✱ Solutions ✱

5.5 Logistic and other growth models – Practice exercises

1. Corn farmers say that their crop is healthy if it is "knee high by the Fourth of July." An equation that relates the height H (in inches) of the corn crop D is days since May 1 is

$$H = 106 - 100 * .989^D$$

(a) According to this equation, how high is corn expected to be on the Fourth of July (day 64)? Is that "knee high"? Let's say that's 18 inches tall.

$H=106-100*.989\wedge \underset{D}{\underline{64}} = 56.732\ldots$ $\boxed{\approx 57 \text{ inches}}$

That is much taller than "knee high" (that's 4'9")

(b) These days with stronger corn from cross-breeding and various seed technologies, the rule ought to be modified to "chest high." Let's say that's 52 inches tall. According to this equation, on approximately what date is the corn projected to be that tall? Use successive approximation to answer.

may 1 = 0
June 1 = 31 → 56 = June 26
July 1 = 61 → July 4 = 64
Aug 1 = 92 → Aug 19 = 110

D	64	50	55	56
H	57	48	51.6	52.1
vs 52	high	low	low	high

$\boxed{\approx 56 \text{ days}}$ → June 26

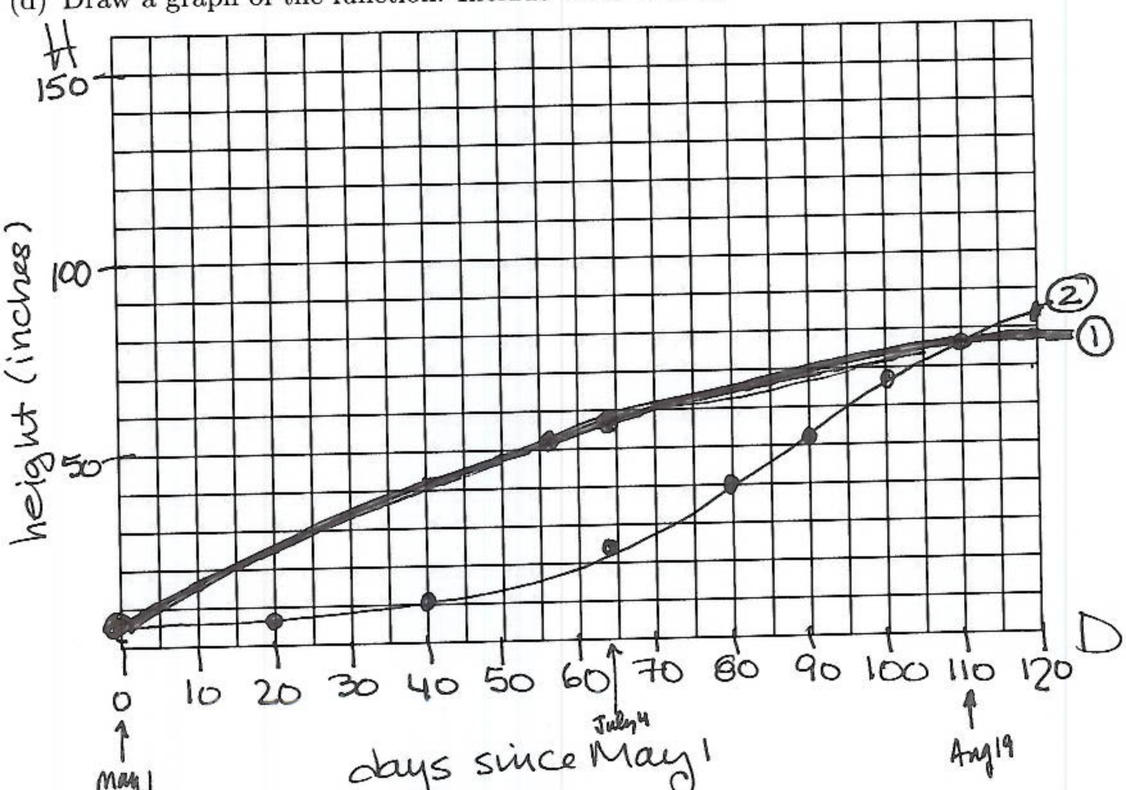
(c) The particular corn matures in approximately 110 days (by August 19). How tall will it be then?

