SOLUTIONS

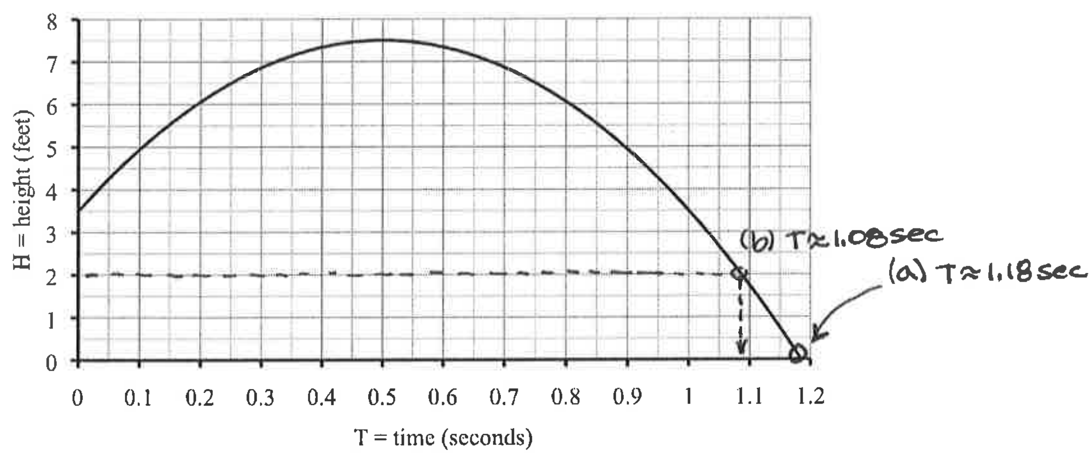
3.5 Solving quadratic equations – Practice exercises

QUADRATIC FORMULA: The equation $aT^2 + bT + c = 0$ has solutions

$$T = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

1. A high-jumper jumps so that the height, H feet, of the point on his back that must clear the bar after T seconds is given by the equation

$$H = 3.5 + 16T - 16T^2$$



(a) When would the high-jumper hit the ground (if there were no pit)? Ouch! Use the QUADRATIC FORMULA to find the answer. Use the graph to check.

H=0

$3.5 + 16T - 16T^2 = 0 \Rightarrow -16T^2 + 16T + 3.5 = 0$

$a = -16,\ b = 16,\ c = 3.5$

note: need parentheses around bottom of fraction

By the Quadratic Formula

note: need parentheses around inside of √

$$T = \frac{-16}{2(-16)} \pm \frac{\sqrt{16^2 - 4(-16)(3.5)}}{2(-16)}$$

$$= \underbrace{(-)16 \div (2 \times (-)16) =}_{.5} \quad \pm \underbrace{\sqrt{(16x^2 - 4 \times (-)16 \times 3.5)} \div (2 \times (-)16) =}_{-.68465}$$

$T \to .5 + (-).68465 = \cancel{-.1846}$

$T \to .5 - (-).68465 = 1.18465 \ \boxed{\approx 1.18 \text{ seconds}}$

The problem continues ...

(b) The high jump pit is 2 feet off the ground. When does the high-jumper land in the pit? Use the QUADRATIC FORMULA to find the answer and the graph to check.

H=2

$$3.5+16T-16T^2=2 \Rightarrow -16T^2+16T+3.5=2 \Rightarrow -16T^2+16T+1.5=0$$

$-2 \quad -2$

$a=-16, b=16, c=1.5$ NEW

$$T=\frac{-16}{2(-16)} \pm \frac{\sqrt{16^2-4(-16)(1.5)}}{2(-16)}$$ By the Quadratic Formula

$$= \underbrace{(-)16\div(2\times(-)16)=}_{.5} \quad \pm \quad \underbrace{\sqrt{}(16x^2-4\times(-)16\times1.5)\div(2\times(-)16)=}_{-.5863...}$$

T → $.5+(-).5863... = -.\cancel{08}63$

T → $.5-(-).5863... = 1.0863.$ $\boxed{\approx 1.08 \text{ seconds}}$

(c) How high a bar can the high-jumper clear? Find the maximum height of that point above ground by evaluating at $T = \dfrac{-b}{2a}$. Use the graph to check.

$$T=\frac{-16}{2(-16)} = (-)16\div(2\times(-)16)= .5$$ as before

T	.5
H	7.5

$3.5+16\times.5-16\times.5\wedge2=$

$\boxed{7.5 \text{ feet}}$ or 7'6"

