solutions

4.3 Intercepts and direct proportionality – Practice exercises

1. Each of the two stories, below, involve how temperature changes over time. It might be confusing to call either variable T, so use H for the time in hours and D for the temperature in degrees (°F). In each case, time should be measured from the start of the story.

   (a) It was really cold at 8:30 this morning when Raina arrived at the office. Luckily the heating system warms things up very quickly, 4°F per hour. By 11:00 a.m. it was a very comfortable 72° F.

   i. Figure out what the temperature was at 8:30 a.m.

   start °f + 4°f/hr * 3.5 hrs = 72°f

   14°F

   start = 72°F − 14°F = **58°F**

   11:00 a.m. − 8:30 a.m. = 3:30 hours = 30 hr, 30 min = 3½ hours = 3.5 hours

   ii. Write an equation illustrating the function.

   H = time (hours since 8:30 a.m.) ~ indep

   D = temperature (°F) ~ dep

   D = 58 + 4H

   (b) While 72°F is a perfectly good temperature for an office, not so for ballroom dancing. When Raina arrived for her practice at 5:30 that evening, she began to sweat before she even took the floor. Turns out the air conditioner had been running since 4:00 p.m. but it only cools down the room 3°F per hour.

   i. Figure out what the temperature was at 4:00 p.m.

   start °f − 3°F/hr * 1.5 hrs = 72°f

   4.5°F

   start = 72°F + 4.5°F = **76.5°F**

   5:30 pm − 4:00 pm = 1:30 hours = 1 hour, 30 min = 1½ hours = 1.5 hours

   ii. Write an equation illustrating the function.

   H = time (hours since 4:00 pm) ~ indep

   D = temperature (°F) ~ dep

   D = 76.5 − 3H

2. Maryn is very happy. Her interior design business is finally showing a profit. She has logged a total of 471 billable hours at \$35 per hour since she started her business. Accounting for start up costs, her net profit is totals \$2,194.

(a) What were Maryn's start up costs?

start up costs were \$14,256

start + \$35/hr × 471 hrs = \$2,194

16,485

start = 2194 − 35×471 = −\$14,256

(b) Identify the slope and intercept (including their units and sign) and explain what each means in terms of the story.

intercept = −\$14,256 starting loss (due to costs)

slope = \$35/hr rate she charge

(c) Calculate what Maryn's profits will be once she has logged a total of 1,000 hours.

−\$14,256 + \$35/hr × 1000 hrs = −14,256 + 35×1000

start

= \$20,744

(d) Name the variables and write an equation relating them.

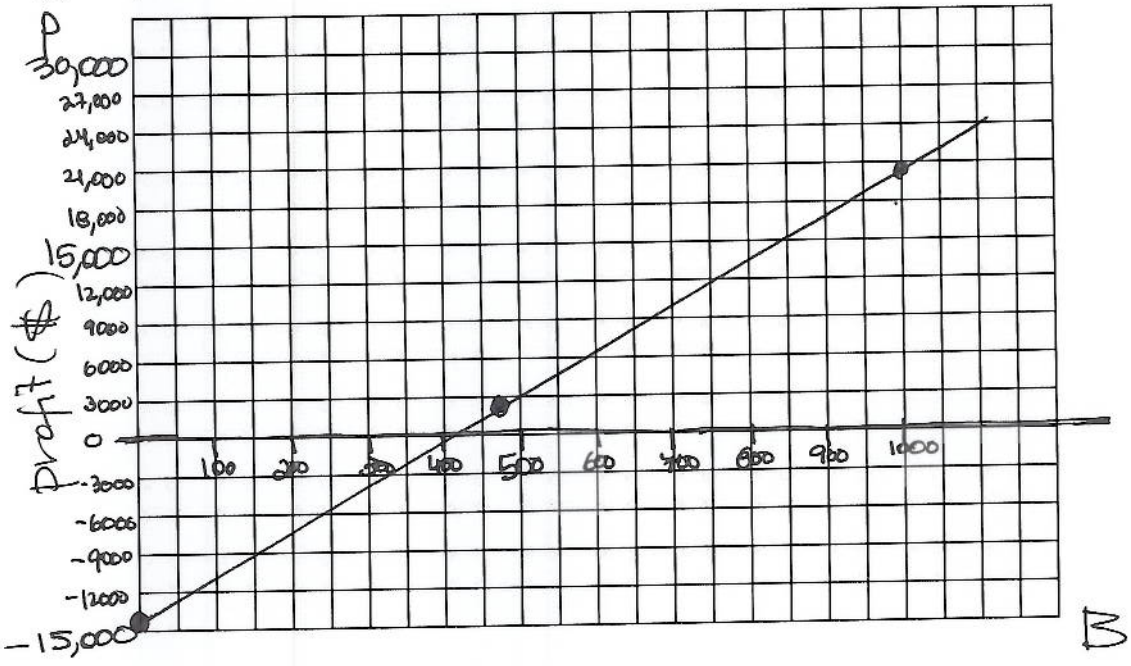
P = Maryn's profit (\$) ~dep

B = how much she works (billable hours) ~indep

P = 35B − 14,256

(e) Graph the function.

B	P
0	−14,256
471	2,194
1000	20,744



billable hours

★ initial weight = end weight ± slope ★ # weeks

3. For each story, find the initial weight of the person and use it to write an equation showing how the person's weight P pounds depends on the time, W weeks.

   (a) Jerome has gained weight since he took his power training to the next level ten weeks ago, at the rate of around 1 pound a week. He now weighs 198 pounds.

initial weight $= 198 - \underbrace{1 \times 10}_{10} =$ 188 pounds

   (b) Vanessa's doctor put her on a sensible diet and exercise plan to get her back to a healthy weight. She will need to lose an average of 1.25 pounds a week to reach her goal weight of 148 pounds in a year. Use 1 year = 52 weeks.

initial weight $= 148 + \underbrace{1.25 \times 52}_{65} =$ 213 pounds

   (c) After the past 6 weeks of terrible migrane headaches, Carlos is down to 158 pounds. He's lost 4 pounds a week.

initial weight $= 158 + \underbrace{4 \times 6}_{24} =$ 182 pounds

   (d) Since she's been pregnant, Zoe has gained the recommended 1/2 pound per week. Now 30 weeks pregnant and 168 pounds, she wonders if she'll ever see her feet again.

initial weight $= 168 - \underbrace{\frac{1}{2} \times 30}_{15} =$ 153 pounds

4. Each story describes a situation that we're assuming is linear. Decide whether it is directly proportional or not. If not, identify what the intercept would mean in the story.

(a) The price of a kiwis depends on how many kiwis you buy.

$/kiwi no fixed cost

directly proportional

(b) The price of a bag of tortillas depends on how many tortillas are in the bag.

10 tortillas $2.95 ? } probably some
30 tortillas $4.95 ? } economy of scale.

Not intercept = per bag cost

(c) The time it takes to vacuum a rug depends on the area of the rug.

Not intercept = time to get out & put away vacuum

save time by doing several rugs at once.

(d) The time it takes to wash dishes depends on how many dirty dishes there are.

directly proportional

No real start up time, just show up & wash

(e) The amount of laundry detergent I have left depends on how many loads of laundry I did.

64oz − 2oz * # loads Not

↑

intercept = amount of laundry detergent I started with