SOLUTIONS



## 4.4 Slopes – Practice exercises

| W | C |
|---|---|
| 70 | 51.25 |
| 125 | 83.70 |

1. For his Oscars party, Harland had 70 chicken wings delivered for \$51.25. For his Super Bowl bash, Harland had 125 chicken wings delivered for \$83.70. In each case, the total cost includes the cost per wing and the fixed delivery charge.

   (a) Find the slope, including units, and explain what it means in the story.

   $\text{slope} = \frac{\text{diff dep}}{\text{diff indep}} = \frac{\$83.70 - \$51.25}{125 - 70 \text{ wings}} = (83.70 - 51.25) \div (125 - 70) = \$.59/\text{wing}$

   It costs 59¢ per wing.

   (b) Find the intercept, including units, and explain what it means in the story.

   $\text{intercept} = \text{dep} - \text{slope} * \text{indep} = 51.25 - .59 \times 70 = \$9.95$

   The delivery charge is \$9.95

   could do $83.70 - .59 \times 129 = 9.95$ instead

   (c) Name the variables and write an equation for the function.

   $W$ = # wings ~ indep

   $C$ = total cost (\$) ~ dep

   $C = 9.95 + .59W$

   check: $9.95 + .59 \times 70 = 51.25$, $9.95 + .59 \times 125 = 83.70$ ☺

   (d) How many wings could Harland order for \$100? Solve your equation.

   $9.95 + .59W = 100$

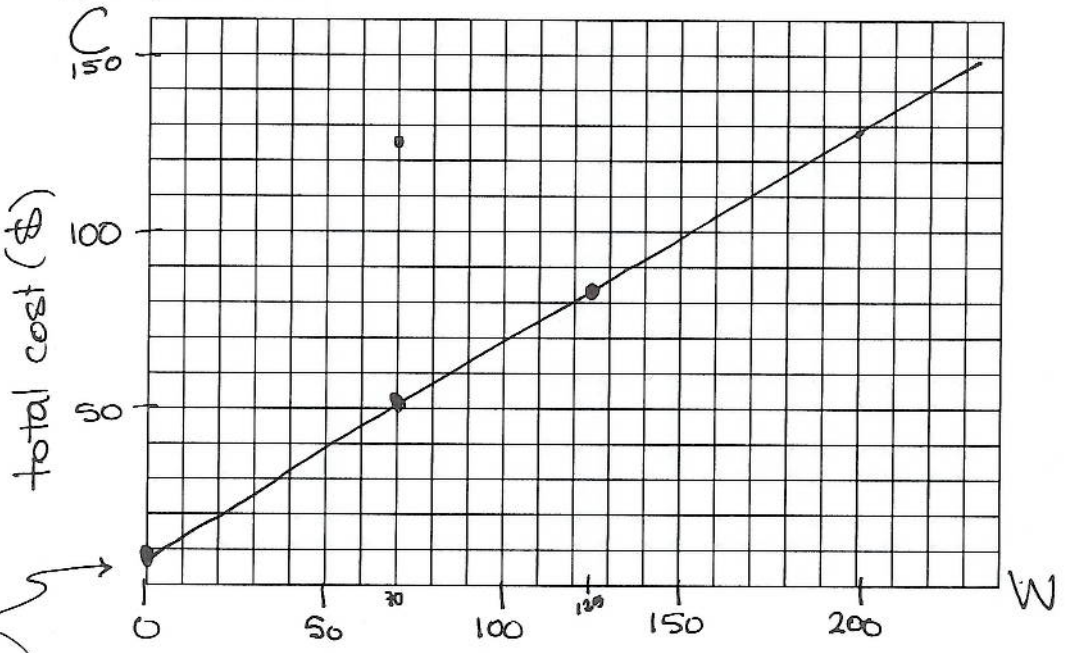
   $-9.95 \quad -9.95$

   $\frac{.59W}{.59} = \frac{90.05}{.59}$

   $W = 152.6\ldots$

   ⇒ Harland can order up to 152 wings for \$100

   (e) Graph and check.




Included intercept!

2. Jana is making belts out of leather strips and a metal clasp. An short belt (as shown) is 24.5 inches long and includes 7 leather strips. An long belt (not shown) is 37.3 inches long and includes 11 leather strips. Each belt includes one metal clasp that is part of the total length. All belts use the same length clasp.

| N | 7 | 11 |
|---|---|---|
| L | 24.5 | 37.3 |

(a) Name the variables, including units.

