

SOLUTIONS

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3.2 Solving linear inequalities – Practice exercises

1. A truck hauling bags of grass seed weighs 3,900 pounds when it's empty. Each bag of seed it carries weighs 4.2 pounds. The equation for the gross weight W pounds is

$$W = 3,900 + 4.2B$$

for B bags of grass seed.

Story also appears in 2.1 #1 and 3.1 #1

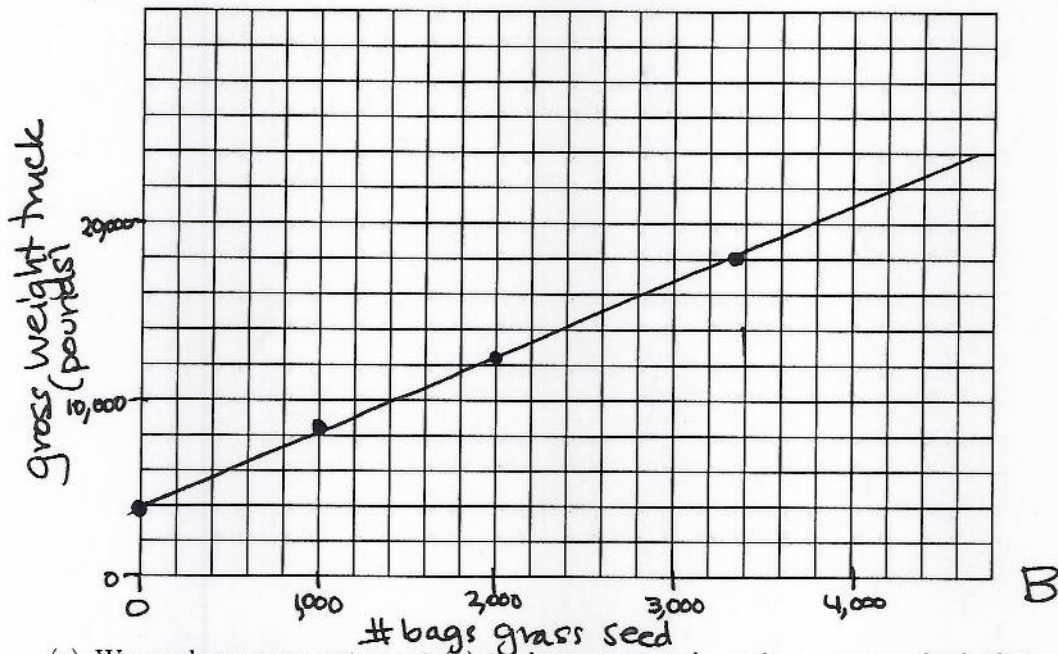
- (a) The state highways have a 18,000 pound gross weight limit. How many bags of grass seed can the truck can haul? Set up and solve an inequality.

$$\begin{array}{r} 3,900 + 4.2B \leq 18,000 \\ - 3,900 \quad - 3,900 \\ \hline 4.2B \leq 14,100 \\ \hline \frac{4.2}{4.2} \quad \frac{14,100}{4.2} \\ B \leq 3,357.14... \end{array}$$

The truck can haul up to 3,357 bags

- (b) Record your answer to part (a) in the table and graph the function.

B	0	1,000	2,000	3,357
W	3,900	8,100	12,300	18,000



- (c) We used our answer to part (a) to draw our graph, so how can we check that answer to make sense? Hint: what shape should the graph be?

It should be a line.

(and it is ✓)

18,000 pounds is the largest weight allowed so $W \leq 18,000$

be sure you used \leq here

3,358 bags would put the truck over 18,000 pounds

2. The altitude, A feet above ground, of an airplane M minutes after it begins its descent is given by the equation

$$A = 32,000 - 1,200M$$

★ Answer each question by evaluating; setting up and solving an equation; or setting up and solving an inequality, whichever is most appropriate.

- (a) At what altitude does the plane begin its descent?

$$\leftarrow M=0$$

$$A = 32,000 - 1,200 \times 0 = \boxed{32,000 \text{ feet}}$$

of course!
that's the
intercept
from the
equation

- (b) How fast is the airplane descending?

