FROM 2.2 A first look at exponential equations

1. At a local college the comprehensive fee is \$37,000. It continues to rise at 5.8% per year.
   - Calculate the annual growth factor.
   - What do you expect the tuition to be in five years?
   - Name the variables, including units, and write an equation describing the dependence.
   - Make a table of values showing the tuition now, in 5 years, 10 years, 20 years, and 50 years (even though that's not realistic).
   - Draw a graph illustrating the function.
2. Bunnies, bunnies, everywhere. My neighborhood is overrun with them. They eat the tops of my tulips in early spring and almost anything I grow in the garden (which is why it's fenced off.) The most recent count was in 2007 and estimated 1,800 rabbits in my neighborhood. Rabbits multiply quickly and with the exception of a few red fox that come up from the river, they have no natural predators. (Cars and small children, maybe.) Anyway, the rabbit population was projected to increase at 13% per year.
   (a) Name the variables.
   (b) Calculate the annual growth factor.
   (c) What does this story suggest the rabbit population was in 2010? In 2013?
   (d) Write an equation relating the variables.
3. A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 6% per day.
   (a) Calculate the daily growth factor and use it to write an equation describing the spread of the virus. *Don't forget to name the variables too.*
4. My savings account earns a modest amount of interest, the equivalent of .75% annually. I have \$12,392.18 in the account now.
   (a) How much interest will I earn this year?
   (b) What will my balance be in three years, assuming I neither deposit nor withdraw money?
   (c) Name the variables and write an equation relating them.
   (d) What would the equation be if I moved all of my money into a certificate of deposit earning the equivalent of .92%?
   (e) What would the equation be if I moved \$10,000 into that certificate of deposit, and kept the rest in savings? *Hint: to get the total balance, add the amount in each account.*

5. **Variation on main example = raise + health care costs** Mai's salary was \$78,000 before she got a 6% raise. Now the economy was not doing as well and she got only a 1.5% raise this year.

    (a) What was her salary after the second raise?

    (b) Her colleague Tom started with a salary of \$78,000 but did not get a raise the first year like Mai did. What percentage raise would he need now in order to have the same final salary as Mai?

    (c) Would Mai's salary have been the more than, less than, or the same as now if she had received the 1.5% raise first and then the 6% raise?

    (d) Which order would you rather have: 6% then 1.5% or 1.5% then 6%? Why?

6. In 1990 it was estimated that 2.5 million households watched reality television at least once a week. Executives predicted that number would increase by 7.2% each year. According to their estimates, how many millions of households watched reality television in 2000? In 2010? *As part of your work, name the variables, find the annual growth factor, and write an exponential equation modeling reality television viewing.*

7. INCREASE double digits and not annual (ideally not even time)

8. The number of school children in the district from a single parent household has been on the rise. In one district there were 1,290 children from single parent households in 2010 and that number was expected to increase about 3% per year.

    (a) Calculate the annual growth factor.

    (b) How many children from single parent households are expected in that district by 2015?

    (c) Name the variables and write an equation relating them.

    (d) Make a table showing the number of school children in the district from a single parent household in 2010, 2015, 2020, and 2030.

    (e) Graph the function.

9. The Data One software company reported earnings of \$42.7 billion in 2007. At that time executives projected 17% increase in earnings annually.

    (a) Name the variables and find an equation relating them.

    (b) According to your equation, what would Data One's earnings be in 2015.

    (c) If Data One reports earnings of \$78.1 billion in 2015, would you say the projected rate of 17% was too high or too low? Explain.

10. In 2005, the Worldwatch Institute estimated that world poultry production was growing at a rate of 1.6% per year. In 2005, poultry production was at 78 million tons.

    (a) Write an equation showing how poultry production is expected to rise. *Don't forget to name the variables.*

(b) Make a table showing the production in 2005, 2010, 2020,and 2050 (at least according to the equation.)

(c) Draw a graph showing how production will change in the future.

11. United States ethanol production has been growing exponentially. In 1990, there were .9 billion gallons of ethanol produced. At that time it was estimated that production would increase 5.5% per year.

    (a) Name the variables including units.

    (b) What is the annual growth factor?

    (c) Write an equation that describes the dependence.

    (d) In 2008 actual production of ethanol was 9.0 billion gallons. Is that production level higher or lower than predicted from your equation? Explain.

    (e) What does your equation predict for 2015?

12.

