Solutions

4.4 Slopes – Practice exercises

Fix typo!!

1. Jana is making belts out of leather strips and a metal clasp. An extra short length belt (as shown) is 24.5 inches long and includes 7 leather strips. An extra long length belt (not shown) is 37.3 inches long and includes 11 leather strips. Each belt includes one metal clasp that is part of the total length. All belts use the same clasp.

(a) Name the variables, including units.

L = length of belt (inches) ~ dep
N = number leather strips) ~ indep

N	L
7	24.5
11	37.3

(b) How long is each leather strip?

slope (rate of change) $= \dfrac{\text{extra dep}}{\text{extra indep}} = \dfrac{37.3 - 24.5 \text{ in}}{11 - 7 \text{ strips}} = (37.3 - 24.5) \div (11 - 7) =$ $\boxed{3.2 \text{ inches}}$ per strip

(c) How long is the metal clasp?

intercept $=$ dep $-$ slope $\times$ indep $= 24.5 \text{ in} - \dfrac{3.2 \text{ in}}{\text{strip}} \times 7$ strips
$= 24.5 - 3.2 \times 7 = \boxed{2.1 \text{ inches}}$

(d) Write an equation relating the variables.

$\boxed{L = 2.1 + 3.2N}$

check: $2.1 + 3.2 \times \underline{7} = 24.5$ ✓
$2.1 + 3.2 \times \underline{11} = 37.3$ ✓

(e) Solve your equation to find the number of leather strips in a $\underbrace{\text{extra extra long}}_{\text{XXL}}$ length belt that's 43.7 inches long.

L = 43.7

$\cancel{2.1} + 3.2N = 43.7$
$-\cancel{2.1} \qquad -2.1$
$\dfrac{\cancel{3.2}N}{\cancel{3.2}} = \dfrac{41.6}{3.2}$
$N = \boxed{13 \text{ leather strips}}$

check: $2.1 + 3.2 \times \underline{13} = 43.7$ ✓

2. The local ski resort is trying to set the price for season passes. They know from past experience that they will sell around 14,000 passes if the season ticket price is \$380. If the price is \$400, they will sell fewer, perhaps only 11,000 passes. You can assume this decrease in demand is linear.

T	D
380	14000
400	11,000

(a) Name the variables. Notice that ticket price is the independent variable.

T = ticket price (\$) ~ indep

D = demand / ~~tickets~~ passes sold (~~tickets~~ passes) ~ dep

(b) How many fewer people purchase season passes for every dollar increase in the price?

$$\text{slope} = \frac{11{,}000 - 14{,}000 \text{ passes}}{\$400 - \$380} = (11000-14000) \div (400-380) = -150 \text{ passes/\$ increase}$$

$\boxed{\text{150 fewer passes}}$

(c) Find the intercept. Explain why this number does not make sense in the problem.

intercept = dep − slope × indep = 14000 − −150 × 380 = $\boxed{\text{71000 passes}}$ if free which is not realistic.

(d) Write an equation for the function, using T for the ticket price, in dollars, and D for the demand (number of tickets sold).

$\boxed{D = 71{,}000 - 150T}$

Check:
$T = 380 \Rightarrow D = 71000 - 150 \times 380 = 14000$ ✓
$T = 400 \Rightarrow D = 71000 - 150 \times 400 = 11000$ ✓

(e) How many season passes will they sell if the price is reduced to \$355?

T=

$D = 71{,}000 - 150 \times 355 = \boxed{17{,}750 \text{ passes}}$

(f) The amount of **revenue** (money they take in) depends both on the ticket price and the number of tickets sold. The equation is $R = TD$, where R is the revenue, in dollars. Calculate the revenue when ticket prices are \$355, \$380, and \$400. *That means multiply the ticket price T times the number of tickets sold D in each case listed.* Of these three prices, which yields the most revenue?

T	D	R=T×D
355	17,750	355×17,750 = \$6,301,250 ← most revenue
380	14,000	380×14,000 = \$5,320,000
400	11,000	400× 11,000 = \$4,400,000

3. For his Oscars party, Harland had 70 chicken wings delivered for \$51.25. For his Super Bowl bash, Harland had 125 chicken wings delivered for \$83.70. The price includes a delivery charge.

(a) Assuming pricing is linear, what does each chicken wing cost?

W	C
70	51.25
125	83.70

$$\text{Slope} = \frac{\$83.70 - \$51.25}{125-70 \text{ wings}} = \frac{(83.70-51.25)}{\div(125-70)} = \boxed{=\$.59/\text{wing}}$$

(b) What is the delivery charge?

$$\text{intercept} = \$51.25 - 70 \times .59 = \boxed{\$9.95}$$

(c) Name the variables and write an equation for the function.

