answer.*

3. The altitude, A feet above ground, of an airplane M minutes after it begins its descent is given by the equation $A = 32,000 - 1,200M$. Answer each question by evaluating, setting up and solving and equation, or setting up and solving an inequality – whichever is appropriate.

   (a) What is the plane's altitude 3 minutes into its descent?

   (b) The plane might be asked to go into a holding pattern (that means flying in a circle instead of landing) when it's between 6,000 and 14,000 feet up. When will the plane be in that altitude range?

   (c) How long does it take the plane to land (assuming it's not asked to go into a holding pattern)?

4. One variety of blueberry plant yields an average of 130 blueberries per season but there's quite a bit of variability from plant to plant. One measure of this variability is called the standard deviation, which is approximated at 16.4 berries. Given a plant yielding B blueberries, we can calculate how usual or unusual that is by computing its "standard Z-score" using the equation

$$Z = \frac{B - 130}{16.4}$$

For example, a plant yielding $B = 130$ blueberries has standard Z-score of 0. A plant yielding $B = 173$ blueberries has standard Z-score of

$$Z = \frac{173 - 130}{16.4} = (\underline{173} - 130) \div 16.4 = 0.671875 \approx 0.67$$

Notice that we needed parentheses around the top (numerator) of our fraction to correct for the order of operations because we want the subtraction to be calculated before the division. (And the standard order of operations is the other way around.)

   (a) Calculate the standard Z-score of a plant yielding 240 blueberries.

   (b) If the standard Z-score for a plant is -0.7, what is the corresponding yield?
   Hint: the negative standard Z-score tells us the answer is below average.

   (c) A standard Z-score above 1.96 is considered extraordinarily plentiful. What yields of blueberries would be considered extraordinarily plentiful?

   (d) A standard Z-score between -1 and +1 are considered ordinary. What yields of blueberries are considered ordinary?

Do you know . . .

- What some common phrases are that indicate an inequality?
- How to represent the idea of "between" using a double-sided inequality?
- Why we "do the same thing to both sides" of an inequality when solving?
- How to solve a linear inequality?
- Why the inequality sign is reversed if we switch sides of the equation?
- When to evaluate versus solve an equation versus solve an inequality?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. The Torkelinsons want to dig a new well for water for their lake cabin. The company charges \$900 just to show up and then \$2 per foot to dig.
   (a) Name the variables and write an equation relating them.
   (b) If they want to stay under a \$1,300 budget, how deep a well can they afford? *Set up and solve an inequality to find the answer.*
6. Something with between. NOT MONEY
7. The cost of vacation to Cork, Ireland from the Minneapolis/St. Paul airport for two people is given by the equation

$$C = 2828 + 310N$$

   where C is the total cost in U.S. dollars and N is the number of days. If Ciara has budgeted up to \$10,000 to take her boyfriend Seamus to Cork to meet Ciara's grandmother, how many days can they afford to spend in Ireland? *Set up and solve an inequality to find the answer.*
8. My bank account balance is -\$1,200. Well, not really. I mean my bank lent me \$1,200 through a line of credit on my account. We've agreed that I will pay back the interest I owe plus \$250 each month until it's paid off. After that I plan to keep depositing the \$250 per month to start saving some money.
   (a) Name the variables and write an equation relating them. Notice that we can completely ignore interest because I pay that in full each month.
   (b) Make a table showing my account balance at the start and after 3 months, 6 months, and one year.
   (c) Draw a graph illustrating the dependence.

(d) Set up and solve an inequality to determine when I will have paid off my line of credit. *That means the account balance will be $0 or more.*

(e) Set up and solve an inequality to determine when I will have saved at least $2,000.

(f) Check your answers against your graph.

9. Something with between. Include drawing graph and sketching in segment of graph corresponding to the soln.

10. A truck hauling bags of grass seed pulls into a weigh station along the highway. This particular truck weighs 3,900 pounds when it's empty. Each bag of seed it carries weighs 4.2 pounds. Set up and solve an inequality to determine the maximum number of bags of grass seed that the truck can carry and stay under the 18,000 pound gross weight limit.

11. Another z-score question?

12. The water in the local reservoir was 47 feet deep but over the past few months there's been so little rain that the depth has fallen 18 inches a week. Officials are worried that if dry conditions continue and the depth continues to fall, then the reservoir will not have enough water to supply the town. Set up and solve an inequality to determine when the depth of the water will drop below 20 feet, which is considered dangerously low.

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

3.3 Solving exponential equations (and logs)

INTRODUCTORY EXAMPLE

- The definition of common logarithms (base 10). Estimating logs by successive approximation, evaluating logs on a calculator, understanding the connection to scientific notation (i.e. $logn$ the exponent of n written in scientific notation), and using logs to solve exponential equations with base 10 (i.e. in form $10^x = v$).
- We derive the log divides formula that $g^T = v$ has solution $t = \frac{logv}{logg}$.. The name log divides formula is our own. Instructors will see that it is a restatement of the definition of a log base b using the change of base formula, but we do not present it that way. (Su – probably should natural logs LN in the exercises). SU: should you revise the formula to have $A = P * g^T$ format?

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
   (a) xx

   (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx
   (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

3.4 Solving power equations (and roots)

SU consider adding more homework problems that include formulas from geometry?

There's an old saying – "when life gives you lemons, make lemonade." But how many lemons do you need? It turns out a reasonable equation describing the yield of lemonade from a single lemon is given by

$$J = 0.0185C^3$$

where J is the juice, measured in tablespoons, and C is the circumference of the lemon, measured in inches. (In case you've forgotten, the circumference is the distance *around* the lemon. Think of taking a piece of string and wrapping it around the middle part of the lemon. Then lay the string on a rule to see how long it is.)

A small lemon might measure 6 inches in circumference. According to our equation, it would yield

$$J = 0.0185(6)^3 = 0.0185 \times \underline{6} \wedge 3 = 3.996 \approx 4 \text{ tablespoons}$$

A regular-sized lemon, say 8 inches in circumference, would yield

$$J = 0.0185(8)^3 = 0.0185 \times \underline{8} \wedge 3 = 9.472 \approx 9.5 \text{ tablespoons}$$

Let's make a table of values and look at a graph of this function. We've added some values (including some unrealistic ones) to see the shape better.

C	0	2	4	6	7	8	9	10
J	0	0.148	1.184	3.996	6.3455	9.472	13.4865	18.5

SU GRAPH

How large a lemon would yield half a cup of juice? Remember 1 cup is 16 tablespoons so

$$\frac{1}{2} \text{ cup} * \frac{16 \text{ tablespoons}}{1 \text{ cup}} = 16 \div 2 = 8 \text{ tablespoons}$$

From our graph it look like 7.5 inches in circumference should be pretty close. Let's use successive approximation to get the answer to two decimal places. We'll round our value of J to two decimal places to fit easily into the table.

C	7	8	7.5	7.6	7.55	7.56	7.57
J	6.35	9.47	7.80	8.12	7.96	7.99	8.03
vs. 8	low	high	low	high	low	not quite	just over

So a lemon with circumference of approximately 7.57 inches should yield about half a cup of juice.

Much as we have learned to love successive approximation (and for good reason as it works on any type of equation), you might be happy to know that there is a way to solve power equations exactly. Start with what we're looking for

$$J = 8$$

Next, use our equation $J = 0.0185C^3$ to get

$$0.0185C^3 = 8$$

We want to find the value of C, so we can divide both sides by 0.0185 to get

$$\frac{\cancel{0.0185}C^3}{\cancel{0.0185}} = \frac{8}{0.0185} = 432.432\ldots$$

Thus we have

$$C^3 = 432.432\ldots$$

How can we find C just knowing C^3? We take cube roots of both sides to get

$$\sqrt[3]{C^3} = \sqrt[3]{432.432\ldots} = 432.432\ldots^{1/3} = 432.432\ldots \wedge (1 \div 3) = 7.56204\ldots$$

As before we see a lemon of about 7.57 inches should yield half a cup of juice.

Perhaps a brief digression on roots is in order. (And, that mysterious $\wedge(1 \div 3)$ business needs some explanation, I expect.) For example, we know

$$2^3 = 2 \wedge 3 = 8$$

which tells us

$$\sqrt[3]{8} = 2$$

The ***cube root*** of a number is whatever you would cube to get that number. Similarly

$$\sqrt[3]{1000} = 10$$

because

$$10^3 = 10 \wedge 3 = 1000$$

Some calculators have a key, or sequence of keys, that will take the cube root. For example, if your calculator has a key labeled $\sqrt[x]{y}$, then you can use it to find the cuberoot by doing

$$\sqrt[3]{8} = 3 \ \sqrt[x]{y} \ 8 = 2$$

Be careful if the key is just labelled $\sqrt{x}$ because that only takes square roots and can't be used for cube roots.

Otherwise you need to know that cube root is equivalent to raising to the 1/3rd power. So you can calculate

$$\sqrt[3]{8} = 8^{1/3} = 8 \wedge (1 \div 3) = 2$$

as we did in our example. Notice how the 1/3 is in parentheses. That's because we want the division before the exponent but the usual order of operations is the other way. If your calculator has a fraction entering key, you might be able to do

$$\sqrt[3]{8} = 8^{1/3} = 8 \wedge 1/3 = 2$$

instead, but don't forget to use the fraction key / not the divides key ÷.

In general, the ***nth root*** of a number is whatever number you would raise to the n power to get the number. In symbols

$$C^n = v \quad \text{means} \quad C = \sqrt[n]{v} = v^{1/n}$$

Back to our lemonade example. Let's practice solving this type of equation. If we wanted a lemon that yields 10 tablespoons of juice we would solve

$$J = 10$$

by putting in our equation we get

$$0.0185C^3 = 10$$

Next, divide each side by 0.0185 to get

$$\frac{\cancel{0.0185}C^3}{\cancel{0.0185}} = \frac{10}{0.0185}$$

so that

$$C^3 = 10 \div 0.0185 = 540.540\ldots$$

Then take cuberoots to get

$$C = \sqrt[3]{540.540\ldots} = 540.540\ldots \wedge (1 \div 3) = 8.14596\ldots \approx 8.1 \text{ inches}$$

As before we see that we solve in the reverse order of operations.

If evaluating, first do $\wedge$ (power), then do $\times, \div$

If solving, first do $\times, \div$, then do $\sqrt[n]{\ }$ (root)

A few pages of calculations into our example and we have only half a cup of lemonade to show for it. How about if we're trying to make a 1/2 gallon pitcher of lemonade? Suppose the store sells lemons by the bag, where all the lemons in the bag are just about the same size (all small, all medium, etc.). How many lemons of a fixed size will it take to make 1/2 gallon of lemonade? We can write a new equation to describe this situation.

