SOLUTIONS

Practice Final Exam A

Relax. You have done problems like these before. Even if these problems look a bit different, just do what you can. If you're not sure of something, please ask! You may use your calculator. Please show all of your work and write down as many steps as you can. Don't spend too much time on any one problem. Do well. And remember, ask me if you're not sure about something.

As you work, make a "don't forget" list of any information you need to look up or ask about.

Caution: These review exercises do not include every possible problem you might be asked on a final exam. For example, there are no problems here from Sections 1.5, 2.5, 3.5, 4.5, and 5.5 so be sure to ask your instructor which of those sections are going to be on your final exam.

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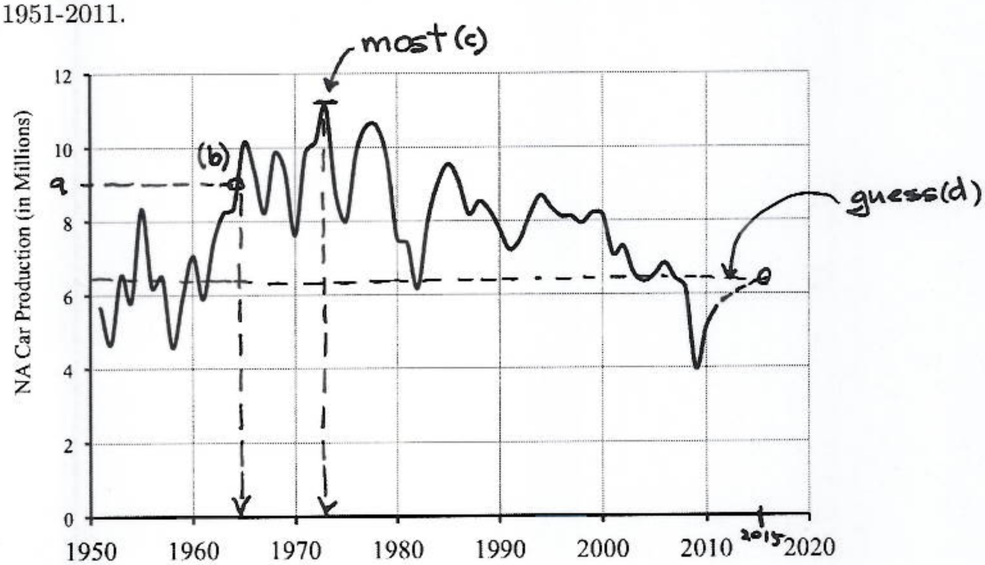
1. The graph shows the number of cars produced in North America (in millions/year) during 1951-2011.



Source: Wards Automotive Yearbooks.

(a) Identify the variables, including units and dependence.

Y = years ~ indep

C = North American car production $\left(\frac{\text{millions cars}}{\text{year}}\right)$ ~ dep

The problem continues ...

(b) Approximately when did North American car production first pass 9 million/year? *Indicate the corresponding point on the graph.*

$\boxed{\approx 1964}$

(c) In which year were the most cars produced? *Again, indicate the point.*

$\boxed{\approx 1973}$

(d) Best as you can tell from your graph, what might be a reasonable estimate of North American car production in 2015? Just guess to the nearest million/year.

$\boxed{\approx 6 \text{ million cars}}$

(e) Calculate the rate of change from 1958 when production was 4.57 million cars/year to 1971 when it was 9.83 cars million/year. What does that tell you about North American car production during 1958-1971?

$$\text{rate of change} = \frac{9.83-4.57 \text{ mil cars}}{1971-1958 \text{ years}} = (9.83-4.57) \div (1971-1958) = .4046\ldots \approx .4$$

$\boxed{\text{production increased} \approx .4 \text{ million cars/year}}$

(f) Now calculate the rate of change of from 1984 when production was 9.03 million cars/year to 2006 when it was 6.84 million cars/year. What does that tell you about North American car production during 1984-2006?

$$\text{rate of change} = \frac{6.84-9.03 \text{ mil cars}}{2006-1984 \text{ years}} = (6.84-9.03) \div (2006-1984) = -.0995\ldots \approx -.1$$

$\boxed{\text{production decreased} \approx .1 \text{ million cars/year}}$

2. Sarah and Koal are bringing a large basket of stuffed animals to the crisis nursery as gifts for the children. They estimate it will cost \$$T$ for S stuffed animals where

$$T = 39.99 + 6.95S$$

(a) Make a table showing the cost if Sarah and Koal include 10, 20, or 40 stuffed animals.

