SOLUTIONS

2.2   A first look at exponential equations – Practice exercises

1. The comprehensive fee at a local private college is $37,000. The fee is projected to increase 5.8% per year.

   (a) Calculate the annual growth factor.

   note: you don't have to ÷ 100% just move decimal point 2 places to left → same answer.

   $$r = 5.8\% = .058 \implies g = 1+r = 1+.058 = \boxed{1.058}$$

   ($\div 100\%$)

   (b) What do you expect the comprehensive fee will be in <u>five years</u>? ←

   $$37000 \times 1.058 \wedge \underline{5} = 49,048.98.. \approx \boxed{\$49,000}$$

   The comp. fee should be around $49,000 in five years.

   (c) Name the variables, including units, and write an equation describing the dependence.

   $$Y = \text{time (years)} \sim \text{indep}$$
   $$C = \text{comp. fee (\$)} \sim \text{dep}$$
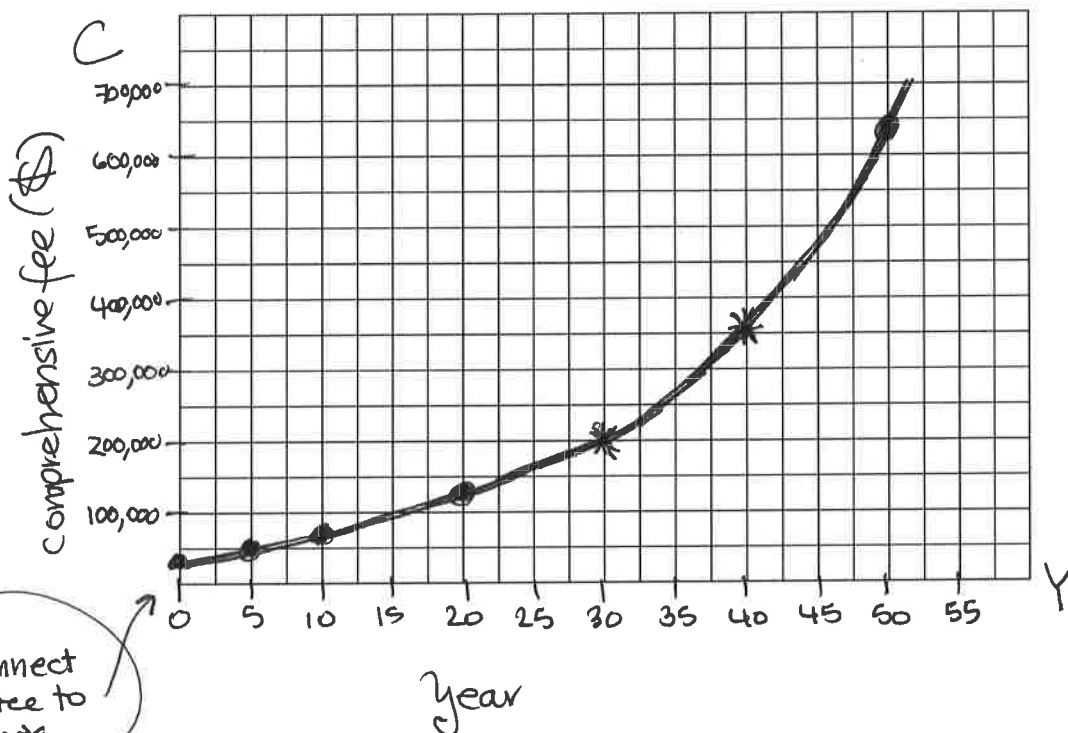 $$\boxed{C = 37000 \times 1.058^{Y}}$$

   be sure Y is written smaller and up higher

   (d) Make a table of values showing the comprehensive fee now, in 5 years, 10 years, 20 years, and 50 years (even though that's not realistic).

   $37000 \times 1.058 \wedge \underline{10} =$

Y	0	5	10	20	50
C	37,000	49,049	65,022	114,265	620,133

   (e) Draw a graph illustrating the function.

   not sure how to connect the dots? feel free to plot extra points ✱

Y	30	40
C	200,803	352,881

2. Bunnies, bunnies, everywhere. They eat the tops of my tulips in early spring and my lilies all summer long. Back in 2007 there were an estimated 1,800 rabbits in my neighborhood. Rabbits multiply quickly, 13% per year by one estimate.

Story also appears in 5.1#3

(a) Name the variables, including dependency.

R = number of rabbits (bunnies) ~ dep

Y = time (years since 2007) ~ indep

don't forget since 2007

(b) Calculate the annual growth factor.

$r = 13\% = .13 \implies g = 1 + r = 1 + .13 = \boxed{1.13}$

($\div 100\%$)

Round to a whole number. We don't want parts of a bunny. :)

(c) What does this story suggest the rabbit population was in 2010? In 2013?

$2010 \implies \begin{matrix} 2010 \\ -2007 \\ \hline Y = 3 \end{matrix} \implies R = 1800 \times 1.13^{\underline{3}} = 2{,}597.2\ldots \approx 2600$ bunnies

$2013 \implies \begin{matrix} 2013 \\ -2007 \\ \hline Y = 6 \end{matrix} \implies R = 1800 \times 1.13^{\underline{6}} = 3{,}747.5\ldots \approx 3750$ bunnies

In 2010 there were about 2600 bunnies and by 2013 there were 3750.