First, convert 1/2 gallon into tablespoons.

$$\frac{1}{2} \text{ gallon } * \frac{4 \text{ quarts}}{1 \text{ gallon}} * \frac{4 \text{ cups}}{1 \text{ quart}} * \frac{16 \text{ tablespoons}}{1 \text{ cup}} = 4 \times 4 \times 16 \div 2 = 128 \text{ tablespoons}$$

Let L be the number of lemons (of a fixed size) and suppose each lemon yields J tablespoons of juice. For example, if $J = 10$ tablespoons, then

$$\frac{128 \text{ tablespoons}}{10 \frac{\text{tablespoons}}{\text{lemon}}} = 128 \div 10 = 12.8 \text{ lemons}$$

so we would need 13 lemons. In general the number of lemons we need is given by the equation

$$L = \frac{128}{J}$$

Look at a table of a few values and graph for this function. Notice that it's decreasing because the larger the lemons, the fewer we need to use. That makes sense, doesn't it?

J	4	6	10	12	16
L	32.0	21.3	12.8	10.7	8

SU NEED GRAPH

Last time I made a pitcher of lemonade it took 9 lemons. How much juice did each yield (assuming they're all about the same size)? Let's solve our equation to find the answer. Start with what we want

$$L = 9$$

and use our equation to get

$$\frac{128}{L} = 9$$

We haven't seen an equation like this before, where the independent variable in in the denominator (bottom) of the fraction, but not to worry. Remembering that $\frac{128}{L}$ means $128 \div L$, we can multiply both sides of the equation by L to get

$$\cancel{L} * \frac{128}{\cancel{L}} = 9 * L$$

so that

$$9L = 128$$

(We switched the variable onto the left-hand side for convenience.) Now the equation looks much more familiar. Dividing both sides by 9 gives us

$$\frac{\cancel{9}L}{\cancel{9}} = \frac{128}{9}$$

so

$$L = \frac{128}{9} = 128 \div 9 = 14.22\ldots \approx 14.2$$

Each lemon must have yielded around 14.2 tablespoons of juice.

And what goes better with lemonade than lemon cake. For that we're going to need some grated lemon peel. As with juice, the amount of lemon peel depends on the size of the lemon. One equation is

$$P = 0.061C^2$$

where C is the circumference of the lemon, measured in inches, as before and P is the amount of lemon peel, measured in tablespoons. For example, a lemon of circumference 7 inches will produce about 3 tablespoons of grated lemon peel because

$$0.061 * 7^2 = 0.061 \times \underline{7} \wedge 2 = 2.989 \approx 3$$

As before, we can look at a table of select values (not all of which are realistic).

C	0	2	4	6	7	8	9	10
P	0	0.244	0.976	2.196	2.989	3.904	4.941	6.1

SU NEED GRAPH

What size lemon would give 4 tablespoons of grated lemon peel? From the graph it looks like just over 8 inches in circumference should do. A quick successive approximation shows it's around 8.1 inches in circumference.

C	8	9	8.1
P	3.904	4.941	4.00221
vs. 4	bit low	high	close

To solve exactly we begin with what we want

$$P = 4$$

and use our equation to get

$$0.061C^2 = 4$$

Then divide both sides by 0.061 to get

$$\frac{\cancel{0.061}C^2}{\cancel{0.061}} = \frac{4}{0.061}$$

which simplifies to

$$C^2 = \frac{4}{0.061} = 4 \div 0.061 = 65.57\ldots$$

Now to undo the square we use regular square roots. There is most likely a special key on your calculator for square roots, but we'll do it the same as any other root here.

$$\sqrt{C^2} = \sqrt{65.57\ldots}$$

so

$$C = \sqrt{65.57\ldots} = 65.57\ldots^{1/2} = 65.57\ldots \wedge (1 \div 2) = 8.097\cdots \approx 8.1$$

as before.

Practice exercises

1. The size of a round pizza is often described by its diameter (distance across). Assuming a 16-inch diameter pizza serves four people, and with a little geometry to help us out, we calculated that a pizza of diameter D inches serves P people where

$$P = 0.015625D^2$$

   (a) Confirm that a 16-inch pizza serves four people.

   (b) According to the equation, how many people does a 12-inch pizza serve? A 14-inch pizza?

   (c) Use successive approximation to estimate the size of a personal pizza (to the nearest inch). *"Personal" means serving one person.*

   (d) Show how to solve the equation to find the size of a personal pizza.

   (e) What super-size should a pizza be to serve 6 people? Show how to solve the equation to find the answer.

2. Recall from Section 2.3 (SU CITE) that the amount of snow in a snowball, C cups, depends on the diameter (distance across) of the snowball, D inches according to the equation
$$C = 0.036D^3$$
If you have a five gallon paint bucket packed with snow and want to make one giant snowball out of it, how big will the snowball be?
   (a) Convert 5 gallons into cups. *Recall 1 gallon = 4 quarts and 1 quart = 4 cups.*

   (b) Show how to use successive approximations to answer the question to the nearest inch.

   (c) Show how to solve the equation to answer the question.

3. Recall from Section 2.3 (SU CITE) that my mom always said to sit close to the lamp when I was reading. The intensity of light L, measured in percentage (%) that you see from a lamp depends on your distance from the lamp, F feet as described by the formula

$$L = \frac{1}{F^2}$$

Set up and solve an equation to complete each part of the table.

F						
L	100	80	60	40	20	10

4. Need problem where, essentially, we find the growth factor of an exponential.

   (a) xx

   (b) xx

Do you know ...

- What we mean by square root, cube root, and nth root?
- How to calculate square roots, cube roots, and nth roots on your calculator?
- How to set up and solve a power equation?
- SU inverse proportions here somewhere? Perhaps say what a power equation is?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

SU fix order here when done to mix it up a bit

5. Recall that the Cadillac Escalade SU FINISH

   (a) xxx

   Jon (NEW NAME) wants to drive 200 miles. How long that will take depends on how fast he drives. CHANGE TO FUND RAISING WALK or maybe bike – SU fix in 2.3 first and then use here.

6. When a rock is dropped from 300 feet its height above ground is given by the equation

   $$H = 300 - 16T^2$$

   where T = time (seconds) and H = height above ground (feet).

   (a) When is the rock 200 feet above ground?
   (b) When is the rock less than 10 feet above ground?
   (c) When does the rock hit the ground?

7. The gas tank of a car is typically designed to hold enough fuel to drive 350 miles. For example, a compact car that gets 35 miles per gallon (mpg) would need a gas tank that holds 10 gallons, but a RV (recreational vehicle) that gets only 10 miles per gallon would need a gas tank that holds 35 gallons. SU – too many car problems. If keep at least use Hummer and Hybrid or something.

   (a) What volume gas tank would a sedan need if it has a fuel efficiency of 25 mpg?
   (b) Name the variables and write an equation relating them.
   (c) SU for later. If my Honda holds 17 gallons of gas, what is the fuel efficiency. SU check that it's not 450 miles instead as these numbers seem off.

8. Inverse to the square (or cube)

9. Cube

10. Cube == finding growth factor of exponential
11. Higher power 10th perhaps
12. Higher power – maybe use compound interest?
13. Higher power == finding growth factor of exponential
14. Perhaps problem on “proportional” and let them discover square by 4 and cube by 8. Another problem could be “inverse proportional” and then direct vs. inverse?

When you’re done . . .

- Don’t forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you’re still stuck, work with a classmate or go to your instructor’s office hours.
- It’s normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The “Do you know?” questions can be a good starting point.

May 16, 2012
SU – in that same section discuss exponents and what they mean. Mention special names like square and cube

3.5 Solving quadratic equations*

Libby likes to juggle. She throws one of her juggling balls high up into the air. The height of the ball changes over time as described by the equation

$$H = 3 + 15T - 16T^2$$

where

$$\begin{aligned} H &= \text{height of ball (feet)} \\ T &= \text{time (seconds)} \end{aligned}$$

Let's make a table and draw a graph illustrating this function. For example, when $T = 0$ second, we have

$$H = 3 + 15(0) - 16(0)^2 = 3 + 15 \times \underline{0} - 16 \times \underline{0} \wedge 2 = 3 \text{ feet}$$

and when $T = 1$ second, we have

$$H = 3 + 15(1) - 16(1)^2 = 3 + 15 \times \underline{1} - 16 \times \underline{1} \wedge 2 = 2 \text{ feet}$$

Huh? I though the ball went up in the air. What's happening here? Oh, I know. it must be falling down by then. Let's look at $T = 0.1$ seconds. Then

$$H = 3 + 15(0.1) - 16(0.1)^2 = 3 + 15 \times \underline{0.1} - 16 \times \underline{0.1} \wedge 2 \approx 4.34 \text{ feet}$$

That makes more sense. As we fill in the table we see how Libby's juggling ball went up in the air and then back down.

T	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
H	3.0	4.34	5.36	6.06	6.44	6.50	6.34	5.66	4.76	3.54	2.00	0.14	~~-2.04~~

Notice as we evaluate at more times for $T = 1.2$ seconds we get

$$H = 3 + 15(0.1) - 16(0.1)^2 = 3 + 15 \times \underline{1.2} - 16 \times \underline{1.2} \wedge 2 \approx -2.04 \text{ feet}$$

We can't have negative feet. It means the ball must have already hit the ground by 1.2 seconds.

SU GRAPH – include Libby and label some key points with shapes to see. Perhaps add those shapes to the table to help codify it better?

A word of caution about this graph. It does not show where the ball travels, but just how it's height changes over time. The specially marked points on the graph correspond to the points in the table/picture. SU – Explain the shapes on the graphs and tables.

How long was the ball in the air? From our table and graph it looks like just over 1.1 seconds. We could refine our answer by successive approximations. The ball will hit the ground when it's height is 0 feet so that $H = 0$. Looks a little strange perhaps, but that's what we want.

T	1.1	1.11	1.105	1.107	1.106
H	0.14	~~-0.06~~	0.0368	~~-0.002~~	0.018
vs. 0	high	low	high	low	good

So the ball hit the ground after approximately 1.106 seconds. That's probably way more precise than we need but there we have it.

In this chapter we've seen how to solve linear, power, and exponential equations. Is there some way to solve this type of equation too?

A little terminology. An equation like Libby's is called a ***quadratic equation***. It can be rewritten in the form

$$H = -16T^2 + 15T + 3$$

where the highest power of the independent variable (T here) is listed first as is the custom. In general a quadratic equation can be written in the form

$$H = aT^2 + bT + c$$

where a, b, and c are constants. For us those constants are

$$a = -16 \quad b = 15 \quad c = 3$$

If there were higher powers of our independent variable in the equation, then it would be called a ***polynomials***. We saw some polynomial equations in Section 2.3 SU CITE including

$$C = 0.036D^3 \text{ (snowball)}$$
$$V = 0.01W^3 - 0.95W^2 + 21W + 153 \text{ (investment stock)}$$
$$S = 10P^9 - 9P^{10} \text{ (orchids)}$$

Quadratics are just polynomials where the highest power is 2. Linears are officially polynomials as well, where the highest power is just 1.

