SOLUTIONS

4.1 Modeling with linear equations – Practice exercises

1. A solar heating system costs approximately $30,000 to install and $150 per year to run. By comparison, a gas heating system costs approximately $12,000 to install and $700 per year to run. *Story also appears in 4.2 Exercises*

   (a) What is the total cost for installing and running a gas heating system for 30 years?

   $\$12{,}000 + \$700/\text{yr} * 30 \text{ yrs}$

   $= 12000 + 700 \times 30 = \boxed{\$33{,}000}$

   (b) Write a linear equation showing how the total cost for a gas heating system depends on the number of years you run it.

   don't forget to name the variables

   T = total cost heating system($) ~dep

   Y = time (years) ~indep

   $\boxed{T = 12{,}000 + 700Y}$

   generalized from (a) note: fits LINEAR EQUATION TEMPLATE

   (c) Write a linear equation showing how the total cost for a solar heating system depends on the number of years you run it.

   $\boxed{T = 30{,}000 + 150Y}$

   (d) How many years of a solar heating system could you get for the cost of a gas heating system lasting 30 years (your answer to part (a))? Set up and solve an equation.

   $$\cancel{30{,}000} + 150Y = 33{,}000$$
   $$\cancel{-30{,}000} \qquad -30{,}000$$
   $$\frac{\cancel{150}Y}{\cancel{150}} = \frac{3{,}000}{150}$$
   $$Y = 20 \text{ years}$$

   You can install and run the solar system for 20 years for what it costs to run the gas system for 30 years

2. Since a very popular e-book reader was released, the price has been decreasing at a constant rate. A blogger developed the following equation representing the price E of the e-book reader in the months M since it was released.

$$E = 359 - 12M$$

(a) Make a table of values for the e-book reader price initially, after 10 months, and after 25 months.

$M=0$

indep →	M	0	10	25
dep →	E	359	239	59

$359 - 12 \times 10 =$

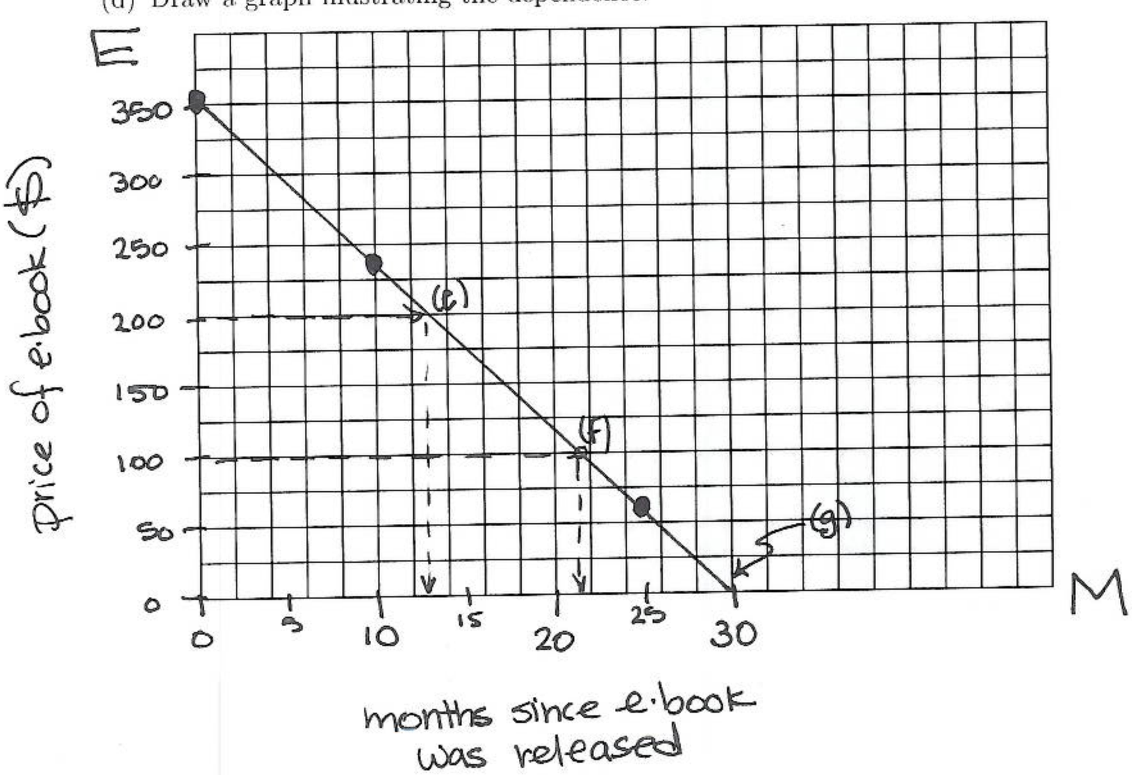
(b) What does the 359 mean in the story and what are its units?

