3.4 Solving exponential equations (and logs)

Formula referenced in the worksheets:

LOG-DIVIDES FORMULA: The equation $g^Y = v$ has solution $Y = \dfrac{\log(v)}{\log(g)}$

1. After his first beer, Stephen's blood alcohol content (BAC) was already .04 and as he continued to drink, his BAC level rose 45% per hour. The equation is

$$S = .04 * 1.45^H$$

where S is Stephen's BAC and H is the time, measured in hours.

Story also appears in 1.1 #4 and 2.4 #11

(a) Make a table showing Stephen's BAC at the start of the problem and each of the next four hours.

(b) At a BAC of .10 it is illegal for Stephen to drive. When will that happen? Set up and solve an equation using the LOG DIVIDES FORMULA. Answer to the nearest minute.

(c) Hopefully Stephen will stop drinking before he reaches a BAC of .20. If not, at the rate he's drinking, when would that be? Set up and solve an equation. Answer to the nearest minute.

2. Chlorine is used to disinfect water in swimming pools. The Chlorine concentration decreases as the pool is used according to the equation

$$C = 2.5(.975)^H$$

where C is the Chlorine concentration in parts per million (ppm) and H hours since the concentration was first measured. *Story also appears in 5.3 #3*

(a) Make a table showing the Chlorine concentration initially and after the swimming pool is used for 3 hours, 10 hours, 15 hours, and 25 hours.

(b) Draw a graph illustrating the function.

The problem continues ...

(c) Chlorine concentrations below 1.5 ppm do not disinfect properly so more Chlorine needs to be added. According to your graph, when will that happen?

(d) Use successive approximate to find when the concentration falls below 1.5 ppm.

(e) Solve the equation to find when the Chlorine concentration falls below 1.5 ppm.

3. Rent in the Riverside Neighborhood is expected to increase 7.2% each year. Average rent for an apartment is currently \$830 per month. Earlier we identified the variables as R for the monthly rent (in \$) and Y for the years. *Story also appears in 1.1 #2*

   (a) Find the annual growth factor.

   (b) Write an equation showing how rent is expected to change.

   (c) Use successive approximation to determine when rent will pass \$1,000/month. Display your work in a table. Round to the appropriate year.

   (d) Show how to solve the equation to calculate when rent will pass \$1,000/month. Display your work in a table. Round to the appropriate year.

   (e) Solve again to determine when rent will reach double what it is now, namely \$1,660/month, assuming this trend continues.

4. Dontrell and Kim borrowed money to buy a house on a 30-year mortgage. After M months of making payments, Dontrell and Kim will still owe \$$D$ where

$$D = 236{,}000 - 56{,}000(1.004)^M$$

D is also known as the **payoff**, how much they would need to pay to settle the debt.

Story also appears in 2.3 #3

(a) How much did Dontrell and Kim originally borrow to buy their house?

(b) They have been in the house for 5 years now and due to a down turn in the housing market, their house is worth only \$150,000. Are they **underwater**, meaning do they owe more than the house is worth?

(c) How much longer would Dontrell and Kim need to stay in their house until they only owe \$150,000? Solve the equation

$$236{,}000 - 56{,}000(1.004)^M = 150{,}000$$

3.5 Solving quadratic equations

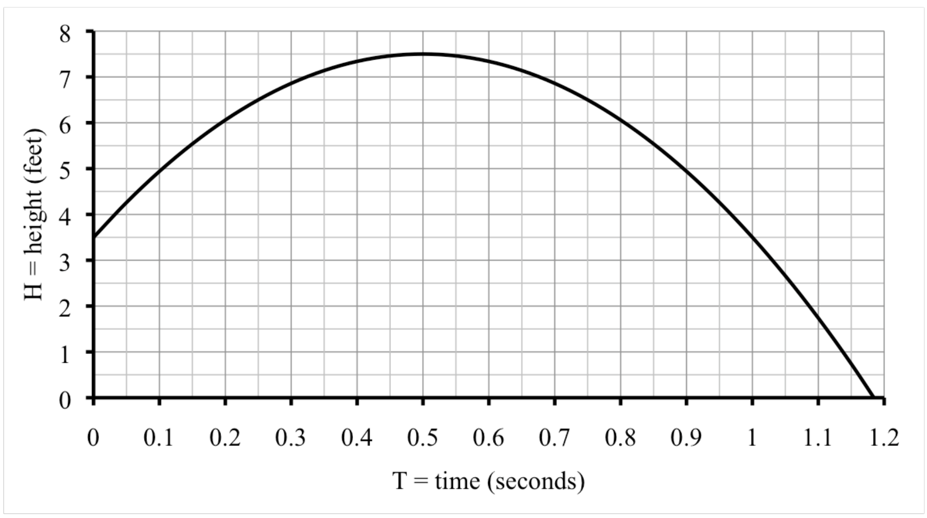
Formula referenced in the worksheets:

QUADRATIC FORMULA: The equation $aT^2 + bT + c = 0$ has solutions

$$T = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

1. A high-jumper jumps so that the height, H feet, of the point on his back that must clear the bar after T seconds is given by the equation

$$H = 3.5 + 16T - 16T^2$$



   (a) When would the high-jumper hit the ground (if there weren't a pit)? Ouch! Use the QUADRATIC FORMULA to find the answer. Use the graph to check.

