SOLUTIONS

2.3 Using equations – Practice exercises

1. Dontrell and Kim borrowed money to buy a house on a 30-year mortgage. After M months of making payments, Dontrell and Kim will still owe $\$D$ where

$$D = 236{,}000 - 56{,}000 * 1.004^M$$

D is also known as the **payoff** (how much they would need to pay to settle the debt).

Story also appears in 3.4 #4

By the way $1.004 \wedge 0 = 1$ so really have $236{,}000 - 56{,}000 * 1 = 180{,}000$

(a) How much did Dontrell and Kim originally borrow to buy their house? What value of M did you evaluate at to answer the question? ← $M=0$

$D = 236{,}000 - 56{,}000 \times 1.004 \wedge 0 = \$180{,}000$

They originally borrowed \$180,000.

(b) Evaluate the equation at $M = 12$ and explain what the answer means in terms of the story. ↳ 12 mo = 1 yr

surprised? most of the payments during the first year are for interest, so still owe nearly \$180,000

$D = 236{,}000 - 56{,}000 \times 1.004 \wedge 12 = \$177{,}252.07$

After 1 year, they still owe \$177,252.07.

(c) After making half the payments, how much money will Dontrell and Kim still owe on the house? Will they have paid more or less (or exactly) half of the loan? *Hint: convert 30 years into months to find the total number of payments. Then divide by 2 to find the halfway point.*

$30 \text{ yrs} * \frac{12 \text{ mo}}{\text{yr}} = 30 \times 12 = 360 \text{ months}$ Halfway $= \frac{360}{2} = 360 \div 2 = 180$ months

$D = 236{,}000 - 56{,}000 \times 1.004 \wedge 180 = \$121{,}116.85$

Half of loan $= \frac{\$180{,}000}{2} = \$90{,}000$ They have paid of less than half!

(d) The very last month they don't actually pay the regular monthly payment, just whatever balance is left on the loan. How much will that be? *Hint: they will have made all but one of the payments.*

$360 - 1 = 359$ months

$D = 236{,}000 - 56{,}000 \times 1.004 \wedge 359 = \$1{,}257.93$

2. "Rose gold" is a mix of gold and copper. We start with 2 grams of an alloy that is equal parts gold and copper and add A grams of pure gold to lighten the color. The percentage of gold in the resulting rose gold alloy, R is given by

$$R = 100\left(\frac{1+A}{2+A}\right)$$

For example, if we add 0.8 grams of pure gold, then $A = .8$ and so the percentage is

$$R = 100\left(\frac{1+.8}{2+.8}\right) = 100 \times (1+\underline{.8}) \div (2+\underline{.8}) = 64.28571428\ldots \approx 64.3\%$$

Story also appears in 4.1 Exercises

A appears twice in the equation, so plug in both places.

(a) Calculate the percentage of gold in the alloy if we add 1.2 grams of pure gold.

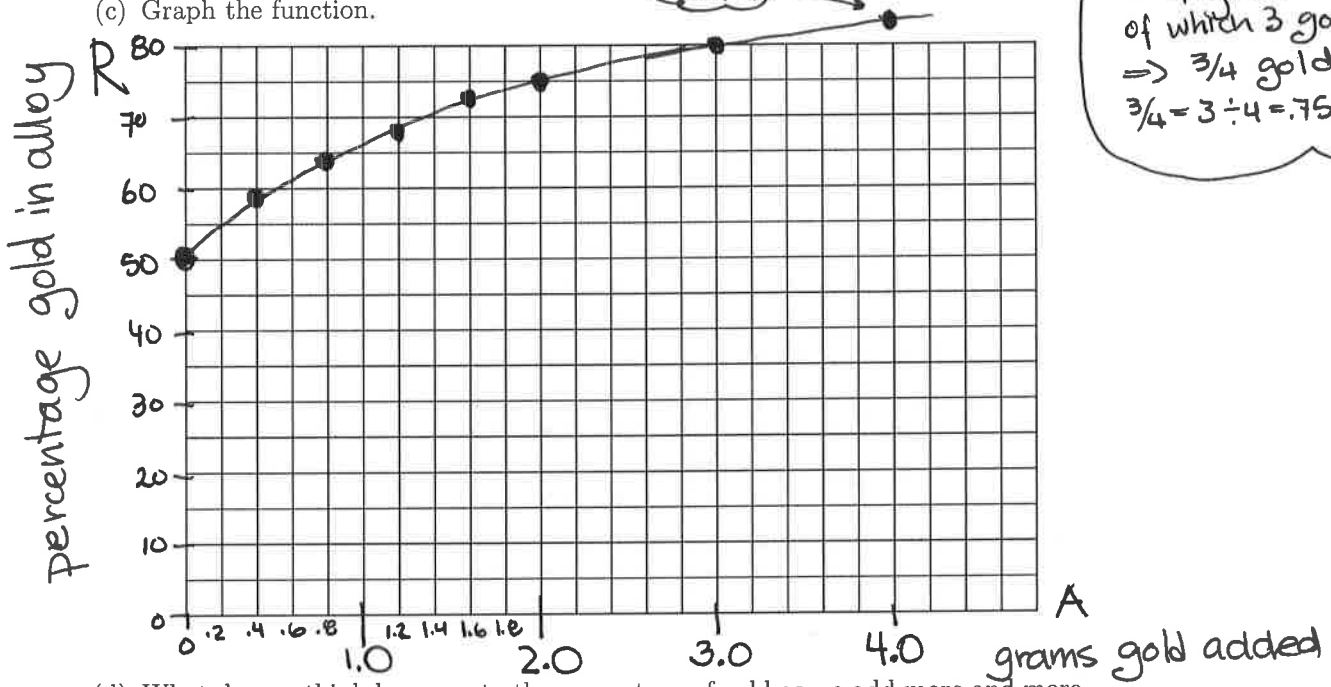
$R = 100 \times (1+\underline{1.2}) \div (2+\underline{1.2}) = 68.75\%$

