SOLUTIONS

1.5 Metric prefixes and scientific notation – Practice exercises

Using this exponential shorthand? Don't forget the parentheses.

1. The GDP (gross domestic product) of the United States was approximately \$15,596 billion in 2011 and the population of the United States was approximately 0.313 billion that year. Source: U.S. Bureau of Economic Analysis, U.S. Census Bureau

   (a) That's a strange way to write the population as 0.313 billion. A more natural unit would be millions. Rewrite the population in millions of people.

   $$.313 \text{ billion} * \frac{1{,}000{,}000{,}000}{1 \text{ billion}} * \frac{1 \text{ million}}{1{,}000{,}000} = .313 \times (10 \wedge 9) \div (10 \wedge 6) = \boxed{313 \text{ million}}$$

   (b) Rewrite the population in people, both in normal decimal notation (that means with all the 0s) and in scientific notation.

   $$.313 \text{ billion} * \frac{1{,}000{,}000{,}000}{1 \text{ billion}} = .313 \times 1{,}000{,}000{,}000 = \boxed{\underbrace{313{,}000{,}000}_{8 \text{ places}}} = \boxed{3.13 \times 10^8}$$

   Remember: should have one digit before the decimal point.

   (c) That's also a strange way to write the GDP as \$15,596 billion. A more natural unit would be **trillions** where

   $$1 \text{ trillion} = 1{,}000{,}000{,}000{,}000$$

   Rewrite the GDP in trillions of dollars.

   $$15{,}596 \text{ billion} * \frac{1{,}000{,}000{,}000}{1 \text{ billion}} * \frac{1 \text{ trillion}}{1{,}000{,}000{,}000{,}000} = 15596 \times (10 \wedge 9) \div (10 \wedge 12) = \boxed{\$15.596 \text{ trillion}}$$

   (d) Rewrite the GDP in dollars, both in normal decimal notation and in scientific notation.

   $$15596 \text{ billion} * \frac{1{,}000{,}000{,}000}{1 \text{ billion}} = 15596 \times 1{,}000{,}000{,}000 = \boxed{\$\underbrace{15{,}596{,}000{,}000{,}000}_{13 \text{ places}}} = \boxed{\$1.5596 \times 10^{13}}$$

   (e) Calculate the GDP **per capita** (meaning per person) by dividing the GDP in dollars by the population in people. Express your answer in \$/person.

   $$\frac{\$15{,}596{,}000{,}000{,}000}{313{,}000{,}000 \text{ people}} \approx \$49{,}827/\text{person} \quad \boxed{\approx \$50{,}000 \text{ produced per person.}}$$

   (f) For practice, repeat your calculation using the numbers in scientific notation. *Because × and ÷ are at the same level in the order of operations, you need to put parentheses around each number in scientific notation before dividing.*

   $$(1.5596 \times 10 \wedge 13) \div (3.13 \times 10 \wedge 8) \approx 49{,}827\ldots \checkmark$$

   Remember: need parentheses to insure ÷ is last operation.

2. Edgar recently changed the cleaning bag on his vacuum cleaner. He became curious about how many particles of dust were in the bag. Edgar did a little research online and found out that the mass of a dust particle is .000 000 000 753 kilograms.

(The strange-looking spaces are to help you see that there are 9 zeros in the number.)

10 places

(a) Write the mass of a dust particle in scientific notation.

$\boxed{7.53 \times 10^{-10} \text{ kg}}$

Remember: the (-) key changes sign +/-

(b) Recall that

kilo	=	1 thousand	=	1,000	=	10^3
milli	=	1 in a thousand	=	.001	=	10^{-3}
micro	=	1 in a million	=	.000001	=	10^{-6}
nano	=	1 in a billion	=	.000 000 001	=	10^{-9}

Express the mass of a dust particle in each of the given units.

i. grams

$$7.53 \times 10^{-16}\,\cancel{kg} * \frac{1000g}{1\,\cancel{kg}} = 7.53 \times 10 \wedge (-)\,10 \times 1000 = \boxed{.000\,000\,753g}$$

ii. milligrams (mg)

$$.000\,000\,753\,\cancel{g} * \frac{1000mg}{\cancel{g}} = .000\,000\,753 \times 1000 = \boxed{.000\,753mg}$$

iii. micrograms (μg)

$$.000\,000\,753\,\cancel{g} * \frac{1{,}000{,}000\,\mu g}{1\,\cancel{g}} = .000\,000\,753 \times 100\,000 = \boxed{.753mg}$$

iv. nanograms (ng)

$$.0000\,000\,753\,\cancel{g} * \frac{1{,}000{,}000{,}000ng}{1\,\cancel{g}} = .000\,000\,753 \times 1000\,000\,000 = \boxed{753ng}$$

(c) Edgar determined that the full vacuum bag weighed 5 pounds. How many dust particles were in the bag? (I am already sneezing.) Use 1 kilogram $\approx$ 2.2 pounds. Express your answer in scientific notation.