The problem continues ...

(b) The high jump pit is 2 feet off the ground. When does the high-jumper land in the pit? Use the Quadratic Formula to find the answer and the graph to check.

(c) How high a bar can the high-jumper clear? Find the maximum height of that point above ground by evaluating at $T = \dfrac{-b}{2a}$. Use the graph to check.

2. The art museum opened in 1920. After an initial rush to see the great holdings, attendance dropped for awhile. But then attendance began to rise again and has risen since. The number of annual visits N is approximated by the equation

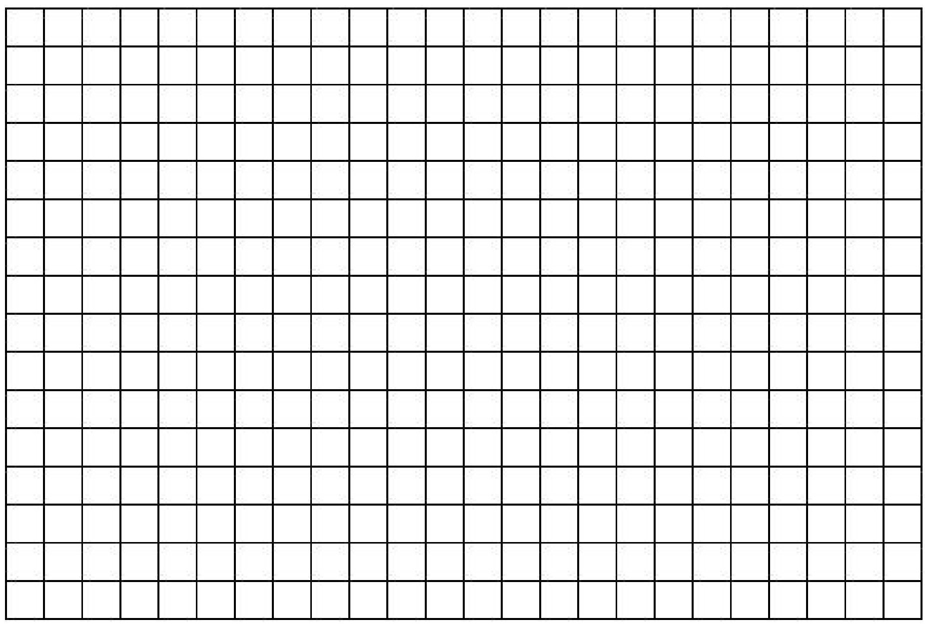
$$N = 51Y^2 - 840Y + 3700$$

where Y is the year since 1920.

(a) Calculate the missing values in the table.

year	1920	1925	1930	1935	1940	1945	1950
Y	0	5	10	15	20	25	30
V	3,700		400	2,575	7,300		24,400

(b) Draw a graph of the function.



(c) In what year did the number of visitors first pass 300,000 in a year? Estimate the value from your graph. Then set up and solve a quadratic equation.

The problem continues . . .

(d) According to this equation, in what year is the number of annual visits the smallest? For that year, what were the number of visits? Use $T = \dfrac{-b}{2a}$

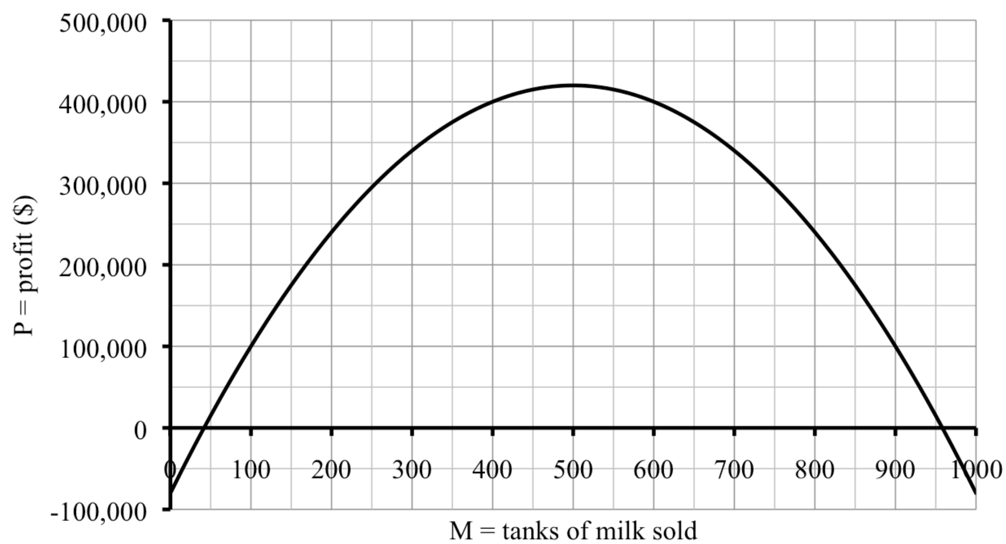
(e) Explain why N never equals 0.

(f) So, what actually happens when you try to use the QUADRATIC FORMULA to solve for $N = 0$?

3. The profit $\$P$ from selling M tanks of milk is described by the equation

$$P = -2M^2 + 2{,}000M - 80{,}000$$

(a) The graph is drawn below. Explain why negative numbers make sense.