W = # wings – indep

C = cost (\$) – dep

$$\boxed{C = 9.95 + .59W}$$

Check:

9.95 + .59 x 70 = 51.25

9.95 + .59 x 125 = 83.70 ✓

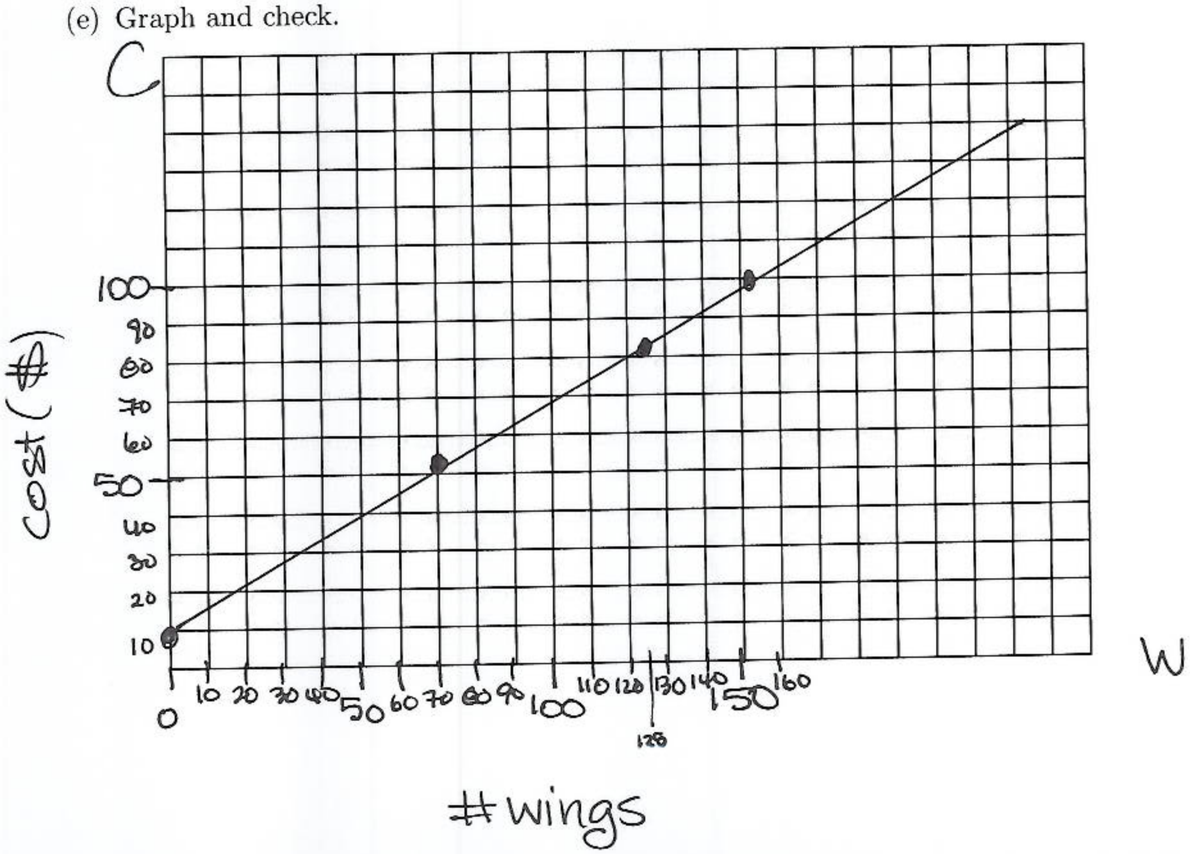
(d) How many wings could Harland order for \$100? Solve your equation.

$$9.95 + .59W = 100$$
$$-9.95 \qquad -9.95$$
$$\frac{.59W}{.59} = \frac{90.05}{.59}$$

→ W = 152.627 . . .

⇒ $\boxed{\text{can afford 152 wings}}$ mmm....

(e) Graph and check.



4. Boy, am I out of shape. Right now I can only press about 15 pounds. (**Press** means lift weight off my chest. Literally.) My trainer says I should be able to press 50 pounds by the end of 10 weeks of serious lifting. I plan to increase the weight I press by a fixed amount each week.

   (a) Name the variables and write an equation for my trainer's projection.
   Hint: you know the intercept.

decided "W" would be too confusing: weight vs weeks!

P = weight I can press (pounds) ~dep

T = time (weeks) ~indep

intercept = 15 pounds

$$\text{slope} = \frac{50-15 \text{ pounds}}{10-0 \text{ weeks}} = (50-15) \div (10-0) = 3.5 \text{ pounds/week}$$

T	P
0	15 ← int.
10	50

$$\boxed{P = 15 + 3.5T}$$

   (b) Make a table showing my trainer's projection for after 0, 5, 10, 15, and 20 weeks.