$$5\,\cancel{\text{pounds}} * \frac{1\,\cancel{kg}}{2.2\,\cancel{\text{pounds}}} * \frac{1 \text{ dust particle}}{7.53 \times 10^{-10}\,\cancel{kg}}$$

don't forget parentheses

$$= 5 \div 2.2 \div (7.53 \times 10 \wedge (-)\,10) =$$

$$= 3.018\ldots \times 10^9 \approx 3 \times 10^9 = 3{,}000{,}000{,}000$$

9 places

$\boxed{\approx \text{3 billion dust particles in the full vacuum bag}}$

3. The list shows the (approximate) mass of the planets in our solar system.

Earth	5.972×10^{24} kg	✓	largest
Jupiter	1.899×10^{27} kg	✓	
Mars	6.417×10^{23} kg	✓	
Mercury	3.302×10^{23} kg	✓	next largest.
Neptune	1.024×10^{26} kg	✓	
Saturn	5.685×10^{26} kg	✓	etc.
Uranus	8.681×10^{25} kg	✓	
Venus	4.868×10^{24} kg	✓	

Source: Wikipedia (Solar System)

(a) Write the mass of Earth and the mass of Mars in standard decimal notation. Which is heavier?

Earth: 5,972,000,000,000,000,000,000,000 kg
Mars: 641,700,000,000,000,000,000,000 kg
Earth is heavier.

(b) List the planets from heaviest (largest mass) to lightest (smallest mass).

Jupiter, Saturn, Neptune, Uranus, Earth, Venus, Mars, Mercury

(c) The mass of astronomical bodies are sometimes measured in **Jupiter mass** abbreviated M_J where $1M_J = 1.899 \times 10^{27}$ kg. Express Earth's mass in M_J. *Because $\times$ and $\div$ are at the same level in the order of operations, you need to put parentheses around each number in scientific notation before dividing.*

$$\text{Earth} = 5.972\times10^{24}\,\cancel{\text{kg}}\ \frac{1\,M_J}{1.899\times10^{27}\,\cancel{\text{kg}}}$$
$$= (5.972\times10^{24}) \div (1.899\times10^{27})$$
$$= .00314\ldots$$
$$\boxed{\approx .003\,M_J}$$

4. Souksavanh is setting up a patient's intravenously (IV) medication. She sets the drip at 42 drops/minute. The drip chamber size is 20 drops/mL. Recall

milli	=	1 in a thousand	=	.001	=	10^{-3}
micro	=	1 in a million	=	.000001	=	10^{-6}

(a) At what rate is the IV fluid being delivered to Souk's patient? Answer in mL/min (that's millileters per minute).

$$\frac{42\ \cancel{drops}}{1\ min} * \frac{1\ mL}{20\ \cancel{drops}} = 42 \div 20 = \boxed{2.1\ mL/min}$$

(b) How fast is the drip measured in μL/sec (that's microliters per second)?

$$\frac{2.1\ \cancel{mL}}{1\ \cancel{min}} * \frac{1\ \cancel{min}}{60\ sec} * \frac{1\ \cancel{L}}{1000\ \cancel{mL}} * \frac{1{,}000{,}000\ \mu L}{1\ \cancel{L}}$$
$$= 2.1 \div 60 \div 1000 \times 1000000 = \boxed{35\ \mu L/sec}$$

(c) If the drip bag holds 1 liter, how long will it take the drip to run? Express your answer in hours and minutes.

$$1\ \cancel{L} * \frac{1000\ \cancel{mL}}{1\ \cancel{L}} * \frac{1\ min}{2.1\ \cancel{mL}} = 1000 \div 2.1 = 476.19\ldots\ \text{minutes}$$

$$476\ min * \frac{1\ hr}{60\ min} = 476 \div 60 = 7.9.. \Rightarrow 7\ \text{hours and} \_\_\ \text{minutes}$$
$$476 - 7\ hrs * \frac{60\ min}{hour} = 476 - 7 \times 60 = 56\ min \qquad \boxed{7:56}$$

(d) The concentration of medication is 1.7 mg/mL (that's milligrams per milliliter). How much medication is in the 1 liter bag? Convert your answer to grams. Explain what you notice.

$$1\ \cancel{L} * \frac{1000\ \cancel{mL}}{1\ \cancel{L}} * \frac{1.7\ \cancel{mg}}{\cancel{mL}} * \frac{1\ g}{1000\ \cancel{mg}} = 1000 \times 1.7 \div 1000 = \boxed{1.7\ grams}$$

Hey! Same number. It's like the "m" (~~for~~ micro) cancelled.

(e) At what rate is the medication being delivered to Souk's patient? Answer in g/min (that's grams per minute).

The 1 liter bag holds 1.7 grams and takes 476 minutes

$$\Rightarrow \frac{1.7\ g}{476\ min} = 1.7 \div 476 \boxed{\approx .0035\ g/min}$$