0.1 Prelude: Approximation and Rounding

Practice exercises

1. Round each number up, down, or off to the precision indicated.

   This problem also appears in Section 1.1 #3.

   (a) My calculations show I need a cross brace around 9.388 feet long. I want the board to be long enough, so round up to the nearest foot.

   (b) Gas mileage is usually rounded down to the nearest one decimal place. What is the gas mileage for a car measured as getting 42.812 miles per gallon? What about a car getting 23.09 miles per gallon?

   (c) The population estimate was 4.2 million people, but revised estimates suggest 4,908,229 people. Report the revised estimate rounded appropriately. What if a different estimate was 4,890,225? Would that change your answer?

2. (a) Callista needs \$117 cash for a mani-pedi at the local salon. The ATM allows her to withdraw multiples of \$20. How much money should she withdraw and how many \$20 bills is that? Did you round up, down, or off?

   (b) Bahari is buying some 8-packs of sparkling water for today's community hour. He expects up to 23 people to be there. He calculates that he will need $23 \div 8 = 2.875$ 8-packs. How many 8-packs should he bring? Did you round up, down, or off?

   (c) Tzuf has \$20 to buy apples for the new year's celebration. A bag of apples costs \$3.49. Tsuf calculates that they can afford $20 \div 3.49 = 5.7306...$ bags. How many bags can they buy? Did you round up, down, or off?

   (d) Eiji read that life expectancy in the United States is 77.28 years whereas in Japan it is 84.62 years. How might he describe these life expectancies in (whole) years? Did you round up, down, or off?

3. Round off the calculated number(s) to give an answer that is reasonable and no more precise than the information given.

   (a) The snow removal budget for the city is currently at \$8.3 million but the city council is requesting a reduction of \$1.15 million per year. We calculate that after three years of cuts, the snow removal budget will be \$4.8079... million.

   (b) A cup of cooked red lentils has around 190 calories and 6.4 grams of dietary fiber, while a cup of cooked chickpeas has around 172 calories and 12.0 grams of dietary fiber. We calculate that lentils provide 0.03368421... grams per calorie whereas chickpeas provide 0.06976744... grams per calorie.

   (c) Hibbing [Minnesota] is the former boyhood home of Bob Dylan, basketball great Kevin McHale and the location of the Hull-Rust-Mahoning Open Pit Iron Mine, which has the largest iron-ore pit in the world. Hibbing is also the birthplace of [baseball star] Roger Maris.

   (source: http://hibbing.areaconnect.com/)

   In 2000 the population of Hibbing, Minnesota was reported at just over 17,000 residents. Based on a projected 0.4% decrease per year, the 2010 population was calculated to be 16,332.110... people.

4. It is easiest to compare the size of decimal numbers when they are written the same precision. For example, \$1.7 million is more money than \$1.34 million because when we write both numbers to two decimal places we see

$$1.7 = 1.70 > 1.34$$

The symbol $>$ means "greater than;" it points to the smaller number. Alternatively, when we expand both numbers we see

$$1{,}700{,}000 > 1{,}340{,}000$$

In each story, write all of the decimal numbers given to the same precision and list the numbers from largest to smallest using $>$ signs.

(a) Dawn tested a water sample from her apartment and found 21.19 ppm of sulfate. She volunteers at a local soup kitchen where the water sample tested at 21.3 ppm. (The abbreviation **ppm** stands for "parts per million. Not to worry – sulfate levels below 250 are considered safe for human consumption.)

(b) There are approximately 1.084 million quarters in circulation in the United States, compared to 1.786 million dimes, 1.6 million \$5 bills, and 1.42 million \$10 bills.

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) What the symbol for "approximately equal to" is?

(b) Why an approximate answer is often as good as we can get?

(c) What the term "precisely" refers to?

(d) What the saying "I'd rather be approximately right than precisely wrong" means?

(e) What the difference is between rounding off, rounding up, and rounding down?

(f) When to round your answer, and when to round your answer up or down (instead of off)?