FROM 3.4 Solving exponential equations (and logs)

1. **2.4 #11** At 8:00 p.m. after his first beer, Tom's blood alcohol content (BAC) was already up to 0.04. As Tom continued to drink, his BAC level rose 45% per hour. Equation is

$$B = .04 * 1.45^{H}$$

   where . . .

   (a) Make a table showing Tom's BAC at each hour from 8:00 p.m. to 2:00 a.m.

   (b) At a BAC of 0.10 it is illegal for Tom to drive. Approximately when does that happen?

2. Chlorine is often used to disinfect water in swimming pools. From an initial Chlorine concentration of 2.5 ppm, the amount drops as the swimming pool is used according to the equation

$$C = 2.5(.975)^{H}$$

   where C = amount of Chlorine (ppm) and H = time used (hours).

   (a) Make a table of values showing the concentration of Chlorine initially and after the swimming pool is used for 3 hours, 10 hours, 15 hours, and 25 hours.

   (b) Draw a graph illustrating the function.

   (c) Chlorine concentrations below 1.5 ppm do not disinfect properly. When the concentration falls below 1.5 ppm, more Chlorine needs to be added to the swimming pool water. According to your graph, after approximately how many hours of use will more Chlorine need to be added?

   (d) Show how to use successive approximate to calculate when the Chlorine concentration will fall below 1.5 ppm.

   (e) Show how to solve the equation to calculate when the Chlorine concentration will fall below 1.5 ppm.

3. **2.2 #** The number of school children in the district from a single parent household has been on the rise. In one district there were 1,290 children from single parent households in 2010 and that number was expected to increase about 3% per year. Earlier, we found the equation was

$$C = 1{,}290 * 1.03^{Y}$$

   where C is the number of children and Y is the years since 2010.

   (a) Use successive approximation to determine when there will be over 3,000 school children in the district from a single parent household. *Display your work in a table. Round your answer to the nearest year.*

   (b) Show how to solve the equation to calculate when there will be over 3,000 school children in the district from a single parent household. *Show how you solve the equation.*

(c) Solve again to determine when there will be over 3,500 children. Check your answer.

4. INSERT FAVORITE NAMES OF BBs. A reporter says that Beanie Babies toys made in 1998 are expected to increase in value according to the equation

$$B = 6(1.07)^Y$$

where B is the value of Beanie Babies (in \$) and Y is the years since 1998.

(a) If the reporter is correct, what was a Beanie Baby toy worth in the year 2010?

(b) According to your equation, when will Beanie Babies made in 1998 be worth over \$100? *Report the actual year.*

(c) Show how you can solve the equation to find the answer.

(d) Because of the recession, the value actually slowed a bit. The revised equation is

$$B = 11.80 * 1.059^Y$$

where now Y is measured from 2008 instead. What is the revised estimate for 2010 and when will it be worth over \$100? Solve an equation to answer. *Report the actual year.*

5. Variation on main example = health care, revisited.

6. Could follow up to 2.4 narrative example and #5 on world country populations.

7. **2.3 #7** Dontrell and Kim borrowed money to buy a house, on a 30-year mortgage. At today's favorable interest rates, they owe \$900 a month. (Plus taxes and insurance.) After M months of making payments, Dontrell and Kim will still owe \$$P$ where

$$P = 225,000 - 45,000(1.004)^M$$

The number P is sometimes referred to as the "payoff" amount since at any time they can pay that much to settle the debt. REWRITE questions so solving here:

(a) How much did Dontrell and Kim originally borrow to buy their house? What value of M did you to answer the question?

(b) Evaluate the equation at $M = 12$ and explain what the answer means in terms of the story.

(c) After making half the payments, how much money will Dontrell and Kim still owe on the house? Will they have paid more or less (or exactly) half of the loan? *Hint: convert 30 years into months to find the total number of payments. Then divide by 2 to find the halfway point.*

(d) The very last month they don't actually pay the full \$900, just whatever balance is left on the loan. How much will that be? *Hint: they will have made all but one of the payments.*

8. **2.4 #9** Suppose a special kind of window glass is 1 inch thick and lets through only 75% of the light. If we use W inches of window glass, it lets L% of the light through where
$$L = 100 * .75^W$$
   (a) What thickness glass should be used to let through less than 10% of the light? *Set up and solve an equation.*
   (b) What about 50%? *Set up and solve an equation.*
   (c) Check the graph (drawn before) to see if your answers make sense.

