Solutions

3.2 Solving linear inequalities – Practice exercises

1. A truck hauling bags of grass seed weighs 3,900 pounds when it's empty. Each bag of seed it carries weighs 4.2 pounds. The equation for the gross weight W pounds is

$$W = 3,900 + 4.2B$$

for B bags of grass seed.   *Story also appears in 2.1 #1 and 3.1 #1*

largest allowed $W \leq 18000$

(a) The state highways have a 18,000 pound gross weight limit. How many bags of grass seed can the truck can haul? Set up and solve an inequality.

$$3,900 + 4.2B \leq 18,000$$
$$-3,900 \qquad\qquad -3,900$$
$$4.2B \leq 14,100$$
$$\frac{4.2B}{4.2} \qquad \frac{14,100}{4.2}$$

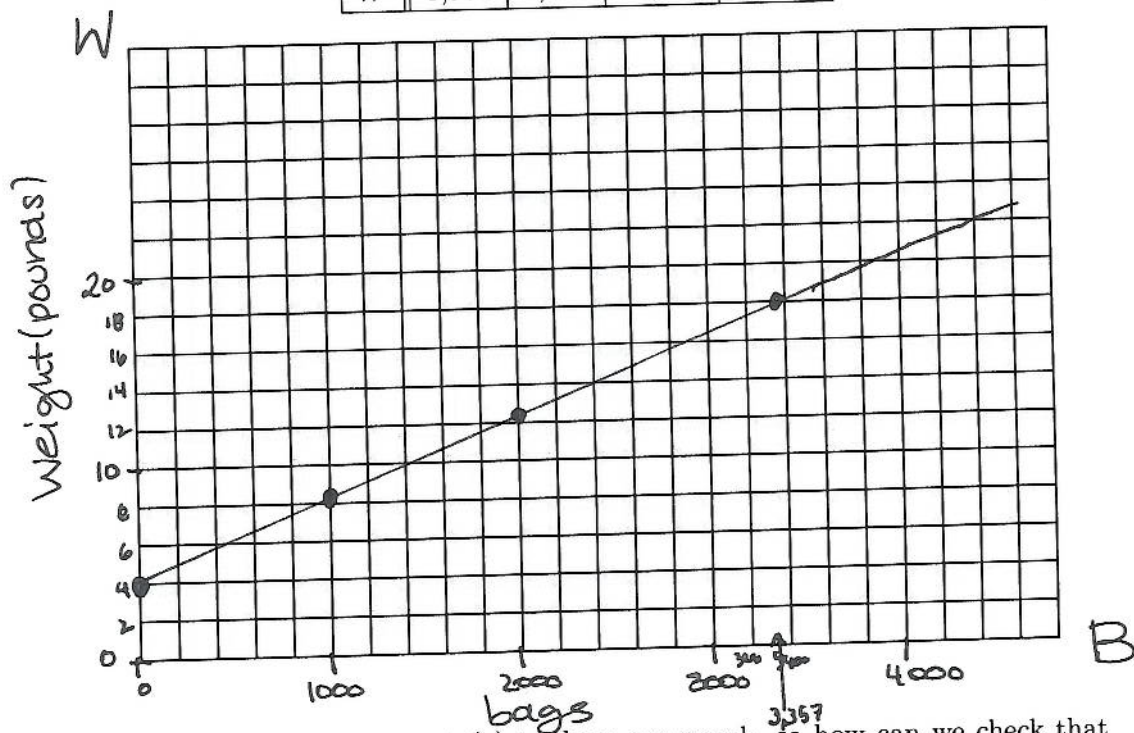$$B \leq 14,100 \div 4.2 = 3,357.14\ldots$$

3,357 bags

↑ round down to stay under 18,000 lbs.

(b) Record your answer to part (a) in the table and graph the function.

B	0	1,000	2,000	18,000 ← 3,357
W	3,900	8,100	12,300	18000

oops!!

weight (pounds)

20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0

1000   2000   3000   3357   4000

bags

(c) We used our answer to part (a) to draw our graph, so how can we check that answer to make sense? *Hint: what shape should the graph be?*

should be a **line** ✓ Yes ☺

2. The altitude, A feet above ground, of an airplane M minutes after it begins its descent is given by the equation

$$A = 32{,}000 - 1{,}200M$$

Answer each question by evaluating; setting up and solving an equation; or setting up and solving an inequality, whichever is most appropriate.

(a) At what altitude does the plane begin its descent?

Start ⇒ M=0 minutes   A=32000 −1200 × 0 = 32000

Of course! That's the intercept ☺   A = 32,000 feet

(b) How fast is the airplane descending?