(g) How to round a decimal to the nearest whole number?

To one decimal place?

To two decimal places?

(h) How precisely to round an answer?

(i) How to compare sizes of decimal numbers?

(j) What the symbol for "greater than" is?

Don't forget!

0.2 Prelude: Arithmetic Operations

Practice exercises

On each problem, write down what you enter into your calculator and don't forget to write the units on your final answer. You are welcome to calculate the answer step-by-step but also challenge yourself to figure out the answer all at once, not hitting = on your calculator until the very end.

1. Tensia loves to garden but can't quite keep up with how many cucumbers are growing.

   - At the start of the week she had 8 cucumbers in her refrigerator.
   - Her son, Néstor took 3 home with him after dinner on Monday.
   - Tensia harvested another 7 cucumbers on Wednesday.
   - Her neighbor Sarah graciously took 4 cucumbers to make pickles.
   - Tensia herself ate 2 cucumbers during the week.

   How many cucumbers does she have left over?

2. Brent's landlord charges $15 per day for late rent.

   (a) What will Brent's late fee be if is 6 days late paying his rent?

   (b) If Brent got a bill showing $195 in late fees, how many days late did he pay his rent?

3. There are 2,624 students at a local university.

   (a) About $\frac{3}{4}$ of those students live on or within a mile of campus. How many students live on or within a mile of campus?

   (b) The university wants to support 40 hours a week of onsite tutoring (in mathematics, writing, etc.) for each the 32 weeks that classes are in session. It costs about \$18/hour to pay the tutors and staff administrators. What is the total cost of tutoring?

   (c) The university is considering adding a tutoring fee to cover the cost of tutoring. If they wanted to cover the total cost of tutoring, what would the cost per student be?

4. A truck hauling grass seed weighs 3,900 pounds when it is empty. Each bag of seed it carries weighs 4.2 pounds. The **gross weight** of the truck is the total weight including the truck and the bags of seed.

   Story also appears in 2.1 #1, 3.2 #1, and 3.1 #1

   (a) How much does 1,300 bags of grass seed weigh?

   (b) What is the gross weight of the truck if it carries 1,300 bags of grass seed?

   (c) You probably entered this calculation as $1300 \times 4.2 = +3900 =$. What happens if you skip the middle = sign and enter $1300 \times 4.2 + 3900$ instead?

   (d) What answer does your calculator give you if you enter $3900 + 4.2 \times 1300$ instead?

   (e) What does part (d) tell you about which operation your calculator did first: the + or the $\times$?

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) When to add, subtract, multiply, or divide numbers?

(b) What is the difference between subtraction and negation?

(c) How to add, subtract, negate, multiply, and divide on a calculator?

(d) How multiplication is related to addition?

(e) What the term "per" indicates?

Don't forget!

0.3 Prelude: Percentages

Practice exercises

On each problem, write down what you enter into your calculator and don't forget to write the units on your final answer. You are welcome to calculate the answer step-by-step but also challenge yourself to figure out the answer all at once, not hitting = on your calculator until the very end.

1. As I write this problem, the population of the world is 8,056,959,718 people (just over 8 billion). It changes by the second, so let's use the round figure of 8,100,000,000.

   (a) I read that the population of Brazil accounts for 2.69% of the world's population. According to that report, what is the population of Brazil? Round your answer to the nearest million.

   (b) If the population of the United States is currently around 334,000,000, what percentage of the world's population is in the United States?

2. In Minneapolis, apartment rent is expected to increase by 16% next year.

   (a) Astra lives in a 1-bedroom apartment where they pay $825 per month in rent. If their rent increased by 16% what would their new rent be?

   (b) Lucky for Astra, their building is subject to rent stabilization laws and so their rent cannot increase by more than 3%. What would their new rent be?

3. The corner by my house is dangerous. One year there were 14 accidents there. The neighbors got together and petitioned to have 4-way stop signs installed.

