Just Enough Algebra

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5.1 Modeling with exponential equations

My grandmother was born in eastern Europe at the end of the 1800s. When she was eight years old her parents brought her and her younger sister and brother to the United States to escape harsh treatment by the government. Both her parents had to work, so my grandmother dropped out of school when she was thirteen years old to take care of the children, which now included another brother and sister.

Time passed and she married a handsome young veteran of World War I, who had also immigrated to the country as a young child. For her wedding dowry his parents bought my grandmother a set of sterling silverware, valued at \$800 in 1920. My grandmother was very proud of her sterling and used it often.

Over the years, the sterling has increased in value, let's say by around 3% per year. In 1957, my grandmother handed it down to my mother as a wedding present. In 1990, I married and my mother handed the sterling down to me. What was it was it worth at those times, and how much should it be insured for through 2015?

Let's write the equation to answer these questions. The variables should be

$$\begin{aligned} S &= \text{value of sterling (\$)} \sim \text{dep} \\ Y &= \text{time (years since 1920)} \sim \text{indep} \end{aligned}$$

We're saying that the sterling increased 3% per year in value. For example, in 1921, the sterling was worth

$$\$800 + 3\% \text{ of } \$800 = 800 + .03 \times 800 = 800 + 24 = \$824$$

Remember the shortcut here?

$$800 \times 1.03 = 824$$

The idea is after one year we have the original \$800 plus 3% more for a grand total of 103% of what we had before. And 103% = 1.03.

After 5 years, the sterling was worth

$$800 * 1.03 * 1.03 * 1.03 * 1.03 * 1.03 = 800 * 1.03^5$$

since multiplying by 1.03 five times is the same as multiplying by 1.03^5. On the calculator we do

$$800 \times 1.03 \wedge 5 = 927.4192594 \approx \$927$$

Generalizing, we get our equation

$$800 \times 1.03 \wedge Y = S$$

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which can be rewritten as

$$S = 800 * 1.03^Y$$

This equation fits our template for an exponential equation

$$\text{dep} \;=\; \text{start} \;*\, \text{growth factor}^{\text{indep}}$$

Quick recap. A function is **exponential** if it corresponds to a fixed percent increase (or decrease). The percent increase is the **growth rate**; in our example, the growth rate is $r = 3\% = .03$. The number we multiply by is the **growth factor** and it is also the base of the power in the equation; in our example, the growth factor is $g = 1.03$. The PERCENT CHANGE FORMULA from Section 2.2 reminds us that

$$g = 1 + r = 1 + .03 = 1.03$$

Let's answer those questions. In 1957, we had $Y = 1957 - 1920 = 37$ years and so

$$S = 800 * 1.03^{37} = 800 \times 1.03 \wedge \underline{37} = 2388.18134\ldots \approx \$2{,}388$$

By 1990, we had $Y = 1990 - 1920 = 70$ years and so

$$S = 800 * 1.03^{70} = 800 \times 1.03 \wedge \underline{70} = 6334.2575\ldots \approx \$6{,}334$$

By 2015, we have $Y = 2015 - 1920 = 95$ years and so

$$S = 800 * 1.03^{95} = 800 \times 1.03 \wedge \underline{95} = 13262.5286\ldots \approx \$13{,}262$$

Let's summarize this information in a table and draw a graph.

year	1920	1921	1925	1957	1990	2015
Y	0	1	5	37	70	95
S	800	824	927	2,388	6,334	13,262

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Actually, the insurance policy allows for up to \$20,000. The curve we drew suggests that the value will be \$20,000 just past $Y = 100$ (the year 2020), probably somewhere around $Y = 110$ (the year 2030).

We can use successive approximation to improve our answer.

year	2020	2030	2029	2028
Y	100	110	109	108
S	15,375	20,663	20,061	19,476
vs. 20,000	low	high	high	low

Seems to around the year 2029, where $Y = 109$, as we had guessed.

Of course, we can solve the exponential equation instead. To find when $S = 20{,}000$ we use our equation $S = 800 * 1.03^Y$ to get

$$800 * 1.03^Y = 20{,}000$$

Divide each side by 800 to get

$$\frac{\cancel{800} * 1.03^Y}{\cancel{800}} = \frac{20{,}000}{800}$$

and so

$$1.03^Y = \frac{20{,}000}{800} = 20{,}000 \div 800 = 25$$

Since we want to solve for the exponent, we use the LOG-DIVIDES FORMULA with growth factor $g = 1.03$ and the value $v = 25$ to get

$$Y = \frac{\log(v)}{\log(g)} = \frac{\log(25)}{\log(1.03)} = \log(25) \div \log(1.03) = 108.89737 \approx 109$$

We rounded up to make sure it would reach the full \$20,000. Since $1920 + 109 = 2029$, we see (again) that the value should reach \$20,000 in the year 2029.

As an aside, look what happens when we calculate the rate of change for this function. For example, during the first five years,

$$\text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{\$927 - \$800}{1925 - 1920} = \frac{\$127}{5 \text{ years}} = 127 \div 5 = \$25.40/\text{year}$$

and from 1925 to 1957,

$$\text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{\$2{,}388 - \$927}{1957 - 1925} = \frac{\$1{,}461}{32 \text{ years}} = 1{,}461 \div 32 \approx \$45.66/\text{year}$$

In the first few years, the value increased an average of \$25.40 a year, but from 1925 to 1957 it increased an average of about \$45.66 per year.

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Were we supposed to get different numbers here? Well, the graph's not a line and it's not a linear equation. That tells us the rate of change isn't going to be constant. So, sure, different numbers are fine. Does it make sense that the rate of change would itself increase? That the value increases at an increasing rate? Yes. Although we are always just adding on 3%, we're taking 3% of larger numbers each year. So more is added each year.

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know ...

- What makes a function exponential?
- The template for an exponential equation? *Ask your instructor if you need to remember the template or if it will be provided during the exam.*
- How to write an exponential equation given the starting amount and percent increase?
- Where the growth factor and starting amount appear in the template of an exponential equation?
- What the graph of an exponential function looks like?
- How to solve an exponential equation using the LOG DIVIDES FORMULA?

  Ask your instructor if you need to remember the LOG DIVIDES FORMULA *or if it will be provided during the exam.*
- How to calculate the rate of change of an exponential function?
- Why the rate of change of an exponential function is not constant?

Exercises

5. Use the equation from this section for the value of the sterling silverware to determine when the sterling was first worth over $5,000.

   (a) First, estimate the answer from our table and graph.

   (b) Next, use successive approximation to refine your answer. Display your work in a table.

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(c) Last, practice setting up and solving an equation using the LOG DIVIDES FORMULA.

6. Mrs. Nystrom's Social Security benefit was \$746.17/month when she retired from teaching in 2009. She had taught in elementary school since I was a girl. Benefits have increased by 4% per year. *Story also appears in 1.1 and 1.2 Exercises*

   (a) Name the variables and write an equation relating them.

