# Applied Algebra

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Fall 2010 edition

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# Preface

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## To the student

## To the instructor

## Special thanks to ...

In this preface, I'd like to thank all the little tiny people in my head who made this book possible, all my students, and also give some sort of introduction to the instructor and students. Perhaps "to the instructor", "to the student", and "acknowledgements" should each be subsections if the preface allows for such things.

SU: check on page number if not done correctly automatically.

# Chapter 1

# Variables

Believe it or not, algebra is useful. Really useful. It's useful in later courses you might take in mathematics, statistics, science, social science, or business. But it's also useful in real life. A lot of what happens in the world around us is easier to understand using algebra. That's what this course is all about: using algebra to answer questions.

In this first chapter, we will introduce the key concepts of variable and function that help us translate between problems stated in words and the mathematics explaining the situation. We explain the important tools of units, tables, and graphs. We also describe how functions change and use the rate-of-change to approximate answers to questions. Throughout this chapter we keep a careful eye on evaluating the reasonableness of answers by connecting what we learn from algebra with our own life experience. After all, an answer to a real problem should make sense, right?

Some of our approach may feel very different to you. It is possibly quite different from what you've seen in mathematics classes before. It might take you a little time to get used to, but it will be worth it.

## 1.1 Variables and functions

Things change; like the price of gasoline, and just about every day it seems. What does it mean when the price of a gallon of gas drops from \$3.999/gal to \$3.699/gal? On one level that means it costs

$$\$3.999 - \$3.699 = \$.30 = 30¢$$

less for one gallon. Does this 30¢ truly matter?

Before we answer that question, are you wondering why there's that extra 9 at the end of the price? We might say a gallon costs \$3.99 but there's really a small 9 following it. Sometimes that 9 is raised up slightly on the gas station sign. You have to read the fine print. What it means is an extra $\frac{9}{10}$¢ for each gallon. So the true price of gas would be \$3.999. So gas costs a tiny bit more than you thought. Good grief.

For the record, the / symbol is short for "per". So when we write \$3.999/gal we mean \$3.999 per gallon. Sometimes we write this as a fraction instead: $\frac{\$3.999}{\text{gallon}}$.

Back to our question. Does 30¢ truly matter to us? Probably not. But, how often do you buy just one gallon of gas? Typically you might put five, or ten, or even twenty gallons of gas into the tank. We want to understand how the price of gasoline influences what it really costs us at the pump. To do that let's compare our costs when we buy ten gallons of gas. (There's no good reason for picking ten except that it's a nice number to work with.)

If gas costs \$3.999/gal and we buy 10 gallons, it costs

$$\frac{\$3.999}{\text{gallon}} * 10 \text{ gallons} = \$39.99.$$

In this text we use the $*$ symbol for multiplication because the standard $\times$ symbol looks too much like an $x$. Your calculator might use $*$ or $\times$ instead.

If gas drops to \$3.699/gal and we buy 10 gallons, it costs

$$\frac{\$3.699}{\text{gallon}} * 10 \text{ gals} = \$36.99$$

instead. That's \$3 less. That amount matters. I mean, for \$3 savings on gas you could buy something else. Especially when you take into account that it's \$3 savings every time you put 10 gallons in the tank.

Gas prices have been changing wildly, and along with them, the price of 10 gallons of gas. In mathematics things that change are called *variables*. The two variables we're focusing on in this story are

$$P = \text{the price of gasoline (\$/gal)}$$
$$C = \text{the total cost (\$)}$$

Notice that we gave each variable a letter name. It is helpful to just use a single letter chosen from the word it stands for. In our example, $P$ stands for "price" and $C$ stands for "cost". In this course we rarely use the letter $x$ simply because so few words begin with $x$. Whenever we name a variable ($P$) we also describe in words what it represents (the price of gasoline), and we state what units it's measured in (\$/gal).

In talking about the relationship between these variables we might say "The cost depends on the price of gas." ($C$ depends on $P$). That tells us that $C$ is the *dependent variable* and $P$ is the *independent variable.* In general, the variable we really care about is the dependent variable, in this case $C$ the total amount of money it costs us. In some situations dependency can be viewed either way; there might not be one correct way to do it.

Given a choice, we usually assign dependence such that given a value of the independent variable, it is easy to calculate the corresponding value for the dependent variable. In our example it's easy to use the price per gallon, $P$, to figure out the total cost, $C$.

We can work backwards – from $C$ to $P$ – but it's not as easy. For example, suppose we buy 10 gallons of gas and it costs \$28.99. We can figure out that the price per gallon must be

$$P = \frac{\$28.99}{10 \text{ gallons}} = 28.99 \div 10 = \$2.899/\text{ gal}$$

Notice that in this example the fraction is shorthand for division.

There's another important way to express this dependence. We say "cost is a function of price" or that $C$ is a *function* of $P$.

There's one more thing to think about when describing the values. From our experience we have a sense of what gas might cost. In my lifetime, I've seen gas prices as low as 35.9¢ /gallon in the 1960s to a high of \$4.099/gallon in 2008. This range of values sounds too specific, so it would sound better to say something more general like

"Gas prices are between \$0/gal and \$5/gal."

The mathematical shorthand for this sentence is

$$0 \leq P \leq 5$$

The inequality symbol $\leq$ is pronounced "less than or equal to". Formally, the range of realistic values of the independent variable is called the *domain* of $P$.

Similarly, in mathematics the things that do not change (at least not during the story) are called *constants.* The one variable in this story is that we're always buying 10 gallons of gas.

We can summarize all of this information as follows.

| | |
|---|---|
| Constant: | 10 gallons |
| Variables: | $P$ = the price of gasoline (\$/gal), indep, $0 \leq P \leq 5$ |
| | $C$ = the total cost (\$), dep |

Let's look at one more situation. The average price of gasoline in Minnesota in 2010 was \$2.900/gal. One year earlier, in 2009, it was \$2.149/gal. We might ask: by what percentage did the price increase this past year and, if it continued to increase at that rate each year, what would the price be in 2011 and 2012?

The price rose \$2.900 − \$2.149 = \$.751 per gallon. *Percent increase* is the proportion of the starting amount that was added, expressed as a percent. That is,

$$\text{percent increase} = \tfrac{\text{change in amount}}{\text{starting amount}}$$

In our situation we have

$$\frac{\text{change in amount}}{\text{starting amount}} = \frac{\$.751/\text{gal}}{\$2.149/\text{gal}} = .751 \div 2.149 = .34946487\ldots \approx .3494$$

Notice that we rounded off our answer. When we want the person reading our calculation to know that we mean approximately .3494, not exactly, we use the *approximately equal to* symbol $\approx$. We save the equal sign, $=$, for when we have not rounded off the number at all.

To find the percentage from the proportion (decimal), we multiply by 100%.

$$.3494 * \frac{100\%}{1} = .3494 * 100 = 34.94\%$$

One way to remember this conversion is that proportions and percentages work just like dollars and cents:

$$.3494 = 34.94\%$$

is just like

$$\$.3494 = 34.94¢$$

So the price of gas increased by approximately 34.94% last year.

If someone asks you would probably round off the answer even more and say that gas prices increased "about 35%". We are keeping more decimal places because we have more calculations to do.

