SOLUTIONS

Practice Exam 5A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

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start = 5

1. Leopard print hat. Originally 5 out of 1,000 women shopping at a major retail store even looked twice. But that number grew and grew, by my estimate around 40% a week, thanks to carefully placed ads in fashion magazines. exponential

   (a) Write an equation illustrating the interest in leopard print hats using W for the time (in weeks) and L for the number of women interested in leopard print hats (women per thousand).

   $r = 40\% = .40$ ($\div 100\%$)

   $\Rightarrow g = 1 + r = 1 + .40 = 1.40$

   used template for exp. eq. $\rightarrow$ $\boxed{L = 5 * 1.40^W}$

   (b) Make a table showing the number of women, per thousand female shoppers, who stop and look at the hat at the start, 1 week, 2 weeks, and 3 weeks after it hits the stores.

W	0	1	2	3
L	5	7	10	14

   $5 \times 1.40 \wedge 3 = 13.72 \approx 14$

   (c) The leopard print hat is considered popular when more than 300 out of 1,000 women try it on. According to the equation, when will the hat be considered popular? Use successive approximation to find the answer to the nearest week and display your work in a table.

W	10	15	12	13
L	145	778	283	397
	LOW	HIGH	LOW	HIGH

   $\boxed{\text{The hat will be considered popular in} \approx 13 \text{ weeks.}}$

   (d) The hat will be considered passé when over 750 out of 1,000 women try it on. I mean – everyone's got one!. According to your equation, when will that happen? Set up and solve an equation, again answering to the nearest week.

   $$\frac{\cancel{5} * 1.40^W}{\cancel{5}} = \frac{750}{5}$$

   $$1.40^W = 150$$

   By the Log-Divider Formula $\rightarrow$

   $$W = \frac{\text{Log}(150)}{\text{Log}(1.40)} = \text{Log}(150) \div \text{Log}(1.40) = 14.89.. \approx 15 \text{ weeks}$$

   $\boxed{\text{The hat will be considered passé after} \approx 15 \text{ weeks.}}$

2. HeeChan bought a classic car in 2003 for investment purposes and has been watching the value increase over the years. Based on the data HeeChan came up with two possible equations

$$\textbf{Logistic:} \quad C = \frac{41{,}000}{1 + 4 * .81^Y}$$

$$\textbf{Saturation:} \quad C = 32{,}000 - 23{,}800 * .85^Y$$

where Y is the years since 2003 and $\$C$ is the value of the car.

(a) How much did HeeChan pay for the car in 2003? $Y=0$

Logistic: $41000 \div (1+4 \times .81 \wedge \underline{0}) = \$8{,}200$

Saturation: $32000 - 23800 \times .85 \wedge \underline{0} = \$8{,}200$

HeeChan paid \$8,200 for the car in 2003.

(b) What does each equation predict for the value of the car now, in 2013? For 2020?

$Y = \begin{array}{r} 2013 \\ -2003 \\ \hline 10 \end{array}$

$Y = \begin{array}{r} 2020 \\ -2003 \\ \hline 17 \end{array}$

	Logistic	Saturation
2013	\$27,585	\$ 27,314
2020	\$36,895	\$30,498

$32000 - 23800 \times .85 \wedge \underline{10} =$

$41000 \div (1+4\times .81 \wedge \underline{17}) =$

(c) What does each equation say will be the eventual value long term? *Hint: if you're not sure try 100 years.*

	Logistic	Saturation
Y=100	40,999.99...	31,999.99...
	≈ \$41,000	≈ \$32,000

aha! those numbers are in the equations

Long term, the logistic predicts a value of \$41,000 whereas the saturation predicts a value of \$32000.

3. The number of geese in the Twin Cities metropolitan area increased from 480 in 1968 to 25,000 in 1994. Although population is sometimes modeled with exponential models, there are many factors that might make an exponential model inappropriate, such as changes in migration, wetlands, and hunting.

   (a) Name the variables.

Y = time (years since 1968) ~ indep
G = goose population (geese) ~ dep

year	Y	G
1968	0	480
1994	26	25,000

1994 − 1968 =

   (b) Write a linear equation modeling the goose population.

$$\text{slope} = \frac{25{,}000 - 480}{26 - 0} = (25000 - 480) \div (26 - 0) = 943.07\ldots \approx 943.1 \text{ geese/year}$$

intercept = 480 geese

$$\boxed{G = 480 + 943.1\,Y}$$

uses linear equation template: dep = start + slope * indep

Check: Y = 26
G = 480 + 943.1 × 26 = 25,000.6
≈ 25,000 ✓

   (c) Now write an exponential equation modeling the goose population.

By the Growth Factor Formula

$$g = \sqrt[t]{\frac{a}{s}} = \sqrt[26]{\frac{25{,}000}{480}} = 26 \,{}^{x}\!\sqrt{\ }\,(25{,}000 \div 480) = 1.1641980\ldots \approx 1.164$$

$$\boxed{G = 480 * 1.164^{Y}}$$

uses exponential equation template: $\text{dep} = \text{start} * g^{\text{indep}}$

Check: Y = 26
G = 480 × 1.164 ^ 26 = 24,889.6
≈ 25,000
notice how much round off error we picked up rounding off g.

To avoid so much round off error, kept more digits for g. Used $G = 480 * 1.164198072^Y$

The problem continues ...

