Workbook for

Just Enough Algebra

Dr. Suzanne Dorée
Professor of Mathematics
Augsburg College
Minneapolis, MN 55454

Fall 2013 edition

Contents

Chapter 1

Variables

1.1 Variables and functions – Practice exercises

1. A 32 pound bag of dog food costs \$29.97, but a 8 pound bag costs \$11.28.

   (a) Identify and name the variables, including the units.

   (b) Which variable is dependent and which is independent?

   (c) What might a 16 pound bag of dog food cost? Explain the reasoning behind your guess.

2. Rent in the Riverside Neighborhood is expected to increase 7.2% each year. Average rent for an apartment is currently \$830 per month. *Story also appears in 3.4 #3*

   (a) Identify and name the variables, including the units.

   (b) Explain the dependence using a sentence of the form "___ is a function of ___"

   (c) Which number is a constant in this story: the percent increase (7.2) or the apartment rent (830)?

   (d) What is a realistic domain for this function? That means, for how many years might this sort of increase in rent continue? Express your answer as an inequality.

   (e) What is the average rent expected to be in 1 year? In 2 years? In 3 years? Note that
   $$7.2\% = \frac{7.2}{100} = 7.2 \div 100 = .072$$
   Try figuring it out.

3. Round each number up, down, or off to the precision indicated.
   For a discussion of rounding, see Prelude: approximation

   (a) My calculations show I'll need a cross brace around 9.388 feet long. I want the board to be long enough, so round up to the nearest foot.

   (b) Gas mileage is usually rounded down to the nearest one decimal place. What is the gas mileage for a car measured as getting 42.812 miles per gallon? What about a car getting 23.09 miles per gallon?

   (c) The original budget estimates for the new community center gym were rounded to the nearest hundred (that means ending in 00), so we want to round our bid of $148,214.79 to the nearest hundred.

   (d) The population estimate was 4.2 million people, but revised estimates suggests 4,908,229 people. Report the revised estimate rounded appropriately.

4. It's about time! In each story, time is one (or both) of the variables. Identify and name the variables, including units and dependence.

   Stories also appear in 1.1 Exercises

   (a) The Nussbaums planted a walnut tree years ago when they first bought their house. The tree was 5 feet tall then and has grown around 2 feet a year.

   (b) After his first beer, Stephen's blood alcohol content (BAC) was already .04 and as he continued to drink, his BAC level rose 45% per hour.

   Story also appears in 2.4 Exercises and 3.4 #1

   (c) When McKenna drives 60 mph (miles per hour) it takes her 20 minutes on the highway to get between exits, but when traffic is bad it can take her an hour.

   (d) The sun set at 6:00 p.m. today and I heard on the radio that it sets about 2 minutes earlier each day this time of year. *Hint: measure the sunset time in minutes after 6:00 p.m.*

1.2 Tables and graphs – Practice exercises

1. My grandfather had \$200 in savings bonds that matured in 1962 when he gave them to me. The bonds continue to earn interest at a fixed rate so I have yet to cash them in. The table shows some values. *Story also appears in 4.1 #3 and 5.3 #1*

year	1962	1970	1980	1990	2000	2010
Y	0	8	18	28	38	48
B	200.00	318.77	570.87	1,022.34	1,830.85	3,278.77

(a) What do Y and B stand for? Include the units and dependence.

(b) What were the savings bonds worth in 1970?

(c) When were the savings bonds worth \$1,022.34?

(d) Approximately when were the savings bonds worth \$1,500?

(e) What do you expect the savings bonds will be worth in 2020?

(f) Graph the function using the information given in the table.

(g) Use the graph to check your answers to the questions.

2. How cold is it? An air temperature of 10°F is cold but manageable. But add a 30 miles per hour wind and, brrr, it feels like it's -12°F (12 below zero). We say the **wind chill** of 10°F with a 30 mph wind is -12°F. The table lists the wind chill for various wind speeds at an air temperature of 10°F. Source: National Weather Service

Wind (mph)	0	5	10	15	20	25	30	35	40	45	50	55	60
Wind chill (°F)	10	1	-4	-7	-9	-11	-12	-14	-15	-16	-17	-18	-19

Story also appears in 4.1 #3

(a) At an air temperature of 10°F with a 20 mph wind, what's the wind chill?

(b) A **cold advisory** is issued whenever the wind chill falls below 0°F. How fast does the wind need to be at an air temperature of 10°F to issue a cold advisory?

(c) Between a wind chill of 0°F and -15°F, schools in our district are open but kids can't go outside for recess. What's the corresponding range of wind speeds at an air temperature of 10°F?

(d) Draw a graph showing how wind chill depends on wind speed and use it to check your answers. Extend the vertical axis both above and below the horizontal axis so you can scale for the negative numbers.

3. Anthony and Christina are trying to decide where to hold their wedding reception. The Metropolitan Club costs \$1,300 for the space and \$92 per person.

   Story also appears in 1.3 #2 and 3.2 #3

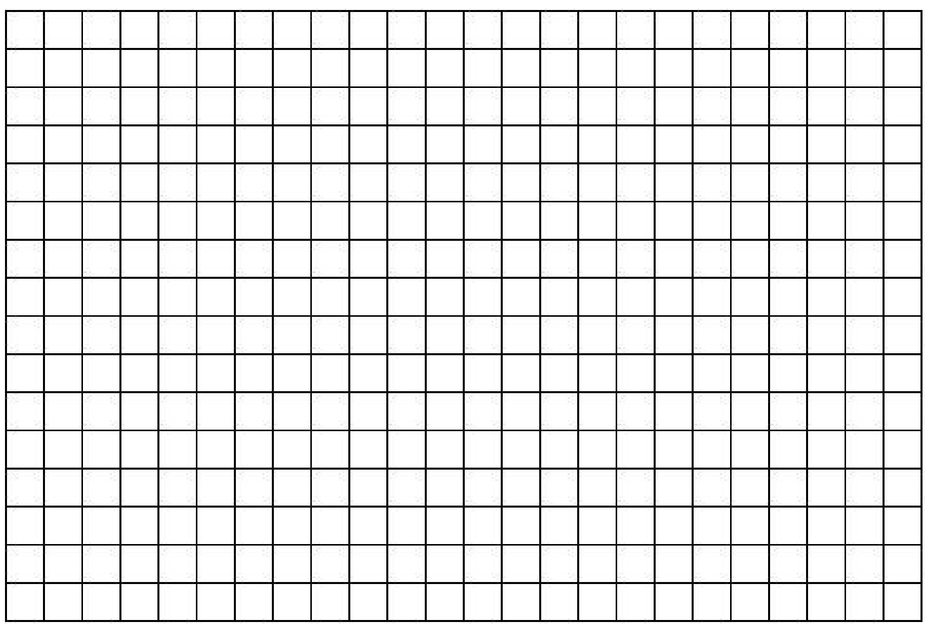
   (a) Identify and name the variables, including units.

   (b) Explain the dependence using a sentence of the form "___ is a function of ___"

   (c) Make a table of showing the cost for 20, 50, 75, 100, or 150 people.

   (d) If Tony and Tina's budget is \$8,000, how many people can they invite to their wedding reception? Give a rough estimate from your table.

   (e) Graph the function.



   (f) Does your estimate agree with your graph? If not, revise.

   (g) Can you figure out from the story exactly how many guests Tony and Tina can invite to their wedding reception and stay within their \$8,000 budget?

4. A mug of coffee costs \$3.45 at Juan's favorite cafe.

Story also appears in 2.1 #4 and 4.2 #2

(a) Juan buys coffee on the way to work every day. How much does Juan spend on coffee in a month? Let's say that's 22 workdays.

(b) If Juan pays \$10 for a discount card, then coffee costs \$2.90/mug instead. How much (total) would Juan spend on coffee in a month if he buys the discount card first? Still use 22 workdays. Include the \$10.

(c) Does the card pay for itself within the month? That means, is the total with the card (including the \$10 for the card) less than the total without the card?

(d) Complete the table, where M is the number of mugs of coffee Juan buys and T is the total cost, in dollars.

M	0	10	22	50
T (regular)				
T (with card)				

(e) Draw a graph illustrating both functions.

(f) What does the point where the two lines cross mean in terms of the story?

1.3 Rate of change – Practice exercises

1. Sweet Rose Bakery makes cakes and cupcakes. Here are their prices.

Cake

Servings	10	20	50
Cost	\$11.95	\$19.95	\$40.95

Cupcakes

Servings	12	24	48
Cost	\$6.95	\$13.90	\$27.80

(a) Calculate the rate of change for cake prices, in \$/person, if there are between 10 and 20 people. Repeat for between 20 and 50 people.

(b) Calculate the rate of change for cupcake prices, in \$/person, if there are between 12 and 24 people. Repeat for between 24 and 48 people.

(c) One the same set of axes, graph how the price depends on the number of people for cake and also for cupcakes. Connect each line or curve smoothly.

(d) The rate of change for cupcakes is constant. Any idea why?

(e) The rate of change for cakes is not constant. Any idea why?

2. Anthony and Christina are trying to decide where to hold their wedding reception. The Metropolitan Club costs $1,300 for the space and $92 per person.

   Story also appears in 1.2 #3 and 3.2 #3

   (a) Make a table showing the cost for 20, 50, 75, or 100 people.

   (b) Calculate the extra cost for each additional person between 20 and 50 people.

   (c) Calculate the extra cost for each additional person between 75 and 100 people.

   (d) What do you notice?

   (e) Explain why the graph of this cost function is a line.

   (f) Is the cost function increasing, decreasing, or neither?

3. Rashad measured his heart rate several times after football practice. Right after practice his heart rate was 178 beats per minute. Two minutes later, it had dropped to 153 beats per minute, and by ten minutes after practice it was down to 120 beats per minute.

   (a) Make a table showing Rashad's heart rate at each time.

   (b) Identify the variables, including units and dependence.

   (c) How quickly was Rashad's heart rate dropping during the first two minutes following practice? *Hint: the units are beats per minute per minute.*

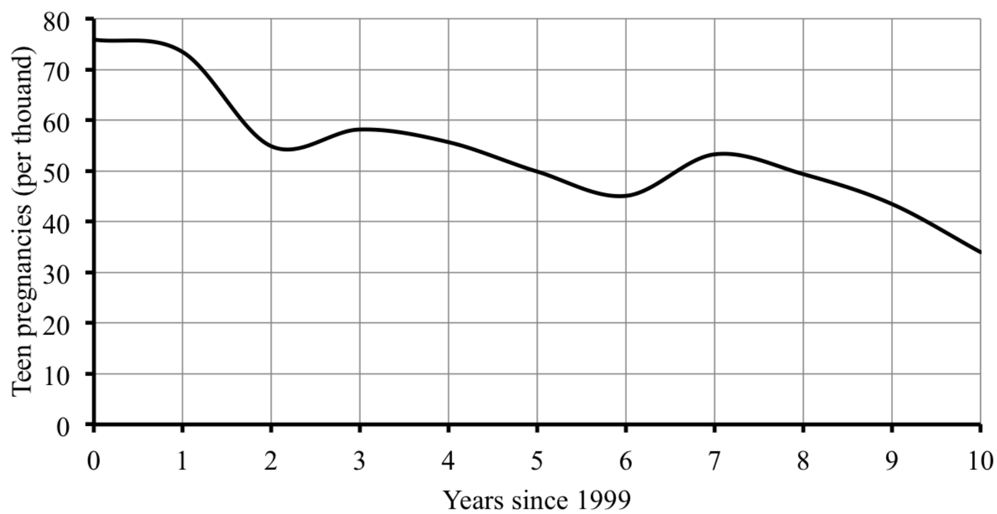
   (d) How quickly was his heart rate dropping during the next time period?

   (e) Rashad does not like hitting the showers until his heart rate is closer to normal, or at least below 100. He usually waits 15 minutes after practice. Do you think that's long enough? Explain.

   (f) Did Rashad's heart rate increase, decrease, or neither?

4. Teen pregnancy rates for Minneapolis (pregnancies per thousand teens) are summarized in the graph and table. Minnesota Department of Health

Year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Teen preg	76.0	73.5	54.9	58.2	55.7	49.9	45.1	53.3	49.4	43.5	34.0



(a) What was the teen pregnancy rate in 2003?

(b) Did the teen pregnancy rate increase or decrease from 2003 to 2004?

(c) While the teen pregnancy rate has generally decreased, from 2001 to 2002 it actually increased. Were there other times when it increased?

(d) When did the teen pregnancy rate first fall below 60 pregnancies per thousand teens?

(e) How fast was the teen pregnancy rate dropping on average per year from 2002 to 2005? How does that compare to 2006 to 2009?

1.4 Units – Practice exercises

1. (a) Compare centimeters (cm) and inches, using that 1 inch $\approx$ 2.54 cm

   i. Which is longer: 1 inch or 1 centimeter?

   ii. Kamari is shopping at an internationally-based retail store. She's looking at a curtain rod that will project 10 cm from the wall. What is that in inches?

   iii. She also wants a basket no more than 1 foot wide or long to fit on her bookcase. How many centimeters are in a foot?

   (b) Compare meters (m) and yards using that 1 yard $\approx$ .9144 m

   i. Which is longer: 1 yard or 1 meter?

   ii. Princeton was watching the Olympics and noticed everything was measured in meters. He's curious how long a football field (100 yards) is in meters.

   iii. Kamari found a really big bath towel she likes. It's 1 meter wide and 1.5 meters long. What are the dimensions in inches? Use that 1 yard = 3 feet.

   (c) Compare kilometers (km) and miles using that 1 mile $\approx$ 1.609 km

   i. Which is longer: 1 mile or 1 kilometer?

   ii. This weekend Princeton and Kamari are doing a 5K run. How many miles long is that? Note: **5K** is short for 5 kilometers.

   iii. Princeton is actually in training for a marathon. How many kilometers is that? Note: a **marathon** is approximately 26.2 miles.

2. (a) Yesterday Cameron worked for 2 hours and 15 minutes (that's 2:15) and then went home and studied for 7 hours and 57 minutes (that's 7:57). Convert each time into decimal hours.

   (b) Ephriam works at a plant that produces very delicate electronic switches. He measured the lifetime for one switch at 4.18 hours. Another had lifetime 19.50 hours. Convert each time into hours and minutes. *That means H:MM format.*

   (c) Phillip measured his office using a digital measure. One wall is 21.8 feet. The other is 10.2 feet. How long is each wall measured in the more usual feet and inches?

   (d) The couch Stetson wanted to buy is 92" long and 44" tall. Convert the length and height to feet and inches.

   (e) Abdi volunteers at a food bank. He noticed that the shelf on the back wall was bowing so he measured its length at 12'5". The formula for load needs the length written as a decimal. Convert the length to a decimal number of feet.

3. Some people say we should drink 8 glasses of water (or other liquids) every day, where a glass is defined as 8 (liquid) ounces.

   (a) Ingrid uses a 20 ounce unbreakable plastic bottle. How many of those bottles full of water does she need to drink each day?

   (b) Siri carries around a insulated water bottle that holds .6 liters. How many of those bottles full of water does she need to drink each day? Use that 1 liter $\approx$ 1.057 quarts and 1 quart $=$ 32 (liquid) ounces.

   (c) To meet the recommendation, how much water would one person drink in an entire year? Give the answer in gallons. Use 1 gallon $=$ 4 quarts.

4. Jenna is studying in Finland this term and rented an older car to drive.

   (a) She learns that no matter what the road signs might say, the maximum speed limit in Finland in winter is never more than 100 km/hr. How fast is that in miles per hour (mph)? Use 1 mile $\approx$ 1.609 km.

   (b) Jenna's car holds 62 liters of gasoline in its tank. How many gallons is that? Use 1 liter $\approx$ 1.057 quarts and 1 gallon = 4 quarts.

   (c) Her car gets 7.6 km/liter. Convert to miles per gallon (mpg).

   (d) Gas prices in Finland were 1.658 €/liter. What's the equivalent price in \$/gal? The symbol € stands for euro. Use 1 € $\approx$ \$1.23.

   (e) What would it cost Jenna, in euros, for a full tank of gas? In dollars?

1.5 Metric prefixes and scientific notation – Practice exercises

1. The GDP (gross domestic product) of the United States was approximately \$15,596 billion in 2011 and the population of the United States was approximately 0.313 billion that year. Source: U.S. Bureau of Economic Analysis, U.S. Census Bureau

   (a) That's a strange way to write the population as 0.313 billion. A more natural unit would be millions. Rewrite the population in millions of people.

   (b) Rewrite the population in people, both in normal decimal notation (that means with all the 0s) and in scientific notation.

   (c) That's also a strange way to write the GDP as \$15,596 billion. A more natural unit would be **trillions** where

   $$1 \text{ trillion} = \ 1{,}000{,}000{,}000{,}000$$

   Rewrite the GDP in trillions of dollars.

   (d) Rewrite the GDP in dollars, both in normal decimal notation and in scientific notation.

   (e) Calculate the GDP **per capita** (meaning per person) by dividing the GDP in dollars by the population in people. Express your answer in \$/person.

   (f) For practice, repeat your calculation using the numbers in scientific notation. *Because* $\times$ *and* $\div$ *are at the same level in the order of operations, you need to put parentheses around each number in scientific notation before dividing.*

2. Edgar recently changed the cleaning bag on his vacuum cleaner. He became curious about how many particles of dust were in the bag. Edgar did a little research online and found out that the mass of a dust particle is .000 000 000 753 kilograms.

   (The strange-looking spaces are to help you see that there are 9 zeros in the number.)

   (a) Write the mass of a dust particle in scientific notation.

   (b) Recall that

kilo	=	1 thousand	=	1,000	=	10^3
milli	=	1 in a thousand	=	.001	=	10^{-3}
micro	=	1 in a million	=	.000001	=	10^{-6}
nano	=	1 in a billion	=	.000 000 001	=	10^{-9}

   Express the mass of a dust particle in each of the given units.

   i. grams

   ii. milligrams (mg)

   iii. micrograms (μg)

   iv. nanograms (ng)

   (c) Edgar determined that the full vacuum bag weighed 5 pounds. How many dust particles were in the bag? (I'm sneezing already.) Use 1 kilogram $\approx$ 2.2 pounds. Express your answer in scientific notation.

3. The list shows the (approximate) mass of the planets in our solar system.

Earth	5.972×10^{24} kg
Jupiter	1.899×10^{27} kg
Mars	6.417×10^{23} kg
Mercury	3.302×10^{23} kg
Neptune	1.024×10^{26} kg
Saturn	5.685×10^{26} kg
Uranus	8.681×10^{25} kg
Venus	4.868×10^{24} kg

Source: Wikipedia (Solar System)

(a) Write the mass of Earth and the mass of Mars in standard decimal notation. Which is heavier?

(b) List the planets from heaviest (largest mass) to lightest (smallest mass).

(c) The mass of astronomical bodies are sometimes measured in **Jupiter mass** abbreviated M_J where $1M_J = 1.899 \times 10^{27}$ kg. Express Earth's mass in M_J.
Because $\times$ and $\div$ are at the same level in the order of operations, you need to put parentheses around each number in scientific notation before dividing.

4. Souksavanh is setting up a patient's intravenously (IV) medication. She sets the drip at 42 drops/minute. The drip chamber size is 20 drops/mL. Recall

$$\begin{array}{lclclcl} \textbf{milli} & = & \text{1 in a thousand} & = & .001 & = & 10^{-3} \\ \textbf{micro} & = & \text{1 in a million} & = & .000001 & = & 10^{-6} \end{array}$$

(a) At what rate is the IV fluid being delivered to Souk's patient? Answer in mL/min (that's millileters per minute).

(b) How fast is the drip measured in μL/sec (that's microliters per second)?

(c) If the drip bag holds 1 liter, how long will it take the drip to run? Express your answer in hours and minutes.

(d) The concentration of medication is 1.7 mg/mL (that's milligrams per milliliter). How much medication is in the 1 liter bag? Convert your answer to grams. Explain what you notice.

(e) At what rate is the medication being delivered to Souk's patient? Answer in g/min (that's grams per minute).

Practice Exam 1A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

---

1. Arva and Ellie began hiking at an elevation of 1,500 feet and climbed at the steady rate of 600 vertical feet per hour.
   (a) Make a table showing their elevation after 1 hour, 2 hours, and 5 hours.

   (b) Name the variables, including units.

   (c) Explain the dependence using a sentence of the form "___ is a function of ___."

   (d) Is the function increasing or decreasing?

   (e) How long does it take them to reach 5,300 feet up? Try to figure out the answer in hours and minutes (H:MM format).

2. The table shows Henry's weight as a baby.

Age (weeks)	0	12	15
Weight (pounds)	8	14	16

(a) How much weight did Henry gain, on average, each week during his first 12 weeks?

(b) During which time interval was Henry gaining weight faster? *Explain.*

(c) Identify the variables, including units and dependence.

(d) Draw a graph illustrating the dependence. Choose a scale that shows up to 20 weeks and 20 pounds.

(e) What might you guess for Henry's weight at 20 weeks?

3. Pramesh's new car used 20.5 gallons of gas for a 715 mile trip.

   (a) How many miles per gallon (mpg) does his car get?

   (b) At that rate, how many gallons of gas would Pramesh use on his 3,200 mile cross-country trip?

   (c) If gas costs $3.799/gallon, how much will gas for that trip cost?

4. Ndwiga is reading an article in the paper about atoms. From his physics textbook he discovered that the size of an atom is .142 nanometers. (That's 0.142 nanometers.)

   (a) Write the size of an atom in meters. Use 1 meter = 1,000,000,000 nanometers. Write your answer in usual decimal notation and in scientific notation.

   (b) Ndwiga would like to know how many atoms across this sheet of paper which is 8.5 inches wide. Use that 1 inch $\approx$ 2.54 cm and 1 meter = 100 cm. Express your final answer in billions of atoms.

Practice Exam 1B

Try taking this version of the practice exam under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

---

1. The amount of money spent on nursing home care for seniors has continued to rise. The table shows the values for select years. Here S is the spending, measured in billions of dollars and Y is the year, measured in years since 1960.

Y	0	10	25	40	52
S	1.0	3.3	33.7	96.6	170.3

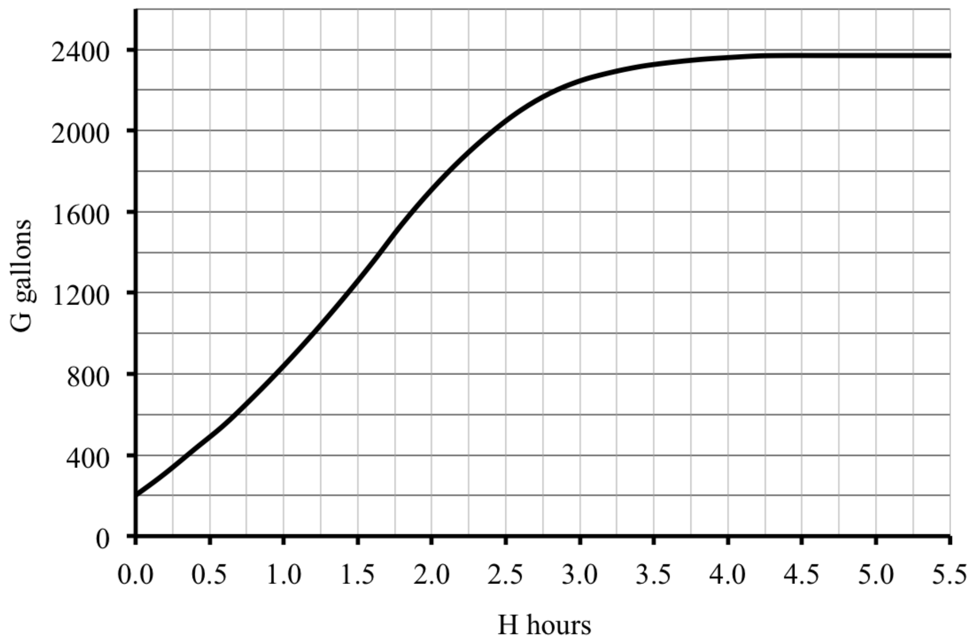
   (a) According to the table, what was the spending in 1970?

