SOLUTIONS

5.4 Linear vs. exponential models – Practice exercises

LINEAR EQUATION TEMPLATE: dep = start + slope * indep

RATE OF CHANGE/SLOPE (OF LINEAR) FORMULA:

$$\text{rate of change} = \frac{\text{1st dep - 2nd dep}}{\text{1st indep - 2nd indep}}$$

EXPONENTIAL EQUATION TEMPLATE: dep = start * growth factor$^{\text{indep}}$

GROWTH FACTOR FORMULA:

If a quantity is growing (decaying) exponentially, then the growth (decay) factor is

$$g = \sqrt[t]{\frac{a}{s}}$$

where s is the starting amount and a is the amount after t time periods.

PERCENT CHANGE FORMULA:

- If a quantity changes by a percentage corresponding to growth rate r, then the growth factor is

$$g = 1 + r$$

- If the growth factor is g, then the growth rate is

$$r = g - 1$$

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start = \$35,000

1. My parents bought the house I grew up in for \$35,000 and sold it 40 years later for \$342,000. True story. (It was before the housing bubble burst.)

   First, assume the value of the house increased exponentially.

   Remember: on a graphing calculator try MATH 5

   (a) Calculate the annual growth factor using the GROWTH FACTOR FORMULA.

year	house
0	35,000 = s
t = 40	342,000 = a

$$g = \sqrt[t]{\frac{a}{s}} = \sqrt[40]{\frac{342{,}000}{35{,}000}} = 40 \times\sqrt{\ } (342{,}000 \div 35{,}000) =$$

$$= \boxed{1.058641589}$$

   (b) In this model, by what percentage did the house value increase each year? *Hint: use the* PERCENT CHANGE FORMULA.

$$g \approx 1.059 \Rightarrow r = g - 1 = 1.059 - 1 = .059 \xrightarrow{\times 100\%} \boxed{5.9\%}$$

The house increased 5.9% in value each year, acc. to the exp. model.

   (c) Write an exponential equation showing how the value of the house increased. Don't forget to name the variables, including units. *Hint: use the* EXPONENTIAL EQUATION TEMPLATE.

H = value of the house (\$) ~ dep

Y = year (years since my parents bought the house) ~ indep

$$\boxed{H = 35{,}000 * 1.058641589^{Y}}$$

notice: * then ^

   (d) Check that your equation gives the correct sold value.

$$35000 \times 1.058641589 \wedge \underline{40} = 341{,}999$$

$$\approx \$342{,}000 \checkmark$$

See why we kept so many digits? g=1.059 would give \$346,662 Way off!

The problem continues ...

Next, assume the value of the house increased linearly instead.

or, subtract the other way to avoid negatives

(e) In this model, by what fixed amount did the house value increase each year? *Hint: calculate the slope using the* RATE OF CHANGE/SLOPE (OF LINEAR) FORMULA.

$$\text{slope} = \frac{1^{st}\text{ dep} - 2^{nd}\text{ dep}}{1^{st}\text{ indep} - 2^{nd}\text{ indep}} = \frac{35{,}000 - 342{,}000}{0-40}$$

$$= (35000 - 342000) \div (0-40) =$$

$$= \boxed{\$7{,}675/\text{year}}$$

The house increased \$7,675 in value each year, according to the linear model.

(f) Using the same variables, write a linear equation showing how the value of the house increased. *Hint: use the* LINEAR EQUATION TEMPLATE.

$$\boxed{H = 35{,}000 + 7{,}675Y}$$

notice: + then *

(g) Check that your equation gives the correct sold value.

$$35000 + 7675 \times \underline{40} = \$342{,}000 \checkmark$$

2. The number of manufacturing jobs in the state has been declining for decades. In 1970, there were 1.2 million such jobs in the state but by 2010 there were only .6 million such jobs. Write J for the number of manufacturing jobs (in millions) and Y for the years since 1970.

First, assume the number of jobs decreased linearly.

(a) Calculate the slope.

Y year	J jobs
(1970) 0	1.2
(2010) 40	.6

$$2010 - 1970 = 40$$

$$\text{slope} = \frac{.6-1.2}{2010-1970} = (.6-1.2) \div (2010-1970) =$$

$$= -.015 \text{ million jobs/year}$$

negative because decreasing

(b) Write a linear equation showing how the number of jobs declined.

$$\boxed{J = 1.2 - .015Y}$$

(c) Check that your equation gives the correct value for 2010.

$1.2 - .015 \times 40 = .6$ ✓

Next, assume the number of jobs decreased exponentially instead.

(d) Calculate the growth factor.

note: smaller # on top since function is decreasing.

$$g = \sqrt[t]{\frac{a}{p}} = \sqrt[40]{\frac{.6}{1.2}} = 40 \times\sqrt{\ } (.6 \div 1.2) = .982820598$$

<1 because decreasing

(e) Write an exponential equation showing how the number of jobs declined.

$$\boxed{J = 1.2 * .982820598^{Y}}$$

(f) Check that your equation gives the correct value for 2010.