Back to Libby. We are trying to figure out when $H = 0$. Putting in our equation we get

$$-16T^2 + 15T + 3 = 0$$

and we want to solve for T. Notice that because T occurs twice in the equation, nothing we have seen to do to both sides of the equation can knock it down to just one T. That means none of our methods so far will work to solve this equation.

Sadly, there is no one-method-fits-all way of solving polynomial equations. In special cases there are various special methods, but no method works for all polynomials.

But it turns out that for quadratics there is a way to solve them. It's called the ***Quadratic Formula***. It tells us the answer, or perhaps I should say possible answer(s) to

$$aT^2 + bT + c = 0$$

are

$$T = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Oh my! First thing to understand in this complicated formula is that we actually get two possible answers

$$T = \frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad T = \frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

Let's figure this out for Libby. Remember we had

$$a = -16 \quad b = 15 \quad c = 3$$

First thing is

$$\frac{-b}{2a} = \frac{-(15)}{2(-16)} = (-)15 \div (2 \times (-)16) = 0.46875$$

Notice how we put parenthses around the denominator (bottom) of our fraction, as always, to preserve the order of operations. To get the negative numbers we hit the one key usually labelled (-). Remember that's the negation key, not the subtraction key, and what looks like parentheses are part of what's printed on the key.

Next part is

$$\frac{\sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{(15)^2 - 4(-16)(3)}}{2(-16)}$$

$$= \sqrt{\ }((15) \wedge 2 - 4 \times (-)16 \times 3) \div (2 \times (-)16)$$

$$= -0.6381431\ldots \approx -0.63814$$

Notice how we have parentheses both around the number we are taking the square root of and, again, the denominator of our fraction. Also there's parentheses around the 15 before it was squared, which we don't really need in this problem, but we would need them if b were negative. Lastly, we're using the squareroot key $\sqrt{\ }$ on a calculator. If you don't have that key, then you'll need do $\wedge(1 \div 2)$ instead.

I'm guessing that you've noticed how complicated these calculations are. It is definitely worth having a calculator with a square root key here. Most students find it takes a lot of practice to get used to these calculations but luckily it's the same calculation for each problem.

Oh, and we're not done yet. Remember there are two possible answers. One is

$$0.46875 + (-0.63814) = -0.16939 \text{ secpmds}$$

which doesn't make any sense because time isn't negative. The other is

$$0.46875 - (-0.63814) = 1.10689 \text{ seconds}$$

We had guessed around 1.106 seconds, so that is definitely the right answer: Libby's ball will hit the ground after 1.10689 seconds. Yeah, too precise. But you get the idea.

Wait a minute! Any good juggler isn't about to let the ball fall on the ground. She's going to catch it again, probably at about 3 feet above ground (which is how high she threw it from). Looking back at our table and graph we see $H = 3$ at $T = 0$ when she threw the ball in the air, and again just after 0.9 seconds.

We could refine our guess with successive approximation. For example, when $T = 0.95$ We find

$$H = 3 + 15(0.95) - 16(0.95)^2 = 3 + 15 \times \underline{0.95} - 16 \times \underline{0.95} \wedge 2 \approx 2.81 \text{ feet}$$

T	0.9	1.0	0.95	0.93	0.94
H	3.54	2.00	2.81	3.1116	2.9624
vs. 3	high	low	low	high	close

We estimate that Libby will catch the ball by 0.94 seconds.

Notice how the ball is falling during this time interval. That means to get a lower height we need to guess a bigger number for time and to get a higher height we need to guess a smaller number for time. That might seem backwards but decreasing functions always work that way. More on the rate of change later.

But first, there is a way to solve the equation to find this answer. We're looking for $H = 3$. Using our equation we get

$$3 + 15T - 16T^2 = 3$$

The Quadratic Formula only works if the equation has one side $= 0$, but we have $= 3$. It might seem that we're out of luck, but don't abandon hope because it's an easy fix. Just subtract 3 from both sides to get

$$\begin{aligned} 3 + 15T - 16T^2 &= \cancel{3} \\ &= \cancel{-3} \\ 15T - 16T^2 &= 0 \end{aligned}$$

So now we have $= 0$. Yes!

As luck would have, the other 3 cancelled too. (That's just because we're looking for the same height that it started at.) We can write the new equation as

$$-16T^2 + 15T = 0$$

from which we see that

$$a = -16 \quad b = 15$$

What about c? Turns out $c = 0$. (Again, because we're looking for the starting height.) To see why, think of our equation as

$$-16T^2 + 15T + 0 = 0$$

.

Now we're set to use the Quadratic Formula. The first fraction is

$$\frac{-b}{2a} = \frac{-(15)}{2(-16)} = (-)15 \div (2 \times (-)16) = 0.46875$$

No surprise here. We used the same values of a and b as before, so we should have the same number here.

Next is the part with the square root.

$$\frac{\sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{(15)^2 - 4(-16)(0)}}{2(-16)}$$

$$= \sqrt{}((15) \wedge 2 - 4 \times (-)16 \times 0) \div (2 \times (-)16)$$

$$= -0.46875$$

That's the negative of the first number. What gives? Take a closer look at the part with the square root. We know 0 times any number is just 0 so that tells us

$$\frac{\sqrt{(15)^2 - 4(-16)(0)}}{2(-16)} = \frac{\sqrt{(15)^2 - 0}}{2(-16)} = \frac{\sqrt{(15)^2}}{2(-16)} = \frac{15}{2(-16)}$$

which is the same as before, except for the - sign.

Don't forget we need to put together these parts to find the possible answers. The sum gives us

$$0.46875 + -0.46875 = 0$$

and the difference gives us

$$0.46875 - -0.46875 = 0.9375$$

Both answers seem to make sense. The ball was 3 feet above ground both at the start of the problem ($T = 0$ seconds) and later one ($T = 0.9375$ seconds. Since we're interested in when Libby caught it on the way down, the answer we want is 0.9375 seconds. That agrees with our estimate from before.

This is a complicated process, but luckily we follow the same steps each time.

Zero Rewrite the equation in the form $H = aT^2 + bT + c = 0$ *If not already =0*

ABC Read off the values of a, b, and c

Fraction Evaluate $\frac{-b}{2a}$

Squareroot Evaluate $\frac{\sqrt{b^2-4ac}}{2a}$

Add/Subtract The possible answers are the sum or difference of those two numbers.

Decide Decide which answer (or none or both) makes sense in the story.

We noted that between 0.9 and 1.0 seconds the function was decreasing. For example we can calculate the rate of change over 0.1 second time intervals to get the speed of the juggling ball. From 0.9 to 1.0 seconds, we have

$$\text{speed} = \text{rate of change} = \frac{\text{change in height}}{\text{change in time}} = \frac{2-3.54}{1.0-0.9} = \frac{-1.54 \text{ feet}}{0.1 \text{ seconds}} = -15.4 \text{ feet/sec}$$

In general we have

T	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
H	3.0	4.34	5.36	6.06	6.44	6.50	6.34	5.66	4.76	3.54	2.00	0.14
speed	–	13.4	10.2	7.0	3.8	0.6	-2.6	-5.8	-9.0	-12.2	-15.4	-18.6

As usual, the positive rate of change correspond to increasing heights; in our story the ball is going up in the air. This happens until around 0.5 seconds or so. The negative rate of change correspond to decreasing heights; in our story the ball is falling back down. This happens starting just before 0.6 seconds.

What happens in the story at the point where the ball stops going up in the air and starts falling down? That must be when the ball is at its highest point. What is the speed at that highest point? Well, I guess 0. For a split second it's almost frozen in midair, neither rising nor falling. If we were able to compute the rate of change for an interval really, really small right then we would find the rate of change ≈ 0.

Turns out it's easy to find that point for a quadratic equation, just plug in the first fraction from the Quadratic Formula! Check it out: when

$$T = \frac{-b}{2a} = \frac{-15}{2(-16)} = (-)15 \div (2 \times (-)16) = 0.46875 \text{ seconds}$$

we get

$$H = 3+15(0.46875)-16(0.46875)^2 = 3+15\times\underline{0.46875}-16\times\underline{0.46875}\wedge 2 = 6.515625\ldots \approx 6.516 \text{ feet}$$

T	0.46875
H	6.516

Libby's ball gets about 6.516 feet up. Converting to more normal units we get

$$.516 \text{ feet} * \frac{12 \text{ inches}}{\text{feet}} = .516 \times 12 = 6.192 \approx 6 \text{ inches}$$

So the juggling ball goes up to about 6'6". (Remember ' is short for feet and " is short for inches.) SU earlier??? Check 1.3 or 1.4

SU – picture showing the roots and vertex from old book. Need a sentence or two explaining it.

One last small note. In this problem the graph of the quadratic was $\cap$ shaped because it was increasing and then decreasing, and we found a maximum value. In other problems the graph of a quadratic might be $\cup$ shaped because it is the other way around – decreasing first and increasing later. In that case evaluating at $T = \frac{-b}{2a}$ would give the minimum value instead. In some problems the domain includes only part of the U shape so it might just increase, or just decrease, on the domain.

Practice exercises

1. A high-jumper jumps so that the height, H feet, of the point on his back that must clear the bar after T seconds is given by the equation:

$$H = 3.5 + 16T - 16T^2$$

   (a) When would the high-jumper hit the ground (if there weren't a pit)? **Ouch!**

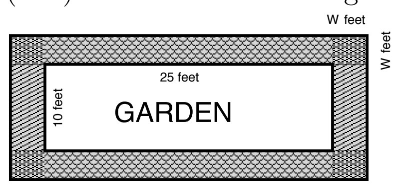
   (b) The high jump pit is 2 feet off the ground. When does the high-jumper land in the pit?

   (c) How high a bar can the high-jumper clear? *Find the maximum height of that point above ground by evaluating at* $T = \frac{-b}{2a}$.

2. Urban gardens are catching on. What was once an abandoned lot down the block is now houses a thriving vegetable and berry garden for the neighborhood that's 10 feet wide and 25 feet long. One of the neighbors works for a landscaper who has volunteered to donate some gravel to make a path around the garden. Typically such a path would be 3 inches deep and the same width all around. The equation (which we derived in Section 2.4 SU CITE) is

$$G = W^2 + 17.5W$$

where W = width of path (feet) and G = amount of gravel (cubic feet).