At this rate, what would the price be the next year, in 2011? To answer that question we know the price in 2010 wss \$2.900/gal. In 2011, it would be 34.94% more, which means it would increase by

$$34.94\% \text{ of } \$2.900/\text{gal} = .3494 * 2.900 = 1.01326 \approx \$1.013/\text{gal}$$

The price in 2011 would therefore be

$$2.900 + 1.013 \approx \$3.913/\text{gal}$$

In case you're curious, there is a quicker way to calculate this answer:

$$2.900 * 1.3494 = 3.91326 \approx \$3.913/\text{gal}$$

Continuing as above, the following year, in 2012, it would be 34.94% more, which means 34.94% more than what it was in 2011. This time we add on 34.94% of the \$3.913/gal.

$$34.94\% \text{ of } \$3.913 = .3494 * 3.913 = 1.3672\ldots \approx \$1.367/\text{gal}$$

The total price in 2012 would therefore be

$$3.913 + 1.367 = \$5.280/\text{gal}$$

As before, we can calculate this answer in one step as:

$$3.913 * 1.3494 \approx \$5.280/\text{gal}$$

A lot of realistic problems involve percentages and so we'll use them often in this course.

*There are two different types of exercises in this textbook. For the "Practice exercises," we have left space for you to write the solutions in the textbook. Think of them as additional examples. We provide full solutions to the Practice Exercises at the end of the textbook. For the regular "Exercises" you will need to write your solutions separately. We provide only the answers, not full solutions, to the Exercises at the end of the textbook.*

## Practice exercises

1. For each story, identify the variables and constants (if any). Include the units, realistic domain, and dependence.

   (a) The cost of holding a wedding reception at the Metropolitan Club is $1,000 down and $75 per person.

   (b) If I drive 60 mph (miles per hour), it takes me 20 minutes on the highway to get between exits, but when traffic is really bad it can take me an hour.

   (c) The sun set at 6:00 p.m. today and I heard on the radio that it sets about 2 minutes later each day this time of year.

   (d) Rent in the Riverside Neighborhood has increased 4.5% each year. Now rent in an apartment complex was $600 per month.

2. Every morning Jill goes for a 45-minute walk.

   (a) Identify the variables and constants in this dependence. Include the units, realistic domain and range, and dependence.

   (b) If Jill walked 2.5 miles, how fast was she walking?

3. The employee-paid cost of health insurance has risen dramatically, increasing by 7% each year since 2003 when it cost $420/month.

   (a) Identify the variables and constants in this dependence. Include the units, realistic domain and range, and dependence.

   (b) If this rate of increase continues, when will or did the employee-paid cost pass $550/month?

4. The Roman Inn charges $19.95 for a "New York" pizza that's 16" diameter and $5.95 for a "personal" pizza that's 6" in diameter.

   (a) Identify the variables and constants in this dependence. Include the units, realistic domain and range, and dependence.

   (b) Approximately how much would you guess the Roman Inn charges for a "regular" pizza that's 12" in diameter?

**Do you know . . .**

- What's the difference between a variable and a constant?
- How variables are named and their units specified?
- What we mean by function or dependence?
- How to distinguish the dependent from the independent variable?
- What's the (realistic) domain for a function?
- How to describe a range of values using an inequality?
- What notations are used for equal values and for and approximate values?
- How to calculate a percent increase?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. For each story, figure out an exact answer to the question if you can. If you can't find an exact answer, make a reasonable guess instead.

   (a) The cost of holding a wedding reception at the Metropolitan Club is \$1,000 down and \$75 per person. If the bill was for \$11,875, how many people were at the wedding dinner?

   (b) If I drive 60 mph (miles per hour), it takes me 20 minutes on the highway to get between exits, but when traffic is really bad it can take me an hour. How slow am I driving when traffic is really bad? *Hint: can you figure out the distance between exits?*

   (c) The sun set at 6:00 p.m. today and I heard on the radio that it sets about 2 minutes later each day this time of year. In how many days will the sun set at 7:30 p.m.?

   (d) Rent in the Riverside Neighborhood has increased 4.5% each year. Now rent in an apartment complex was \$600 per month. What will rent be in three years? *Hint: 4.5% = .045, don't forget the 0.*

6. The temperature was 40 degrees at noon yesterday but it dropped 3 degrees an hour in the afternoon.

   (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

   (b) When did the temperature drop below freezing (32 degrees)?

7. In 1990 the Lefèvre's property tax was \$450 but it doubled every year thereafter.

   (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.

(b) What was the tax up to by 1994? *Note: the city did provide tax credit to offset this rapid increase.*

8. A phone call on Kyle's phonecard costs \$.48 for the first minute (connection fee) and \$.02/minute thereafter.
   (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.
   (b) If Kyle used \$1.82 for a phone call, how long was the phone call?
9. A twenty pound bag of dog food costs \$12.49, but a five pound bag costs \$3.79.
   (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.
   (b) What does a ten pound bag probably cost?
10. Social Security benefits have increased by 3% per year. In 1995 my grandmother's benefit was \$246.17/month.
    (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.
    (b) When did her benefit pass \$300/month?
11. The bookstore charges 85¢ for a pack of gum.
    (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.
    (b) How many packs can Kawena buy for \$3.00?
12. Gilberto's car was worth \$22,500 when he bought it new. Now it's ten years old and worth only \$7,500.
    (a) Identify the variables and constants (if any), including the units, realistic domain, and dependence.
    (b) When will his car be practical worthless (under \$500)?

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

## 1.2 Tables and Graphs

Lung cancer, chronic bronchitis, bad breath, stains on your clothes, and the expense. These are just a few of the consequences of smoking cigarettes. With what we know now about the dangers of smoking, are there more or fewer people smoking than there were ten years ago, fifty years ago, or even one hundred years ago?

A good way to look at these numbers is to compare the number of cigarettes smoked per person per year. How would we calculate that? Well, a person who doesn't smoke at all would count as 0 cigarettes per year. A person who smokes "a pack a day" would count as 7, 300 cigarettes smoked per year because

$$\frac{1 \text{ pack}}{\text{day}} * \frac{20 \text{ cigarettes}}{\text{pack}} * \frac{365 \text{ days}}{\text{year}} = \frac{7,300 \text{ cigarettes}}{\text{year}}$$

If we add up all the cigarettes smoked in one year and divide by the total number of people, we get the number of cigarettes per person per year. Get it?

Here's some information from the Center for Disease Control (http://www.cdc.gov) for the United States. There are more data on their web site, but this list of partial data is good enough for us to get started.

| Year | 1900 | 1910 | 1920 | 1930 | 1940 | 1950 | 1960 | 1970 | 1980 | 1990 | 2000 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cig/person/yr | 54 | 128 | 649 | 1,452 | 2,374 | 3,247 | 3,908 | 4,192 | 3,933 | 2,967 | 1,128 |

What's changing are the number of cigarettes smoked per person per year and time. So, those are our variables. The smoking rate is a function of year, and it's what we care about, so it's the dependent variable. Time, as measured in years, is the independent variable. As before, it is convenient to measure time in years since 1900.

$$C = \text{smoking rate (cigarettes per person per year), dep}$$
$$Y = \text{time (years since 1900), indep}$$

Officially we should rewrite our table as:

| $Y$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | 54 | 128 | 649 | 1,452 | 2,374 | 3,247 | 3,908 | 4,192 | 3,933 | 2,967 | 1,128 |

Notice that the variables are listed in the table, with the independent listed first (on top) and the dependent listed second (on bottom). There are two standard formats for tables used.