The e·book originally cost \$359.
Units = \$ Note: intercept

(c) What does the 12 mean in the story and what are its units?

The e·book is projected to drop \$12/mo in price. Units = \$/mo Note: slope

(d) Draw a graph illustrating the dependence.



The problem continues ...

(e) After approximately how many months was the price of the e-book reader expected to be down to $200? Set up and solve an equation.

$\cancel{359} - 12M = 200$
$-\cancel{359} \qquad -359$
$\frac{-\cancel{12}M}{-\cancel{12}} = \frac{-159}{-12}$
$M = 13.25$

The price was expected to drop to $200 after just over 13 months

(f) Sareth decided to purchase a e-book reader when the price fell below $100. How many months after its release did the price of the e-book reader fall below that level? Set up and solve an inequality.

$\cancel{359} - 12M < 100$
$-\cancel{359} \qquad -359$
$\frac{-\cancel{12}M}{-\cancel{12}} < \frac{-259}{-12}$
$M > 21.58$

Remember: ÷negative so inequality is reversed

The price was expected to drop below $100 after 22 months

(g) If you can believe what you read in blogs, the manufacturer will soon be giving away the e-book reader for free, since they make money on the e-book sales themselves. How many months after it was released would that happen, according to our equation? Set up and solve an equation.

E = $0

$\cancel{359} - 12M = 0$
$-\cancel{359} \qquad -359$
$\frac{-\cancel{12}M}{-\cancel{12}} = \frac{-359}{-12}$
$M = 29.916\ldots$

The price is projected to be "free" after 30 months

Check answers w/ our graph!

rate of change $= \dfrac{\text{diff dep}}{\text{diff indep}}$

3. Can you tell from the table which of these functions are linear? Use the rate of change to help you decide. Remember that these numbers may have been rounded.

(a) Savings bonds from grandpa. *Story also appears in 1.2 #1 and 5.3 #1*

Year	1962	1970	1980	1990	2000	2010
Value bond ($)	200.00	318.77	570.87	1,022.34	1,830.85	3,278.77

① roc $= \dfrac{\$318.77 - \$200}{1970-1962} = (318.77-200) \div (1970-1962) = \$14.85/\text{yr}$

② roc $= \dfrac{\$570.87 - \$318.77}{1980-1970} = (570.87-318.77) \div (1980-1970) = \$25.22/\text{yr}$

roc is different $\longrightarrow$ $\boxed{\text{not linear}}$

(b) Wind chill at 10°F. *Story also appears in 1.2 #2*

Wind (mph)	0	10	20	30	40
Wind chill (°F)	10	-4	-9	-12	-15

Each additional 10mph has different drop in wind chill −14, −5, −3... $\longrightarrow$ $\boxed{\text{not linear}}$

–OR–

① roc $= \dfrac{-4-10}{10-0} = ((-)4-10) \div 10 = -1.4°\text{F/mph}$

② roc $= \dfrac{-9--4}{20-10} = ((-)9-(-)4) \div (20-10) = -1.3°\text{F/mph}$

roc is different $\longrightarrow$ $\boxed{\text{not linear}}$

(c) Pizza. *Story also appears in 2.4 #1 and 3.3 #1*

Size (inches)	8	14	16
People	1	3	4

① roc $= \dfrac{3-1}{14-8} = (3-1) \div (14-8) = .33\ldots \approx .33$ people/inch

② roc $= \dfrac{4-3}{16-14} = (4-3) \div (16-14) = .5$ people/inch

roc is different $\Longrightarrow$ $\boxed{\text{not linear}}$

(d) Water in the reservoir. *Story also appears in 2.1 #2 and 3.2 Exercises*

Week	1	5	10	20
Depth (feet)	45.5	39.5	32	17

guessing this one is linear since (a), (b), (c) were not :)

① roc $= \dfrac{39.5-45.5}{5-1} = (39.5-45.5) \div (5-1) = -1.5$ feet/year

② roc $= \dfrac{32-39.5}{10-5} = (32-39.5) \div (10-5) = -1.5$ feet/year :)

