SOLUTIONS

Practice Exam 5B

Try taking this version of the practice exam under testing conditions: no book, no notes, no classmate's help, no electronics (computer, cell phone, television). Give yourself one hour to work and wait until you have tried your best on all of the problems before checking any answers.

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1. The number of school children in the district whose first language in not English has been on the rise. The equation describing the situation is

$$C = 673(1.043)^Y \quad g$$

where C is the number of school children in the district whose first language is not English, and Y is the number of years (from now).

(a) Make a table showing the number of school children in the district whose first language is not English now, in one year, in two years, and in ten years. *Don't forget now too.*

Y	0	1	2	10
C	673	702	732	1,025

673 × 1.043 ^ 1 =

(b) What percent increase is implicit in this equation?

By the Percent Change Formula

$g = 1.043 \implies r = g - 1 = 1.043 - 1 = .043 \xrightarrow{\times 100\%} \boxed{4.3\%}$

(c) Use successive approximation to determine when there will be over 1,700 school children in the district whose first language is not English Display your work in a table. Round your answer to the nearest year.

Y	20	25	23	22
C	1,562	1,928	1,772	1,699

$\boxed{\approx 23 \text{ years}}$

(d) Show how to solve the equation to calculate when there will be over 1,700 school children in the district whose first language is not English. Show how you solve the equation.

$$\frac{\cancel{673} * 1.043^Y}{\cancel{673}} = \frac{1{,}700}{673}$$

By the Log-Divides Formula

$$1.043^Y = 2.526002\ldots$$

$$Y = \frac{\log(2.526002\ldots)}{\log(1.043)} = \log(\text{ANS}) \div \log(1.043) =$$

$$= 22.009\ldots \quad \boxed{\approx 23 \text{ years}}$$

2. The lottery jackpot started at \$600,000. After 17 days the jackpot had increased to \$2.1 million. The lottery is designed so that the jackpot grows exponentially.

(a) Name the variables including units.

J = value of lottery jackpot (\$) ~ dep
D = time (days) ~ indep

	D	J
	0	600,000 = s
t = 17		2,100,000 = a

2.1 mil = 2.1 x 10^6 =

(b) Write an equation describing the jackpot. *Hint: find the daily growth factor.*

By the Growth Factor Formula

$$g = \sqrt[t]{\frac{a}{s}} = \sqrt[17]{\frac{2{,}100{,}000}{600{,}000}} = 17 \text{ x}\!\sqrt{\ } (2100000 \div 600000)$$

$$= 1.076475135\ldots$$

keep many digits here to avoid too much rounding error

$$\boxed{J = 600{,}000 * 1.076475135^{D}}$$

Check: 600000 x 1.076475135^17
= 2,100,000.008 ✓

(c) By what percentage does the jackpot increase each day?

$$g \approx 1.076 \Rightarrow r = g - 1 = 1.076 - 1 = .076 = \boxed{7.6\%}$$

x100%

By the Percent Change Formula

(d) What will the jackpot be after 20 more days (i.e. after 37 days total)?

D = 37

600,000 x 1.076475135 ^ 37 = 9,168,522.40...

$\boxed{\approx \$9.2 \text{ million}}$

3. The creeping vine is taking over Fiona's front lawn. Write V for the area covered by the vine (in square feet) and Y for the years since she moved in to the house.

double means x2

(a) When Fiona moved in there maybe 3 square feet covered by vine. She believes it has doubled each year since. Write an exponential equation showing how the area covered by the vine is a function of time in this case. *Stuck? Try doing the table first.*

start = 3 sqft
$g=2$

$$\boxed{V=3*2^Y}$$

(b) At some point the vine will take over the entire lawn, so perhaps a saturation model would be better. That equation might be

$$V = 170 - 167 * .8^Y$$

Another equation would be a logistic model. Perhaps

$$V = \frac{129}{1 + 42 * .34^Y}$$

Fill in the corresponding rows of the table for each model.