(a) How many cubic feet of gravel would they need to make the path 2 feet wide? 3 feet wide? 4 feet wide?

(b) Gravel is measured by the "yard," which is short for cubic yard. How many cubic feet are in one cubic yard?

(c) If the neighbor donates three yards of gravel, how wide a path can they build? Set up and solve a quadratic equation to find the answer to two decimal places.

(d) Convert your answer to feet and inches.

3. The profit $\$P$ from selling M tanks of milk is described by the equation

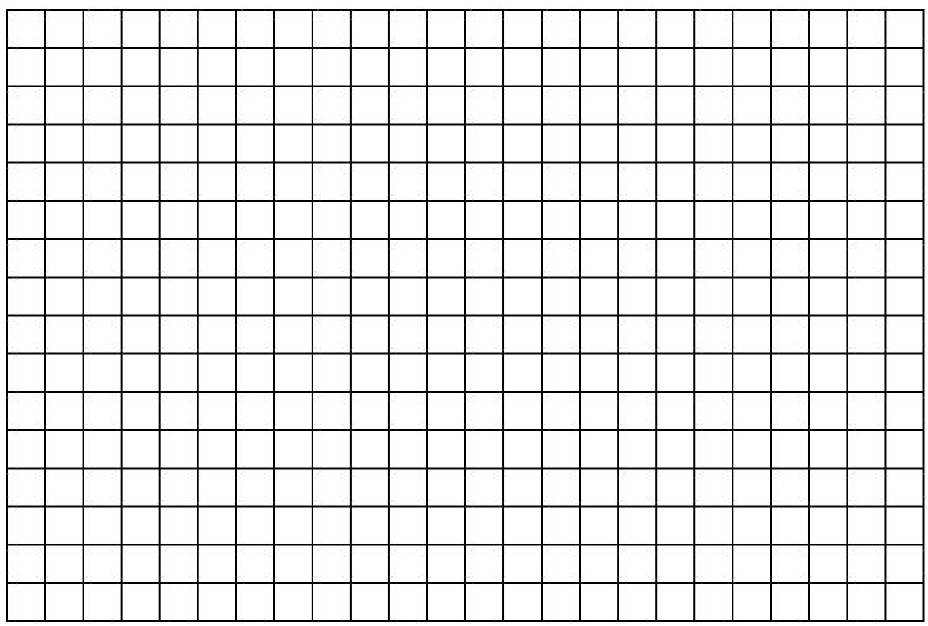
$$P = -2M^2 + 2000M - 80,000$$

(a) Complete the following table showing the profit of selling tanks of milk.

M	0	100	200	300	400	500	600	700	800	900
P			240,000	340,000		420,000	400,000		240,000	100,000

(b) Explain why negative numbers make sense in this problem.

(c) Draw a graph showing how the profit depends on the amount of milk sold.



(d) How much milk must be sold for the company to “break even” (means having \$0 profit.)? Set up and solve a quadratic equation to answer the question.

(e) Does your answer agree with your table and graph? Explain.

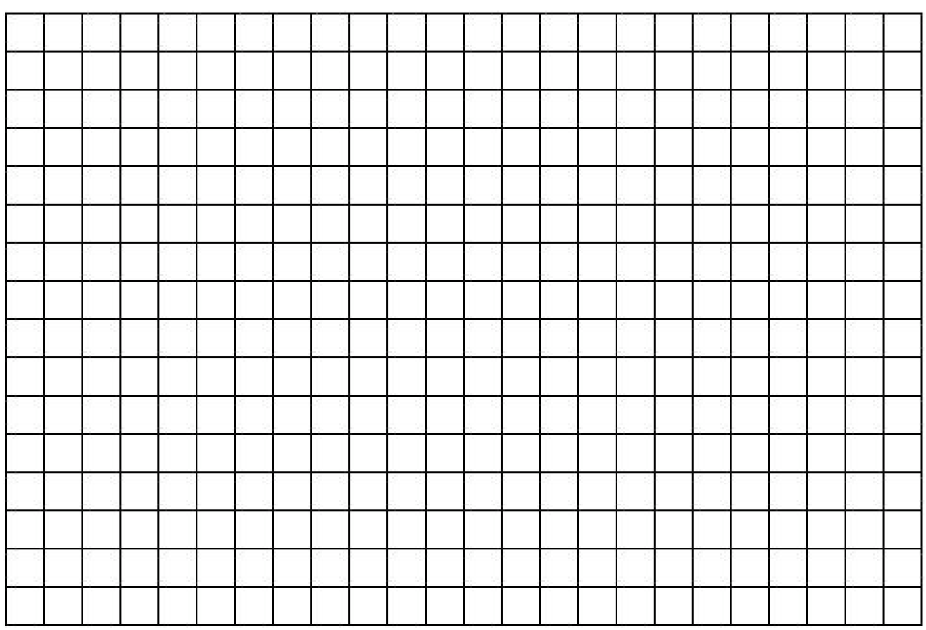
4. A company produces backpacks. The more they make, the less it costs for each one. The cost per backpack is given by the equation

$$C = 0.01B^2 - 1.2B + 50$$

where C = cost per backpack (\$ per backpack) and B = number of backpacks.

(a) How many backpacks do they need to produce in order to hold costs to \$20/backpack? Set up and solve a quadratic equation to find the answer.

(b) Make a table of values and draw a graph of the function. Does your answer agree with your table and graph?



Do you know ...

- What is a quadratic equation? A polynomial?
- When do we use the Quadratic Formula?
- What do we do to solve a quadratic equation when it isn't set equal to zero?
- How to find the values of a, b, c in the formula?
- How to evaluate the formula (using your calculator)? *Ask your instructor if you should memorize the Quadratic Formula or it will be printed on your exam.*
- Why there are (usually) two solutions to a quadratic equation?
- How to decide which solution(s) from the Quadratic Formula are correct?
- What is the general shape of the graph of a quadratic equation?
- What value do we use for the independent variable to find the highest (or lowest) value in a quadratic equation?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. Remember that the height H feet of Libby's juggling ball T seconds after she throws it in the air is described by the equation

$$H = 3 + 15T - 16T^2$$

Answer each of the following question by setting up and solving a quadratic equation. Look back at the graph from earlier to make sure your answers make sense.

(a) When is the ball 5 feet above ground? Why do both answers make sense in the story?

(b) When is the ball 30 inches above ground?

(c) When is the ball 8 feet above ground? Explain why it makes sense in the story that you can't solve this quadratic equation.

6. The breaking distance for the Cadillac Escalade (see Section 2.3 SU CITE) is given by

$$F = 0.04 * S^2$$

where S = speed of car (miles per hour) and F = breaking distance (feet). We are interested in calculating the speed the Escalade can go and still stop within 100 feet after the driver hits the brake.

(a) Use successive approximation to estimate the answer to the nearest miles per hour. Display your work in a table.

(b) Solve the equation to find the answer using the standard method of solving a power equation.

(c) Show how to use the Quadratic Formula to solve the equation. *Hint:* $b = 0$

7. The stopping distance for the Cadillac Escalade (see Section 2.3 SU CITE) is given by

$$D = 1.5S + 0.04S^2$$

where S = speed of car (miles per hour) and D = stopping distance (feet). Recall that the stopping distance includes both the distance travelled before the driver hits the brake and the braking distance. We are interested in calculating the speed the Escalade can go and still stop within 100 feet.

(a) Use successive approximation to estimate the answer to the nearest miles per hour. Display your work in a table.

(b) Show how to use the Quadratic Formula to solve the equation.

8. The amount of a certain product that consumers want to buy depends on the price of that product. When the produce sells for $\$P$ a typical consumer will want to buy Q of them. For this particular (mysterious) product it is determined that

$$Q = 20 + 5P - P^2$$

for $0 \leq P \leq 10$.

(a) Make a table of values and graph showing this demand function.

(b) At what price does this equation say the consumer will no longer want to buy any of the product? Set up and solve a quadratic equation to find the answer. *The stated domain allowed for values up to* $P = \$10$, *but really the domain of this function should be restricted to this value.*

(c) At what price will the typical consumer want to buy the most of the product?

9. Mrs. Weber's cooking class came up with the equation

$$M = 7 + 4S + 1.2S^2$$

to approximate the grilling time of a steak depending on its thickness. Here M is the number of minutes to grill the steak and S is the thickness of the steak (in inches). SU CITE 2.3 also 2.4 SU FIX EQN!

(a) If we want to make sure the steak will cook in under half an hour, what thickness steak can we have? *Set up and solve a quadratic equation to find the answer. Check against our results from Sections 2.3 and 2.4 – SU CITE*

10. A rabbit jumps so that her height is given by the formula

$$R = 17.6S - 22S^2$$

where R = height of rabbit (feet) and S = time (seconds).

  (a) At what height did the rabbit start her jump?
  (b) How long is the rabbit in the air? First use successive approximation to estimate the answer. Then use the Quadratic Formula to find the answer. *Hint:* $c = 0$
  (c) When is the rabbit 1 foot off the ground?
  (d) Can the rabbit jump over a 3 foot fence?
  Use whatever method you want, but explain your reasoning.

11. In the urban garden story (From practice exercises – SU CITE)

$$G = W^2 + 17.5W$$

where W = width of path (feet) and G = amount of gravel (cubic feet), how wide a path can they build if the neighbor is able to donate 100 cubic feet of gravel instead?

12. In the milk example

$$P = -2M^2 + 2000M - 80,000$$

where \$$P$ is the profit from selling M tanks of milk, how many tanks of milk would they need to sell to keep profits over \$400,000? *Your answer should be in the form of an inequality.*

13. In the backpack example

$$C = 0.01B^2 - 1.2B + 50$$

where C = cost per backpack (\$ per backpack) and B = number of backpacks, what is the minimum price per backpack? *Hint: make a table and evaluate at* $T = \frac{-b}{2a}$.

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

Practice exams on Solving equations

Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

Practice exam 3– version I

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

Practice exam 3 – version II

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

Chapter 4

A closer look at linear equations

Yada yada

]

4.1 Modeling with linear equations

Yada yada

4.2 Systems of linear equations

INTRODUCTORY EXAMPLE

SU – consider adding an example from linear programming?

SU – start with equilibrium price, shortage and surplus

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second
   (a) xx
   (b) xx

3. Third

    (a) xx

    (b) xx

4. Fourth

    (a) xx

    (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

    (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

4.3 Intercepts and direct proportionality

SU: Su – should now include direct proportion and discussion of proportionality.

It was chilly at 7:30 this morning when I arrived at my office, only 58°F. Luckily the heating system warms things up very quickly, 5°F per hour. What type of function describes this situation? First, since the time and temperature are both changing in this problem, those are our variables – we can't call them both "T" though. Let's use

$$H = \text{time (hours since 7:30 a.m.)}$$
$$T = \text{temperature (°F)}$$

The constant rate of change of 5°F/hour means that that T depends linearly on H. Remember that for linear equations, the rate of change is also called the ***slope***. So the slope is 5°F/hour.

