Chapter 4

A closer look at linear equations

And topping the algebra charts at #1 for the past two millennia and counting, it's . . . linear equations. Why? First, many everyday and scientific events are naturally linear. Second, many decidedly nonlinear functions can be reasonably approximated by a linear function. Third, sometimes we don't actually have an equation that perfectly fits our information (data) at all, so a reasonably close equation is better than none at all and linear equations often do a good job as a first approximation. Last, and perhaps most importantly, we can solve linear equations. Easily. Exactly. Every time.

We have encountered linear equations often in this text already, so this chapter begins with a review section on everything we have seen: modeling with linear equations; interpreting linear equations, especially slope and intercept; graphing lines; solving linear equations and inequalities; and what it means for a function to be linear in the first place.

Then we dive deeper. For example, the second section of this chapter explores situations that involve comparing or coordinating several linear equations at once in a system. After that, we take a closer look at slope and intercepts to revisit modeling with linear equations in contexts where we are not told the slope and intercept. We pause to look at when a linear function is a direct proportion. We end with a section on the idea of fitting a line to data.

You might have seen most, if not all, of the ideas and techniques in this chapter. Why are we studying linears yet again, then? Because they're #1.

4.1 Modeling with linear equations

A family with one full-time worker earning minimum wage cannot afford the local fair-market rent for a two-bedroom apartment anywhere in the United States. Even families earning above minimum can struggle to rent an apartment for less than 30% of their income. As a result, many people need affordable housing. There are various local, state, and federally funded programs as well as non-profit agencies working to increase availability.

Source: U.S. Department of Housing and Urban Development

In our city there are about 64,100 apartments considered affordable. So the city partnered with local developers to build 7,800 more apartments each year. Our variables are

$$\begin{aligned} A &= \text{affordable housing (apartments)} \sim \text{dep} \\ Y &= \text{time (years from now)} \sim \text{indep} \end{aligned}$$

Assuming things proceed as planned, after 5 years there would be

$$64{,}100 \text{ apts } + 5 \text{ years} * \frac{7{,}800 \text{ apts}}{\text{year}} = 64{,}100 + \underline{5} \times 7{,}800 = 103{,}100 \text{ apartments}$$

Generalizing, we get our equation

$$64{,}100 + Y * 7{,}800 = A$$

which can be rewritten as

$$A = 64{,}100 + 7{,}800Y$$

This equation fits our template for a linear equation

$$\text{dep} \;=\; \text{start} \;+ \text{slope} * \text{indep}$$

Quick recap. A function is **linear** if its graph is a line, and **nonlinear** otherwise. The rate of change measures the steepness of the graph for any function, but a line is the same steepness everywhere, so the rate of change, or **slope** of a line is constant. Our example is linear because the slope of 7,800 apartments per year is constant. Our starting or fixed amount is the **intercept**. In our example it's 64,100 apartments. The dependent variable and the intercept always have the same units – apartments in our example. But

$$\text{units for slope} = \frac{\text{units for dep}}{\text{units for indep}}$$

so, in our example slope is measured in apartments *per year*. These units can help you identify the slope and intercept in a story – so keep a look out.

How many years will it take the city to reach 150,000 apartments at this rate? After ten years, for example, there would still not be enough affordable apartments because

$$A = 64{,}100 + 7{,}800 * 10 = 64{,}100 + 7{,}800 \times \underline{10} = 142{,}100 \text{ apartments}$$

Continuing successive approximation we get

Y	0	5	10	11	12
A	64,100	103,100	142,100	149,900	157,700
vs. 150,000	low	low	low	low	high

This city will reach 150,000 affordable apartments within 12 years.

Of course, we could solve a linear equation instead. We want $A = 150{,}000$. Using our equation $A = 64{,}100 + 7{,}800Y$ we get

$$64{,}100 + 7{,}800Y = 150{,}000$$

However, since we want *at least* 150,000 affordable apartments, an inequality is even better. Let's practice that.

$$64{,}100 + 7{,}800Y \geq 150{,}000$$

Subtract 64,100 from each side to get

$$\begin{array}{lcl} \cancel{64{,}100} + 7{,}800Y & \geq & 150{,}000 \\ -\cancel{64{,}100} & & -64{,}100 \end{array}$$

