3.3 Solving power equations (and roots)

Practice exercises

1. **2.4#1** A pizza of diameter D inches serves P people where

$$P = 0.015625D^2$$

   (a) Show how to solve the equation to find the size of a personal pizza. Means serves one person.

   (b) What super-size should a pizza be to serve 6 people? Show how to solve the equation to find the answer.

2. The weight of a wood cube (W ounces) is a function of the length of the side (E inches) according to the equation

$$W = .76E^3$$

   (a) How much does a 2-inch cube weigh? A 3-inch cube?

   (b)

3. **2.4 #2** The gas tank of a car is typically designed to hold enough fuel to drive 350 miles. For example, a hybrid that gets 35 miles per gallon (mpg) would need a gas tank that holds 10 gallons, but a RV (recreational vehicle) that gets only 10 miles per gallon would need a gas tank that holds 35 gallons. We saw earlier that the size tank, G gallons is a function of the fuel efficiency, F mpg according to the equation

$$G = \frac{350}{F}$$

   (a) My Honda Accord's tank holds about 16 gallons. According to the equation, what is the corresponding fuel efficiency? *Set up and solve the equation. Start solving by multiplying both sides by F. There is no power, so you won't have to take a root.*

   (b) My ex-husband's Honda Civic's tank holds only 13 gallons. According to the equation, what is the corresponding fuel efficiency. *Set up and solve the equation.*

4. Moose bought a commemorative football jersey for \$250 fourteen years ago. Now he's planning to sell it and is interested in what the effective return on his investment might be for various prices. The equation is

$$J = 150g^{12}$$

   where $\$J$ is the current value of the jersey and g is the annual growth factor. The effective return is r where $r = g - 1$ and is normally written as a percentage. Find the effective return if the current value is

(a) \$290 (b) \$350 (c) \$400

For each part, first solve for g, then calculate r, and write as a percentage.

Exercises

5. **FIVE** Recall our formula $Z = .018C^2$ where C is the circumference of the lemon, in inches, and Z is the amount of lemon zest, in tablespoons.

   (a) Use the information we found earlier to draw a graph of the function. Include values $0 \leq C \leq 10$.

   (b) Set up and solve an equation to find the size lemon needed for 1 tablespoon of zest.

   (c) Suppose the formula holds for grapefruit too. I don't know of any recipe that calls for grapefruit zest; it is very bitter! But grapefruit is notorious for interacting with certain medications, and so we're collecting some zest for an experiment. Let's say we need 1/4 cup of zest. How large a grapefruit will we need? Set up and solve an equation to answer. Useful fact: 1cup = 16 tablespoons

6. **2.4 #8** Wind turbines are used to generate electricity. For a particular wind turbine, the equation

$$W = 2.4S^3$$

   can be used to calculate the amount of electricity generated (W watts) for a given wind speed (S mph), over a fixed period of time.

   (a) Set up and solve an equation to determine the wind speed that will generate 12,500 watts of electricity.

   (b) Ditto 45,000 watts.

7. **2.3 #8** Mom always said to sit close to the lamp when I was reading. The intensity of light L, measured in percentage (%) that you see from a lamp depends on your distance from the lamp, F feet as described by the formula

$$L = \frac{1}{F^2}$$

   (a) I am most comfortable reading in good light, say 70% intensity. According to the equation, how far away can I sit from the lamp? Use successive approximation to guess the answer to the nearest 1/10 foot. Then set up and solve an equation. Answer to the nearest inch. *For 70%, use $L = .70$.*

   (b) For reading a magazine it's really fine as long as I have 35% intensity. According to the equation, how far away can I sit from the lamp? Use successive approximation to guess the answer to the nearest 1/10 foot. Then set up and solve an equation. Answer to the nearest inch. *For 35%, use $L = .35$.*

8. NEED higher power

9. **FOUR** If you drop a rock from a high place, the distance that it falls in T seconds is given by the equation $R = 16T^2$ where R is measured in feet.

   (a) How far does the rock fall in 2 seconds? In 4 seconds?

   (b) Is the rock falling faster during the first two seconds ($T = 0$ to $T = 2$) or during the second two seconds ($T = 2$ to $T = 4$)?

   (c) The rock was originally dropped from 300 feet above ground. Will it have hit the ground by 4 seconds after it was dropped?

   (d) If you evaluate at $T = 5$, what value of R do you get and what does it mean in the story, again assuming the rock is dropped from 300 feet up.