(d) Write an equation relating the variables.

$R = 1800 * 1.13^{Y}$

start = 8

3. A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 16% per day.

Story also appears in 5.1 #2 and 5.5 textbook

(a) Calculate the daily growth factor and use it to write an equation describing the spread of the virus. Don't forget to name the variables too.

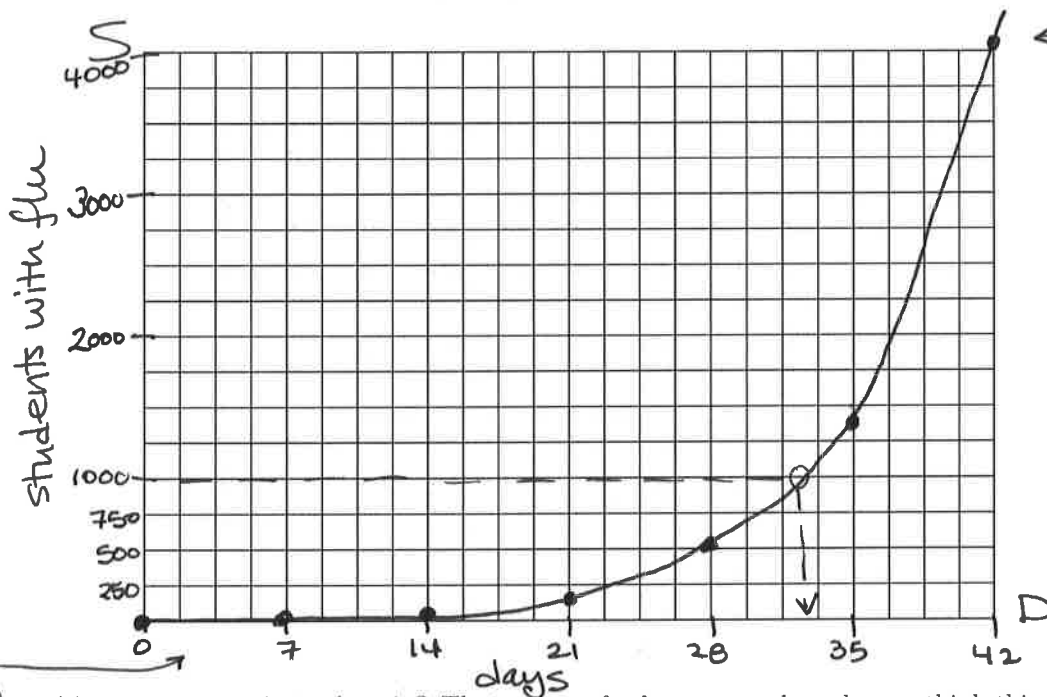$$r = 16\% \overset{\div 100\%}{=} .16 \rightarrow g = 1 + r = 1 + .16 = 1.16$$

S = number of students with flu (students) ~dep

D = time (days since first diagnosed) ~indep

$$\boxed{S = 8 \times 1.16^D}$$

(b) Make a table and graph for the six weeks following the initial diagnosis. (That means use 0, 7, 14, 21, 28, 35, and 42 days.)

D	0	7	14	21	28	35	42
S	8	23	64	181	510	1,443	4,077

oops! just a bit above graph grid

normally would not use awkward scale (each box is 7/4 = 1.75 days) but all info is multiples of 7 so it works

(c) What is a realistic domain? That means, for how many days do you think this model is reasonable? To keep a sense of scale, there are 1,094 students currently living in the dorms.

Certainly not everyone will catch the flu. so < 1000. Maybe ≈ 30 days (1 month)?

$$\boxed{0 \leq D \leq 30}$$

4. My savings account earns a modest amount of interest, the equivalent of .75% annually. I have \$12,392.18 in the account now.

   (a) How much interest will I earn this year?

looks strange but thats correct.

$$r = .75\% = .0075 \quad (\div 100\%)$$

$$\text{interest} = .0075 \times \$12{,}392.18$$
$$= 92.941\ldots \approx \$92.94$$

   (b) What will my balance be in three years, assuming I neither deposit nor withdraw money?

$$r = .0075 \implies g = 1 + r = 1 + .0075 = 1.0075$$

$$\text{balance} = 12{,}392.18 \times 1.0075 \wedge 3 = 12{,}673.10\ldots$$
$$\approx \$12{,}673.10$$

   (c) Name the variables and write an equation relating them.

$$B = \text{savings account balance (\$)} \sim \text{dep}$$
$$Y = \text{time (years)} \sim \text{indep}$$
$$B = 12{,}392.18 \times 1.0075^Y$$

   (d) What would the equation be if I moved all of my money into a certificate of deposit earning the equivalent of .92%?

$$r = .92\% = .0092 \implies g = 1.0092$$
$$(\div 100\%)$$

new balance $\implies$
$$B = 12{,}392.18 \times 1.0092^Y$$

   (e) What would the equation be if I moved \$10,000 into that certificate of deposit, and kept the rest in savings? *Hint: to find the total balance, add the amounts.*

$$\begin{array}{r} 12{,}392.18 \\ -\ 10{,}000.00 \\ \hline 2{,}392.18 \end{array}$$

total balance $\implies$
$$B = 10{,}000 \times 1.0092^Y + 2{,}392.18 \times 1.0075^Y$$