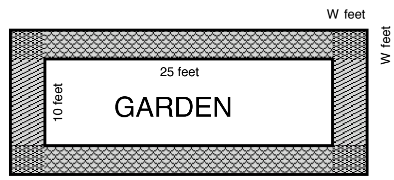
(b) How much milk must be sold for the company to **break even**, meaning having $0 profit? Guess from the graph and check using the equation.

(c) For practice, set up and solve a quadratic equation to find the break even point.

The problem continues . . .

(d) How many tanks of milk would they need to sell to keep profits over \$400,000? Set up and solve a quadratic equation to find the answer. Then check that it agrees with your graph. Your answer should be in the form of an inequality.

4. Urban community gardens are catching on. What was once an abandoned lot down the block is now a thriving 10'×25' vegetable and berry garden for the neighborhood. One neighbor volunteered to donate gravel to make a path around the garden. The path will be 3 inches deep and the same width all around.



The amount of gravel we need (G cubic feet) is given by the equation

$$G = W^2 + 17.5W$$

where W is the width of the path in feet. For example, a path 4 feet wide requires 86 cubic feet of gravel, as you can check. *Story also appears in 2.3 #6 and 2.4 #12*

(a) If the neighbor donates 60 cubic feet of gravel, how wide a path can they build? Set up and solve a quadratic equation to find the answer to two decimal places in feet. Then convert your answer into inches.

(b) Gravel is measured by the **yard**, which is short for cubic yard. There are 27 cubic feet in 1 cubic yard. If the neighbor donates three yards of gravel, how wide a path can they build? Set up and solve a quadratic equation to find the answer to two decimal places in feet. Then convert your answer into inches.

(c) What would it mean to solve the equation to find where $G = 0$? Can you tell what the answer is from the equation (without actually solving)?

Chapter 4

Practice exercises for A closer look at linear equations

4.1 Modeling with linear equations

1. A solar heating system costs approximately \$30,000 to install and \$150 per year to run. By comparison, a gas heating system costs approximately \$12,000 to install and \$700 per year to run. *Story also appears in 4.2 #8*

   (a) What is the total cost for installing and running a gas heating system for 30 years?

   (b) Write a linear equation showing how the total cost for a gas heating system depends on the number of years you run it.

   (c) Write a linear equation showing how the total cost for a solar heating system depends on the number of years you run it.

   (d) How many years of a solar heating system could you get for the cost of a gas heating system lasting 30 years (your answer to part (a))? Set up and solve an equation.

2. Since the Amazon Kindle was released in February 2009, the price has been decreasing at a constant rate. In fact, in February 2011, a blogger developed the following equation representing the price P of the Kindle in the months M since it was released in February 2009:

$$P = 359 - 12M$$

   (a) Make a table of values for the Kindle price initially, 10 months, and 25 months since February 2009.

   (b) What does the 359 mean in the story and what are its units?

   (c) What does the 12 mean in the story and what are its units?

   (d) Draw a graph illustrating the dependence.

The problem continues ...

(e) Approximately how many months after February 2009 is the price of the Kindle expected to be \$200? Set up and solve an equation.

(f) I will purchase a Kindle if the price falls below \$100. When will the price fall below that level? Set up and solve an inequality..

(g) If you can believe what you read in blogs, Amazon will soon be giving away the Kindle for free, since they make money on the book sales themselves. When would that happen, according to our equation? Set up and solve an equation.

3. Can you tell from the table which of these functions is linear? Use the rate of change to help you decide. Remember numbers may have been rounded.

   (a) Savings bonds from grandpa. *Story also appears in 1.2 #1 and 5.3 #1*

Year	1962	1970	1980	1990	2000	2010
Value bond ($)	200.00	318.77	570.87	1,022.34	1,830.85	3,278.77

   (b) Wind chill at 10°F. *Story also appears in 1.2 #2 and 2.1 #10*

Wind (mph)	0	10	20	30	40
Wind chill (°F)	10	-4	-9	-12	-15

   (c) Pizza *Story also appears in 2.4 # 1*

Size (inches)	8	14	16
People	1	3	4

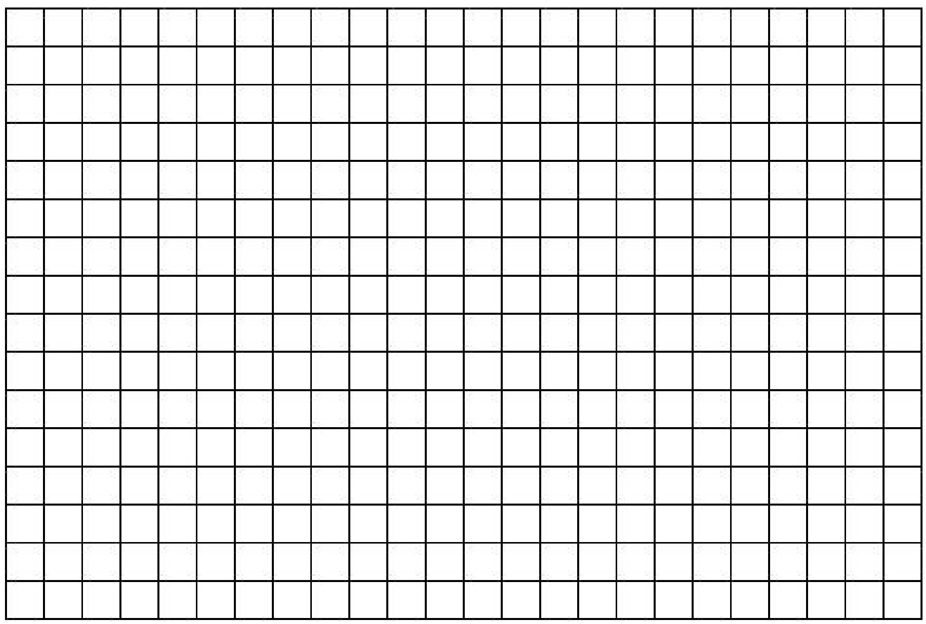
   (d) Water in the reservoir *Story also appears in 2.1 #2 and 3.2 #10*

