SOLUTIONS

4.3 Intercepts and direct proportionality – Practice exercises

1. Each of the two stories, below, involve how temperature changes over time. It might be confusing to call either variable T, so use H for the time in hours and D for the temperature in degrees (°F). In each case, time should be measured from the start of the story.

   (a) It was really cold at 8:30 this morning when Raina arrived at the office. Luckily the heating system warms things up very quickly, 4°F per hour. By 11:00 a.m. it was a very comfortable 72° F.

   i. Figure out what the temperature was at 8:30 a.m.

   11:00 a.m − 8:30 a.m = 2:30 hrs = 2 hrs, 30 min = 2.5 hours

   30 min * 1 hour / 60 min = 30 ÷ 60 = .5 hrs

   start + 4°F/hr * 2.5 hrs = 72°F

   4 × 2.5 = 10°F, −10°F

   −10°F ⇒ It was 62°F at 8:30 a.m.

   OR start = dep − slope * indep = 72 − 4 × 2.5 = 62 ✓

   ii. Write an equation illustrating the function.

   don't forget to identify when H starts

   H = time (hours since 8:30 a.m.) ~ indep

   D = temperature of Raina's office (°F) ~ dep

   $D = 62 + 4H$

   since H starts at 8:30 a.m., starting temp is 62°F

   (b) While 72°F is a perfectly good temperature for an office, not so for ballroom dancing. When Raina arrived for her practice at 5:30 that evening, she began to sweat before she even took the floor. Turns out the air conditioner had been running since 4:00 p.m. but it only cools down the room 3°F per hour.

   i. Figure out what the temperature was at 4:00 p.m.

   5:30 p.m − 4:00 p.m = 1:30 hrs = 1 hr, 30 min = 1.5 hours

   start − 3°F/hr * 1.5 hrs = 72°F

   −4.5°F, +4.5°F

   +4.5°F ⇒ It was 76.5°F at 4:00 p.m.

   OR start = dep − slope * indep = 72 − −3 × 1.5 = 76.5 ✓

   ii. Write an equation illustrating the function.

   H = time (since 4:00 p.m.) ~ indep

   D = temperature of dance studio (°F) ~ dep

   $D = 76.5 - 3H$

   slope is negative because studio is dropping in temperature

2. Maryn is very happy. Her interior design business is finally showing a profit. She has logged a total of 471 billable hours at \$35 per hour since she started her business. Accounting for start up costs, her net profit is totals \$2,194.

(a) What were Maryn's start up costs?

start + \$35/hr × 471 hrs = \$2,194

\$16,485

⇒ start = 2,194 − 16,485 = **\$14,291**

OR start = dep − slope × indep
= 2194 − 35 × 471
= −14,291 ✓

(b) Identify the slope and intercept (including their units and sign) and explain what each means in terms of the story.

intercept = −\$14,291 represents the initial loss because of the start up costs

slope = \$35/hr represents the rate Maryn's bills for her work.

(c) Calculate what Maryn's profits will be once she has logged a total of 1,000 hours.

\$35/hr × 1000 hrs − \$14,291 = 35 × 1000 − 14,291 = **\$20,709**

(d) Name the variables and write an equation relating them.