2. The art museum opened in 1920. After an initial rush to see the great holdings, attendance dropped for awhile. But then attendance began to rise again and has risen since. The number of annual visits N is approximated by the equation

$$N = 51Y^2 - 840Y + 3{,}700$$

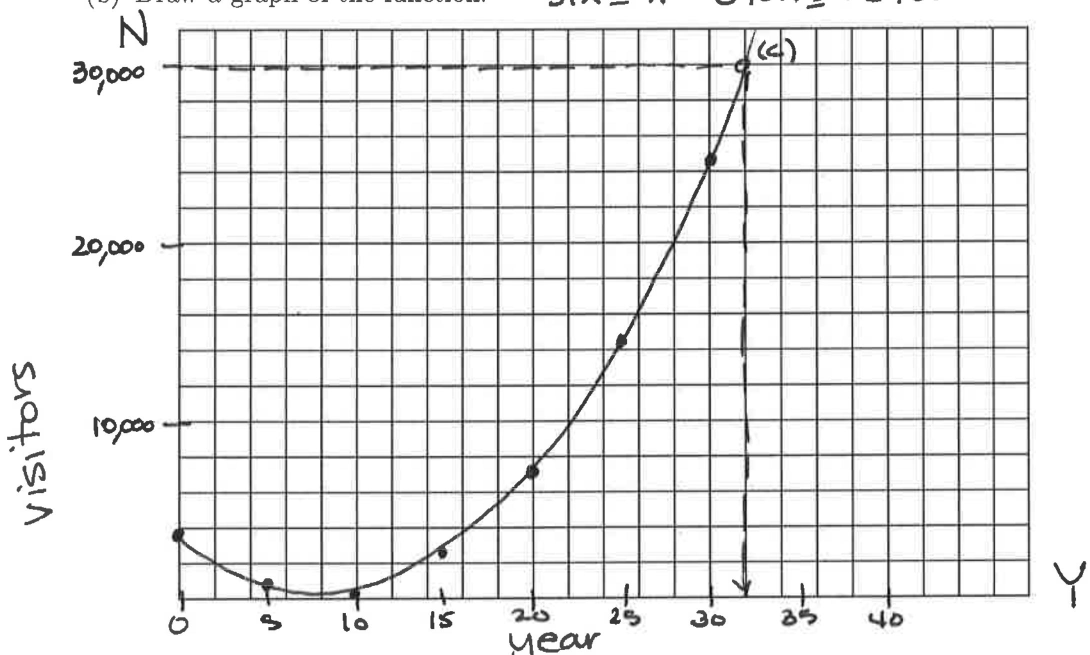
where Y is the year since 1920.

(a) Calculate the missing values in the table.

year	1920	1925	1930	1935	1940	1945	1950
Y	0	5	10	15	20	25	30
N	3,700	775	400	2,575	7,300	14,575	24,400

$51 \times 5\, x^2 - 840 \times 5 + 3700 =$

(b) Draw a graph of the function.



(c) In what year did the number of visitors first pass 30,000 in a year? Estimate the value from your graph. Then set up and solve a quadratic equation.

≈ 32 years $\quad 1920 + 32 \rightarrow \boxed{1952}$

By the Quadratic Formula

$51Y^2 - 840Y + 3700 = 30{,}000 \Rightarrow 51Y^2 - 840Y - 26{,}300 = 0$

$-30{,}000 \qquad -30{,}000$

$a = 51, b = -840, c = -26{,}300$

$$Y = \frac{-(-840)}{2(51)} \pm \frac{\sqrt{(-840)^2 - 4(51)(-26300)}}{2(51)}$$

notice $(-840)^2 = ((-)840)x^2$ so, yes double parentheses needed

$= \underbrace{(-)(-)840 \div (2 \times 51) =}_{8.2353} \quad \pm \quad \underbrace{\sqrt{(((-)840)x^2 - 4 \times 51 \times (-)26300) \div (2 \times 51)} =}_{24.1559}$

Yes, could just do 840 since $(-)(-) = +$

8.2353 ± 24.1559

Y

$8.2353 + 24.1559 = 32.3912 \Rightarrow \approx 32$ years $\rightarrow \boxed{1952}$ ✓

$8.2353 - 24.1559 = -15.\cancel{9}206$

The problem continues ...

(d) According to this equation, in what year was the number of annual visits the smallest? For that year, what were the number of visits? Use $T = \dfrac{-b}{2a}$.

$$Y = \frac{-(-840)}{2(51)} = (-)(-)840 \div (2\times 51) = 8.2353 \leftarrow \text{as before}$$

Y	8.2353
N	241.17

$\leftarrow 51 \times \underline{8.2353}\, x^2 - 840 \times \underline{8.2353} + 3700 =$

$\boxed{\approx 241 \text{ visitors}}$

(e) Explain why N never equals 0.