9. **2.2** # We saw that poultry population was estimated to grow according to the equation
$$P = 78 * 1.016^Y$$
where P is the poultry population in million tons and Y is the years starting in 2005.
   (a) When will production rise above 95 million tons? *Set up and solve an equation. Then use some other method to check.*
   (b) 120 million tons?

10. The number of school children in the district whose first language in not English has been on the rise. The equation describing the situation is
$$C = 673(1.043)^Y$$
where C is the number of school children in the district whose first language is not English, and Y is the number of years (from now).
   (a) Make a table showing the number of school children in the district whose first language is not English now, in one year, in two years, and in ten years. *Don't forget now too.*
   (b) Use successive approximation to determine when there will be over 1,700 school children in the district whose first language is not English. *Display your work in a table. Round your answer to the nearest year.*
   (c) Show how to solve the equation to calculate when there will be over 1,700 school children in the district whose first language is not English. *Show how you solve the equation.*
   (d) When will the number pass 1,000? Set up and solve an equation. Then check your answer.

11.

12.

FROM 5.1 Modeling with exponential equations

1. The population of Buenos Aires, Argentina in 1950 was estimated at 5.0 million and expected to grow at 1.8%.
   (a) Name the variables.
   (b) What is the annual growth factor?
   (c) Write an equation estimating the population of Buenos Aires.
   (d) Make a table of values showing the population of Buenos Aires every 20th year from 1950 to 2030, according to the equation.
   (e) Graph the function.
   (f) By how many people has the population been increasing during each 20 year period? Add these calculations to your table. *Notice how this answer changes because the rate of change is not constant.*
   (g) The actual population of Buenos Aires in the year 2000 was around 12.6 million and by 2010 it was around 15.2. (This now includes a larger surrounding area as the city has spread beyond historical limits.) How does that compare to the estimates?

2. **2.2** # A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 6% per day. Earlier we found the equation
$$N = 8 * 1.06^D$$
where D is the number of days (since the first diagnosis) and N is the total number of students who had the flu. There are 1,094 students currently living in the dorms.
   (a) Make a table and graph for the six weeks following the initial diagnosis.
   (b) Use successive approximations to estimate when the number of infected students reaches 100.
   (c) Use the LOG DIVIDES FORMULA to solve your equation.
   (d) What is a realistic domain? That means, for how many days do you think this model is reasonable?

3. Third: another start and percent increase where over 10% , do different set of stuff

4. Carbon dioxide is a greenhouse gas in our atmosphere. Increasing carbon dioxide concentrations are related to global climate change. In 1980, the carbon dioxide concentration was 338 parts per million carbon dioxide (that is on average, one million molecules of air contained 338 molecules of carbon dioxide). At that time it was assumed that carbon dioxide concentrations would increase .42% per year.
   (a) Name the variables including units.
   (b) Assuming the growth is exponential as predicted, write an equation that describes the dependence.

(c) The carbon dioxide concentration in 2008 was 385 parts per million. Is that count higher or lower than predicted from your equation? Explain.

(d) Does that mean that carbon dioxide increased at a higher or lower rate than .42%? Explain.

5. **Variation on main example = value of sterling**

6.

7.

8. The number of players the hit game app Draw Something has been growing exponentially according to the equation $P = 2(1.57)^W$ where P is the number of players in millions and W is time in weeks.

   (a) Make a table showing the number of players after 0 weeks, 2 weeks, 4 weeks, and 6 weeks.

   (b) Use successive approximation to determine when there will be over 60 million players of Draw Something. (Round your answer to the nearest week.)

   (c) Show how to solve the equation to determine when there will be over 60 million players. Record your answer to two decimal places.

9.

10.

11. In 2006 there were about 5.2 million people living in the state of Minnesota. Predicted growth rates vary, perhaps around .5% per year.

   (a) Based on these figures, about how many people will be living in the state of Minnesota in 2010? In 2020?

   (b) Identify the variables and constants (if any), including the units, realistic domain and range, and dependence. Fix wording

   (c) Write an equation showing how Minnesota's population is a function of the year.

   (d) Make a table of values showing the projected population every two years from 2006 to 2020.

   (e) Draw a graph illustrating the dependence.

12.

FROM 5.2 Exponential growth and decay

1. A signal one unit strong is sent down a fiber optic cable. It decreases by 2% each mile it travels.

   (a) Make a table showing the strength of the signal of the first five miles.