   (b) Use your equation to estimate her benefit in the year 2020.

   (c) Set up and solve an equation to determine when her benefit will pass \$900/month.

   (d) Repeat for \$1,000/month.

7. The number of players of a wildly popular mobile app drawing game has been growing exponentially according to the equation

$$N = 2 * 1.57^W$$

   where N is the number of players (in millions) and W is the number of weeks since it caught on. *Story also appears in 5.1 #3 and 5.3 Exercises*

   (a) Make a table showing the number of players after 0 weeks, 2 weeks, 4 weeks, and 6 weeks.

   (b) Use successive approximation to determine when there will be over 60 million players. Round your answer to the nearest week.

   (c) Show how to solve the equation to determine when there will be over 60 million players. Record your answer to two decimal places.

   (d) Use your answer to (a), (b), and (c) to graph the function.

8. In 2006 there were about 5.2 million people living in the state of Minnesota. Predicted growth rates vary, perhaps around .5% per year.

   (a) What is the annual growth factor? Careful, the growth rate is .5%

   (b) Based on these figures, about how many people will be living in the state of Minnesota in 2010? In 2020?

   (c) Write an equation showing how Minnesota's population is a function of the year. Don't forget to name the variables.

   (d) Make a table of values showing the projected population every two years from 2006 to 2020.

   (e) Draw a graph illustrating the dependence.

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(f) Set up and solve an equation to determine when Minnesota's population is expected to be double the population from 2006.

9. Um Archivo data consultant group reported earnings of \$42.7 billion in 2012. At that time executives projected 17% increase in earnings annually. Based on that information, we wrote the equation

$$U = 42.7 * 1.17^Y$$

where U is Um Archivo's reported earnings (in \$billions) and Y is the years since 2012. *Story also appears in 2.2 Exercises*

(a) According to your equation, in what year would Um Archivo's reported earnings pass \$60 billion? Set up and solve an equation. Then check your answer.

(b) Repeat for \$100 billion.

10. In 1990 it was estimated that 2.5 million households watched reality television at least once a week. Executives predicted that number would increase by 7.2% each year. According to their estimates, how many millions of households watched reality television in 2000? In 2010? As part of your work, name the variables, find the annual growth factor, and write an exponential equation modeling reality television viewing.

Story also appears in 5.2 #3

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5.2 Exponential growth and decay

It is 2:00 a.m. and Joe is up studying. The dorm has quieted down, but Joe's feeling mighty jittery. He drank 5 large mugs of coffee in the past few hours and all that caffeine is peaking in his system now. At around 200 mg per mug, Joe wonders when his caffeine levels will drop down to where he can sleep a little.

First things first: staying up that late to study is probably a bad idea. I mean, who can think properly at 2:00 in the morning? And, how tired is Joe going to be by the time his test rolls around? Plus, we know that

$$5 \text{ mugs} * \frac{200 \text{ mg}}{\text{mug}} = 5 \times 200 = 1{,}000 \text{ mg}$$

which is a lot of caffeine, probably more than he needed to stay awake.

At this point Joe is stuck so let's help him. Let's say that at 2:00 a.m. he has 1,000 mg of caffeine in his blood. Joe searches online and discovers that 13% of the caffeine should leave his body each hour and below 300 mg he should be fine. When will that happen?

We know how percent increase works, but here the caffiene is leaving his body according to a percent decrease. I guess we need to figure it out one step at a time. After one hour (by 3:00 a.m.), Joe will have

$$1{,}000 \text{ mg} - 13\% \text{ of } 1{,}000 \text{ mg} = 1{,}000 - .13 \times 1{,}000 = 1{,}000 - 130 = 870 \text{ mg}$$

By 4:00 a.m. (after 2 hours), Joe will have

$$870 \text{ mg} - 13\% \text{ of } 870 \text{ mg} = 870 - .13 \times 870 = 870 - 113.1 = 756.9 \text{ mg}$$

Wait a minute. When we calculated 13% decrease on 1,000 mg we got 870 mg. That's 87% of 1,000. Yeah, that's right, take off 13% and you should be left with 87% of what you started with because 100% − 13% = 87%. So we could have calculated

$$.87 \times 1{,}000 = 870 \text{ mg}$$

and then

$$.87 \times 870 = 756.9 \text{ mg}$$

Aha, to find the amount after a 13% decrease we just multiply by .87.

Still nowhere near 300 mg so fast-forward. For example, after 5 hours (at 7:00 a.m.), we need to multiply 1,000 by .87 five times

$$1000 * .87 * .87 * .87 * .87 * .87 = 1{,}000 * .87^5$$

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where we use a power to abbreviate repeatedly multiplying. So

$$1000 * .87^5 = 1{,}000 \times .87 \wedge 5 = 498.420920\ldots \approx 498.4 \text{ mg}$$

The bad news is that it's 7:00 a.m. and Joe is still too jittery to sleep. The good news is that we can write the equation. The variables are

$$J = \text{Joe's caffeine level (mg)} \sim \text{dep}$$
$$H = \text{time (hours since 2:00 a.m.)} \sim \text{indep}$$

Our equation must be

$$J = 1{,}000 * .87^H$$

Notice this equation fits our template for an exponential equation.

$$\text{dep} \;=\; \text{start} \;*\text{growth factor}^{\text{indep}}$$

A little terminology here. When a function is exponential but decreasing, it's called **exponential decay**. It sounds a little odd to say "growth factor" if the quantity is getting smaller so we sometimes say **decay factor** instead. We know from the Percent Change Formula that the growth factor (g) can be found from the growth rate (r) by the formula $g = 1 + r$. If we think of 13% decrease as negative growth rate, $r = -13\% = -.13$, then the formula still works to find the decay factor (g)

$$g = 1 + r = 1 + -.13 = 1 - .13 = .87$$

Back to jittery Joe. Let's summarize what we've found and add a few more times to see when Joe's caffeine level should fall below 300 mg.

time	2:00	3:00	4:00	5:00	6:00	7:00	8:00	9:00	10:00	11:00
H	0	1	2	3	4	5	6	7	8	9
J	1,000	870	756.9	658.5	572.9	498.4	433.6	377.3	328.2	285.5
vs. 300	high	high	high	high	high	high	high	high	high	low

That means Joe should be able to fall asleep by around 11:00 a.m. Exactly when his exam starts. Sorry, Joe.

We could have solved the equation instead. We were looking for $J = 300$. Using our equation $J = 1{,}000 * .87^H$ we get

$$1{,}000 * .87^H = 300$$

Divide each side 1,000 to get

$$\frac{\cancel{1{,}000} * .87^H}{\cancel{1{,}000}} = \frac{300}{1{,}000}$$

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which simplifies to

$$.87^H = \frac{300}{1{,}000} = 300 \div 1{,}000 = .3$$

We find ourselves in the familiar situation – solving to find the exponent. Logs to the rescue. By the THE LOG-DIVIDES FORMULA with growth factor $g = .87$ and value $v = .3$ we get

$$Y = \frac{\log(v)}{\log(g)} = \frac{\log(.3)}{\log(.87)} = \log(.3) \div \log(.87) = 8.64537506\ldots \approx 9$$

which corresponds to 11:00 a.m. Same answer. Much quicker.

