SOLUTIONS

1.5 Metric prefixes and scientific notation – Practice exercises

Common metric prefixes:

giga	=	1 billion	=	1,000,000,000	=	10^9
mega	=	1 million	=	1,000,000	=	10^6
kilo	=	1 thousand	=	1,000	=	10^3
centi	=	1 in a hundred	=	.01	=	10^{-2}
milli	=	1 in a thousand	=	.001	=	10^{-3}
micro	=	1 in a million	=	.000001	=	10^{-6}
nano	=	1 in a billion	=	.000 000 001	=	10^{-9}

1. Souksavanh is setting up a patient's intravenously (IV) medication. She sets the drip at 42 drops/minute. The drip chamber size is 20 drops/mL.

   (a) Souk needs to know a few conversions.

   i. How many milliliters (mL) are in a liter? $\boxed{1{,}000 \text{ mL}}$

   ii. How many microliters (μL) are in a liter? $\boxed{1{,}000{,}000\ \mu\text{L}}$

   iii. How many milligrams (mg) are in a gram? $\boxed{1{,}000\text{g}}$

   Use these numbers to answer the following questions.

   (b) At what rate is the IV fluid being delivered to Souk's patient? Answer in mL/min. $\frac{42\ \cancel{\text{drops}}}{\text{min}} * \frac{\text{mL}}{20\ \cancel{\text{drops}}} = 42 \div 20 = \boxed{2.1 \text{ mL/min}}$

   (c) How fast is the drip measured in μL/sec?

   $$\frac{2.1\ \cancel{\text{mL}}}{\cancel{\text{min}}} * \frac{\cancel{\text{L}}}{1{,}000\ \cancel{\text{mL}}} * \frac{1{,}000{,}000\ \mu\text{L}}{\cancel{\text{L}}} * \frac{1\ \cancel{\text{min}}}{60 \text{ sec}} = 2.1 \div 1{,}000 \times 1{,}000{,}000 \div 60 = \boxed{35\ \mu\text{L/sec}}$$

   (d) If the drip bag holds 1 liter, how long will it take the drip to run? Express your answer in hours and minutes.

   $$1\ \cancel{\text{L}} * \frac{\text{min}}{2.1\ \cancel{\text{mL}}} * \frac{1{,}000\ \cancel{\text{mL}}}{1\ \cancel{\text{L}}} = 1{,}000 \div 2.1 = 476 \text{ min}$$

   $$476 \text{ min} * \frac{1 \text{ hour}}{60 \text{ min}} = 476 \div 60 = 7.93..$$

   $$.93.. \text{ hr} * \frac{60 \text{ min}}{1 \text{ hour}} = .93.. \times 60 \approx 56 \text{ m}$$

   $\boxed{7 \text{ hrs}, 56 \text{ min}}$

   (e) The concentration of medication is 1.7 mg/mL. How much medication is in the 1 liter bag? Convert your answer to grams. Explain what you notice.

   $$1\ \cancel{\text{L}} * \frac{1.7\ \cancel{\text{mg}}}{\cancel{\text{mL}}} * \frac{1{,}000\ \cancel{\text{mL}}}{1\ \cancel{\text{L}}} * \frac{1 \text{ gram}}{1{,}000\ \cancel{\text{mg}}} = 1.7 \times 1{,}000 \div 1{,}000 = \boxed{1.7 \text{ grams}}$$ same #

   (f) At what rate is the medication being delivered to Souk's patient? Answer in grams/min.

   $$\frac{1.7 \text{ grams}}{476 \text{ min}} = 1.7 \div 476 = \boxed{.0035\ \text{g/min}}$$

2. The list shows the (approximate) mass of the planets in our solar system.

Earth	5.972×10^{24} kg ✓ — largest
Jupiter	1.899×10^{27} kg ✓
Mars	6.417×10^{23} kg ✓
Mercury	3.302×10^{23} kg ✓ — next largest.
Neptune	1.024×10^{26} kg ✓
Saturn	5.685×10^{26} kg ✓ etc.
Uranus	8.681×10^{25} kg ✓
Venus	4.868×10^{24} kg ✓

Source: Wikipedia (Solar System)

(a) Write the mass of Earth and the mass of Mars in standard decimal notation. Which is heavier?

Earth: 5,972,000,000,000,000,000,000,000 kg
Mars: 641,700,000,000,000,000,000,000 kg
Earth is heavier.

(b) List the planets from heaviest (largest mass) to lightest (smallest mass).

Jupiter, Saturn, Neptune, Uranus, Earth, Venus, Mars, Mercury

(c) The mass of astronomical bodies are sometimes measured in **Jupiter mass** abbreviated M_J where $1M_J = 1.899 \times 10^{27}$ kg. Express Earth's mass in M_J. *Because $\times$ and $\div$ are at the same level in the order of operations, you need to put parentheses around each number in scientific notation before dividing.*

$$\text{Earth} = 5.972 \times 10^{24}\,\cancel{\text{kg}}\ \frac{1\,M_J}{1.899 \times 10^{27}\,\cancel{\text{kg}}}$$
$$= (5.972 \times 10^{24}) \div (1.899 \times 10^{27})$$
$$= .00314\ldots$$
$$\boxed{\approx .003\,M_J}$$

3. Edgar recently changed the cleaning bag on his vacuum cleaner. He became curious about how many particles of dust were in the bag. Edgar did a little research online and found out that the mass of a dust particle is .000 000 000 753 kilograms.