S	10	20	40
T	109.49	178.99	317.99

$\nwarrow$ $39.99 + 6.95 \times \underline{10} =$

(b) Included in the cost is a new toy box for the animals. What does it cost?

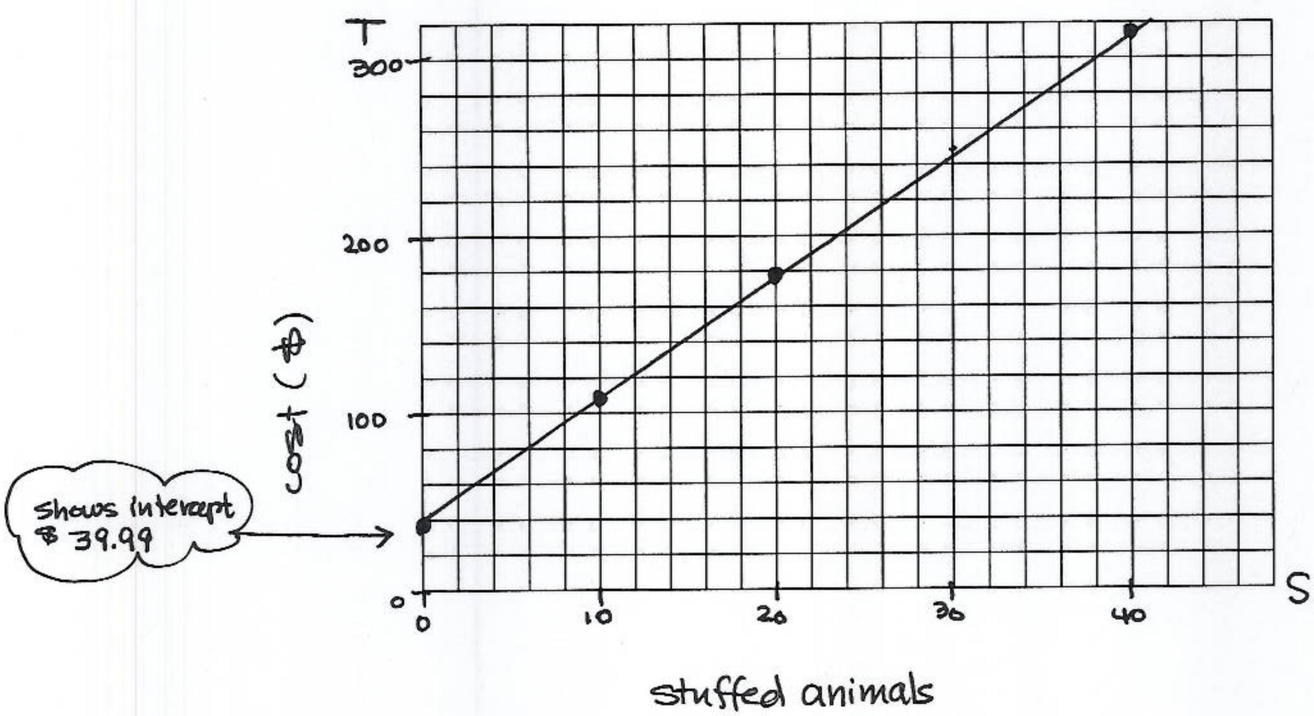
$S = 0 \Rightarrow T = 39.99 + 6.95 \times \underline{0} = \boxed{\$39.99}$

(c) What does the 6.95 represent and what are its units?

the cost per stuffed animal, \$

linear equation
⇒ graph is LINE

(d) Draw a detailed graph, starting at 0.



The problem continues ...

