Just Enough Algebra

Dr. Suzanne Dorée
Professor of Mathematics
Augsburg College
Minneapolis, MN 55454

Fall 2012 edition

Contents

Preface . . . v

1 Variables **1**

Prelude: approximation . . . 2

1.1 Variables and functions . . . 4

1.2 Tables and graphs . . . 10

1.3 Rate of change . . . 19

1.4 Units . . . 25

1.5 Metric prefixes and scientific notation . . . 31

2 Equations **41**

2.1 A first look at linear equations . . . 42

2.2 A first look at exponential equations . . . 49

2.3 Using equations . . . 57

2.4 Approximating solutions of equations . . . 65

2.5 Finance formulas . . . 72

3 Solving equations **81**

3.1 Solving linear equations . . . 82

3.2 Solving linear inequalities . . . 89

3.3 Solving power equations (and roots) . . . 96

3.4 Solving exponential equations (and logs) . . . 104

3.5 Solving quadratic equations . . . 111

4 A closer look at linear equations **121**

4.1 Modeling with linear equations . . . 122

4.2 Systems of linear equations . . . 128

4.3 Intercepts and direct proportionality . . . 135

4.4 Slopes . . . 141

4.5 Fitting lines to data . . . 148

5 A closer look at exponential equations **159**
5.1 Modeling with exponential equations 160
5.2 Exponential growth and decay 166
5.3 Growth factors 172
5.4 Linear vs. exponential models 178
5.5 Logistic and other growth models 185

A Answers to exercises **191**
A.1 Variables 192
A.2 Equations 195
A.3 Solving equations 199
A.4 A closer look at linear equations 202
A.5 A closer look at exponential equations 206

4.1 Modeling with linear equations

A family with one full-time worker earning minimum wage cannot afford the local fair-market rent for a two-bedroom apartment anywhere in the United States. Even families earning above minimum can struggle to rent an apartment for less than 30% of their income. As a result, many people need affordable housing. There are various local, state, and federally funded programs as well as non-profit agencies working to increase availability.

Source: U.S. Department of Housing and Urban Development

In our city there are about 64,100 apartments considered affordable. So the city partnered with local developers to build 7,800 more apartments each year. Our variables are

$$\begin{aligned} A &= \text{affordable housing (apartments)} \sim \text{dep} \\ Y &= \text{time (years from now)} \sim \text{indep} \end{aligned}$$

Assuming things proceed as planned, after 5 years there would be

$$64{,}100 \text{ apts } + 5 \text{ years} * \frac{7{,}800 \text{ apts}}{\text{year}} = 64{,}100 + \underline{5} \times 7{,}800 = 103{,}100 \text{ apartments}$$

Generalizing, we get our equation

$$64{,}100 + Y * 7{,}800 = A$$

which can be rewritten as

$$A = 64{,}100 + 7{,}800Y$$

This equation fits our template for a linear equation

$$\text{dep} \;=\; \text{start} \;+ \text{slope} * \text{indep}$$

Quick recap. A function is **linear** if its graph is a line, and **nonlinear** otherwise. The rate of change measures the steepness of the graph for any function, but a line is the same steepness everywhere, so the rate of change, or **slope** of a line is constant. Our example is linear because the slope of 7,800 apartments per year is constant. Our starting or fixed amount is the **intercept**. In our example it's 64,100 apartments. The dependent variable and the intercept always have the same units – apartments in our example. But

$$\text{units for slope} = \frac{\text{units for dep}}{\text{units for indep}}$$

so, in our example slope is measured in apartments *per year*. These units can help you identify the slope and intercept in a story – so keep a look out.

How many years will it take the city to reach 150,000 apartments at this rate? After ten years, for example, there would still not be enough affordable apartments because

$$A = 64{,}100 + 7{,}800 * 10 = 64{,}100 + 7{,}800 \times \underline{10} = 142{,}100 \text{ apartments}$$

Continuing successive approximation we get

Y	0	5	10	11	12
A	64,100	103,100	142,100	149,900	157,700
vs. 150,000	low	low	low	low	high

This city will reach 150,000 affordable apartments within 12 years.

Of course, we could solve a linear equation instead. We want $A = 150{,}000$. Using our equation $A = 64{,}100 + 7{,}800Y$ we get

$$64{,}100 + 7{,}800Y = 150{,}000$$

However, since we want *at least* 150,000 affordable apartments, an inequality is even better. Let's practice that.

$$64{,}100 + 7{,}800Y \geq 150{,}000$$

Subtract 64,100 from each side to get

$$\begin{array}{lcl} \cancel{64{,}100} + 7{,}800Y & \geq & 150{,}000 \\ -\cancel{64{,}100} & & -64{,}100 \end{array}$$

which simplifies to

$$7{,}800Y \geq 85{,}900$$

Divide each side by 7,800 to get

$$\frac{\cancel{7{,}800}\, Y}{\cancel{7{,}800}} \geq \frac{85{,}900}{7{,}800}$$

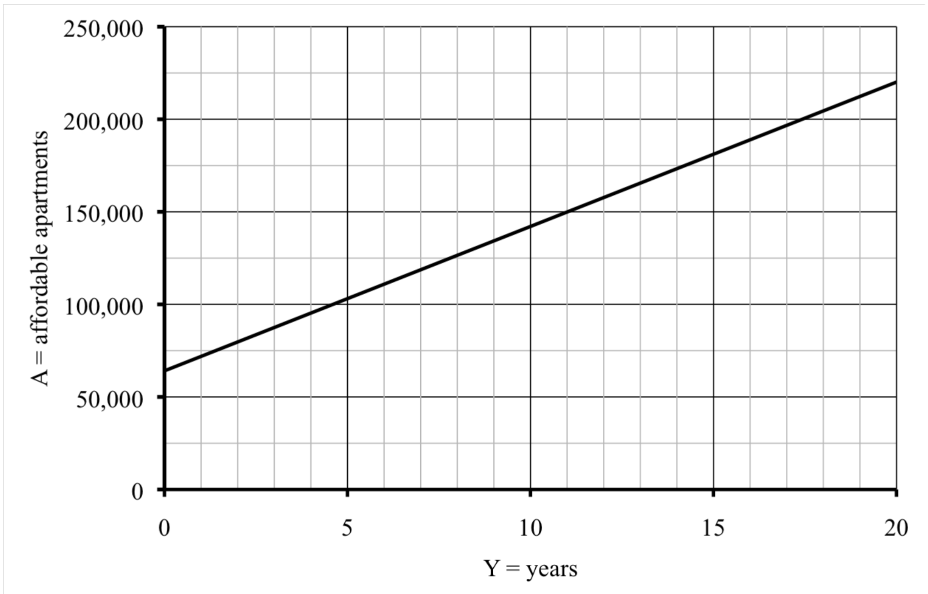
which simplifies to

$$Y \geq \frac{85{,}900}{7{,}800} = 85{,}900 \div 7{,}800 = 11.0128205...$$

To be sure $Y \geq 11.0128205...$ we need to round up to get

$$Y \geq 12$$

Let's confirm our findings on the graph.



As expected, the graph is a line. And we see that the city should reach its goal of 150,000 affordable apartments in 12 years, or slightly before then.

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know . . .

- What makes a function linear?
- What the slope of a linear function means in the story and what it tells us about the graph?
- What the intercept of a linear function means in the story and what it tells us about the graph?
- The template for a linear equation? *Ask your instructor if you need to remember the template or if it will be provided during the exam.*
- How to write a linear equation given the starting amount (intercept) and the rate of change (slope)?
- Where the slope and intercept appear in the template of a linear equation?

- What the graph of a linear function looks like?
- How to solve a linear equation?
- Why the rate of change of a linear function is constant?

If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

Exercises

5. We looked at the city's plan to increase the number of affordable apartments. From a current estimate of 64,100 apartments classified as "affordable," they hoped to build 7,800 per year. At that rate, they can reach a total of 150,000 apartments in 12 years.

   (a) Things change. Revised estimates call for only 6,200 new apartments each year. At that rate, when will the city reach the 150,000 apartments goal? Using the same variables as in this section, set up and solve an equation.

   (b) More bad news. The definition of "affordable" has changed again, so the new count shows only 48,700 apartments on the list. And still only 6,200 new apartments each year. Now when will the city reach the 150,000 apartments goal? Set up and solve an equation.

   (c) In light of the new definition and, consequently, only 48,700 apartments currently on the list, the city has received additional funding to up the number of apartments built each year. They would like to return to their goal of having 150,000 affordable apartments in 12 years. How many apartments do they need to add each year to reach that goal? Figure out the answer however you like, but check that it works.

6. At a local **state university**, the tuition each student pays is based on the number of credit hours that student takes plus fees. The university charges $870 per credit hour plus a $560 fee. The fee is paid once regardless of how many credits are taken.

   (a) Name the variables and write an equation relating them.

   (b) Find the slope and intercept and explain what each means in terms of the story.

   (c) Make a table of values showing the tuition cost for 3 credits, 12 credits, or 16 credits.

   At the local **community college**, the tuition each student pays is based only on the number of credits. The college charges $415 per credit.

(d) Using the same variables as before, write an equation relating them for the community college.

(e) Find the slope and intercept and explain what each means in terms of the story.