$H=106-100*.989\wedge \underline{110} = 76.379\ldots$ $\boxed{\approx 76 \text{ inches}}$ (thats 6'4")

(d) Draw a graph of the function. Include when $D = 0$.

$H=106-100*.989\wedge \underline{0}=6$

date	D	H
may 1	0	6
(b) June 26	56	52
(a) July 4	64	57
(c) Aug 19	110	76



2. An alternative equation for corn height is

$$H = \frac{200}{1 + 70 * .965^D}$$

(a) According to this new equation, how high is corn expected to be on the Fourth of July (day 64)? Is that "knee high" (18 inches tall)?

D

$H = 200 \div (1 + 70 \times .965 \wedge \underline{64}) = 24.513\ldots$ $\boxed{\approx 24 \text{ inches}}$

(2 feet)

that's a little taller than "knee high"

(b) According to this new equation, on approximately what date is the corn projected to be "chest high" (52 inches tall)? Use successive approximation to answer.

$200 \div (1 + 70 \times .965 \wedge \underline{100}) =$

D	64	100	80	90	89
H	24	67	40	52.1	50.7
VS 52	Low	high	low	yes!	

$\boxed{\approx 90 \text{ days}}$

July 30

recall Aug 1 = 92
⇒ July 31 = 91 ⇒ July 30 = 90 :)

(c) The particular corn matures in approximately 110 days (by August 19). How tall will it be then?

$H = 200 \div (1 + 70 \times .965 \wedge \underline{110}) = 83.672\ldots$ $\boxed{\approx 84 \text{ inches}}$

(7 feet)

HARD TO GUESS
SHAPE OF GRAPH
W/O MORE PTS

$200 \div (1 + 70 \times .965 \wedge \underline{0}) = 2.81\ldots$

(d) Add the graph of this function to your graph of the original equation.

date	May 1	July 4	July 30	Aug 19				
D	0	64	90	110	20	40	80	100
H	3	24	52	84	6	11	40	67

3. Following the 2011 Japanese earthquake and tsunami there was concern of radiation leaking from nuclear power plants. Suppose that a monitoring station recorded radiation approximated by the equation

$$R = \frac{.162}{1 + 3319 * .3127^T}$$

where R is radiation measured in milliSieverts (mSv) and T is time in hours.

(a) How much radiation was detected at the start? After 24 hours? 48 hours?

$T=0 \Rightarrow R = .162 \div (1+3319 \times .3127 \wedge \underline{0}) = .00004879..$ mSv
$T=24 \Rightarrow R = .162 \div (1+3319 \times .3127 \wedge \underline{24}) = .16199999..$ mSV
$T=48 \Rightarrow R = .162 \div (1+3319 \times .3127 \wedge \underline{48}) = .162$ mSV

(b) Approximately when did the radiation level off? (Display your work in a table.) What was the largest amount of radiation at that time? $\boxed{\approx .162 \text{ mSv}}$

T	10	20	15	14	12	13
R	.157...	.16199...	.16198...	.16195..	.1615...	.1618...

$\boxed{\approx 15 \text{ days}}$

(c) The normal level of radiation that a person is exposed to around 2.4 mSv during an entire year. What is that normal level of radiation measured in mSv/day? Use 1 year = 365 days. Source: Wikipedia (Sievert)

$$\frac{2.4 \text{ mSV}}{1 \cancel{\text{year}}} * \frac{1 \cancel{\text{year}}}{365 \text{ days}} = 2.4 \div 365$$
$$= \boxed{.006575.../\text{day}}$$

(d) At its largest amount (where it leveled off), did the radiation the exceed normal daily levels? If so, by how many times normal?

That means divide your answer to (b) by your answer to (c).