Week	1	5	10	20
Depth (feet)	45.5	39.5	32	17

4. Plumbers are really expensive, so I've been shopping around. James charges \$50 to show up plus \$120 per hour. Jo is just getting started in the business. She charges \$45 to show up plus \$55 per hour. Mario advertises "no trip charge" but his hourly rate is \$90 per hour. Not to be outdone, Luigi offers to unclog any drain for \$150, no matter how long it takes. For each plumber, the table lists the corresponding equation and several points. In each equation, the plumber charges \$$P$ for T hours of work. *Story also appears in 2.1 #5*

Plumber	James	Jo	Mario	Luigi
Equation	$P = 50 + 120T$	$P = 45 + 55T$	$P = 90T$	$P = 150$
0 hours	\$50	\$45	\$0	\$150
2 hours	\$290	\$155	\$180	\$150
4 hours	\$530	\$265	\$360	\$150

(a) Use the points given to plot each of the four lines on the same set of axes. Label each line with the plumber's name.



(b) What do you notice about Luigi's line?

(c) List the plumbers in order from steepest to least steep line. What does that mean in terms of the story?

(d) Now list the plumbers in order from smallest to largest intercept of their line. What does that mean in terms of the story?

4.2 Systems of linear equations

1. Madison want to buy a new car, either the Toyota Prius, priced at \$26,100, or the Volkswagen Jetta, priced at \$23,700. Annual fuel costs for the Toyota Prius are currently \$1,100. For the Jetta, annual fuel costs are currently \$1,800. The total cost of each car will depend on how many years she keeps it.

   (a) Name the variables.

   (b) Write a linear equation for the total cost (including purchase price and fuel costs) of the Prius and write another linear equation for the total cost of the Jetta, each as a function of how long she keeps it. Assume fuel costs are constant.

   (c) Make a table comparing the total costs for the Jetta and for the Prius if Madison keeps the car for 3, 5, or 10 years.

   (d) Set up and solve a system of linear equations to determine the **payoff time**, or the number of years for which the total costs of each car are equal.

   (e) Based on what you've learned, **fill in the blank.**

      The more expensive Toyota Prius pays off if Madison is going to keep it for ___ years or more.

2. A mug of coffee costs \$3.45 at Juan's favorite cafe, unless he buys their discount card for \$10 in which case a mug costs \$2.90. Or, he can buy a membership for \$59.99 and then coffee is only \$1/mug. If we let M represent the number of mugs of coffee he buys and T represent the total cost in dollars, then the equations are:

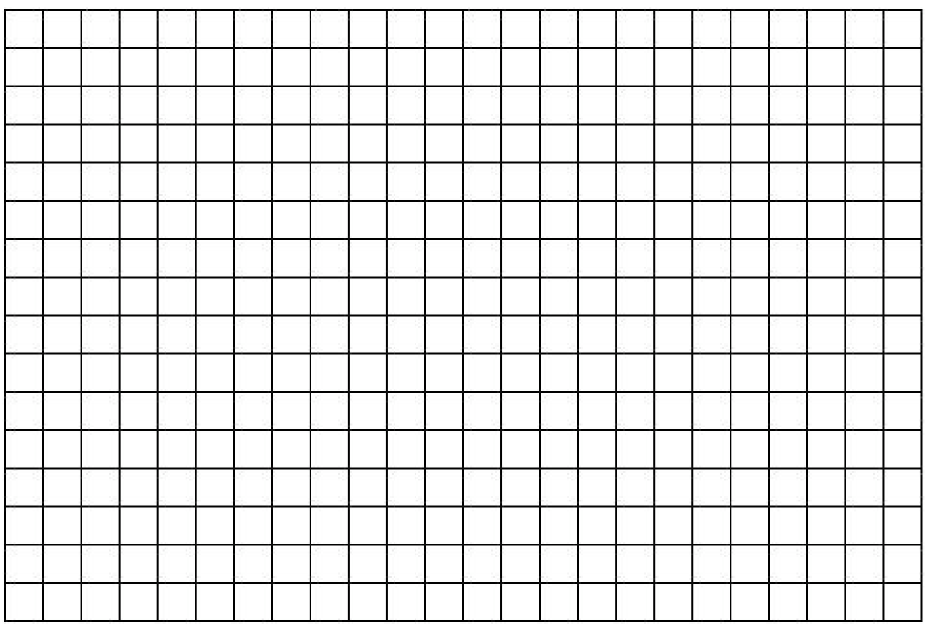
No Card:	$T = 3.45M$
With Card:	$T = 10.00 + 2.90M$
Member:	$T = 59.99 + 1.00M$

Story also appears in 1.2 #4 and 2.1 #4

(a) Compare the total costs for all three options.