   (e) When does the rock hit the ground? *Set up and solve an equation. What value of R do you solve for?*

   (f) Now suppose we have a new variable, H, which represents the height of the rock after T seconds and still assuming that the rock was originally dropped from 300 feet above ground, write a new equation for H as a function of T.

   (g) Show how to set up and solve an equation using this new equation to find when the rock hits the ground. *What value of H do you solve for?*

10. **TEN** NEED another cube

11. **ELEVEN** NEED higher power

12. **2.3 #12** Wynter has a pretty decent job. He is paid a salary of \$780 per week but is hours vary week-to-week. Even though Wynter is not paid by the hour, he can figure out what his hourly wage would be depending on the number of hours he works using the equation

$$E = \frac{780}{H}$$

    where if he works H hours, then he's earning the equivalent of \$$E$/hour.

    (a) Wynter was complaining that things have been so busy lately at work that he's earning the equivalent of only \$9.25/hr. How many hours a week does that correspond to?

    (b) Wynter was hoping to earn the equivalent of \$14/hour. How many hours a week does that correspond to?

3.5 Solving quadratic equations*

Practice exercises

1. A high-jumper jumps so that the height, H feet, of the point on his back that must clear the bar after T seconds is given by the equation

$$H = 3.5 + 16T - 16T^2$$

SU PROVIDE GRAPH

(a) When would the high-jumper hit the ground (if there weren't a pit)? **Ouch!** Use the QUADRATIC FORMULA to find the answer.

(b) The high jump pit is 2 feet off the ground. When does the high-jumper land in the pit? Use the QUADRATIC FORMULA to find the answer.

(c) How high a bar can the high-jumper clear? Find the maximum height of that point above ground by evaluating at $T = \frac{-b}{2a}$.

2. The number of people visiting national parks each year has been increasing for many years. The number of annual visits is approximated by the equation

$$N = .0913Y^2 - 1.7Y + 10.1$$

where N is number of annual visits, in millions, and Y is the year since 1920.

(a) Calculate the missing values in the table.

Y	0	5	10	15	20	25	30
V	10		2	5	13		41

(b) Draw a graph of the function

(c) According to this equation, in what year is the number of annual visits the smallest? For that year, what were the number of visits?

(d) Explain why N never equals 0.

(e) So, what actually happens when you try to use the QUADRATIC FORMULA to solve for $N = 0$?

(f) In what year did the number of visitors first pass 700 million in a year? Estimate the value from your graph. Then set up and solve a quadratic equation.

3. The profit $\$P$ from selling M tanks of milk is described by the equation

$$P = -2M^2 + 2000M - 80,000$$

(a) Complete the following table showing the profit of selling tanks of milk.

M	0	100	200	300	400	500	600	700	800	900
P			240,000	340,000		420,000	400,000		240,000	100,000

(b) Explain why negative numbers make sense in this problem.

(c) Draw a graph showing how the profit depends on the amount of milk sold.

(d) How much milk must be sold for the company to "break even" (means having \$0 profit.)? Set up and solve a quadratic equation to answer the question.

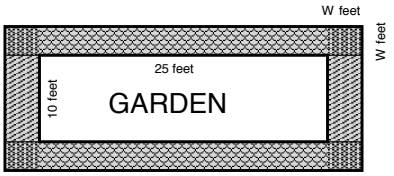
(e) Does your answer agree with your table and graph? Explain.

(f) How many tanks of milk would they need to sell to keep profits over \$400,000? *Your answer should be in the form of an inequality.*

4. **2.3#6, then 2.4 #12** Urban community gardens are catching on. What was once an abandoned lot down the block is now a thriving 10'×25' vegetable and berry garden for the neighborhood. One of the neighbors works for a landscaper who has volunteered to donate some gravel to make a path around the garden that's 3 inches deep and the same width all around. The amount of gravel we need (G cubic feet) is given by the equation

$$G = W^2 + 17.5W$$

where W is the width of the path in feet.



(a) If the landscaper donates 60 cubic feet of gravel, how wide a path can they build? *Set up and solve a quadratic equation to find the answer to two decimal places in feet. Then convert your answer into inches.*

(b) Gravel is measured by the "yard," which is short for cubic yard. There are 27 cubic feet in 1 cubic yard. If the landscaper donates three yards of gravel, how wide a path can they build? *Set up and solve a quadratic equation to find the answer to two decimal places in feet. Then convert your answer into inches.*

(c) What would it mean to solve the equation to find where $G = 0$? Can you tell what the answer is from the equation (without actually solving)?