Yes $\begin{cases} \text{normal} \approx .066 \\ \text{leveled @} \approx .162 \end{cases}$

$.162 \div .006575. = 24.6\ldots$ $\boxed{\approx 25 \text{ times normal level}}$

4. Jason works at a costume shop selling Halloween costumes. The shop is busiest during the fall before Halloween. An equation that describes the number of daily visitors V the shop receives D days from August 31 is the following:

Calculate as

$$EQ1 \longrightarrow \quad V = \frac{430}{1 + 701 * .81^D} \qquad 430 \div (1 + 701 \times .81 \wedge \_\_) =$$

An alternative equation is

$$EQ2 \longrightarrow \quad V = 700 - 690 * .985^D \qquad 700 - 690 \times .985 \wedge \_\_ =$$

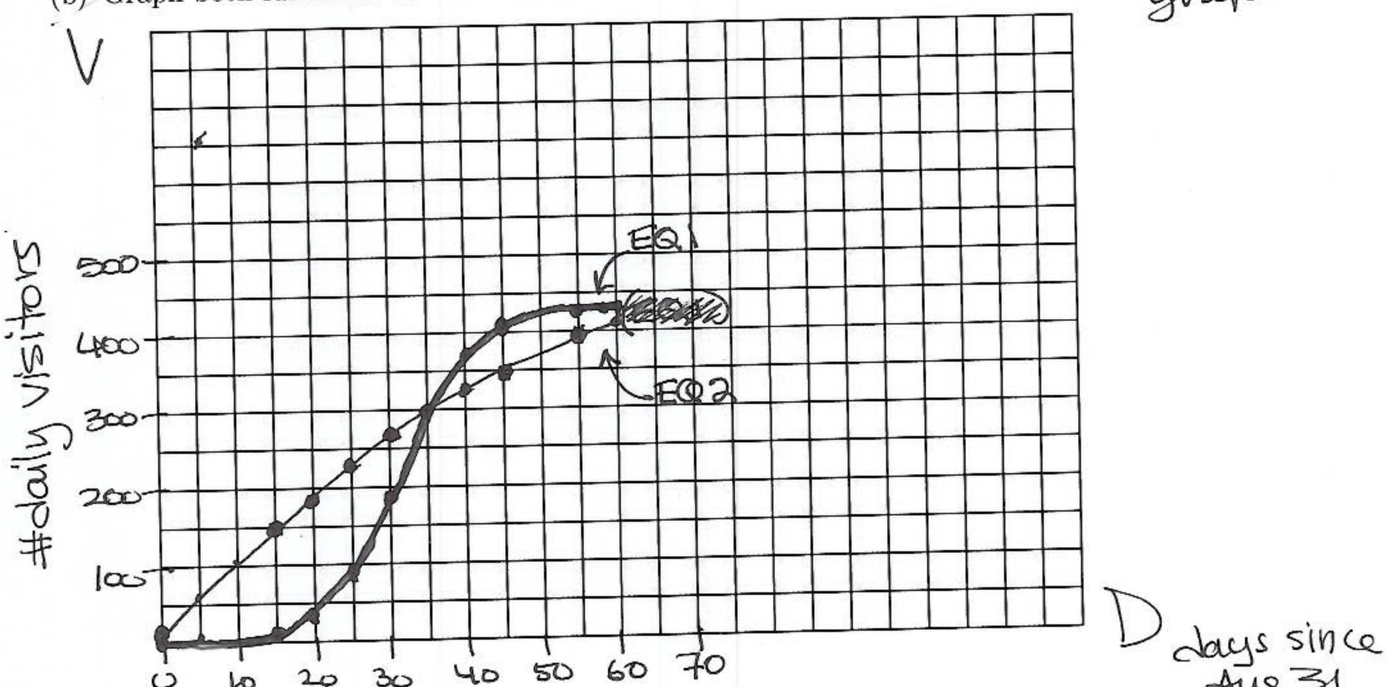
(a) Make a table showing what each equation predicts for August 31, September 15, September 30, October 15, October 25, October 28, and October 31.
Hint: those days are numbered 0, 15, 30, 45, 55, 58, and 61.

D	0	15	30	45	55	58	61
V(EQ 1)	1	14	190	408	427	428	429
V(EQ2)	10	150	262	350	400	413	426

D	20	25	35	40
V(EQ1)	38	93	299	373
V(EQ2)	190	227	293	323

to help see graph

(b) Graph both functions on the same set of axes.



(c) Which function is more consistent with a major advertising campaign that aired starting the first week of September? Explain.