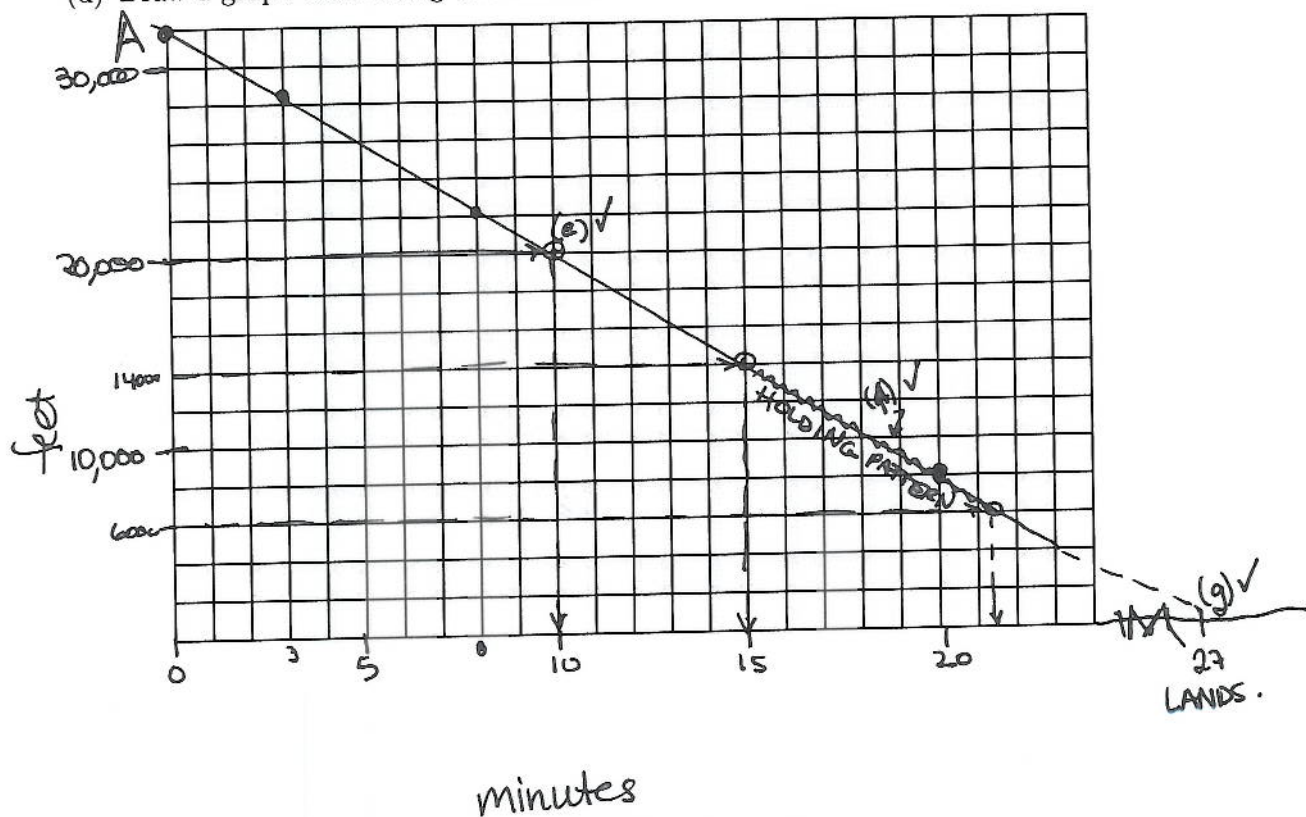— 1200 feet/minute

(c) What is the airplane's altitude 3 minutes into its descent? 8 minutes? 20 minutes? Display your answers in a table.

M	0	3	8	20
A	32000	28,400	22,400	8000

↑ from earlier

↖ 32000 − 1200 × 3 =

(d) Draw a graph illustrating the function.

feet

A

30,000

20,000

14000

10,000

6000

(e)✓

HOLDING PATTERN (f)✓

(g)✓

LANDS.

0   3   5   8   10   15   20   27

minutes

The problem continues ...

(e) For how many minutes of its descent is the airplane above 20,000 feet?

smallest allowed $A \geqslant 20,000$

$$32000 - 1200M \geq 20,000$$
$$-32000 \qquad\qquad -32,000$$

$$-1200M \geq -12,000$$
$$\overline{-1200} \qquad \overline{+1200}$$

$\div$ negative switches $\geq$ to $\leq$

$$M \leq \boxed{10 \text{ minutes}}$$

check graph ✓

(f) The airplane might be asked to go into a **holding pattern** (that means flying in a circle instead of landing) when it's between 6,000 and 14,000 feet up. When will the plane be in that altitude range?

between: $6000 \leq A \leq 14000$

$$6000 \leq 32000 - 1200M \leq 14000$$
$$-32000 \quad -32000 \qquad\qquad -32000$$

$$\underset{-25,000}{-26,000} \leq -1200M \leq -18000$$
$$\overline{-1200} \qquad \overline{-1200} \qquad \overline{-1200}$$