   (b) Name the variables, including units, and write an equation relating them.

   (c) The signal will need a booster when it has fallen to under .75 units. How far along the cable should the booster be placed? Set up and solve an equation.

   (d) Draw a graph illustrating the relationship.

   (e) What's the half-life (or should we say half-distance) of a signal? That means, how far can it travel without dropping below 50%? (Won't actual happen because we'd boost the signal.)

2. A recent news report stated that cell phone usage is growing exponentially in developing countries. In one small country, 50,000 people owned a cell phone in the year 2000. At that time it was estimated that usage would increase at 1.4% percent per year.

   (a) Name the variables including units.

   (b) Assuming the growth is exponential, write an equation for the function.

   (c) At this rate, how many years would it take for the number of people owning a cell phone to double? That's called the **doubling time**. Show how to set up and solve an equation to find the answer.

   (d) In 2011, about 682,000 people owned a cellphone. Is that count higher or lower than predicted from your equation? Explain.

   (e) Based on the 2011 data, would you say that cell phone usage was growing slower or faster than1 .4%?

3.

4. You and two buddies each invite 10 people to "like" your online group. Suppose everyone accepts and then they each invite 10 people. And then everyone accepts and they each invite 10 people. And so on. Of course, there is likely to be substantial overlap, but for the moment pretend that there isn't.

   (a) There are 3 friends to start. They each invite 10 in the first round, so a total of 30 new people "like" your online group. How many new people "like" your group in the second round? The third?

   (b) Name the variables and write an equation showing how the total number of new people increases in each round.

   (c) Draw a graph showing the number of new people in each of the first six rounds.

   (d) What is the total number of people who "like" your online group after six rounds. *Hint: add*

(e) Comment on why our assumption is unrealistic.

5. **Variation on main example = jittery joe** Joe's friend NAME starts the day by downing two cans of Red Bull, containing a total of 160 mg of caffeine. Her body eliminates the caffeine at a slightly slower rate of 12% each hour.

   (a) Name the variables and write an equation to model this situation.

   (b) What's the half-life of caffeine for NAME?

   (c) NAME heard that drinking a glass of water an hour can help eliminate caffeine faster. If so, would the half-life be shorter or longer? Explain.

6. A patient is given an initial dose of 120 mg of a medicine. It has now reached full levels, and is expected to drop around 5% per hour. What is the half-life of the medicine? *As part of your work, name the variables, write an equation, solve your equation, and check your answer.*

7. The population of bacteria in a culture dish began at 2,000 and has tripled every day since.

   (a) Write an equation, including variables.

   (b) Draw a graph, including a table of reasonable values.

   (c) Use successive approximations to find when there were first over 1 million bacteria. Give your answer to the nearest hour.

   (d) Use theLOG DIVIDES FORMULA to solve the equation exactly.

8. If you have a heart attack and your heart stops beating, the amount of time it takes paramedics to restart your heart with a defibrillator is critical. According to a medical report, each minute that passes decreases your chance of survival by 10%. Assume that this statement means the decrease is exponential and that the survival rate is 100% if the defibrillator is used immediately.

   (a) Name the variables and write an equation.

   (b) If it takes the paramedics 2 minutes to use the defibrillator, what is your survival rate?

   (c) When does the survival rate drop below 50%? Display your work in a table.

   (d) Use the Log Divides Formula to solve the equation exactly.

9. Tenzin bought a house for $291,900 but the housing market collapsed and his house value dropped 5.1% each year.

   (a) Name the variables and write an equation relating them.

   (b) At this rate, how many years would it take for the value of Tenzin's house to drop below $240,000? Use successive approximation to guess the year.

   (c) Now set up and solve an equation.

10. INCREASE

11. In 1980, downtown companies did a combined business income of \$7.8 billion. Since then profit declines and companies leaving downtown have resulted in a net decrease of 1.4% per year.

    (a) According to this statement, what is combined business income of downtown companies in 2012?

    (b) Name the variables and write an equation relating them.

12. Since 1970, each year's model of computer has twice the memory of the previous year's model. (Say that a computer in 1970 had 1 unit of memory).

    (a) Identify and name the variables, including their units.

    (b) According to your equation, how many units of memory in 1990, 2005, 2020?

    (c) Write an equation describing the dependence.

    (d) Graph the function.

    (e) According to your graph, in what year was memory 1 million times that of 1970?

    (f) Set up and solve an equation to find the answer.