Quick side note. We could have rounded to 8.64 hours and then converted units to get

$$.64 \text{ hours} * \frac{60 \text{ minutes}}{1 \text{ hour}} = .64 \times 60 = 38.4 \approx 39 \text{ minutes}$$

Counting from 2:00 a.m., we see that Joe's caffeine levels drop below 300 mg at 10:39 a.m. Since we are approximating throughout the problem, we should round to 11:00 a.m. anyway.

Let's calculate the rate of change and think about what it means. During the first hour,

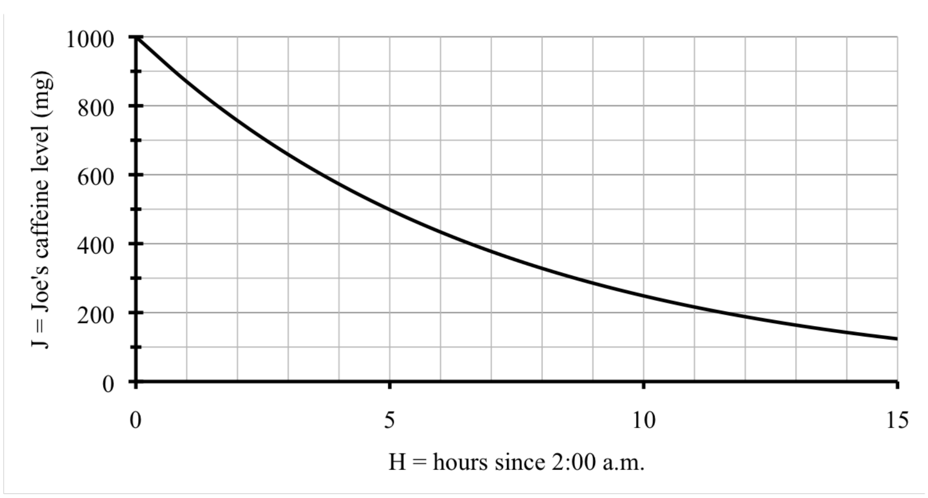
$$\text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{870 - 1{,}000 \text{ mg}}{3{:}00 \text{ a.m.} - 2{:}00 \text{ a.m.}} = \frac{-130 \text{ mg}}{1 \text{ hour}} = -130 \text{ mg/hour}$$

Was the rate of change supposed to be negative? Sure. Joe's caffeine level is dropping. Any decreasing function has a negative rate of change. And, as exam time approaches,

$$\text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{285.5 - 328.2 \text{ mg}}{11{:}00 \text{ a.m.} - 10{:}00 \text{ a.m.}} = \frac{-42.7 \text{ mg}}{1 \text{ hour}} \approx -43 \text{ mg/hour}$$

Joe's caffeine level was dropping faster at first and is not dropping as fast now.

A glance at the graph confirms our findings.



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One last thing. There's another way to describe the decrease here. When our story began Joe's caffeine level was around 1,000 mg and after 5 hours is was at 498.4 mg. That's just about 500 mg, or half of what he started with. We say the **half-life** of caffeine is around 5 hours.

Doesn't sound very important but check this out. Start with 1,000 mg. After 5 hours, there's 500 mg left. (Okay, approximately.) Now go another 5 hours, which means 10 hours total. Evaluate our equation $J = 1{,}000 * .87^H$ when $H = 10$ to get

$$J = 1{,}000 * .87^{10} = 1{,}000 * .87 \wedge \underline{10} = 248.423414\ldots \approx 250 \text{ mg}$$

That means half of what was left is now gone. Go another 5 hours. Lose another half. Check for yourself:

$$J = 1{,}000 * .87^{15} = 1{,}000 * .87 \wedge \underline{15} = 123.819426\ldots \approx 125 \text{ mg}$$

And so on. Cool.

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know ...

- How to write an exponential equation given the starting amount and percent decrease?
- What "half-life" means?
- What "doubling time" means?
- What the graph of exponential growth and exponential decay look like?
- Why the rate of change for exponential decay is negative?

  If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

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Exercises

5. Joe's girlfriend Ceyda starts the day by downing two cans of Red Bull, containing a total of 160 mg of caffeine. Her body eliminates the caffeine at a slightly slower rate of 12% each hour.

   (a) Name the variables and write an equation to model this situation.

   (b) What's the half-life of caffeine for Ceyda? Set up and solve an equation.

   (c) Ceyda heard that drinking a glass of water an hour can help eliminate caffeine faster. If so, would the half-life be shorter or longer? Explain.

6. The population of bacteria in a culture dish begins at 2,000 and will triple every day.

   (a) Name the variables, including units and dependence.

   (b) Make a table showing the number of bacteria at the start and after 1 day, 2 days, and 3 days.

   (c) Write an equation illustrating bacterial growth.

   (d) Your equation should fit the template for an exponential equation. What is the daily growth factor?

   (e) Use your equation to extend your table to include 10 days, 20 days, and 30 days. Be careful to report the large numbers appropriately. *Hint: scientific notation*

   (f) The dish can support around 1 million bacteria. When does that happen? Give your answer to the nearest hour.

   (g) Draw a graph, including a table of reasonable values. Remember, the dish can only support up to 1 million bacteria, so your graph should go up to 1 million.

7. Tenzin bought a house for $291,900 but the housing market collapsed and his house value dropped 4.1% each year.

   (a) Name the variables and write an equation relating them.

   (b) At this rate, how many years would it take for the value of Tenzin's house to drop below $240,000? Use successive approximation to guess the year.

   (c) Now set up and solve an equation.

8. One modern technique for cleaning waste water involves the use of constructed (man-made) wetlands. Wetlands act as a natural biofilter for various contaminants in the waste water. After each month in the wetlands, only about $1/4$ of the contaminants remain in any given sample. Suppose a sample had 8 grams of contaminants before processed in the constructed wetlands.

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(a) How much would remain in the sample after 1 month? 2 months? 3 months?

(b) Name the variables and write an equation relating them.

(c) Your equation should fit the template for an exponential equation. What is the monthly decay factor?

(d) According to your equation, when will the contaminants fall below 1 mg? 1 μg? Remember 1 gram = 1,000 mg and 1 gram = 1,000,000 μg

(e) Draw a graph illustrating the waste water treatment process for the first 6 months.

9. Hibbing, Minnesota is the hometown of baseball star Roger Maris, basketball great Kevin McHale, the Greyhound Bus lines, the Hull-Rust-Mahoning Open Pit Iron Mine and, perhaps most famously, the childhood home of songwriter Bob Dylan. It is not a big town. In 2000 the population was reported at 17,071 residents, with an expected decrease of around .4% per year.