Cups	0	10	20	30
With Card				
No Card				
Member				

(b) Draw a graph showing all three options.



(c) Which option is least expensive if Juan plans to buy

- A small number of mugs of coffee:
- A medium number of mugs of coffee:
- A large number of mugs of coffee:

The problem continues . . .

(d) Set up and solve a system of linear equations to compare with and without the discount card.

(e) Set up and solve a system of linear equation to compare the discount card to the membership.

(f) Describe in words what you've learned.

3. Ahmed planted two shrubs in the backyard on May 1. The virburnum was 16.9 inches tall and expected to grow .4 inches each week this summer. The weigela was 20.3 inches tall but only expected to grow .2 inches per week this summer. If we let S represent the total height of the shrub in inches after W weeks, then the equations are:

Virburnum:	$S = 16.9 + .4W$
Weigela:	$S = 20.3 + .2W$

Story also appears in 4.1 #4

(a) Compare the height of the shrub on the given dates.

date	May 1	June 12	July 10	Sept 4
W	0	6	10	18
S (virburnum)	16.9	19.3	20.9	24.1
S (weigela)	20.3	21.5	22.3	23.9

(b) When will the shrubs be the same height? Continue successive approximation to find the answer to the nearest week.

(c) Set up and solve an equation to find the day when the two shrubs are the same height. Report the actual calendar date.

4. The **supply** of flour is the amount of flour produced. It depends on the price of flour. A high price encourages producers to make more flour. If the price is low, they tend to make less of it. The dependence of the supply of flour S (in loads) on the price P (in \$/ pound) is given by the equation

$$\textbf{Supply:} \quad S = .8P + .5$$

The **demand** of flour is the amount of flour consumers want to buy. It also depends on the price of flour. If flour sells for a high price, then consumers will buy less. If flour sells for a low price instead, then consumers will buy more. The dependence of the demand of flour D (in loads) on the price P (in \$/pound) is given by the equation

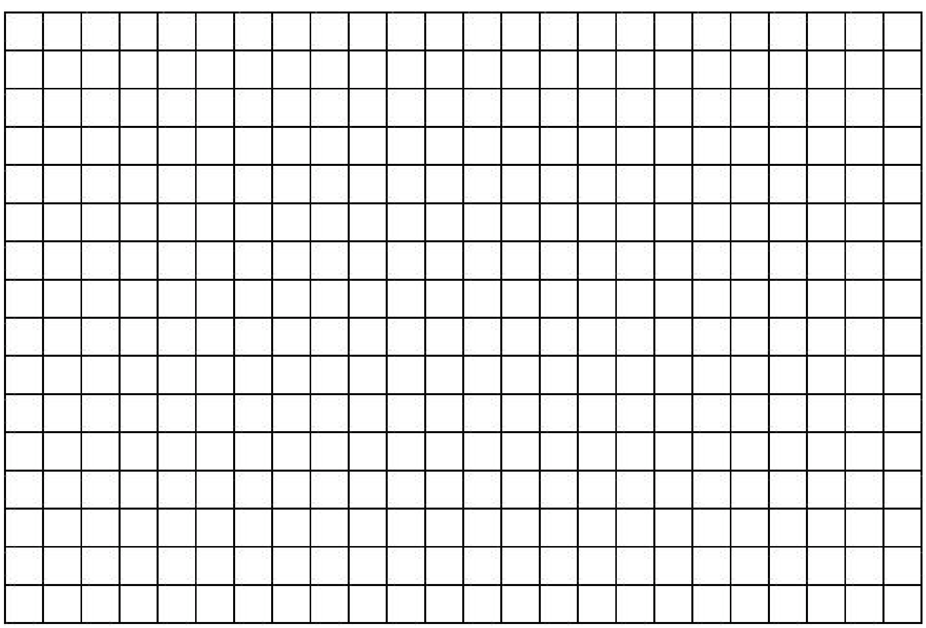
$$\textbf{Demand:} \quad D = 1.5 - .4P$$

The **equilibrium price** of flour is the price where the supply equals the demand.

(a) What happens if flour is priced at \$1.00/pound? That is, how much flour will be produced and how much will consumers want?

(b) What happens if flour is priced at \$0.50/pound? That is, how much flour will be produced and how much will consumers want?

(c) Graph each dependence on the same set of axes. What is the equilibrium price, approximately, according to your graph?



The problem continues ...

(d) Set up and solve an equation to find the equilibrium price of flour.

(e) When more of a product is produced than consumers want to buy, we have a **surplus** of the product. For practice, set up and solve an inequality to find the range of price values for which there wil be a surplus of flour.

(f) When less of a product is produced than consumers want to buy, we have a **shortage** of the product. For practice, set up and solve an inequality to find the range of price values for which there wil be a shortage of flour.

4.3 Intercepts and direct proportionality

1. Each of the two stories, below, involve how temperature changes over time. Best to not call either variable T, so use H for the time in hours and D for the temperature in degrees (°F). In each case, time should be measured from the start of the story.

