* Solutions *

5.4 Linear vs. exponential models – Practice exercises

1. My parents bought the house I grew up in for \$35,000 and sold it 40 years later for \$342,000. True story. (It was before the housing bubble burst.)

First, assume the value of the house increased exponentially.

(a) Calculate the annual growth factor.

Try MATH 5 on a graphing calculator

year	house
0	35,000 = s
t = 40	342,000 = a

$$g = \sqrt[t]{\frac{a}{s}} = \sqrt[40]{\frac{342{,}000}{35{,}000}} = 40 \sqrt[x]{\ } (342{,}000 \div 35{,}000) =$$

$$= 1.058641589$$

(b) In this model, by what percentage did the house value increase each year?

$g \approx 1.059 \quad r = g - 1 = 1.059 - 1 = .059 \xrightarrow{\times 100} \boxed{\approx 5.9\%}$

(c) Write an exponential equation showing how the value of the house increased. Don't forget to name the variables, including units.

Uses exponential template: dep = start * g^{indep}

H = house value (\$) ~ dep

Y = year (years since bought house)

$H = 35000 * 1.058641589^{Y}$

if used 1.059 get 346,662

(d) Check that your equation gives the correct sold value.

$35000 \times 1.058641589 \wedge \underline{40} = 341{,}999$ ✓

Next, assume the value of the house increased linearly instead.

(e) In this model, by what fixed amount did the house value increase each year? *Hint: calculate the slope.*

$$\text{slope} = \frac{\text{diff dep}}{\text{diff indep}} = \frac{\$342{,}000 - 35{,}000}{40 \text{ years}} = \$7{,}675/\text{year}$$

$(342000 - 35000) \div 40 =$

(f) Write a linear equation showing how the value of the house increased.

Uses linear template: dep = start + slope * indep

$H = 35000 + 7675Y$

(g) Check that your equation gives the correct sold value.

$35000 + 7675 \times \underline{40} = 342{,}000$ ✓

2. The number of manufacturing jobs in the state has been declining for decades. In 1970, there were 1.2 million such jobs in the state but by 2010 there were only .6 million such jobs. Write J for the number of manufacturing jobs (in millions) and Y for the years since 1970.

First, assume the number of jobs decreased linearly.

(a) Calculate the slope.

Y year	J jobs
(1970) 0	1.2
(2010) 40	.6

$$\begin{array}{r} 2010 \\ -1970 \\ \hline 40 \end{array}$$

$$\text{slope} = \frac{.6-1.2}{2010-1970} = (.6-1.2) \div (2010-1970) =$$

$$= -.015 \text{ million jobs/year}$$

(b) Write a linear equation showing how the number of jobs declined.

$$J = 1.2 - .015Y$$

(c) Check that your equation gives the correct value for 2010.

$1.2 - .015 \times 40 = .6$ ✓

Next, assume the number of jobs decreased exponentially instead.

(d) Calculate the growth factor.

note: smaller # on top since function is decreasing.

$$g = \sqrt[t]{\frac{a}{p}} = \sqrt[40]{\frac{.6}{1.2}} = 40 \times \sqrt{\ } \ (.6 \div 1.2) = .982820598$$

<1 because decreasing

(e) Write an exponential equation showing how the number of jobs declined.

$$J = 1.2 * .982820598^{Y}$$

(f) Check that your equation gives the correct value for 2010.

$1.2 \times .982820598 \wedge \underline{40} = .599999... \approx .6$ ✓

If rounded g with only get approx answer here.