FROM 5.3 Growth factors

1. GFF percent increase

2. Every plant and animal contains a certain amount of radioactive carbon-14. This radioactivity is not dangerous. As long as the plant or animal is alive, the carbon-14 stays as is. When the plant or animal dies, the carbon-14 begins to turn into carbon-12. By comparing the amount of carbon-14 versus carbon-12 in a specimen such as a fossil, scientists cna determine how long the plant or animal died. This technique won scientists the Nobel Prize. The half-life of carbon-14 is about 5,730 years. Assume that a specimen is found that should have initially contained 500 milligrams of carbon-14.

   (a) Find the annual "growth" factor.

   (b) Name the variables and write an equation describing the dependence.

   (c) How many milligrams of carbon-14 should remian after 1,000 years? After 10,000 years? After 100,000 years? After 1 million years?

   (d) If the specimen is determined to have 320 milligrams of carbon-14, how old is it? (That is, approximately how long ago did it die?) Display your work in a table.

   (e) Solve the equation exactly.

3. For each equation, find the growth rate (percent increase or percent decrease) and state the units. (For example, something might "grow 2% per year" while something else might "drop 7% per hour") SU YOU ARE HERE

   (a) **3.4** # The number of school children in the district whose first language in not English has been on the rise. The equation describing the situation is

   $$C = 673(1.043)^Y$$

   where C is the number of school children in the district whose first language is not English, and Y is the number of years (from now).

   (b) **3.4** # Chlorine is often used to disinfect water in swimming pools. From an initial Chlorine concentration of 2.5 ppm, the amount drops as the swimming pool is used according to the equation

   $$C = 2.5(.975)^H$$

   where C = amount of Chlorine (ppm) and H = time used (hours).

   (c) NOT TIME?

   (d) SOMETHING less than 1% NOT TIME

4. Find the annual growth factor and growth rate for each of the following stories.

   (a) Donations to the food shelf have increased 35% per year for the past few years.

   (b) People picking up food at the food shelf has increased exponentially too, from 120 per week in 2005 to 630 per week in 2011.

(c) The crime rate has dropped 3% each year recently.

(d) The creeping vine taking over my lawn doubles in area each year.

(e) Attendance at parent volunteer night has done so well it has doubled every 3 years.

(f) The new stop sign has decreased accidents exponentially, from 40 in 2008 to 17 in 2013.

5. Variation on main example = obesity

6. xx

7. Wetlands help support fish populations, various plant and animal populations, control floods and erosion from nearby lakes and streams, filter water, and help preserve our supply of ground water. Minnesota wetlands acreage in 1850 was 18.6 million acres. By 2003, that number had dropped to 9.3 million acres.

   (a) Assuming the acreage decreased exponentially, name the variables, find the annual decay factor and write an exponential equation showing how Minnesota wetlands have decreased.

   (b) With some effective management, many wetlands have been restored. By 2012, it's up to about 10.6 million acres. Assuming acreage has increased exponentially from 2003, name the variables (you may now want to start the years in 2003), find the growth factor and write an exponential equation showing how Minnesota wetlands have been restored.

8. Claire's silver tea set was worth $560 when new. Now, 30 years later, she had it appraised for $3,700.

   (a) Calculate the annual growth factor, assuming the value of Claire's tea set increased exponentially.

   (b) What should she expect the set to be worth in another 10 years? *As part of your work, name the variables and write an equation relating them.*

9. One of the toxic radioactive elements produced by nuclear power plants is strontium-90. A large amount of strontium-90 was released in the nuclear accident at Chernobyl in the 1980's. The clouds carried the strontium-90 great distances. The rain washed it down into the grass, which was eaten by cows. People then drank the milk from the cows. Unfortunately, strontium-90 causes cancer. Strontium-90 is particularly dangerous because it has a half-life of approximately 28 years, which means that every 28 years half of the existing strontium-90 changes into a safe product (zirconium-90); the other half remains strontium-90. Suppose that a person drank milk containing 100 milligrams of strontium-90.

   (a) After 28 years, how many milligrams of strontium-90 remains in the person's body?

   (b) After 56 years, how many milligrams of strontium-90 remains?

(c) Find the average annual percentage decrease of strontium-90.

(d) Name the variables and write an equation relating them.

(e) Suppose that any amount under 20 milligrams of strontium-90 is considered "acceptable" in humans. How long will this person have to wait until their level is acceptable? (Do you think they will be alive?) Display your work in a table.