(f) Make a table of values showing the tuition cost for 3 credits, 12 credits, or 16 credits.

(g) Graph both functions on the same axes.

(h) What do you notice about the graph that confirms the community college is always cheaper?

7. Can you tell from the table which of these functions is linear? Use the rate of change to help you decide. Remember that numbers may have been rounded.

(a) Ahmed's virburnum shrub. *Story also appears in 4.2 #3*

Week	0	6	10	18
Height (inches)	16.9	19.3	20.9	24.1

(b) Rose gold *Story also appears in 2.3 #2*

Grams of gold added	0	.4	.8	1.4	1.6
Percent gold in alloy	50.0	58.3	64.3	70.6	72.2

(c) Sea-ice (in millions of square miles)

Year	1980	1990	2000	2012
Sea-ice	3.10	2.66	2.23	1.70

(d) Wild rice *Story also appears in 4.5 Exercises*

Hint: rewrite the table in order by temperature first.

Temperature (°F)	39	42	41	35	47	45
Acres	2,300	1,950	1,425	2,015	1,233	1,256

8. The temperature was 40 degrees at noon yesterday but it dropped 3 degrees an hour in the afternoon. Earlier we found the temperature, T°F depends on the time, H hours after noon according to the equation

$$T = 40 - 3H$$

Story also appears in 1.1 and 1.2 Exercises

(a) When does the temperature drop below freezing (32° F)? Set up and solve the relevant inequality. Report your answer as an actual time (to the minute)

(b) When does the temperature drop below zero (0° F)? Same instructions.

9. Shanille is collecting rare books. She inherited 382 books and buys another 3 books every month.

   (a) Make a table showing the number of rare books in Shanille's collection at the start, after 1 month, after 12 months, and after 3 years.

   (b) Name the variables and write an equation relating them.

   (c) Solve your equation to determine when Shanille will reach her goal of 1,000 rare books.

   (d) Graph and check.

4.2 Systems of linear equations

A local factory produces small locks for industrial use. The old machine has seen better days and Quia Xun, the manager, is shopping around for a new machine. She's narrowed it down to two options. The first option is replace the old machine with a new model (Machine #1) for \$3,200. The second is a larger unit (Machine #2) priced at \$5,400. In each case, the price includes installation and the standard service contract. The reason she is considering the more expensive machine is Machine #2 runs at a cost of only \$.80 per lock, whereas the replacement Machine #1 runs at a cost of \$1.25 per lock.

Since Machine #1 is less expensive, Quia Xun knows it is the right choice if the factory only produces a small number of locks. But since Machine #2 costs less per lock to run, she knows it will pay off if the factory makes a large number of locks. She would like to understand the total expenditure better, particularly the number of locks at which it would be worthwhile to invest in the more expensive machine.

Since Quia Xun is interested in how the total expenditure, including both the purchase price and the running cost, depend on the number of locks produced, the variables are

$$\begin{aligned} L &= \text{amount produced (locks)} \sim \text{indep} \\ E &= \text{total expenditure (\$)} \sim \text{dep} \end{aligned}$$

She recognizes that total expenditure is a linear function of the purchase price and the running cost for each machine. In each case, the starting amount (intercept) is the purchase price: \$3,200 for Machine #1 and \$5,400 for Machine #2. The slope (rate of change) is the constant running cost: \$.80 per lock for Machine #1 and \$1.25 per lock for Machine #2. Using the template for a linear equation

$$\text{dep} \;=\; \text{start} \;+\; \text{slope} * \text{indep}$$

she writes the equations

$$\begin{aligned} &\textbf{Machine \#1:} \quad \text{E} = 3{,}200 + 1.25\text{L} \\ &\textbf{Machine \#2:} \quad \text{E} = 5{,}400 + \;\; .80\text{L} \end{aligned}$$

Since there are two linear equations and we are interested in a comparison, we have a **system (of linear equations)**.

To begin the comparison, Quia Xun starts with figuring out what the expenditure to produce 2,000 locks would be for each machine.

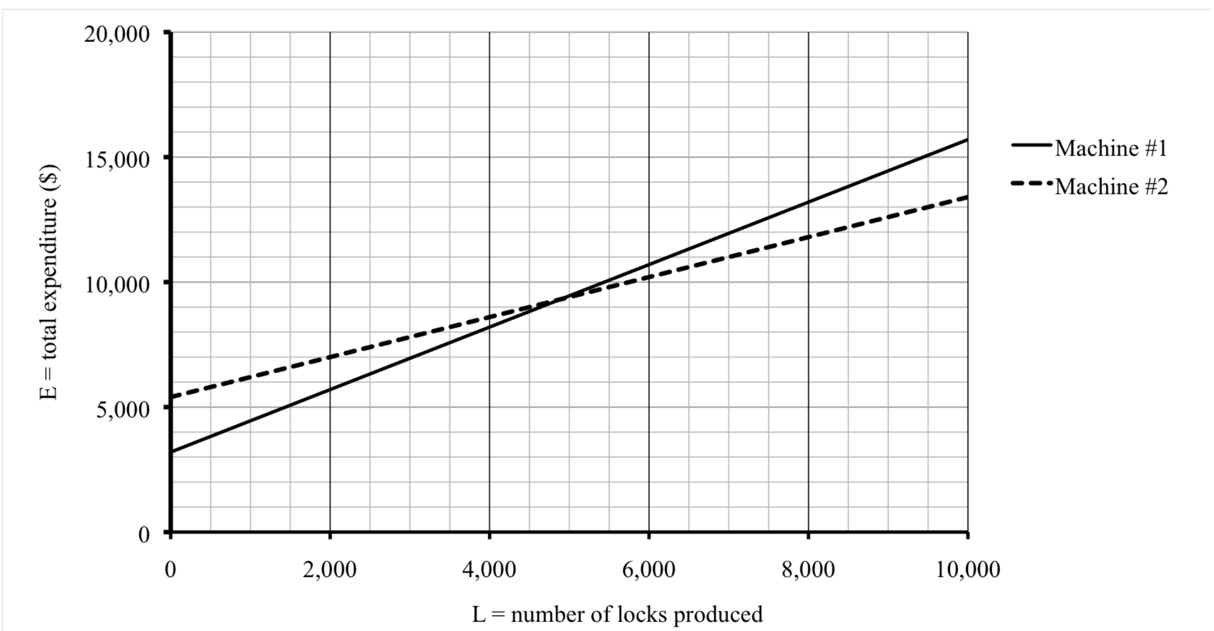
$$\begin{aligned} &\textbf{Machine \#1:} \qquad E = 3{,}200 + 1.25 \times \underline{2{,}000} = \$5{,}700 \\ &\textbf{Machine \#2:} \qquad E = 5{,}400 + 0.80 \times \underline{2{,}000} = \$7{,}000 \end{aligned}$$

If the factory were only going to make only 2,000 locks, then Machine #2 would not be worth it. She calculates a few more examples to see what the cutoff would be.

L	2,000	4,000	6,000	8,000	10,000
Machine #1: E	5,700	8,200	10,700	13,200	15,700
Machine #2: E	7,000	8,600	10,200	11,800	13,400

Even at 4,000 locks, Machine #1 is the better deal. By 6,000 locks, Machine #2 becomes the better deal. Somewhere in between it switches.

Quia Xun makes a quick graph to see what's going on. On the graph, whichever line is lower corresonds to the lower total expenditure and whichever line is higher corresponds to the higher total expenditure. As suspected, for a smaller number of locks the line for Machine #1 is lower on the graph, so that's the better deal. For a larger number of locks it switches and the line for Machine #2 is lower on the graph, since that's the better deal instead. Where they switch corresponds to the point on the graph where the two lines cross, somewhere around 5,000 locks.



A quick successive approximation narrows down the answer.

L	4,000	6,000	5,000	4,500	4,800	4,900
E (for Machine #1)	8,200	10,700	9,450	8,825	9,200	9,325
E (for Machine #2)	8,600	10,200	9,400	9,000	9,240	9,320
Less expensive option	#1	#2	#2	#1	#1	#2

So the choice changes somewhere between 4,800 and 4,900 locks.

There is a way for Quia Xun to solve the problem symbolically; we refer to this process as **solving the system**. She wants to find the number of locks where

$$\text{cost of Machine \#1} = \text{cost of Machine \#2}$$

Using her equations $E = 3{,}200 + 1.25L$ for Machine #1 and $E = 5{,}400 + .80L$ for Machine #2 she has

$$3{,}200 + 1.25L = 5{,}400 + .80L$$

She wants to find the value of L that makes both sides the same number. To solve, Quia Xun subtracts 3,200, the smaller of the two purchase prices, from each side to get

$$\begin{aligned} \cancel{3{,}200} + 1.25L &= 5{,}400 + .80L \\ -\cancel{3{,}200} \qquad &\qquad -3{,}200 \end{aligned}$$

which simplifies to

$$1.25L = 2{,}200 + .80L$$

Pause for a minute. What does that \$2,200 mean in the story? It's the extra cost of buying Machine #2 because \$5,400 − \$3,200 = \$2,200.