(e) If Sarah and Koal spent \$262.39, how many stuffed animals were in the toy box they gave to the crisis nursery? Show how to set up and solve an equation to answer the question.

$$\cancel{39.99} + 6.95S = 262.39$$
$$\cancel{-39.99} \qquad\qquad -39.99$$

$$\frac{\cancel{6.95}S}{\cancel{6.95}} = \frac{222.4}{6.95}$$

$$S = \boxed{\text{32 stuffed animals}}$$

(f) Solve the inequality

$$200 \le 39.99 + 6.95S \le 300$$

What does the answer mean in terms of the story?

$$200 \le \cancel{39.99} + 6.95S \le 300$$
$$-39.99 \qquad \cancel{-39.99} \qquad\qquad -39.99$$

$$\frac{160.01}{6.95} \le \frac{\cancel{6.95}S}{\cancel{6.95}} \le \frac{260.01}{6.95}$$

$$23.02\ldots \le S \le 37.41\ldots$$

$$\boxed{\text{To stay in a bouget of \$200–\$300, Sarah \& Koal can include 24–37 stuffed animals.}}$$

3. My favorite little black dress is machine washable. Unfortunately each time I wash it the color fades a little. The intensity of black color remaining, B, is a function of the number of times I have washed the dress, W, according to the equation

$$B = 100 * .985^W$$

(a) It will still look new as long as the intensity stays above 90%. Set up and solve an equation to figure out how many times I can wash the dress and keep it looking new. Then check some other way.

$$\frac{\cancel{100} * .985^W}{\cancel{100}} = \frac{90}{100}$$

need to solve for exponent ⇒ use the Log-Divides Formula

$$.985^W = .9 \rightarrow W = \frac{\log(.9)}{\log(.985)} = \log(.9) \div \log(.985) = 6.971\ldots \Rightarrow \boxed{\text{can wash 6 times}}$$

check: 100 x .985^6 = 91.3%

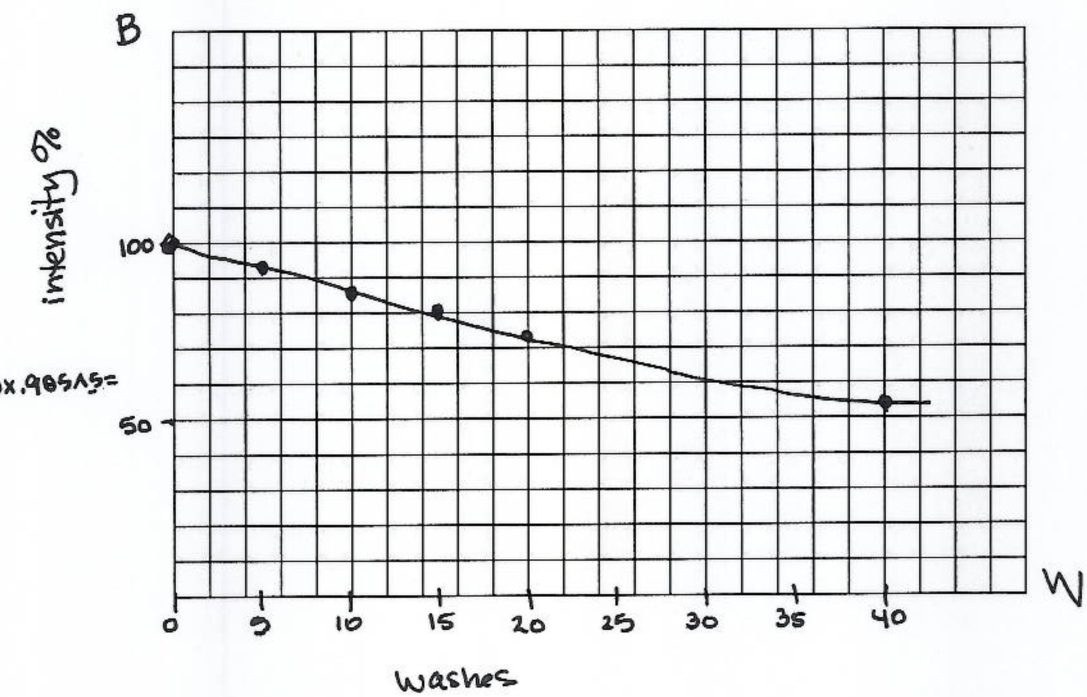
(b) By the time only 75% of the color remains, the dress will look too faded to wear formally. How many washes before then? Find the answer to the nearest number of washes by any method you prefer.

$$\frac{\cancel{100} * .985^W}{\cancel{100}} = \frac{75}{100}$$

would also be okay to use succ. approx →

$$.985^W = .75 \rightarrow W = \frac{\log(.75)}{\log(.985)} = \log(.75) \div \log(.985) = 19.03\ldots \Rightarrow \boxed{\text{after 19 washes, it will look too faded to wear.}}$$

(c) Draw a graph showing how the color of my favorite little black dress fades.



