SOLUTIONS

0.9 Prelude: Logarithms

μL stands for microliters where μ is the Greek letter mu.

Practice exercises

1. The number of bacteria in a dish increases 10-fold each day. Note: 10-fold means $\times 10$. Suppose we had 1 microliter of bacteria at the start of the first day. That means after D days there will be 10^D microliters of bacteria.

   (a) How many bacteria (in microliters) will there be after 1 day? After 2 days? After 3 days?

   $10^1 = 10\wedge 1 = \boxed{10 \mu L}$
   $10^2 = 10\wedge 2 = \boxed{100 \mu L}$
   $10^3 = 10\wedge 3 = \boxed{1{,}000 \mu L}$

   (b) In how many days will the bacteria have reached 1 liter, which is million microliters?

   We want $10^D = 1$ million $= 1{,}000{,}000 = 10^6$

   so $D = \boxed{6 \text{ days}}$

   (c) How can we use logs to find the answer?

   $\log(1{,}000{,}000) = 6$ days

2. The problem continues ...

   (a) How many days(from the start) does it take to reach the 25 millilter capacity of the petri dish, which is 25,000 microliters? Guess and check to find the answer to 1 decimal place. We want $10^D = 25{,}000$

   $10^4 = 10\wedge 4 = 10{,}000$ small
   $10^5 = 10\wedge 5 = 100{,}000$ big
   $10^{4.5} \approx 31{,}622$ big
   $10^{4.3} \approx 19{,}952$ small
   $10^{4.4} \approx 25{,}118$
   $\boxed{\text{Approx } 4.4 \text{ days}}$

   (b) How can we use logs to find the answer?

   $\log(25{,}000) = \boxed{4.3979\ldots \text{ days}} \approx 4.4$ days

   (c) Convert your answer to days & hours format (meaning d days and h hours).

   4 days & ? hours

   $.3979 \text{ days} * 24 \frac{\text{hours}}{\text{day}} = .3979 \times 24 = 9.5$ hours

   $\boxed{4 \text{ days } \& \; 9\tfrac{1}{2} \text{ hours}}$

3. The equation $pH = -\log(H^+)$ tells us the pH of a substance (on a scale from 0 to 14) based on its molar hydrogen ion concentration H^+. Don't let the notation here scare you: pH is a single quantity and H^+ has nothing to do with exponents or adding.

   For example, lemon juice has $H^+ = .0025$ and so the pH of lemon juice is

$$-\log(.0025) = (\text{-})\log(.0025) = 2.6020599913 \approx 2.6$$

   (a) Coca-cola has $H^+ = .000\ 398$. Find the pH of orange juice. Note: the funny spaces are to help you read the number.

   $-\log(.000\ 398) = 3.4001... \approx \boxed{3.4}$

   (b) Hair shampoo has $H^+ = .000\ 003\ 162$ Find the pH of hair shampoo.

   $-\log(.000\ 003\ 162) = 5.5000... \approx \boxed{5.5}$

   (c) Household bleach has $H^+ = 1.1 \times 10^{-13}$ Find the pH of bleach.

   $-\log(1.1 \times 10^{-13}) = 12.9586.. \approx \boxed{12.9}$

   (d) Materials with pH values between 0-5 are **acidic**, between 9-14 are **basic**, and between 5-7 are **neutral**. Which of the above materials are acid, basic, and neutral?

   orange juice pH ≈ 3.4 acidic
   shampoo pH ≈ 5.5 neutral
   bleach ph ≈ 12.9 basic

4. In Minneapolis, apartment rent is expected to increase by 16% next year.

   Story also appears in 0.3 #2

   (a) Astra lives in a 1-bedroom apartment where they pay $825 per month in rent. If their rent increased by 16% in how many years would their rent be doubled to $1,650. As we'll see later, the answer is $\dfrac{\log(2)}{\log(1.16)}$. Don't forget to the close the parentheses.

   $\log(2) \div \log(1.16) = 4.6701... \approx \boxed{5 \text{ years}}$

   (b) Lucky for Astra, their building is subject to rent stabilization laws and so their rent cannot increase by more than 3%. In how many years would their rent double under this cap? The answer is $\dfrac{\log(2)}{\log(1.03)}$.

   $\log(2) \div \log(1.03) = 23.4497... \approx \boxed{23 \text{ years}}$

   might be reasonable to round up to 24 years