What next? This equation has the variable L on each side. Quia Xun needs to combine them somehow. Here's how to do that. Subtract $.80L$ from each side. Look closely. She is subtracting $.80L$, not just .80. We get

$$\begin{aligned} 1.25L &= 2{,}200 + \cancel{.80L} \\ -.80L \qquad &\qquad -\cancel{.80L} \end{aligned}$$

How do we simplify $1.25L - .80L$? Think about what these numbers represent in the story. The cost was \$1.25 per lock versus \$.80 per lock. The difference is \$1.25-\$.80 = \$.45 per lock. So that means

$$1.25L - .80L = .45L$$

Think: 125 apples − 80 apples = 45 apples. She can now simplify her equation to get

$$.45L = 2{,}200$$

Ah, she can solve this equation just by dividing each side by .45 to get

$$\frac{\cancel{.45}L}{\cancel{.45}} = \frac{2{,}200}{.45}$$

which simplifies to

$$L = \frac{2{,}200}{.45} = 2{,}200 \div .45 = 4888.88888\ldots \approx 4{,}889 \text{ locks}$$

If they plan to produce 4,889 locks or more, Quia Xun should go ahead and buy the more expensive machine, Machine #2. Yeah, that's what we guessed – just under 4,900 locks is the payoff.

She solved an equation here, but Quia Xun really wanted to know when

$$\text{Machine \#1} \geq \text{Machine \#2}$$

so she could have solved the inequality

$$3{,}200 + 1.25L \geq 5{,}400 + 0.80L$$

instead. Check that the same steps give

$$L \geq 4,889 \text{ locks}$$

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know . . .

- How to compare two linear functions using a table?
- How to graph two linear functions on the same axes?
- What the solution of a linear system means in terms of the story?
- Where to look on a graph to see the solution of a linear system?
- How to successively approximate the solution of a linear system?
- How to solve a linear system?
- When to use inequality instead of an equation for a linear system?

If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

Exercises

5. Just when Quia Xun thought she had things figured out, another possible option emerged. She can outsource her lock production to a different plant at the cost of \$1.55 per lock. While that's more expensive per lock, it avoids having to buy a machine at all. Recall her first two options were

   Machine #1: E = 3,200 + 1.25L
   Machine #2: E = 5,400 + .80L

   where E is the total expenditure to produce L locks on that machine.

   (a) Write an equation showing how E depends on L if Quia Xun decides to outsource lock production.

   (b) For what number of locks does outsourcing make financial sense versus Machine #1? Set up and solve a system of equations.

   (c) Repeat for Machine #2.

6. Don't ask why I know this, but it takes me 80 seconds per square foot to wash a floor using a rag. If I use a mop, it's slightly quicker at 75 seconds per square foot, but another 3 minutes at the end to wring out the mop. (The rag just pops in the washing machine.)

   (a) Name the variables and write an equation for each option: rag versus mop.
   Hint: time is the dependent variable

   (b) Set up and solve the system to determine the area of a room where either option takes the same amount of total time.

   (c) What do you suggest for each room? (Note: find the area by multiplying the length times the width.) My bathroom floor is 5'×5'. My kitchen floor is 8'×10'. The laundry room is 10'×14'.

7. Maria needed to replace the light bulb in the hallway. When she went to the home improvement store she was overwhelmed with the choices of light bulbs. One option is a compact fluorescent light (CFL) bulb. A CFL bulb costs \$1. This fixture will cost \$.95 per month to run with a CFL bulb. A different option Maria is considering is a light-emitting diode (LED) instead of a bulb. A LED costs \$24 but reduces the cost for the fixture to \$.60 per month.

   (a) Name the variables and write an equation for each option: LED versus CFL.

   (b) Compare the total cost for each bulb for 1, 6, 18, and 36 months.

(c) Draw a graph showing both functions on the same axes.

(d) Set up and solve a system of linear equations to determine the payback time.

(e) Based on what you've learned, fill in the blank and circle the correct word.

The more expensive LED pays off if Maria is going to use it for ___ or more months.

8. The community center offers exercise classes on a pay-as-you-go basis. It normally costs \$20 to register and then \$15 per exercise class. Alternatively, you can pay \$150 to become a member, and then \$10 per exercise class. If we let E represent the number of exercise classes I attend and T represent the total cost in dollars, then the equations are:

Pay as-you-go: $T = 20 + 15E$
Member: $T = 150 + 10E$

(a) Create a table of reasonable values and draw a graph showing both options.

(b) According to your graph, what is the break even point?

(c) Use successive approximations to find the break even point.

(d) Set up and solve a system of linear equations to determine the exact break even point

(e) Describe in words what you've learned about whether or not to buy membership.

9. The weekly supply S and demand D for corn-on-the-cob (in hundreds) at a local market are given by the equations

Supply: $S = 1.5P + 6$
Demand: $D = 11 - .9P$

where P is the price per unit, in dollars per dozen.

(a) Is there a shortage or surplus if corn is priced at \$3.25/dozen? What if it's priced at \$1.75/dozen?

(b) Set up and solve an equation to find the equilibrium price of corn.

(c) Make a small table of values and graph. Does your answer to (b) agree with your graph? Explain.

10. A solar heating system costs approximately \$30,000 to install and \$150 per year to run. By comparison, a gas heating system costs approximately \$12,000 to install and \$700 per year to run. Earlier we wrote equations showing how the total cost \$$T$ depends on the time, Y years.

$$\textbf{Solar:}\quad T = 30{,}000 + 150Y$$
$$\textbf{Gas:}\quad T = 12{,}000 + 700Y$$

Story also appears in 4.1 #1

(a) Set up an solve a system to determine the payback time of installing the solar heating system.

(b) How does the payback time change if the state offers a \$7,000 rebate? That means, in effect, that the solar system costs \$7,000 less to install. Write a new equation for the solar heating system. Then set up and solve the new system.

(c) How high a rebate would the state have to offer to insure a payback time of 15 years? *Hint: compare the costs of gas and solar heating systems at 15 years.*

4.3 Intercepts and direct proportionality

Kaleb runs 8½ minute miles, which means it takes him around 8.5 minutes to run each mile. Yesterday he was out for about 30 minutes and ran the 2.8 mile loop by our house. That strikes me as curious because if he ran 2.8 miles at 8.5 minutes per mile that should take

$$\frac{8.5 \text{ minutes}}{\text{mile}} * 2.8 \text{ miles} = 8.5 \times 2.8 = 23.8 \approx 24 \text{ minutes}$$

But Kaleb took 30 minutes. That's 6 minutes longer than expected. Well, technically 6.2 minutes since

$$30 - 23.8 = 6.2 \approx 6 \text{ minutes}$$

but let's work with 6 since the 30 was only approximate to begin with.

The point is, what's up with that missing 6 minutes? Oh, I bet I know what it is. Ever since Kaleb turned fifty years old, he's been having trouble with his knees. I bet he's finally stretching like his doctor ordered. Must be around 6 minutes of stretches after each run.

Since Kaleb's total time is function of how far he runs, our variables are

$$\begin{aligned} T &= \text{total time (minutes)} \sim \text{dep} \\ D &= \text{distance (miles)} \sim \text{indep} \end{aligned}$$

Notice that we are determining how the time depends on the distance, so the time T is our dependent variable. Often time is the independent variable, but not so here.

For the sake of this problem, we assume Kaleb runs a steady 8.5 minutes per mile so the rate of change is constant. The equation must be linear and so it fits the template

$$\text{dep} \ = \ \text{start} \ + \text{slope} * \text{indep}$$

The slope is 8.5 minutes per mile. The 6 minutes Kaleb spends stretching is the intercept, even though it's named "start" in the template and Kaleb is actually stretching at the end of his run. A better name might be "fixed." Whatever you call it, the equation is

$$\textbf{Kaleb:} \quad T = 6 + 8.5D$$

As a quick check, for that 2.8 mile run we have $D = 2.8$ and so

$$T = 6 + 8.5 * 2.8 = 6 + 8.5 \times \underline{2.8} = 29.8 \approx 30 \text{ minutes}$$

By the way, there's a shorter way to find the intercept. The intercept is the "starting value," or in this case the time spent stretching. So we take the total time and then subtract out the time spent running

$$\text{intercept } = 30 - 8.5 \times 2.8 = 6.2 \approx 6 \text{ minutes}$$

In general,

$$\text{intercept} = \text{dep} - \text{slope} * \text{indep}$$

Kaleb's daughter Muna runs considerably faster, 7 minute miles, and she's not into stretching at all. For her to run the 2.8 mile loop by our house, it would take

$$\frac{7 \text{ minutes}}{\text{mile}} * 2.8 \text{ miles} = 7 \times 2.8 = 19.6 \text{ minutes}$$

