Workbook for

Just Enough Algebra

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Fall 2012 edition (Aug 24 version)

Contents

5.1 Modeling with exponential equations

1. The population of Buenos Aires, Argentina in 1950 was estimated at 5.0 million and expected to grow at 1.8%. Source: Mongabay

   (a) Name the variables.

   (b) What is the annual growth factor?

   (c) Write an equation estimating the population of Buenos Aires.

   (d) Make a table of values showing the estimated population of Buenos Aires every 20th year from 1950 to 2030.

   (e) By how many people has the population been increasing during each 20 year period? Add these numbers to your table. *As expected, these numbers change because the rate of change is not constant.*

   (f) The actual population of Buenos Aires in the year 2000 was around 12.6 million and by 2010 it was around 15.2. (This now includes a larger surrounding area as the city has spread beyond historical limits.) How does that compare to the estimates?

2. A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 16% per day. Earlier we found the equation

$$N = 8 * 1.16^D$$

where D is the number of days (since the first diagnosis) and N is the total number of students who had the flu. *Story also appears in 2.2 #3 and 5.5*

   (a) Use successive approximations to estimate when the number of infected students reaches 100. Display your guesses in a table.

   (b) Use the LOG DIVIDES FORMULA to solve your equation.
   The formula is reprinted in Section 3.4.

   (c) There are 1,094 students currently living in the dorms. Suppose ultimately 250 students catch the flu. According to your equation, when would that happen? Show how to solve your equation.

   (d) It is not realistic to expect that everyone living in the dorms will catch the flu, but what does the equation say? Set up and solve an equation to find when all 1,094 students would have the flu. (Again, this is not realistic.)

3. Bunnies, bunnies, everywhere. Earlier we found the equation

$$B = 1{,}800 * 1.13^{Y}$$

where B is the number of bunnies and Y is the years since 2007.

Story also appears in 2.2 #2

(a) Make a table showing the number of bunnies in 2007, 2010, 2013, and 2020.

(b) Draw a graph showing how the bunny population grew.

(c) When will the population pass 5,000 bunnies? Guess from the graph. Then refine your answer using successive approximation.

(d) Show how to solve your equation to get the answer.

4. Carbon dioxide is a greenhouse gas in our atmosphere. Increasing carbon dioxide concentrations are related to global climate change. In 1980, the carbon dioxide concentration was 338 ppm (parts per million). At that time it was assumed that carbon dioxide concentrations would increase .42% per year.

Source: Earth Systems Research Laboratory, NOAA

   (a) Name the variables including units.

   (b) Assuming the growth is exponential as predicted, write an equation that describes the increase in Carbon dioxide concentrations.

   (c) The carbon dioxide concentration in 2008 was 385 ppm. Is that count higher or lower than predicted from your equation? Explain.

   (d) Does that mean that carbon dioxide increased at a higher or lower rate than .42%? Explain.

5.2 Exponential growth and decay

1. A signal one unit strong is sent down a fiber optic cable. It decreases by 2% each mile it travels.

   (a) Make a table showing the strength of the signal of the first five miles.

   (b) Name the variables, including units, and write an equation relating them.

   (c) The signal will need a booster when it has fallen to under .75 units. How far along the cable should the booster be placed? Set up and solve an equation.

The problem continues ...

(d) What's the half-life (or should we say half-distance) of a signal? That means, how far can it travel without dropping below 50%? (That won't actual happen because we'd boost the signal.) Again, set up and solve an equation.

(e) Draw a graph illustrating the relationship.

(f) Indicate the points on your graph where you can check your answers to parts (c) and (d).

2. A recent news report stated that cell phone usage is growing exponentially in developing countries. In one small country, 50,000 people owned a cell phone in the year 2000. It was estimated that usage would increase at 1.4% percent per year.

   (a) Name the variables including units.

   (b) Assuming the growth is exponential, write an equation for the function.