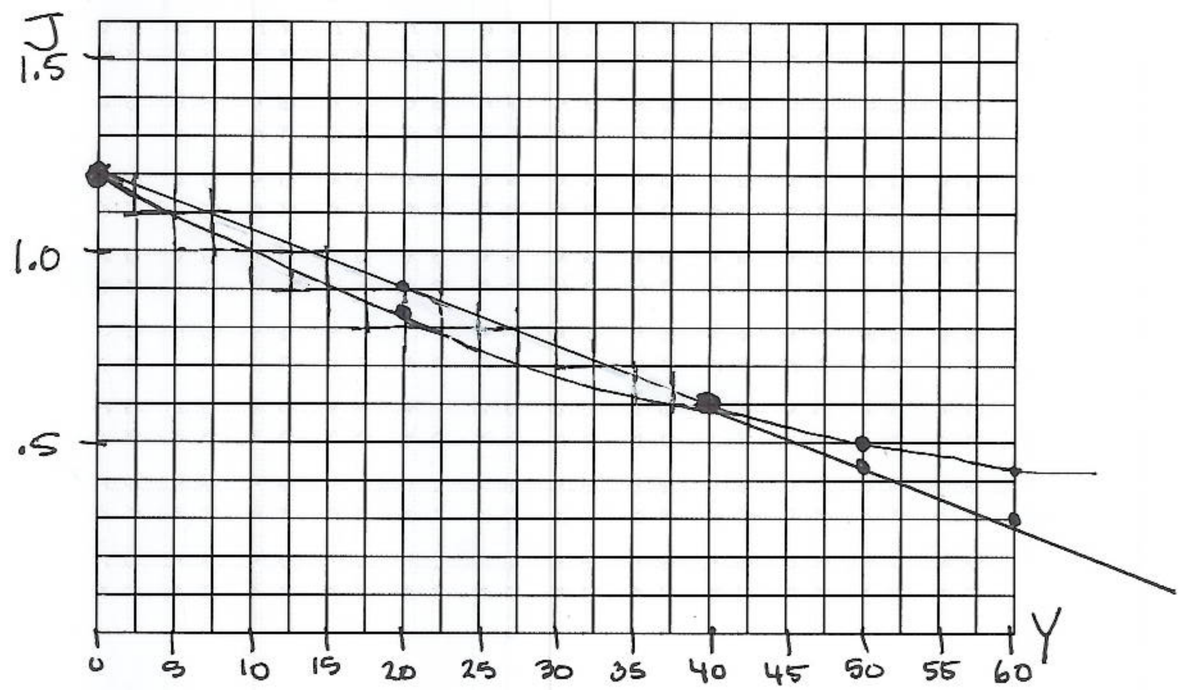
The problem continues ...

Now, compare the models.

(g) Complete the table of values.

year	1970	1990	2010	2020	2030
Y	0	20	40	50	60
J (if linear) $J=1.2-.015Y$	1.2	.9	.6	.45	.3
J (if exponential) $J=1.2 \times .982820598^Y$	1.2	.848... $\approx .85$	.6	.504... $\approx .50$	.424... $\approx .42$

(h) Draw a graph showing both models.



(i) Which model has better news for 2030?

.42 is larger than .3 $\Rightarrow$ more jobs

$\Rightarrow$ exponential has better news for 2030

3. In December 2010, a popular mobile app game featuring animated birds launched from slingshots had 50 million downloads. Six months later (May 2011), the game had 200 million downloads. Let D denote the number of downloads of the game (in millions) and M the months since December 2010.

   (a) Suppose that the number of downloads have been increasing at a *constant rate each month.* What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$).

→ linear

	M month	D downloads
Dec 2010	0	50 mil
May 2011	6	200 mil

slope $= \frac{200-50}{6-0} = (200-50) \div 6 = 25$ mil/yr

$$\boxed{D = 50 + 25Y}$$

Nov 2011 $\Rightarrow M = 11 \rightarrow D = 50 + 25 \times \underline{11} =$ 325 million

   (b) Suppose that the number of downloads have been increasing at a *fixed percentage each month.* What type of equation is suggested here? Write that equation and use it to estimate the number of downloads in November 2011 (when $M = 11$).

→ exponential

$g = \sqrt[t]{\frac{a}{s}} = \sqrt[6]{\frac{200}{50}} = 6 \times\sqrt{\ } (200 \div 50) = 1.25992105$

$$\boxed{D = 50 * 1.25992105^{Y}}$$

Nov 2011 $\Rightarrow M = 11 \rightarrow D = 50 \times 1.25992105 \wedge \underline{11} \approx$ 635 million

B

4. Bus fares are up to \$2.25 per ride during rush hour. Two different plans of increasing fares are being debated: 10¢ per year or 2.5% per year.

