$\boxed{\text{SOLUTIONS}}$

4.2 Systems of linear equations – Practice exercises

1. Madison wants to buy a new car, either Car A: a hybrid priced at $26,100, or Car B: a high-efficiency gas car priced at $23,700. Annual fuel costs for Car A are currently $1,100. For Car B annual fuel costs are currently $1,800. The total cost of each car will depend on how many years she keeps it.

   (a) Name the variables.

   T = total cost ($) ~ dep

   Y = time (years) ~ indep

   (b) Write a linear equation for the total cost (including purchase price and fuel costs) of Car A and write another linear equation for the total cost of Car B each as a function of how long she keeps it. Assume fuel costs are constant.

   Car A: $T = 26{,}100 + 1{,}100Y$

   Car B: $T = 23{,}700 + 1{,}800Y$

   (c) Make a table comparing the total costs for the two cars if Madison keeps the car she buys for 3, 5, or 10 years.

Y	3	5	10
Car A	29,400	31,600	37,100
Car B	29,100	32,700	41,700

   (d) Set up and solve a system of linear equations to determine the **payoff time**, or the number of years for which the total costs of each car are equal.

$$26{,}100 + 1{,}100Y = 23{,}700 + 1{,}800Y$$
$$-23{,}700 \qquad\qquad -23{,}700$$
$$2{,}400 + 1{,}100Y = 1{,}800Y$$
$$-1{,}100Y \qquad\qquad -1{,}100Y$$
$$\frac{2{,}400}{700} = \frac{700Y}{700}$$
$$Y = 3.4285\ldots \text{ years } \boxed{\approx 3 \text{ years, } 5 \text{ months}}$$

$$.4285\ldots \text{yrs} * \frac{12\text{ mo}}{1\text{ yr}} = .4285\ldots \times 12 = 5.142\ldots \approx 5\text{ mo}$$

   (e) Based on what you have learned, fill in the blank.

   The more expensive hybrid pays off if Madison is going to keep it for ~~4~~ years or more.

   $3\frac{1}{2}$

2. A mug of coffee costs \$3.45 at Juan's favorite cafe, unless he buys their discount card for \$10 in which case a mug costs \$2.90. Or, he can buy a membership for \$59.99 and then coffee is only \$1/mug. If we let M represent the number of mugs of coffee he buys and T represent the total cost in dollars, then the equations are:

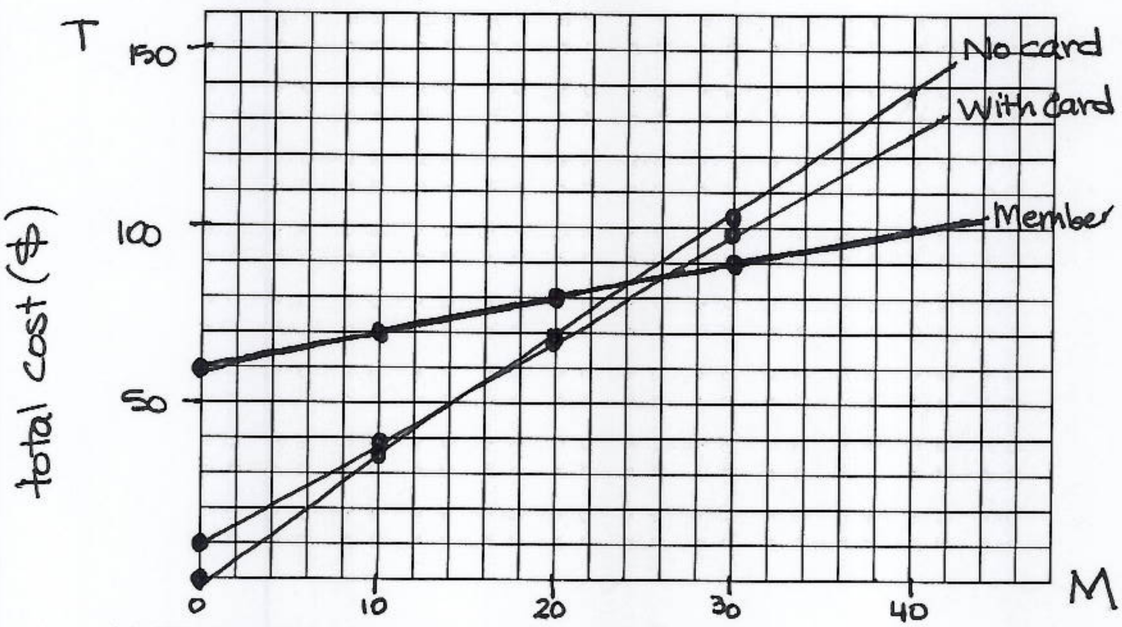
No card:	$T = 3.45M$
With card:	$T = 10.00 + 2.90M$
Member:	$T = 59.99 + 1.00M$