   (b) According to the table, what was the spending in 1985?

   (c) Calculate the rate of change of spending over the period 1970 to 1985. Don't forget to state the units.

   (d) In approximately what year did spending first pass $50 billion?

2. Trish is filling a swimming pool with water. The graph below shows how many gallons of water (G) are in the pool after H hours. Use the graph to answer the following questions.



(a) How much water was in the swimming pool already when Trish began?

(b) How much water was in the swimming pool after 3 hours?

(c) After how many hours were there 1,000 gallons of water in the swimming pool?

(d) Was Trish filling the pool faster at 2 hours or at 2.5 hours? Explain how you see that on the graph.

(e) After (about) how many hours did Trish stop filling the swimming pool? Explain how you see that on the graph.

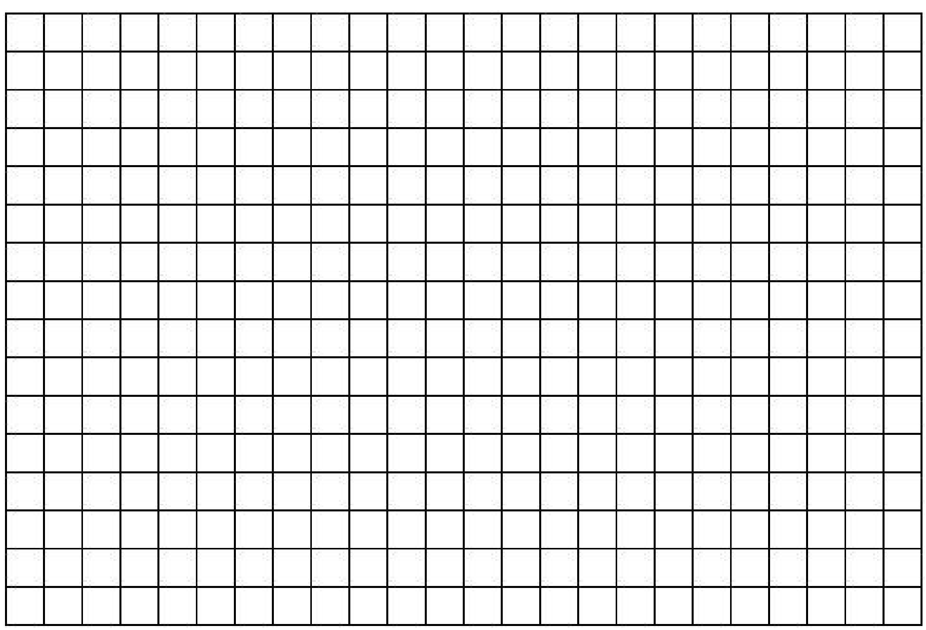
3. In 1990 the Lefèvre's property tax was \$450 but it doubled every year thereafter.

   (a) Name the variables, including units.

   (b) Which is the independent variable and which is the dependent variable?

   (c) Make a table showing the property tax each year from 1990 to 1994.

   (d) Draw a graph illustrating the dependence.



4. The distance from the Earth to the Moon is approximately 384,000,000 meters.

Source: Wikipedia (Lunar distance)

(a) Express this distance in scientific notation.

(b) Express this distance in kilometers (km), using 1 km = 1,000 meters.

(c) Express this distance in miles, using the conversion 1 mile $\approx$ 1.609 km.

(d) If you could drive to the moon at 55 mph, how long would it take to get there? Express your answer in terms of months, using 1 month $\approx$ 30 days.

Chapter 2

Equations

2.1 A first look at linear equations – Practice exercises

1. A truck hauling bags of grass seed pulls into a weigh station along the highway. Trucks are weighed to determine the amount of highway tax. This particular truck weighs 3,900 pounds when it's empty. Each bag of seed it carries weighs 4.2 pounds. For example, a truck is carrying 1,000 bags of grass seed weighs

$$3{,}900 \text{ pounds } + \frac{4.2 \text{ pounds}}{\text{bag}} * 1{,}000 \text{ bags} = 3{,}900 + 4.2 \times 1{,}000 = 8{,}100 \text{ pounds}$$

   In official trucking lingo, we say the **curb weight** of 3,900 pounds plus the **load weight** of 4,200 pounds results in a **gross weight** of 8,100 pounds. So, now you know. *Story also appears in 3.1#1 and 3.2 #1*

   (a) Calculate the gross weight of the truck if it contains 2,000 bags of grass seed.

   (b) Name the variables, including units, and write an equation showing how the gross weight of the truck is a function of the number of bag seed it contains.

   (c) Identify the slope and intercept, along with their units, and explain what each means in terms of the story.

   (d) The bags of grass seed are piled on wood **pallets** (sturdy platforms) to make them more stable for moving. How much does the truck weigh if it is carrying 12 pallets of grass seed bags, where each pallet weighs 15 pounds and holds 96 bags of seed?

2. The water in the local reservoir was 47 feet deep but there's been so little rain that the depth has fallen 18 inches a week over the past few months. Officials are worried if dry conditions continue the reservoir will not have enough water to supply the town.

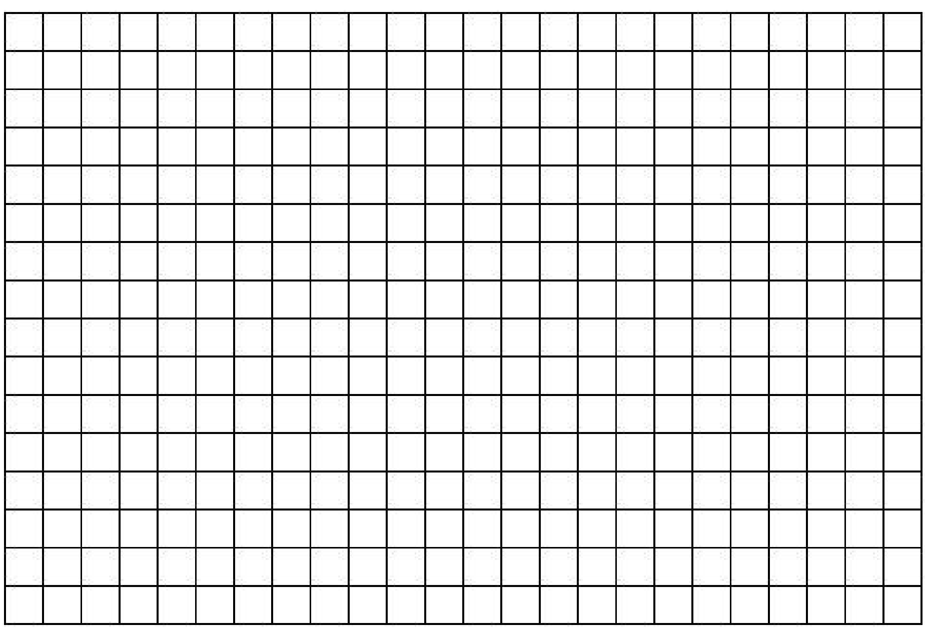
Story also appears in 3.2 Exercises and 4.1 #3

(a) Name the variables and write an equation relating them. First convert 18 inches to feet.

(b) Identify the slope and intercept, along with their units, and explain what each means in terms of the story.

(c) Make a table of values showing the projected depth of the reservoir after 1 week, 5 weeks, 10 weeks, and 20 weeks if the current trend continues.

(d) Draw a graph illustrating the function.



3. I was short on cash so I got a **line of credit** (short term loan) on my bank account, of which I spent \$2,189.57. That means my account balance is −\$2,189.57. I will pay back the interest plus an extra \$250 each month. When the loan is paid off, I plan to continue to deposit \$250 per month to start saving.

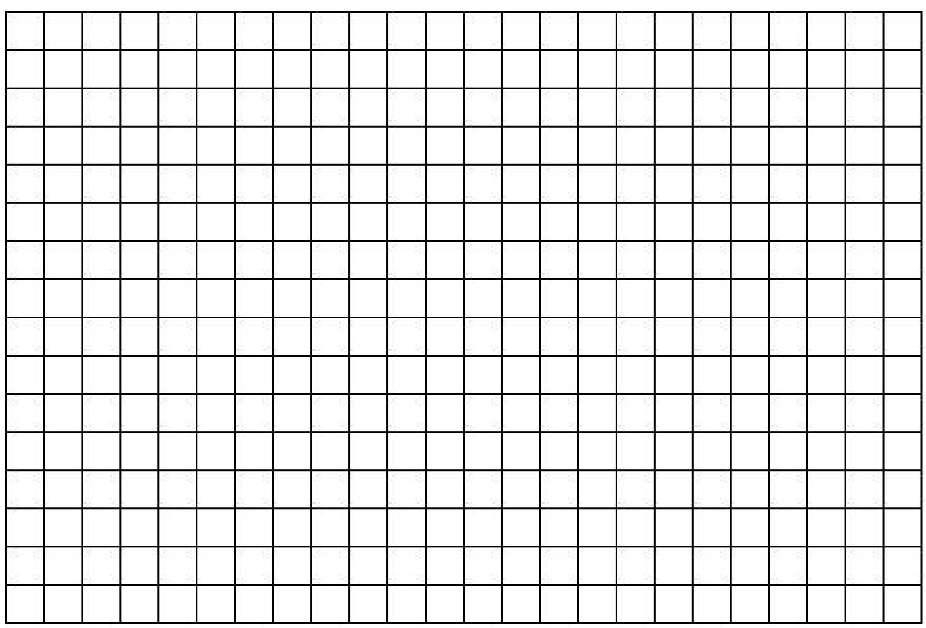
Story also appears in 3.2 Exercises

   (a) Write an equation showing my account balance, $\$B$, in M months. Ignore the interest.

   (b) Identify the slope and intercept, along with their units, and explain what each means in terms of the story.

   (c) Make a table of values showing my account balance now, after 4 months, and at the end of a year.

   (d) Draw a graph showing my account balance over this coming year.



   (e) About how many months will it take to pay off my line of credit?

4. A mug of coffee costs \$3.45 at Juan's favorite cafe, unless he buys their discount card for \$10 in which case each mug costs \$2.90. *Story also appears in 1.2 #4 and 4.2 #2*

   (a) Name the variables, including units.

   (b) Write an equation describing how the total cost depends on how many mugs of coffee Juan buys without the discount card.

   (c) Write an equation describing how the total cost depends on how many mugs of coffee Juan buys if he buys the discount card.

   (d) How would the equation change if the cafe offers a new annual membership card that cost \$59.99 but entitles Juan to buy coffee for only \$1 per mug all year?

2.2 A first look at exponential equations – Practice exercises

1. The comprehensive fee at a local private college is \$37,000. The fee is projected to increase 5.8% per year.

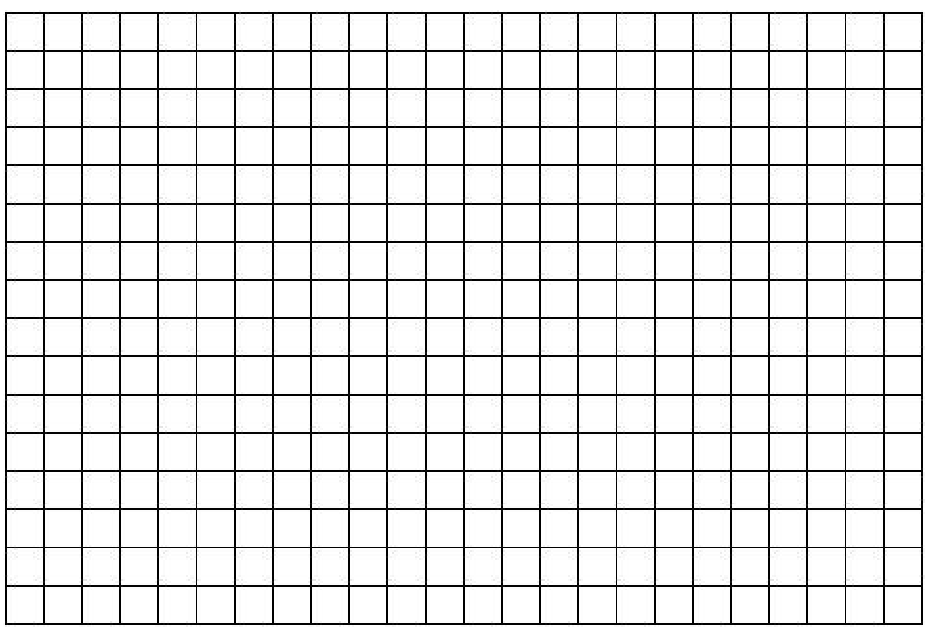
   (a) Calculate the annual growth factor.

   (b) What do you expect the comprehensive fee will be in five years?

   (c) Name the variables, including units, and write an equation describing the dependence.

   (d) Make a table of values showing the comprehensive fee now, in 5 years, 10 years, 20 years, and 50 years (even though that's not realistic).

   (e) Draw a graph illustrating the function.



2. Bunnies, bunnies, everywhere. They eat the tops of my tulips in early spring and my lilies all summer long. Back in 2007 there were an estimated 1,800 rabbits in my neighborhood. Rabbits multiply quickly, 13% per year by one estimate.

   Story also appears in 5.1#3

   (a) Name the variables, including dependency.

   (b) Calculate the annual growth factor.

   (c) What does this story suggest the rabbit population was in 2010? In 2013?

   (d) Write an equation relating the variables.

3. A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 16% per day.

   Story also appears in 5.1 #2 and 5.5 textbook

   (a) Calculate the daily growth factor and use it to write an equation describing the spread of the virus. Don't forget to name the variables too.

   (b) Make a table and graph for the six weeks following the initial diagnosis. (That means use 0, 7, 14, 21, 28, 35, and 42 days.)

   (c) What is a realistic domain? That means, for how many days do you think this model is reasonable? To keep a sense of scale, there are 1,094 students currently living in the dorms.

4. My savings account earns a modest amount of interest, the equivalent of .75% annually. I have \$12,392.18 in the account now.

   (a) How much interest will I earn this year?

   (b) What will my balance be in three years, assuming I neither deposit nor withdraw money?

   (c) Name the variables and write an equation relating them.

   (d) What would the equation be if I moved all of my money into a certificate of deposit earning the equivalent of .92%?

   (e) What would the equation be if I moved \$10,000 into that certificate of deposit, and kept the rest in savings? *Hint: to find the total balance, add the amounts.*

2.3 Using equations – Practice exercises

1. Dontrell and Kim borrowed money to buy a house on a 30-year mortgage. After M months of making payments, Dontrell and Kim will still owe $\$D$ where

$$D = 236{,}000 - 56{,}000 * 1.004^M$$

D is also known as the **payoff** (how much they would need to pay to settle the debt).

Story also appears in 3.4 #4

(a) How much did Dontrell and Kim originally borrow to buy their house? What value of M did you evaluate at to answer the question?

(b) Evaluate the equation at $M = 12$ and explain what the answer means in terms of the story.

(c) After making half the payments, how much money will Dontrell and Kim still owe on the house? Will they have paid more or less (or exactly) half of the loan? *Hint: convert 30 years into months to find the total number of payments. Then divide by 2 to find the halfway point.*

(d) The very last month they don't actually pay the regular monthly payment, just whatever balance is left on the loan. How much will that be? *Hint: they will have made all but one of the payments.*

2. "Rose gold" is a mix of gold and copper. We start with 2 grams of an alloy that is equal parts gold and copper and add A grams of pure gold to lighten the color. The percentage of gold in the resulting rose gold alloy, R is given by

$$R = 100\left(\frac{1+A}{2+A}\right)$$

For example, if we add 0.8 grams of pure gold, then $A = .8$ and so the percentage is

$$R = 100\left(\frac{1+.8}{2+.8}\right) = 100 \times (1+\underline{.8}) \div (2+\underline{.8}) = 64.28571428\ldots \approx 64.3\%$$

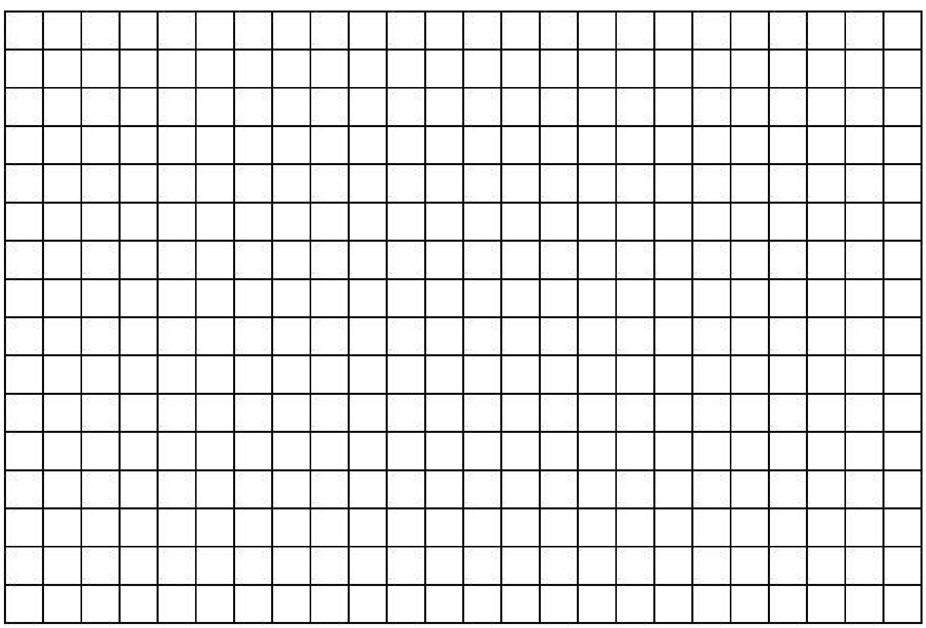
Story also appears in 4.1 Exercises

(a) Calculate the percentage of gold in the alloy if we add 1.2 grams of pure gold.

(b) Fill in that and the rest of the missing values.

A	0	.4	.8	1.2	1.6	2	3	4
R	50.0		64.3		72.2		80.0	

(c) Graph the function.



(d) What do you think happens to the percentage of gold as we add more and more pure gold? Try adding 10 grams, and then try adding 100 grams to check.

3. Monty hopes to grow orchids but they are fragile plants. He will consider his greenhouse a success if at least nine of the ten orchids survive. Assuming the orchids each survive at rate S, the probability his greenhouse is a success, P, is given by

$$P = 10S^9 - 9S^{10}$$

Story also appears in 2.4 #3

   (a) If the orchids are perfect ($S = 1$), what is the probability of a successful greenhouse? Explain how this answer makes sense in the story.

   (b) If the orchids are complete duds ($S = 0$), what is the probability of a successful greenhouse? Explain how this answer makes sense in the story.

   (c) Make a table comparing the probability of a successful greenhouse if the orchids each survive at rate $S = 0, .5, .8, .9, .95$, or 1.

   (d) Draw a graph of the function.

4. Valerie plans to do a 3-day, 50-mile walk to raise money for breast cancer research, in honor of her aunt. Valerie's friends have pledged a total of $93 per mile.

   (a) Valerie hopes to walk all 50 miles. If so, how much money will she raise?

   (b) She might have to stop sooner, however. Name variables and write an equation showing how the money Valerie raises is a function of how far she is able to walk.

   (c) How long will it take Valerie to walk the full 50 miles if she's able to keep a pace of 3.2 miles per hour? *Write your answer in H:MM format.*

   (d) Name the new variables and write a new equation showing how the time it takes Valerie to walk the full 50 miles depends on her pace.

   (e) Good news. Valerie walked the full 50 miles at a pace of 3.2 miles per hour. Way to go, girl! How much money did she raise each hour?

   Hint: Divide your answers from (a) by your answer from (c) to get $/hour.

2.4 Approximating solutions of equations – Practice exercises

1. The size of a round pizza is described by its **diameter** (distance across). Assuming a 16-inch diameter pizza serves four people, and with a little geometry to help us out, we calculated that a pizza of diameter D inches serves P people where

$$P = .015625D^2$$

Story also appears in 3.3 #1 and 4.1 #3

(a) Confirm that a 16-inch pizza serves four people.

(b) How many people does a 12-inch pizza serve? A 14-inch pizza?

(c) Graph the function. Include what happens when $D = 0$.

(d) A **personal** pizza is sized to serve one person. Use successive approximation to estimate the diameter of a personal pizza to the nearest inch.

(e) What diamter should an extra large pizza be to serve 6 people? Answer to the nearest $1/10$ inch.

2. Suppose a car gas tank is designed to hold enough fuel to drive 350 miles. (That's fairly average.) A hybrid car with fuel efficiency of 50 miles per gallon (mpg) would only need a 7 gallon gas tank, but a recreational vehicle that gets only 10 mpg would need a 35 gallon gas tank. *Story also appears in 3.3 #3*

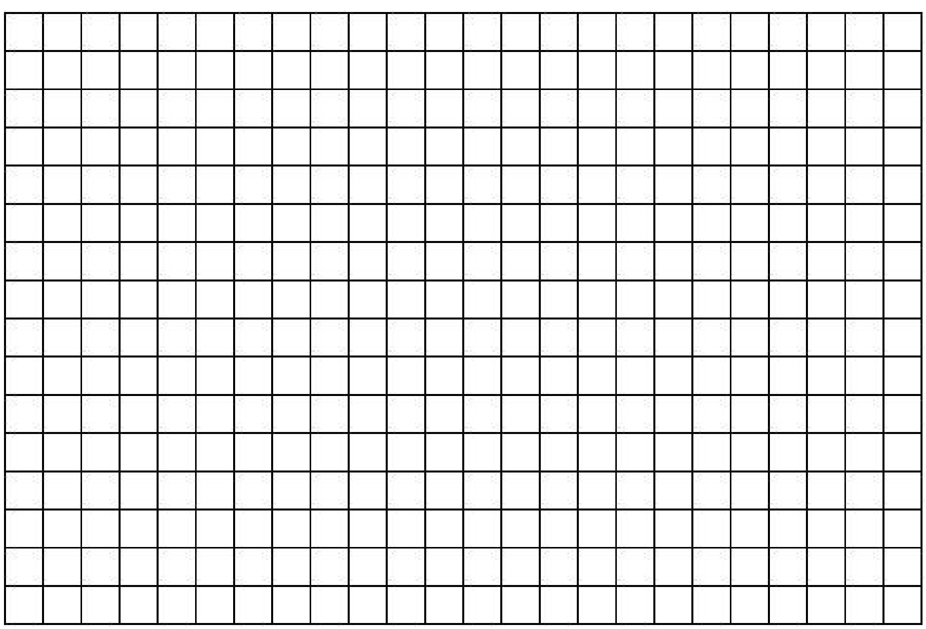
   (a) Name the variables including units. The way the story is stated, the size tank is a function of the fuel efficiency.

   (b) Write an equation describing this function.

   (c) My Honda Accord's tank holds about 16 gallons. Approximate the corresponding fuel efficiency to one decimal place.

   (d) My ex-husband's Honda Civic's tank holds only 13 gallons. Approximate the corresponding fuel efficiency to one decimal place.