(b) Fill in that and the rest of the missing values.

A	0	.4	.8	1.2	1.6	2	3	4
R	50.0	58.33	64.3	68.75	72.2	75.0	80.0	83.33

Yes! make sense because start with 1 gram copper and 1 gram gold add 2 more grams gold = 4 grams alloy of which 3 gold => 3/4 gold 3/4 = 3 ÷ 4 = .75 = 75%

(c) Graph the function.



(d) What do you think happens to the percentage of gold as we add more and more pure gold? Try adding 10 grams, and then try adding 100 grams to check.

As we add more gold, the percentage will get closer to 100%

Check:

A	10	100
R	91.67	99.02

$100 \times (1+\underline{100}) \div (2+\underline{100}) = 99.0196\ldots$

3. Monty hopes to grow orchids but they are fragile plants. He will consider his greenhouse a success if at least nine of the ten orchids survive. Assuming the orchids each survive at rate S, the probability his greenhouse is a success, P, is given by

$$P = 10S^9 - 9S^{10}$$ *Story also appears in 2.4 #3*

(a) If the orchids are perfect ($S = 1$), what is the probability of a successful greenhouse? Explain how this answer makes sense in the story.

$P = 10 \times 1 \wedge 9 - 9 \times 1 \wedge 10 = \boxed{1}$ $\xrightarrow{\times 100\%}$ 100% chance success

If the orchids are perfect, the greenhouse is sure to be a success.

(b) If the orchids are complete duds ($S = 0$), what is the probability of a successful greenhouse? Explain how this answer makes sense in the story.

$P = 10 \times 0 \wedge 9 - 9 \times 0 \wedge 10 = \boxed{0}$ 0% chance success.

