SOLUTIONS

Practice Exam 4A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

---

1. Forde collects miniature cars, each weighing 1.76 ounces. His car box weighs 4 ounces when empty. The total weight T ounces of Forde's car box depends on the number of cars C according to the equation

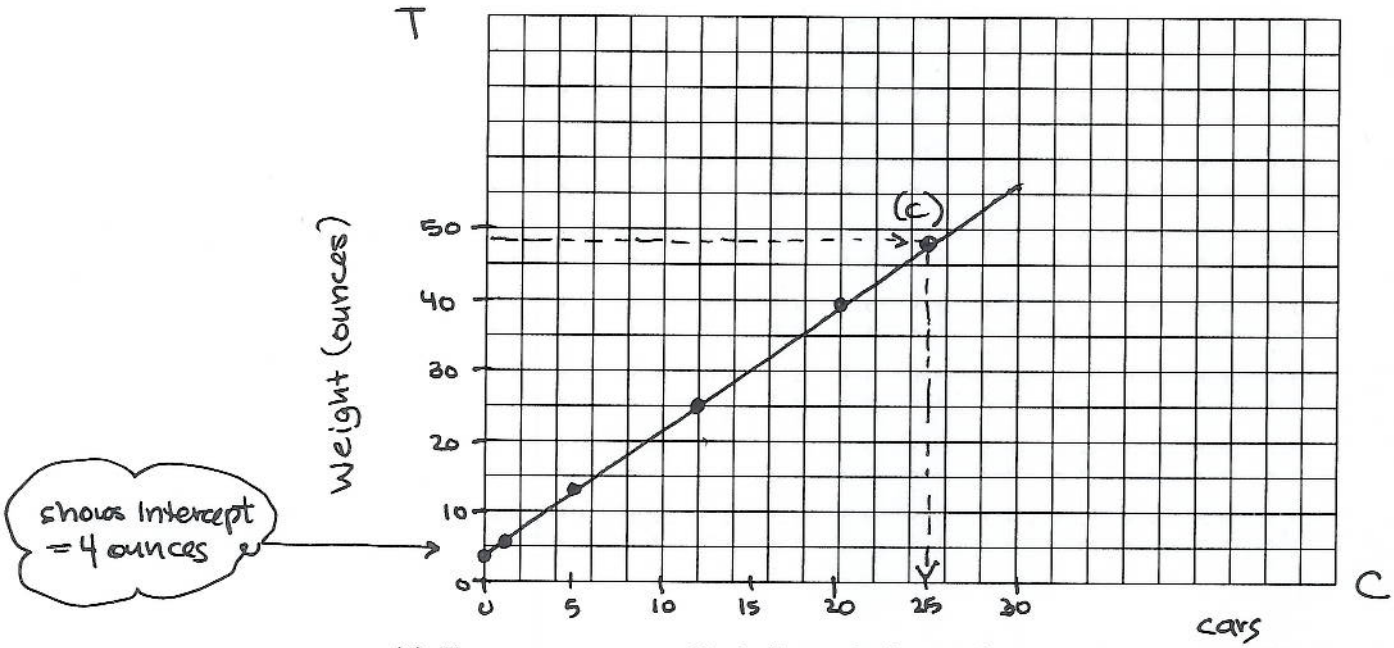
$$T = 4 + 1.76C$$

(a) Make a table of values showing the weight if it contains 1, 5, 12, or 20 cars.

C	1	5	12	20
T	5.76	12.80	25.12	39.20

← $4+1.76\times 20=$

(b) Draw a graph illustrating the dependence.



(c) How many cars can Forde fit in the box and stay under 3 pounds (that's 48 ounces)? *Figure out the answer and mark the corresponding point on your graph.*

C	25
T	48

← $4+1.76\times 25=$

Forde can put 25 cars in the box.

seems like comparing 2 models ⇒ linear system?

2. Will women every run the marathon as fast as men do? The world records are getting close. In 2012 the men's record was 2:03:38 and the women's record was 2:15:25. That's only about 12 minutes apart! On the other hand, the record is changing very slowly. Estimates for the men's time shows about 13 seconds drop per year on average. Estimates for the women's time shows about 26 seconds drop per year on average. } intercepts } slopes

Source: Wikipedia (Marathon World Record Progression)

(a) Write an equation for each function: men's and women's. Use T for the marathon times (in seconds) and Y for the years (measured in years since 2012). Note that 2:03:38 = 7,418 seconds and 2:12:25 = 7,945 seconds.

used linear equation template: dep = start + slope * indep

men's record :	$T = 7{,}418 - 13Y$
women's record :	$T = 7{,}945 - 26Y$

(b) Use successive approximate to estimate when the women's record might equal the men's record. Display your guesses in a table.