   (e) Draw a graph showing how the size tank depends on the fuel efficiency.



3. Monty hopes to grow orchids but they are fragile plants. He will consider his greenhouse a success if at least nine of the ten orchids survive. Assuming the orchids each survive at rate S, the probability his greenhouse is a success, P, is given by

$$P = 10S^9 - 9S^{10}$$

Story also appears in 2.3 #3

   (a) Monty can buy orchids each with survival rate of $S = .8$. Is that enough to give probability $P \geq .8$ of a successful greenhouse?

   (b) What quality of orchids would Monty need to have probability $P \geq .8$ of a successful greenhouse? *Answer to two decimal places.*

   (c) What quality of orchids would Monty need to have probability $P \geq .95$ of a successful greenhouse? *Answer to three decimal places.*

4. After China, India, and the United States, the next five most populous countries (in 2011) are Indonesia, Brazil, Pakistan, Nigeria, and Bangladesh. Their projected growth rates and corresponding equation are listed below. Here Q is the population measured in millions and Y is the years since 2011. Source: CIA Factbook

4^{th}	Indonesia	pop. 248 million	growth rate 1.04%	$Q = 248 * 1.0104^Y$
5^{th}	Brazil	pop. 205 million	growth rate 1.10%	$Q = 205 * 1.0110^Y$
6^{th}	Pakistan	pop. 190 million	growth rate 1.55%	$Q = 190 * 1.0155^Y$
7^{th}	Nigeria	pop. 170 million	growth rate 2.55%	$Q = 170 * 1.0255^Y$
8^{th}	Bangladesh	pop. 161 million	growth rate 1.58%	$Q = 161 * 1.0158^Y$

(a) Which of these countries is projected to have the largest population in 2020? In 2030? In 2050?

(b) Explain why Bangladesh's population will not overtake Nigeria's, assuming these projections are accurate.

(c) Approximately when will Brazil's population top 500 million? Will Nigeria get there first? Display your work in a table.

2.5 Finance formulas – Practice exercises

COMPOUND INTEREST FORMULA: $a = p\left(1 + \dfrac{r}{12}\right)^{12y}$

EQUIVALENT APR FORMULA: $\text{APR} = \left(1 + \dfrac{r}{12}\right)^{12} - 1$

FUTURE VALUE ANNUITY FORMULA: $a = p * \dfrac{\left(1 + \frac{r}{12}\right)^{12y} - 1}{\frac{r}{12}}$

LOAN PAYMENT FORMULA: $p = \dfrac{a * \frac{r}{12}}{1 - \left(1 + \frac{r}{12}\right)^{-12y}}$

where

a = account balance or loan amount ($)
p = initial deposit (principal), regular deposit, or regular payment ($)
y = time invested (years)
r = interest rate compounded monthly (as a decimal)

---

1. Use the indicated formulas to help Kiran figure out her finances.

   (a) Kiran deposited $2,500 in a money market account that earned 7% interest compounded monthly. Use the COMPOUND INTEREST FORMULA to calculate her account balance after 4 years.

   (b) What is the equivalent APR on Kiran's money market account? Use the EQUIVALENT APR FORMULA.

   (c) Kiran puts $400 a month in her retirement account that amazingly also earns 7% interest compounded monthly. Use the FUTURE VALUE ANNUITY FORMULA to determine how much Kiran will have in her retirement account in 28 years.

   (d) Kiran would really like to buy a new hybrid car that sells for $23,500. Sadly Kiran's credit rating is not very good, so the best the dealership offers is a loan at (you guessed it) 7% interest compouned monthly. Use the LOAN PAYMENT FORMULA to calculate her monthly car payments on a six year loan.

2. Tim and Josh are saving for their kids' college in fifteen years. The account pays the equivalent of 5.4% interest compounded monthly (taking into consideration various tax incentives).

   (a) Make a table comparing how much they will have after fifteen years if they contribute \$100 per month vs. \$500 per month vs. \$1,000 per month. Use the FUTURE VALUE ANNUITY FORMULA.

   (b) Tim's parents decide to put \$15,000 into the account now. How much will that deposit be worth in fifteen years? Use the COMPOUND INTEREST FORMULA.

3. Use the EQUIVALENT APR FORMULA to find the APR for each of the following published interest rates (compounded monthly) offered by recent credit card companies.

   (a) 9%

   (b) 12.8%

   (c) 20.19%

4. Cesar and Eliana are looking at three different houses to buy. The first, a large new townhouse, for \$240,000. The second, a small but charming bungalow, for \$260,000. The third, a large 2-story house down the block, for \$280,000.

   (a) Calculate the monthly payment for each house for a 30-year mortgage at 3.5% interest compounded monthly. Use the LOAN PAYMENT FORMULA.

   Townhouse

   Bungalow

   2-Story

   (b) Describe the effect on Cesar and Eliana's monthly payment of each \$20,000 increase in the house price at this interest rate.

Practice Exam 2A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

---

1. United States ethanol production has been growing exponentially. In 1990, there were 0.9 billion gallons of ethanol produced. At that time it was estimated that production would increase 5.5% per year. Source: Renewable Fuels Association

   (a) Name the variables, including units.

   (b) What is the annual growth factor?

   (c) Write an equation that describes the function.

   (d) In 2008 actual production of ethanol was 9.0 billion gallons. Is that production level higher or lower than predicted from your equation? Explain.

   (e) When does your equation predict that ethanol production was (or will be) 9.0 billion gallons? Use successive approximation. Display your guesses in a table. Report the actual year.

2. An insurance **deductible** is the amount you pay for any claim before the insurance company starts paying. Lee's automobile insurance starts at \$500, but they take off \$10 for each month where he has no accidents or tickets.

   (a) Name the variables.

   (b) Make a table showing the deductible after 6 months, 1 year, or 3 years without an accident or ticket.

   (c) When would the deductible **vanish**? (Meaning, when is it \$0?)

   (d) Write an equation showing how the deductible decreases.

   (e) What is the slope and what does it mean in the story?

   (f) What is the intercept and what does it mean in the story?

3. Our investment club has been tracking the performance of a biofuel company's stock over the past year. Using an econometrics software package, we found the equation

$$V = .00004W^3 + .01W^2 - .9W + 31$$

which describes the value of each share of stock \$$V$ as a function of the week W, starting exactly one year ago.

   (a) Complete the following table of values.

W	0	13	26	39	52
V	31.00	21.08			16.86

   (b) Draw a graph showing how the value changed during the past year.

The problem continues ...

(c) According to the table, what is the value of the stock when we began tracking it? What is it worth now?

(d) We're thinking of buying some stock now, and selling it in 10 weeks. Does the equation say that's a good idea? Explain. *Hint: 10 weeks from now is not* $W = 10$ *because we started counting weeks one year ago.*

(e) Looking back over the past year, how low did the value of the stock get? Use successive approximation to estimate to the nearest cent.

4. (a) Cicely wants to buy a new car that costs \$19,400. The dealership offers 6.18% compounded monthly for a 5 year loan. What will Cicely's monthly payment be? Use the LOAN PAYMENT FORMULA.

   (b) What is the equivalent APR Cicely is paying? Use the EQUIVALENT APR FORMULA. *Don't forget to report the percentage.*

   (c) Cicely is working on her monthly budget. She has only \$230 per month left after those car payments. If she puts that money into a bank account each month earning 2.91% interest compounded monthly how much will she have after 5 years when the car is paid off? Use the FUTURE VALUE ANNUITY FORMULA.

   (d) In 2011, Cicely was cleaning out the basement and found some savings bonds with face value \$1,600 that matured in 1972 and have been earning 3% interest compounded monthly ever since. What were they worth? Use the COMPOUND INTEREST FORMULA.

Practice Exam 2B

Try taking this version of the practice exam under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

---

1. The Skärstroms want to dig a new well for water for their lake cabin. The company charges $900 to bring the equipment on site and draw the permit and then $2 per foot to dig.

   (a) What would a 100 foot deep well cost?

   (b) Name the variables and write an equation relating them.

   (c) Make a table showing the total cost for a well 100, 250, or 400 feet deep.

2. Xander grows tomatoes in his garden. He's noticed that a typical plant yields 5 pounds of tomatoes. He's been experimenting with the impact of liquid food on plant yield and estimates that each drop increases yield by 14%.

   (a) Calculate the growth factor and write an equation showing how yield for each tomato plant depends on the number of drops of liquid food. Use Y for the yield (in pounds) and D for the amount of liquid food (in drops).

   (b) Xander uses 10 drops of food on one of his tomato plans and uses all of the tomatoes from that plant to make salsa. If each pound of tomatoes makes around a pint of salsa, how much salsa will Xander have (from that one plant)?

   (c) Convert your answer into gallons. Use 1 gallon = 4 quarts and 1 quart = 2 pints. Olé!

The problem continues ...

(d) Make a table showing Xander's projections for yield for each tomato plant if he uses 0, 1, 2, 5, or 10 drops of liquid food.

(e) Graph the function.

3. Skye and her sister Clover started a t-shirt printing company. To produce a particular t-shirt it costs \$350 in materials and labor to set up a silkscreen and then \$7.50 for each shirt made to cover materials and printing. The average cost per t-shirt $\$C$ is a function of N, the number of t-shirts printed. The equation for this function is

$$C = \frac{350 + 7.50N}{N}$$

(a) Evaluate this formula when $N = 50$ and explain what the value of C you get means in the story.

(b) Make a table showing the average cost per t-shirt if Skye and Clover make 1, 20, 50, 100, or 300 t-shirts.

(c) Approximately how many t-shirts would they need to make to keep the average cost per shirt under \$10? Use successive approximation and display your guesses in a table.

The problem continues . . .

Skye designs the shirts and runs the press. Clover is the brains behind sales. She would like to price the shirts at \$12.95 each. The sisters will make a profit of $\$P$ where

$$P = 5.45N - 350$$

(d) This is a linear equation. What is the slope, what are its units, and what does it mean in the story?

(e) What is the intercept, what are its units, and what does it mean in the story?

(f) How many t-shirts do they sisters need to sell to make \$1,000 profit? Use successive approximation and display your guesses in a table.

4. (a) Kotoyo's uncle won \$100,000 on a game show. If he invests it in a fund that's expected to earn 5.7% interest compounded monthly, how much will he have after 5 years? Use the COMPOUND INTEREST FORMULA.

   (b) Kotoyo's grandmother has been contributing \$150 a month into a college fund for Kotoyo for the past 8 years. The account pays 4% interest compounded monthly. How much is in the account now? Use the FUTURE VALUE ANNUITY FORMULA.

   (c) Kotoyo owes \$8,742 on her credit card. They charge her 16% interest compounded monthly. What would her monthly payment be if she wants to pay it off in 5 years? Use the LOAN PAYMENT FORMULA.

   (d) What is the equivalent annual percentage rate (APR) of Kotoyo's credit card? Use the EQUIVALENT APR FORMULA. *Don't forget to report the percentage.*

Chapter 3

Solving equations

3.1 Solving linear equations – Practice exercises

1. A truck hauling bags of grass seed weighs 3,900 pounds when it's empty. Each bag of seed it carries weighs 4.2 pounds. The equation for the gross weight W pounds is

$$W = 3{,}900 + 4.2B$$

for B bags of grass seed. *Story also appears in 2.1 #1 & 3.2 #1*

   (a) Set up and solve an equation to determine the number of bags of grass seed being carried by the truck with gross weight of 14,500 pounds.

   (b) Do the same for a truck with gross weight 8 tons. A **ton** is 2,000 pounds

2. Is laughter really the best medicine? A study examined the impact of comedy on anxiety levels. Subjects' anxiety levels were rated on a scale of 1 to 5 before and after the study. Levels averaged 4.3 before the study. There was no significant change in subjects in the control group. Subjects who watched the comedy videos showed a noticeable difference, and it depended on the hours of comedy watched. Anxiety levels fell an average of .098 (on the 1 to 5 scale) for each hour of comedy watched.

   (a) Make a table showing average anxiety levels for subjects who watched comedy videos for 0 hours (control group), 2 hours, 10 hours, and 20 hours, according to these findings.

   (b) Use successive approximation to guess the number of hours watching comedy needed to lower the average anxiety level below 2 (on that scale of 1 to 5).

   (c) Name the variables and write an equation relating them. Anxiety is measured on a unitless scale.

   (d) Solve your equation to determine the number of hours watching comedy needed to lower the average anxiety level below 2.

3. Lizbeth wants to send her mom truffles for Mother's Day. It cost $\$C$ to send a box of T truffles where

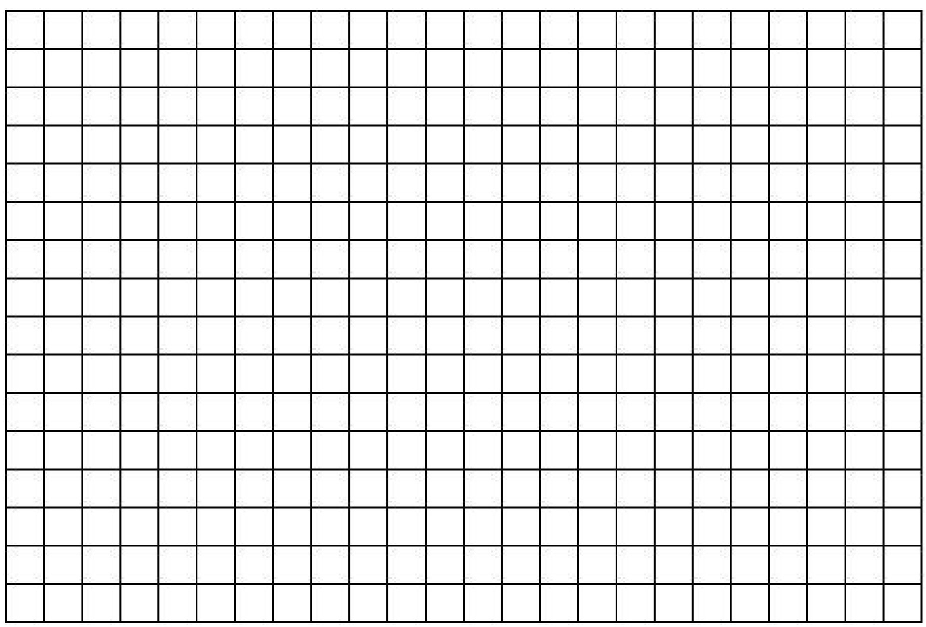
$$C = 1.90T + 7.95$$

   (a) Make a table of values showing the charges for a box of 8 truffles, 12 truffles, or 30 truffles.

   (b) What are the units on 1.90 and what does it mean in the story?

   (c) What are the units on 7.95 and what does it mean in the story?

   (d) Draw a graph illustrating the cost of sending truffles. Include $T = 0$.



   (e) If Lizbeth was charged \$53.55 for the box of truffles she sent her mom, how many truffles were there? *Set up and solve an equation to answer the question.*

4. The local burger restaurant had a promotion this summer. They reduced the price on a bacon double cheeseburger by 2¢ for each degree in the daily high temperature. The equation is

$$B = 7.16 - .02H$$

where $\$B$ is the price of the bacon double cheeseburger and H is the daily high temperature, in °F. *Story also appears in 2.1 Exercises*

(a) What is the usual price of a bacon double cheeseburger?

(b) Ronald paid \$5.34 for a bacon double cheeseburger on Tuesday. How hot was the temperature that day? Set up and solve an equation.

(c) What was the high temperature on Sunday when Wendy bought a bacon double cheeseburger for only \$5.70? Set up and solve an equation.

(d) Leroy is holding out for a \$5 burger. What temperature will make Leroy's wish to come true? Set up and solve an equation.

3.2 Solving linear inequalities – Practice exercises

1. A truck hauling bags of grass seed weighs 3,900 pounds when it's empty. Each bag of seed it carries weighs 4.2 pounds. The equation for the gross weight W pounds is

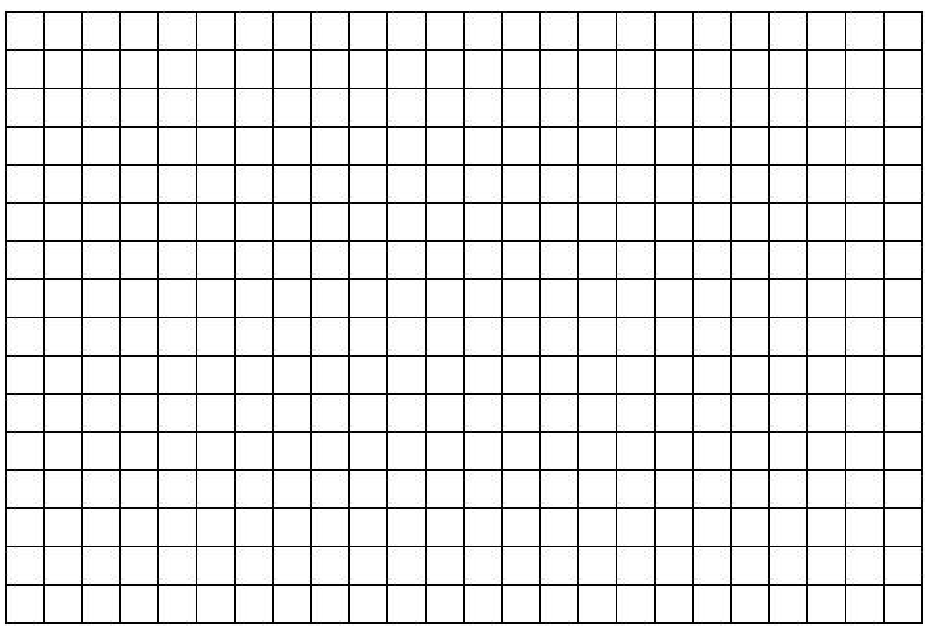
$$W = 3{,}900 + 4.2B$$

for B bags of grass seed. *Story also appears in 2.1 #1 and 3.1 #1*

(a) The state highways have a 18,000 pound gross weight limit. How many bags of grass seed can the truck can haul? Set up and solve an inequality.

(b) Record your answer to part (a) in the table and graph the function.

B	0	1,000	2,000	
W	3,900	8,100	12,300	18,000



(c) We used our answer to part (a) to draw our graph, so how can we check that answer to make sense? *Hint: what shape should the graph be?*

2. The altitude, A feet above ground, of an airplane M minutes after it begins its descent is given by the equation

$$A = 32{,}000 - 1{,}200M$$

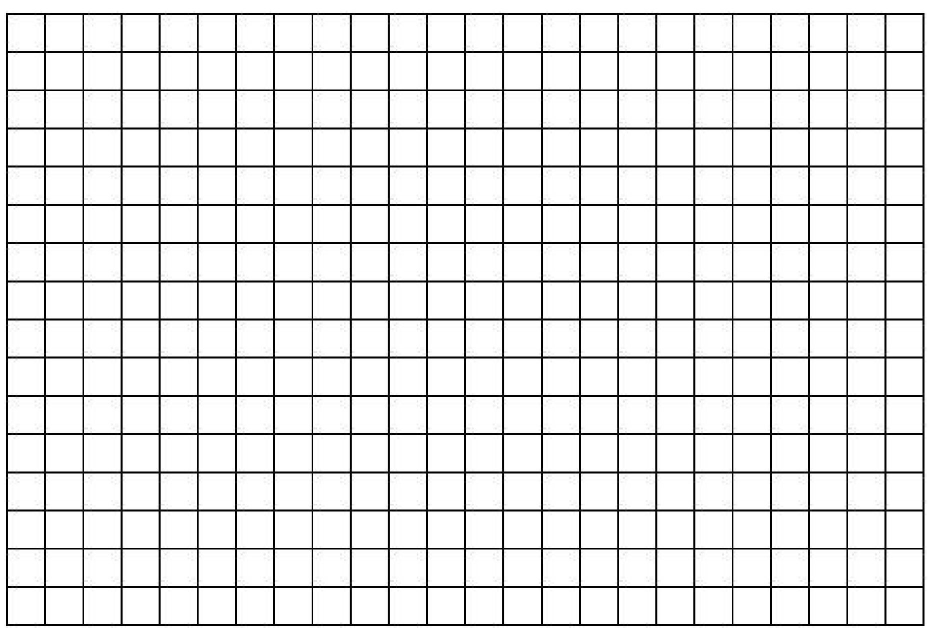
Answer each question by evaluating; setting up and solving an equation; or setting up and solving an inequality, whichever is most appropriate.

(a) At what altitude does the plane begin its descent?

(b) How fast is the airplane descending?

(c) What is the airplane's altitude 3 minutes into its descent? 8 minutes? 20 minutes? Display your answers in a table.

(d) Draw a graph illustrating the function.



The problem continues ...

(e) For how many minutes of its descent is the airplane above 20,000 feet?

(f) The airplane might be asked to go into a **holding pattern** (that means flying in a circle instead of landing) when it's between 6,000 and 14,000 feet up. When will the plane be in that altitude range?

(g) How long does it take the airplane to land, assuming it's not asked to go into a holding pattern?

3. Anthony and Christina are trying to decide where to hold their wedding reception. For each possible site, write an equation using T for the total cost of their wedding reception (in dollars) and G for the number of guests. Then set up and solve an inequality to calculate the number of guests Tony and Tina can afford on their \$8,000 budget.

   (a) The Metropolitan Club costs \$1,300 for the space and \$92 per person.

   Story also appears in 1.2 #3 and 1.3 #2

   equation:

   number of guests:

   (b) Black Elk Park charges \$500 to rent the pavilion and the family can bring in picnic food for \$65 per person.

   equation:

   number of guests:

   (c) The Dabbling Duck Inn charges \$1,400 for the space and \$80 per person for their local specialties.

   equation:

   number of guests:

   (d) Pranzo Ristorante has only a \$300 room rental fee but averages \$145 per person, including wine.

   equation:

   number of guests:

4. One variety of blueberry plant yields an average of 130 blueberries per season but there's quite a bit of variability from plant to plant. One measure of this variability is the standard deviation, which is approximated at 16.4 berries. Given a plant yielding B blueberries, we can calculate how usual or unusual that is by computing its **(standard) z-score** using the equation

$$Z = \frac{B - 130}{16.4}$$

For example, a plant yielding $B = 130$ blueberries has z-score of 0. A plant yielding $B = 173$ blueberries has z-score of

$$Z = \frac{173 - 130}{16.4} = (\underline{173} - 130) \div 16.4 = .671875 \approx .67$$

Answer each question by evaluating; setting up and solving an equation; or setting up and solving an inequality, whichever is appropriate.

(a) Calculate the z-score of a plant yielding 140 blueberries.

(b) If the z-score for a plant is -.7, what is the corresponding yield?
Hint: the negative z-score tells us the answer is below average.

(c) A plant with z-score above 1.96 is considered **plentiful**. What yields of blueberries would be considered plentiful?

(d) A plant with z-score between -1 and +1 is considered **ordinary**. What yields of blueberries woul be considered ordinary?

3.3 Solving power equations (and roots) – Practice exercises

ROOT FORMULA: The equation $C^n = v$ has solution $C = \sqrt[n]{v}$

---

1. A pizza of diameter D inches serves P people where

$$P = .015625D^2$$

Story also appears in 2.4 #1

(a) Set up and solve an equation using the ROOT FORMULA to find the diameter of a personal pizza ($P = 1$). Answer to the nearest inch.