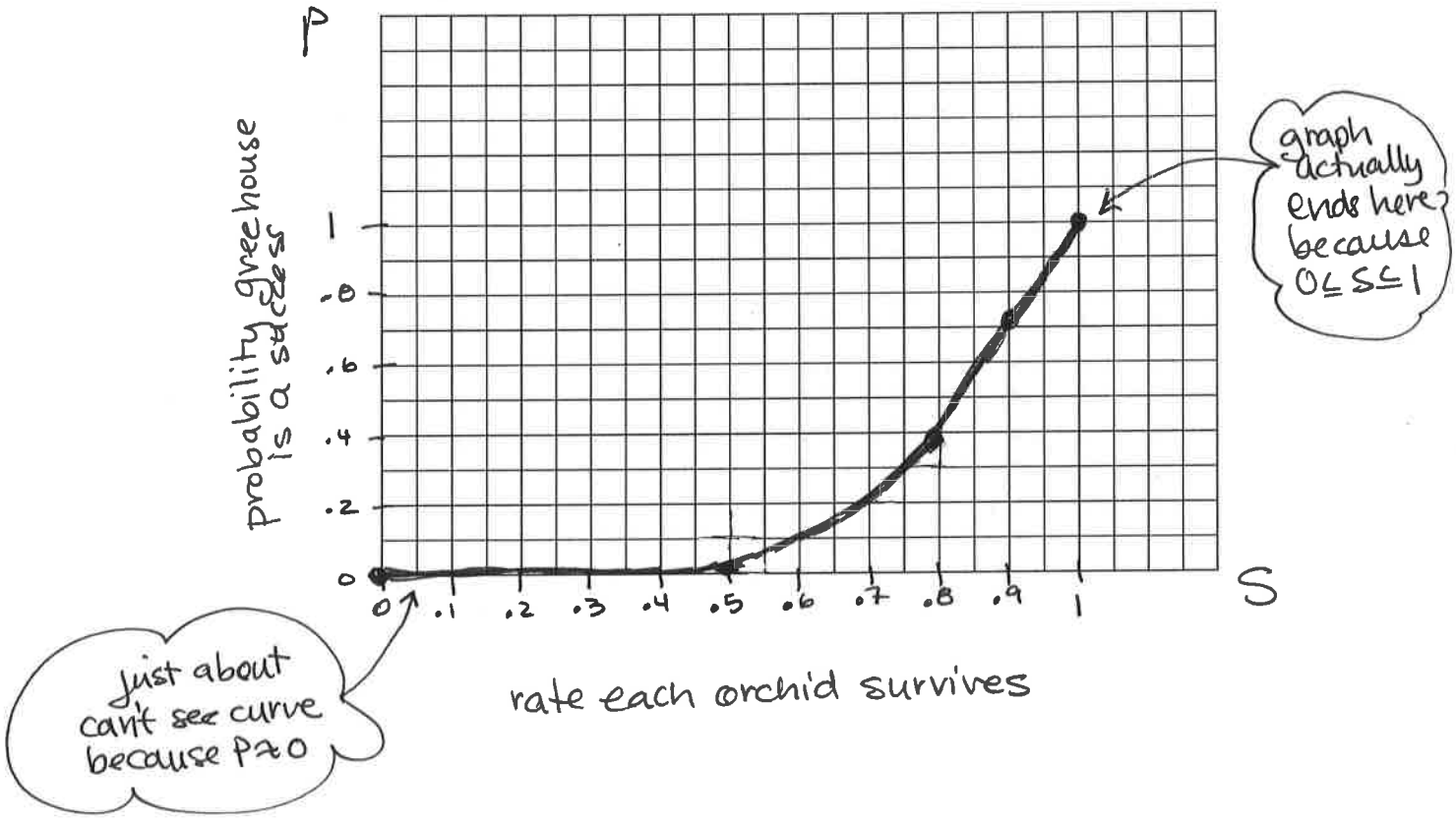
If the orchids are duds, the greenhouse will not succeed.

(c) Make a table comparing the probability of a successful greenhouse if the orchids each survive at rate $S = 0, .5, .8, .9, .95$, or 1.

S	0	.5	.8	.9	.95	1
P	0	.0107	.3758	.7361	.9139	1

$10 \times .5 \wedge 9 - 9 \times .5 \wedge 10 =$

(d) Draw a graph of the function.



4. Valerie plans to do a 3-day, 50-mile walk to raise money for breast cancer research, in honor of her aunt. Valerie's friends have pledged a total of \$93 per mile.

   (a) Valerie hopes to walk all 50 miles. If so, how much money will she raise?

   $50 \text{ miles} * \frac{\$93}{\text{mile}} = 50 \times 93 = \boxed{\$4{,}650}$

   She will raise \$4,650 if she walks all 50 miles.

   (b) She might have to stop sooner, however. Name variables and write an equation showing how the money Valerie raises is a function of how far she is able to walk.

   maybe D for distance & M for money would've been better choice? I used M for miles & D for dollars.

   M = distance Valerie walks (miles) ~ indep

   D = money Valerie raises (\$) ~ dep

   $\$D = M \text{ miles} * \frac{\$93}{\text{mile}} \Rightarrow \boxed{D = 93M}$

   remember: Write the number before the variable.

   (c) How long will it take Valerie to walk the full 50 miles if she's able to keep a pace of 3.2 miles per hour? *Write your answer in H:MM format.*

   $50 \text{ miles} * \frac{1 \text{ hour}}{3.2 \text{ miles}} = 50 \div 3.2 = 15.625 \text{ hours}$

   $= 15 \text{ hours and } \underline{38} \text{ minutes}$

   $.625 \text{ hours} * \frac{60 \text{ min}}{1 \text{ hour}} = .625 \times 60 = 37.5 \text{ min}$ ↑

   $= \boxed{15:38}$

   (d) Name the new variables and write a new equation showing how the time it takes Valerie to walk the full 50 miles depends on her pace.

   what's S for? speed.

   T = time it takes Valerie to walk 50 miles (hours) ~ dep

   S = pace (mph) ~ indep

   $\text{Thrs} = 50 \text{ miles} * \frac{1 \text{ hour}}{S \text{ miles}} \Rightarrow \boxed{T = \frac{50}{S}}$

   (e) Good news. Valerie walked the full 50 miles at a pace of 3.2 miles per hour. Way to go, girl! How much money did she raise each hour?

   Hint: Divide your answers from (a) by your answer from (c) to get \$/hour.

   (a) → \$4,650

   (c) → 15.625 hrs

   $= 4650 \div 15.625 = \boxed{\$297.60/\text{hour}}$