41 + 2012 = 2053

If records continue to drop at these rates, the women's record might equal the men's by 2053

Y	0	20	50	40	41
men's	(7418)	7,158	6,768	(6,898)	(6,885)
women's	7945	7,425	(6,645)	6,905	6,879
				CLOSEST!	

(c) Set up and solve a system to estimate when the women's record might equal the men's record.

men's record = women's record

$7{,}418 - 13Y = 7{,}945 - 26Y$

$-7{,}418 \qquad -7{,}418$

$-13Y = 527 - 26Y$

$+26Y \qquad +26Y$

$\frac{13Y}{13} = \frac{527}{13}$

$Y = 40.538\ldots \approx 41 \Rightarrow \boxed{2053}$ as estimated ☺

$/album ⇒ Slope

3. An online music club charges a monthly enrollment fee plus $.95 per album you download. Last month Andrew downloaded 31 albums for a total cost of $49.00.

fixed, upfront cost ⇒ Intercept

(a) What is the monthly enrollment fee?

$\text{fee} = \$49.00 - \frac{\$.95}{\text{album}} \times 31 \text{ albums}$

Used formula: intercept = dep − slope × indep

$= 49.00 - .95 \times 31 = \$19.55$

A	T
0	?
31	49.00

The monthly enrollment fee must be $19.55.

(b) Name the variables, including units, and write an equation relating them.

A = # albums Andrew downloads in a month ~indep
T = total cost for that month ($) ~dep

$T = 19.55 + .95A$

Used linear equation template: dep = start + slope × indep

Check: $19.55 + .95 \times 31 = \$49.00$ ✓

(c) If the bill next month is for $87.95, how many albums did Andrew download? Show how to solve the equation

$19.55 + .95A = 87.95$
$-19.55 \qquad -19.55$

$\frac{.95A}{.95} = \frac{68.40}{.95}$

$A = 72$ albums

Check: ☺ $19.55 + .95 \times 72 = 87.95$

Andrew downloaded 72 albums that month.

4. A report shows September sea-ice declining in the Northern hemisphere. In 1980 the extent of the sea-ice was 3.1 million square miles. By 2012, the sea-ice extended only 1.7 million square miles. For this problem, suppose that the area of sea-ice decreases linearly.
Source: National Snow and Ice Data Center

(a) Name the variables, including units.

Y = year (years since 1980) ~ indep
S = amount of Sept. sea-ice in N.H. (million square miles) ~ dep

year	Y	S
1980	0	3.1
2012	32	1.7

↑ 2012 − 1980 =

note: Y = 0 ⇒ S = 3.1 mil sq miles is the Intercept

(b) What is the rate of sea ice decrease?

rate ⇒ roc / slope

$$\text{Slope} = \frac{1.7 - 3.1 \text{ mil sq miles}}{32 - 0 \text{ years}} = (1.7 - 3.1) \div 32 = -.04375 \text{ mil sq miles per year}$$

note: $\frac{-.04375 \text{ million sq miles}}{\text{year}} * \frac{1{,}000{,}000 \text{ sq miles}}{1 \text{ million sq miles}} = -.04375 \times 1000000 = -43{,}750$ square miles/year

(c) Write a linear equation relating your variables.

$$S = 3.1 - .04375Y$$

used linear equation template: dep = start + slope * indep

check: $3.1 - .04375 \times 32 = 1.7$ ✓

(d) Scientists are concerned that if the September sea-ice falls between 200,000 and 500,000 square miles, then other climate feedbacks will lead to no more sea-ice in September. According to your equation, in what years is this expected to occur? Set up and solve an inequality to answer the question.