   (a) What is the annual decay factor?

   (b) Name the variables and write an equation relating them.

   (c) Based on these estimates, what was the anticipated population of Hibbing in 2010?

   (d) The actual 2010 U.S. Census estimate of Hibbing's population was 16,361 people. Was the decrease slower or faster than expected? Was the decay rate more or less than .4%? Explain.

10. Donations to a local food shelf have increased 35% over last year. There were 3,400 pounds of food donated last year. *Story also appears in 5.3 #4*

   (a) Name the variables, measuring time since last year. (Yes, it's a little awkward.)

   (b) Write an exponential equation relating the variables, assuming the rate of increase continues.

   (c) How much was donated this year? How much is expected next year? The year after?

   (d) At this rate, what is the doubling time? That means, how long would it be until twice as much is donated. Set up and solve an equation to answer. Report your answer to the nearest month.

   (e) The agency estimates that around 12,000 pounds of food is needed each year to meet the needs of the people they serve. In how many years will donations reach that level? (In your answer, say how many years from now.)

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5.3 Growth factors

Obesity among children ages 6-11 continues to increase. From 1994 to 2010, the proportion of children classified as obese rose from an average of 1.1 out of every ten children in 1994 to around 2 out of every ten children in 2010.

Source: Center for Disease Control and Prevention

Assuming that the prevalence of childhood obesity increases exponentially, what is the annual percent increase and what does the equation project for the year 2020? Well, unless we are able to make drastic improvements in how children eat and how much they exercise.

Because we are told obesity is increasing exponentially we can use the template for an exponential equation.

$$\text{dep} \ = \ \text{start} \ * \text{growth factor}^{\text{indep}}$$

The variables are

$$C = \text{obese children (out of every ten)} \sim \text{dep}$$
$$Y = \text{year (years since 1994)} \sim \text{indep}$$

The starting amount is 1.1 children out of every ten in 1994 so our equation is of the form

$$C = 1.1 * g^Y$$

Trouble is we don't actually know what the growth factor g is. Yet.

We do know that in 2010 we have $Y = 2010 - 1994 = 16$ years and $C = 2$. We can put those values into our equation to get

$$1.1 * g^{16} = 2$$

No good reason for switching sides, just wanted to have the variable on the left. That's supposed to be true but we don't know what number g is so we can't check. Argh.

Oh, wait a minute. The only unknown in that equation is the growth factor g. What if we solve for g? First, divide each side by 1.1 to get

$$\frac{\cancel{1.1} * g^{16}}{\cancel{1.1}} = \frac{2}{1.1}$$

which simplifies to

$$g^{16} = \frac{2}{1.1} = 2 \div 1.1 = 1.818181818\ldots$$

Since we want to solve for the base (not the exponent), we have a power equation. We use the ROOT FORMULA with power $n = 16$ and value $v = 1.818181818$ to get

$$g = \sqrt[n]{v} = \sqrt[16]{1.818181818} = 16 \sqrt[x]{} \ 1.818181818 = 1.038071653 \approx 1.0381$$

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Want a quicker way to find the growth factor? Forget the entire calculation we just did. It all boils down to two steps:

$$\frac{2}{1.1} = 2 \div 1.1 = 1.818181818\ldots$$

and then

$$g = \sqrt[16]{1.818181818} = 16 \sqrt[x]{} 1.818181818 = 1.038071653 \approx 1.0381$$

We can even do this calculation all at once as

$$g = \sqrt[16]{\frac{2}{1.1}} = 16 \sqrt[x]{} (2 \div 1.1) = 1.038071653 \approx 1.0381$$

Notice we added parentheses because the normal order of operations would do the root first and division second. We wanted the division calculated before the root.

Here's the easy version in a formula.

GROWTH FACTOR FORMULA

If a quantity is growing (or decaying) exponentially, then the growth (or decay) factor is

$$g = \sqrt[t]{\frac{a}{s}}$$

where s is the starting amount and a is the amount after t time periods.

We knew from the beginning that our equation was in the form $C = 1.1 * g^Y$. Now that we found the growth factor $g \approx 1.0381$ we get our final equation

$$C = 1.1 * 1.0381^Y$$

For example, we can check that in 2010, we have $Y = 16$ still and so

$$C = 1.1 * 1.0381^{16} = 1.1 \times 1.0381 \wedge \underline{16} = 2.000874004 \approx 2 \quad \checkmark$$

You might wonder why we didn't just round off and use the equation

$$C = 1.1 * 1.04^Y$$

Look what happens when we evaluate at $Y = 16$ then. We would get

$$C = 1.1 * 1.04^{16} = 1.1 \times 1.04 \wedge \underline{16} = 2.06027937 \approx 2.1$$

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Not a big difference (2.1 vs. 2.0) but enough to encourage us to keep extra digits in the growth factor in our equation. Lesson here is: don't round off the growth factor too much.

Back to the more reliable equation

$$C = 1.1 * 1.0381^{Y}$$

We can now answer the two questions. First, in 2020 we have $Y = 2020 - 1994 = 26$ and so

$$C = 1.1 * 1.0381^{26} = 1.1 \times 1.0381 \wedge \underline{26} = 2.908115507 \approx 2.9$$

According to our equation, by 2020 there would be approximately 2.9 obese children for every ten children.

The other question was what the annual percent increase is. Think back to an earlier example. Remember that Jocelyn was analyzing health care costs in Section 2.2? They began at \$2.26 million and grew 6.7% per year. She had the equation

$$H = 2.26 * 1.067^{Y}$$

So the growth factor $g = 1.067$ in the equation came from the growth rate $r = 6.7\% = .067$. Our equation modeling childhood obesity is

$$C = 1.1 * 1.0381^{Y}$$

The growth factor of $g = 1.0381$ in our equation must come must come from the growth rate $r = .0381 = 3.81\%$. Think of it as converting to percent $1.0381 = 103.81\%$ and then ignoring the 100% to see the 3.81% increase. Childhood obesity has increased around 3.81% each year. Well, on average.

Here's the general formula relating the growth rate and growth factor.

PERCENT CHANGE FORMULA: (updated version)

- If a quantity changes by a percentage corresponding to growth rate r, then the growth factor is

$$g = 1 + r$$

- If the growth factor is g, then the growth rate is

$$r = g - 1$$

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Let's check. We have $g = 1.0381$ and so the growth rate is

$$r = g - 1 = 1.0381 - 1 = .0381 = 3.81\%$$

Not sure we really need these formulas, but there you have it.

By the way, formula works just fine if a quantity decreases by a fixed percent. One example we saw was Joe, who drank too much coffee. The growth (or should I say decay) factor was $g = .87$. That corresponds to a growth (decay) rate of

$$r = g - 1 = .87 - 1 = -.13 = -13\%$$

Again, the negative means that we have a percent decrease.

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know ...