Horizontal table format:

| independent variable | | | | |
|---|---|---|---|---|
| dependent variable | | | | |

Vertical table format:

| independent variable | dependent variable |
|---|---|
| | |
| | |
| | |
| | |

In a table, the placement of the independent variable first (either top or left) and the dependent variable second (either bottom or right) is a mathematical convention. That's not when a bunch of math folks get together but a *mathematical convention* is a custom, practice, or standard used within the mathematical community. Though based on reason, it often involves some arbitrary choice, making it impossible to figure out. So, whenever some practice is introduced to you as a "convention", you will need to memorize it.

The years are listed from 1900 to 2000, but we might be interested in future projections perhaps through the year 2050. Remember that we are measuring time in years since 1900. Thus in 1900 we had $Y = 0$, in 2000 we had $Y = 100$, and to get to 2050 we'll need $Y = 150$. Our domain is $0 \leq Y \leq 150$.

The number of cigarettes smoked per person per year was never 0 in the years but we'll begin at 0 anyway. The largest number listed in the table is 4,192 but it might have been higher at some point so to be safe we'll say $0 \leq C \leq 4,500$.

$$C = \text{smoking rate (cigarettes per person per year), dep, } 0 \leq C \leq 4,500$$

$$Y = \text{time (years since 1900), indep, } 0 \leq Y \leq 150$$

We can tell a lot of information from this table. For example, how many cigarettes per person per year were smoked in 1980? The answer appears in the table, a whopping 3,933 cigarettes per person that year.

When did the consumption first pass 3,000? That answer does not appear in the table, but we can use the information in the table to make a good guess. In 1940 ($Y = 40$), there were an average of 2,374 cigarettes per person per year and by 1950 ($Y = 50$) there were 3,247. Somewhere between 1940 and 1950 the number first climbed above 3,000. More specifically, the number we're looking for (3,000) is closer to the 1950 figure (3,247) than to the 1940 figure (2,374). So, it would be reasonable to guess close to 1950. I'd say 1947. Of course, you might guess 1946 or 1948, or even 1949 and those would be good guesses too. Interestingly, the full table of data from the CDC shows that consumption first topped 3,000 as early as 1944. Here's an example where the history tells you more than the mathematics as cigarette consumption rose sharply during World War II.

When did the consumption drop below 3,000 again? This answer also does not appear in the table, but falls somewhere between 1980 ($Y = 80$), when consumption was 3,933, and 1990 ($Y = 90$), when consumption was 2,967. Here I'd guess just before 1990, say in 1989. This time that guess does agree with the full table of data from the CDC.

Tables are useful because they contain specific numbers, but it can be difficult to guess or see general trends. For that, a picture is worth a thousand words (or numbers, in this case).

SU: insert plotted points here (no connect the dots yet). Y-horiz count by 10s to 150 and C-vert count by 500s to 4500.

To draw this graph, we began by labeling the axes, with the independent variable (year $Y$) on the horizontal axis and the dependent variable (consumption $C$) on the vertical axis. That's another mathematical convention you'll need to learn.

SU: insert diagram like from old text about dep/indep variables (top old 1.3)

In any graph, we mark the axes in an even scale. That means each box or grid mark counts for the same amount. The domain for the cigarette function is $0 \leq Y \leq 150$ and so we needed to chose a scale that would fit all those values. Counting by 10s worked just fine. The range is $0 \leq C \leq 4,500$. Now counting by 10s would not have worked. Even to count by 100s we would have needed 45 grid marks, which wouldn't fit on the page either. Here we decided to count by 500s. In general, I like to count by round numbers (2s, 5s, 10s, etc.) because it makes guessing in between easier. Even if it would fit nicely to count by something like 300s, I'd rather use 200s or 500s instead. Similarly, you'll not see me count by 7s when 5s or 10s will work more easily. Officially the domain, range, and corresponding labels for the axes can start at any number, but I try to always start at 0. For now, I encourage you to do the same.

To plot each point we moved right to that $Y$-value, and then moved up to that $C$-value. The way the scale worked, the $Y$-values landed exactly on grid marks, but for the $C$-values we had to guess where to place the point. For example, in 1940 we had $Y = 40$ and $C = 2,374$. That $C$-value likes between 2,000 and 2,500, just above the un-marked half-way point of 2,250. In 1930 we had $Y = 30$ and $C = 1452$. That $C$-value lies between the grid lines for 1000 and 1500, but is almost equal to 1,500. We draw the point just a tiny bit below the gridline for 1,500. The point basically lies on that line.

To turn this plot of points into a graph, we need to draw in the curve showing the overall tendency. We can start by "connecting the dots" – drawing a line between each pair of points. But that isn't exactly right. As we can see, the values go up, level off, and then drop back down. It was probably more of a continuous trend and so the graph should be smoother. We can round off the corners to get a graph like this instead:

SU: graph with dots smoothly connected, but not exceeding the max.

When we draw in this smooth curve for the graph, what we are really doing is making a whole lot of guesses all at once. For example, earlier we guessed that consumption first passed 3,000 in around 1947. What does the graph show? If we look where $Y = 47$ the point is about at height $C = XX$.

SU: same graph, but now draw in line up from 47, then back left to the axis to show C value.

Can you tell from the graph what period of time $C > 3,400$? It looks like $x \leq Y \leq x$, or from years 19xx to 19xx, approximately.

SU: same graph, but now draw a horizontal line across at 3,400, then drop two verticals down to show the Y values.

Don't forget that when we drew in that curve it was really just a guess. We're sure about the points we plotted, but we're only guessing about where to draw the curve in. That means we're not sure about the other points. If we knew a lot more points we could have a more accurate graph. As I mentioned before there really is more data available from the CDC. Look at how the graph from the full data compares to our guess. In particular, notice the sharp increase during World War II

(1939-1945) that we mentioned earlier.

SU draw same graph, but now superimpose the graph of the full CDC data.

For now I encourage you to draw graphs by hand, ideally on graph paper where it is easy to see the gridlines. At some point in time it would be good to learn how to use some technology to draw graphs too. Graphing calculators, spreadsheet programs (like Excel™), or computer algebra systems (like Maple™ or Mathematica™) all draw graphs well. All the graphs in this text were done using Excel. But there's a reason I suggest you draw graphs by hand for now, and possibly for this whole course, even if you already know how to use the technology: you'll really understand graphs better.

There are four key steps in drawing a graph (by hand or by technology):

1. **Label** the axes.
2. **Scale** the axes.
3. **Plot** the points.
4. **Connect** the points smoothly.

SU: say something like this somewhere: Again, we could have done this calculation all at once as

$$1,000 + 75 * 200 = 16,000.$$

AND this too: but the units for the $P$ was just "people". That may seem a little odd, but whenever you have a variable that counts something, like the number of people, the unit is what it counts.

## Practice exercises

1. In 1962 my grandmother opened a savings account for me and put in \$2,000. Since then no-one has put in any money nor taken any money out, but the account earns interest each year. The table lists the account balance at various times since then. SU ADD 2010

| year | 1962 | 1970 | 1980 | 1990 | 2000 |
|---|---|---|---|---|---|
| $Y$ | 0 | 8 | 18 | 28 | 38 |
| $B$ | \$2,000 | \$3,187 | \$5,709 | \$10,223 | \$18,308 |

(a) What are the variables in this story, denoted by $Y$ and $B$ in the table? Include the units, realistic domain, and dependence.