1200 feet/min

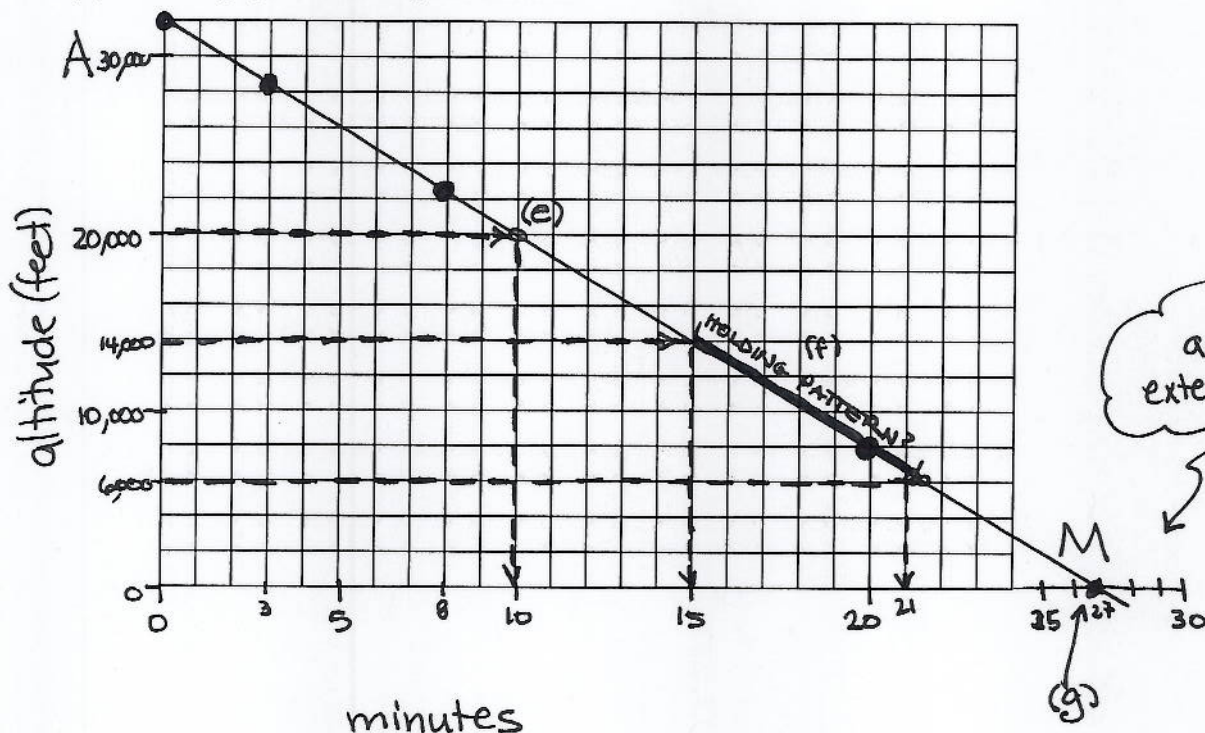
that's the slope
from the equation
without - sign

- (c) What is the airplane's altitude 3 minutes into its descent? 8 minutes? 20 minutes? Display your answers in a table.

M	0	3	8	20
A	32,000	28,400	22,400	8,000

$\leftarrow 32,000 - 1,200 \times 3 =$

- (d) Draw a graph illustrating the function.



The problem continues ...

- (e) For how many minutes of its descent is the airplane above 20,000 feet?

20,000 feet is the smallest altitude here so $A \geq 20,000$

could use $>$ instead of \geq

$$\begin{array}{r} 32,000 - 1,200M \geq 20,000 \\ -32,000 \qquad \qquad -32,000 \\ \hline -1,200M \geq -12,000 \\ \hline \quad \quad -1,200 \quad \quad -1,200 \\ \hline \end{array}$$

notice \div negative reversed sign

$$M \leq 10 \text{ minutes}$$

The airplane is above 20,000 for 10 minutes.

- (f) The airplane might be asked to go into a **holding pattern** (that means flying in a circle instead of landing) when it's between 6,000 and 14,000 feet up. When will the plane be in that altitude range?

"between" signals double inequality $6,000 \leq A \leq 14,000$

$$\begin{array}{r} 6,000 \leq 32,000 - 1,200M \leq 14,000 \\ -32,000 \quad -32,000 \qquad \qquad -32,000 \\ \hline -26,000 \leq -1,200M \leq -18,000 \\ \hline \quad \quad -1,200 \quad \quad -1,200 \quad \quad -1,200 \\ \hline \end{array}$$

again switches

$$21.666... \geq M \geq 15$$

The airplane will be at the range after 15 to 21 minutes.

- (g) How long does it take the airplane to land, assuming it's not asked to go into a holding pattern?

