Solutions

5.1 Modeling with exponential equations – Practice exercises

1. The population of Buenos Aires, Argentina in 1950 was estimated at 5.0 million and expected to grow at 1.8% each year. Source: Mongabay

   start

   r

   (a) Name the variables.

   P = population of Buenos Aires (million) ~ dep

   Y = year (years since 1950) ~ indep

   (b) What is the annual growth factor?

   $r = 1.8\% \underset{\div 100}{=} .018 \quad g = 1 + r = 1 + .018 = \boxed{1.018}$

   $\boxed{\text{fits exponential template: } dep = start * g^{indep}}$

   (c) Write an equation estimating the population of Buenos Aires over time.

   $P = 5.0 * 1.018^{Y}$

   (d) Make a table of values showing the estimated population of Buenos Aires every 20^{th} year from 1950 to 2030.

year	Y	P	rate of change
1950	0	5.0	
			$\frac{7.14 - 5.0}{20-0} \approx .107$ millions people per year
1970	20	7.14	
			$\frac{10.21 - 7.14}{40-20} \approx .154$
1990	40	10.21	
			$\frac{14.58 - 10.21}{60-40} \approx .216$
2010	60	14.58	
			$\frac{20.83 - 14.58}{80-60} \approx .313$
2030	80	20.83	

   5.0×1.018^80 =

   (e) By how many people has the population been increasing during each 20 year period? Add these numbers to your table. As expected, these numbers change because the rate of change is not constant.

   (f) The actual population of Buenos Aires in the year 2000 was around 12.6 million and by 2010 it was around 15.2 million. How does that compare to the estimates?

   $Y = 2000 - 1950 = 50 \quad P = 5.0 * 1.018 \wedge 50 \approx 12.2$ vs. 12.6 actual

   2010 $P \approx 14.58$ vs 15.2 actual.

   $\boxed{\text{The actual numbers were higher than estimated.}}$

2. A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 16% per day. Earlier we found the equation

$$N = 8 * 1.16^D$$

where D is the number of days (since the first diagnosis) and N is the total number of students who had the flu. *Story also appears in 2.2 #3 and 5.5*

(a) Use successive approximations to estimate when the number of infected students reaches 100. Display your guesses in a table.

8×1.16^10 =

D	0	10	~~50~~	20	15	17	18
N	8	35.29	~~13,365~~	155.68	74.12	99.74	115.70
VS 100	LOW	LOW	NO WAY!	HIGH	LOW	ALMOST	OVER.

$\Rightarrow \boxed{\approx 18 \text{ days}}$

(b) Use the LOG DIVIDES FORMULA to solve your equation.

$$\frac{\cancel{8} * 1.16^D}{\cancel{8}} = \frac{100}{8}$$

$$1.16^D = 12.5$$

By the Log-Divides Formula $g^T = V$ has solution $T = \frac{\log(V)}{\log(g)}$

$$D = \frac{\log(12.5)}{\log(1.16)} = \log(12.5) \div \log(1.16) = 17.01\ldots \boxed{\approx 18 \text{ days}}$$

(c) There are 1,094 students currently living in the dorms. Suppose ultimately 250 students catch the flu. According to your equation, when would that happen? Show how to solve your equation.

$$\frac{\cancel{8} * 1.16^D}{\cancel{8}} = \frac{250}{8}$$

$$1.16^D = 31.25$$

By the Log-Divides Formula

$$D = \frac{\log(31.25)}{\log(1.16)} = \log(31.25) \div \log(1.16) = 23.19\ldots \boxed{\approx 24 \text{ days}}$$

(d) It is not realistic to expect that everyone living in the dorms will catch the flu, but what does the equation say? Set up and solve an equation to find when all 1,094 students would have the flu. (Again, this is not realistic.)

$$\frac{\cancel{8} * 1.16^D}{\cancel{8}} = \frac{1094}{8}$$

$$1.16^D = 136.75$$

By the Log-Divides Formula

$$D = \frac{\log(136.75)}{\log(1.16)} = \log(136.75) \div \log(1.16) = 33.13\ldots \boxed{\approx 34 \text{ days}}$$