which simplifies to

$$7{,}800Y \geq 85{,}900$$

Divide each side by 7,800 to get

$$\frac{\cancel{7{,}800}\,Y}{\cancel{7{,}800}} \geq \frac{85{,}900}{7{,}800}$$

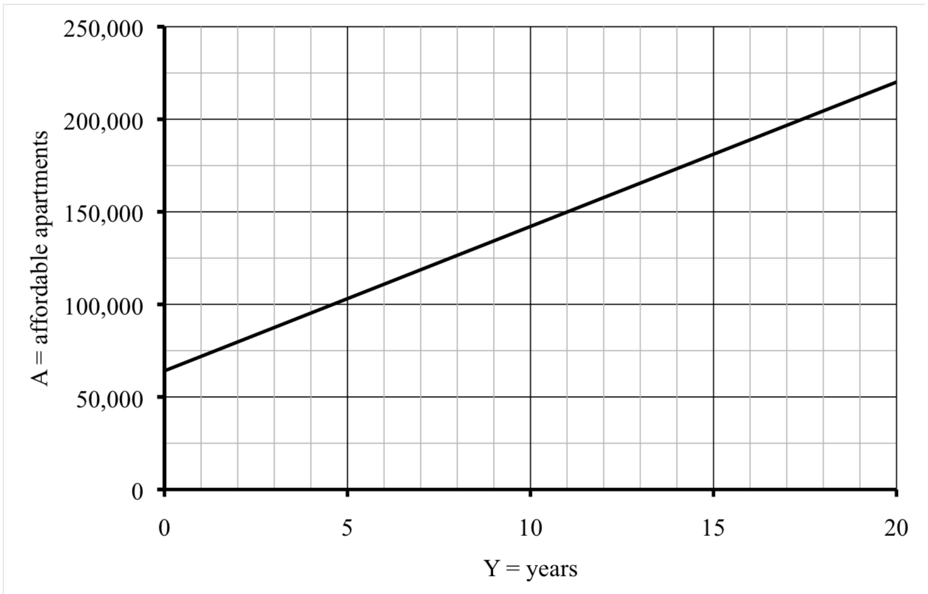
which simplifies to

$$Y \geq \frac{85{,}900}{7{,}800} = 85{,}900 \div 7{,}800 = 11.0128205...$$

To be sure $Y \geq 11.0128205...$ we need to round up to get

$$Y \geq 12$$

Let's confirm our findings on the graph.



As expected, the graph is a line. And we see that the city should reach its goal of 150,000 affordable apartments in 12 years, or slightly before then.

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know . . .

- What makes a function linear?
- What the slope of a linear function means in the story and what it tells us about the graph?
- What the intercept of a linear function means in the story and what it tells us about the graph?
- The template for a linear equation? *Ask your instructor if you need to remember the template or if it will be provided during the exam.*
- How to write a linear equation given the starting amount (intercept) and the rate of change (slope)?
- Where the slope and intercept appear in the template of a linear equation?

- What the graph of a linear function looks like?
- How to solve a linear equation?
- Why the rate of change of a linear function is constant?

If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

Exercises

5. We looked at the city's plan to increase the number of affordable apartments. From a current estimate of 64,100 apartments classified as "affordable," they hoped to build 7,800 per year. At that rate, they can reach a total of 150,000 apartments in 12 years.

   (a) Things change. Revised estimates call for only 6,200 new apartments each year. At that rate, when will the city reach the 150,000 apartments goal? Using the same variables as in this section, set up and solve an equation.

   (b) More bad news. The definition of "affordable" has changed again, so the new count shows only 48,700 apartments on the list. And still only 6,200 new apartments each year. Now when will the city reach the 150,000 apartments goal? Set up and solve an equation.

   (c) In light of the new definition and, consequently, only 48,700 apartments currently on the list, the city has received additional funding to up the number of apartments built each year. They would like to return to their goal of having 150,000 affordable apartments in 12 years. How many apartments do they need to add each year to reach that goal? Figure out the answer however you like, but check that it works.

6. At a local **state university**, the tuition each student pays is based on the number of credit hours that student takes plus fees. The university charges $870 per credit hour plus a $560 fee. The fee is paid once regardless of how many credits are taken.

   (a) Name the variables and write an equation relating them.

   (b) Find the slope and intercept and explain what each means in terms of the story.

   (c) Make a table of values showing the tuition cost for 3 credits, 12 credits, or 16 credits.

   At the local **community college**, the tuition each student pays is based only on the number of credits. The college charges $415 per credit.

(d) Using the same variables as before, write an equation relating them for the community college.

(e) Find the slope and intercept and explain what each means in terms of the story.

(f) Make a table of values showing the tuition cost for 3 credits, 12 credits, or 16 credits.

(g) Graph both functions on the same axes.

(h) What do you notice about the graph that confirms the community college is always cheaper?

7. Can you tell from the table which of these functions is linear? Use the rate of change to help you decide. Remember that numbers may have been rounded.

(a) Ahmed's virburnum shrub. *Story also appears in 4.2 #3*

Week	0	6	10	18
Height (inches)	16.9	19.3	20.9	24.1

(b) Rose gold *Story also appears in 2.3 #2*

Grams of gold added	0	.4	.8	1.4	1.6
Percent gold in alloy	50.0	58.3	64.3	70.6	72.2

(c) Sea-ice (in millions of square miles)

Year	1980	1990	2000	2012
Sea-ice	3.10	2.66	2.23	1.70

(d) Wild rice *Story also appears in 4.5 Exercises*

Hint: rewrite the table in order by temperature first.

Temperature (°F)	39	42	41	35	47	45
Acres	2,300	1,950	1,425	2,015	1,233	1,256

8. The temperature was 40 degrees at noon yesterday but it dropped 3 degrees an hour in the afternoon. Earlier we found the temperature, T°F depends on the time, H hours after noon according to the equation

$$T = 40 - 3H$$

Story also appears in 1.1 and 1.2 Exercises

(a) When does the temperature drop below freezing (32° F)? Set up and solve the relevant inequality. Report your answer as an actual time (to the minute)

(b) When does the temperature drop below zero ($0°$ F)? Same instructions.

9. Shanille is collecting rare books. She inherited 382 books and buys another 3 books every month.

   (a) Make a table showing the number of rare books in Shanille's collection at the start, after 1 month, after 12 months, and after 3 years.

   (b) Name the variables and write an equation relating them.

   (c) Solve your equation to determine when Shanille will reach her goal of 1,000 rare books.

   (d) Graph and check.