Story also appears in 1.2 #4 and 2.1 #4

(a) Compare the total costs for all three options.

	Mugs	0	10	20	30	
Indep →						3.45 x $\underline{30}$ =
dep	No card	0	34.50	69.00	103.50	
dep	With card	10.00	39.00	68.00	97.00	10.00+2.90 x $\underline{30}$ =
dep	Member	59.99	69.99	79.99	89.99	← 59.99+1.00 x $\underline{30}$ =

(b) Draw a graph showing all three options.



(c) Which option is least expensive if Juan plans to buy

mugs of coffee

- A small number of mugs of coffee: Nocard
- A medium number of mugs of coffee: With card
- A large number of mugs of coffee: Member

The problem continues ...

(d) Set up and solve a system of linear equations to compare total cost with no card to the total cost with the card.

no card = with card

$$3.45M = 10.00 + \cancel{2.90M}$$

$$-2.90M \qquad\qquad -\cancel{2.90M}$$

$$.55M = 10.00$$

$$\frac{\cancel{.55}}{} \qquad \frac{}{.55}$$

$$M = 18.18\ldots$$

(e) Set up and solve a system of linear equation to compare the total cost with the card to the total cost with the membership.

with card = with membership

$$\cancel{10.00} + 2.90M = 59.99 + 1.00M$$

$$-\cancel{10.00} \qquad\qquad -10.00$$

$$2.90M = 49.99 + \cancel{1.00M}$$

$$-1.00M \qquad\qquad -\cancel{1.00M}$$

$$\frac{1.90M}{\cancel{1.90}} = \frac{49.99}{1.90}$$

$$M = 26.31\ldots$$

(f) Describe in words what you've learned.

# mugs	best choice is:
0-18	no card
19-26	with card
27+	membership

3. Ahmed planted two shrubs in the backyard on May 1. The virburnum was 16.9 inches tall and expected to grow .4 inches each week this summer. The weigela was 20.3 inches tall but only expected to grow .2 inches per week. If we let S represent the total height of the shrub in inches after W weeks, then the equations are:

Virburnum: $S = 16.9 + .4W$
Weigela: $S = 20.3 + .2W$

Story also appears in 4.1 exercises

(a) Compare the height of the shrub on the given dates.

	date	May 1	June 12	July 10	Sept 4	
indep →	W	0	6	10	18	
dep	S (virburnum)	16.9	19.3	20.9	24.1	$16.9 + .4 \times \underline{18} =$
dep	S (weigela)	20.3	21.5	22.3	23.9	← $20.3 + .2 \times \underline{18} =$

(b) When will the shrubs be the same height? Continue successive approximation to find the answer to the nearest week.