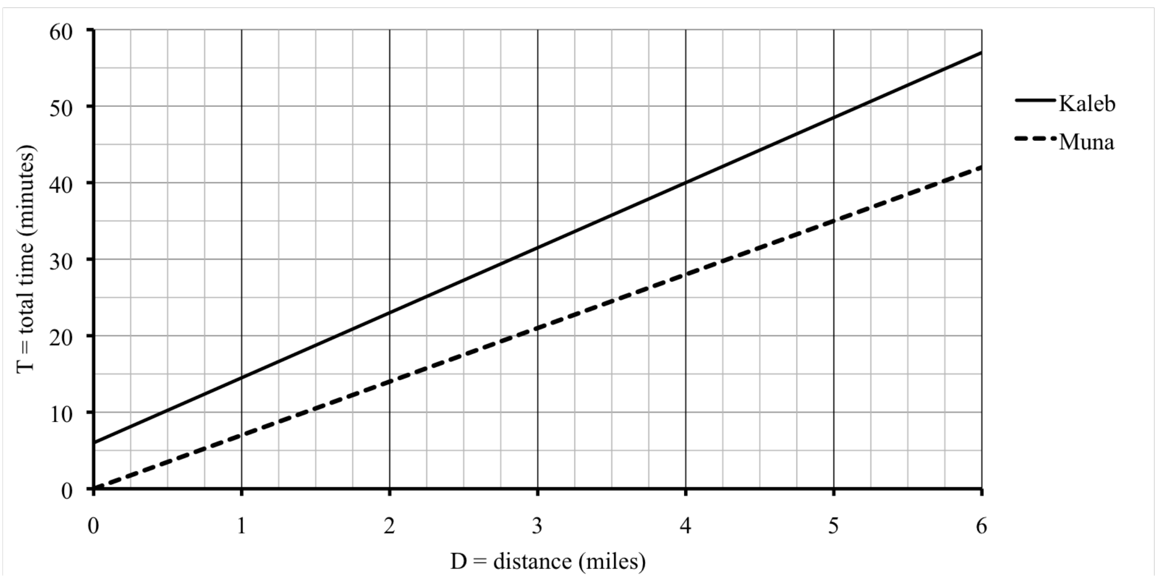
That means while her dad would take 30 minutes to run the loop and do his stretches, Muna can run it in just under 20 minutes.

The equation for Muna is

$$\textbf{Muna:} \quad T = 7D$$

The slope is 7 minutes per mile. What's the intercept for this equation? There's no time for stretching in her equation, so it's like $T = 0 + 7D$. The intercept is 0 minutes.

Compare the graphs. Each intercept shows where that line meets the vertical axis. Kaleb's crosses at 6 minutes, but Muna's crosses at 0 minutes, at the origin (where the two axes cross).



By the way, Muna's equation $T = 7D$ is a **direct proportionality** because the only thing happening is that the independent variable is being scaled by a **proportionality constant**, $k = 7$. Any direct proportionality fits this template.

DIRECT PROPORTIONALITY TEMPLATE: $\quad \text{dep} = k * \text{indep}$

To understand the proportionality, recall that Muna can run 2.8 miles in 19.6 minutes. What happens if she goes for a run twice as long? Then she would be running $2 \times 2.8 = 5.6$ miles. Her time would be

$$T = 7 * 5.6 = 7 \times \underline{5.6} = 39.2 \text{ minutes}$$

Notice that $2 \times 19.6 = 39.2$. So, it would take her twice the time to run twice the distance. This general idea – that you get twice the value of the dependent variable if you have twice the value of the independent variable – characterizes direct proportions. We sometimes say that Muna's time is **proportional** to how far she runs. Nothing special about twice here, as it would take her three times the time to run three times the distance, etc.

Not so for Kaleb. Remember it takes him 29.8 minutes to run that 2.8 miles. If he runs twice the distance, which is 5.6 miles, it takes

$$T = 6 + 8.5 * 5.6 = 6 + 8.5 \times \underline{5.6} = 53.6 \text{ minutes}$$

which is not quite twice the time, since $2 \times 29.8 = 59.6$ minutes. The key is that Kaleb does not stretch twice, only once, for the longer run so double the distance does not count the 6 minutes again. Kaleb's equation is not a direct proportionality. Another way to say that is that Kaleb's time is not proportional to how far he runs. It is a function of how far he runs, yes, but not proportionally so.

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know ...

- What the intercept of a linear function means in the story and what it tells us about the graph?
- How to calculate the intercept given the slope and an example (another point on the graph)?
- Why an intercept might not make sense, for example if it's outside the domain of the function?
- When a linear function is a direct proportion?
- Why you cannot reason proportionally if the linear function is not a direct proportion?

- What the graph of a direct proportion looks like?

  If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

Exercises

5. Different runners run at different paces. And take a different amount of extra time to warm-up and/or cool down. The table lists six runners, their training time to run a 5K (rounded to the nearest minute), and their pace (in minutes per mile).

Name	Yannick	Olga	Aziz	Hitomi	Galen	Fiona
Pace	8.2	8.6	9.5	10	10	11.2
5K time	32	35	33	36	31	44
extra time	?	?	?	?	?	?

   (a) We are interested in each runner's extra time, but first convert 5K, which is short for 5 kilometers, to miles using 1 mile $\approx$ 1.609 kilometers.

   (b) Now, determine the extra (warm up/cool down) time for each runner and list your answer in the table. Report your answer to the nearest minute.

   (c) List the runners in order from least to most warm up/cool down time.

6. At 10:00 a.m. we've got snowy skies and 4 inches of new snow on the ground. It's coming down fast out there at $^2/_3$ of an inch per hour.

   (a) Name the variables, measuring time in hours since 10:00 a.m.

   (b) Write an equation illustrating the dependence.

   (c) When did the snowstorm start?

   (d) Name a new variable for time measured this time in hours since the snowstorm started.

   (e) Write an equation illustrating the dependence using this new variable instead.

   (f) Check that this equation confirms 4 inches of new snow at 10:00 a.m.

   (g) Explain why the two equations have different intercepts.

7. The public beach near Paloma's house has lost about 3'9" feet a year of beach depth (measured from the dunes to the high water mark) due to erosion since they started keeping records 60 years ago. Currently it's 210 feet deep.

   Story also appears in 1.3 Exercises

(a) The county is considering filling in sand to offset the erosion, back to the historical mark (60 years ago). How deep was it then? Notice that you need to convert 3'9" to (decimal) feet first.

(b) Name the variables and write an equation relating them, assuming the county does not fill in the beach now. Measure time from 60 years ago.

(c) The country agrees to start filling in sand when the depth drops below 180 feet. How many (more) years will that take to happen? First estimate the answer using successive approximation. Then set up and solve an inequality to find the answer.

(d) Draw a graph showing the sand erosion over the past 60 years and including the next 20 years, assuming the county does not do any filling.

(e) Identify the slope and intercept and explain their meaning in the story.

8. Clyde is loading bricks weighing 4.5 pounds each onto his wheelbarrow. The wheelbarrow weighs 89 pounds when it has 16 bricks in it. (That weight includes both the bricks and the wheelbarrow itself.)

   (a) How much would Clyde's wheelbarrow weigh if it were empty?

   (b) Name the variables and write an equation relating them.

   (c) How much (total) will the wheelbarrow weigh if he loads a total of 30 bricks?

   (d) Clyde continues loading bricks until the wheelbarrow full of bricks weighs 206 pounds. How many bricks are in it?

   (e) Graph and check.

9. The city offers bus "convenience" passes – 20 rides for \$12.95 or 80 rides for \$51.80.

   (a) Calculate the rate of change.

   (b) Is there a convenience charge?

   (c) What is the name for this type of function?

10. To make cookies it takes a few minutes to prepare the dough. After that it takes 12 minutes per batch to bake in the oven. Last time I made 3 batches of cookies and it took a total of 54 minutes.

    (a) How long does it take me to prepare the dough?

    (b) How long would it take me to make 10 batches of cookies for the cookie swap? Assume the time to prepare the dough remains the same and only one batch bakes in the oven at a time.

(c) Name the variables and write an equation describing the function.

(d) Identify the slope and intercept and explain their meaning in the story.

4.4 Slopes

Last week our supplier delivered 13 cases of paper for the office and charged us \$534.87. This week, they delivered 20 cases of paper for \$814.80. We assume that their charge includes a fixed delivery fee and per case cost, so the dependence must be linear. We would like to understand their pricing scheme better by writing the equation.

What to do? We can name the variables and put the information we are given into a table. That's a start. The variables must be

$$\begin{aligned} C &= \text{total charge (\$)} \sim \text{dep} \\ N &= \text{number of cases delivered (cases)} \sim \text{indep} \end{aligned}$$

and we know

N	13	20
C	534.87	814.80

Let's see. The fixed delivery fee that we don't know is the intercept. The per case cost that we also don't know is the slope. To write the linear equation we need to know both.

The slope is just the rate of change, so we can figure out the slope just from the information in our table.

$$\begin{aligned} \text{slope} &= \text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{\$814.80 - \$534.87}{20 - 13 \text{ cases}} \\ &= \frac{\$279.93}{7 \text{ cases}} = 279.93 \div 7 = \$39.99 \text{ per case} \end{aligned}$$

or, all at once, as

$$\begin{aligned} \text{slope} &= \text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{\$814.80 - \$534.87}{20 - 13 \text{ cases}} \\ &= (814.80 - 534.87) \div (20 - 13) = \$39.99 \text{ per case} \end{aligned}$$

Either way, each case costs \$39.99 and the slope is \$39.99 per case.