- How to find the growth/decay factor given the starting amount and another point of information?
- How to find the growth/decay factor given the doubling time or half-life?
- When we use the PERCENT CHANGE FORMULA, and when we use the GROWTH FACTOR FORMULA instead? *Ask your instructor if you need to remember the* PERCENT CHANGE FORMULA *and* GROWTH FACTOR FORMULA *or if they will be provided during the exam.*
- How to evaluate the PERCENT CHANGE FORMULA and GROWTH FACTOR FORMULA using your calcuator?
- How to read the starting amount and percent increase/decrease from the equation?

If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

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Exercises

5. Estimates for childhood obesity for 2010 were revised to 2.1 out of every ten children. (The 1994 figure of 1.1 out of every ten children remains accurate.)

   (a) Calculate the revised growth factor. What is the revised percent increase?

   (b) Revise your equation.

   (c) Use your new equation to project childhood obesity rates for 2020.

   (d) Graph both the original and revised estimates on the same set of axes.

6. For each equation, find the growth rate (percent increase or percent decrease) and state the units. (For example, something might "grow 2% per year" while something else might "drop 7% per hour")

   (a) The light $L\%$ that passes through panes of glass W inches thick is given by the equation
   $$L = 100 * .75^W$$
   Story also appears in 2.4 and 3.4 Exercises

   (b) The population of bacteria (B) in a culture dish after D days is given by the equation
   $$B = 2{,}000 * 3^D$$
   Story also appears in 5.2 Exercises

   (c) The remaining contaminants (C grams) in a waste water sample after M months of treatment is given by
   $$C = 8 * .25^M$$
   Story also appears in 5.2 Exercises

7. Years ago, Whitney bought an antique mahogany table worth \$560. Now, 30 years later, she had the table appraised for \$3,700.

   (a) Calculate the annual growth factor, assuming the value of Whitney's table has increased exponentially.

   (b) What should she expect the set to be worth in another 10 years? As part of your work, name the variables and write an equation relating them.

8. The opiate drug morphine leaves the body quickly. After 72 hours about 10% remains. A patient receives 100 mg of morphine.

   (a) How much morphine will remain in the patient's body after 72 hours?

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(b) Convert 72 hours to days.

(c) Find the daily decay factor using the Growth Factor Formula.

(d) What is the corresponding percent decrease?

(e) Name the variables and write an equation relating them. Check that 72 hours gives you the same answer as in part (a).

(f) What is the half-life of morphine? Set up and solve an appropriate equation.

(g) Draw a graph showing this patient's morphine levels for 10 days following the injection.

9. Unemployment figures were just released. At last report there were 20,517 unemployed adults and now, 10 months later, we have 39,061 unemployed adults.

(a) Calculate the monthly growth factor, assuming unemployment increases exponentially.

(b) Write an equation relating the variables.

(c) According to your equation, what is the expected number of unemployed adults 6 months from now. *Notice: the report was issued 10 months ago.*

(d) Make a table of values and draw a graph showing the number of unemployed adults for the past 10 months and the next 2 years.

10. Wetlands help support fish populations, various plant and animal populations, control floods and erosion from nearby lakes and streams, filter water, and help preserve our supply of ground water. Minnesota wetlands acreage in 1850 was 18.6 million acres. By 2003, that number had dropped to 9.3 million acres.

Source: Minnesota Department of Natural Resources

(a) Assuming the acreage decreased exponentially, name the variables, find the annual decay factor and write an exponential equation showing how Minnesota wetlands have decreased.

(b) With some effective management, many wetlands have been restored. By 2012, it's up to about 10.6 million acres. Assuming acreage has increased exponentially from 2003, name the variables (you may now want to start the years in 2003), find the growth factor and write an exponential equation showing how Minnesota wetlands have been restored.

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5.4 Linear vs. exponential models

Sofía bought her car new for \$22,500. Now the car is fairly old and just passed 109,000 miles. Sofía looked online and estimates the car is still worth \$5,700. She wonders when the car would be practically worthless, meaning under \$500.

We can describe the variables in this story.

$$M = \text{mileage (thousand miles)} \sim \text{indep}$$
$$C = \text{value of car (\$)} \sim \text{dep}$$

Notice we are measuring the mileage in thousands. The information we are given is

M	0	109
C	22,500	5,700

But what's the equation? Hmm. Don't know for sure what type of equation might work here. Tell you what, let's compare what a linear and exponential model would tell us about the value of the car.

First, linear. The template is

$$\text{LINEAR EQUATION TEMPLATE:} \quad \text{dep} = \text{start} + \text{slope} * \text{indep}$$

The starting value of Sofía's car is \$22,500 so we just need to find the slope. We expect the slope to be negative because her car is worth less the more she drives it.

$$\begin{aligned} \text{slope} &= \text{rate of change} = \frac{\text{change dep}}{\text{change indep}} = \frac{\$5{,}700 - \$22{,}500}{109 \text{ thousand miles}} = \frac{-\$16{,}800}{109 \text{ thousand miles}} \\ &= (\text{-})16{,}800 \div 109 = -154.1284404\ldots \approx -\$154/\text{thousand miles} \end{aligned}$$

Her car loses value at a rate of around \$154 for each thousand miles she drives.

We are ready to write the linear equation.

$$\textbf{linear:} \quad C = 22{,}500 - 154M$$

As usual we wrote this with subtraction instead of adding the negative slope. Quick check: when $M = 109$ we get

$$C = 22{,}500 - 154 * 109 = 22{,}500 - 154 \times \underline{109} = 5{,}714 \approx \$5{,}700 \quad \checkmark$$

Remember we don't expect the exact answer because we rounded off the slope.

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When will Sofía's car be worth under \$500 according to this linear equation? Let's solve to find out. When $C = 500$, use our linear equation to get

$$22{,}500 - 154M = 500$$

Subtract 22,500 from each side and simplify to get

$$-154M = -22{,}000$$

Now divide each side by -154 and simplify to get

$$M = \frac{-22{,}000}{-154} = \text{(-) } 22{,}000 \div \text{(-)}154 = 142.738095\ldots \approx 143$$

According to the linear equation, Sofía's car will be worth under \$500 at about 143,000 miles. Since her car already has 109,000 miles on it, that means in another 143,000 − 109,000 = 34,000 miles. For a typical driver that's two or three more years.

Next, let's take a look at the exponential model. Here goes. The template is

$$\text{Exponential Equation Template:} \quad \text{dep} = \text{start} * \text{growth factor}^{\text{indep}}$$

We know everything except the growth factor. We expect it to be less than 1 because her car is worth less the more she drives it. Perhaps we should say "decay" factor here since the function is decreasing. The starting amount is $s = 22{,}500$ and the ending amount is $a = 5{,}700$ after $t = 109$ thousand miles. By the Growth Factor Formula we have

$$g = \sqrt[t]{\frac{a}{s}} = \sqrt[109]{\frac{5{,}700}{22{,}500}} = 109 \sqrt[x]{\ } (5{,}700 \div 22{,}500) = 0.98748222\ldots \approx .9875$$

We are ready to write the exponential equation.

$$\textbf{exponential:} \quad C = 22{,}500 * .9875^M$$

Quick check: when $M = 109$ we get

$$C = 22{,}500 * .9875^{109} = 22{,}500 \times .9875 \wedge \underline{109} = 5711.19365\ldots \approx \$5{,}700 \quad \checkmark$$

Again, we don't expect the exact answer because we rounded off the decay factor.