   Y

   (a) Make a table comparing these plans over the next **decade** (ten years).

linear

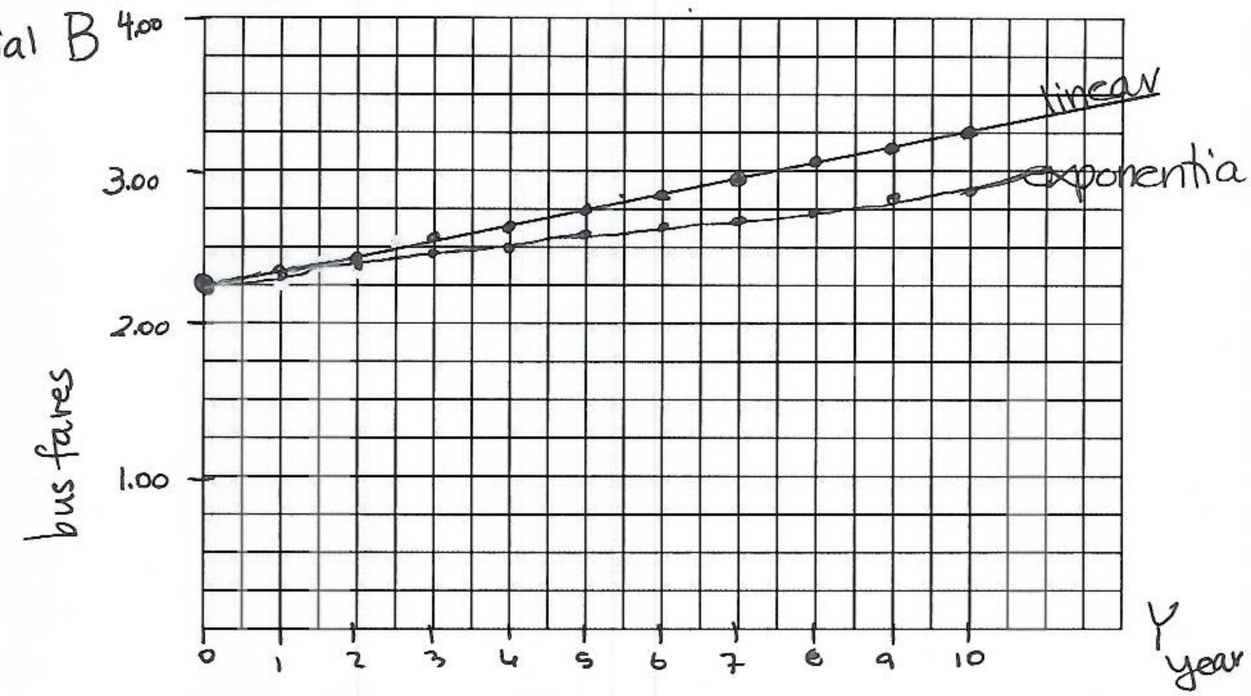
$B = 2.25 + .10Y$

$B = 2.25 \times 1.025^Y$

exponential

Y	0	1	2	3	4	5	6	7	8	9	10
B @ 10¢/yr	2.25	2.35	2.45	2.55	2.65	2.75	2.85	2.95	3.05	3.15	3.25
B @ 2.5%/yr	2.25	2.31	2.36	2.42	2.48	2.54	2.61	2.67	2.74	2.81	2.88

   (b) Draw a graph showing both options.



   (c) As a city council representative, you want to support the plan that your constituents prefer. If most of your constituents ride the bus, which plan should you support?

   exponential

   (d) If most of your constituents are members of the same union as the bus drivers (who count on solid earnings from the bus company to keep their jobs), then which plan should you support?

   linear

The problem continues ...

(e) Which type of equation is being suggested in each plan? Write the equations. Don't forget to name the variables, including units.

10¢ year linear: $B = 2.25 + .10Y$

2.5%/year exponential $B = 2.25 * 1.025^Y$

Where B = bus fare ($) ~ dep
Y = year (years from now) ~ indep