Now that we know the slope, we can find the intercept. At \$39.99 per case we would expect 13 cases to cost

$$13 \text{ cases } * \frac{\$39.99}{\text{case}} = 13 \times 39.99 = \$519.87$$

But the story tells us 13 cases cost \$534.87. The difference \$534.87 − \$519.87 = \$15 must be the delivery fee which is the intercept. Remember

$$\text{intercept} = \text{dep} - \text{slope} * \text{indep} = 534.87 - 39.99 \times 13 = \$15$$

Why did we use 13 cases? No good reason. Look what happens if we use 20 cases at \$814.80 instead.

$$\text{intercept} = \text{dep} - \text{slope} * \text{indep} = 814.80 - 39.99 \times 20 = \$15$$

Yup. Still \$15 delivery fee.

The equation is linear so it fits our template

$$\text{dep} = \text{start} + \text{slope} * \text{indep}$$

and now that we know the slope and intercept, we can put those in to get our equation.

$$C = 15 + 39.99N$$

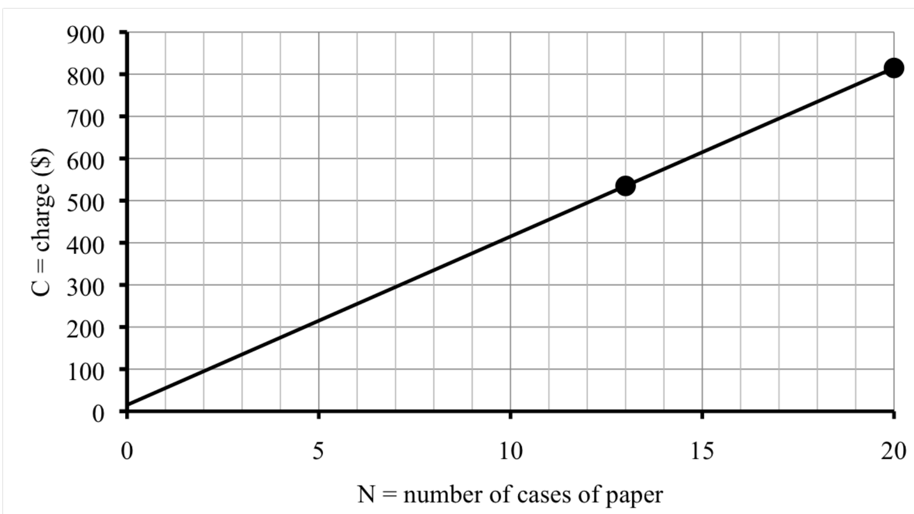
Let's check. When $N = 13$ we get

$$C = 15 + 39.99 * 13 = 15 + 39.99 \times \underline{13} = \$534.87 \quad \checkmark$$

and when $N = 20$ we get

$$C = 15 + 39.99 * 20 = 15 + 39.99 \times \underline{20} = \$814.80 \quad \checkmark$$

You can also check that the graph goes through the original two points we were given. The intercept is \$15, but because of the scale it shows up as barely above \$0 on our graph.



The supplier also picks up recyclable paper and boxes. The normally charge \$18 per pickup but under a new reuse incentive program, they discount a little for each box that's in good enough condition to use again. This week's recycling charge was only \$7.60 because we returned the previous 13 boxes all in good shape.

Now we're interested in how the recycling charge depends on the number of boxes in good condition that we return. The new variables are

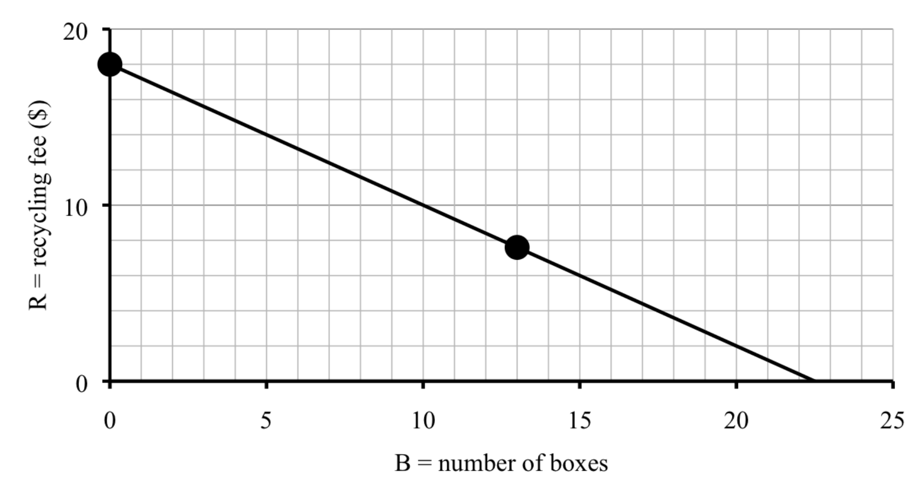
$$R = \text{recycling charge (\$)} \sim \text{dep}$$
$$B = \text{number of boxes returned (boxes)} \sim \text{indep}$$

and we know

B	0	13
R	18	7.60

See how we used $B = 0$ for the situation where no boxes are returned? Clever.

We can draw the graph using just these two points. (But we'll check later, once we have the equation, to be sure.)



Since there is a fixed discount per box, we again have a linear function. We know the intercept is the normal recycling fee of \$18. We need to find the slope.

$$\begin{aligned} \text{slope} &= \text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{\$7.60 - \$18}{13 - 0 \text{ cases}} \\ &= (7.60 - 18) \div (13 - 0) = -\$.80 \text{ per box} \end{aligned}$$

It might look funny to get a negative, but it's to be expected. They are subtracting for each good box returned. The discount is 80¢ per box and so the equation is

$$R = 18 - .8B$$

Check when $B = 13$ we have

$$R = 18 - .8 * 13 = 18 - .8 \times \underline{13} = \$7.60 \quad \checkmark$$

What's the most boxes you could get credit for? Probably the most they discount is the full \$18, which would mean that $R = 0$. That means we want to solve $18 - .8B = 0$.

Check that we get $B = 22.5$ boxes, which means that 22 boxes would be almost \$0 and for 23 boxes, they should pick up for free. We can check that 22 boxes gives

$$R = 18 - .8 * 22 = 18 - .8 \times \underline{22} = \$.40$$

and 23 boxes gives

$$R = 18 - .8 * 23 = 18 - .8 \times \underline{23} = -\$.40 \implies \text{free}$$

Well, unless they're nice and give us cash back.

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know ...

- Which types of situations are linear?
- What the slope of a linear function means in the story and what it tells us about the graph?
- How to calculate the slope between two points?
- What is means if the slope is negative?
- How to find the equation of a line through two points?
- How to find a linear function given two examples in a story?
- If both the slope and intercept are unknown, which is easier to calculate first?

  If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

Exercises

5. I just saw an advertisement for the same paper we use at the office for only \$4.25 per ream at a supply store. (Ream? Yes. That's 500 sheets of paper, usually wrapped in paper.) Is that a good deal?

   (a) There are 10 reams in a case. What is the advertised price come to per case?

(b) I'm not sure I want to go get a case of paper myself because a case of paper is pretty heavy to lift. Paper is sold by the weight. Thick, heavier paper is considered fancier than lighter paper. The office uses a multipurpose paper called "92" meaning it weighs 92 grams per square meter which comes out to around 5 grams per sheet. How much does a case weigh? Use 1 kilogram $\approx$ 2.2 pounds.

(c) But, at the office we pay a delivery charge. Compare the cost of having just one case delivered versus my buying one case at the store. Recall that the office pays \$15 delivery fee and \$39.99 per case.

(d) Write new equation for paper cost assuming I pick it up at the store. Use N for the number of cases of paper and C is the total cost, in dollars. *Hint: this equation is a direct proportionality.*

(e) Compare total cost if get 4 cases either delivered or from store. Repeat 13 cases. Recall that the equation for delivered paper is $C = 15 + 39.99N$.

(f) Graph both functions together on the same axes.

(g) Set up and solve inequality for when delivered is cheaper.

6. The amount of garbage generated in the United States has increased steadily, from 88.1 million tons in 1960 to 254.2 million tons in 2006.

Source: Environmental Protection Agency

Story also appears in 4.5 Exercises

(a) Assume the amount of garbage increases linearly, by how much has garbage increased each year?

(b) Name the variables, including units, and write a linear equation relating them.

(c) According to your equation, how much garbage was projected for 2010? For 2020?

(d) If this trend continues, when will the amount of garbage generated exceed 300 million tons? Show how to set up and solve an inequality to find the answer. Be sure to state the actual year.

(e) A 2010 report listed the amount of garbage at 249 million tons. Compare this information to your previous answer. What are some possible explanations for why this amount was less than expected (and actually decreased from 2006)?

7. Now that he is retired, Elmer gets a pension check from the Railroad Company each month. There's a set amount he gets each month but the company deducts a fixed percentage of whatever outside income he earns. Elmer works part-time at the local

hardware store. In February he earned \$444.10 at the hardware store and his pension check that month was \$886.23. In March he worked much less, earning only \$179.30 at the hardware store; his pension check that month was \$912.71

(a) What percentage of his income from the hardware store is deducted from his pension check? *Calculate the fraction of a dollar deducted for each dollar earned. Convert your answer to percent.*

(b) If Elmer doesn't work in April, how much will his pension check be?

(c) Write an equation showing how Elmer's pension check is affected by his income from the hardware store. Use H for his income from the hardware store and P for his pension check, both in dollars.