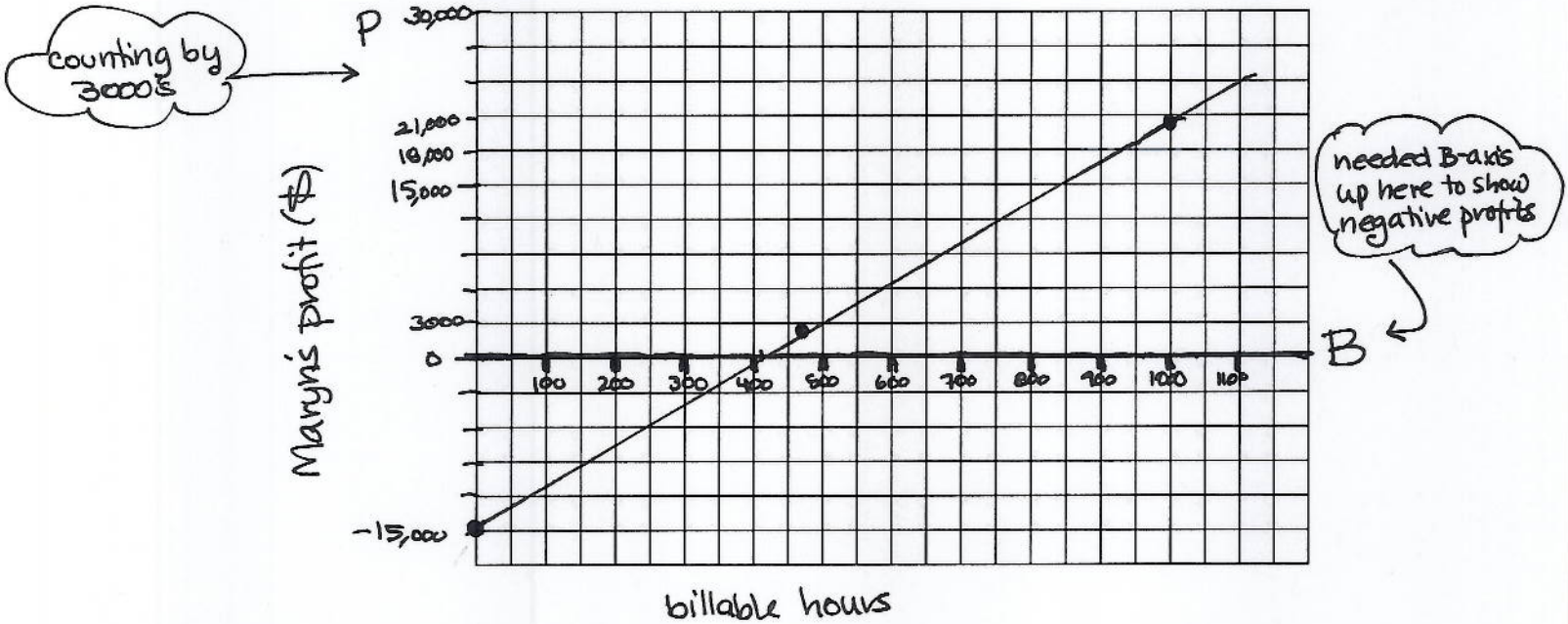
P = Maryn's profit (\$) ~ dep

B = time she bills (hours) ~ indep

P = 35B − 14,291

the equation can also be written P = −14,291 + 35B

(e) Graph the function.



Summary of info:

B	0	471	1000
P	−14,291	2,194	20,709
	from (a)	from story	from (c)

initial weight = end weight ± slope * # weeks

3. For each story, find the initial weight of the person and use it to write an equation showing how the person's weight P pounds depends on the time, W weeks.

   (a) Jerome has gained weight since he took his power training to the next level ten weeks ago, at the rate of around 1 pound a week. He now weighs 198 pounds.

initial weight $= 198 - \underbrace{1 \times 10}_{10} =$ 188 pounds

$P = 188 + 1W$ or just $P = 188 + W$

   (b) Vanessa's doctor put her on a sensible diet and exercise plan to get her back to a healthy weight. She will need to lose an average of 1.25 pounds a week to reach her goal weight of 148 pounds in a year. Use 1 year = 52 weeks.

initial weight $= 148 + \underbrace{1.25 \times 52}_{65} =$ 213 pounds

$P = 213 - 1.25W$

   (c) After the past 6 weeks of terrible migrane headaches, Carlos is down to 158 pounds. He's lost 4 pounds a week.

initial weight $= 158 + \underbrace{4 \times 6}_{24} =$ 182 pounds

$P = 182 - 4W$

   (d) Since she's been pregnant, Zoe has gained the recommended 1/2 pound per week. Now 30 weeks pregnant and 168 pounds, she wonders if she'll ever see her feet again.

initial weight $= 168 - \underbrace{\frac{1}{2} \times 30}_{15} =$ 153 pounds

$P = 153 + .5W$

$1/2 = 1 \div 2 = .5$ ☺

4. Each story describes a situation that we arre assuming is linear. Decide whether it is **proportional**, meaning the intercept equals zero. If it is not proportional, explain what the intercept would mean in the story.

   (a) The price of kiwis depends on how many kiwis you buy. ***Kiwi** is a fruit.*

   Cost = # kiwis × #/kiwi ⇒ no fixed cost

   proportional

   (b) The price of a bag of tortillas depends on how many tortillas are in the bag.

   some economy of scale 100-tortilla bag much less than 5 of 20-tortilla bags.

   not

   (c) The time it takes to vacuum a rug depends on the area of the rug.

   intercept = time to get out/return vacuum

   not

   (d) The time it takes to wash dishes depends on how many dirty dishes there are.

   no start-up time

   proportional

   (e) The amount of laundry detergent I have left depends on how many loads of laundry I did.

   amount left = starting amount − detergent/load × # loads

   ↑ intercept

   not