SOLUTIONS

3.4 Solving exponential equations (and logs) – Practice exercises

LOG-DIVIDES FORMULA: The equation $g^Y = v$ has solution $Y = \dfrac{\log(v)}{\log(g)}$

1. After his first beer, Stephen's blood alcohol content (BAC) was already .04 and as he continued to drink, his BAC level rose 45% per hour. The equation is

$$S = .04 * 1.45^H$$

where S is Stephen's BAC and H is the time, measured in hours.

Story also appears in 1.1 #4 and 2.4 Exercises

(a) Make a table showing Stephen's BAC at the start of the story and each of the next four hours.

H	0	1	2	3	4
S	.04	.06	.08	.12	.18

$.04 \times 1.45 \wedge 1 = .058 \approx .06$

(b) At a BAC of .10 it is illegal for Stephen to drive. When will that happen? Set up and solve an equation using the LOG DIVIDES FORMULA. Answer to the nearest minute.

$10 \div .04 =$

$\dfrac{\cancel{.04} * 1.45^H}{\cancel{.04}} = \dfrac{.10}{.04}$

$1.45^H = 2.5$

By the Log Divides Formula with $g = 1.45$ and $V = 2.5$,

$H = \dfrac{\log(2.5)}{\log(1.45)} = \log(2.5) \div \log(1.45) = 2.466\ldots = \boxed{2 \text{ hrs}, 28 \text{ min}}$

$.466 \text{hr} * \dfrac{60 \text{min}}{\text{hr}} = .466 \times 60 = 27.9\ldots \approx 28 \text{ min}$

(c) Hopefully Stephen will stop drinking before he reaches a BAC of .20. If not, at the rate he's drinking, when would that be? Set up and solve an equation. Answer to the nearest minute.

$\dfrac{\cancel{.04} * 1.45^H}{\cancel{.04}} = \dfrac{.20}{.04}$

$20 \div .04 =$

$1.45^H = 5$

By the Log Divides Formula with $g = 1.45$ and $V = 5$

$H = \dfrac{\log(5)}{\log(1.45)} = \log(5) \div \log(1.45) = 4.3315\ldots = \boxed{4 \text{hrs}, 20 \text{ min}}$

$.3315 \text{hr} * \dfrac{60 \text{min}}{\text{hr}} = .3315 \times 60 = 19.8\ldots \approx 20 \text{ min}$

2. Chlorine is used to disinfect water in swimming pools. The chlorine concentration decreases as the pool is used according to the equation

$$C = 2.5 * .975^H$$

where C is the chlorine concentration in parts per million (ppm) and H hours since the concentration was first measured. *Story also appears in 5.3 #3*

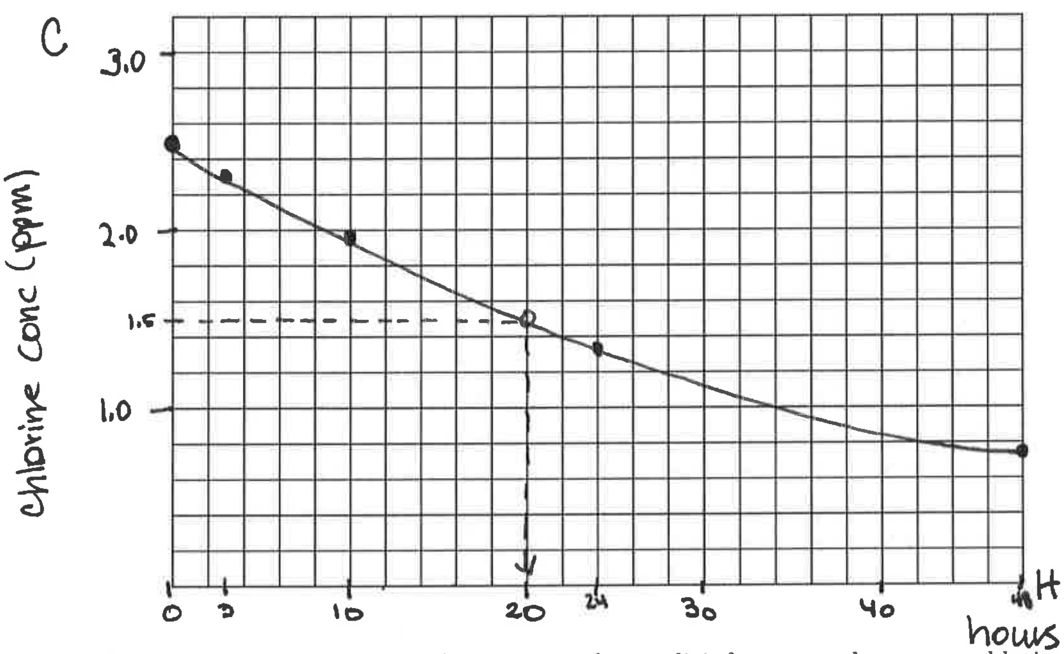
(a) Make a table showing the chlorine concentration initially and after the swimming pool is used for 3 hours, 10 hours, 24 hours, and 48 hours.

initially

H	0	3	10	24	48
C	2.5	2.32	1.94	1.36	.74

2.5 x .975^3 = 2.317...

(b) Draw a graph illustrating the function.



(c) Chlorine concentrations below 1.5 ppm do not disinfect properly so more chlorine needs to be added. According to your graph, when will that happen?

$\boxed{\approx 20 \text{ hours}}$

The problem continues . . .

(d) Use successive approximate to find when the concentration falls below 1.5 ppm.