Because the smallest number of visitors is $N \approx 241$.

(f) So, what actually happens when you try to use the QUADRATIC FORMULA to solve for $N = 0$?

By the Quadratic Formula

$$51Y^2 - 840Y + 3700 = 0$$

$a = 51$, $b = -840$, $c = 3700$

$$Y = \frac{-(-840)}{2(51)} \pm \frac{\sqrt{(-840)^2 - 4(51)(3700)}}{2(51)}$$

$$= \underbrace{(-)(-)840 \div (2\times 51) =}_{8.2353} \pm \underbrace{\sqrt{\,((-)840)\, x^2 - 4 \times 51 \times 3700) \div (2\times 51) =}_{\text{ERROR}}$$

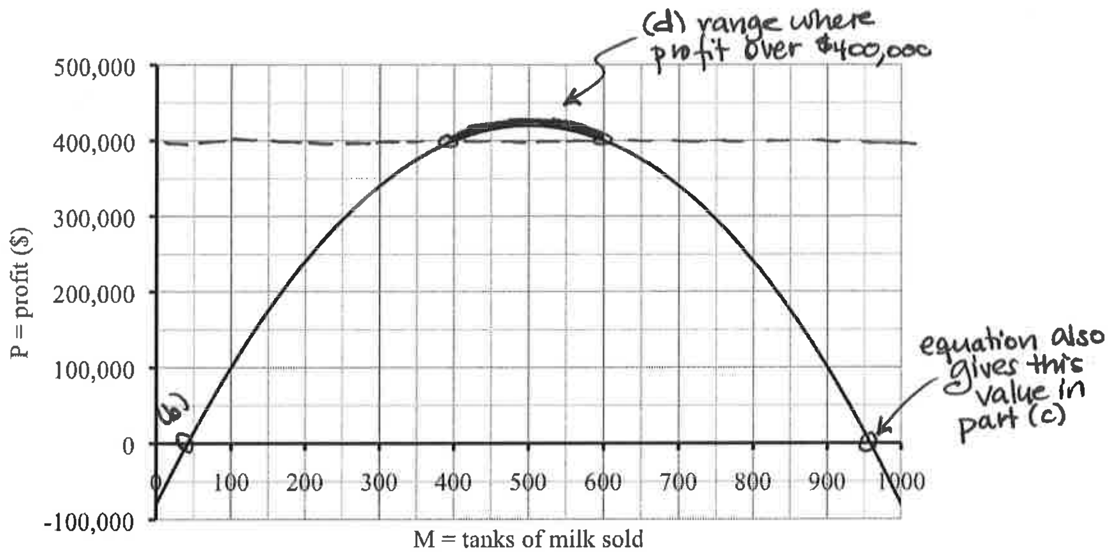
Note: $(-840)^2 - 4(51)(3700)$
$= ((-)840)\, x^2 - 4 \times 51 \times 3700 =$
$= -49{,}200$ so would be $\sqrt{-49{,}200}$
the squareroot of a negative number does not exist!

3. The profit $\$P$ from selling M tanks of milk is described by the equation

$$P = -2M^2 + 2{,}000M - 80{,}000$$

(a) The graph is drawn below. Explain why negative numbers make sense.

You can lose \$



(b) How much milk must be sold for the company to **break even**, meaning having \$0 profit? Guess from the graph and check using the equation.

guess ≈ 45 tanks

M	45	42	41
P	5,950	472	$\cancel{-1362}$

$-2 \times 45x^2 + 2000 \times 45 - 80{,}000 =$

closer.

$\boxed{\approx 42 \text{ tanks}}$

(c) For practice, set up and solve a quadratic equation to find the break even point.

P=0

$-2M^2 + 2000M - 80{,}000 = 0$

$a = -2,\ b = 2000,\ c = -80{,}000$

By the Quadratic Formula

$$M = \frac{-2000}{2(-2)} \pm \frac{\sqrt{2000^2 - 4(-2)(-80000)}}{2(-2)}$$

$= \underbrace{(-)2000 \div (2 \times (-)2) =}_{500} \quad \pm \quad \underbrace{\sqrt{(2000\ x^2 - 4 \times (-)2 \times (-)80000)} \div (2 \times (-)2) =}_{-458.3}$

$M \to 500 + (-)458.3 = 41.7\ \boxed{\approx 42 \text{ tanks}}$

$M \to 500 - (-)458.3 = 958.3$ (see graph)

The problem continues ...