   (a) The city estimated that the installed stop signs would reduce accidents at least 40%. If that happens, how many accidents would we expect the next year?

   (b) The national average shows that the new signs could reduce accidents up to 62%. If that happens instead, how many accidents would we expect the next year?

   (c) If there were 6 accidents the next year, is that in the range you figured out? What percent decrease does that correspond to?

4. My savings account earns a modest amount of interest, the equivalent of .75% annually. I have $12,392.18 in the account now. *Story also appears in 2.2#4*

   (a) How much interest will I earn this year?

   (b) How much will my account balance be at the end of the year?

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) What the words "per" and "cent" mean in the word "percent."

(b) How to convert a fraction or decimal to a percent?

(c) How to convert a percent to a decimal?

(d) How to calculate a percentage of a number?

(e) How to calculate the result of a percent increase or a percent decrease?

Don't forget!

0.4 Prelude: Order of Operations

Practice exercises

On each problem, write down what you enter into your calculator and don't forget to write the units on your final answer. Challenge yourself to use one-line calculations. You are welcome to calculate the answer step-by-step to check.

1. *Story also appears in 1.2 #4, 2.1 #4, and 4.2 #2*

   (a) A mug of coffee costs \$3.45 at Juan's favorite cafe. If Juan orders 25 coffees each month, how much will he pay (total) for the month?

   (b) The cafe offers a deal each month, where if you buy their \$10 discount card, then you only pay \$2.90 per mug of coffee during the month. If Juan buys the discount card and orders 25 coffees, how much will he pay (total) for the month?

   (c) Comparing answers to parts (a) and (b) is it worthwhile for Juan to buy the card? Discuss.

2. "Rose gold" is a mix of gold and copper.

   Story also appears in 2.3 #2 and 4.1 Exercises

   (a) If we mix 2 grams of gold with 2 grams of copper, what is the percentage of gold in the resulting alloy?

   (b) If instead we mix 2 grams of gold with 7 grams of copper, what is the percentage of gold in the resulting alloy?

3. *Stories also appear in 4.3 #3*

   (a) Vanessa's doctor put her on a sensible diet and exercise plan to get her back to a healthy weight. She currently weighs 213 pounds. She will need to lose an average of 1.25 pounds a week to reach her goal weight in a year. What is her goal weight? Use 1 year = 52 weeks.

   (b) Since she has been pregnant, Zoe has gained the recommended 1/2 pound per week. She weighed 153 at the start of her pregnancy. What does she weigh now at 30 weeks pregnant?

4. *Stories also appear in 4.3 #3*

   (a) Jerome has gained weight since he took his power training to the next level ten weeks ago, at the rate of around 1 pound a week. He is now 198 pounds. What was his original weight?

   (b) After the past 6 weeks of terrible migrane headaches, Carlos is down to 158 pounds. He has lost 4 pounds a week. What did Carlos weigh when 6 weeks ago?

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) How a calculator will evaluate an expression that has several different operations, such as $2.1 + 7 \times 1.1$?

(b) What is the order of operations in general?

(c) A good way to remember PEMDAS?

(d) Why you need to know what the order of operations is?

(e) When might you need to override the order of operations?

(f) How to override the order of operations using parentheses?

Don't forget!

0.5 Prelude: Fractions

Practice exercises

1. There are 2,624 students at a local university.

   (a) Of those students, 673 of those students placed into this algebra class. What fraction of students placed into algebra?

   (b) The Dean said that approximately 1 in 4 students, or $\frac{1}{4}$ of all students, placed into algebra. Is that correct? Check by determining if your answer to part (a) $\approx \frac{1}{4}$ by comparing decimal approximations.

2. Gas mileage is usually rounded down to the nearest one decimal place. Gas mileage is measured in miles per gallon (mpg).

   (a) Xu does gig work delivering take-out food from local restaurants. He started the week with a full tank of gas and drove 319 miles. When he went to fill the tank, he needed 11.3 gallons. What was Xus gas mileage?