100x.985^5=

W	B
0	100
5	92.7
10	85.9
15	79.7
20	73.9
40	54.6

picked values

4. Brock is working as the equipment manager at a local gym. They need to replace several weight machines. One option will cost \$475 per month to rent the machines plus a delivery/removal fee of \$300. The other option is to buy the machines for \$23,600 and pay \$92/month for a service contract.

   (a) What should Brock recommend if they plan to have the machines for 3 years? $3\text{yrs} * \frac{12\text{mo}}{\text{yr}} = 36\text{ months}$

   RENT: $475 \times 36 + 300 = \$17,400$

   BUY: $23,600 + 92 \times 36 = \$26,912$

   Brock should just rent the machines.

   (b) Name the variables and write an equation for each option.

   W = total cost of weight machines (\$) ~ dep

   M = time used (months) ~ indep

   $$\boxed{\begin{array}{l}\text{RENT: } W = 300 + 475M \\ \text{BUY: } W = 23,600 + 92M\end{array}}$$

   (c) Set up and solve a system of equations to determine when the options cost the same.

   $$\cancel{300} + 475M = 23,600 + 92M$$
   $$-\cancel{300} \qquad\qquad -300$$
   $$475M = 23,300 + \cancel{92M}$$
   $$-92M \qquad\qquad -\cancel{92M}$$
   $$\frac{\cancel{383}M}{\cancel{383}} = \frac{23,300}{383}$$
   $$M = 60.83\ldots \text{ months} * \frac{1\text{yr}}{12\text{mo}} = 60.83 \div 12 = 5.06\ldots$$

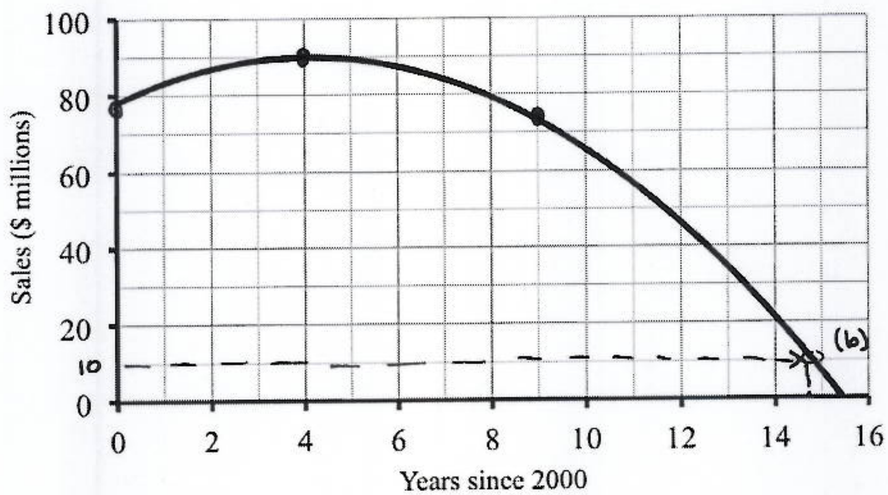
   They cost the same if they use the machines for ≈ 5 years.

   (d) What does the answer tell Brock?

   If they plan to use the machines for more than 5 years, Brock should buy the machines. For less time, it would be cheaper to rent.

5. Dwight's company was doing great business in 2000, but a few years later sales began to drop, and have only gotten worse. Their sales \$$S$ in millions Y years from 2000 is given by the following equation

$$S = 78.1 + 5.75Y - 0.7Y^2$$



(a) According to this equation, what were the company's sales in 2000, 2004, 2009? *You may confirm your answers with the graph, but use the equation to calculate.*

year	2000	2004	2009
Y	0	4	9
S	78.1	89.9	73.1

$78.1 + 5.75 \times 9 - .7 \times 9^2 =$

(b) The company decided to declare bankruptcy when sales fell below \$10 million. In what year was that? Find the answer to the nearest year, showing work to justify your answer. Also, indicate the point on the graph where you can check. *You may use successive approximations or the appropriate formula.*

