SOLUTIONS

2.5 Finance formulas – Practice exercises

COMPOUND INTEREST FORMULA: $a = p\left(1+\frac{r}{12}\right)^{12y}$

EQUIVALENT APR FORMULA: $\mathrm{APR} = \left(1+\frac{r}{12}\right)^{12} - 1$

FUTURE VALUE ANNUITY FORMULA: $a = p * \frac{\left(1+\frac{r}{12}\right)^{12y} - 1}{\frac{r}{12}}$

LOAN PAYMENT FORMULA: $p = \frac{a * \frac{r}{12}}{1-\left(1+\frac{r}{12}\right)^{-12y}}$

where

$a =$ account balance or loan amount ($)
$p =$ initial deposit (principal), regular deposit, or regular payment ($)
$y =$ time invested (years)
$r =$ interest rate compounded monthly (as a decimal)

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1. Use the indicated formulas to help Kiran figure out her finances.

   (a) Kiran deposited \$2,500 in a money market account that earned 7% interest compounded monthly. Use the COMPOUND INTEREST FORMULA to calculate her account balance after 4 years.

   $p = \$2500,\quad r = \frac{7\%}{100} = .07,\quad y = 4$ years

   $a = 2500\left(1 + \frac{.07}{12}\right)^{12\times 4}$

   $= 2500 \times (1 + .07 \div 12) \wedge (12\times 4) = 3305.1346\ldots$

   $\boxed{\approx \$3,305.13}$

   (b) What is the equivalent APR on Kiran's money market account? Use the EQUIVALENT APR FORMULA.

   $r = .07$

   $APR = \left(1 + \frac{.07}{12}\right)^{12} - 1 = (1 + .07 \div 12) \wedge 12 - 1 = .07229\ldots$

   $\approx \underbrace{.0723}_{\times 100\%} = \boxed{7.23\%}$

   (c) Kiran puts \$400 a month in her retirement account that amazingly also earns 7% interest compounded monthly. Use the FUTURE VALUE ANNUITY FORMULA to determine how much Kiran will have in her retirement account in 28 years.

   $p = \$400,\quad r = .07,\quad y = 28$ years

   $a = 400\,\dfrac{\left(1 + \frac{.07}{12}\right)^{12\times 28} - 1}{\frac{.07}{12}}$

   $= 400 \times ((1 + .07 \div 12) \wedge (12 \times 28) - 1) \div (.07 \div 12) = \boxed{\$415,475.29}$

   (d) Kiran would really like to buy a new hybrid car that sells for \$23,500. Sadly Kiran's credit rating is not very good, so the best the dealership offers is a loan at (you guessed it) 7% interest compouned monthly. Use the LOAN PAYMENT FORMULA to calculate her monthly car payments on a six year loan.

   $a = \$23,500,\quad r = .07,\quad y = 6$ years

   $p = \dfrac{23,500 \times \frac{.07}{12}}{1 - \left(1 + \frac{.07}{12}\right)^{-12\times 6}}$

   $= 23,500 \times .07 \div 12 \div (1 - (1 + .07 \div 12) \wedge ((-)12 \times 6)) =$

   $\boxed{\approx \$400.65}$

2. Tim and Josh are saving for their kids' college in fifteen years. The account pays the equivalent of 5.4% interest compounded monthly (taking into consideration various tax incentives).

   (a) Make a table comparing how much they will have after fifteen years if they contribute \$100 per month vs. \$500 per month vs. \$1,000 per month. Use the FUTURE VALUE ANNUITY FORMULA.

$$r = 5.4\% \xrightarrow[\div 100\%]{} .054 \qquad y = 15 \text{ years}$$

$$a = p * \frac{\left(1+\frac{.054}{12}\right)^{12\times 18} - 1}{\frac{.054}{12}} = p \times \left(\left(1+.054\div 12\right)\wedge(12\times 18)-1\right)\div(.054\div 12) =$$

P	100	500	1000
a	27,640	138,203	276,406

check \$1000/mo is worth 10 times \$100/mo ☺

   (b) Tim's parents decide to put \$15,000 into the account now. How much will that deposit be worth in fifteen years? Use the COMPOUND INTEREST FORMULA.

$$r = .054 \qquad y = 15 \text{ years} \qquad P = 15{,}000$$

$$a = 15{,}000\left(1+\frac{.054}{12}\right)^{12\times 15} = 15000 \times (1+.054\div 12)\wedge(12\times 15) =$$

$$= \boxed{\$33{,}657.41}$$

3. Use the EQUIVALENT APR FORMULA to find the APR for each of the following published interest rates (compounded monthly) offered by recent credit card companies.

   (a) 9% $r = 9\% \xrightarrow[\div 100\%]{} .09$ $\quad APR = (1+.09\div 12)\wedge 12 - 1 = .093806\ldots$

$$\approx .0938 \xrightarrow[\times 100\%]{} \boxed{9.38\%}$$

   (b) 12.8% $r = 12.8\% \xrightarrow[\div 100\%]{} .128$ $\quad APR = (1+.128\div 12)\wedge 12 - 1 = .135782\ldots$

$$\approx .1358 \xrightarrow[\times 100\%]{} \boxed{13.58\%}$$

   (c) 20.19% $r = 20.19\% \xrightarrow[\div 100\%]{} .2019$

$$APR = (1+.2019\div 12)\wedge 12 - 1 = .221671\ldots \approx .2217 \xrightarrow[100\%]{} \boxed{22.17\%}$$

Check: in each case APR is slightly larger ✓ than the published rate.

4. Cesar and Eliana are looking at three different houses to buy. The first, a large new townhouse, for \$240,000. The second, a small but charming bungalow, for \$260,000. The third, a large 2-story house down the block, for \$280,000.

$y = 30$ years

(a) Calculate the monthly payment for each house for a 30-year mortgage at 3.5% interest compounded monthly. Use the LOAN PAYMENT FORMULA.

$r = \frac{3.5\%}{100\%} = .035$

Townhouse $a = \$240{,}000$

$$p = \frac{240{,}000 * \frac{.035}{12}}{1-\left(1+\frac{.035}{12}\right)^{-12\times30}}$$

$= 240{,}000 \times .035 \div 12 \div (1-(1+.035\div12)\wedge((-)12\times30)) =$

$= 1{,}077.70725 \boxed{\approx \$1{,}077.71}$

Bungalow $a = \$260{,}000$

$$p = \frac{260{,}000 * \frac{.035}{12}}{1-\left(1+\frac{.035}{12}\right)^{-12\times30}}$$

$= 260{,}000 \times .035 \div 12 \div (1-(1+.035\div12)\wedge((-)12\times30)) =$

$= 1{,}167.5161\ldots \boxed{\approx \$1{,}167.52}$

2-Story $a = \$280{,}000$

$$p = \frac{280{,}000 * \frac{.035}{12}}{1-\left(1+\frac{.035}{12}\right)^{-12\times30}}$$

$= 280{,}000 \times .035 \div 12 \div (1-(1+.035\div12)\wedge((-)12\times30)) =$

$= 1{,}257.3251\ldots \boxed{\approx \$1{,}257.33}$

(b) Describe the effect on Cesar and Eliana's monthly payment of each \$20,000 increase in the house price at this interest rate.

\$1,167.52
− \$1,077.71
\$ 89.81

\$1,257.33
− \$1,167.52
\$ 89.81

≈ \$90/mo.

Each \$20,000 increase in price adds about \$90/month to their mortgage payment