   (b) Margaret and Cathy are on a cross-country trip. They've driven from Minnesota to Maine (approximately 1,430 miles). They have bought gas a few times along the way: 12.7 gallons, then 14.0 gallons, then 13.1 gallons, and then 12.4 gallons. What was Margaret and Cathy's gas mileage?

   (c) How could you do the calculation in part (b) one line on your calculator by using parentheses?

3. In January 2015, Graham had 47 albums in his vinyl collection. By September 2023 (that's 8 years, 9 months later), he had 783 albums. Approximately how many albums per month did Graham buy?

   (a) Figure out the answer step by step.

   (b) Now try to combine all of your calculations into one line on your calculator. Hint: write as a fraction first.

4. It took Mariam 3 hours to complete the reading for her Religion class. The reading was 102 pages long.

   (a) How fast did she read measured in pages per hour? Write the answer as a fraction and as a decimal.

   (b) Reading speed is often measured in words per minute. Assuming there are approximately 500 words per page, calculate Mariam's reading speed step by step.

   (c) How could you do the calculation in part (b) one line on your calculator by using parentheses? Hint: the "hours" cancel!

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) How we represent a part of a whole as a fraction?

(b) How to multiply fractions?

(c) What"canceling" a factor means?

(d) How fractions are related to division?

(e) How to calculate the decimal approximation of a fraction?

(f) How to compare two fractions using their decimal approximations?

(g) How the units of a fraction are determined?

(h) When we need to use parentheses around the top (numerator) and bottom (denominator) to evaluate a fraction?

Don't forget!

0.6 Prelude: Powers and roots

Practice exercises

1. Jody is using small wooden balls to make noses for her knitted gnomes. She figured out that she can calculate the weight of each ball (in ounces) as $.2 \times B \wedge 3$ where B is the diameter of the ball (in inches).

   (a) What does a 2.5 inch diameter wooden ball weigh?

   (b) Jody is considering building a giant gnome for her office. The nose will be a wooden ball weighing 1 pound. She calculates that the diameter of the ball will be $\sqrt[3]{80}$. How big is that?

2. The size of a round pizza is described by its diameter. It turns out that we can calculate how many people are served by a pizza of diameter D inches as $.015625 \times D \wedge 2 =$. For example, a 16-inch diameter pizza serves $.015625 \times 16 \wedge 2 = 4$ people. (The mysterious number .015625 comes from a little geometry and pizza science.)

   Story also appears in 2.4 #1 and 3.3 #1.

   (a) How many people would be served by a 12-inch pizza?

   (b) A personal pizza is designed to serve one person. It turns out the diameter of a personal pizza is $\sqrt{64}$. Calculate the diameter of a personal pizza using the square root key (or just the root key) on your calculator.

   (c) An extra large pizza serves 6 people. It turns out the diameter of an extra large pizza is $\sqrt{384}$. Calculate the diameter of a personal pizza using the square root key (or just the root key) on your calculator.

3. A signal sent down a fiber optic cable decreases by 2% per mile. That means after M miles, its strength is $\underbrace{.98 \times .98 \times \cdots \times .98}_{M \text{ times}} = .98 \wedge M$. What is the signal strength after 10 miles? After 20 miles? Note: your answers should be decimal numbers less than 1.

4. Otis invested \$500,000 and estimates his investment will double in value every 10 years.

   (a) Calculate the value of Otis's investment after 10, 20, 30, and 40 years.

   (b) If Kricia invested \$230,000 instead, what would her investment be worth after 40 years? Try to use a power to help answer the question. Hint: how many times will the value of her investment double?

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) What the square, cube, or higher power of a number means?

(b) How to calculate powers of a number using a calculator?

(c) What the square root, cube roots, or higher root of a number means?

(d) How to calculate roots of a number using a calculator?

Don't forget!