(f) Draw a graph illustrating the amount of strontium-90 remaining for 100 years after ingestion.

(g) Solve the equation exactly.

10. INCREASE

11. Unemployment figures were just released. At last report there were 20,517 unemployed adults and now, 10 months later, we have 39,061 unemployed adults.

    (a) Calculate the monthly growth factor, assuming unemployment increases exponentially.

    (b) Write an equation relating the variables.

    (c) According to your equation, what is the expected number of unemployed adults 6 months from now. *Notice: the report was issued 10 months ago.*

12.

FROM 5.4 Linear vs. exponential models

1. My parents bought the house I grew up in for $35,000 and sold it 40 years later for $342,000. True story. (It was before the housing bubble burst.)
   (a) Name the variables including units.
   (b) Find the annual growth factor, assuming the house value increased **exponentially**.
   (c) In this **exponential** model, by what percentage did the house value increase each year?
   (d) Write an **exponential** equation illustrating this dependence.
   Check that your equation gives the correct amount for 40 years.
   (e) What does your **exponential** equation say the house had been worth after 30 years?
   (f) Now assume the house value increased **linearly** instead. In this **linear** model, by what fixed amount did the house value increase each year? *Hint: calculate the slope.*
   (g) Write the **linear** equation illustrating this dependence.
   Check that your equation gives the correct amount for 40 years.
   (h) What does your **linear** equation equation say the house had been worth after 30 years?

2. The number of jobs available for people without a college degree has been declining steadily. In 1970, there were 1.2 million such jobs in the cities but by 2010 there are only 0.6 million such jobs. Suppose that the number of jobs decreases exponentially.

   First, assume that sales grow linearly.
   (a) Calculate the slope.
   (b) Write a linear equation showing how the number of jobs has declined.
   (c) Check that your equation gives the correct value for 2010.

   Next, assume that sales grow exponentially instead.
   (d) Calculate the growth factor.
   (e) Write an exponential equation showing how the number of jobs has declined.
   (f) Check that your equation gives the correct value for 2010.

   Now, compare the models.
   (g) Complete the table of values.

year	2007	2010	2012	2020	2030
x	0	10	20	40	55
x (if linear)					
x (if exponential)					

(h) Draw a graph showing both models.

3. In December 2010, the popular mobile app game Angry Birds had 50 million downloads. Six months later (May 2011), the game had 200 million downloads. Let D denote the number of downloads of Angry Birds (in millions) and M the months since December 2010.

   (a) Suppose that the number of downloads have been increasing at a *constant rate each month.* What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$)

   (b) Suppose that the number of downloads have been increasing at a *fixed percentage each month.* What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$)

4. Bus fares are up to \$1.40 per ride during rush hour. Two different plans of increasing fares are being debated: 10¢per year or 3% per year.

   (a) Make a table comparing these two plans over the next decade. *A decade is ten years.*

   (b) Graph?

   (c) As a city council representative, you want to support the plan that your constituents prefer. If most of your constituents ride the bus, which plan should you support?

   (d) If most of your constituents are members of the same union as the bus drivers (who count on solid earnings from the bus company to keep their jobs), then which plan should you support?

   (e) Which type of equation is being used in each plan?

5. (a) Sarah weighed 145 pounds when she started her diet three months ago. Now she weighs 133 pounds. How much longer will she have to stay on her diet to reach her goal weight of 120 pounds? Make an initial guess of how long it will take her, and use a linear and an exponential model to predict how long she will be on her diet. *That means, write both equations and solve each to find the answer.*

   (b) Sarah's diet-buddy Dan started at 202 pounds and now weighs 184 pounds after 3 months. Dan's goal weight is 178 pounds. How long will it take him? Again, make an initial guess, and then use a linear and an exponential model to predict. *Again, write and then solve both equations.*

   (c) Explain why weight loss might be similar to a car depreciating in value. Think about what happens at the start of a diet and what happens long run.

6. The number of geese in the Twin Cities metropolitan area increased from 480 in 1968 to 14,000 in 1984. Although population is sometimes modeled with exponential models, there are many factors that might make an exponential model inappropriate, such as changes in migration, wetlands, and hunting.

(a) Name the variables and write a linear equation and an exponential equation modeling the relationship.

(b) Graph each of the functions over the period from 1968 to 2003 on the same set of axes.