   (a) It was really cold at 8:30 this morning when Raina arrived at the office. Luckily the heating system warms things up very quickly, 4°F per hour. By 11:00 a.m. it was a very comfortable 72° F.

      i. Figure out what the temperature was at 8:30 a.m.

      ii. Write an equation illustrating the function.

   (b) While 72°F is a perfectly good temperature for an office, not so for ballroom dancing. When Raina arrived for her practice at 5:30 that evening, she began to sweat before she even took the floor. Turns out the air conditioner had been running since 4:00 p.m. but it only cools down the room 3°F per hour.

      i. Figure out what the temperature was at 4:00 p.m.

      ii. Write an equation illustrating the function.

2. Maryn is very happy. Her interior design business is finally showing a profit. She has logged a total of 471 billable hours at \$35 per hour since she started her business. Accounting for start up costs, her net profit is totals \$2,194.

   (a) What were Maryn's start up costs?

   (b) Identify the slope and intercept (including their units and sign) and explain what each means in terms of the story.

   (c) Calculate what Maryn's profits will be once she has logged a total of 1,000 hours.

   (d) Name the variables and write an equation relating them.

   (e) Graph the function.

3. For each story, find the initial weight of the person and use it to write an equation showing how the person's weight P pounds depends on the time, W weeks.

   (a) Jerome has gained weight since he took his power training to the next level ten weeks ago, at the rate of around 1 pound a week. He now weighs 198 pounds.

   (b) Vanessa's doctor put her on a sensible diet and exercise plan to get her back to a healthy weight. She would need to lose an average of 1.25 pounds a week to reach her goal weight of 148 pounds in a year (which is 52 weeks).

   (c) After the past 6 weeks of terrible migrane headaches, Carlos is down to 158 pounds. He's lost 4 pounds a week.

   (d) Since she's been pregnant, Zoe has gained the recommended $1/2$ pound per week. Now 30 weeks pregnant and 168 pounds, she wonders if she'll ever see her feet again.

4. Each story describes a situation that we're assuming is linear. Decide whether it is directly proportional or not. If not, identify what the intercept would mean in the story.

   (a) The price of a bag of kiwis depends on how many pounds of kiwis are in the bag.

   (b) The price of a bag of tortillas depends on how many tortillas are in the bag.

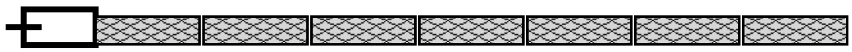
   (c) The time it takes to vacuum a rug depends on the area of the rug.

   (d) The time it takes to wash dishes depends on how many dirty dishes there are.

   (e) The amount of laundry detergent I have left depends on how many loads of laundry I did.

4.4 Slopes

1. Jana is making belts out of leather strips and a metal clasp. An extra short length belt (as shown in the picture) is 24.5 inches long and includes 7 leather strips. An extra long length belt (not shown) is 37.3 inches long and includes 13 leather strips. Each belt includes one metal clasp that is part of the total length. All belts use the same clasp.



   (a) Name the variables, including units.

   (b) How long is each leather strip?

   (c) How long is the metal clasp?

   (d) Write an equation relating the variables.

   (e) Solve your equation to find the number of leather strips in a extra extra long length belt that's 43.7 inches long.

2. The local ski resort is trying to set the price for season passes. They know from past experience that they will sell around 12,000 passes if the season ticket price is \$380. If the price is \$400, they will sell fewer, perhaps only 10,000 passes. You can assume this decrease in demand is linear.

   (a) Name the variables. Notice that ticket price is the independent variable.

   (b) How many fewer people purchase season passes for every dollar increase in the price?

   (c) Find the intercept. Explain why this number does not make sense in the problem.

   (d) Write an equation for the function, using T for the ticket price, in dollars, and D for the demand (number of tickets sold).

   (e) How many season passes will they sell if the price is reduced to \$355?

   (f) The amount of **revenue** (money they take in) depends both on the ticket price and the number of tickets sold. The equation is $R = TD$, where R is the revenue, in dollars. Calculate the revenue when ticket prices are \$355, \$380, and \$400. *That means multiply the ticket price T times the number of tickets sold D in each case listed.* Of these three prices, which yields the most revenue?

3. For his Oscars party, Harland had 70 chicken wings delivered for \$51.25. For his Super Bowl bash, Harland had 125 wings delivered for \$83.70. The price includes delivery.

   (a) Assuming pricing is linear, what does each chicken wing cost?

   (b) What is the delivery charge?

   (c) Name the variables and write an equation for the function.

   (d) How many wings could Harland order for \$100? Solve your equation.

   (e) Graph and check.

4. Boy, am I out of shape. Right now I can only press about 15 pounds. (**Press** means lift weight off my chest. Literally.) My trainer says I should be able to press 50 pounds by the end of 10 weeks of serious lifting. I plan to increase the weight I press by a fixed amount each week.