Use the table to answer the following questions:

(b) What was my account balance in 1970?

(c) When was my account balance \$10,223?

(d) Approximately when did my account balance equal \$7,000?

(e) What do you expect the account balance was in 2008? SU FIX

(f) When do you expect the account balance to be \$25,000?

(g) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points.

(h) Use the graph to check your answers to the questions. Modify your answers if necessary.

2. How cold is it? In cold climates we know that it's not just the temperature, but also the wind, that governs how cold it feels outside. For example, at an air temperature of 10°F with no wind it feels like an honest 10°F. Cold but manageable. But add a 30 miles per hour wind and brrr, it feels like it's -12°F (12° below 0°F). At that temperature you can get frostbite in under an hour of exposure. We say the "wind chill" of 10°F with a 30 mph wind is -12°F. The National Weather Service lists the wind chill at 10°F air temperature for various wind speeds (from http://www.weather.gov/om/windchill/)

| Wind (mph) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Wind chill (°F) | 10 | 1 | -4 | -7 | -9 | -11 | -12 | -14 | -15 | -16 | -17 | -18 | -19 |

(a) At an air temperature of 15°F, what's the wind chill when the wind is blowing 20 mph?

(b) A "cold advisory" is issued whenever the wind chill falls below 0°F. How fast does the wind need to be to issue a cold advisory?

(c) Between a wind chill of 0°F and -15°F, schools in our district are open but kids can't go outside for recess. What's the corresponding range of wind speeds?

(d) Draw a graph illustrating the dependence and use it to check your answers.

3. The cost of holding a wedding reception at the Metropolitan Club is \$1,000 down and \$75 per person.

   (a) Identify the variables and constants in this dependence. Include the units, realistic domain, and dependence.

   (b) Make a table of showing the cost for 20, 50, 75, 100, or 150 people.

   (c) Draw a detailed graph illustrating the dependence.

   (d) If our budget is \$10,000, how many people can we have at the reception? First estimate the answer from your table. Then confirm, or adjust, your answer based on your graph. Can you figure it out exactly from the story?

4. My neighborhood theater offers a "movie pass" for \$24.00. With it, tickets cost \$4.50 each all year long. Without the movie pass, tickets cost \$8.00 each.

   (a) If you're going to buy 5 movie tickets in a year, would you buy the pass?

   (b) How many tickets would you need to buy in a year to make the pass worth it? Make a table and graph to support your reasoning.

**Do you know . . .**

- Where the independent and dependent variables appear in a table and in a graph?
- How to guess values from a table or from a graph?
- How to make a graph from a table?
- What we mean by scaling an axis evenly?
- How to make a table and then a graph from a story?
- Why we draw in a smooth curve connecting the points?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. The table adapted from http://www.crh.noaa.gov shows the "heat index" as a function of humidity at an air temperature of 88°F. With up to about 40% humidity, 88°F feels like it's 88°F. But if the humidity rises to 60%, then it feels like it is 95°F; that is, the heat index is 95°F.

| humidity (%) | 50 | 60 | 70 | 85 | 90 | 95 |
|---|---|---|---|---|---|---|
| heat index (°F) | 91 | 95 | 100 | 110 | 113 | 117 |

All of the following questions refer to situations when the air temperature is 88°F.

(a) What is the heat index when the humidity level is 70%?

(b) At what humidity level does 88°F feel more like 98°F?

(c) Heat exhaustion is likely to occur when the heat index reaches 105°F. At what humidity level will heat exhaustion likely occur?

(d) The heat index is considered danger in the range from 105°F to 129°F. What range of humidity levels are considered dangerous?

(e) What do you think the heat index would be at 99% humidity?

(f) Identify the variables, including units, realistic domain, and dependence.

(g) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points.

(h) Use your graph to check your answers to the questions. Modify your answers if necessary.

6. Your local truck rental agency lists what it costs to rent a truck (for one day) based on the number of miles you drive the truck.

| distance driven (miles) | 50 | 100 | 150 | 200 |
|---|---|---|---|---|
| rental cost ($) | 37.50 | 55.00 | 72.50 | 90.00 |

Use the table to find or reasonably guess the answers to the following questions.

(a) How much does it cost to rent a truck if you drive it 100 miles?

(b) How many miles did you drive a truck costing \$90.00 to rent?

(c) If you rent a truck and drive it 75 miles, how much do you think it will cost?

(d) If you rent a truck and drive it 10 miles, how much do you think it will cost?

(e) If you rent a truck and it costs \$60.00, about how many miles was it driven?

(f) Identify the variables, including units, realistic domain, and dependence.

(g) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points.

(h) Use your graph to check your answers to the questions. Modify your answers if necessary.

7. The table lists the United Nations' estimates of population of the Earth, in billions, for select years since 1800. (from www.un.org/) *Their report was released in 1999 to coincide with the world population crossing the 6 billion mark.*

| year | 1800 | 1850 | 1900 | 1950 | 1970 | 1990 | 2000 |
|---|---|---|---|---|---|---|---|
| population | 0.98 | 1.26 | 1.65 | 2.52 | 3.70 | 5.270 | 6.06 |

Use the table to find or reasonably guess the answers to the following questions.

(a) What was the population of Earth in 1850?

(b) What do you think the population of Earth was in 1860?

(c) What do you think the population of Earth was in 1960?

(d) In what year do you think the population of Earth first exceeded 2 billion?

(e) In what year do you think the population of the world will exceed 7 billion?

(f) Identify the variables, including units, realistic domain, and dependence.

(g) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points.

(h) Use your graph to check your answers to the questions. Modify your answers if necessary.

8. The temperature was 40 degrees at noon yesterday but it dropped 3 degrees an hour in the afternoon.

(a) Make a table of reasonable values.

(b) Draw a graph illustrating the dependence

(c) According to your table and graph, when did the temperature drop below freezing (32 degrees)?

(d) Compare your answer to your estimate in Exercise 1.1.3

9. In 1990 the Lefèvre's property tax was \$450 but it doubled every year thereafter.
   (a) Make a table of reasonable values.
   (b) Draw a graph illustrating the dependence.
   (c) According to your table and graph, what was the tax up to by 1994?
   (d) Compare your answer to Exercise 1.1.4
10. A phone call on Kyle's phonecard costs \$.48 for the first minute (connection fee) and \$.02/minute thereafter.
   (a) Make a table of reasonable values.
   (b) Draw a graph illustrating the dependence.
   (c) According to your table and graph, if Kyle used \$1.82 for a phone call, how long was the phone call?
   (d) Compare your answer to Exercise 1.1.5
11. Social Security benefits have increased by 3% per year. In 1995 my grandmother's benefit was \$246.17/month.
   (a) Make a table of reasonable values.
   (b) Draw a graph illustrating the dependence.
   (c) According to your table and graph, when did her benefit pass \$300/month?
   (d) Compare your answer to Exercise 1.1.7
12. The bookstore charges 85¢ for a pack of gum.
   (a) Make a table of reasonable values.
   (b) Draw a graph illustrating the dependence.
   (c) According to your table and graph, how many packs can Kawena buy for \$3.00?
   (d) Compare your answer to Exercise 1.1.8

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 1.3 Rate of change

A diver bounces on a 3-meter springboard. Up she goes. A summersault, a twist, then whoosh, into the water. The table shows the diver's height, measured as $H$ meters above the water, as a function of time, $T$ seconds.

| $T$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 |
|---|---|---|---|---|---|---|---|---|
| $H$ | 3.00 | 3.88 | 4.38 | 4.48 | 4.20 | 3.52 | 2.45 | 1.00 |

In case you're wondering, 3 meters is nearly 10 feet up and the highest height listed, 4.48 meters, is close to 15 feet above the water. More on how we figured those numbers out in the next section, but thought you might like to know.