(c) Research indicates that the Twin Cities metropolitan area could support 472,000 geese. Using your first linear model and then your exponential model, determine when the population will reach 472,000 by setting up and solving the appropriate equations.

(d) The goose population in 1994 was 24,960. Determine which model fits better.

7. DECREASE GRAPH!!

8. Chlorofluorocarbons (CFCs 11 and 12) are greenhouse gases that result from our use of refrigeration, air conditioning, aerosols, and foams. In 1960, the concentration of CFC-11 in the northern hemisphere was 11.1 parts per trillion (ppt), meaning on average, there are 11 CFC molecules in a trillion (=1,000,000,000,000) molecules of air. In 1980 the concentration of CFC-11 in the northern hemisphere was 177 ppt. Let C denote the concentration of CFC-11 in the northern hemisphere (in ppt or parts per trillion) and Y the year, measured in years since 1960.

   (a) Suppose that this concentration has been increasing at a *constant rate each year*.

      i. By how many parts per trillion per year have CFC-11 concentrations increased?
      ii. Write an equation illustrating this model.
      iii. According to this equation, how much will the concentration of CFC-11 be by the year 2009?
      iv. What type of equation is being used here?

   (b) Assume instead that the concentration of CFC-11 has been increasing *a fixed percentage each year*.

      i. What is the annual growth factor that CFC-11 concentrations increased?
      ii. Write an equation illustrating this model.
      iii. According to this new equation, how much will the concentration of CFC-11 be by the year 2009?
      iv. What type of equation is being used here?

9. In 1995 the average price of a movie ticket was $4.35, and in 2010 the average price of a movie ticket was $7.85.39. The variables are T, the average price of a movie ticket in dollars and Y the number of years since 1995.

   (a) Write a linear equation that fits this information and use it to estimate the average price of a movie ticket in 2015 and 2025.

   (b) Write an exponential equation that fits this information and use it to estimate the average price of movie ticket in 2015 and 2025.

(c) Draw a graph of each function on the same set of axes. (Include also what each equation said about ticket prices in the year 2000.)

(d) The actual average movie ticket price in the year 2000 was \$5.39. Which model predicted a closer value – linear or exponential?

10. Sales of hybrid cars in the United States have continued to increase. In 1999, 17 (yes, seventeen!) hybrid cars were purchased. In 2002, **34,521** hybrid cars were purchased. Let H denote the number of hybrid cars purchased in the United States and Y the year, measured in years since 1999.

    (a) Suppose that the hybrid car sales have been increasing *at a constant rate each year.* Write the appropriate equation and use it to estimate sales in 2010.

    (b) Assume instead that hybrid car sales have been increasing *a fixed percentage each year.* Write the appropriate equation and use it to estimate sales in 2010.

    (c) Actual sales of hybrid cars in 2010 were around. Comment. (And, was it because of the recession? Okay, don't answer that.)

11. DECREASE

12. The number of asthma sufferers worldwide in 1990 was 84 million and 130 million in 2001. Let A be the number of people with asthma (in millions) and Y the year, measured in years since 1990. Compare what the linear and exponential models project for the year 2015 and 2030. Include a graph showing both functions on the same axes.

FROM 5.5 Logistic and other growth models

1. Corn farmers say that their crop is healthy if it is "knee high by the Fourth of July." An equation that relates the height H (in inches) of the corn crop as it matures is the following, where D is days since May 1:

$$W = 106 - 100 * .989^D$$

   (a) According to this equation, how high is corn expected to be on the Fourth of July (day 64)? Is that "knee high"? Let's say that's 18 inches tall.

   (b) These days with stronger corn from cross-breeding and various seed technologies, the rule ought to be modified to "chest high." Let's say that's 52 inches tall. According to this equation, on approximately what date is the corn projected to be that tall? Use successive approximation to answer.

   (c) The particular corn matures in approximately 110 days (by August 19). How tall will it be then?

   (d) Draw a graph of the function.

2. An alternative equation for corn height is

$$C = \frac{200}{1 + 70 * .965^D}$$

   (a) According to this new equation, how high is corn expected to be on the Fourth of July (day 64)? Is that "knee high" (18 inches tall)?

   (b) According to this new equation, on approximately what date is the corn projected to be "chest high" (52 inches tall)? Use successive approximation to answer.

   (c) The particular corn matures in approximately 110 days (by August 19). How tall will it be then?

   (d) Add the graph of this function to your graph of the original function.