We're measuring time since 7:30 a.m when I arrived at the office. At that time the temperature was 58°F. So that's our starting temperature. Remember that for linear equations, the starting value of the dependent variable is also called the ***intercept***. The intercept is 58°F.

One last thing to remember. A linear equation has the general form

$$\text{dep. var.} = \text{intercept} + \text{slope} * \text{indep. var.}$$

Putting all these pieces together we find that the temperature of my office as a function of time is given by the linear equation

$$T = 58 + 5H$$

Does that make sense? We know that the temperature started at 58°F. After one hour it was 5°F warmer, which is 63°F. After another hour it was again 5°F warmer, which is 68°F. And so on – up to 73°F after 3 hours and then 78°F after 4 hours. Let's put this information in a table.

H	0	1	2	3	4
T	58	63	68	73	78

Now we can check that our equation makes sense. After 4 hours we should evaluate our equation at $H = 4$ to get

$$T = 58 + 5(4) = 58 + 5 \times 4 = 78$$

Actually 78°F is pretty warm, so I probably would have turned off the heat once the temperature reached 70°F. When was that? From our table we see it's between $H = 2$ and $H = 3$ or, in real time, between 9:30 and 10:30 a.m. (Just added 7:30 + 2:00 = 9:30 and 7:30 + 3:00 = 10:30, in case you're wondering.)

We can easily solve our equation to get a more specific answer. As always, begin with what we want

$$T = 70°\text{F}$$

Use our equation, $T = 58 + 5H$ to get

$$58 + 5H = 70$$

Next, subtract 58 from each side to get

$$\begin{aligned} \cancel{58} + 5H &= 70 \\ -\cancel{58} \quad & \quad -58 \end{aligned}$$

which simplifies to

$$5H = 12$$

Last, divide each side by 5 to get

$$\frac{\cancel{5}H}{\cancel{5}} = \frac{12}{5}$$

which simplifies to

$$H = 2.4$$

I should turn the heat off after 2.4 hours.

When exactly is that, you ask? Well,

$$0.4 \text{ hours} = 0.4 \text{ hours} * \frac{60 \text{ minutes}}{1 \text{ hour}} = 0.4 \times 60 = 24 \text{ minutes}$$

I should turn off the heat after 2 hours and 24 minutes. Since we start counting time at 7:30 a.m., that means

$$7{:}30 + 2{:}24 = 9{:}54 \text{ a.m.}$$

I should turn off the heat at 9:54 a.m., or just before 10:00 a.m. Makes sense.

Wouldn't you know it, but it was chilly in my apartment when I got home at 5:00 that evening. I turned on the heat which is rather slow. It normally raises the temperature by only 2°F per hour. Now it's 8:00 p.m. and it still feels a little cool, only 67°F. Assuming my heat was working normally, what was the temperature when I got home? And when will be be 70°F as I like it?

Let's begin by naming the variables and write an equation relating them. Just like with my office, the constant rate of change (slope) of 3°F/hour tells us that the equation is going to be linear. We need to adjust our variables a little bit. Notice that we measure time from when I arrived home at 5:00 p.m.

$$\begin{aligned} P &= \text{time (hours since 5:00 p.m.)} \\ T &= \text{temperature (°F)} \end{aligned}$$

We can summarize the information we were given in the table. Notice that we are not told the starting temperature (intercept). In fact, that's one of the numbers we're trying to figure out. The example of 67°F at 8:00 p.m. corresponds to a point on the graph.

time	5:00 p.m.	8:00 p.m.
P	0	3
T		67

What was the temperature when I arrived home? We can work backwards to figure that out. Since it was 67°F at 8:00 p.m. and the temperature had been increasing at the rate of 2°F/hour, it must have been 65°F at 7:00 p.m., 63°F at 6:00 p.m., and 61°F when I arrived home at 5:00 p.m. The intercept must be 61°F. Notice what we calculated

$$\text{intercept } = 67 - 2 - 2 - 2 = 61$$

There's a shorter way to do this calculation

$$\text{intercept } = 67 - 3 \times 2 = 61$$

In general we can find the intercept this way.

$$\text{Intercept} \quad = \quad \text{value of dep. var.} \;-\; \text{value of indep.var.} \;*\; \text{rate of change}$$

Now that we know the slope and intercept, we can find the equation. We know

$$\text{dep. var.} = \text{intercept} + \text{slope} * \text{indep. var.}$$

So our equation is

$$P = 61 + 2H$$

Let's quickly check that this equation works. When $H = 3$ we get

$$61 + 2(3) = 61 + 2 \times 3 = 67$$

as expected.

One more question to answer. When will the temperature in my apartment reach 70°F? We know from our table that it will take more than 3 hours. Let's solve the equation to find a more precise time. We want

$$P = 70$$

Use our equation $P = 61 + 2H$, we get

$$61 + 2H = 70$$

Next, subtract 61 from each side to get

$$\begin{aligned} \cancel{61} + 2H &= 70 \\ -\cancel{61} \quad & \quad -61 \end{aligned}$$

which simplifies to

$$2H = 9$$

Last, divide each side by 2 to get

$$\frac{\cancel{2}H}{\cancel{2}} = \frac{9}{2}$$

which simplifies to

$$H = 4.5$$

It will take 4.5 hours for my apartment to heat up to a comfy 70°F.

Notice 4.5 hours means 4 and a half hours. That's 4 hours and 30 minutes. So, does that mean I need to wait that long for it to be warm enough in my apartment? Well, no. The value $H = 4.5$ corresponds to four and half hours since when I got home at 5:00. So it should be warm enough at 9:30 p.m. You can check: 5:00 + 4:30 = 9:30. Since it's already 8:00 p.m., I need to wait another hour and a half. Of course, by then I'm probably given up on my cold apartment and headed under a warm blanket to watch t.v.

SU – look at narrative in the old Section 2.2 The rise over run picture, etc.

Practice exercises

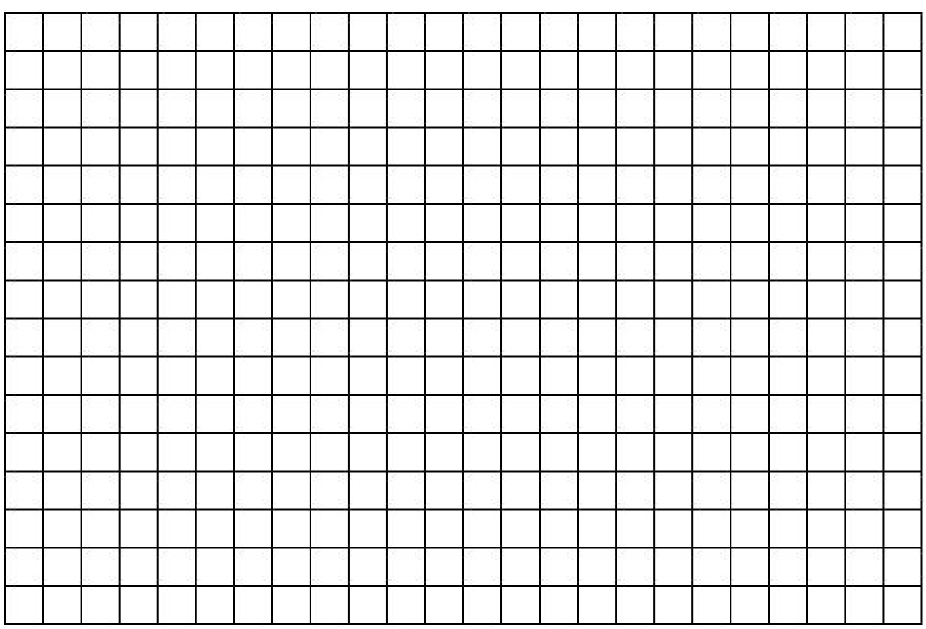
1. An air conditioning unit can lower the temperature in a classroom 4° every hour. After the air conditioner has been running for $1\frac{1}{2}$ hours, the temperature in the room was 72°.

   (a) What was the starting temperature of the room?

   (b) Name the variables and write an equation relating them.

   (c) Solve your equation to determine when the temperature will reach 67°.

   (d) Draw a graph of the function.



2. Holiday Express offers vacation packages to Punta Cana. They charge \$1,270 per person for round-trip airfare plus five nights at an all-inclusive resort, based on double occupancy. Normally they charge \$144 per night per person.

   (a) At this rate, what are they charging for the airfare?

   (b) Name the variables including units, and write the equation.

   (c) What is the slope and what is the intercept?

   (d) According to your equation, what would it cost for round-trip airfare plus seven nights?

3. Third

    (a) xx

    (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- How to recognize whether a function is linear from its story, table, or graph?
- Where the intercept appears in a linear equation?
- Where the intercept appears on the graph of a linear function?
- What the intercept means in context (and how to identify it within a story)?
- How to figure out the intercept given the slope and an example (another point on the graph)?
- Why an intercept might not make sense, for example if it's outside the domain of the function? SU FIX THESE

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. For \$24.00 you can buy a movie pass which entitles you to buy movie tickets for only \$6.00 each.
   (a) Explain why this dependence is linear and identify the slope and intercept from the story.
   (b) Name the variables and write an equation illustrating the dependence.
   (c) Solve your equation to find the number of movie tickets you can buy for \$100, assuming you first use some of the money to buy the pass.
6. Suppose that the total cost for each type of heating system is as given. Notice that the rate of change, in \$/year, is assumed to be constant.
   (a) Name the variables.
   (b) Write an equation for the gas heating system assuming installation cost of \$12,000 and \$700/year to operate.
   (c) Write an equation for the electric heating system assuming installation cost of \$5,000 and \$1,100/year to operate.
   (d) Write an equation for the solar heating system assuming installation cost of \$30,000 and \$150/year to operate.
   SU this example is from Bolker's book and is likely hopelessly out of date.
7. A jet airplane uses a fixed quantity of fuel for takeoff and landing (combined). When it's in the air, it uses about 2.7 gallons per mile of fuel. A 400 mile trip uses 1,455 gallons of fuel.
   (a) Name the variables.

(b) What is the combined fixed quantity of fuel uses for takeoff and landing (combined)?

(c) Write an equation illustrating the dependence.

(d) How many gallons of fuel does a 260 mile trip use?

(e) What are the slope and intercept and what do they represent in the story.

8. At 10:00 a.m. we've got snowy skies and 4 inches of new snow on the ground. It's coming down fast out there – 2/3 of an inch an hour.

   (a) Name the variables, measuring time in hours since 10:00 a.m.