How fast is she moving? During the first 0.2 seconds, her height changes from 3.00 meters to 3.88 meters. She rose $3.88 - 3.00 = 0.88$ meters in that 0.2 seconds. Measured in meters per second, her speed is

$$\frac{0.88 \text{ meters}}{0.2 \text{ seconds}} = 4.4 \text{ meters/sec.}$$

Her speed is called the *rate of change*, and is calculated as

$$\text{Rate of change} = \frac{\text{change in height}}{\text{change in time}}$$

$$= \frac{3.88 - 3.00 \text{ meters}}{0.2 \text{ seconds}} = \frac{0.88 \text{ meters}}{0.2 \text{ seconds}} = 4.4 \text{ meters/sec.}$$

What about during the next 0.2 seconds? Does she move faster, slower, or the same? This time her height changed from 3.88 meters to 4.38 meters. In these 0.2 seconds she rose $4.38 - 3.88 = .50$ meters. That's less than before (since $0.50 < 0.88$), which means so she is going slower. Officially, we can calculate that her speed is

$$\text{Rate of change} = \frac{\text{change in height}}{\text{change in time}}$$

$$= \frac{4.38 - 3.88 \text{ meters}}{0.4 - 0.2 \text{ seconds}} = \frac{0.50 \text{ meters}}{0.2 \text{ seconds}} = 2.5 \text{ meters/sec.}$$

To calculate the rate of change we figured out the top of the fraction (0.50) and the bottom of the fraction (0.2), and then divided $0.50 \div 0.2 = 2.5$. There is a way to do the entire calculation at once on the calculator, but you need to use parentheses:

$$(4.38 - 3.88) \div (0.4 - 0.2) =$$

We need those parentheses to force the calculator to do the subtractions first and division second. The usual order of operations would do it the other way around: multiplication and division before addition and subtraction.

Notice that the top of our fraction is

height at 0.4 seconds - height at 0.2 seconds

and the bottom of our fraction is

$$0.4 \text{ seconds - } 0.2 \text{ seconds}$$

It's important that they match up – the 0.4s first on top and bottom and the 0.2s second on top and bottom.

During the next time interval she's moving even slower.

$$\text{Rate of change} = \frac{4.48 - 4.38 \text{ meters}}{0.6 - 0.4 \text{ seconds}} = 0.5 \text{ meters/sec.}$$

And look what happens when we calculate her speed during the next time interval.

$$\text{Rate of change} = \frac{4.20 - 4.48 \text{ meters}}{0.8 - 0.6 \text{ seconds}} = \frac{-.28 \text{ meters}}{0.2 \text{ seconds}} = -1.4 \text{ meters/sec.}$$

What does a negative speed mean? During this time interval her height drops. She's headed down towards the water. Her speed is 1.4 meters/sec downward. The negative tells us her height is falling. We can add these speeds in our table.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | 0.0 | | 0.2 | | 0.4 | | 0.6 | | 0.8 | | 1.0 | | 1.2 | | 1.4 |
| $H$ | 3.00 | | 3.88 | | 4.38 | | 4.48 | | 4.20 | | 3.52 | | 2.45 | | 1.00 |
| speed | | 4.4 | | 2.5 | | 0.5 | | -1.4 | | -3.4 | | -5.35 | | -7.25 | |

To be perfectly correct, these are her "average" speeds over the interval. Instead of saying "rate of change" people will often say "average rate of change," but the formula is the same.

Over any interval where increasing the independent variable corresponds to an increase in the dependent variable, we say the function is *increasing*. The diver's height is increasing for $0 \leq T \leq 0.6$ seconds. It is possible that she continues to rise a little longer, but we can't tell from just the numbers in our table.

On the other hand, over any interval where increasing the dependent variable corresponds to a decrease in the dependent variable, we say the function is *decreasing*. The diver's height is decreasing for $0.8 \leq T \leq 1.4$ seconds. It is possible that her height starts decreasing sooner, and it certainly continues decreasing until she hits the water, but we don't know exactly when.

When does the diver's height stop increasing and start decreasing? When she's at the highest height, some time between 0.6 and 0.8 seconds into her dive. Before then her rate of change is positive. After that time her rate of change is negative. So, at the highest height her rate of change is probably equal to zero. Does that make sense? Think about watching a diver on film in very slow motion. Up, up she goes, then almost a pause at the top, and then down, down, into the water. At the top of her dive it's as if she stands still for an instant. That would correspond to zero speed.

We can use the rate of change to estimate values missing from the table. For example, let's guess her height at 0.3 seconds. During the time interval between 0.2 and 0.4 seconds, we figured out that her average speed was 2.5 meters/sec. From 0.2 to 0.3 is 0.1 extra seconds. During that 0.1 extra second, she goes about

$$0.1 \text{ extra seconds } * \frac{2.5 \text{ meters}}{\text{second}} = 0.25 \text{ extra meters,}$$

so her height would be approximately

$$3.88 \text{ meters} + 0.25 \text{ extra meters} = 4.13 \text{ meters.}$$

We expected something in between 3.88 and 4.38 meters. In fact, since 0.3 was right in the middle of 0.2 and 0.4, we actually get the average $\frac{3.88+4.38}{2} = 4.13$. If it wasn't right in the middle we wouldn't get the average, but a more weighted average. By the way, we could do this estimate all at once as

$$\begin{aligned}
\text{Estimated value} &= \text{original value} + \text{extra independent variable} \ * \ \text{rate of change} \\
&= 3.88 \text{ meters} + 0.1 \text{ extra seconds} \ * \frac{2.5 \text{ meters}}{\text{second}} \\
&= 3.88 \text{ meters} + 0.25 \text{ extra meters} \\
&\approx 4.13 \text{ meters.}
\end{aligned}$$

A photographer snapped a picture at exactly 1.03 seconds. How high was the diver then? We expect the answer to be just a little bit less than her height at 1.0 seconds, which was 3.52 meters, but not nearly as low as after 1.2 seconds, which was 2.45 meters. Let's see what the rate of change estimate is. First,

$$0.03 \text{ extra seconds} \ * \frac{-5.35 \text{ meters}}{\text{second}} = -0.1605 \text{ extra meters,}$$

which means about 0.1605 lower. Her height would be approximately

$$3.52 \text{ meters} - 0.1605 \text{ extra meters} = 3.3595 \text{ meters} \approx 3.36 \text{ meters.}$$

As before we can calculate this estimate in one fell swoop as

$$\begin{aligned}
\text{Estimated value} &= 3.52 \text{ meters} + 0.03 \text{ extra seconds} \ * \frac{-5.35 \text{ meters}}{\text{second}} \\
&= 3.3595 \text{ meters} \\
&\approx 3.36 \text{ meters.}
\end{aligned}$$

Notice that we rounded off to two decimal places for our approximation because all the numbers in the table were rounded off. That's a reasonable answer, much closer to her height at 1.0 seconds (3.52 meters) than her height at 1.2 seconds (2.45 seconds).