.5 million square miles

200,000 sq mi * (1 mil sq miles / 1,000,000 sq mi) = 200,000 ÷ 1,000,000 = .2 million sq miles

$$.2 \leq 3.1 - .04375Y \leq .5$$

$$-3.1 \quad -3.1 \quad -3.1$$

$$-2.9 \leq -.04375Y \leq -2.6$$

$$\frac{}{-.04375} \quad \frac{}{-.04375} \quad \frac{}{-.04375}$$

÷ negative so inequality is reversed

$$66.28\ldots \geqslant Y \geqslant 59.42\ldots$$

⇒ 60–66 years

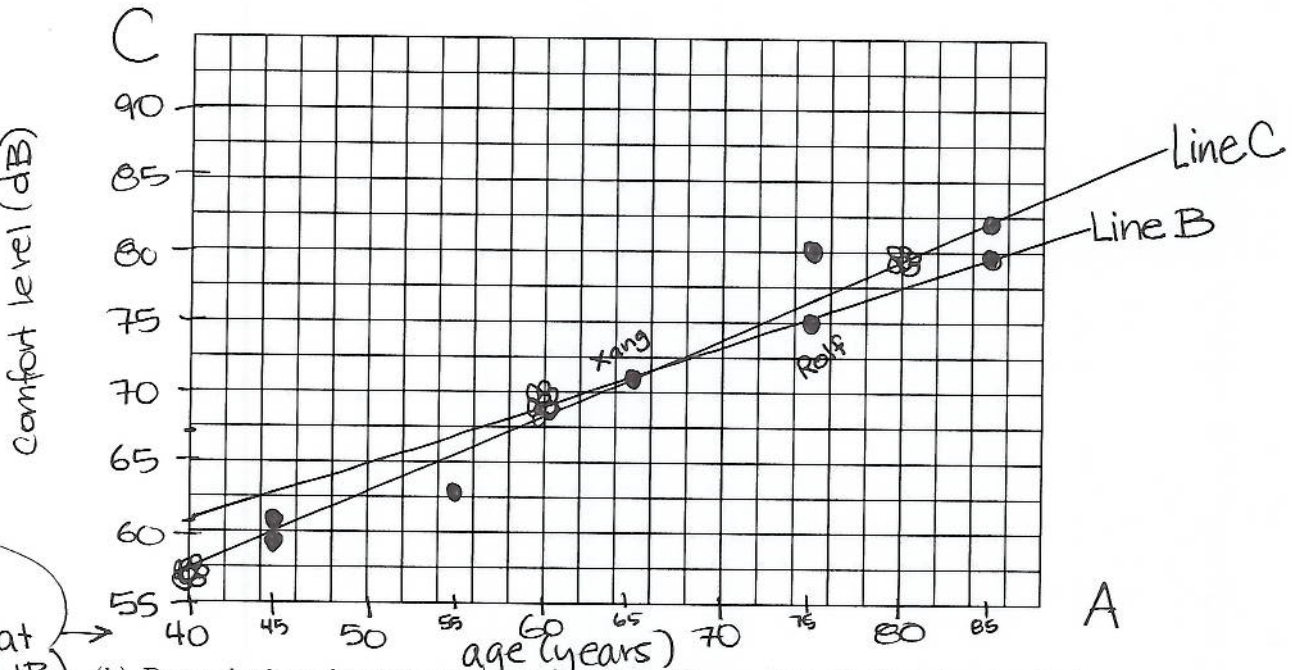
60 + 1980 = 2040
66 + 1980 = 2046

It's expected 2040 − 6

5. As people age, they begin to experience hearing loss. A study was done to determine the "comfort level" of sound for people of different ages, meaning the loudest sound (in decibels) that the person could listen to comfortably. The data are given in the table below.

Name	Akbar	Javier	Walter	Xang	Rolf	Derrick	Iago	Raheem
Age	45	45	55	65	75	75	85	85
Comfort level	58	61	63	71	75	80	82	79

(a) Make a scatterplot showing the data. Scale your axes to start at 40 years and start the level at 55 decibels. Spread out your scale to get a large, detailed graph.



note: instructions suggested start at 40 years old & 55 dB to insure spread out scatter plot.

(b) Draw the line through the points listed for Xang and Rolf. Explain why that line does not fit the data well. *Label this line B.*

Line B does not seem steep enough.

At first it's above points and later it's below points. We wanted more intermixed.

(c) The "best-fitting line" from statistics has equation

$$C = 34.315 + .5556A$$

where A is the person's age (in years) and C is the comfort level (in decibels). Make a table showing the values of C when $A = 40, 60$, and 80. Use those points to add this "best-fitting line" to your graph.

A	40	60	80
C	56.559	67.681	78.763
	≈ 57	≈ 68	≈ 79

34.315 + .5556 × 40 =

These points are shown as ✿ on the graph so we don't confuse them with actual data points