   (b) Write an equation illustrating the dependence.

   (c) When did the snowstorm start?

   (d) Name a new variable for time measured this time in hours since the snowstorm started.

   (e) Write an equation illustrating the dependence using this new variable instead.

   (f) Check that this equation confirms 4 inches of new snow at 10:00 a.m.

   (g) Explain why the two equations have different intercepts.

9. To make cookies it takes a few minutes to prepare the dough. After that it takes 8 minutes per batch to bake in the oven. Last time I made 4 batches of cookies and it took a total of 45 minutes.

   (a) How long does it take me to prepare the dough?

   (b) How long would it take me to make 10 batches of cookies for the cookie swap? Assume the time to prepare the dough remains the same and only the baking time increases.

   (c) Name the variables and write an equation illustrating the dependence.

   (d) What is a realistic domain for the function?

   (e) Identify the slope and intercept and explain their meaning in the story.

10. SU – probably kill this example. See the next one instead. Sharonda recently went on a diet. She mentioned that she now weighs 137 pounds after 12 weeks. She claims to have lost 3 pounds per week.

    (a) At this rate when will she reach her goal weight of 125 pounds?

    (b) What did she originally weigh?

    (c) Name the variables and write an equation for the function.

    (d) Show how to solve your equation to find when Sharonda will reach her goal weight of 125 pounds.

    (e) What is a realistic domain for this function?

11. In each of the following situations, find the initial weight of the person. Also, name the variables and write an equation illustrating the dependence.

(a) After 2 weeks of the flu, Shawn weighs 178 pounds. He's lost 3 pounds per week.

(b) Jeremy hopes to lose 5 pounds per week on his new diet to reach his goal weight of 180 in only 6 weeks.

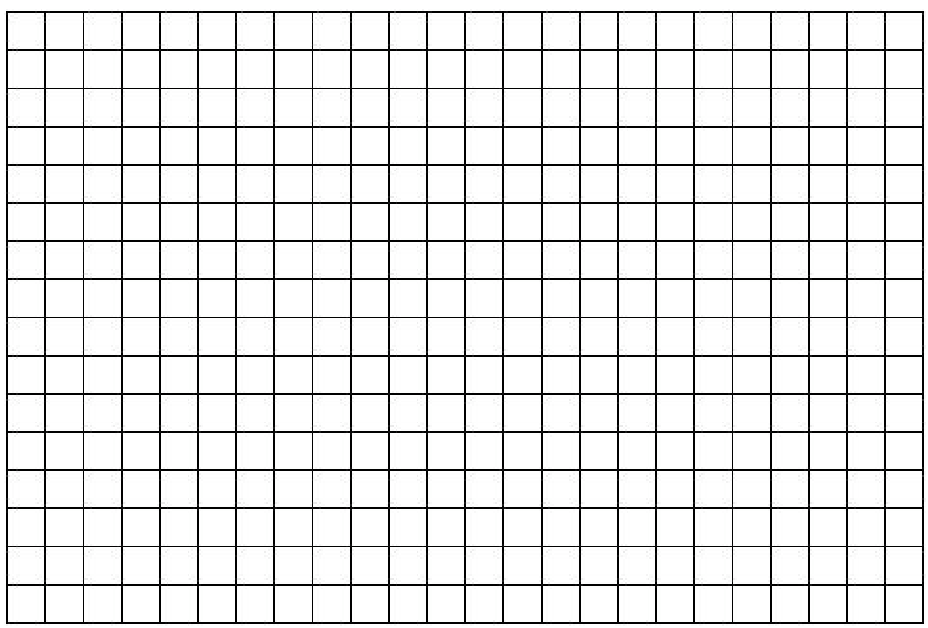
(c) Since she's been pregnant, Zoe has gained the recommended 1/2 pound per week. Now 30 weeks pregnant and 152 pounds, she wonders if she'll ever see her feet again.

12. Anthony's candy bucket weighs 4 ounces when empty. Each Snickers Almond bar he puts in his bucket weighs 1.76 ounces. The total weight T ounces of Anthony's candy bucket depends on the number of Snickers Almond bars B in it according to the equation

$$T = 4 + 1.76B$$

(a) Make a table of values showing the weight of the bucket if it contains 1, 5, 12, or 20 candy bars.

(b) Draw a graph illustrating the dependence.



(c) If Isabella's candy bucket can hold up to 3 pounds (that's 48 ounces), approximately how many candy bars can it hold?

Say what the answer is and mark the point on your graph that shows the answer.

(d) Solve the inequality $4 + 1.76B \leq 48$.

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.

- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

4.4 Slopes

Change to some gal in town on business. My mother took a taxicab from the airport to downtown Minneapolis. The meter showed that the taxicab travelled 4.2 miles and the fare was SU FIX. On the way back to the airport, she asked the taxicab driver to take the scenic route around the lakes. That time the meter showed the taxicab had travelled 7.6 miles and the fare was SU FIX. Taxicab fares are based on a linear model of a starting charge and an additional price per mile.

How much per mile?

How much starting charge?

Name variables and write equation.

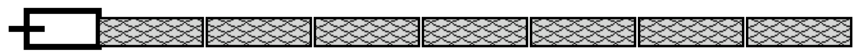
Draw graph.

Travel guide says fares to hotels usually range between $x to $y dollars. What distances correspond? Set up and solve inequality.

Really they say first tenth of mile and then per tenth of mile thereafter. How do that?

Practice exercises

1. Jana is making belts out of leather strips and a metal clasp. An extra short length belt (as shown in the picture) is 24.5 inches long and includes 7 leather strips. An extra long length belt (not shown) is 37.3 inches long and includes 11 leather strips. Each belt includes one metal clasp that is part of the total length.



   (a) Name the variables, including units.

   (b) Display the information from the story in a table.

   (c) How long is each leather strip?
   If you are not sure, you are welcome to find the equation in part (e) first.

   (d) How long is the metal clasp?

   (e) Write an equation relating the variables.

   (f) Solve your equation to find the number of leather strips in a extra extra long length belt that's 43.7 inches long? *For full credit show how to solve your equation. If you figure out the answer another way, you are only eligible for partial credit.*

2. In 2000, the median income for a college graduate was about \$31,700. By 2006 it was up to about \$34,300.

   (a) By how much has it increased each year, on average? *Note: in this context the phrase "on average" means that you should assume the increase is **linear**.*

   (b) Name the variables, including units, and write a linear equation relating them. *Hint: measure the years since 2000.*

   (c) According to your equation, at this rate when will the median income be over \$38,000?

3. Third – neg slope and pos intercept

   (a) xx

   (b) xx

4. The average 5'7" tall male college athlete weighs 160 pounds while the average 6' tall male college athlete weighs 195 pounds. Assume that weight depends linearly on height (which is probably not true, but let's assume so anyway.)

   (a) Name the variables, including units, and a realistic domain.

   (b) Write an equation illustrating the dependence.

   (c) According to your equation, how tall on average is a male college athlete who weighs 175 pounds?

   (d) Explain why the intercept does not make sense in this problem.

   (e) Draw a graph illustrating the dependence. *Graph over the realistic domain. Do not start at 0.*

Do you know . . .

- How to find the equation of a line through two points?
- How to find a linear function given two examples in a story?
- If both the slope and intercept are unknown, which is easier to calculate first?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

SU ORDER THESE TO HAVE first problem w/ intercept known, next few walking through the intercept/slope in steps, then another intercept known, then a bunch with just – find equation. Be sure there's a mix of negative slopes in here. Might be fun to throw in one where the intercept turns out to be zero. OR do that in last section!

SU need some graphs in here too. Don't forget

5. The amount of garbage generated in the United States has increased steadily, from 88.1 million tons in 1960 to 254.2 million tons in 2006.
   (a) Assume the amount of garbage increases linearly, by how much has garbage increased each year?
   (b) Name the variables, including units, and write a linear equation relating them. *Hint: measure the years since 1960.*
   (c) According to your equation, what would predict the amount of garbage generated in 2010 will be?
   (d) If this trend continues, when will the amount of garbage generated exceed 280 million tons?
   Show how to set up and solve an inequality to find the answer. Or you may find the answer some other way for possible partial credit. Be sure to state the actual year.

6. xx
   (a) xxx

7. My gas bill last month was \$249.00 SU FIX and I used 140 therms of gas. This month my gas bill was \$385 and I used 225 therms of gas. The bill is based on a fixed fee plus a charge per therm used.
   (a) What is the charge per therm used?
   (b) What is the fixed fee?
   (c) Name the variables and write an equation relating them.

(d) The gas company offered to put me on a budget plan of \$217 per month. How many therms does that assume? Solve your equation to find the answer.

8. In the year 1960, the average person in the United States drank 14 gallons of pop. By 1990, it was up to 50 gallons. SU check data and update. Do I want it to be something about fat or calories. Or "supersize" hamburgers?

   (a) xxx

9. The infamous Koolbar problem (negative demand)

   (a) xxx

10. The infamous History text problem.

    (a) xxx

11. Data about the average square feet in a new house built?

    (a) xxx

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

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4.5 Fitting lines to data*

INTRODUCTORY EXAMPLE

Su: now ;use technology to find the equation of best fit line. Maybe add drawing residuals and correlation.

Perhaps introduce secant lines to curves and the idea of convexity (+ is over, - is under) estimate using linear interpolation.

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

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Practice exams on Linear equations

Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

Practice exam 4– version I

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

Practice exam 4 – version II

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts
   (b) Listed here

Chapter 5

A closer look at exponential equations

Yada yada

5.1 Modeling with exponential equations

INTRODUCTORY EXAMPLE

This is a review of what we have done so far.

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

    (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

5.2 Exponential growth

This section is now limited to growth ONLY. Introduce the growth factor formula. Explain how to find r from g. Do example with “doubling time”. Include examples where solve equations.

INTRODUCTORY EXAMPLE

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

    (a) xx

    (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know ...

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

5.3 Exponential decay

INTRODUCTORY EXAMPLE

This section is now limited to decay ONLY. Revisit $g = 1 + \frac{r}{100}$ explaining if r is negative how that works. Repeat the growth factor formula, but now for decay. Explain how to find r from g. Do example with “half life”. Include examples where solve equations.

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

5.4 Linear vs. exponential models

INTRODUCTORY EXAMPLE

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

5.5 Logistic growth (and other models)*

INTRODUCTORY EXAMPLE

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

Practice exams on Exponential equations

Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

Practice exam 5– version I

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

Practice exam 5 – version II

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

Practice final exams

Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself two hours to work and wait until you have tried your best on all of the problems before checking any answers.

Practice final exam– version I

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

Practice final exam – version II

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

Appendix A

More about . . .