   (c) At this rate, how many years would it take for the number of people owning a cell phone to double? That's called the **doubling time**. Show how to set up and solve an equation to find the answer.

   (d) In 2011, about 682,000 people owned a cellphone. Is that count higher or lower than predicted from your equation? Explain.

   (e) Based on the 2011 data, would you say that cell phone usage was growing slower or faster than 1.4%?

3. If you have a heart attack and your heart stops beating, the amount of time it takes paramedics to restart your heart with a defibrillator is critical. According to a medical report, each minute that passes decreases your chance of survival by 10%. Assume that this statement means the decrease is exponential and that the survival rate is 100% if the defibrillator is used immediately. Source: American Red Cross

   (a) Name the variables and write an equation.

   (b) If it takes the paramedics 2 minutes to use the defibrillator, what is your survival rate?

   (c) When does the survival rate drop below 50%? Use successive approximation to estimate to the nearest minute. Display your work in a table.

   (d) Solve your equation.

4. You and two buddies each invite 10 people to "like" your online group. Suppose everyone accepts and then they each invite 10 people. And then everyone accepts and they each invite 10 people. And so on. Of course, there is likely to be substantial overlap, but for the moment pretend that there isn't. *Story also appears in 5.3 #x and 5.5 #x*

   (a) There are 3 friends to start. They each invite 10 in the first round, so a total of 30 new people "like" your online group. How many new people "like" your group in the second round? The third?

   (b) Name the variables and write an equation showing how the number of new people increases in each round.

   (c) Make a table showing the 3 original friends who "like" your group, the numbers you found in part (a), and also the number of new people who "like" your group after four and five rounds.

   (d) What is the *total* number of people who "like" your online group after five rounds. *Hint: add*

   (e) Comment on why our assumption is unrealistic.

5.3 Growth factors

Formulas referenced in the worksheets:

PERCENT CHANGE FORMULA: (updated version)

- If a quantity changes by a percentage corresponding to growth rate r, then the growth factor is
$$g = 1 + r$$
- If the growth factor is g, then the growth rate is
$$r = g - 1$$

GROWTH FACTOR FORMULA

If a quantity is growing (or decaying) exponentially, then the growth (or decay) factor is
$$g = \sqrt[t]{\frac{a}{s}}$$
where s is the starting amount and a is the amount after t time periods.

1. In 1962, my grandfather had savings bonds that matured to \$200. He gave those to my mother to keep for me. These bonds have continued to earn interest at a fixed, guaranteed rate so I have yet to cash them in. The table lists the value at various times since then.

year	1962	1970	1980	1990	2000	2010
Y	0	8	18	28	38	48
B	200.00	318.77	570.87	1,022.34	1,830.85	3,278.77

Story also appears in 1.2 #1 and 4.1 #3

(a) Use the GROWTH FACTOR FORMULA to find the annual growth factor for the time period from 1962 to 1970.

(b) Repeat for 1970 to 1980.

(c) What do you notice? What in the story told you that would happen?

(d) What is the corresponding interest rate?

(e) Write an equation for the value of bonds over time.

(f) Use your equation to check the information for 1990, 2000, and 2010.

The problem continues ...

(g) In what year will the bond be worth over \$5,000? Set up and solve an equation to decide.

(h) Draw a graph using the data in the table, but not your answer to part (e). Include another year that is later than your answer to (e).

(i) Does your answer to (e) agree with your graph? If not, fix what's incorrect.

2. Have you read news stories about archaeological digs where a specimen (like a bone) is found that dates back thousands of years? How do scientists know how old something is? One method uses the radioactive decay of carbon. After an animal dies the carbon-14 in its body very slowly changes to carbon-12. By comparing how much carbon-14 is in the specimen to how much was in the animal when alive, scientist can estimate how long the animal has been dead. Clever, huh? Actually, it's so clever that Willard Libby won the Nobel Prize in Chemistry for it. The key information to know is that the half-life of carbon-14 is about 5,730 years. For this problem, suppose a specimen is found that should have contained 300 milligrams of carbon-14 when alive.