$1.2 \times .982820598 \wedge \underline{40} = .599999... \approx .6$ ✓

If rounded g with only get approx answer here.

The problem continues ...

Now, compare the models.

(g) Complete the table of values.

$1.2 - .015 \times \underline{20} =$

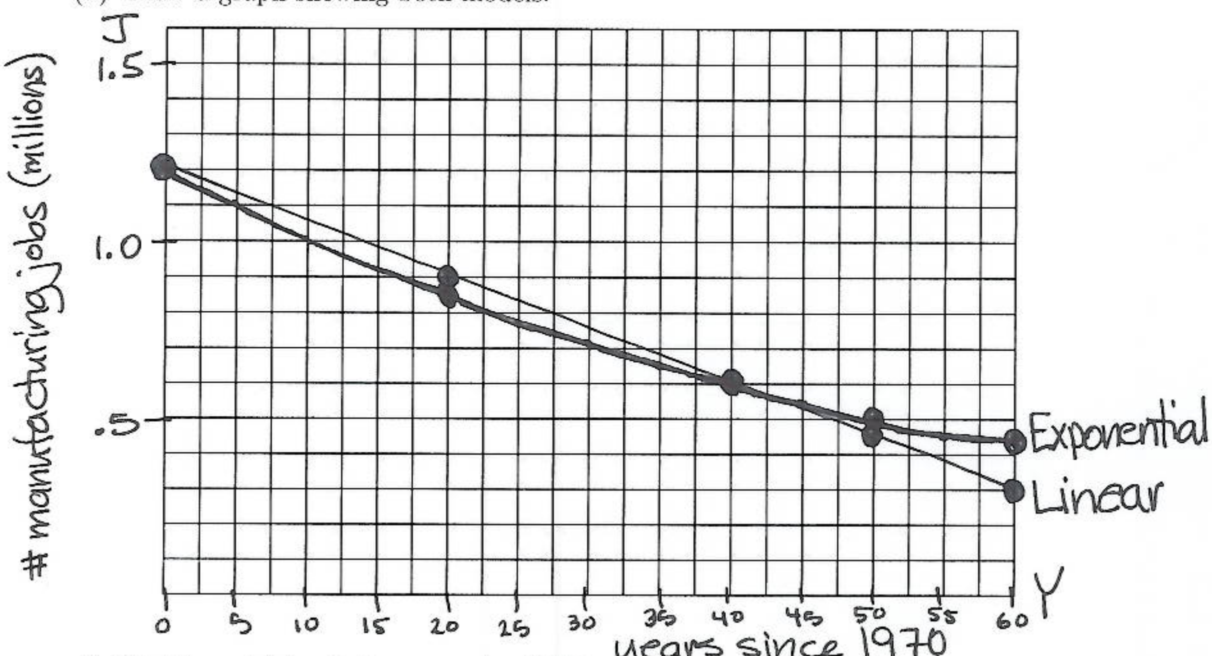
$J = 1.2 - .015Y \Rightarrow$

$J = 1.2 * .982820598^Y$

year	1970	1990	2010	2020	2030
Y	0	20	40	50	60
J (if linear)	1.20	.90	.60	.45	.30
J (if exponential)	1.20	.85	.60	.50	.42

$1.2 \times .982820598 \wedge \underline{20} = .848... \approx .85$

(h) Draw a graph showing both models.



(i) Which model has better news for 2030?

The exponential model predicts more jobs in 2030 than the linear model. That's good news :)

3. In December 2010, a popular mobile app game featuring animated birds launched from slingshots had 50 million downloads. Five months later (May 2011), the game had 200 million downloads. Let D denote the number of downloads of the game (in millions) and M the months since December 2010.

M	D
0	50
5	200

(a) Suppose that the number of downloads have been increasing at a *constant rate each month*. What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$).

$\boxed{\text{Linear}}$ start = 50 million downloads

slope $= \frac{200-50}{5-0} = (200-50) \div (5-0) = 30$ million downloads per month

$$\boxed{D = 50 + 30M}$$

Nov 2011 $\Rightarrow M = 11$

$D = 50 + 30 \times \underline{11} = \boxed{\text{380 million downloads}}$

(b) Suppose that the number of downloads have been increasing at a *fixed percentage each month*. What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$).