N = number of leather strips ~indep
L = total length of belt (inches) ~dep

(b) How long is each leather strip?

wanted inches per strip ⇒ SLOPE

$\frac{37.3-24.5 \text{ in}}{11-7 \text{ strips}} = (37.3-24.5) \div (11-7) = 3.2$ inches/strip

Each leather strip is 3.2 inches long.

(c) How long is the metal clasp?

wanted fixed length of clasp ⇒ INTERCEPT

$24.5 - 3.2 \times 7 = 2.1$ inches

The metal clasp is 2.1 inches long.

(d) Write an equation relating the variables.

$L = 2.1 + 3.2N$

Check:
$2.1 + 3.2 \times 7 = 24.5$ ✓
$2.1 + 3.2 \times 11 = 37.3$ ✓

(e) Solve your equation to find the number of leather strips in a extra long belt that's 43.7 inches long.

When problem says "solve" need to show this method

$2.1 + 3.2N = 43.7$
$-2.1 \quad\quad -2.1$

$\frac{3.2N}{3.2} = \frac{41.6}{3.2}$

$N = 13$ leather strips

An extra long belt would require 13 leather strips.

3. The local ski resort is trying to set the price for season passes. They know from past experience that they will sell around 14,000 passes if the season ticket price is \$380. If the price is \$400, they will sell fewer, perhaps only 11,000 passes. You can assume this decrease in demand is linear.

| T | D |
|---|---|
| 380 | 14,000 |
| 400 | 11,000 |

(a) How many fewer people purchase season passes for every dollar increase in the price?

want drop in passes per \$1 increase price ⟹ Slope

$$\frac{11{,}000 - 14{,}000 \text{ passes}}{\$400 - \$380} = (11000-14000)\div(400-380) = -150 \text{ passes/\$}$$

For each dollar increase, 150 fewer people buy the ski pass.

(b) Find the intercept. Explain why this number does not make sense in the problem.

intercept = dep − slope * indep = 14,000 − (−150) * 380
= 14000 − (−)150 × 380 = 71,000 passes

The intercept corresponds to \$0 which is probably not realistic since they want to make money.

(c) Write an equation for the function, using $T$ for the ticket price, in dollars, and $D$ for the demand (number of tickets sold).

$D = 71{,}000 - 150T$

check: 71000 − 150 × 380 = 14000
71000 − 150 × 400 = 11000

Even though the intercept is unrealistic, it makes sense in the equation.

(d) How many season passes will they sell if the price is reduced to \$355?

T = 355

$D = 71000 - 150 \times 355 = 17{,}750$ passes

If the price is \$355, they should sell 17,750 passes.

(e) The amount of **revenue** (money they take in) depends both on the ticket price and the number of tickets sold. The equation is $R = TD$, where $R$ is the revenue, in dollars. Calculate the revenue when ticket prices are \$355, \$380, and \$400. *That means multiply the ticket price $T$ times the number of tickets sold $D$ in each case listed.* Of these three prices, which yields the most revenue?

| T | D | R = TD |
|---|---|---|
| 355 | 17,750 | $355 \times 17{,}750 = \$6{,}301{,}250$ |
| 380 | 14,000 | $380 \times 14{,}000 = \$5{,}320{,}000$ |
| 400 | 11,000 | $400 \times 11{,}000 = \$4{,}400{,}000$ |

The price of \$355 generates the most revenue of these three prices.

| T | P |
|---|---|
| now 0 | 15 |
| 10 | 50 |

4. Boy, am I out of shape. Right now I can only press about 15 pounds. (**Press** means lift weight off my chest. Literally.) My trainer says I should be able to press 50 pounds by the end of 10 weeks of serious lifting. I plan to increase the weight I press by a fixed amount each week.

   (a) Name the variables and write an equation for my trainer's projection.
   *Hint: you know the intercept.*

P = weight I can press (pounds) ~dep
T = time (weeks) ~indep
intercept = weight I can lift now = 15 pounds
slope = $\frac{50-15 \text{ pounds}}{10-0 \text{ weeks}}$ = (50-15) ÷ 10 = 3.5 pounds/week

$\boxed{P = 15 + 3.5T}$

   (b) Make a table showing my trainer's projection for after 0, 5, 10, 15, and 20 weeks.

| T | 0 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| P | 15 | 32.5 | 50 | 67.5 | 85 |

15 + 3.5 × 5 =

   (c) Years ago I could press 90 pounds. At this rate, when will I be able to press (at least) 90 pounds again? Set up and solve an inequality.