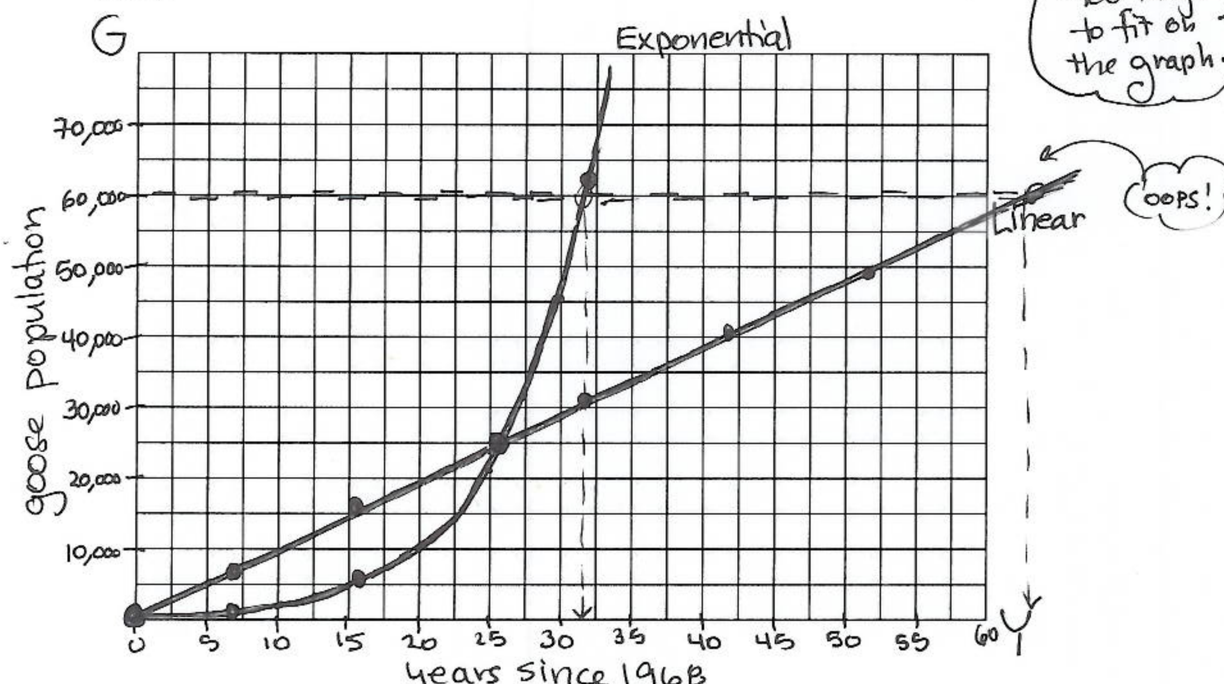
1975 − 1968 =

(d) Compare the models projections for 1968, 1975, 1984, 1994, 2000, 2010, and 2020. Summarize your findings in a table.

	year	1968	1975	1984	1994	2000	2010	2020
	Y	0	7	16	26	32	42	52
G	linear	480	7,082	15,570	25,000	30,659	40,090	49,521
G	exponential	480	1,391	5,466	25,000	62,245	284,689	1.3 million

(e) Graph each function over the period from 1968 to 2020 on the same set of axes. *Test-taking tip: even if you have trouble with the equations, you should be able to plot the information given in the story and sketch in the appropriate shape curves.*

these last two values are too large to fit on the graph.



(f) Research indicates that the Twin Cities metropolitan area could support 60,000 geese. Use your graph to estimate when that will happen.

linear ≈ 63 yrs = $\begin{array}{r}1968\\+63\end{array}$ = $\boxed{2031}$

exponential ≈ 32 yrs = $\begin{array}{r}1968\\+32\end{array}$ = $\boxed{2000}$

(g) The actual goose population in 2010 was around 50,000. Which model was closer?

in table

linear estimate for 2010 was 40,090 geese, which is lower than the actual 50,000 geese, but much closer than the exponential estimate which was 284,689 geese.

4. One of the toxic radioactive elements produced by nuclear power plants is strontium-90. A large amount of strontium-90 was released in the nuclear accident at Chernobyl in the 1980's. The clouds carried the strontium-90 great distances. The rain washed it down into the grass, which was eaten by cows. People then drank the milk from the cows. Unfortunately, strontium-90 causes cancer. Strontium-90 is particularly dangerous because it has a half-life of approximately 28 years, which means that every 28 years half of the existing strontium-90 changes into a safe product; the other half remains strontium-90. Suppose that a person drank milk containing 100 milligrams of strontium-90. ← start = 100mg

SOURCE: "Explorations in College Algebra," by Kime and Clark

(a) After 28 years, how many milligrams of strontium-90 remains in the person's body? After 56 years?

notice 28 + 28 = 56

Y	0	28	56
S	100	50	25

$100 \xrightarrow{1/2} 50 \xrightarrow{1/2} 25$

Half-life is 28 years. That means every 28 years, half of what you started with remains.

(b) Find the annual percentage decrease of strontium-90.

By the Growth Factor Formula

$$g = \sqrt[t]{\frac{a}{s}} = \sqrt[28]{\frac{50}{100}} = 28 \times\!\sqrt{\ } (50 \div 100) = .9755486\ldots \approx .9755$$

$$\Rightarrow r = g - 1 \approx .9755 - 1 = -.0245 \xrightarrow{\times 100\%} = \boxed{-2.45\%}$$

(c) Name the variables and write an equation relating them.

Y = time since injested Strontium-90 (years) ~ indep

S = amount of Strontium-90 (mg) ~ dep

$$\boxed{S = 100 * .9755^{Y}}$$

used exponential equation template

Check:
100 x .9755 ^ 28 = 49.9... ≈ 50 ✓
100 x .9755 ^ 56 = 24.9... ≈ 25 ✓

(d) Suppose that any amount under 20 milligrams of strontium-90 is considered "acceptable" in humans. Will it have reached acceptable levels after 70 years?

Y = 70, S = 100 x .9755 ^ 70 = 17.616... < 20mg

Yes, it will have reached acceptable levels 70 years after the accident.