graph $\Rightarrow Y \approx 15 \Rightarrow$ $\boxed{\text{In 2015}}$

Y	S
14	21.4
15	6.8

rounded up to be sure below \$10 mil

note: could use Quadratic Formula to solve

$78.1 + 5.75Y - .7Y^2 = 10$

If you want to do that for practice, note that

$a = -.7$, $b = 5.75$, $C = 68.1$

did -10 first

$Y = 4.107 \pm 10.011$

$= -5.904$ or $14.118 \Rightarrow 15$ ✓

6. Infants are regularly checked to make sure they are growing accordingly. The World Health Organization publishes growth charts to evaluate infant weight W in kilograms at a given age M in months since birth (for up to three years). An equation that describes an average infant boy is the following:

$$W = 15 - 11.5 * .932^M$$

(a) According to this equation, what is the average infant boy weight at birth, 4 months, a year, and 2 years? → $2\text{yr} * \frac{12\text{mo}}{1\text{yr}} = 24$ months

↳ 12 months

M	0	4	12	24
W	3.5	6.32	10.1	12.9

← 15−11.5 × .932 ^ 24 =

(b) Convert your answer for 4 months to pounds and ounces using

$$1 \text{ kilogram} \approx 2.2 \text{ pounds} \quad \text{and} \quad 1 \text{ pound} = 16 \text{ ounces}$$

Hint: first convert to pounds. Then convert just the decimal part to ounces.

$6.32 \cancel{\text{kg}} * \frac{2.2 \text{ pounds}}{1 \cancel{\text{kg}}} = 6.32 \times 2.2 = 13.904\ldots$ pounds

$= \boxed{13 \text{ pounds}, 14 \text{ ounces}}$

ANS−13 = →

$.904\ldots \cancel{\text{pounds}} * \frac{16 \text{ ounces}}{\cancel{\text{pound}}} = .904.. \times 16 = 14.46\ldots$ ounces

≈ 14 ounces

7. Gail ~~calculated that~~ number of pieces of fudge F she can cut from a square that's D inches ~~on~~ each side is given ~~by~~ the formula

$$F = 1.5625D^2$$

(a) Make a table showing the number of pieces of fudge from a square with edge: 5 inches, 10 inches, and 2 feet. Include the value for a 0 inch square too.

D	0	5	10	24
F	0	39.06	156.25	900

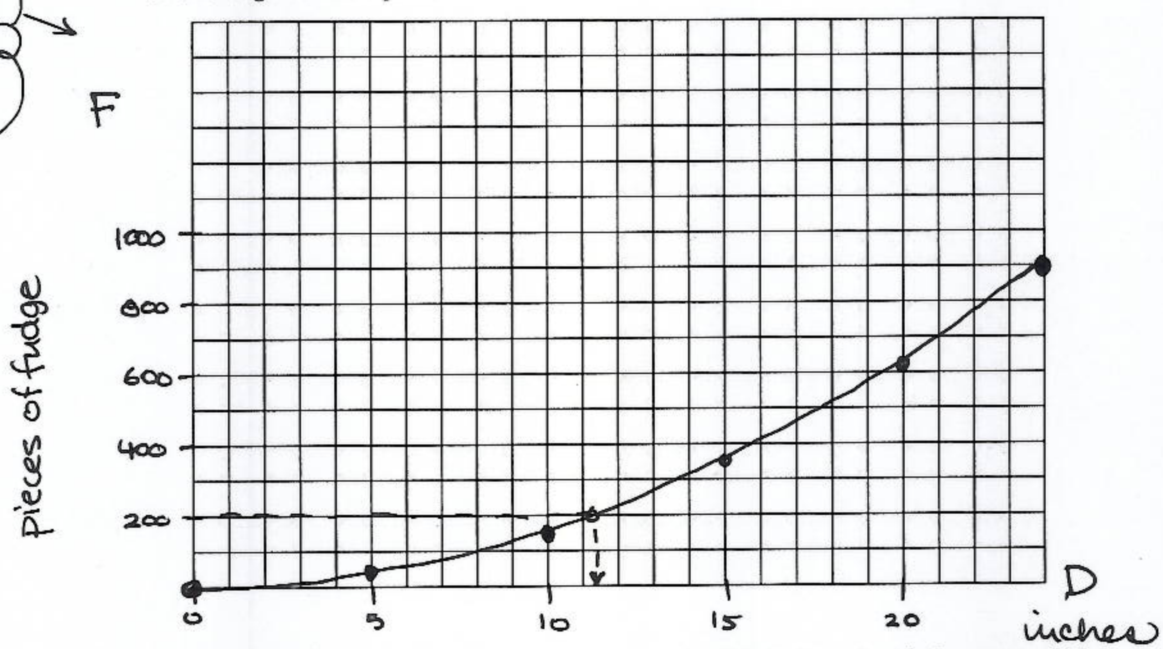
$2\,\cancel{ft} \times \frac{12\,in}{1\,\cancel{ft}} = 24\,in$