You have been using and learning mathematics all of your life. Sometimes in school. Other times at home or on the job. But if you are like most people, you have likely forgotten many details over the years. Moreover, there are probably concepts that you never really did understand.

Mathematics that you use everyday is probably very familiar to you. Perhaps you have taken another mathematics course in college already, or had other classes where you have reviewed some mathematics. Or maybe you were reminded of some things when you helped a child with her homework. Only one thing is certain – each of us remembers, knows, and understands a different collection of mathematics.

Where should a mathematics book begin, then? What mathematics can we assume most students already know? What can we review quickly? What is new to most students? In this course we take the approach that reviewing at the time we need a piece of information works well for most people. (That philosophy is known as "just in time" instruction.) Throughout this text we try not to assume too much and try to include all of the mathematics you need to know.

There are times, however, when we likely review a topic too quickly for some students. If you find yourself in that situation, consider this Appendix your safety net. Hopefully you will find here enough additional instruction and practice with the core prerequisite knowledge for those times. If not, be sure to ask your instructor for advice.

A reminder that since everything you need to know is in the main text, if the course is going well you might find you never use this Appendix. But, it's here if you need it.

Pretests on More about ...

SU mention that this is really a pretest. Not that you have to know 100% of this material at the get-go, but it's a good indicator of readiness if you've at least seen this stuff.

Try taking these practice exams under testing conditions: no book, no notes, no class-mate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

Practice exam 3– version I

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

Practice exam 3 – version II

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

A few formulas from our book:

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

A.1 Approximation

How tall is that Maple tree? If you think about it, it is not obvious how to measure the height of a tree. We could measure to the highest leaf, but it seems odd to say that the tree is shorter if a leaf falls off. Or we could measure to the top of a branch, but it might bend lower in the wind. Or we could measure to the top of a thick enough branch, whatever that means. The point is that we don't know how to measure the height of a tree that precisely. By the way, the word **precisely** refers to the number of decimal digits.

Could the Maple tree be 93.2 feet tall? No way – that is too precise. Is 93 feet tall correct? Maybe, but we could be off by a couple of feet depending on where we measure. Perhaps we can hedge slightly and call it 95 feet tall. Hopefully that's reasonable. Or maybe we should play it really safe and say it is not quite 100 feet tall.

The point is: there is no such thing as "the" right answer. When we ask a real world question, we want a real world answer. The answer depends on the question.

While it is good to keep as many digits as possible during calculations, in practice, at the end of a problem we always approximate the answer by **rounding** – finding the closest number of a given precision. The height of 93.2 feet was likely rounded to the nearest tenth (one decimal place). We rounded to the nearest whole number to get 93 feet. The point is that 93.2 is closer to 93.0 than it is to 94.0, so our answer is 93.0 or 93 feet.

Perhaps this is a good place to mention the notation. We write

$$93.2 \approx 93$$

to indicate that we have rounded. The symbol $\approx$ means "approximately equal to."

How much to round off the answer depends on the question. To begin you should apply your common sense. Your answer should definitely sound natural, something you might actually say to a friend or your boss. But there's also one more rule to know: your answer should not be more precise than the information in the problem.

For example, suppose we read that the comprehensive fee at a local university is around \$23,000 and projected to increase by 12% per year. We might calculate that in four years fee would be \$36,190.945... The ... indicate that we have not copied all the digits from the calculator. (More in the next section on how we calculated that number). We could round to the nearest penny and say "around \$36,190.95" or we could round to the nearest dollar and say "around \$36,191." The numbers we are given (\$23,000 and 12%) have only two digits that matter, however, so we should actually round off and say "just over \$36,000."

By the way, when we refer to digits that matter, we are really referencing **significant digits**. That theory explains how combining numbers influences the number of digits in the answer that are correct, which is why we wait until our final answer to round. In this text we do not follow those rules exactly, but you should be aware that some areas of study (such as Chemistry) do.

You might be surprised to learn that approximate answers are not only good enough; they are often best. For one thing, in practice we want a round number so it is easy to understand and work with our answer. A rounded answer is just approximate. Also often the numbers we are given in a problem were rounded or approximated – for the record, that fee was really \$23,058, not \$23,000. When we start with approximate numbers, then no matter how precise the mathematics we use, we can only get approximate answers.

Last, in much of this course the methods we will use to calculate answers are , themselves, approximate. We might *suppose* that tuition increases exactly 12% each year, when we know in reality that the percent will likely vary. That is an example of using an **approximate model**. We might have an actual model but use some numerical or graphical technique for solving. That is an example of using an **approximation technique.** In either case, if the model or technique we use is approximate, then our answer can only be either.

There is an old saying we try to live by in this course.

> I'd rather be approximately right than precisely wrong.

One more subtlety. We have been rounding to the nearest number of a given precision. That process is also known as **rounding off**. There are times when we will need to **round up** – to the next highest number of a given precision, or **round down** – to the next lowest number of a given precision.

For example, during Happy Hour at a local restaurant, buffalo wings sell for 60¢ per wing. Your buddy only has \$7. After a quick calculation on his cell phone he decides to order a dozen wings. Your buddy probably calculated

$$7 \div .60 = 11.6666666... \approx 12$$

(More in later sections about how to calculate these numbers.) Trouble is he cannot afford a dozen wings, because they would cost \$7.20. (You can check $12 \times .60 = \$7.20$.) Not to mention the tax, tip, and three beers (so far) that he downed. Good thing you can point him to the bank machine so he can get cash and you won't have to pay his tab (again). What's the trouble here? Besides ignoring tax, tip, and those beers he rounded off when he should have rounded down

$$7 \div .60 = 11.6666666... \approx 11$$

It should be clear form the story whether you will need to round off, round up, or round down. Again, our mantra is: the answer depends on the question.

Practice exercises

1. Round (off) each number as indicated.

   (a) Round 19.388 feet to the nearest foot. *That means nearest whole number.*

   (b) Round \$17.9892 to the nearest penny. *That means to two decimal places.*

   (c) Round 72,498 marbles to the nearest thousand.

   (d) Round 4,908,229 people to the nearest million. *That means ending in six 0s.*

2. Round off the calculated number(s) to give an answer that is reasonable and no more precise than the information given.

   (a) The snow removal budget for the city is currently at \$8.3 million but the city council is requesting a reduction of \$1.15 million per year. We calculate that after three years of cuts, the snow removal budget will be \$4.85 million.

   (b) A cup of cooked red lentils has around 190 calories and 6.4 grams of dietary fiber, while a cup of cooked chickpeas has around 172 calories and 12.0 grams of dietary fiber. We calculate that lentils provide 0.03368421... grams per calorie whereas chickpeas provide 0.06976744... grams per calories.

   (c) > Hibbing [Minnesota] is the former boyhood home of Bob Dylan, basketball great Kevin McHale and the location of the Hull-Rust-Mahoning Open Pit Iron Mine, which has the largest iron-ore pit in the world. Hibbing is also the birthplace of [baseball star] Roger Maris.
       >
       > (source: http://hibbing.areaconnect.com/)

       In 2000 the population of Hibbing, Minnesota was reported at just over 17,000 residents. Based on a projected 0.4% decrease per year, the 2010 population was calculated to be 16,332.110... people.

3. It is easiest to compare the size of decimal numbers, if they are written the same precision. For example, \$1.7 million is more money than \$1.34 million because when we write both numbers to two decimal places we see

$$1.7 = 1.70 > 1.34$$

The symbol > means "greater than;" it points to the smaller number. Alternatively, when we expand both numbers we see

$$1{,}700{,}000 > 1{,}340{,}000$$

In each story, write all of the decimal numbers given to the same precision and list the numbers from largest to smallest using > signs.

(a) Dawn tested a water sample from her apartment and found 21.19 ppm of sulfate. She volunteers at a local soup kitchen where the water sample tested at 21.3 ppm. (The abbreviation **ppm** stands for "parts per million. Not to worry – sulfate levels below 250 are considered safe for human consumption.)

(b) There are approximately 1.084 million quarters in circulation in the United States, compared to 1.786 million dimes, 1.6 million \$5 bills, and 1.42 million \$10 bills.

(c) The width of a human hair is around .00012 meters. A virus can measure around .00000002 meters. Red blood cells are about .000003 meters in diameter. An e-coli bacteria measures approximately .00000025 meters across.

These numbers might be difficult to read, so we can line them up vertically and rewrite them with space every three digits.

Human hair	.000 12	meters
Virus	.000 000 02	meters
Red blood cells	.000 003	meters
E-coli bacteria	.000 000 25	meters

(In practice these widths are usually reported using different units – 120 microns for a human hair, 20 nanometers for a virus, etc.)

4. Body Mass Index (or BMI for short) is one indicator of whether a person is a healthy weight. BMI between 18.5 and 24.9 are considered "normal".

   Jarron is 6 foot 4 inches tall, which he calculated is approximately 1.93 meters. He weights 202 pounds, which he calculated was approximately 91.625 kilograms. He would like to calculate his BMI directly.

   (a) Jarron entered the following keystrokes on a scientific calculator:

   $$91.625 \div 1.93 \wedge 2 =$$

   and got the answer

   $$\text{Jarron's BMI } = 24.747969...$$

   Is his BMI considered "normal"?

   More later on where this calculation comes from. If your calculator does not have the $\wedge$ key, look for y^x key instead.

   (b) Suppose Jarron had rounded off his height to 1.9 meters and his weight to 92 kilograms. Calculate his BMI by entering the following keystrokes on a scientific calculator:

   $$92 \div 1.9 \wedge 2 =$$

   What do you get? Round your answer to one decimal place. Is Jarron's BMI considered "normal"?

   (c) What would you tell Jarron?

Do you know . . .

- When to round your answer?
- When to round your answer up or down (instead of off)?
- How to round a decimal to the nearest whole number? To one decimal place? To two decimal places?
- What the difference is between rounding off, rounding up, and rounding down?
- What the term "precisely" refers to?
- How precisely to round an answer?
- What the symbol for "approximately equal to" is?
- Why an approximate answer is often as good as we can get?
- What the saying "I'd rather be approximately right than precisely wrong" means?
- How to compare sizes of decimal numbers?
- What the symbol for "greater than" is?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. Round (off) $\$148,214.779$ as indicated
   (a) To the nearest penny (two decimal places).
   (b) To the nearest dollar.
   (c) To the nearest thousand.
   (d) To the nearest ten thousand. *That means ending in 0,000*
6. Round up/down problem directly.
7. Gas mileage is usually reported to the nearest tenth (one decimal place). Rewrite each given number.
   (a) 42.889 miles per gallon
   (b) 23.01 miles per gallon
   (c) 24.99 miles per gallon
   (d) 37 miles per gallon.
8. Joshua is trying to plot points on a graph. The scale on the graph is labelled by $100s so he needs the numbers rounded to the nearest $10. For example, he needs to know that $\$247 \approx \$250$ while $\$73 \approx \$70$.