W	10	18	15	17
S-Virburnum	20.9	24.1	22.9	23.7
S-Weigela	22.3	23.9	23.3	23.7

smaller # is circled

The shrubs will be the same height 17 weeks after May 1

(c) Set up and solve an equation to find the day when the two shrubs are the same height. In what month does that happen? $\boxed{\text{Aug 28}}$

Virburnum = Weigela

$$16.9 + .4W = 20.3 + .2W$$
$$-16.9 \qquad\qquad -16.9$$

$$.4W = 3.4 + .2W$$
$$-.2W \qquad\qquad -.2W$$

$$\frac{.2W}{.2} = \frac{3.4}{.2}$$

$W = 17$ weeks ✓

$17 \text{ weeks} * \frac{7 \text{ days}}{1 \text{ week}}$

$= 17 \times 7 = 119$ days

date	day	
May 1	0	+30
May 31	30	+30
June 30	60	+31
July 31	91	+31
Aug 31	122	
Aug 30	121	
Aug 29	120	
Aug 28	119	

4. The **supply** of flour is the amount of flour produced. It depends on the price of flour. A high price encourages producers to make more flour. If the price is low, they tend to make less of it. The dependence of the supply of flour S (in loads) on the price P (in \$/pound) is given by the equation

$$\textbf{Supply:} \quad S = .8P + .5$$

The **demand** of flour is the amount of flour consumers want to buy. It also depends on the price of flour. If flour sells for a high price, then consumers will buy less. If flour sells for a low price instead, then consumers will buy more. The dependence of the demand of flour D (in loads) on the price P (in \$/pound) is given by the equation

$$\textbf{Demand:} \quad D = 1.5 - .4P$$

The **equilibrium price** of flour is the price where the supply equals the demand.

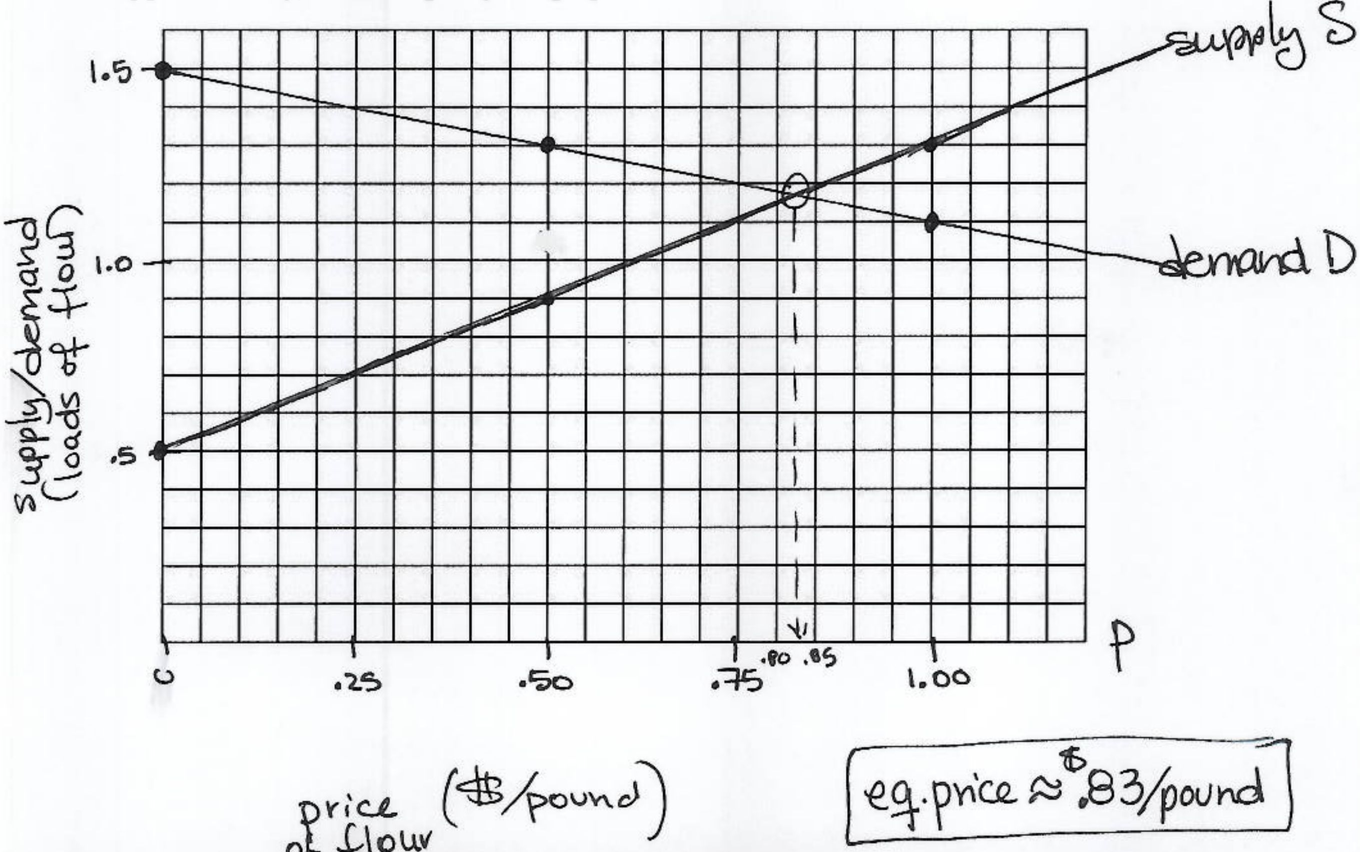
(a) What happens if flour is priced at \$1.00/pound? That is, how much flour will be produced and how much will consumers ~~want~~ demand?

