SOLUTIONS

5.1 Modeling with exponential equations – Practice exercises

1. The population of Buenos Aires, Argentina in 1950 was estimated at 5.0 million (start) and expected to grow at 1.8% each year. Source: Mongabay

   (a) Name the variables.

   P = population of Buenos Aires (million people) ~ dep
   Y = year (years since 1950) ~ indep

   (b) What is the annual growth factor?

   $r = 1.8\% = .018$ ($\div 100\%$) $\implies g = 1 + r = 1. + .018 = \boxed{1.018}$

   (c) Write an equation estimating the population of Buenos Aires over time.

   fits exponential equation template: $\text{dep} = \text{start} * g^{\text{indep}}$ →

   $\boxed{P = 5.0 * 1.018^Y}$ ← be sure Y is raised to show exponential.

   (d) Make a table of values showing the estimated population of Buenos Aires every 20$^{\text{th}}$ year from 1950 to 2030.

year	Y	P	annual rate of change
1950	0	5.00	
1970	20	7.14	$\frac{7.14-5.00}{20-0} =$ .107 mil people/yr
1990	40	10.21	$\frac{10.21-7.14}{40-20} =$ .154 mil people/yr
2010	60	14.58	$\frac{14.58-10.21}{60-40} =$ .216 mil people/yr
2030	80	20.83	$\frac{20.83-14.58}{80-60} =$ .313 mil people/yr

   $5.0 \times 1.018 \wedge 20 =$

   $(10.21-7.14) \div (40-20) =$

   (e) By approximately how many people has the population been increasing per year over each 20 year period? Add these numbers to your table. As expected, these numbers change because the rate of change is not constant. ✓

   (f) The actual population of Buenos Aires in the year 2000 was around 12.6 million and by 2010 it was around 15.2 million. How does that compare to the estimates?

   $2000 - 1950 = 50$

   2000: estimate $= 5.0 \times 1.018 \wedge 50 \approx 12.2$ million people
   The actual number of 12.6 million is higher.

   2010: estimate = 14.58 million people
   The actual number of 15.2 million is higher.

   $\boxed{\text{The population was higher than predicted.}}$

2. A flu virus has been spreading through the college dormitories. Initially 8 students were diagnosed with the flu, but that number has been growing 16% per day. Earlier we found the equation

$$N = 8 * 1.16^D$$

where D is the number of days (since the first diagnosis) and N is the total number of students who had the flu. *Story also appears in 2.2 #3 and 5.5*

(a) Use successive approximations to estimate when the number of infected students reaches 100. Display your guesses in a table.

$8 \times 1.16 \wedge 10 =$

D	0	10	~~50~~	20	15	17	18
N	8	35.29	~~13,365~~	155.68	74.12	99.74	115.70
VS 100	LOW	LOW	NO WAY!	HIGH	LOW	ALMOST	OVER.

$\Rightarrow \boxed{\approx 18 \text{ days}}$

(b) Use the LOG DIVIDES FORMULA to solve your equation.

$$\frac{\cancel{8} * 1.16^D}{\cancel{8}} = \frac{100}{8}$$

$$1.16^D = 12.5$$

By the Log-Divides Formula $g^T = v$ has solution $T = \frac{\log(v)}{\log(g)}$

$$D = \frac{\log(12.5)}{\log(1.16)} = \log(12.5) \div \log(1.16) = 17.01\ldots \boxed{\approx 18 \text{ days}}$$

(c) There are 1,094 students currently living in the dorms. Suppose ultimately 250 students catch the flu. According to your equation, when would that happen? Show how to solve your equation.

$$\frac{\cancel{8} * 1.16^D}{\cancel{8}} = \frac{250}{8}$$

$$1.16^D = 31.25$$

By the Log-Divides Formula

$$D = \frac{\log(31.25)}{\log(1.16)} = \log(31.25) \div \log(1.16) = 23.19\ldots \boxed{\approx 24 \text{ days}}$$

(d) It is not realistic to expect that everyone living in the dorms will catch the flu, but what does the equation say? Set up and solve an equation to find when all 1,094 students would have the flu. (Again, this is not realistic.)

$$\frac{\cancel{8} * 1.16^D}{\cancel{8}} = \frac{1094}{8}$$

$$1.16^D = 136.75$$

By the Log-Divides Formula

$$D = \frac{\log(136.75)}{\log(1.16)} = \log(136.75) \div \log(1.16) = 33.13\ldots \boxed{\approx 34 \text{ days}}$$

3. Bunnies, bunnies, everywhere. Earlier we found the equation

$$B = 1{,}800 * 1.13^{Y}$$

where B is the number of bunnies and Y is the years since 2007.