3. Bunnies, bunnies, everywhere. Earlier we found the equation

$$B = 1{,}800 * 1.13^Y$$

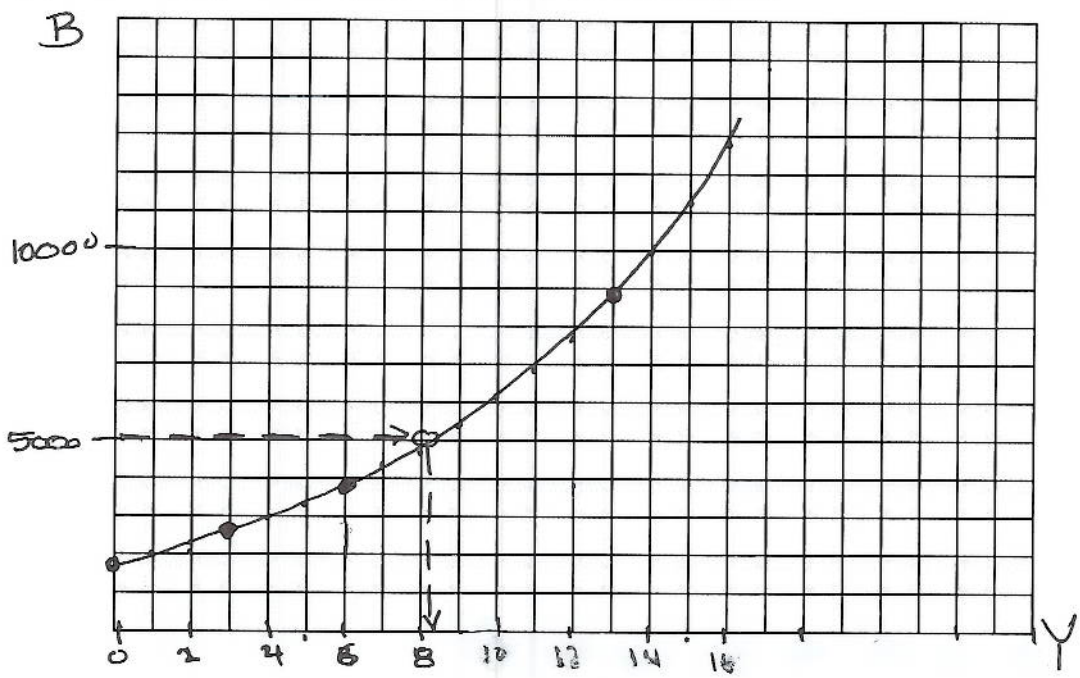
where B is the number of bunnies and Y is the years since 2007.

Story also appears in 2.2 #2

(a) Make a table showing the number of bunnies in 2007, 2010, 2013, and 2020.

year	2007	2010	2013	2020
Y	0	3	6	13
B	1800	2,597	3,748	8,816

(b) Draw a graph showing how the bunny population grew.



(c) When will the population pass 5,000 bunnies? Guess from the graph. Then refine your answer using successive approximation.

graph $\Rightarrow \approx 8$ years $=$ 2007 + 8 = $\boxed{2015}$

Y	8	9
B	4785	5407 ✓

(d) Show how to solve your equation to get the answer.

$$\frac{1800 * 1.13^Y}{1800} = \frac{5000}{1800}$$

$$1.13^Y = 2.777\ldots$$

By the Log-Divides Formula

$$Y = \frac{\log(2.777\ldots)}{\log(1.13)}$$

$$= \log(2.777\ldots) \div \log(1.13) =$$

$$= 8.359\ldots \boxed{\approx 8\tfrac{1}{2} \text{ years}}$$

start

4. Carbon dioxide is a greenhouse gas in our atmosphere. Increasing carbon dioxide concentrations are related to global climate change. In 1980, the carbon dioxide concentration was 338 ppm (parts per million). At that time it was assumed that carbon dioxide concentrations would increase .42% per year. r

Source: Earth Systems Research Laboratory, NOAA

(a) Name the variables including units.

C = carbon dioxide concentrations (ppm) ~ dep

Y = year (years since 1980) ~ indep

(b) Assuming the growth is exponential as predicted, write an equation that describes the increase in carbon dioxide concentrations.

fits template exponential. $\text{dep} = \text{start} * g^{\text{indep}}$

$r = .42\% \xrightarrow{\div 100} = .0042 \Rightarrow g = 1 + r = 1 + .0042 = 1.0042$

$\boxed{C = 338 * 1.0042^{Y}}$

(c) The carbon dioxide concentration in 2008 was 385 ppm. Is that count higher or lower than predicted from your equation? Explain.

$Y = 2008 - 1980 = 28$

$C = 338 * 1.0042 \wedge 28 = 380.086\ldots$

$\boxed{\approx 380\text{ppm}}$ ← predicted

actual → 385ppm larger

(d) Does that mean that carbon dioxide increased at a higher or lower rate than .42%? Explain.

Must be increasing at a higher rate than expected.