(d) How many tanks of milk would they need to sell to keep profits over \$400,000? Set up and solve a quadratic equation to find the answer. Then check that it agrees with your graph. Your answer should be in the form of an inequality.

$$-2M^2+2000M-80{,}000=400{,}000$$
$$-400{,}000 \qquad -400{,}000$$

$$\Rightarrow -2M^2+2000M-480{,}000=0$$

$a=-2,\ b=2000,\ c=-480{,}000$ NEW

By the Quadratic Formula

$$M=\frac{-2000}{2(-2)} \pm \frac{\sqrt{2000^2-4(-2)(-480{,}000)}}{2(-2)}$$

$$=\underbrace{(-)2000\div(2\times(-)2)}_{500} \pm \underbrace{\sqrt{(2000x^2-4\times(-)2\times(-)480{,}000)}\div(2\times(-)2)=}_{100}$$

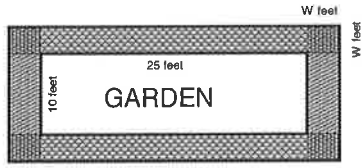
$$500 \pm 100$$

$$M \begin{cases} 500+100=600 \\ 500-100=400 \end{cases}$$

$$\boxed{400\le M\le 600}$$

> They would need to sell between 400 and 600 tanks.

4. Urban community gardens are catching on. What was once an abandoned lot down the block is now a thriving 10'×25' vegetable and berry garden for the neighborhood. One neighbor volunteered to donate gravel to make a path around the garden. The path will be 3 inches deep and the same width all around.



notice: W^2 means $1 \cdot W^2$ so coefficient is $a=1$

The amount of gravel we need (G cubic feet) is given by the equation

$$G = W^2 + 17.5W$$

where W is the width of the path in feet. For example, a path 4 feet wide requires 86 cubic feet of gravel, as you can check. *Story also appears in 2.3 and 2.4 Exercises*

(a) If the neighbor donates 60 cubic feet of gravel, how wide a path can they build? Set up and solve a quadratic equation to find the answer to two decimal places in feet. Then convert your answer into inches.

$W^2+17.5W=60 \Rightarrow W^2+17.5W-60=0 \Rightarrow a=1, b=17.5, c=-60$

$-60 \quad -60$

$W = \frac{-17.5}{2(1)} \pm \frac{\sqrt{17.5^2-4(1)(-60)}}{2(1)}$ ← By the Quadratic Formula

$= \underbrace{(-)17.5 \div (2\times1) =}$ $\pm \underbrace{\sqrt{(17.5x^2-4\times1\times(-)60)} \div (2\times1) =}$

Convert answer ← $= -8.75 \pm 11.69$ $\quad W \to -8.75+11.69 = \boxed{2.94 \text{ feet}}$, $\quad W \to -8.75-11.69 = -\cancel{20.44}$

Then 2.94 feet
= 2 feet, 11 inches
$.94 ft \times \frac{12 in}{ft} = .94\times12 =$
$= 11.28$ in ≈ 11 inches
$\Rightarrow \boxed{2'11''}$

(b) Gravel is measured by the **yard**, which is short for cubic yard. There are 27 cubic feet in 1 yard of gravel. If the neighbor donates three yards of gravel, how wide a path can they build? Set up and solve a quadratic equation to find the answer to two decimal places in feet. Then convert your answer into inches.

3 yards gravel $\times \frac{27 \text{ cubic feet}}{\text{yard}}$
$= 3\times27 = 81$ cubic feet $\Rightarrow$ $W^2+17.5W=81 \Rightarrow W^2+17.5W-81=0 \Rightarrow a=1, b=17.5, c=-81$

$-81 \quad -81$

$W = \frac{-17.5}{2(1)} \pm \frac{\sqrt{17.5^2-4(1)(-81)}}{2(1)}$ ← By the Quadratic Formula

$= \underbrace{(-)17.5 \div (2\times1) =}$ $\pm \underbrace{\sqrt{(17.5x^2-4\times1\times(-)81)} \div (2\times1) =}$

convert answer ← $= -8.75 \pm 12.55$ $\quad W \to -8.75+12.55 = \boxed{3.8 \text{ feet}}$, $\quad W \to -8.75-12.55 = -\cancel{31}.3$

3.8 feet
= 3 feet, 10 inches
$.8 ft \times \frac{12 in}{ft} = .8\times12 =$
$= 9.6$ in ≈ 10 inches
$\Rightarrow \boxed{3'10''}$

(c) What would it mean to solve the equation to find where $G = 0$? Can you tell what the answer is from the equation (without actually solving)?

$W=0$ No gravel $\Rightarrow$ no path!

$\boxed{0 \text{ Inches}}$