   Source: Wikipedia (Radiocarbon Dating)

   (a) Find the annual "growth" factor.

   (b) Name the variables and write an equation describing the dependence.

   (c) How many milligrams of carbon-14 should remain after 1,000 years? After 10,000 years? After 100,000 years?

   (d) Draw a graph that shows up to 10,000 years.

The problem continues . . .

(e) How many milligrams of carbon-14 should remain after 1 million years? Explain the answer your calculator gives you.

(f) If the specimen is determined to have 100 milligrams of carbon-14, how old is it? That is, approximately how long ago did it die? Display your work in a table.

(g) Solve the equation exactly.

3. For each equation, find the growth rate and state its units. For example, something might "grow 2% per year" while something else might "drop 7% per hour"

   (a) The number of households watching reality television R (in millions) was estimated by the equation
   $$R = 2.5 * 1.072^Y$$
   where Y is the years since 1990. *Story also appears in 5.1 # 6*

   (b) Chlorine is often used to disinfect water in swimming pools, but the concentration of Chlorine C (in ppm) drops as the swimming pool is used for H hours according to the equation
   $$C = 2.5 * .975^H$$
   Story also appears in 3.4 #2

   (c) The number of players of the hit game Draw Something has been growing exponentially according to the equation
   $$P = 2 * 1.57^W$$
   where P is the number of players (in millions) and W is the number of weeks since it caught on. *Story also appears in 5.1 # 8*

4. Find the annual growth factor g and annual growth rate r for each story. Don't forget to include the negative sign for decay rates.

   (a) Donations to the food shelf have increased 35% per year for the past few years.

   $g =$
   $r =$

   (b) People picking up food at the food shelf has increased exponentially too, from 120 per week in 2005 to 630 per week in 2011.

   $g =$
   $r =$

   (c) The crime rate has dropped 3% each year recently.

   $g =$
   $r =$

   (d) The creeping vine taking over my lawn doubles in area each year.

   $g =$
   $r =$

   (e) Attendance at parent volunteer night has done so well it has doubled every 3 years.

   $g =$
   $r =$

   (f) The new stop sign has decreased accidents exponentially, from 40 in 2008 to 17 in 2013.

   $g =$
   $r =$

5.4 Linear vs. exponential models

1. My parents bought the house I grew up in for \$35,000 and sold it 40 years later for \$342,000. True story. (It was before the housing bubble burst.)

   First, assume the value of the house increased exponentially.

   (a) Calculate the annual growth factor.

   (b) In this model, by what percentage did the house value increase each year?

   (c) Write an exponential equation showing how the value of the house increased. Don't forget to name the variables, including units.

   (d) Check that your equation gives the correct sold value.

   Next, assume the value of the house increased linearly instead.

   (e) In this model, by what fixed amount did the house value increase each year? *Hint: calculate the slope.*

   (f) Write a linear equation showing how the value of the house increased.

   (g) Check that your equation gives the correct sold value.

2. The number of manufacturing jobs in the state has been declining for decades. In 1970, there were 1.2 million such jobs in the state but by 2010 there were only .6 million such jobs. Write J for the number of manufacturing jobs (in millions) and Y for the years since 1970.

   First, assume the number of jobs decreased linearly.

   (a) Calculate the slope.

   (b) Write a linear equation showing how the number of jobs declined.

   (c) Check that your equation gives the correct value for 2010.

   Next, assume the number of jobs decreased exponentially instead.

   (d) Calculate the growth factor.

   (e) Write an exponential equation showing how the number of jobs declined.

   (f) Check that your equation gives the correct value for 2010.