$1.5625 \times 24 \wedge 2 =$

calculated two more points to help show graph:

D	15	20
F	351.6	625

(b) Draw a graph showing how the number of pieces of fudge depends on the length of the edge of the square.



(c) According to your graph, what size square should Gail make if she wants 200 pieces of fudge?

F=200

$\boxed{\approx 11\text{-}12 \text{ inches}}$

(d) Now set up and solve an equation to find the answer to the nearest one decimal place.

$$\frac{1.5625D^2}{1.5625} = \frac{200}{1.5625}$$

$$D^2 = 128$$

$$D = \sqrt{128} = \sqrt{\ }128 = 11.31\ldots \boxed{\approx 11.3 \text{ inches}}$$

8. In 2000 there were an estimated 20,851,820 Texans. The population of Texas has grown around 1.89% per year since then.

source: United States Census Bureau

(a) Name the variables and write an equation relating them.

P = population of Texas (people) ~ dep

Y = year (years since 2000) ~ indep

% increase
⇒ EXPONENTIAL EQUATION

$r = 1.89\% \underset{\div 100\%}{=} .0189 \Rightarrow g = 1 + r = 1 + .0189 = 1.0189$

$$\boxed{P = 20{,}851{,}820 * 1.0189^{Y}}$$

fits exponential equation template:
$\text{dep} = \text{start} * g^{\text{indep}}$

(b) According to your equation, what was the population of Texas in 2010?

$Y = \begin{array}{r} 2010 \\ -2000 \\ \hline 10 \end{array}$ $\quad P = 20{,}851{,}820 \times 1.0189 \wedge \underline{10} = \boxed{25{,}145{,}460 \text{ people}}$

(c) The U.S. Census Bureau counted 25,145,561 Texans in 2010. Does that mean the actual growth rate was slightly more or slightly less than 1.89%? Explain.

more than 25,145,460

⇒ grew slightly faster than expected

⇒ $\boxed{\text{growth rate was slightly higher than } 1.89\%}$

9. Ericson has been lifting weight for the past 12 weeks. He has increased his curl weight by about 1.5 pounds per week and is up to 30 pounds.

(a) What weight could Ericson curl 12 weeks ago?

$30 - 1.5 \times 12 = \boxed{12 \text{ pounds}}$

now　　increase　　was originally

(b) Name the variables and write a linear equation relating them.

constant rate of 1.5 pounds/week ⟹ LINEAR (which they said ☺)

C = Ericson's curl weight (pounds) ~ dep

W = time (weeks) ~ indep

$$\boxed{C = 12 + 1.5W}$$

fits linear equation template:
dep = start + slope * indep

Check: $12 + 1.5 \times \underline{12} = 30$ ✓

(c) At this rate when will Ericson be able to curl his goal of at least 45 pounds? Set up and solve an inequality.

$$\cancel{12} + 1.5W \geq 45$$

$$-\cancel{12} \qquad -12$$

$$\frac{\cancel{1.5}W}{\cancel{1.5}} \geq \frac{33}{1.5}$$

$$W \geq 22 \text{ weeks}$$

$- \underline{12 \text{ weeks already}}$

10 more weeks

He'll be up to 45 pounds in another 10 weeks.