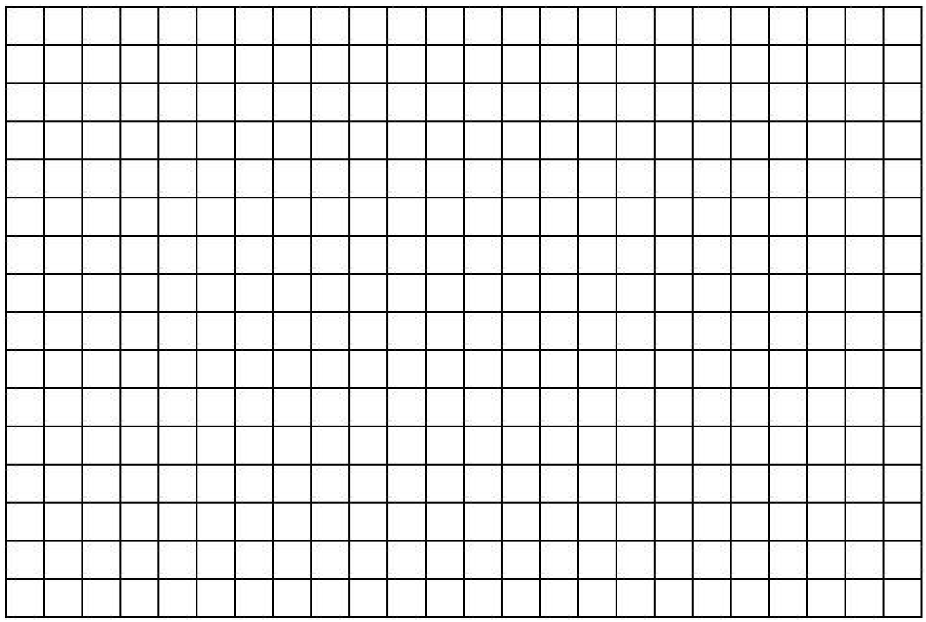
(b) Set up and solve an equation using the ROOT FORMULA to find the diameter of an extra large pizza to serve 6 people. Answer to the nearest 1/10 inch.

2. The weight of a wood cube is a function of the length of the sides. A cube with sides each E inches long has weight W ounces according to the equation

$$W = .76E^3$$

   (a) What is the weight of a cube with sides 2 inches long? 3 inches?

   (b) Draw a graph showing how the weight depends on the side length. Include $E = 0$.



   (c) Set up and solve an equation to find the length of the side of a wood cube weighing 8 ounces.

   (d) Repeat for 1 pound (that's 16 ounces).

3. Suppose a car gas tank is designed to hold enough fuel to drive 350 miles. (That's fairly average.) That means the size tank, G gallons, is a function of the fuel efficiency, F miles per gallon (mpg), according to the equation

$$G = \frac{350}{F}$$

Story also appears in 2.4 #2

   (a) My Honda Accord's tank holds about 16 gallons. According to the equation, what is the corresponding fuel efficiency? Set up and solve the equation. Start solving by multiplying both sides by F. *Note: you won't have to take a root.*

   (b) My ex-husband's Honda Civic's tank holds only 13 gallons. According to the equation, what is the corresponding fuel efficiency. Set up and solve the equation.

4. Moose bought a commemorative football jersey for \$250 fourteen years ago. Now he's planning to sell it and is interested in what the **effective return** (equivalent annual percent increase) on his investment might be for various prices. If $\$J$ is the current value of the jersey and g is the annual growth factor, then

$$J = 150g^{12}$$

For each part, first solve for g using the ROOT FORMULA, then calculate $r = g - 1$. The effective return is r written as a percentage.

(a) Find the effective return if the current value is \$290.

(b) Find the effective return if the current value is \$350.

(c) Find the effective return if the current value is \$400.

3.4 Solving exponential equations (and logs) – Practice exercises

---

LOG-DIVIDES FORMULA: The equation $g^Y = v$ has solution $Y = \dfrac{\log(v)}{\log(g)}$

---

1. After his first beer, Stephen's blood alcohol content (BAC) was already .04 and as he continued to drink, his BAC level rose 45% per hour. The equation is

$$S = .04 * 1.45^H$$

where S is Stephen's BAC and H is the time, measured in hours.

Story also appears in 1.1 #4 and 2.4 Exercises

(a) Make a table showing Stephen's BAC at the start of the story and each of the next four hours.

(b) At a BAC of .10 it is illegal for Stephen to drive. When will that happen? Set up and solve an equation using the LOG DIVIDES FORMULA. Answer to the nearest minute.

(c) Hopefully Stephen will stop drinking before he reaches a BAC of .20. If not, at the rate he's drinking, when would that be? Set up and solve an equation. Answer to the nearest minute.

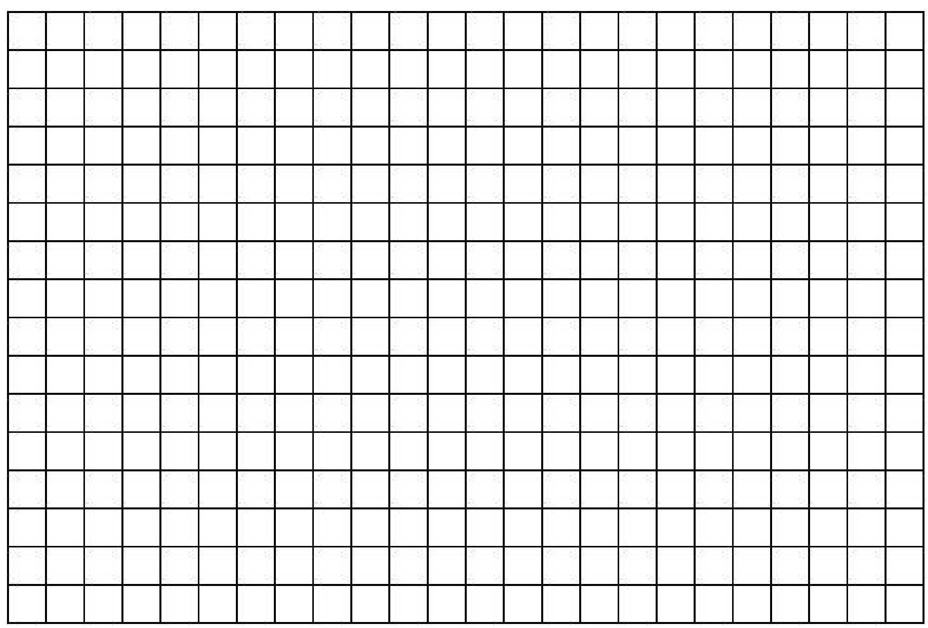
2. Chlorine is used to disinfect water in swimming pools. The chlorine concentration decreases as the pool is used according to the equation

$$C = 2.5 * .975^H$$

where C is the chlorine concentration in parts per million (ppm) and H hours since the concentration was first measured. *Story also appears in 5.3 #3*

(a) Make a table showing the chlorine concentration initially and after the swimming pool is used for 3 hours, 10 hours, 24 hours, and 48 hours.

(b) Draw a graph illustrating the function.



(c) Chlorine concentrations below 1.5 ppm do not disinfect properly so more chlorine needs to be added. According to your graph, when will that happen?

The problem continues . . .

(d) Use successive approximate to find when the concentration falls below 1.5 ppm.

(e) Solve the equation to find when the chlorine concentration falls below 1.5 ppm.

(f) Solve the equation to find when the chlorine concentration would fall below 0.1 ppm (essentially no chlorine) assuming no chlorine was added earlier. Show how to solve the equation to find the answer (and check it!).

(g) Report your answer to the nearest day.

3. Rent in the Riverside Neighborhood is expected to increase 7.2% each year. Average rent for an apartment is currently \$830 per month. Earlier we identified the variables as R for the monthly rent (in \$) and Y for the years. *Story also appears in 1.1 #2*

   (a) Find the annual growth factor.

   (b) Write an equation showing how rent is expected to change.

   (c) Use successive approximation to determine when rent will pass \$1,000/month. Display your work in a table. Round to the appropriate year.

   (d) Show how to solve the equation to calculate when rent will pass \$1,000/month. Round to the appropriate year.

   (e) Solve again to determine when rent will reach double what it is now, namely \$1,660/month, assuming this trend continues.

4. Dontrell and Kim borrowed money to buy a house on a 30-year mortgage. After M months of making payments, Dontrell and Kim will still owe $\$D$ where

$$D = 236{,}000 - 56{,}000 * 1.004^M$$

D is also known as the **payoff** (how much they would need to pay to settle the debt).

Story also appears in 2.3 #3

(a) How much did Dontrell and Kim originally borrow to buy their house?

(b) They have been in the house for 5 years now and due to a downturn in the housing market, their house is worth only \$150,000. Are they **underwater**, meaning do they owe more than the house is worth?

(c) How much longer would Dontrell and Kim need to stay in their house until they only owe \$150,000? That means you need to solve the equation

$$236{,}000 - 56{,}000(1.004)^M = 150{,}000$$

3.5 Solving quadratic equations – Practice exercises

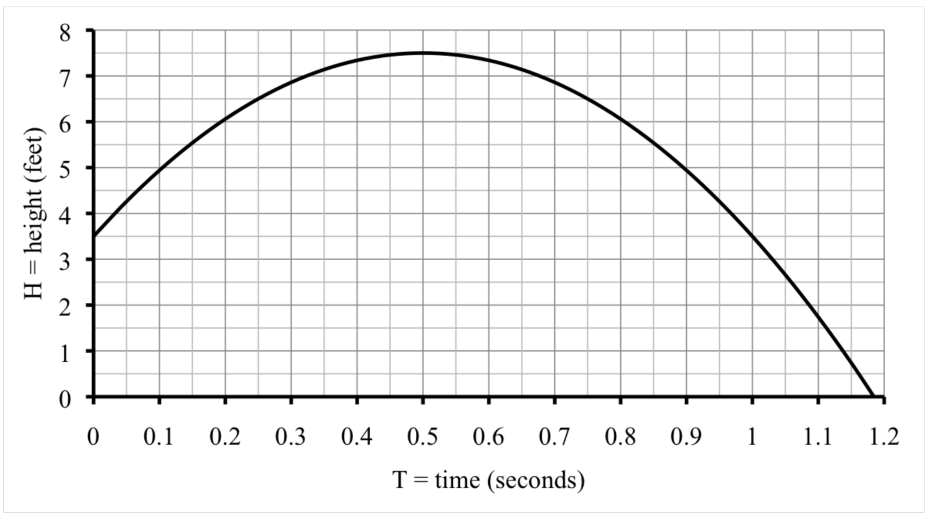
QUADRATIC FORMULA: The equation $aT^2 + bT + c = 0$ has solutions

$$T = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

---

1. A high-jumper jumps so that the height, H feet, of the point on his back that must clear the bar after T seconds is given by the equation

$$H = 3.5 + 16T - 16T^2$$



   (a) When would the high-jumper hit the ground (if there were no pit)? Ouch! Use the QUADRATIC FORMULA to find the answer. Use the graph to check.

The problem continues ...

(b) The high jump pit is 2 feet off the ground. When does the high-jumper land in the pit? Use the Quadratic Formula to find the answer and the graph to check.

(c) How high a bar can the high-jumper clear? Find the maximum height of that point above ground by evaluating at $T = \dfrac{-b}{2a}$. Use the graph to check.

2. The art museum opened in 1920. After an initial rush to see the great holdings, attendance dropped for awhile. But then attendance began to rise again and has risen since. The number of annual visits N is approximated by the equation

$$N = 51Y^2 - 840Y + 3{,}700$$

where Y is the year since 1920.

(a) Calculate the missing values in the table.

year	1920	1925	1930	1935	1940	1945	1950
Y	0	5	10	15	20	25	30
N	3,700		400	2,575	7,300		24,400

(b) Draw a graph of the function.

(c) In what year did the number of visitors first pass 30,000 in a year? Estimate the value from your graph. Then set up and solve a quadratic equation.

The problem continues ...

(d) According to this equation, in what year was the number of annual visits the smallest? For that year, what were the number of visits? Use $T = \dfrac{-b}{2a}$.

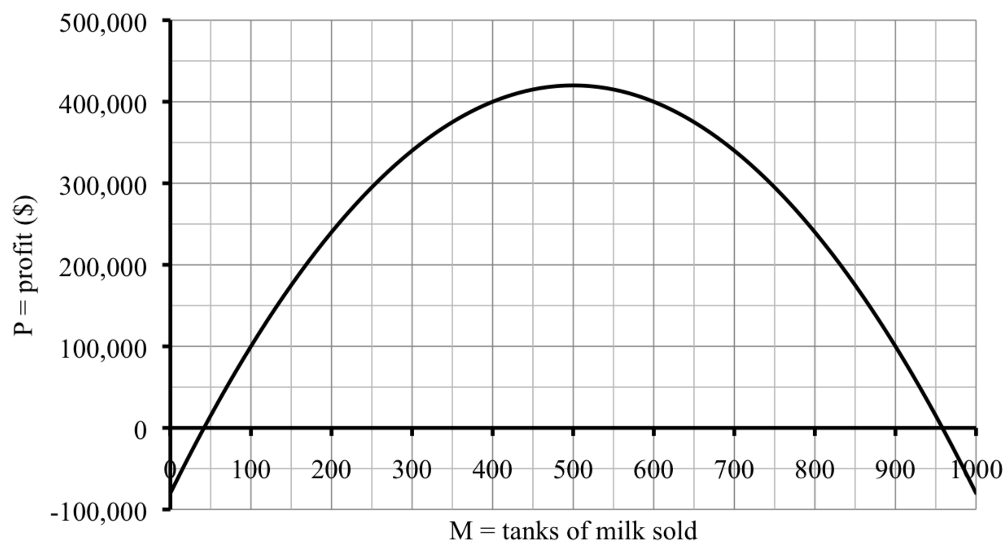
(e) Explain why N never equals 0.

(f) So, what actually happens when you try to use the QUADRATIC FORMULA to solve for $N = 0$?

3. The profit $\$P$ from selling M tanks of milk is described by the equation

$$P = -2M^2 + 2{,}000M - 80{,}000$$

(a) The graph is drawn below. Explain why negative numbers make sense.



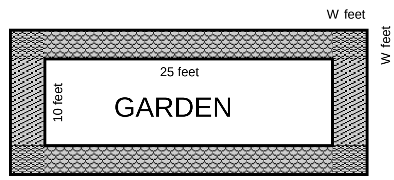
(b) How much milk must be sold for the company to **break even**, meaning having \$0 profit? Guess from the graph and check using the equation.

(c) For practice, set up and solve a quadratic equation to find the break even point.

The problem continues . . .

(d) How many tanks of milk would they need to sell to keep profits over \$400,000? Set up and solve a quadratic equation to find the answer. Then check that it agrees with your graph. Your answer should be in the form of an inequality.

4. Urban community gardens are catching on. What was once an abandoned lot down the block is now a thriving 10'×25' vegetable and berry garden for the neighborhood. One neighbor volunteered to donate gravel to make a path around the garden. The path will be 3 inches deep and the same width all around.



The amount of gravel we need (G cubic feet) is given by the equation

$$G = W^2 + 17.5W$$

where W is the width of the path in feet. For example, a path 4 feet wide requires 86 cubic feet of gravel, as you can check. *Story also appears in 2.3 and 2.4 Exercises*

(a) If the neighbor donates 60 cubic feet of gravel, how wide a path can they build? Set up and solve a quadratic equation to find the answer to two decimal places in feet. Then convert your answer into inches.

(b) Gravel is measured by the **yard**, which is short for cubic yard. There are 27 cubic feet in 1 yard of gravel. If the neighbor donates three yards of gravel, how wide a path can they build? Set up and solve a quadratic equation to find the answer to two decimal places in feet. Then convert your answer into inches.

(c) What would it mean to solve the equation to find where $G = 0$? Can you tell what the answer is from the equation (without actually solving)?

Practice Exam 3A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

Over 50 points on this exam are for solving equations and inequalities. Be sure you understand what you need to show for full credit. Using a different method will get little to no partial credit.

As you work, make a "don't forget" list of any information you need to look up or ask about.

---

1. The Skärstroms want to dig a new well for water for their lake cabin. The company charges \$900 to bring the equipment on site and draw the permit and then \$2 per foot to dig. That means

$$W = 900 + 2F$$

where F is the depth of the well (in feet) and W is the cost of the well (in \$).

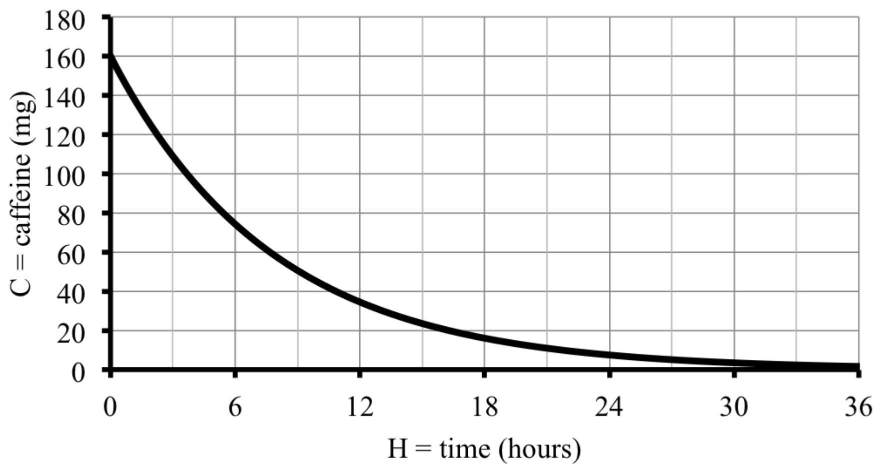
(a) In their neck of the woods, wells often run 200 feet deep. According to the equation, how much would that cost?

(b) The Skärstroms have budgeted between \$1,500 and \$1,800 for the well. Set up and solve a chain of inequalities to find the corresponding range of depths.

(c) No such luck. The company had to drill much deeper than hoped to find water. The final result, wonderfully cold and clear drinking water. And, a hefty bill for \$2,072. How deep is their well? Set up and solve an equation.

2. Ceyda starts the day by downing two cans of Red Bull, containing a total of 160 mg of caffeine. Her body eliminates the caffeine at a rate of 12% each hour. That means the amount of caffeine (C mg) after H hours is given by the equation graphed below

$$C = 160 * .88^H$$



   (a) According to the equation, how much caffeine is in her body initially, after 2 hours, 5 hours, and 24 hours? Display your answers in table.

   (b) Show show to use successive approximation to find when there will Ceyda's caffeine level first drops below 20 mg. Answer to the nearest hour.

   (c) Set up and solve an equation to determine when Ceyda's caffeine level first drops below 20 mg. Round your answer to two decimal places.

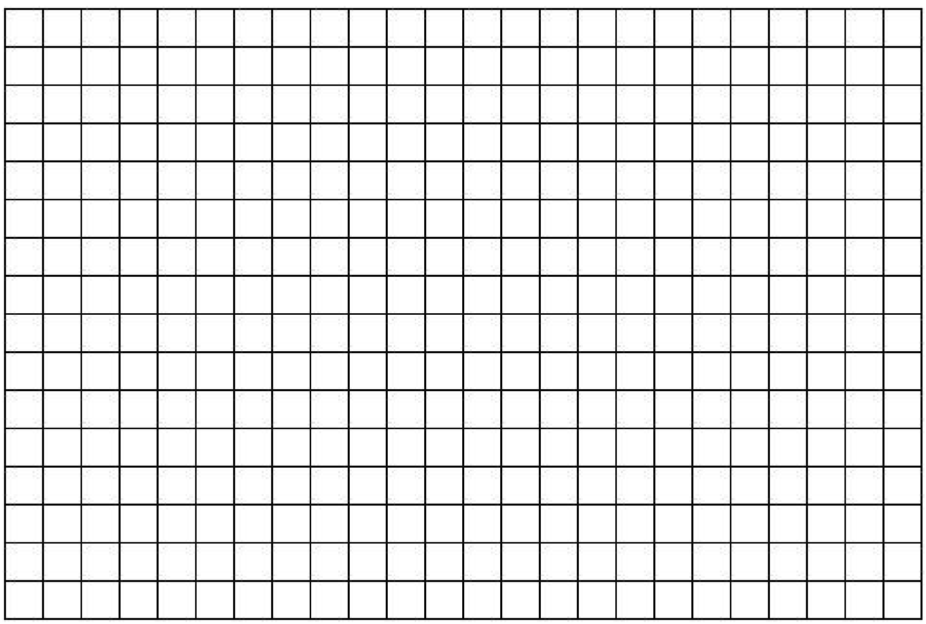
3. Jorge deposited \$1,500 in an high yield money market account and plans to leave it there for 5 years. The value of his account after 5 years \$$A$ will depend on the growth factor g as given by the equation

$$A = 1{,}500g^5$$

(a) Use the equation to complete the table.

g	1.02	1.03	1.05	1.10
A	1,656.12			2,415.77

(b) Draw a graph showing how A depends on the growth factor g. Start the g-axis at 1.00, instead of 0.



(c) Use your graph to estimate the growth factor if the value of Jorge's account after 5 years is \$1,780.

(d) Now set up and solve an equation to find the answer.

(e) What is the corresponding **return on investment**? That means calculate $r = g - 1$. The return on investment is r written as a percentage.

4. A rabbit jumps so that her height above ground is given by the formula

$$R = 17.6S - 22S^2$$

where R = height of rabbit (feet above ground) and S = time (seconds).

(a) At what height did the rabbit start her jump?

(b) Can the rabbit jump over a 3 foot fence? Calculate the exact maximum height of the rabbit to decide.

(c) How long does it take the rabbit to get 2 feet in the air and when is she at that height again? Set up and solve the appropriate equation to find the answers.

Practice Exam 3B

Try taking this version of the practice exam under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

Caution: Usually more than half points on this exam are for solving equations and inequalities. Be sure you understand what you need to show for full credit. Using a different method, or not showing enough work might get little to no partial credit.

---

1. Goldie the Goldfish, Pinches the Lobster, Quackers the Duck, Speedy the Turtle. These first generation Beanie Babies toys were anticipated to increase in value according to the equation

$$B = 6 * 1.08^{Y}$$

where B is the value of Beanie Babies (in \$) and Y is the years since 1994.

These names are registered trademarked.

(a) Make a table showing the anticipated value in 1994, 2004, 2010, and 2025.

(b) Draw a graph showing how the value of the Beanie Babies increased.

The problem continues ...

(c) Set up and solve an equation to determine when will Beanie Babies made in 1994 were anticipated to be worth over \$40. Report the actual year.

2. Best we can tell, the floor of our front porch was 7'2" above ground when the house was built. It has been slowly sinking ever since. The contractor estimated that it's dropped .45 inches per year. The equation is

$$P = 86 - .45A$$

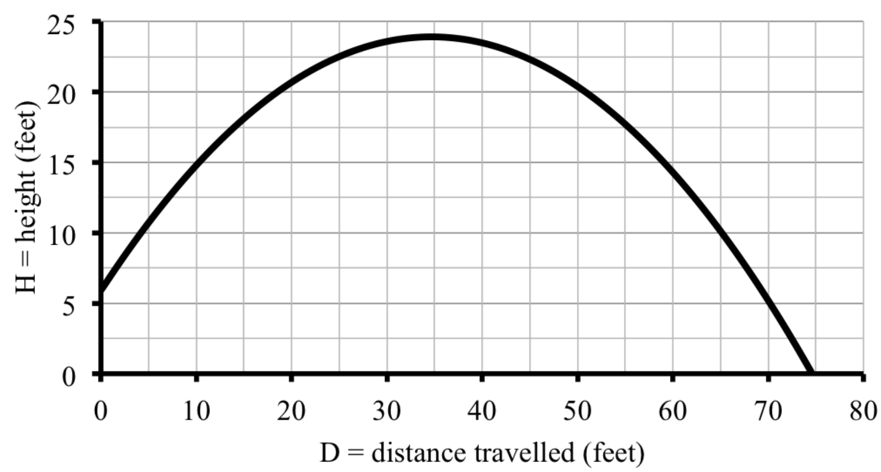
where P is the height of the porch (in inches) and A is the age of the house (in years).

