SOLUTIONS

Practice Exam 2A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

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1. United States ethanol production has been growing exponentially. In 1990, there were 0.9 billion gallons of ethanol produced. At that time it was estimated that production would increase 5.5% per year.

start → 0.9 billion gallons

Source: Renewable Fuels Association

(a) Name the variables, including units.

E = ethanol produced (billion gallons) ~ dep

Y = time (years since 1990) ~ indep

(b) What is the annual growth factor?

$r = 5.5\% \underset{\div 100\%}{=} .055 \Rightarrow g = 1 + r = 1 + .055 = \boxed{1.055}$

fits template for exponential equation: $\text{dep} = \text{start} * g^{\text{indep}}$

(c) Write an equation that describes the function.

$\boxed{E = .9 * 1.055^{Y}}$

(d) In 2008 actual production of ethanol was 9.0 billion gallons. Is that production level higher or lower than predicted from your equation? Explain.

$Y = 2008 - 1990 = 18$ $E = .9 \times 1.055 \times 18 = 2.35\ldots \approx 2.4$ billion gallons

The actual production of 9.0 bil gal was much $\boxed{\text{higher}}$ than predicted.

(e) When does your equation predict that ethanol production was (or will be) 9.0 billion gallons? Use successive approximation. Display your guesses in a table. Report the actual year.

Y	18	30	50	40	45	42	43	44
E	2.35	4.48	13.08	7.66	10.01	8.52	8.99	9.49
VS 9.0	LOW	LOW	HIGH	LOW	HIGH	LOW	LOW/CLOSE	HIGH

$1990 + 44 = 2034$

$\boxed{\text{Ethanol production was predicted to reach 9.0 bil gal around 2034.}}$

2. An insurance **deductible** is the amount you pay for any claim before the insurance company starts paying. Lee's automobile insurance starts at \$500, but they take off \$10 for each month where he has no accidents or tickets.

   (a) Name the variables.

   D = amount of deductible (\$) ~ dep

   M = time without an accident or ticket (months) ~ indep

   (b) Make a table showing the deductible after 6 months, 1 year, or 3 years without an accident or ticket.

   1 yr = 12 mo

   $3\text{ yr} \times \frac{12\text{ mo}}{\text{yr}} = 3 \times 12 = 36\text{ mo}$

M	6	12	36
D	440	380	140

   $500 - 6 \times 10 =$

   (c) When would the deductible **vanish**? (Meaning, when is it \$0?)

   guess $\boxed{\text{50 months}}$

   or, do succ. approx.

   check: $500 - 50 \times 10 = 0$ ✓

   (d) Write an equation showing how the deductible decreases.

   $\boxed{D = 500 - 10M}$

   can find equation by generalizing examples or recognize − \$10/month constant rate of change so linear ⇒ fits template dep = start + slope × indep

   (e) What is the slope and what does it mean in the story?

   $\boxed{-\$10/\text{month}}$

   It's the rate at which deductible is reduced.

   (f) What is the intercept and what does it mean in the story?

   $\boxed{\$500}$

   It's the deductible before any reductions

3. Our investment club has been tracking the performance of a biofuel company's stock over the past year. Using an econometrics software package, we found the equation

$$V = .00004W^3 + .01W^2 - .9W + 31$$

1 year = 52 weeks

which describes the value of each share of stock $\$V$ as a function of the week W, starting exactly one year ago.

$.00004 \times \underline{26} \wedge 3 + .01 \times \underline{26} \wedge 2 - .9 \times \underline{26} + 31 =$

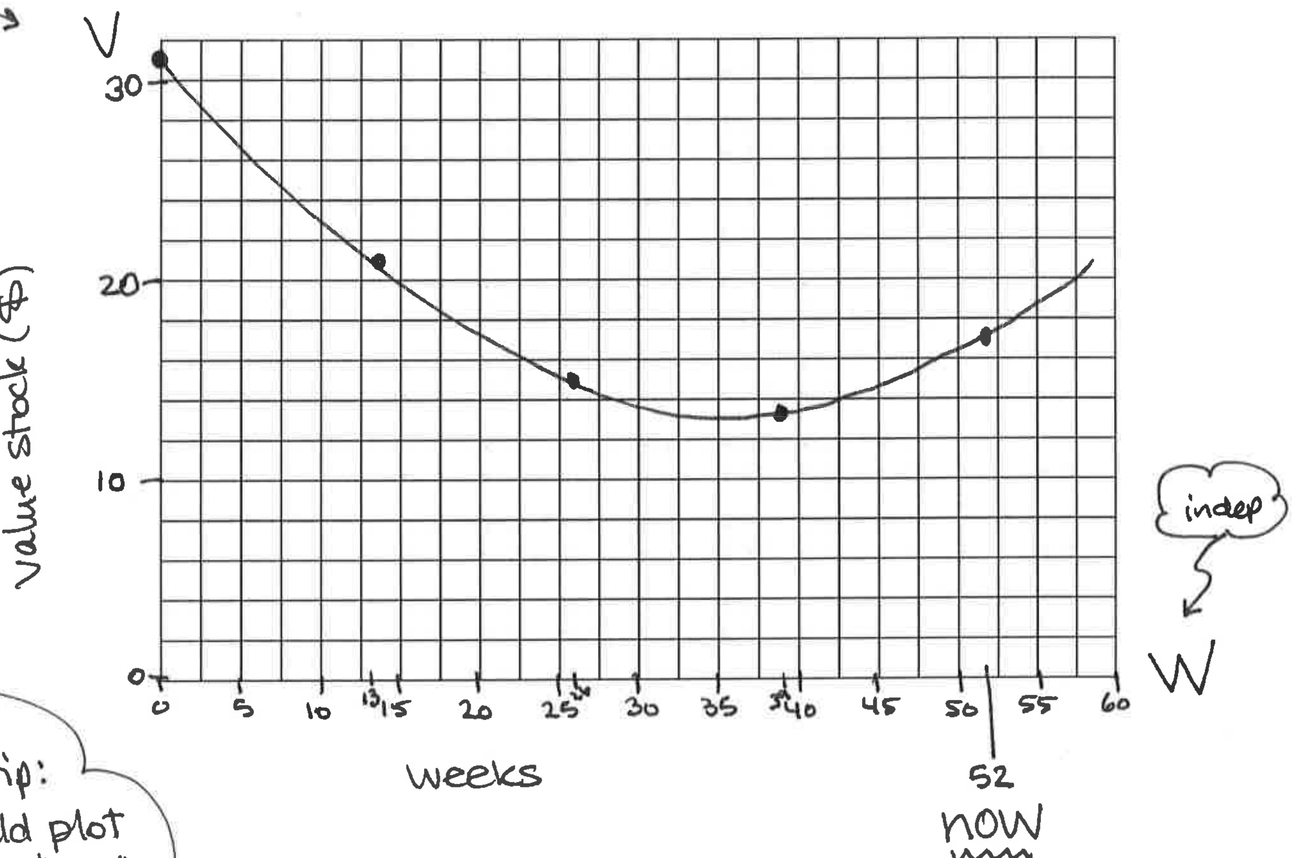
(a) Complete the following table of values.