$S = .8 \times 1.00 + .5 = 1.3$ loads
$D = 1.5 - .4 \times 1.00 = 1.1$ load
More flour produced than bought (SURPLUS)

(b) What happens if flour is priced at \$0.50/pound? That is, how much flour will be produced and how much will consumers ~~want~~ demand?

$S = .8 \times .50 + .5 = .9$ loads
$D = 1.5 - .4 \times .50 = 1.3$ loads
More flour wanted than produced. (SHORTAGE)

(c) Graph each dependence on the same set of axes. What is the equilibrium price, approximately, according to your graph?



$\boxed{\text{eq. price} \approx \$.83/\text{pound}}$

The problem continues ...

(d) Set up and solve an equation to find the equilibrium price of flour.

$$.8P + .\cancel{5} = 1.5 - .4P$$
$$-.\cancel{5} \quad -.5$$
$$.8P = 1.0 - .\cancel{4}P$$
$$+.4P \qquad +.\cancel{4}P$$
$$\frac{1.\cancel{2}P}{\cancel{1.2}} = \frac{1.0}{1.2} \qquad P = .83333\ldots \approx \boxed{\$.83/\text{pound}}$$

(e) When more of a product is produced than consumers want to buy, we have a **surplus** of the product. Solve an inequality to find the range of price values for which there will be a surplus of flour. Compare your answer to part (d).

supply > demand

$$.8P + .\cancel{5} > 1.5 - .\cancel{4}P$$
$$-.\cancel{5} \quad -.5$$
$$+.4P \qquad +.\cancel{4}P$$
$$\frac{1.\cancel{2}P}{\cancel{1.2}} > \frac{1.0}{1.2}$$
$$P > .8333\ldots$$

answer (d)

$\boxed{\text{surplus if price is over \$.83/pound}}$

(f) When less of a product is produced than consumers want to buy, we have a **shortage** of the product. Solve an inequality to find the range of price values for which there will be a shortage of flour. Compare your answer to parts (d) and (e).

supply < demand

$$.8P + .\cancel{5} < 1.5 - .\cancel{4}P$$
$$-.\cancel{5} \quad -.5$$
$$+.4P \qquad +.\cancel{4}P$$
$$\frac{1.\cancel{2}P}{\cancel{1.2}} < \frac{1.0}{1.2}$$
$$P < .8333\ldots$$

answer (d)

$\boxed{\text{shortage if price is under \$.83/pound}}$