0.7 Prelude: Algebraic Notation

Practice exercises

1. Since she has been pregnant, Zoe has gained the recommended $\frac{1}{2}$ pound per week. She weighted 153 pounds at the start of her pregnancy. That means when she is W weeks pregnant, that Zoe weighs

$$153 + \frac{1}{2}W$$

What does this expression say Zoe will weigh when she's 40 weeks pregnant?

Story also appears in 0.4 #3 and 4.3 #3

2. Jody is using small wooden balls to make noses for her knitted gnomes. She figured out that she can calculate the weight of each ball (in ounces) as $.2 \times B \wedge 3$. Write this expression in algebraic notation.

Story also appears in 0.6 #1

3. Astra lives in a 1-bedroom apartment where they pay \$825 per month in rent. Thanks to new rent stabilization laws, Astra's rent can only increase 3% per year. That means after Y years, their rent will be at most

$$825(1.03^Y)$$

What does this expression say her rent could be in 5 years?

Story also appears in 0.3 #2

4. "Rose gold" is a mix of gold and copper. If we mix 2 grams of gold with C grams of copper, the percentage of the resulting alloy that is gold is given by the expression

$$\frac{200}{2+C}$$

What does this expression say the percentage of gold will be if we add 7 grams of copper?

Story also appears in 0.4 #2, 2.3 #2, and 4.1 Exercises

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) Where multiplication can be hidden in algebraic notation?

(b) How powers are written in algebraic notation?

(c) How division is written in algebraic notation?

(d) What the word evaluate means?

(e) How to evaluate an algebraic expression on your calculator?

Don't forget!

0.8 Prelude: Scientific Notation

Practice exercises

1. In each story, write out the highlighted numbers (with all the zeros).

   (a) Melvin was looking populations based on the 2020 Census and saw the population of Saint Paul, MN listed as $\mathbf{3.10942 \times 10^5}$ **people**. Hint: you can check the answer to this part by evaluating on your calculator.

   (b) The gross domestic product (GDP) measures the market value of all final goods and services produced by an economy. The United States GDP is approximately $\$\mathbf{2.332 \times 10^{13}}$. *Story also appears in 1.5 #1*

   (c) The Earth weighs approximately $\mathbf{5.972 \times 10^{24}}$ **kilograms**.
   Story also appears in 1.5 #3

2. In each story, write out the highlighted numbers (with all the zeros).

   (a) Alpaca have very fine hairs (which can be spun into yarn to make very soft sweaters). The width of an alpaca hair is around $\mathbf{2.5 \times 10^{-7}}$ **meters**. Hint: you can check the answer to this part by evaluating on your calculator.

   (b) A dust particle weighs approximately $\mathbf{7.53 \times 10^{-10}}$ **grams**.
   Story also appears in 1.5 #2

   (c) A proton (part of an atom) has mass of about $\mathbf{1.67262 \times 10^{-27}}$ **kilograms**.
   Story also appears in 1.5 #7

3. In each story, evaluate the number and report your answer in scientific notation.

   (a) Bunnies, bunnies, everywhere. In 2007 there were 1800 and that number was predicted to increase 13% each year. I was trying to predict the number of rabbits in 2023 (after 16 years) but I accidentally typed in 166 years by mistake

   $$1800 * 1.13 \wedge 166 =$$

   Report the answer I got in scientific notation. (Yes, this is a gigantic number. The exponential model I used doesn't actually make sense for that many years.)

   Story also appears in 2.1 #2 and 5.1 #3

   (b) A signal is sent down a fiber optic cable. Its strength decreases by 2% each mile it travels. We can calculate the signal strength after 1000 miles by evaluating

   $$.98 \wedge 1000 =$$

   Report the answer you get in scientific notation. (Yes, this is a teeny number. In reality there would be signal booster installed along the route.)

   Story also appears in 5.2 #1

4. In each story, write out the highlighted number (with all the 0s). Note that **million** is short for $\times 10^6$, **billion** is short for $\times 10^9$, and **trillion** is short for $\times 10^{12}$.

   (a) There are approximately **1.084 million quarters** in circulation in the United States. *Story also appears in 0.1 #4*

   (b) The population of the world is approximately **8.1 billion people**.