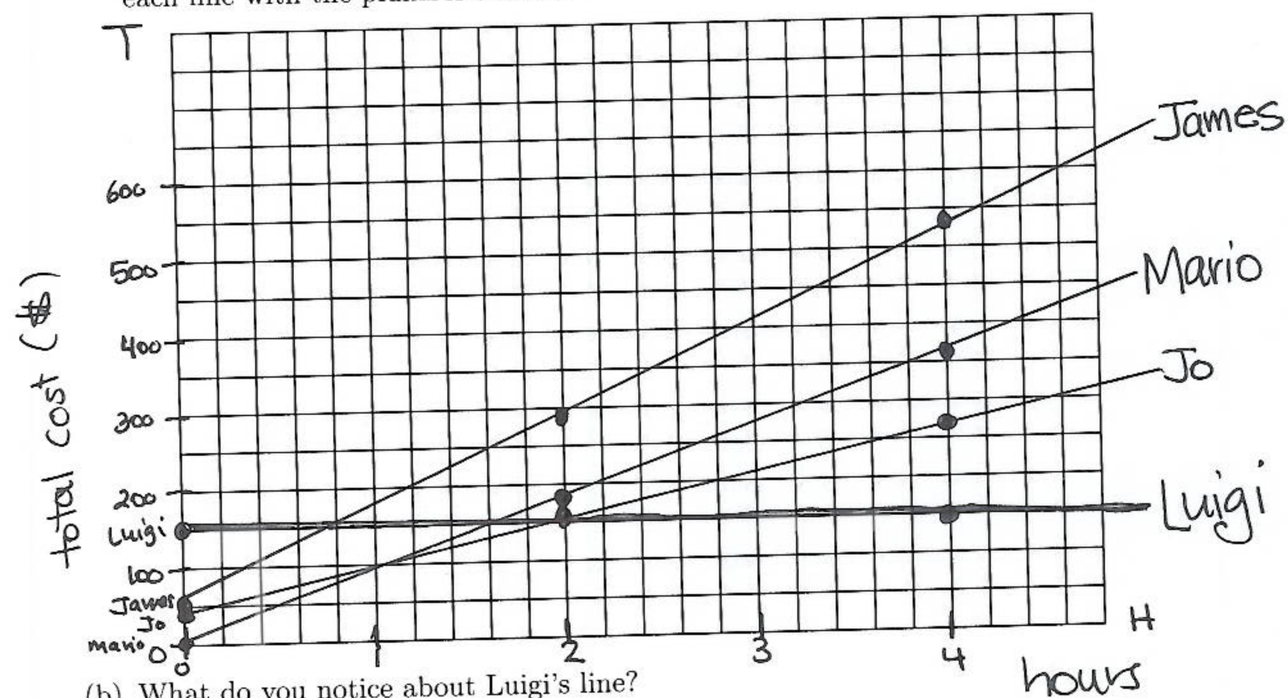
③ roc $= \dfrac{17-32}{20-10} = (17-32) \div (20-10) = -1.5$ feet/year :)

roc is constant $\Longrightarrow$ $\boxed{\text{linear}}$

4. Plumbers are really expensive, so I've been shopping around. James charges \$50 to show up plus \$120 per hour. Jo is just getting started in the business. She charges \$45 to show up plus \$55 per hour. Mario advertises "no trip charge" but his hourly rate is \$90 per hour. Not to be outdone, Luigi offers to unclog any drain for \$150, no matter how long it takes. For each plumber, the table lists the corresponding equation and several points. In each equation, the plumber charges $\$P$ for T hours of work. *Story also appears in 2.1 Exercises*

Plumber	James	Jo	Mario	Luigi
Equation	$P = 50 + 120T$	$P = 45 + 55T$	$P = 90T$	$P = 150$
0 hours	\$50	\$45	\$0	\$150
2 hours	\$290	\$155	\$180	\$150
4 hours	\$530	\$265	\$360	\$150

(a) Use the points given to plot each of the four lines on the same set of axes. Label each line with the plumber's name.



(b) What do you notice about Luigi's line?

It's horizontal (flat).

(c) List the plumbers in order from steepest to least steep line. What does that mean in terms of the story?

slope →

James, Mario, Jo, Luigi most \$/hr → least \$/hr

(d) Now list the plumbers in order from smallest to largest intercept of their line. What does that mean in terms of the story?

intercept →

Mario, Jo, James, Luigi least trip charge \$ → most trip charge \$