(d) Elmer would like to earn enough at the hardware store to make at least \$1,200 total per month. Using T for the total Elmer earns in a month (in dollars), write an equation for T as a function of H. *Hint: start with* $T = H + P$, *then use your equation for* P *from part (c) to write everything with* H *instead.*

(e) Now set up and solve an inequality to determine how much Elmer needs to earn at the hardware store to make at least \$1,200 total per month.

(f) If Elmer earns \$8.15 per hour, how many hours does he need to work at the hardware store to make at least \$1,200 total per month, accounting for his income from the hardware store and his pension check?

8. Your local truck rental agency lists what it costs to rent a truck (for one day) based on the number of miles you drive the truck. They use a linear pricing model.

distance driven (miles)	50	100	150	200
rental cost (\$)	37.50	55.00	72.50	90.00

If you rent a truck and drive it 10 miles, how much do you think it will cost? As part of your work, name the variables and write a linear equation relating them.

Story also appears in 1.2 and 1.3 Exercises

9. In 2008, the median household income was about \$50,303. By 2010 it was down to about \$49,445. Source: U.S. Census Bureau

(a) By how much has it decreased each year, on average? The phrase "on average" means that you should assume the decrease is linear.

(b) Name the variables and write a linear equation relating them.

(c) At this rate when will the median family fall below \$48,000? Set up and solve an inequality.

(d) Graph and check.

10. Buoy instruments in the oceans report changes in the sea level. In 2005 the sea level (averaged across all the oceans) was 51.7 millimeters above the historical sea level. In 2012 the sea level was 73.4 millimeters above the historical sea level. You can assume the increase is linear. Source: National Aeronautics and Space Administration

    (a) Name the variables, including units.
    (b) Display the information from the story in a table.
    (c) What is the rate of increase for the sea level?
    (d) Write an equation relating the variables.
    (e) In what year will the sea level be 80 millimeters above the historical level?

4.5 Fitting lines to data

Thanh has an internship studying road salt usage in a northern metropolitan area. Road salt is used to melt ice and snow on paved streets. Because it can damage vegetation and influence both surface water (lakes) and ground water, and because it costs money to run the trucks that apply the salt, people are interested in the amount of road salt used.

One data set compares road salt usage per county. Thanh learned from county officials that road salt use varies widely from county to county, but, not surprisingly, it depends heavily on the length of road in the county. So, the variables are

$$L = \text{road length (lane miles)} \sim \text{indep}$$
$$S = \text{road salt applied (tons per year)} \sim \text{dep}$$

A **lane mile** is the area of road one mile long and one lane wide. Now you know.

Thanh also learned that while road salt use is a function of lane miles, it is not proportional as there are more complicated factors involved. Still, he would like to model road salt use as a function of road length. Here are the data for counties in the metro area.

County	A	C	D	H	R	T	W
L	710	420	800	1,420	720	510	480
S	14,700	3,900	11,600	15,500	9,400	5,000	9,700

To develop his model Thanh imagined a new county, County X, that had 600 lane miles of road. In looking at the data, he finds two counties with close to 600 lane miles.

	County T	County X	County A
L	510	600	710
S	5,000	?	14,700

Based on this data, Thanh expects County X would use between 5,000 and 14,700 tons/year of road salt. Since 600 is closer to 510 than to 710, he starts with a guess of around 9,000 tons/year of road salt.

To improve this estimate, Thanh decides to use a linear mode, hoping that will account for both road length influence and fixed factors. He begins by finding the slope.

$$\begin{aligned} \text{slope} &= \text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{14{,}700 - 5{,}000 \text{ tons/year}}{710 - 510 \text{ lane miles}} \\ &= (14{,}700 - 5{,}000) \div (710 - 510) = 48.5 \text{ tons/year per lane mile} \end{aligned}$$

Next he calculates the intercept.

$$\text{intercept} = \text{dep} - \text{slope} * \text{indep} = 5{,}000 - 48.5 \times 510 = -19{,}735 \text{ tons/year}$$

He was not expecting a negative value but decides to use it anyway. Using the template for a linear equation

$$\text{dep} = \text{start} + \text{slope} * \text{indep}$$

Thanh gets

$$S = -19{,}735 + 48.5L$$

which he rewrites as

$$S = 48.5L - 19{,}735$$

As a check, for $L = 710$ lane miles, he gets

$$S = 48.5 \times \underline{710} - 19{,}735 = 14{,}700 \text{ tons/year} \quad \checkmark$$

More importantly, for 600 miles his equation gives the estimate of

$$S = 48.5 \times \underline{600} - 19{,}735 = 9{,}365 \text{ tons/year}$$

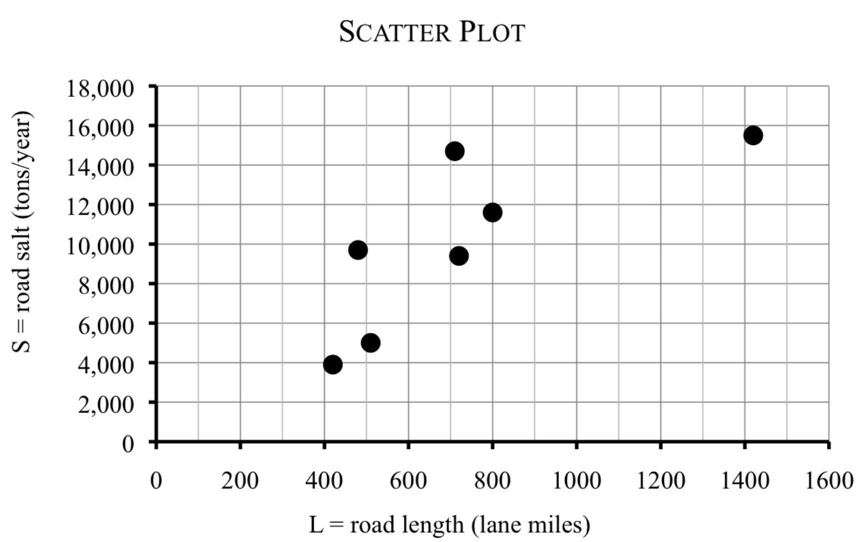
Thanh rounds this estimate to 9,400 tons/year of road salt for County X, which is close to his initial guess of 9,000 tons/year.

Next, Thanh imagines another new county, County Y, that has 500 lane miles of road. He looks to the data for counties with close to 500 lane miles.

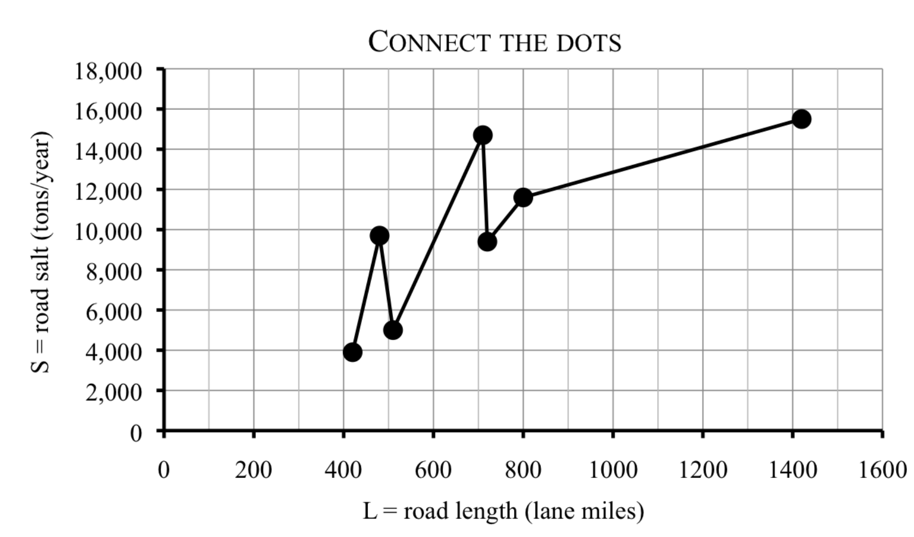
	County W	County Y	County T
L	480	500	510
S	9,700	?	5,000

Wait a minute. The county with fewer roads used more salt? That doesn't make sense.

Thanh decides to look at all the data at once in a scatter plot.