(a) Round each number to the nearest \$10:
   i. \$589
   ii. \$41
   iii. \$190
   iv. \$2

(b) Order those numbers from largest to smallest.

9. Another problem where decide round up or down

10. Souksavanh is trying adjust a patient's medication to deliver 15 μg/min. If she runs the drip at 9.1 ml/hour, medication will be delivered at 14.76 μg/min which is too low. If she runs the drip at 9.3 ml/hour, medication will be delivered at 15.09 μg/min which is too high.

   (a) Which of these values are between 9.1 and 9.3 ml/hour:

   9.18 ml/hour, 9.22 ml/hour, 9.07 ml/hour, 9.41ml/hour?

   (b) If she runs the drip at 9.2 ml/hour, medication will be delivered at 14.93 μg/min which is still too low. Souk would like to try a rate between 9.2 and 9.3 ml/hour. What rate can she try? *That means, identify a number between 9.2 and 9.3. Hint: Try thinking of them as 9.20 and 9.30.*

   (c) She has narrowed it down to between 9.24 and 9.25 ml/hour (though perhaps the drip can't be controlled that precisely). What can she try? *That means, identify a number between 9.24 and 9.25. Hint: Try thinking of them to three decimal places.*

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

A.2 Arithmetic operations

Numbers, numbers everywhere – how do we put them together to get a final answer?

As a first example, Zahra needs to complete 200 hours of classroom observation before she is eligible to student teach. She logged 45 hours last spring, another 42 hours this past fall, and is on pace to finish 51 hours this spring. How many more hours will she need next fall to finish here 200 hours?

Let's begin by figuring out how many hours Zahra finished before this semester. She did 45 hours and then another 42 hours. We add to get the total number of hours.

$$45 + 42 = 87 \text{ hours}$$

If we add in the number of hours from this spring, her total will be

$$87 + 51 = 138 \text{ hours}$$

We assume you are using a calculator to add these numbers. It is a good habit to write down the keystrokes you do. Perhaps you did this sequence of keystrokes.

$$45 + 42 =$$

and then

$$87 + 51 =$$

That works, but it is not necessary to type the 87 in yourself. Instead try this sequence of keystrokes.

$$45 + 42 = +\ 51 =$$

Did you get 138 again? When you typed that first equal sign the calculator should have displayed 87, and after the second equal sign, the final answer of 138. That works too, but the first equal sign is not needed. Try this shortest (best!) sequence of keystrokes.

$$45 + 42 + 51 =$$

Hopefully you got 138 again.

How many more hours does Zahra need to reach the goal of 200 hours? We are looking for the number of hours where

$$138 + \ ? = 200$$

We find the missing time by subtracting

$$200 - 138 = 62 \text{ hours}$$

You can check that

$$138 + 62 = 200$$

or, starting from the beginning

$$45 + 42 + 51 + 62 = 200$$

Any way you calculate it, Zahra is going to have a busy fall.

Here's another example. Cole was shocked by his credit card bill. It shows a previous balance of \$529.16, credit for a payment of \$200, finance charge of \$42.78, a late fee of \$30 (ouch!), a credit for \$17.43 for a return he made, and \$618.25 in new charges.

Cole would like to practice his math and check the balance on his bill. One way to calculate his balance is to add the charges while subtracting the credits. Cole calculates

$$529.16 - 200 + 42.78 + 30 - 17.43 + 618.25 = \$1002.76$$

Sometimes credits are represented by negative numbers. Another way Cole could calculate the answer is to add the mix of positive numbers (charges) and negative numbers (credits).

$$529.16 + \text{-}200 + 42.78 + 30 + \text{-}17.43 + 618.25 = \$1002.76$$

You may notice that the sign $-$ subtraction and - used for negation look very similar. On the calculator these are two different keys. The subtraction key reads just $-$. The negation key often reads (-) and is done before the number. This does not mean you type in parentheses, just hit the key that is labeled (-) already. In this notation, we can write what Cole entered as

$$529.16 + (\text{-})200 + 42.78 + 30 + (\text{-})17.43 + 618.25 =$$

Try it.

(If your calculator does not have a key labeled (-), look for a key labeled +/- instead. That is not three keys, just one labeled $+/-$ To emphasize that it is one key, we just write $\pm$. Often that key needs to follow the number, so enter the following keystrokes.

$$529.16 + 200 \pm + 42.78 + 30 + 17.43 \pm + 618.25 =$$

You should get \$1002.76 again.)

There are times when it is convenient to rewrite a sum in a different order. That can get tricky if there are both + and -. Rewriting each subtraction as addition of a negative keeps the minus signs where they belong. For example, Cole might have calculated

$$529.16 + 42.78 + 30 + 618.25 - 200 - 17.43 =$$

Notice how the subtractions stay with the payment of \$200 and credit of \$17.43.

Another example. Too much snow this winter and the rainiest May anyone can remember. The river has been rising rapidly, 10 inches a day some say, for the past 3 weeks. How much has the river risen in total?

We need to deal with some units here. (For more information on Units, read the section.) How many days is 3 weeks? There are 7 days in a week so in three weeks there are

$$7 + 7 + 7 = 21 \text{ days}$$

Since multiplication is short for addition, we can calculate this number more quickly.

$$3 \times 7 = 21$$

(Most calculators have a $\times$ key, but in some computer programs $*$ is used instead.)

At 10 inches per day, the river has risen

$$10 + 10 + 10 + \ldots + 10 \text{ inches } \textit{(21 times)}$$

which we calculate as

$$21 \times 10 = 210 \text{ inches}$$

The river has risen 210 inches.

The river has risen 210 inches, you say? Hmm. How many feet is that? There are 12 inches in a foot so we want

$$12 \times \ ? = 210$$

We find the missing rise by dividing

$$210 \div 12 = 17.5 \text{ feet}$$

(Many calculators use a key labeled / instead of the more old fashioned $\div$. We use the notation $\div$ since the slash has so many different meanings and is easily misread. Just remember to do / whenever you see $\div$ in this book. Try

$$210/12 =$$

You should get 17.5 again.)

Any way you calculate it, the river has risen 17.5 feet, or nearly 18 feet. Sadly it turns out that is 6 feet above flood stage for this stretch of river, so the flooding is causing a lot of damage.

Two more examples. There are two other situations in which we divide. The first is fractions. If we are given a fraction, like $\frac{2}{3}$ of residents have Internet access, we might want to have a decimal approximation to do further calculations. We find it by dividing

$$\frac{2}{3} = 2 \div 3 = 0.666666666... \approx .67$$

The bar in between the 2 and 3 in the fraction stands for division. It might help to remember this connection visually.

$$\frac{\bullet}{\bullet}$$

So if there are 14,573 residents, and $\frac{2}{3}$ have Internet access, then we can multiply to find the number of residents with Internet access.

$$14573 \times 0.7 = 9763.91 \approx 9764$$

Since .67 only has two digits that matter (and we rounded up to get it), we would round our answer down to compensate and say there were around 9700.

Can we say that there 9,764 residents with Internet access? If we had used .0.666666666 instead we would have calculated

$$14573 \times 0.666666666 = 9715.33332... \approx 9715$$

Sounds like we should play it safe and say over 9,700 residents have internet access. The most accurate calculation would be to do the multiplication and division all at once.

$$14573 \times 2 \div 3 = 9715.33333... \approx 9715$$

It would be acceptable to estimate at 9,715 residents.

Another cause for division is the word "per". What is her pace, in minutes per mile if Karleen runs 6.3 miles in 50 minutes?

$$50 \div 6.3 = 7.936507937... \approx 7.9$$

With the units we would write

$$50 \text{ minutes} \div 6.3 \text{ miles } \approx 7.9 \text{ minutes per mile}$$

Karleen runs just under 8 minutes per mile.

Practice exercises

e saw that the snow removal budget for the city is currently at \$8.3 million but the city council is requesting a reduction of \$1.15 million per year. They calculated that after three years of cuts, the snow removal budget will be $\$4.85 \approx \$4.9$ million. One way to calculate the \$4.85 million is to

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

Do you know . . .

- When to add, subtract, multiply, or divide numbers?
- What is the difference between subtraction and negation?
- How to add, subtract, negate, multiply, and divide on a calculator?
- How multiplication is related to addition?
- How fractions are related to division?
- What the term "per" indicates?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

    (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

A.3 Percentages

INTRODUCTORY EXAMPLE – not sure this chapter needs to follow lead of others. Could be quite brief.

How to convert between decimal and percent. How to calculate percentage of. How to calculate percentage increase and decrease (long and short, but only one time period – 1.05, .95 not to powers).

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

A.4 Power operations

INTRODUCTORY EXAMPLE – not sure this chapter needs to follow lead of others. Could be quite brief.

Power – as repeated multiplication (just positive integer exponents, include powers of 10), how to find powers on calculator

Root – what root means (as soln base), how to find roots on calculator (various ways)

Logarithms – what log means (as soln power), how to find logs on calculator, connection to number of digits/exponent in scientific notation?

: $\wedge$, $\sqrt{\ }$, and log

Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third
   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know ...

- How powers are related to multiplication?
- What a root means?
- What a logarithm means?
- When to raise a number to a power, take a root, or take a logarithm?
- How to raise to a power, take roots, and take logarithms on a calculator?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

A.5 Order of operations

Now combine, only in calculator notation. Addition Subtraction, Negation Multiplication Division – no fraction notation yet! Power – just positive integer exponents Root –

Explain how need parentheses to override. Justify distribution over addition, but not over multiplication

INTRODUCTORY EXAMPLE – not sure this chapter needs to follow lead of others. Could be quite brief.

Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

    (a) xx

    (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
   (a) xx

   (b) xx

Do you know ...

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx
   (a) xxx

When you're done ...

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

A.6 Algebraic notation

INTRODUCTORY EXAMPLE – not sure this chapter needs to follow lead of others. Could be quite brief.

Key ideas

Unwritten multiplication – both number letter and number parentheses

Fractions – how bar means division, how relates to order of operations, perhaps do some examples (or leave room for) from other sections?

Exponential notation (vs. to the)

How we tend to write with fewest symbols (b - a vs. -a + b), and numbers before letters, and alphabetical when choice

Practice converting from expression in algebraic notation to calculator arithmetic.

Perhaps also practice evaluating formulas?

Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

Do you know . . .

- Questions?

If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.

Exercises

5. xx

   (a) xxx

When you're done . . .

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 16, 2012

Solutions to practice exercises and practice exams

SU: handwrite and scan in here

Answers to exercises

Solutions to the practice exercises appear in an earlier section.

SU: type in here – preferably with links to the actual problems so don't have to hand enter?