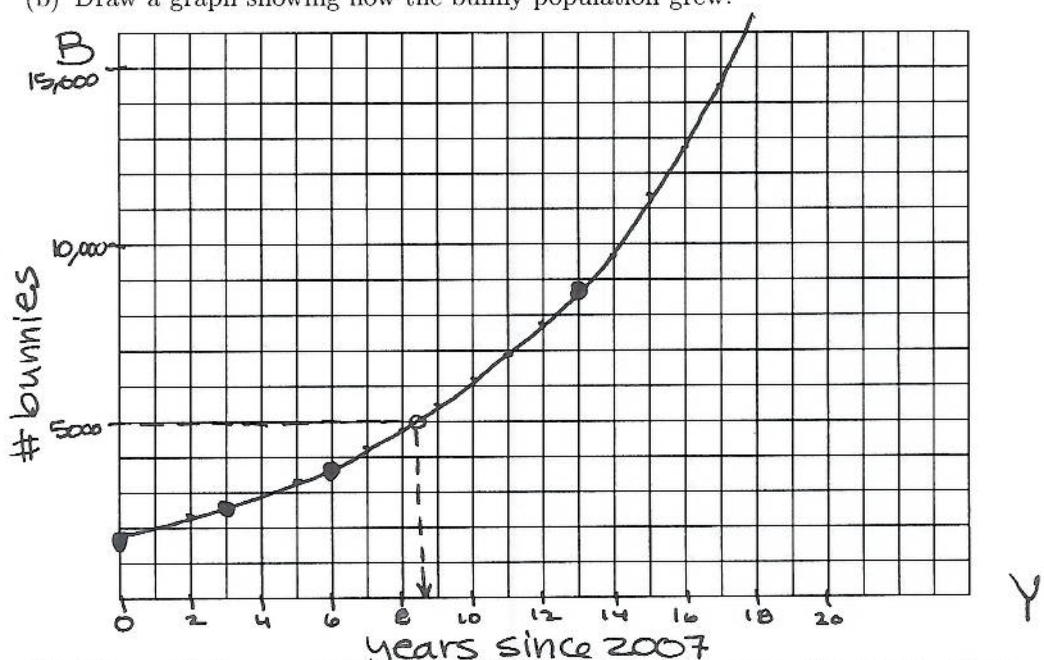
2010−2007= *Story also appears in 2.2 #2*

(a) Make a table showing the number of bunnies in 2007, 2010, 2013, and 2020.

year	2007	2010	2013	2020
Y	0	3	6	13
B	1,800	2,597	3,748	8,816

1800 × 1.13^13 =

(b) Draw a graph showing how the bunny population grew.



(c) When will the population pass 5,000 bunnies? Guess from the graph. Then refine your answer using successive approximation.

guess just over 8 years, say 2007+9 = $\boxed{2016}$

Y	8	9
B	4,785	5,407

✓ $\boxed{\text{The bunny population will pass 5,000 in 2016}}$

(d) Solve your equation and check that you get the same answer.

$$\frac{1{,}8\cancel{00} * 1.13^{Y}}{1{,}\cancel{800}} = \frac{5{,}000}{1{,}800}$$

$$1.13^{Y} = 2.777\ldots$$

By the Log Divides Formula,

$$Y = \frac{\log(2.777\ldots)}{\log(1.13)} = \text{LOG(ANS)} \div \text{LOG}(1.13) = 8.359\ldots$$

$\Rightarrow$ just over 8 yrs $\Rightarrow$ $\boxed{2016}$ ✓

4. Carbon dioxide is a greenhouse gas in our atmosphere. Increasing carbon dioxide concentrations are related to global climate change. In 1980, the carbon dioxide concentration was 338 ppm (parts per million). At that time it was assumed that carbon dioxide concentrations would increase .42% per year.

Source: Earth Systems Research Laboratory, NOAA

(a) Name the variables including units.

C = carbon dioxide concentrations (ppm) ~ dep
Y = year (years since 1980) ~ indep

(b) Assuming the growth is exponential as predicted, write an equation that describes the increase in carbon dioxide concentrations.

$r = .42\% \xrightarrow{\div 100\%} = .0042 \implies g = 1 + r = 1 + .0042 = 1.0042$

$\boxed{C = 338 \times 1.0042^{Y}}$ ← fits exponential equation template: $dep = start \times g^{indep}$

(c) The carbon dioxide concentration in 2008 was 385 ppm. Is that count higher or lower than predicted from your equation? Explain.

$2008 - 1980 = 28$

$C = 338 \times 1.0042 \wedge \underline{28} = 380.086\ldots \approx 380$ ppm

$\boxed{\text{The actual concentration of 385 ppm is larger than predicted}}$

(d) Does that mean that carbon dioxide increased at a higher or lower rate than .42%? Explain.

$\boxed{\text{Carbon dioxide must be increasing at a higher rate than expected.}}$