SOLUTIONS

0.6 Prelude: Powers and roots

Practice exercises

1. Jody is using small wooden balls to make noses for her knitted gnomes. She figured out that she can calculate the weight of each ball (in ounces) as $.2 \times B \wedge 3$ where B is the diameter of the ball (in inches).

Don't worry— the calculator knows to do ^3 (the exponent) before .2x (the multiplication)

(a) What does a 2.5 inch diameter wooden ball weigh?

$.2 \times 2.5 \wedge 3 =$ **3.125 ounces**

(b) Jody is considering building a giant gnome for her office. The nose will be a wooden ball weighing 1 pound. She calculates that the diameter of the ball will be $\sqrt[3]{80}$. How big is that?

$\sqrt[3]{80} = 3\ \sqrt{x}y\ 80 = 4.3088\ldots \approx$ **4.3 inches**

2. The size of a round pizza is described by its diameter. It turns out that we can calculate how many people are served by a pizza of diameter D inches as $.015625 \times D \wedge 2 =$. For example, a 16-inch diameter pizza serves $.015625 \times 16 \wedge 2 = 4$ people. (The mysterious number .015625 comes from a little geometry and pizza science.)

Story also appears in 2.4 #1 and 3.3 #1.

(a) How many people would be served by a 12-inch pizza?

$.015625 \times 12 \wedge 2 = 2.25$ **Just over 2 people**

If your calculator has a special square root key try $\sqrt{}\ 64 = 8$ instead.

(b) A personal pizza is designed to serve one person. It turns out the diameter of a personal pizza is $\sqrt{64}$. Calculate the diameter of a personal pizza using the square root key (or just the root key) on your calculator.

$\sqrt{64} = 2\ \sqrt{x}y\ 64 =$ **8 inches**

(c) An extra large pizza serves 6 people. It turns out the diameter of an extra large pizza is $\sqrt{384}$. Calculate the diameter of a personal pizza using the square root key (or just the root key) on your calculator.

$\sqrt{384} = 2\ \sqrt{x}y\ 384 = 19.5959\ldots \approx$ **20 inches**

3. A signal sent down a fiber optic cable decreases by 2% per mile. That means after M miles, its strength is $\underbrace{.98 \times .98 \times \cdots \times .98}_{M \text{ times}} = .98 \wedge M$. What is the signal strength after 10 miles? After 20 miles? Note: your answers should be decimal numbers less than 1.

10 miles: $.98 \wedge 10 = .8170\ldots \approx \boxed{.82}$

20 miles: $.98 \wedge 20 = .6676\ldots \approx \boxed{.67}$

means ×2

4. Otis invested \$500,000 and estimates his investment will double in value every 10 years.

   (a) Calculate the value of Otis's investment after 10, 20, 30, and 40 years.

10 years: $500000 \times 2 = 1{,}000{,}000 = \boxed{\$1 \text{ million}}$

20 years: $1{,}000{,}000 \times 2 = 2{,}000{,}000 = \boxed{\$2 \text{ million}}$

30 years: $2{,}000{,}000 \times 2 = 4{,}000{,}000 = \boxed{\$4 \text{ million}}$

40 years: $4{,}000{,}000 \times 2 = 8{,}000{,}000 = \boxed{\$8 \text{ million}}$

   (b) If Kricia invested \$230,000 instead, what would her investment be worth after 40 years? Try to use a power to help answer the question. Hint: how many times will the value of her investment double?

40 years: $230{,}000 \times \underbrace{2 \times 2 \times 2 \times 2}_{4 \text{ times}}$

$= 230000 \times 2 \wedge 4$

$= \boxed{\$3{,}680{,}000}$