T	0	5	10	15	20
P	15	32.5	50	67.5	85

15 + 3.5 x 5 =

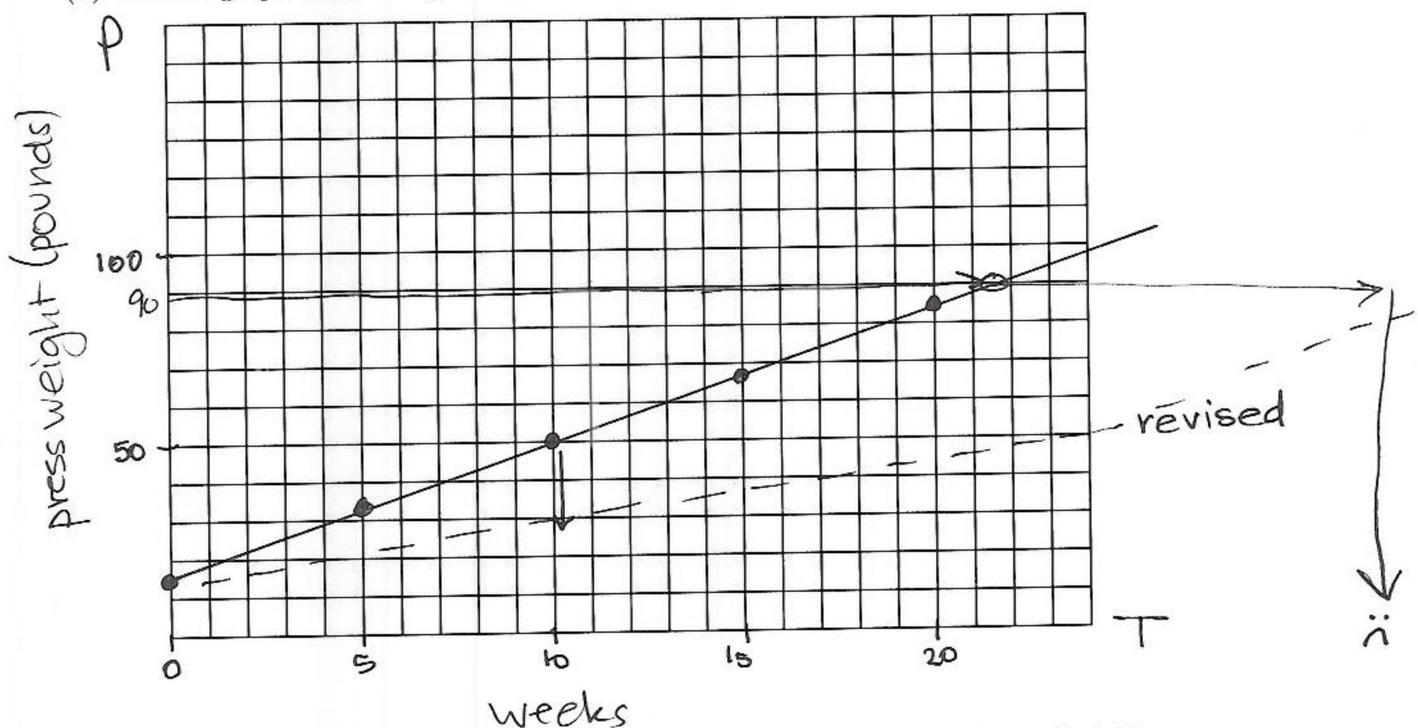
   (c) Years ago I could press 90 pounds. At this rate, when will I be able to press (at least) 90 pounds again? Set up and solve an inequality.

$$15 + 3.5T \geq 90$$
$$-15 \qquad -15$$
$$3.5T \geq 75$$
$$\frac{3.5T}{3.5} \geq \frac{75}{3.5}$$
$$T \geq 21.42\ldots$$

$\boxed{\text{in} \approx 22 \text{ weeks}}$

The problem continues ...

(d) Draw a graph illustrating the function.



(e) I am skeptical. I don't think I'll be able to press 50 pounds by the end of 10 weeks. If I revise my equation, will the new slope be larger or smaller?
Hint: try sketching in a possible revised line on your graph assuming that after 10 weeks I will press much less than 50 pounds.

less steep $\Rightarrow$ $\boxed{\text{smaller slope}}$

(f) Will my revised projections mean I'll reach that 90-pound goal sooner or later? Explain. *Hint: extend your graph.*

$\boxed{\text{much later!}}$