SOLUTIONS

5.5 Logistic and other growth models – Practice exercises

1. Corn farmers say that their crop is healthy if it is "knee high by the Fourth of July." An equation that relates the height H (in inches) of the corn crop D is days since May 1 is

$$H = 106 - 100 * .989^D$$

(a) According to this equation, how high is corn projected to be on June 1 (day 31)?

$D=31$, $H = 106 - 100 \times .989 \wedge \underline{31} = 35.028\ldots$ $\boxed{\approx 35.0 \text{ inches}}$

(b) According to this equation, how high is corn projected to be on the Fourth of July (day 64)? Is that "knee high" (18 inches tall)? That's much taller than "knee high"

$D=64$, $H = 106 - 100 \times .989 \wedge \underline{64} = 56.732\ldots$ $\boxed{\approx 56.7 \text{ inches}}$

(c) With stronger corn these days, the rule ought to be "chest high (52 inches) by the Fourth of July." According to this equation, when is the corn projected to be that tall? Use successive approximation to answer.

D	31	64	50	55	56
H	35.0 LOW	56.7 HIGH	48.5 LOW	51.6 ALMOST	52.2 YES!

It should be "chest high" by day 56 = June 26

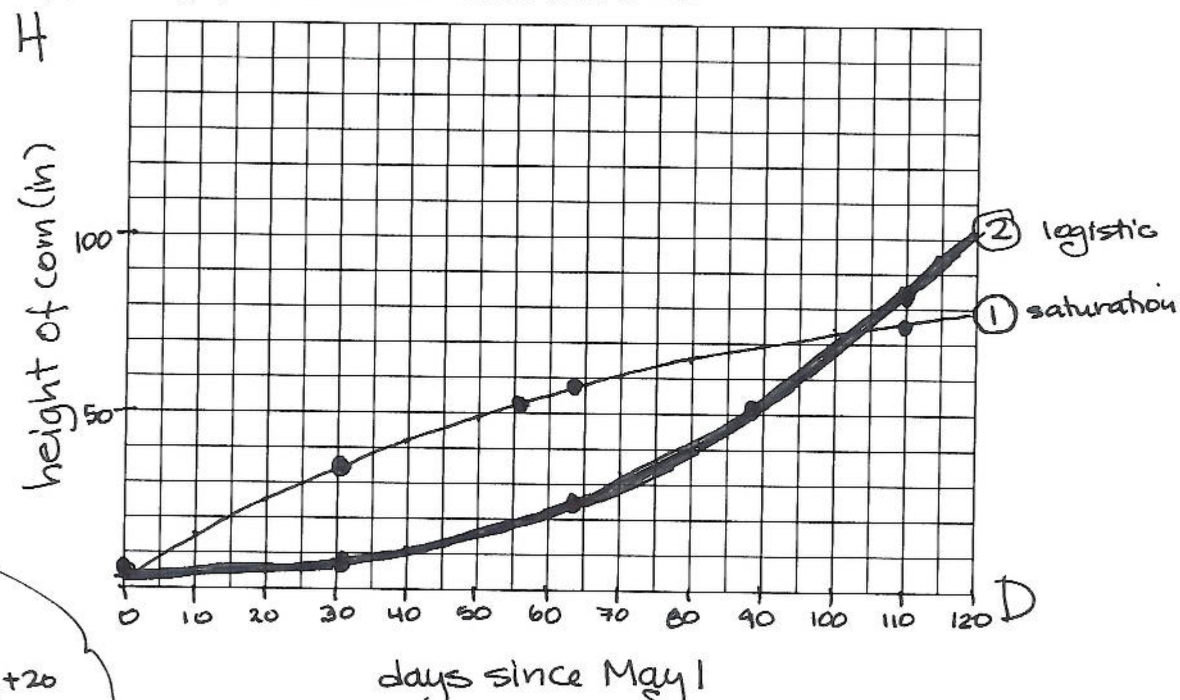
see note at bottom.

(d) The corn matures in 110 days. How tall will it be then?

$D=110$, $H = 106 - 100 \times .989 \wedge \underline{110} = 76.379\ldots$ $\boxed{\approx 76.4 \text{ inches}}$

(e) Draw a graph of the function. Include when $D = 0$.

$D=0$,
$H = 106 - 100 \times .989 \wedge 0$
$= 6''$



Note:
June 1 = day 31 +20
June 21 = day 51 +5
June 26 = day 56

By the way (a) = 35.0 inches is just under 3' tall
(b) = 56.7 inches is 4'9" tall,
(d) = 76.4 inches is 6'4" tall.
check!

2. An alternative equation for corn height is

$$H = \frac{200}{1 + 70 * .965^D}$$

(a) According to this new equation, how high is corn projected to be on June 1 (day 31)?

$H = 200 \div (1 + 70 \times .965 \wedge \underline{31}) = 8.265\ldots$ $\boxed{\approx 8.3 \text{ inches}}$

(b) According to this new equation, how high is corn projected to be on the Fourth of July (day 64)? Is that "knee high" (18 inches tall)?

$H = 200 \div (1 + 70 \times .965 \wedge \underline{64}) = 24.513\ldots$ $\boxed{\approx 24.5 \text{ inches}}$

That's a bit taller than "knee high"

(c) According to this new equation, on approximately what date is the corn projected to be "chest high" (52 inches tall)? Use successive approximation to answer.

D	64	100	80	90	89
H	24.5 LOW	67.0 HIGH	39.6 LOW	52.1 YES?	50.8 LOW

$\Rightarrow$ day 89

June 1 = day 31 + 30 days in June

July 1 = day 61 + 28 days more

July 29 = day 89

$\boxed{\text{It should be "chest high" by July 29.}}$

(d) The corn matures in 110 days. How tall will it be then, according to this new equation?