When will Sofía's car be worth under \$500 according to this exponential equation? Let's solve to find out. When $C = 500$, use our exponential equation to get

$$22{,}500 * .9875^M = 500$$

Divide each side by 22,500 and simplify to get

$$.9875^M = \frac{500}{22{,}500} = 500 \div 22{,}500 = .02222222\ldots$$

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By the THE LOG-DIVIDES FORMULA with growth factor $g = .9875$ and the value $v = .02222222$ we get

$$M = \frac{\log(v)}{\log(g)} = \frac{\log(.022222222)}{\log(.9875)} = \log(.02222222) \div \log(.9875) = 302.6256856 \approx 300$$

According to the exponential equation, Sofía's car will be worth under \$500 at about 300,000 miles. Hard to imagine the car would last that long. Essentially the exponential model says the car will always be worth at least \$500, if only for parts, I guess. Quite different from our answer from the linear equation.

Time to compare models. Which one makes more sense? First things first, the car already has a lot of miles on it. Don't know what make or model the car is, but another couple of years seems a reasonable time until is worth under \$500. That's what the linear equation projects. On the other hand, the exponential model project it will hold that value for a long time, essentially for parts. That makes sense too.

Wait a minute. Does a car lose the same value for each thousand miles it's driven? That's what it means to be linear. Every thousand miles, same decrease. Nah, that's not right. Once the car is old, another 1,000 miles or so probably won't affect the value at all. Also, when a car is new, once you drive it off the lot and then that strange vinyl smell wears off and it's officially "used," the car is worth a lot less. Even if it hasn't been driven much at all. What would each model say the car was worth soon after Sofía bought it, say with 10,000 miles on it? With $M = 10$, the estimates are

linear: $C = 22{,}500 - 154 * 10 = 22{,}500 - 154 \times \underline{10} = \$20{,}960$

exponential: $C = 22{,}500 * .9875^{10} = 22{,}500 \times .9875 \wedge \underline{10} \approx \$19{,}841$

The lower value, from the exponential equation, seems more reasonable.

Here are a few more values and the graph. The graph is shows both the line and exponential curve have intercept just over \$22,000, which should be \$22,500. The line and curve intersect again between 100,000 and 120,000 miles (close to the exact mileage of 109,000) at right under \$6,000 (close to the exact value of \$5,700).

M	0	10	50	80	109	200	250
C (if linear)	22,500	20,960	14,800	10,180	5,714	~~- 8,300~~	~~-16,000~~
C (if exponential)	22,500	19,841	11,996	8,225	5,711	1,818	969

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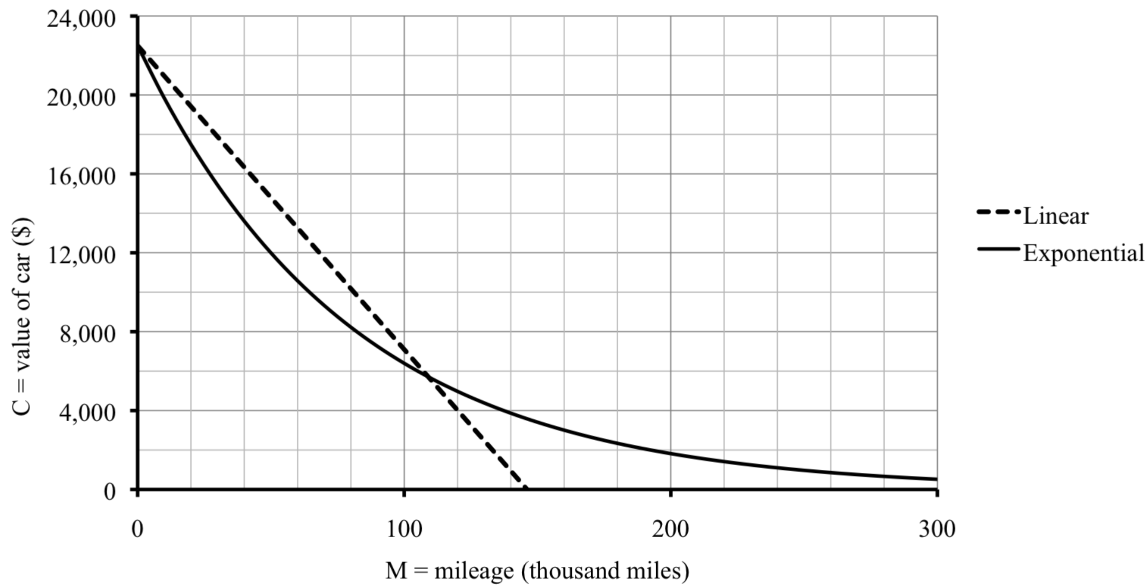

There's no way of knowing whether the function is linear or exponential. It is probably not exactly either one. But if we have to pick, the exponential model seems closer to reality.

Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know . . .

- When we might think a model might be linear?
- The template for a linear equation?
- How to find the linear equation between two points (a start and end value)?
- When we might think a model might be exponential?
- The template for an exponential equation?
- How to find the exponential equation between two points (a start and end value)?
- Why we compare linear and exponential models?

If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

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Exercises

5. (a) Gina weighed 165 pounds when she started her diet three months ago. Now she weighs 153 pounds. How much longer will she have to stay on her diet to reach her goal weight of 120 pounds? Make an initial guess of how long it will take her, and use a linear and an exponential model to predict how long she will be on her diet. *That means, write both equations and solve each to find the answer.*

   (b) Gina's diet-buddy Casey started at 253 pounds and now weighs 235 pounds after 3 months. Casey's goal weight is 178 pounds. How long will it take him? Again, make an initial guess, and then use a linear and an exponential model to predict. *Again, write and then solve both equations.*

   (c) Explain why weight loss might be similar to a car depreciating in value. Think about what happens at the start of a diet and what happens long run.

6. Chlorofluorocarbons (CFC) are greenhouse gases that result from our use of refrigeration, air conditioning, aerosols, and foams. In 1960, the concentration of CFC in the northern hemisphere was 11.1 parts per trillion (ppt), meaning on average, there are 11.1 CFC molecules in a trillion molecules of air. In 1980 the concentration was 177 ppt. Use C for the concentration in the northern hemisphere in ppt and Y for the year, measured in years since 1960. U.S. Department of Energy

   (a) Suppose that this concentration has been increasing at a *constant rate each year*.

      i. By how many parts per trillion per year have CFC concentrations increased?
      ii. Write an equation illustrating this model.
      iii. According to this equation, how much will the concentration of CFC be by the year 2015?
      iv. What type of equation is being used here?