When airplane "lands" it's 0 feet above ground so $A = 0$

$$\begin{array}{r} 32,000 - 1,200M = 0 \\ -32,000 \qquad \qquad -32,000 \\ \hline -1,200M = -32,000 \\ \hline \quad \quad -1,200 \quad \quad -1,200 \\ \hline \end{array}$$

check graph!

$$M = 26.67... \approx 27 \text{ minutes to land}$$

★ Make sure you set-up and solved in each case. See instructions marked ★ earlier.

3. Anthony and Christina are trying to decide where to hold their wedding reception. For each possible site, write an equation using T for the total cost of their wedding reception (in dollars) and G for the number of guests. Then set up and solve an inequality to calculate the number of guests Tony and Tina can afford on their \$8,000 budget.

largest allowed
 $T \leq 8000$

- (a) The Metropolitan Club costs \$1,300 for the space and \$92 per person.

Story also appears in 1.2 #3 and 1.3 #2

equation: $T = 1300 + 92G$

number of guests: $1300 + 92G \leq 8000$
 -1300
 $92G \leq 6700$
 $\frac{92G}{92} \leq \frac{6700}{92}$
 $G \leq 72.82...$
 round down to stay within budget
72 guests

- (b) Black Elk Park charges \$500 to rent the pavilion and the family can bring in picnic food for \$65 per person.

equation: $T = 500 + 65G$

number of guests: $500 + 65G \leq 8000$
 -500
 $65G \leq 7500$
 $\frac{65G}{65} \leq \frac{7500}{65}$
 $G \leq 115.38...$
115 guests

- (c) The Dabbling Duck Inn charges \$1,400 for the space and \$80 per person for their local specialties.

equation: $T = 1400 + 80G$

number of guests: $1400 + 80G \leq 8000$
 -1400
 $80G \leq 6600$
 $\frac{80G}{80} \leq \frac{6600}{80}$
 $G \leq 82.5$
82 guests

- (d) Pranzo Ristorante has only a \$300 room rental fee but averages \$145 per person, including wine.

equation: $T = 300 + 145G$

number of guests: $300 + 145G \leq 8000$
 -300
 $145G \leq 7700$
 $\frac{145G}{145} \leq \frac{7700}{145}$
 $G \leq 53.10...$
53 guests

4. One variety of blueberry plant yields an average of 130 blueberries per season but there is quite a bit of variability from plant to plant. One measure of this variability is the standard deviation, which is approximated at 16.4 berries. Given a plant yielding B blueberries, we can calculate how usual or unusual that is by computing its (standard) z -score using the equation

$$Z = \frac{B - 130}{16.4}$$

For example, a plant yielding $B = 130$ blueberries has z -score of 0. A plant yielding $B = 138$ blueberries has z -score of

$$Z = \frac{138 - 130}{16.4} = (138 - 130) \div 16.4 = .04878... \approx .48$$

Did you follow these instructions?

Answer each question by evaluating; setting up and solving an equation; or setting up and solving an inequality, whichever is appropriate.

- (a) Calculate the z -score of a plant yielding 140 blueberries.

$$z = \frac{140 - 130}{16.4} = (140 - 130) \div 16.4 = .0656... \approx .06$$

Note: the near 0 z -score says the yield is nearly average

16.4 on bottom of fraction means $\div 16.4$ so $\times 16.4$ will cancel!

- (b) If the z -score for a plant is $-.7$, what is the corresponding yield?

Hint: the negative z -score tells us the answer is below average.

$$\frac{B - 130}{16.4} = -.7 \times 16.4$$

$$B - 130 = -11.48$$

$$B = 118.52 \approx 119 \text{ blueberries}$$

Check: below average ✓

- (c) A plant with z -score above 1.96 is considered plentiful. What yields of blueberries would be considered plentiful?

$$\frac{B - 130}{16.4} \geq 1.96 \times 16.4$$

$$B - 130 \geq 32.144$$

$$B \geq 162.144...$$

163 blueberries (or more) is considered plentiful

rounded up to be sure ≥ 1.96

- (d) A plant with z -score between -1 and $+1$ is considered ordinary. What yields of blueberries would be considered ordinary?

$$-1 \leq \frac{B - 130}{16.4} \leq 1 \times 16.4$$

$$-16.4 \leq B - 130 \leq 16.4$$

$$113.6 \leq B \leq 146.4$$

between 114 and 146 blueberries is considered ordinary.