10. In the United States in 1970, the average person ate 2,169 calories per day. By 2008 that number had increased to 2,674 calories per day. Let C be the amount a typical person eats (in calories per day) and Y the year, measured in years since 1970. Compare what the linear and exponential models project for the years 2015 and 2030. Include both equations and a graph showing both functions on the same axes.

source: United States Department of Agriculture

$2008 - 1970 = 38$

year	Y	C
1970	0	2169 start
2008	38	2,674
2015	45	LINEAR: 2,767 EXPL: 2,779
2030	60	LINEAR: 2,966 EXPL: 3,108

LINEAR:

slope = rate of change = $\dfrac{2674-2169\text{ cal}}{38\text{ yrs}} = (2674-2169)\div 38 = \approx 13.29$ cal/yr

equation: $\boxed{C = 2169 + 13.29Y}$

check: $2169 + 13.29 \times \underline{38} = 2{,}674.02$ ✓

In 2015, $Y=45 \rightarrow C = 2169 + 13.29 \times \underline{45} = 2767.05$ cal

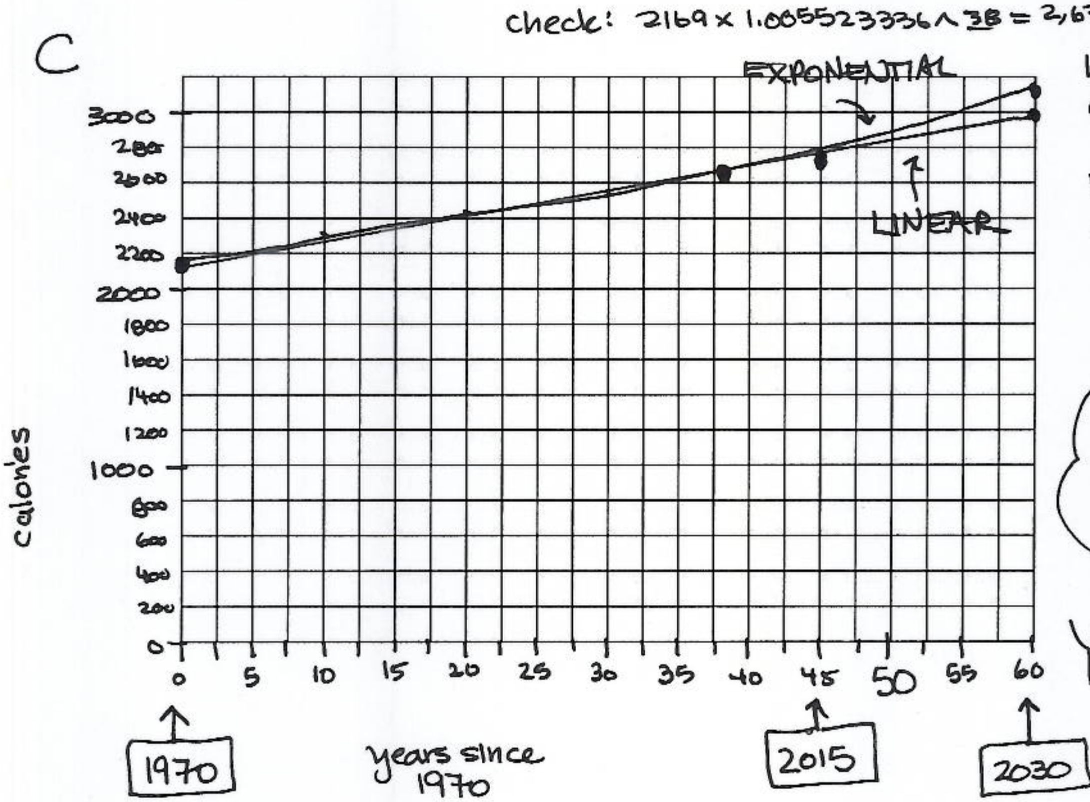
In 2030, $Y=60 \Rightarrow C = 2169 + 13.29 \times \underline{60} = 2966.4$ cal

EXPONENTIAL:

growth factor $g = \sqrt[t]{\dfrac{a}{s}} = \sqrt[38]{\dfrac{2674}{2169}} = 38 \sqrt[x]{\ } (2674 \div 2169) = 1.005523336$

equation: $\boxed{C = 2169 * 1.005523336^{Y}}$

check: $2169 \times 1.005523336 \wedge \underline{38} = 2{,}674.000\ldots$ ✓



In 2015, $Y=45 \Rightarrow$
$C = 2169 \times 1.005523336 \wedge \underline{45} =$
$= 2{,}779.11\ldots$

In 2030, $Y=60 \Rightarrow$
$C = 2169 \times 1.005523336 \wedge \underline{60} =$
$= 3{,}018.48\ldots$

on this scale it's difficult to distinguish exponential vs. linear