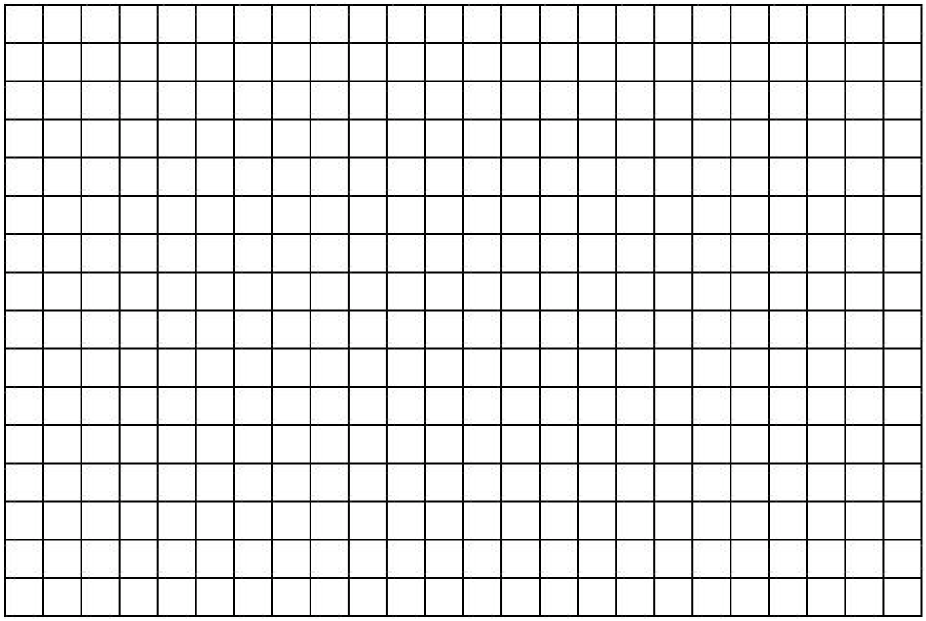
   (a) Name the variables and write an equation for my trainer's projection.
   Hint: you know the intercept.

   (b) Make a table showing my trainers projection for after 0, 5, 10, 15, and 20 weeks.

   (c) Years ago I could press 90 pounds. At this rate, when will I be able to press (at least) 90 pounds again? Set up and solve an inequality.

The problem continues ...

(d) Draw a graph illustrating the function.



(e) I am skeptical. I don't think I'll be able to press 50 pounds by the end of 10 weeks. If I revise my equation, will the new slope be larger or smaller? *Hint: try sketching in a possible revised line on your graph assuming that after 10 weeks I will press much less than 50 pounds.*

(f) Will my revised projections mean I'll reach that 90-pound goal sooner or later? Explain. *Hint: extend your graph.*

4.5 Fitting lines to data

1. Noel is considering investing in a company's stock so he looked up a few values.

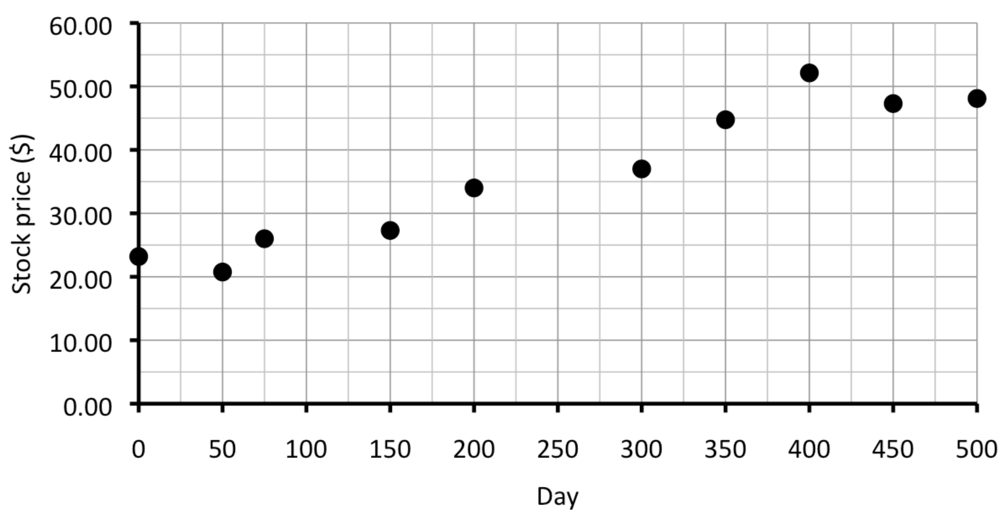
Day	0	300	500
Value ($)	23.19	37.00	48.10

   (a) Calculate the rate at which the stock prices changed during the first 300 days.

   (b) Calculate the rate at which the stock prices changed from Day 300 to Day 500.

   (c) Is this growth linear?

   (d) The scatter plot shows additional values of the stock Noel is considering buying.



   (e) Draw in a line that through the points for Day 300 and Day 500. Label this line #1. Explain why that line does not fit the data well.

   (f) Draw in a line that fits the data better. It does not need to go through any of the points exactly. Label that line #2.

2. The table shows the GPA (grade point average) of ten students compared to the number of hours per week each student works at a part time job. The variables we used are H for the time worked at job (hours / week) and G for the grades GPA (on scale of 0.0 to 4.0)

H	0	0	10	12	14	15	16	18	20	20
G	3.72	3.91	3.43	2.79	3.08	2.62	2.44	3.17	3.00	2.55

(a) Make a scatter plot of the points. Start the G-axis at 2.0.