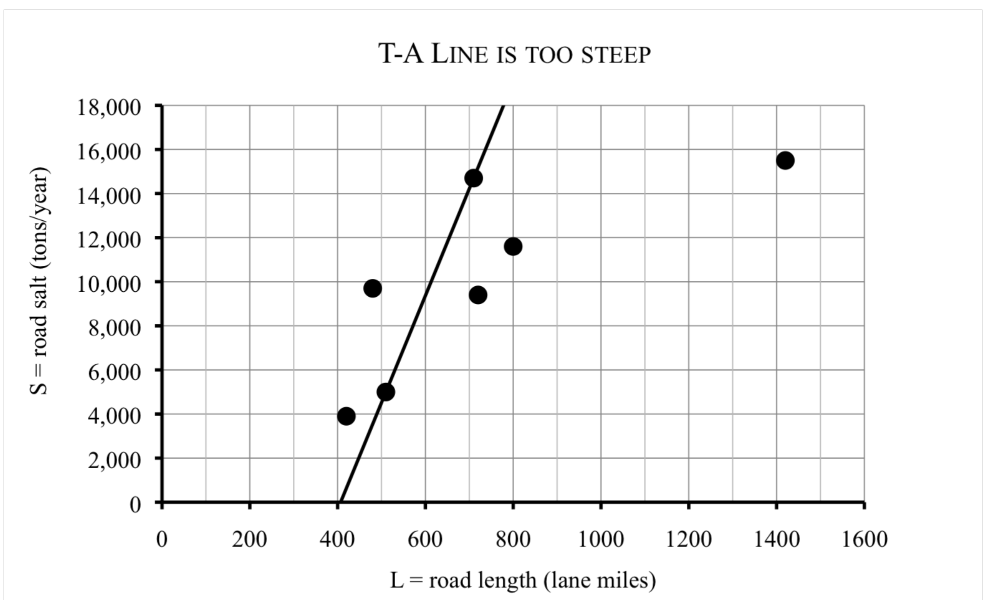


When Thanh was using the nearby points to estimate for Counties X and Y, it's as if he were connecting the dots with line segments on the graph. Notice that the line that goes through 500 lane miles is decreasing, just like Thanh saw in his table.



Thanh suspects that this connect-the-dots model is too heavily influenced by individual county road-salting habits. He would like a way to get one line to use for everything, knowing full well that one line cannot possibly go through all of the data points.

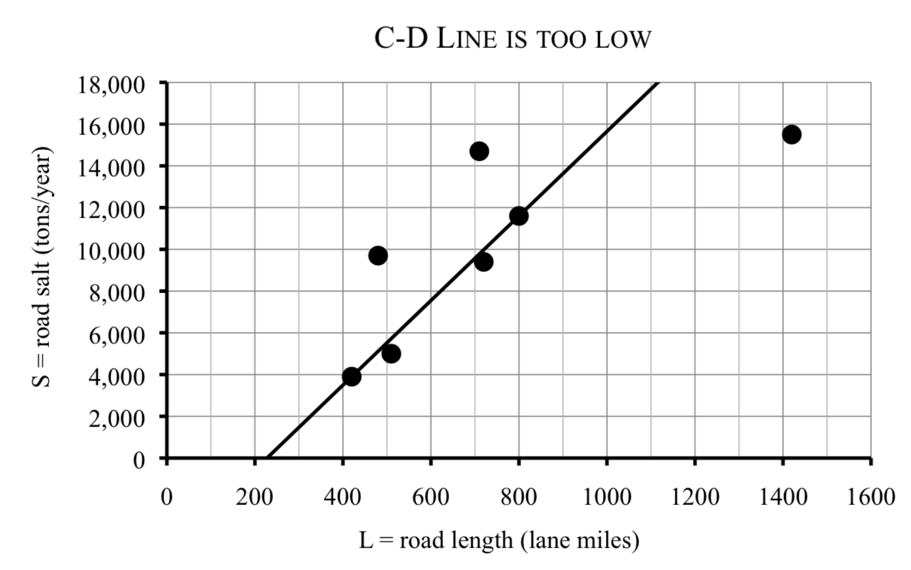
Which line to use? One option would be to stick with the line he found through the points for Counties T and A. He redraws the scatter plot to show that line. Because the intercept is negative, it doesn't show up on his graph. The line seems to be too low at first and too high later. The problem is that this line is too steep (has too large a slope).



Thanh decides to try a line that is less steep. After drawing in a few lines, he decides to try the line between the points for Counties C and D instead, which has equation

$$\textbf{C-D line:} \quad S = 20.26L - 4{,}610$$

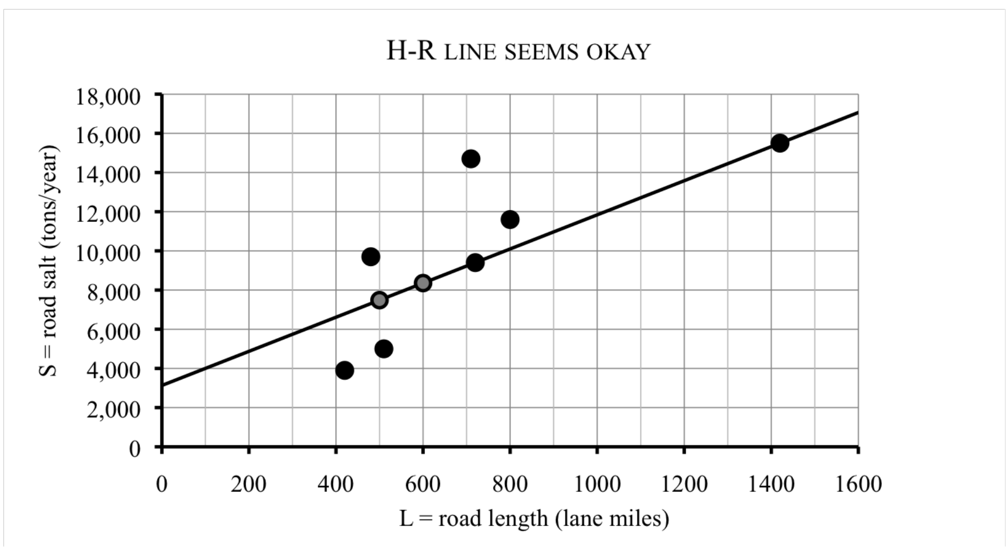
Unfortunately this line seems too low. (Again the negative intercept isn't visible.)



Neither of these lines came close to the point for County H on the far right, so Thanh considers one more line, this time through County H and County R, which has equation

$$\textbf{H-R line:} \quad S = 8.71L + 3{,}130$$

This line has a positive intercept just above 3,000 tons/year, as you can see on the graph.



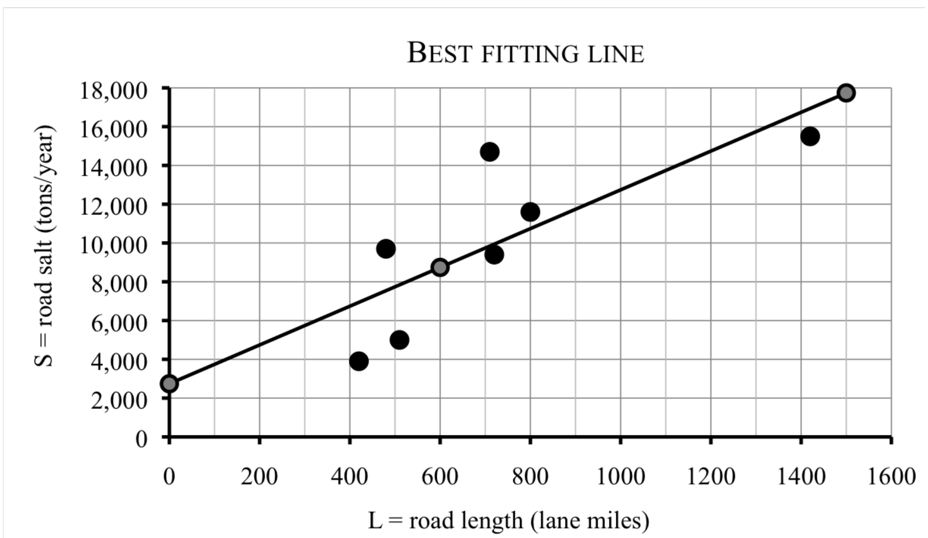
Thanh thinks the H-R line is reasonable, but it makes him wonder how to decide if one line is better than another. Generally speaking the **best fitting line** makes the space between the line and the data points as small as possible. (There is actually a much more official definition.) After using a little statistical software, Thanh determines that for this data set, the official best fitting line has equation

$$\textbf{Best fitting line:} \quad S = 10.0L + 2{,}741$$

Thanh wants to add this line to his graph so first he calculates a few values. While it's true that any two points would do, he played it safe and plotted three points, being sure to use 0 in order to find the intercept.

L	0	600	1,500
S	2,741	8,741	17,741

He graphs this line and notices it is very similar to the H-R line, just a tiny bit higher and a tiny bit steeper. The points from the table are highlighted on the graph just to help you see how we graphed the line. Remember, those aren't actual data points.