$H = 200 \div (1 + 70 \times .965 \wedge \underline{110}) = 83.672\ldots$ $\boxed{\approx 83.7 \text{ inches}}$

Just under 7 feet

(e) Add the graph of this function to your graph of the original equation on the previous problem. Again, include when $D = 0$.

$H = 200 \div (1 + 70 \times .965 \wedge \underline{0}) = 2.8 \text{inches}$

3. Back in 1975 when my aunt and uncle bought their house upstate New York, there was a small pond in the yard. They enlarged it and stocked it with 10 small fish. The number of fish F increased over time, approximately according to the equation

$$F = \frac{1{,}000}{1 + 99 * .65^Y}$$

where Y measures the years since 1975.

(a) Make a table showing the fish population in 1975, 1990, 2000, and 2013.

1990 − 1975 = 15

year	1975	1990	2000	2013
Y	0	15	25	38
F	10 ☺	866	998	1,000

rounded to whole # fish

$1000 \div (1 + 99 \times .65 \wedge 0) =$

(b) By the time there were over 500 fish in the pond, you could catch them with your bare hands. In approximately what year did that happen?

Y	0	15	10	12	11
F	10 LOW	866 HIGH	429 LOW	640 HIGH	536 HIGH

$\Rightarrow Y \approx 11$
$+ 1975 =$

$\boxed{\text{By 1986}}$

(c) In approximately what year did the fish population reach its capacity? Use successive approximations and display your calculations in a table.

note: will never actually = 1000 so answer when 1st rounds off to 1000 ☺

→ capacity = top of fraction = 1,000 fish.

Y	38	25	30	28	29
			999.75	999.42	999.6
F	1000 THERE	998 LOW	1000 THERE	999 ALMOST	1000 YES!

$Y \approx 29$
$+1975$

$\boxed{\text{By 2004}}$

4. Jason works at a costume shop selling Halloween costumes. The shop is busiest during the fall before Halloween. An equation that describes the number of daily visitors V the shop receives D days from August 31 is the following:

$$1 \quad V = \frac{430}{1 + 701 * .81^D}$$

An alternative equation is

$$2 \quad V = 700 - 690 * .985^D$$

(a) Make a table showing what each equation predicts for August 31, September 15, September 30, October 15, October 25, October 28, and October 31.
Hint: those days are numbered 0, 15, 30, 45, 55, 58, and 61.

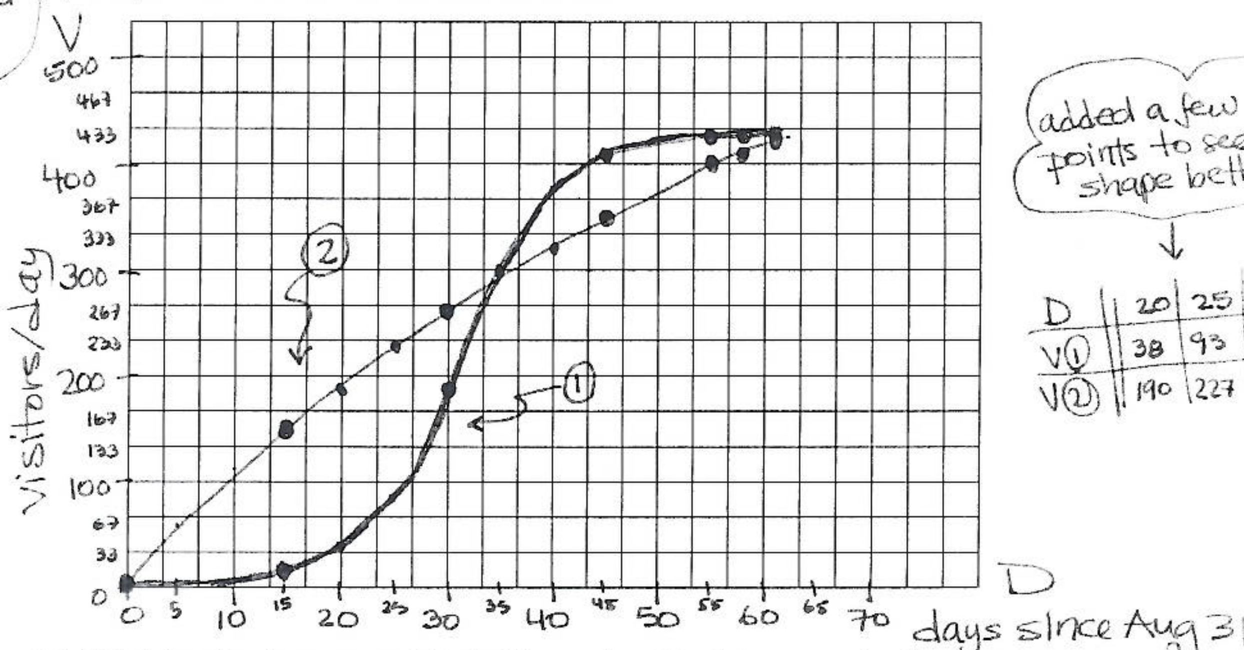
430÷(1+701X.81∧61)=

D	0	15	30	45	55	58	61
V1	1	14	190	408	427	428	429
V2	10	150	262	350	406	413	426

700−690x.985∧61

(b) Graph both functions on the same set of axes.

Chose awkward scale to show graphs large enough



added a few more points to see shape better

D	20	25	35	4
V1	38	93	299	37
V2	190	227	293	32

(c) Which function is more consistent with a major advertising campaign that aired starting the ~~first~~ second week of September? Explain.

The first equation 1, the logistic, is more consistent with an ad campaign early September. There were almost no visitors during the start of Sept but then the number of visitors increased rapidly, perhaps due to the ads.