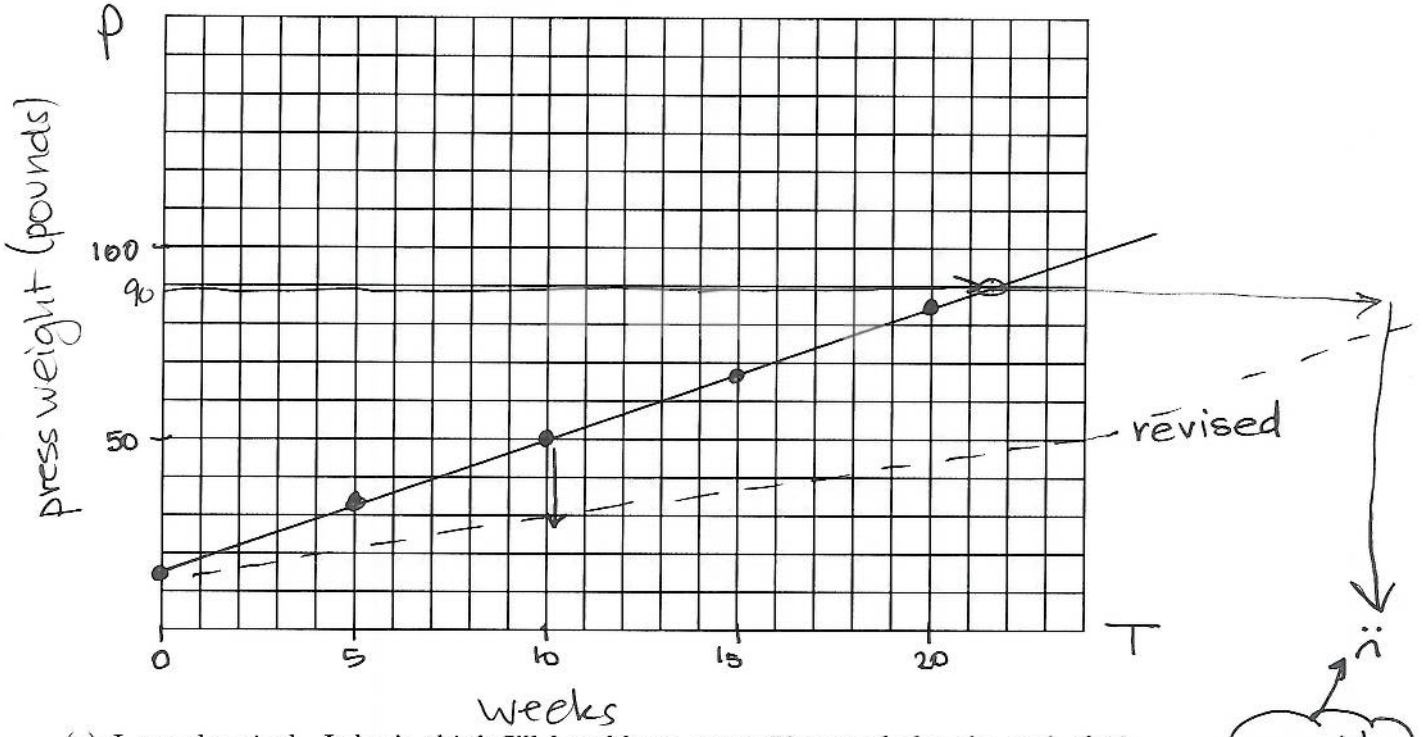
$15 + 3.5T \geq 90$
$-15 \quad -15$
$3.5T \geq 75$
$\frac{3.5T}{3.5} \geq \frac{75}{3.5}$

$T \geq 21.42\ldots \approx 22$ weeks

At this rate, I will be able to press 90 pounds again in 22 weeks.

*The problem continues . . .*

(d) Draw a graph illustrating the function.




(e) I am skeptical. I don't think I'll be able to press 50 pounds by the end of 10 weeks. If I revise my equation, will the new slope be larger or smaller?

*Hint: try sketching in a possible revised line on your graph assuming that after 10 weeks I will press much less than 50 pounds.*

less steep $\Rightarrow$ smaller slope


(f) Will my revised projections mean I'll reach that 90-pound goal sooner or later? Explain. *Hint: extend your graph.*

much later!