   (The strange-looking spaces are to help you see that there are 9 zeros in the number.)

   10 places

   (a) Write the mass of a dust particle in scientific notation.

   $\boxed{7.53 \times 10^{-10}\text{ kg}}$

   (b) Express the mass of a dust particle in a each of the following units:

   i. grams

   $7.53 \times 10^{-10}\,\cancel{\text{kg}} * \frac{1{,}000\text{g}}{1\cancel{\text{kg}}} = 7.53 \times 10 \wedge (-)10 \times 1000 = \boxed{.000\,000\,753\text{ g}}$

   $= 7.53 \times 10^{-7}\text{g}$

   ii. milligrams (mg)

   $7.53 \times 10^{-7}\,\cancel{\text{g}} * \frac{1{,}000\text{mg}}{\cancel{\text{g}}} = 7.53 \times 10 \wedge (-)7 \times 1000 = \boxed{.000753\text{mg}}$

   iii. micrograms (μg)

   $7.53 \times 10^{-7}\,\cancel{\text{g}} * \frac{1{,}000{,}000\mu\text{g}}{1\cancel{\text{g}}} = 7.53 \times 10 \wedge (-)7 \times 1{,}000{,}000 = \boxed{.753\mu\text{g}}$

   iv. nanograms (ng) where

   $$\textbf{nano} = \text{1 in a billion} = .000\,000\,001 = 10^{-9}$$

   $7.53 \times 10^{-7}\,\cancel{\text{g}} * \frac{1{,}000{,}000{,}000\text{ng}}{1\cancel{\text{g}}} = 7.53 \times 10 \wedge (-)7 \times 1{,}000{,}000{,}000$

   $= \boxed{753\text{ng}}$

   (c) Edgar determined that the full vacuum bag weighed 5 pounds. How many dust particles were in the bag? (I'm sneezing already.) Use 1 kilogram ≈ 2.2 pounds. Express your answer in scientific notation.

   $5\text{ }\cancel{\text{pounds}} * \frac{2.2\text{ kg}}{1\text{ }\cancel{\text{pound}}} * \frac{1\text{ dust particle}}{7.53 \times 10^{-10}\text{ kg}}$

   $= 5 \times 2.2 \div (7.53 \times 10 \wedge (-)10) = 1.46\ldots \times 10^{10}$

   $\approx 14{,}600{,}000{,}000$ $\boxed{\approx 15 \text{ billion dust particles}}$

4. The GDP (gross domestic product) of the United States was approximately \$15,596 billion in 2011 and the population of the United States was approximately 0.313 billion that year. Source: U.S. Bureau of Economic Analysis, U.S. Census Bureau

   (a) That's a strange way to write the population of ~~\$15,596~~ .313 billion. A more natural unit would be millions. Rewrite the population in millions of people.

   $.313 \text{ billion} = .313 \times 10^9 = \underbrace{313{,}000{,}000}_{\text{9 places}} = \boxed{313 \text{ million}}$

   (b) Rewrite the population in people, both in normal decimal notation (that means with all the 0s) and in scientific notation.

   $\boxed{\underbrace{313{,}000{,}000}_{\text{8 places}} \text{ people}} = \boxed{3.13 \times 10^8 \text{ people}}$

   (c) That's also a strange way to write the GDP of ~~0.313~~ \$15,596 billion. A more natural unit would be **trillions** where

   $$1 \text{ trillion} = 1{,}000{,}000{,}000{,}000$$

   Rewrite the GDP in trillions of dollars.

   $\$15{,}596 \text{ billion} = \$15{,}596 \times 10^9 = 15{,}596{,}000{,}000{,}000$
   $= \boxed{\$15.596 \text{ trillion}}$

   (d) Rewrite the GDP in dollars, both in normal decimal notation and in scientific notation.

   $\boxed{\underbrace{\$15{,}596{,}000{,}000{,}000}_{\text{13 places}}} = \boxed{\$1.5596 \times 10^{13}}$

   (e) Calculate the GDP **per capita** (meaning per person) by dividing the GDP in dollars by the population in people. Express your answer in \$/person.

   $\frac{15{,}596{,}000{,}000{,}000}{313{,}000{,}000} = 15{,}596{,}000{,}000{,}000 \div 313{,}000{,}000$
   $\approx \boxed{\$49{,}827/\text{person}}$

   (f) For practice, repeat your calculation using the numbers in scientific notation. *Because × and ÷ are at the same level in the order of operations, you need to put parentheses around each number in scientific notation before dividing.*

   $\frac{\$1.5596 \times 10^{13}}{3.13 \times 10^8} = (1.5596 \times 10\wedge 13) \div (3.13 \times 10 \wedge 8) =$
   $\approx \boxed{\$49{,}827/\text{person}}$

maybe should round to $\approx$ \$50,000/person