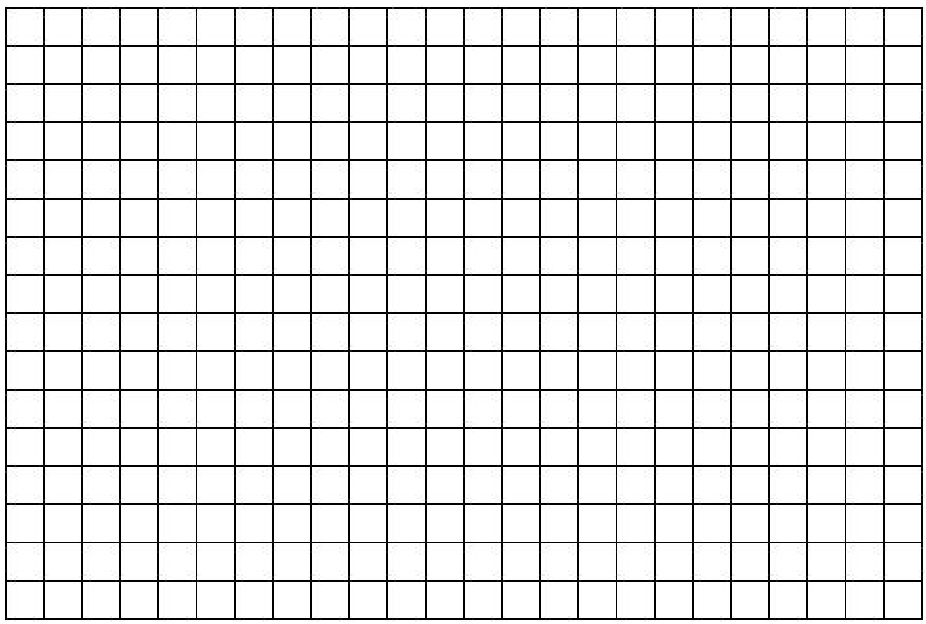
The problem continues . . .

Now, compare the models.

(g) Complete the table of values.

year	1970	1990	2010	2020	2030
Y	0	20	40	50	60
J (if linear)					
J (if exponential)					

(h) Draw a graph showing both models.



(i) Which model has better news for 2030?

3. In December 2010, the popular mobile app game Angry Birds had 50 million downloads. Six months later (May 2011), the game had 200 million downloads. Let D denote the number of downloads of Angry Birds (in millions) and M the months since December 2010.

   (a) Suppose that the number of downloads have been increasing at a *constant rate each month*. What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$).

   (b) Suppose that the number of downloads have been increasing at a *fixed percentage each month*. What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$).

4. Bus fares are up to \$2.25 per ride during rush hour. Two different plans of increasing fares are being debated: 10¢ per year or 2.5% per year.

   (a) Make a table comparing these plans over the next **decade** (ten years).

   (b) Draw a graph showing both options.

   (c) As a city council representative, you want to support the plan that your constituents prefer. If most of your constituents ride the bus, which plan should you support?

   (d) If most of your constituents are members of the same union as the bus drivers (who count on solid earnings from the bus company to keep their jobs), then which plan should you support?

The problem continues ...

(e) Which type of equation is being suggested in each plan? Write the equations. Don't forget to name the variables, including units.

5.5 Logistic and other growth models

1. Corn farmers say that their crop is healthy if it is "knee high by the Fourth of July." An equation that relates the height H (in inches) of the corn crop as it matures is the following, where D is days since May 1:

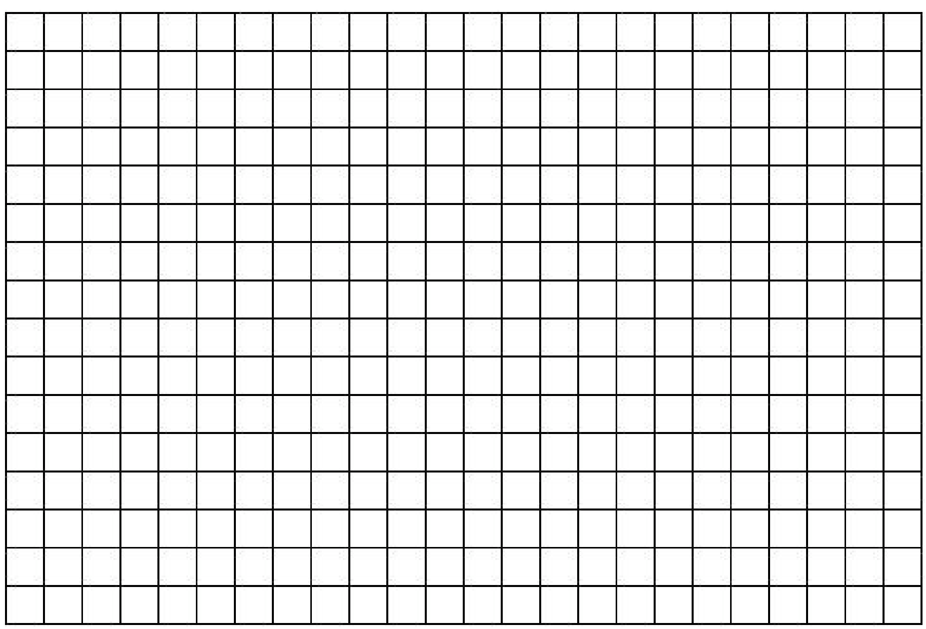
$$H = 106 - 100 * .989^D$$

   (a) According to this equation, how high is corn expected to be on the Fourth of July (day 64)? Is that "knee high"? Let's say that's 18 inches tall.

   (b) These days with stronger corn from cross-breeding and various seed technologies, the rule ought to be modified to "chest high." Let's say that's 52 inches tall. According to this equation, on approximately what date is the corn projected to be that tall? Use successive approximation to answer.

   (c) The particular corn matures in approximately 110 days (by August 19). How tall will it be then?

   (d) Draw a graph of the function.



2. An alternative equation for corn height is

$$H = \frac{200}{1 + 70 * .965^D}$$

   (a) According to this new equation, how high is corn expected to be on the Fourth of July (day 64)? Is that "knee high" (18 inches tall)?

   (b) According to this new equation, on approximately what date is the corn projected to be "chest high" (52 inches tall)? Use successive approximation to answer.

   (c) The particular corn matures in approximately 110 days (by August 19). How tall will it be then?

   (d) Add the graph of this function to your graph of the original function.

3. Following the 2011 Japanese earthquake and tsunami there was concern of radiation leaking from nuclear power plants. A monitoring station near the Fukushima Daiichi recorded radiation according to the following equation, where R is radiation measured in milliSieverts (mSv for short) and T is time in hours:

$$R = \frac{.162}{1 + 3{,}319 * .3127^T}$$

   (a) How much radiation was detected at the start? After 24 hours? 48 hours?

   (b) At how many mSv did radiation levels level off?

   (c) A typical person is exposed to around 2.4 mSv in a year. What is that normal level of radiation measured in mSv/day? (Use 365 days/year.)

   (d) At its highest value (where it leveled off), did the detected radiation exceed normal levels? If so, by how many times normal? *That means divide your answer to (b) by your answer to (c).*

4. Jason works at a costume shop selling Halloween costumes. The shop is busiest during the fall before Halloween. An equation that describes the number of daily visitors V the shop receives D days from August 31 is the following:

$$V = \frac{430}{1 + 701 * .81^D}$$

an alternative equation is

$$V = 700 - 690 * .985^D$$

   (a) Make a table showing what each equation predicts for August 31, September 15, September 30, October 15, October 25, October 28, October 31.

   Hint: those days are numbered 0, 15, 30, 45, 55, 58, and 61.

   (b) Graph both functions on the same set of axes.

   (c) Which function is more consistent with a major advertising campaign that aired starting the first week of September? Explain.