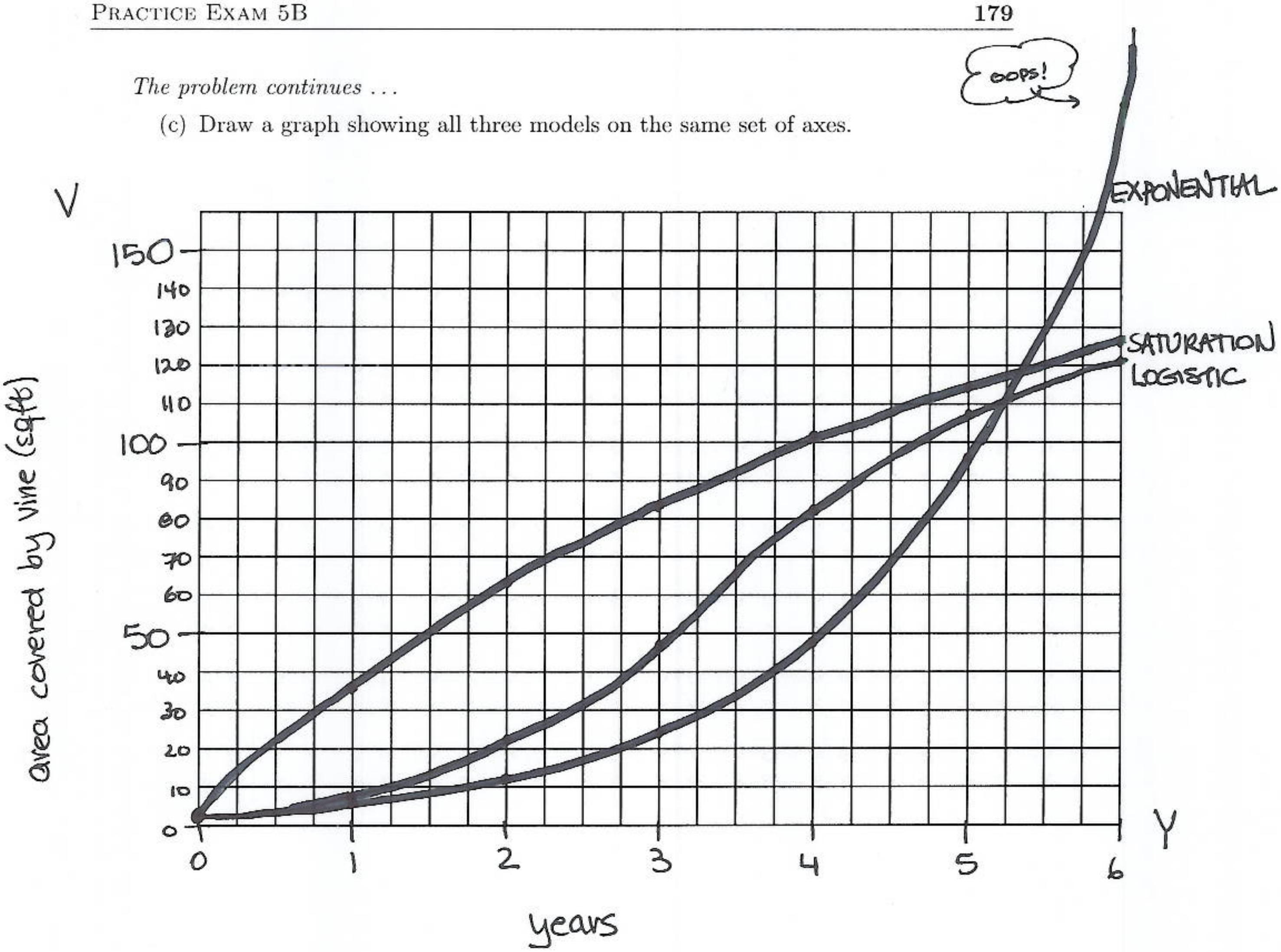
years	0	1	2	3	4	5	6
area exponential	3	6	12	24	48	96	192
area saturation	3	36.4	63.1	84.5	101.6	115.3	126.2
area logistic	3	8.4	22.0	48.7	82.6	108.3	121.1

$170-167\times.8\wedge 1 =$

$129\div(1+42\times.34\wedge 1)=$

The problem continues ...

(c) Draw a graph showing all three models on the same set of axes.



year	Y	M
1955	0	52=s
2000	45 ↑t	23=a

2000
-1955
45

start = 52 deaths/th. births

4. Many different agencies and governments are working to lower infant mortality. Infant mortality is measured in deaths per thousand births. The world infant mortality rate in 1955 was around 52 (per ~~thousand~~ deaths births). By the year 2000, it was down to around 23.

Source: Wikipedia (Infant Mortality)

(a) Name the variables.

M = infant mortality (deaths per thousand births) ~ dep

Y = time (years since 1955) ~ indep

(b) Write a linear equation modeling infant mortality.

decreasing, so expected −

$$\text{slope} = \frac{23-52}{45-0} = (23-52) \div 45 = -.6444\ldots.$$

$$\boxed{M = 52 - .6444Y}$$

check: $52 - .6444 \times 45 = 23.002$ ✓

fits Linear equation template: dep = start + slope * indep

(c) Now write an exponential equation modeling infant mortality.

decreasing, so expected < 1

$$\text{growth factor } g = \sqrt[t]{\frac{a}{s}} = \sqrt[45]{\frac{23}{52}} = 45 \sqrt[x]{\ } (23 \div 52) = .9820355\ldots$$

$$\boxed{M = 52 * .9820355^{Y}}$$

check: $52 \times .9820355 \wedge 45 = 22.96\ldots \approx 23$ ✓

fits Exponential equation template: dep = start * g^{indep}

(d) Compare the models projections for 1955, 1970, 1990, 2000, 2010, and 2020. Summarize your findings in a table.

1970−1955=15

year	1955	1970	1990	2000	2010	2020
Y	0	15	35	45	55	65
linear M	52.0	42.3	29.4	23.0	16.5	10.1
exponential M	52.0	39.6	27.5	23.0	19.1	16.0

$52 - .6444 \times 65 =$

$52 \times .9820 \wedge 65 =$

(e) The actual rates were 40 deaths per thousand births in 1970 and 28 deaths per thousand births in 1990. Which model fits this additional data better?

$\boxed{\text{exponential}}$