   (b) Assume instead that the concentration of CFC has been increasing *a fixed percentage each year*.

      i. What is the annual growth factor that CFC concentrations increased?
      ii. Write an equation illustrating this model.
      iii. According to this new equation, how much will the concentration of CFC be by the year 2015?
      iv. What type of equation is being used here?

7. According to an online calculator, the recommended daily caloric intake for a woman Diane's height, weight, and activity level was 1,900 calories/day when she was 30 years old and 1,840 calories per day when she was 40 years old.

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(a) Name the variables and summarize the information in a table. Start her age at 30 years old (so you will know the intercept).

(b) Use a linear model to estimate the recommended daily caloric intake for Diane when she was 20 years old and now that she's 50 years old. According to this model, what will it be when she's 80 years old? *Note: because we started the age at 30 years old, you will need to use -10 to get her age of 20 years old. Looks weird, but 20 is 10 years before our starting value 30 so officially that's a negative. It works fine.*

(c) Repeat using an exponential model instead.

(d) Thoughts?

8. In 1995 the average price of a movie ticket was \$4.35, and in 2011 the average price of a movie ticket was \$7.93. The variables are T, the average price of a movie ticket in dollars and Y the number of years since 1995.

Source: National Association of Theatre Owners

(a) Write a linear equation that fits this information and use it to estimate the average price of a movie ticket in 2015 and 2025.

(b) Write an exponential equation that fits this information and use it to estimate the average price of movie ticket in 2015 and 2025.

(c) Draw a graph of each function on the same set of axes. (Include also what each equation said about ticket prices in the year 2000.)

(d) The actual average movie ticket price in the year 2000 was \$5.39. Which model predicted a closer value – linear or exponential?

9. The number of asthma sufferers worldwide in 1990 was 84 million and 130 million in 2001. Let A be the number of people with asthma (in millions) and Y the year, measured in years since 1990. Compare what the linear and exponential models project for the year 2015 and 2030. Include a graph showing both functions on the same axes.

10. Sales of hybrid cars in the United States have continued to increase. In 1999, 17 (yes, seventeen) hybrid cars were sold. By 2002 that number was up to 34,521 hybrid cars sold. Write H for the number of hybrid cars sold and Y for the year, measured in years since 1999. Source: Earth Policy Institute

(a) Suppose that the hybrid car sales have been increasing *at a constant rate each year.* Write the appropriate equation and use it to estimate sales in 2011.

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(b) Assume instead that hybrid car sales have been increasing *a fixed percentage each year.* Write the appropriate equation and use it to estimate sales in 2011. Comment.

(c) Actual sales of hybrid cars in 2010 were around 13,500. Why might the number be lower than the linear model predicted?

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5.5 Logistic and other growth models

A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing rapidly. After 2 weeks, there were 64 students with the flu. We are interested in predicting how many students will catch the flu over the next 6 weeks or so. To get a sense of scale, there are 1,094 students currently living in the dorms.

The variables are

$$D = \text{time since first cases (days)} \sim \text{indep}$$
$$F = \text{total number of students with the flu (students)} \sim \text{dep}$$

One model estimates that the number of students diagnosed with the flu was growing 16% per day. (If this story sounds familiar, it's because the story also appears in the practice exercises 2.2 #3 and 5.1 #2.) The corresponding equation is

$$\textbf{exponential:} \quad F = 8 * 1.16^D$$

As a check, at 14 days there were

$$F = 8 * 1.16^{14} = 8 \times 1.16 \wedge \underline{14} = 63.900143\ldots \approx 64 \text{ students}$$

We rounded the numbers in our table to the nearest person.

D	0	7	14	21	28	35	42
F (**exponential**)	8	23	64	181	510	1,442	4,077

While at first the exponential model seems reasonable, it quickly gets too large to make sense. After all, there are only 1,094 students currently living in the dorms so the numbers we found at 5 and 6 weeks (also known as 35 and 42 days) are totally unrealistic. The exponential model is based on the assumption that the rate of change of the number of new cases is proportional to the number of **infected** students: those who already have the flu.

There are both advantages and disadvantages of the exponential model. To it's credit, the exponential model captures the reality of the first few weeks, where the flu spreads very rapidly. But, the exponential model misses several basic facts. First, as more students catch the flu, the number of new cases decreases in part because sick people are already surrounded by sick people so there aren't new people to get sick. Second, for whatever reasons, not everyone is going to catch the flu no matter how exposed they are. We would like to have an alternative model that keeps what works (rapid increase at first) but deals

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better with the long term (the growth slows down and not everyone catches the flu). There are two different models we consider that have these properties: saturation and logistic.

The first example is a **saturation** model. Basically it assumes that the rate of change of the number of new cases is proportional to the number of **susceptible** students: those who are likely to catch the flu but haven't already. Since at the beginning many susceptible students don't have the flu, it spreads very quickly, even faster than the exponential does. But once most susceptible students have caught the flu, the number of new cases dwindles.

Leaving out the details of how we found it, a possible saturation equation for our example is

$$\textbf{saturation:} \quad F = 96 - 88 * .93^{D}$$

As a check, initially there were

$$F = 96 - 88 * .93^{0} = 96 - 88 \times .93 \wedge \underline{0} = 8 \text{ students} \quad \checkmark$$

and at 14 days there were

$$F = 96 - 88 * .93^{14} = 96 - 88 \times .93 \wedge \underline{14} = 64.1401334\ldots \approx 64 \quad \checkmark$$

We rounded the numbers in our table to the nearest person.

D	0	7	14	21	28	35	42
F (**saturation**)	8	43	64	77	85	89	92

The saturation model predicts that 92 students (total) will have (or have had) the flu over the next 6 weeks.

The second example is a **logistic** (or **S-curve**) model. Basically it assumes that the rate of change of the number of new cases is jointly proportional to the number of infected students and the number of susceptible students. It acknowledges the heavy influence the number of infected students have initially on the growth, but balances it with the limiting influence of the diminishing number of susceptible students over time.

It turns out that a possible logistic equation for our example is

$$\textbf{logistic:} \quad F = \frac{129}{1 + 15 * .825^{D}}$$

For example, initially there were

$$F = \frac{129}{1 + 15 * .825^{0}} = 129 \div (1 + 15 \times .825 \wedge \underline{0}) = 8.0625000\ldots \approx 8 \text{ students} \quad \checkmark$$

and at 14 days there were

$$F = \frac{129}{1 + 15 * .825^{14}} = 129 \div (1 + 15 \times .825 \wedge \underline{14}) = 64.0212993\ldots \approx 64 \text{ students} \quad \checkmark$$

Notice how we need parentheses around the bottom of our fraction, as usual, to override the normal order of operations. We rounded the numbers in the table to the nearest person.