Exercises

5. **FIVE** Claude is an excellent juggler. Remember that the height H feet of Claude's beanbag T seconds after he throws it in the air is described by the equation $H = 3 + 15T - 16T^2$. Answer each of the following question by the suggested method and then look back at the graph from earlier to make sure your answers make sense.

(a) Use the QUADRATIC FORMULA to find when is the bean bag is 5 feet above ground? Why do both answers make sense in the story?

(b) When is the beanbag 8 feet above ground? Try to use the QUADRATIC FORMULA to find the answer. What happens? Explain why it makes sense in the story that you can't solve this quadratic equation.

(c) Claude decided that the beanbag was too high in the air, so he modified his throw slightly. Now the height is given by $H = 3 + 14T - 16T^2$. What is the maximum height the beanbag will reach now? *Hint: use the idea of the example to avoid successive approximations.*

6. **Example from 2.3** The stopping distance for the Cadillac Escalade is given by

$$D = .04S^2 + 1.47S$$

where S = speed of car (miles per hour) and D = stopping distance (feet). Jeff took 183 feet to stop. How fast was he going?

(a) Use successive approximation to estimate the answer to the nearest miles per hour. *Display your work in a table.*

(b) Show how to use the QUADRATIC FORMULA to solve the equation.

7. A company produces backpacks. The more they make, the less it costs for each one. The cost per backpack is given by the equation

$$C = 0.01B^2 - 1.2B + 50$$

where C = cost per backpack ($ per backpack) and B = number of backpacks.

(a) How many backpacks do they need to produce in order to hold costs to $20/backpack? Set up and solve a quadratic equation to find the answer.

(b) Make a table of values and draw a graph of the function. Does your answer agree with your table and graph?

(c) What is the minimum price per backpack? *Hint: evaluate at* $T = \frac{-b}{2a}$.

8. **2.3 #10** Mrs. Weber's cooking class came up with the equation

$$M = 1.2F^2 + 4F + 7$$

to approximate the grilling time of a piece of fish depending on its thickness. Here M is the number of minutes to grill the fish and F is the thickness of the fish (in inches).

(a) If we want to make sure the fish will cook in under 20 minutes, what thickness steak can we have? *Set up and solve a quadratic equation to find the answer.*

(b) Ditto 10 minutes.

9. **2.4 #10** Sales of compact discs (CDs) have been declining recently. The money M (in billions of dollars) made from CD sales Y years from 1996 is given by the following equation

$$M = 104.4 + 11.5Y - 1.4Y^2$$

(a) A company that manufactures the materials for compact disks will terminate production if sales fall below 20 billion dollars. In what year will that be? Show how to solve using the QUADRATIC FORMULA.

10. A rabbit jumps so that her height is given by the formula

$$R = 17.6S - 22S^2$$

where R = height of rabbit (feet) and S = time (seconds).

(a) At what height did the rabbit start her jump?

(b) Can the rabbit jump over a 3 foot fence? First, guess the answer by successive approximation. Next, set up and solve a quadratic equation. Last, find the maximum height of the rabbit.

(c) How long is the rabbit in the air? Can you guess the answer from your work so far? Then use the Quadratic Formula to find the answer. *Hint:* $c = 0$

11. SU John says to check the table values!! Revenue from airline travel has been increasing. The total revenue R made is given by the equation

$$R = 22.16Y^2 - 71.71Y + 135.3$$

where R is number of annual revenue, in billions of dollars, and Y is the year since 2000.

(a) Calculate the missing values in the table.

Y	0	1	2	3	4	5	6
R	1,235		1,179	1,217	1,299		1,597

(b) Draw a graph showing how airline travel revenue has changed over time.

(c) According to this equation, in what year was the revenue the smallest? What was the revenue for that year? Show how to use the appropriate formula to calculate the revenue, according to the equation. *Be sure to show some work.*

12. The path of shot putter Randy Barnes' record-breaking put in 1990 closely followed the parabolic arch given by the equation

$$H = -0.0149D^2 + 1.043D + 6.6$$

where D is the distance travelled horizontally, and H is the height above the ground of the shot, both in feet?

(a) How high does the shot get?

(b) How far away did the shot land?

(c) What is the average rate of change of height versus distance over the first inch of horizontal distance?