How long is the diver in the air? At 1.4 seconds she's 1.00 meter up, so she must enter the water soon after that. We can use the rate of change to estimate her height after 1.5 and 1.6 seconds to see. We don't know the average speed past 1.4 seconds, so we'll just have to use the closest value we know, her speed was -7.25 meters/sec during the preceding interval. Bear in mind that we're really guessing about that, and so our estimate is even less accurate than usual. For both estimates we start with 1.00 meter at 1.4 seconds.

$$\begin{aligned}
\text{Estimated height at 1.5 seconds} &= 1.00 \text{ meters} + 0.1 \text{ extra seconds} \ * \frac{-7.25 \text{ meters}}{\text{second}} \\
&= 0.275 \text{ meters} \\
&\approx 0.3 \text{ meters.}
\end{aligned}$$

$$\begin{aligned}
\text{Estimated height at 1.6 seconds} \quad &= \quad \text{1.00 meters} + \text{0.2 extra seconds} * \frac{-7.25 \text{ meters}}{\text{second}} \\
&= \quad -0.45 \text{ meters} \\
&\Longrightarrow \quad \text{already hit the water.}
\end{aligned}$$

Here is a graph showing the diver's height. The variables are

$$T = \text{time (seconds), indep, } 0 \leq T \leq 1.6$$
$$H = \text{diver's height (meters), dep, } 0 \leq H \leq 5$$

As usual we drew in a smooth curve connecting the points, which illustrates our best guesses for the points we don't know. We also drew in the straight lines connecting each pair of points. As you can see, the first line segment is steepest – that's where the rate of change was 4.4 meters/sec. The next line segment was less steep – that's where the rate of change was less, down to 2.5 meters/sec. The third line segment is almost flat – that's where the rate of change was only 0.5 meters/sec.

We notice the same connection between the rate of change and steepness of the curve for the decreasing portion, only this time all the rate of changes are negative. The first downhill line segment is fairly flat – that's where the rate of change was -1.4 meters/sec. The next downhill line segment was much steeper – that's where the rate of change was -3.4 meters/sec. The next two downhill line segments were each steeper yet – this time with rates of change -5.35 and -7.25 meters/sec.

In each case we can visualize the rate of change as the steepness of the graph.

SHOULD WE INCLUDE THIS: When the rate of change is constant, the graph is a line and the function is called *linear*. OR MAYBE WITHIN THE PRACTICE PROBLEM ABOUT WEDDING?

DO WE DO ENOUGH INTERPOLATION IN THIS EXAMPLE – check old solutions from first version of 1.3Epsilon

Whenever we use the rate of change to estimate values it's as if we're assuming the rate of change is constant, at least for that interval of values. If the function really is linear and the graph is a line, then this estimate is excellent. If the function is pretty far off from linear and the graph curves a lot, then this estimate might not be as accurate. Something to keep in mind.

There's a formal name for what we're doing here. When we use the rate of change to estimate a number in between two numbers that we know it's called *linear interpolation.* When we estimate a number beyond what we know (smaller than the smallest number or larger than the largest number), it's called *linear extrapolation.* For both terms, the word "linear" reminds us that it only works perfectly for a line. In general interpolation is often a reasonable guess. Extrapolation can be pretty far off, but sometimes it's the only guess we have.

Sometimes we just want to know when a function is increasing, decreasing, at a maximum or minimum, how steep it is, or how much it seems to change. Even if there aren't specific numbers on the graph, we can sometimes learn a lot about a story. The next example looks at this sort of numberless graphs, sometimes called *qualitative* graphs.

## Practice exercises

1. The table shows the costs for various sizes of sheet cakes available from the bakery.

| Number of people | 10 | 20 | 50 |
|---|---|---|---|
| Price of cake ($) | $11.95 | $19.95 | $40.95 |

(a) On average, how much does sheet cake cost for each additional person if there are between 10 and 20 people?

(b) On average, how much does sheet cake cost for each additional person if there are between 20 and 50 people?

(c) Why do you think the average price per person drops?

The next table shows the costs for various orders of cupcakes available from the bakery.

| Number of people | 12 | 24 | 48 |
|---|---|---|---|
| Price of cupcakes ($) | $6.95 | $13.90 | $27.80 |

(d) On average, how much does a cupcake cost for each additional person if there are between 12 and 24 people?

(e) On average, how much does a cupcake cost for each additional person if there are between 24 and 48 people?

(f) Why do you think the average price per person doesn't change?

(g) For 30 people, which is less expensive – sheet cake or cupcakes? *Explain.*

(h) For 18 people, which is less expensive – sheet cake or cupcakes? *Explain.*

2. As we read about in the last section the cost of holding a wedding reception at the Metropolitan Club is \$1,000 down and \$75 per person.

   (a) Calculate the extra cost, on average, for each additional person between 20 and 50 people.

   (b) Calculate the extra cost, on average, for each additional person between 75 and 100 people.

   (c) What do you notice?

   (d) Look back at the graph of the function. Explain why it is a line.

   (e) Is the cost function increasing, decreasing, or neither?

3. A twenty pound bag of dog food costs \$12.49, but a five pound bag costs \$3.79.

   (a) Calculate the rate of change. What does it mean in terms of the story?

   (b) Use the rate of change to estimate the cost of a 10 pound bag of dog food. Compare your answer to Exercise 1.1.6

   (c) Draw a graph illustrating the dependence using all three points calculated here. Do they fall on a line?

4. Suzanne's weight has certainly changed over the years. She sketched this graph to show how her weight has changed, but she deliberately didn't scale the axes because she didn't want to tell everyone her age and weight. The variables are

$$A = \text{Suzanne's age (years), indep, } 0 \leq T \leq ?$$
$$W = \text{Suzanne's weight (pounds), dep, } 0 \leq H \leq ?$$

Graph of Su's weight goes here

(a) Has she been gaining or losing weight lately?

(b) What might explain the two tall hill shapes in the graph?

(c) Did she ever get back to her pre-baby weight?

(d) After her last major weight loss, did she gain all the weight back?

**Do you know . . .**

- How to calculate rate of change between two points?
- What the rate of change means in the story?
- How we can use the rate of change to estimate values?
- When a function is increasing or decreasing, and the connection to the rate of change?
- Why the rate of change is zero at the maximum (or minimum) value of a function?
- What the connection is between rate of change and the steepness of the graph?
- How to sketch or read trends from a qualitative graph?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. The tables in the previous exercise showed the cost of sheet cake and cupcakes.

   (a) Identify the variables, including units, realistic domain and range, and dependence.
   (b) Draw a detailed graph illustrating the dependence based on the points given in the table. Be sure your axes are labeled and evenly scaled. Sketch in a smooth curve connecting the points for the sheet cake and another for the cupcakes.
   (c) Explain why the cupcake curve is a straight line, but the sheet cake curve is really curved.
   (d) According to your graph, which is less expensive for 30 people – sheet cake or cupcakes?
   (e) According to your graph, which is less expensive for 18 people – sheet cake or cupcakes?
   (f) Compare your answers to Exercise 1.3.1

6. It began to snow early one morning. At 7:00 a.m. there was no snow on the ground. By noon there was 2 inches of snow. By 3:00 p.m. there was a total of 4 inches.