H	20	21
C	1.51	1.47

$\uparrow$ 2.5×.975^20=

$\boxed{\approx 21 \text{ hours}}$

(e) Solve the equation to find when the chlorine concentration falls below 1.5 ppm.

$$\frac{\cancel{2.5} * .975^H}{\cancel{2.5}} = \frac{1.5}{2.5} \quad 1.5 \div 2.5 =$$

$$.975^H = .6$$

By the Log Divides Formula with $g = .975$ and $v = .6$

$$H = \frac{\log(.6)}{\log(.975)} = \log(.6) \div \log(.975) = = 20.17\ldots \boxed{\approx 21 \text{ hours}}$$

(f) Solve the equation to find when the chlorine concentration would fall below 0.1 ppm (essentially no chlorine) assuming no chlorine was added earlier. Show how to solve the equation to find the answer (and check it!).

$$\frac{2.\cancel{5} * .975^H}{\cancel{2.5}} = \frac{0.1}{2.5} \quad .1 \div 2.5 =$$

$$.975^H = .04$$

By the Log Divides Formula With $g = .975$ and $v = .04$

$$H = \frac{\log(.04)}{\log(.975)} = \log(.04) \div \log(.975) = = 127.13\ldots \boxed{\approx 128 \text{ hours}}$$

check:
2.5×.975^128 = .0978...
≈ .1 ✓

(g) Report your answer to the nearest day.

$$128 \text{ hours} * \frac{1 \text{ day}}{24 \text{ hours}} = 128 \div 24 = 5.33.. \boxed{\approx 5 \text{ days}}$$

start r

3. Rent in the Riverside Neighborhood ~~is~~ expected to increase 7.2% each year. Average rent for an apartment is currently $\boxed{\$830 \text{ per month.}}$ Earlier we identified the variables as R for the monthly rent (in \$) and Y for the years. *Story also appears in 1.1 #2*

(a) Find the annual growth factor.

$r = 7.2\% = .072$

$g = 1 + r = 1 + .072 = \boxed{1.072}$

fits template exponential equation

dep = start * g^indep

(b) Write an equation showing how rent is expected to change.

$$\boxed{R = 830 * 1.072^Y}$$

(c) Use successive approximation to determine when rent will pass $1,000/month. Display your work in a table. Round to the appropriate year.

R=1000

(e) (d)

Y	0	10	5	3	2
R	830	1,663	1,175	1,022	953
vs 1000	LOW	HIGH	HIGH	HIGH	LOW

$830 \times 1.072 \wedge \underline{10} =$

$\boxed{\approx 3 \text{ years}}$

(d) Show how to solve the equation to calculate when rent will pass $1,000/month. Display your work in a table. Round to the appropriate year.

R=1000

$$\frac{\cancel{830} * 1.072^Y}{\cancel{830}} = \frac{1,000}{830} = 1000 \div 830 = 1.2048\ldots$$

$$1.072^Y = 1.2048\ldots \xrightarrow[\text{Log-Divides Formula}]{g=1.072\ \ v=1.2048} Y = \frac{\log(1.2048\ldots)}{\log(1.072)}$$

$$= \log(\text{ANS}) \div \log(1.072) =$$

$$= 2.679\ldots \boxed{\approx 3 \text{ years}}$$

(e) Solve again to determine when rent will reach double what it is now, namely $1,660/month, assuming this trend continues.

R=1,660

$$\frac{83\cancel{0} * 1.072^Y}{830} = \frac{1,660}{830} = 1660 \div 830 = 2$$

$$1.072^Y = 2 \xrightarrow[\text{Log-Divides Formula}]{g=1.072\ \ v=2} Y = \frac{\log(2)}{\log(1.072)}$$

$$= \log(2) \div \log(1.072) =$$

$$= 9.969\ldots \boxed{\approx 10 \text{ years}}$$

4. Dontrell and Kim borrowed money to buy a house on a 30-year mortgage. After M months of making payments, Dontrell and Kim will still owe $\$D$ where

$$D = 236{,}000 - 56{,}000 * 1.004^M$$

D is also known as the **payoff** (how much they would need to pay to settle the debt).

Story also appears in 2.3 #3

(a) How much did Dontrell and Kim originally borrow to buy their house?

M=0

$D = 236{,}000 - 56{,}000 * 1.004 \wedge 0 = \boxed{\$180{,}000}$

M= 5 years * 12 months / year = 5x12 = 60 months

(b) They have been in the house for 5 years now and due to a downturn in the housing market, their house is worth only \$150,000. Are they **underwater**, meaning do they owe more than the house is worth?

They owe: $D = 236{,}000 - 56000 \times 1.004 \wedge 60 = \$164{,}844$

House worth: \$150,000

$\boxed{\text{Yes, they are underwater}}$

(c) How much longer would Dontrell and Kim need to stay in their house until they only owe \$150,000? That means you need to solve the equation

$$236{,}000 - 56{,}000(1.004)^M = 150{,}000$$

$$\cancel{236{,}000} \qquad -236{,}000 \qquad -236{,}000$$

* They've been in the house 5 years already

$$\frac{-\cancel{56{,}000}(1.004)^M}{-\cancel{56{,}000}} = \frac{-86{,}000}{-56{,}000} = (-)86000 \div (-)56000 = 1.5357\ldots$$

$$1.004^M = 1.5357\ldots$$

Log-Divides Formula

$g = 1.004 \quad v = 1.5357\ldots$

$$M = \frac{\log(1.5357\ldots)}{\log(1.004)} = \log(\text{ANS}) \div \log(1.004) =$$

$$= 107.46 \approx 108 \text{ months} * \frac{1 \text{ year}}{12 \text{ months}}$$

$$= 108 \div 12 = 9 \text{ years} \Rightarrow \boxed{4 \text{ more years}}$$