   (a) Make a table showing the height of the front porch when the house was built, when it was 20 years old, and when it was 50 years old.

   (b) The floor of our front porch is currently 48 inches above ground. Set up and solve an equation to figure out how old our house is.

   (c) Once the porch is below 40 inches, we will have to do something to fix it. Set up and solve an inequality to find the answer to the nearest year. Remember it's already an old house, so figure out how many years *from now.*

3. A shot put (large metal ball) is thrown closely following the parabolic arch given by the equation
$$H = -.015D^2 + 1.04D + 5.9$$
where D is the distance travelled horizontally, and H is the height above the ground of the shot, both in feet. The path of the shot put is graphed below.



(a) How far away did the shot put land? Estimate the answer from the graph. Then show how to set up and solve an equation to find the answer to two decimal places.

(b) How high does the shot put get? Show how to find the exact answer. Then compare to the graph.

4. The amount of snow in a snowball, C cups, depends on the diameter (distance across) of the snowball, D inches according to the equation

$$C = 0.036D^3$$

(a) How many cups of snow are needed to make a snowball that's 3 inches across? 5 inches across?

(b) How many cups of snow are needed to make a snowball that's 2 feet across? Give your answer in gallons. Use that 1 gallon = 4 quarts and 1 quart = 4 cups.

(c) Karen has a 5 gallon paint bucket packed with snow and wants to make a giant snowball out of it. How big will the snowball be? Show how to use successive approximation to find the answer to the nearest inch. Display your calculations in a table.

(d) Now set up and solve an equation to find the answer to the nearest inch.

Chapter 4

A closer look at linear equations

4.1 Modeling with linear equations – Practice exercises

1. A solar heating system costs approximately \$30,000 to install and \$150 per year to run. By comparison, a gas heating system costs approximately \$12,000 to install and \$700 per year to run. *Story also appears in 4.2 Exercises*

   Source: "Using Algebra" by Ethan Bolker

   (a) What is the total cost for installing and running a gas heating system for 30 years?

   (b) Write a linear equation showing how the total cost for a gas heating system depends on the number of years you run it.

   (c) Write a linear equation showing how the total cost for a solar heating system depends on the number of years you run it.

   (d) How many years of a solar heating system could you get for the cost of a gas heating system lasting 30 years (your answer to part (a))? Set up and solve an equation.

2. Since a very popular e-book reader was released, the price has been decreasing at a constant rate. A blogger developed the following equation representing the price E of the e-book reader in the months M since it was released.

$$E = 359 - 12M$$

(a) Make a table of values for the e-book reader price initially, after 10 months, and after 25 months.

(b) What does the 359 mean in the story and what are its units?

(c) What does the 12 mean in the story and what are its units?

(d) Draw a graph illustrating the dependence.

The problem continues . . .

(e) After approximately how many months was the price of the e-book reader expected to be down to \$200? Set up and solve an equation.

(f) Sareth decided to purchase a e-book reader when the price fell below \$100. How many months after its release did the price of the e-book reader fall below that level? Set up and solve an inequality.

(g) If you can believe what you read in blogs, the manufacturer will soon be giving away the e-book reader for free, since they make money on the e-book sales themselves. How many months after it was released would that happen, according to our equation? Set up and solve an equation.

3. Can you tell from the table which of these functions are linear? Use the rate of change to help you decide. Remember that these numbers may have been rounded.

   (a) Savings bonds from grandpa. *Story also appears in 1.2 #1 and 5.3 #1*

Year	1962	1970	1980	1990	2000	2010
Value bond ($)	200.00	318.77	570.87	1,022.34	1,830.85	3,278.77

   (b) Wind chill at 10°F. *Story also appears in 1.2 #2*

Wind (mph)	0	10	20	30	40
Wind chill (°F)	10	-4	-9	-12	-15

   (c) Pizza. *Story also appears in 2.4 #1 and 3.3 #1*

Size (inches)	8	14	16
People	1	3	4

   (d) Water in the reservoir. *Story also appears in 2.1 #2 and 3.2 Exercises*

Week	1	5	10	20
Depth (feet)	45.5	39.5	32	17

4. Plumbers are really expensive, so I've been shopping around. James charges \$50 to show up plus \$120 per hour. Jo is just getting started in the business. She charges \$45 to show up plus \$55 per hour. Mario advertises "no trip charge" but his hourly rate is \$90 per hour. Not to be outdone, Luigi offers to unclog any drain for \$150, no matter how long it takes. For each plumber, the table lists the corresponding equation and several points. In each equation, the plumber charges $\$P$ for T hours of work. *Story also appears in 2.1 Exercises*

Plumber	James	Jo	Mario	Luigi
Equation	$P = 50 + 120T$	$P = 45 + 55T$	$P = 90T$	$P = 150$
0 hours	\$50	\$45	\$0	\$150
2 hours	\$290	\$155	\$180	\$150
4 hours	\$530	\$265	\$360	\$150

(a) Use the points given to plot each of the four lines on the same set of axes. Label each line with the plumber's name.

(b) What do you notice about Luigi's line?

(c) List the plumbers in order from steepest to least steep line. What does that mean in terms of the story?

(d) Now list the plumbers in order from smallest to largest intercept of their line. What does that mean in terms of the story?

4.2 Systems of linear equations – Practice exercises

1. Madison wants to buy a new car, either the Toyota Prius, priced at \$26,100, or the Volkswagen Jetta, priced at \$23,700. Annual fuel costs for the Toyota Prius are currently \$1,100. For the Jetta, annual fuel costs are currently \$1,800. The total cost of each car will depend on how many years she keeps it.

   (a) Name the variables.

   (b) Write a linear equation for the total cost (including purchase price and fuel costs) of the Prius and write another linear equation for the total cost of the Jetta, each as a function of how long she keeps it. Assume fuel costs are constant.

   (c) Make a table comparing the total costs for the Jetta and for the Prius if Madison keeps the car for 3, 5, or 10 years.

   (d) Set up and solve a system of linear equations to determine the **payoff time**, or the number of years for which the total costs of each car are equal.

   (e) Based on what you've learned, fill in the blank.

   The more expensive Toyota Prius pays off if Madison is going to keep it for ___ years or more.

2. A mug of coffee costs \$3.45 at Juan's favorite cafe, unless he buys their discount card for \$10 in which case a mug costs \$2.90. Or, he can buy a membership for \$59.99 and then coffee is only \$1/mug. If we let M represent the number of mugs of coffee he buys and T represent the total cost in dollars, then the equations are:

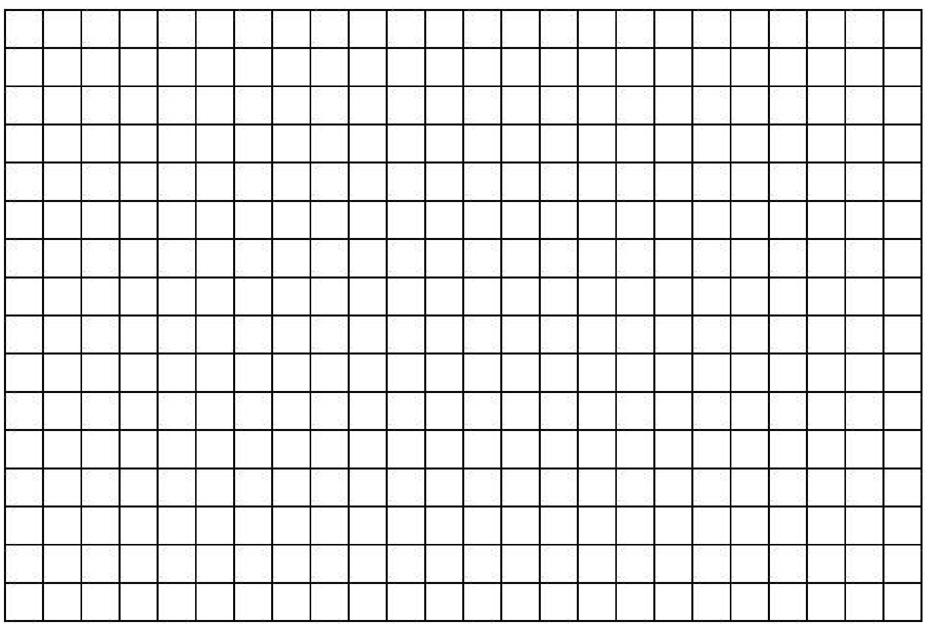
   No card: $T = 3.45M$
   With card: $T = 10.00 + 2.90M$
   Member: $T = 59.99 + 1.00M$

   Story also appears in 1.2 #4 and 2.1 #4

   (a) Compare the total costs for all three options.

Mugs	0	10	20	30
No card				
With card				
Member				

   (b) Draw a graph showing all three options.



   (c) Which option is least expensive if Juan plans to buy

   - A small number of mugs of coffee:
   - A medium number of mugs of coffee:
   - A large number of mugs of coffee:

The problem continues ...

(d) Set up and solve a system of linear equations to compare total cost with no card to the total cost with the card.

(e) Set up and solve a system of linear equation to compare the total cost with the card to the total cost with the membership.

(f) Describe in words what you've learned.

3. Ahmed planted two shrubs in the backyard on May 1. The virburnum was 16.9 inches tall and expected to grow .4 inches each week this summer. The weigela was 20.3 inches tall but only expected to grow .2 inches per week. If we let S represent the total height of the shrub in inches after W weeks, then the equations are:

   Virburnum: $S = 16.9 + .4W$
   Weigela: $S = 20.3 + .2W$

   Story also appears in 4.1 exercises

   (a) Compare the height of the shrub on the given dates.

date	May 1	June 12	July 10	Sept 4
W	0	6	10	18
S (virburnum)				
S (weigela)				

   (b) When will the shrubs be the same height? Continue successive approximation to find the answer to the nearest week.

   (c) Set up and solve an equation to find the day when the two shrubs are the same height. In what month does that happen?

4. The **supply** of flour is the amount of flour produced. It depends on the price of flour. A high price encourages producers to make more flour. If the price is low, they tend to make less of it. The dependence of the supply of flour S (in loads) on the price P (in $/pound) is given by the equation

$$\textbf{Supply:} \quad S = .8P + .5$$

The **demand** of flour is the amount of flour consumers want to buy. It also depends on the price of flour. If flour sells for a high price, then consumers will buy less. If flour sells for a low price instead, then consumers will buy more. The dependence of the demand of flour D (in loads) on the price P (in $/pound) is given by the equation

$$\textbf{Demand:} \quad D = 1.5 - .4P$$

The **equilibrium price** of flour is the price where the supply equals the demand.

(a) What happens if flour is priced at $1.00/pound? That is, how much flour will be produced and how much will consumers demand?

(b) What happens if flour is priced at $0.50/pound? That is, how much flour will be produced and how much will consumers demand?

(c) Graph each dependence on the same set of axes. What is the equilibrium price, approximately, according to your graph?

The problem continues ...

(d) Set up and solve an equation to find the equilibrium price of flour.

(e) When more of a product is produced than consumers want to buy, we have a **surplus** of the product. Solve an inequality to find the range of price values for which there will be a surplus of flour. Compare your answer to part (d).

(f) When less of a product is produced than consumers want to buy, we have a **shortage** of the product. Solve an inequality to find the range of price values for which there will be a shortage of flour. Compare your answer to parts (d) and (e).

4.3 Intercepts and direct proportionality – Practice exercises

1. Each of the two stories, below, involve how temperature changes over time. It might be confusing to call either variable T, so use H for the time in hours and D for the temperature in degrees (°F). In each case, time should be measured from the start of the story.

   (a) It was really cold at 8:30 this morning when Raina arrived at the office. Luckily the heating system warms things up very quickly, 4°F per hour. By 11:00 a.m. it was a very comfortable 72° F.

   i. Figure out what the temperature was at 8:30 a.m.

   ii. Write an equation illustrating the function.

   (b) While 72°F is a perfectly good temperature for an office, not so for ballroom dancing. When Raina arrived for her practice at 5:30 that evening, she began to sweat before she even took the floor. Turns out the air conditioner had been running since 4:00 p.m. but it only cools down the room 3°F per hour.

   i. Figure out what the temperature was at 4:00 p.m.

   ii. Write an equation illustrating the function.

2. Maryn is very happy. Her interior design business is finally showing a profit. She has logged a total of 471 billable hours at \$35 per hour since she started her business. Accounting for start up costs, her net profit is totals \$2,194.

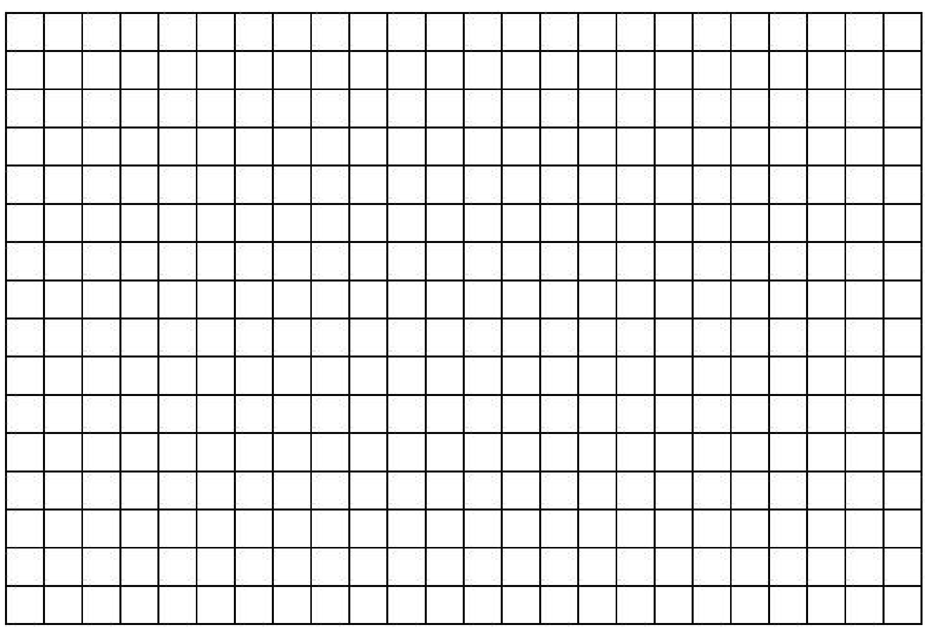
   (a) What were Maryn's start up costs?

   (b) Identify the slope and intercept (including their units and sign) and explain what each means in terms of the story.

   (c) Calculate what Maryn's profits will be once she has logged a total of 1,000 hours.

   (d) Name the variables and write an equation relating them.

   (e) Graph the function.



3. For each story, find the initial weight of the person and use it to write an equation showing how the person's weight P pounds depends on the time, W weeks.

   (a) Jerome has gained weight since he took his power training to the next level ten weeks ago, at the rate of around 1 pound a week. He now weighs 198 pounds.

   (b) Vanessa's doctor put her on a sensible diet and exercise plan to get her back to a healthy weight. She will need to lose an average of 1.25 pounds a week to reach her goal weight of 148 pounds in a year. Use 1 year = 52 weeks.

   (c) After the past 6 weeks of terrible migrane headaches, Carlos is down to 158 pounds. He's lost 4 pounds a week.

   (d) Since she's been pregnant, Zoe has gained the recommended $1/2$ pound per week. Now 30 weeks pregnant and 168 pounds, she wonders if she'll ever see her feet again.

4. Each story describes a situation that we're assuming is linear. Decide whether it is directly proportional or not. If not, identify what the intercept would mean in the story.

   (a) The price of a kiwis depends on how many kiwis you buy.

   (b) The price of a bag of tortillas depends on how many tortillas are in the bag.

   (c) The time it takes to vacuum a rug depends on the area of the rug.

   (d) The time it takes to wash dishes depends on how many dirty dishes there are.

   (e) The amount of laundry detergent I have left depends on how many loads of laundry I did.

4.4 Slopes – Practice exercises

1. For his Oscars party, Harland had 70 chicken wings delivered for \$51.25. For his Super Bowl bash, Harland had 125 chicken wings delivered for \$83.70. In each case, the total cost includes the cost per wing and the fixed delivery charge.

   (a) Find the slope, including units, and explain what it means in the story.

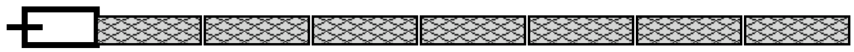
   (b) Find the intercept, including units, and explain what it means in the story.

   (c) Name the variables and write an equation for the function.

   (d) How many wings could Harland order for \$100? Solve your equation.

   (e) Graph and check.

2. Jana is making belts out of leather strips and a metal clasp. An short belt (as shown) is 24.5 inches long and includes 7 leather strips. An long belt (not shown) is 37.3 inches long and includes 11 leather strips. Each belt includes one metal clasp that is part of the total length. All belts use the same length clasp.



   (a) Name the variables, including units.

   (b) How long is each leather strip?

   (c) How long is the metal clasp?

   (d) Write an equation relating the variables.

   (e) Solve your equation to find the number of leather strips in a extra long belt that's 43.7 inches long.

3. The local ski resort is trying to set the price for season passes. They know from past experience that they will sell around 14,000 passes if the season ticket price is \$380. If the price is \$400, they will sell fewer, perhaps only 11,000 passes. You can assume this decrease in demand is linear.

   (a) How many fewer people purchase season passes for every dollar increase in the price?

   (b) Find the intercept. Explain why this number does not make sense in the problem.

   (c) Write an equation for the function, using T for the ticket price, in dollars, and D for the demand (number of tickets sold).

   (d) How many season passes will they sell if the price is reduced to \$355?

   (e) The amount of **revenue** (money they take in) depends both on the ticket price and the number of tickets sold. The equation is $R = TD$, where R is the revenue, in dollars. Calculate the revenue when ticket prices are \$355, \$380, and \$400. *That means multiply the ticket price T times the number of tickets sold D in each case listed.* Of these three prices, which yields the most revenue?

4. Boy, am I out of shape. Right now I can only press about 15 pounds. (**Press** means lift weight off my chest. Literally.) My trainer says I should be able to press 50 pounds by the end of 10 weeks of serious lifting. I plan to increase the weight I press by a fixed amount each week.

   (a) Name the variables and write an equation for my trainer's projection.
   Hint: you know the intercept.

   (b) Make a table showing my trainer's projection for after 0, 5, 10, 15, and 20 weeks.

   (c) Years ago I could press 90 pounds. At this rate, when will I be able to press (at least) 90 pounds again? Set up and solve an inequality.

The problem continues ...

(d) Draw a graph illustrating the function.

(e) I am skeptical. I don't think I'll be able to press 50 pounds by the end of 10 weeks. If I revise my equation, will the new slope be larger or smaller?

Hint: try sketching in a possible revised line on your graph assuming that after 10 weeks I will press much less than 50 pounds.

(f) Will my revised projections mean I'll reach that 90-pound goal sooner or later? Explain. *Hint: extend your graph.*

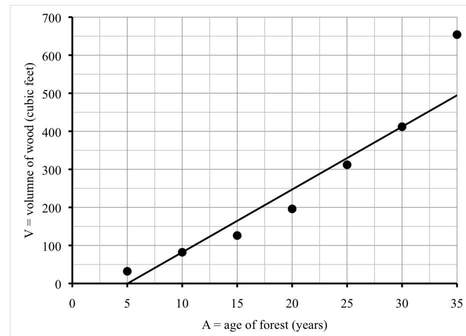
4.5 Fitting lines to data – Practice exercises

1. The scatter plot shows the total volume of wood, V cubic feet, in managed forests of different ages, A years.

   (a) For each line, state some reason why the fit is not good. (We know the line will not go through all, or even most, of the points, so that is not the problem. Instead look at slope/steepness, intercept/height, etc.)

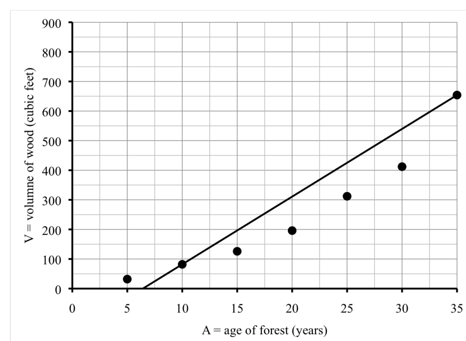
LINE A

LINE B



LINE C

LINE D



   (b) Which of these four lines do you think fits best, and why?

2. Noel is considering investing in a company's stock so he looked up a few values.

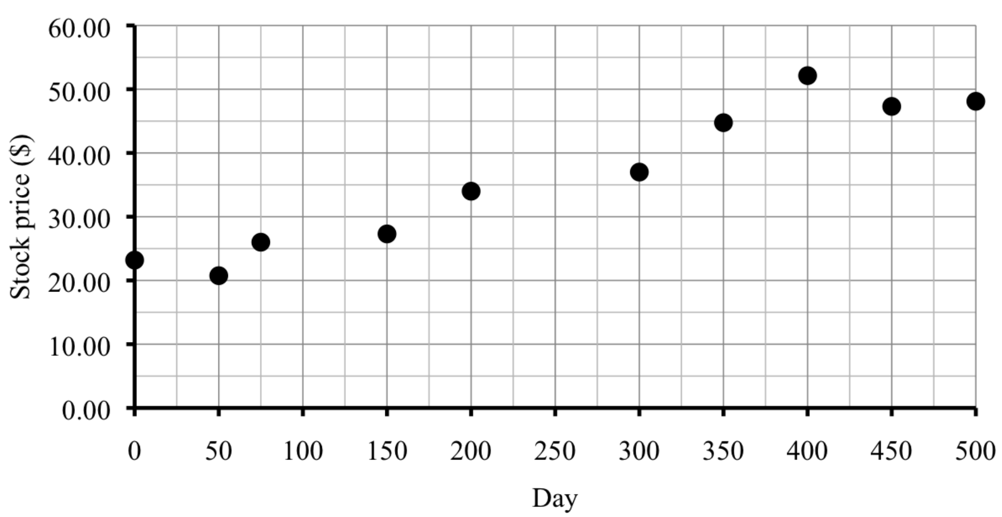
Day	0	300	500
Value ($)	23.19	37.00	48.10

(a) Calculate the daily rate stock prices changed during the first 300 days.

(b) Calculate the daily rate stock prices changed from Day 300 to Day 500.

(c) Is this growth linear?

(d) The scatter plot shows additional values of the stock Noel is considering buying.



Draw in a line that through the points for Day 300 and Day 500. Label this line #1. Explain why that line does not fit the data well.

(e) Draw in a line that fits the data better. It does not need to go through any of the points exactly. Label that line #2.

3. Is it true that students who work part-time have lower grades? Do the number of hours matter? The table shows the grade point average (GPA) of ten students compared to the number of hours per week each student works at a part time job. The variables we used are H for the time worked at job (hours/week) and G for the grades GPA, on the usual scale of 0.0 to 4.0.

H	0	0	10	12	14	15	16	18	20	20
G	3.72	3.91	3.43	2.79	3.08	2.62	2.44	3.17	3.00	2.55

(a) Make a scatter plot of the points. Start the G-axis at 2.0.