(b) Find the equation of the line that goes through the first and last point listed.
Hint: the first point tells you the intercept.

The problem continues . . .

(c) Draw this line on your graph and label it line A.

(d) Use your equation for line A to figure out what you would expect the GPA of a student working a 30 hour per week job to be.

(e) The best fitting line from statistics has equation $G = 3.7597 - .0551H$. Make a table of values for this equation using $H = 0, 10, 20$ hours.

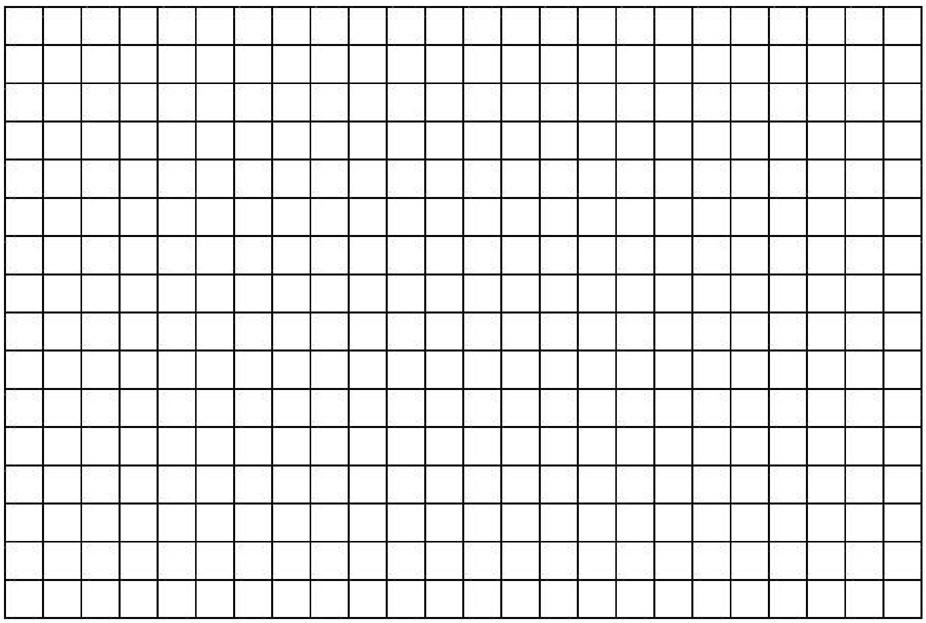
(f) Use that table of values to graph this best fitting line on that same set of axes. Label it line B.

(g) According to line B, what's the most hours a student should work to be able to maintain a 3.5 GPA? Solve an equation, then check on your graph.

3. Mia and Mandi and opened a candy shop this January. The table shows their monthly sales profit. Except for some seasonal fluctuation, Mia and Mandi generally expect your profits to rise steadily while their business is getting established.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Sales Profit ($)	3,394	4,702	3,683	4,840	5,632	4,432	4,649	4,590

(a) Make a scatter plot. Begin the profit axis at $3,000.



(b) Name the variables and write an equation for the line through January and August. Add this line (#1) to your graph. This line is too low.

The problem continues . . .

(c) Write an equation for the line through March and July. Notice that you need to find the intercept this time. Add this line (#2) to your graph. This line is too steep.

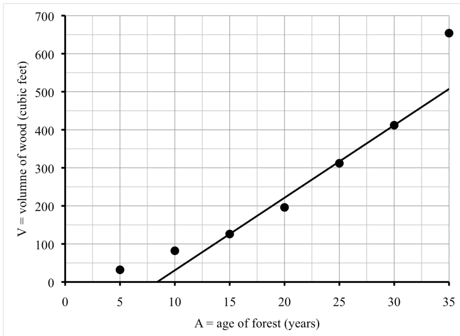
(d) Neither of these lines go anywhere near the data for February, April, and May, because those are outliers. Any idea why those months had much higher candy sales than the other months?

(e) What does each equation give as an estimate for September's sales?

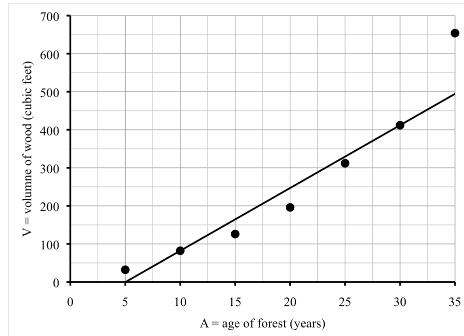
(f) Explain why Mia and Mandi should not use either of these lines to estimate October's sales.

4. The scatter plot shows the total volume of wood, V cubic feet, in managed forests of different ages, A years. For each line, decide if it's a good fit or not. Explain.

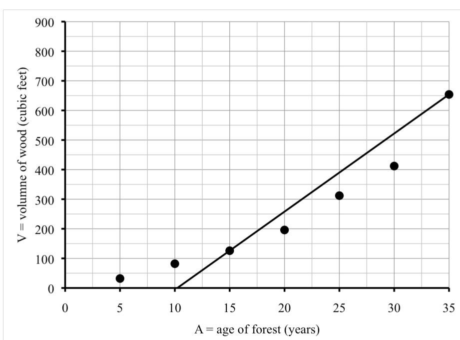
LINE A



LINE B



LINE C



LINE D