   (a) Calculate the rate of snowfall during the time periods given – from 7:00 a.m.-noon and from noon-3:00 p.m.
   (b) During which time period, morning or afternoon, was the snow falling faster? *Explain.*
   (c) Estimate the amount of snowfall that was on the ground at 8:00 a.m., at 2:30 p.m., and at 4:00 p.m. *Explain your reasoning.*
   (d) Identify the variables, including units, realistic domain and range, and dependence.
   (e) Is the snow cover increasing, decreasing, or neither?

7. After aerobics class was over, Katie measured her heart rate several times while she was cooling down. Right after class her heart rate was 178 beats per minute. Two minutes later, it had dropped to 153 beats per minute, and by ten minutes after class had ended it was down to 120 beats per minute.

(a) Make a table showing how Katie's heart rate.

(b) Identify the variables, including units, realistic domain and range, and dependence.

(c) How quickly was Katie's heart rate dropping, on average, during the first two minutes following class? *Hint: the units are beats per minute per minute. Looks funny, but that's what it is.*

(d) How quickly was Katie's heart rate dropping, on average, during the next time period?

(e) Katie feels comfortable taking a shower once her heart rate is closer to normal, below 100. She usually waits 15 minutes after class. Do you think that's long enough? Explain.

(f) Is Katie's heart rate increasing, decreasing, or neither?

8. The table shows Henry's weight as a baby.

| | | | |
|---|---|---|---|
| Age (weeks) | 0 | 12 | 15 |
| Weight (pounds) | 8 | 14 | 16 |

(a) How much weight did Henry gain, on average, each week during his first 12 weeks?

(b) During which time interval was Henry gaining weight faster? *Explain.*

(c) What might you guess for Henry's weight at 20 weeks? *Explain your reasoning.*

(d) Identify the variables, including units, realistic domain and range, and dependence.

(e) Is Henry's weight increasing, decreasing, or neither?

9. Chaoxiang is considering investing in a certain company's stock. He looked up a table of values over the past 500 days. A few of those values are recorded in the table.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Day | 0 | 50 | 75 | 150 | 200 | 300 | 350 | 400 | 450 | 500 |
| Value ($) | 29.00 | 24.75 | 23.19 | 20.75 | 21.00 | 26.00 | 30.75 | 37.00 | 44.75 | 54.00 |

(a) Identify the variables, including units, realistic domain and range, and dependence.

(b) Draw a detailed graph illustrating the dependence based on the points given in the table.

(c) Calculate the rate of change between 75 and 150 days. Sketch in the line segment connecting those two points on the graph. Was the stock price increasing or decreasing over that time period?

(d) Calculate the rate of change between 400 and 450 days. Sketch in the line segment connecting those two points on the graph. Was the stock price increasing or decreasing over that time period?

(e) Approximately when was the rate of change equal to zero? What does that tell us in terms of the story?

(f) Assuming current trends continue, do you think it's a good idea that Chaoxiang buy the stock now?

10. Your local truck rental agency lists what it costs to rent a truck (for one day) based on the number of miles you drive the truck.

| distance driven (miles) | 50 | 100 | 150 | 200 |
|---|---|---|---|---|
| rental cost ($) | 37.50 | 55.00 | 72.50 | 90.00 |

(a) Calculate the rate of change for each time period.

(b) Use the rate of change to estimate the cost of renting a truck to drive 75 miles. Compare your answer to Exercise 1.2.3

(c) Use the rate of change to estimate the cost of renting a truck to drive 10 miles. Compare your answer to Exercise 1.2.3

(d) Is truck rental pricing linear? *Explain.*

(e) Is the rental cost increasing, decreasing, or neither?

11. The table lists the United Nations' estimates of population of the Earth, in billions, for select years since 1800. (from www.un.org/) *Their report was released in 1999 to coincide with the world population crossing the 6 billion mark.*

| year | 1800 | 1850 | 1900 | 1950 | 1970 | 1990 | 2000 |
|---|---|---|---|---|---|---|---|
| population | 0.98 | 1.26 | 1.65 | 2.52 | 3.70 | 5.270 | 6.06 |

(a) Calculate the rate of change for each time period.

(b) Look back at your guess of when the population of the Earth first exceeded 2 billion. (Exercise 1.2.4) Use the rate of change to estimate the population of the Earth in that year. Is it close to 2 billion?

(c) Look back at your guess of when the population of the Earth first exceeded 7 billion. (Exercise 1.2.4) Use the rate of change to estimate the population of the Earth in that year. Is it close to 7 billion?

(d) Is the population function linear? *Explain.*

(e) Is the population increasing, decreasing, or neither?

12. Gilberto's car was worth $22,500 when he bought it new. Now it's ten years old and worth only $7,500.

(a) Calculate the rate of change.

(b) Look back at your guess of when when his car will be practical worthless (under $500). (Exercise 1.1.9) Use the rate of change to estimate the the value of Gilberto's car in that year. Is it close to $500?

(c) That estimate is based on the assumption that the value of the car decreasing linearly. In fact, the value of a car drops sharply at first, and then more gradually over time. Given this new information, redraw a graph illustrating the dependence.

(d) Add the line connecting the two given points to your graph. The rate of change estimate was based on that line. How does our estimate from the rate of change (line) compare to the estimate of the graph?

13. "My computer has crashed one time too many. Now it's really going to crash!" Tony proclaimed as he stood on the top of Science Building and hurled his computer up in the air – carefully checking first that no-one was walking by, of course. He watched it fall back down and then it crashed into thousands of pieces on the sidewalk. The graph shows how the height of Tony's computer changed over time.

    (a) Which point on the graph shows the highest height of the computer? What is the rate of change at that point?
    (b) Which point on the graph shows when the computer hit the ground? Was the rate of change positive or negative right before then?
    (c) Which point on the graph shows when Tony let the computer pass him on the way down?

14. The Greenrug company makes weather-resistant carpeting. The graph shows how their profit is a function of the amount of carpeting they sell each month.

    (a) Which point on the graph shows the start-up costs each month?
    (b) Which point on the graph shows the break-even point (when the company started to make money)?
    (c) Which parts of the graph correspond to increasing profits?
    (d) Which point on the graph shows the maximum profit?
    (e) As the amount of carpet sold increase past the maximum profit point, what happens to the rate of change?

15. Sketch a qualitative graph showing each of the following:

(a) The temperature over the course of a year in Minnesota.
(b) The temperature over the course of a year in Las Vegas, Nevada.
(c) How your weight has changed in your lifetime.
(d) How the length of time it takes to get to school (or work, or the mall) depends on the time of day.

**When you're done ...**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010
Examples from the old 2.3 that could be here, or could wait until later.

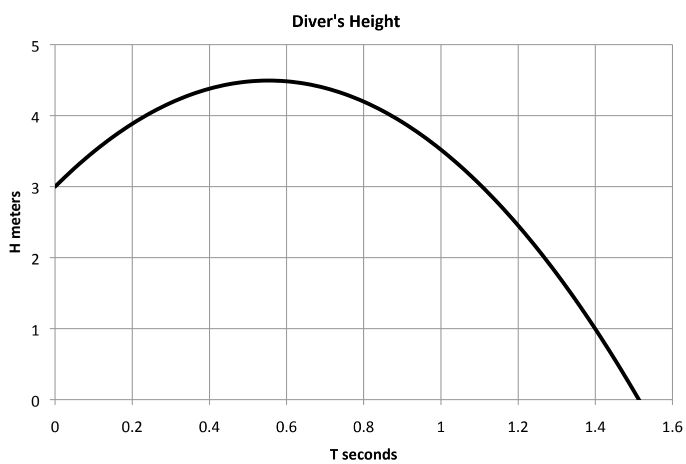
A well-drilling company charges its customers by the number of feet it digs into the ground to hit water. A well that is 350 deep costs $2,200 but a well that's only 200 feet deep only costs $1,300.