(b) Find the equation of the line that goes through the first and last point listed. *Hint: the first point tells you the intercept.*

(c) Draw this line on your graph and label it line A.

The problem continues . . .

(d) Use your equation for line A to figure out what you would expect the GPA of a student working a 30 hour per week job to be.

(e) It turns out, the best fitting line has equation $G = 3.7597 - .0551H$. Make a table of values for this equation using $H = 0, 10, 20$ hours.

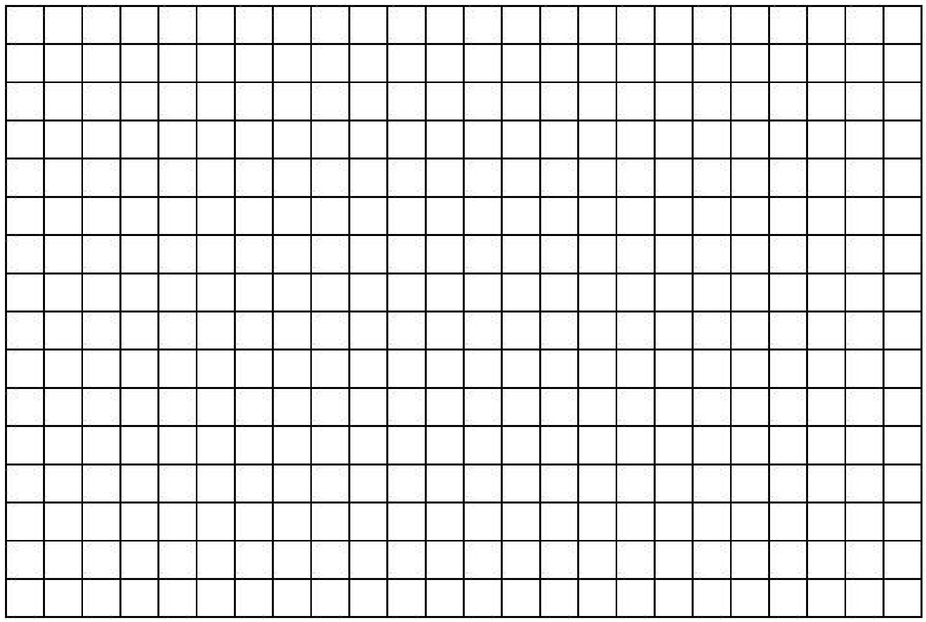
(f) Use that table of values to graph this best fitting line on that same set of axes. Label it line B.

(g) According to line B, what's the most hours a student should work to be able to maintain a 3.5 GPA? Solve an equation, then check on your graph.

4. Mia and Mandi and opened a candy shop this January. The table shows their monthly sales profit. Except for some seasonal fluctuation, Mia and Mandi generally expect your profits to rise steadily while their business is getting established.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Sales Profit ($)	3,394	4,702	3,683	4,840	5,632	4,432	4,649	4,590

(a) Make a scatter plot. Begin the profit axis at $3,000.



(b) Name the variables and write an equation for the line through January and August. Add this line (#1) to your graph. This line is too low.

The problem continues ...

(c) Write an equation for the line through March and July. Notice that you need to find the intercept this time. Add this line (#2) to your graph. This line is too steep.

(d) Neither of these lines go anywhere near the data for February, April, and May, because those are outliers. Any idea why those months had much higher candy sales than the other months?

(e) What does each equation give as an estimate for September's sales?

(f) Explain why Mia and Mandi should not use either of these lines to estimate October's sales.

Practice Exam 4A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

---

1. Forde collects miniature cars, each weighing 1.76 ounces. His car box weighs 4 ounces when empty. The total weight T ounces of Forde's car box depends on the number of cars C according to the equation

$$T = 4 + 1.76C$$

   (a) Make a table of values showing the weight if it contains 1, 5, 12, or 20 cars.

   (b) Draw a graph illustrating the dependence.

   (c) How many cars can Forde fit in the box and stay under 3 pounds (that's 48 ounces)? *Figure out the answer and mark the corresponding point on your graph.*

2. Will women every run the marathon as fast as men do? The world records are getting close. In 2012 the men's record was 2:03:38 and the women's record was 2:15:25. That's only about 12 minutes apart! On the other hand, the record is changing very slowly. Estimates for the men's time shows about 13 seconds drop per year on average. Estimates for the women's time shows about 26 seconds drop per year on average.

Source: Wikipedia (Marathon World Record Progression)

   (a) Write an equation for each function: men's and women's. Use T for the marathon times (in seconds) and Y for the years (measured in years since 2012). Note that 2:03:38 = 7,418 seconds and 2:12:25 = 7,945 seconds.

   (b) Use successive approximate to estimate when the women's record might equal the men's record. Display your guesses in a table.

   (c) Set up and solve a system to estimate when the women's record might equal the men's record.

3. An online music club charges a monthly enrollment fee plus \$.95 per album you download. Last month Andrew downloaded 31 albums for a total cost of \$49.00.

   (a) What is the monthly enrollment fee?

   (b) Name the variables, including units, and write an equation relating them.

   (c) If the bill next month is for \$87.95, how many albums did Andrew download? Show how to solve the equation

4. A report shows September sea-ice declining in the Northern hemisphere. In 1980 the extent of the sea-ice was 3.1 million square miles. By 2012, the sea-ice extended only 1.7 million square miles. For this problem, suppose that the area of sea-ice decreases linearly. Source: National Snow and Ice Data Center

   (a) Name the variables, including units.

   (b) What is the rate of sea ice decrease?

   (c) Write a linear equation relating your variables.

   (d) Scientists are concerned that if the September sea-ice falls between 200,000 and 500,000 square miles, then other climate feedbacks will lead to no more sea-ice in September. According to your equation, in what years is this expected to occur? Set up and solve an inequality to answer the question.

5. As people age, they begin to experience hearing loss. A study was done to determine the "comfort level" of sound for people of different ages, meaning the loudest sound (in decibels) that the person could listen to comfortably. The data are given in the table below.

Name	Akbar	Javier	Walter	Xang	Rolf	Derrick	Iago	Raheem
Age	45	45	55	65	75	75	85	85
Comfort level	58	61	63	71	75	80	82	79

(a) Make a scatterplot showing the data. Scale your axes to start at 40 years and start the level at 55 decibels. Spread out your scale to get a large, detailed graph.

(b) Draw the line through the points listed for Xang and Rolf. Explain why that line does not fit the data well. *Label this line B.*

(c) The "best-fitting line" from statistics has equation

$$C = 34.315 + .5556A$$

where A is the person's age (in years) and C is the comfort level (in decibels). Make a table showing the values of C when $A = 40, 60,$ and 80. Use those points to add this "best-fitting line" to your graph.

Practice Exam 4B

Try taking this version of the practice exam under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

---

1. The Vang family want to buy a new washing machine. The first model costs \$645 and then \$13.29 per month to run or another, more efficient, model costs \$940 and then \$7.82 per month to run. If M is the number of months and V is the Vang family's total cost (in \$), then the equations and some comparable values (to the nearest \$) are:

First model: $V = 645 + 13.29M$
Second model: $V = 940 + 7.82M$

M	12	36	60
First model:	804.48	1,123.44	1,442.40
Second model:	1,033.84	1,221.52	1,409.20

(a) Draw a graph illustrating both equations. *Be sure to include the intercepts.*

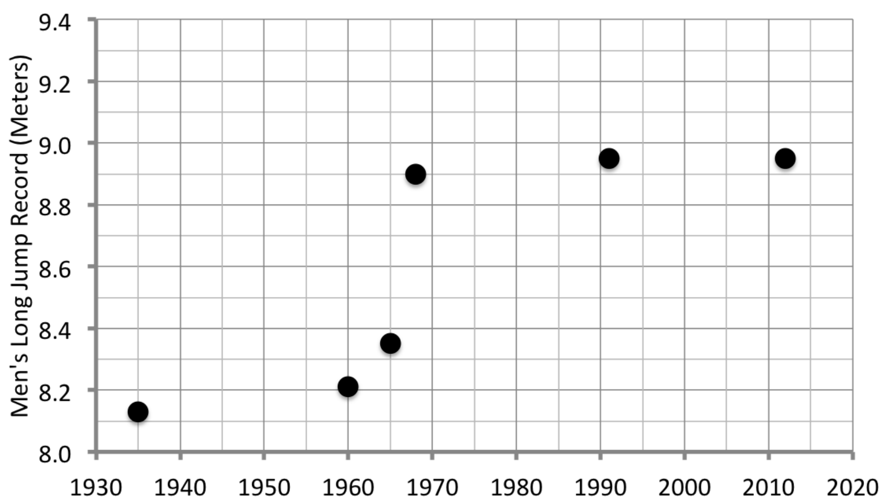
(b) According to your graph, approximate what is the **payback time** (the number of months for which the total costs of each washing machine are equal)? Answer and indicate the point on the graph where you can check.

The problem continues ...

(c) Set up and solve a system of linear equations to find the payback time.

(d) If the manufacturer offers a \$25 rebate on the more efficient model, how does that change the payback time? Adjust your equation and set up and solve a new system. OR carefully explain some other way of figuring it out.

2. It's been a long time since anyone broke the record for the men's long jump. In 1935 Jesse Owens jumped 8.13 meters. The record was next broken 25 years later (in 1960) by Ralph Boston who jumped 8.21 meters. He broke his own record several times over the next few years, including being surpassed briefly by Igor Ter-Ovanesyan. Ralph's final record was 8.35 meters in 1965. Not to be outdone, Igor tied the record in 1967. Then in 1968, Bob Beamon jumped 8.90 meters. That record held for 23 years, until Mike Powell jumped 8.95 meters in 1991 (much to Carl Lewis' dismay). Powell's record still stands 21 years later, in 2012. Source: Wikipedia (Long Jump)



(a) Draw in the line connecting the data from 1935 and 1991. Use it to predict the long jump record in 2020.

(b) Draw in the line connecting the data from 1968 and 1991. Use it to predict the long jump record in 2020.

(c) Which of your lines do you prefer, and why?

3. Arjun just graduated from college but is living with his uncle for the summer to save money. They agreed that Arjun would do chores and some light renovations instead of paying rent. Arjun has been doing around 5 hours of work a week for the past 8 weeks, but still owes his uncle another 30 hours of work.

   (a) What was the original agreement? That means, how many hours of work did Arjun promise his uncle?

   (b) Name the variables and write an equation relating them, assuming Arjun continues to do 5 hours a week of work.

   (c) How many more weeks will it take Arjun to finish the work he promised? Show how to solve the equation.

4. The local zoning commission is considering a plan to expand housing in the city, as measured in the number of residential units. But with more residential units come more shops, offices, schools, recreational facilities, churches, and other commercial property. Currently the city has 3,500 residential units and 1,575 acres of commercial property. If the proposal is passed and completed, the city will have a new total of 3,600 residential units and 1,620 acres of commercial property. You can assume this increase is linear.

   (a) Name the variables and summarize the given information in a table.

   (b) How many new acres of commercial property are there for each new residential unit built?

   (c) Write an equation relating the variables. *Hint: first find the intercept.*

   (d) If the city decides to limit the amount of land to 1,600 acres of commercial property, approximately how many residential units can there be? Use successive approximation, displaying your guesses in a table.

   (e) Now answer the question by setting up and solving an inequality.

Chapter 5

A closer look at exponential equations

5.1 Modeling with exponential equations – Practice exercises

1. The population of Buenos Aires, Argentina in 1950 was estimated at 5.0 million and expected to grow at 1.8% each year. Source: Mongabay

   (a) Name the variables.

   (b) What is the annual growth factor?

   (c) Write an equation estimating the population of Buenos Aires over time.

   (d) Make a table of values showing the estimated population of Buenos Aires every 20^{th} year from 1950 to 2030.

   (e) By approximately how many people has the population been increasing per year over each 20 year period? Add these numbers to your table. As expected, these numbers change because the rate of change is not constant.

   (f) The actual population of Buenos Aires in the year 2000 was around 12.6 million and by 2010 it was around 15.2 million. How does that compare to the estimates?

2. A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 16% per day. Earlier we found the equation

$$N = 8 * 1.16^D$$

where D is the number of days (since the first diagnosis) and N is the total number of students who had the flu. *Story also appears in 2.2 #3 and 5.5 textbook*

(a) Use successive approximations to estimate when the number of infected students reaches 100. Display your guesses in a table.

(b) Use the Log Divides Formula to solve your equation.

(c) There are 1,094 students currently living in the dorms. Suppose ultimately 250 students catch the flu. According to your equation, when would that happen? Show how to solve your equation.

(d) It is not realistic to expect that everyone living in the dorms will catch the flu, but what does the equation say? Set up and solve an equation to find when all 1,094 students would have the flu. (Again, this is not realistic.)

3. Bunnies, bunnies, everywhere. Earlier we found the equation

$$B = 1{,}800 * 1.13^{Y}$$

where B is the number of bunnies and Y is the years since 2007.

Story also appears in 2.2 #2

(a) Make a table showing the number of bunnies in 2007, 2010, 2013, and 2020.

(b) Draw a graph showing how the bunny population grew.

(c) When will the population pass 5,000 bunnies? Guess from the graph. Then refine your answer using successive approximation.

(d) Solve your equation and check that you get the same answer.

4. Carbon dioxide is a greenhouse gas in our atmosphere. Increasing carbon dioxide concentrations are related to global climate change. In 1980, the carbon dioxide concentration was 338 ppm (parts per million). At that time it was assumed that carbon dioxide concentrations would increase .42% per year.

   Source: Earth Systems Research Laboratory, NOAA

   (a) Name the variables including units.

   (b) Assuming the growth is exponential as predicted, write an equation that describes the increase in carbon dioxide concentrations.

   (c) The carbon dioxide concentration in 2008 was 385 ppm. Is that count higher or lower than predicted from your equation? Explain.

   (d) Does that mean that carbon dioxide increased at a higher or lower rate than .42%? Explain.

5.2 Exponential growth and decay – Practice exercises

1. A signal is sent down a fiber optic cable. It decreases in strength by 2% each mile it travels. (Say it was one unit strong to start.)

   (a) Make a table showing the strength of the signal over the first five miles.

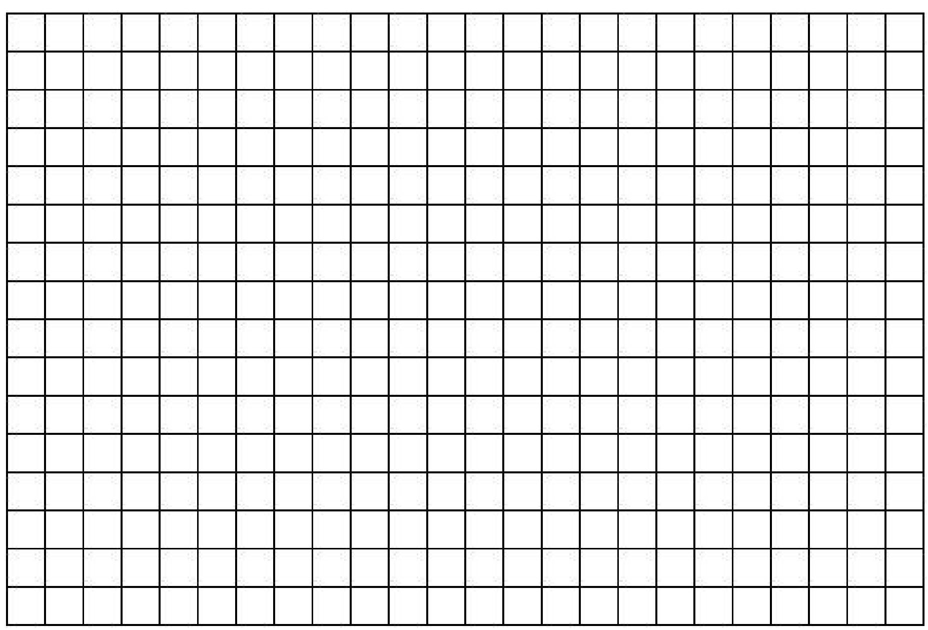
   (b) Name the variables, including units, and write an equation relating them.

   (c) The signal will need a **booster** (something to make the signal stronger again) when it has fallen to under .75 units. How far along the cable should the booster be placed? Set up and solve an equation.

The problem continues . . .

(d) What's the half-life (or should we say half-distance) of a signal? That means, how far can it travel without dropping below 50%? (That won't actual happen because we'd boost the signal.) Again, set up and solve an equation.

(e) Draw a graph illustrating the relationship.



(f) Indicate the points on your graph where you can check your answers to parts (c) and (d).

2. A recent news report stated that cell phone usage is growing exponentially in developing countries. In one small country, 50,000 people owned a cell phone in the year 2000. It was estimated that usage would increase at 1.4% percent per year.

   (a) Name the variables including units.

   (b) Assuming the growth is exponential, write an equation for the function.

   (c) At this rate, how many years would it take for the number of people owning a cell phone to double? That's called the **doubling time**. Show how to set up and solve an equation to find the answer.

   (d) In 2011, about 682,000 people owned a cellphone. Is that count higher or lower than predicted from your equation? Explain.

   (e) Based on the 2011 data, would you say that cell phone usage was growing slower or faster than 1.4%?

3. If a person has a heart attack and his or her heart stops beating, the amount of time it takes paramedics to restart his or her heart with a defibrillator is critical. Each minute that passes decreases the person's chance of survival by 10%. Assume that this statement means the decrease is exponential and that the survival rate is 100% if the defibrillator is used immediately. Source: American Red Cross

   (a) Name the variables and write an equation.

   (b) If it takes the paramedics 2 minutes to use the defibrillator, what is the person's chance of survival?

   (c) When does the survival rate drop below 50%? Use successive approximation to estimate to the nearest minute. Display your work in a table.

   (d) Solve your equation.

4. You and two buddies each invite 10 people to "like" your online group. Suppose everyone accepts and then they each invite 10 people. And then everyone accepts and they each invite 10 people. And so on. Of course, there is likely to be substantial overlap, but for the moment pretend that there isn't.

   (a) There are 3 friends to start. In the first round they each invite 10 friends, so a total of 30 new people "like" your online group in the first round. How many new people "like" your group in the second round? The third?

   (b) Name the variables and write an equation showing how the number of new people increases in each round. Think of the original 3 friends as round 0.

   (c) Make a table showing this information. Continue your table to include the number of new people who "like" your group in the fourth and fifth rounds.

   (d) What is the *total* number of people who "like" your online group after five rounds. *Hint: add*

   (e) Comment on why our assumption is unrealistic.

5.3 Growth factors – Practice exercises

PERCENT CHANGE FORMULA:

- If a quantity changes by a percentage corresponding to growth rate r, then the growth factor is
$$g = 1 + r$$
- If the growth factor is g, then the growth rate is
$$r = g - 1$$

GROWTH FACTOR FORMULA:

If a quantity is growing (decaying) exponentially, then the growth (decay) factor is

$$g = \sqrt[t]{\frac{a}{s}}$$

where s is the starting amount and a is the amount after t time periods.

---

1. Find the annual growth factor g and annual growth rate r (as a percent) for each story. Don't forget to include the negative sign for decay rates.

   Hint: first decide if you can use the PERCENT CHANGE FORMULA *or if you will need to use the* GROWTH FACTOR FORMULA.

   (a) Donations to the food shelf have increased 35% per year for the past few years.

   $g =$

   $r =$

   (b) People picking up food at the food shelf has increased exponentially too, from 120 per week in 2005 to 630 per week in 2011.

   $g =$

   $r =$

The problem continues ...

(c) The crime rate has dropped 3% each year recently.

$g =$
$r =$

(d) The creeping vine taking over Fiona's lawn doubles in area each year.

$g =$
$r =$

(e) Attendance at parent volunteer night has doubled every 3 years.

$g =$
$r =$

(f) The new stop sign has decreased accidents exponentially, from 40 in 2008 to 17 in 2013.

$g =$
$r =$

2. In 1962, my grandfather had savings bonds that matured to \$200. He gave those to my mother to keep for me. These bonds have continued to earn interest at a fixed, guaranteed rate so I have yet to cash them in. The table lists the value at various times since then.

year	1962	1970	1980	1990	2000	2010
Y	0	8	18	28	38	48
B	200.00	318.77	570.87	1,022.34	1,830.85	3,278.77

Story also appears in 1.2 #1 and 4.1 #3

(a) Use the GROWTH FACTOR FORMULA to find the annual growth factor for the time period from 1962 to 1970.

(b) Repeat for 1970 to 1980.

(c) What do you notice? What in the story told you that would happen?

(d) What is the corresponding interest rate?

(e) Write an equation for the value of bonds over time.

(f) Use your equation to check the information for 1990, 2000, and 2010.

The problem continues ...

(g) In what year will the bond be worth over \$5,000? Set up and solve an equation to decide.

(h) Draw a graph using the data in the table, but not your answer to part (g). Include another year that is later than your answer to part (g).

(i) Does your answer to part (g) agree with your graph? If not, fix what's incorrect.

3. Have you read news stories about archaeological digs where a specimen (like a bone) is found that dates back thousands of years? How do scientists know how old something is? One method uses the radioactive decay of carbon. After an animal dies the carbon-14 in its body very slowly decays. By comparing how much carbon-14 remains in the bone to how much carbon-14 should have been in the bone when the animal was alive, scientist can estimate how long the animal has been dead. Clever, huh? Actually, it's so clever that Willard Libby won the Nobel Prize in Chemistry for it. The key information to know is that the half-life of carbon-14 is about 5,730 years. For this problem, suppose a bone is found that should have contained 300 milligrams of carbon-14 when the animal was alive. Source: Wikipedia (Radiocarbon Dating)

   (a) Find the annual "growth" factor. Keep as many digits as possible for your calculations.

   (b) Name the variables and write an equation describing the dependence.

   (c) How many milligrams of carbon-14 should remain in this bone after 1,000 years? After 10,000 years? After 100,000 years?

   (d) How many milligrams of carbon-14 should remain in this bone after 1 million years? Explain the answer your calculator gives you.

The problem continues . . .

(e) Draw a graph that shows up to 10,000 years.

(f) If the bone is determined to have 100 milligrams of carbon-14, how old is it? That is, approximately how long ago did it die? Start by estimating the answer from your graph. Then revise your estimate using successive approximation. Display your work in a table.

(g) Solve the equation exactly.

4. For each equation, find the growth rate and state its units. For example, something might "grow 2% per year" while something else might "drop 7% per hour"

   (a) The number of households watching reality television R (in millions) was estimated by the equation
   $$R = 2.5 * 1.072^Y$$
   where Y is the years since 1990. *Story also appears in 5.1 Exercises*

   (b) Chlorine is often used to disinfect water in swimming pools, but the concentration of chlorine C (in ppm) drops as the swimming pool is used for H hours according to the equation
   $$C = 2.5 * .975^H$$
   Story also appears in 3.4 #2

   (c) The number of players of a wildly popular mobile app drawing game has been growing exponentially according to the equation
   $$N = 2 * 1.57^W$$
   where N is the number of players (in millions) and W is the number of weeks since people started playing the game. *Story also appears in 5.1 Exercises*

5.4 Linear vs. exponential models – Practice exercises

Linear equation template: $\text{dep} = \text{start} + \text{slope} * \text{indep}$

Rate of Change/Slope (of Linear) Formula:

$$\text{rate of change} = \frac{\text{1st dep - 2nd dep}}{\text{1st indep - 2nd indep}}$$

Exponential equation template: $\text{dep} = \text{start} * \text{growth factor}^{\text{indep}}$

Growth Factor Formula:

If a quantity is growing (decaying) exponentially, then the growth (decay) factor is

$$g = \sqrt[t]{\frac{a}{s}}$$

where s is the starting amount and a is the amount after t time periods.