$\div$ negative switches $\leq$ to $\geq$

check graph ✓

$$\underset{21 \approx 20.33...}{22 \approx 21.67} \geq M \geq 15$$

$\boxed{\text{between 15 and 22 minutes after begin descent}}$

(g) How long does it take the airplane to land, assuming it's not asked to go into a holding pattern?

means on ground $\Rightarrow A = 0$ *feet*

$$32000 - 1200M = 0$$
$$-32000 \qquad\qquad -32,000$$

$$-1200M = -32,000$$
$$\overline{-1200} \qquad \overline{-1200}$$

$$M = 26.67... \qquad \boxed{\approx 27 \text{ minutes to land}}$$ check graph ✓

✦ Make sure you set-up and solved. (See instructions on previous page)

3. Anthony and Christina are trying to decide where to hold their wedding reception. For each possible site, write an equation using T for the total cost of their wedding reception (in dollars) and G for the number of guests. Then set up and solve an inequality to calculate the number of guests Tony and Tina can afford on their $8,000 budget.

largest allowed $T \le 8000$

(a) The Metropolitan Club costs $1,300 for the space and $92 per person.

Story also appears in 1.2 #3 and 1.3 #2

equation: $T = 1300 + 92G$

number of guests:
$$1300 + 92G \le 8000$$
$$-1300 \qquad\qquad -1300$$
$$92G \le 6{,}700$$
$$\frac{92G}{92} \le \frac{6{,}700}{92}$$

$G \le 72.82\ldots$
round down to stay within budget

72 guests

(b) Black Elk Park charges $500 to rent the pavilion and the family can bring in picnic food for $65 per person.

equation: $T = 500 + 65G$

number of guests:
$$500 + 65G \le 8000$$
$$-500 \qquad\qquad -500$$
$$\frac{65G}{65} \le \frac{7500}{65}$$

$G \le 115.38\ldots$

115 guests

(c) The Dabbling Duck Inn charges $1,400 for the space and $80 per person for their local specialties.

equation: $T = 1400 + 80G$

number of guests:
$$1400 + 80G \le 8000$$
$$-1400 \qquad\qquad -1400$$
$$\frac{80G}{80} \le \frac{6600}{80}$$

$G \le 82.5$

82 guests

(d) Pranzo Ristorante has only a $300 room rental fee but averages $145 per person, including wine.

equation: $T = 300 + 145G$

number of guests:
$$300 + 145G \le 8000$$
$$-300 \qquad\qquad -300$$
$$\frac{145G}{145} \le \frac{7700}{145}$$

$G \le 53.10\ldots$

53 guests

4. One variety of blueberry plant yields an average of 130 blueberries per season but there's quite a bit of variability from plant to plant. One measure of this variability is the standard deviation, which is approximated at 16.4 berries. Given a plant yielding B blueberries, we can calculate how usual or unusual that is by computing its (standard) z-score using the equation

$$Z = \frac{B - 130}{16.4}$$

For example, a plant yielding $B = 130$ blueberries has z-score of 0. A plant yielding $B = 173$ blueberries has z-score of

$$Z = \frac{173 - 130}{16.4} = (173 - 130) \div 16.4 = .671875 \approx .67$$

Answer each question by evaluating; setting up and solving an equation; or setting up and solving an inequality, whichever is appropriate. ___ oops!!.

(a) Calculate the z-score of a plant yielding 140 blueberries.

$$Z = \frac{140 - 130}{16.4} = (140 - 130) \div 16.4 = .0656\ldots$$

$$\boxed{\approx .066}$$

(b) If the z-score for a plant is -.7, what is the corresponding yield? $Z = -.7$
   Hint: the negative z-score tells us the answer is below average.

$$16.4 \times \left(\frac{B - 130}{16.4}\right) = (-.7) \times 16.4$$

$$B - 130 = (-).7 \times 16.4 = -11.48$$
$$+130 \qquad\qquad +130$$

$B = 118.52$

$\boxed{\approx 119 \text{ blueberries}}$

(c) A plant with z-score above 1.96 is considered extraordinarily plentiful. What yields of blueberries would be considered extraordinarily plentiful?

smallest allowed $Z \geq 1.96$

$$16.4 \times \left(\frac{B - 130}{16.4}\right) \geq (1.96) \times 16.4$$

$$B - 130 \geq 1.96 \times 16.4 = 32.144$$
$$+130 \qquad\qquad +130$$

$B \geq 162.144$ round up

$\boxed{163 \text{ blueberries or more}}$

(d) A plant with z-score between -1 and +1 are considered ordinary. What yields of blueberries are considered ordinary?

between $-1 \leq Z \leq 1$

$$-1 \leq \left(\frac{B - 130}{16.4}\right) \leq 1$$
$$\times 16.4 \quad \times 16.4 \qquad\qquad \times 16.4$$

$$-16.4 \leq B - 130 \leq 16.4$$
$$+130 \qquad +130 \qquad +130$$

$$113.6 \leq B \leq 146.4$$

$\boxed{\text{between } 114 \text{ and } 146 \text{ blueberries}}$