1. Calculate the additional charge for each additional foot of depth.
2. What do you expect a 400 foot deep well would cost?
3. Identify the variables, including units, realistic domain and range, and dependence.

Sofía was been baking cookies for her church bake sale. Last night she made 12 batches of chocolate cookies; it took her 2 hours and 40 minutes from start to finish including making the dough and baking each batch. The night before she made 7 batches of chocolate cookies; it took her 1 hour, 40 minutes. Each time she first made the dough for all the batches at once, and then she baked the cookies one batch at a time.

1. How long does each batch of cookies bake in the oven? *Hint: how much time does each additional batch of cookies add to the overall time?*
2. How long do you think it would take Sofía to prepare the dough and bake another 9 batches tonight?
3. If Sofía has only 2 hours, how many batches of cookies can she make?
4. Identify the variables, including units, realistic domain and range, and dependence.

4 in



## 1.4 Units

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 1.5 Scientific notation and logarithms

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## Practice exams on Variables

*Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.*

### Practice exam 1– version I

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

## Practice exam 1 – version II

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

# Chapter 2

# Equations

Yada yada

## 2.1 A first look at linear equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

**Do you know ...**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

    (a) xxx

**When you're done ...**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 2.2 A first look at exponential equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 2.3 Using equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

    (a) xx

    (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 2.4 Approximating solutions of equations

INTRODUCTORY EXAMPLE

### Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
    (a) xx

    (b) xx

**Do you know ...**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
    (a) xxx

**When you're done ...**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## Practice exams on Equations

*Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.*

### Practice exam 2– version I

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

## Practice exam 2 – version II

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

# Chapter 3

# Solving equations

Yada yada

## 3.1 Solving linear equations and inequalities

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

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## 3.2 Solving exponential equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

**Do you know ...**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

    (a) xxx

**When you're done ...**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

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## 3.3 Roots and solving power equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

   (a) xx

   (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

   (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 3.4 Solving other equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

    (a) xx

    (b) xx

4. Fourth
    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 3.5 ⋆ Solving quadratic equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## Practice exams on Solving equations

*Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.*

### Practice exam 3– version I

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

## Practice exam 3 – version II

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

May 10, 2010

# Chapter 4

# A closer look at linear equations

Yada yada

## 4.1 Modeling with linear equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 4.2 Slopes

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
   (a) xx

   (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
   (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 4.3 Fitting lines to data

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 4.4 Systems of linear equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

**Do you know ...**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

    (a) xxx

**When you're done ...**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
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- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## Practice exams on Linear equations

*Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.*

### Practice exam 4– version I

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

## Practice exam 4 – version II

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

May 10, 2010

# Chapter 5

# A closer look at exponential equations

Yada yada

## 5.1 Modeling with exponential equations

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

    (a) xx

    (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth

    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx

    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 5.2 Growth factors

INTRODUCTORY EXAMPLE

### Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third
    (a) xx
    (b) xx

4. Fourth
   (a) xx

   (b) xx

**Do you know ...**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
   (a) xxx

**When you're done ...**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 5.3 Exponential growth and decay

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

   (a) xx

   (b) xx

2. Second

    (a) xx

    (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
   (a) xx

   (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
   (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 5.4 Linear vs. exponential models

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

    (a) xx

    (b) xx

4. Fourth
   (a) xx

   (b) xx

**Do you know ...**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
   (a) xxx

**When you're done ...**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## Practice exams on Exponential equations

*Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.*

### Practice exam 5– version I

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

## Practice exam 5 – version II

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

May 10, 2010

# Chapter 6

# $\star$ Formulas

6.1 Formulas from finance (e.g. Compound interest, equivalent rates, annuities (both savings and loans), discounts.)

6.2 Formulas from social sciences and statistics (e.g. revenue/cost/profit and other economics formulas. various ratios used in business, common formulas from psychology or sociology, z-score, pooled variance, confidence intervals and other statistics formulas)

6.3 Formulas from natural and health sciences (e.g. pH and other log scales, multi-variable formulas, body mass index type formulas, bio, chem, physics, earth science, exercise/nutrition, environmental)

6.4 Formulas from geometry (e.g. Pythag, similar triangles, polygons, areas and volumes - maybe also include fine arts here? Construction formulas (e.g. handicap ramp))

6.5 Formulas from trigonmetry (e.g. right triangle definition of sine-cosine-tangent, law of cosines, inverse to solve for angle, unit circle definition of sine-cosine, radians to degrees conversion, period/amplitude formulas about graphs of sine and cosinesonly.)

*In all of these problems there is NO choice of appropriate formula, but rather in every case we provide the students with precisely the proper formula to use. Leave it to QF courses to worry about what to use when. This is a very important note for instructors! Also, exams probably should not introduce new formulas unless they're cashews-to-peanuts away.*

## 6.1 Formulas from finance

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second
    (a) xx
    (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 6.2 Formulas from social sciences and statistics

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

    (a) xx

    (b) xx

4. Fourth
   (a) xx

   (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
   (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 6.3 Formulas from natural and health sciences

INTRODUCTORY EXAMPLE

### Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

    (a) xx

    (b) xx

3. Third

    (a) xx

    (b) xx

4. Fourth
    (a) xx

    (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
    (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

## 6.4 Formulas from geometry

INTRODUCTORY EXAMPLE

## Practice exercises

1. First

    (a) xx

    (b) xx

2. Second

   (a) xx

   (b) xx

3. Third

   (a) xx

   (b) xx

4. Fourth
   (a) xx

   (b) xx

**Do you know . . .**

- Questions?

*If you're not sure, work the rest of exercises and then return to these questions afterwards. Or, ask your instructor or a classmate for help.*

## Exercises

5. xx
   (a) xxx

**When you're done . . .**

- Don't forget to check your answers with those in the back of the textbook.
- Not sure if your answers are close enough? Compare with a classmate or ask the instructor.
- Getting the wrong answers or stuck on a problem? Re-read the section and try the problem again. If you're still stuck, work with a classmate or go to your instructor's office hours.
- It's normal to find some parts of some problems difficult, but if all the problems are giving you grief, be sure to talk with your instructor or advisor about it. They might be able to suggest strategies or support services that can help you succeed.
- Make a list of key ideas or processes to remember from the section. The "Do you know?" questions can be a good starting point.

May 10, 2010

# Practice final exams

*Try taking these practice exams under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself two hours to work and wait until you have tried your best on all of the problems before checking any answers.*

## Practice final exam– version I

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here
   (a) With all of it's subparts

   (b) Listed here

## Practice final exam – version II

*Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.*

*A few formulas from our book:*

FORMULAS PRINTED ON EXAM GO HERE

---

1. First problem goes here

   (a) With all of it's subparts

   (b) Listed here

# Solutions to practice exercises

SU: handwrite and scan in here

# Answers to exercises

*Solutions to the practice exercises appear in an earlier section.*

SU: type in here – preferably with links to the actual problems so don't have to hand enter?