Percent Change Formula:

- If a quantity changes by a percentage corresponding to growth rate r, then the growth factor is
$$g = 1 + r$$
- If the growth factor is g, then the growth rate is
$$r = g - 1$$

---

1. My parents bought the house I grew up in for \$35,000 and sold it 40 years later for \$342,000. True story. (It was before the housing bubble burst.)

   First, assume the value of the house increased exponentially.

   (a) Calculate the annual growth factor using the Growth Factor Formula.

   (b) In this model, by what percentage did the house value increase each year? *Hint: use the* Percent Change Formula.

   (c) Write an exponential equation showing how the value of the house increased. Don't forget to name the variables, including units. *Hint: use the* Exponential Equation Template.

   (d) Check that your equation gives the correct sold value.

The problem continues ...

Next, assume the value of the house increased linearly instead.

(e) In this model, by what fixed amount did the house value increase each year? *Hint: calculate the slope using the* RATE OF CHANGE/SLOPE (OF LINEAR) FORMULA.

(f) Using the same variables, write a linear equation showing how the value of the house increased. *Hint: use the* LINEAR EQUATION TEMPLATE.

(g) Check that your equation gives the correct sold value.

2. The number of manufacturing jobs in the state has been declining for decades. In 1970, there were 1.2 million such jobs in the state but by 2010 there were only .6 million such jobs. Write J for the number of manufacturing jobs (in millions) and Y for the years since 1970.

   First, assume the number of jobs decreased linearly.

   (a) Calculate the slope.

   (b) Write a linear equation showing how the number of jobs declined.

   (c) Check that your equation gives the correct value for 2010.

   Next, assume the number of jobs decreased exponentially instead.

   (d) Calculate the growth factor.

   (e) Write an exponential equation showing how the number of jobs declined.

   (f) Check that your equation gives the correct value for 2010.

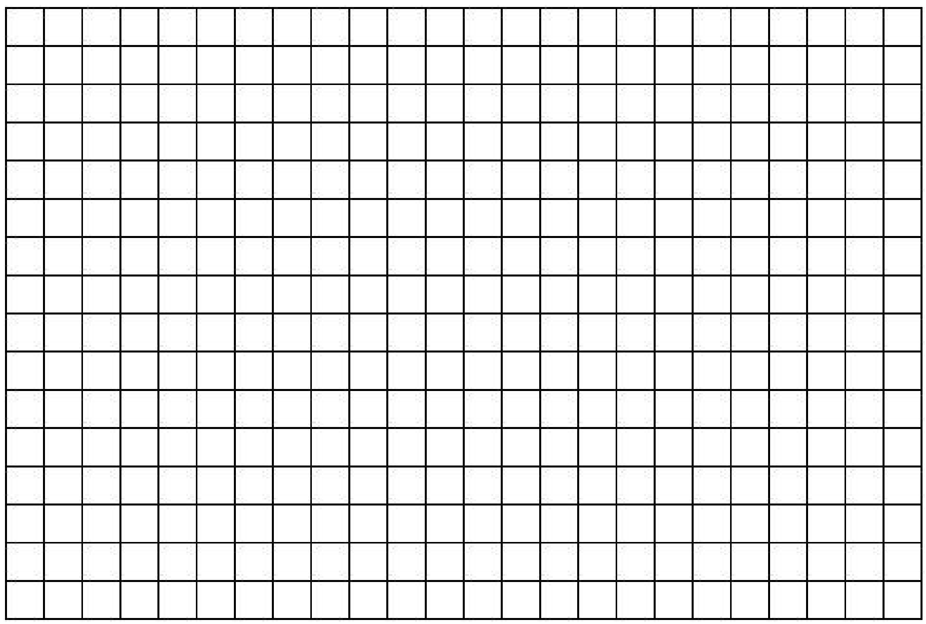
The problem continues ...

Now, compare the models.

(g) Complete the table of values.

year	1970	1990	2010	2020	2030
Y	0	20	40	50	60
J (if linear)					
J (if exponential)					

(h) Draw a graph showing both models.



(i) Which model has better news for 2030?

3. In December 2010, a popular mobile app game featuring animated birds launched from slingshots had 50 million downloads. Five months later (May 2011), the game had 200 million downloads. Let D denote the number of downloads of the game (in millions) and M the months since December 2010.

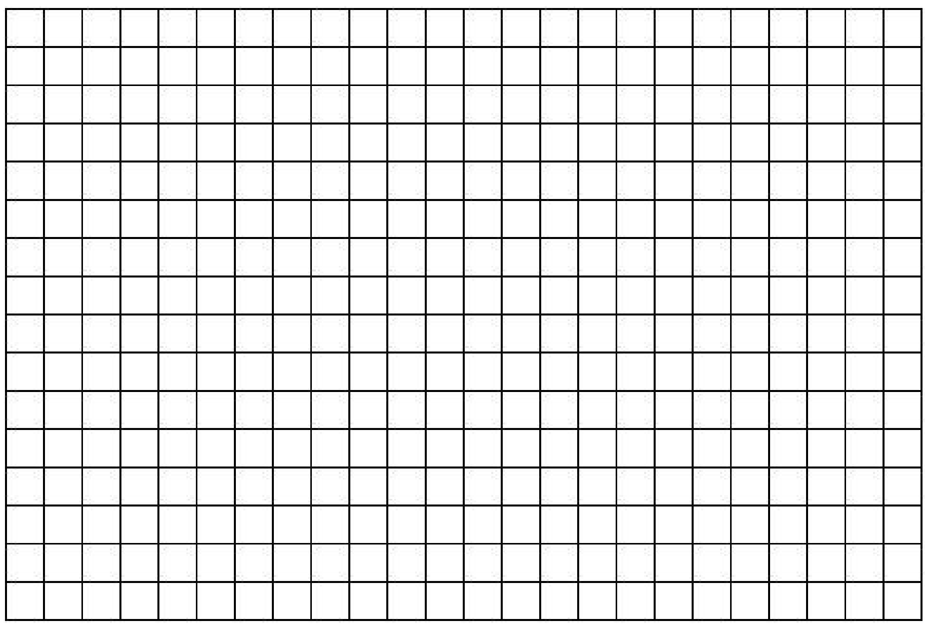
   (a) Suppose that the number of downloads have been increasing at a *constant rate each month.* What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$).

   (b) Suppose that the number of downloads have been increasing at a *fixed percentage each month.* What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$).

4. Bus fares are up to \$2.25 per ride during rush hour. Two different plans of increasing fares are being debated: 10¢ per year or 2.5% per year.

   (a) Make a table comparing these plans over the next **decade** (ten years).

   (b) Draw a graph showing both options.



   (c) As a city council representative, you want to support the plan that your constituents prefer. If most of your constituents ride the bus, which plan should you support?

   (d) If most of your constituents are members of the same union as the bus drivers (who count on solid earnings from the bus company to keep their jobs), then which plan should you support?

The problem continues . . .

(e) Which type of equation is being suggested in each plan? Write the equations. Don't forget to name the variables, including units.

5.5 Logistic and other growth models – Practice exercises

1. Corn farmers say that their crop is healthy if it is "knee high by the Fourth of July." An equation that relates the height H (in inches) of the corn crop D is days since May 1 is

$$H = 106 - 100 * .989^D$$

   (a) According to this equation, how high is corn projected to be on June 1 (day 31)?

   (b) According to this equation, how high is corn projected to be on the Fourth of July (day 64)? Is that "knee high" (18 inches tall)?

   (c) With stronger corn these days, the rule ought to be "chest high (52 inches) by the Fourth of July." According to this equation, when is the corn projected to be that tall? Use successive approximation to answer.

   (d) The corn matures in 110 days. How tall will it be then?

   (e) Draw a graph of the function. Include when $D = 0$.

2. An alternative equation for corn height is

$$H = \frac{200}{1 + 70 * .965^D}$$

   (a) According to this new equation, how high is corn projected to be on June 1 (day 31)?

   (b) According to this new equation, how high is corn projected to be on the Fourth of July (day 64)? Is that "knee high" (18 inches tall)?

   (c) According to this new equation, on approximately what date is the corn projected to be "chest high" (52 inches tall)? Use successive approximation to answer.

   (d) The corn matures in 110 days. How tall will it be then, according to this new equation?

   (e) Add the graph of this function to your graph of the original equation on the previous problem. Again, include when $D = 0$.

3. Back in 1975 when my aunt and uncle bought their house upstate New York, there was a small pond in the yard. They enlarged it and stocked it with 10 small fish. The number of fish F increased over time, approximately according to the equation

$$F = \frac{1{,}000}{1 + 99 * .65^{Y}}$$

where Y measures the years since 1975.

   (a) Make a table showing the fish population in 1975, 1990, 2000, and 2013.

   (b) By the time there were over 500 fish in the pond, you could catch them with your bare hands. In approximately what year did that happen?

   (c) In approximately what year did the fish population reach its capacity? Use successive approximations and display your calculations in a table.

4. Jason works at a costume shop selling Halloween costumes. The shop is busiest during the fall before Halloween. An equation that describes the number of daily visitors V the shop receives D days from August 31 is the following:

$$V = \frac{430}{1 + 701 * .81^D}$$

An alternative equation is

$$V = 700 - 690 * .985^D$$

(a) Make a table showing what each equation predicts for August 31, September 15, September 30, October 15, October 25, October 28, and October 31.

Hint: those days are numbered 0, 15, 30, 45, 55, 58, and 61.

(b) Graph both functions on the same set of axes.

(c) Which function is more consistent with a major advertising campaign during the second week of September? Explain.

Practice Exam 5A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

---

1. Leopard print hat. Originally 5 out of 1,000 women shopping at a major retail store even looked twice. But that number grew and grew, by my estimate around 40% a week, thanks to carefully placed ads in fashion magazines.

   (a) Write an equation illustrating the interest in leopard print hats using W for the time (in weeks) and L for the number of women interested in leopard print hats (women per thousand).

   (b) Make a table showing the number of women, per thousand female shoppers, who stop and look at the hat at the start, 1 week, 2 weeks, and 3 weeks after it hits the stores.

   (c) The leopard print hat is considered popular when more than 300 out of 1,000 women try it on. According to the equation, when will the hat be considered popular? Use successive approximation to find the answer to the nearest week and display your work in a table.

   (d) The hat will be considered passé when over 750 out of 1,000 women try it on. I mean – everyone's got one! According to your equation, when will that happen? Set up and solve an equation, again answering to the nearest week.

2. HeeChan bought a classic car in 2003 for investment purposes and has been watching the value increase over the years. Based on the data HeeChan came up with two possible equations

$$\textbf{Logistic:} \quad C = \frac{41{,}000}{1 + 4 * .81^Y}$$

$$\textbf{Saturation:} \quad C = 32{,}000 - 23{,}800 * .85^Y$$

where Y is the years since 2003 and $\$C$ is the value of the car.

(a) How much did HeeChan pay for the car in 2003?

(b) What does each equation predict for the value of the car now, in 2013? For 2020?

(c) What does each equation say will be the eventual value long term? *Hint: if you're not sure try 100 years.*

3. The number of geese in the Twin Cities metropolitan area increased from 480 in 1968 to 25,000 in 1994. Although population is sometimes modeled with exponential models, there are many factors that might make an exponential model inappropriate, such as changes in migration, wetlands, and hunting.

   (a) Name the variables.

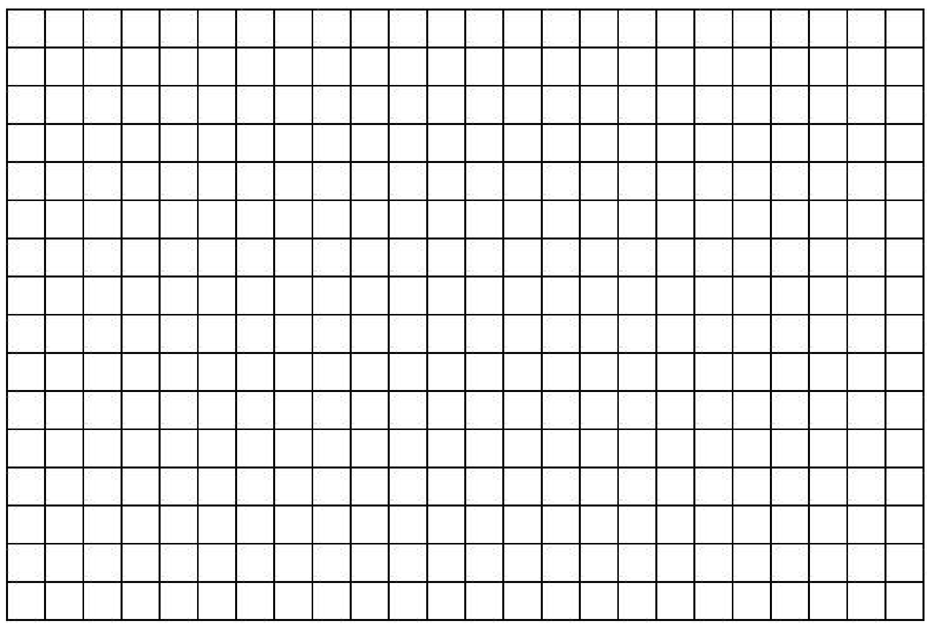
   (b) Write a linear equation modeling the goose population.

   (c) Now write an exponential equation modeling the goose population.

The problem continues . . .

(d) Compare the models projections for 1968, 1975, 1984, 1994, 2000, 2010, and 2020. Summarize your findings in a table.

(e) Graph each function over the period from 1968 to 2020 on the same set of axes. *Test-taking tip: even if you have trouble with the equations, you should be able to plot the information given in the story and sketch in the appropriate shape curves.*



(f) Research indicates that the Twin Cities metropolitan area could support 60,000 geese. Use your graph to estimate when that will happen.

(g) The actual goose population in 2010 was around 50,000. Which model was closer?

4. One of the toxic radioactive elements produced by nuclear power plants is strontium-90. A large amount of strontium-90 was released in the nuclear accident at Chernobyl in the 1980's. The clouds carried the strontium-90 great distances. The rain washed it down into the grass, which was eaten by cows. People then drank the milk from the cows. Unfortunately, strontium-90 causes cancer. Strontium-90 is particularly dangerous because it has a half-life of approximately 28 years, which means that every 28 years half of the existing strontium-90 changes into a safe product; the other half remains strontium-90. Suppose that a person drank milk containing 100 milligrams of strontium-90.

SOURCE: "Explorations in College Algebra," by Kime and Clark

(a) After 28 years, how many milligrams of strontium-90 remains in the person's body? After 56 years?

(b) Find the annual percentage decrease of strontium-90.

(c) Name the variables and write an equation relating them.

(d) Suppose that any amount under 20 milligrams of strontium-90 is considered "acceptable" in humans. Will it have reached acceptable levels after 70 years?

Practice Exam 5B

Try taking this version of the practice exam under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

---

1. The number of school children in the district whose first language in not English has been on the rise. The equation describing the situation is

$$C = 673(1.043)^Y$$

where C is the number of school children in the district whose first language is not English, and Y is the number of years (from now).

   (a) Make a table showing the number of school children in the district whose first language is not English now, in one year, in two years, and in ten years. *Don't forget now too.*

   (b) What percent increase is implicit in this equation?

   (c) Use successive approximation to determine when there will be over 1,700 school children in the district whose first language is not English Display your work in a table. Round your answer to the nearest year.

   (d) Show how to solve the equation to calculate when there will be over 1,700 school children in the district whose first language is not English. Show how you solve the equation.

2. The lottery jackpot started at \$600,000. After 17 days the jackpot had increased to \$2.1 million. The lottery is designed so that the jackpot grows exponentially.

   (a) Name the variables including units.

   (b) Write an equation describing the jackpot. *Hint: find the daily growth factor.*

   (c) By what percentage does the jackpot increase each day?

   (d) What will the jackpot be after 20 more days (i.e. after 37 days total)?

3. The creeping vine is taking over Fiona's front lawn. Write V for the area covered by the vine (in square feet) and Y for the years since she moved in to the house.

   (a) When Fiona moved in there maybe 3 square feet covered by vine. She believes it has doubled each year since. Write an exponential equation showing how the area covered by the vine is a function of time in this case. *Stuck? Try doing the table first.*

   (b) At some point the vine will take over the entire lawn, so perhaps a saturation model would be better. That equation might be

$$V = 170 - 167 * .8^Y$$

   Another equation would be a logistic model. Perhaps

$$V = \frac{129}{1 + 42 * .34^Y}$$

   Fill in the corresponding rows of the table for each model.

years	0	1	2	3	4	5	6
area exponential							
area saturation							
area logistic							

The problem continues ...

(c) Draw a graph showing all three models on the same set of axes.

4. Many different agencies and governments are working to lower infant mortality. Infant mortality is measured in deaths per thousand births. The world infant mortality rate in 1955 was around 52 (per thousand births). By the year 2000, it was down to around 23. Source: Wikipedia (Infant Mortality)

   (a) Name the variables.

   (b) Write a linear equation modeling infant mortality.

   (c) Now write an exponential equation modeling infant mortality.

   (d) Compare the models projections for 1955, 1970, 1990, 2000, 2010, and 2020. Summarize your findings in a table.

   (e) The actual rates were 40 deaths per thousand births in 1970 and 28 deaths per thousand births in 1990. Which model fits this additional data better?

Chapter 6

Review

Practice Final Exam A

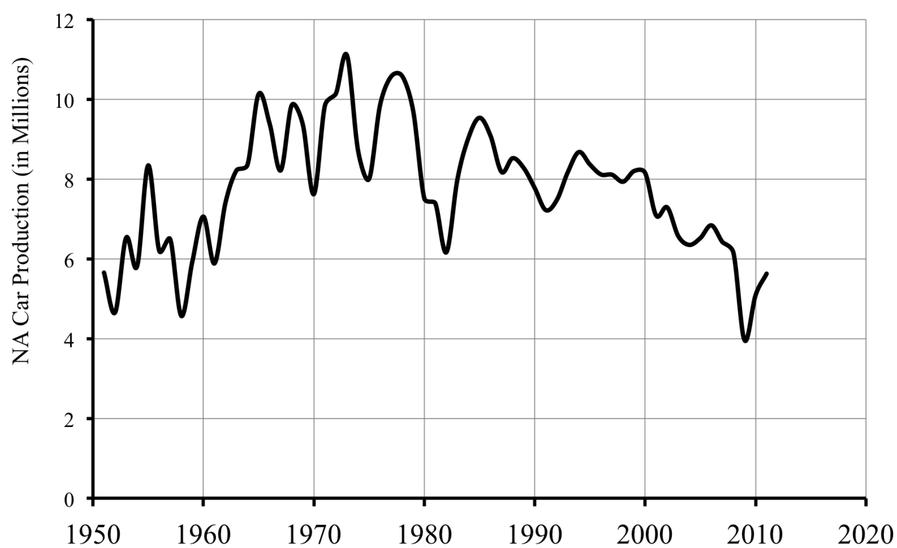
Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

Caution: These review exercises do not include every possible problem you might be asked on a final exam. For example, there are no problems here from Sections 1.5, 2.5, 3.5, 4.5, and 5.5 so be sure to ask your instructor which of those sections are going to be on your final exam.

---

1. The graph shows the number of cars produced in North America (in millions/year) during 1951-2011.



Source: Wards Automotive Yearbooks.

   (a) Identify the variables, including units and dependence.

The problem continues . . .

(b) Approximately when did North American car production first pass 9 million/year? *Indicate the corresponding point on the graph.*

(c) In which year were the most cars produced? *Again, indicate the point.*

(d) Best as you can tell from your graph, what might be a reasonable estimate of North American car production in 2015? Just guess to the nearest million/year.

(e) Calculate the rate of change from 1958 when production was 4.57 million cars/year to 1971 when it was 9.83 cars million/year. What does that tell you about North American car production during 1958-1971?

(f) Now calculate the rate of change of from 1984 when production was 9.03 million cars/year to 2006 when it was 6.84 million cars/year. What does that tell you about North American car production during 1984-2006?

2. Sarah and Koal are bringing a large basket of stuffed animals to the crisis nursery as gifts for the children. They estimate it will cost $\$T$ for S stuffed animals where

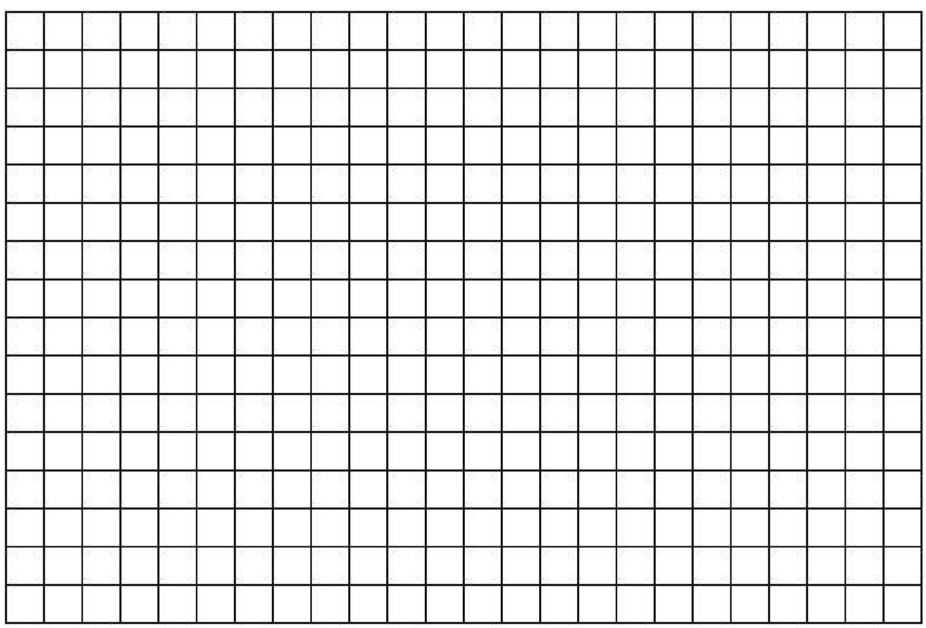
$$T = 39.99 + 6.95S$$

(a) Make a table showing the cost if Sarah and Koal include 10, 20, or 40 stuffed animals.

(b) Included in the cost is a new toy box for the animals. What does it cost?

(c) What does the 6.95 represent and what are its units?

(d) Draw a detailed graph, starting at 0.



The problem continues ...

(e) If Sarah and Koal spent \$262.39, how many stuffed animals were in the toy box they gave to the crisis nursery? Show how to set up and solve an equation to answer the question.

(f) Solve the inequality

$$200 \leq 39.99 + 6.95S \leq 300$$

What does the answer mean in terms of the story?

3. My favorite little black dress is machine washable. Unfortunately each time I wash it the color fades a little. The intensity of black color remaining, B, is a function of the number of times I have washed the dress, W, according to the equation

$$B = 100 * .985^{W}$$

   (a) It will still look new as long as the intensity stays above 90%. Set up and solve an equation to figure out how many times I can wash the dress and keep it looking new. Then check some other way.

   (b) By the time only 75% of the color remains, the dress will look too faded to wear formally. How many washes before then? Find the answer to the nearest number of washes by any method you prefer.

   (c) Draw a graph showing how the color of my favorite little black dress fades.