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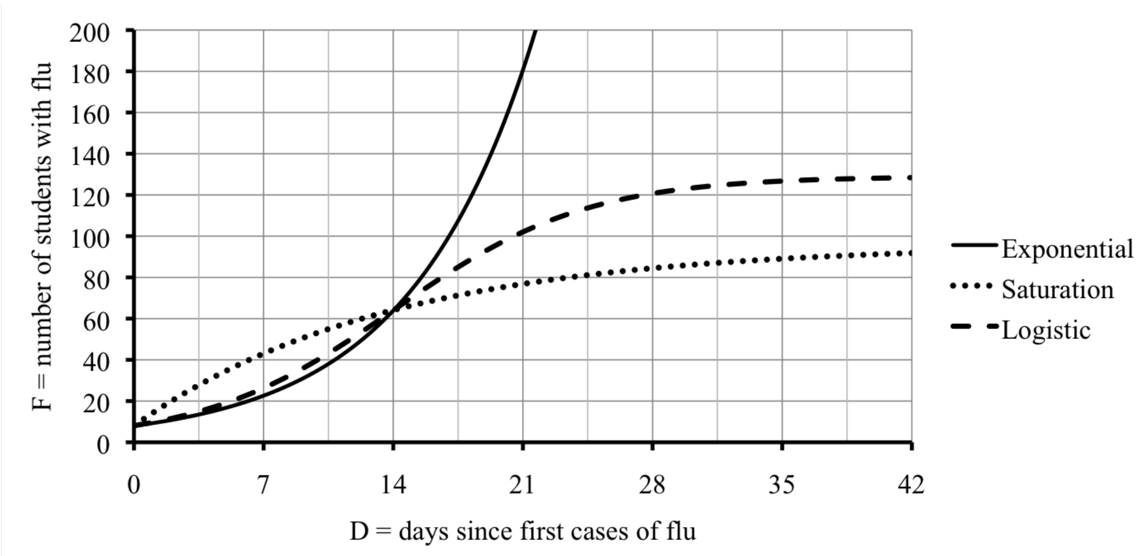
D	0	7	14	21	28	35	42
F (**logistic**)	8	26	64	102	121	127	128

The logistic model projects that 128 students (total) will have (or have had) the flu over the next 6 weeks, considerably more than projected by the saturation model.

Here are all three models on the same graph.



As you can see from the graph, both the saturation and logistic curves level off as expected. One way to estimate those **limiting values** (or **carrying capacity**) is to evaluate the functions at large values, say 60 days, 100 days, and (the unrealistic) 1,000 days.

D	60	100	1,000
F (**exponential**)	~~$\approx$ 59,000~~	~~$\approx$ 22 million~~	~~$\approx 1.35 \times 10^{33}$~~
F (**saturation**)	94.86	95.94	96.00
F (**logistic**)	128.98	128.99	129.00

We crossed out the unrealistic values from the exponential equation. So, if the saturation model is accurate, then we should expect around 96 total cases. But, if the logistic model is accurate, then we should expect around 129 total cases instead.

Look back at the equations:

$$\textbf{saturation:} \quad F = 96 - 88 * .93^D$$

$$\textbf{logistic:} \quad F = \frac{129}{1 + 15 * .825^D}$$

The limiting values were there all along!

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Homework

Start by doing Practice exercises #1-4 in the workbook.

Do you know . . .

- Why we might use a logistic or saturation model, instead of an exponential model?
- The difference between a logistic and saturation model?
- What the limiting value of a logistic function means in the story and what it tells us about the graph?
- How to find the limiting value of a logistic function?
- What the graph of a logistic function looks like?
- What the limiting value of a saturation function means in the story and what it tells us about the graph?
- How to find the limiting value of a saturation function?
- What the graph of a saturation function looks like?

  If you're not sure, work the rest of exercises and then return to these questions. Or, ask your instructor or a classmate for help.

Exercises

5. In our example in this section, we made several tables of values. Go back and check that they are correct.

6. Mari volunteers answering calls for in the office of her local state government representative. The office has been receiving a lot of calls recently with about BPA, a chemical found in plastics. The callers want their representative to support a bill banning BPA. An equation that describes the number of total number of calls over time is the following:
$$C = \frac{837}{1 + 118 * .8025^D}$$
where D is the time since January 1 (in days), and C is the total number of calls.

   (a) According to this equation, how many calls (total) will Mari's office get by February 1 (day 31), March 1 (day 59), April 1 (day 90), May 1 (day 120), and Nov 8 (day 311)?

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(b) During which months did most of the calls come in?

(c) Draw a graph illustrating the function.

(d) Describe what happened over time.

7. Even though all the callers support the bill, Mari isn't sure whether the calls represent the local constituents. Perhaps only supporters are calling her office, for example. So, she asks her pollster, Paul, to add this question to the list for his daily survey. Based on that survey, Paul estimates the percentage P of local constituents who support the bill on day D by the equation

$$P = 100 - 87.3 * .992^D$$

(a) According to this equation, what percentage of callers supported the bill on January 1 (day 0), March 1 (day 59), Aug 1 (day 212), Oct 1 (day 273) and Nov 8 (day 311)?

(b) What does your equation say the percentage would be on day 500 (which probably isn't realistic in this problem)? How about day 1,000?

(c) Use successive approximations to estimate when the percentage supporting the bill first reached majority (50%).

(d) Set up and solve an equation to find when the percentage supporting the bill first reached majority (50%).

8. Infants are regularly checked to make sure they are growing accordingly. The World Health Organization publishes growth charts to evaluate infant weight W in kilograms at a given age M in months since birth. An equation that describes an average infant boy is the following:

$$W = 15 - 11.5 * .932^M$$

(a) According to this equation, what is the average infant boy weight at birth, 1 month, 4 months, and a year?

(b) Convert your answers to pounds and ounces using

$$1 \text{ kilogram } \approx 2.2 \text{ pounds} \quad \text{and} \quad 1 \text{ pound } = 16 \text{ ounces}$$

Hint: first convert to pounds. Then convert just the decimal part to ounces.

(c) The equation is valid for $0 \leq M \leq 36$, or up to three years old. Draw a graph that includes your points from earlier and the values at 3, 4, 5, and 6 years. Can you explain why the equation doesn't make sense after around 3 years?

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9. The lake by Rodney's condo was stocked with bass (fish) 10 years ago. There were initially 400 bass introduced. The carrying capacity of the lake is estimated at 4,000 bass. Two potential models for the number of bass (B) over time, where Y measures the years from when the lake was stocked 10 years ago are

saturation: $$B = 4{,}000 - 3{,}600 * .78^Y$$

logistic: $$B = \frac{4{,}000}{1 + 9 * .78^Y}$$

Story also appears in 3.3 Exercises

(a) Make a table showing the bass population projected by each model, including 10 years ago, now, in 10 more years, in 20 more years, and in 30 more years.

(b) Draw a graph showing both curves.

(c) Which model shows the lake reaching (near) capacity sooner: the saturation model or the logistic model?

(d) If the current bass population in the lake by Rodney's house is around 2,500 fish, which model is more realistic?

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