3. Following the 2011 Japanese earthquake and tsunami there was concern of radiation leaking from nuclear power plants. A monitoring station near the Fukushima Daiichi recorded radiation according to the following equation, where R is radiation measured in milliSieverts (mSv for short) and T is time in hours:

$$R = \frac{.162}{1 + 3{,}319 * .3127^T}$$

   (a) How much radiation was detected at the start (0 hours)? After 24 hours? 48 hours?

   (b) At how many mSv did radiation levels level off?

   (c) A typical person is exposed to around 2.4 mSv in a year. How many is that in a day? (Use 365 days/year.)

(d) Did the detected radiation exceed normal (at its highest value where it leveled off)? If so, by how many times normal? *That means divide your answer to (b) by your answer to (c).*

4. Jason works at a costume shop selling Halloween costumes. The shop is busiest during the fall before Halloween. An equation that describes the number of daily visitors V the shop receives D days from August 31 is the following:

$$V = \frac{430}{1 + 701 * .81^D}$$

an alternative equation is

$$V = 700 - 690 * .985^D$$

(a) Make a table showing what each equations projects forAugust 31, September 15, September 30, October 15, October 25, October 28, October 31

(b) Graph both functions on the same set of axes.

(c) Which function is more consistent with a major advertising campaign that aired starting the first week of September? Explain.

5. Variation on main example – use Bolker's fish in pond (cite source)

6. Mari volunteers answering calls for in the office of her local state government representative. The office has been receiving a lot of calls recently with about BPA, a chemical found in plastics. The callers want their representative to support a bill banning BPA. An equation that describes the number of total number of calls over time is the following:

$$C = \frac{837}{1 + 118 * .8025^D}$$

where D is the time since January 1 (in days), and C is the total number of calls.

(a) According to this equation, how many calls (total) will Mari's office get by February 1 (day 31), March 1 (day 59), April 1 (day 90), May 1 (day 120), and Nov 8 (day 311)?

(b) During which months did most of the calls come in?

(c) Draw a graph illustrating the function.

(d) Describe what happened over time.

7. Even though all the callers support the bill, Mari isn't sure whether the calls represent the local constituents. Perhaps only supporter are calling her office, for example. So, she asks her pollster, Paul, to add this question to the list for his daily survey. Based on that survey, Paul estimates the percentage P of local constituents who support the bill on day D by the equation

$$P = 100 - 87.3 * .992^D$$

(a) According to this equation, what percentage of callers supported the bill on January 1 (day 0), March 1 (day 59), Aug 1 (day 212), Oct 1 (day 273) and Nov 8 (day 311)?

(b) What does your equation say the percentage would be on day 500 (which probably isn't realistic in this problem)? How about day 1,000?

(c) Use successive approximations to estimate when the percentage supporting the bill first reached majority (50%).

(d) Set up and solve an equation to find when the percentage supporting the bill first reached majority (50%).

8. Infants are regularly checked to make sure they are growing accordingly. The World Health Organization publishes growth charts to evaluate infant weight W in kilograms at a given age M in months since birth. An equation that describes an average infant boy is the following:

$$W = 15 - 11.5 * .932^M$$

(a) According to this equation, what is the average infant boy weight at birth, 1 month, 4 months, and a year?

(b) Convert your answers to pounds and ounces using

$$1 \text{ kilogram } \approx 2.205 \text{ pounds} \quad \text{and} \quad 1 \text{ pound } = 16 \text{ ounces}$$

Hint: first convert to pounds. Then convert decimal part to ounces.

(c) The equation is valid for $0 \leq M \leq 36$, or up to three years old. Draw a graph that includes your points from earlier and the values at 3, 4, 5, and 6 years. Can you explain why the equation doesn't make sense after around 3 years?

9. **2.2** # A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 6% per day. Earlier we used the exponential model

$$N = 8 * 1.06^D$$

where D is the number of days (since the first diagnosis) and N is the total number of students who had the flu. There are 1,094 students currently living in the dorms.

(a) Consider the alternative saturation model SU FILL IN. According to this model, approximately when will the number of infected students reach 100? What will be the infected population six weeks after the first diagnosis. Draw a quick graph.

(b) Consider the alternative logistic model SU FILL IN.According to this model, approximately when will the number of infected students reach 100? What will be the infected population six weeks after the first diagnosis. Draw a quick graph.

(c) Which model do you think does the best job of predicting infected populations: exponential, saturation, or logistic? Explain.

10.

11.

12.