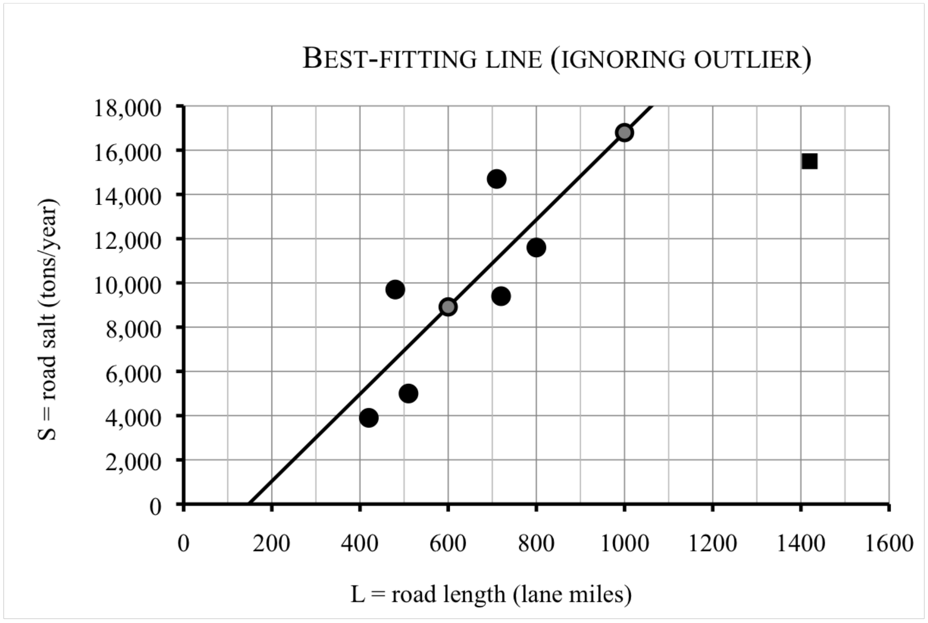


Thanh is bothered by the fact that County H seems to be off on its own. The largest city in this area is in County H. Between the budget crunch and the nature of the urban landscape, the city tends to use much less road salt than the surrounding areas. So County H really isn't very typical at all. In statistics, this sort of value is known by the descriptive term **outlier** (as in "it lies way out there.")

So Thanh decides to look at the statistically best-fitting line ignoring County H this time. Back to his software and he finds

$$\textbf{Best fitting line (ignoring outlier):} \quad S = 19.7L - 2{,}905$$

This line is less steep than the T-A line and higher than the C-D line. Seems perfect.



Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know ...

- What a scatter plot is?
- Why we might begin the scale for a scatter plot somewhere other than 0?
- Why we would approximate data with a linear function?
- How to decide visually whether a line is a reasonable approximation of the data?
- The name for a point that falls very far away from an approximating line?
- How to graph a line from its equation by creating a table first.
- Why even the best fitting line doesn't go through most of the data points?

If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

Exercises

5. Look back at Thanh's data in the section.

   (a) Check that the C-D line has equation $S = 20.26L - 4{,}610$

   (b) Check that the H-R line has equation $S = 8.71L + 3{,}130$

   (c) The best-fitting line (ignoring the outlier) had equation $S = 19.7L - 2{,}905$. Make a table of values for $L = 600$ and 1,000 lane miles and use these values to check the graph Thanh drew. (They are highlighted on the graph.)

6. Wild rice is a native plant that grows in lakes in the upper Midwest. The table shows how the annual acreage of wild rice has varied with the average spring temperature in various years. The variables are T for the temperature measured in °F and W for the wild rice yield, measured in acres. In case you're curious, the year is included as well, but it's not one of the variables we're interested in.

year	1985	1989	1993	1997	2001	2005	2009
T	39	42	41	35	47	45	42
W	2,300	1,950	1,425	2,015	1,233	1,256	1,345

Story also appears in 4.1 Exercises

   (a) Make a scatter plot of the points. Make your graph as large as possible by starting your temperature axis at 35°F and your acreage axis at 1,000 acres.

   (b) Find the equation of the line through the data from 1997 and 2001. Use T for the average temperature (in °F) and W for the acres of wild rice.

   (c) Based on your line, what might you expect the acreage of wild rice to be in a year when the average temperature is 46°F? 40°F? Use your equation to answer the questions.

   (d) Draw that line on your scatter plot. Comment.

   (e) The best fitting line has equation

$$W = 5{,}072.2 - 82.41T$$

   Make a table of values and use it to graph that line as well.

   (f) If you use the best fitting line, how would that change your estimate for the acreage of wild rice in a year when the average temperature is 46°F? 40°F?

7. The amount of garbage generated in the United States has increased steadily, from 88.1 million tons in 1960 to 254.2 million tons in 2006. Earlier we did linear model.

But, in fact, the amount of garbage has not increased exactly linearly. The table shows data for select years, where Y measures years since 1960 and G is the amount of garbage (in millions of tons).

year	1960	1970	1980	1990	2000	2006	2010
Y	0	10	20	30	40	46	50
G	88.1	121.1	151.6	205.2	239.1	254.2	249.0

Source: Environmental Protection Agency

Story also appears in 4.4 Exercises.

(a) Make a large scatter plot of the points, beginning at the year 1960 and extending to at least 2030.

(b) Draw in the line through the points from 1960 and 2006. (We found that equation in 4.4 Exercises.)

(c) Draw in the line through the points from 2000 and 2006. Would this line predict that garbage will reach 300 million tons sooner or later than the previous prediction? Use the graph to explain.

8. My mechanic, Paye, believes that frequent oil changes reduce the amount of maintenance on a car. To prove his point, Paye showed me a table of customers with the number of yearly oil changes and the cost of their engine repairs.

N	1	2	3	4	5	6	7
R	725	500	415	300	275	100	150

where N is the number of oil changes per year and R is the cost of repairs, in dollars.

(a) Make a large scatter plot of the points.

(b) Draw in the line through the points for 3 and 5 oil changes.

(c) Write the equation for that line. Use your equation to predict the cost of engine repairs for a customer who does no oil changes, and one who does 8 oil changes.

(d) The best fitting line has equation approximately

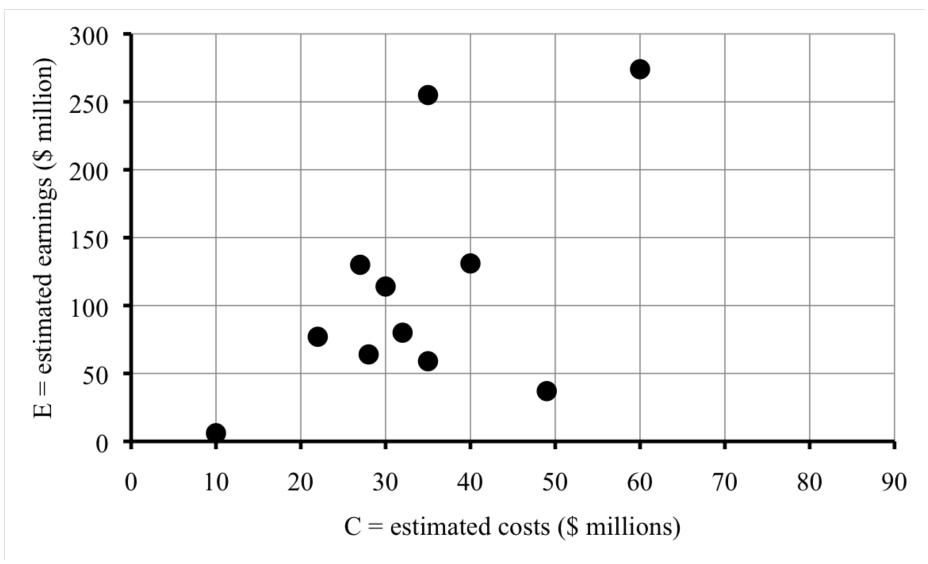
$$R = 732.86 - 95.179N$$

Plot three points on that line and use them to draw it on your scatter plot.

(e) What does the best fitting line predict the cost of engine repairs for a customer who does no oil changes, and one who does 8 oil changes?

9. The table and scatterplot shows the estimated costs and earnings for various sports-themed movies.

Movie	Estimated cost	Estimated earnings
A League of Their Own	$40 million	$131 million
Dodgeball	$30 million	$114 million
Invictus	$49 million	$37 million
Jerry Maguire	$60 million	$274 million
Miracle	$28 million	$64 million
Nacho Libre	$32 million	$80 million
Remember the Titans	$27 million	$130 million
Run Fatboy Run	$10 million	$6 million
Secretariat	$35 million	$59 million
The Blind Side	$35 million	$255 million
The Rookie	$22 million	$77 million



(a) Draw the line (A) that goes through the points for "Remember the Titans" and "A League of Their Own". Explain why this line does not fit the data well.

(b) Draw the line (B) that goes through the points for "Run Fatboy Run" and "Jerry Maguire". Explain why this line does not fit the data well.

(c) Draw the line (C) that goes through the points for "The Rookie" and "A League of Their Own". This line fits the data pretty well I think.

(d) Why might you expect the slope of good fitting line to be positive?

10. The annual consumption of meat in millions of metric tons is given in the following table. Here C is for chicken, B is for beef, and Y measures years since 1975.

Source: U.S. Department of Agriculture

year	1975	1985	1990	1995	2000	2005	2009	2012
Y	0	10	15	20	25	30	34	37
C	3.6	6.1	7.7	9.4	11.5	13.4	12.9	13.3
B	12.1	11.8	11.0	11.7	12.5	12.7	12.2	11.4

(a) Make a scatter plot of the chicken consumption (use $*$ to label each point) and red meat consumption (use $\circ$ to label each point).

(b) Sketch a line through each data set that seems to reasonably approximate the dependence.

(c) When were chicken and red meat consumption equal?

(d) The best fitting line for chicken is

$$C = 3.6375 + .2854Y$$

and the best fitting line for beef is

$$B = 11.774 + .007Y$$

Set up and solve a system of linear equations to find the year when chicken and red meat consumption will likely be equal. How does this answer compare to your estimate?

(e) The slope of the best fitting beef line is .007 million tons/year. What does that tell you about beef consumption over the past 40 years?