4. Brock is working as the equipment manager at a local gym. They need to replace several weight machines. One option will cost \$475 per month to rent the machines plus a delivery/removal fee of \$300. The other option is to buy the machines for \$23,600 and pay \$92/month for a service contract.

   (a) What should Brock recommend if they plan to have the machines for 3 years?

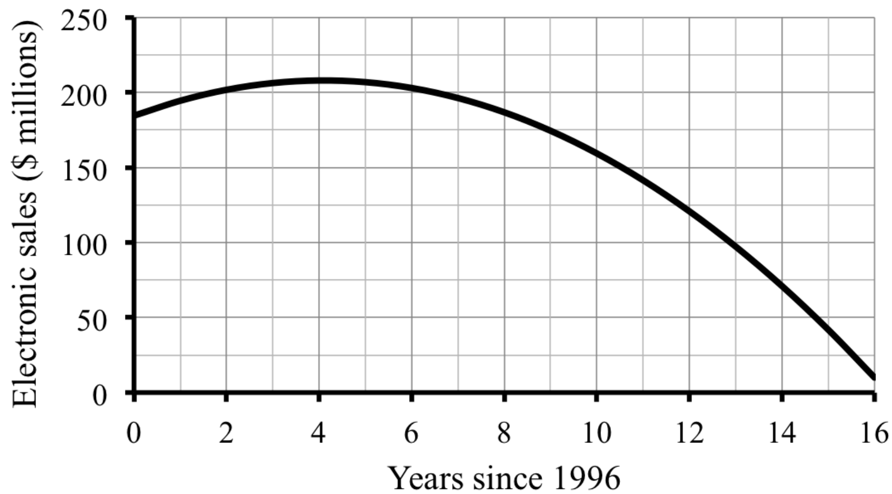
   (b) Name the variables and write an equation for each option.

   (c) Set up and solve a system of equations to determine when the options cost the same.

   (d) What does the answer tell Brock?

5. Dwight's company was doing great business in 2000, but a few years later sales began to drop, and have only gotten worse. Their sales $\$S$ in millions Y years from 2000 is given by the following equation

$$S = 78.1 + 5.75Y - 0.7Y^2$$



(a) According to this equation, what were the company's sales in 2000, 2004, 2009? *You may confirm your answers with the graph, but use the equation to calculate.*

(b) The company decided to declare bankruptcy when sales fell below $10 billion. In what year was that? Find the answer to the nearest year, showing work to justify your answer. Also, indicate the point on the graph where you can check. *You may use successive approximations or the appropriate formula.*

6. Infants are regularly checked to make sure they are growing accordingly. The World Health Organization publishes growth charts to evaluate infant weight W in kilograms at a given age M in months since birth (for up to three years). An equation that describes an average infant boy is the following:

$$W = 15 - 11.5 * .932^M$$

   (a) According to this equation, what is the average infant boy weight at birth, 4 months, a year, and 2 years?

   (b) Convert your answer for 4 months to pounds and ounces using

$$1 \text{ kilogram } \approx 2.2 \text{ pounds} \quad \text{and} \quad 1 \text{ pound } = 16 \text{ ounces}$$

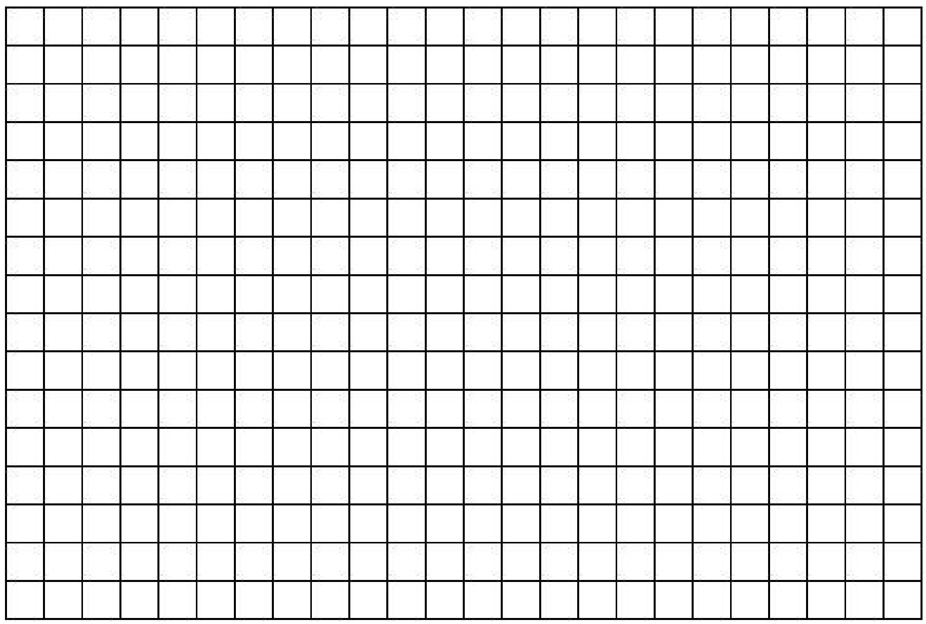
   Hint: first convert to pounds. Then convert just the decimal part to ounces.

7. Gail calculated that number of pieces of fudge F she can cut from a square that's D inches on each is given by the formula

$$F = 1.5625D^2$$

(a) Make a table showing the number of pieces of fudge from a square with edge: 5 inches, 10 inches, and 2 feet. Include the value for a 0 inch square too.

(b) Draw a graph showing how the number of pieces of fudge depends on the length of the edge of the square.



(c) According to your graph, what size square should Gail make if she wants 200 pieces of fudge?

(d) Now set up and solve an equation to find the answer to the nearest one decimal place.

8. In 2000 there were an estimated 20,851,820 Texans. The population of Texas has grown around 1.89% per year since then.

source: United States Census Bureau

(a) Name the variables and write an equation relating them.

(b) According to your equation, what was the population of Texas in 2010?

(c) The U.S. Census Bureau counted 25,145,561 Texans in 2010. Does that mean the actual growth rate was slightly more or slightly less than 1.89%? Explain.

9. Ericson has been lifting weight for the past 12 weeks. He has increased his curl weight by about 1.5 pounds per week and is up to 30 pounds.

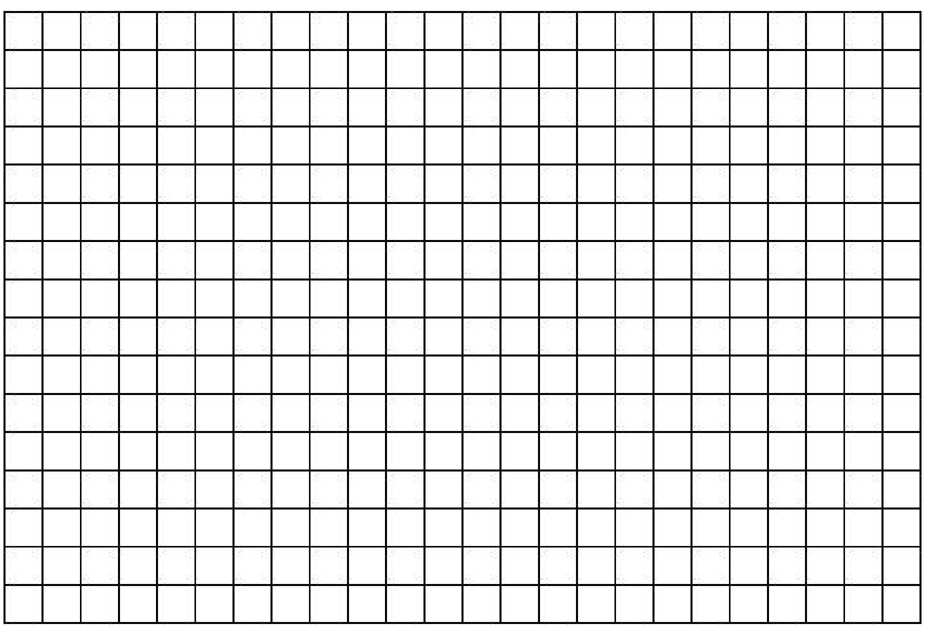
   (a) What weight could Ericson curl 12 weeks ago?

   (b) Name the variables and write a linear equation relating them.

   (c) At this rate when will Ericson be able to curl his goal of at least 45 pounds? Set up and solve an inequality.

10. In the United States in 1970, the average person ate 2,169 calories per day. By 2008 that number had increased to 2,674 calories per day. Let C be the amount a typical person eats (in calories per day) and Y the year, measured in years since 1970. Compare what the linear and exponential models project for the years 2015 and 2030. Include both equations and a graph showing both functions on the same axes.

source: United States Department of Agriculture

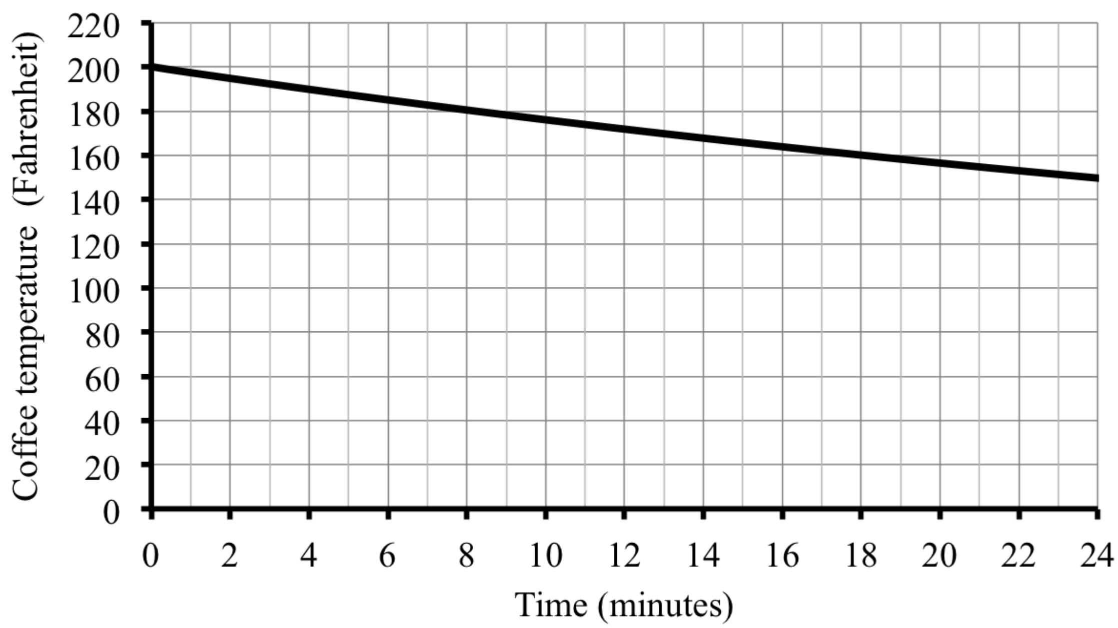


Practice Final Exam B

Try taking this version of the practice exam under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself two hours to work and wait until you have tried your best on all of the problems before checking any answers.

---

1. I love coffee. But not when it gets cold. The graph shows how my cup of coffee cools.



(a) Identify the variables, including units and dependence.

(b) Answer each question and indicate the point on the graph you use.

i. How hot is my coffee right after I pour it (before it starts cooling)?

ii. I simply will not drink my coffee once it's cooler than 150°F. How long does it take for my coffee to cool off that much?

iii. I prefer to drink my coffee between 160°F and 180°F. What's the corresponding time period during which I should drink my coffee?

2. Jolene is driving up to Duluth to visit her aunt. Unfortunately it's snowing so traffic is going slowly. Her distance D miles to Duluth is described by the equation

$$D = 100 - 45H$$

where H is the hours since 12:00 noon when Jolene started driving.

(a) Identify the intercept, including units, and explain what it means in the story.

(b) Identify the slope, including units, and explain what it means in the story.

(c) Jolene plans to call her aunt once she's under 20 miles from Duluth. When will that be? Show how to set up and solve an inequality to answer the question. Find the exact time, to the nearest minute.

3. There are sure a lot of squirrels in my neighborhood. The equation

$$S = 4{,}000 * 1.12^{Y}$$

estimates the number of squirrels (S) where Y is the years since 2005.

(a) Make a table showing the number of squirrels in 2005, 2008, 2013, and 2017.

(b) Draw a graph showing how the squirrel population grew.

The problem continues ...

(c) When will the population pass 10,000 squirrels? Guess from the graph. Then refine your answer using successive approximation.

(d) Show how to solve the equation to determine when there will be 10,000 squirrels.

(e) There were 10,000 in 2011, so our equation is a bit off. Assuming there were still 4,000 squirrels in 2005, revise the equation. *Hint: find the new growth factor.*

4. Gail calculated that the number of calories C in a cube of fudge depends on how large the cube is, say E inches long on each edge. A possible equation is

$$C = 90E^3$$

   (a) How many calories are in a cube of fudge that's 1 inch on each edge?

   (b) What size cube of fudge has 200 calories? Answer to the nearest tenth (that means to 1 decimal place), showing work to justify your answer.

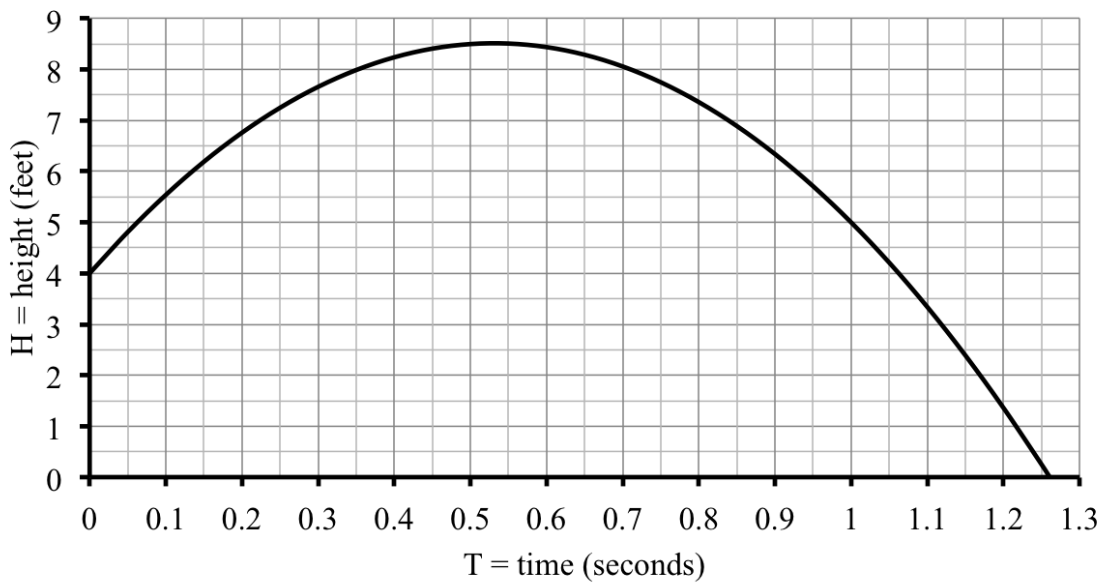
   You may use successive approximations or the appropriate formula.

   (c) Convert your answer to millimeters (mm) using 1 inch $\approx$ 2.54 cm and 1 cm = 10 mm.

   Test-taking tip: No answer for part (b)? Write down a guess and convert that.

5. The height H feet of a ball T seconds after it is thrown straight up in the air is given by the equation

$$H = 4 + 17T - 16T^2$$



(a) According to the equation, how high up was the ball to start, after 0.5 seconds, and after 1 second? *Use the equation to evaluate and check against the graph.*

(b) Calculate the speed (rate of change) between 0.7 seconds and 0.8 seconds.

The problem continues ...

(c) Convert your answer from part (b) to mph. Use 1 mile = 5,280 feet.

Test-taking tip: No answer for part (b)? Write down a guess and convert that.

(d) When will the ball hit the ground? Find the answer to the nearest hundredth (that means to 2 decimal places), showing work to justify your answer.

You may use successive approximations or the appropriate formula.

6. A local sporting goods store does custom embroidered jerseys for \$29 each plus \$1.75 per letter. Or you can order the same jerseys online for \$18 each plus \$2.35 per letter, but it costs another \$4.95 for shipping per jersey. If we write L for the number of letters on the jersey and T for the total cost (in \$), then those equations are

Local shop:	$T = 29 + 1.75L$
Online:	$T = 22.95 + 2.35L$

   (a) If a player named **Redeisheimer** (12 letters) wants a jersey, which option costs least?

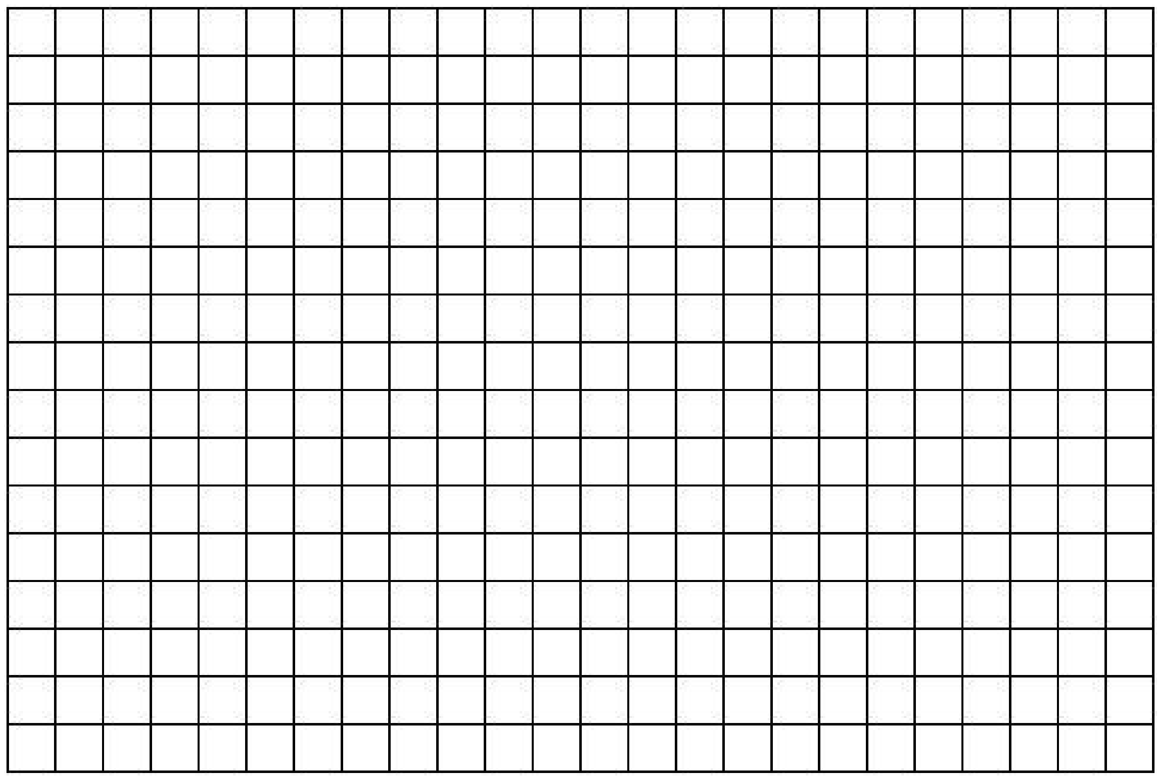
   (b) Make a table showing the cost for players named: **Buls** (4 letters), **Schaaf** (6 letters), and **Johnston** (8 letters).

   (c) Set up and solve a system of equations to determine when the options cost the same.

   (d) Summarize your findings in words.

The problem continues . . .

(e) Graph both functions on the same set of axes. *Don't forget 0 letters.*



(f) Indicate the point on your graph where you can check your solution to part (c). *If it doesn't agree, check your work and/or your graph again.*

Just two more problems. Good work! Keep going.

7. For their holiday party at the office, Adriana had a tray of 200 empanadas delivered for \$196. They were so good that she had a tray of 72 empanadas delivered to bring to her sister's on Christmas Eve, which cost \$78.24. Assume that the total cost is a linear function of the number of empanadas.

   Test-taking tip: Note sure about parts (b) and (c)? Try finding the equation first.

   (a) Name the variables, including units and dependence.

   (b) What does each empanada cost?

   (c) What is the delivery charge?

   (d) Write an equation for the function.

8. Bus fares are up to \$3.00 per ride during rush hour. Two different plans of increasing fares are being debated: 15¢ per year or 4.5% per year.

   (a) Name the variables, including units and dependence.

   (b) Write an equation describing bus fares over the next few years, assuming they increase 10¢ per year. What is this type of function called?

   (c) Write an equation describing bus fares over the next few years, assuming they increase 4.5% per year. What is this type of function called?

The problem continues ...

(d) Compare what each equation tells you bus fares would be in 1 year, 5 years, and 20 years. List your answers in a table.

(e) Graph both options on the same set of axes. *Don't forget now.*

Templates

Templates are used to write equations from given or calculated information, to read constants from the equations, or simply to identify the type of equation in order to use type-specific formulas or information.

LINEAR EQUATION TEMPLATE: $\text{dep} = \text{start} + \text{slope} * \text{indep}$

POWER EQUATION TEMPLATE: $\text{dep} = k * \text{indep}^n$

QUADRATIC EQUATION TEMPLATE: $\text{dep} = a * \text{indep}^2 + b * \text{indep} + c$

EXPONENTIAL EQUATION TEMPLATE: $\text{dep} = \text{start} * \text{growth factor}^{\text{indep}}$

Formulas

Formulas are used for many purposes, including finding constants or solving specific types of equations. Various disciplines, such as finance, have specific formulas to evaluate quantitiies.

Formulas used to find constants:

RATE OF CHANGE/SLOPE (OF LINEAR) FORMULA:

$$\text{rate of change} = \frac{\text{1st dep - 2nd dep}}{\text{1st indep - 2nd indep}}$$

INTERCEPT (OF LINEAR) FORMULA:

$$\text{intercept} = \text{dep} - \text{slope} * \text{indep}$$

PERCENT CHANGE FORMULA:

- If a quantity changes by a percentage corresponding to growth rate r, then the growth factor is
$$g = 1 + r$$
- If the growth factor is g, then the growth rate is
$$r = g - 1$$

GROWTH FACTOR FORMULA:

If a quantity is growing (decaying) exponentially, then the growth (decay) factor is

$$g = \sqrt[t]{\frac{a}{s}}$$

where s is the starting amount and a is the amount after t time periods.

Formulas used to find solve specific types of equations:

ROOT FORMULA: The equation $C^n = v$ has solution $C = \sqrt[n]{v}$

LOG-DIVIDES FORMULA: The equation $g^Y = v$ has solution $Y = \dfrac{\log(v)}{\log(g)}$

QUADRATIC FORMULA: The equation $aT^2 + bT + c = 0$ has solutions

$$T = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Formulas from finance:

COMPOUND INTEREST FORMULA: $a = p\left(1 + \dfrac{r}{12}\right)^{12y}$

EQUIVALENT APR FORMULA: $\text{APR} = \left(1 + \dfrac{r}{12}\right)^{12} - 1$

FUTURE VALUE ANNUITY FORMULA: $a = p * \dfrac{\left(1 + \frac{r}{12}\right)^{12y} - 1}{\frac{r}{12}}$

LOAN PAYMENT FORMULA: $p = \dfrac{a * \frac{r}{12}}{1 - \left(1 + \frac{r}{12}\right)^{-12y}}$

where

a = account balance or loan amount ($)
p = initial deposit (principal), regular deposit, or regular payment ($)
y = time invested (years)
r = interest rate compounded monthly (as a decimal)