$\boxed{\text{Exponential}}$ start = 50 million downloads

$g = \sqrt[t]{\frac{a}{s}} = \sqrt[5]{\frac{200}{50}} = 5 \times\!\sqrt{\ } (200 \div 50) = 1.319507911$

$$\boxed{D = 50 * 1.319507911^{M}}$$

Nov 2011 $\Rightarrow M = 11$

$D = 50 \times 1.319507911 \wedge \underline{11} = 1{,}055.606\ldots$

$\boxed{\approx 1{,}060 \text{ million downloads}}$

$r = 2.5\% = .025$ ÷100%

$\rightarrow g = 1 + r = 1 + .025 = 1.025$

did +$.10 each year

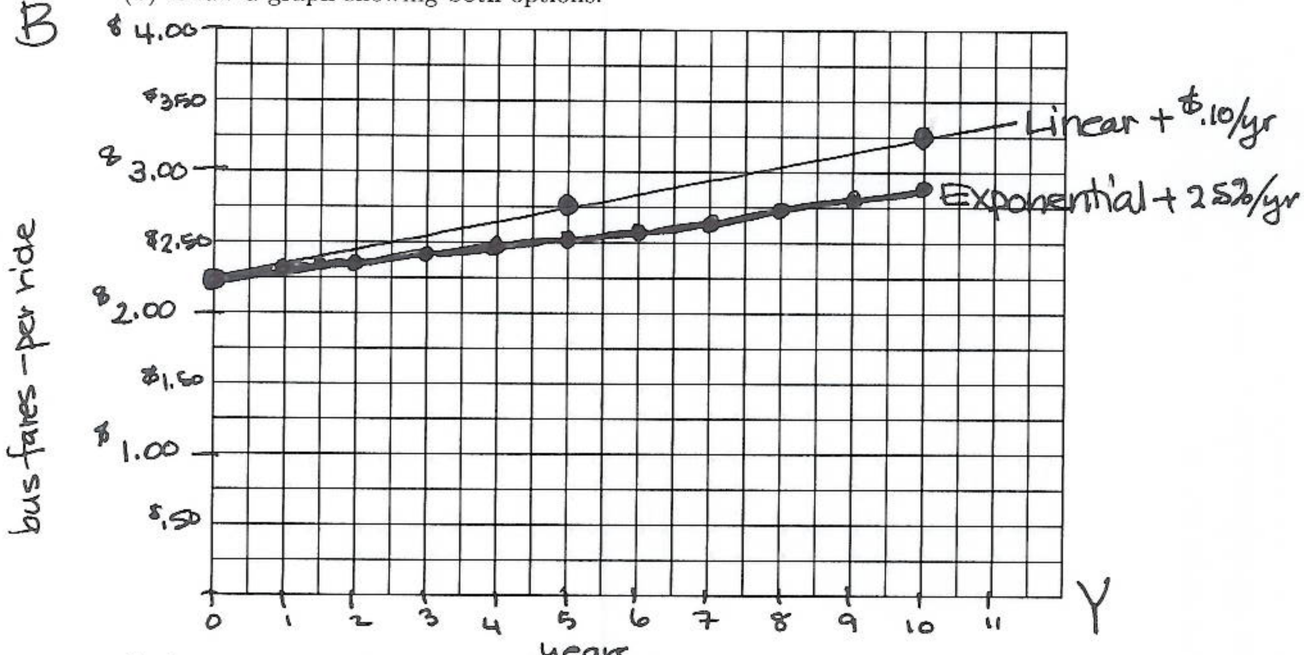
4. Bus fares are up to $2.25 per ride during rush hour. Two different plans of increasing fares are being debated: 10¢ per year or 2.5% per year.

   (a) Make a table comparing these plans over the next **decade** (ten years).

	year	0	1	2	3	4	5	6	7	8	9	10
bus fare	+$.10/yr	2.25	2.35	2.45	2.55	2.65	2.75	2.85	295	3.05	3.15	3.25
	+2.5%/yr	2.25	2.31	2.36	2.42	2.48	2.54	2.61	2.67	2.74	2.81	2.88

×1.025 ×1.025 ×1.025 etc.

   (b) Draw a graph showing both options.



   (c) As a city council representative, you want to support the plan that your constituents prefer. If most of your constituents ride the bus, which plan should you support?

Exponential, because it keeps fares lower.

   (d) If most of your constituents are members of the same union as the bus drivers (who count on solid earnings from the bus company to keep their jobs), then which plan should you support?

Linear, because it raises fares enough to hopefully increase revenue (without making it so expensive that ridership drops)

The problem continues ...

(e) Which type of equation is being suggested in each plan? Write the equations. Don't forget to name the variables, including units.

- 10¢ per year = fixed increase each year ⇒ $\boxed{\text{Linear}}$

  slope = \$.10/yr start = \$2.25/ride

  B = bus fare (\$/ride) ~ dep

  Y = time (years) ~ indep

  $$\boxed{B = 2.25 + .10\,Y}$$

- 2.5% per year = fixed percentage increase each year ⇒ $\boxed{\text{Exponential}}$

  g = 1.025 start = \$2.25/ride

  $$\boxed{B = 2.25 * 1.025^{Y}}$$

from before:
$r = 2.5\% \underset{\div 100\%}{=} .025$
$g = 1 + r = 1 + .025$
$= 1.025$