indep → W; dep → V

W	0	13	26	39	52
V	31.00	21.08	15.06	13.48	16.86

(b) Draw a graph showing how the value changed during the past year.

dep



test-taking tip: at home could plot more points to "see" shape but during test just connect and move on :)

The problem continues ...

(c) According to the table, what is the value of the stock when we began tracking it? What is it worth now?

start → $W=0$, $V=\$31.00$

now → $W=52$, $V=\$16.86$

$\boxed{\text{The stock was worth \$31 a year ago and only \$16.86 now.}}$

(d) We're thinking of buying some stock now, and selling it in 10 weeks. Does the equation say that's a good idea? Explain. *Hint: 10 weeks from now is not $W=10$ because we started counting weeks one year ago.*

$W=\underbrace{52}_{\text{now}}+10=62$

$V=.00004\times \underline{62}\wedge 3+.01\times \underline{62}\wedge 2-.9\times 62+31$

$=23.173\ldots \approx \$23.17$

Stock should be worth \$23.17 in 10 weeks and only costs \$16.86 now. $\boxed{\text{Yes, that would be good.}}$

(e) Looking back over the past year, how low did the value of the stock get? Use successive approximation to estimate to the nearest cent.

guess from graph

between the two smaller values

W	39	35	37	36	36.5	36.8	36.9
V	13.48	13.46	13.416	13.426	13.417	13.415	13.415

$.00004\times \underline{35}\wedge 3+.01\times \underline{35}\wedge 2-.9\times \underline{35}+31=$

$\boxed{\text{The lowest value of the stock was \$13.41}}$

there's a parenthesis and then a negative

4. (a) Cicely wants to buy a new car that costs \$19,400. The dealership offers 6.18% compounded monthly for a 5 year loan. What will Cicely's monthly payment be? Use the LOAN PAYMENT FORMULA.

$a = \$19{,}400 \quad r = 6.18\% \xrightarrow{\div 100\%} .0618 \quad y = 5 \text{ years}$

$$p = \frac{19400 * \frac{.0618}{12}}{1-\left(1+\frac{.0618}{12}\right)^{-12\cdot 5}} = (19400 \times .0618 \div 12) \div (1-(1+.0618\div 12)\wedge(-12\times 5))$$

$$= 376.6822\ldots \approx \boxed{\$376.68}$$

Cicely's car payment will be \$376.68/month.

(b) What is the equivalent APR Cicely is paying? Use the EQUIVALENT APR FORMULA. *Don't forget to report the percentage.*

$$APR = \left(1+\frac{.0618}{12}\right)^{12} - 1 = (1+.0618\div 12)\wedge 12 - 1$$

$$= .063580\ldots \approx .0636 \xrightarrow{\times 100\%} \boxed{6.36\%}$$

She's paying the equivalent of 6.36% APR.

(c) Cicely is working on her monthly budget. She has only \$230 per month left after those car payments. If she puts that money into a bank account each month earning 2.91% interest compounded monthly how much will she have after 5 years when the car is paid off? Use the FUTURE VALUE ANNUITY FORMULA.

$p = \$230 \quad r = 2.91\% \xrightarrow{\div 100\%} .0291 \quad y = 5 \text{ years}$

$$a = 230\,\frac{\left(1+\frac{.0291}{12}\right)^{5\times 12} - 1}{\frac{.0291}{12}} = 230 \times ((1+.0291\div 12)\wedge(5\times 12) - 1) \div (.0291 \div 12) =$$

$$= 14{,}835.145\ldots \approx \boxed{\$14{,}835.15}$$

Cicely will have saved about \$14,835.15.

(d) In 2011, Cicely was cleaning out the basement and found some savings bonds with face value \$1,600 that matured in 1972 and have been earning 3% interest compounded monthly ever since. What were they worth? Use the COMPOUND INTEREST FORMULA.

$p = \$1600 \quad y = \begin{array}{r} 2011 \\ -1972 \\ \hline 39 \text{ years} \end{array} \quad r = 3\% \xrightarrow{\div 100\%} .03$

$$a = 1600\left(1+\frac{.03}{12}\right)^{12\times 39} = 1600 \times (1+.03\div 12)\wedge(12\times 39) =$$

$$= 5{,}147.6667\ldots \approx \boxed{\$5{,}147.67}$$

The bonds were worth \$5,147.67 in 2011.