   Story also appears in 0.3 #1

   (c) One way that the United States government borrow money is Treasury bonds (T-bonds). There are approximately **$24 trillion** worth of T-bonds currently.

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) What million, billion, and trillion mean?

(b) Why scientific notation is used?

(c) The standard format for scientific notation?

(d) That a positive exponent corresponds to a big number and a negative exponent corresponds to a tiny number?

(e) How to convert from scientific notation to decimal?

(f) How your calculator reports numbers in scientific notation, and what (might be) different when you're reporting that number?

Don't forget!

0.9 Prelude: Logarithms

Practice exercises

1. The number of bacteria in a dish increases 10-fold each day. Note: 10-fold means $\times 10$. Suppose we had 1 microliter of bacteria at the start of the first day. That means after D days there will be 10^D microliters of bacteria.

   (a) How many bacteria (in microliters) will there be after 1 day? After 2 days? After 3 days?

   (b) In how many days will the bacteria have reached 1 liter, which is million microliters?

   (c) How can we use logs to find the answer?

2. The problem continues ...

   (a) How many days(from the start) does it take to reach the 25 millilter capacity of the petri dish, which is 25,000 microliters? Guess and check to find the answer to 1 decimal place.

   (b) How can we use logs to find the answer?

   (c) Convert your answer to days & hours format (meaning d days and h hours).

3. The equation $pH = -\log(H^+)$ tells us the pH of a substance (on a scale from 0 to 14) based on its molar hydrogen ion concentration H^+. Don't let the notation here scare you: pH is a single quantity and H^+ has nothing to do with exponents or adding.

   For example, lemon juice has $H^+ = .0025$ and so the pH of lemon juice is

   $$-\log(.0025) = (\text{-})\log(.0025) = 2.6020599913 \approx 2.6$$

   (a) Coca-cola has $H^+ = .000\ 398$. Find the pH of orange juice. Note: the funny spaces are to help you read the number.

   (b) Hair shampoo has $H^+ = .000\ 003\ 162$ Find the pH of hair shampoo.

   (c) Household bleach has $H^+ = 1.1 \times 10^{-13}$ Find the pH of bleach.

   (d) Materials with pH values between 0-5 are **acidic**, between 9-14 are **basic**, and between 5-7 are **neutral**. Which of the above materials are acid, basic, and neutral?

4. In Minneapolis, apartment rent is expected to increase by 16% next year.

   Story also appears in 0.3 #2

   (a) Astra lives in a 1-bedroom apartment where they pay \$825 per month in rent. If their rent increased by 16% in how many years would their rent be doubled to \$1,650. As we'll see later, the answer is $\dfrac{\log(2)}{\log(1.16)}$. Don't forget to the close the parentheses.

   (b) Lucky for Astra, their building is subject to rent stabilization laws and so their rent cannot increase by more than 3%. In how many years would their rent double under this cap? The answer is $\dfrac{\log(2)}{\log(1.03)}$.

When you're done ...

- ☐ Check your solutions. Still confused? Work with a classmate, instructor, or tutor.
- ☐ Try the **Do you know** questions. Not sure? Read the textbook and try again.
- ☐ Make a list of key ideas and process to remember under **Don't forget!**
- ☐ Do the textbook exercises and check your answers. Not sure if you are close enough? Compare answers with a classmate or ask your instructor or tutor.
- ☐ Getting the wrong answers or stuck? Re-read the section and try again. If you are still stuck, work with a classmate or go to your instructor's office hours or tutor hours.
- ☐ It is normal to find some parts of exercises difficult, but if most of them are a struggle, meet with your instructor or advisor about possible strategies or support services.

Do you know ...

(a) What a logarithm (base 10) means?

(b) How to evaluate logarithms (base 10) on a calculator?

(c) Which size numbers have a positive log and which have a negative log (base 10)?

(d) The connection between logarithms (base 10) and scientific notation.

Don't forget!